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Abstract

This research paper explores the possibilities of expanding the Fama and French three-factor and Five-Factor Models (1992, 2015) with the liquidity factor. By using Fama and Macbeth (1973) regressions, Fama and French (1992, 2015) single sorted portfolios, and Fama and French (1992, 2015) double sorted portfolios several results are found. The sample contains all U.S. publicly traded stocks, during the 52 year period of 1970 till 2022. The Fama and Macbeth (1973) regressions finds a coefficient of -0.3274% per month for the liquidity factor, which indicates an underperformance of the more liquid stocks. The Fama and French (1992, 2015) single sorted long-short portfolios find alpha of -0.6713% and -0.4517% per month for the three and five-factor models, which are significant on the 5% level. Additional robustness tests of the value weighted and signal weighted returns find mixed results. Finally, the Fama and French (1992, 2015) double sorted portfolios found monthly alphas, controlled for the three and five-factor factors respectively of 0.3737% and 0.3149% for the liq-size, 0.5917% and 0.3837% for the liq-btmt, 0.3781% and 0.1294% (insignificant) for the liq-rmb, and finally 0.6551% and 0.5313% for the liq-cma double sorted portfolios. Further investigating the limits to arbitrage in literature, lead to the conclusion that these significant outcomes are easier to attain on paper, compared to trading them in real-life, since the least liquid stocks do have the highest trading costs and short positions are very costly.

Keywords: Liquidity, Fama and French(1992, 2015), Datar (1998), U.S stock market

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Ⅰ: Introduction

In the first month of the COVID-19 stock market crash, the stock market trading volumes and thus the stock market liquidity quickly shrunk. This meant that very few trades were happening and that the price discovery of both stocks and bonds was very slow. At some point the stock market liquidity was so close to drying up that the FED considered directly buying stocks, on top of their usual measures to revive the economy (like dropping the interest rates).Research by Tiwari et al. (2022) investigated the relation between the outbreak of COVID-19 and stock market liquidity and found a causal linear (negative) relationship between the number of COVID-19 infections and stock market liquidity within the emerging and developed equity markets. In hindsight, the stock market had a very quick recovery following the March crash, due to large quantitative easing packages from the FED. If investors would have invested while the market was illiquid, this could have been a potentially great investment, since a new bull market started only a few months after the initial crash. While there could be many more reasons for the swift stock market recovery, it is still a very interesting question what role illiquidity plays in the stock market, and more specifically the illiquidity risk factor and its explanatory power in stock returns.

The liquidity risk factor has been discussed in asset pricing literature since the start of asset pricing theory. This factor has often been in the shadow of the biggest, and most popular asset pricing models, like the CAPM of Sharpe (1964), the three-factor model of Fama and French (1992), and more recently the five-factor model of Fama and French (2015). The liquidity factor of Amihud and Mendelson (1986) received a lot of attention in the late 80s, before the three-factor model of Fama and French (1992) was published. Ever since then, researchers have debated whether the liquidity risk factor should be added as an explanatory factor for stock returns. This paper aims to answer that question, using different asset pricing methodologies, in order to show the explanatory power of the liquidity risk factors. A Fama and Macbeth (1973) regression will be used to explore the relationship between liquidity and the cross-section of stock returns, controlling for the Fama and French (1992, 2015) three and five-factor models, without imposing decile breakpoints. Also, this paper will follow the methodology of Fama and French (1992, 2015) to also make a Factor model, using the long-short strategy proposed in their 1992 paper. Finally, both the single sorted and double sorted factor portfolios and their returns will be looked at, to deepen the analysis of the explanatory power of the liquidity factor.

Further motivation for examining the liquidity factor is caused by the recent gaps in the literature on liquidity. These gaps might exist because of the increasing liquidity of the general market, as trading volumes have been growing since many brokers offer commission free stock trading. However, this does not directly imply that the liquidity factor has ceased to exist. After the Fama and French (1992) three-factor model, there were asset pricing papers that attempted to expand this model with a liquidity factor. Some were successful, others using a less effective proxy for liquidity did not find significant results. One of the papers that found significant coefficients for the illiquidity risk factor was the paper of Datar et al. (1998).Their paper is the basis for our research paper to re-examine the liquidity factor. This will be expanded on by adding more than 20 years of new data to the model and again investigating the significance of the liquidity factor. Furthermore, since the paper of Datar et al. (1998), the Fama and French (2015) five-factor model has been published. This new and improved model adds two additional factors to their 1992 three-factor model. There have been no attempts to also expand this model with the illiquidity factor. Furthermore, there have been relatively little attempts to expand the five-factor model with an illiquidity risk factor to potentially improve the explanatory power of this asset pricing model. This is a gap in the asset pricing literature that this paper will attempt to reduce. Finally, a section of this paper will be dedicated to the real life limitations to arbitrage from this strategy. This research aims to have a social impact, aiming to bring new and updated insights to retail investors by looking into the various weighting methods that they can use to invest in factor portfolios. Furthermore, it aims to include the most recently available data, with data till the end of 2022, to give some more insights into the strength or weakness of the illiquidity factor.

To conclude the introduction, the structure of this paper will be explained. In the next section, a review of the relevant literature on asset pricing models and the most prominent findings from other researchers will be provided. Furthermore, it will be argued why certain control variables are used and how this research paper is related to the existing literature. Next, an outline of the data and the data collection process will be given, discussing what timeframe and index constituents were used in this research. In the methodology section several research hypotheses will be formulated that lay the foundation of this research paper, and it will be explained what variables are used and how they are calculated. Afterwards, in the discussion and limitations section, the findings of this paper and their potential shortcomings and weaknesses will be discussed. Finally, this paper will end with a conclusion about these findings.

Ⅱ: Literature Review

Financial researchers have long been interested in understanding the factors that drive the returns of the stock markets. One of the most influential asset pricing models to have been written, the CAPM, or Capital Asset Pricing Model of Sharpe (1964), was a popular attempt to do so during the 60s and 70s. In short, the CAPM suggests that the expected return on a risky asset should be proportional to its relative market risk, measured by the asset's Beta. The model argues that investors demand higher returns to compensate for holding riskier assets. Thus, the higher the market risk of an asset, the higher its expected return should be, according to this model. The CAPM has been widely used in academic research since its introduction. However, over time, the relevance of this model changed from being a practical model to being more influential as a theoretical model. This is mainly because it is argued that this model has many assumptions that may not always hold in reality. such as the assumption of a perfectly efficient market.

Since the CAPM there have been many attempts to expand the model. One of these attempts is the creation of factor models based on certain characteristics of stocks. Looking to find what coefficients influence stock returns in a factor asset pricing model, a paper was published by Fama and Macbeth (1973). In this paper, the Fama–Macbeth regression is introduced. This is a two-step procedure that can be used to estimate the parameters of asset pricing models such as the capital asset pricing model (CAPM). The method consists of estimating the Betas and risk premia for any factors that are expected to determine asset prices. This research will also make use of this methodology, to assess the illiquidity factor. Ever since it was published, the methodology of the Fama and Macbeth (1973) regression has been widely used in subsequent research to investigate the pricing of various factors. While the results of this paper were not influential enough to really dethrone the CAPM, they did lay the foundation for a new model that would be created by Fama in 1992. Following the Fama and Macbeth (1973) methodology, the first question that will be answered in this research paper is as follows:

What is the explanatory power of the illiquidity risk factor running a Fama and Macbeth (1973) regression on the three and five-factors in the U.S. stock market?

The CAPM model stood strong in the asset pricing literature for a long time, but around the 80s and beginning of the 90s, more models were built that slowly deteriorated the strength of Sharpes (1964) CAPM. One of the first models that tried to use liquidity as a factor for explaining asset returns was the model of Amihud and Mendelson (1986). Their paper was one of the first to find evidence that the illiquidity factor is positively related to expected returns, after controlling for other factors such as the market risk, firm size, and book-to-market ratios. Furthermore, there were plenty of reasons economists could think of that would explain why this liquidity premium did exist, and also why it would persist over time. The main argument that is given for the illiquidity premium is that less liquid assets are seen as riskier for investors to buy. Therefore, the investors that do decide to buy them require a higher expected return, to compensate for holding them in their portfolios long-term.

A few years after the model of Amihud and Mendelson (1986) was published, another, new and well respected paper was published in the asset pricing literature: the Fama and French (1992) three-factor model, which included market risk, size, and the book-to-market equity factors. The Fama and French (1992) model looked at five potential risk factors (market ß, size, B/M, financial leverage, and E/P^2) and found that size and value are the best proxies for explaining stock returns. This model provided a better explanation of stock returns than the traditional CAPM, and also managed to show some of the weaknesses of the CAPM. This weakness was shown by the finding that small companies have higher returns versus big companies, even though the market Betas were very similar in their dataset. Furthermore, it was found that value companies outperform growth companies. Over time, researchers started to prefer this model over the CAPM. Furthermore, an important difference between the CAPM and Fama and French (1992) is found in the portfolio construction. The Fama and French (1992) model used long-short portfolios that put the characteristics into quintiles and based their results on those quintile differences. With the Fama and French (1992) model becoming a standard in asset pricing theory, the liquidity factor seemed to lose attention. Even after the three-factor model was published by Fama and French (1992), there was still debate within the asset pricing literature about what model best explained stock returns. Shortly after the publication, potential extensions of the Fama and French (1992) three-factor model were already being proposed, like the model of Carhart (1997). This model used the three-factor model as the basis and added a momentum factor.

With the aim of further improving this model, a new paper was published by Datar et al. (1998). This paper proposed a variation on the paper of Amihud and Mendelson (1986), which used a different liquidity proxy. Amihud and Mendelson (1986) used the average daily ratio of absolute stock return divided by dollar trading volume. They first found evidence that the illiquidity factor is positively related to expected returns. Datar et al. (1998) then further investigated the illiquidity premium. They decided to use another proxy to decide on what makes a stock liquid or illiquid, the turnover ratio. This was calculated by dividing the number of shares traded by the number of shares outstanding. There still is discussion about what illiquidity proxy best captures the coefficients and significance of the factor within the asset pricing literature, which is why different variations were used in different research papers. Further differences from the Amihud and Mendelson (1986) paper were that Datar et al. (1998) changed the control variables that were used in their research. Following the recent influences of Fama and French (1992) in the asset pricing literature, they controlled their model for the well-known firm-size, book-to-market ratio and the firm beta. They found evidence for the illiquidity premium, which inspired more research to be conducted towards this factor. This research paper builds further onto this evidence, by using the same illiquidity measure.

In 2002, Amihud published a research paper that further analyzed the illiquidity factor. In this paper it was shown that over time, the expected market illiquidity positively affects excess stock returns. This suggests that expected excess stock returns could be partly due to an illiquidity premium. The illiquidity measure used in this paper is the average of the daily ratio of absolute stock return to dollar volume. The controls used in this paper were not exactly the ones which were used by Fama and French (1992) in cross-section asset pricing estimation. However, Amihud (2002) did decide to include the size factor as a control factor again. The reason he gave for not including the ratio of book-to-market equity, BE/ME, is that his study contains only NYSE stocks for which the BE/ME was found to have no significant effect. This was first found in the research paper of Loughran (1997). A similar conclusion about the liquidity factor was reached by Pastor and Stambaugh (2003), who showed that stocks with high trading volume and low transaction costs tend to have lower expected returns than those with low trading volume and high transaction costs. The Pastor and Stambaugh (2003) liquidity factor relied on the principle that 'order flow', meaning how many people are buying and selling a stock, induces greater return reversals when the liquidity of a stock is lower. If a lot of buying and selling of a stock is happening at time *t*, this paper implies that there is high order flow in the market, and hence, a higher expected return reversal at time $t+1$. From 1966 through 1999, the average return on stocks with high sensitivities to liquidity exceeded that of stocks with low sensitivities by 7.5 percent annually. The results of Pastor and Stambaugh (2003) were adjusted for exposures to the market return as well as size, value, and momentum factors. They furthermore suggested including a liquidity factor in the Fama-French-Carhart model, turning this four-factor model into a five-factor model that included a liquidity factor.

Many years later, Fama and French (2015) introduced their latest model. They published a paper describing a five-factor model that included their original three factors (market, size, and book-to-market) as well as adding the profitability and investment factor. There has been a long period between their three and five-factor model, but this extension seems to have improved the explanatory power of the asset pricing models. Nevertheless, there is an ongoing academic debate about how much of an improvement this new model is, and that is yet to be fully decided.

Since the five-factor model was published, there hasn't been a replication and extension of the paper of Datar et al. (1998) for the five-factor Model. Furthermore this research will take a look at the prolonged timeline. Since 2000, the investment landscape for investors has evolved, also with the growing popularity of trading algorithms, used by both institutional investors and retail investors. This research will assess whether the illiquidity factor still holds up when including more than 20 years of new data. Finally, Fama and French (2015) briefly discussed the possibility of a liquidity factor in addition to their three- or five-factor model, and stated that in literature, the liquidity factor of Pástor and Stambaugh (2003) became a somewhat common addition. They did not include this factor into their model, because the regression slope coefficients were close to zero in their research. This might have changed with the addition of new data since 2015.

Among the many factors that have been proposed to expand the original CAPM, liquidity has stood the test of time and kept being proposed as a factor in the asset pricing literature. Many different factors have been tested on their strength as a determinant of stock returns. The importance of liquidity in financial markets has been recognized for a long time, but the extension of one of today's leading asset pricing models with the liquidity measure of Datar et al. (1998) has yet to be studied by academics. Using a much more recent dataset, this paper will give insights into the current state of the liquidity factor, and how it behaves when controlled for the Fama and French (1992, 2015) Factor Models.

This paper will go beyond making a combination of two different asset pricing papers, by adding an additional weighting technique. As can be seen in most asset pricing literature, like the papers of Fama and French (1992, 2015), the value weighted returns, and equal weighted returns are the only shown returns for the quintiles and regressions. When looking into other asset pricing literature, like the paper of Asness (2013), it can be seen that a different weighting methodology is applied to the results. In this case, a ranking based weighting is created that gives signal based weights to all observations. This method works great to combat outliers, since an outlier's ranking is only one rank higher compared to the last observation. Next, the weighting of the portfolio is turned into a net zero investment, or long-short portfolio, by subtracting the mean ranking of every variable. In short, this means that the middle numbers get a near 0 weighting, and the more extreme outcomes get a relatively sizable long, or relatively sizable short position. The exact explanation of this weighting is further explained in the methodology. The alternative weighting method of Asness (2013) is added to this paper, because this ranking technique could influence the coefficients found in this research, and the significance of the results. This is mainly because of the method's ability to combat outliers, which is not taken into account by the equal and value weighted portfolios. This gives the following three ways in which the regression results will be weighted:

- Value weighted, meaning that the stocks are weighted based on their market capitalization relative to the total market capitalization of the sample.
- Equal weighted, meaning that all stocks have the same weighting, which means that relatively more weight is put on the small firms, since there are more small companies.
- Signal weighted, as it was used in the paper of Asness (2013). Here, each stock gets a number ranking, based on the relative height of the specific factor.

A combination of the previously discussed illiquidity risk factor papers, in combination with the old and new Fama and French (1992, 2015) model created the following main research question that will be answered in this research paper using the three weighting methods:

What is the explanatory power of the illiquidity risk factor running a Fama and French (1992, 2015) regression on the three and five-factor model in the U.S. stock market using different weighting techniques?

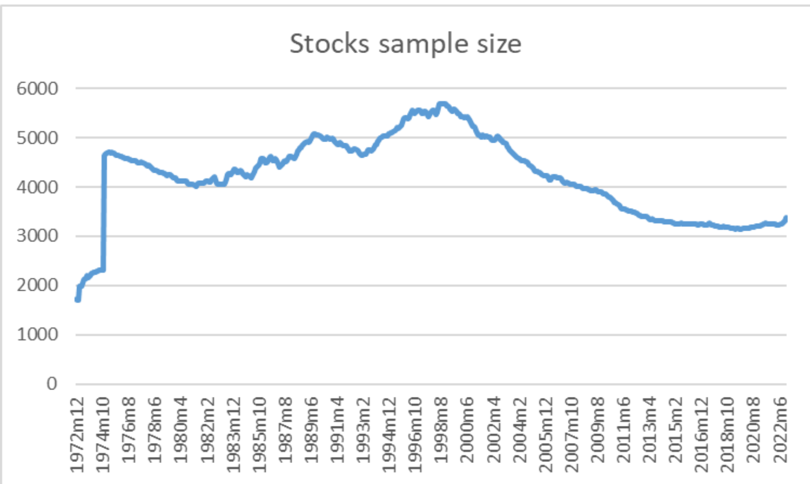
Besides analyzing the liquidity risk factor, further focus of this paper will be put into the potential limits to arbitrage from this strategy. Since the illiquidity factor implies that you need to be able to invest in relatively illiquid stocks, it's important to also look at the feasibility of this strategy. A further look into the limits to arbitrage is done by investigating the double sorted portfolios, based on the liquidity-size / liquidity-book to market / liquidity-RMW / liq-CMA combinations. The idea behind this is that with the single sorted portfolios, a potentially incorrect view is given of the liquidity returns. For example, with the single sorted portfolio based on liquidity, a combination between the size factor and liquidity factor is shown in the same quintile returns. This paper aims to further isolate the illiquidity factor returns, and look into how this strategy might be further limited to arbitrage if a premium is found in the most illiquid stocks. This could be done by looking at the double sorted returns of all control variables. A similar methodology using double sorted portfolios has been used in the paper of Fama and French (1992). Fama and French (1992) made double sorted portfolios based on the market Beta and Size. They did so because they saw that when common stock portfolios are formed on size alone, there seems to be evidence of the CAPMs prediction: average return is positively related to Beta. This was not the effect Fama and French were expecting, as this was actually supporting the CAPM model instead of the size factor in their three-factor model. The solution they used was allowing for variation in both the Size and Beta quintiles at the same time. This allowed the researchers to look at both factors individually and the patterns that were visible in the stock returns. Thus, after subdividing the portfolio with these rules, a strong relation between average return and size was found, and no relation between average return and Beta. This finding was the basis of how the Fama and French (1992) three-factor model is an improvement of the CAPM. This paper will look at the limits to arbitrage, by finding out if the more liquid or illiquid stocks lead to higher returns on the control factors. If this paper finds that the illiquid stocks have higher premia, this could be caused by the limits to arbitrage, since these premia might not be easily arbitraged away in the real world. These limits to arbitrage will be further investigated in this research. How the double sorted regression is performed is further explained in the methodology section of this paper. These discussed potential limitations to arbitrage will be answered by the third question of this research paper, which is:

What potential limitations to arbitrage from our strategy can be found and what is the explanatory power of the illiquidity risk factor running a double sorted regression on the liquidity -Fama and French (1992, 2015) factors?

Ⅲ: Data

This paper will use the Kenneth R. French data library to obtain the monthly coefficients of the Mkt-Rf, SMB, HML, RMW, CMA, and RF variables. Obtaining the coefficients from the data library ensures that the control factors are following the Fama and French (1992, 2015) methodology. This, together with the quarterly and annual data, will be turned into monthly data, since this is the standard in financial research. Furthermore, this research will use the WRDS/CRSP database for the illiquidity factor, the monthly stock returns, and for our self-replicated Fama and French (1992, 2015) control factors when this research performs a Fama and Macbeth (1973) regression.

A 52 year time period is analyzed in this paper, using financial data from December 1970 up to December 2022, as this is a timeframe that is long enough to derive significant conclusions from. Companies that merge, get delisted, or discontinue will stay in the dataset, in order to prevent survivorship bias in the results. Furthermore, all U.S. companies listed on the U.S. public stock market are included in the research, meaning this paper includes all companies from the NYSE, the NASDAQ, and the AMEX. Including all U.S. companies available also ensures that this paper can derive the correct conclusions about the full U.S. stock market. Taking a look at the descriptive statistics of the data obtained through the WRDS/CRSP database, it can be seen that most of the monthly variables have between 3-4 million observations over the 52 year time period. Furthermore, observing figure 1, it can be seen that the amount of observations is relatively low in the first years. The first month of the dataset, December 1970 includes approximately 1700 U.S. companies, while the more recent time periods, like December 2022 include approximately 5700 U.S. companies. Since this paper regresses the data per month using means, this does not have a significant impact on our results. Nevertheless, it can be argued that the 1700 companies in this research paper's sample are plenty to draw conclusions from: every quintile still consists of hundreds of observations, making a single observation or outlier not influential to the general results.



Graph 1: The amount of observations tabulated per month. The trend is that it linearly increases over time, since more recent data is more easily accessible.

Descriptive Statistics

Variable	Obs	Mean	Std. Dev.	Min	Max
Turnover incl. outliers	3,895,087	.182	2.753	2e-7	2939.133
Turnover excl. outliers	3,817,815	.129	.257	0.0002	8.406
RET-RF	2,455,903	.009	.181	-.995	23.997
Market Value	2,457,368	2.537e+09	2.022e+10	10562.5	2.902e+12
Book To Market	1,898,095	.799	3.019	-906.639	134.68
Operating Profitability	1,715,084	.044	16.751	-3349	10422.728
Asset Growth	1,779,575	1.331	31.961	0.00018	10174.72

Table 1: The descriptive statistics of the variables used in this research paper. In the first column, the amount of observations (Obs) per variable is shown. Further, it shows the mean per variable, calculated over all observations. Next, the standard deviation of that mean is given. Finally, the minimum and maximum values out of all observations listed.

Furthermore, the correlations between the variables in this research paper show that most factors have relatively low correlations. The highest correlation that can be seen in this matrix is between the liquidity proxy and the market capitalization. This correlation being relatively high is not surprising. The paper of Amihud and Mendelson (1986) already found that the size effect is closely linked to liquidity risk, which they measured using the bid-ask spread. Also, looking at the asset pricing theory supporting the size factor, it is often claimed that smaller firms are riskier than larger firms on average because less information is available on the companies, but also because the trading volumes of these companies is much lower, adding to the risk experienced by the investor. The negative correlation between market capitalization and the book-to-market ratio of -0.331 is also quite sizable. The correlation matrix in the paper of Datar et al. (1998) reported a correlation of -0.373, which is very similar. This correlation appears to not have changed much in the extended time period.

Matrix of correlations

Variables	(1)	(2)	(3)	(4)	(5)	(6)
(1) RET-RF	1.000					
(2) LN (liquidity)	-0.010	1.000				
(3) LN (market cap)	0.040	0.467	1.000			
(4) LN (BTMT)	0.025	-0.331	-0.350	1.000		
(5) RMW	-0.000	0.001	0.004	0.001	1.000	
(6) CMA	-0.006	0.030	0.005	-0.035	0.000	1.000

Table 2: The Correlation Matrix of the variables used in this research paper. RET-RF represents the monthly returns minus the risk-free rate, LN liquidity represents the turnover ratio following Datar et al. (1998), the LN market capitalization and LN book-to-market are based on Fama and French (1992). RMW represents the Robust Minus Weak Operating profitability factor and CMA the Conservative Minus Aggressive Investment strategy, from the Fama and French (2015) model.

Ⅳ: Methodology

Before diving into the results of this research paper, it is important to explain how the results were obtained in the first place. As stated in the data section, everything is gathered from the The Kenneth R. French data library in combination with the WRDS/CRSP database. After merging the separate datafiles, several steps are followed in order to stay as close as possible to the research methodologies of both Fama and French (1992, 2015) and Datar et al. (1998).

The variables

First, the construction of the control variables in this research paper will be explained. For the first results, this paper is looking at the excess market returns, and controlling for the three-factor Fama and French (1992) model.

The Excess return on the market portfolio = Returns of the broad market - Risk-Free rate (1)

To construct this factor, meaning the excess return on the market, the value-weighted return of all US firms, listed on either the NYSE, AMEX, or NASDAQ with a CRSP share code of 10 or 11 is downloaded from the CRSP database. Furthermore, the Risk-Free rate is gathered by Fama and French (1992, 2015) from the Ibbotson Associates database. Additionally, the excess returns on the market portfolio were lagged by one month relative to the excess stock returns. This is done following the Fama and French (1992, 2015) papers, because the stock market returns must be a consequence of the changed independent variables, meaning the returns must be 1 month later. This factor shows that if this paper does find an outperformance, this will thus also mean that it is an outperformance relative to the general stock market. The second control variable is the size factor, using the market capitalization:

*Market Capitalization = Stock Price * Shares Outstanding (2)*

$$SMB_{(B/M)} = 1/3\ (Small\ Value + Small\ Neutral + Small\ Growth) - 1/3\ (Big\ Value + Big\ Neutral + Big\ Growth).$$

$$SMB_{(OP)} = 1/3\ (Small\ Robust + Small\ Neutral + Small\ Weak) - 1/3\ (Big\ Robust + Big\ Neutral + Big\ Weak).$$

$$SMB_{(INV)} = 1/3\ (Small\ Conservative + Small\ Neutral + Small\ Aggressive) - 1/3\ (Big\ Conservative + Big\ Neutral + Big\ Aggressive).$$

$$SMB = 1/3\ (SMB_{(B/M)} + SMB_{(OP)} + SMB_{(INV)})\ (3)$$

Using the formula above, it is possible to find the monthly Market Capitalization per firm. This can then be used to calculate the average return on the three ‘small’ portfolios based on their market capitalization, minus the average return on the three ‘big’ portfolios based on their market capitalization. Furthermore, the Small Minus Big (SMB) factor will be lagged by one month, relative to the stock returns, like with the market proxy. The third control variable is the value factor, using the book-to-market ratio of every firm:

Book-to-Market Ratio = (Total Assets - Total Liabilities) / Market Capitalization (4)

The High Minus Low Factor (HML) = 1/3 (Small Value + Medium Value + Big Value) - 1/3 (Small Growth + Medium Growth + Big Growth) (5)

To construct this factor, it is first important to calculate the book value of every firm. This can be done by subtracting the total liabilities from the total assets. Furthermore, this paper is again using the market capitalization of the previous equation, now as the denumerator. Also again, the book-to-market ratio is lagged, this time by six months, relative to the stock returns. This is done following the Fama and French (1992, 2015) papers. The idea behind this is that assets and liabilities are usually only released quarterly, or once a year. This six month lag ensures that the investors do know the book-to-market value, when they are trading on it. This book-to-market factor is used to invest into ‘value’ stocks, relative to the worse performing ‘growth’ stocks. This paper will define ‘value’ portfolios as the stocks with a high book-to-market ratio, and ‘growth’ portfolios as the low book-to-market ratio. This research will be continued by further explaining the second group of control variables, based on the Fama and French (2015) model. Two new control variables are added in their model, that made the coefficients of their model more significant and that had more explanatory power relative to the three-factor model. To construct these factors, this paper will follow the Fama and French (2015) methodology:

The Profitability Factor (RMW) = (Annual revenues – COGS - interest expenses - Selling, General, and Administrative expenses) / Book equity (6)

RMW = 1/2 (Small Robust + Big Robust) - 1/2 (Small Weak + Big Weak) (7)

To construct the profitability factor, this paper will again follow the calculations used in Fama and French (2015). This means using the annual revenue, as reported in the annual/quarterly report. Since almost no companies disclose this information monthly, or even quarterly, this variable is lagged to be the same for the entire year, depending on its June value. This assures that the operating profitability is already reported to investors, so that they are actually already able to trade on the knowledge of the profitability factor’s value. On average, this also means that there is a six month lag implemented for the Operating Profitability. After gathering the annual revenue, the Cost Of Goods

Sold is subtracted from the revenue. These two variables are mandatory for constructing this variable, and if one or both of the variables are missing, no profitability is included in the sample. Finally, if available, this paper will subtract the Selling, General, and Administrative Expenses. In many cases, this is not available, or included in other reported data, and then the Selling, General, and Administrative Expenses cannot be subtracted. Next, this is divided by the book equity, as discussed in the previous section. The Operating Profitability factor suggests that stocks with a higher operating profitability ratio perform better than less profitable stocks. This means that companies that are focused on making a profit right now, and not in the future, should outperform the general market ceteris paribus. This research will use one more control variable. This is, again, a variable that is often only reported quarterly, or yearly. This means that this factor also remains the same for a 12 month period. The calculation done to create this factor is:

The Investment Factor (CMA) = Total Assets (t-1) / Total Assets (t-2) (8)

CMA = 1/2 (Small Conservative + Big Conservative) - 1/2 (Small Aggressive + Big Aggressive) (9)

For the final control variable, the main focus shifts to the total assets. The Fama and French (2015) CMA factor findings suggest that stocks with a greater total asset growth ratio have lower excess returns. This means that this paper will control for the total assets growth, to correctly isolate the illiquidity factor returns. These control variables are chosen for this research paper, because the Fama and French (1992, 2015) firm characteristics are very popular to control asset pricing models and have already been proven to have a significant influence on stock returns in many different asset pricing papers. Furthermore, expanding their research with new and different portfolio weightings might improve the strength of these papers' findings and add some new insights into the Fama and French (1992, 2015) models.

Next to the control variables previously discussed, this research will have to establish how liquidity, and thus the liquidity premium, is defined. To achieve this, the paper of Datar et al. (1998) is followed. This paper had made a variation on the previously published Amihud and Mendelson's (1986) illiquidity measure. Their predecessors, Amihud and Mendelson, first found evidence that the illiquidity factor is positively related to expected returns, after controlling for the effect of other factors such as market risk, firm size, and book-to-market ratios. This result could also be explained using economic rationale: namely by the idea that less liquid assets are riskier for investors to buy and therefore require higher expected returns to compensate for holding them long-term in their portfolio. Datar et al. (1998) further investigated this measure, where they used an alternative measure of the illiquidity, namely the turnover ratio. This can be calculated as follows:

The Illiquidity Factor (IML) = the number of shares traded / the number of shares outstanding (10)

Here, IML stands for Illiquid Minus Liquid, and is represented by the above calculation. Shares traded is equal to the monthly trading volume, as reported from the WRDS/CRSP database. By default, WRDS/CRSP report this data rounding the monthly trading volumes off by the hundreds of shares. This restriction is therefore also included in this research paper. Furthermore, the numbers of shares outstanding are measured by including all publicly held shares of a company's common stock.

Exclusion of observations

After explaining the construction of the control variables, it is important to note what observations are excluded from the analysis. This is done by combining some exclusions of the methodologies of the Fama and French (1992, 2015) models and the methodology of Datar et al. (1998).

The first group of observations that are excluded are the stocks that do not have a share code 10 or 11. A share code in CRSP/WRDS is a two-digit code describing the type of shares that are traded. This is done in both Fama and French (1992, 2015) models, in order to keep only common shares in the sample, and to exclude funds and such from the data. The first digit describes the type of security that is traded. In this case, the first 1 represents ordinary common shares. The second digit, and then specifically, the 0 and 1, are securities which have not been further defined (0) and securities which need not be further defined (1). This means that only the ordinary shares are included, and funds, REITs and shares from outside the U.S. are excluded.

The second exclusion used in our research sample is also from the Fama and French (1992, 2015) methodology. Here, this paper followed their example and decided to leave out the financial companies from the dataset. This is due to the relatively high leverage that is being used in the financial companies, which include mainly banks and insurance companies. It is explained by Fama and French (1992, 2015), that leaving those companies in the dataset would lead to a bias, since our measure for the book value, total assets minus total liabilities, would have most banks and insurance companies end up with a very low book value. This in turn makes them more included in one specific quintile, leading to the previously stated bias.

The third exclusion in the dataset of this research paper, is the exclusion of the first 24 months of every company's stock returns from the dataset. This is done to follow the Fama and French (1992) methodology. The idea behind excluding this data from our research, is further explained in the paper of Banz and Breen (1986). This paper finds that some studies are using accounting and price data from the COMPUSTAT database. Their paper compared this database to another and found that the COMPUSTAT database could introduce a look-ahead bias and an ex-post-selection bias into the study that used them. The solution offered in the Fama and French (1992) paper is to exclude the first 2 years of data from the sample, to prevent this survivorship bias, since this bias was found to almost never persist for longer than 24 months.

Finally, the highest 1% and the lowest 1% of Turnover Ratios are trimmed from the dataset.

This is done because these outliers tend to have a very big upwards effect, relative to the trimmed sample. By doing this, this paper follows the Datar et al. (1998) methodology, that also excludes these observations. The expectation is that this helps to attain more comparable results to the original paper.

Weighting techniques and break-even costs

This research paper is not only interested in looking into the liquidity factor, but also wants to add some additional robustness tests, beyond some of the tests that are shown in the paper of Fama and French (1992, 2015). This will be done by taking a look at three different weightings, of which one was not included in their 1992 and 2015 original papers. The first factor weighting discussed is the equal weighted results.

Equal weighted returns = $\sum(\text{ mean of } R_{i,t}) / \sum(N_t)$ *(11)*

The equal weighted returns are a fairly simple weighting method. In this formula the first part represents the sum of the returns of all stocks and securities at time period t. The second part of the formula divides this number by the total number of stocks at time period t. Subsequently, this generates a return weighting that gives an equal weight to all stocks. If a company has just been added to the U.S. publicly traded companies, it gets the same weighting with their returns as the biggest companies in the U.S. market. Furthermore, research by Plyakha et al. (2012) found that the equal weighted portfolio with monthly rebalancing tends to outperform value weighted and price weighted portfolios in terms of total mean return and Sharpe ratio. This could offer another interesting result by looking at the differences between the equal weighted, value weighted and signal weighted returns in the sample. The second factor weighting that will be discussed is the value weighted results.

Value weighted returns = $\sum(\text{mean of } R_{i,t}) / \sum(\text{market cap. stock } i \text{ / total market capitalization U.S. index})$ *(12)*

The value weighted return is already a bit more complex relative to the equal weighted return. In this formula the first part again represents the sum of the returns of all stocks and securities at time period t. This time, the second part of the equation represents the market capitalization of the security i, relative to the total market capitalization of all U.S. stock in our sample. By using this division, security i gets a weighting relative to their proportion of the total market capitalization in all U.S. stocks. This also means that the changes in stock returns of a very small company are much less influential on the returns, relative to the much bigger companies. Furthermore, a study by Frankfurter and Vertes (1990) found that for all portfolio sizes, the risk measures of market-value-based portfolios are lower than equal-value-based portfolios. It was concluded that this is because risk is inversely related to size. Thus, market value weights understate risk much more for randomly selected portfolios compared to equal weighted portfolios. This means that by using a combination of these two weightings, some robustness is already added due to the varying characteristics of the weighting methods. Finally, a third weighting method is added to further examine the influence of different portfolio weightings on our model.

Signal weighting = $w_{it}^{S} = c_t(\mathbf{rank}(S_{it}) - \Sigma_i \mathbf{rank}(S_{it})/N),$ *(13)*

The first step of this formula shows the calculation on how to weight the returns of our portfolio. The aim is to create a long-short portfolio that has no net long or short position in the market. This means that the total weight is equal to 0. The calculation of the signal weighting uses the following variables. The first variable in the formula is Ct. This is simply a scaling factor, that makes the total position equal to one dollar long, and one dollar short, which results in the previously discussed net investment of 0. The rest of the equation calculates the rank of every signal (so individually for SMB, BTMT, RMW, CMA a rank is calculated per month) and subtracts the mean ranking of that signal for that month. Finally, this outcome is divided by the total amount of securities that are given a ranking.

Signal returns = $r_t^{S} = \Sigma_i w_{it}^{S} r_{it},$ *(14)*

Next, the returns are calculated using the previously found weightings. This is done by multiplying the found monthly signal weights of security i times the monthly returns of security i. All returns are then combined together to get a monthly signal weighted stock return.

Finally, this paper will calculate the break-even costs for 10, 50, 100, and 4127 companies, for the single sorted portfolios. This last number might seem random, but is based on the average monthly number of stocks in our sample. This number is used to show clearly how much the annual trading costs (total fee of buying long position + annual fee of short position) of the found strategy could be before the strategy turns unprofitable. The following formula can be used to calculate the break even costs:

Break-even costs = Mean Excess Return of strategy / number of stocks annually turned over (15)

Here, the mean excess returns represent the found alphas of the regressions previously discussed. For the number of stocks annually turned over, the number of stocks are multiplied with the mean annual turnover ratio, which is based on the number of quintile changes within our sample. Furthermore, this paper will take into account the fact that a change from a long to a short position is in fact two individual trades, giving this extra weighting for the average turnover ratio used in this research.

The analysis

The Fama and Macbeth (1973) regression

The first regression that is performed in this asset pricing study is the Fama and Macbeth (1973) regression. This research paper will use the results of this regression to analyze the potential explanatory power of the liquidity factor in the Fama and French (1992, 2015) three- and five-factor model. The Fama and MacBeth (1973) regression is a two-step procedure that is performed to estimate the parameters of asset pricing models, such as the capital asset pricing model (CAPM). With this method, the betas and risk premia for any factors that are expected to determine asset prices can be estimated. For this research paper, this means that it will be used to look at the explanatory power of the liquidity factor, controlling for the Fama and French three-factor model, and controlling for the Fama and French five-factor model.

The two steps done in the Fama-Macbeth regression are as follows: the first step is a cross-sectional regression. In this case, it means that each of the *n* asset returns is regressed against the *m* proposed risk factors used in the research, for every time period *t*. This is done to determine each asset's beta exposure to these factors. The formula that was used for the first step of the Fama and Macbeth (1973) regression is:

$$R_{(n,\,t)} = \alpha_{n\,(t)} + \delta_{n(t),\,X1} X_{1(t)} + \ldots + \delta_{n(t),\,Xm} X_{m(t)} + \varepsilon_{(n,t)} \quad (16)$$

Where the dependent variable of interest, returns, is denoted in the formula as R. Furthermore, *n* represents the number of asset returns against the *m* proposed risk factors. The returns are regressed on the *m* independent variables, which are the Fama and French (1992, 2015) factors and the main variable of interest, liquidity (also see the section 'the variables'). This is done to find the cross-sectional regression coefficients of the independent variables, which can be used in the second step of our analysis. The second step that is performed in a Fama and Macbeth (1973) regression is performing a time-series regression. This means regressing the average of the dependent variable against the averages of the independent variables, for each of the *T* time periods. By taking the average asset returns for every time period, and regressing it on the average estimated beta coefficients for every variable from step one this can be done. The average risk premium is then found for each of the factors in our dataset, over the specified period of our sample. After this second step of the Fama and Macbeth regression, this paper will look at the results which provide us the coefficients and p-values. A first idea whether the variables in our model are actually related to stock returns can be found here. Important to note is that this paper deviates from the Fama and Macbeth (1973) methodology by using the MSCI US index as an proxy for the U.S. stock market return, and subtracting the risk-free rate from that. In the original paper, all returns in the dataset were combined, before any exclusions were made to the dataset. This small deviation is chosen because it appears to have no downsides, since both proxies are very similar. The hypothesis that will be answered by this research paper is:

H_0: A higher stock liquidity has no effects on stock returns in the Fama and Macbeth (1973) regression controlling for the Fama and French (1992, 2015) three and five-factor model.
H_a: A higher stock liquidity has effects on stock returns in the Fama and Macbeth (1973) regression controlling for the Fama and French (1992, 2015) three and five-factor model.

A second and more recent methodology will also be used to find the coefficients and significance of the (control) factors. By creating long-short portfolios of the liquidity variable, and analyzing the difference in average stock returns the liquidity factor can be further tested. By doing this, the paper will use two ways to find a significant alpha in our strategies.

The Fama and French (1992, 2015) single sorted factors:

This research paper has the main goal of reexamining the (il)liquidity factor, in the context of the Fama and French (1992, 2015) three- and five-factor models and taking a further look at the limits to arbitrage from this strategy. In order to evaluate the success of our strategy, a long-short strategy is applied that has a net zero investment. This is realized by following the next methodology.

First of all, to get a long-short strategy, it is important to create different quintiles, for different groups of characteristics of a stock. In this research 3 quintiles are generated for every of the previously discussed variables. These quintiles have breakpoints at the 30% and 70% marks. This means that this paper created groups of the 0-30% values of a characteristic, 30-70%, and 70%-100% values. This is replicated from the Fama and French (1992, 2015) methodology, in order to attain similar groups of stocks. This leads to approximately 500k observations for the smaller quintiles, and approximately 800k observations for the big quintile in our sample. The quintiles are best explained with an example. If a company has a relatively low market capitalization, it could be put into the first quintile of the size factor. Medium sized companies are added to the second quintile of the size factor, and the biggest companies are in the third quintile. Furthermore these quintiles are all generated annually. At the month of June, the market capitalization quintiles are generated for the entire year. After 12 months, the quintiles are again recalibrated, to update them with the most current changes in market capitalization. This is done following the Fama and French (1992, 2015) methodology. The reason to only use the annually set quintiles instead of monthly, is to keep out noise and randomness in the estimates, due to short-term fluctuations in the monthly data. By using annual data, it is easier to isolate the long-term relationships between variables.

Lastly, this paper will be controlling all coefficients using the Newey and West (1987) standard errors. This is because the return and control variables might be persistent. Even though the two variables are probably only correlated for one time period, it could be possible that it is multiple periods. This research will control for a potential autocorrelation of up to 6 months of data. By using the Newey and West (1987) standard errors instead of running a standard regression, it is more likely to reject our hypothesis, because the standard errors will be bigger. This does add additional strength to the results of this paper, since the findings and conclusions are more robust.

The results will be acquired using two types of portfolios. The first method that will be applied is a single sorted portfolio. This means that results are sorted based on the (il)liquidity factor, and the difference in portfolio returns between the first and third quintile of this factor is assessed. If the constant (meaning alpha) of the regression is statistically different from zero, it could be concluded that the factor gives us a profitable strategy. For the single sorted portfolio, this paper will use the (il)liquidity measure to sort into quintiles, to consequently investigate the returns per group. This involves doing the following calculation:

LMI (Liquid Minus Illiquid) = Liquid (quintile 3) - Illiquid (quintile 1) (17)

The above calculation uses the quintiles discussed before, where the first group contains the most illiquid stocks, denoted as Illiquid, based on our liquidity proxy. The third group contains the most liquid stocks, and is denoted as Liquid. Basing our expectations on the Datar et al. (1998) paper, this paper would expect a significant and negative coefficient for the LMI factor. This is because the illiquid stocks should have a premium on top of them for being more difficult to trade. The found alpha of this long-short position will be controlled for the Fama and French (1992, 2015) control variables. This gives the following regression:

$$Y = \alpha_1 + \beta_1 *SMB_t + \beta_2 *HML_t + \beta_3 *RMW_t + \beta_4 *CMA_t + \varepsilon_t \quad (18)$$

Here, the parameter α indicates the predicted returns when the explanatory variables are equal to zero. Furthermore, SMB represents the Small Minus Big factor, HML represents the High Minus Low factor, RMW represents the Robust Minus Weak factor and CMA represents the Conservative Minus Aggressive factorThe hypothesis that will be answered by this research paper is:

H_0: A higher stock liquidity has no effects on stock returns in the single sorted regression controlling for the Fama and French (1992, 2015) three- and five-factor model.
H_a: A higher stock liquidity has effects on stock returns in the single sorted regression controlling for the Fama and French (1992, 2015) three- and five-factor model.

The Fama and French (1992, 2015) double sorted factors:

The second method used in this cross-sectional asset pricing study is to make double-sorted portfolios. This technique sorts the variables into portfolios that are based on the ranking of two variables at the same time. This paper will make combinations of the liquidity factor, with every control variable of the Fama and French (1992, 2015) models. As an example, this research will create a double sorted ranking, on both the liquidity factor, and the size factor at the same time. Using this methodology, this creates 9 different portfolios, with each different mean excess returns (3*3, meaning three liquidity quintiles and 3 size quintiles combinations). This paper will use these more specialized asset returns to derive conclusions about the explanatory power of the liquidity factor in our sample, in combination with the other factors. There have been several research papers before that used double sorted portfolios in combination with a liquidity factor. One of the first papers to make use of double sorted portfolios that included a liquidity factor, was the paper of Amihud and Mendelson's (1986). In this paper, a double sorted portfolio is created consisting of their liquidity measure, the bid-ask spread and relative risk factor, the beta over the period of 1961-1980. Another paper, that was published after the Fama and French (1992) paper, did decide to make use of these new factors that were found. The paper of Pástor and Stambaugh (2003) looked at the combination of Liquidity risk and expected stock returns. As control variables, they used the CAPM model of Sharpe (1964), the Fama and French (1992) three-factor model, and the Carhart (1997) four-factor model. This research paper will follow a methodology similar to these papers, but will also create double sorted portfolios on the liquidity-profitability, and liquidity-investment portfolios for the Fama and French (2015) five-factor models.

The easiest way to explain what double sorted portfolios are created is by sharing the formulas that are used. The LMI as discussed before with the single sorted portfolios, represents the Liquid Minus Illiquid factor. SMB represents the Small Minus Big factor. This factor looks at the differences in returns between the smallest and biggest firms in our dataset. HML represents the High Minus Low book-to-market firms, where a high book-to-market firm can be seen as an 'value' stock, and the low book-to-market firm can be seen as a 'growth' stock. The RMW factor represents Robust Minus Weak Operating Profitability firms. Here, the companies with the highest operating profitability are expected to outperform the market. Finally, the CMA factor represents the Conservative Minus Aggressive Investment firms. This looks at the total asset growth of companies at the years t-1 and t-2, and looks at the ratio increase. The consensus is that lower asset growth companies outperform the market.

*SMB - LMI Doublesort = ½ * LMI + ½ * SMB , where:*
LMI= 1/2 (Illiquid Small + Illiquid Big) - 1/2 (Liquid Small + Liquid Big)
SMB = 1/2 (Illiquid Small + Liquid Small) (19)

*HML - LMI Doublesort = ½ * LMI + ½ * HML , where:*
LMI = 1/2 (Illiquid Growth+ Illiquid Value) - 1/2 (Liquid Growth + Liquid Value)
HML = 1/2 (Illiquid Value+ Liquid Value) - 1/2 (Illiquid Growth+ Liquid Growth) (20)

*RMW - LMI Doublesort = ½ * LMI + ½ * RMW , where:*
LMI = 1/2 (Illiquid Weak + Illiquid Robust) - 1/2 (Liquid Weak + Liquid Robust)
RMW = 1/2 (Illiquid Robust + Liquid Robust) - 1/2 (Illiquid Weak + Liquid Weak) (21)

*CMA - LMI Doublesort = ½ * LMI + ½ * CMA , where:*
LMI=1/2(Illiquid Conservative+Illiquid Aggressive)- 1/2 (Liquid Conservative + Liquid Aggressive)
CMA = 1/2 (Illiquid Aggressive + Liquid Aggressive) - 1/2 (Illiquid Conservative + Liquid Conservative) (22)

The aim is to not just look at the single sorted portfolios, but also analyze the double sorted portfolios. Looking at single sorted portfolios, the securities returns are only sorted on the liquidity risk factor. This means that previously proven factors like the size factor, would also be included in the same quintile returns as the liquidity factor. This single sorted approach does not allow us to fully isolate the effect of illiquidity risk on expected returns, because the illiquidity risk factor would be contaminated with the size factor. It would be less clear what the effect is from the illiquidity factor, and what part of the returns comes from other factors. In a double sorted portfolio approach you sort securities based on both illiquidity risk and another variable, for example, the size factor. Doing this, groups of quintile returns are created that have different combinations of these two risks. By double sorting on both liquidity and a Fama & French (1992, 2015) factor, the liquidity factor can be investigated more isolated, without the control variables influencing the returns too much. This also means that the quintile returns might be very different compared to the single sorted returns. This approach allows us to investigate the interaction between illiquidity risk and size and how it affects expected returns. Furthermore, it might be interesting to look at the differences in alphas between the single sorted and double sorted portfolios. The found alpha of the double sorted strategy in this long-short position will again be controlled for the same Fama and French (1992, 2015) control variables. This gives the following regression:

$$Y = \alpha_1 + \beta_1 *SMB_t + \beta_2 *HML_t + \beta_3 *RMW_t + \beta_4 *CMA_t + \varepsilon_t \ (18)$$

A potential finding is that our strategy is very dependent on the highly illiquid stocks. This means that higher transaction costs might be incurred if this strategy is applied in the real world. Additionally, specifically for the liquidity-size double sort, it would be interesting to look at the results. In the research paper of Ibbotson et al. (2013), the effect on returns from different levels of liquidity across all size quintile portfolios using a turnover proxy was studied. They found that within each size quintile portfolio, low liquidity portfolios generally earned higher returns than the high liquidity portfolios. Furthermore, it could be seen that the size impact was inconsistent across various liquidity portfolios. This could be due to the size and liquidity factors both being correlated with each other, and perhaps both trying to explain a partially overlapping risk factor. The hypothesis that will be answered by this research paper is:

H_0: A higher stock liquidity has no effects on stock returns in the double sorted regression controlling for the Fama and French (1992, 2015) three- and five-factor model.
H_a: A higher stock liquidity has effects on stock returns in the double sorted regression controlling for the Fama and French (1992, 2015) three- and five-factor model.

Ⅴ: Results

The Fama and Macbeth (1973) regression

As further explained in the methodology, two types of regressions will be performed to look at the liquidity factor and its explanatory power in stock returns. The Fama and Macbeth (1973) regression is the first analysis that will be discussed. This regression is used to estimate the coefficients for asset pricing models, like for example the CAPM or as is done in this case, the three- and five-factor model. Thus, we performed the Fama and Macbeth (1973) regression with the control variables being the market return (proxied by the MSCI U.S. Index), the natural logarithm of the market capitalization, and the natural logarithm of the book-to-market ratio, the operating profitability and the total asset growth ratio..

The Fama and Macbeth regression looks at every time period individually and makes a separate regression based on those values. After doing that for every time period, the average of all those regressions over all time periods is taken to find the coefficients that are stated in table 3 below. As stated before, table 3 shows the coefficients of the Fama and Macbeth (1973) regression which is is:

$$R_{(n,\ t)} = \underline{\alpha}_{n\ (t)} + \delta_{n(t),X1} X_{1(t)}\ +\ ...\ + \delta_{n(t),\ Xm} X_{m(t)} + \varepsilon_{(n,t)}\ (16)$$

Where the dependent variable of interest, returns, is denoted in the formula as R. Furthermore, *n* represents the number of asset returns against the *m* proposed risk factors. The returns are regressed on the *m* independent variables, which are the Fama and French (1992, 2015) factors and the main variable of interest, liquidity. These control variables are further explained in the methodology. The second step that is performed in a Fama and Macbeth (1973) regression is performing a time-series regression. This means regressing the average of the dependent variable against the averages of the independent variables, for each of the *T* time periods, which gave us the coefficients in table 3.

The second column of table 3 shows the coefficients for a Fama and Macbeth (1973) regression on the liquidity factor, controlling for the Fama and French (1992) factors. The third column does a similar regression, but this time it is controlled for the Fama and French (2015) five-factor model. Taking an in-depth look at the first column of the found coefficients, it can be seen that the coefficient of the liquidity factor on the monthly stock return is very small and slightly negative. Furthermore, it is statistically significant (t = -4.34). The coefficient of -0.003274, which is equal to -0.3274% per month, means an underperformance of the more liquid stocks. The interpretation of this factor is, since the liquidity variable is log-transformed, while the return variable is not, as follows. For every 1% increase in the liquidity variable, the return variable increases by -0.003274% per month. Thus, with these initial results, a negative significant relationship between liquidity and stock returns is found.

Looking further at Fama and French (1992) control variables, it can be seen that all coefficients are significant at the 10% level, while only the MKT-Rf variable is not significant at the 5% level. Generally, this shows that the control variables are important in our research, and do explain the stock returns well. A surprising finding that can be seen in the control variables is that the market capitalization variable is positive and significant. With a coefficient of 0.004142, or 0.4142%, and t-statistic of -9.33, this can be interpreted as follows. For every 1% increase in the market capitalization, the return variable increases by 0.004142% per month. This is the opposite of what would be expected based on the literature. In most research papers, it is found that when the size of a firm increases, the stock returns tend to become lower, as was found in the paper of Fama and French (1992). Furthermore, the constant or alpha is statistically significant in all cases. This means that our control variables were not successful in explaining all variation in the stock returns. As a result, there might still be an additional factor that could further explain the stock returns, or that there might be a better fitting proxy instead of the factors used in this research paper. Finally, it can be seen that the R^2 is relatively low. The r-squared is around .03483, which reveals that approximately 3.5% of the variability observed is explained by the first regression model.

Taking an in-depth look at the second column of the found coefficients, it can be seen that the results of the first three factors of the Fama and French (2015) model are very similar to the previously discussed Fama and French (1992) model. This is to be expected, since the five-factor model is an extension of the three-factor model. Next, the RMW coefficient in table 3 has a coefficient of 0.003234, or 0.3234% per month, with a t-statistic of -4.03. This result is as expected based on the previously discussed literature. As stated in the methodology, the higher operating profitability ratio companies are expected to outperform the lower ratio companies. Using the Fama and Macbeth (1973) regression, it is found that for every 1% increase in the operating profitability ratio, the return variable increases by 0.003234% per month. Next, the CMA factor has a coefficient of -0.004423, or 0.4423% per month. With a t-statistic of -5.08, it can be concluded that this result is also significant at the 5% level. Furthermore, it is found that for every 1% increase in the investment ratio, the return variable increases by -0.004423% per month. This follows the expectations from the methodology, since the companies with the lowest asset growth have the highest stock returns. In general, these results follow the findings of Fama and French (1992, 2015). Finally, looking at the alpha of our strategy in table 3, it is found that it is statistically significant for the five-factor model, meaning that our control variables were not successful in explaining all variation in the stock returns. The R^2 for the five-factor model is still very low. With an r-squared of around 0.04779, this shows that only approximately 4.8% of the variability observed is explained by the second regression model. Nevertheless, this is still an improvement from the three-factor model, since the adjusted R^2 is higher. Still, this model does not sufficiently explain the stock returns. A second regression method will now be discussed.

Name	Returns FF3	Returns FF5
LN Liquidity	-0.003274***	-0.002561***
	(-4.336254)	(-2.963352)
MKT-Rf	0.002939*	0.002597*
	(-1.819443)	(-1.694031)
LN Market Cap	0.004142***	0.004045***
	(-9.332048)	(-8.784961)
LN BTMT	0.005859***	0.005481***
	(-8.447799)	(-7.869683)
RMW		0.003234***
		(-4.039720)
CMA		-0.004423***
		(-5.082998)
Alpha	-0.073926***	-0.064539***
	(-6.201540)	(-5.324899)
----------	----------	----------
r2	.03483	.04779
r2_a	.03326	.04433
N	1770190	1428985
* p<0.10,	** p<0.05,	*** p<0.01

Table 3: This shows the coefficients of a Fama and Macbeth (1973) regression. The first column shows the coefficients controlling for the Fama and French (1992) three-factor Model, explained in calculations (1), (2), and (4). The second column shows the coefficients controlling for the Fama and French (2015) five-factor model additionally using calculation (6) and (8). The r2 stands for R-squared (r^2) and is a statistical measure that represents the proportion of the variance for a dependent variable that's explained by an independent

variable or variables in a regression model. The r2_a stands for the adjusted R-squared(r2_a), which is a corrected goodness-of-fit measure for linear models. Finally, N represents the amount of observations used in this research. The T-statistics are denoted, using * p<0.10, ** p<0.05, *** p<0.01.

The Fama and French (1992, 2015) single sorted factors:

The equal weighted alpha

The second regressions that are run are based on grouping stocks on certain characteristics. For the single sorted portfolios, this paper will group the returns only based on the liquidity factor. Three quintiles are generated. The first quintile, group one, contains stocks with the lowest liquidity. This will be done following the paper of Datar et al. (1998) by using their liquidity ratio. This means using the ratio of trading volume and shares outstanding. After sorting these portfolios, the difference in returns between these groups is analyzed, specifically the difference between group three and group one. Based on the literature previously discussed, a negative coefficient could be expected. As explained in the methodology, this paper will find an alpha using calculation (17):

LMI (Liquid Minus Illiquid) = Liquid (quintile 3) - Illiquid (quintile 1) (17)

The above calculation uses the quintiles discussed before, where the first group contains the most illiquid stocks, denoted as Illiquid, based on our liquidity proxy. The found alpha of this long-short position will be controlled for the Fama and French (1992, 2015) control variables. This gives the following regression:

$$Y = \alpha_1 + \beta_1 *SMB_t + \beta_2 *HML_t + \beta_3 *RMW_t + \beta_4 *CMA_t + \varepsilon_t \quad (18)$$

Here, the parameter α indicates the predicted returns when the explanatory variables are equal to zero. Furthermore, SMB represents the Small Minus Big factor, HML represents the High Minus Low factor, RMW represents the Robust Minus Weak factor and CMA represents the Conservative Minus Aggressive factor. All factors are further explained in the ‘the variables’ section. Furthermore, note that for the three factor model control variables, the last two variables are not included. This same regression is run for the value weighted returns, and the signal weighted returns.

Looking at the results in table 4a, it can be seen that three different regressions are performed. The first column is a simple test to find if the difference between quintile 3 and 1 of the liquidity ratio is significant, without controlling yet for any related variables. This paper finds an alpha of -0.004950, meaning an outperformance of -0.4950% per month. With a t-statistic of -2.949090, it can be concluded that this difference is significant. This first result does not yet provide enough information. This is because different influential factors are not taken into account by this model.

The second column of table 4a shows the coefficients with the Fama and French (1992) control variables can be seen. Again, this paper focuses on the alpha of this strategy. With a coefficient of -0.006713, meaning a monthly -0.6713% return for the most liquid stocks in group three compared to the illiquid stocks in group one, and a t-statistic of -5.29, this analysis received the expected results. This alpha can be interpreted as follows. On average, in the sample, the stocks with the lowest liquidity ratio tend to outperform the stocks with the highest liquidity ratio, by a 0.6713% monthly stock return per month. These results confirm some of the findings in earlier research papers, like the paper of Datar et al. (1998).

Finally, the third column of the results in table 4b shows the coefficients with the Fama and French (2015) control variables. Additionally to the three factors both RMW and CMA are significant at the 5% level. Furthermore the alpha of the long-short portfolio is equal to -0.004517, or -0.4517% per month. This is a small decrease in value of the alpha, as compared to the three-factor model previously discussed. This reduction could be explained by the fact that there was some alpha unexplained by the three-factor model, that might be explained by the RMW or CMA factor. Furthermore, looking at the t-statistic of the alpha, it can be seen that this is equal to -3.46. It can therefore be concluded that the long-short portfolio does lead to a significant outperformance on the 5% level. Finally, the r2, meaning the R^2 measure, shows interesting results. As one would expect, the R^2 does increase with the addition of the two control variables. More interestingly, for the r2_a, meaning the adjusted R^2, it is found that the addition of these two factors, does improve the goodness of fit of the second model compared to the first model. The adjusted R-squared method is a modified version of the R-squared method that takes into account the number of independent variables. However, it is important to note that an increase in the adjusted R-squared does not always mean that the second model is the better model overall. It mainly means that the second model fits the pure data better, without taking factors like the theoretical soundness of the model into account.

Name	**q. diff (1)**	**Newey FF3 q. diff (2)**	**Newey FF5 q. diff (3)**
MKT-Rf		**0.004661*** **	**0.004230*** **
		(-11.252220)	**(-10.963283)**
SMB		**0.003097*** **	**0.001831*** **
		(-5.555083)	**(-3.261779)**
HML		**-0.004264*** **	**-0.003315*** **
		(-6.040416)	**(-5.628975)**
RMW			**-0.004352*** **

			(-6.366380)
CMA			**-0.002439****
			(-2.312858)
Alpha	**-0.004950*****	**-0.006713*****	**-0.004517*****
	(-2.949090)	**(-5.299592)**	**(-3.469424)**
----------	----------	----------	----------
r2		**.5968**	**.6463**
r2_a		**.5948**	**.6434**
N	**601**	**601**	**601**
----------	----------	----------	----------
	*** p<0.10,**	**** p<0.05,**	***** p<0.01**

Table 4a: This shows the equal weight results of the three performed regressions. Column one is without control variables. The second column shows the coefficients controlling for the Fama and French (1992) three-factor Model, explained in calculations (1), (3), and (5). The third column shows the coefficients controlling for the Fama and French (2015) five-factor model additionally using calculation (7) and (9). The r2 stands for R-squared (r^2) and is a statistical measure to represent the proportion of the variance for a dependent variable that is explained by an independent variable in a regression model. The r2_a stands for the adjusted R-squared(r2_a), which is a corrected goodness-of-fit measure for linear models. Finally, N represents the number of months that are taken into consideration in this research. The T-statistics are denoted, using * p<0.10, ** p<0.05, *** p<0.01.

Additionally, figure 2 is added to show the three liquidity quintiles and their corresponding logarithmic returns. As stated before, the strategy of this research paper is to take a long position in quintile 1, and a short position in quintile 3. With this, the U.S stock market proxy that is used in this paper is added. It can be seen that specifically the third quintile returns are much lower compared to the general market proxy. Therefore, this also shows that a big part of the outperformance of the found strategy is because of the short positions' profits. This finding is in line with a similar finding of Lu et al. (2017). This paper looked at long-short portfolios based on market anomalies, and stated that few studies directly quantify the impact of shorting on long-short strategies, largely due to the complexity of the shorting costs. Among size, value, and momentum strategies, they found that when deducting shorting costs, essentially all the profits of long-short portfolios using these factors was lost. This paper will further examine the actual tradability of the liquidity factor in the 'limits to arbitrage' section.

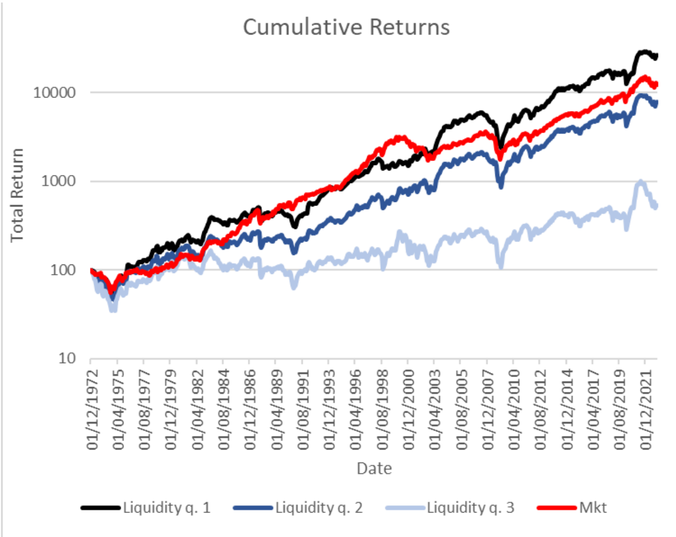


Figure 2: This displays the cumulative returns over time of the three liquidity quintiles, on a logarithmic scale. The performance of the first quintile (lowest liquidity) is shown in black. The performance of the second quintile (medium liquidity) is shown in dark blue. The performance of the third quintile (highest liquidity) is shown in light blue. Finally, the market performance (from MSCI U.S. index) is shown in red. The time period is from December 1972 till December 2022.

The value weighted alpha

Table 4b shows the results of the value weighted regressions. Here, instead of weighting every stock equally, another weighting depending on the market capitalization is used. This means that the biggest companies, like Apple and Microsoft, have a much bigger impact on the returns, compared to a very new and small company. This specifically has a big impact on the illiquidity factor. The paper of Limkriangkrai et al. (2008) looked at a combination of the illiquidity factor and both the equal and value weighted portfolio returns. This paper found that the value-weighted return calculation provides less difference between the illiquid and liquid stocks than the equal-weighted version. They further explain this by the fact that illiquid stocks tend to be small stocks, which receive less weight in the value-weighted return calculation. Their results also showcase this, because their null hypothesis of the liquidity factor not existing is rejected in most instances for the equal weighted returns, but is rejected in only two instances for the value-weighted portfolios. These results are only applicable for the Australian stock market returns, so it is interesting to note the differences that might apply to this U.S. stock market based research.

The first column of table 4b is a simple test to look at the difference between quintile 3 and 1 of the liquidity ratio. The interpretation is similar to the interpretation of table 4a, discussed in the equal weighted section. However, it is surprising that a positive coefficient is found, contrary to the negative coefficient in the equal-weighted returns. In the second column of table 4b, the coefficients of the Fama and French (1992) three-factor control variables are listed. The focus is on the alpha of this strategy. With a coefficient of 0.002576, meaning a monthly 0.2576% excess return, and a t-statistic of -1.944138, this paper has not found significant results for the value-weighted three-factor model, on the 5% significance level. This also means that the coefficient cannot be interpreted. These results are similar to the value-weighted findings of the paper of Limkriangkrai et al. (2008), which also had significant results for the equal-weighted returns, but often insignificant results for the value-weighted portfolios.

Finally, the third column of table 4b shows the coefficients when the Fama and French (2015) five-factor model is used as control variables. The alpha of the long-short portfolio is equal to 0.005252, or 0.5252% per month. With a t-statistic of -3.84, this is significant at the 5% level. This finding is the opposite of the findings in the literature review of this paper. Though unexpected, this type of result is probable to happen. When the paper of Momani (2018) revisited the Pástor and Stambaugh (2003) paper with much more recent data, while also using the value weighted returns, Momani (2018) found that only five out of the 30 created portfolios were significant at the 5% significance level. Furthermore, many other portfolios were insignificant, or had coefficients that were the opposite of what is expected based on the most popular illiquidity factor papers. This shows that the value-weighted returns can give surprising results, because of the different weighting that puts more focus on the higher market capitalization firms. It is also interesting to look at the r2, meaning the R^2 measure. Similar to the equal-weighted returns, the R^2 does increase with the addition of 2 control variables. The same goes for the r2_a, meaning the adjusted R^2. It can be found that the addition of these 2 factors, does improve the goodness of fit of the second model compared to the first model. Furthermore, this paper compares the equal-weighted and value-weighted r2 and r2_a coefficients. Using the value-weighted returns, both measures have less explanatory power compared to the equal-weighted r2 and r2_a. This can be interpreted to be a consequence of the different weighting technique. Since more weighting is given to the big companies, and the illiquidity factor is less prevalent in those companies, the explanatory power of the entire model becomes lower. This is seen in the comparative r2 measures (VW FF5: 0.5807, EW FF5 0.6463) and r2_a measures (VW FF5: 0.5771, EW FF5: 0.6434).

Name	q. diff (1)	Newey FF3 q. diff (2)	Newey FF5 q. diff (3)
MKT-Rf		0.004221***	0.003675***
		(-10.155661)	(-9.784038)

SMB		0.004487***	0.002942***
		(-5.922071)	(-4.978626)
HML		-0.003644***	-0.002439***
		(-6.112423)	(-4.047509)
RMW			-0.004872***
			(-7.473817)
CMA			-0.003461***
			(-3.585703)
Alpha	0.004487**	0.002576*	0.005252***
	(-2.536781)	(-1.944138)	(-3.843740)
----------	----------	----------	----------
r2	0	.528	.5807
r2_a	0	.5256	.5771
N	601	601	601
----------	----------	----------	----------
	* p<0.10,	** p<0.05,	*** p<0.01

Table 4b: This shows the value weight results of the three performed regressions. Column one is without control variables. The second column shows the coefficients controlling for the Fama and French (1992) three-factor Model, explained in calculations (1), (3), and (5). The third column shows the coefficients controlling for the Fama and French (2015) five-factor model additionally using calculation (7) and (9). The r2 stands for R-squared (r^2) and is a statistical measure to represent the proportion of the variance for a dependent variable that is explained by an independent variable in a regression model. The r2_a stands for the adjusted R-squared(r2_a), which is a corrected goodness-of-fit measure for linear models. Finally, N represents the number of months that are taken into consideration in this research. The T-statistics are denoted, using * p<0.10, ** p<0.05, *** p<0.01.

The signal weighted alpha

Table 4c shows the results of the signal-weighted regressions. As discussed in the methodology section, the signal-weighting comes from calculations (13) and (14). This means that every factor is ranked, and given a weight relative to that rank. This weighting is re-evaluated every month. The paper of Asness (2013) stated that using the ranks of the signals as portfolio weights helps mitigate the influence of outliers, but portfolios constructed using the raw signals are similar and generate a slightly better performance. This means that because this research paper had already dropped the top and bottom 1% of observations following the methodology of Datar et al. (1998), it is very certain with the signal weighting that our results are not influenced by the most extreme observations. In the research paper of Asness (2013), it is shown that the signal-weighted factor portfolios outperform simple portfolio sort spreads because the security weights are a positive (linear) function of the signal, as opposed to the coarseness of only classifying securities into three groups. In addition, the factors are better diversified since more securities in the cross section are given a nonzero weight and the weights are less extreme, because of the ranking format.

The first column of table 4c is again a simple test to look at the difference between quintile 3 and 1 of the liquidity ratio. This paper finds an alpha of 0.012343, meaning an outperformance of the most liquid stocks of just 1.2343% per month. Again, it is important to look further at the results when using the set control variables. The second column in table 4c shows the coefficients when the Fama and French (1992) three-factor model is used as control variables. With a coefficient of -0.000174, meaning a monthly -0.0174% return, and a t-statistic of -0.138050, this paper has not found significant results on the 5% significance level when controlling for the three-factor model. This also means that the coefficient cannot be interpreted. This might be caused by the ranking method, which gives relatively low weighting to outliers, since they are just one rank higher than the previous observation. The combination of findings for the value-weight and signal-weight returns, where both the signal-weighted and value-weighted portfolios do not have the expected, or significant results, do bring up some additional questions. Most importantly, since the results of the equal-weighted portfolio cannot be replicated when the weightings are more towards the big companies (with value-weighting) or when the outliers are made less influential (with signal-weighting), this raises the question if this outperformance is actually tradable for an investor. This is in line with some of the findings of Godfrey and Brooks (2015). Their paper focused on a previous finding in literature, that stated that high credit risk stocks earn lower returns compared to low credit risk stocks. They argued that this finding was based on rational expectations, and decided to look at it using four different limits to arbitrage. Their paper demonstrates that the negative pricing of credit stocks is driven by the underperformance of stocks which have both high credit risk and have suffered recent relative underperformance. Furthermore, interesting about this research is that the earlier found poor performance can be explained by a mixture of the four limits-to-arbitrage factors. These are illiquidity, in combination with idiosyncratic risk, turnover, and bid-ask spreads. Collectively, these impede the correction of mispricing by arbitrageurs, especially on the short leg of the trade, where commonly reported returns are found to be unattainable.

The third column of table 4c shows the coefficients when the Fama and French (2015) five-factor model is used as control variables. The alpha of the long-short portfolio is equal to 0.001598, or 0.16% per month. With a t-statistic of -1.30, it is not significant at the 5% level. This finding can thus not be interpreted. It is also interesting to look at the r2. Contrary to the equal-weighted and value-weighted returns, the R^2 does not increase with the addition of two control variables. The same goes for the r2_a. It can be found that the addition of these two factors does not improve the goodness of fit of the second model compared to the first model by much. It is further found that the signal-weighted returns do have a much higher explanatory power compared to our previous models, but there is no difference between the three- and five-factor models. Comparing the three r2 measures, it shows that the signal-weighting clearly has the highest explanatory power, with a value of 0.9051 compared to the value-weighted r2 of 0.5807 and the equal-weighted r2 of 0.6463, even though these results were not significant, which is a surprising finding. As with the three-factor model, since the results of the equal-weighted portfolio cannot be replicated easily, the previously found significant alphas can be questioned due to limits to arbitrage. This will be further investigated in the last section of the results of this research, by looking at the double sorted portfolios, in combination with limits to arbitrage literature. This will give further insight into the behavior of the illiquidity factor, in combination with the Fama and French (1992, 2015) models.

Name	**q. diff (1)**	**Newey FF3 q. diff (2)**	**Newey FF5 q. diff (3)**
MKT-Rf		0.016255***	0.014630***
		(-57.503)	(-52.312460)
SMB		0.012824***	0.011818***
		(-30.161)	(-27.676245)
HML		0.004065***	0.001706***
		(-9.841)	(-3.286300)
RMW			-0.003507***
			(-6.361167)

CMA			0.000208
			(0.252956)
Alpha	0.012343***	-0.000174	0.001598
	(-3.272282)	(-0.138050)	(-1.300803)
----------	----------	----------	----------
r2	0	.9008	.9051
r2_a	0	.9003	.9043
N	581	601	581
----------	----------	----------	----------
	* p<0.10,	** p<0.05,	*** p<0.01

Table 4c: This shows the signal weight results of the three performed regressions. Column one is without control variables. The second column shows the coefficients controlling for the Fama and French (1992) three-factor Model, explained in calculations (1), (3), and (5). The third column shows the coefficients controlling for the Fama and French (2015) five-factor model additionally using calculation (7) and (9). The r2 stands for R-squared (r^2) and is a statistical measure to represent the proportion of the variance for a dependent variable that is explained by an independent variable in a regression model. The r2_a stands for the adjusted R-squared(r2_a), which is a corrected goodness-of-fit measure for linear models. Finally, N represents the number of months that are taken into consideration in this research. The T-statistics are denoted, using * p<0.10, ** p<0.05, *** p<0.01.

Limits to arbitrage from the illiquidity factor

The theoretical framework of limits to arbitrage, described by Shleifer and Vishny (1997), is that arbitrage in financial markets is costly to execute, risky, and tends to be undertaken by participants who operate with limited capital and whose shareholders may withdraw capital from arbitrageurs' operations if they stop believing in the investment. They further describe the arbitrageurs as often exposed to fluctuations in asset prices which might move away from fundamental values, so that arbitrage trades may show losses in the short-term, and the threat of capital withdrawal by shareholders in such circumstances makes them more cautious in entering into arbitrage trades. They argue that since arbitrageurs are typically not well-diversified, high volatility arising from noise trader sentiment makes arbitrage unattractive. It can even be that because of that financial market, anomalies are therefore more likely to persist where market factors make arbitrage more risky or costly to execute.

This framework at first seems to go against the Efficient Market Hypothesis (EMH) of Fama (1970), which states that financial markets are efficient and that asset prices reflect all available information. This consequently implies that it is impossible to earn long-term excess returns by trading on information that is already known to the market. This does not mean that prices are always and at all moments accurate or that they cannot deviate from their true value in the short-term. It can be argued that the presence of limits to arbitrage can help explain why prices deviate from their true value in the short-term, even in an efficient market. The difference that is stated by the EMH, is that in the long run these mispricings should be corrected by arbitrageurs that find ways to overcome these limits to the possibility of arbitrage.

To this, the paper of Miller (1977) adds that arbitrage is more likely to be constrained on the leg of the trade requiring a short sale. This is because selling a share short requires first that it can be borrowed from a willing counterparty, and that the facility for stock lending in sufficient size may be limited. Looking specifically at the illiquidity factor, this factor focuses specifically on the stocks that are the least liquid and have a low turnover ratio. The model previously described, assumes that these short positions are possible, and have no costs attached to them. Even though retail investors are more and more able to trade in stocks without commissions, this assumption still seems too soon for all investors, and might never be attainable for institutional investors trading in much bigger quantities. These constraints to trade the short positions were already previously discussed in relation to the findings of Godfrey and Brooks (2015), with Miller (1977) adding to these findings.

The paper of Godfrey and Brooks (2015) distinguishes between three categories of limits-to-arbitrage factors:

- The factors that impede the adoption of short positions but not long positions, and so hinder the correction of overpricing but not underpricing: these include short selling costs.
- Secondly, factors that impede the adoption of long positions but not short positions, and hinder the correction of underpricing. The factors in this category may include concentration limits by asset managers, but these are not likely to be significant.
- Thirdly, limits-to-arbitrage factors that are symmetrical in impeding arbitrage positions in either direction. These include high leverage costs; wide bid-ask spreads; high illiquidity; low turnover; and high idiosyncratic volatility.

This section will focus mainly on the third type of factors, since the illiquidity factor has an impact on both the short and long position costs. As for the short position taken in our strategy, it might be very expensive to hold a short position. A recent example of these high shorting costs in combination with low liquidity can be derived from the famous GameStop short-squeeze. Here, the paper of Hilliard and Hilliard (2021) reported that the borrowing fees were 20 to 40 percent per annum in the pre-squeeze period, and were at the highest in the squeeze period at up to 90 percent per annum.

Similarly high trading costs for some of the illiquid stocks in this paper's strategy could lead to it being unprofitable. Also, for the long position in our strategy, a potential weakness is that it is expensive to acquire the stocks. The strategy of this paper mainly focuses on going long in the illiquid stocks. However, if it is much more expensive to buy (and later again sell) these stocks, this means that the profits seen in our model might not be actually attainable.This paper will look at the double sorted portfolios of the liquidity factor with each of the Fama and French (1992, 2015) factors in order to see what patterns can be found that might be a limit to the possibilities to arbitrage from the found strategy. This is done by sorting portfolios based on both liquidity and a second Fama and French (1992, 2015) factor, such as market capitalization or book-to-market. By doing this, the paper uses two ways to compare the returns.

First, it will look at the 9 different created portfolios, and analyze the general excess stock return differences of the quintiles. If it is found that the most illiquid stocks consistently outperform the more liquid stocks when double sorting, this could suggest that the illiquidity factor is not subsumed by the other control variables. Furthermore, there is reason to suspect that the illiquidity factor is much easier to attain on paper, compared to the real-world trading experience. This is because some of the assumptions that are made, like the non-existing shorting costs, do not go up in the real world. Another problem with the liquidity factor is that the bid-ask spreads of the stocks are often very big, meaning the trading costs of these stocks might be very high. This is even worse when trying to buy large quantities of a certain stock with a low trading volume. Both arguments suggest that the illiquidity factor could perhaps not be arbitraged away over the long term, in the real world, which might explain the persistence of this factor in asset pricing literature.

The second way of looking at the different portfolio returns is to analyze the alpha’s of the double sorted strategies, and control them again for the Fama and French (1992, 2015) factors. This paper will analyze the alpha’s to find if the strategy is still profitable to follow after double sorting the portfolio, which further isolates and separates the effects of the liquidity factor, from the Fama and French (1992, 2015) factors. The found alpha of the double sorted strategy in this long-short position will be controlled for the same Fama and French (1992, 2015) control variables. This gives the following regression:

$$Y = \alpha_1 + \beta_1 *SMB_t + \beta_2 *HML_t + \beta_3 *RMW_t + \beta_4 *CMA_t + \varepsilon_t \quad (18)$$

Finally, a short analysis is done of the annual trading costs that could be induced without the strategy losing profitability.

The Fama and French (1992, 2015) double sorted factors:

The liquidity - size double sort

From the double sorted combinations analyzed in this paper, the liquidity-size double sort has been the most popular in literature. This is mainly because there is still debate about how these variables interact with each other, and if one factor (partly) subsumes the other. The paper of Keene and Peterson (2007) found that there is a relation between the size and liquidity factors. For small firms, the average size of the firm increases from low liquidity to high liquidity firms. However, that pattern does not exist for big firms, when moving from low liquidity to high liquidity firms. Moving from small to big firms, the liquidity proxy does again increase. This suggests that, on average, big firms are more liquid than small firms. This finding could also partly be used to explain the difference between the equal-weighted returns and the value-weighted returns in the long-short strategy previously discussed. Because value-weighting puts more focus on the big companies, more liquid companies are heavily weighted in the returns. Consequently, the results become less significant and coefficients might change. Another paper that double sorted the liquidity-size factors, is the paper of Ibbotson et al. (2013). One of the goals of this paper was to determine whether liquidity is effectively a proxy for size. In order to test this, their paper constructed equally-weighted double-sorted portfolios in market capitalization and turnover quartiles. Their empirical study found that looking at the different size quintiles, the low liquidity portfolios generally earned higher stock market returns compared to the high liquidity portfolios. Furthermore, in their results, the size impact is generally inconsistent across the liquidity levels of the double sorted portfolios.

Looking at the double sorted liquidity-size portfolios in Appendix table 10, it can be seen that the most illiquid stocks do have the highest returns with double sort. This follows the general findings of this paper so far. Furthermore, for the size quintiles, it is found that the size factor is clearly visible in the lowest liquidity and medium liquidity portfolios. The size factor seems to disappear in the group of most liquid stocks, since the mean monthly returns of portfolio (3,3) are higher than the (3,2) portfolio. It can thus be concluded from the 3*3 double sorted portfolio on size and liquidity, that the size effect does not hold across all liquidity quintiles in the double sorted portfolio with liquidity. This is concluded specifically from the highest turnover ratio quintile, that shows that the liquidity premium is not the simple projection of size premium. This also means that this paper found that the illiquidity factor strategy described in this paper uses the most difficult and expensive to trade stocks to profit from, at least when looking at the double sorted liquidity-size factors. This paper found that the combination of both factors, does lead to the highest excess stock returns.

Furthermore, looking at the long-short portfolios of table 5, the excess returns that can be attained are listed when controlling for the Fama and French (1992, 2015) factors. The strategy gives a monthly excess market return of 0.3737% and 0.3149% per month. Both findings are also significant with t-statistics of (-4.207) and (-2.470). These findings can thus be interpreted. This means that the long-short position on the liquidity-size portfolio does still lead to an outperformance of the stock market, similar to what was found on the single sorted portfolio. Another additional reason for the high returns, specifically on the lowest liquidity, lowest market capitalization (1,1) portfolio, is given by the paper of Pástor and Stambaugh (2003). Their paper found that small and illiquid stocks might be those stocks whose values are most affected by drops in market wide liquidity, of particular concern for the investors concerned with the overall liquidity of their portfolios. When market liquidity declines, many investors sell stocks and buy bonds and those investors might prefer to sell liquid stocks in order to save on transaction costs. This is what potentially causes investors to move to assets with greater liquidity. Further in the paper, it is suggested that investors in smaller firms do require higher returns for accepting the liquidity risk. This does again give an argument for the real world limitations to arbitrage from this strategy, showing one of the additional factors that influence the applicability of this strategy in the real world.

Name	q. diff (1)	Newey FF3	Newey FF5
MKT-Rf		-0.002398***	-0.002197***
		(-8.780)	(-8.4899)
SMB		0.002097***	0.002138***
		(-3.813)	(-3.564)
HML		0.002794***	0.001614***
		(-5.621)	(-2.764)
RMW			0.000357
			(-0.7177)
CMA			0.001961
			(-2.470)
Alpha	0.003558****	0.003737***	0.003149**
	(-3.383)	(-4.207)	(-2.470)
----------	----------	----------	----------
r2	0		

r2_a	0		
N	601	601	601
----------	----------	----------	----------
	* p<0.10,	** p<0.05,	*** p<0.01

Table 5: The equal weight results of the double sorted liq-size regressions. Column one contains the regression without control variables. The second column shows the coefficients controlling for the Fama and French (1992) three-factor Model, explained in calculations (1), (3), and (5). The third column shows the coefficients controlling for the Fama and French (2015) five-factor model additionally using calculation (7) and (9). The r2 stands for R-squared (r^2) and is a statistical measure that represents the proportion of the variance for a dependent variable that's explained by an independent variable in a regression model. The r2_a stands for the adjusted R-squared(r2_a), which is a corrected goodness-of-fit measure for linear models. Finally, N represents the number of months that are taken into consideration in this research and the T-statistics are denoted, using * p<0.10, ** p<0.05, *** p<0.01.

The liquidity - book-to-market double sort

The second double sorted combination is that of the liquidity-book-to-market. The paper of Ibbotson et al. (2013) also investigated this relationship, and reported their findings. One of the goals of their paper was to address the question of how liquidity differs from value in their behavior. This was done by constructing a liquidity factor and comparing it with the Fama and French (1992) book-to-market factor. In their research, a combination of 16 value and liquidity portfolios was created. It was found that among the high growth stocks, the low-liquidity stock portfolio had an annualized geometric mean (compound) return of 9.99% whereas the high-liquidity stock portfolio had a return of 2.24%. This showed that the liquidity factor again played a big role in the stock returns in combination with the book-to-market ratio. Furthermore, among the high-value stocks, the low-liquidity stocks had a 18.43% return whereas high-liquidity stocks had a return of 9.98%. The paper concluded that value and liquidity are distinctly different ways of picking stocks. The best return comes from combining high-value stocks with low-liquidity stocks; the worst return comes from combining high-growth stocks with high-turnover stocks. Ibbotson et al. (2013) their paper is fairly similar in research approach, so it will be used to find similarities and differences with our research. Another paper that discussed the liquidity - value interaction is the paper of Asness (2013). This paper finds significant evidence that liquidity risk is negatively related to value across asset classes. They show that this link is also present in other markets and asset classes, and that value returns are significantly negatively related to liquidity risk globally. Furthermore, they found that value performs poorly when funding liquidity rises, which occurs during times when borrowing is easier. However, later in the same paper, it is also said that because value loads negatively on liquidity risk, the positive premium associated with value is still a deep puzzle to explain.

Looking at the double sorted liquidity-btmt portfolios in Appendix table 11, this paper can further investigate the interaction between liquidity and the other factors in our model. It is found that the most illiquid stocks do have the highest monthly stock returns in the double sorted portfolios. This follows the general findings of this paper so far. Furthermore, the book-to-market quintiles show that the book-to-market factor is consistently visible in all quintile portfolios. A potential reason why this effect might be more visible compared to the size factor, is because the book-to-market ratio is less related to the liquidity factor. There is less debate about how one of these factors might subsume the other, which is why the factor results of both are more clear. It can thus be concluded that a very similar relationship to the paper of Ibbotson et al. (2013) is found, when exploring the double sorted liquidity - btmt 3*3 portfolios. Again, the least liquid, and thus most difficult to trade stocks, get the highest stock returns. Also, the lowest liquidity quintile, in combination with the highest book-to-market quintile has the best excess stock returns. This shows that the combination of both factors does lead to the best stock returns.

Furthermore, looking at the long-short portfolios of table 6, it can be seen what excess returns are found, when controlling for the Fama and French (1992, 2015) factors. This paper's strategy leads to a monthly excess market return of 0.5917% and 0.3837% per month. Both findings are also significant with t-statistics of (4.673) and (2.7523). These findings can thus be interpreted. This means that the long-short position on the liquidity-btmt portfolio does also lead to an outperformance of the stock market. Furthermore, it can be seen that the excess returns become lower with the addition of the RMW and CMA control factors. This might imply that some of the excess returns of the three-factor model could be explained by these additional factors.

Name	q. diff (1)	Newey FF3	Newey FF5
MKT-Rf		-0.004328***	-0.003892***
		(-10.262)	(-9.8325)
SMB		-0.003203***	-0.002073***
		(-5.82)	(-3.6179)
HML		0.002570***	0.001586**
		(-3.382)	(-2.384)
RMW			0.003816***

			(-5.703)
CMA			0.002696**
			(-2.387)
Alpha	0.003791***	0.005917***	0.003837***
	(2.3721)	(4.673)	(2.7523)
----------	----------	----------	----------
r2	0		
r2_a	0		
N	601	601	601
----------	----------	----------	----------
	* p<0.10,	** p<0.05,	*** p<0.01

Table 6: The equal weight results of the three double sorted liq-btmt regressions. Column one contains the regression without control variables. The second column shows the coefficients controlling for the Fama and French (1992) three-factor Model, explained in calculations (1), (3), and (5). The third column shows the coefficients controlling for the Fama and French (2015) five-factor model additionally using calculation (7) and (9). The r2 stands for R-squared (r^2) and is a statistical measure that represents the proportion of the variance for a dependent variable that's explained by an independent variable in a regression model. The r2_a stands for the adjusted R-squared(r2_a), which is a corrected goodness-of-fit measure for linear models. Finally, N represents the number of months that are taken into consideration in this research and the T-statistics are denoted, using * p<0.10, ** p<0.05, *** p<0.01.

The liquidity-profitability double sort

The third double sorted combination is the liquidity-operating profitability. Since the five-factor model of Fama and French (2015) has been published relatively recently, less research has been done on the interaction between liquidity and operating profitability. The paper of Skočir and Lončarski (2018) tried to extend the Fama and French (2015) five-factor model with three additional factors that are popular in asset pricing literature. These factors are momentum, liquidity and default risk. Their paper found that the slopes of the RMW and CMA factors absorb most of the variation in the liquidity factor return. These strongly positive RMW and CMA factor slopes indicate, according to them, a potentially strong relationship between stock liquidity, profitability and investment intensity. This further means that the double sorted portfolios based on the LMI-RMW and LMI-CMA factors could see one of the factors being (partly) subsumed with the other. Another paper that looked at a profitability factor is the paper of Novy-Marx (2013). This paper looked at a variation, the gross profitability factor and found that it performs relatively better than the other strategies that are based on a ‘quality’ factor, especially among large-cap U.S. stocks. Furthermore, their profitability factor has roughly the same power as the book-to-market factor in predicting the cross section of average returns. They analyzed portfolios double sorted on size and profitability, and found that the profitability factor’s power is economically significant even among the largest, most liquid stocks. This paper will also look further into this profitability factor and will focus on the liquidity-profitability relationship. The portfolio findings of this paper can be compared to the profitability-liquidity double sorted findings of Novy-Marx (2013), even though their profitability proxy is a bit different. This paper will find if the profitability factor is influenced in combination with the illiquidity factor. Furthermore this paper will find if there is still a significant alpha that can be attained controlling for the Fama and French (1992, 2015) factors.

Looking at the double sorted liquidity-profitability portfolios in Appendix table 12, it is found that the most illiquid stocks have the highest monthly stock returns in the double sorted portfolios. Furthermore, looking at the operating profitability quintiles, it can be seen that the operating profitability factor is not behaving as was found by Fama and French (2015). As the operating profitability increases (moving from q1 to q3), the excess monthly return decreases. Though surprising, this was also found in the equal-weighted single sorted portfolio previously discussed. Important to notice is that the liquidity factor remains visible in combination with the other variables, while some of the used control variables do lose some strength. Again, the least liquid, and thus most difficult to trade stocks, get the highest stock returns. Furthermore, unexpectedly, the lowest liquidity quintile, in combination with the lowest profitability ratio quintile has the best stock returns. This means this paper also finds different results to the findings in the paper of Novy-Marx (2013). The finding also goes against our expectations based on the literature, that would suggest that the lowest liquidity, highest operating profitability stocks would have the highest stock performance. An exact explanation for this return is difficult to give, but it could be something interesting to further investigate in future research.

Furthermore, looking at the long-short portfolios of table 7, the excess returns that are attained can be seen when controlling for the Fama and French (1992, 2015) factors. It is found that our strategy leads to a monthly excess market return of 0.3781% and 0.1294% per month. Here, only the Fama and French (1992) three-factor model findings are significant with a t-statistic of (3.7626). The Fama and French (2015) five-factor model is insignificant with a t-statistic of (1.5404). This paper can thus only interpret the findings of our first model. This means that the long-short position on the liquidity-btmt portfolio does only lead to an outperformance in the stock market for the three-factor model. Furthermore, it can be seen that the excess returns become lower with the addition of the RMW and CMA control factors, even until the alpha of the strategy is significantly different from 0, taking away all profits.

Name	q. diff (1)	Newey FF3	Newey FF5
MKT-Rf		-0.002130***	-0.001776***
		(-6.795)	(-7.514)
SMB		-0.004107***	-0.002240***
		(-8.733)	(-6.085)
HML		0.002681***	0.002247***
		(-4.715)	(-5.025)
RMW			0.006082***
			(-13.043)
CMA			0.001141
			(-1.507)
Alpha	0.002815**	0.003781***	0.001294
	(2.2963)	(3.7626)	(1.5404)
----------	----------	----------	----------
r2	0		
r2_a	0		
N	601	601	601
----------	----------	----------	----------
	* p<0.10,	** p<0.05,	*** p<0.01

Table 7: The equal weight results of the three double sorted liq-rmw regressions. Column one contains the regression without control variables. The second column shows the coefficients controlling for the Fama and French (1992) three-factor Model, explained in calculations (1), (3), and (5). The third column shows the

coefficients controlling for the Fama and French (2015) five-factor model additionally using calculation (7) and (9). The r2 stands for R-squared (r^2) and is a statistical measure that represents the proportion of the variance for a dependent variable that's explained by an independent variable in a regression model. The r2_a stands for the adjusted R-squared(r2_a), which is a corrected goodness-of-fit measure for linear models. Finally, N represents the number of months that are taken into consideration in this research and the T-statistics are denoted, using * p<0.10, ** p<0.05, *** p<0.01.

The liquidity - investment double sort

The final double sorted liquidity - Fama and French (1992, 2015) factor combination is the liquidity-investment double sort. The paper of Skočir and Lončarski (2018) tried to extend the Fama and French (2015) five-factor model with three popular additional factors. As stated before, this paper found that the slopes of the RMW and CMA factors absorb most of the variation in the liquidity factor return. This further means that the double sorted portfolios based on the CMA factors, could see this factor being (partly) subsumed in the liquidity factor, or it being the other way around. This paper aims to add to the limited existing literature on CMA liquidity double sorts, by further investigating the returns of these factors.

Looking at the double sorted liquidity-investment portfolios in Appendix table 13, this interaction between liquidity and CMA can be further analyzed. It is found that the most illiquid stocks have the highest monthly stock returns in the double sorted portfolios. Looking further, to the second liquidity quintile, the movements are relatively modest and small, though still present. This follows the general findings of this paper so far. Furthermore, in the investment quintiles, it is found that the investment factor is behaving as expected based on Fama and French (2015). As the investment increases (moving from q1 to q3), the monthly returns decrease. This means that, the higher the asset growth companies have, the lower the excess returns are. Like in the previous double sorted portfolios, it is concluded that persistently the liquidity factor remains visible. This also again finds that the least liquid, and thus most difficult to trade stocks, get the highest stock returns in our sample. This makes our model in theory very strong and profitable, but the model's profitability in real-world trading is still up for debate, because of the difficulty to trade on this factor. Furthermore, the lowest liquidity quintile, in combination with the lowest investment quintile has the best stock returns. This shows that the combination of both factors does lead to the best excess stock returns.

Furthermore, looking at the long-short portfolios of table 8, the alphas are listed when controlling for the Fama and French (1992, 2015) factors. It can be seen that this paper's strategy leads to a very high excess market return of 0.6551% and 0.5313% per month respectively. Here, both alphas are significant with t-statistics of (8.7621) and (7.273). This paper can thus interpret the findings of these models. This means that the long-short position on the liquidity-btmt portfolio does lead to an outperformance in the stock market for the three- and five-factor model. Furthermore, it can be seen that the excess returns become lower with the addition of the RMW and CMA control factors, showing their explanatory power in the model.

Name	q. diff (1)	Newey FF3	Newey FF5
MKT-Rf		-0.002174***	-0.001739***
		(-10.228)	(-8.8399)
SMB		-0.000037	-0.000027
		(-0.094)	(0.077)
HML		0.003280***	0.001647***
		(-8.846)	(-4.369)
RMW			0.00057
			(-1.310)
CMA			0.003842***
			(-7.452)
Alpha	0.006219**	0.006551***	0.005313***
	(6.5896)	(8.7621)	(7.273)
----------	----------	----------	----------
r2	0		
r2_a	0		
N	601	601	601
----------	----------	----------	----------
	* p<0.10,	** p<0.05,	*** p<0.01

Table 8: The equal weight results of the three double sorted liq-cma regressions.Column one contains the regression without control variables. The second column shows the coefficients controlling for the Fama and French (1992) three-factor Model, explained in calculations (1), (3), and (5). The third column shows the

coefficients controlling for the Fama and French (2015) five-factor model additionally using calculation (7) and (9). The r2 stands for R-squared (r^2) and is a statistical measure that represents the proportion of the variance for a dependent variable that's explained by an independent variable in a regression model. The r2_a stands for the adjusted R-squared(r2_a), which is a corrected goodness-of-fit measure for linear models. Finally, N represents the number of months that are taken into consideration in this research and the T-statistics are denoted, using * p<0.10, ** p<0.05, *** p<0.01.

The real world limits of this strategy

The main argument that explains why the illiquidity premium still exists, and has not been arbitraged away, is because it is costly to arbitrage away. This potential weakness has been discussed many times in our analysis, but a separate section looking at the actual trading costs could provide more information. The aim is to show if these found alphas, and thus profits, can stay profitable while incurring trading costs. As shown in the double sorted portfolio section, the liquidity premium cannot easily be attained by investing in only the most liquid stocks. If this strategy was able to only invest in liquid stocks, this would have suggested that the liquidity factor is solely a risk premium, meaning an additional return that is given for investing in that specific characteristic. Instead, this paper saw in the portfolios that it were the most illiquid stocks in all double sorted portfolios that had the highest stock returns. In general, it can be stated that these stocks are the most difficult to trade, based on their low liquidity quintile. This means that some of the assumptions of this paper are unrealistic, since they do not deal with the actual trading restrictions. It will be shortly analyzed in this section if this premium is perhaps only profitable on paper. It is beyond the scope of this paper to use the actual trading costs and shorting costs of every stock to give an exact answer to this question. A short analysis is performed to provide additional insights into the annual trading costs that could be incurred without the strategy becoming unprofitable relative to the general market.

The investment strategy of this paper does not rely on a high number of yearly trades. The previous results that were discussed almost all suggested that our strategy has a positive alpha, and this is attained with a rebalancing frequency of only once a year. By setting our research paper up to an investment strategy that is changed only annually, the possibility of performing the strategy in the real world becomes more likely, since trading costs shrink the profits. Furthermore, it can be found in figure 3, that the annual turnover ratio is relatively low for this strategy, specifically compared to strategies like the momentum factor that are more dependent on monthly data. This paper shows a liquidity strategy that can be managed relatively passively. The fractions of stocks in our portfolios that changed quintile for the subsequent year are on average only 33.9% of stocks. This is very similar to the number of quintile changes in the paper of Ibbotson (2013), which found that in their portfolios 62.93% of the stocks stayed in the same quartile. This means that relatively little portfolio changes have to be made annually to follow our strategy. It is also found in figure 3 that the turnover ratio is relatively stable over time, not being influenced too much by any specific economic booms or busts during the researched timeframe.

Finally, using table 9, the annual trading costs that can be incurred without the investment strategy losing profitability is given. It is found that only with very low annual trading costs, this strategy could be applicable in the real world. However, this does come at a cost, since the diversification is very limited for the smallest portfolios of 10 stocks. Furthermore, it might still be very difficult to hold a short position in a stock for an entire year, where an average of an 3% annual shorting fee would already make the strategy unprofitable. This is still without even taking into account the costs of buying the illiquid stocks for a long position. These short positions of the most liquid stocks are relatively costly, and the consequences of more extreme cases, like margin calls, are not taken into account by this paper. Furthermore, knowing that this strategy takes a long position in the most illiquid stocks, adds additional questions to the feasibility of this strategy. It might be very costly, specifically when taking larger positions which institutional investors often have to do, into these small and often low trading volume stocks. This might lead to higher actual buying (selling) prices, due to the additional buying (selling) pressure big orders have on illiquid stocks. Overall, this paper concludes that the strategy does generate a significant alpha that is clearly visible and tradable on paper. One potential explanation that is given for the persistence of the illiquidity premium only in the literature, is that it might be too difficult to profit from it with real world trading costs. The question that remains, is if this anomaly is also profitable using an even more realistic setting, taking into account the actual costs. This could add additional robustness to the findings of this paper, and the related papers.

Stocks in Portf.	10	50	100	4127
Ann. Turnover	3.389	16.945	33.889	1398.608
Ann. BE Costs % FF3	2.377	0.475	0.238	0.006
Ann. BE Costs % FF5	1.599	0.320	0.160	0.004

Table 9: The maximum trading costs for the equal weighted strategies, depending on the number of stocks in the portfolio. The annual turnover represents the number of stocks that will be traded annually, using the mean turnover of our strategy. The annual break even costs are calculated using calculation (15).

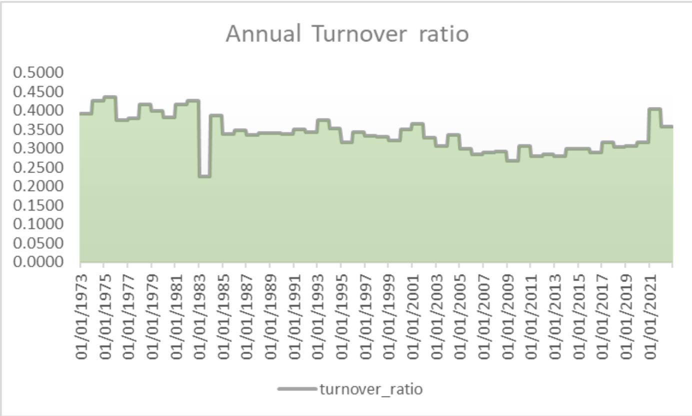


Figure 3: This displays the annual turnover ratio over time. The time period is from December 1972 till December 2022. The turnover ratio is calculated where a change in quintile is one trade, but movements from quintile 1-3 or 3-1 are recorded as two trades (selling the long (short) position and buying the short(long) position).

Ⅵ: Discussion and Limitations

Several weaknesses of this paper, specifically with regards to the actual trading costs, have already been discussed in the previous section. This is why this discussion section will mainly focus on potential limitations on how the research is performed. The results section of this paper started with the Fama and Macbeth (1973) regression being performed. Here, as pointed out in the methodology, the MSCI U.S. index was used as the market proxy, instead of the combination of all assets used in the rest of this research. This might have had a slight influence on the results, but seeing as both are a broad combination of most U.S. market constituents, the effects of this are presumably limited. Furthermore, the size factor, proxied by the natural logarithm of the market capitalization, had a positive and significant coefficient. This is surprising, since many other research papers have found a negative relation between size and stock returns, like the paper of Amihud and Mendelson (1989). Their paper found that excess return is negatively correlated with size, when they researched a model that also included an illiquidity factor in it (bid-ask spread).

The next section of this research paper discussed the single sorted portfolios. Here, the paper analyzed the illiquidity factor with three different weighting techniques, with varying results. The equal-weighted portfolio returns were the only results that were as anticipated beforehand. Specifically, looking at the value-weighted returns, it was unexpected to find a positive alpha for the illiquidity factor. The weighting method appears to have a huge impact, that changes the coefficients a lot relative to the equal-weighted results. Further research could be performed to further analyze the effects reported in this paper and how this difference could be explained. Next, looking at the signal-weighted returns, the findings were not as expected. This measure generally gives relatively little impact to outliers, which was concluded by this paper to be very influential in the returns. This was also the found explanation for the insignificant results for the signal-weighting. It could be interesting for future research to further investigate effects of the signal-weighting, and how it performs with different liquidity measures. Another limitation to this paper is the amount of data that was lacking for the signal-weighted ranking. Specifically, for the Fama and French (2015) signal model, the limitation was set by this paper that all four values, for the market capitalization, book-to-market, profitability, and investment factors needed to be present. Even though all factors individually contained approximately 500.000+ observations, the requirement that all four factors needed to be present lead to only ~2500 observations creating this four-factor signal-weighted result. This also meant that observations before 1975 had to be excluded for this weighting, because too few observations were available after the first 24 months of observations were deleted.

The final section of this paper discussed the double sorted portfolios of the liquidity - Fama and French (1992, 2015) combinations. All combinations of double sorted portfolios attained a positive alpha and were highly significant, except the five-factor controlled Liq-RMW double sort. Interesting here was that the alpha was significant for the three-factor portfolio but not for the five-factor model. Thus, it can be known that it has something to do with the addition of the RMW and CMA factors. Table 12 of the Appendix shows clearly that the quintiles do not behave as expected. It could be interesting to dive further into this specific double sorted result, since there has not been much literature dedicated to this double sorted interaction. A direction for future research could be the double sorted liquidity - Fama and French (2015) RMW/CMA combinations. This could further improve the robustness of this papers' findings.

Ⅶ: Conclusion

This research paper explores the possibilities of expanding the Fama and French three-factor and five-factor models (1992, 2015) with the liquidity factor. By using the Fama and Macbeth (1973) regressions, Fama and French (1992, 2015) single sorted portfolios, and Fama and French (1992, 2015) double sorted portfolios, several results are found. The sample contains all U.S. publicly traded stocks, during the 52 year period of 1970 until 2022. The Fama and Macbeth (1973) regressions finds a coefficient of -0.3274% per month for the liquidity factor, which indicates an underperformance of the more liquid stocks. The Fama and French (1992, 2015) single sorted long-short portfolios find an alpha of -0.6713% and -0.4517% per month for the three- and five-factor models, which are significant on the 5% level. Additional robustness tests of the value-weighted and signal-weighted returns find mixed results. Finally, the Fama and French (1992, 2015) double sorted portfolios found monthly alpha's, controlled for the three- and five-factor factors respectively of 0.3737% and 0.3149% for the liq-size, 0.5917% and 0.3837% for the liq-btmt, 0.3781% and 0.1294% (insignificant) for the liq-rmb, and finally 0.6551% and 0.5313% for the liq-cma double sorted portfolios. Further investigating the limits to arbitrage in the literature, lead to the conclusion that these significant outcomes are easier to attain on paper than in real-life, since the least liquid stocks do have the highest trading costs and short positions are very costly. Finally, the limitations section offers ideas for further research. Specifically interesting are the findings of the single-sorted portfolios. Here, this paper looked at three different weighting techniques, with varying significance of the results. The weighting method appears to have a large impact, changing the coefficients that were found between the equal-weighted, value-weighted and signal-weighted results. Further research could be done to further investigate the effects found and how they could be explained.

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Ⅸ: Appendix:

Monthly	**Excess**	**Returns**	
	Size 1	**2**	**3**
Liq 1	**0.013550 *****	**0.008700 *****	**0.007731 *****
2	**0.011993 *****	**0.008925 *****	**0.007524 *****
3	**0.007439 ***	**0.005337**	**0.006434****

Table 10: This shows the monthly excess portfolio returns of the various portfolio combinations of the liq-size double sorts. The returns of the three liquidity quintiles can be compared by looking up & down in a certain column. The returns of the three size quintiles can be compared by looking left & right in a certain row. The T-statistics are denoted, using * p<0.10, ** p<0.05, *** p<0.01.

Monthly	**Excess**	**Returns**	
	BTMT 1	2	3
Liq 1	0.007388***	0.009145***	0.013610***
	(-3.3827)	(-4.7460)	(-6.0062)
2	0.006162***	0.009015***	0.013032***
	(-2.6346)	(-3.9974)	(-4.8763)
3	0.002939	0.007631**	0.010477***
	(-0.9360)	(-2.5751)	(-3.1797)

Table 11: This shows the monthly excess portfolio returns of the various portfolio combinations of the liq-btmt double sorts. The returns of the three liquidity quintiles can be compared by looking up & down in a certain column. The returns of the three btmt quintiles can be compared by looking left & right in a certain row. The T-statistics are denoted, using * p<0.10, ** p<0.05, *** p<0.01.

Monthly	Excess	Returns	
	RMW 1	2	3
Liq 1	0.012146***	0.011026***	0.011473***
	(-4.4795)	(-5.437)	(-5.5419)
2	0.010455***	0.009032***	0.009379***
	(-3.3835)	(-4.0217)	(-4.246)
3	0.005836	0.007230**	0.007009**
	(-1.5559)	(-2.4592)	(-2.4304)

Table 12: This shows the monthly excess portfolio returns of the various portfolio combinations of the liq-rmw double sorts. The returns of the three liquidity quintiles can be compared by looking up & down in a certain column. The returns of the three rmw quintiles can be compared by looking left & right in a certain row. The T-statistics are denoted, using * p<0.10, ** p<0.05, *** p<0.01.

Monthly	Excess	Returns	
	CMA 1	**2**	**3**
Liq 1	**0.015379*** **	**0.009397*** **	**0.007088*** **
	(-6.1403)	**(-5.2540)**	**(-3.4064)**
2	**0.013877*** **	**0.009063*** **	**0.005697** **
	(-5.0896)	**(-4.3504)**	**(-2.3651)**
3	**0.012452*** **	**0.008774*** **	**0.003203**
	(-3.5844)	**(-3.107)**	**(-1.0169)**

Table 13: This shows the monthly excess portfolio returns of the various portfolio combinations of the liq-cma double sorts. The returns of the three liquidity quintiles can be compared by looking up & down in a certain

column. The returns of the three cma quintiles can be compared by looking left & right in a certain row. The T-statistics are denoted, using * $p<0.10$, ** $p<0.05$, *** $p<0.01$.