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The Evolution of the Idiosyncratic Volatility Anomaly: Dynamics of Time and Size Effect

Master Thesis: Financial Economics

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Abstract

This paper shows that the idiosyncratic volatility anomaly may exist relative to the Fama-French three and five factor models, but its presence is highly dependent on circumstances like the calculation methods, time periods and stock samples being used to investigate the anomaly. When using value-weighted returns, the anomaly is stronger for the smallest quintiles of size of stocks. On the other hand, when using equal-weighted returns the anomaly is stronger for the largest quintiles of size of stocks. Furthermore, for both value- and equal-weighted returns the anomaly seems to have diminished in magnitude over time, and is mostly fueled by economic uncertainty when looking at recession years. This paper underlines the findings by Bali and Cakici (2008) that differences in methodology can highly influence the existence and significance of a cross-sectional relation between idiosyncratic volatility and expected returns.

Table of Contents

1 Introduction

When investing in any type of asset, a person will always consider the risks and returns before making a decision whether to invest or not. This relationship between risk and reward is one of the most fundamental ones within financial economics. As the Capital Asset Pricing Model (CAPM) by Sharpe (1964) states, when a person is willing to take more risk, this person should have a higher expected return. However, over the years more and more evidence emerged that the CAPM does not always hold within the stock market. Black et al. (1972) were one of the first to provide such evidence that within the US stock market the risk/reward concept of CAPM did not hold. Later, the efficient market theory and the CAPM have been challenged by other investing strategies which seem to statistically outperform the market. Fama and French (1993) added the size and value factors to their model in their paper. This was the beginning of a hunt for the so-called anomalies: profitable trading strategies. In fact, evidence has been found that stocks with high volatility have lower returns than stocks with low volatility. This completely contradicts the CAPM model and goes against the common concept of risk and reward. It is, however, good to mention there is a difference between the regular low-risk effect and the idiosyncratic-risk effect. Within the low-risk effect, also known as the low-volatility anomaly, systematic risk is taken into account. This is measured via the correlation with the market, also known as the beta. Talking about idiosyncratic-risk, we talk about unsystematic risk which is risk specific to the company or asset. In other words, this is the residual that cannot be explained by systematic risk.

In early research of idiosyncratic volatility, evidence was found by Merton (1987) that idiosyncratic risk and expected returns were positively related. He concluded that investors demand higher returns for higher idiosyncratic risk. This was supported by other literature, like the paper from Malkiel and Xu (1997). Using portfolios sorted on idiosyncratic volatility they also found that portfolios with the highest idiosyncratic volatility, resulted in the highest returns. Later, Ang et al. (2006) built further on this assumption, but found contradicting evidence that the idiosyncratic volatility and expected returns are actually negatively related. They concluded this after testing relative to the three factor model by Fama and French (1993). This contradiction is known as *the idiosyncratic risk puzzle*. The study by Ang et al. (2006) was later followed by other papers that confirmed similar findings. In the years after, Blitz and van Vliet (2007), Bali and Cakici (2008) and Ang et al. (2009) confirmed the existence of the idiosyncratic volatility anomaly and confirmed that stocks with low volatility earn high risk-adjusted returns after sorting on past volatility.

It seems there has been a gradual shift from concluding a positive relationship to a negative relationship between idiosyncratic risk and stock returns. However, even research from the 21st century concluded the that the idiosyncratic volatility anomaly did not exist. For example, Goyal and Santa Clara (2003) and Fu (2009) actually found a significant positive relationship between idiosyncratic volatility and expected returns. Most research that has been done around the idiosyncratic volatility anomaly were all based on stock data prior to the early 2000s. For this research, I will use a sample of all US Stocks (NYSE, NASDAQ, AMEX) from CRSP, with a sample period of 1963-2021. Using daily returns of all stocks, both equal-weighted and value-weighted, I am able to get a clear picture on the relationship between idiosyncratic volatility and expected returns. First, after calculating the idiosyncratic volatility by taking the standard deviation of the residuals from a time-series regression of daily excess returns relative to the Fama-French three factor and five factor model, I sort the stocks into quintile portfolios based on idiosyncratic volatility. Then, I also create quintiles sorted on time to see if the anomaly was present for each quintile. I compare the monthly excess returns of the highest quintile with respect to the lowest. Using a large timeframe and by double sorting, I will be able to test these anomaly's with more recent data than when these anomalies were originally discovered. Also, by testing against the Fama-French three and five factor model, and using value-weighted and equal-weighted returns, for each of these four pairings it will show how the choice of calculation will influence the results. Bali and Cakici (2008) showed that differences in methodology can highly influence the existence and significance of a cross-sectional relation between idiosyncratic volatility and expected returns. With regards to the evolution of idiosyncratic volatility, this paper will give a new look into how the idiosyncratic risk puzzle changed over time. As this suggested anomaly has now been known to more investors, one could expect this anomaly has diminished in magnitude over time. Therefore the first hypothesis will be as follows:

Hypothesis 1: *The magnitude of the idiosyncratic volatility anomaly is less present in modern markets.*

For the second hypothesis, I would like to incorporate the size effect. Banz (1981) discovered this effect and found that smaller firms had higher risk-adjusted returns than larger firms in the sample period of 1963-1975. This was later confirmed by Fama and French (1992) and is one of the factors within the Fama and French three factor and five factor model. After the size effect was founded, several studies have been done around the relationship between size and the idiosyncratic volatility anomaly. Ang et al. (2006) found a significant negative relationship between idiosyncratic volatility and size. They found that the volatility effect is strongest among small stocks and weakest among large stocks. Bali and Cakici (2008) have even shown that the idiosyncratic volatility effect disappears after excluding the smallest size decile of stocks from a portfolio. This would mean that with stock with a higher market cap, the idiosyncratic volatility anomaly would be less and less present. Furthermore they found that the idiosyncratic volatility anomaly was only present when using value-weighted returns. The second hypothesis therefore will be as follows:

Hypothesis 2: *The size effect diminishes the magnitude of the idiosyncratic volatility anomaly.*

To test for the size effect I double sort on idiosyncratic volatility and size. The size is measured by the lagged market cap of the stock. As done for the first hypothesis, testing against the Fama-French three and five factor model, and using value-weighted and equal-weighted returns, for each of these four pairings it will show how the choice of calculation will influence the results. Also, for each quintile and each portfolio I will use a trading strategy as done by Ang et al. (2006). The strategy goes long on the quintile with the highest idiosyncratic volatility and short on the quintile with the lowest idiosyncratic volatility. After every month the portfolios are rebalanced and sorted into new quintiles of idiosyncratic volatility. This strategy will result in excess strategy returns and an alpha which will be tested using Newey-West t-statistics.

For robustness tests, I estimate idiosyncratic volatility using multiple estimation periods. I use a one month lag, but also a three-month and twelve-month rolling period to calculate the idiosyncratic volatility and test what the effect is on the equal-weighted and value-weighted returns. I find that the idiosyncratic volatility anomaly calculated by the Fama-French three and five factor models can be and has been present under certain circumstances that are highly dependent on the calculation methods and samples being used to investigate the anomaly. However, there is not a noticeable difference between the two models by Fama and French, indicating the two additional factors of the five factor model have no additional explanatory power beyond the three factor model. Using a longer rolling estimation period to calculate the idiosyncratic volatility, the effect becomes weaker. Furthermore, when using value-weighted returns, the anomaly is only present for the smallest quintiles of size of stocks. On the other hand, when using equal-weighted returns the anomaly is present for the largest quintiles of size of stocks. For both equal- and value-weighted returns the anomaly seems to have diminished in magnitude over time, and the presence of this seems to be fueled by economic uncertainty when looking at years of recession.

This paper contributes to a broader understanding of the idiosyncratic volatility anomaly in the stock market. Using daily returns on a sample of almost 60 years, using multiple methodologies and applying different robustness checks, this paper tries to get a clearer picture of when and to what extent the volatility anomaly exists and to what extent it does not. The paper is structured as follows. Following the introduction, the literature overview will contain relevant research regarding the low-volatility anomaly, idiosyncratic volatility and the size effect. Next, the data and methodology will be explained and discussed. After that, the results will be shown. Finally, the paper will end with the conclusion.

2 Literature

2.1 Classical Economic Theory

The Capital Asset Pricing Model by Sharpe (1964) and Lintner (1965) is one of the most widely known models to measure the expected returns in relation to risk. As of today, this model is still widely used within the world of financial economics. In simple words, this model shows that an increase in the systematic risk will increase the expected return. This theory says that the cross-sectional variation in stock returns is not affected by any other variable than the systematic risk. It is being assumed in this model that investors are well-diversified, which causes idiosyncratic risk to be diversified as well. In other words, within CAPM, idiosyncratic risk is not priced. The risk in this model is measured by the beta of the asset. Within the stock market, this means that when a stock is riskier than the market, the beta will increase. The positive relationship between the beta and expected return is something that later has been questioned by other research. One of the first ones to question this relationship were Black, Jensen, Scholes et al. (1972). They tested the CAPM model by looking at the slope of the beta using a time-series. They found in their research that stocks with high beta tend to have lower returns than stocks with low beta. This is contradicting with the CAPM, which states otherwise. This finding caused many researchers to look deeper into this anomaly.

Banz (1981) further challenged the CAPM, as he found that returns on small stocks (measured by bottom 20% of NYSE market capitalization) outperform the returns on large stocks. This finding is the foundation to what is known as the size effect. Fama and French (1992) extended the work of Black, Jensen, Scholes et al. (1972) and Banz (1981) and they found that there was no positive relationship between the beta and the returns. In fact, they found a flat relationship between the market volatility and stock returns. This was the basis for their Fama and French three factor model, where they added the size factor and the value factor to the CAPM model, which only consists of a market factor.

As discussed earlier, the concept of risk is not just systematic risk, but also idiosyncratic risk, also known as company-specific risk. One of the first researchers to study idiosyncratic risk was Merton (1987). He found that investors do not hold portfolios that are as well-diversified as CAPM would suggest. In fact, due to factors like transaction costs and information asymmetry, he suggests investors would demand higher expected returns for this idiosyncratic risk. He found a positive relationship between idiosyncratic volatility and expected returns. Also other researchers found a positive relationship. Lehmann (1990) studied residual risk and found that idiosyncratic volatility and expected returns are positively related, albeit within the context of the CAPM model. Malkiel and Xu (1997) similarly found a positive relationship between the idiosyncratic volatility and stock returns. These studies are in line with the concept of classical economic theory, where higher risk should be rewarded with higher returns.

2.2 Idiosyncratic risk puzzle

Idiosyncratic volatility has been a heavily discussed topic in further research. Contradictory to Merton (1987), literature had different findings on the relationship between idiosyncratic volatility and future returns. In their seminal paper, Ang et al. (2006) not only found that the high idiosyncratic volatility stocks had low returns, but also found that the relationship between the idiosyncratic volatility and expected return was negative. The so-called *idiosyncratic risk puzzle* was founded. They measured this anomaly by using the IVOL (Idiosyncratic Volatility) measure. After sorting stocks into quintile portfolios ranked on idiosyncratic volatility, the stocks with the lowest idiosyncratic volatility outperformed the stocks with the highest idiosyncratic volatility. This measure uses daily return data from the past month to estimate the IVOL effect. Furthermore, the result remained robust after controlling for factors like value, momentum and size.

Similarly, Blitz and van Vliet (2007) continued research by Ang et al. (2006) and also concluded that stocks with low volatility earn high risk-adjusted returns after sorting on past volatility. This effect was measured not only in US markets, but also in Europe and Japan. According to their research, the reasoning for this effect cannot be explained by value and size. Later, Blitz et al. (2013) did similar research in emerging markets, and found that the relationship between idiosyncratic risk and returns is negative within these markets. Interestingly they also found that the anomaly has become stronger over time in these emerging markets. They did however, not find any correlation between emerging and developing markets, which suggests that the drivers for the low-volatility effect are different for each market.

Bali and Cakici (2008) looked at the cross-sectional relation between idiosyncratic volatility and expected stock returns. They found that the methodology of the research is of great importance. Aspects like data frequency, the weighting scheme and the breakpoints being used within sorting play a big role in the existence and significance of the idiosyncratic risk anomaly. In their research they found that the idiosyncratic volatility anomaly was only present when using value-weighted returns. Furthermore, they conclude that the anomaly is only significant in small-cap stocks and becomes insignificant when the 20% of smallest stocks are being removed. Contrary, Chen et al. (2020) found that when penny stocks were removed from the sample, the idiosyncratic volatility anomaly actually becomes stronger and more robust. They even claim that market microstructure noise like the small sized penny stocks weakens the idiosyncratic volatility instead of being the cause of it.

As more recent literature based on Ang et al. (2006) was pointing towards a negative relationship between idiosyncratic volatility and stock returns, Fu (2009) had a different conclusion and found a significant positive relationship. Fu (2009) used a different measure for the idiosyncratic volatility than Ang et al. (2006). Instead of using the one-month lagged idiosyncratic volatility, he used the exponential generalized autoregressive conditional heteroskedastic (EGARCH) model to estimate the idiosyncratic volatility. By using this model he found a positive relationship between the stock returns and the idiosyncratic volatility after regressing against the three factor model by Fama and French (1993). Later however, Guo et al. (2014) criticized this methodology, by stating the EGARCH model may be subject to substantial look-ahead bias. According to them, this bias was actually the main driver of the positive relationship between idiosyncratic risk and returns. Fink et al. (2012) corrected for this bias and did not find a positive relationship between the idiosyncratic volatility and expected returns using this model. In one of the more recent papers on idiosyncratic volatility, Liu (2021) decomposed the idiosyncratic volatility of stock returns into short-run and long-run components. He found a significant negative relationship between conditional long-run idiosyncratic volatility and expected stock returns, and a positive relationship between short-run idiosyncratic volatility. This shows that idiosyncratic variations over short and long horizons can significantly impact cross-sectional stock returns.

2.3 Explanations for the anomaly

After the findings on the low volatility anomaly, Frazzini and Pedersen (2014) took it a step further by identifying the Betting-Against-Beta (BAB) factor. To explore the low volatility anomaly further, they created a long low-beta and short high-beta portfolio which resulted in higher expected returns. They showed this in multiple asset classes and multiple regions. The effect may be explained due to excess demand for high-beta stocks by leverage constrained investors, which causes the price for these high-beta stocks to go up and expected returns to go down. Vice versa, this causes higher expected returns for the low-beta stocks. Liu et al. (2018) extended the research by Frazzini and Pedersen (2014) by comparing the Betting-Against-Beta factor with the IVOL factor from Ang et al. (2006). They found that the beta anomaly arises from the positive correlation with the idiosyncratic volatility. This relationship between this so-called IVOL factor and alpha is negative and stronger among overpriced stocks, which is in line with earlier research by Stambaugh et al. (2015). After controlling for idiosyncratic volatility or excluding overpriced stocks with high idiosyncratic volatility, the beta anomaly becomes insignificant. Asness et al. (2020) continued this research by introducing a new factor, the Betting-Against-Correlation (BAC) factor. This new factor is used to differentiate between leverage constraints and behavioral explanations. They find that the main driver of the low-risk effect is the volatility. Leverage constraints explain the positive risk-adjusted returns based on systematic risk, while idiosyncratic risk factors can be explained by behavior as shown by Ang et al. (2006) and Ang et al. (2009).

Blitz and van Vliet (2007) think the effect can be explained due to factors like leverage and low liquidity of high-risk stocks. Also behavioral biases can play a part in the volatility effect. Based on earlier research by Shefrin and Statman (2000), they suggest that private investors seem to overpay for risky stocks as they are perceived as lottery tickets. They are being perceived like this as they have a very low probability of a very large payoff, like buying an actual ticket for a lottery. Therefore, as investors prefer these assets, these risky high volatile stocks will become overpriced and in the end will lower the expected returns. Based on the cumulative prospect theory by Tversky and Kahneman (1992) there is also reasoning for the effect. Within this theory it is being perceived that investors attach objective probabilities to expected returns, which causes them to think the chance of success is higher than it actually is. Bali et al. (2011) concluded similarly that investors are overconfident in thinking they are able to select stocks that will outperform the market. By preferring lottery-like assets with a small probability for a large payoff, the price of these assets get influenced and future returns will decrease. Earlier, this phenomenon was also found found by Kumar (2009) in relation to stocks, where investors have a preference for lottery-type stocks. These are defined as low-priced stocks with high idiosyncratic volatility and high idiosyncratic skewness. Looking at behavioral biases within decision making under risk and uncertainty, the behavioral biases have also been given as reasoning by Baker et al. (2011). They focused more on the institutional investors, and pointed out that they intend to be biased towards high-volatility stocks due to factors like overconfidence.

Hsu et al. (2013) also look at explanations for the low-volatility anomaly after empirically finding the presence of it in developed and emerging markets. They suggest that the bias of analysts plays a role in the overpricing of high-volatile stocks. According to their study, sell-side analysts are overestimating the growth forecast of earnings of high volatile stocks. In return, this overestimation causes the stock price to increase as investors are not able to correct for this bias and overpay for the stock. Another explanation is that analysts overestimate growth forecasts for stocks where they have positive information of.

From a macroeconomic point of view, Chen and Petkova (2012) argue that the idiosyncratic volatility anomaly can be explained by the average market variance. During economic uncertainty, when market variance is higher, investors would prefer stocks with high idiosyncratic risk as they have higher expected returns during such a period. Therefore it is being argued that it is the economic uncertainty in macroeconomics that is a missing factor and could be causing the idiosyncratic volatility anomaly to exist. Recent study by Li et al. (2021) also finds that economic uncertainty is an important driver to the idiosyncratic volatility puzzle. They argue that the uncertainty-averse investor contributes to the existence of the puzzle. Only when economic uncertainty increases, the idiosyncratic volatility returns become negative. These findings were consistent with research by Bali et al. (2017) who also found that economic uncertainty is a negatively priced factor in the cross-section of stock returns. Although this paper does not have the intention to search for a missing factor, as per the first hypothesis I will be checking for different years of economic uncertainty to see if this could potentially play a role in the existence of idiosyncratic volatility.

3 Data and Methodology

3.1 Data

For this paper, daily stock return data is obtained from the Center of Research in Security Prices (CRSP). These returns are adjusted for corporate action events, like stock splits, dividends and spin-offs. The sample contains all common stocks (share codes 10 and 11) traded on NYSE, NASDAQ and AMEX. The sample period being used is from July 1963 to December 2021. This range is chosen as CRSP data starts in July 1963, so this is the largest available sample of daily returns. The total amount of observations is around 68,000,000. Then, the daily returns are adjusted for delisting's as done by Shumway (1997). Also, stocks with an average price below 5$ have been removed. This is done because those stocks can be marked as penny stocks and are usually very illiquid so are likely to cause a bias in the sample. This is in line with existing literature like Bali and Cakici (2008) who also correct for this. Daily returns are chosen above monthly returns as I want to compute the idiosyncratic volatility per month by using the daily excess returns. Furthermore, this is in line with key papers on idiosyncratic volatility by Ang et al. (2006) and Ang et al. (2009). It is of importance to note that Bali and Cakici (2008) found that differences in methodology can highly influence the existence and significance of a cross-sectional relation between idiosyncratic risk and expected returns. The use of daily versus monthly returns can already make a huge impact. Therefore it is good to keep in mind that other approaches might significantly change the results, as we have seen in the idiosyncratic puzzle.

3.2 Methodology

I will test the anomaly against the Fama-French three factor model and the Fama-French five factor model. As the five factor model was introduced in 2015, in recent years more and more literature is adapting towards the Fama-French five factor model. Older literature and almost all research on the idiosyncratic volatility anomaly has only used the Fama-French three factor model, so this research will be one of the first to also test against the five factor model. Other studies like Merton (1987) use the CAPM model, however in CAPM there is only one single factor, the beta. As the Fama-French models include more factors, this is seen as a better predictor. The factor data has been obtained from Kenneth

French's website. Furthermore, the risk-free rate has also been obtained from Kenneth French's website. They use the one-month Treasury bill rate in their data. Following the approach by Ang et al. (2006) the idiosyncratic volatility anomaly will be tested by taking the standard deviation of the residuals from a time-series regression of daily excess returns relative to the Fama-French three factor model. As mentioned, one of the differences of this paper is that I will extent this research by also testing it against the Fama-French five model. This model is as follows:

$$R_{i,t} = \alpha_i + \beta_{MKT,i}\, MKT_t + \beta_{SMB,i}\, SMB_t + \beta_{HML,i}\, HML_t + \beta_{RMW,i}\, RMW_t + \beta_{CMA,i}\, CMA_t + \varepsilon_{i,t} \quad (1)$$

$$IVOL = \sqrt{Var(\varepsilon_{i,t,\tau})} \quad (2)$$

Here the daily excess returns $R_{i,t}$ of stock i in month t are calculated by subtracting the daily stock returns by the risk-free rate. MKT is the market factor, which are the excess market returns, SMB is size (small size over big size stocks), HML is book-to-market (value over growth stocks), RMW is operating profitability (robust over weak operating profitability portfolios) and CMA is investment (conservative over aggressive investment portfolios). The idiosyncratic volatility (IVOL) is being regressed for each stock i and each month t separately for all trading days τ during that month. The other Fama-French model, the three factor model, is as follows:

$$R_{i,t} = \alpha_i + \beta_{MKT,i}\, MKT_t + \beta_{SMB,i}\, SMB_t + \beta_{HML,i}\, HML_t + \varepsilon_{i,t} \quad (3)$$

Here we see that the big difference with the five factor model is that the three factor model does not include the RMW and CMA factors. These two factors were only recently introduced by Fama and French (2015). After calculating the monthly idiosyncratic volatility, the data has been collapsed to monthly stock returns with a unique idiosyncratic volatility per stock per month. I will regress for both equal-weighted returns and value-weighted returns. This is being done, as for smaller firms there might be more extreme returns compared to larger sized firms. Also, this is in line with Ang et al. (2009), who use these returns. Value-weighted returns are calculated by using the lagged market cap as a weight variable. Then, the lagged market caps per observation are divided by the total lagged market caps of all observations at that specific date. That generates a weight, which is then multiplied by the excess returns.

After calculating the excess returns and the idiosyncratic volatility, all stocks will be sorted into quintile portfolios, ranked on the idiosyncratic volatility. This means Q1 has the lowest idiosyncratic volatility and Q5 has the highest. For both equal-weighted returns and value-weighted returns of the idiosyncratic volatility computed using the Fama-French three factor and five factor model, the quintiles are created. In relation to the first hypothesis, I divide the total sample of both equal- and value-weighted returns into five different time quintiles. Via this way I can test if the idiosyncratic volatility anomaly has changed over time. The time periods are divided into groups of approximately 11-12 years. Quintile 1 is July 1963-1974, followed by 1975-1986, 1987-1998, 1999-2010 and 2011-2021. Furthermore I will also look at years of recession within the U.S. economy to see if the idiosyncratic volatility anomaly was stronger in those periods. I will look at five years of (partial) recession: 1974, 1982, 1990, 2001, 2008. These years are (partially) stated as recession years by the National Bureau of Economic Research, and are characterized by a significant decline in economic activity.

For the second hypothesis I take into account size. The size variable is defined by the market capitalization of the stock. For this purpose I use the lagged market cap to account for size, as returns are based on the difference compared to the previous observation. Also for size I divided the data into quintiles from lowest size to highest size. Next, I double sort on size and idiosyncratic volatility to see how each portfolio differs from each other with regards to the excess monthly returns. This results in a 5x5 matrix, of 25 different portfolios.

Besides creating quintiles on idiosyncratic volatility, size and also time, I also use a trading strategy in order to test for the relationship between idiosyncratic volatility and returns. The strategy goes long on the quintile with the highest idiosyncratic volatility and short on the quintile with the lowest idiosyncratic volatility. After every month the portfolios are rebalanced and sorted into new quintiles of idiosyncratic volatility. Then again, I go long on the highest quintile and short on the lowest. This will be repeated for every month and is similar to the approach by Ang et al. (2006). This strategy will result in excess strategy returns and an alpha which will be tested using Newey-West t-statistics. The Newey-West t-statistic tests for heteroscedasticity and autocorrelation. I will also apply this strategy for the second hypothesis looking at the size effect.

3.3 Robustness checks

For robustness checks, I will not only use the one month idiosyncratic volatility over the past month ($\sigma_i(t-1,t)$), but also test for past three $(t-3,t)$ and twelve $(t-12,t)$ months of idiosyncratic volatility, which is similar to the approach by Ang et al. (2009). They do report significant differences in using different formation periods of computing idiosyncratic volatility. Furthermore, to reduce skewness and kurtosis, I will also do the tests with the natural logarithm of idiosyncratic volatility as done by Goyal and Santa Clara (2003). I will test if the change of one standard deviation increase of idiosyncratic volatility will change the magnitude of the average excess return per quintile.

4 Results

Table 4.1: Summary Statistics

This table shows the summary statistics for the sample used for this research. There are six main factors for this research: Value-weighted FF3 IVOL, value-weighted FF5 IVOL, equal-weighted FF3 IVOL, equal-weighted FF5 IVOL, value-weighted monthly excess returns, equal-weighted monthly excess returns. The summary statistics include the mean, median, min, max, standard deviation, 25th percentile, 75th percentile, and the skewness.

	Mean	Median	Min	Max	Std. Dev.	P25	P75	Skewness
FF5 value ivol	0.00057	0.00023	0	0.321	0.00213	0.000166	0.00038	24.79
FF3 value ivol	0.00057	0.00023	0	0.321	0.00213	0.000164	0.00038	24.79
FF5 equal ivol	0.02777	0.02166	0.0000296	3.247	0.02511	0.014151	0.03368	11.83
FF3 equal ivol	0.02776	0.02165	0.0000002	3.217	0.02512	0.014148	0.03368	11.82
Value excess return	0.00016	0.00937	-0.836	1.820	0.00937	-0.00018	0.00247	13.49
Equal excess return	0.00556	-0.0007	-1.000	18.947	0.11579	-0.38818	0.04152	7.24

First, we take a look at the summary statistics in Table 4.1 We see there is no difference in the idiosyncratic volatility between the three five factor models. However, we do see differences between the equal-weighted and value-weighted portfolios. For the equal-weighted IVOL and monthly excess return, the mean, median, max and standard deviation are much higher than for the value-weighted. This is mainly due to the fact that the value-weighted excess returns and idiosyncratic volatility have higher skewness. As a small group of large cap stocks outweigh the large group of small-cap stocks, the small-cap stocks get very low value-weighted returns. Even though the penny stocks were excluded from the sample, the difference between equal-weighted and value-weighted returns remained. The very small value-weighted returns then also cause the average idiosyncratic volatility (IVOL) to become smaller. Therefore it is better to look at the economic meaning of the results by interpreting the numbers given a one standard deviation increase of idiosyncratic volatility. As explained earlier, per monthly excess return and firm, the idiosyncratic volatility has been divided into five quintiles. The lowest quintile, Q1, consists of the lowest idiosyncratic volatility and the highest quintile, Q5 consists of monthly excess returns with the highest idiosyncratic volatility. In Table 4.2 we can see a summary, regressions results and the difference/delta of a one standard deviation increase of the five factor idiosyncratic volatility on the value-weighted monthly excess returns.

Table 4.2: Magnitude FF5 value-weighted

This table reports the expected change in results after a one standard deviation change in the Fama and French five factor idiosyncratic volatility of monthly value-weighted returns. For every month the stocks are sorted into quintiles based on idiosyncratic volatility of the past month. Every month the portfolios and quintiles are rebalanced. Significance at a 5% level or better is indicated in bold.

Quintiles	1	2	3	4	5
FF5 value IVOL coeff	**0.1154165**	**0.0658134**	**-0.0121016**	**0.0417391**	**0.130552**
	(11.64)	(12.81)	(-2.36)	(3.53)	(20.42)
IVOL mean	0.0001921	0.000218	0.0002336	0.0002877	0.0019427
IVOL SD	0.0000998	0.000122	0.0001292	0.0001384	0.0045077
Excess return mean	0.0000465	0.000024	0.0000196	0.000052	0.0006312
Δ SD	0.24771111	0.33455145	-0.079771771	0.11109022	0.932334047

We can see that for the lowest quintile, one standard deviation increase of IVOL, results in an increase of the average return by 0.2477, or 24.77%. Only for the third quintile this number is negative. For the highest quintile this number is the highest at 0.9323, or 93.23%. Here, we see the clear difference between the lowest and highest quintile and the magnitude of an increase of one standard deviation of idiosyncratic volatility on the value-weighted monthly excess returns. So far there is no indication of the presence of the idiosyncratic volatility anomaly. After this, we repeat the same test for the equal-weighted monthly excess returns.

Table 4.3: Magnitude FF5 equal-weighted

This table reports the expected change in results after a one standard deviation change in the Fama and French five factor idiosyncratic volatility of monthly equal-weighted returns. For every month the stocks are sorted into quintiles based on idiosyncratic volatility of the past month. Every month the portfolios and quintiles are rebalanced. Significance at a 5% level or better is indicated in bold.

Quintiles	1	2	3	4	5
FF5 equal IVOL coeff	0.3026329	0.4998175	0.6094092	0.605195	0.1509442
	(10.05)	(16.71)	(22.07)	(-3.61)	(16.09)
IVOL mean	0.107549	0.165355	0.225857	0.311653	0.569998
IVOL SD	0.0040111	0.0049908	0.0066938	0.0094758	0.0364725
Excess return mean	0.0081258	0.0102247	0.0113263	0.010858	0.0084931
Δ SD	0.14938724	0.24396698	0.360158507	0.528154981	0.648209998

For the equal-weighted returns there are different results. Here, the positive delta for a one standard deviation increase in the lowest quintile of IVOL on the equal-weighted monthly excess returns becomes larger and larger the higher the quintile. On the other hand, the delta for Q5 is lower than for the value-weighted returns, indicating that the magnitude of higher returns after a one standard deviation increase is less. Nonetheless, also for the equal-weighted returns, there is no sign of the idiosyncratic volatility anomaly so far, as for that matter we would expect to see a decrease once we go higher into the quintiles.

Next, we applied a trading strategy on the entire sample of the four portfolios of FF3 and FF5 equal- and value-weighted returns. I applied the long/short trading strategy onto each quintile of size. Each month for each quintile, we go long on the stocks with the highest past one month idiosyncratic volatility and short on the lowest idiosyncratic volatility. After each month the portfolio's will be rebalanced and the method is repeated.

Table 4.4: One-month idiosyncratic volatility Long/Short strategy

This table reports the trading strategy excess returns and alphas of the Fama and French five factor and three factor value-weighted and equal-weighted returns. For every month the stocks are sorted into quintiles based on idiosyncratic volatility of the past month. Every month the portfolios and quintiles are rebalanced. Alpha significance at a 5% level or better is indicated in bold, using Newey-West adjusted t-stats in brackets and calculated using 12 lags.

	FF5 value	FF3 value	FF5 equal	FF3 equal
Strategy ret	0.0292%	0.0293%	0.1837%	0.0184%
Alpha	**0.0339%**	**0.0339%**	**0.8310%**	**0.8301%**
	(21.69)	(21.69)	(46.6)	(46.35)

We see that the long/short strategy results in positive monthly excess returns for all four portfolios. For the value-weighted portfolio based on the FF5-factors, the returns are equal to the FF3-factor portfolio. Both are slightly higher than the equal-weighted excess returns, which are also similar for both models. The alpha's for the equal-weighted excess returns on the other hand are much higher. However, for all outcomes we see no negative numbers yet. As in the previous table 4.2 and 4.3 there is yet no signs of the idiosyncratic volatility anomaly.

To get more insight into the idiosyncratic volatility anomaly and more insight into the quintiles, the portfolios are double sorted. First I divided the sample into quintile time periods. This will allow to see for hypothesis one if and how the volatility anomaly evolved over time. For both value-weighted and equal-weighted returns and both FF3- and FF5-factors I divided the sample into five time periods. To test for the volatility anomaly I again sorted on idiosyncratic volatility by dividing the sample into quintiles per FF3 and FF5 value- and equal-weighted monthly excess returns. There I looked at the difference in monthly excess returns of the highest quintile of idiosyncratic volatility compared to the lowest quintile. In below table, this difference is expressed in percentages. This is the difference in excess monthly returns between the fifth and the first quintile for the four datasets.

Table 4.5: One-month Idiosyncratic Volatility effect over time

This table reports the excess returns of highest quintile with respect to the lowest quintile of one month idiosyncratic volatility per period in time. By double sorting on idiosyncratic volatility and time periods, the excess returns were generated. Significance at a 5% level or better is indicated in bold.

	FF5 value	FF3 value	FF5 equal	FF3 equal
Total	**0.0585%**	**0.0585%**	0.0367%	0.0367%
	(-31.94)	(-31.98)	(-1.18)	(-1.18)
1963-1974	**-0.0281%**	**-0.0281%**	**-0.4333%**	**-0.4335%**
	(-4.32)	(-4.32)	(-6.19)	(-6.19)
1975-1986	**0.0635%**	**0.0636%**	0.0468%	0.0483%
	(19.06)	(19.08)	(-0.83)	(-0.86)
1987-1998	**0.0854%**	**0.0855%**	**-0.1859%**	**-0.1866%**
	(-33.78)	(-33.79)	(-3.13)	(-3.14)
1999-2010	0.0014%	0.0016%	**0.4926%**	**0.4879%**
	(-0.32)	(-0.37)	(-6.42)	(-6.36)
2011-2021	**0.1579%**	**0.1579%**	0.0961%	0.1020%
	(-29.57)	(-29.57)	(-1.19)	(-1.27)

In the table above we see a few interesting trends. First, for the total sample in all portfolios, the monthly excess returns are positive. However, we do see a trend of increasing excess returns. In 1963-1974 all four models had negative excess returns, and also in 1987-1998 for equal-weighted returns the excess returns were significantly negative. Since the 21st century there have not been negative excess returns, showing an indication that the idiosyncratic volatility anomaly has diminished over time using the one-month estimation. There are only minor differences between the five factor model and the three factor model for both returns, indicating there is not a difference between the two models. With respect to the first hypothesis, solely based on this information we do see signs that the idiosyncratic volatility anomaly is less present in modern markets.

So far only for a few specific time quintiles and portfolios there are signs of a presence of the idiosyncratic volatility anomaly. To get a better understanding we will look at the idiosyncratic volatility in recession years. As earlier mentioned, we will look at 1974,1982,1990,2001,2008 as these years were (almost) full recession years as by the National Bureau of Economic Research. Following research by Chen and Petkova (2012), we might expect market variance to have some explanatory power in the idiosyncratic volatility effect. During economic recession, markets are more volatility and the variance is higher. Below we see the results of conducting the same analysis as in Table 4.5, except this time we specified for the specific years.

Table 4.6: One-month Idiosyncratic Volatility effect during recessions

This table reports the excess returns of the highest quintile with respect to the lowest quintile of one month idiosyncratic volatility for the recession years 1974, 1982, 1990, 2001, 2008. By sorting on idiosyncratic volatility and only selecting the specific time period, the excess returns were generated. Significance at a 5% level or better is indicated in bold.

	FF5 value	FF3 value	FF5 equal	FF3 equal
1974	**-0.3309%**	**-0.3310%**	0.0369%	0.0473%
	(-18.12)	(-18.14)	(-0.17)	(-0.17)
1982	**0.0731%**	**0.0731%**	-0.3083%	-0.2846%
	(-5.80)	(-5.8)	(-1.49)	(-1.38)
1990	**-0.0861%**	**-0.0863%**	**-2.1899%**	**-2.2033%**
	(-8.54)	(-8.56)	(-9.34)	(-9.40)
2001	**-0.1459%**	**-0.1462%**	**1.2962%**	**1.3075%**
	(-8.70)	(-8.72)	(-3.96)	(-4.00)
2008	**-0.3925%**	**-0.3916%**	**-3.5504%**	**-3.5746%**
	(-21.68)	(-21.63)	(-12.69)	(-12.77)

During the recession years we see differences in the monthly excess returns compared to the time periods as stated in Table 4.5. In 1974, we see significant lower returns for the value-weighted portfolios in comparison to the overall period of 1963-1974. In 1982 we see somewhat similar results for value-weighted returns, and lower, though not significant returns for the equal-weighted models. In 1990 we again see a big negative difference for both the value-weighted and the equal-weighted returns in comparison to the larger timeframe of 1987-1998. In 2001 we see mixed results for the value- and equal-weighted portfolios. In 2008 we see a big negative difference compared to the larger timeframe of 1999-2010. Altogether, for all four portfolios we see large negative excess returns for the highest quintile compared to the lowest quintile in this year. Since also for most other recession years the excess returns are lower than the total sample and the associated timeframes, the idiosyncratic volatility anomaly is showing its first signs of existence. In 1990 and 2008 for all four and in 1974 and 2001 for at least one portfolio we can even state there is a presence of an idiosyncratic volatility anomaly for that specific year.

After looking at the different time periods and the few years of recession, not finding too many signs of a presence of the idiosyncratic volatility anomaly, size will be incorporated into this research to answer the second hypothesis. All four portfolios/datasets of FF5 and FF3 value- and equal-weighted returns have been divided into five quintiles on size. Then, the excess returns of the difference between the highest and the lowest quintile of idiosyncratic volatility were compared within each quintile.

Table 4.7: One-month Idiosyncratic Volatility effect on size

This table reports the excess returns of the highest quintile with respect to the lowest quintile of one month idiosyncratic volatility per quintile of size. By double sorting on idiosyncratic volatility and size, the excess returns are taken for each level of size. Significance at a 5% level or better is indicated in bold.

Size	FF5 value	FF3 value	FF5 equal	FF3 equal
1	**-0.0061%**	**-0.0061%**	**0.8180%**	**0.8158%**
	(-13.56)	(-13.52)	(7.78)	(7.76)
2	-0.0001%	-0.0005%	**-0.2900%**	**-0.2869%**
	(-1.64)	*(-1.01)*	(-3.72)	(-3.68)
3	**-0.0033%**	**-0.0029%**	**-0.5652%**	**-0.5667%**
	(-4.70)	(-4.17)	(-7.88)	(-7.90)
4	**-0.0065%**	**-0.0065%**	**-0.9296%**	**-0.9367%**
	(-10.18)	(-10.26)	(-13.89)	(-14.00)
5	**0.0604%**	**0.0608%**	**-1.5291%**	**-1.5308%**
	(3.83)	(3.86)	(-20.52)	(-20.56)

Above, we see differences between the results of the value-weighted monthly excess returns and the equal-weighted excess returns. For the value-weighted monthly excess returns we can see that the returns of the highest quintile with respect to the lowest quintile of idiosyncratic volatility are the highest in the fifth quintile. This is indicating that for the largest sized stocks, the stocks with the highest idiosyncratic volatility have positive excess returns compared to the lowest quintile. For both the five factor and the three factor value-weighted portfolio there is a presence of the idiosyncratic volatility anomaly at the first four quintiles, with only an exception of significance at the second quintile. This trend is in line with Bali and Cakici (2008) who concluded that the smaller sized stocks caused the anomaly for the value-weighted returns. For the equal-weighted portfolios, we can see the opposite. There, with every increase in size, the highest idiosyncratic volatility quintile excess returns in comparison to the lowest quintile become less. For only the lowest quintile of size, we can see significant positive excess returns. This shows that, with equal-weighted excess returns there is a presence of the idiosyncratic volatility anomaly when excluding the smallest sized stocks.

To get more insight into the effect of size on the idiosyncratic volatility anomaly, I also applied the long/short trading strategy onto each quintile of size. Each month for each quintile, we go long on the stocks with the highest idiosyncratic volatility and short on the lowest idiosyncratic volatility. After each month the portfolio's will be rebalanced and the method is repeated. I also looked at the alpha's of the trading strategy with respect to the Fama-French models.

Table 4.8: One-month Idiosyncratic Volatility trading strategy on size

This table reports the trading strategy excess returns and alphas of the Fama and French five factor and three factor value-weighted and equal-weighted returns. For every month the stocks are double sorted into quintiles based on idiosyncratic volatility of the past month and size. Every month the portfolios and quintiles are rebalanced. Alpha significance at a 5% level or better is indicated in bold, using Newey-West adjusted t-stats in brackets and calculated using 12 lags.

Size	FF5 value	FF3 value	FF5 equal	FF3 equal
1	-0.0031%	-0.0030%	0.4090%	0.4079%
Alpha	-0.0022%	-0.0022%	**1.2662%**	**1.2680%**
	(-0.83)	(-0.83)	(32.16)	(32.20)
2	-0.0004%	-0.0003%	-0.1450%	-0.1435%
Alpha	0.0011%	0.0013%	**0.7551%**	**0.7522%**
	(0.77)	(0.91)	(21.53)	(21.43)
3	-0.0017%	-0.0015%	-0.2826%	-0.2834%
Alpha	**0.0019%**	**0.0020%**	**0.5507%**	**0.5527%**
	(2.24)	(2.45)	(14.49)	(14.53)
4	-0.0032%	-0.0033%	-0.4648%	-0.4684%
Alpha	**0.0065%**	**0.0065%**	**0.3646%**	**0.3613%**
	(14.68)	(14.66)	(8.19)	(8.11)
5	0.0302%	0.0304%	-0.7645%	-0.7654%
Alpha	**0.0446%**	**0.0445%**	-0.0121%	-0.0158%
	(22.39)	(22.31)	(-0.17)	(-0.22)

The trading strategy results in similar results as the effect shown in Table 7. For the value-weighted portfolios, we see positive excess returns in the highest quintile of size. Also, the alpha's are increasing with every quintile and are positive for all of them. For the equal-weighted returns we see the opposite and see negative excess returns from the trading strategy, except for the lowest quintile. Here the alpha's are decreasing the higher we go into the quintiles, however they do remain positive all the time. Therefore, these findings suggest that in most scenarios there could be a presence of the idiosyncratic volatility anomaly, however given the positive alpha it suggests there are other factors driving the returns. For the one-month past idiosyncratic volatility we can therefore conclude the results are contradicting. The value-weighted returns imply that the idiosyncratic volatility, to a very low degree, has a higher presence in low sized stocks, while for equal-weighted returns the anomaly has a higher presence in larger sized stocks.

For the robustness checks, like Ang et al. (2009) I test the equal-weighted and value-weighted returns on past three and twelve months of idiosyncratic volatility. First, for the three month idiosyncratic volatility I conducted similar research as for the one month time interval. Per quintile of idiosyncratic volatility, I check what the effect on average returns is of a one standard deviation increase of idiosyncratic volatility. As can be seen in Table 6.1 in Appendix, similarly to the one month idiosyncratic volatility we see positive delta's. For the value-weighted returns they are all higher, implying a larger increase of monthly excess returns for a one standard deviation increase of idiosyncratic. For the equal-weighted returns they are also mostly higher, with only in the third and fourth quintile a lower impact. This shows that using a rolling estimation of three months to calculate idiosyncratic volatility it has a higher positive magnitude.

Looking at the different time periods in Table 6.2 in Appendix, for value-weighted returns the results are very similar compared to the one-month idiosyncratic volatility. There remains an increasing trend of excess returns per month. For the equal-weighted returns we also see similar results compared to the one-month idiosyncratic volatility, though on average the returns are for both slightly higher. In combination with the higher impact after a one standard deviation increase, it is showing signs that a longer period to estimate the idiosyncratic volatility decreases the anomaly. Also in the years of recession we see similar results, as can be seen in Table 6.3 in Appendix. The trading strategy as done for the one-month idiosyncratic volatility gives similar results for the three-month idiosyncratic volatility. As can be seen in Table 6.5 in Appendix, we can see that for all portfolios there are again positive excess returns. In line with other the other measures for the three-month idiosyncratic volatility they remain positive. Also the alpha's remain positive, albeit slightly lower for the equal-weighted portfolios. When incorporating the size effect for the three-month idiosyncratic volatility, compared to the one-month idiosyncratic volatility, the excess returns on the all portfolios are slightly lower, except the highest quintile of size. Also the alpha's are slightly lower, though they remain positive in all quintiles. These results indicate that using three-month idiosyncratic volatility does not change the overall trend for value-weighted and equal-weighted returns, however does seem to slightly strengthen the anomaly in most quintiles of size. Though the magnitude and overall trading strategies would indicate a weaker idiosyncratic volatility anomaly for a three-month idiosyncratic volatility estimation period, it seems size is more sensitive to a longer period. On the other hand, all four portfolios have better excess returns in the fifth quintile, indicating that for the highest quintile of size the idiosyncratic volatility anomaly is weaker when using a three month estimation compared to one month.

The robustness check is being continued by looking at the idiosyncratic volatility of the past twelve months, instead of one and three. We repeat the same procedure that has been done for the one-month and three-month estimation period. Again, we start by looking at the magnitude of a standard deviation increase in idiosyncratic volatility per quintile. Here, we see mixed, but overall similar results to the one-month and three-month periods. For FF5 value-weighted lower quintiles one and two we see higher delta's, and for FF5 equal-weighted returns we see higher delta's for quintiles three and four as can be seen in Appendix table 6.7. Compared to the shorter estimation period of idiosyncratic volatility we can conclude however that when using a longer period of past idiosyncratic volatility a one standard deviation increase in idiosyncratic volatility has a larger positive impact on average returns. For the trading strategy we see much higher excess returns, especially for the equal weighted portfolios. Here the excess returns by going long in the highest quintile of idiosyncratic volatility and short in the lowest quintile resulted into significant excess returns of 0.1691% and 0.1686%. This, compared to only 0.0309% for value weighted returns. It's also higher than the trading strategy using one-month and three-month idiosyncratic volatility. The alpha's also increase when the estimation period increases, with still much larger alpha's for the equal-weighted returns than for the value-weighted.

Looking at the trend over the different time periods when comparing the highest with the lowest quintile of idiosyncratic volatility, we see slightly higher excess returns for the value-weighted returns, but significant much higher returns for the equal-weighted returns. When we compare above results for the equal-weighted portfolios to the three-month time period, the Fama-French five factor went from 0.0509% (not significant) to 0.3382%. Still, the trend remains positive and the last two time periods have higher excess returns compared to the first two time periods, indicating that the idiosyncratic volatility anomaly is less present in modern markets. The same goes for the value-weighted returns, were we see a trend of higher returns in later time periods. The impact on a longer estimation period for idiosyncratic volatility does not have as much impact as on equal-weighted returns. For recession years we see similar trends as for the one-month and three-month idiosyncratic volatility, as can be seen in Table 6.9 in Appendix. Changing from the three month to the twelve month period for idiosyncratic volatility shows similar effects on size. For value-weighted returns we see similar results to three-month idiosyncratic volatility, and even lower for the lowest quintile, for equal-weighted returns the difference is larger in positive sense, as can be seen in Table 6.11 in Appendix. Also the trading strategy performs better in Table 6.12 in Appendix. We can conclude that taking twelve month idiosyncratic volatility as a measure reduces the idiosyncratic volatility effect compared to a three-month estimation. The one-month idiosyncratic volatility has the highest indications of presence of the anomaly.

To reduce skewness in the idiosyncratic volatility I also test by using the natural logarithm of idiosyncratic volatility. Testing for the magnitude of a one standard deviation increase of one month idiosyncratic volatility on FF5 and FF3 value- and equal-weighted returns, the results are similar to the standard idiosyncratic volatility anomaly. As can be seen in table 6.13 in Appendix, for value-weighted returns the magnitude is slightly larger and for equal-weighted it is slightly smaller.

5 Conclusion

This paper has found that the presence and magnitude of the idiosyncratic volatility anomaly being calculated by the Fama-French three and five factor model highly depends on the measurement and calculation methods being used to investigate the anomaly. First, I tried to calculate the idiosyncratic volatility anomaly using equal-weighted and value-weighted returns for both the three and five factor model and apply a trading strategy to this as was done by Ang et al. (2006). This did not immediately show the presence of the idiosyncratic volatility anomaly. However, when double sorting on time and idiosyncratic volatility , there were periods in time where the idiosyncratic volatility anomaly was present when using a one-month, three-month and twelve-month estimation period by comparing the highest quintile of idiosyncratic volatility relative to the lowest. All of these periods where the idiosyncratic volatility anomaly was present were before the 21st century, and more present in equal-weighted returns when using both the Fama-French five factor model and three factor model. Furthermore, we see that for all estimation periods of idiosyncratic volatility the excess returns of the last quintile period for value-weighted returns are slightly higher than the average and the first quintile periods are lower. Therefore, with regards to the first hypothesis, there is a trend visible and we can conclude that within both value-weighted and equal-weighted returns the magnitude of the idiosyncratic volatility anomaly has diminished over time. This is strongest visible using one-month idiosyncratic volatility. For longer estimation periods and using value-weighted returns there is weaker evidence that the anomaly has become weaker over time. Testing for economic uncertainty by looking at years of recession, we see that for value-weighted returns for almost all years of recession the excess returns were lower than the associated time quintile, indicating that the idiosyncratic volatility anomaly is more present in these years. This holds when using a longer estimation period and even becomes slightly stronger. Therefore it seems that economic uncertainty expressed in years of recession could be a driver of the idiosyncratic volatility anomaly. To some extent this confirms findings by Chen and Petkova (2012) who claim that the higher average market variance during economic uncertainty can explain the idiosyncratic volatility anomaly.

Using the trading strategy by going long in the highest quintile of idiosyncratic volatility and short in the lowest, we found that for all estimation periods the five factor model did not have a notable difference compared to the three factor model. Using a twelve month estimation period for idiosyncratic volatility gave the highest excess returns, indicating that a longer time period to estimate idiosyncratic volatility would weaken the anomaly. When including size into the research, especially for equal-weighted returns we see higher excess returns and alphas when using a longer estimation period. After double sorting on size and idiosyncratic volatility we found that there is again a difference between equal-weighted and value-weighted returns. After both comparing the highest quintile relative to the lowest and applying a trading strategy by going long in the highest quintile and short in the lowest, the anomaly was present in both equal-weighted and value-weighted returns. For value-weighted returns, in quintiles 1,2,3 and 4 of all estimation periods there were negative excess returns. Only in the highest quintile of size the anomaly was never present. The alpha's however, remained positive when significant, indicating that the trading strategy is moved by other factors beyond the Fama-French factors. However, these results indicate that for value-weighted returns the anomaly is more common in small sized stocks which is in line with the findings of Bali and Cakici (2008). For equal-weighted returns the story is different. There, it is the other way around and does only the first quintile of size generate positive significant excess returns in the trading strategy. The higher we go in size the lower the alpha's and the larger the effect of the idiosyncratic volatility anomaly becomes. We can therefore say that the size effect diminishes the magnitude of the idiosyncratic volatility anomaly when using value-weighted returns. However, we cannot say this for equal-weighted returns. I find that for almost all calculations, both for equal- and value-weighted returns there is almost no difference between the two Fama-French models. We can therefore conclude that the two additional factors of the five factor model have no additional explanatory power beyond the three factor model.

As for most research on the idiosyncratic volatility anomaly, there are limitations to the conclusions. Bali and Cakici (2008) already found that differences in methodology can highly influence the existence and significance of a cross-sectional relation between idiosyncratic risk and expected returns. In this research we only looked at the Fama-French three and Fama-French five factor model. I could have used more models like the CAPM, EGARCH or 6-factor model. Furthermore I could have used more trading strategies with different estimation periods, holding periods and/or other robustness checks like checking on other factors besides the size effect. It also shows how hard it is to find a clear answer to the existence of this anomaly. As so many small factors can change the outcome of the results, it is hard to judge when an anomaly is indisputable. Therefore, somehow symbolic and in line with the idiosyncratic anomaly puzzle we cannot simply say that the anomaly is present or not. It depends on the sample, the estimation period of idiosyncratic volatility and the usage of value-weighted or equal-weighted returns within the Fama-French three and five factor model whether or not the anomaly is present within a quintile of time period and/or size. We can however say that over the course of the past sixty years, the presence of the idiosyncratic volatility anomaly has weakened in the stock market, but can still be fueled in times of recession.

6 Appendix

Table 6.1: Magnitude FF5 three-month IVOL

This table reports the expected change in results after a one standard deviation change in the Fama and French five factor and three factor three month idiosyncratic volatility of monthly value-weighted returns and equal-weighted returns. For every month the stocks are sorted into quintiles based on idiosyncratic volatility of the past three months. Every month the portfolios and quintiles are rebalanced. Significance at a 5% level or better is indicated in bold.

Quintiles	1	2	3	4	5
FF5 value IVOL coeff	**0.3801986**	**0.1607325**	**0.050972**	**0.0882014**	**0.1984938**
	(39.82)	(28.71)	(9.01)	(6.55)	(29.65)
IVOL mean	0.0002011	0.0002198	0.0002337	0.000288	0.0019544
IVOL SD	0.0000887	0.0001042	0.0001094	0.0001224	0.0043584
Ret mean	0.0000498	0.0000244	0.000018	0.0000476	0.0006383
Delta SD	0.67718104	0.686406824	0.309796489	0.2268036	1.355342908

Quintiles	1	2	3	4	5
FF5 equal IVOL coeff	**0.4151176**	**0.5464644**	**0.5964577**	**0.6139404**	**0.2301928**
	(14.53)	(18.47)	(21.54)	(24.17)	(18.18)
IVOL mean	0.0119993	0.0176059	0.0234573	0.0314394	0.0529838
IVOL SD	0.0039191	0.004834	0.0064793	0.0090504	0.0277293
Ret mean	0.0082797	0.0099458	0.109039	0.108919	0.0087884
Delta SD	0.1964911	0.265600445	0.035442625	0.051014113	0.726307998

Table 6.2: Three-month Idiosyncratic Volatility effect over time

This table reports the excess returns of highest quintile with respect to the lowest quintile of three month idiosyncratic volatility per period in time. By double sorting on idiosyncratic volatility and time periods, the excess returns were generated. Significance at a 5% level or better is indicated in bold.

	FF5 value	FF3 value	FF5 equal	FF3 equal
Total	**0.0589%**	**0.0589%**	0.0509%	0.0504%
	(31.74)	(31.78)	(1.64)	(1.62)
1963-1974	**-0.0324%**	**-0.0324%**	**-0.3740%**	**-0.3768%**
	(-4.87)	(-4.87)	(-5.25)	(-5.29)
1975-1986	**0.0646%**	**0.0647%**	-0.0318%	-0.0352%
	(19.18)	(19.2)	(-0.56)	(-0.62)
1987-1998	**0.0863%**	**0.0863%**	**-0.2116%**	**-0.2096%**
	(33.62)	(33.61)	(-3.54)	(-3.51)
1999-2010	0.0011%	0.0013%	**0.5925%**	**0.5929%**
	(0.25)	(0.29)	(7.71)	(7.72)
2011-2021	**0.1603%**	**0.1605%**	0.1301%	0.1300%
	29.68	29.71	(1.64)	(1.64)

Table 6.3: Three-month Idiosyncratic Volatility effect during recessions

This table reports the excess returns of the highest quintile with respect to the lowest quintile of three month idiosyncratic volatility for the recession years 1974, 1982, 1990, 2001, 2008. By sorting on idiosyncratic volatility and only selecting the specific time period, the excess returns were generated. Significance at a 5% level or better is indicated in bold.

	FF5 value	FF3 value	FF5 equal	FF3 equal
1974	**-0.3353%**	**-0.3312%**	**-0.4282%**	**-0.4428%**
t-value	(-18.29)	(-18.14)	(-1.99)	(-2.06)
1982	**0.0750%**	**0.0731%**	**-0.5235%**	**-0.5112%**
t-value	(5.91)	(5.8)	(-2.53)	(-2.47)
1990	**-0.0927%**	**-0.0863%**	**-2.5362%**	**-2.5309%**
t-value	(-9.12)	(-8.56)	(-10.8)	(-10.77)
2001	**-0.1433%**	**-0.1462%**	**1.4735%**	**1.4680%**
t-value	(-8.51)	(-8.72)	(4.49)	(4.48)
2008	**-0.4160%**	**-0.3916%**	**-3.9707%**	**-3.9190%**
t-value	(-22.89)	(-21.63)	(-14.16)	(-13.97)

Table 6.4: Three-month Idiosyncratic Volatility Long/Short strategy

This table reports the trading strategy excess returns and alphas of the Fama and French five factor and three factor value-weighted and equal-weighted returns. For every month the stocks are sorted into quintiles based on idiosyncratic volatility of the past twelve months. Every month the portfolios and quintiles are rebalanced. Alpha significance at a 5% level or better is indicated in bold, using Newey-West adjusted t-stats in brackets and calculated using 12 lags.

	FF5 value	FF3 value	FF5 equal	FF3 equal
Strategy ret	0.0294%	0.0295%	0.0254%	0.0252%
Alpha	**0.0344%**	**0.0344%**	**0.8534%**	**0.8528%**
	(21.77)	(21.76)	(46.71)	(46.69)

Table 6.5: Three-month Idiosyncratic Volatility effect on size

This table reports the excess returns of the highest quintile with respect to the lowest quintile of three month idiosyncratic volatility per quintile of size. By double sorting on idiosyncratic volatility and size, the excess returns are taken for each level of size. Significance at a 5% level or better is indicated in bold.

Size	FF5 value	FF3 value	FF5 equal	FF3 equal
1	**-0.0066%**	**0.0064%**	**0.7580%**	**0.7555%**
	(-13.21)	(-12.9)	(6.70)	(6.67)
2	**0.0014%**	**0.0013%**	**-0.3608%**	**-0.3614%**
	(2.06)	(1.98)	(-4.46)	(-4.47)
3	**-0.0020%**	**-0.0019%**	**-0.6626%**	**-0.6538%**
	(-2.27)	(-2.18)	(-8.91)	(-8.79)
4	**-0.0088%**	**-0.0086%**	**-0.9859%**	**-0.9945%**
	(-12.87)	(-12.68)	(-13.99)	(-14.11)
5	**0.0508%**	**0.0509%**	**-1.6543%**	**-1.6550%**
	(2.70)	(2.70)	(-19.55)	(-19.57)

Table 6.6: Three-month Idiosyncratic Volatility trading strategy on size

This table reports the trading strategy excess returns and alphas of the Fama and French five factor and three factor value-weighted and equal-weighted returns. For every month the stocks are double sorted into quintiles based on idiosyncratic volatility of the past three months and size. Every month the portfolios and quintiles are rebalanced. Alpha significance at a 5% level or better is indicated in bold, using Newey-West adjusted t-stats in brackets and calculated using 12 lags.

Size	FF5 value	FF3 value	FF5 equal	FF3 equal
1	-0.0033%	-0.0032%	0.3790%	0.3777%
Alpha	-0.0027%	-0.0025%	**1.2811%**	**1.2809%**
	(-0.80)	(-0.78)	(33.10)	(33.09)
2	0.0007%	0.0007%	-0.1804%	-0.1807%
Alpha	0.0023%	0.0022%	**0.7623%**	**0.7625%**
	(1.03)	(1.05)	(21.77)	(21.77)
3	-0.0010%	-0.0010%	-0.3313%	-0.3269%
Alpha	**0.0030%**	**0.0030%**	**0.5600%**	**0.5592%**
	(2.39)	(2.36)	(13.99)	(13.97)
4	-0.0044%	-0.0043%	-0.4929%	-0.4972%
Alpha	**0.0069%**	**0.0069%**	**0.3746%**	**0.3699%**
	(13.91)	(13.87)	(7.44)	(7.34)
5	0.0254%	0.0255%	-0.8271%	-0.8275%
Alpha	**0.0482%**	**0.0482%**	-0.0875%	-0.0871%
	(24.42)	(24.38)	(-0.98)	(-0.97)

Table 6.7: Magnitude FF5 twelve-month IVOL

This table reports the expected change in results after a one standard deviation change in the Fama and French five factor and three factor three month idiosyncratic volatility of monthly value-weighted returns. For every month the stocks are sorted into quintiles based on idiosyncratic volatility of the past month. Every month the portfolios and quintiles are rebalanced. Significance at a 5% level or better is indicated in bold.

Quintiles	1	2	3	4	5
FF5 value IVOL coeff	**0.5573917**	**0.2896472**	**0.0945839**	0.0225588	**0.185283**
	(52.56)	(40.61)	(11.17)	(1.30)	(25.7)
IVOL mean	0.0002072	0.0002215	0.0002348	0.0002924	0.0020082
IVOL SD	0.0000732	0.0000823	0.0000847	0.0001051	0.0042845
Ret mean	0.0000431	0.0000239	0.0000217	0.0000534	0.0006642
Delta SD	0.94666061	0.997404375	0.369182319	0.044399436	1.195189722

Quintiles	1	2	3	4	5
FF5 equal IVOL coeff	**0.4359675**	**0.4592455**	**0.4312708**	**0.5465833**	**0.3435891**
	(14.07)	(13.88)	(14.14)	(19.31)	(19.39)
IVOL mean	0.0128263	0.0181483	0.0237589	0.0312668	0.0494173
IVOL SD	0.0035982	0.0043853	0.0059236	0.0082126	0.0207381
Ret mean	0.0082484	0.0095864	0.0103241	0.0105427	0.0116302
Delta SD	0.19018213	0.210081917	0.247447788	0.425779924	0.612662303

Table 6.8: Twelve-month Idiosyncratic Volatility effect over time

This table reports the excess returns of highest quintile with respect to the lowest quintile of twelve month idiosyncratic volatility per period in time. By double sorting on idiosyncratic volatility and time periods, the excess returns were generated. Significance at a 5% level or better is indicated in bold.

	FF5 value	FF3 value	FF5 equal	FF3 equal
Total	**0.0618%**	**0.0619%**	**0.3382%**	**0.3373%**
t-value	(31.49)	(29.92)	(10.58)	(10.56)
1963-1974	**-0.0527%**	**-0.0527%**	-0.0586%	-0.0543%
t-value	(-7.03)	(-7.04)	(-0.76)	(-0.71)
1975-1986	**0.0690%**	**0.0691%**	**0.2536%**	**0.2528%**
t-value	(19.56)	(19.59)	(4.43)	(4.42)
1987-1998	**0.0940%**	**0.0941%**	0.0800%	0.0794%
t-value	(34.02)	(34.04)	(1.30)	(1.29)
1999-2010	0.0015%	0.0015%	**0.8769%**	**0.8721%**
t-value	(0.32)	(0.33)	(11.29)	(11.23)
2011-2021	**0.1686%**	**0.1687%**	**0.3458%**	**0.3465%**
t-value	(29.74)	(29.77)	(4.30)	(4.31)

Table 6.9: Twelve-month Idiosyncratic Volatility effect during recessions

This table reports the excess returns of the highest quintile with respect to the lowest quintile of twelve month idiosyncratic volatility for the recession years 1974, 1982, 1990, 2001, 2008. By sorting on idiosyncratic volatility and only selecting the specific time period, the excess returns were generated. Significance at a 5% level or better is indicated in bold.

	FF5 value	FF3 value	FF5 equal	FF3 equal
1974	**-0.3412%**	**-0.3414%**	**-0.7154%**	**-0.7020%**
	(-18.22)	(-18.24)	(-3.3)	(-3.24)
1982	**0.0791%**	**0.0790%**	-0.1522%	-0.1484%
	(5.98)	(5.97)	(-0.73)	(-0.71)
1990	**-0.0948%**	**-0.0947%**	**-2.1328%**	**-2.1354%**
	(-8.93)	(-8.93)	(-8.87)	(-8.88)
2001	**-0.1410%**	**-0.1408%**	**1.6068%**	**1.6096%**
	(-8.08)	(-8.07)	(4.84)	(4.85)
2008	**-0.4339%**	**-0.4340%**	**-4.1394%**	**-4.1516%**
	(-23.02)	(-23.03)	(-14.42)	(-14.46)

Table 6.10: Twelve-month Idiosyncratic Volatility Long/Short strategy

This table reports the trading strategy excess returns and alphas of the Fama and French five factor and three factor value-weighted and equal-weighted returns. For every month the stocks are sorted into quintiles based on idiosyncratic volatility of the past twelve months. Every month the portfolios and quintiles are rebalanced. Alpha significance at a 5% level or better is indicated in bold, using Newey-West adjusted t-stats in brackets and calculated using 12 lags.

	FF5 value	FF3 value	FF5 equal	FF3 equal
Strategy ret	0.0309%	0.0309%	0.1691%	0.1686%
Alpha	**0.0355%**	**0.0355%**	**0.9939%**	**0.9931%**
	(21.23)	(21.22)	(51.88)	(51.84)

Table 6.11: Twelve-month Idiosyncratic Volatility Volatility effect on size

This table reports the excess returns of the highest quintile with respect to the lowest quintile of twelve month idiosyncratic volatility per quintile of size. By double sorting on idiosyncratic volatility and size, the excess returns are taken for each level of size. Significance at a 5% level or better is indicated in bold.

Size	FF5 value	FF3 value	FF5 equal	FF3 equal
1	**-0.0124%**	**-0.0121%**	**0.9078%**	**0.9056%**
	(-16.15)	(-15.95)	(6.84)	(6.82)
2	**0.0048%**	**0.0047%**	**-0.2020%**	**-0.2092%**
	(5.97)	(5.97)	(-2.26)	(-2.34)
3	**-0.0023%**	**-0.0024%**	**-0.3078%**	**-0.3025%**
	(-2.57)	(-2.73)	(-3.92)	(-3.85)
4	**-0.0077%**	**-0.0075%**	**-0.7334%**	**-0.7344%**
	(-10.24)	(-9.98)	(-9.82)	(-9.84)
5	**0.0521%**	**0.0525%**	**-1.3638%**	**-1.3587%**
	(2.36)	(2.38)	(-14.01)	(-13.97)

Table 6.12: Twelve-month Idiosyncratic Volatility trading strategy on size

This table reports the trading strategy excess returns and alphas of the Fama and French five factor and three factor value-weighted and equal-weighted returns. For every month the stocks are double sorted into quintiles based on idiosyncratic volatility of the past twelve months and size. Every month the portfolios and quintiles are rebalanced. Alpha significance at a 5% level or better is indicated in bold, using Newey-West adjusted t-stats in brackets and calculated using 12 lags.

Size	FF5 value	FF3 value	FF5 equal	FF3 equal
1	-0.0062%	-0.0060%	0.4539%	0.4528%
Alpha	-0.0055%	-0.0054%	**1.3644%**	**1.3634%**
	(-0.75)	(-0.75)	(33.59)	(33.54)
2	0.0024%	0.0027%	-0.1010%	-0.1046%
Alpha	0.0040%	0.0039%	**0.9541%**	**0.9534%**
	(1.25)	(1.27)	(25.68)	(25.66)
3	-0.0011%	-0.0012%	-0.1539%	-0.1513%
Alpha	**0.0028%**	**0.0027%**	**0.7596%**	**0.7622%**
	(2.30)	(2.21)	(17.71)	(17.76)
4	-0.0038%	-0.0037%	-0.3667%	-0.3672%
Alpha	**0.0073%**	**0.0073%**	**0.5318%**	**0.5313%**
	(12.72)	(12.73)	(9.54)	(9.42)
5	0.0260%	0.0262%	-0.6819%	-0.6794%
Alpha	**0.0486%**	**0.0484%**	0.0248%	0.0260%
	(23.81)	(23.74)	(0.22)	(0.23)

Table 6.13: Magnitude FF5 one-month log IVOL

This table reports the expected change in results after a one standard deviation change in the Fama and French five factor and three factor one-month natural logarithm of idiosyncratic volatility of monthly value-weighted returns and equal-weighted returns. For every month the stocks are sorted into quintiles based on log idiosyncratic volatility of the past month. Every month the portfolios and quintiles are rebalanced. Significance at a 5% level or better is indicated in bold.

Quintiles	1	2	3	4	5
FF5 value IVOL coeff	**0.0000385**	**0.0000217**	**0.0000037**	**0.0000305**	**0.0008045**
	(16.4)	(14.79)	(-2.27)	(6.91)	(26.86)
IVOL mean	-8.654385	-8.534007	-8.457585	-8.231831	-6.899794
IVOL SD	0.4223678	0.4271998	0.4060579	0.3713202	0.9609655
Ret mean	0.0000465	0.000024	0.0000196	0.000052	0.0006312
Delta SD	0.34970237	0.386259819	0.076653787	0.217793579	1.224804729

Quintiles	1	2	3	4	5
FF5 equal IVOL coeff	**0.0032519**	**0.0090398**	**0.0144748**	**0.0190971**	**0.0120877**
t-value	(9.69)	(16.44)	(21.19)	(23.21)	(61.21)
IVOL mean	-4.596308	-4.141202	-3.828846	-3.508845	-2.969698
IVOL SD	0.3599284	0.2714554	0.270495	0.2772992	0.4196522
Ret mean	0.0081258	0.102247	0.113263	0.010858	0.0084931
Delta SD	0.14404135	0.023999751	0.034568756	0.4877151	0.597264826

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