Erasmus School of Economics

How well Value at Risk and Average Value at Risk methods perform under Black Swan events

ERASMUS UNIVERSITY ROTTERDAM

Master Thesis Strategy Economics

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Abstract

This project investigates the performance of two widely used risk measurement methods, Value-at-Risk (VaR) and Average Value-at-Risk (AVaR), under extreme events known as Black Swans. This is achieved using a parametric, a historical approach, and a Markov Monte Carlo simulation on three distinct portfolios at two significance levels (95 and 99%). The study is structured around three hypotheses, which assume (i) the non-normality of returns in the financial market, (ii) the impact of diversification on returns distribution, and (iii) the systematic impact of the method used to achieve an (A)VaR value. The results indicate that (A)VaR is effective in a stable market; traditional (A)VaR measures underestimate the potential losses under extreme events. The historical approach at the 99%-level generates the most promising results for March 2020 estimation for each portfolio. As such, the historical approach offers some compelling opportunities to serve as a method of detecting Black Swan occurrence. Overall, the project provides valuable insights into the limitations of traditional risk measures and the need for a more flexible and dynamic approach to risk management. This paper highlights the importance of understanding the distribution of losses under extreme events to accurately assess tail risk.

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1 Introduction

Most crises are unpredictable by nature and lead to surprises that fall beyond the control of major decision-makers (Sotiropoulos et al., 2013). Over the last few decades, tools have been created to mitigate and value risk around financial activities (BIS, 2013). Global markets invariably operate under multiple types of uncertainties that can impact the financial positions of their stakeholders. In finance, these uncertainties are usually called risk exposure (Tirole, 2010). Most economic actors aiming for wealth accumulation accept a calculated risk exposure as it often promises higher returns. It is referred to as the risk premium, which represents the extra payment an investors receive for tolerating the extra risk in a given investment over that of a risk-free asset. This is where investment risk analysis becomes crucial for investors to make informed decisions (BCBS, 1996). However, some shocks remain unforeseen, which can significantly impact investors' returns. For example, the Covid-19 pandemic has forced widespread lockdowns and restrictions, leading to a massive market crash and unusual market volatility (Young, 2020). Risk management is not only a topic that investment institutions frequently discuss, but it establishes the foundation for the economic security of the public in the future (Penikas, 2015). This includes important aspects such as savings, the pension system, currency stability, and inflation. Negligence in risk management is one of the central catalysers of the 2008 sub-prime crisis, which annihilated more than $3.5 trillion for pension funds in Organisation for Economic Co-operation and Development (OECD) countries (Yermo & Salou, 2010).

Hence, this paper will aim to improve the understanding of the most widely used risk metric. Understanding the most common risk measurement methods is crucial to see how efficiently they protect investors and financial players against heavy losses. This becomes pivotal considering Kahneman et al.'s (1990) publication which shows that investors are risk-averse and heavily engaged in hedging their portfolios against market crashes. Understanding the most common risk measurement methods and their comparative effectiveness is vital for many stakeholders in public markets. Since the 80s, financial regulators have used Value-at-Risk (VaR) to assess bank assets and default risks (Goodhart, 2011). Value-at-Risk is a statistical measure used to estimate the potential loss on an investment over a given time period, at a certain level of confidence. In 1980, The Securities and Exchange Commission (SEC) was the first to implement Value-at-Risk as a regulatory tool requiring United States' (US) financial service firms to hold enough capital to cover their potential losses with 95% confidence over a thirty-day interval (Goodhart, 2011). Later Basel II and Basel III agreements made this Value-at-Risk (VaR) and Average Value-at-Risk (AVaR) the standards measurements for risk exposure of financial service firms (BCBS, 2004; BCBS, 2011; J. M. Chen, 2013; Penikas, 2015). Average Value-at-Risk is a risk assessment measure that calculates the expected loss of an investment beyond the corresponding Value-at-Risk (Alexander, 2009). It provides a more comprehensive and conservative estimate of potential losses. Average Value-at-Risk is often used in financial risk management to answer the question: "*what would be my loss in the worst case scenario?"* (Alexander, 2009). Following, banking regulators recommendations VaR and AVaR have become the bedrock of the modern financial system (Goodhart, 2011).

On the other hand, risk management has been developed to reduce investors' exposure to financial losses and market crashes. Crashes represent extreme and unexpected events that cause the market to fall, resulting in notable value loss for most financial stakeholders. Because of this, they are on the far left of the probability distribution curve, often designated as tail risk. This paper will investigate events at the far post of this risk tail, extremely negative events that recent literature refers to as Black Swans. Popularized by Taleb's (2007) book "The Black Swan: The Impact of the Highly Improbable", these events are rare but can have far-reaching consequences on financial markets and economies. Such events can dramatically impact investors savings and the global economy. By utilizing sophisticated risk management techniques, financial institutions can better prepare for potential Black Swan events and mitigate their impact on the market (Bordo, 2008). A more profound comprehension of Value at Risk and Average Value at Risk can aid in safeguarding investors' wealth against unexpected and severe events, ultimately contributing to improved global financial stability (Bordo, 2008).

1.1 Project Purpose and Scope

The objective of this project will be to assess and compare the effectiveness of VaR and AVaR during extreme market situations such as Black Swans. The period of interest in this project will be the latest event which could qualify as a Black Swan: the 2020 Covid-19 pandemic. Specifically, this study aims to evaluate three different techniques for calculating Value at Risk and Average Value at Risk: (i) parametric, (ii) historical and (iii) semi-parametric. Using some European equity indexes as portfolios This paper will assess the accuracy of these methods in measuring overall risk levels at 95% and 99% confidence intervals over a 30-day.

1.2 Theoretical Relevance and Literature Gap

Overall, a considerable amount of publications exist on Value-at-Risk, Average Value-at-risk and Black Swans. Abad & Benito's (2013) article compares VaR estimates for the UK FTSE100, US Dow Jones

Industrial Average (DJAI), S&P 500, Japanese Nikkei 225 and Hong Kong Hang Seng (HSI) indexes. Adams & Thornton (2014) researched VaR measurement using two parametric approaches on the Dow Jones index. Aniūnas et al. (2009) and Danielsson and De Vries (1997) publications focus on VaR and the forex exchange market for US dollars against foreign currency exchange rates (Euro not included). This is also the case for Aussenegg & Miazhynskaia (2006), who have investigated a broader scope of assets by adding Brent oil prices, the S&P500 index and US treasury bonds. Cabedo & Moya (2003) studied VaR with Brent oil prices, from January 1992 to December 1999, with semi-parametric methods. Duplessy (2020) Researched Black Swans' impact on Canadian stocks VaR measurement. However, research focused on the European stock exchange is largely missing. Only Glasserman et al. (2002) and Vlaar (2000) show VaR studies researching the European market. The first one studies semi-parametric VaR simulation for heavy-tailed models using European puts and calls portfolios (Glasserman et al., 2002). The second analyses VaR for 25 portfolios comprising Dutch fixed-interest securities (Vlaar, 2000). This underlines a research gap in terms of geographic location; it can be seen that most publications focus on the US market. This is explainable by the recent financial shocks and the availability of quality data that have led many academic researchers to focus on the US and its stock exchange indexes.

Furthermore, the few papers based on the European market are generally limited assets specific or single-country case studies (Glasserman et al., 2002; Vlaar, 2000). Therefore, there is a strong need for more research on the global European equity market. This gap in the academic literature around the European stock exchange can be explained by the multiple recent changes in the membership composition of the European Union (EU) over the last two decades[1]. Contrary to the US market, the integrated financial European market is still very young and fragmented, with every country still having a national stock exchange centre; Euronext was only created in 2001. Finally, this gap in the literature is intriguing as it provides a significant theoretical foundation based on the US financial market, which requires further investigation to determine its applicability to the financial market of the European continent.

A second potential gap comes from the nature of the event researched in this paper. The interest in the so-called Black Swans has recently gained significant attention (Adams & Thornton, 2014; Aven, 2013; Morales & Andreosso-O'Callaghan, 2020; Yarovaya et al., 2021). This can be attributed to the

[1] 5th and 6th EU enlargements: Cyprus, Czech Republic, Estonia, Hungary, Latvia, Lithuania, Malta, Poland, Slovakia, Slovenia, Bulgaria, Romania (2004 and 2007); 7th EU enlargement: Croatia (2013).

popularisation of the terminology via Taleb's (2007) best-selling books. *Black Swans* are defined as an unexpected event which arises from an unknown- unknowns (Aven, 2013). *Unknown-unknowns* refer to things that we don't know we don't know (Taleb, 2013). In a risk context, these are particularly challenging to manage because, there is no awareness of their existence, so it is challenging to mitigate them (Taleb, 2013). However, It is still controversial to consider Covid-19 as a Black Swan, as it is hard to see a consensus among the experts and many opinions differ (Ahmad et al., 2021; Drake, 2021; Inayatullah & Black, 2020; Phadnis et al., 2021; Yarovaya et al., 2021; Phan & Wood, 2020). Consequently, as no clear consensus seems to exist, the classification of the Covid-19 pandemic as a Black Swan will be discussed in the literature review section of this paper.

Figure 1-1:

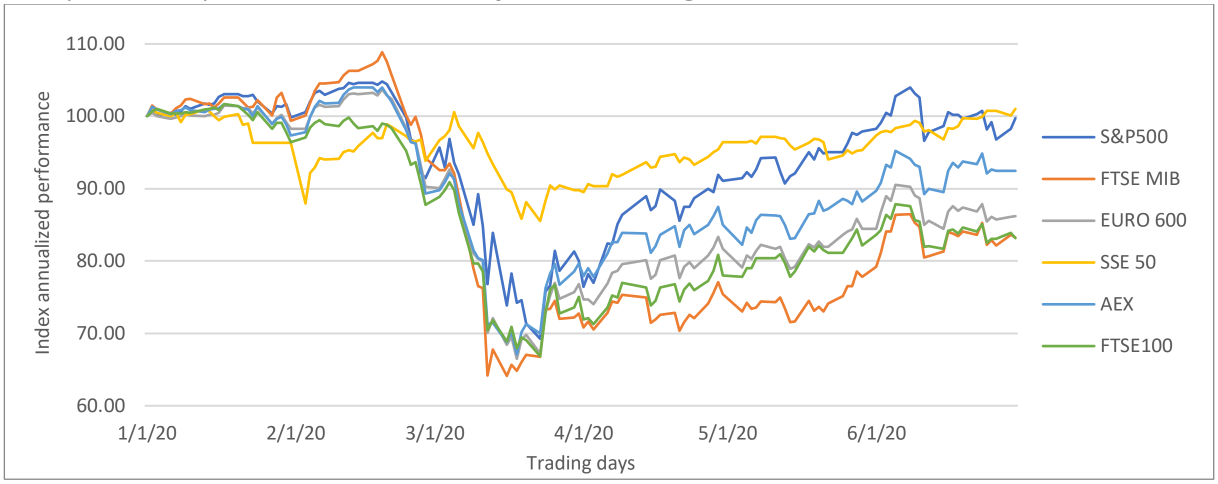
Volatility Index (VIX) Prices in USD (Jan 2020 – June 2020)

Note: Excel's computation based on Yahoo Finance's closing price for the Volatility index (^VIX) in USD from 01/01/2020 to 31/06/2020.

Combining these two sections is the first step in defining the scope of this research. To develop the period of interest, we see the first reported cases of Covid-19 in Italy towards the end of February 2020 (Chen et al., 2020). This effect was compounded by the lack of timely and coordinated policy reaction, leading to a significant downturn in global markets by the beginning of March 2020 (Gopinath, 2020). This becomes clearer by looking at the price evolutions of the volatility index (Figure 1-1) and the evolution of global stock exchanges indices (Figure 1-2). Hence, this paper will research the Value-at-risk and Average value-at-risk during the Covid-19 financial crash of March 2020. To my knowledge, this research paper is the first analysis focusing on the impact of Covid-19 as a Black Swan event on the European stock exchange market and the efficacity of VaR and AVaR tools to assess these risks.

Figure 1-2:

Multiple Blue Ship Indexes Annualized Performance During H1-2020



Note: Excel's computation based on Yahoo Finance's daily returns derivate from closing price for the S&P500 index (^GSPC) in USD, FTSE MIB index (^FTSEMIB.MI) in EUR, EURO 600 index (^STOXX) in EUR, SSE 50 index (^000016.SS) in CNY, AEX index (^AEX) in EUR, and FTSE100 index (^FTSE) in GBP from 01/01/2020 to 31/06/2020.

1.3 Research Question

Looking at the current academic research and existing literature, one can identify a grey area requiring more research. How VaR and AVaR perform in the European equity market? Moreover, how useful are they under Black Swan events? A comparative study of the different VaR and AVaR measurements methods on the EU stock exchange under such extreme events would improve the literature coverage. Bearing that in mind, the research question explored in this paper will be:

How efficient are the Value-at-Risk (VaR) and Average Value-at-Risk (AVaR) methodologies under Black Swan occurrence?

This report will have three focuses to find an answer to this question. Each of these area of focus, will have a dedicated hypothesis to be validate or reject, will be presented and developed in the literature review section.

First, some attention will be given to the European returns distribution function. In 1965, Samuelson (1965) was the first to suggest that equity returns followed a normal distribution. While renowned researchers like Fama (1970), Mandelbrot (1963) and Markowitz (1991) suggest modified returns distribution, the normality of the stock's returns is still essential in mathematical finance (Adams &

Thornton, 2014). In the context of (A)VaR, returns distribution is important because the accuracy of the parametric and semi-parametric will depend on its unbiasedness. As the normal distribution appears dominant, it will be the chosen distribution for the parametric and semi-parametric (A)VaR. However, investigating if this choice leads to bias in the obtained results is crucial.

Second, investors and pension funds typically invest in a variety of assets in order to achieve diversification and manage risk. This is because investors are risk-averse and heavily interested in hedging their portfolios against the downside risk (Kahneman et al.; 1990). The diversification theory first presented by Markowitz (1952) in the 50s remains the bedrock of many investment strategies for regular pension funds, investment firms and private savers. This risk reduction strategy became extremely popular for its theoretical simplicity and famous advocates such as Warren Buffett (Damodaran, 2007; Buffett & Cunningham, 2013). However, looking back on the literature linking (A)VaR and Black Swan, many papers have chosen to investigate forex values or global exchange index prices. No research has been founded on more common diversified portfolios. Most investors who invest their savings do not invest all their assets in Forex values or Brent oil prices. To deepen learnings from the first research objective, investigations on how diversification levels impacts the distribution of returns will be conducted.

Finally, as this paper aims to assess the efficiency measurement of (A)VaR in Black Swan's context, this report will compare the predicted to the actual losses experienced during March 2020. Special attention will be given to detect if the results exhibit systematic differences in their value. The goal will be to identify if the computation method displays consistent differences, translating unequal efficiencies to evaluate risk level.

2 Background

This chapter presents a general discussion on the relevant historical concepts needed in order to understand this paper and the associated literature. The first section will discuss the global consequences of the SARS-CoV-2 virus, with a focus on to the European region. In the following section, the term Black Swan will be introduced more extensively, along with the history of this unknown-unknown event. In the end of the section an attempt to close the debate on whether the Covid-19 crisis should be categorised as a Black Swan will be presented. The third section will discuss banking policy evolutions which led to risk valuations evolution. A significant part of this section will expose how the various Basel committee agreements pushed the democratisation of Value-at-Risk.

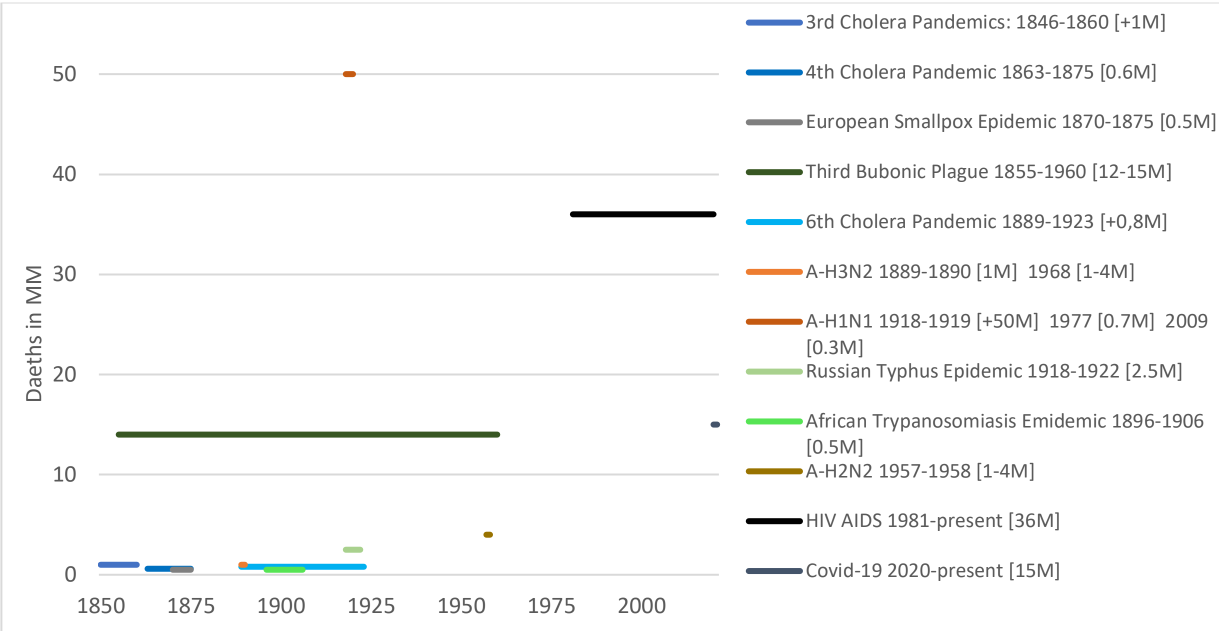
2.1 Covid-19 pandemic Historic and Facts

This section will examine how the SARS-CoV-2 virus has impacted the world, with a particular emphasis on the European region.

The Covid-19 pandemic started in Wuhan (China) in December 2019. In Europe, the first confirmed case of Covid-19 was reported in France on January 24th, 2020 (Walsh, 2020). However, Italy experienced the first major outbreak in Europe, starting on February 20th in the Lombardy region (Walsh, 2020). The virus quickly spread across Europe, particularly in Spain, Italy, France and Germany. On March 11th, the World Health Organisation declared Covid-19 a pandemic, and many European countries implemented strict measures to limit the spread of the virus (Jebril, 2020; J. Chen et al., 2020). Borders were closed, schools were shut down, and large gatherings were banned (J. Chen et al., 2020).

The pandemic overwhelmed European healthcare systems, and the mortality rate was high, particularly among seniors and individuals with health conditions (Gopinath, 2020; Lupu & Tiganasu, 2022). A number that describes the magnitude of the event well is the number of excess deaths, estimated to be above 15M worldwide (World Health Organization, 2022a; World Health Organization, 2022b). This makes SARS-COV2 the third deadliest epidemic of the last two decades; see Figure 3-1 (Drake, 2021).

Figure 2-1:
Timeline of major epidemics since 1850. minimum number of deaths attributable & duration



Note: **Red** for viral influenza derivate diseases and **black** for other viral diseases; **Blue** for microbial cholera derivate diseases and **green** for other microbial derivate diseases. Source: Drake's (2021) article.

The pandemic significantly impacted European society, with many people losing their jobs and businesses closing down (Gopinath, 2020). Furthermore, scientists have identified multiple long-lasting losses due to this event (Gopinath, 2020; The World Bank et al., 2021). Gopinath (2020) argues that the pandemic impacted the economic opportunity of an entire generation of students over the long term. With a significant increase in dropout rates, this generation of students is expected to lose an estimated $17 trillion in future earnings (10% of global GDP) following a World Bank et al. press release (2021). In addition, the pandemic led to a harmful financial crisis in Europe due to widespread business closures, job losses, and a decrease in consumer spending, resulting in a sharp decline in economic activity.

The financial crisis has impacted multiple sectors, including tourism, hospitality, and aviation(Gopinath, 2020). European governments have implemented measures to mitigate the economic impact of the pandemic, including financial support for businesses and individuals, tax deferrals, and monetary policy measures such as interest rate reductions and quantitative easing (Creel et al., 2020). The European Union has also introduced a recovery plan, including a significant stimulus package to support regional economic recovery (Creel et al., 2020). Nevertheless, the ongoing Covid-19 pandemic continues to pose a significant challenge to the European economy, and it remains to be seen how long-lasting the economic impacts of the pandemic will be.

2.2 Black Swans

This section, will introduce the term Black Swan along with its history. Followed by a debate and conclusions on whether the Covid-19 crisis should be categorised as a Black Swan.

2.2.1 The Basics of Black Swans

The first ever written traces of Black Swans come from Juvenal, a I[st] century Roman poet, who wrote, *«rara avis in terris nigroque simillima cygno* », which translates to "a rare bird upon earth, rare as a black swan" (Juvénal, 1929, Satire VI). This illustrative ironic expression became common in 16th-century London as a statement of something impossible. This was due to the empirical observation that all observed swans were white up to this point. However, in 1697, Willem de Vlarningh, a Dutch explorer, arrived in today's Swan River in Australia and discovered the first of many real black swans (Martins, 2022). The concept of Black Swans evolved from something impossible to a perceived impossibility that might later be disproven. This discovery gave birth to a classic example in elementary philosophy, illustrated in David Hume's quote: « No amount of observations of white swans can allow the inference that all swans are white, but the observation of a single black swan is sufficient to refute that conclusion » (Alquié, 2010).

Taleb's (2007) book further popularised the concept of Black Swan events, especially in finance. He defined Black Swans as events with the following three attributes : (i) it is an outlier, as it lies outside the spectrum of regular expectations because nothing in the past can convincingly point to its possibility, (ii) it has an extreme impact, (iii) despite its outlier status, humans develop explanations for its occurrence after the fact, making it explainable and predictable (Taleb, 2007). On the other hand, Terje Aven (ex-president of the Society for Risk Analysis) has researched Black Swans, especially in a risk context and has suggested a simplified definition: "*an extreme, surprising event relative to the present knowledge*" (Aven, 2013).

2.2.2 Is the Covid-19 pandemic categorizable as Black Swans

Many actors have debated whether the Covid-19 pandemic should be categorised as a Black Swan. This paper will base its argumentation on the three attributes put forward in Taleb's (2007) book.

2.2.2.a Is the Covid-19 Pandemic an outlier?

Scientists have warned the public about a potential global pandemic for years (Menzel, 2017; Treverton et al., 2012). Even Bill Gates (2015) asserts that the "*greatest threat to the world*" to be "*not missiles, but* *microbes*", warning the public in 2015 of the impact of a potential highly infectious virus in his TED Talk: "The Next Outbreak? We're Not Prepared".

However, a pandemic outbreak's exact timing and location are hardly predictable. While the emergence of a pandemic was not unexpected, the severity and global impact of the Covid-19 pandemic have been largely unprecedented, which makes the 2020 pandemic an outlier among historical epidemics due to its unprecedented growth. Few experts have forecasted such rapid expansion and global diffusion. Most expert scenarios pointed out that the next major viral pandemic source would probably be due to terrorists, war intentions or climate change (Fan et al., 2018; Olga, 2013; Treverton et al., 2012).

So, *yes*, the emergence of a pandemic was predictable, as outbreaks of infectious diseases have occurred throughout history. However, the specific emergence of Covid-19 was not predictable, along with its rapid growth. Therefore, it can be concluded that the Covid-19 pandemic is an outlier.

2.2.2.b Has the Covid-19 Pandemic had an extreme impact?

Taleb's (2007) second Black Swan requirement is to have an extreme impact. While the 2020 pandemic is not even close to being the deadliest virus outbreak humanity has faced, no one questions that the current Pandemic has had an extreme impact on both populations and national economies. As previously mentioned the long-lasting impacts Gopinath (2020) and The World Bank et al. (2021) highlighted. The high death rate is very self-ruling of the disproportionate impact of Covid-19, with 15'000'000 deaths worldwide (World Health Organization, 2022a; World Health Organization, 2022b). Finally, the United Nations (UN) is raising the alarm about the rising concern of debt crisis for developing countries due to the Covid-19 pandemic (Spiegel et al., 2020). Hence after a decade of rising debt risk, the public debt in emerging markets has climbed to levels not seen in the last 50 years (World Bank 2021).

In conclusion, *yes*, the Covid-19 pandemic has had an extreme impact on the European markets and the World, with far-reaching consequences for health, economics, and politics.

2.2.2.c Did researchers develop explanations for the Covid-19 Pandemic a posteriori?

The last characteristic of Taleb's (2007) Black Swan is that humans tend to develop explanations for its occurrence after the fact, making it explainable and predictable in the future. We may require some assumptions to answer this third point, as 2020 can be seen as still very recent, and many post-Covid trends are yet to be defined.

Multiple researchers have already started, for example, Walsh's (2020) article "Covid-19, could not have been more predictable" and Youg's (2020) publication « A global pandemic of this scale was inevitable ». However, it can be seen that before February 2020, there was a gap in publications and articles concerning the emergence of what will happen. This, combined with human psychology findings, proves that the propensity to normalise the occurrence of an event in its aftermath can be attributed to human behaviour and a blind spot in our cognition (American Psychology Association, 2021). This demonstrate that outcome information creates biased judgments about what was previously believed, in this case, about the covid 19 pandemic (Giroux et al., 2022)

Thus, *yes*, the third Taleb's characteristic is also verified. While researchers have developed explanations for the Covid-19 pandemic both in real-time and in *a posteriori*, our understanding of the virus and its impact continues to evolve as more data and evidence become available daily. This paper can be used for this section as an example of *a posteriori* explanation for the Covid-19 Pandemic.

To conclude, while observers have asserted that the Covid-19 pandemic was highly predictable, the magnitude and speed of the Covid-19 outbreak caught many countries and institutions off guard. Following Taleb's (2007) definitions of the Black Swan, this paper positions itself as a solid advocator for categorising the Covid-19 Pandemic as a Black Swan. The pandemic has disrupted many aspects of our modern society (public health, economics, supply chains, politics and more) with far-reaching consequences. Therefore, the COVID-19 pandemic can be seen as a classic example of a Black Swan event.

2.3 Basel Committee

The Basel Committee[2] was created In 1974 by the central bank governors of the Group of Ten[3] countries in the aftermath of significant disruptions in international currency and banking markets due to the 1973 oil crisis, The British Secondary banking crisis of 1973–1975, and the Herstatt Bank Crisis (Goodhart, 2011; Penikas, 2015). The Committee was established to improve the quality of banking supervision globally and to act as a platform for regular cooperation among member countries on matters related to banking

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[2] Initially named the Committee on Banking Regulations and Supervisory Practices

[3] The G10: It is a group of 11, originally 10, countries developed economies, whose central bank officials meet to discuss making money available to the IMF for loans to its members. Members: Belgium, Canada, France, Germany, Italy, Japan, the Netherlands, Sweden, Switzerland, the UK, and the US. (Cambridge Business English Dictionary)

supervision, thus enhancing financial stability. The Committee held its inaugural meeting in February 1975 and has since convened three to four times annually. Over time, the Committee has grown from the G10 to include 45 institutional members (Goodhart, 2011). The Committee has played a significant role in setting global standards for bank regulation, mainly through the publication of three critical accords on capital adequacy, commonly referred to as Basel I, Basel II, and Basel III (Bertholon-Lampiris, 2015; Chen, 2013).

2.3.1 Basel I: The Basel Capital Accord

In January 1996, the Committee published the *Amendment to the Capital Accord to incorporate market risks* (or Market Risk Amendment) to take effect by the end of 1997. The crucial aspect of this Amendment that interests us today is that banks were, for the first time, allowed to use internal models to measure their market risk capital requirements (Goodhart, 2011). Following the Basel I Agreement, the Value at Risk (VaR) methodology has become one of the most prominent tools to calculate the risk incurred by a company or a portfolio (Penikas, 2015). Combining the publications of RiskMetrics and the Basel I agreement, VaR has become *de facto* the risk exposure assessment tool within the finance industry.

2.3.2 Basel II: the new capital framework

In 1999, the Committee started to work on a revised capital framework, known as "Basel II", structured around three pillars:

1. Minimum capital requirements, which expand the standardised Basel I rules,
2. Supervisory review of an institution's capital adequacy and internal assessment process,
3. Effective use of disclosure as a lever to strengthen market discipline and encourage sound banking practices. (Bank for International Settlements, 2013).

Published in 2004, this new framework aimed to improve regulatory capital requirements and underlying risks of the recent financial innovations. The changes intend to encourage continuous improvement in risk measurement and control. During almost six years of intensive preparation, the Basel Committee extensively consulted the financial sector's stakeholders (representatives, supervisory agencies, central banks and outside observers) to develop increasingly risk-sensitive requirements (Dan*i*elsson et al., 2001).

2.3.3 Basel III: Responding to the sub-prime financial crisis

Between 2014 and 2017, the Committee’s redacted response to the apparent weaknesses of Basel II, Basel III is intended not as a substitute but as a more robust and profound complement (Bertholon-Lampiris, 2015). These reform packages will take effect from 1 January 2023 and will span over five years. The highlighted purpose is to:

1. Improve the shock absorption ability of the banking sector from financial and economic stress,
2. Enhance the risk management and governance,
3. Upgrade banks' transparency and disclosures.

Even before Wall Street institutions collapsed in September 2008, multiple scholars had begun to point out an apparent need to strengthen the Basel II framework (Benati & Rizzi, 2007; Dowd, 2006; Yamai & Yoshiba, 2005). The financial and banking sector entered the crisis with too much leverage and inadequate liquidity buffers (Bordo, 2008). These weaknesses were accentuated by poor governance and risk assessment from multiple financial institutions worldwide and inappropriate incentive structures (Erkens et al., 2012).

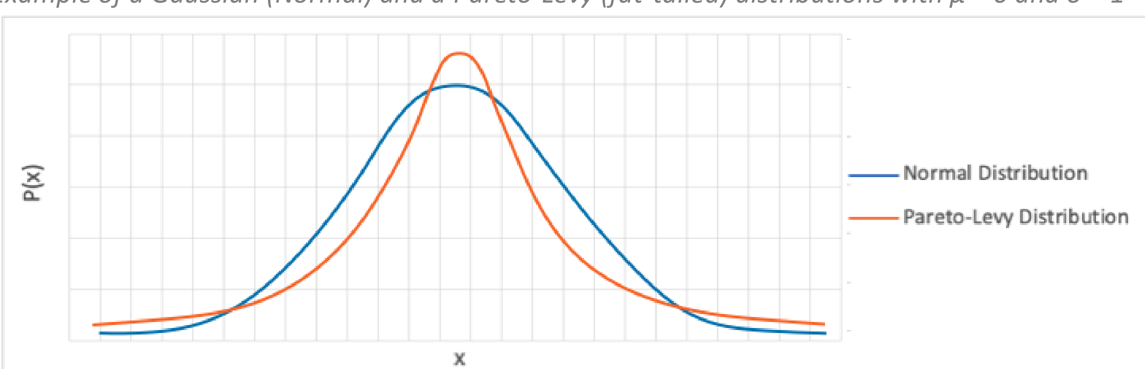
3 Literature Review

The following chapter is divided in three main section. The first section discuss the literature around the distribution of financials returns. The second section focus on portfolio parameters that could influence portfolio distribution behavior. Finally, the last section present the different method of (AVaR) computation and their current evolutions. Along these three section three crucial hypothesis are presented, which will structure the analysis required to answer the paper's research question.

3.1 Financial market distribution

A century ago, Bachelier (1900) introduced Brownian motion in finance (or Wiener process), which is the basis of most price models in finance. He is the first to theorise that stock market prices followed a Normal distribution. Later, Samuelson's (1965) publication asserts that it is the price's variation and not the price, which follows a Normal distribution. He asserts that the prices would be distributed according to a Lognormal distribution. Since then, most finance theories have assumed that prices are Normally distributed. However, the results of Normality distribution verification do not generally coincide with the Bachelier's (1900) and Samuelson's (1965) proposals. Some researchers have observed the non-normal character of the distributions of price variations (Fama, 1965; Mandelbrot, 1960). Mandelbrot (1960) found that prices in the financial market do not follow a Gaussian distribution but rather a Pareto-Lévy distribution. Similarly, Fama (1965), in his analysis of the daily return rates of the 30 Dow Jones stocks from 1957-1962, noted that their distributions deviated from the normal distribution. His paper asserts that securitie return distribution is closer to a stable Paretian style distributions with infinite variances than to the normal distribution (Fama, 1965).

Figure 3-1:
Example of a Gaussian (Normal) and a Pareto-Levy (fat-tailed) distributions with μ = 0 and σ = 1



Note: Excel's computation based on Normal and Levy distribution function with $\mu = 0$ and $\sigma = 1$

Although these two distributions do not appear to have huge divergences, their implications are significantly different. The main graphically identifiable difference lies in the magnitude of the centric part and the tails (both right and left). Moreover, Mandelbrot (1960) show that the Pareto-Lévy distribution, in opposition to the normal distribution, has a theoretically infinite variance. When the variance is infinite, it means that there is no upper limit to how much the data points can deviate from the mean. These findings indicated that the central limit theorem did not apply to estimate financial returns on his period of interest.

Mandelbrot (1960) pointed out that the individual effects constituting a price change did not have finite variance but were nevertheless independent. This suggests that the price change should belong to the stable family of distributions, which Lévy showed as the only possible limiting distribution for sums of independent random variables. In most financial models, the use of the Normality assumption for returns is widespread but results in underestimating extreme events (Damodaran, 2007; Fama, 1970; Mandelbrot, 1960; Mina & Yi Xiao, 2001). An event such as the October 1987 Wall Street crash is so rare that its occurrence is almost impossible in a Gaussian universe. In other words, the tails of the distribution are often underestimated in this risk model.

Gaussian distributions are frequently used in finance, but since many have demonstrated the non-normality of markets, authors have proposed many alternatives to the normal distribution. Excessive kurtosis is one of the features that has captured the curiosity of several authors who have proposed different alternative density specifications to account for these properties.

For Tucker and Pond (1988), the distribution of returns is more general than a Normal distribution. Furthermore, he noted a variation in the price process parameters over time, wich suggest that the true distribution of returns could be subject to change overtime. Blattberg and Gonedes (1974) studied the return rate of the Dow Jones stocks, for which they found that the Normal distribution achieved strong estimates over weekly and monthly returns. However, when using daily rates of return, their distribution resembles a Student's law (Blattberg and Gonedes, 1974). Clark (1973) refers to the lognormal distribution, McDonald and Xu (1995) to the generalised Beta distribution, while Eberlein et al. (1995) proposed the hyperbolic distributions, which are characterised by their logarithmic density.

In conclusion, the normality of the financial markets is still a subject of debate within the financial communities. As much financial theory relies on the assumption of Normal distribution (including VaR & AVaR) and considerable research has highlighted the non-normality of the market, some settings investigating this debate are crucial to highlight some potential bias presence. Moreover, as one of the researched methods heavily relies on the normality assumption, establishing the robustness of this assumption is thus crucial. For those reasons, this paper will investigate the distribution of daily returns observed during the period of interest on the Euronext stock exchange. It is expected to find that the normality assumption should not hold.

Hypothesis 1: Stock returns on the European market do not follow a Normal distribution

3.2 Diversification and Distribution

Every Stakeholder faces a constant risk of significant losses, additionally, many financial actors have observed a rise in financial instability, largely due to the globalisation of financial trade and the emergence of complex financial products (Daníelsson et al., 2001). This gives importance to the question, what do cause stock price to constantly adjust. On global markets, prices are expected to vary due to (i) Firm related news and (ii) Market-related news. These two types of news represent firm- and market-specific risks (Elton & Gruber, 1997). As a result, when constructing a financial portfolio, investors need to review the company and the market in which it operates. Furthermore, the Efficient Market Theory (EMH) suggests that firm-related risks can be diversified, reducing firm-specific news impact on the investment. A highly diversified portfolio is a portfolio that “erases” the firm-related news fluctuation by bundling together a high number of assets (Perignon & Smith, 2008). This has the effect of diluting all firm-specific risks within a big pool of different securities. Such well-diversified indices are often seen as lower-risk investments and are heavily advocated by famous investors such as Warren Buffet (Buffett & Cunningham, 2013). Furthermore, Elton and Gruber (1997) pushed the theory of diversification and argue that best highly diversified portfolio could, erase all firm specific risk and include only systemic risk (market related).

On the other hand, research have shown that Large capitalisation companies tend to grow faster than medium and smaller capitalisation companies (B. Scott et al., 1998; Switzer, 2010). B. Scott et al. (1998) shows that value-stock (Large-Cap) strategies outperform growth-stock (Small-Cap) strategies in the long run (B. Scott et al., 1998). However, in their paper, they mentioned that smaller-cap outperform other during economic crashes. Switzer (2010) study shows that Canadian Small-Cap achieve significantly positive performance compared to Large-Cap during recessions, post-2001 period. These Blue ship companies are less prone to volatility in extreme financial events and market downturns (Elton & Gruber, 1997; Harris & Spivey, 1990). Typically, companies with large market capitalisation are established and financially sound businesses at the apex of their economic model, generating consistent cash flows. These large companies are closely tied to the overall market trends as they serve as market leaders and benchmarks for their respective sectors.

On the other hand, technologically focused companies are at the forefront of innovation and are expected to generate more significant profits but a higher level of volatility (William Schwert, 2002; Sadorsky, 2003). Researchers have shown that technology equities often offer more severe results (positive or negative) than other industries' stocks (Nekhili & al., 2021).

Researchers have for a long time studied which characteristic to use to assess the risk of portfolios. To quantify the inherent riskiness of a portfolio, researchers use most of the time variance based model (Carr & Wu, 2009; Dew-Becker et al., 2017; Aniūnas et al., 2009). As the variance is obtained with the expected square deviation from the mean value, there are no differences between favourable and unfavourable deviations (Tirole, 2010). This is why many risk measurement tools are used in portfolio analysis, as investors are primarily interested in protecting themselves from losses (Kahneman et al., 1990). Hult et al. (2012) suggested an alternative method to calculate the risk premium of a portfolio by considering both risks and potential rewards. However, while this approach has some merits, it is difficult to use effectively for managing risk or assessing the overall risk position of a company. It also does not align with this research project's analysis and optimisation goals.

In conclusion, the assets composing a portfolio are determinants of its core characteristics (e.g. mean of returns, variance, risk premium). This supposed that portfolios with different holdings should display different behaviour. More diversified portfolios are expected to display lower variance at the cost of a lower mean of returns. Furthermore, it is expected to detect differences in the normality distribution of returns based on portfolio composition. More stable portfolios should have returns closer to the normal distribution compared to higher variance portfolios.

Hypothesis 2: Diversified portfolios have returns distribution closer to the Normal curve (with a smaller mean and variance)

3.3 Value at Risk

Value-at-Risk (VaR) can be summarized as the quantitative answer to the investor's question: "What is the downside risk of this investment?". Developed as a portfolio theory by Markowitz (1952), VaR has become one of the most adopted measures of risk in the financial sector (Dowd, 2006). Value-at-Risk measures the potential value loss of a position over a specific period and confidence level. Investment professionals mostly use this to assess and quantify the potential loss of a position over an arbitrary period. As VaR focuses on downside risk, its relevance is linked to what Kahneman et al. (1990) describe as risk aversion. In addition to this, the implementation of VaR also came from the pressure regulators who were willing to stabilise banking system in the 1990s following some market failures[4] and extreme volatility due to exponential growth of trading (Basle Committee on Banking Supervision, 1996; Embrechts et al., 2002; Goodhart, 2011).

In 1994, J.P. Morgan made available for the public RiskMetrics developed in partnership with Reuters, providing individual investors extensive access to data across a range of security and asset classes (Mina & Yi Xiao, 2001). Risk metrics included calculation of Value-at-Risk, based on historical method at first (Mina & Yi Xiao, 2001). Today, RiskMetrics belongs to MSCI, and its aim is still to improve the transparency of market risks and provide investors with better information on risk management. Sollis (2009) sets three essential components of VaR: (i) a defined loss level, (ii) a confidence level, and (iii) a specific period. As aforementioned, approaches to VaR are divided into three methods categories. First, the parametric approach is calculated analytically based on the assumptions of the probability distributions of returns within the market. Second, non-parametric methods relies on historical data. The latter, often called a hybrid method, uses simulation find an hypothetical portfolio returns, which will then rely on the historical approach to compute a VaR (Ammann & Reich, 2001; Cabedo & Moya, 2003; Metropolis & Ulam, 1949).

3.3.1 Comparison of the three Value at Risk methods

As previously mentioned, VaR has become a standard tool in the financial industry but has several significant drawbacks. One of the critical limitations of VaR is that it ignores the part of the distribution

[4] E.g. Saving and Loan Crisis, more than 1000 savings and loan institutions failed (US – 1986 to 1995); Barings Bank failure due to a single rogue trader (UK – 1995); Daiwa Bank concealing $1.1 billion in losses due to a unauthorized trading by a single employee (JPN – 1995); Bailout of LT Capital Management (US – 1998)

located beyond the p-level. This means risk managers may underestimate or miss significant risks, particularly catastrophic ones in the left tail (Hult et al., 2012).

Furthermore, VaR is usually calculated over a short period, such as daily or weekly, due to the focus of financial institutions on hedging risks on a short-time basis (Basak & Shapiro, 2001). Regulatory authorities also require firms to report their risk exposure at regular intervals. Consequently, portfolio managers are evaluated based on their near-term VaR, which incentivises them to disregard the potential for extensive losses under unusual unfavourable circumstances. This disregard for loss magnitude beyond the VaR threshold exposes portfolios to outsize losses, as could be expected in the event of a Black Swan (Adams & Thornton, 2014; Basak & Shapiro, 2001).

Initially, the first VaR using a parametric method relied on a Normal distribution to approximate the returns' dynamics (Alexander, 2009; J.P. Morgan & Reuters, 1996). In the 90s, some researchers proposed alternative distribution to model financial returns (Fama, 1970; Mandelbrot, 1960). Newly proposed distributions were characterised by heavier tails than the Normal curve. These new models were justified by an above-normal occurrence of extreme events (Danielsson & de Vries, 1997; Aven, 2013; Eberlein & Keller, 1995; McDonald & Xu, 1995). Later, in early 2020, another type of distribution was proposed, the multivariate normal distribution. (Billio & Pelizzon, 2000; Hull & White, 1998). A multivariate normal distribution has the advantage of having its dependence structure defined uniquely by a correlation matrix. This property is shared only by the elliptical distribution family, including the multivariate normal. Lopez & Walter (2000) have shown that the multivariate normal distribution often achieves better empirical results than the Pareto-levy-based model (Lopez & Walter, 2000). Following this new theory on returns distribution, Glasserman et al. (2002) proposed a method for determining VaR based on multivariate t-distributions. In opposition to this new but more complex model Rockafellar and Urysaev (2000) and Artzner et al. (1999) both show that VaR achieves very efficient results when computed using the correct standard deviation.

Some researchers have proposed a promising alternative for VaR modelling in the presence of Black Swans with the introduction of Extreme Value Theory (McNeil & Frey, 2000; De Haan et al., 2007). Extreme Value Theory (EVT) offers a model that includes the event at both distribution tails. EVT's key advantage is that it produces a more precise VaR at a lower confidence interval (De Haan et al., 2007). Furthermore, EVT separates the distribution tails from the central part, allowing for a better fit to the leptokurtic return distribution. However, despite its solid theoretical underpinning, EVT has numerous limitations, such as complex calibration and the need for several years of daily observations, which makes it less agile than other methods (De Haan & Ferreira, 2006; McNeil & Frey, 2000).

To address VaR's weakness, Rockafellar and Urysaev (2000) and also Artzner et al. (1999) proposed a new metric in the early 2000s, the Average Value at Risk (AVaR). The nature of AVaR is different from VaR. Average Value-at-Risk calculates the average loss an investment or portfolio will experience once the VaR threshold is attained. In other words, it measures the expected loss, given that the loss exceeds the VaR threshold (Adams & Thornton, 2014; Chang et al., 2019). One of the motivation behind this new method has been motivated by the multiple research how have highlight that the estimates of VaR are generally imprecise and become more so as we move towards the tail of the distribution compared to AVaR. Even though those extreme events, like crashes and bubbles, are rare but represent a significant change in investors' wealth. AVaR is seen more coherent than VaR because it better adapted to specific mathematical properties that VaR does not, such as subadditivity and convexity. Subadditivity is a distribution property which assumes that the joint probability of two events is less than or equal to the sum of their individual probabilities. Convexity is a distribution property where distribution behaviour allows the probability of extreme values to be higher than that of intermediate values. Multiple publications have highlighted AVaR to have better properties than VaR, thanks to better coherence and convexity (Artzner et al.,1999; Chang et al., 2019; Embrechts et al., 2002).

Finally, the Basel III agreement proposes a change in their risk assessment tool recommendation from the Value at Risk (VaR) method to the Average Value-at-Risk (AVaR) to ensure a better assessment of tail risk events. This decision is motivated by the fact that Average Value-at-Risk is a more comprehensive measure that estimates the maximum potential loss and considers the expected loss beyond that point (Adams & Thornton, 2014). Following the publications of Chang et al. (2019) and Chen (2013), banking system regulators have considered AVaR a more reliable risk measure than VaR because it provides additional information about the severity of losses beyond the VaR threshold.

The third hypothesis tested in this research will be whether we can detect systematic differences in the VaR and AVaR values. Given the nature of the abovementioned method, we expect to find recurrent differences between the parametric and historical methods, with the semi-parametric lying between the two. This is due to the market's potential non-normality, which will include heavy bias in the calculations. Therefore, the computations and results used for this hypothesis resolution will provide pivotal material to answer the research question correctly.

Hypothesis 3: Value-at-Risk and Average Value-at-risk achieve systematically different results depending on the technique used (parametric – non-parametric – semi-parametric).

4 Methodology

The following passage focuses on the methodology and techniques use to produce the results. First, the choice of which data source and portfolio to use is discussed. Next, the period of interest is fixed, determining which timespan to use in this paper. Then this chapter presents the statistical tools and techniques used to investigate the first and second hypothesis. The latter part of this chapter examines the method for calculating Value-at-Risk (VaR) and Average Value at Risk (AVaR) for each approach. Finally, these techniques and computations lead to the needed results to resolve the third hypothesis.

4.1 Data

Unless otherwise noted, all data given in this report were obtained from Yahoo Finance. Quantitative data are exported as CSV documents and then imported into Microsoft Excel or SPSS Statistics for calculations and statistical tests.

4.1.1 Financial Portfolios

This report will research VAR and AVAR under Black Swan occurrence based on three portfolios and two significance levels. These portfolios will have three distinct characteristics. The first will have to be highly diversified, the second will have to be composed of large capitalisation assets, and the third will have to represent a high growth-focused portfolio. For these reasons, it has been chosen to research these three indexes (i) Europe 600, (ii) Europe 50, (iii) Europe 600 Technology.

4.1.1.a Stoxx Europe 600

The STOXX Europe 600 or STOXX 600 is a stock market index composed of the 600 most common and largest European stock market capitalizations. The index has a fixed number of 600 constituents, including the largest capitalized companies in 17 European countries, covering approximately 90% of the free market capitalization of the European stock market (STOXX, 2023b).

Table 4-1:
STOXX EURO 600 Top Ten Holdings

Name	Sector	Country	Weight %
NESTLE	Food, Beverage and Tobacco	CH	3,10%
ASML HLDG	Technology	NL	2,45%
LVMH MOET HENNESSY	Consumer Products and Services	FR	2,10%
NOVO NORDISK B	Health Care	DK	2,09%
ROCHE HLDG P	Health Care	CH	2,02%
SHELL	Energy	GB	1,92%
ASTRAZENECA	Health Care	GB	1,87%
NOVARTIS	Health Care	CH	1,86%
TOTALENERGIES	Energy	FR	1,50%
LINDE	Chemicals	DE	1,50%

Note: Retrieved from STOXX (2023b) on March 2023

4.1.1.b Stoxx Europe 50

The EURO STOXX 50 is a stock market index composed of 50 stocks from 11 countries within the Eurozone, representing top companies in various sectors, designed by STOXX. These are the largest and most traded equities on the Euronext stock exchange. In addition, EURO STOXX 50 index is a highly liquid financial product worldwide (STOXX, 2023).

Table 4-2:

STOXX EURO 50 Top Ten Holdings

Name	Sector	Country	Weight %
ASML HLDG	Technology	NL	7,81%
LVMH MOET HENNESSY	Consumer Products and Services	FR	6,71%
TOTALENERGIES	Energy	FR	4,80%
LINDE	Energy	IE	4,79%
SAP	Technology	DE	3,57%
SIEMENS	Industrial Goods and Services	DE	3,39%
SANOFI	Health Care	FR	3,31%
L'OREAL	Consumer Products and Services	FR	2,96%
ALLIANZ	Insurance	DE	2,84%
SCHNEIDER ELECTRIC	Industrial Goods and Services	FR	2,72%

Note: Retrieved from STOXX (2023b) on March 2023

4.1.1.c STOXX Europe 600 Technology

The STOXX Europe 600 Technology is a stock market index composed of technology companies members of the STOXX Europe 600. In 2023, the portfolio was composed of 34 holdings across 14 European countries (STOXX, 2023c).

Table 4-3:
STOXX EURO 600 Technology Top Ten Holdings

Name	Sector	Country	Weight %
ASML HOLDING NV	Information Technology	NL	29,99%
SAP	Information Technology	DE	15,04%
PROSUS NV	Consumer Discretionary	NL	10,27%
INFINEON TECHNOLOGIES AG	Information Technology	DE	7,10%
CAPGEMINI	Information Technology	FR	4,95%
STMICROELECTRONICS NV	Information Technology	NL	4,70%
AMADEUS IT GROUP SA	Information Technology	ES	4,29%
DASSAULT SYSTEMES	Information Technology	FR	4,05%
HEXAGON CLASS B	Information Technology	SE	3,50%
ASM INTERNATIONAL NV	Information Technology	NL	2,25%

Note: Retrieved from STOXX (2023b) on March 2023

Tables 4-1, 4-2 and 4-3 illustrate the top ten assets of the three researched portfolios. Unsurprisingly, the ten most significant firms in the Euro 600 and Euro 50 are similar. This is because the Euro 50 comprises the 50 largest corporations of the Euro 600. Moreover, these 50 companies evolve across the 20 super sectors in nine Eurozone countries. However, it is noteworthy to compare the weightings of the top ten holdings within each index: (i) 20,41% for the Euro 600, (ii) 42,90% for the Euro 50, and (iii) 86,14% for the Euro 600 Technology. These three very distinct values highlight that the three studied portfolios represent three distinct degrees of diversification. Euro 600 is the most diversified, and Euro 600 Tech is the least diversified. This is crucial because it gives the opportunity to study the diversification effect on the portfolios.

4.2 Period of Interest

Before discussing the methods used to assess the stock market normality, a historical period of interest has to be fixed. To study the normality of returns, this paper will follow the methodology used in Adams and Thornton's (2014) publication. In their publication, they empirically tested the theoretical normality assumption of stock returns for the Dow Jones (U.S. stocks) market for the last 30 years (Adams and Thornton, 2014).

Before discussing the methods used to assess the stock market normality, a historical interest period must be fixed. This paper follows the methodology used in Adams and Thornton's (2014) publication to study the returns' normality. Their publication empirically tested the theoretical normality assumption of stock returns for the Dow Jones (US Industrial Index) market for 30 years (Adams & Thornton, 2014).

One could argue that including every known historical result will result in a more accurate distribution because the underlying data will affect the overall distribution curve. However, it is worth mentioning that this study used data from the last ten years. Multiple reasons motivated this choice. First, this decision has been driven by the desire to focus on the specific event of the Covid-19 crisis. As this paper focuses on a specific event, it has more sense to capture the trends that lead to and results of its occurrence than extensive historical research. Secondly, an extensive time frame will fail to consider new assets or market risks, making it challenging to interpret novel business sectors. This is expected to be more apparent in the third researched portfolio (Euro600 Technology), which contains many young technology companies. Thirdly, the market of interest here is the European stock exchange market, which is far more recent and, until recently, completely fragmented compared to the US stock exchange. Finally, starting in 2013 is motivated by the desire to avoid including the 2010-2012 eurozone crisis period as this would have a high probability of including external bias (Morlino & Sottilotta, 2020). This is why the last ten years' timeframe will be preferred from January 1st 2013 to December 31st 2022. Furthermore, the presence of the seven years from January 1st 2013 to December 31st 2022 will allow us to investigate the potential impact of Covid and post Covid period on the distribution of the chosen ten-year period of interest.

4.3 Normality of the returns

Now that the period of interest is fixed and the logical sequence of the analysis is established, multiple statistical tests can be used to assess the normality of returns of a financial asset (Adams & Thornton, 2014; Stokie, 1982). Some focus will be set on the Euro 600 portfolio, as it is the highest diversified portfolio that best represents the global European stock market. This part will allow us to investigate the first hypothesis, “Stock returns on the European market do not follow a normal distribution”. Next, we will discuss whether we see significant differences between the Euro 600 and the two other portfolios. We will try here to reveal if Large Cap focus or Growth focus portfolio configuration significantly impacts its distribution. This section will be dedicated to answering the second hypothesis “Diversified portfolios have returns’ distribution closer to the normal curve (with smaller mean and variance)”.

When assessing the normality of returns, it is best to use multiple methods and consider the results collectively (Doulah, 2019). This is because no single method is foolproof; each has strengths and weaknesses. This is why statisticians are using a combination of methods in order to get a complete picture of the distribution of returns (Adams & Thornton, 2014; Doulah, 2019; Mishra et al., 2019; Stokie, 1982). In this paper, four methods will be used to assess the normality of the returns: (i) Histogram, (ii) Descriptive statistics with Kurtosis and Skewness measurement, (iii) Q-Q plots, and (iv) Kolmogorov-Smirnov Test.

4.3.1 Histograms

In this paper, the histogram will be manually computed using a ten and seven years of returns for the three portfolios on Excel. A *histogram* is a graph that visually represents the frequency distribution of a set of continuous data. When assessing the normality of returns, the returns of the financial asset will be plotted on a histogram. If the histogram appears (approximately) bell-shaped and symmetric, this indicates normality. A histogram can be a quick and easy way to get a visual sense of the distribution of returns. However, it can be subjective, as the shape of the histogram can affect the researcher's gut feelings or the number of bins used.

4.3.2 Skewness and Kurtosis

These two measures will be derived along with more global descriptive statistics tests using ten years and seven years of returns for the three researched portfolios on SPSS statistics. Skewness measures the degree of asymmetry in the distribution, while Kurtosis evaluates the degree of peakedness of the distribution. In their publication, Hair et al. (2021) indicate that symmetrical distribution should have a skewness value between [-1;+1]. Further, they indicate that values outside the]-2;+2[scope should indicate substantial non-normality (Hair et al., 2021). This aligns with Byrne's book (2016), which indicates that a normal distribution should have a skewness between -2 and +2.

Moreover, when perfectly Normal, the Kurtosis should equal 0 (Byrne, 2016; Hair et al., 2010; Hair et al., 2021). Hair et al. (2010; 2021) indicate that well-fitted normal distribution should have Kurtosis between -3 and +3. If Kurtosis is above +3, it indicates that a distribution is peaked, while having a Kurtosis under -3 indicates a distribution or is too flat compared to a perfect normal distribution (Hair et al., 2021). However, Byrne (2016), Hair et al. (2010) and Hair et al. (2021) all agree that data set with sufficient size and kurtosis values between [-7; +7] can be considered normal, kurtosis value outside these boundaries indicates a substantial departure from the normal distribution. The skewness and Kurtosis are helpful because they provide indications about the shape of the distribution beyond just normality (George & Mallery, 2019). Nevertheless, they can be affected by outliers and may not be as reliable, especially if the sample size is small (George & Mallery, 2019).

4.3.3 Q-Q Plot

A Quantile-Quantile plot is a graphical representation used to compare the distribution of a sample to a normal distribution (Doulah, 2019). The Q-Q plot is created by plotting the observed quantiles of the sample data against the quantiles of a perfect normal distribution with the same mean and standard deviation. Then, if the sample is normally distributed, the points on the Q-Q plot will follow a straight line. On the other hand, if the points deviate from a straight line, it indicates non-normality (Ford, 2015; Kratz & Resnick, 1996). The Q-Q plot is a valuable tool because it can detect departures from normality in the distribution's tails, which other tests may not be able to pick up (Doulah, 2019; Stokie, 1982). This paper will manually compute the histogram using ten-year and seven-year returns for the three portfolios in Excel.

4.3.4 Kolmogorov-Smirnov Test

The Kolmogorov-Smirnov test uses a non-parametric analysis method to assess normality (Hanusz & Tarasińska, 2015; Massey, 1951). In this test, the data’s empirical distribution function (EDF) is compared to the cumulative distribution function (CDF) of a theoretical normal distribution. If the difference between the two is small, the data is likely normally distributed. On the other hand, if the p-value of the test is less than the chosen significance level, it suggests that the data does not follow a normal distribution.

In this paper, the Shapiro-Wilk test will not be performed and used as an assessment test for the normality of the returns. The Shapiro-Wilk test is widely used because it is relatively simple and can handle small to moderate sample sizes (Hanusz & Tarasińska, 2015). However, it can be susceptible to departures from normality and may result in false rejections if the sample size is large enough (Hanusz & Tarasińska, 2015). Therefore, this study will not use the SW normality test because the sample size here can be considered very large (2559 observations for the 2013-2022 period).

The Kolmogorov-Smirnov test is less sensitive to sample size than the Shapiro-Wilk test and is relatively easy to use. However, it can be less powerful than other tests and may not be as effective at detecting departures from normality in the tails of the distribution (Hanusz & Tarasińska, 2015). However, as this paper includes multiple normality tests, KS will not be used alone, minimising potential weakness..

4.4 Value-at-Risk and Average Value-at-Risk Calculations

This section will cover the calculation method for the two risk measurements. The chased purpose here is to discuss the calculation methods and technics required to research and answer the third hypothesis: "Value-at-Risk and Average Value-at-risk achieve systematically different results depending on the technique used (parametric – nonparametric – semiparametric)". In addition, a comparison between the expected losses of VaR and AVaR will be made to evaluate how well the two methods performed to see if these risk methods have been effective during the economic downturn of March 2020. This will be crucial material to answer the research question correctly.

This paper rely on the Value-at-Risk definitions and computations from Alexander's (2009) book: Market Risk Analysis, Value at Risk Models (Vol. 4). In his book, VaR is defined as « the loss, in present value terms, due to market movements, that we are reasonably confident will not be exceeded if the portfolio is held static over a certain period» (Alexander, 2009). In his publication, it has been made explicit that the Value-at-Risk metric depends on two primary parameters, the risk horizon (h) and the significance level (α). Also, Alexander (2009) highlight that even if VaR is meant to be expressed in value (e.g. euros) terms, for comparison across multiple assets and long period measured in relative terms would be preferred (e.g. returns percentage). This will be the case in this paper; the returns percentage will be used to be easily compared between the different research assets. The following section will review how the three methods were followed to obtain VaR and AVaR metrics.

4.4.1 Parametric method

To compute VaR using a parametric method, the following method will be applied using Excel. For this method, the market returns will be assumed as normally distributed.

It is crucial to mention that the normality assumption of returns does not always hold. Multiple papers reject this assumption and suggest alternative distributions to the normal (Barndorff-Nielsen & Shephard, 2001; Eberlein & Keller, 1995; Hull & White, 1998; Mandelbrot, 1960). However, as Hull & White (1998) and Alexander (2009) mention in their respective publications, VaR computed with a Normal (parametric) method is still a very commonly used and relatively simple approach.

After gathering the historical data on the daily closing price for the three researched assets from 2013 to 2022. The daily returns will be computed as follows:

$$\frac{P_t}{P_{t-1}} - 1 \tag{1}$$

Where P represents the closing price value, because this paper intends to study the performance of the VaR and AVaR over the March 2020 period, the risk horizon h research here will be one month. To compute the VaR and AVaR values on the 1st of March 2020 and see how they compare to the actual returns performance of that month. The Monthly returns will have to be calculated so Pt will be the closing price at time t and Pt-1 the closing price value one month prior. The next set will be to obtain the Annualized expected returns R and annual standard deviation SD. Assuming Normality of returns, these values can be used to model the returns as a normal distribution. Then using the NORM.S.INV function in Excel, the z-score can be computed for each corresponding confidence level.

$$= Norm.s.inv\ (\alpha) \tag{2}$$

Once the z-score computed, it can be used to calculate the VaR at the desired risk horizon (*h*).

$$VaR = [\ R * \frac{h}{y}\ ] - [\ SD\ *\ z\ *\ \sqrt{\frac{h}{y}}\ ] \tag{3}$$

Once the z-score is obtained, it is used to calculate the VaR at the desired time horizon. Here y represent the annual trading days. This will give the VaR at the desired confidence level and time horizon. To compute the AVaR another Z score have to be computed.

$$Z = \frac{1}{\alpha} * \frac{1}{\sqrt{2\pi}} * e^{-\frac{z^2}{2}} \tag{4}$$

The AVaR will then be calculated using the same formula as the **Error! Reference source not found.** but w ith the newly calculated Zreplacing z.

In this paper, the parametric methods will follow a conventional Normal distribution setting for two main reasons. First, an article by Mandelbrot (1999), argues that it was “foolish” for short periods (daily to annual) stock returns different distribution as it was crucial to use a model which can be applied to all assets’ classes, aside from the period length observed (Adams & Thornton, 2014; Mandelbrot, 1999).

However, Adams & Thornton (2014) conclude that the distribution of returns observed on the S&P500 appears “pinched” around the mean. They assert that this excludes mesokurtic distribution with thin tails, similar to the normal distribution, and shows leptokurtic (fat-tailed) characteristics. The second reason is that Adams & Thornton (2014) argue that the most appropriate distribution should be with a means that it is highly concentrated in its centre and has fatter tails than a normal distribution, as well as a left-skewed pattern (Adams & Thornton, 2014). This joins Mandelbrot’s (1960) suggestions. Both conclude that Normal distribution assumptions meet the requirement for daily VaR observation; however, they lead to optimistic value-at-risk measurement in extreme events.

Nevertheless, VaR using normal distribution remains very common in the banking industry (Penikas, 2015; Vlaar, 2000). Furthermore, as this paper dedicates a significant part to testing the normality of returns, having a VaR and AVaR using this distribution could be necessary. Nonetheless, this paper will give some interest in verifying if their conclusion made on the US equity market can be exported to the European stock exchanges. Some attention will be given if their observation on the returns distribution are similar and if the utilisation of a “simple” normal distribution also leads to over-optimistic value in the occurrence of extreme events.

4.4.2 Non-parametric method

Researchers and publications often describe the historical method as the easiest computation method for calculating Value at Risk (Alexander, 2009; Hendricks, 1996; Sollis, 2009). Also, very few variants exist for this method, with only some alternatives advocating for a weighting of past data to give more importance to recent results (Sollis, 2009; Taylor, 2008; Mayer et al., 2020). This paper will not weight our data here as this is the most common method used, and no consensus arose among the financial community on which weighting should be preferred (Sollis, 2009; Taylor, 2008; Rockafellar & Uryasev, 2000).

To compute the Value-at-Risk (VaR) using the historical method, the following method will be applied using Excel. As the daily returns value exposed in Equation 1, the monthly returns for each trading day will be computed from January 1st 2013 to December 31st 2019. Then the results are sorted in ascending order (most minor to most significant). The VaR value can then be determined as the α percentile of the sorted monthly returns, using Equation 5 in Excel:

$$VaR = Percentile.Inc(Sorted\ Monthly\ Returns; \alpha\ ) \tag{5}$$

Here, it is vital to remember that the historical method only considers past returns and assumes that future returns will follow a similar pattern, which is only sometimes the case. The AVaR will be obtained using the definition from the CFA institute[5], " Designed to measure the risk of extreme losses, AVaR is an extension of VaR that gives the total amount of loss given a loss event" (Kidd, 2012). This means that AVaR can be obtained by measuring the average value of losses beyond the VaR threshold. Obtained with a relatively "simple" average of the value remaining behind the obtained VaR measure will be performed in Excel. It is worth noting that regulators require historical VaR and AVaR calculations to be based on at least one year of data, which will be the case in this paper (Basle Committee on Banking Supervision, 1996; J.P. Morgan & Reuters, 1996; Penikas, 2015).

The computation obtained will be based on past returns (7 years timespan pre-Covid) without any dependency, not a parametric distribution, removing parametric distribution influence. Hence providing crucial material to understand further the potential bias influence of usage of normality assumption on non-normal returns

4.4.3 Semi-Parametric Method

Based on Ammann & Reich's (2001) paper has indicated that Partial Monte Carlo, which relies on Normal achieves equivalent approximation to more complex methods. In their conclusion, they assert for portfolios without substantial option components, without excessive time horizon (under 25 trading days) and confidence interval (not above the 99% confidence interval), basic simulation models achieve comparable results (Ammann & Reich; 2001). With this optic in mind, Value-at-Risk (VaR) will be computed using a semi-parametric method, a basic Monte Carlo simulation in Excel. The historical annual returns and standard deviation will be used to simulate the future performance of the three researched portfolios on 10'000 data points each. Then VaR and AVaR can be computed similarly to the non-parametric method. This method combines techniques and components from previous methods to compute the risk value. The semi-parametric computation uses some parametric concepts to generate simulation and a VaR computation methodology similar to the historical methods.

[5] The Chartered Financial Analyst (CFA) Institute is a global organisation of investment professionals that provides certification programs and promotes ethical and professional standards in the financial industry. It is a worldwide accepted standard.

5 Results

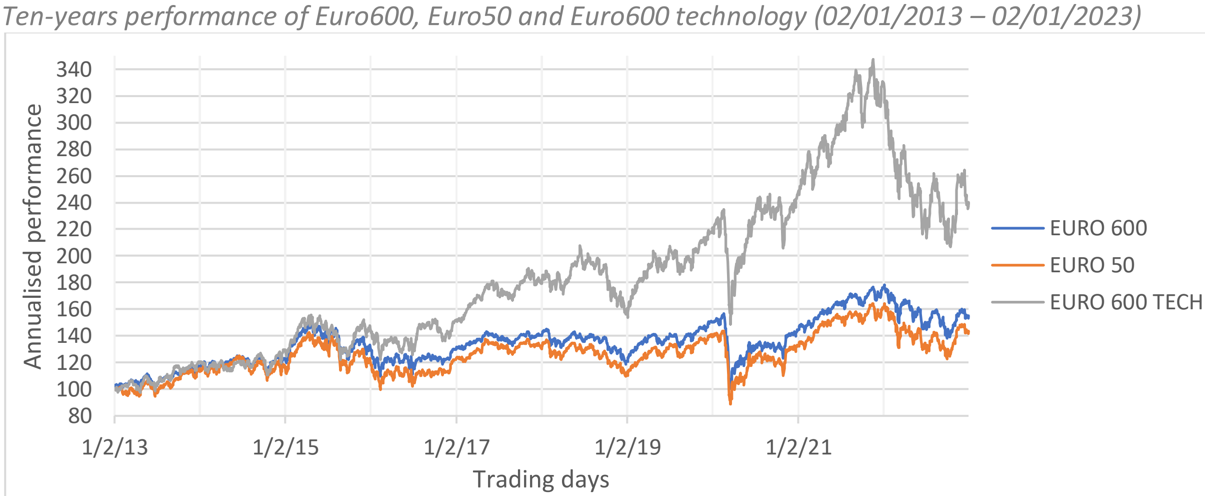
The following chapter focuses on the computation and interpretation of the results. This section is divided into three main parts. First, some attention is given to the historical component of the three selected portfolios under a ten-year and seven-year time span (2013-2023 & 2013-2019). The second part is dedicated to the analysis of the returns distributions. Finally, the normality or non-normality of the financial market returns is reviewed. Moreover, some attention is given to whether the characteristics of a portfolio's constituent significantly influence its distribution. This latter part examine the normality of the return distribution through their descriptive statistics, histograms, QQ plots, Kolmogorov-Smirnov and Shapiro-Wilk tests. The last section is reserved for the risk metrics value results. The performance of the VaR and AVaR is compared to the actual loss that occurred during March 2020.

5.1 Normality Assumption & Financial Markets

To Begin with, Figure 5-1 displays the ten years' performance of the three chosen portfolios. It becomes evident that their movements are quite similar. It can be noticed that Euro 600 and Euro 50 follow each other quite closely while Euro 600 technology surged significantly. It illustrates well the growth experienced by the portfolios and the sharp fall experienced during March 2020.

Figure 5-1:

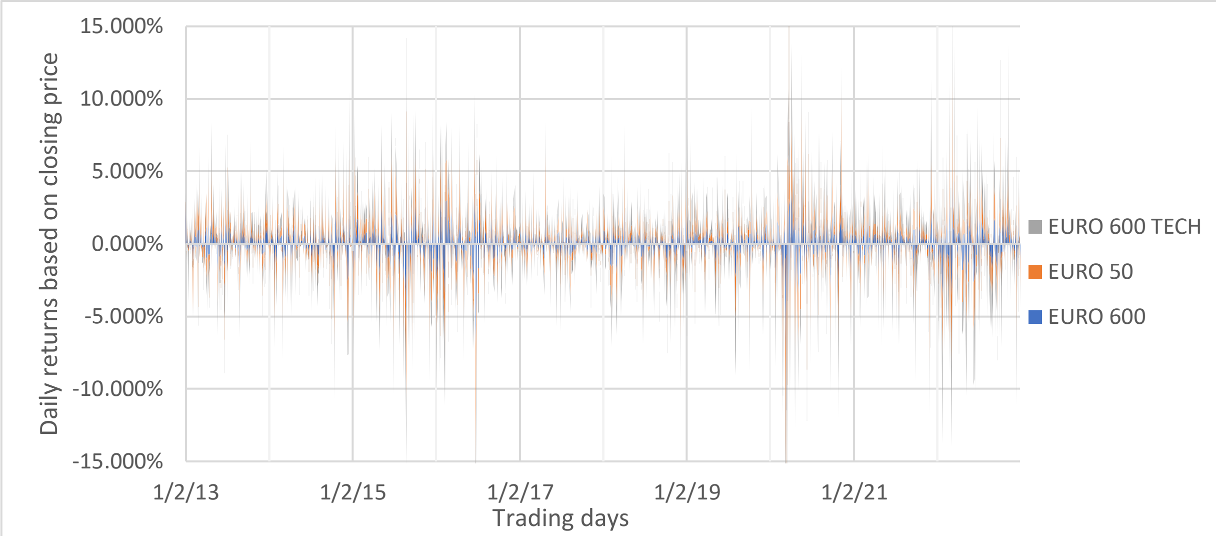
Ten-years performance of Euro600, Euro50 and Euro600 technology (02/01/2013 – 02/01/2023)



Note: Excel's computation based on daily returns calculated from Yahoo Finance's closing price for the Euro 600 (^STOXX), Euro 50 (^EUEA), Euro600 technology (^EXV3).

In analysing the impact of the Black Swan event of March 2020, Figure 5-2 highlights the sharp spike in volatility experienced during this period. This corroborates the previously displayed Figure 1-1, which was highlighting the spike in the volatility index experienced in early March 2020.

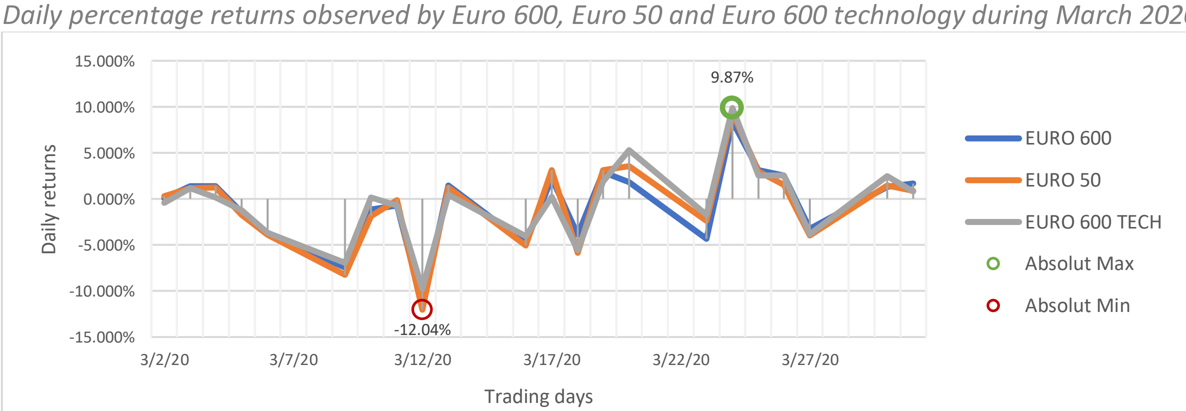
Figure 5-2:
Ten-year daily returns of Euro600, Euro50 and Euro600 technology (02/01/2013 – 02/01/2023)



Note: Excel's computation based on daily returns calculated from Yahoo Finance's closing price for the Euro 600 (^STOXX), Euro 50 (^EUEA), Euro 600 technology (^EXV3).

Moreover, to better understand how each portfolio performed during this event, a closer examination of the price development and returns during March would be helpful. The percentage of daily returns, as shown in Figure 5-3, reveals that all three portfolios experienced significant volatility during March 2020, with returns fluctuating between -12% and +10%. With a hard decline until the 12th, followed by a period that stabilised volatility and the recovery with positive returns on the 24th. The absolute worst daily return is -12.04% for the Euro 50 on the 12th of March 2020, and the absolute maximum is +9,87% for the Euro 60 technology on the 24th of March 2020.

Figure 5-3:
Daily percentage returns observed by Euro 600, Euro 50 and Euro 600 technology during March 2020



Note: Excel's computation based on daily returns calculated from Yahoo Finance's closing price for the Euro 600 (^STOXX), Euro 50 (^EUEA), Euro 600 technology (^EXV3).

Figure 5-3 and Table 5-1 provide information on the loss experienced for March 2020. From March 9th to March 18th, all portfolios experienced high volatility, with returns dominantly negative. Regarding the most considerable loss and total loss, the Euro600 and Euro50 index had similar results, losing approximately 25% over three weeks (March 1st to March 23rd). However, the Technology index had comparatively less severe losses, with a total loss of approximately 22.5% on March 23rd (compared to ~25.5% for the two other portfolios). The pandemic has incentivised shifts towards technology-based solutions such as e-commerce, online payments, remote work, and semiconductor products, which has benefited technology focus companies, thus explaining this result. Additionally, many technology companies, such as ASML, SAP or Prosus, are among the largest holdings in the Euro600 and Euro50 index, which helped minimise the losses incurred during the pandemic.

Table 5-1:
Loss incurred during March 2020

Descriptive	EURO 600	EURO 50	EURO 600 TECH
March 1st-31th (%)	-14,87%	-16,34%	-11,16%
March 1st-23th (%)	-25,41%	-25,53%	-22,55%
March 23th-31th (%)	14,13%	12,34%	14,71%

Note: Excel's computation based on Yahoo Finance's closing price for the Euro 600 (^STOXX), Euro 50 (^EUEA), Euro 600 technology (^EXV3).

5.1.1 Histograms

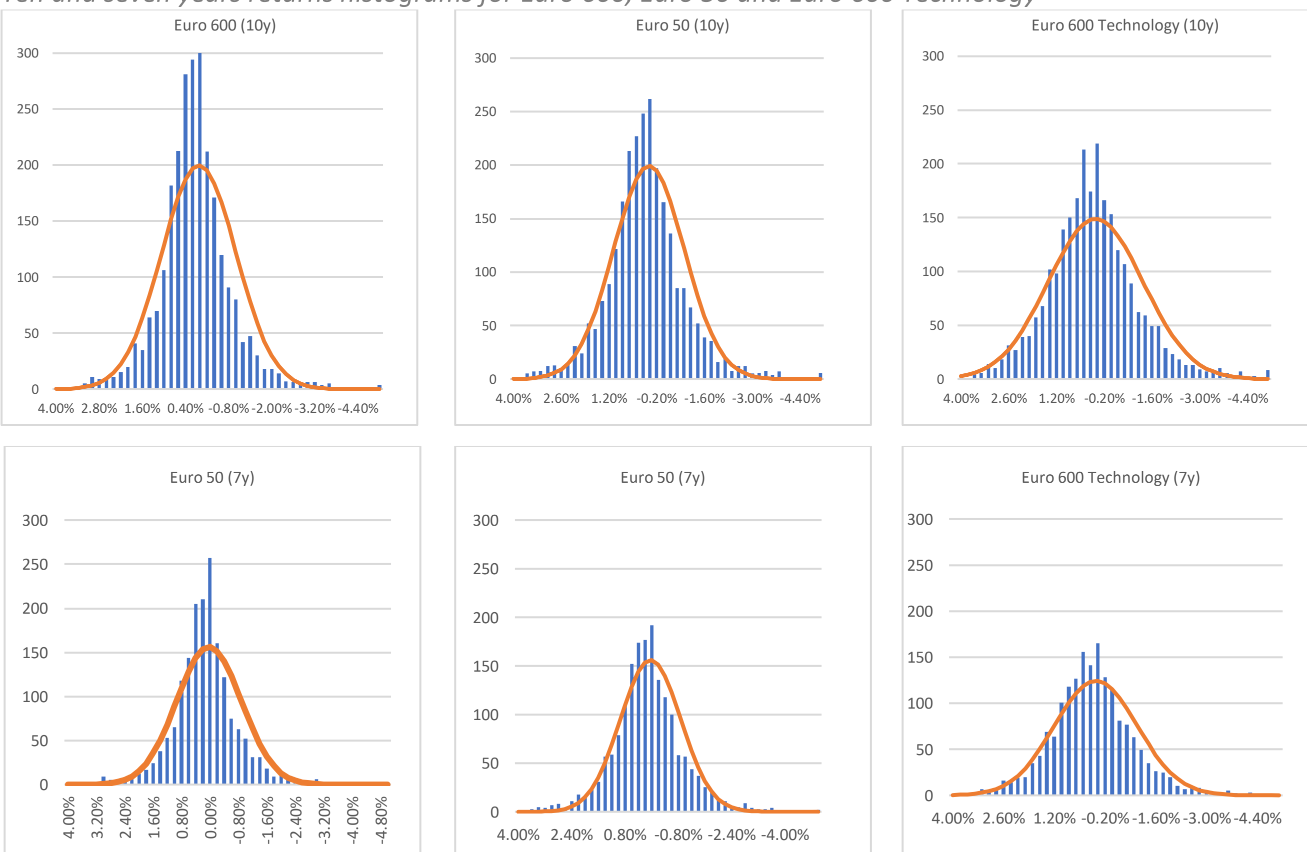
Figure 5-4 shows the histograms of the three researched portfolios with the corresponding Normal curve. Focusing first on the shape of the Euro 600 portfolio, it can be seen that the overall shape appears more peaked than the standard Normal distribution. The observed behaviour displays characteristics which suggest leptokurtosis presence, especially for the Euro600 but also in a smaller magnitude in the other two portfolios. A leptokurtic distribution can be visually detected through its pointer peak and thicker tails compared to a normal distribution. It can be seen that the observed distribution has a higher concentration of observations around the mean and more extreme values in the tails than a Normal distribution curve.

Moreover, a slight skewness to the left can be seen. Looking at the second histogram, Euro 50, we see a slightly less pointy distribution; nevertheless still pointier than the Normal curve. The slight left skewness is still present. However, this trend seems to continue, with the third portfolio coming closer to the Normal curve. Lastly, all the portfolio presents what resembles outliers or fat-tails.

Based on the observed histograms for the ten years, we can expect to find a distribution that deviates from a perfect Normal distribution framework. Moreover, a certain degree of skewness and kurtosis should be detected, especially kurtosis, which is apparent and should be distinctively higher for the Euro 600.

Figure 5-4:

Ten and seven years returns histograms for Euro 600, Euro 50 and Euro 600 Technology



Note: Excel's computation based on daily returns calculated from Yahoo Finance's closing price for the Euro 600 (^STOXX), Euro 50 (^EUEA), Euro 600 technology (^EXV3). 10 years period observed goes form 01/01/2013 to 31/12/2022; 7 years period observed goes form 01/01/2013 to 31/12/2019.

Skewness & KurtosisTable 5-2 displays the descriptive statistics of daily returns for Euro 600, Euro 50 and Euro 600 technology on a ten-year and seven-year timespan.

First, on the left side of the table, ten years timespan, focusing on the Euro600 portfolio to start. The Euro 600 portfolio has the lowest variance and standard deviation; it comes at the expense of a lower mean return relative to the other portfolios. On the other hand, Euro 600 Technology generated a higher return over the ten years, with a mean return of 0.049% and a maximum return of 9,872%, but this comes with much higher standard deviation and variance compared to the two others. Euro50, with a more similar composition to the Euro 600, has values closer to the first portfolio. The correlation between the returns of the Euro 600 and the Euro 50 is remarkably high, at 95.59%, indicating that the returns of the two portfolios closely track each other. By contrast, Euro 600 Technology's correlation with Euro 600 is 82,57%, indicating that its returns follow the global European market portfolio to a smaller extent. When comparing the mean of returns, we see that Euro 600 has the lowest and Euro 600 Technology has the highest. This trend is the same for the standard deviation, which ascends between Euro 600 and Euro 600 Technology.

Second, it is interesting to see if these characteristics remain the same even when March 2020 and its aftermath are removed from the scope of interest (as seen in Table 5-2). To begin with, we see that the standard deviation decrease when the 2020-2022 period is removed for all three portfolios. Also, the rank between them remains unchanged (Euro 600 lowest σ and Euro 600 tech highest σ). This highlight that the 2020-2022 period was a substantial-high variance period. This makes sense when it has been observed that this period was subject to high market volatility (Biermann, 2023; Mensi et al., 2021). On the other hand, the means of returns do not follow their expected movements. While the Euro 600 and Euro 600 technology means of returns increase ($\Delta_{Euro600}$ = +0.002% and $\Delta_{Euro600Tech}$ = +0.003%), the Euro 50 do the opposite (Δ_{Euro50} = -0.001%) when excluding the 2020-2022 period. This suggests that returns experienced by the Euro 50 for the period 2020-2022 were above the previous seven-year average. This trend was not expected; this will be further discussed in the discussion section of this paper.

Table 5-2:

Table 5-2 displays the descriptive statistics of daily returns for Euro 600, Euro 50 and Euro 600 technology on a ten-year and seven-year timespan.

First, on the left side of the table, ten years timespan, focusing on the Euro600 portfolio to start. The Euro 600 portfolio has the lowest variance and standard deviation; it comes at the expense of a lower mean return relative to the other portfolios. On the other hand, Euro 600 Technology generated a higher return over the ten years, with a mean return of 0.049% and a maximum return of 9,872%, but this comes with much higher standard deviation and variance compared to the two others. Euro50, with a more similar composition to the Euro 600, has values closer to the first portfolio. The correlation between the returns of the Euro 600 and the Euro 50 is remarkably high, at 95.59%, indicating that the returns of the two portfolios closely track each other. By contrast, Euro 600 Technology's correlation with Euro 600 is 82,57%, indicating that its returns follow the global European market portfolio to a smaller extent. When comparing the mean of returns, we see that Euro 600 has the lowest and Euro 600 Technology has the highest. This trend is the same for the standard deviation, which ascends between Euro 600 and Euro 600 Technology.

Second, it is interesting to see if these characteristics remain the same even when March 2020 and its aftermath are removed from the scope of interest (as seen in Table 5-2). To begin with, we see that the standard deviation decrease when the 2020-2022 period is removed for all three portfolios. Also, the rank between them remains unchanged (Euro 600 lowest σ and Euro 600 tech highest σ). This highlight that the 2020-2022 period was a substantial-high variance period. This makes sense when it has been observed that this period was subject to high market volatility (Biermann, 2023; Mensi et al., 2021). On the other hand, the means of returns do not follow their expected movements. While the Euro 600 and Euro 600 technology means of returns increase ($\Delta_{Euro600}$ = +0.002% and $\Delta_{Euro600Tech}$ = +0.003%), the Euro 50 do the opposite (Δ_{Euro50} = -0.001%) when excluding the 2020-2022 period. This suggests that returns experienced by the Euro 50 for the period 2020-2022 were above the previous seven-year average. This trend was not expected; this will be further discussed in the discussion section of this paper.

Table 5-2:

Descriptive Statistics of daily returns for Euro 600, Euro 50 and Euro 600 Technology on a 10 & 7 years timespan

Descriptive statistics	10 years (01.01.2013 - 31.12.2022)		7 years (01.01.2013 - 31.12.2019)	
EURO 600	Statistic	Std. Error	Statistic	Std. Error
Mean	0,025%	0,020%	0,027%	0,022%
95% Conf. Inter. for Mean - Lower Bound	-0,015%		-0,015%	
95% Conf. Inter. for Mean - Upper Bound	0,065%		0,069%	
Median	0,050%		0,034%	
Variance	0,011%		0,008%	
Std. Deviation	1,040%		0,912%	
Minimum	-11,478%		-7,034%	
Maximum	8,405%		4,196%	
Skewness	-0,83	0,05	-0,44	0,06
Kurtosis	10,86	0,10	4,52	0,12
EURO 50				
Mean	0,027%	0,024%	0,026%	0,025%
95% Conf. Inter. for Mean - Lower Bound	-0,020%		-0,024%	
95% Conf. Inter. for Mean - Upper Bound	0,074%		0,076%	
Median	0,057%		0,055%	
Variance	0,015%		0,012%	
Std. Deviation	1,225%		1,074%	
Minimum	-12,037%		-8,445%	
Maximum	9,324%		4,917%	

Skewness	-0,53	0,05	-0,41	0,06
Kurtosis	8,45	0,10	3,93	0,12
EURO 600 Technology				
Mean	0,049%	0,027%	0,052%	0,027%
95% Conf. Inter. for Mean - Lower Bound	-0,004%		-0,002%	
95% Conf. Inter. for Mean - Upper Bound	0,103%		0,105%	
Median	0,099%		0,092%	
Variance	0,019%		0,013%	
Std. Deviation	1,392%		1,144%	
Minimum	-9,776%		-5,386%	
Maximum	9,872%		5,061%	
Skewness	-0,22	0,05	-0,25	0,06
Kurtosis	4,06	0,10	1,58	0,12

Note: Derivate from SPSS Statistics calculations based on daily returns from closing price of the Euro 600 (^STOXX), Euro 50 (^EUEA), Euro 600 technology (^EXV3); 10 years period observed goes form 01/01/2013 to 31/12/2022 (n = 2559); 7 years period observed goes form 01/01/2013 to 31/12/2019 (n = 1787).

The third part of this subsection will be dedicated to interpreting the excess kurtosis and skewness results from Table 5-2. It can be seen that the three skewness values are all comprised of [-1;+1], which indicates a substantial sign of normality. However, it is worth mentioning that all the skewness values are negative, highlighting a tendency to be slightly weighted on the left of the mean. It can be seen that when the 2020-2022 period is excluded (right side of the graph), the Euro 600 and Euro 50 see their skewness value increasing, as they were negative, which means they get closer to 0, which indicates a less skewed distribution. However, the opposite appears for the Euro 600 technology, which has a higher left skewness value before 2020. These unexplained results will be further discussed in the discussion section of this paper.

On the other hand, we see that the three kurtosis values are not within the [-3;+3] scope recommended by Hair et al. (2010; 2021). All three portfolios have kurtosis results above the +3 value on the ten years section. A positive kurtosis above 3 indicates that the tails are "heavier" than for a normal distribution (Hair et al., 2010; Hair et al., 2021). Moreover, we can see that the kurtotic tendency is inversely proportional to the degree of diversification, with the Euro 600 index displaying the largest kurtosis value. On the other hand, the Euro 600 technology has a value of more than half the size of its counterparts but is still above the +3 limits.

Nonetheless, the kurtosis value for the Euro 600 technology on the 10-year timespan is ~4.1, which is in the [-7;+7] bounders, indicating a normal distribution can be assumed. What is very interesting here is to see the impact on the kurtosis value when excluding the 2020-2022 period of the statistics computations. It can be seen that all values undergo a significative reduction. All three portfolios a divided by more than two ($\Delta_{Euro600}$ = -58%; Δ_{Euro50} = -54%; $\Delta_{Euro600Tech}$ = -61%). This suggests that the 2020-2022 period had increased the number of outliers and data points in the distribution tails. This makes sense when looking back on Figures 5-1 and 5-2; the Covid and Post-Covid periods present signs of high volatility and unstable financial markets, which lead to increased returns kurtosis. Nevertheless, the second part of the table (right side) display kurtosis value all positive but within the [-7;+7] boundaries, with Euro 600 technology even under the +3 threshold.

Considering this, assuming a normal distribution with the second timespan period (7y: 2013-2019) is not false, even though a slight leptokurtosis tendency can be detected (all positive values). These results suggest that a Normal parametric method on the 7y time period should be statistically correct. Also, it is worth noting that a tendency for leptokurtic behaviour and slight left skewness left is detected. These results are comparable to the Adams and Thornton (2014) and Mandelbrot (1999) findings mentioned earlier (see sections 2.1 and 4.4.1), which assert that S&P500 returns distribution to have Normal characteristics with, however, a leptokurtic presence and as sight skewed left.

5.1.2 Quantile-Quantile plot

QQ plot is a valuable tool for visualising how well a sample of data fits a specific distribution, such as the normal distribution (Ford, 2015; Kratz & Resnick, 1996). To summarise, the closer the observation points follow the 45-degree line, the more likely the sample comes from a normal distribution. Figure 5-5 QQ plots have been drawn for the two timeframes, seven and ten years. First, the distribution of the Euro 600 will be analysed, as this portfolio can be used as a proxy for the global European market. Here, it can be noted that points fall along a line in the middle section of the Euro600 graph. However, they curve off in the extremities. “Normal Q-Q plots that exhibit this behaviour usually mean your data have more extreme values than expected if they truly came from a Normal distribution” (Ford, 2015). This effect of deviation from the Normal line can be seen as more important on the left side of the graph. This is because they are highlighting a more considerable deviation from the Normal curve for the more extreme adverse events.

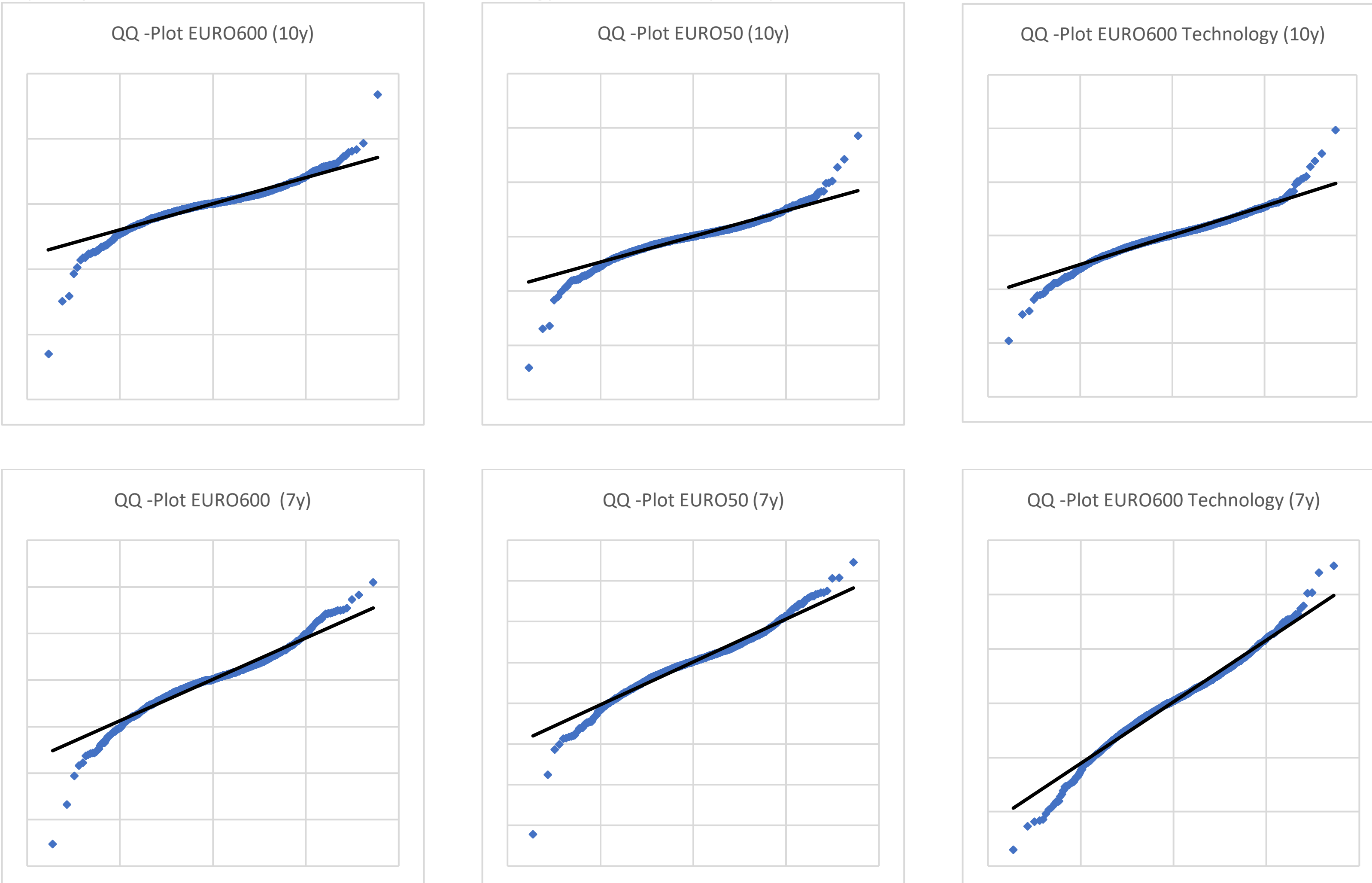
When analysing the two other portfolios, Euro 50 and Euro 600 Technology, it can be seen that the second one follows a distribution very similar to the Euro 600 while the other display different behaviour. The Euro 600 Technology portfolio seems to follow more than the 45° line. Deviation from the line can also be seen on the outer part of the line. However, on the 10y Euro 600 technology, in opposition two the two first portfolios, a more considerable deviation is present on the right side of the graph.

Comparing the 10-year distributions to their 7years homologues, the right side of the distribution falls back more on the distribution line. Also, on both sides of the extreme values, the data points seem less distant from the 45° line.

In conclusion, while the normality assumption is widely used in financial mathematics, it is critical to be aware of its limitations and to evaluate its validity when working with financial data. In this case, the QQ-plot results indicate that a normal distribution is a reasonable approximation, although with some deviations from normality. The slight deviation at the extreme sides of the graph indicates the potential presence of heavy tails in our distribution models. The potential presence of these tails should be acknowledged and recognised to avoid biases. These results can be compared with the previous section (5.1.2), where a slight leptokurtosis behaviour with some left skewness tendency has also been detected.

Figure 5-5:

QQ plots for Euro 600, Euro 50 and Euro 600 technology on a 10 and 7 years periods



Note: Excel's computation based on daily returns calculated from Yahoo Finance's closing price for the Euro 600 (^STOXX), Euro 50 (^EUEA), Euro 600 technology (^EXV3). 10 years period observed goes form 01/01/2013 to 31/12/2022. 7 years period observed goes form 01/01/2013 to 31/12/2019.

5.1.3 Kolmogorov-Smirnov Test

Table 5-3 presents the results of the Kolmogorov-Smirnov test, which is used to determine if a normal distribution can represent a set of data. The test was performed on the three investment portfolios for ten and seven years. All six tests did not pass, as the p-values were below 0.05 with a confidence interval of 95%. The Kolmogorov-Smirnov value below 0.05 is indicative of a significant failure of fit. The D value is the largest observed difference with ECDF. When looking at the D value, they are all relatively small (max= 0,082; min=0,048). This indicates that the delta between the normal distribution and the sample distribution is minima.

Table 5-3:

Kolmogorov-Smirnov normality test on 10 years and 7 years periods

	10 y (01.01.2013 - 31.12.2022)			7 y (01.01.2013 - 31.12.2019)		
	Kolmogorov-Smirnov[a]			Kolmogorov-Smirnov[a]		
	D	df	Sig.	D	df	Sig.
Euro 600	0,082	2559	<,001	0,074	1787	<,001
Euro 50	0,074	2559	<,001	0,061	1787	<,001
Euro 600 Tech	0,057	2559	<,001	0,048	1787	<,001

Note: Derivate from SPSS Statistics computation made from daily returns based on closing price of the Euro 600 (^STOXX), Euro 50 (^EUEA), Euro 600 technology (^EXV3); 10 years period observed goes form 01/01/2013 to 31/12/2022 (2559 data points); 7 years period observed goes form 01/01/2013 to 31/12/2019 (n = 1787).

This could be why there is such a hard rejection of normality from the KS test while the other measurements are more nuanced. To begin with, researchers have shown that the K-S test can be sensitive to sample size. Wood (1978) has highlighted that a larger sample size increases the likelihood of normality rejection (even when Δ with normality is small). With an enormous sample size, Kolmogorov-Smirnov becomes overpowered and rejects even minimal distribution differences (Wood, 1978). Therefore if the sample size is large, the test may reject normality even if the histogram looks normal; this behaviour was already mentioned in Massey's (1951) publications, the founding paper of the K-S test.

Moreover, it has been proved that the K-S test can be sensitive to outliers (Wood, 1978). If the sample contains outliers, the test may reject normality even if the bulk of the data follows a normal distribution. Lastly, if the sample has significant skewness or kurtosis, the K-S test may reject normality even if the other tests and histograms indicate some normal behaviour.

5.1.4 Agostino-Pearson Test

In the light of these results, an additional test will be performed. This helps understand if the normality assumption should be firmly rejected as the KS test suggests or if the situation is more ambiguous, as suggested by the three first sections. Looking back on the literature and the descriptive statistics obtained for three data samples, another test could be applied—the Agostino-Pearson test, which is based on the skewness and kurtosis of the sample data. The Agostino-Pearson test is commonly used in fields such as finance, biology, and engineering to check for normality assumptions in statistical analyses.

Doulah (2019), to compare 27 normality tests, indicated that the Agostino-Pearson, Jarque Bera, and Kurtosis tests have better power than other tests for large samples size. In his conclusion, he mentions that the Agostino-Pearson performance is best when the sample size is moderate to large, and the data are (close to) normally distributed. In a similar fashion Stokie (1982) asserts that Agostino-Pearson is more accurate than KS and SW for sample sizes above n>200, especially for samples with symmetric distribution and low kurtosis values (Stokie; 1982). Yap et al. (2011) conclude that the D'Agostino and SW tests have the best power for samples without extreme kurtosis presence. For Yap et al. (2011), the choice between the two should depend on the sample size, n<50 for SW and n>50 for Agostino-Pearson.

The results of the Agostino-Pearson Test can be seen in Table 5-4. It is worth mentioning that as Stokie (1982) and Doulah (2019) have highlighted, that A-P test is very sensitive to outliers a Tukey Fence outliers detection has been applied, and outliers have been put aside.

Table 5-4:
Agostino-Pearson normality test on 10 years and 7 years periods

	10 y (01.01.2013 - 31.12.2022)			7 y (01.01.2013 - 31.12.2019)		
	Agostino-Pearson			Agostino-Pearson		
	Statistics	df	Sig.	Statistics	df	Sig.
Euro 600	3,024	2407	0,2204	2,422	1679	0,298
Euro 50	1,777	2414	0,4114	2,230	1696	0,3279
Euro 600 Tech	1,144	2450	0,5643	1,035	1722	0,5959

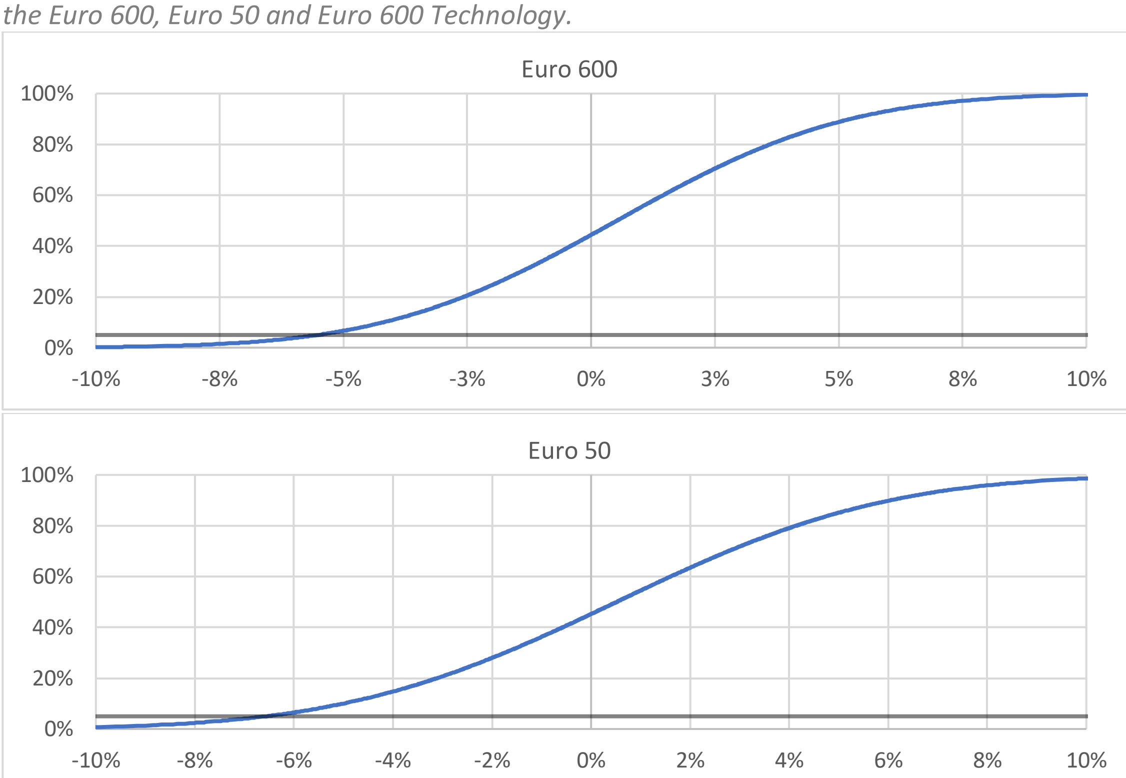
Note: Derivate from SPSS Statistics computation made from daily returns based on closing price of the Euro 600 (^STOXX), Euro 50 (^EUEA), Euro 600 technology (^EXV3); Outliers detection methods used: Tukey Fence, k=1.5; 10 years period observed goes form 01/01/2013 to 31/12/2022 (2559 data points); 7 years period observed goes form 01/01/2013 to 31/12/2019 (n = 1787).

Results show that the Agostino-Pearson test shows a non-significant difference from the normal distribution. This does not prove that the normality assumption (H0) is correct, only that it cannot be confidently rejected. However, outliers have been rejected, which logically increases the p-value. To avoid p-hacking[6], the A-P test results will only be used to nuance the KS test. The A-P test strengthens the suggestion that the sample size, the presence of skewness, kurtosis or outliers, could have overpowered the K-S test. The conclusion on the normality assumption and the effect of diversification on the portfolio distribution will be discussed in the Discussion section.

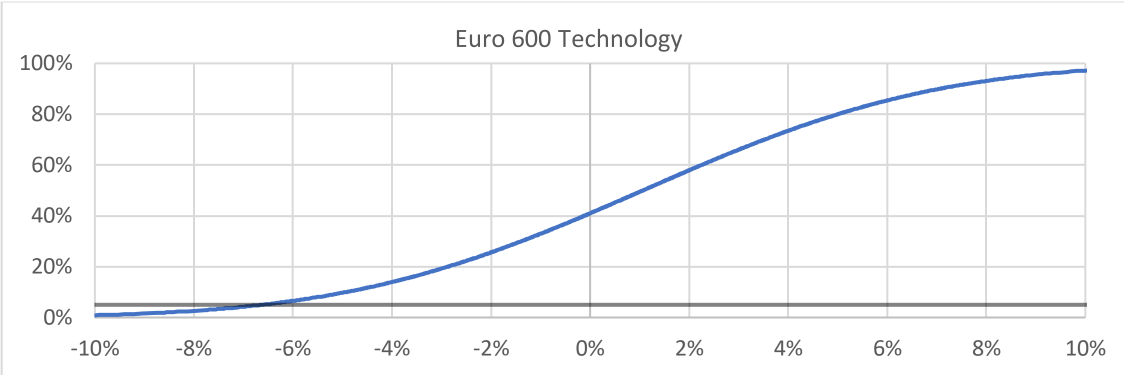
5.2 Value-at-Risk and Average Value-at-Risk

First, to get an overall idea of the value VaR, the Empirical Cumulative Distribution Function (ECDF) of the three portfolios were plotted, see Figure 5-6. Analysing these figures, the 95% VaR should end up slightly above -5% for the Euro 600, -7% for the Euro 50 and -7% for the Euro 600 Tech. Additionally, a slight tendency for a positive mean of returns became more apparent for all portfolios, as all three portfolios display a majority of positive monthly returns.

Figure 5-6:
Empirical cumulative distribution function (ECDF) of monthly returns between 2013 and 2022 for the Euro 600, Euro 50 and Euro 600 Technology.



[6] P-value hacking is a term used when statistician adapt data analysis in order to discover patterns which would be presented as statistically significant, when in reality, there is no underlying effect.



Note: Excel's computation based on daily returns calculated from Yahoo Finance's closing price for the Euro 600 (^STOXX), Euro 50 (^EUEA), Euro 600 technology (^EXV3). 10 years period observed goes form 01/01/2013 to 31/12/2022.

Table 5-4 displays the results for VaR and AVaR (95% and 99%) for each of the three methods (parametric, non-parametric, semi-parametric). Furthermore, the absolute difference between the computed VaR and AVaR results and the experienced returns in March 2020 (01/03/2020 - 31/03/2020) has been computed. The smaller the delta value is, the closer the modelled value is to the actual returns.

Table 5-5:

VaR and AVaR for parametric, historical and semi-parametric methods at 95% and 99% p-levels

	Euro 600			Euro 50			Euro600 Tech		
	Parametric	Historic	Monte Carlo	Parametric	Historic	Monte Carlo	Parametric	Historic	Monte Carlo
Actual loss	-14,87%	-14,87%	-14,87%	-16,34%	-16,34%	-16,34%	-11,16%	-11,16%	-11,16%
VaR 95	-6,16%	-5,91%	-6,12%	-7,41%	-7,33%	-7,38%	-7,57%	-7,19%	-7,51%
Δ Actual loss	8,71%	8,96%	8,75%	8,93%	9,01%	8,96%	3,59%	3,97%	3,65%
VaR 99	-8,92%	-9,43%	-8,96%	-10,66%	-11,17%	-10,68%	-11,13%	-11,65%	-11,11%
Δ Actual loss	5,95%	5,44%	5,91%	5,68%	5,17%	5,66%	0,03%	0,49%	0,05%
AVaR 95	-7,85%	-8,02%	-7,79%	-9,40%	-9,54%	-9,41%	-9,75%	-9,78%	-9,69%
Δ Actual loss	7,02%	6,85%	7,08%	6,94%	6,80%	6,93%	1,41%	1,38%	1,47%
AVaR 99	-10,29%	-10,55%	-10,21%	-12,27%	-12,35%	-12,24%	-12,90%	-12,94%	-12,91%
Δ Actual loss	4,58%	4,32%	4,66%	4,07%	3,99%	4,10%	1,74%	1,78%	1,75%

Note: Excel's computation based on daily returns calculated from Yahoo Finance's closing price for the Euro 600 (^STOXX), Euro 50 (^EUEA), Euro 600 technology (^EXV3).

First, it is worth mentioning that all three portfolios the VaR and AVaR values at both (95% and 99%) levels produce very similar results. This could be indicative of low bias presence and a lower risk of computation mistakes presence. Globally, the third portfolio, Euro 600 Technology, received the smallest differences compared to the others. This can be explained by the fact that the growth portfolio generates the largest VaR and AVaR values and experienced the smallest loss during March 2020 (compared to the Euro 600 and Euro 50 performances). It is apparent that the 99% level produced a smaller difference for all measures.

In more details, the -6.16% value displayed in the top-left corner of Table 5 4, represents the parametric value-at-risk for Euro 600, indicating that in 95% of outcomes, the monthly returns would be above a 6.16% loss. When analysed row by row, Table 5 4 demonstrates that the parametric technique achieved the lowest value-at-risk for the 95% threshold, whereas the historical approach has lowest VaR values for the 99% level. computations. Interestingly, among all portfolios, the Euro 600 Technology portfolio has the highest value-at-risk and the lowest difference for both confidence levels. Next, looking into the 99% part of the value at risk we see a shift of trend. The historical method has now the lowest values, which was the opposite than for the 95% level. Concerning the Monte Carlo approach produced metrics which are located between the historical and parametric (one exception for the 99% VaR of the Euro600 Tech). Another intriguing Monte Carlo behaviour is that all its values are constantly closer to the parametric values.

Next, in the lowest part of Table 5 4, the results for the Average Value-at-Risk can be seen. For example, the -7.85% on the middle left of Table 5 4, represents the parametric value of the Average Value-at-Risk for Euro 600. This value should be interpreted as the average returns beyond the 95% level (if this threshold was to be exceeded). In other words, for the Euro 600 portfolio when a month has a loss greater than 7,85% which represent the 5% worst historical monthly returns, it will loose on average 10,29%.

The most interesting is that the historical method have across all portfolio and significance level lower AVaR that the two other methods. Also, here again, the Monte Carlo approach achieved values very close to the parametric results. However, it can be seen that 4 out of 6 times Monte Carlo method produce higher average-value at-risk. The conclusions that can be retrieve from these VaR and AVaR results will be discuss in more detail in the Discussion section.

6 Discussion

Divided into three parts, this section further discusses the obtained results. Each part focus on specific results valuable to resolve the three presented hypotheses of this paper. The first section investigates the researched normality of the financial market. Next, the diversification theory and its influence on returns distribution is debated. Finally, the last section explores whether VaR and AVaR have systematically achieved different results depending on the technique used.

6.1 Normality of the financial market

This section discusses the results and their impact on the first hypothesis, “Stock returns on the European market do not follow a normal distribution”. Multiple results exposed in this paper will be compared. By aggregating these various measurements together, the objectify is to get an accurate view of whether or not the European financial markets follows some normal behaviour during the observed period (2013-2019 & 2013-2022). As mentioned in the methodology section, the Euro 600 index will serve as a proxy to test the normality of returns on the global European market. This is motivated by the fact that the Euro 600 index covers 17 European countries and captures 90% of the European stock market free float.

First, Figure 5-2 highlights the higher volatility of the returns for 2020-2022. Figure 5-2 suggests that the overall return distribution could significantly change between the 10-year and 7-year periods. Second, Figure 5-4, which shows the histogram of returns for the 2013-2022 and 2013-2019 periods, suggests that the distribution needs to fit a perfect standard curve for the 10y. However, it comes close to Normal for seven years. It can be seen that the histogram in Figure 5-4 has a leptokurtosis deviation, especially for the 10y section. Third, Table-5-2 shows the presence of negative skewness and positive kurtosis. Skewness and kurtosis decrease when 2020-2022 are excluded from the scope of research. All skewness values are between the [-1;+1] barriers for both periods, which raise no red flags about a potential non-normality of returns.

Nonetheless, all skewness measurements are negative, translating a trend for a “light skewness” toward the left side of the returns’ mean. On the other hand, for the ten years, kurtosis levels are all above +3 (two above +7), especially the Euro 600, valued at (+10,86), confirming the observed behaviour on the histograms. The kurtosis level for Euro 600 in the ten years indicates a non-normal distribution behaviour. However, once the 2020-2022 period is removed from the computation scope, all the kurtosis measurements improve significantly (all three metrics decrease). On the seven years, all kurtosis values fall under the +7 threshold; Euro 600 is valued at +4,52, an acceptable value for an assumption of normality. Next, Figure 5-5 shows the QQ plots for the three observed portfolios on the two distinct periods; there is concluded that the behaviour on the centre part of the distribution follows quite closely a Normal curve. However, extreme values deviate significantly from the 45° line on both distribution sides. Putting together the results mentioned above, we can conclude that global returns for 2013-2019 (pre-Covid) have shown some normality, especially during a more stable period. However, extreme returns look more recurrent than a normal bell shape suggests. The 2020-2022 period, where higher volatility has dramatically affected the potential normality of the market.

Finally, the results obtained by the Kolmogorov-Smirnov test and the Agostino-Pearson Test have to be discussed. The first one strongly rejects the potential normality of the market for both periods. However, it can be seen through the D value that once again, when the 2020-2022 period is excluded, the returns distribution comes closer to a theoretical normal distribution (D-value decrease from 0.082 to 0.074). Nonetheless, all K-S tests strongly reject the hypothesis of a Normal distribution shape. Following that, the weaknesses of the K-S tests in this paper situation have been discussed. Based on that, it has been decided to run an additional test to test if this strong rejection was due to an overpowered K-S test or an abnormal departure from normality undetected in the previous test. The Agostino-Pearson Test came with results that do not exclude normality for 2013-2019. These results must be nuanced as they have been obtained without numerous outliers.

To conclude, when added together, these results should help assess the validity of the first hypothesis: "Stock returns on the European market do not follow a normal distribution". In the strictest sense, the only answer to this statement is "yes". This strengthens recent research which rejects the normality of returns (Adams & Thornton, 2014; Iorgulescu & Altăr, 2008; Mandelbrot, 1999); and goes against the core theories of Bachelier (1900) and Samuelson (1965), which advocate for either a normal distribution of prices or price variances (returns). This is especially true when considering that not a single test suggests a strong normality distribution. Across all normality tests, stock returns do not fit a perfectly normal behaviour; some tests (QQ-plot, histogram, kurtosis, Agostino-Pearson Test) suggest that for a stable period (e.g. 2013-2019), normality can be assumed for the centre part of the distribution. However, extreme sides deviating from the theoretical distribution cannot be neglected. For example, multiple tests suggest leptokurtic distribution due to the heavier tails than a Normal distribution curve suggested. Based on this, it can be concluded that the European stock market has returns for the observed period that follow Fama's (1965) and Mandelbrot's (1960; 1999) findings, which assert that thick tails, skewness, kurtosis characterise the U.S. stock market. This suggests that approximating the returns distribution during stable periods with the Normal law should not include too many biases. However, normal distribution appears to be a weak approximate method for studying events in the distribution's tails, making it a poor tool for risk estimations (hence VaR and AVaR).

One important matter has to be mentioned, this paper research daily returns, which has been shown to be less normally distributed than larger time horizons. Further research could be made to verify the theory suggested by Blattberg and Gonedes (1974) hold on the European market. Their publication showed that Dow Jones using a normal distribution was statistically robust on weekly and monthly returns (Blattberg & Gonedes, 1974).

6.2 Portfolio Diversification and return distribution

The following section discusses the results and their impact on the second hypothesis, "Diversified portfolios have returns' distribution closer to the normal curve (with smaller mean and variance)". The first part of this section discusses the diversification theory and the results observed in this research. Next, the unexpected results from Table 5-2 on means of returns is discussed. Then, the observed effect of diversification on the distribution shape is presented, especially kurtosis measurements. Finally, combining these multiple results, a conclusion will be drawn on supporting or refuting hypothesis two.

To begin with, when an investor builds a portfolio well diversified, they reduce their risk exposure (Markowitz, 1952). As a result, the returns of the portfolio will be less influenced by the performance of a single asset and more by the performance of the portfolio as a whole. In other words, diversification leads to a more stable distribution of returns over time (Evans & Archer, 1968; Perignon & Smith, 2008). Table 5-2 have shown that the means and standard deviation of returns for the 10-year observed period followed the expected behaviour. In our sample, for the 2013-2022 period, the mean of returns and the standard deviation is inversely proportional to the level of diversification. This observed behaviour confirms Markowitz's (1952) theory that diversification stabilises the portfolio's returns. Also, the higher returns experienced by the Euro 600 Technology strengthen the findings of Nekhili & al. (2021), Sadorsky (2003) and William Schwert (2002), which have suggested that Technology stock offer higher growth than other industry.

Second, the results from Table 5-2 over the seven years (2013-2019) have to be discussed. On the right side of Table 5-2, the Euro 600 portfolio achieved higher returns than the Euro 50. This is without having a higher variance and standard deviation. This suggests that returns experienced by the Euro50 before Covid were worse than the more diversified portfolio Euro 600 while not having a more stable variance. This does not align with Evans & Archer's (1968), Markowitz's (1952) Perignon & Smith's (2008) theories. Also, as this change for the ten years, it can be concluded that 2020-2022 was highly profitable for the Euro50 compared to the Euro600. This was unexpected as it goes against some of the current literature , (B. Scott et al., 1998; Switzer, 2010). B. Scott et al. (1998) and Switzer (2010) suggested that small-Cap outperformed Large-Cap. These two publications were based on multiple international markets; it should be applicable to the European situation (Scott et al., 1998; Switzer, 2010). This suggests that financial market behaviour may have changed. Some explanations can be found using Biermann's (2023) results. Biermann (2023) suggests that large-cap and small-cap behaviour have changed in the last decade. He has divided the economic cycle into four stages: slowdown, recession, recovery and expansion (Biermann, 2023). His findings suggest that expansion and slowdown are profitable to Large Cap, while recession and recovery profits small and mid-cap companies. His paper shows that Covid created a recession (March 2020-June 2020); however, the global economy follows a mix of slowdown and expansion cycles associated with significant advantages for large-cap companies, a theory also defended by Harris & Spivey (1990). Biermann's (2023) paper shows that 2010-2016 was predominantly a recovery period which is a cycle favourable to mid and small-cap companies. This economic cycle sequence justifies quite nicely these unexpected findings on the portfolio means of returns.

Third, according to foundational publications like Evans & Archer's (1968) "Diversification and the Reduction of Dispersion: An Empirical Analysis", a well-diversified portfolio should have a smoother distribution of returns over time than a portfolio with a concentrated mix of assets. This means the portfolio is less likely to experience extreme highs or lows and more likely to generate consistent returns over the long term. These have been verified with the kurtosis measurement results in Table 5-2, which have shown that the most diversified portfolio (Euro 600) had higher kurtosis measurement (most peaked distribution). In contrast, the least diversified portfolio (Euro 600 Technology) had the least peaked distribution (smaller kurtosis value). These findings suggest that a well-diversified portfolio does not have a smoother distribution of returns. Instead, they have a more peaked distribution, concentrating a higher proportion of the returns around their means of returns. This can be further reinforced by the observation of Figure 5-4, where it can be seen that the least diversified portfolio has a distribution closer to the normality tests, while higher diversification portfolios a “pinched” in the middle. Given those results, it can be concluded that diversification can have anti-normal and pro-leptokurtic effects, staking up more returns closer to the mean than the normal distribution. This effect was also observed by Perignon and Smith (2008), which established an inverse relationship between the number of securities included in a portfolio and the level of portfolio dispersion.

To conclude, here again, diversification has been detected as a great tool to mitigate variance hence risk. Moreover, the situation between Euro 600 and Euro 50 in 2013-2019 shows that sometimes well-diversified portfolios can offer greater returns while having lower variance (hence risk), which can benefit investors greatly. However, this goes against Elton & Gruber’s (1997) and Evans & Archer’s (1968) publications, which asserted that diversification came at the cost of the lower mean of returns to achieve lower volatility. The histogram and kurtosis measurements show that the more a portfolio is diversified, the more its returns concentrate around its mean, reducing the probability of returns away from their mean. However, it has been theorised that diversification has an anti-normal and pro-leptokurtic effect on the distribution of returns. These findings suggest that using financial models based on belle shape normal distribution for high diversification comes with a substantial bias of the returns distribution.

6.3 Value at risk and extreme events

The following section discusses the results and their impact on the third hypothesis, “Value-at-Risk and Average Value-at-risk achieve systematically different results depending on the technique used (parametric – non-parametric – semi-parametric)”. Finally, combining these different results, a conclusion is presented on supporting or refuting hypothesis two.

Before elaborating on the numbers obtained in the Results section, it is crucial to mention that Value-at-Risk and Average Value-at-Risk are not directly similar or substitutes. For example, VaR represents the expected loss at a given quantile, whereas AVaR represents the expected loss beyond that quantile (J. M. Chen, 2013; Chang et al., 2019; Kidd, 2012). This explains why Average Value-at-Risk numbers are always more significant, making direct comparison difficult. Furthermore, it is crucial to understand that this report will not provide an immediate comparison between both methods but would rather provide a framework to understand and assess the efficiency of the chosen methodology.

6.3.1 Parametric

The parametric technique provided more significant risk value estimations at 95% level than the historical technique (VaR and AVaR are negative values; the lower the value, the higher the risk). However, this is the opposite for the 99% level. This switch in the ranking VaR value between the historical and the parametric method suggests that the tails of the probability distribution for the historical method are larger than those modelled by the parametric method. These heavier-than-normal tails can be graphically observed in Figure 5-4, where outliers exceed the Normal curve at both distribution extremes. As mentioned in the corresponding section, this suggests the presence of outliers or heavier tails than a typical normal distribution. These are signs that the parametric approach's modelled display left tail thinner than the observed historical returns.

On the other hand, the normal distribution used in this paper excludes the presence of outliers and abnormal losses in its tails. In other words, the left tail of the distributions is larger than this method implies, which seems to minimise the exposure to certain fat tail risks. This results in higher estimates overall at the 99% level, which suggest a lower risk for the parametric than the historical method at this level.

These thin tails substantially affect the Average Value-at-Risk because its value represents the integral from the tail's leftmost point to the target quantile. Conversely, with thinner tails, the area under the distribution curve is underestimated, resulting in more significant AVaR estimates (which indicate more negligible risk, as these are negative values). These explain why the parametric approach has AVaR above the historic matching values at both levels and for all portfolios.

6.3.2 Historical

The historical method entails creating a distribution based on previous findings. It is worth noting that both the historical and parametric approaches, to differing degrees, rely on historical data. The parametric technique infers which distribution to utilise and its properties based on past returns (mean and st. dev). On the other hand, the calculation of the historical techniques relies on past returns to establish VaR and AVaR values.

As mentioned above historical method has fatter tails than the parametric method has suggested. This theory is reinforced by the Average value-at-risk, which for every portfolio and at both confidence levels (95% and 99%) has a lower value than the other methods. Based on the Average Value-at-Risk definition mentioned above, this indicates that the areas beyond the value at risk are larger when accounted for with the historical method.

6.3.3 Monte Carlo

The Monte Carlo simulation gives results that follow a normal distribution with few outliers by sampling from numerous simulated normal distributions (10’000 iterations in this case) but do not account for skewness and kurtosis. As a result, a primary Gaussian distribution would not predict extreme outcomes and has thin tails. It explains why the Monte Carlo method for VaR and AVaR never achieves the lowest values at 95% and 95% confidence levels. Moreover, due to its heavily relying on normal distribution, this method achieves results similar to the parametric method used in this paper. Moreover, the period used to simulate the 10’000 returns does not contain any black swans. Historical data from periods with significant market swings, such as the 2008 Global Financial Crisis, could be advantageous to improve this method.

Nevertheless, this will significantly change the return and volatility parameters. Simulating more extensive risk-level scenarios and including more adverse events in the simulation. There is, however, a trade-off between including dramatic market fluctuations in the dataset and accurately capturing current stock performance.

6.3.4 Systematic results of Value-at-risk and Average Value-at-Risk

The third hypothesis resolution will now be discussed. Hypothesis 3: *Value-at-Risk and Average Value-at-risk achieve systematically different results depending on the technique used (parametric – non parametric – semi parametric*). The concise answer to this statement is, y*es*, Value-at-Risk (VaR) and Conditional Value-at-Risk (AVaR) can achieve systematically different results depending on the technique used. Table 5-5, which displays the VaR and AVaR computations results, shows signs that the method used significantly impacts the results obtained for VaR and AVaR. Parametric methods have achieved the lowest VaR value at 95% (greater risk), followed by the Monte Carlo. However, for the 99% level, the trend is not the same; the historical method has the lowest VaR value (greater risk). At a 99% confidence level, parametric methods underestimate the risk with the highest VaR values (except for Euro 600 technology). The parametric method chosen in this paper fails systematically to capture the tails of the distribution correctly. On the other hand, the non-parametric method is more accurate than its alternatives in capturing the distribution’s tails. The semi-parametric method is very sensitive to the choice of functional form and achieves results close to the parametric technique.

7 Conclusion

This chapter present the conclusive remarks of this paper. The first section present a summary of the hypothesis resolutions. Secondly, the research question will be answered based on the hypothesis conclusions and the results obtained. Finally, the last section of this chapter present suggestion for further research and policy improvements based on this paper findings

7.1 Hypothesis

The first hypothesis tested whether stock returns on the European market do not follow a normal distribution. This paper compared various returns measurements on the Euro 600 index for 2013-2019 and 2013-2022. The results show higher volatility of returns for 2020-2022. Furthermore, a leptokurtosis deviation in the histogram of returns, particularly for the ten years, has been detected. Table 5-2 showed some negative skewness and positive kurtosis, indicating a trend for "light skewness" to the left side of the returns' mean and a non-normal distribution behaviour in the ten years. However, the study concluded that normality could be assumed for a stable period, such as 2013-2019, especially for the centre part of the distribution. On the other hand, extreme sides deviating from the theoretical distribution must be addressed to study extreme events. Therefore, the European stock market does not follow a normal distribution, supporting previous research findings rejecting the normality of returns.

The second hypothesis leads to studying diversification's impact on the distribution of returns and its relationship with the Normal curve. It has been found that diversified portfolios lead to a more concentrated distribution of returns, supporting Markowitz's theory. The results also show that diversified portfolios (Euro 600) achieved higher returns with lower variance and standard deviation than Euro 50 for 2013-2019. However, Euro 50 catchup and beat Euro 600 average returns during the post-Covid period. The study found that diversification can have anti-normal and pro-leptokurtic effects, leading portfolios to more peaked distribution and thickening returns around their means.

The third hypothesis investigates if the different methods used for Value-at-Risk (VaR) and Average Value-at-Risk (AVaR) yield systematically different results. This paper highlights that (A)VaR and are not similar or substitutes, as VaR represents expected loss at a given quantile, while AVaR represents expected loss beyond that quantile. The parametric technique yielded more significant risk value estimations at 95% than the historical technique, but the opposite was observed at 99%. The Monte Carlo method gave results similar to the parametric method, and research suggests that a significant complexity is required for the accuracy of this method.

7.2 Research Question

This section will focus on answering the research question, How efficient are the Value-at-Risk (VaR) and Average Value-at-Risk (AVaR) methodologies under Black Swan occurrence?

While it may appear at first that Value-at-Risk and Average Value-at-Risk methods failed to perform efficiently during March 2020, as they suggested lower risk levels than actual returns. This failure may have arisen from the mismatch between the characteristics of the observed events and the core definitions of the risk tools themselves rather than a simple lack of efficiency. Below is a discussion of how efficiently each method appeared to have performed in this study.

First, the parametric method chosen in this paper is a poor tool for studying extreme events. This statement is based on the first hypothesis, which has been reconfirmed. Although the normality tests showed that the returns distribution exhibits some similarities with the Normal distribution curve, there is an apparent lack of fit at the distribution's tails. This suggests that (A)VaR, including a parametric approach based on Normal distribution, can be remarkably efficient in stable market conditions and biased during extreme volatility periods. Other publications have already mentioned this (Abad & Benito, 2013; Artzner et al., 1999; Rockafellar & Urysaev, 2000). For example, Abad & Benito (2013) conclude that value-at-risk measures perform better during more stable periods and are weaker instruments during highly volatile periods. As this paper studies Black Swans, which are seen as the most extreme event, using a parametric method based on normal distribution shows minimal efficiency. However, this paper does not rule out that with a more adapted distribution, the parametric approach could be the most efficient estimation technique; literature has shown that leptokurtic distribution achieves better VaR estimates as they account for financial returns' skewness and fat-tail, leads to the most promising results. The next section of this chapter discusses alternatives and suggestions that could be further tested.

Second, the historical method is the least biased (A)VaR computation in this paper due to the lack of fit of the parametric chosen distribution. This is because it is the only method which accounts for kurtosis and skewness. The computed technique is very close to the first VaR model published by J.P. Morgan & Reuters (1996). Nevertheless, a theoretical argument rules the historic method out of the best VaR method to study Black Swan position; experts define Black Swan events as unknown-unknown events (Aven Taleb).

This suggests that past data and observations should have a low claim in studying the next Black Swan. However, historical-based (A)VaR could be a great tool to detect when a situation falls out of what can be historically expected. According to this viewpoint, the 99% historical VaR could be a great whistle-blower for detecting potential Black swans. In Table 5-2, it can be seen that all 99% VaR values are above the actual returns. This suggests that the market has fallen outside its usual behaviour, which suggests that the European market is reacting to an extreme, surprising event relative to the present knowledge, which is how Aven (2013) defines a Black Swan.

Lastly, the results obtained for the Monte Carlo suggest that this approach heavily depends on the goodness of fit of the distribution used to simulate the financial returns. Similarly to the parametric computations, a Normal distribution-based model fits the returns distribution well in the centre of the distribution but failed to capture the tail behaviour on the extreme sides of the returns distribution. The heavy tails and outliers on the extreme sides of the returns distribution require a distribution that accounts for financial returns' skewness and kurtosis. However, the second hypothesis shows that a portfolio's distribution depends on its assets. This paper has given some attention to diversification factors, but it inferred that other portfolio parameters could influence its distribution (e.g. class of assets: bond, put/calls, derivatives or assets liquidity). Monte Carlo technique would require a high level of complexity to be accurate. As it is highly dependent on the distribution used to simulate the numerous returns. Hence, the simulation must be based on the most accurate possible distribution to achieve precise measurement. As mentioned above, a portfolio distribution can be influenced by its composition and diversification. This suggests that each industry and asset class would require adapted modelling. This idea is not new; it has been present in Pritzker's (2006) publication, which has achieved a precise VaR (99% for a 1-day time horizon) for the forex oil pricing. Pritzker (2006) and Ammann & Reich (2001) both agree that semi-parametric VaR, with complex but well-fitted returns distribution model, can achieve extremely precise VaR for short periods (under ten trading days).

In conclusion, this paper shows that with low complexity (A)VaR models, the historical model has delivered the most reliable risk values. However, with returns distribution models that fit better actual returns, distribution parametric and semi-parametric have the potential to be a more logical choice to study extreme events.

7.3 Improvement and Further Research

Following the conclusion of this paper, additional research can be performed to advance the field of (A)VaR associated with Black Swans.

A comparison of different methods for calculating financial return distribution is needed to advance research. Researchers have advocated for multiple solutions. This includes Pareto-Levy-based distribution, advocated by Fama (1970) and Mandelbrot (1963). On the other hand, there are more modern multivariate-based models (Abad & Benito, 2013; Billio & Pelizzon, 2000; Glasserman et al., 2002; Hull & White, 1998). For example, Abad & Benito (2013) and Glasserman et al. (2002) defend the Student t-distribution as an excellent return probability distribution. Finally, extreme value theory has been presented by some research as more adequate for studying the risks of improbable events (De Haan & Ferreira, 2006; McNeil & Frey, 2000).

Additionally, exploring how the composition of a portfolio impacts the distribution of returns could also be an area for further research beyond the anti-Normal and Pro-leptokurtic effects of diversification. Some other characteristics could be explored. For instance, a study could investigate the impact of different types of assets, such as options, bonds, and derivatives.

Finally, it would be helpful to examine various types of Black Swans and determine if the nature of the event affects the precision of the risk measures.

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