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Master Thesis

How to approach the Idiosyncratic Momentum Anomaly

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Abstract

The idiosyncratic momentum anomaly can be calculated using different methods. This thesis studies four beta estimation methods: realised beta, linear regression beta, random walk beta and DCC GARCH beta. It is shown that more sophisticated models don't always outperform simpler methods. The realised beta, which is one of the simpler methods, performs best with a Sharpe ratio of 0.227. In addition, different factor models are used to estimate the idiosyncratic momentum anomaly. The idiosyncratic momentum anomaly performs best constructed with the factor model by Daniel et al., resulting in a Sharpe ratio of 0.243.

Keywords: Idiosyncratic momemtum, Beta estimation, factor models

Contents

1 Introduction

Throughout time, people have tried to predict financial markets as accurately as possible. during this time, research has emerged showing patterns in the market. Firm characteristics patterns, human behaviour patterns, and return patterns are just a few examples. Jegadeesh and Titman (1993) were one of the first researchers to show a pattern in returns. A pattern that had nothing to do with the underlying firm, but with the returns of the asset. A pattern in which past winners tend to outperform past losers. This was called the (relative) momentum anomaly. A strategy with a 12.01% yearly return over a net zero investment portfolio in the studied time period. However, there was one major downside to this anomaly, its exposure to crash risk. When the market experiences a style reversal, it is exposed to the factors that perform the worst, resulting in major losses. Gutierrez Jr and Prinsky (2007) and Blitz et al. (2011) find a momentum strategy that has similar returns, but reduces volatility by half. The idiosyncratic momentum anomaly. Instead of comparing a stock's momentum to that of other stocks, it calculates momentum as the difference between the realised return and expected return. The major question this study aims to investigate is, how do you calculate the expected return. This introduces the research question of this thesis.

Hypothesis: The most accurate calculation of the expected return will lead to the best performance of the idiosyncratic momentum anomaly.

This thesis presents significant scientific contributions by investigating alternative methods for estimating the idiosyncratic momentum anomaly. Prior research has primarily focused on factor models to estimate idiosyncratic returns. However, the influence of the construction of beta on the idiosyncratic momentum anomaly has not been studied. In this study, three new beta estimation techniques were examined, and the range of factor models was extended. As a result, this thesis aims to fill this gap in the literature. Furthermore, the findings of this study have societal significance for investors seeking to implement the idiosyncratic momentum anomaly effectively, as it shows which methods perform best.

There are two main parts to calculate returns. Which factor model is used, and how to estimate the parameters of that factor model. How to estimate the parameters of the factor model is the main research topic of this paper. The main parameter is beta. This study will test four different beta estimation methods on the CAPM and see how this affects the performance of the idiosyncratic momentum anomaly. First, the linear regression beta, which is most commonly used in previous literature, using an ordinary least squares regression to estimate beta. Second, the realised beta, which is the linear relationship between the market portfolio and an individual asset. Third, the random walk beta, which assumes that beta follows a random walk. Fourth, the DCC GARCH beta, which is a more sophisticated method allowing for different weights over time and allowing time-varying correlation between the market portfolio and an individual asset.

Subhypothesis 1: A more accurate method of estimating beta will increase the performance of the idiosyncratic momentum anomaly.

Secondly, which factor model is used to calculate expected returns, will be touched upon. Every year new factors emerge, and with that new factor models. For this study, seven factor models are studied. 4 of which showcased by Blitz et al. (2020) and three new ones that were published recently and gathered scientific attention. The most traditional is the CAPM, which only uses the market portfolio as a factor. Then there are the Fama and French 3-, and 5-factor models (Fama and French, 1993)(Fama and French, 2015). These use factors based on the characteristics of the firm. The Q- and Q5-factor models are based on profitability and investment factor as a basis, with the Q5-factor model adding an expected growth factor (Hou et al., 2015)Hou et al. (2021). The Daniel et al. (2020) factor model, which incorporates behavioural factors. Lastly, the Stambaugh and Yuan (2017) factor model, which incorporates factors of mispricing. In the paper of Hou et al. they compare different factor models and find that the Q5-factor model performs best in their tests[1]. All factor models have different characteristics and some predict expected returns better than others. In general it can be said from this research that the more recent factor models do a better job at predicting stock returns. This introduces the second subhypothesis.

Subhypothesis 2: The most accurate factor models are better suited for the idiosyncratic momentum anomaly, as they predict stock returns better.

In this thesis I use stock information from the using ranging from January 1925 up until December 2021. The assets of the NYSE, AMEX, and NASDAQ are used. All factors for the factor models are retrieved from the corresponding author's website.

For the methodology I start with the realised momentum, this follows the work of Jegadeesh and Titman by calculating the sum of returns over the past 12 months excluding the most recent month. After that, stocks are placed in decile portfolios, and the winner minus loser (WML) portfolio is constructed by subtracting the loser portfolio from the winner portfolio. Then the idiosyncratic momentum anomaly is constructed using linear regression and the CAPM. Here, first a 3-year window is used to construct the estimated alpha and beta. A 1-year period is used to calculate the idiosyncratic momentum over the past year. The paper will continue with the other three methods of estimating beta. The realised beta is calculated by dividing the past assets return over the market return over the estimation period. The random walk beta is estimated by using the realised beta at the previous observation. Lastly the DCC GARCH is used to estimate beta. First, the GARCH model is fitted to estimate the conditional variance of each asset return. The DCC model is then applied to estimate the time-varying conditional correlation between asset returns varying over time. Finally, the DCC coefficients are used as measures of beta for each asset. Once the betas are estimated, the idiosyncratic momentum is calculated identical to the traditional method. For the factor models the traditional method of calculating betas is used: the linear regression. The factor models are used to estimate expected returns. With the expected and realised returns, the idiosyncratic momentum is calculated using the different factor models.

For the results, the same order as the methodology is followed. Here I will discuss the results of the WML portfolio and in the results go into more detail. The relative momentum anomaly performed

[1]It must be noted that this was the paper where they introduced this factor so it can be seen as less objective

similar to previous research with an average return of 1.46% and a standard deviation of 7.01%. The traditional idiosyncratic momentum anomaly performed slightly worse compared to previous research with an average monthly return of 1.03% and a standard deviation of 4.55%. It was clear that there was a strong reduction in volatility, resulting in a higher Sharpe ratio. Both anomalies were robust to various empirical tests and remained statistically significant at 5%. Only in the spanning test when adding the oposing anomaly to the regression, this was reduced to 10% significance. Continuing with the other estimation methods, the realised beta showed promising results with an average return of 1.20% and a standard deviation of 5.22%. The random Walk beta was underperforming with 0.44% and a standard deviation of 3.06%. Lastly the DCC beta was underwhelming with a monthly return of 0.89% and a standard deviation of 5.09%. The realised beta and DCC beta were robust to various empirical tests however the random walk beta was not. Overall the realised beta performed best both on return and Sharpe ratio. Finally the different factor models. Here all returns were relatively close. The lowest volatility was achieved by the FF3-factor model. The highest return was achieved by the Daniel et al. factor model. This was accompanied with the highest sharpe ratio, making the Daniel et al. factor model the best performing factor model for the idiosyncratic momentum anomaly based on these results.

2 Literature review

2.1 Relative momentum anomaly

Past winners tend to outperform past losers. This describes the relative momentum anomaly[2][3], a theory first discussed by Levy (1967), and later extensively worked out by Jegadeesh and Titman (1993). A phenomenon in which stocks with the highest returns over the previous six to twelve months have, on average, higher returns than stocks with the lowest returns over the same period. Although the relative momentum anomaly used to be very persistent, a recent study by Dolvin and Foltice (2017) found that the relative momentum anomaly was unable to generate significant returns during the 2007-2015 period. The authors even stated that ”institutional investors seeking to profit from traditionally based momentum trading strategies may need to rethink their approaches”. On top of the decreased performance, the relative momentum anomaly is less systematic relative to other anomalies. This can be attributed to the fact that it is solely dependent on the returns of an asset and not on the characteristics of the underlying. Portfolios sorted on return have significant exposure to systematic factors (Kothari and Shanken, 1992). This can be explained intuitively: during times when the market is in a bull (bear) market, high (low) beta stocks tend to outperform low (high) beta stocks. This means that the relative momentum anomaly on average goes long on high (low) beta assets and short on low (high) beta assets. Thus, indirectly gaining exposure to systematic factors. This introduces a problem with the relative momentum anomaly. When the market experiences rapid turns of direction, the relative momentum anomaly is heavily loaded on assets that tend to perform worst during the style reversal. This is particularly strong when the style is swapped from a bear market to a bull market. An example is the Fama-French momentum factor [4] making a -83% return in 2009 when the stocks that previously performed worst strongly outperformed past winners. Exposure to the reversal anomaly is a major drawback of the relative momentum anomaly, often described as crash risk.

2.2 Idiosyncratic momentum anomaly

Fund managers have a strong incentive to reduce exposure to crash risk. A sharp decline in performance can result in a large outflow of funds as investors exhibit return chasing behaviour (Frazzini and Lamont, 2008). This strong outflow can danger a manager’s job and/or damage his reputation. Resulting in the need to reduce the strategy’s volatility. Grundy and Martin (2001) show that hedging the dynamic exposure to the size and market factors, reduces the volatility of the relative momentum anomaly while preserving comparable returns. However, Daniel and Moskowitz (2016) argue that this outperformance is driven by forward-looking betas. They show that when using ex ante betas, the performance improvement does not hold. Gutierrez Jr and Prinsky (2007) propose an additional way of looking at momentum. Relative momentum as described by Jegadeesh and Titman (1993) looks at a stock’s momentum relative

[2] Levy (1967) calls this relative strength

[3] Jegadeesh and Titman (1993) call this the momentum anomaly

[4] As can be seen on **http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html**

to the momentum of other stocks, but Gutierrez Jr and Prinsky (2007) look at a stock's individual momentum. The idiosyncratic momentum[5], where the momentum is defined as the error in the forecast return, the residual. This allows for a comparison of the return of an asset with its previous returns. They find that the idiosyncratic momentum anomaly yields returns similar to the relative momentum anomaly for the first 12 months. After a year, the relative momentum anomaly suffers from a reversal and yields negative returns. The idiosyncratic momentum anomaly does not suffer from the reversal effect and remains positive on average for a holding period of up to five years. Blitz et al. (2011) add to the idiosyncratic momentum anomaly[6] with evidence that the volatility of the idiosyncratic momentum anomaly is significantly lower than the relative momentum anomaly. This results in a Sharpe ratio twice as high. In addition to lower volatility, the idiosyncratic momentum factor is not directly linked to systematic factors. This results in the idiosyncratic momentum factor being less exposed to crash risk than the momentum factor. Based on these facts, the idiosyncratic momentum factor historically outperformed the relative momentum factor based on the risk-return trade-off.

2.3 Estimating beta

The model used for forming the idiosyncratic momentum factor plays a crucial role in the performance of the idiosyncratic momentum anomaly. The way used to calculate the betas in the model ultimately determines the value of the idiosyncratic momentum. The studies mentioned earlier use linear regression to estimate parameters; however, there are alternatives that are more sophisticated and perhaps more accurate. First discussing the linear regression. Univariate regressions are the most widely used method to estimate betas. In the regression, the relationship between an asset and a portfolio is measured by their covariance over the variance of the portfolio (Fama and MacBeth, 1973). When describing the relationship to the market, the portfolio will be the market portfolio. This equation is based on the hypothesis that the relationship between asset returns and the market is linear. Fama and MacBeth tested this hypothesis by adding a non-linear estimator which turned out to be insignificant and thus used the linear relationship. However, over time, evidence showed that the security market line is too flat (Frazzini and Pedersen, 2014). That is, when stocks have a relatively low (high) beta, on average, the estimate of their beta is too high (low). Thus, I explore different methods of estimating parameters. I follow the work of Hollstein and Prokopczuk (2016) who give an overview of the different methods to use when estimating beta.

The first option that the authors provide is to use the realised beta. In this option, the beta is defined as how many times the returns of an asset are needed to get the equivalent return of the market. Andersen et al. (2006) show that only under weak conditions this estimate of beta is consistent with the true underlying beta. A major advantage of the realised beta approach is how simple it is to estimate, as it consists solely of a division of previous observations. Compared to other approaches, no variances, covariances, or correlations are required. The authors, however, note that realised betas display much

[5]Gutierrez Jr and Prinsky (2007) call this the abnormal momentum anomaly
[6]Blitz et al. (2011) call this the residual momentum anomaly

less persistence and predictability compared to realised variances and covariances. The upside is that the variation in the market variance and individual assets, but no variation in betas, is exactly what is expected in a factor volatility model. Thus, the realised beta approach is significant to investigate.

The work of Hollstein and Prokopczuk continues with a discussion of what they call historical beta. I will briefly explain the historical beta. It uses ordinary least-squares linear regression to estimate beta. It follows the work of Fama and MacBeth (1973) where they regress the assets excess return on the markets excess return. Where beta is the covariance between the asset and the market divided by the variance in excess returns of the market. This is the method used for the base case and the same method that previous literature used. Thus, it is already known in previous estimations. Therefore, the other methods will be compared with the historical beta and will show whether an improvement is achieved. Throughout this paper, this method will also be called the traditional method, referring to previous literature.

The third method described by Hollstein and Prokopczuk is Dynamic Conditional Correlation beta, DCC beta in short. Here they follow Engle (2002) to estimate beta. Until now, all estimations of parameters have equally valued the information over the estimation period. Meaning, an observation at the beginning of the estimation period has the same influence on the predicted parameter at t=0 as the latest information in the estimation period. However, as one can understand, more recent observations are more likely to influence an outcome than older observations. A way to solve this problem is using a generalised auto-regressive conditional heteroskedasticity (GARCH) model. This model combines the estimated average variance with a certain amount of variance lags[7]. In this way, the recent variance has more influence. Therefore GARCH models are a popular choice to estimate volatility more accurately. Multivariate GARCH was a theoretical solution; this links assets to ensure that a sudden change in volatility is due to the change in volatility of the assets and not to a change in volatility in the market. This turned out to be very difficult to implement. Asset allocation and risk management are strongly dependent on correlations; however, in this setting, a large number of correlations and volatilities must be estimated. The number of correlations can be is shown in Equation 1.

$$N_{corr} = \frac{N_{assets}^2 - N_{assets}}{2} \tag{1}$$

Consider a portfolio of three assets, this requires computing three pairwise correlations[8]. However, if we instead consider a larger universe of assets, such as the S&P 500, the number of pairwise correlations needed grows significantly. For the S&P 500, this translates to 124,750 pairwise correlations[9]. As the number of assets increases, the number of pairwise correlations required grows exponentially. Thus, Engle came up with the Dynamic Conditional Correlation GARCH (DCC GARCH) model to solve this issue. DCC Garch is an extension of the standard GARCH model that allows time-varying correlations between residuals of multiple time series. In a normal GARCH model, the variance predicted is merely a function of its residuals. The DCC GARCH model uses two time series and is modelled in such a

[7] Often only one lag is used

[8] (3*3-3)/2=3

[9] (500*500-500)/2=124750

way that the function of their residuals also uses the other time series' residuals and the conditional correlation between them. The DCC GARCH model includes two levels of estimation. In the first stage, the GARCH model is estimated separately for each time period and its volatility is obtained. In the second step, the relationship between the residuals is modelled using a dynamic regression model, where lagged residuals and correlations are included as regressor variables. The correlation coefficients are then used to calculate the covariance matrix, which is used to estimate the covariance of each time series for the next period. The main advantage of the DCC GARCH model over a standard GARCH model is that it allows for time-varying correlations. The standard GARCH model assumes that the residuals are not time-related, which is not always true for financial data. This can lead to poor beta prediction and poor risk management. By capturing changes in the relationship patterns between the market portfolio and an individual asset, DCC GARCH can provide more reliable beta estimates. This model offers the flexibility of univariate GARCH, while avoiding the complexity of traditional multivariate GARCH models. This simplicity is achieved by employing the market portfolio as a proxy for the relationship between an individual asset and all others. As a result, the number of parameters estimated in the correlation process is independent of the number of series that are correlated, significantly reducing the computational burden and speeding up the estimation process. For instance, if there are 500 assets in the S&P500, only the correlation between each asset and the market portfolio needs to be estimated, resulting in a total of 500 correlations. This results in a reduction of 99.60% in the number of correlations estimated in the outlined example. This approach enables the use of a GARCH model to estimate beta in a multivariate setting, which is essential for estimating the iMOM anomaly.

The fourth method of estimating beta is the Random Walk model, which is based on the theory that betas follow a random walk. This means that there is no trend or pattern in observations and that the best estimation for predicting beta is the last observed observation. This theory dates back to 1953 Kendall and Hill (1953), and although it may be outdated, it can still serve as a useful baseline for testing the precision of beta estimates. If the Random Walk model is found to accurately predict beta, it would suggest that beta is more random than expected. However, more sophisticated methods for estimating beta have been developed, such as the CAPM and GARCH models. These models are better able to capture the complexities of stock market behaviour and produce more reliable beta estimates. Nevertheless, the Random Walk model is still relevant in finance and can provide a useful starting point for understanding and analysing asset pricing models. Furthermore, the Random Walk model can be used as a null hypothesis in statistical tests of asset pricing models.

The work of Hollstein and Prokopczuk continued with more sophisticated methods to estimate betas, but these methods use daily observations and option data. As the idiosyncratic momentum anomaly is an investment strategy traditionally studied on monthly data, the more sophisticated methods are not well-suited for this thesis. Additionally, the required level of mathematics and econometrics is beyond the scope of my level of expertise, making it infeasible to master these concepts in the given timeframe. Therefore, more sophisticated methods for estimating beta will be left open for further research.

2.4 Factor models

For the estimation of residuals, Gutierrez Jr and Prinsky (2007) use a traditional CAPM regression, a Fama-French 3-factor model as described by Fama and French (1993) and a mean return model. Blitz et al. (2011) use a Fama-French 3-factor model. The results show that the performance of the idiosyncratic momentum anomaly is dependent on the chosen factor model. Nevertheless, none of the factor models used in spanning tests can explain the outperformance of the idiosyncratic momentum anomaly. Blitz et al. (2020) expand the number of factor models tested using the factor models proposed by Hou et al. (2015) and Stambaugh and Yuan (2017). Blitz et al. conclude that the alpha remains statistically significant and the models do not explain the outperformance. However, the results in alpha are different and show that the choice of factor model is important. In recent years various other factor models have emerged that have not been used to generate the idiosyncratic momentum factor in scientific literature. The models include the Fama and French (2015) 5-factor model, the Daniel et al. (2020) 3-factor model and the Hou et al. (2021) 5-factor model. This thesis will study whether these factor models can explain the idiosyncratic momentum anomaly and which model is best suited to form the idiosyncratic momentum factor. The reason for choosing these models is that they are the most popular factor models that have emerged in recent years. This thesis will also compare the newer factor models with the factor models already studied.

3 Data

For the stock data, the CRSP database is used to obtain US stock prices, holding period returns, and delisting returns. Missing holding period returns are merged with delisting returns if available. This prevents data from suffering from survivorship bias, as bankrupt companies are not automatically given a return of -100%. Only stocks from the NYSE, AMEX, and NASDAQ are used, similar to Blitz et al. (2011). The reason for using only American stocks is that the focus of this thesis is to expand the methods used in the previous literature and not to explore different markets. Only common shares are used, with sharecodes 10 and 11. Financial companies are excluded based on their SIC code (between 6000 and 6999) due to their inherently different capital structure. Financial firms tend to have more leverage and this leverage often means for non-financial firms that they are in distress, whereas for financial firms this is a part of their business model. Penny stocks are excluded, defined as stocks with a share price at the beginning of the month smaller than 1 dollar. Additionally, microcap stocks are excluded from equally weighted portfolios defined as stocks below the 20th percentile of stocks based on their market cap. This is done to prevent microcaps from overinfluencing the results. The microcaps are excluded based on their values at the start of the month so that the information is known at the start of the period to the investor. I examine the period between January 1925 and December 2021. The reason for this time frame is the availability of data as of writing this thesis. I will obtain the Fama-French factors from the Kenneth R. French. library[10]. The factors of the Q- and Q5 factor model are obtained from the authors'

[10] **http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html**

website global-Q[11]. The factors of (Daniel et al., 2020) are obtained from the author's website[12]. lastly, the factors of the (Stambaugh and Yuan, 2017) model are obtained from the Wharton open library[13].

4 Methodology

In this section, I will discuss the methodology used in this paper. First, I will briefly summarize the methods used by previous authors and compare my result to theirs to asses a baseline. Afterwards I will expand the methodology on the different beta estimation methods. Starting with the realised beta, then the random walk beta and ending with the DCC GARCH beta due to its complexity. Lastly, I will explain the methodology on the different factor models.

4.1 Replicating previous research

Starting with the relative momentum anomaly. Portfolios following the example of Jegadeesh and Titman (1993) are first being constructed. Portfolios are rebalanced after each month. Each asset gets distributed to a portfolio based on its cumulative 12 month past return, excluding the most recent month (t-12 to t-2). The most recent month is excluded to prevent the influence of the short term reversal effect as studied by Jegadeesh (1990). In formula:

$$MOM = \sum_{t=-12}^{-2} R_{i,t} \tag{2}$$

Where $R_{i,t}$ stands for asset i's logarithmic return at time t. The fact that the log returns are additive is used and, therefore, the sum of returns suffices to get the past total return. To get logarithmic returns, simple returns are converted with the following formula:

$$R_{i,t} = log(r_{i,t} + 1) \tag{3}$$

Where R stands for the log returns and r stands for the simple returns. After constructing the relative momentum, 10 portfolios are created, sorted on their relative momentum. In these portfolios equally weighted returns are calculated. The results can then be compared to the study by Jegadeesh and Titman, as well as the more recent studies by Gutierrez Jr and Prinsky (2007) and Blitz et al. (2011).

Similarly to the relative momentum anomaly, the idiosyncratic momentum anomaly is constructed. However, first, idiosyncratic returns must be calculated. Here, the CAPM is used to estimate the alpha and market beta of the model. The reason for choosing the CAPM over, e.g. the FF3 model[14], is that for the different estimation methods of beta, also a CAPM is used and this makes comparison more

[11]**http://global-q.org/index.html**
[12]**http://www.kentdaniel.net/data.php**
[13]**https://finance.wharton.upenn.edu/ stambaug/**
[14]As done by Blitz et al. (2020)

accurate. In formula:

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_{mkt,i} \times (r_{mkt,t} - r_{f,t}) + \epsilon_{i,t} \tag{4}$$

Where $r_{i,t}$ stands for asset i's return at time t. $r_{f,t}$ is the risk free rate. α_i is the constant of the asset i. The β are the estimated relations of asset i with the market. $r_{mkt,t} - r_{f,t}$ is the excess market return at time t. Lastly, $\epsilon_{i,t}$ is the error term. Once the parameters are estimated, the returns are forecasted. Then the actual returns are subtracted with the forecasted returns to get the idiosyncratic return. in formula:

$$e_{i,t} = r_{i,t} - r_{f,t} - (\widehat{\alpha_i} + \widehat{\beta_{mkt,i}} \times (r_{mkt,t} - r_{f,t})) \tag{5}$$

Where $e_{i,t}$ are the idiosyncratic returns. The hat above alpha and beta signifies that the parameter is estimated. After calculating the idiosyncratic returns, each asset gets distributed to a portfolio based on their past 12 month idiosyncratic momentum, excluding the most recent month (t-12 to t-1). In formula:

$$iMOM_{i,t} = \frac{\sum_{t=-12}^{-2} e_{i,t}}{\sqrt{\sum_{t=-12}^{-2} (e_{i,t} - \bar{e_i})^2}} \tag{6}$$

Where $\bar{e_i}$ is the average idiosyncratic return. The returns are standardized to provide a better estimate and reduce noise. The result will be compared to a non-standardised idiosyncratic momentum anomaly to make sure that the results are not driven by the standardisation of the anomaly. Additionally, the performance of the standardized idiosyncratic momentum will be compared with a standardized relative momentum strategy. After constructing both the relative momentum portfolios and the idiosyncratic momentum portfolios, a portfolio is constructed for both factors where the 10th decile portfolio is subtracted with the 1st decile portfolio to create a zero net investment portfolio. This portfolio is the winners minus the loser portfolio, and this will be called the relative momentum portfolio, relative winners minus losers (rWML). For the idiosyncratic momentum portfolio, it is called the idiosyncratic winners minus losers (iWML) portfolio.

4.2 Estimating beta

This thesis focusses on the estimation of Beta. This is depicted in equation 5 as $\widehat{\beta_{mkt,i}}$. There are various ways to estimate this beta, and this thesis will follow the work of Hollstein and Prokopczuk (2016). Here, the historical beta, the realised beta, the random walk beta, and finally the dynamic conditional beta will be discussed.

4.2.1 Historical beta

First, historical beta will be discussed. Hollstein and Prokopczuk Call this historical beta but it is the method of using an ordinary least square regression (OLS) analysis to estimate the parameters alpha and beta. Ordinary least squares is a method that minimises the sum of square errors (SSE) between the observed value and the predicted value based on the independent variable. The ordinary least-squares regression can be seen as a line between observed values in the estimation window that has the lowest sum of squared errors. In matrix form:

$$Y = X\beta + \epsilon \tag{7}$$

Where Y is the $N \times 1$ matrix of the predicted returns of stocks. X is the $N \times 2$ matrix representing the independent variable with the first column being the risk free rate and the second column representing the expect market return. β is the 2×1 matrix with the first row representing the intercept representing the risk-free rate and the second row representing the slope. The OLS regressions minimises the SSE, which is the sum of squared differences between forecasted Y and actual Y. Which can be expressed in formula as:

$$SSE = \sum (Y - X\beta)^2 \tag{8}$$

The SSE is minimised, and this gives the estimated β. Beta is estimated as follows:

$$\beta_{j,t} = \frac{cov(r_{j,t}, r_{M,t})}{var(r_{M,t})} \tag{9}$$

Where $cov(r_{j,t}, r_{M,t})$ is the covariance between the excess return of asset j at time t and the excess market return at time t. The variation in this method is estimated as the average squared deviation around the mean. For the DCC GARCH we will see a different method for calculating variances. OLS regression is performed using the statistical software package STATA.

4.2.2 Realised beta

The Realised beta is relatively straightforward. The log returns of the asset in question are divided by the log returns of the market to get the direct relationship. Log returns are used as they are additive and thus over a period of time can be added together. The calculation of the realised beta can be seen in Equation 10.

$$\beta^R_{j,t} = \frac{\sum_{t=1}^{N} R_{j,t} R_{M,t}}{\sum_{t=1}^{N} R^2_{M,t}} \tag{10}$$

The variables $r_{j,t}$ and $r_{M,t}$ refer to the excess return of the asset j and the market M at time t respectively. N stands for the number of observations used for the estimation period. A numeric example would be that if the average returns of asset i are 2 over the estimation period. And the average returns of the market would be 1, that would result in a beta of 2. So, no variances or covariances are used. The realised beta uses the excess returns and predicts that the beta at time t is the same as the average over the estimation period. Realised beta beta relies heavily on the following assumption:

$$E(r_{i,t}) = \beta_i[E(r_{m,t})] \tag{11}$$

Showing the relationship between the beta, expected market return and the expected return of asset i.

4.2.3 Dynamic conditional beta

Third, the dynamic conditional beta is a more sophisticated approach for estimating beta. The first step is to estimate the covariance and variance of the asset. After estimating the covariances and variances, the beta can be estimated using the traditional historical beta formula.

To estimate the univariate volatility models, it is assume that the return at time t, denoted as r_t, is the average return denoted as μ, plus an error term ϵ_t that follows a normal distribution with mean 0 and variance σ_t^2, as shown in Equation 12.

$$r_t = \mu + \epsilon_t \tag{12}$$

Next, the univariate volatility model based on the work of Glosten et al. (1993) is estimated. This model assumes that the variance at time t, denoted as h_t^2, depends on the variance at time t-1, denoted as ϵ_{t-1}^2, and the variance predicted at t-1, denoted as h_{t-1}^2. The variance at time t-1 is used to capture the slow development of h_t^2 over time, while ϵ_{t-1}^2 represents the influence of the most recent observation, as shown in Equation 13.

$$h_t^2 = \omega + \alpha(\epsilon_{t-1}^2) + \beta h_{t-1}^2 \tag{13}$$

Here, ω, α, and β are parameters estimated by the model. The term ω is the constant variance component, α is the parameter that captures the impact of the squared lagged error term, and β is the parameter that captures the persistence of the volatility. To estimate the parameters, the maximum likelihood method is used to find the values that maximise the likelihood function. The likelihood function measures the probability of observing the data given the parameters. Finally, the dynamic conditional beta is calculated using the estimated univariate volatility model and the covariance between the asset and the market. This approach is based on the work of Engle (2002). The dynamic conditional beta is given by Equation 14.

$$\beta_t = \frac{\gamma_t}{\sum_{i=1}^{t} \gamma_i} \tag{14}$$

Here, γ_t is the time-varying covariance between the asset and the market at time t. The time-varying covariance is estimated using the DCC-GARCH. Once the univariate volatility model is estimated for each asset, the DCC-GARCH model can be used to estimate the time-varying conditional correlation between the assets. The DCC-GARCH model is a multivariate volatility model that captures the correlation between assets by modelling the conditional covariance matrix. The model assumes that the returns of the assets follow a multivariate normal distribution with a time-varying mean and conditional covariance matrix. The conditional covariance matrix is modeled using a GARCH-type specification as follows:

$$Q_t = D_t R_t D_t, \tag{15}$$

where Q_t is the conditional covariance matrix at time t, D_t is a diagonal matrix with the standard deviations of the returns of each asset at time t on its diagonal and R_t is the conditional correlation matrix at time t.

The parameters of the DCC-GARCH model are estimated using maximum likelihood estimation (MLE). The log-likelihood function is maximized with respect to the parameters α_i, β_j, and ω_{ij} using the Broyden-Fletcher-Goldfarb-Shanno algorithm for numerical optimization (Dennis and Moré, 1977). The method uses iteration which is capped at a maximum of 100 iterations. This is done to optimize the speed of the model. Not all assets reach convergence, meaning that parameters were estimated below an acceptable threshold. Assets that did not reach convergence are excluded as their parameters are not trustworthy.

Once the parameter estimates are obtained, the log-likelihood function can be computed as: where ϵ_t is the standardised error term, defined as $\epsilon_t = r_t - \mu_t/h_t$, and μ_t and h_t are the conditional mean and conditional standard deviation of the asset return at time t, respectively. The log-likelihood function is then optimised using the maximum likelihood estimator (MLE). The resulting estimates of the parameters are used to construct the time-varying conditional beta of the asset, which is given by:

$$\beta_t = \frac{\sigma_{i,t}}{\sigma_{m,t}} \rho_{i,m,t} \tag{16}$$

where $\sigma_{i,t}$ and $\sigma_{m,t}$ are the conditional standard deviations of the asset return and market return, respectively, and $\rho_{i,m,t}$ is the conditional correlation between the two returns.

This dynamic conditional beta model is more sophisticated than the traditional static beta model, as it takes into account the time-varying nature of the conditional covariance between the asset return and market return.

In summary, the DCC GARCH model provides a way to estimate the time-varying conditional covariance and correlation between asset returns and market returns, which can be used to estimate the dynamic conditional beta of the asset. The estimation process involves estimating the univariate GARCH models for asset and market returns, estimating the DCC model for the conditional covariance matrix, and optimising the log-likelihood function to obtain parameter estimates. The resulting estimates are used to construct the time-varying conditional beta, which, in theory, provides a more accurate measure of the asset's systematic risk than the traditional static beta model.

4.2.4 Random Walk beta

Lastly, the Random Walk method. The way to estimate these methods is more straightforward, as can be seen in the work of Hollstein and Prokopczuk (2016). The random walk is estimated as can be seen in Equation 17

$$\beta_{j,t} = \frac{r_{j,t-1}}{r_{M,t-1}} \tag{17}$$

Variable $\beta_{j,t}$ is the beta of the asset j at time t.The explanation of the Random Walk is simple; it assumes that the relationship of the market to the individuals return are the same as yesterdays relation of the market to the assets return.

4.3 Factor models

After observing the impact of different estimations of beta on the idiosyncratic momentum, the significance of selecting appropriate factor models will be explored. As previously demonstrated, the idiosyncratic momentum anomaly was evaluated using a CAPM. Continuing, the results of various other factor models will be compared to CAPM and each other in this study. To estimate the factor models, the methodology employed in Equation 4 and Equation 5 will be replicated. Idiosyncratic returns obtained from Equation 6 will be used in the analysis. First, all factor models will be listed in equation form.

Starting with the standard FamaFrench 3-factor model (FF3):

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_{mkt,i} \times (r_{mkt,t} - r_{f,t}) + \beta_{hml,i} \times r_{hml,t} + \beta_{smb,i} \times r_{smb,t} + \epsilon_{i,t} \tag{18}$$

Here r_{HML} is the return of the High Minus Low (HML) factor. r_{SMB} is the return of the Small Minus Big (SMB) factor. Lastly the betas are all corresponding to their factor.

Second, the FamaFrench 5-factor model (FF5):

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_{mkt,i} \times (r_{mkt,t} - r_{f,t}) + \beta_{hml,i} \times r_{hml,t} + \beta_{smb,i} \times r_{smb,t} + \beta_{RMW,i} \times r_{RMW,t} + \beta_{CMA,i} \times r_{CMA,t} + \epsilon_{i,t} \tag{19}$$

This factor expands the FF3 model by adding two additional factors; Robust Minus Weak (RMW) and Conservative Minus Aggressive (CMA). r_{RMW} stands for the return on the RMW factor and r_{CMA} represents the return on the CMA factor.

Third the Q-factor model (Q):

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_{mkt,i} \times (r_{mkt,t} - r_{f,t}) + \beta_{ME,i} \times r_{ME,t} + \beta_{IA,i} \times r_{IA,t} + \beta_{ROE,i} \times r_{ROE,t} + \epsilon_{i,t} \tag{20}$$

Their model consists, in addition to the CAPM, of a size (ME) factor, an investment (IA) factor, and a Return on Equity (ROE) factor. r stands for the corresponding return to that factor.

Fourth is the Q5-factor model (Q5):

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_{mkt,i} \times (r_{mkt,t} - r_{f,t}) + \beta_{ME,i} \times r_{ME,t} + \beta_{IA,i} \times r_{IA,t} + \beta_{ROE,i} \times r_{ROE,t} + \beta_{EG,i} \times r_{EG,t} + \epsilon_{i,t} \tag{21}$$

This model expands the Q model by adding an expected growth factor (EG),

Fifth, the Stambaugh-Yuan (SY) model:

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_{mkt,i} \times (r_{mkt,t} - r_{f,t}) + \beta_{smb,i} \times r_{smb,t} + \beta_{MGMT,i} \times r_{MGMT,t} + \beta_{PERF,i} \times r_{PERF,t} \epsilon_{i,t} \tag{22}$$

This model consists of a market factor, a size factor, and two mispricing factors. The first is constructed out of factors influenced by management, thus denoted as the management (MGMT) factor. The second is more related to performance and, hence, is denoted as the performance factor (PERF).

Lastly, Daniel et al. model (D) is considered:

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_{mkt,i} \times (r_{mkt,t} - r_{f,t}) + \beta_{PEAD,i} \times r_{PEAD,t} + \beta FIN, i \times r_{FIN,t} + \epsilon_{i,t} \tag{23}$$

Their model expands the CAPM with two factors, the financing (FIN) factor and the performance (PERF) factor.

Exactly like the CAPM, after the parameters are estimated, and used to forecast returns as done with the CAPM in equation 4 but adjusted for their factors. Then the forecated returns are subtracted from the actual returns to get the idiosyncratic returns.

5 Results

In this section, the findings of this study will be examined. First, the summary statistics of the relative momentum anomaly and the traditional idiosyncratic momentum anomaly will be discussed. This will be followed up with empirical results that contain the GRS test, Fama-Macbeth regressions, and the spanning test. After the base case has been established, the results will be compared with the summary statistics of the different beta estimation methods. Lastly, the factor models used in the previous literature are compared with newer factor models to test for improvement.

5.1 Descriptive statistics

Table 1: Summary statistics relative momentum

This table shows the characteristics of the relative momentum portfolios. Data ranges from January 1925 to December 2021. Due to 1 year formation period of portfolios and one month needed to calculate returns from price, returns are from February 1926 to December 2021. Mean is the excess returns defined as return on the asset minus the risk free rate. Returns are in percentages. sd stands for standard deviation. min stands for minimum excess returns generated during a 1 month period. max stands for maximum excess returns generated during a 1 month period. N is the amount of observations in the sample.

	mean	sd	Sharpe	min	max	skewness	kurtosis
P1	0.031	10.265	0.003	-40.487	98.161	1.820	17.706
P2	0.506	8.336	0.061	-37.596	79.788	1.551	18.018
P3	0.649	7.441	0.087	-33.081	61.176	1.323	16.424
P4	0.717	6.887	0.104	-32.601	61.269	1.019	15.516
P5	0.803	6.503	0.123	-30.301	62.070	1.018	16.956
P6	0.919	6.327	0.145	-31.261	56.077	0.658	14.087
P7	0.959	6.039	0.159	-29.165	46.462	0.259	10.650
P8	1.109	6.221	0.178	-30.197	53.102	0.312	11.177
P9	1.272	6.462	0.197	-32.140	38.986	-0.219	6.812
P10	1.492	7.609	0.196	-34.354	48.465	-0.092	6.871
WML	1.460	7.007	0.208	-66.782	32.185	-2.663	25.340
N	1141						

Table 1 shows the summary statistics of the relative momentum. For the relative momentum anomaly, there is, in line with previous literature, such as Jegadeesh and Titman (1993), a monotonically increasing pattern from losers (P1) to winners (P10) for the mean. The volatility of the lowest relative momentum portfolio is highest, with a standard deviation of 10.27%. Other portfolios are close in volatility to each other with a standard deviation of around 7%. This leads to the Sharpe ratio also monotonically increasing from losers to winners. For the skewness and kurtosis we see characteristics that are typical to stock returns. A positive skewness is expected as there is a downside limit to returns (-1) whereas the upside potential is infinite. This leads to stronger positive outliers and a positive skewness. A high kurtosis indicates that the observations are centred around the mean with strong outliers, which is common with stock returns. When looking at the WML portfolio, it can be seen that it has an average monthly return of 1.46% which is higher than previous literature such as Blitz et al. (2020) and Gutierrez Jr and Prinsky (2007). For the WML portfolio, the main drawback of the relative momentum anomaly can also be seen, which is the crash risk. The lowest return on the WML portfolio is -66.8%. Using an average of 1.40% monthly returns, it would take over six years to recover from this crash[15]. Thus, a strategy less susceptible to this risk could drastically smoothen the returns. When looking at the skewness and kurtosis, a negative skewness and high kurtosis is observed. The negative skewness follows because P10 has low skewness compared to P1, therefore, when subtracting P1 from P10, the negative skewness follows because the outliers that were first positive on average are now negative. The kurtosis is high, which can be explained by the fact that some outliers cancel each other out, as outliers across portfolios tend to happen at the same time. concluding, skewness and kurtosis are as expected. Crash risk is visible and the WML generates a positive return on average in line with previous literature.

Table 2 shows the summary statistics of the idiosyncratic momentum portfolios. There is a steady increase in returns as the idiosyncratic momentum increases. This is in line with previous literature. The volatility of the portfolios is close to equal. The most notable is the volatility of the WML portfolio, whose volatility is significantly lower. More importantly, the volatility of the idiosyncratic momentum WML portfolio is also much lower than that of the relative momentum WML portfolio. The average returns of the WML portfolio for the idiosyncratic momentum strategy are 1.03%, which is lower than the average return of the WML portfolio of the relative momentum strategy. However, due to the volatility being much lower, it has a higher Sharpe ratio, namely 0.226. The skewness and kurtosis show the same characteristics for the idiosyncratic momentum portfolios as the relative momentum portfolios. Thus, the same conclusions are drawn. When looking at the minimal and maximal returns, the minimal return of the idiosyncratic momentum WML portfolio is closer to zero than the relative momentum WML portfolio. The maximum of the two WML portfolios is close to equal. Based on these summary statistics, I conclude that, on a stand-alone basis, the idiosyncratic momentum anomaly is a strategy that outperforms the relative momentum anomaly on both risk and Sharpe ratio. One of the critiques of the idiosyncratic momentum anomaly is that the outperformance based on the Sharpe ratio comes from the way the idiosyncratic momentum anomaly is constructed. Namely standardizing the factor. However, in the Appendix, from Tables 17 and 18 we can see that this is not the case. The relative momentum

[15] This scenario is purely hypothetical, it could be faster or slower

Table 2: Summary statistics idiosyncratic momentum

This table shows the characteristics of the idiosyncratic momentum portfolios. Data ranges from January 1925 to December 2021. Due to 3 year estimation period of parameters and 1 year formation period of portfolios returns, results range from February 1928 to December 2021. Mean is the excess return defined as return on the asset minus the risk free rate. sd stands for standard deviation. min stands for minimum excess returns generated during a 1 month period. max stands for maximum excess returns generated during a 1 month period. N is the number of observations in the sample.

	mean	sd	Sharpe ratio	min	max	skewness	kurtosis
P1	0.396	7.899	0.050	-29.956	62.811	1.268	13.809
P2	0.552	7.230	0.076	-29.353	60.397	0.941	12.675
P3	0.688	7.136	0.096	-30.895	56.064	1.095	14.384
P4	0.779	6.888	0.113	-33.591	64.688	0.994	15.741
P5	0.897	6.940	0.129	-31.450	59.738	1.200	15.730
P6	0.954	7.033	0.136	-33.797	65.618	1.338	18.971
P7	1.038	6.875	0.151	-31.137	60.172	0.824	14.034
P8	1.163	6.977	0.167	-30.317	57.326	0.912	14.105
P9	1.237	7.121	0.174	-32.855	66.673	1.131	17.171
P10	1.429	6.719	0.213	-32.214	48.792	0.109	9.118
WML	1.033	4.555	0.227	-41.331	29.916	-1.260	17.224
N	1105						

strategy performs worse when standardised, and the idiosyncratic momentum anomaly performs slightly better when not standardised. Making not standardizing the relative momentum anomaly optimal while standardizing the idiosyncratic momentum anomaly suboptimal. For this research, to make comparisons to previous literature, the standardised version of the idiosyncratic momentum anomaly will be used.

When looking at figure 1, it can be seen, that over time the investor is compensated with higher returns for the relative momentum anomaly. What also is pictured clearly is that in times of a style reversal the relative momentum suffers more than the idiosyncratic momentum anomaly. For example looking at the great depression period around 1930 and the financial crisis around 2008, we see that the relative momentum suffers from much higher losses than the idiosyncratic momentum anomaly. Thus, previous literature is confirmed in both cases.

In this section, the summary statistics of both the relative momentum and idiosyncratic momentum portfolios has been presented. The results showed a monotonically increasing pattern of mean returns from losers to winners for both strategies, as well as expected characteristics such as positive skewness and high kurtosis. However, the idiosyncratic momentum strategy outperformed the relative momentum strategy on both risk and Sharpe ratio. Additionally, it was found that standardizing the idiosyncratic momentum anomaly was suboptimal, while not standardizing the relative momentum anomaly was optimal. These findings contribute to the literature on momentum trading strategies and suggest that the idiosyncratic momentum strategy may be a better option for investors seeking higher returns with lower risk.

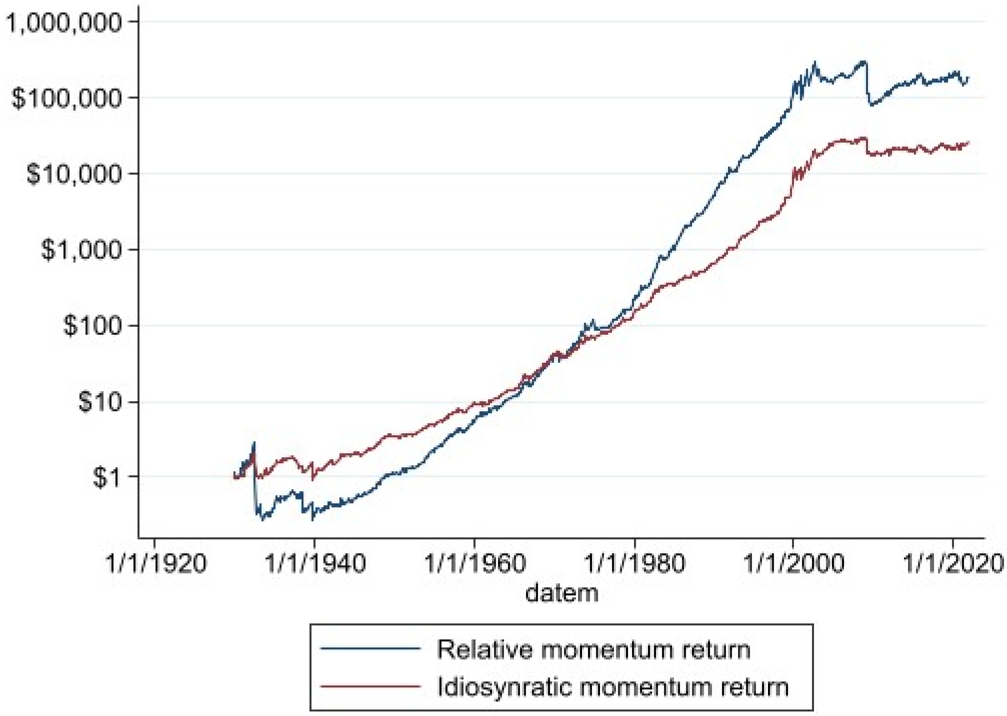

Figure 1: Cumulative return for the relative and idiosyncratic WML portfolios

5.2 Empirical results

To test whether the WML portfolios identified show a unique type of risk, regressions on other factors including the CAPM, Fama-French 3-factor model (FF3), and Fama-French 5-factor model (FF5) are run. When alpha remains significant, it can confidently be said that the other factors do not fully explain the outperformance found in the market. This will be done by a CAPM regression, a regression on the Fama-French 3-factor model (FF3) and the Fama-French 5-factor model (FF5). These regressions will look as follows, respectively:

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_{mkt,i} \times (r_{mkt,t} - r_{f,t}) + \epsilon_{i,t} \tag{24}$$

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_{mkt,i} \times (r_{mkt,t} - r_{f,t}) + \beta_{hml,i} \times r_{hml,t} + \beta_{smb,i} \times r_{smb,t} + \epsilon_{i,t} \tag{25}$$

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_{mkt,i} \times (r_{mkt,t} - r_{f,t}) + \beta_{hml,i} \times r_{hml,t} + \beta_{smb,i} \times r_{smb,t} + \beta_{rmw,i} \times r_{rmw,t} + \beta_{cma,i} \times r_{cma,t} + \epsilon_{i,t} \tag{26}$$

From these equations, alphas and their corresponding t statistics are obtained to test whether they are statistically significant.

The results of found alphas for the relative momentum strategy can be seen in Table 3. For the relative momentum strategy, it can be seen that alphas are mostly statistically significant. All alphas of P1, P10 and WML are statistically significant at the 1% confidence levels. This means that the risk factors applied to the found returns cannot fully explain the outperformance of the market. Noticeable is that the alpha is not decreasing with a more sophisticated model. This can indicate that the outperformance

Table 3: Alpha regressions relative momentum

This table shows factor regressions on the formed residual momentum portfolios. Alpha is the constant in the regression that cannot be explained by factors in the model. Data ranges from January 1925 to December 2021. Due to 1 year formation period of portfolios returns are from January 1926 to December 2021. Alpha returns are in whole numbers.t statistics in parentheses, *. $p < .10$, ** $p < 0.05$, *** $p < 0.01$

	CAPM alpha	t-stat	FF3 alpha	t-stat	FF5 alpha	t-stat
P1	-1.06***	(-6.40)	-1.35***	(-7.48)	-1.04***	(-5.84)
P2	-0.43***	(-3.71)	-0.57***	(-5.19)	-0.42***	(-3.82)
P3	-0.21**	(-2.25)	-0.29***	(-3.53)	-0.25***	(-2.88)
P4	-0.09	(-1.19)	-0.17***	(-2.90)	-0.18***	(-3.04)
P5	0.03	(0.48)	-0.03	(-0.57)	-0.08	(-1.57)
P6	0.17**	(2.52)	0.07*	(1.78)	0.02	(0.49)
P7	0.24***	(3.79)	0.20***	(4.66)	0.14***	(3.25)
P8	0.38***	(5.27)	0.34***	(6.59)	0.30***	(5.70)
P9	0.55***	(6.10)	0.51***	(6.93)	0.52***	(6.99)
P10	0.69***	(5.45)	0.64***	(5.77)	0.75***	(6.80)
WML	1.75***	(8.89)	1.99***	(8.20)	1.80***	(7.22)
GRS	pval: 0.000	(7.939)	pval: 0.000	(8.723)	pval: 0.000	(7.473)

Table 4: Alpha regressions idiosyncratic momentum

This table shows factor regressions on the formed residual momentum portfolios. Alpha is the constant in the regression that cannot be explained by factors in the model. Data ranges from January 1925 to December 2021. Due to 3 year formation period of portfolios and one month needed to generate returns from prices, results range from February 1928 to December 2021. Alpha returns are in whole numbers.t statistics in parentheses, *. $p < .10$, ** $p < 0.05$, *** $p < 0.01$

	CAPM alpha	t-stat	FF3 alpha	t-stat	FF5 alpha	t-stat
P1	-0.49***	(-4.61)	-0.61***	(-5.39)	-0.52***	(-4.52)
P2	-0.27***	(-3.11)	-0.42***	(-5.01)	-0.42***	(-4.87)
P3	-0.13	(-1.63)	-0.30***	(-4.47)	-0.30***	(-4.41)
P4	-0.01	(-0.17)	-0.15***	(-2.61)	-0.18***	(-3.04)
P5	0.10	(1.23)	-0.00	(-0.03)	-0.04	(-0.70)
P6	0.14*	(1.80)	0.06	(1.24)	0.03	(0.58)
P7	0.25***	(3.05)	0.13***	(2.86)	0.11**	(2.38)
P8	0.37***	(4.33)	0.28***	(5.54)	0.27***	(5.33)
P9	0.43***	(4.84)	0.46***	(7.29)	0.48***	(7.54)
P10	0.68***	(7.46)	0.71***	(8.48)	0.68***	(8.13)
WML	1.17***	(8.72)	1.32***	(7.67)	1.21***	(6.87)
GRS	pval: 0.000	(85.864)	pval: 0.000	(61.004)	pval: 0.000	(47.726)

is not related to the factors added in the model.

Looking at Table 4, similar results are observed for the idiosyncratic momentum anomaly as for the relative momentum anomaly. P1 and P10 and the WML portfolio have alphas significant at the 1% level. The outperformance of the WML portfolio for the idiosyncratic momentum anomaly is lower than that of the WML portfolio for the relative momentum anomaly. This can be attributed to portfolio 1 having a higher performance in the idiosyncratic momentum anomaly. Nevertheless, all are alphas significant and are an outperformance of approximatley 1.2% monthly on average high. Overall, the added factors of the FF3 and FF5 models cannot fully explain the outperformance of the market, supporting the conclusion that the WML portfolios exhibit a different kind of risk.

5.2.1 GRS test

In addition to the inclusion of factors in the regression analysis, a Gibbons, Ross, and Shanken (GRS) test was conducted to examine the average joint pricing errors for the momentum anomaly portfolios Gibbons et al. (1989). The rejection of the null hypothesis in the GRS test suggests that the models used in this study require further improvement. The GRS test results provide a stronger indication of the model's explanatory power than individual tests as it considers the correlation between portfolios. The asymptotic GRS test was utilized as the sample size was large, and it does not require the assumption of normality in the errors but instead relies on the convergence of alpha and beta to a normal distribution.

The results of the relative and idiosyncratic momentum anomaly portfolios are presented in Tables 3 and 4, respectively. For the relative momentum anomaly, the null hypothesis was rejected in all three models. These results indicate that the models fail to explain the outperformance of alpha, suggesting room for improvement. Interestingly, the test statistic did not decrease across models, contrary to what might be expected. Although the FF5 model offered the best explanatory power for the relative momentum anomaly, the overall performance was still inadequate.

For the idiosyncratic momentum anomaly, the pattern of decreasing test statistics across models was observed. However, compared to the relative momentum anomaly, the test statistics were considerably larger, indicating the robustness of the alpha found in this anomaly. It is noteworthy that the p-value for both anomalies was far below 0.000, indicating their robustness and consistency with previous literature findings Blitz et al. (2020). Therefore, it is concluded that the GRS test does not aid in explaining the outperformance of either anomaly.

5.2.2 Fama-Macbeth regressions

Another popular test to test the persistence of anomalies is the Fama and MacBeth (1973) cross-section test. Stock returns are regressed each month on various elements to gather a time-series of coefficients. Afterwards, averages and corresponding t-statistics of the result are calculated. The cross-section test, tests whether there is a premium in return associated with increasing a unit exposure to a factor. The null hypothesis is that, under the CAPM, only beta should explain returns and all other factors influencing the return should be zero. Therefore one would expect the other factors to be insignificant under the null-hypothesis. The persistence of both the relative momentum strategy and the idiosyncratic momentum anomaly is tested. As can be seen in Table 5, starting with the CAPM. All standard errors are adjusted Newey and West (1987) standard errors with a maximum lag of 3 months. Individually, both the relative momentum and the idiosyncratic momentum remain statistically significant at the 1% level. However, when both are added to the model, only the idiosyncratic momentum anomaly remains statistically significant, signalling that the idiosyncratic momentum anomaly is more robust. The model is expanded

Table 5: Fama-Macbeth Regressions

This table shows the results from the Fama-Macbeth regressions on the returns. All results are on individual stocks. Results are between January 1963 and December 2021. Beta is estimated using a univariate regression over a 36 month time period on a CAPM. size is the natural logarithm of a firm's market capitalization. bm is the natural logarithm of the book to market ratio defined as a firms book equity for the fiscal year ending in t-1 and the market cap at the end of the previous month. Inv is the percentage of a firms growth in total assets at the end of of the fiscal year ending t-1. Prof is the firms profitability defined as the ratio of operating profits and book equity at the end of the previous fiscal year. Regressions one to nine are all returns regressed on the given factors. All standard errors are Newey and West (1987) calculated standard errors with a maximum of three lags. t statistics are in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

	1	2	3	4	5	6	7	8	9
cons	0.81***	0.93***	0.83***	5.52***	4.75***	5.27***	5.53***	4.81***	5.31***
	(5.63)	(6.76)	(5.92)	(9.18)	(7.20)	(9.24)	(9.23)	(7.29)	(9.28)
beta	0.04	0.14	0.04	0.01	0.06	0.03	-0.09	-0.03	-0.05
	(0.42)	(1.13)	(0.40)	(0.11)	(0.60)	(0.38)	(-0.95)	(-0.31)	(-0.58)
size				-0.12***	-0.10**	-0.11***	-0.12***	-0.10**	-0.11***
				(-3.04)	(-2.48)	(-2.84)	(-3.08)	(-2.48)	(-2.83)
bm				0.45***	0.36***	0.44***	0.44***	0.37***	0.44***
				(7.24)	(5.90)	(7.26)	(7.35)	(5.92)	(7.21)
inv							-0.03	0.00	-0.01
							(-0.60)	(0.06)	(-0.26)
prof							0.26***	0.27***	0.23***
							(4.12)	(3.65)	(3.25)
rMOM	0.74***		0.26	0.88***		0.77***	0.95***		0.77***
	(4.95)		(1.22)	(6.48)		(4.17)	(7.17)		(4.15)
iMOM		0.24***	0.24***		0.23***	0.10**		0.25***	0.12**
		(7.74)	(5.04)		(6.52)	(2.09)		(6.78)	(2.48)
r2	.03833	.0314	.04452	.06091	.05913	.06595	.07618	.07465	.0809
r2_a	.0369	.02989	.04228	.05823	.05627	.06241	.07261	.07076	.0764

with the logarithmic size and logarithmic book to market characteristics of the firm. Size takes on a negative sign, as expected, due to the size effect. Both the relative and idiosyncratic momentum remain statistically significant at 1% and when combined at 5%. Lastly, whether the relative and idiosyncratic momentum anomalies remain significant in the FF5 model is examined. This is done by adding the investment and profitability factors to the model. Again, it can be seen that both remain statistically significant at 5%. With these results, I conclude that even in a FF5 model, there is a premium in returns associated with taking exposure to these anomalies, making them robust.

Table 6: Spanning tests Relative momentum

This table shows spanning tests performed on the WML portfolio of the idiosyncratic momentum anomaly. Data ranges from January 1963 until December 2021. Returns are in percentages. t statistics in parentheses. * p < .10, ** p < 0.05, *** p < 0.01.

	(1) rMOM	(2) rMOM	(3) rMOM	(4) rMOM	(5) rMOM
MktRF		-0.234**	-0.293***	-0.225**	-0.061
		(-2.32)	(-3.46)	(-2.53)	(-1.18)
SMB			-0.065	-0.004	-0.214***
			(-0.35)	(-0.03)	(-3.48)
HML			-0.521**	-0.756***	-0.309***
			(-2.34)	(-3.60)	(-3.30)
CMA				0.531	-0.057
				(1.50)	(-0.40)
RMW				0.242	0.207**
				(0.67)	(2.00)
iMOM					1.212***
					(18.84)
_cons	1.455***	1.802***	1.995***	1.796***	0.334*
	(7.03)	(7.78)	(8.68)	(5.85)	(1.79)
N	702	702	702	702	702

5.2.3 Spanning Tests

In this section, the results of the spanning tests performed on the relative and idiosyncratic momentum WML portfolios will be studied in order to assess their robustness. Spanning tests are a statistical technique used to evaluate the relationship between two or more variables in a financial model. The purpose is to determine whether the inclusion of additional variables can account for the alpha of the dependent variable. In this study, the dependent variables are the relative and idiosyncratic momentum WML portfolios, as presented in Table 6 and Table 7, respectively. The null hypothesis is that the other factors can explain the alpha of the dependent variables, which is rejected if the alpha remains statistically significant even after the addition of other factors.

Table 6 presents the results of the spanning tests on the relative momentum WML portfolio. The table shows that the outperformance of this portfolio is approximately 1.46%. Interestingly, the addition of other factors does not seem to explain the outperformance as expected. Further analysis reveals that the market factor and the HML factor are statistically significant at 5% across regressions 2, 3, and 4, but the market factor is no longer significant in Model 5. This may be due to the strong relationship between the relative and idiosyncratic momentum factors, as evidenced by a t-statistic of over 18 in Model 5. This finding is consistent with previous research indicating that the relative momentum factor is not robust when the idiosyncratic momentum factor is included, as the two factors are highly correlated due to significant overlap between their high portfolio assets.

Table 7: Spanning tests Idiosyncratic momentum

This table shows spanning tests performed on the WML portfolio of the idiosyncratic momentum anomaly. Data ranges from January 1963 until December 2021. Returns are in percentages. t statistics in parentheses. * $p < .10$, ** $p < 0.05$, *** $p < 0.01$.

	(1) iMOM	(2) iMOM	(3) iMOM	(4) iMOM	(5) iMOM
MktRF		-0.135**	-0.189***	-0.135**	0.000
		(-2.00)	(-3.17)	(-2.22)	(0.00)
SMB			0.163	0.173	0.175***
			(1.05)	(1.46)	(3.89)
HML			-0.163	-0.369**	0.087
			(-1.00)	(-2.31)	(1.39)
CMA				0.485**	0.165*
				(2.08)	(1.75)
RMW				0.029	-0.117
				(0.10)	(-1.23)
rMOM					0.603***
					(24.50)
_cons	1.037***	1.283***	1.322***	1.207***	0.124
	(7.52)	(7.60)	(7.58)	(5.17)	(1.03)
N	702	702	702	702	702

Table 7 presents the results of the spanning tests on the idiosyncratic momentum WML portfolio. The table shows that the outperformance of this portfolio is approximately 1.03%. The addition of the CAPM, FF3, and FF5-factors does not explain the outperformance, and the alpha only increases. However, the addition of the relative momentum factor makes the alpha statistically insignificant. This contrasts with the findings of Blitz et al. (2020), where the idiosyncratic momentum factor remains statistically significant after the inclusion of the relative momentum factor. This discrepancy may be due to differences in the construction of the idiosyncratic momentum factor, as this study uses the CAPM instead of the FF3 model. The high correlation between the relative and idiosyncratic momentum factors suggests that they can explain each other relatively well. Thus, based on the spanning tests, momentum is a phenomenon that cannot be explained by more traditional factors that are widely known in the finance industry.

5.3 Constructing beta

In this section, the different estimation methods' results will be discussed. This section will follow a similar line as the previous section starting with the descriptive statistics and following with the empirical methods. The results will be compared to the traditional method as presented above.

Table 8: Fama Macbeth Regressions

This table shows the Fama Macbeth regressions on the returns. rBeta is the realised beta. rwBeta is the Random Walk beta. DCC is the Dynamic Conditional correlation beta. Period is from February 1928 until December 2021. Returns are in percentages. t statistics in parentheses * p < .10, ** p < 0.05, *** p < 0.01

	RET	RET	RET
cons	0.88^{***}	1.00^{***}	0.74^{***}
	(6.08)	(4.94)	(6.10)
rBeta	0.18^{**}		
	(2.18)		
rwBeta		0.06	
		(1.46)	
DCC_beta			0.28^{**}
			(2.40)
r2	.01997	.01761	.03371

5.3.1 Constructing beta descriptive statistics

Starting with assessing the performance of different beta estimation methods in predicting returns. To assess whether taking exposure to beta results in a return premium, a Fama-French two-step regression is utilized. The results of the Fama-Macbeth regressions are summarized in Table 8. Both the realised beta and the DCC beta are statistically significant at the 5% level of significance. In contrast, the random walk beta is not significant, indicating poor predictive power. One possible explanation for this is that the random walk beta can take extreme values when the market return approaches zero, due to the way it is calculated:

$$\beta_{RW} = \lim_{r_m \to \pm 0} \frac{r_i}{r_m} \tag{27}$$

Anomalously high returns on a particular stock, relative to the overall market, can result in a calculated random walk beta that is substantially elevated. For instance, if a stock experiences a gain of 50% in the previous month while the market only yields a return of 0.1%, the resulting beta value would be 500, an extreme and atypical outcome. Such extreme values are improbable to be repeated, and thus predicted returns based on this random walk beta may be subject to significant mispricing.

Now, the idiosyncratic momentum factor per asset is constructed using a CAPM model and estimated betas. This allows assets to be divided into portfolios as previously done with the univariate regressions in Section 5.1. Table 9 shows the results of the portfolio classification. Each beta estimation method will be discussed and their corresponding result on the idiosyncratic momentum anomaly portfolios individually. The focus will be on the WML portfolio. Comparisons will be made with Table 2.

First the realised beta. With the WML portfolio having a return of 1.20% monthly, it is higher than that of the idiosyncratic momentum anomaly of 1.03%. This difference can be attributed to the fact that P1 of the realised beta estimation portfolio has a lower return, making a short position in this portfolio

Table 9: Portfolio returns of different beta estimations

This table shows the descriptive statistics of the different methods for estimating beta. Period is from January 1925 until December 2021. Due to 3 years estimation period and one month price to return conversion, results start at February 1928. Returns are in percentages.

	Realised Beta Idiosyncratic Momentum Portfolios					
	mean	sd	min	max	skewness	kurtosis
P1	0.256	8.026	-31.374	65.037	1.294	12.826
P2	0.466	7.770	-32.853	77.167	1.383	17.487
P3	0.634	7.466	-32.200	61.601	1.191	14.759
P4	0.697	7.273	-31.540	55.141	0.892	13.120
P5	0.812	7.171	-33.002	54.894	0.973	14.265
P6	0.917	7.061	-32.523	57.286	0.806	13.574
P7	1.035	6.986	-31.690	60.672	0.985	15.490
P8	1.162	6.876	-32.772	56.246	0.494	11.801
P9	1.242	6.619	-30.884	48.725	0.119	9.229
P10	1.457	6.406	-30.941	41.207	0.024	8.546
WML	1.201	5.222	-39.076	28.630	-1.422	13.440
N	1105					

	Random Walk beta Beta Idiosyncratic Momentum Portfolios					
	mean	sd	min	max	skewness	kurtosis
P1	0.645	7.276	-32.187	52.844	0.744	11.120
P2	0.737	7.211	-33.125	59.486	0.777	13.186
P3	0.820	7.272	-33.387	66.710	0.903	14.527
P4	0.857	7.231	-31.467	62.109	0.998	14.688
P5	0.889	7.085	-34.466	58.230	0.793	13.274
P6	0.850	6.962	-29.546	54.415	0.689	12.006
P7	0.878	7.142	-30.130	53.641	0.971	13.584
P8	0.921	6.808	-30.087	49.937	0.495	9.919
P9	1.000	6.840	-29.680	56.170	0.808	12.706
P10	1.082	6.794	-30.457	55.643	0.786	14.154
WML	0.436	3.062	-25.727	18.592	-0.401	12.196
N	1105					

	DCC Beta Beta Idiosyncratic Momentum Portfolios					
	mean	sd	min	max	skewness	kurtosis
P1	0.513	7.734	-30.303	47.335	0.885	9.748
P2	0.644	7.348	-33.027	54.050	0.801	11.820
P3	0.717	7.057	-29.902	58.183	1.105	13.919
P4	0.832	7.097	-34.413	66.426	1.340	18.093
P5	0.929	6.687	-30.828	54.718	0.653	12.379
P6	0.952	6.839	-29.458	62.842	1.125	16.447
P7	1.018	6.688	-33.050	55.631	0.840	15.056
P8	1.187	6.611	-31.991	54.291	0.616	12.819
P9	1.263	6.719	-32.040	56.960	0.714	13.929
P10	1.400	6.627	-30.030	55.168	0.289	11.332
WML	0.886	5.090	-41.294	31.916	-0.763	11.421
N	1105					

generate higher returns. The standard deviation is higher as well at 5.22%, meaning its risk is higher. Combine these two and you get a Sharpe ratio of 0.230, which is almost equal to the Sharpe ratio of

0.227 of the univariate beta idiosyncratic momentum WML portfolio.

Second, there is the Random Walk beta. Where the results of the realised beta portfolios were relatively close to the univariate beta portfolios, the random walk beta is much more different. Most importantly is that the average monthly return of only 0.44% is not even half that of the univariate beta WML portfolio. It should be noted that the standard deviation is also lower, with only 3.06%. What stands out is that the minimum and maximum returns are also closer to zero. Therefore concluding that the portfolio sorts based on the Random Walk beta are less prone to crash risk. lastly the Sharpe ratio is also a lot lower with only 0.142, meaning that the reduced risk is not enough to compensate for the lower returns. The general conclusion I take off these results is that the Random Walk beta is less accurate in predicting returns[16], and therefore does a worse job in exploiting the idiosyncratic momentum anomaly. This is invigorated by the fact that portfolios P3 to P8 only differ slightly and all portfolios are much closer than the portfolios that followed out of the other estimation techniques. Therefore, the conclusion is that the Random Walk beta is not suited to form the idiosyncratic momentum anomaly. There is not enough reduced risk to compensate for the lower returns based on Sharpe ratio.

Third, the portfolios were formed on the DCC beta. The results are somewhat disappointing. It performs worse than traditional method for estimating the idiosyncratic momentum anomaly. The returns of the WML portfolio are lower, 0.89% compared to 1.03% in the traditional method. The standard deviation is higher at 5.09%. Resulting in a much lower Sharpe ratio of only 0.174. The conclusion is therefore clear: the DCC beta should not be used for estimating the idiosyncratic returns based on these results. However, there are some notes that are important to address and that might make this method more plausible in the future. The first is that to estimate the DCC beta, a lot of data is needed. Even with daily data, it often does not reach convergence. For monthly data, a 60-month window is used with a minimum of 24 observations. Making convergence harder to achieve and reducing the size of the data set. Second is that the amount of Iterations was capped at 100 to the speed of the model[17]. One could increase the amount of iterations to improve the chance of finding convergence. This is possible as in practice a data set of 1925 until the present will be redundant and a researcher can chose a much smaller time window for their research.

In Figure 2 the cumulative returns over the entire studied period are shown for the idiosyncratic momentum anomaly using the different beta estimation techniques. A logarithmic y-axis is used due to the cumulative returns growing exponentially over time, making the different WML portfolios more comparable. This graph is a visual representation of Tables 2 and 9. We see that overall they show similar patterns. When all portfolios crash at the same time and show large gains at approximately the same time. Remarkable is that the DCC beta WML portfolio has a much stronger crash compared to the other methods after the 2008 financial crisis. A possible explanation could be that due to the DCC GARCH method adjusting quicker to style reversals, a quick style reversal happening twice might cause the DCC GARCH WML portfolio to be exposed first in the crash, and then in the reversal from bear to bull market.

[16] As also can be seen in Table 8

[17] The current model took approximately 40 hours to complete, making time optimization necessary.

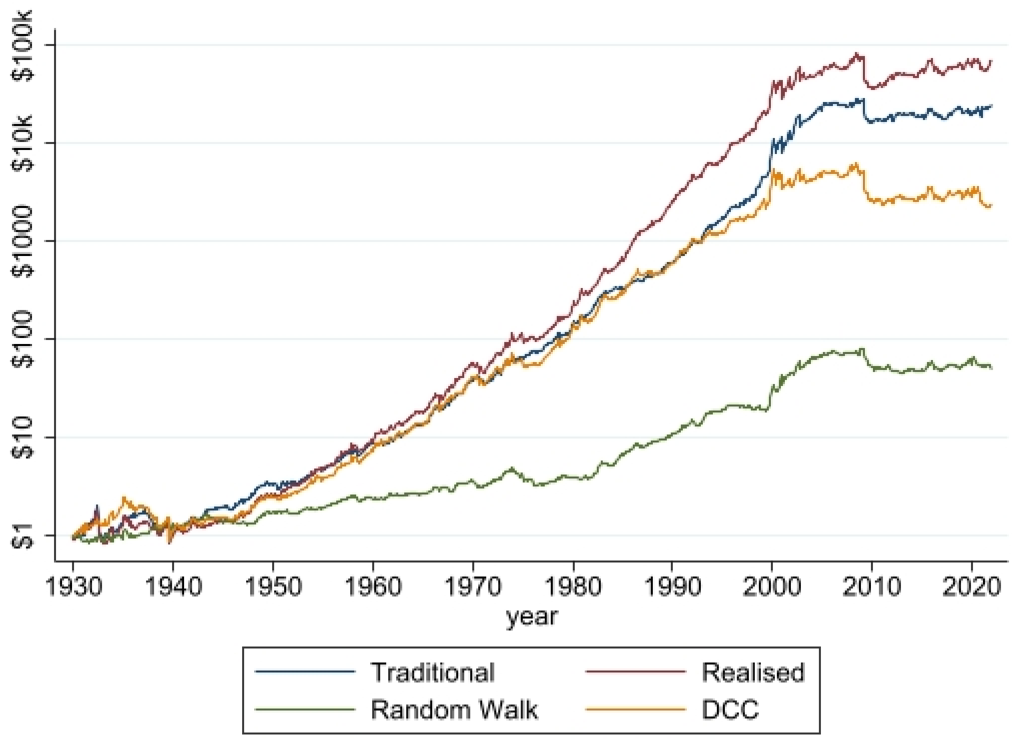

Figure 2: Cumulative returns for the idiosyncratic momentum anomaly using different beta formations

5.3.2 Empirical results betas

To test the robustness of the results for the different beta estimation methods, empirical tests are performed similar to those previously performed for traditional idiosyncratic and relative momentum factors. The alpha regressions, GRS test and spanning tests will be performed. As I am most interested in comparing the different idiosyncratic momentum methods, the results will be compared to Table 4. For the empirical methods, the exact same tests will be performed as performed previously on the traditional relative and idiosyncratic momentum anomalies. In the next section the found differences and similarities will be discussed.

Starting with the idiosyncratic momentum factor created using the realised beta. The results are shown in Table 10. Compared to Table 4, the realised beta has, in all factor models, a larger difference between the winner and loser alphas. This results in greater outperformance in the WML portfolio. The WML alphas are larger. Noteworthy is that the realised WML alphas show a similar pattern to the traditional WML alphas. The CAPM has the lowest alpha, the FF3 model the highest, and, lastly, the FF5 alpha is between these two. Important to note is that all WML alphas, as well as those of P1 and P10 are statistically significant at the 1% confidence level. Based on these notes I conclude that the alphas are more robust than the traditional idiosyncratic momentum factor.

Second, the robustness of the Random Walk beta is studied. The results can be seen in Table 11. The pattern of Table 8 and Table 9 continues. The Random Walk beta performed poorly in these tests and continues to do so in Table 11. The Loser portfolio has higher returns than for the Random

Table 10: realised beta alphas

This table shows factor regressions on the formed idiosyncratic momentum portfolios formed by using the realised beta method. Alpha is the constant in the regression that cannot be explained by factors in the model. Data ranges from January 1963 to December 2021. Alpha returns are in percentages. t statistics in parentheses * $p < .10$, ** $p < 0.05$, *** $p < 0.01$

	CAPM alpha	t-stat	FF3 alpha	t-stat	FF5 alpha	t-stat
P1	-0.63***	(-3.86)	-0.84***	(-6.47)	-0.74***	(-5.57)
P2	-0.37***	(-2.76)	-0.56***	(-5.93)	-0.51***	(-5.29)
P3	-0.20*	(-1.67)	-0.37***	(-4.74)	-0.35***	(-4.33)
P4	-0.09	(-0.86)	-0.26***	(-4.04)	-0.25***	(-3.85)
P5	0.04	(0.39)	-0.10**	(-1.99)	-0.12**	(-2.17)
P6	0.10	(0.98)	-0.03	(-0.75)	-0.02	(-0.52)
P7	0.29***	(2.92)	0.20***	(4.10)	0.22***	(4.38)
P8	0.45***	(4.18)	0.37***	(7.29)	0.39***	(7.64)
P9	0.58***	(5.25)	0.54***	(8.50)	0.55***	(8.57)
P10	0.80***	(6.64)	0.80***	(9.32)	0.79***	(8.93)
WML	1.44***	(7.18)	1.64***	(8.49)	1.53***	(7.69)
GRS	pval: 0.000	(64.12)	pval: 0.000	(95.87)	pval: 0.000	(90.23)

Table 11: Random walk beta alphas

This table shows factor regressions on the formed idiosyncratic momentum portfolios formed by using the random walk beta method. Alpha is the constant in the regression that cannot be explained by factors in the model. Data ranges from January 1963 to December 2021. Alpha returns are in percentages. t statistics in parentheses * $p < .10$, ** $p < 0.05$, *** $p < 0.01$

	CAPM alpha	t-stat	FF3 alpha	t-stat	FF5 alpha	t-stat
P1	-0.16	(-1.34)	-0.31***	(-3.68)	-0.29***	(-3.40)
P2	-0.06	(-0.57)	-0.19***	(-2.86)	-0.17**	(-2.48)
P3	0.03	(0.31)	-0.08	(-1.28)	-0.05	(-0.90)
P4	0.06	(0.62)	-0.04	(-0.76)	-0.02	(-0.29)
P5	0.13	(1.24)	0.01	(0.11)	0.04	(0.86)
P6	0.09	(0.82)	-0.05	(-0.95)	-0.03	(-0.53)
P7	0.09	(0.79)	-0.05	(-0.94)	-0.02	(-0.41)
P8	0.20*	(1.83)	0.06	(1.23)	0.09	(1.62)
P9	0.22**	(2.07)	0.11*	(1.95)	0.13**	(2.29)
P10	0.37***	(3.49)	0.28***	(4.88)	0.28***	(4.65)
WML	0.52***	(4.54)	0.59***	(5.17)	0.57***	(4.83)
GRS	pval: 0.002	(28.57)	pval: 0.000	(35.31)	pval: 0.000	(33.53)

Walk beta idiosyncratic momentum anomaly than the anomaly has formed on the traditional method or the realised beta. Vice versa is true for the winner portfolio. This results in a very poor result. The combination of these factors results in a WML portfolio that performs significantly worse. It should be noted that, despite its relatively poor performance, it is still possible to generate significant returns even at the 1% confidence level. This signals that despite the performance dropping as beta is estimated using the Random Walk method, there is still an underlying foundation of the idiosyncratic momentum anomaly that can generate returns, as this test proves that the outperformance is not driven by any of the Fama French factors[18]. Nevertheless, despite the outperformance being statistically significant, there are better ways for forming the idiosyncratic momentum factor than using the Random Walk method based on these results.

Table 12: DCC GARCH beta alphas

This table shows factor regressions on the formed idiosyncratic momentum portfolios formed by using the DCC GARCH beta method. Alpha is the constant in the regression that cannot be explained by factors in the model. Data ranges from January 1963 to December 2021. Alpha returns are in percentages.

	CAPM alpha	t-stat	FF3 alpha	t-stat	FF5 alpha	t-stat
P1	-0.22	(-1.53)	-0.45***	(-3.78)	-0.47***	(-3.85)
P2	-0.13	(-1.09)	-0.33***	(-3.70)	-0.41***	(-4.45)
P3	-0.03	(-0.28)	-0.21***	(-2.82)	-0.29***	(-3.83)
P4	0.10	(1.00)	-0.08	(-1.30)	-0.18***	(-2.93)
P5	0.20**	(2.26)	0.05	(0.77)	-0.07	(-1.26)
P6	0.22**	(2.57)	0.07	(1.39)	-0.03	(-0.52)
P7	0.29***	(3.58)	0.17***	(3.32)	0.09*	(1.85)
P8	0.53***	(6.32)	0.45***	(8.17)	0.39***	(6.99)
P9	0.54***	(5.44)	0.50***	(7.08)	0.44***	(6.21)
P10	0.75***	(6.43)	0.75***	(8.09)	0.74***	(7.73)
WML	0.97***	(4.86)	1.20***	(6.36)	1.22***	(6.23)
GRS	pval: 0.000	(54.73)	pval: 0.000	(83.33)	pval: 0.000	(71.57)

The last beta method examined is the DCC GARCH method. The results are shown in Table 12. Unlike Table 10, not all the alphas for P1 and P10 are statistically significant at 1%. For the CAPM Alpha of P1, the result is not significantly different from zero. This can be attributed to the fact that the CAPM Alpha of P1 in the DCC GARCH method lies closer to zero, -0.16 versus -0.63. When looking at the WML portfolio, contrary to expectations, the outperformance of alpha increases as the model becomes more sophisticated. This is against expectations as one would expect that adding factors helps explain the outperformance of alpha. When looking at absolute returns of the WML portfolio, the WML portfolios performs significantly better than the idiosyncratic momentum formed using Random Walk beta estimation. However, the factor formed using realised beta outperforms the DCC beta. Compared to the traditional method, the results are very close. Due to the more complex method used to construct the DCC beta, one might think that the more simplistic traditional method might have the advantage. While there are cases where this definitely is true, such as time limited situations, there is not enough evidence to deem the DCC beta unusable. There are situations in which investors can use this method where it may be beneficial. The possibilities lie in shorter investment horizon strategies. This is due to the fact that DCC GARCH requires a high number of observations to be used. When using daily data instead of monthly data, we see that DCC GARCH tends to perform better as seen in

[18] I do however not fully contribute the outperformance to the underlying Idiosyncratic momentum mechanism as there may be uncaptured factors that drive the outperformance

Hollstein and Prokopczuk (2016). So when looking at the DCC beta method and taking into account that the situation is suboptimal, the fact it still generates comparable returns to the traditional method is promising. Therefore I would conclude that this thesis shows that the DCC GARCH method is a method that shows potential in the monthly period and will likely perform even better when using daily returns as strategy due to the nature of the method.

5.3.3 Spanning tests

In this section, the results of spanning tests performed on the different WML portfolios of the various beta construction methods are discussed. The results will be compared with Table 7.

Table 13: Realised beta spanning tests

This table shows spanning tests performed on the WML portfolio of the idiosyncratic momentum anomaly formed on the realised beta. Due to the factors only being available from 1963, data ranges form January 1963 until December 2021. t statistics in parentheses. * $p < .10$, ** $p < 0.05$, *** $p < 0.01$.

	WML	WML	WML	WML	WML
MktRF		-0.138*	-0.187***	-0.155**	0.012
		(-1.80)	(-2.96)	(-2.29)	(0.54)
SMB			-0.126	-0.078	-0.075**
			(-0.81)	(-0.56)	(-2.26)
HML			-0.532***	-0.632***	-0.070
			(-3.24)	(-3.68)	(-1.56)
CMA				0.217	-0.178**
				(0.83)	(-2.47)
RMW				0.193	0.013
				(0.66)	(0.21)
rMOM					0.744***
					(42.89)
_cons	1.356***	1.438***	1.641***	1.528***	0.191**
	(6.97)	(8.06)	(9.04)	(6.29)	(2.35)

Following the order as has been done throughout this paper, the first method examined is the Realised beta. The results of the spanning tests performed on the WML portfolio formed on the realised beta can be seen in Table 13. Looking across all regressions, it can be seen that the most influencial factor is the rMOM factor. This is not a surprise due to the similar nature of the underlying mechanism that drives the idiosyncratic WML factor and the realised WML factor. It should be noted that the HML factor is statistically significant until the rMOM factor is added. Most important out of the spanning tests is that the constant remains statistically significant at 5%. This means that the added factors are not able to fully explain the returns of the WML portfolio. The idiosyncratic momentum factor formed on the realised beta is robust and prices a potential risk not covered by the other factors. This is different from the traditional method tested in Table 7, here the rMOM was able to explain most of the iMOM returns. Important is that this is only the case in this study; Blitz et al. (2020) show in their paper that the traditional method remains statistically significant even when adding the rMOM factor. This can be explained by the fact that they form their iMOM factor with a FF3 method and this paper uses a CAPM regression to make it comparable to the different beta estimation techniques. returning to the realised beta method, it again shows dominance over the traditional method, making it robust and it can be seen as an improvement. Therefore after all tests performed I conclude that forming portfolios using realised beta, despite its more simplistic approach, performs better than the traditional method for forming the idiosyncratic momentum anomaly.

Table 14: Random Walk beta spanning tests

This table shows spanning tests performed on the WML portfolio of the idiosyncratic momentum anomaly formed on the random walk beta. Due to the factors only being available from 1963, data ranges form January 1963 until December 2021. t statistics in parentheses. * $p < .10$, ** $p < 0.05$, *** $p < 0.01$.

	WML	WML	WML	WML	WML
MktRF		-0.093**	-0.131***	-0.126***	-0.065**
		(-2.03)	(-3.46)	(-3.18)	(-2.02)
SMB			0.053	0.063	0.064
			(0.80)	(0.91)	(1.35)
HML			-0.205*	-0.221**	-0.015
			(-1.82)	(-2.53)	(-0.24)
CMA				0.033	-0.111
				(0.19)	(-1.04)
RMW				0.044	-0.022
				(0.24)	(-0.20)
rMOM					0.272***
					(8.00)
_cons	0.469***	0.524***	0.590***	0.568***	0.080
	(3.40)	(3.87)	(4.49)	(3.56)	(0.64)

Second, the Random Walk spanning tests. The results are shown in Table 14. These results will be discussed shortly as the beta estimation method has already been proven to be less useful in previous tests. The alpha is approximately one third of the realised beta alpha. However, it is statistically significant at 1% until the rMOM factor is added to the regression. But other than this it is an inferior method to the realised beta on all fronts, hence proven to be a less effective method for estimating the idiosyncratic momentum. Therefore, my advice to investors would be not to use the Random Walk method to calculate beta to form the idiosyncratic returns based on the results shown.

Lastly, the spanning tests performed on the DCC GARCH beta estimation method. The results are shown in Table 15. Results are very similar to the traditional method. The outperformance remains

Table 15: DCC beta spanning tests

This table shows spanning tests performed on the WML portfolio of the idiosyncratic momentum anomaly formed on the DCC GARCH beta. Due to the factors only being available from 1963, data ranges form January 1963 until December 2021. t statistics in parentheses. * $p < .10$, ** $p < 0.05$, *** $p < 0.01$.

	WML	WML	WML	WML	WML
MktRF		-0.123	-0.210***	-0.208***	-0.057*
		(-1.56)	(-3.24)	(-2.83)	(-1.71)
SMB			-0.016	-0.030	-0.027
			(-0.09)	(-0.21)	(-0.52)
HML			-0.653***	-0.668***	-0.159**
			(-4.23)	(-3.84)	(-2.33)
CMA				0.042	-0.316***
				(0.17)	(-3.36)
RMW				-0.060	-0.223***
				(-0.22)	(-2.64)
rMOM					0.673***
					(22.28)
_cons	0.904***	0.976***	1.210***	1.222***	0.012
	(4.43)	(5.09)	(6.29)	(5.02)	(0.11)

statistically significant at 1% until the rMOM factor is added. It is noteworthy that outperformance is actually increasing across the first 4 regressions as more factors are added. This can be attributed to the fact that except for the CMA factor, all have negative sings. The reason for this can be that the low portfolio is more exposed to these factors than the high portfolio, resulting in a net negative exposure. The DCC GARCH shows signs of possible benefits due to the limitations as described in the previous section. Although the setting is not optimal for the DCC GARCH, it performs relatively similar to the traditional method, making it interesting to see for further research how the idiosyncratic momentum anomaly in combination with DCC GARCh based on daily data performs. For now, however, we must conclude that the DCC GARCH method is performing worse than the realised beta method. When this is combined with the complexity of the DCC garch method, the realised beta has the benefit in both the performance and simplicity.

5.4 Factor models

As mentioned in the literature review, whether the construction of the idiosyncratic momentum with different factor models can improve the performance of the anomaly is tested. As can be seen in Table 16, there are some major differences in the outcome in the way the idiosyncratic momentum anomaly is constructed. Focussing on the WML portfolio as this is the net zero investment portfolio. When looking

Table 16: Different factor models for the construction of idiosyncratic momentum

This table shows a comparison of constructing the idiosyncratic momentum anomaly using different factor models to calculate the idiosyncratic returns. The period is January 1968 until December 2016. Mean and standard deviation (sd) are in percentages. Sr stands for the Sharpe ratio. P1 is the portfolio with the lowest idiosyncratic returns, P10 is the portfolio with the highest idiosyncratic returns. WML is the winner minus losers portfolio.

		P1	P10	WML
CAPM	mean	0.234	1.400	1.170
	sd	6.720	6.056	4.472
	Sr	0.034	0.231	0.262
FF3	mean	0.328	1.207	0.879
	sd	6.750	6.165	3.895
	Sr	0.049	0.196	0.226
FF5	mean	0.337	1.194	0.857
	sd	6.600	6.032	3.486
	Sr	0.051	0.198	0.246
Q	mean	0.339	1.383	1.044
	sd	6.768	6.179	4.219
	Sr	0.050	0.224	0.247
Q5	mean	0.299	1.380	1.081
	sd	6.703	6.186	4.061
	Sr	0.045	0.223	0.266
D	mean	0.340	1.444	1.070
	sd	6.532	5.935	3.965
	Sr	0.052	0.243	0.270
SY	mean	0.232	1.315	1.102
	sd	6.697	6.024	4.033
	Sr	0.035	0.218	0.273

at the mean return of the portfolios, the CAPM has the highest return. Followed by the SY model and Q5 model. Based on the return, it can be seen that FF3 and FF5 have the lowest return by more than 0.1 percentage points difference. Remarkable is that while the FF5 model has the lowest return, it also achieves the lowest standard deviation. An investor looking to minimize risk might therefore be interested in using the FF5 model when constructing an idiosyncratic momentum strategy. The CAPM, while having the highest return, also has the highest standard deviation, emphasizing the risk-return tradeoff. Therefore the Sharpe ratio can help in comparing the different models in their risk-return tradeoff. Looking at the Sharpe ratio, the SY model performs best. The standard deviation of the SY model is lower than that of the CAPM, while maintaining high returns, making it the best option in the Sharpe ratio. The FF3 model achieves the lowest Sharpe ratio, while being the main model used in the previous literature. This signifies the importance of the factor model used when constructing the idiosyncratic momentum anomaly.

6 Conclusion and Discussion

In this Section, the conclusion and discussion of this paper will be discussed. First, I will start with highlighting the main findings in the conclusion and give an answer to the research question. After that, in the discussion, limitations and possible continuations for future research will be addressed.

6.1 Conclusion

In the results, I have shed light on the possibilities in using the idiosyncratic momentum. First, we have seen that, based on the Sharpe ratio and volatility, the idiosyncratic momentum anomaly outperforms the relative momentum anomaly. While the average returns are slightly lower, the decrease in risk shows the trade-off investors can make. The largest critique from relative momentum is crash risk. I have shown that this risk is significantly lower for the idiosyncratic momentum anomaly.

The first decision studied in this paper is the way beta is calculated for the factor model. I tested four methods on the CAPM to see which performed best. First, the descriptive statistics showed that the realised beta was the best based on returns; it did, however, also have the highest returns. Second was the method using univariate regressions. Subsequently, empirical tests were performed to see whether the different methods are robust to other factors. Here the Realised beta method exhibit that it was most robust, remaining statistically significant even when the relative momentum factor was added. The contrary was true for the other methods. Therefore, the conclusion was drawn that the realised beta showed the best characteristics in this study. Answering the first subhypothesis: ”A more accurate method of estimating beta will increase the performance of the idiosyncratic momentum anomaly.”, this was incorrect, as the traditional method is more accurate in estimating beta according to Hollstein and Prokopczuk (2016), but was outperformed by the realised beta. This signals that simplicity can prevail over sophisticated methods for the idiosyncratic momentum anomaly.

The second important decision in constructing the idiosyncratic momentum factor is which asset model to use to construct the forecast returns. This paper sheds light on various factor models and used those to calculate the idiosyncratic returns. Of the factor models tested, the CAPM was able to generate the highest returns. This was accompanied by also having the highest volatility. The lowest volatility was achieved by the FamaFrench 5 factor model, but consequently also had the lowest return. The factor model that exhibited the best risk return trade-off, translated into the highest Sharpe ratio, was the Daniel et al. model. Answering the second subhypothesis: ”The most accurate factor models are better suited for the idiosyncratic momentum anomaly, as they predict stock returns better.”, here there is no clear answer. While Daniel et al. performed best, second place was obtained by the CAPM, which is not the most accurate according to Hou et al. (2021), signalling the exact opposite.

In conclusion, there are many ways to construct the Idiosyncratic momentum anomaly. Thus, the study has attempted to shed light on a couple of these possibilities; however, the possibilities are endless. This study concluded that, of the factor models tested, the Stambaugh and Yuan model performed best and of the beta estimation techniques, the performance of the realised beta was most promising. To answer the research question of this thesis: ”The most accurate calculation of the expected return will lead to the best performance of the idiosyncratic momentum anomaly.”, was seemed not true, there is a strong pattern in simplistic models having a great performance on the idiosyncratic momentum anomaly. In conclusion, this study advises using a realised beta for beta estimation and the Daniel et al. model as a factor model.

6.2 Discussion

This paper has some limitations that will be discussed in this section. I will start with limitations to the beta method model segment. Then I will follow up with a discussion on the factor model segment.

First, the beta-forming segment, the amount of beta-formations studied. The Hollstein and Prokopczuk paper showed beta estimation methods that outperformed the beta estimation methods studied in this paper. These methods use daily data and are more sophisticated. Additionally, these methods use advanced econometric methods which are out of my league of expertise. My suggestion for future research would be to test whether these methods could help increase the performance of the idiosyncratic momentum anomaly.

To add to the use of daily data, the limitation of DCC GARCH is that it also performs better using more observations. I used a windows of 5 years, however that is only 60 observations. For the DCC GARCH often a year of daily data is used which is roughly 250 observations, making the chance of convergence through iteration significantly higher. In my sample, it was often the case that convergence was not achieved, resulting in unusable betas. For future research, I would suggest using a smaller dataset to combine this with daily data. Time-wise this was not achieveable to me due to time it took for the model to run. With only 60 observations, due to the size of my sample, it took approximately 40 hours. Add in the fact that you would sometimes have to make an adjustment and you are looking at weeks when using daily data. This was an oversight in planning. Therefore reducing the amount of assets in your sample, or shortening the studied period could allow for use of daily.

Third, the time period studied is shorter than the entire period that was studied for the descriptive statistics of the idiosyncratic and relative momentum anomaly. This is due to the availability of factors provided by the authors. There is the possibility to construct the factors yourself; however, this was beyond the scope of this thesis due to many factors being dependent on accounting data, which is more complex. This made it so that the most recent 4 years were not studied, which might be most important to investors. While the period of approximately 40 years that were studied should give a general picture, this should be seen as a limitation to recent years.

Fourth, we have the factor models studied. A study by Hou et al. (2017) tested 437 different anomalies. That shows how large the universe of different factors in investing is. While it must be noted that not all survived the different tests of the authors, there are many who did. Of these factors, there are a lot of different factor models that are not studied in this thesis. In addition, as time progresses, new factor models are likely to emerge. This makes that the idiosyncratic momentum might be estimated more accurately in the future and the possibilities of the idiosyncratic momentum anomaly are constantly expanding.

As described above, there are still improvements that can be made, as well as expanding the research into new topics. Which will be interesting to follow closely as time progresses.

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Appendices

Appendix A:

Table 17: Comparison relative momentum anomaly standardization

This table shows the influence of standardization of the relative momentum factor on portfolio performance.

	(Not standardized)		(Standardized)	
	mean	sd	mean	sd
P1	0.031	10.265	0.127	8.557
P2	0.506	8.336	0.450	8.271
P3	0.649	7.441	0.585	7.755
P4	0.717	6.887	0.674	7.401
P5	0.803	6.503	0.837	7.150
P6	0.919	6.327	0.953	7.006
P7	0.959	6.039	1.079	6.790
P8	1.109	6.221	1.128	6.567
P9	1.272	6.462	1.276	6.268
P10	1.492	7.6096	1.442	5.895
WML	1.460	7.007	1.315	5.682
N	1141		1141	

Table 18: Comparison idiosyncratic momentum anomaly standardization

This table shows the influence of standardization of the idiosyncratic momentum factor on portfolio performance.

	(Not standardized)		(Standardized)	
	mean	sd	mean	sd
P1	0.263	8.776	0.396	7.899
P2	0.602	7.099	0.552	7.230
P3	0.727	6.680	0.688	7.136
P4	0.670	6.725	0.779	6.888
P5	0.911	6.387	0.897	6.940
P6	0.957	6.557	0.954	7.033
P7	1.047	6.564	1.038	6.875
P8	1.034	6.479	1.163	6.977
P9	1.156	6.932	1.237	7.121
P10	1.383	8.247	1.429	6.719
WML	1.161	4.535	1.033	4.555
N	1115		1105	