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ESG-Integrated Strategies in Long-Term Dynamic Asset Allocation

Master Thesis Quantitative Finance

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Abstract

This paper analyses the ESG-integrated long-term portfolio choice models among the riskless asset, long-term bond, corporate bond and eight ESG portfolios obtained through K-means sorting or simple sorting based on the stocks' ESG component scores. The optimal asset allocation is determined using the analytical buy&hold (Viceira, 2001) and numerical dynamic (Binsbergen & Brandt, 2007) solutions, based on a return simulation process generated with the VAR(1) framework of Stambaugh (1997). The results show that the dynamic, K-means sorted model achieves the best average return performance, where the most ESG-compliant portfolios automatically receive the highest allocations, thus showing that ESG is still not fully priced into the strategic asset allocation. But the ESG-restricted variant of the portfolio choice models effectively reduce the allocation towards lower ESG portfolios, as these portfolios can still provide higher returns at some periods while increasing the allocation's ESG score. When the ESG threshold is increased and becomes more binding, the asset allocations are impacted more, resulting in lower returns as the flexibility to adjust towards changing investment opportunities diminishes. Lastly, investors with higher Environmental or Social ESG component preferences can expect higher returns compared to the average ESG or Governance component investors, due to the best performing ESG portfolios being linked with higher Environmental and Social component scores.

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1 Introduction

Pension funds are responsible for one of the most important financial decisions an individual needs to make: choosing an optimal strategic asset allocation over time to maximise a participant's retirement income. The Dutch pension system is ranked second in the world (Mercer, 2021), as the system is built on solidarity schemes, collective risk-sharing and stable capital inflows into the pension funds. But an important development in the pension fund industry towards sustainable investing within the domains of Environmental, Social and Governance (ESG) issues has been happening, also known as ESG investing. There is substantial evidence that ESG impacts the financial performance of companies positively and that it does not necessarily come at the cost of return (Wagenaar, 2018). The Dutch pension funds pioneered the movement of the pension fund industry towards ESG investing as early as in 2017, by implementing a socially responsible investment policy and numerous funds committing to the Dutch climate agreement. ESG investing is typically employed by screening the ESG scores of the portfolio's assets, to determine whether the asset is "sustainable" enough to be taken into the investment portfolio (Partners, 2020). This for example has led to ABP, one of the largest Dutch pension funds, to announce that all unsustainable energy investments will be reversed (ABP, 2021).

Furthermore, the current pension system has come under pressure due to the low interest rate environment and longer life expectancies. This resulted in the Dutch pension reform, where pension funds have to shift to a Defined-Contribution (DC) plan. Under a DC plan, only the contribution or premium paid towards the pension is known, while the eventual height of a participant's pension depends on the fund's return performance. This essentially shifts the risk from employer to employee, though research has shown that DC pension plans lead to a higher utility compared to the current defined-benefited (DB) pension plans, as the investment choice can be tailored easier towards personal preferences (Potter van Loon & Grooters, 2018).

Given that including ESG into the investment process is a relatively recent procedure employed by financial institutions, research into ESG investing and more specifically how it should be integrated into the long-term strategic asset allocation, known as an ESG-integrated asset-management study, has not been investigated yet. To further supplement this area of research, the following research question is studied:

How do ESG-integrated long-term portfolio choice models perform compared to the return-only models and how sensitive are they towards different ESG preferences?

The analysis of this topic is addressed through using an ESG-integrated portfolio choice model where the Environmental (E), Social (S) and Governance (G) components of ESG are embedded into the model as a constraint, whereby the portfolio return is maximized while being subject to a certain ESG threshold score $\overline{ESG}$. Some research argues that portfolios considering ESG do not necessarily have a lower expected return and may even outperform return-only portfolios (Shen et al., 2019), though other research has shown that unsustainable stocks, such as sin stocks, can also generate larger abnormal returns (Hong & Kacperczyk, 2009). Given that ESG data is more widely available for companies traded on the more developed financial markets of North America, Europe and Asia, a collection of 6045 companies from these regions with 18 years of ESG data is used. These stocks are then used to construct 8 portfolios, referred to as ESG portfolios, using the K-means unsupervised clustering technique based on the companies' E, S and G component scores and also comparing this to a simple sorting procedure. Besides the ESG portfolios, the optimal asset allocation is determined along with the riskless asset, the long-term bond and the corporate bond. As such, the portfolio represents asset classes which differ widely in their risk and return trade-off. The optimal allocation is obtained by employing two popular long-term portfolio choice frameworks: the approximate analytical framework of Viceira (2001) and the numerical dynamic framework of Binsbergen & Brandt (2007).

The ESG-integrated version of above asset allocation models are compared to the performance of the return-only version of above models. Given that the future return of the assets are uncertain, the returns are assumed to be predictable based on financial and climate-related state variables, whose dynamics are modelled and simulated using restricted and unrestricted vector auto-regressive models as proposed by Stambaugh (1997). By splitting the scenario set into a training and testing set with a 80%-20% split, the out-of-sample performance of the models is measured using the Certainty Equivalent Rate, the mean and volatility of the terminal wealth, the Sharpe ratios, the turnover rates and the allocation's ESG score across different investment horizons, up to 15 years.

The above research question facilitates the research into ESG investing for long-term portfolio management. ESG investing in long-term portfolio management has been shown to assign larger weights to more sustainable assets when a climate change risk factor is incorporated as a state variable into the return simulation process (Shen et al., 2019). Nevertheless, the incorporation of ESG directly into the optimal allocation model has not been studied yet in a long-term portfolio choice model setting, and thus aids pension funds in determining the best method for ESG to be included in the portfolio. Secondly, this paper further fills the gap in research with regards to the included assets in the models: rather than using (sustainable) exchange-traded equity funds, a sample of 6045 stocks is employed to construct equity portfolios. To the current knowledge, such a big equity asset space has not been studied yet in a long-term portfolio choice setting. Besides the scientific relevance, this paper is also socially relevant. The research analyses how pension funds should incorporate ESG, such that the pressure from the stakeholders to have sustainable investments is satisfied and may perhaps even lead to higher abnormal returns, which benefits all individuals participating in the pension scheme. Moreover, the ESG-integrated model is a flexible framework, where the ESG components can be replaced with the preferred ESG indicators/components of the pension funds.

The portfolio choice models reveal that the optimal asset allocations reflect common risk-return trade-offs and account well for the term structure of the assets, with high demand for the riskless asset at shorter horzions and larger allocations for equity at longer horizons. The Dynamic, K-means sorted portfolio choice model reaches the highest Certainty Equivalent Rate and thus achieves the best return performance on average compared to other models. In contrast with the simple sorting procedure, The K-means sorting procedure creates ESG portfolios which have more closely aligned stocks in terms of ESG clustered together, thus accounting better for the ESG score developments over time. When the optimal asset allocation is subjected to the ESG restriction, the allocation is barely impacted as the most ESG-compliant portfolios are already included in the optimal asset mix under return-only models, though it is effective at reducing the demand for lower ESG portfolios, as these are only included in the allocation at a reduced rate under ESG-restricted models, in the case they offer superior risk-adjusted returns in specific periods. But as the ESG threshold increases and thus becomes more binding, it in general leads to lower return performance as the optimal asset allocation loses its ability to efficiently get allocations which hedge expected return movements. Lastly, investors with higher Environmental and/or Social ESG component preferences are found to achieve a higher average return performance compared to the average or Governance component investors, as the best performing ESG portfolios are more closely related with higher Environmental or Social component scores.

The remainder of this paper consists of the literature review in section 2, the methodology in section 3, the data in section 4, the results in section 5 and the conclusion and discussion in section 6.

2 Literature Review

2.1 ESG Investing

Portfolio management has been a longstanding field of research within the finance industry, where asset managers have excelled in dealing with various financial risks such as interest rate and inflation risks. However, climate risks remain poorly understood and unaccounted for, which can significantly impact the financial performance of financial institutions that are concentrated in certain sectors and regions (Alogoskoufis et al., 2021). To better mitigate climate risks, more asset managers are applying sustainable investment strategies, defined as "an investment discipline that considers environmental, social and governance criteria to generate long-term competitive financial returns and positive societal impact" (US$\|SIF$, 2021), with investments totalling over \$ 35 trillion as of 2019 (Jessop, 2021).

But the effect of ESG investing on return remains a topic of debate: positive excess returns are found in stocks with higher employee satisfaction and/or higher governance scores (Edmans, 2011; Sloan, 1996) and if the unknown climate-related risk factor is accounted for through climate-related state variables like temperature change, then ESG investing does not sacrifice return (Shen et al., 2019), but may produce significantly different portfolio allocations (Qi & Li, 2020). But on the other hand, Hong & Kacperczyk (2009) find evidence that Sin stocks can also achieve higher abnormal returns. But overall, Friede et al. (2015) conclude that ESG investing is empirically well-founded across 2000 ESG studies, but no clarification is given as to when ESG investing is more profitable than return-only strategies.

Besides the ambiguity surrounding sustainable investing, ESG investing often involves ESG screening strategies, where 'ESG-poor' stocks are completely excluded. While this a simple method, Amel-Zadeh & Serafeim (2018) argue that such ESG strategies are ineffective and problems of portfolio incompleteness can arise (Qi & Li, 2020). To overcome these issues, Amel-Zadeh & Serafeim (2018) recommend the use of ESG-integrated strategies, where ESG is directly used in the investment decision. An example of that is the ESG-efficient frontier of Pedersen et al. (2021), where ESG scores influence investor preferences and correlate with firm fundamentals in the ESG-tangency portfolio. This frontier can simultaneously explain that ESG can act as a positive predictor of future returns when ESG is not yet fully priced in, while also explaining that ESG can also have a neutral impact when investor demand for ESG is low. Following Amel-Zadeh & Serafeim (2018) and Pedersen et al. (2021), this paper also aims at integrating ESG directly into the long-term strategic asset allocation.

2.2 Mean-Variance Models and Long-term Portfolio Choice Models

A vast amount of research by academics and portfolio managers has gone into finding the best risk-adjusted return for investors. The pioneering portfolio optimisation framework for a short-term investment horizon originates from the work of Markowitz (1952), through only considering the portfolio return's mean and variance in Mean-Variance (M-V) models. But this model only incorporates one-period ahead state information and it cannot explain how to make long-term investment decisions, as it fails to account for the term structure of assets and that investors move to less risky assets as retirement approaches. This led to new research areas known as lifecycle investing and long-term portfolio choice problems. A closed-form solution for such an asset allocation problem is first proposed by Merton (1969) in continuous-time, while a discrete-time using the lognormal-power utility framework is proposed by Samuelson (1969). Besides the long-term horizon, their models also explain the asset allocation between different asset classes with different risk profiles. Assuming that investment opportunities are constant, by simulating returns that are independently and identically distributed (i.i.d.) (Samuelson, 1969), both papers conclude that the optimal weight in risky assets is constant across the full period, and that the optimal risky asset weight depends on its price for risk and the risk aversion parameter.

Above papers essentially find that the dynamic and myopic strategies are equivalent when returns are i.i.d., occurring when investment opportunities are stochastic but unhedgeable (Brandt, 2010). But research has emerged that equity and bond returns are predictable for longer horizons (Breen et al., 1989; Balvers et al., 1990; Cochrane, 1991), where a significant predictable component can be found with predictor variables that alter over business-cycles, such as price-earnings ratios and credit spreads, when parameter uncertainty is incorporated (Cochrane, 1999; Pesaran & Timmermann, 1995). This return predictability is shown to be exploitable after transaction costs (Pesaran & Timmermann, 1995; Campbell & Viceira, 1999), though no robust out-of-sample forecasting model has been found that consistently performs well (Pesaran & Timmermann, 1995; Welch & Goyal, 2008), thus leading to theoretically inconclusive findings (Fama, 1991).

The discrete-time portfolio choice model is extended by Viceira (2001) by adding return predictability, non-tradable labour income and retirement into the lognormal-power utility framework, through the use of state variables. This makes investment opportunities time-varying, meaning that portfolio weights are adjusted to reflect future circumstances. Through using Bellman equations and the log-linear approximation of the portfolio return, the paper derives an approximate optimal consumption and portfolio rule in each state, which can be solved with the first-order conditions of the Bellman equations, which is a necessary condition for optimality (Campbell & Viceira, 1999). Unlike before, this framework designates a more important role for bonds in the optimal portfolio for conservative investors, by accounting for the risk factor. Retirement also now plays a more important role as this model assigns larger weights to riskier asset classes to hedge against expected return movements at longer horizons, though this may not hold if multiple risky asset classes are considered, such as real estate (Barberis, 2000; Hoevenaars et al., 2007). Given the convenient characteristics of the approximate analytical solution of Viceira (2001), it is one of the models to be used in this paper to obtain the (ESG-restricted) optimal asset allocation over time.

The downside of the analytical solution is that its FOCs consist of nonlinear equations with high-order integrals, which become cumbersome to solve analytically (Brandt, 2010). To fix this problem, Binsbergen & Brandt (2007) propose a numerical method which relies on the recursive use of approximated optimal portfolio weights, from which the expected terminal wealth is maximised. By iterating over a grid of portfolio allocations, which can incorporate short-selling restrictions, the portfolio choice problem becomes computationally feasible while creating bounds on the error accumulations across time periods, leading to superior results (Binsbergen & Brandt, 2007). Given these convenient features, the numerical framework is also employed alongside the analytical solution in this paper. Besides this numerical method, further extensions have been studied, such as including liabilities subject to interest rate risk under a DB pension plan (Hoevenaars et al., 2007), accounting for incomplete markets known as the Martingale approach (He & Pearson, 1991) or using different utility functions, such as the hyperbolic absolute risk aversion (Bajeux-Besnainou et al., 2003) or Epstein-Zin functions, which incorporates the elasticity of inter-temporal substitution between consumption and the willingness to take on risk (Epstein & Zin, 1991). In the context of ESG, Shen et al. (2019) also include temperature change as a state variable and finds that the model assigns a larger weight to greener investments. This feature is thus also considered in this paper.

As future returns are uncertain, all the portfolio choice models work with simulated return scenarios to find the asset allocation which maximises the average terminal wealth. When returns are i.i.d., returns tend to be generated using a Brownian motion. But in case of predictable returns, literature tends to simulate returns with the vector auto-regressive model (Sims, 1980), using the dynamics and auto-correlation among the return and state variables. But a typical problem with return data is the unequal sample sizes, which may harm parameter estimation. To avoid truncating historic data and to reduce the number of parameters to be estimated, Stambaugh (1997) introduces an efficient estimator using unrestricted and restricted vector auto-regressive models, where the shorter sampled variables cannot affect the expectation of the longer sampled variables. Since equity returns have a shorter sample size, the framework of Stambaugh (1997) is employed to perform the return simulation process in this paper.

3 Methodology

The goal of the portfolio choice models within this paper is to optimise the portfolio allocation, using the approximate analytical and numerical solutions. Besides the return-only variant, ESG-integrated version of these portfolio choice models are also employed to increase the holdings of green(er) assets. Do note that these models are only suitable for well diversified portfolios and for long-term investors focusing on the intertemporal risk-return trade-off.

3.1 State Variables, ESG Portfolios and Bond Returns

3.1.1 ESG Portfolio Creation Process

The optimal asset allocation is determined among the riskless asset ($r_{f,t}$), the long-term bond ($r_{lb,t}$), the corporate bond ($r_{cor,t}$) and the ESG portfolios ($r_{x,t}$, where $x \in \{c1, c2, c3, c4, c5, c6, c7, c8\}$). To decrease the dimensionality in the equity spectrum, stocks are placed into equally-weighted ESG portfolios based on a K-means unsupervised learning procedure using the stocks' ESG component scores $ESG_{i,j,t}$, where $j \in \{E, S, G\}$ denotes the Environmental, Social and Governance component scores of stock i at time t. With numerous clustering techniques being available, K-means clustering is chosen as it is a simple algorithm and has a computational advantage when dealing with a high number of features. To benchmark this clustering algorithm and see whether it can be beneficial, a comparison with a simple portfolio sorting procedure is made.

The simple sorting procedure splits the portfolio at time t using the medians of $ESG_{i,j,t-1}$ for all ESG components $j \in \{E, S, G\}$. As a split is performed over three ESG components, 8 equally-weighted, well-diversified portfolios are created in every time period, where the portfolio's return is determined by the equally-weighted stocks' excess return at a monthly frequency. To prevent data-leakage, the ESG portfolio sorting at time t is performed using the ESG components from time $t-1$, as the ESG component scores are not known yet at time t. But due to stock return data starting in 1997 while the ESG data begins in 2004, the ESG scores of 2004 are used for the portfolio sorting between 1997 and 2004. For this time frame, it is unavoidable to have data leakage and be subjected to the look-ahead bias. But as ESG data collection is becoming more mainstream and standardised, this problem will be less relevant in the future.

The K-means method is an unsupervised learning technique developed by Hartigan & Wong (1979), which can classify unlabelled data into a pre-defined number of non-overlapping and unique clusters. Among the stocks' features $ESG_{i,t}$, K-means optimises the centre of the clusters, known as centroids, such that when a stock is assigned to a cluster, the within-group sum of squared (WGSS) euclidean norm between the stocks and corresponding cluster's centroid is minimised. Using $i \in \{1, \dots, N\}$, $j \in \{E, S, G\}$, $k \in \{1, \dots, K\}$ and $t \in \{1, \dots, T\}$, where N is number of stocks, K is the number of clusters and T is final year of the ESG data, the K-means technique can be solved with the following NP-hard problem:

$$\textbf{argmin} \sum_{i=1}^{N} \sum_{k=1}^{K} \sum_{t=1}^{T} \sum_{j \in \{E,S,G\}} z_{i,k,t} \left\| (ESG_{i,j,t} - \mu_{k,j,t}) \right\|^2, \text{s.t.} \sum_{k=1}^{K} z_{i,k,t} = 1 \; \forall \; i, t, \qquad (1)$$

where $z_{i,k,t}$ is a binary variable, set to 1 when stock i is placed into cluster k at time t and 0 otherwise, and $\mu_{k,j,t}$ is the mean of cluster k for ESG component j at time t. An intuitive heuristic using the Expectation-Maximisation (EM) method can be applied to solve this NP-hard problem. First, as K-means is sensitive to different feature scales, the ESG components are standardised. The heuristic is then initialised by setting $\mu_{k,j,t}$ randomly within reasonable bounds of the ESG components. In the Expectation-step, all stocks are assigned to its closest cluster k and $z_{i,k,t}$ is updated, achieved by choosing the cluster for each stock with the lowest WGSS. The Maximization-step then follows, where the means $\mu_{k,j,t}$ are updated given the revised $z_{i,k,t}$. The E- and M-steps are then repeated until convergence is reached, which is guarenteed with the EM-method. The pseudocode of this heuristic is provided below:

Algorithm 1 K-means clustering EM heuristic

Data : $ESG_{i,t} = [E_{i,t}, S_{i,t}, G_{i,t}]$, where $i \in \{1, ..., N\}$ and $t \in \{1, ..., T\}$
Set $\mu_{k,j,t}$ to be random means of $ESG_{i,j,t}$ for cluster $k \in \{1, ..., K\}$ and $j \in \{E, S, G\}$
Repeat
E-step
for $i = 1, \ldots, N$ and $t = 1, \ldots, T$ **do**
Update $z_{i,k,t} = \begin{cases} 1, \text{ if } k = \mathbf{argmin}_k \left\| \sum_{j \in \{E,S,G\}} (ESG_{i,j,t} - \mu_{k,j,t}) \right\|^2, \\ 0, \text{ otherwise} \end{cases}$
End for
M-step
for $k = 1, \ldots, K$ and $t = 1, \ldots, T$ **do**
Update $\mu_{k,j,t} = \frac{\sum_{i=1}^{N} z_{i,k,t} ESG_{i,j,t}}{\sum_{i=1}^{N} z_{i,k,t}}, \forall j, k, t$
End for
Until Convergence
Return $z_{i,k,t}$, $\mu_{k,j,t}$, $\forall i, j, k, t$

Given that this convergence only leads to local minima, the K-means algorithm is re-initialised using 10 different seed numbers for $\mu_{k,j,t}$, after which the solution with the lowest WGSS is chosen. This increases the likelihood of being closer to the global minimum. Furthermore, to benchmark with the simple sorting procedure, the number of clusters is matched and thus set to 8, which also leads to well-diversified portfolios under K-means sorting. However, compared to the simple procedure, the K-means algorithm rather looks at the ESG trend over time. Thus, when the cluster allocation takes place at time t, $ESG_{i,p}$ are used as features, $\forall p < t$ such that data leakage is avoided and stocks are clustered with other stocks that have similar ESG performance over time. Lastly, as with the simple sorting procedure, the ESG data of 2004 is used in the K-means algorithm to determine the ESG portfolios between 1997 and 2004.

3.1.2 Log Excess Returns of ESG Portfolios and Bonds

To determine the optimal inter-temporal asset allocation, the log excess returns of all the included equity and bond assets are used at a monthly frequency, where the returns do not follow a stochastic Brownian motion process but are rather assumed to be predictable to a certain extent. The choice for log returns is made as this also makes cumulative returns log-normally distributed, which holds useful properties for the lognormal-power utility frameworks of the analytical and the numerical solutions. For ESG portfolio x, the log excess return at time t, $r_{x,t}$, is derived as in (2), where the risk-free rate $r_{f,t}$ is subtracted from the log return, obtained using the closing stock prices $P_{i,t}$ for stock i at time t and $t-1$. For the bonds, the log yield $y_{b,t}$ is obtained using $y_{b,t} = log(1 + Y_{b,t})$, where $Y_{b,t}$ is the nominal yield of bond b at time t, where $b \in \{f, lb, cor\}$ denotes the riskless asset, the long-term bond and the corporate bond respectively. The log excess bond returns at a monthly frequency are then acquired using $y_{b,t}$ within the log-linear approximation as in (3), following Hoevenaars et al. (2007) and Viceira (2001). In (3), the $\frac{1}{12}$ term scales the annualised yields towards a monthly return and $D_{b,t}$ denotes the bond's duration, following $D_{b,t} = \frac{1-(1+Y_{b,t})^{-M_b}}{1-(1+Y_{b,t})^{-1}}$, where M_b is bond b's time to maturity.

$$r_{x,t} = log(\frac{P_{x,t}}{P_{x,t-1}}) - r_{f,t}, \qquad (2)$$

$$r_{b,t+1} \approx \frac{1}{12} y_{b,t+1} - D_{b,t}(y_{b,t+1} - y_{b,t}) - r_{f,t}. \qquad (3)$$

3.1.3 State Variables

Given that returns of equity and bonds are assumed to be predictable to a certain extent, the yield spread (YS_t), the credit spread (CS_t), the price-earnings ratio (PE_t), the ex-post real rate (rr_t), the temperature change (ΔT_t) and the inflation rate (π_t) are used as state variables to quantify the state of the financial markets and climate change. YS_t is attained by taking the difference between the log yields of the long-term bond and the riskless asset. CS_t is obtained through the difference between the log yields of the corporate bond and the long-term bond.

Moreover, the PE ratio is calculated by taking the log of the ratio of the current price over the lagged mean of earnings ratio from the past 10 years, following the method of (Diris et al., 2015). Lastly, the ex-post real rate rr_t is obtained through the difference of the riskless asset's log yield and the log inflation rate. The log inflation rate π_t is retrieved by taking the log difference between the consumer price index CPI_t at times t and $t-1$.

Lastly, similar to Shen et al. (2019), temperature change is included as an external climate risk influence on asset returns. ΔT_t is included because it is also a fairly stochastic variable, where the earth's average temperature change is forecasted to vary between 1.4°C (degrees Celsius) and 5.8°C in 2100 (Gitay et al., 2002). Following Lemoine (2021), ΔT_t is modelled by using the difference of the 12-month moving average temperature T_t at time t and $t-1$, $\Delta T_{t+1} = T_{t+1} - T_t$.

3.2 Return Dynamics and Simulation Process

3.2.1 The Vector Auto-Regressive Model

To represent the dynamic behaviour among multiple return and state variables through linear functions of the lagged components of the variables, a vector auto-regressive (VAR) model is used as developed by Sims (1980). Where C is the number of ESG portfolios, B the number of bond assets, V the number of state variables and T the total number of time series observations, let Y_t be the (C+B+V)x1 vector of log excess asset returns and the state variables. The VAR model with lag order p, VAR(p), can then be expressed as:

$$Y_t = \mu + \sum_{i=1}^{p} A_i Y_{t-i} + \epsilon_t, \tag{4}$$

where μ, Y_t and ϵ_t represent a (C+B+V)x1 vector of constants, endogenous variables and white noise errors respectively for all t. White noise errors imply that $E(\epsilon_t) = 0$, $\Sigma_\epsilon = E(\epsilon_t \epsilon_t')$ is non-singular and that errors are homoskedastic and cross-sectionally correlated but not serially correlated, insinuating that ϵ_t and ϵ_v are independent when t≠v. A_i is a (C+B+V)x(C+B+V) matrix of coefficients for lag order i and the variance-covariance matrix of ϵ_t is $\Sigma_\epsilon = FDF'$, where F is a lower triangular matrix with ones on the diagonal and D is a diagonal matrix. Do note that by not allowing innovations to be cross-sectionally correlated, this implies that the coefficients are not time-varying, similar to Hoevenaars et al. (2007) and Campbell & Viceira (2005). A conscious choice for not having time-varying parameters is made, as the sample size does not suffice for reliable parameter estimation.

3.2.2 The Restricted and Unrestricted Vector Auto-Regressive Model

As will be noted in the data section, the ESG portfolios have a smaller sample size (starting in 1997) compared to the bond return data and the remaining state variables (starting in 1982). To circumvent the problem of unreliable parameter estimates for the shorter sampled variables and that useful historic data is truncated from the longer sampled variables, restrictions are imposed on the VAR(p) estimation process, following Stambaugh (1997). In Stambaugh (1997), Y_t is split into the longer sampled variable matrix $Y_{1,t}$ and the shorter sampled variable matrix $Y_{2,t}$. Thus, let $Y_{1,t} = [r_{f,t}, r_{lb,t}, r_{cor,t}, YS_t, CS_t, rr_t, PE_t, \Delta T_t, \pi_t]$ and $Y_{2,t} = [r_{c1,t}, r_{c2,t}, r_{c3,t}, r_{c4,t}, r_{c5,t}, r_{c6,t}, r_{c7,t}, r_{c8,t}]$. The estimation restriction is imposed on $Y_{2,t}$: the ESG portfolios in $Y_{2,t}$ can have no dynamic feedback on $Y_{1,t}$ and thus have zero explanatory power. As such, truncate-sample estimators are avoided and information from the earlier estimation period is not ignored, while allowing for the dynamics of $Y_{2,t}$ to be partially driven by $Y_{1,t}$.

Following the method of Stambaugh (1997), $Y_{2,t}$ is modelled with an unrestricted VAR(1) model, as in (5) where μ and $\epsilon_{1,t}$ are (B+V)x1 vectors of the constants and white noise residuals at time t following $\epsilon_{1,t} \sim \mathcal{N}(0, \Sigma_{\epsilon_1})$ respectively where Σ_{ϵ_1} is the (B+V)x(B+V) variance-covariance matrix, and A_1 is a (B+V)x(B+V) coefficient matrix. Moreover, lag order of one is chosen, as it is a common choice in literature to limit the number of coefficients to be estimated (Hoevenaars et al., 2007; Shen et al., 2019).

$$Y_{1,t} = \mu + A_1 Y_{1,t-1} + \epsilon_{1,t} \tag{5}$$

$$Y_{2,t} = \alpha + B_0 Y_{1,t} + B_1 Y_{1,t-1} + H_1 Y_{2,t-1} + \epsilon_{2,t} \tag{6}$$

The above described restrictions on the dynamics of $Y_{2,t}$ are incorporated in a restricted VAR(1) model, as in (6), where α and $\epsilon_{2,t}$ are Cx1 vectors of constants and white noise errors at time t

respectively, B_0 and B_1 are both unrestricted Cx(B+V) coefficient matrices where B_0 estimates the instantaneous effect of $Y_{1,t}$ on $Y_{2,t}$ and B_1 estimates the lagged effect of $Y_{1,t}$ and H_1 is a restricted CxC diagonal coefficient matrix, indicating that variables in $Y_{2,t}$ can only affect themselves and not the other assets. The innovation follows $\epsilon_{2,t} \sim \mathcal{N}(0, \Sigma_{\epsilon_2})$, where Σ_{ϵ_2} is a diagonal CxC variance-covariance matrix. This restricted VAR(1) model can be further expanded out, such that it only contains the lagged components of $Y_{1,t}$ and $Y_{2,t}$:

$$Y_{2,t} = \alpha + B_0\mu + (B_0A_1 + B_1)Y_{1,t-1} + H_1Y_{2,t-1} + B_0\epsilon_{1,t} + \epsilon_{2,t}. \tag{7}$$

Using (7), the restricted and unrestricted VAR(1) models can be rewritten as a complete VAR(1) model in matrix notation:

$$Y_t = \Phi_0 + \Phi_1 Y_{t-1} + u_t, \text{ where} \tag{8}$$

$$Y_t = \begin{bmatrix} Y_{1,t} \\ Y_{2,t} \end{bmatrix}, \Phi_0 = \begin{bmatrix} \mu \\ \alpha + B_0\mu \end{bmatrix}, \Phi_1 = \begin{bmatrix} A_1 & 0 \\ B_0A_1 + B_1 & H_1 \end{bmatrix}, u_t = \begin{bmatrix} \epsilon_{1,t} \\ B_0\epsilon_{1,t} + \epsilon_{2,t} \end{bmatrix},$$

representing the (C+B+V)xT matrix of endogenous variables, the (C+B+V)x1 vector of constants, the (C+B+V)x(C+B+V) slope coefficient matrix and the (C+B+V)xT matrix of white noise residuals, respectively. The innovations follow a multivariate Gaussian distribution, such that $u_t \sim \mathcal{N}(0, \mathbf{\Sigma})$, where the variance-covariance matrix $\mathbf{\Sigma}$ is defined as in (9). This VAR(1) framework uses the available data optimally and ensures that $\mathbf{\Sigma}$ is semi-definite. The derivation of all the $\mathbf{\Sigma}$ elements can be found in section A.1 in Appendix A.

$$\mathbf{\Sigma} = \begin{bmatrix} \Sigma_{\epsilon_1} & \Sigma_{\epsilon_1}B_0' \\ B_0\Sigma_{\epsilon_1} & B_0\Sigma_{\epsilon_1}B_0' + \Sigma_{\epsilon_2} \end{bmatrix} \tag{9}$$

Some state variables like PE_t may be persistent and non-stationary, but the VAR coefficients are only consistent when all endogenous variables are stationary. Therefore $Y_{1,t}$ and $Y_{2,t}$ are tested for stationarity with the Augmented Dickey-Fuller (ADF) test, with H_0 that a unit root (non-stationarity) is present and H_α that a unit root is not present. Though as stated in Stambaugh (1997), the non-stationarity problem in this VAR(1) framework remains minimal. As such, the sole purpose of the ADF test is to identify the (near) non-stationary endogenous variables, whose coefficients may be slightly inconsistent.

This VAR(1) process is then used to create a set of 10000 simulated scenarios of the state and return variables using a Monte-Carlo simulation method with the VAR(1) model. This is done by sampling the residual u_t from the multivariate Gaussian distribution to obtain a simulated forecast for Y_{t+i} for $1 \leq i \leq \tau$ with the estimated VAR(1) parameters. These simulated forecasts are constructed for a maximum horizon τ of 15 years. This horizon is chosen as the horizon effects become negligible after about 15 years, where a differentiation between short-term and long-term allocations cannot be made anymore (Campbell & Viceira, 2005). Based on this scenario set, a certain portfolio choice model is then determined to be optimal, by finding the allocation which achieves the highest on-average risk-adjusted return across the scenarios.

3.3 Long-Term Portfolio Choice Models

The aim of this section is to propose an optimal strategic asset allocation to maximise the investor's return but also ESG-integrated allocation models, such that the holdings in greener assets can be increased while efficiently maintaining long-term return perspectives. As the current pension reforms are shifting the Dutch pension industry towards DC pension schemes, only asset-only management portfolio choice models are considered where long-term liability risks can be neglected (Markowitz, 1952). This also means studying an asset-only approach becomes more relevant, as asset-liability management studies are only appropriate for DB pension schemes, which are being phased out in the Netherlands.

3.3.1 The Wealth Function, Portfolio Return and Utility Function

The strategic asset allocation determines the optimal allocation among the included assets. For all the considered models, the goal is to maximise the terminal wealth $W_{t+\tau}$ for an investment horizon τ. The terminal wealth is the accumulated wealth over the horizon, following:

$$W_{t+1} = (1 + R_{p,t+1})W_t, \tag{10}$$

where $R_{p,t+1}$ is the portfolio excess return between time t and t+1, defined as follows:

$$R_{p,t+1} = (1 - \iota'\alpha_t)R_{f,t+1} + \alpha_t' R_{A,t+1} = R_{f,t+1} + \alpha_t'(R_{A,t+1} - R_{f,t+1}), \tag{11}$$

where α_t is a (C+B^*)x1 vector of portfolio weight for all the included assets except the risk-free asset at time t and $R_{A,t+1}$ is the (C+B^*)xT vector of excess returns of the assets, excluding the risk-free asset. Note that $B^* = B - 1$, which simplifies the notation that excludes the riskless asset from the bond asset class set B. To determine the value of the terminal wealth of the investors, the power utility function is considered due to its suiting properties when combined with log-normal returns and it being a popular option in literature (Campbell & Viceira, 2002):

$$U_\gamma(W_t) = \begin{cases} \log(W_t), \text{if } \gamma = 1, \\ \frac{W_t^{1-\gamma}}{1-\gamma}, \text{otherwise}, \end{cases} \tag{12}$$

where γ is the constant relative risk aversion (CRRA) parameter. The power utility is a concave function, assuming CRRA and quantifies the trade-off between the expected return and risk through γ. Given that the sensitivity towards the risk-return trade-off is not considered in this paper, γ is set to 5, as commonly used in literature and known to represent an investor with moderate risk-aversion. For the exact derivation and meaning of γ, please refer to section A.2 in Appendix A. Also, the power utility function is homothetic when assuming that the investor has no labor income, which makes the utility function independent from the initial wealth. As such, the initial wealth W_t is standardized at time t by setting it to 1.

3.3.2 The Return-Only Approximate Analytical Portfolio Choice Model

The approximate analytical portfolio choice model is the first model that is discussed, as derived by Campbell & Viceira (2002). To find the optimal asset allocation, a utility criterion function in terms of the terminal wealth $W_{t+\tau}$ is formulated, which follows the recursive wealth function as in (10). Thus the optimal strategic asset allocation for a horizon of τ maximises the expected terminal utility of $W_{t+\tau}$ at time t + τ:

$$V_t(\tau, W_t, SV_t) = \max_{\{\alpha_z\}_{z=t}^{t+\tau-1}} \mathbb{E}_t\Big[U(W_{t+\tau})\Big]. \tag{13}$$

Such an optimisation problem assumes that all the wealth is reinvested throughout time and that the full terminal wealth is used to purchase a pension at time $t + \tau$. It makes use of the Bellman equations $V_t(\tau, W_t, SV_t)$, which serve as the expected utility of $W_{t+\tau}$, conditional on the state variables SV_t. The Bellman equations can be expanded in terms of the Bellman equations of the future periods $t + 1, \dots, t + \tau$, due to the power utility function from (12) and the recursiveness of W_t:

$$\begin{aligned} V(\tau, W_t, SV_t) = \max_{\alpha_t} \mathbb{E}_t\Big[\max_{\{\alpha_z\}_{z=t+1}^{t+\tau-1}} \mathbb{E}_{t+1}\Big[\frac{W_{t+\tau}^{1-\gamma}}{1-\gamma}\Big]\Big] &= \max_{\alpha_t} \mathbb{E}_t\Big[\frac{W_{t+1}^{1-\gamma}}{1-\gamma} \max_{\{\alpha_z\}_{z=t+1}^{t+\tau-1}} \mathbb{E}_{t+1}\Big(\prod_{s=t+1}^{t+\tau-1}(R_{s+1}^p)\Big)\Big] \\ &= \max_{\alpha_t} \mathbb{E}_t\Big[U(W_{t+1})V(\tau-1, W_{t+1}, SV_{t+1})\Big], \end{aligned} \tag{14}$$

where the optimal asset allocation at time t depends on the utility obtained from W_{t+1} at time t + 1 and $\psi(\tau - 1, SV_{t+1})$ is the value function in terms of the investment horizon and the future states, defined by SV_{t+1}. To prevent data leakage, the optimal asset allocation is obtained at time t using the state variables at time $t - 1$. These Bellman equations are used to consecutively determine the asset allocation over time for $V(1, W_{t+\tau-1}, SV_{t+\tau-1}), \dots, V(\tau, W_t, SV_t)$, in a backward recursion fashion.

To obtain the optimal asset allocation, cumulative returns should be used to identify the risk-return trade-off among the assets. Given that the log excess returns are log-normally distributed, this gives the convenient feature that cumulative returns are also log-normally distributed. As such, the portfolio excess return from (11) is transformed into a log portfolio excess return, in terms of the individual assets' log excess return. However, a one-on-one relation between the log portfolio returns and the individual log returns does not exist, but can be approximated using a second-order log-linear Taylor approximation (Campbell & Viceira, 2002):

$$r_{p,t+1} - r_{f,t+1} \approx \alpha_t'(r_{A,t+1} - r_{f,t+1}\iota) + \frac{1}{2}\alpha_t'\sigma_t^2 - \frac{1}{2}\alpha_t'\Sigma_t\alpha_t, \tag{15}$$

where $r_{A,t}$ is the log excess return, Σ_t is the (C+B^*)x(C+B^*) variance-covariance matrix of the assets and σ_t^2 is a (C+B^*)x1 vector of Σ_t's diagonal elements. Note that a closed-form solution does not exist, but as the number of asset increases, the approximation becomes more precise.

A known problem is that investment opportunities could be overestimated in optimal asset allocations and the portfolio weights essentially become error-maximisers, where portfolio weights can be extreme due to parameter uncertainty/estimation error in expected returns and the (co-)variance of returns (Michaud, 1989; Jobson & Korkie, 1980). To reduce this problem, the approximate analytical solution invests in a buy&hold fashion with constant proportions $\alpha_t^{(\tau)}$ over the investment horizon τ, where the initial allocation is re-balanced in every time period. Fixed allocations are actually more closely associated with the pension fund industry, as their strategic asset allocation is reviewed approximately every three years. For a fixed $\alpha_t^{(\tau)}$, the τ-period portfolio return follows:

$$r_{p,t+\tau}^{(\tau)} = \sum_{j=1}^{\tau} r_{p,t+j} = r_{f,t+\tau}^{(\tau)} + \alpha_t^{(\tau)\prime}\Big(r_{A,t+\tau}^{(\tau)} + \frac{\tau}{2}\sigma_A^2\Big) - \frac{\tau}{2}\alpha_t^{(\tau)\prime}\Sigma_A\alpha_t^{(\tau)}, \tag{16}$$

where $r_{f,t+\tau}^{(\tau)}$ is the τ-period risk-free log return represented by the riskless asset, $r_{A,t+\tau}^{(\tau)}$ is the (C+B^*)x1 vector of τ-period log returns of the assets, Σ_A is the (C+B^*)x(C+B^*) variance-covariance matrix of the assets and σ_A^2 is the (C+B^*)x1 vector consisting of the diagonal elements of Σ. This result becomes useful, because the maximisation problem in (13) can be transformed using the lognormal-power utility framework such that the problem reduces to:

$$\max_{\{\alpha_z\}_{z=t}^{t+\tau-1}} \mathbb{E}_t\Big[U(W_{t+\tau})\Big] = \max_{\alpha_{t+\tau}^{(\tau)}} \mathbb{E}_t\Big[r_{p,t+\tau}^{(\tau)}\Big] + \frac{1}{2}(1-\gamma)Var_t\Big[r_{p,t+\tau}^{(\tau)}\Big], \tag{17}$$

where γ is the power utility's CRRA factor. Using the expression of $r_{p,t+\tau}^{(\tau)}$ in (16), the mean and variance of the τ-period portfolio returns become:

$$\mathbb{E}_t\Big[r_{p,t+\tau}^{(\tau)}\Big] = \tau\Big(\mu_{f,t+\tau}^{(\tau)} + \alpha_t^{(\tau)\prime}(\mu_t^{(\tau)} + \frac{1}{2}\sigma^2) - \frac{1}{2}\alpha_t^{(\tau)\prime}\Sigma\alpha_t^{(\tau)}\Big), \tag{18}$$

$$Var_t\Big[r_{p,t+\tau}^{(\tau)}\Big] = \tau\Big(\sigma_f^{(\tau)2} + 2\alpha_t^{(\tau)\prime}\sigma_{A,f}^{(\tau)} + \alpha_t^{(\tau)\prime}\Sigma^{(\tau)}\alpha_t^{(\tau)}\Big), \tag{19}$$

where $\mu_{f,t}^{(\tau)}$ is the annualised τ-period mean return of the riskless asset, $\mu_{A,t}^{(\tau)}$ is the (C+B^*)x1 vector of annualised τ-period mean returns of the assets following $\mu_{A,t}^{(\tau)} = \frac{1}{\tau}\mathbb{E}_t[r_{A,t+\tau}^{(\tau)}]$, $\Sigma^{(\tau)}$ is the (C+B^*)x(C+B^*) annualised τ-period variance-covariance matrix defined by $\Sigma^{(\tau)} = \frac{1}{\tau}Var_t[r_{A,t+\tau}^{(\tau)}]$ and $\sigma_{A,f}^{(\tau)}$ is the (C+B^*)x1 covariance vector between the assets and the riskless asset.

Following Campbell & Viceira (2005), the first two moments of the τ-period asset returns, $\mu_{A,t+\tau}^{(\tau)}$ and $\Sigma_A^{(\tau)}$ respectively, can be consistently estimated from the VAR(1) framework as in (8), for a horizon of τ. This has implications for the term structure of the assets: by dynamically modelling the return and state variable dynamics, the conditional (co-)variances are not flat anymore, implying that the assets' risk differs across horizons. The τ-period forecasted expected returns $\hat{Y}_{T+\tau|T}$ are then constructed using Φ_0 and Φ_1 from the VAR(1) model of (8). This expectation is then used to construct the expected cumulative return $\mu^{(\tau)}_{A,T+\tau}$, by simply summing up the τ-period ahead expected return forecasts over the investment period, as in (21).

$$\hat{Y}_{t+j|t} = \sum_{i=0}^{j-1} \Phi_1^i \Phi_0 + \Phi_1^j Y_T \quad (20) \qquad \mu^{(\tau)}_{A,T+\tau} = \sum_{j=1}^{\tau} \hat{Y}_{T+j|T} = \sum_{j=1}^{\tau} \Big(\sum_{i=0}^{j-1} \Phi_1^i \Phi_0 + \Phi_1^j Y_T \Big) \tag{21}$$

Furthermore, the conditional (co-)variances over the different investment horizons are obtained as in (22).

$$\Sigma_A^{(\tau)} = \sum_{j=1}^{\tau} \Big(\Big(\sum_{i=0}^{j-1} \Phi_1^i \Big) \Sigma_A \Big(\sum_{i=0}^{j-1} \Phi_1^i \Big) \Big) \tag{22}$$

The optimal asset allocation, with constant proportions over the τ future periods, can then be derived by plugging in the results of (18) and (19) into the maximisation problem of (17) and setting the partial derivative function in terms of $\alpha_t^{(\tau)}$ equal to zero. This results in the following solution to the maximisation problem:

$$\alpha_t^{(\tau)} = \frac{1}{\gamma} \Big(\Big(1 - \frac{1}{\gamma}\Big) \Sigma_A^{(\tau)} + \frac{1}{\gamma} \Sigma_A \Big)^{-1} \Big(\mu^{(\tau)}_{A,t} + \frac{1}{2} \sigma_A^2 + \Big(1 - \gamma\Big) \sigma^{(\tau)}_{A,f} \Big), \tag{23}$$

where the solution essentially gives the optimal constant portfolio weights, where γ, $\mu^{(\tau)}_{A,t}$, Σ_A, $\Sigma_A^{(\tau)}$, σ_A^2 and $\sigma^{(\tau)}_{A,f}$ are still defined as before. The exact derivation of $\alpha_t^{(\tau)}$ can be found in section A.3 in Appendix A.

3.3.3 The ESG-Restricted Analytical Portfolio Choice Model

The approximate analytical solution in (23) is solely dependent on the risk and return of the assets. In an effort to make the portfolio achieve a higher ESG score, an ESG-integrated portfolio choice model is proposed which accommodates pension funds with an alternative policy towards reaching their return and ESG objectives: enforce a minimum ESG threshold score such that low ESG score stocks are not unequivocally excluded from the portfolio but rather efficiently reduce the holdings of such stocks, as it may still generate favorable risk-adjusted returns. The proposition is to add a constraint to the maximisation problem, where the portfolio's ESG score should satisfy a certain ESG threshold score $\overline{ESG}$:

$$\begin{aligned} \max_{\{\alpha_z\}_{z=t}^{t+\tau-1}} \quad & \mathbb{E}_t \Big[U(W_{t+\tau}) \Big] = \max_{\alpha^{(\tau)}_{t+\tau}} \mathbb{E}_t \Big[r^{(\tau)}_{p,t+\tau} \Big] + \frac{1}{2}(1-\gamma) Var_t \Big[r^{(\tau)}_{p,t+\tau} \Big], \\ \text{s.t.} \quad & \sum_{j \in \{E,S,G\}} w_j \alpha_t^{(\tau)\prime} ESG_{j,T} \geq \overline{ESG}, \text{for } \forall\, t, \end{aligned} \tag{24}$$

where $ESG_{j,T}$ is the matrix containing either the environmental, social and governance ESG component score of the ESG portfolios at time T, where $j \in \{E, S, G\}$, and w_j is the weight assigned to the ESG component, where $\sum_{j \in \{E,S,G\}} w_j = 1$. It is initially assumed that the investor equally values the ESG components, thus all w_j is set to 0.33. Also, $\overline{ESG}$ is initially set to 65: this is the 75th percentile of all the ESG scores and thus a score high enough to increase the holdings in the greener ESG portfolios. Note that this maximisation is still assumed to produce a buy&hold allocation, as in the return-only solution.

For ease of notation, the constraint can be rewritten to $\sum_{j\in\{E,S,G\}} w_j \alpha_t^{(\tau)\prime} ESG_{j,T} = \alpha_t^{(\tau)\prime} \sum_{j\in\{E,S,G\}} w_j ESG_{j,T} = \alpha_t^{(\tau)\prime} ESG_T$. Using this, the maximisation problem is also rewritten using the method of Lagrange multipliers, where the Lagrangian function $\Lambda(\alpha_t^{(\tau)}, \lambda)$ becomes:

$$\max_{\alpha_{esg,t}^{(\tau)}} \Lambda(\alpha_t^{(\tau)}, \lambda) = \max_{\alpha_{esg,t}^{(\tau)}} \mathbb{E}_t\Big[r_{p,t+\tau}^{(\tau)}\Big] + \frac{1}{2}(1-\gamma)Var_t\Big[r_{p,t+\tau}^{(\tau)}\Big] - \lambda\Big(\alpha_{esg,t}^{(\tau)}{}' ESG_T - \overline{ESG}\Big), \tag{25}$$

where λ denotes the lagrange multiplier of the ESG constraint. λ can be interpreted as a shadow price of ESG on the optimal wealth: the accumulated wealth from the optimal allocation may increase or decrease in value, depending on the direction of λ, if the allocation's ESG score deviates from $\overline{ESG}$ where potentially better risk-adjusted returns may be achieved for a slightly higher/lower ESG score. To derive the optimal ESG-integrated portfolio weights $\alpha_{esg,t}^{(\tau)}$, the partial derivatives of $\Lambda(\alpha_{esg,t}^{(\tau)}, \lambda)$ with respect to $\alpha_{esg,t}^{(\tau)}$ and λ are set to zero, which produces the following two expressions:

$$\alpha_{esg,t}^{(\tau)} = \frac{1}{\gamma}\Big(\Big(1-\frac{1}{\gamma}\Big)\Sigma^{(\tau)} + \frac{1}{\gamma}\Sigma\Big)^{-1}\Big(\mu_{A,t}^{(\tau)} + \frac{1}{2}\sigma^2 + \Big(1-\gamma\Big)\sigma_{A,f}^{(\tau)} - \lambda(ESG_T)\Big), \tag{26}$$

$$\alpha_{esg,t}^{(\tau)}{}' ESG_T = \overline{ESG}. \tag{27}$$

As can be seen, $\alpha_{esg,t}^{(\tau)}$ is similar to the return-only optimal portfolio weight from (23), but has an additional term $-\lambda(ESG_T)$, representing the ESG-correction term towards the optimal allocation, and the optimal allocation's ESG score needs to be equal to $\overline{ESG}$. By multiplying (26) from the left-hand-side (LHS) with ESG_T' and substituting (27) into (26), the optimal λ^* can be obtained and becomes:

$$\lambda^* = \frac{ESG_T'\Big(\frac{1}{\gamma}\Sigma + (1-\frac{1}{\gamma})\Sigma^{(\tau)}\Big)^{-1}\Big(\mu_{A,t+\tau}^{(\tau)} + \frac{1}{2}\sigma^2 + (1-\gamma)\sigma_{A,f}^{(\tau)}\Big) - \gamma\overline{ESG}}{ESG_T'\Big(\frac{1}{\gamma}\Sigma + (1-\frac{1}{\gamma})\Sigma^{(\tau)}\Big)^{-1} ESG_T}. \tag{28}$$

When this λ^* is substituted into (26), the optimal ESG-integrated portfolio weights $\alpha_{esg,t}^{(\tau)*}$ are obtained. For the details and exact derivation of λ^*, please refer to section A.4 in Appendix A. But this optimal allocation is unconstrained and can lead to extreme leveraged and short positions. As commonly performed at pension funds, the optimal portfolio weights are subjected to the short-selling constraint, following the procedure of Kole et al. (2006).

3.3.4 The Numerical Dynamic Portfolio Choice Model

Another common approach to long-term portfolio choice models is the numerical procedure. Firstly, the analytical solution is not exact: it depends on the log-linear approximation of log portfolio returns which may not hold when N is small. Secondly, the Bellman equations in (14) turn into a system of high-order integrated nonlinear equations, which may not be straightforward to solve (Brandt, 2010). Lastly, given that the analytical solution produces constant/buy&hold portfolio weights, it means that the value functions become independent of the state variables, implying that the allocation is myopically visioned. All these implications of the analytical solution can be addressed using the numerical approach of Binsbergen & Brandt (2007). This framework is primarily used to model dynamic investment strategies, where the allocation can alter freely in each period such that allocation adapts efficiently to the changing investment opportunities, quantified by the state with SV_t.

Unlike in the analytical solution, the numerical approach accounts for return predictability. As seen in the pseudocode below, the dynamic strategy optimises the expected terminal utility by changing the allocation in each time period, where the optimal allocation is made dependent on the simulated return and state variable scenarios from the VAR process, where S is the number of scenarios and s is a scenario, where $s \in \{1, \ldots, S\}$. The numerical approach finds the optimal asset allocation by iterating over a grid of portfolio allocations and finding the allocation with the highest average utility across the simulated scenarios. To also hedge against changing investment opportunities, across-path regressions are employed, where for each allocation on the grid g, the accumulated utility $U^{dyn}_{s,g,t}$ at time t is the dependent variable and the state variables SV_{t-1} act as the independent variables, taken from time $t-1$ to prevent data leakage. This regression estimates the conditional expected utility across the scenarios with a backward recursion, from time $T+\tau$ to T. Due to the backward recursion, the asset allocation with the highest conditional utility in each scenario is stored for all t, as it is used as input for the accumulated utility in the previous time periods (which is the next period in the backward recursion) as part of the optimal wealth function $W^*_{s,t}$, as seen in step 5 of the pseudocode. However at time T, the final period in the backward recursion, SV_{T-1} is known and thus equivalent across all the scenarios. As such, an across-path mean of the utility $\mathbb{E}^{dyn}_t[U(W_t|SV_t, \alpha_g)]$ is taken to find the optimal asset allocation which performs the best on average over all the scenarios. Please refer to the pseudocode for a clearer description of the numerical method.

Certain additional restrictions on α_t should be enforced to make the numerical approach feasible and logical from a pension fund perspective. The following three restrictions are applied:

$$\sum_i \alpha_{i,t} = 1, \; \forall \, t \text{ and } i \in r_t, \quad (29) \qquad \alpha_{i,t} \geq 0, \; \forall \, t \text{ and } i \in r_t, \quad (30)$$

$$|\alpha_{i,t-1} - \alpha_{i,t}| \leq 8\%, \; \forall \, t \text{ and for } j \in P, \quad (31)$$

where first the sum of portfolio weights should be equal to one. Secondly, no short positions and thus no leveraged positions are allowed for any asset. Lastly, the allocation for any asset should remain smooth and cannot alternate excessively, enforced by allowing the allocation to change at most 8% between consecutive periods. But with 11 included assets, the allocation grid grows exponentially large. Thus, finding the 'most' optimal asset allocation is sacrificed in favor of decreasing the allocation grid. This is achieved by only exploring increments in allocations

Algorithm 2 Numerical method for dynamic asset allocation

1: **Data :** Return R_t scenario sets, state variables SV_t, asset allocation grid $\alpha_1, \ldots, \alpha_G$ that satisfy the constraints in (29)-(31) and adjust the grid of portfolio weights if the ESG restriction needs to be included from (24).
2: **for** $t = T + \tau - 1, \ldots, 1$ **do**
3:     **for** $g = 1, \ldots,$ allocations in grid **do**
4:         Set $\alpha_t = \alpha_g$ for all s and obtain wealth $W_{s,t}$, as in (13)
5:         Retrieve dynamic utility per scenario $U^{dyn}_{s,g,t} = \frac{1}{1-\gamma}(W_{s,t})^{1-\gamma} \cdot \prod_{i=t+1}^{T+\tau-1}(W^*_{s,i})^{1-\gamma}$
6:         Extract fitted utility $\hat{U}^{dyn}_{1:S,g,t}$ from across-path regression $\hat{U}^{dyn}_{1:S,g,t} = \hat{c} + \hat{\delta}' SV_{1:S,t} + \varepsilon_t$
7:         Obtain across-path dynamic utility mean $\mathbb{E}^{dyn}_{g,t}[U(W_t|SV_t, \alpha_g)] = \frac{1}{S}\sum_{s=1}^{S} \hat{U}^{dyn}_{s,g,t}$
8:     Extract allocation that maximises utility per scenario s from $\hat{U}^{dyn}_{1:S,g,t}$ at time t, α^*_g, which
9:     you use for W^*_t in line 5 to determine dynamic utilities further in the backward recursion.
10:     Choose the allocation that maximises the conditional expected utility from all
11:     across-path means, which is the optimal asset allocation at time t.

of ± 4% between consecutive periods. Besides these constraints, the numerical solution is also extended towards an ESG-integrated asset allocation, by applying the same minimum ESG threshold constraint to the grid of asset allocations, as for the analytical solution in (24).

3.3.5 Performance Metrics

Throughout the methodology, 8 different portfolio choice models are described, analysed across different investment horizons between 1 and 15 years, which are benchmarked against two naïve $\frac{1}{N}$ strategies: the fair $\frac{1}{N}$ model where all asset classes (equity, riskless asset, long-term bond and corporate bond) receive the same weight and the $\frac{1}{N}$ model where all assets are assigned the same weight. The fair $\frac{1}{N}$ model essentially results in a less risky portfolio, as the ESG portfolios receive a smaller weight. The performance of these models are based on the Certainty Equivalent Rate, the Sharpe ratio, the mean and volatility of the terminal wealth, the allocation's turnover and the average ESG score across time. To benchmark the models consistently, the portfolio choice models' performance is determined out-of-sample: from the simulated scenarios, a randomised 80/20 train/test split is made, where the optimal allocation is obtained using the training set while the performance metrics benchmark the models using the test set.

The first performance metric is the Certainty Equivalent Rate (CER). The CER is the certain amount for which the investor's utility is equivalent to the expected utility from the uncertain outcome of the return scenarios. It considers the wealth's utility per scenario, something which is not recognised when only looking at the terminal wealth. By initialising the wealth at time T with $W_{s,T} = 1$ and using the wealth's recursion as in (10), the utility of the terminal wealth $W_{s,T+\tau}$ is obtained for each scenario s, using the power utility function. To obtain the CER, the inverse of the power utility function in terms of the terminal wealth's expected utility (EU) is taken, as shown in the following expressions:

$$EU = \frac{1}{S_{test}} \sum_{s=1}^{S_{test}} U_\gamma(W_{s,T+\tau}), \quad (32) \qquad CER = U_\gamma^{-1}(EU). \quad (33)$$

Besides using the terminal wealth for the CER, the mean and the standard deviation of the terminal wealth are also extracted. The next performance metric is the turnover, used to quantify the trading volume of a certain model and gives a good estimate of the level of transaction costs a certain investment strategy accumulates. The turnover is attained as follows:

$$\text{Turnover} = \frac{1}{\tau} \sum_{t=T}^{T+\tau} \sum_{j=1}^{N} (|\alpha_{j,t+1} - \alpha_{j,t+}|), \quad (34)$$

where $\alpha_{j,t+1}$ is the optimal weight of asset j at time $t+1$ and $\alpha_{j,t+}$ is the asset allocation before re-balancing at time $t+1$, obtained by multiplying the optimal asset allocation with $\frac{r_{j,t}}{r_{p,t}}$, which is the contribution of asset j's return to the portfolio return $r_{p,t}$ at time t. Note that a higher turnover implies that the allocation requires more trading and thus higher transaction costs, often seen as a disadvantage of the dynamic allocation. Moreover, the Sharpe Ratio (SR) is also considered, one of most common tools to compare the risk-adjusted returns of different models. In this paper, the average SR across the scenarios is taken, thus following:

$$SR_\tau = \frac{1}{S_{test}} \sum_{s=1}^{S_{test}} \frac{r_{p,s,T->T+\tau} - r_{f,s,T->T+\tau}}{\sigma_{p,s,T->T+\tau}}, \quad (35)$$

where the numerator is the cumulative excess portfolio return for scenario s and investment horizon τ and the denominator is the standard deviation of the portfolio's excess return for the same horizon and scenario. Lastly, the models' ESG score over time is explored, following a straightforward procedure: the score is obtained by multiplying the optimal asset allocation with the ESG score, weighted by the weight assigned to each ESG component. This thus follows the following expression: $ESG_{t+i} = w_E \alpha_{t+i}^{*}{}' E_T + w_S \alpha_{t+i}^{*}{}' S_T + w_G \alpha_{t+i}^{*}{}' G_T$.

3.4 Sensitivity Analysis

The following section explores the sensitivity of the portfolio choice models towards changing ESG demands and preferences. Note that the below analyses are only benchmarked against the best-performing return-only portfolio choice model, based on the above performance metrics.

3.4.1 Threshold ESG Score

The ESG-integrated portfolio choice models establish the optimal allocation by adhering to a minimum threshold ESG score, $\overline{ESG}$, initially set to 65. However, each investor may desire different values of $\overline{ESG}$. As such, $\overline{ESG}$ is varied between 35 and 95 with an increment of 10. This should reveal how the optimal portfolio reacts to changing ESG demands.

3.4.2 Investor's ESG preferences

For the ESG-integrated models, it is assumed that the investor equally values all ESG components. However, as with risk-aversion, every investor has unique preferences towards the ESG components, which may impact the optimal asset allocation and thus its risk-return performance. Thus, a set of six exemplary ESG-minded investor with different ESG preferences are analysed, as displayed in Table 1. It contains investors with 'extreme' ESG preferences towards a single ESG component and investors which have interests in all ESG components but with higher preferences towards a certain ESG component.

Table 1: The six ESG investor with different ESG preferences for the sensitivity analysis

	w_E	w_S	w_G
investor 1	0.7	0.2	0.1
investor 2	0.1	0.8	0.1
investor 3	0.2	0.1	0.7
investor 4	1	0	0
investor 5	0	1	0
investor 6	0	0	1

As with the changing threshold scores, these investors are compared with each other in terms of the above-mentioned performance metrics, using the best-performing portfolio choice model when looking at the risk-return performance of all the above portfolio choice models.

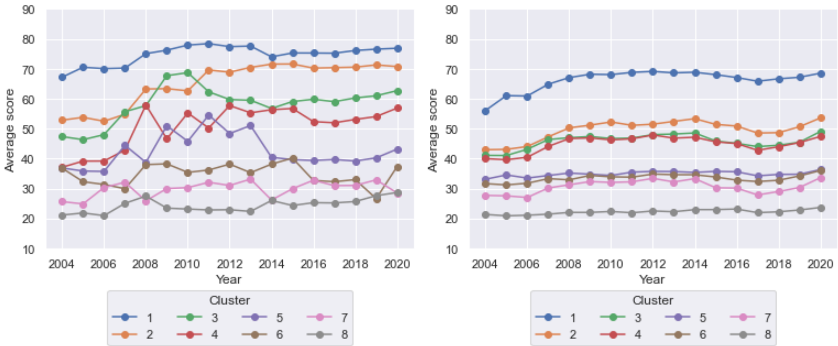
4 Data

4.1 Historical Data

The optimal asset allocation is determined across the riskless asset, the long-term bond, the corporate bond and the 8 ESG portfolios. The riskless asset $r_{f,t}$ is represented by the 3-month Treasury-bill (T-bill), as it is one of the safest assets and provides a good hedge against inflation. The long-term bond $r_{lb,t}$ is proxied by the 10-year US zero-coupon bond, such that interest rate and annuity risks can be accounted for. The corporate bond $r_{cor,t}$ follows the Moody's Baa corporate bond, as used in Shen et al. (2019). The monthly nominal yields of these bonds are collected from the Federal Reserve Economic Data (FRED) of Saint Louis website (The Federal Reserve System (US), 2023). Even though $r_{f,t}$, $r_{lb,t}$ and $r_{cor,t}$ date from 1982, 1962 and 1919 respectively, the overlapping starting date of 1982 is chosen which still provides a sufficiently large sample size.

For the ESG portfolios, stocks from the developed financial markets of North America, Europe and Asia that have any annual E, S and G component data ranging from 2004 up until 2020 available in the Thomson Reuters database are considered. It is the largest ESG database, covering 80% of the global market cap (Refinitiv, 2022), providing compressed E, S and G component scores between 0 and 100. The precise topics for each ESG component can be found in Figure B.1 in Appendix B. As the ESG portfolios are created every year based on the stocks' ESG component scores, a stock is only included if it has ESG scores in the year before the portfolio creation period. The monthly closing stock price is collected via the Yahoo Finance Python API *yfinance* (Aroussi, 2019) from 1997 onwards and merged with the ESG data based on the ISIN codes. This amounts to 6045 stocks, where also the survivorship, selection and forward-looking biases are accounted for. Table B.2 in Appendix B also shows a further breakdown of the number of stocks per sector (as defined by Thomson Reuters), per region.

To fulfill the assumption that returns are predictable, the following state variables are used in the VAR(1) model and also sampled at a monthly frequency: the price-earnings ratio PE_t, represented by the PE ratio of the S&P 500 index, is obtained from Bloomberg. This PE ratio is included as it has predictive power in stock returns (Campbell & Shiller, 1988) and the S&P 500 index captures most of the worldwide market cap. Second, the yield spread YS_t is used as it can predict interest rate dynamics (Campbell & Shiller, 1991) and future bond returns (Brandt & Santa-Clara, 2006). Furthermore, the credit spread CS_t is also employed, as it can explain bond and corporate bond returns (Shen et al., 2019). In addition, the ex-post real rate rr_t is used as in Shen et al. (2019), extracted using the 3-month T-bill and the log inflation rate π_t. π_t is obtained from the Consumer Price Index CPI_t from the FRED website with a 3-month lag, to account for lag biases. Lastly, to integrate climate risk factors, temperature change ΔT_t is retrieved with the monthly land-surface average temperature anomalies, relative to the 1951-1980 average temperature from the Berkeley Earth website (Berkeley, 2023).



(a) K-means sorting

(b) Simple sorting

Figure 2: The average total ESG score for the ESG portfolios under K-means and simple sorting.

Based on the K-means and simple sorting procedures, the 8 ESG portfolios' total ESG scores over time are displayed in Figure 2, ranked 1 to 8 from most ESG-compliant to least. The distinct E, S and G component scores can be found in Figures B.3 and B.4 in Appendix B. It can be seen that generally all ESG portfolios enjoy an increasing trend, which fades away after 2010, with all the portfolios' ESG scores becoming less volatile. ESG portfolios 1 and 2 have ESG scores mostly between 65 and 80 from 2010 onwards, the average portfolios 3 and 4 are the best improving portfolios but remain around 60 in 2020, while portfolios 7 and 8 never exceed scores of 35 and 30, respectively. Compared to K-means sorting, simple sorting notably produces relatively lower ESG scores, where the most ESG-compliant portfolios fluctuate only around 70, while portfolio 8 has a score of about 20. It is also seen that portfolios 2 to 4 and portfolios 5 to 7 all have similar ESG scores over time. As seen in Table B.5 in Appendix B, the industry distribution among the ESG portfolios does reveal that the worst ESG portfolios are more represented by 'Industrial' and 'Consumer Discretionary', while 'Financials','Health Care' and 'Real Estate' industry stocks occupy more the centered ESG portfolios.

Table 2: Descriptive statistics for the monthly excess log returns of the ESG portfolios under K-means and simple sorting, the other asset classes and the state variables.

	Mean	**Std. dev.**	**SR**	**Skew**	**Kurt**	**Min**	**Max**
K-means ESG portfolios							
Cluster 1 ($r_{c1,t}$)	0.004	0.064	0.059	-2.213	12.572	-0.433	0.186
Cluster 2 ($r_{c2,t}$)	0.005	0.060	0.082	-0.680	3.522	-0.282	0.262
Cluster 3 ($r_{c3,t}$)	0.007	0.058	0.127	-0.908	4.532	-0.273	0.209
Cluster 4 ($r_{c4,t}$)	0.005	0.054	0.096	-0.976	2.439	-0.236	0.138
Cluster 5 ($r_{c5,t}$)	0.005	0.057	0.082	-0.338	1.036	-0.189	0.186
Cluster 6 ($r_{c6,t}$)	-0.001	0.075	-0.011	-0.749	10.041	-0.419	0.455
Cluster 7 ($r_{c7,t}$)	0.005	0.065	0.073	0.528	6.890	-0.212	0.446
Cluster 8 ($r_{c8,t}$)	0.002	0.065	0.027	-0.796	3.770	-0.324	0.222
Simple sorting ESG portfolios							
Cluster 1 ($r_{c1,t}$)	0.007	0.069	0.098	-0.372	3.841	-0.308	0.325
Cluster 2 ($r_{c2,t}$)	0.002	0.074	0.023	-0.725	3.079	-0.342	0.231
Cluster 3 ($r_{c3,t}$)	0.010	0.057	0.174	0.526	8.981	-0.237	0.406
Cluster 4 ($r_{c4,t}$)	0.004	0.060	0.068	-0.820	1.433	-0.211	0.144
Cluster 5 ($r_{c5,t}$)	0.003	0.060	0.043	-1.072	3.687	-0.297	0.158
Cluster 6 ($r_{c6,t}$)	0.004	0.065	0.057	-0.177	1.871	-0.235	0.220
Cluster 7 ($r_{c7,t}$)	0.005	0.068	0.069	-1.066	6.793	-0.410	0.243
Cluster 8 ($r_{c8,t}$)	0.001	0.060	0.012	-0.610	1.464	-0.263	0.145
Return of other asset classes							
Riskless asset ($r_{f,t}$)*	0.003	0.003	0.000	0.709	0.468	0.000	0.015
Long-term bond ($r_{lb,t}$)	0.003	0.022	0.139	0.088	0.775	-0.075	0.089
Corporate bond ($r_{cor,t}$)	0.006	0.021	0.263	-1.079	7.954	-0.156	0.077
State variables							
real rate (rr_t)	0.034	0.029	-	0.481	-0.511	-0.008	0.128
yield spread (YS_t)	0.017	0.011	-	-0.109	-0.936	-0.007	0.037
credit spread (CS_t)	0.022	0.007	-	1.432	4.291	0.007	0.059
PE ratio (PE_t)	18.667	4.838	-	0.230	-0.208	7.310	30.770
temperature change (ΔT_t)	0.001	0.037	-	-0.005	0.769	-0.140	0.150
log inflation rate (π_t)	0.002	0.003	-	-1.109	9.603	-0.018	0.014

Note. * The riskless asset is denoted as a normal return, as $r_{f,t}$ is used as the risk-free rate for excess returns.
Note. Historic data of equity starts in 1997 while bond and state variable historic data in 1982.

The descriptives of the return and state variables are displayed in Table 2 over the historical data. The ESG portfolios' average monthly excess return range between -0.1% and 1%, with the minimum from cluster 6 under K-means sorting and the maximum from cluster 3 under simple sorting. Cluster 3 records the highest excess return being 70 and 100 basis points respectively, whereas the most ESG-compliant portfolio 1 lacks behind by 30 basis points. As expected, equity is more risky than other asset classes, as seen by the higher volatility and returns ranging between -43% and 40%. When comparing the extreme clusters 1 and 8, cluster 1's SR exceeds cluster 8's SR by at least 4% under both sortings. It can also be observed that the centered ESG portfolios have relatively lower volatility, as cluster 3 has the highest risk-adjusted return with SRs of 12.7% and 17.4% due to its lower volatility. The upper tail portfolios (clusters 1 to 3) do have a higher risk-adjusted return compared to the lower tail portfolios (cluster 6 and 8), except for cluster 2. ESG investing thus does not sacrifice return performance relative to the lower tail ESG portfolios, but the higher ESG portfolios do not achieve higher risk-adjusted returns compared to the centered ESG portfolios, as seen with cluster 3. In combination with the conclusion from Figure 2 that cluster 3 has the best improvement in ESG score over time, the historic data suggests that (strongly) improving ESG portfolios are rewarded with the best risk-adjusted returns.

Among the remaining asset classes, it can be observed that all the bonds have considerably lower volatility, where the returns range between -15.6% and 8.9%, while the riskless asset does not even have any downside risk, which is in line of expectations that bonds are less risky than equity. The lower volatility results in lower average returns, with $r_{lb,t}$ having the lowest with 30 basis points and $r_{cor,t}$ being the highest at 60 basis points, which still exceeds 6 out of 8 ESG portfolios. The very low volatility of bonds results in high SRs for the corporate bond (26.3%), outperforming all the ESG portfolios, and for the long-term bond (13.9%). Nevertheless, it can be argued that the SRs of the ESG portfolios are not at a higher level, due to these portfolios being highly diversified.

Table 3: The correlation matrix for excess log returns of the K-means clustered ESG stock portfolios, the remaining asset classes and the state variables

	$r_{c1,t}$	$r_{c2,t}$	$r_{c3,t}$	$r_{c4,t}$	$r_{c5,t}$	$r_{c6,t}$	$r_{c7,t}$	$r_{c8,t}$	$r_{f,t}$	$r_{lb,t}$	$r_{cor,t}$	rr_t	YS_t	CS_t	PE_t	ΔT_t	π_t
$r_{c1,t}$	1.00																
$r_{c2,t}$	0.70	1.00															
$r_{c3,t}$	0.77	0.71	1.00														
$r_{c4,t}$	0.73	0.66	0.65	1.00													
$r_{c5,t}$	0.62	0.69	0.58	0.57	1.00												
$r_{c6,t}$	0.62	0.58	0.60	0.52	0.48	1.00											
$r_{c7,t}$	0.56	0.55	0.58	0.55	0.54	0.49	1.00										
$r_{c8,t}$	0.70	0.62	0.66	0.68	0.62	0.57	0.57	1.00									
$r_{f,t}$	-0.06	-0.08	-0.11	-0.16	-0.04	-0.11	-0.02	-0.09	1.00								
$r_{lb,t}$	-0.24	-0.30	-0.27	-0.29	-0.32	-0.22	-0.24	-0.32	0.06	1.00							
$r_{cor,t}$	0.29	0.05	0.20	0.22	0.05	0.16	0.12	0.23	-0.14	0.37	1.00						
rr_t	0.04	0.00	-0.01	-0.04	-0.01	-0.03	0.06	0.03	0.95	0.01	-0.08	1.00					
YS_t	-0.07	-0.02	0.02	0.05	0.01	0.03	-0.02	-0.01	-0.64	-0.09	0.00	-0.66	1.00				
CS_t	-0.17	-0.13	-0.11	-0.10	-0.07	-0.12	-0.16	-0.12	-0.43	0.22	0.09	-0.48	0.45	1.00			
PE_t	0.13	0.12	0.11	0.06	0.11	0.10	0.04	0.17	0.35	-0.12	-0.03	0.35	-0.25	-0.31	1.00		
ΔT_t	0.03	0.07	-0.03	0.04	0.04	0.02	0.04	0.04	0.03	0.02	0.07	0.06	-0.04	0.01	-0.05	1.00	
π_t	-0.04	-0.01	-0.09	-0.08	-0.04	-0.06	-0.06	-0.11	0.08	-0.07	-0.14	-0.04	-0.07	-0.32	0.15	-0.09	1.00

The correlations of the K-means sorted excess return and state variable data are presented in Table 3, with the correlations under simple sorting to be found in Table C.1 in Appendix C. In general, all ESG portfolios are positively and highly correlated with each other between 0.48 and 0.77, where the correlation is lower when the portfolios' ESG scores are closer towards each other. Besides equity, all ESG portfolios are negatively correlated with the long-term bond and riskless asset, showing that these bonds become a safe asset in times of downturns in the equity markets, but the correlations are rather negligible and minimally -0.32. However, all ESG portfolios are positively correlated with the corporate bond, suggesting that the corporate bond is a riskier investment product and also becomes less attractive during economic downturns. Between the bond classes, the long-term bond is positively correlated to both bonds, while the corporate bond is negatively correlated with the riskless asset at a low level of -0.14. In terms of the state variables, the correlation with return variables are not significantly different from zero, with the exception of the riskless asset: $r_{f,t}$ is positively correlated with PE_t and rr_t but negatively with YS_t and CS_t. Note the correlation with rr_t of 0.95 is exceptionally high and it may create collinearity issues within the VAR(1) model. Lastly, ΔT_t can be noted to not have any significant correlation with any of the financial variables.

Table 4: The first-order autocorrelation matrix for excess log returns of the K-means clustered ESG stock portfolios, the remaining asset classes and the state variables

	$r_{c1,t}$	$r_{c2,t}$	$r_{c3,t}$	$r_{c4,t}$	$r_{c5,t}$	$r_{c6,t}$	$r_{c7,t}$	$r_{c8,t}$	$r_{f,t}$	$r_{lb,t}$	$r_{cor,t}$	rr_t	YS_t	CS_t	PE_t	ΔT_t	π_t
$r_{c1,t-1}$	0.21	0.07	0.14	0.23	0.06	0.11	0.10	0.22	-0.01	-0.24	0.31	0.05	-0.06	-0.33	0.15	0.00	0.07
$r_{c2,t-1}$	0.18	0.04	0.13	0.14	0.15	0.13	0.14	0.23	-0.05	-0.21	0.20	0.02	-0.01	-0.25	0.13	0.02	0.00
$r_{c3,t-1}$	0.09	0.02	0.05	0.13	0.04	0.05	0.05	0.13	-0.06	-0.25	0.19	-0.01	0.04	-0.24	0.10	0.00	0.02
$r_{c4,t-1}$	0.12	0.07	0.11	0.17	0.09	0.07	0.16	0.16	-0.09	-0.18	0.21	-0.03	0.05	-0.22	0.06	0.01	0.00
$r_{c5,t-1}$	0.14	0.07	0.11	0.18	0.12	0.05	0.11	0.19	-0.05	-0.17	0.15	0.01	0.02	-0.17	0.12	0.02	-0.01
$r_{c6,t-1}$	0.16	0.05	0.10	0.13	0.08	-0.05	0.06	0.15	-0.06	-0.14	0.18	-0.03	0.04	-0.21	0.10	0.00	0.04
$r_{c7,t-1}$	0.12	0.00	0.04	0.12	0.05	0.01	0.05	0.14	0.01	-0.15	0.18	0.07	-0.02	-0.26	0.04	-0.01	-0.01
$r_{c8,t-1}$	0.08	0.02	0.03	0.12	0.07	-0.01	0.06	0.12	-0.01	-0.23	0.14	0.03	0.01	-0.24	0.15	-0.03	0.01
$r_{f,t-1}$	-0.01	-0.04	-0.06	-0.09	0.00	-0.07	0.03	-0.01	0.95	0.02	-0.14	0.94	-0.63	-0.39	0.35	0.04	0.06
$r_{lb,t-1}$	0.01	-0.05	0.05	0.01	0.01	-0.04	-0.02	0.06	0.03	0.05	0.23	0.02	-0.08	0.18	-0.11	0.12	-0.24
$r_{cor,t-1}$	0.00	-0.02	-0.01	0.04	-0.02	-0.05	-0.03	0.01	-0.11	-0.12	0.22	-0.11	0.02	-0.01	-0.02	0.11	0.07
rr_{t-1}	0.00	-0.04	-0.07	-0.08	-0.03	-0.08	0.01	-0.02	0.97	0.01	-0.10	0.99	-0.66	-0.46	0.35	0.06	0.03
YS_{t-1}	-0.07	-0.04	0.01	0.06	-0.04	0.03	-0.03	-0.02	-0.67	0.08	0.10	-0.65	0.97	0.45	-0.27	-0.03	-0.07
CS_{t-1}	-0.01	-0.01	0.03	0.05	0.05	0.00	-0.05	0.04	-0.48	-0.01	0.22	-0.50	0.49	0.94	-0.28	0.03	-0.33
PE_{t-1}	-0.01	-0.03	-0.03	-0.06	-0.02	-0.03	-0.07	0.04	0.36	-0.09	-0.07	0.36	-0.24	-0.32	0.97	-0.06	0.18
ΔT_{t-1}	-0.03	0.03	-0.08	0.00	0.02	-0.02	0.00	-0.04	0.04	0.11	0.10	0.05	-0.06	0.02	-0.04	0.34	-0.06
π_{t-1}	0.02	0.03	0.02	-0.03	0.05	0.05	0.04	-0.01	0.13	0.07	-0.09	0.04	-0.07	-0.26	0.14	-0.19	0.46

Given that a VAR(1) model is used to simulate the asset returns, it is useful to look at the first-order (cross-)autocorrelations under K-means sorting, as displayed in Table 4. The autocorrelations under simple sorting can be found in Table C.2 in Appendix C. By inspecting the diagonal, most assets have a low autocorrelation, because equity returns are known to be mainly exposed to idiosyncratic shocks and thus difficult to predict. Only the riskless asset has a high autocorrelation of 0.95, because unlike equity, bond returns are more persistent and thus more predictable with past returns, which is in line with the common stylized facts of bond yields/returns. Besides the riskless asset, the real rate, yield spread, credit spread and PE ratio are also persistent state variables with high autocorrelations. Among the cross-autocorrelations of the ESG portfolios, it can be noted that ESG portfolios do not cross-correlate well with other ESG portfolio returns as the highest autocorrelation does not exceed 0.32, though it can be noted that clusters 1, 4 and 8 are two to three times more cross-autocorrelated compared to the remaining ESG portfolios. As also highlighted among the correlations, $r_{f,t}$ and rr_t are highly autocorrelated with each other, being 0.94 and 0.99 respectively, which again poses risks towards having multicollinearity issues in the VAR(1) model. Furthermore, in contrast to return variables, state variables are more persistent as the autocorrelations are higher as well as cross-autocorrelations. These findings show that a VAR(1) model may have some difficulty uncovering (significant) relationships between the return variables, but perhaps may have more significant coefficients among the state variables.

4.2 Simulated Data

As stated, the restricted and unrestricted VAR(1) model is used to simulate monthly forecasted returns and state variables for a horizon of 15 years. The simulated variables are aggregated at an annual frequency through accumulating returns throughout the year. However, for the yield spread, credit spread and PE ratio, as these variables cannot be accumulated over time, the average across the year is taken. The simulated cumulative return paths of the equity (for simplicity, the average return across the ESG portfolios is taken), the riskless asset, the long-term bond and the corporate bond asset classes can be seen in Figure C.3 in Appendix C, which clearly portrays the riskiness of each asset class.

Table 5: Descriptive statistics of the simulated annual excess log returns under K-means sorting, the other asset classes and the state variables of the training and testing set.

	Train set			**Test set**		
	Mean	**Std. dev.**	**SR**	**Mean**	**Std. dev.**	**SR**
K-means ESG portfolios						
Cluster 1 ($r_{c1,t}$)	0.043	0.059	0.735	0.043	0.059	0.739
Cluster 2 ($r_{c2,t}$)	0.079	0.048	1.656	0.080	0.047	1.703
Cluster 3 ($r_{c3,t}$)	0.077	0.049	1.554	0.078	0.049	1.605
Cluster 4 ($r_{c4,t}$)	0.070	0.047	1.474	0.071	0.047	1.516
Cluster 5 ($r_{c5,t}$)	0.052	0.051	1.027	0.052	0.050	1.037
Cluster 6 ($r_{c6,t}$)	0.002	0.061	0.026	0.002	0.062	0.029
Cluster 7 ($r_{c7,t}$)	0.033	0.098	0.342	0.031	0.095	0.326
Cluster 8 ($r_{c8,t}$)	0.020	0.058	0.355	0.022	0.058	0.382
Return of other asset classes						
Riskless asset ($r_{f,t}$)	0.031	0.013	0.000	0.031	0.013	0.000
Long-term bond ($r_{lb,t}$)	0.009	0.012	0.724	0.009	0.012	0.764
Corporate bond ($r_{cor,t}$)	0.031	0.017	1.814	0.032	0.017	1.888
State variables						
real rate (rr_t)	0.161	0.186	-	0.157	0.180	-
yield spread (YS_t)	0.014	0.004	-	0.014	0.005	-
credit spread (CS_t)	0.022	0.003	-	0.022	0.003	-
PE ratio (PE_t)	21.677	1.946	-	21.754	1.999	-
temperature change (ΔT_t)	0.040	0.050	-	0.038	0.051	-
log inflation rate (π_t)	0.024	0.006	-	0.024	0.006	-

Using the training and testing sets, the descriptives of the annualised simulated variables under K-means sorting are displayed in Table 5. The more detailed descriptives can be found in Tables C.4 and C.5 in Appendix C. It can be seen that given the relatively large sample sizes of the training and testing sets, the sets approximately converge to the same mean and stdev for all variables. Regarding the ESG portfolios, rather than cluster 3 as in the historic data, now cluster 2 obtains the highest average annual excess return of 7.9%, closely followed by cluster 3 sitting at 7.7%. The lowest recorded annual excess return is from cluster 6, sitting at 20 basis points. This is way below other portfolios, likely due to the portfolio performing poorly at the end of its sample, thus heavily impacting all its simulated paths. While the centered ESG portfolios enjoy lower volatility compared to the tail ESG portfolios in the historic data, the upper tail ESG portfolios now match their volatility, but the lower tail ESG portfolios remain with higher volatility in combination with low returns, thus making their risk-adjusted return still less attractive to invest into compared to the remaining ESG portfolios. Lastly, compared to their historic mean estimates, only clusters 2 and 4 manage to obtain a higher simulated excess return, while the rest of the ESG portfolios achieve a lower simulated average return.

With the remaining asset classes, the corporate bond still remains an attractive investment opportunity compared to some ESG portfolios, with a return of 3.1% on average, but with a much lower volatility and thus resulting again in the highest SR. The average annual excess return of the long-term bond sits at 90 basis points, which is in line with the historic data that long-term bonds offer low but stable returns. In terms of the state variables, the inflation rate, yield spread and credit spread remain the same, while the real rate almost halves compared to its historic mean estimate. The PE ratio's mean increases from 18.7 to 21.7, while temperature change increases three to four times compared to its historic estimate. This is likely influenced by the larger recorded temperature changes witnessed in the last few years due to climate change. Nevertheless, this higher simulated temperature change falls in line with a more pessimistic climate scenario, and thus assimilates a future of more severe climate change.

5 Results

As introduced in the methodology, the approximate analytical solution deriving the buy&hold allocation and the numerical solution obtaining a dynamic allocation are used. For each of these models, a return-only and an ESG-integrated asset allocation are explored, where the ESG-integrated solution needs to fulfill the ESG threshold score $\overline{ESG}$, set to 65 for the main results. Furthermore, for each of above models, the ESG portfolios are either generated by means of the K-means or simple sorting procedure. With all the possible combinations, this results in 8 portfolio choice models which are studied below in terms of the performance metrics and benchmarked against the standard and fair $\frac{1}{N}$ strategies.

5.1 VAR(1) Model Estimation Results

The used scenario set to obtain the optimal asset allocation for all the models is obtained through the VAR(1) framework of Stambaugh (1997). While this framework can handle non-stationary variables, the variables still undergo a ADF test for stationarity, where the results are shown in Table D.1 in Appendix D. All variables are found to be stationary at a 1% level, except for $r_{f,t}$ and rr_t not rejecting the H_0 of non-stationarity at a 10% level. Also considering that $r_{f,t}$ and rr_t are strongly correlated, $r_{f,t}$ is removed from the VAR(1) model and modelled with a univariate AR(1) model. This is done to avoid multicollinearity issues while still having the influence of $r_{f,t}$ through rr_t in the VAR(1) model, as well as keeping the $r_{f,t}$ dynamics consistent with the VAR(1) assumption: all variable dynamics should be modelled with its lagged values.

Table 6: The restricted and unrestricted VAR(1) and AR(1) model coefficients and significance

Restricted VAR(1) and riskless asset's AR(1) models									
	Lagged $Y_{1,t}$ coefficients: A_1								AR(1) coefficients
	rr_t	YS_t	CS_t	PE_t	ΔT_t	π_t	$r_{lb,t}$	$r_{cor,t}$	$r_{f,t}$
μ	0.00 (0.00)	0.00 (0.00)***	0.00 (0.00)**	0.33 (0.40)	0.02 (0.01)	0.00 (0.00)**	0.01 (0.01)	-0.01 (0.01)*	0.00 (0.00)***
rr_{t-1}	0.98 (0.01)***	0.01 (0.01)**	0.00 (0.01)	-0.55 (2.17)	-0.05 (0.07)	0.01 (0.00)	0.09 (0.05)**	0.11 (0.04)***	-
YS_{t-1}	0.03 (0.02)	0.95 (0.01)***	0.01 (0.01)	-10.37 (4.74)**	-0.05 (0.16)	0.00 (0.01)	0.25 (0.10)**	0.18 (0.09)**	-
CS_{t-1}	-0.05 (0.03)	0.11 (0.03)***	0.94 (0.02)***	16.73 (8.90)*	-0.28 (0.30)	-0.05 (0.02)***	-0.11 (0.19)	0.52 (0.16)***	-
PE_{t-1}	0.00 (0.00)	0.00 (0.00)**	0.00 (0.00)*	0.97 (0.01)***	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)*	0.00 (0.00)	-
ΔT_{t-1}	0.00 (0.00)	-0.01 (0.00)**	0.00 (0.00)	0.71 (1.27)	0.34 (0.04)***	0.00 (0.00)	0.06 (0.03)**	0.04 (0.02)*	-
π_{t-1}	0.35 (0.06)***	0.13 (0.06)**	0.08 (0.05)*	1.46 (19.33)	-1.80 (0.65)***	0.42 (0.04)***	0.78 (0.40)*	0.21 (0.35)	-
$r_{lb,t-1}$	0.02 (0.01)**	-0.01 (0.01)	-0.01 (0.01)**	-0.63 (2.52)	0.06 (0.08)	-0.03 (0.01)***	0.14 (0.05)***	0.29 (0.05)***	-
$r_{cor,t-1}$	-0.02 (0.01)**	0.01 (0.01)	-0.02 (0.01)***	3.08 (2.51)	0.00 (0.08)	0.02 (0.01)***	-0.09 (0.05)*	0.13 (0.05)***	-
$r_{f,t-1}$	-	-	-	-	-	-	-	-	0.94 (0.02)***
Unrestricted VAR(1) model									
	$r_{c1,t}$	$r_{c2,t}$	$r_{c3,t}$	$r_{c4,t}$	$r_{c5,t}$	$r_{c6,t}$	$r_{c7,t}$	$r_{c8,t}$	
α	0.03 (0.02)	0.03 (0.02)	0.03 (0.02)	0.02 (0.02)	0.01 (0.02)	0.03 (0.03)	0.06 (0.03)**	-0.02 (0.02)	
	Lagged $Y_{2,t}$ coefficients: H_1								
$r_{c1,t-1}$	0.01 (0.09)	-0.09 (0.08)	-0.02 (0.08)	0.08 (0.08)	-0.28 (0.09)	0.09 (0.11)	-0.24 (0.11)	-0.03 (0.09)	
$r_{c2,t-1}$	0.02 (0.08)	-0.16 (0.07)	0.01 (0.07)	-0.15 (0.07)	0.09 (0.08)	0.15 (0.10)	0.02 (0.10)	0.09 (0.08)	
$r_{c3,t-1}$	-0.14 (0.08)	-0.01 (0.08)	-0.12 (0.08)	-0.05 (0.07)	-0.04 (0.08)	-0.01 (0.11)	-0.08 (0.10)	-0.13 (0.08)	
$r_{c4,t-1}$	-0.06 (0.08)	0.07 (0.08)	0.06 (0.08)	0.01 (0.07)	0.09 (0.08)	-0.04 (0.11)	0.27 (0.10)***	-0.01 (0.08)	
$r_{c5,t-1}$	0.00 (0.07)	0.05 (0.07)	0.05 (0.07)	0.07 (0.06)	0.05 (0.07)	-0.08 (0.09)	0.05 (0.09)	0.06 (0.07)	
$r_{c6,t-1}$	0.09 (0.05)*	0.05 (0.05)	0.05 (0.05)	0.03 (0.04)	0.05 (0.05)	-0.20 (0.06)	0.02 (0.06)	0.05 (0.05)	
$r_{c7,t-1}$	0.03 (0.06)	-0.04 (0.05)	-0.04 (0.05)	0.00 (0.05)	-0.03 (0.06)	-0.04 (0.07)	-0.08 (0.07)	0.02 (0.06)	
$r_{c8,t-1}$	0.01 (0.07)	0.04 (0.06)	-0.02 (0.06)	0.04 (0.06)	0.08 (0.07)	-0.03 (0.09)	0.02 (0.08)	0.02 (0.07)	
	Contemporaneous and lagged effect of $Y_{1,t}$ on $Y_{2,t}$: $B_0A_1 + B_1$								
rr_{t-1}	0.04 (26.53)	-0.21 (24.45)	-0.06 (23.84)	0.00 (1075.17)	-0.07 (58.23)	0.49 (31.52)	0.25 (49.48)	0.02 (42.70)	
YS_{t-1}	-0.12 (26.49)	-0.30 (24.41)	-0.40 (23.80)	0.00 (1073.69)	0.09 (58.15)	1.03 (31.47)	-0.21 (49.40)	0.07 (42.64)	
CS_{t-1}	0.01 (15.79)	0.25 (14.55)	-0.22 (14.18)	0.00 (642.08)	-0.12 (34.72)	0.68 (18.75)	0.31 (29.48)	0.01 (25.44)	
PE_{t-1}	-0.05 (0.00)	0.27 (0.00)***	-0.26 (0.00)	0.00 (0.13)	0.02 (0.01)***	-0.66 (0.00)	0.16 (0.01)***	-0.05 (0.01)	
ΔT_{t-1}	0.12 (0.10)	-0.38 (0.09)	0.75 (0.09)***	0.00 (3.88)	0.01 (0.21)	1.77 (0.11)***	0.28 (0.18)	0.05 (0.15)	
π_{t-1}	-0.14 (26.58)	0.52 (24.49)	-0.73 (23.88)	0.00 (1073.92)	-0.02 (58.24)	1.22 (31.58)	-0.06 (49.51)	0.03 (42.73)	
$r_{lb,t-1}$	0.31 (0.58)	0.08 (0.52)	-0.59 (0.46)	0.00 (119.04)	-0.01 (4.19)	1.28 (0.49)***	0.13 (2.70)	0.06 (2.36)	
$r_{cor,t-1}$	0.05 (0.33)	0.12 (0.30)	0.32 (0.27)	0.00 (52.49)	-0.07 (1.92)	-0.04 (0.31)	0.50 (1.28)	-0.03 (1.12)	

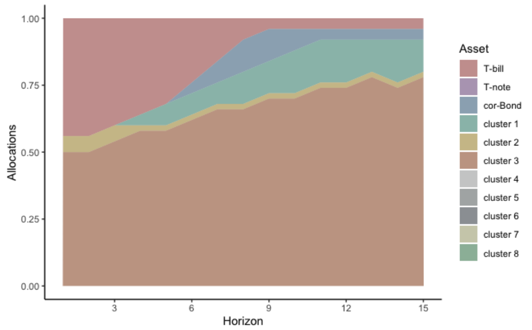
Note. Standard errors are in parentheses. * = $p < 0.1$, ** = $p < 0.05$, *** = $p < 0.01$.

The estimates of these models are reported in Table 6. Firstly, the AR(1) coefficient of $r_{f,t}$ equals 0.94, so $r_{f,t}$ is fairly persistent. The dynamics of $Y_{1,t}$ are modelled with A_1, where the real rate rr_t (through which $r_{f,t}$ transmits into the VAR(1) model) is also persistent and significantly influenced by inflation π_t and the long-term and corporate bond returns. However, rr_t itself is only significant towards the yield spread YS_t and the corporate bond return $r_{cor,t}$, meaning its importance in the dynamics of other variables is minimal. It is also interesting to note that YS_t is significantly impacted by all state variables and that π_t significantly impacts both bond returns and vice versa, likely linked to the monetary policy effects, where increasing interest rates are used as a tool to reduce high inflation in overheated economies. Regarding the bond returns, both bond returns have a significant relationship with multiple state variables, but the corporate bond is not significant towards the long-term bond returns, while there is significance in reverse direction. Lastly, temperature change ΔT_t is not very persistent with a coefficient with 0.34 while only significantly impacting the bond returns, though at a minimal magnitude with coefficients of 0.06 and 0.04 for $r_{lb,t}$ and $r_{cor,t}$ respectively. These estimates show that bond returns are significantly impacted by most state variables, though shocks in temperature change have a minimal influence.

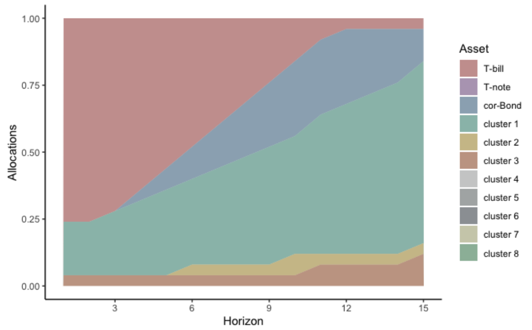
For the unrestricted VAR(1) model, where the short sampled variable variable matrix $Y_{2,t}$ (consisting of only ESG portfolios) can be influenced with their lagged values through H_1 and the combined contemporaneous and lagged effects of $Y_{1,t}$ on $Y_{2,t}$, transmitted through $B_0A_1+B_1$. Please refer to Table D.2 in Appendix D for the detailed coefficients of B_0 and B_1. In H_1, the only significant dynamics among the ESG portfolios are cluster 4 on cluster 7 (coefficient of 0.27, significant at the 1% level) and cluster 6 on cluster 1 (coefficient of 0.09, significant at the 10% level). But further looking at the effects of $Y_{1,t}$ on $Y_{2,t}$, it also reveals that not many coefficients are significant at the 10% level; the PE ratio significantly influences ESG portfolios 2, 5 and 7 and ΔT_t impacts clusters 3 and 6 with high coefficients of 0.75 and 1.77 respectively. The high coefficient of 1.77 is due to cluster 6 having the lowest mean return historically, thus making any changes in ΔT_t having a relatively larger impact on the returns. Besides ΔT_t, the long-term bond is also significant towards cluster 6, with also a high coefficient of 1.28. But all in all, the lack of significance across all ESG portfolios reinforces that equity returns remain mostly influenced by idiosyncratic shocks, and thus difficult to forecast. This is in contrast with bond returns, where state variables do have more of a significant influence on their returns.

5.2 Optimal Portfolio Choice Model Results

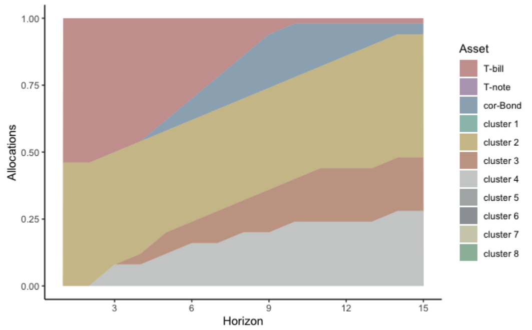
For the above-mentioned 8 portfolio choice models, the optimal asset allocations over an investment horizon of 1 to 15 years are disclosed in Figure 3. Note that the asset allocations for the $\frac{1}{N}$ models are not displayed, as the allocation remains fixed and thus does not show any interesting developments across the different investment horizons.



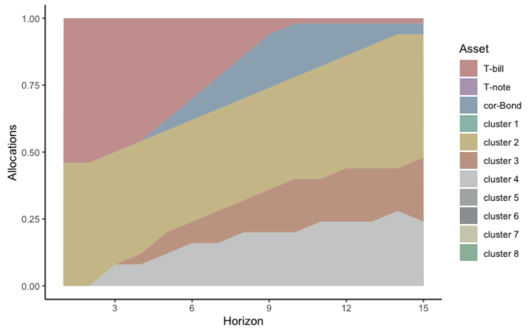
(a) Buy&hold, Simple sorting, return-only



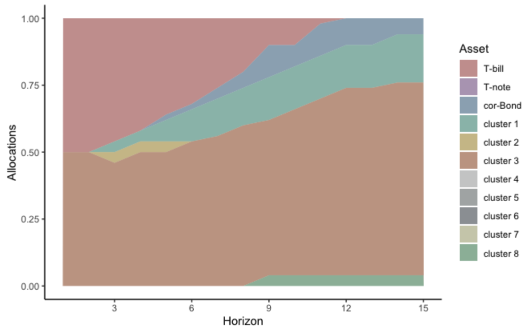
(b) Buy&hold, Simple sorting, ESG restricted



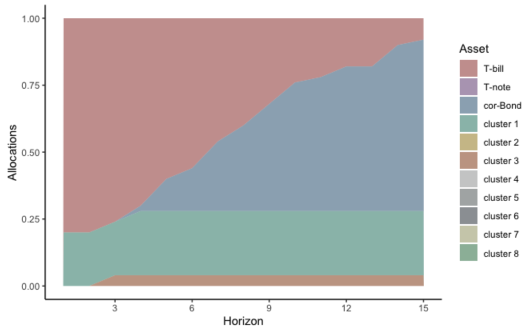
(c) Buy&hold, K-means sorting, return-only



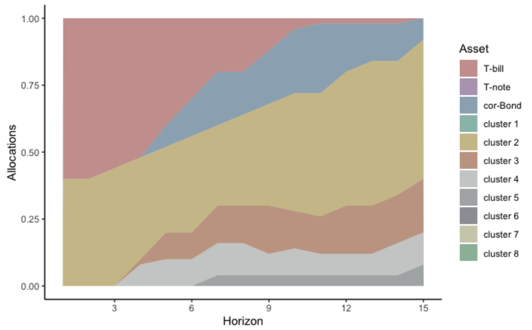
(d) Buy&hold, K-means sorting, ESG restricted



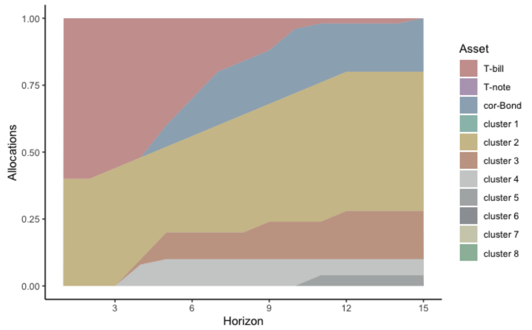
(e) Dynamic, Simple sorting, return-only



(f) Dynamic, Simple sorting, ESG-restricted



(g) Dynamic, K-means sorting, return-only



(h) Dynamic, K-means sorting, ESG-restricted

Figure 3: The optimal asset allocation of the portfolio choice models across the investment horizons

Across all the asset allocations can be seen that the demand for the riskless asset is high at a short horizon and then gradually decreases towards an allocation of close to 0%. This shows that the models account for the term structure of the assets and thus rather invest more into safer assets with shorter investment horizons. The allocation in the riskless asset then mostly shifts towards equity for longer horizons, where the demand for equity keeps rising with the investment horizon, accounting for an equity allocation of 80 to 96% of the total allocation at the terminal horizon. This is also in line with expectations, as the term structure of equity shows that equity becomes less risky over longer horizons, due to equity exhibiting the mean-reverting property, which the investor uses to hedge against these expected return movements. Furthermore, part of the allocation from the riskless asset also moves towards the corporate bond at longer horizons, though it reaches an allocation of at most 25% at horizon 12 under the buy&hold, simple sorting, ESG-restricted model in Figure 3b. But a remarkable observation is that the long-term bond is completely ignored across all strategies. Even though it could provide a hedge against interest rate and annuity risks at longer horizons, its low mean excess return likely plays too big of a role in the bond not being attractive enough at medium/longer horizons, even compared to riskier assets but with better return prospects, such as the corporate bond or some ESG portfolios.

When comparing the return-only and the ESG-integrated models with each other, interesting similarities and differences can be observed. Under the K-means sorting strategies (see Figures 3c, 3d, 3g and 3h), the optimal asset allocation only gets slightly adjusted, though the demand for lower ESG portfolios (especially clusters 3, 4 and 5) does efficiently decrease while demand for cluster 2 increases. But under the simple sorting models (see Figures 3a, 3b, 3e and 3f), the ESG-integrated asset allocation looks significantly different. Under the dynamic model in Figure 3f, most of the allocation shifts towards the corporate bond while simultaneously decreasing the equity allocation to not exceed 25%, with the model becoming more invested into the most ESG-compliant portfolio, cluster 1. But under the buy&hold strategy in Figure 3b, the equity allocation does keep the increasing trend to reach an equity allocation of approximately 85%, but with a clear allocation shift from cluster 3 to cluster 1 to be noted, such that the threshold score can be fulfilled. These allocations infer that ESG-integrated models are successful in reducing the demand for lower tail ESG portfolios such that the asset allocation fulfills the ESG restriction. Nevertheless, the optimal solution still efficiently allocates some demand for these ESG portfolios when they offer better return performance for specific horizons.

When further examining the K-means sorting models, a growing allocation over the horizons goes towards equity, where cluster 2 receives a 40 to 45% allocation per period, followed by a smaller allocation towards cluster 3, which only start to be interesting to invest into from horizon 3. Furthermore, the lower ESG portfolios 4 and 5 also receive allocations from horizon 3 onwards, but this combined allocation remains below 10% under the dynamic strategy, while this allocation rises to 25% under the buy&hold strategy. But under the buy&hold strategy, a 5% higher allocation for equity is observed compared to the dynamic models, because the buy&hold

models have to account for a higher equity allocation to be desired in future investment periods, which may produce over-allocations in the ESG portfolios towards the end of the investment horizon. Regarding the simple sorting portfolio choice models, when the allocation gets shifted towards equity at larger horizons, it is all invested into the top 3 ESG portfolios; most of the allocation goes towards cluster 3 due to its superior return performance, followed by cluster 1 and 2 in the respective order. When comparing with the K-means sorting asset allocations, it can be noted that cluster 1 is completely neglected under K-means sorting, suggesting that upper tail ESG portfolios are more demanded under simple sorting strategies. While lower tail ESG portfolios are mostly excluded from the portfolio, it is interesting to see in Figure 3e that cluster 8 can still occassionally provide some return performance under return-only models, as the ESG scores of cluster 8 are too poor to be included in ESG-integrated models.

The reason as to why the ESG-integrated asset allocation under simple sorting significantly changes compared to the return-only allocation, whereas the allocation differences under K-means sorting remain minuscule, can be seen in the equity allocation and the optimal allocation's corresponding ESG scores over time in Figure 4.

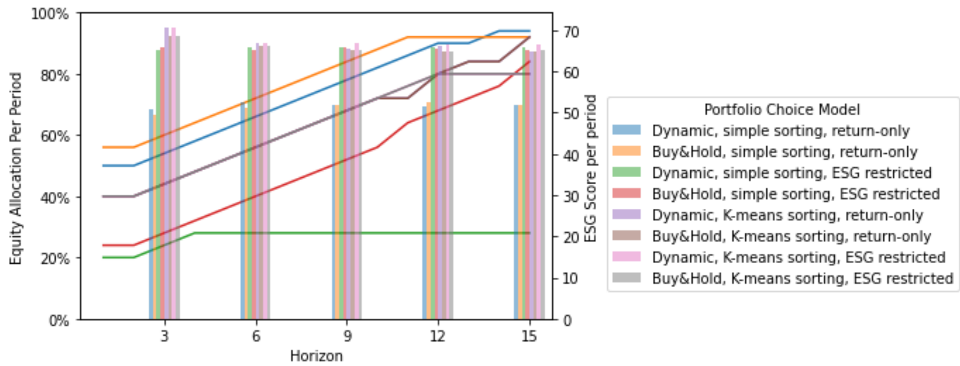


Figure 4: The equity allocation and corresponding ESG scores of optimal portfolio choice models across the investment horizons

As already observed in the allocation plots, under ESG-restricted strategies, it can be seen that the equity allocations under K-means sorting (see grey and pink lines) are much higher compared to ESG-restricted simple sorting strategies (see red and green lines): the allocation decreases by about 40% for the buy&hold strategy while under ESG-restricted dynamic strategies, the equity allocation does not even exceed 25%. For return-only strategies, the demand for equity is about 15 to 20% lower for simple sorting strategies. The reason for this can be found in the total portfolio's ESG score, as reported on the right Y-axis of Figure 4. K-means sorted return-only strategies report a score around 65 over the years while under simple sorting strategies, the optimal ESG scores lies around 50. This means that $\overline{ESG} = 65$ is more binding towards the simple sorting models, thus creating a larger impact on the allocation, as upper tail ESG portfolios have to receive a higher allocation. For ESG-restricted K-means sorted strategies, since the ESG constraint is only binding in a couple of periods, it results only in minimal differences in the equity allocation and the portfolio's ESG score, as seen in Figure 4.

Given the asset allocations for the portfolio choice models, this produces the following out-of-sample mean terminal wealth (at the terminal horizon), their standard deviation/volatility and their turnover rate, as presented in Table 7.

Table 7: The mean and standard deviation of the terminal wealth and the turnover of the portfolio choice models

Strategy	Mean Terminal Wealth	Stdev Terminal Wealth	Turnover
Fair $\frac{1}{N}$	1.314	0.192	0.075
$\frac{1}{N}$	1.523	0.629	0.114
Buy&hold, simple sorting, return-only	2.622	1.712	0.045
Buy&hold, simple sorting, ESG-restricted	1.926	1.301	0.068
Buy&hold, K-means sorting, return-only	2.759	1.441	0.08
Buy&hold, K-means sorting, ESG-restricted	2.772	1.458	0.079
Dynamic, simple sorting, return-only	2.327	1.222	0.07
Dynamic, simple sorting, ESG-restricted	1.639	0.356	0.072
Dynamic, K-means sorting, return-only	2.433	0.929	0.088
Dynamic, K-means sorting, ESG-restricted	2.387	0.869	0.088

The highest mean terminal wealth is reached by the buy&hold, K-means sorting, return-only strategy, with a terminal wealth of 2.8, implying a 180% return on the initial wealth of 1. In general, the return-only strategies achieve higher terminal wealth compared to their corresponding ESG-restricted strategies, except for the buy&hold K-means sorting strategies, which has approximately the same mean terminal wealth as the return-only counterpart due to the optimal asset allocation being similar. Under the simple sorting strategies, the mean terminal wealth drops by 30% for $\overline{ESG} = 65$ compared to K-means sorting strategies, while both $\frac{1}{N}$ benchmark strategies obtain a substantially lower terminal wealth, with an average of 1.31 and 1.52 for the fair $\frac{1}{N}$ and the $\frac{1}{N}$ strategies respectively. Such poor performance by the benchmark models can be clarified by the bond allocations being 27% and 75% for the $\frac{1}{N}$ and the fair $\frac{1}{N}$ benchmark strategies. Such high fixed bond allocations are too conservative for longer horizons; this makes the strategies less risky, but comes at a higher cost on return compared to the optimal portfolio choice model, which has a higher risk-adjusted return due to the riskier equity allocation.

When looking at the volatility, it confirms that the $\frac{1}{N}$ strategy has the lowest stdev of 0.192, while the highest stdev is obtained by the the buy&hold, simple sorting, return-only strategy, with 1.71. Further distinctions in volatility cannot be found, though the buy&old strategies are in general more volatile compared to the dynamic models, likely due to the lack of adaptability of buy&hold strategies towards the changing investment opportunities.

Lastly, looking at the turnover, the highest turnover is seen with the $\frac{1}{N}$ strategy with 0.11, followed by the dynamic, K-means sorting, return-only and ESG-restricted strategies with a rate of 0.088. The lowest turnover is found with the buy&hold, simple sorting, ESG-restricted strategy at 0.045. Furthermore, the results also show that simple sorted models have up to 25% lower turnover rates compared to K-means sorted strategies. But in general, it can be concluded that dynamic strategies have slightly higher turnover rates compared to buy&hold models; given that the asset allocation is re-assessed in each period, this obviously involves more frequent trading and thus a higher turnover. However, as shown with the $\frac{1}{N}$ model, maintaining a fixed allocation strategy can also be costly, if frequent excessive re-balancing is required.

Based on these outcomes, it can be concluded that by deploying K-means sorted strategies, a higher terminal wealth and thus better return performance can be expected on average, though the investor may be exposed towards more volatility and higher transaction costs, which is less suitable for conservative investors. Furthermore, under K-means sorting, the buy&hold strategies also show on average better return performance compared to dynamic strategies, though such strategies experience higher volatility.

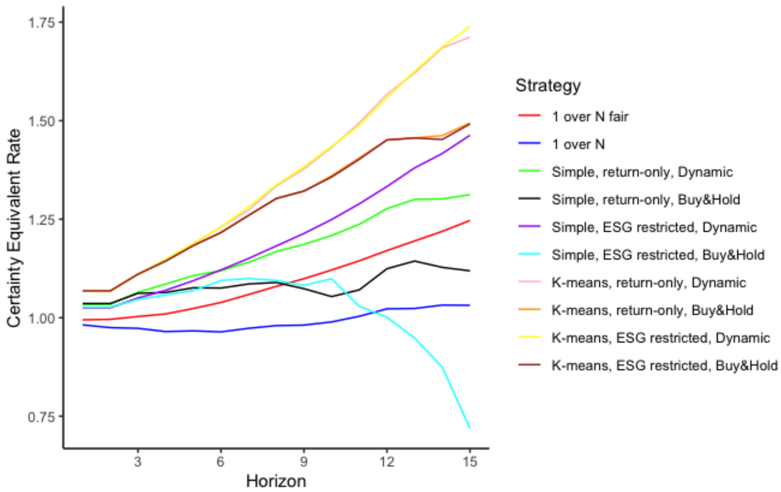


Figure 5: The Certainty Equivalent Rates of the optimal portfolio choice models

The next discussed metric is the Certainty Equivalent Rate (CER) of the models, which is the certain amount for which the investor's utility is equivalent to the expected utility from the uncertain outcome of the simulated return scenarios. The out-of-sample CER across the investment horizons is displayed in Figure 5. The highest terminal CER of 1.74 is achieved by the dynamic, K-means sorted and ESG-restricted portfolio choice model, closely followed by the return-only variant. This means that over an investment horizon of 15 years using this model, you could gain the same utility by receiving a 75% return with 100% certainty compared to the uncertain wealth outcome of investing with this strategy. These two models are then followed by the buy&hold, K-means sorted strategies (both return-only and ESG-restricted) with a CER of 1.5. It then follows that the simple sorted models perform significantly poorer than the K-means sorted models, where the terminal CER is between 0.7 and 1.3, except for the dynamic, simple sorted, ESG-restricted model having a CER of 1.4. A likely reason for the over-performance of the K-means sorted models is due to the K-means sorting procedure better allocating stocks into portfolios with more similar stocks in terms of ESG. To add to this, when inspecting the stock distribution among the clusters in Tables B.5 and B.6 in Appendix B, the upper tail ESG portfolios under simple sorting are more diversified than the upper tail ESG portfolios under K-means sorting. These aspects lead to lower returns under the simple sorting models, because the portfolio choice models automatically invest into the upper tail ESG portfolios due to the ESG constraint, while these more diversified upper tail ESG portfolios have lower return expectations compared to their counterparts under K-means sorting.

But as with the K-means sorted models, the dynamic allocation models also perform better than the buy&hold strategies under simple sorting. This is due to buy&hold strategies being mostly invested into ESG portfolio 1, an ESG portfolio which performs poorly at the longer horizons based on the simulations. This indicates that dynamic strategies can achieve higher returns through adapting to such changing investment opportunities, while the buy&hold strategies fail to accustom to such drastic return declines, which heavily impacts the CER.

Another observation is that the ESG-restricted models under simple sorting have a significantly lower CER compared to their return-only counterparts; this is linked to the return-only solution under simple sorting having an ESG score of 50. This makes $\overline{ESG} = 65$ strongly binding and thus significantly impacts the optimal asset allocation. Lastly, comparing the CER performance of the $\frac{1}{N}$ strategies to the remaining portfolio choice models, the naïve portfolio strategies do not perform well at longer horizons: the fair $\frac{1}{N}$ and $\frac{1}{N}$ strategies have a minimum drop of 50% in average return performance compared to K-means sorted models, having terminal CERs of 1.22 and 1.03 respectively. With these CERs, the naïve strategies only beat the buy&hold, ESG-restricted simple sorted model, but are still significantly outperformed by the dynamic and most buy&hold return-only strategies.

All in all, it can be concluded that for the given simulation process that assumes return predictability to some degree, the included assets, the CRRA preferences of the investor being $\gamma = 5$ and the studied performance metrics, the dynamic K-means sorted portfolio choice model offers the best return performance compared to simple sorted and/or buy&hold models. This has several reasons; firstly, its optimal asset allocation is less aggressive through a slightly lower equity allocation and thus accounts better for the term structures of the risky assets. Secondly, given a dynamic allocation set-up, the asset allocation can adapt to the changing investment opportunities and thus hedge better against expected return movements and excessive downturns in specific assets. When comparing the return-only and ESG-restricted strategies, the largest allocations are automatically dedicated towards greener ESG portfolios, with only minimal allocations given to lower tail ESG portfolios. This consequently means that ESG-restricted models are similar to the optimal return-only asset allocations, though it does show that the demand of less ESG-compliant portfolios is efficiently reduced. The ESG-integrated models thus prove to be effective at efficiently reducing the demand for lower tail ESG portfolios in order to increase the portfolio's ESG score, though still keeping some allocation when their return performance may be superior in some periods. As a result, this implies that the ESG risk factor is still not fully priced into long-term portfolio choice models, as by investing more into greener assets, a superior return performance can still be expected.

5.3 ESG Sensitivity Analysis Results

In this section, the sensitivity towards $\overline{ESG}$ and the ESG preferences of the investor is analysed, executed on the best performing portfolio choice model: the dynamic, K-means sorting model.

5.3.1 Sensitivity Towards ESG Threshold Score

Initially, $\overline{ESG} = 65$ is set to be equivalent to the 75th quantile of the ESG scores. To quantify the possible effects on return performance with different ESG demands, the portfolio choice model is optimised for different $\overline{ESG}$ values between 35 and 95 with an increment of 10. This results in the following empirical distributions of the terminal wealth:

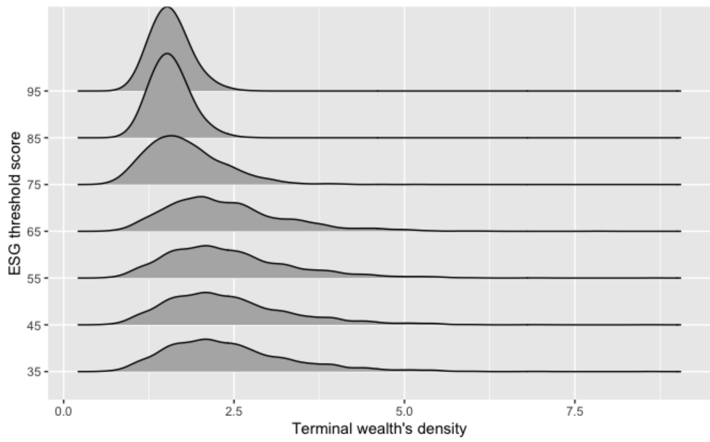
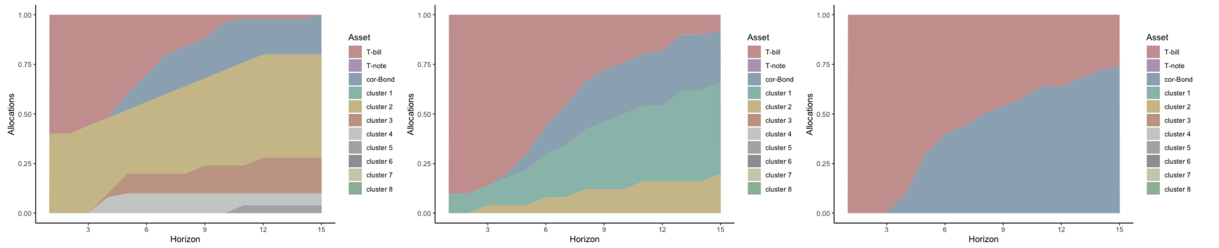


Figure 6: The empirical distributions of terminal wealth of the optimal portfolio choice models across different ESG threshold scores

It serves as a good reminder that the dynamic, K-means sorted, return-only model obtains an average ESG score of around 65. Thus when $\overline{ESG} < 65$ as seen in Figure 6, the distribution does not differ much, due to the optimal asset allocation not changing as the ESG restriction is not binding for these thresholds. At $\overline{ESG} = 65$, the ESG restriction is binding only in some periods, which creates a slightly more skewed distribution towards the right. But the impact on the allocations becomes greater for $\overline{ESG} \geq 75$, as the mean terminal wealth is lower, the distributions are less skewed to the right and the distributions' tails are less fat. It also reveals that the mean terminal wealth is lower when the threshold score increases and thus becomes more binding. The empirical distributions also shift more towards being normally distributed, due to the changing optimal allocation, as clearly seen for $\overline{ESG} \in \{65, 75, 95\}$ in Figure 7.



(a) 65 (b) 75 (c) 95

Figure 7: The optimal asset allocations of the portfolio choice models across different ESG threshold scores

For $\overline{ESG} = 65$, as previously seen in Figure 3, the equity allocation ranges between 50 and 77%, mostly allocated towards clusters 2 and 3 with some minimal demand for lower ESG portfolios 4 and 5. With the more stringent $\overline{ESG} = 75$, the demand for equity decreases, as it now ranges between an allocation of 10 to 60%, and the high $\overline{ESG}$ only permits upper tail ESG portfolios 1 and 2 to be invested in. This decreased equity demand leads to higher allocations into the riskless asset of up to 85% at shorter horizons and the demand for corporate bonds doubling at longer horizons. But when looking at the extreme $\overline{ESG} = 95$, such extreme restrictions even prevent any equity allocations to occur, resulting in a bond-only portfolio achieving lower return rates, but at lower risk levels. This shows that as the threshold becomes more binding, the investor loses its adaptability towards the changing investment opportunities, where even the upper tail ESG portfolios are invested less into compared to the bond assets at shorter horizons, due to the under-performance of those ESG portfolios at shorter horizons and the mean-reverting property of equity only showing its advantageous effects at longer horizons.

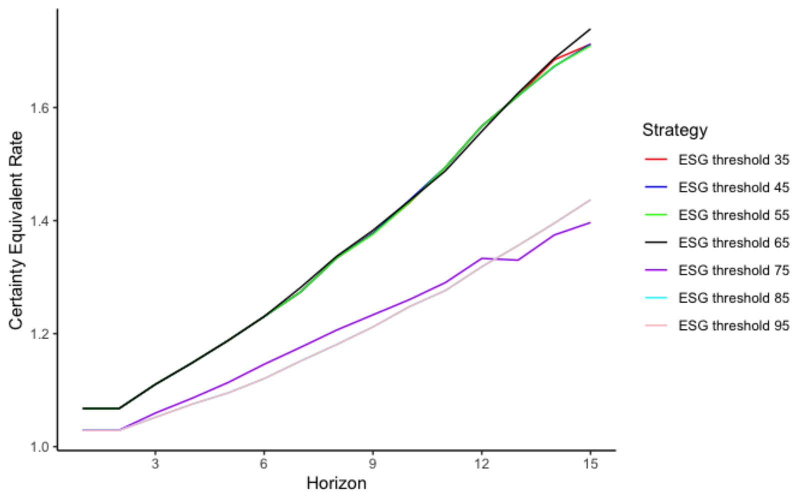
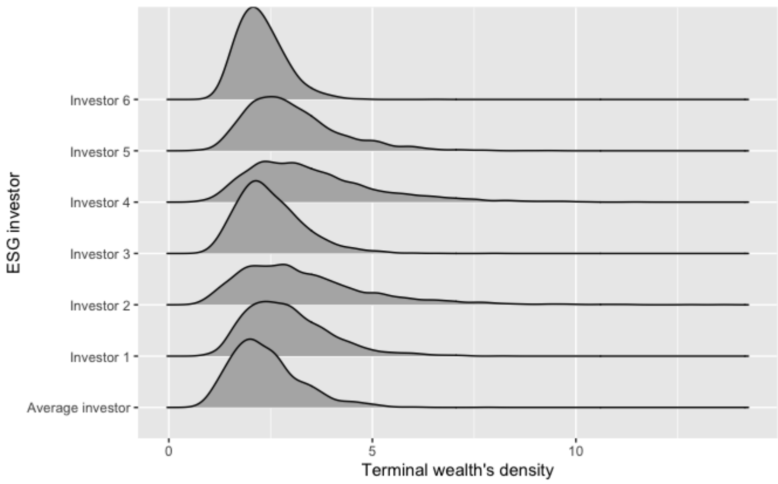


Figure 8: The Certainty Equivalent Rates of the optimal portfolio choice models across different ESG threshold scores

Using the optimal asset allocations for different threshold scores, the corresponding CER performance is displayed in Figure 8. As expected for $\overline{ESG} \leq 65$, the similar asset allocations practically result in identical CERs, where the best CER of 1.76 is obtained for $\overline{ESG} = 65$ (see black line), due to small differences at horizons 14 and 15. This contrasts with the CERs for $\overline{ESG} \geq 75$, which lie considerably lower compared to $\overline{ESG} \leq 65$ and the growth rate over the horizons seems to be lower, where the gap in performance is about 30% at the terminal horizon. This concludes that different ESG threshold demands do impact the optimal allocation, with the general rule that the expected return decreases as the ESG threshold increases, mainly due to the investor being less flexible in adapting the asset allocations towards the changing investment opportunities, as less ESG portfolios fulfilling the ESG requirement remain available.

5.3.2 Sensitivity Towards Investor's ESG Preferences

Besides $\overline{ESG}$, an average ESG investor is assumed, where the ESG component weights W_E, W_E and W_G are all set to 0.33. This is now relaxed by analysing the effect on the optimal asset allocation by considering investors with different ESG preferences, as seen in Figure 9b.



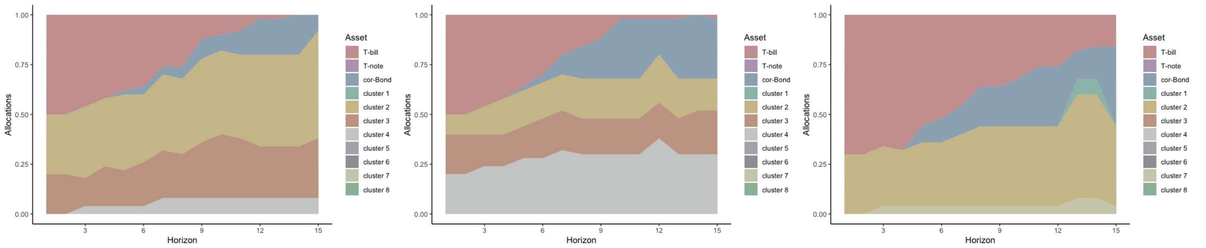
(a) The empirical distributions of terminal wealth

	w_E	w_S	w_G
Investor 6	0	0	1
Investor 5	0	1	0
Investor 4	1	0	0
Investor 3	0.2	0.1	0.7
Investor 2	0.1	0.8	0.1
Investor 1	0.7	0.2	0.1
Av. Investor	0.33	0.33	0.33

(b) The different ESG investors

Figure 9: The empirical distributions of terminal wealth under the optimal portfolio choice models for different ESG investors

The empirical distributions of the terminal wealth for these investors are shown in Figure 9a. The mean terminal wealth of the average investor is lower while also having less upside potential, compared to other ESG investors. Such an investor is the E component investor (investors 1 and 4), where higher mean terminal wealth can be achieved while having fatter tails and skewness to the right, meaning that more upside potential can be realised. Similar findings for the S component investors (investors 2 and 5) can be noted, though the distribution of these investors are less skewed and more centered, implying a more conservative asset allocation. But G component investors (investors 3 and 6) can in general expect less return, as the mean terminal wealth is visibly lower with the performance deteriorating as w_G becomes larger. The differences in these empirical distributions can be further explained by looking at the optimal asset allocations in Figure 10 of the E, S and G component investors. The optimal asset allocation of the remaining ESG investors can be found in Figure D.3 in Appendix D.



(a) Investor 4: E component (b) Investor 5: S component (c) investor 6: G component

Figure 10: The optimal asset allocation of the portfolio choice models for different ESG investors

It clearly shows that the demand for the riskless asset and the corporate bond is lower for the E and S component investors, while the equity allocation is up to 20% higher in each period compared to the G component investor, observed by the narrower spread in the empirical distribution. Furthermore, The equity allocations mostly remains below 50% for the G component investor and even declines towards 45% at horizon 15. This suggests that the G component investor's lower return perspectives are attributed to the lower equity allocation, but this observation is further explained by the equity composition: the G component investor mainly invests into ESG portfolio 2 and to a certain extent into clusters 1 and 7. While the E and S component investors have a monotonically increasing equity allocation rising towards 65 to 90% at horizon 15, these investors also predominantly allocate towards cluster 2. However, a substantial part is also allocated towards cluster 3, the ESG portfolio with the highest mean return as seen in Table 2. These allocations imply that E and S component investors have more upside potential due to the larger equity allocation, in combination with allocating towards better performing ESG portfolios, which hold higher E and S component scores.

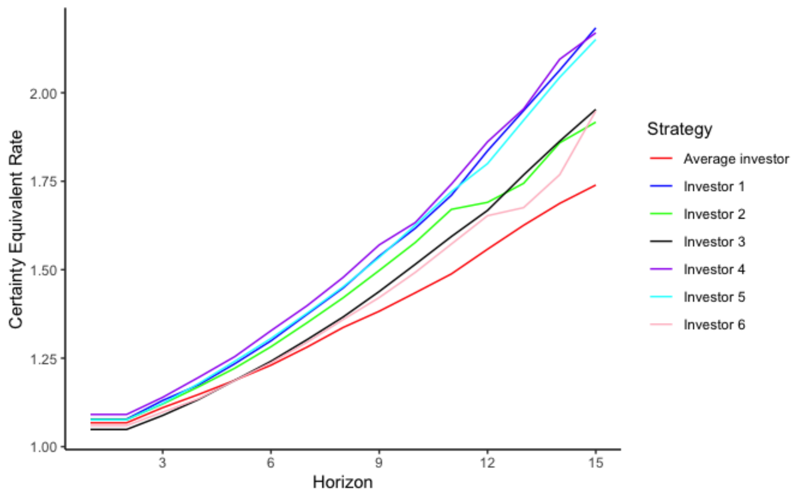


Figure 11: The Certainty Equivalent Rates of the optimal portfolio choice models for different ESG investors

In terms of performance, the CERs of the ESG investors are presented in Figure 11. With the insights obtained from the empirical distributions and the asset allocations in Figures 9 and 10, the CER results confirm that investors with higher E and/or S component preferences realise higher certainty amounts, with investors 1 and 4 (E component investors) and investor 5 (S component investor) all achieving a CER of 2.20 at the terminal horizon. The lowest CER is recorded by the average investor with a CER of 1.73, suggesting that equally valuing all ESG components comes at a cost of 45% in terms of average return performance relative to E and S component investors. For the G component investors (investors 3 and 6), a CER of 1.86 is maximally achieved, though investor 3, which has a lower preference towards G (lower w_G value) than investor 6, performs slightly better across all horizons. Relative to the E and S component investors, this is equivalent to an under-performance of about 35% at the final horizon.

These results thus show that the extent to which ESG is priced into the optimal portfolio choice model is determined by the ESG preferences: an investor who equally values all ESG components underperforms on average, due to ESG having a more neutral impact on return. On the other hand, investors that prioritise the E or the S component can expect to realise higher average returns. In this situation, the ESG portfolios under K-means sorting with higher E and S component scores achieve higher risk-adjusted returns compared to ESG portfolios with higher G component scores. However, the optimal asset allocations also show that Governance dominated or average ESG models may be more suitable for risk-averse investors, due to the majority of the portfolio being allocated towards the less volatile bond assets.

6 Conclusion And Discussion

6.1 Conclusion

The research question "*How do ESG-integrated long-term portfolio choice models perform compared to the return-only models and how sensitive are they towards different ESG preferences?*" has been examined for the strategic asset allocation among the riskless asset, the long-term bond, the corporate bond and the 8 ESG portfolios, created with K-means and simple sorting procedures based on the E, S and G ESG component scores of 6045 stocks. The optimal allocations are obtained with the return-only analytical buy&hold solution of Viceira (2001) and the numerical dynamic solution of Binsbergen & Brandt (2007), where also the ESG-integrated variants of these models are employed to determine the impact of demanding a certain ESG threshold score $\overline{ESG}$ on the models' performance. Furthermore, given that investors have different ESG preferences, a sensitivity analysis of the optimal asset allocation towards $\overline{ESG}$ and the ESG component weights is conducted.

The general results show that most portfolio choice models show logical allocations in terms of the assets' term structure: for short horizons, the demand for the highly volatile equity portfolios is lower as the investor wants to invest safer through higher allocations in the riskless asset and the corporate bond. At longer horizons, equity becomes less risky due to its mean-reverting property and thus is demanded more, while the corporate bond also shows favorable risk-adjusted returns at longer horizons. But among all the models, the dynamic, K-means sorted model achieves the highest average return performance, due to multiple reasons: Firstly, K-means sorted portfolios cluster stocks which are more closely related in terms of ESG, whereas for simple sorted portfolios, this leads to overly diversified portfolios in the most ESG-compliant portfolios with lower return potential. Secondly, simple sorted models have higher equity allocations, leaving them more exposed towards the volatile ESG portfolios, while K-means sorted models offer more conversative allocations and in general a better allocation within the equity space that in the end offers a better return performance. Thirdly, dynamic models can adapt better to changing investment opportunities and can hedge better against expected return movements.

But the most important finding of this paper is that most of the allocation consists of the most ESG-compliant portfolios under both the return-only and ESG-integrated models, though the lower tail ESG portfolios' demand gradually decreases, showing that the ESG-integrated models are efficiently allowing such portfolios to still provide some better risk-adjusted return performance in some periods while making the allocation 'greener'. But the allocation under the K-means sorted models are barely impacted, as the $\overline{ESG} = 65$ is not strongly binding. This implies that ESG-restricted models do not underperform compared to return-only models when the ESG constraint is not fully binding, as ESG is still not fully priced into the optimal allocation. But when $\overline{ESG}$ becomes more binding, the optimal asset allocation is impacted more, as the investor loses adaptability towards the changing investment opportunities due to ESG portfolio allocations becoming more limited, leading to lower return performance. In terms of ESG preferences, it is found that investors with higher E and/or S component preferences can expect a higher return performance compared G component investors, as the ESG portfolios with higher E and/or S scores achieve better risk-adjusted returns.

The findings of this paper are compelling with regards to previous research. In the context of ESG investing in portfolio choice models, Shen et al. (2019) similarly find that eco-investing does not result in a lower return performance. This contrasts though with Hong & Kacperczyk (2009), who argue that sin stocks can achieve higher abnormal returns, whereas this paper rather shows that lower tail ESG portfolios can occasionally offer better return performance, which the portfolio choice models can efficiently use in some periods. Furthermore, Qi & Li (2020) find that ESG-integrated strategies result in significantly different portfolio allocations, while this paper only finds this result when the ESG constraint becomes more binding. Lastly, the findings that the riskless asset allocation is larger at shorter horizons and equity allocation is larger at longer horizons are in line with Campbell & Viceira (1999), where it is found that larger weight is dedicated towards bonds when the term structure of the assets over time is accounted for.

6.2 Discussion

The framework used in this paper for the strategic asset allocation runs into some limitations. The biggest limitation is that ESG scores are not standardised. While this facilitates the flexibility of integrating custom ESG metrics, until ESG metrics are not standardised, there is no way around using truly reliable ESG scores for studying its impact on return performance. Secondly, when using ESG-restricted models, an unrealistic assumption of the ESG scores remaining constant in the future is used, as it is not yet empirically well-founded as to how ESG scores should be forecasted. When research into ESG forecasting becomes more mature, it may be an interesting addition to model ESG scores stochastically as well.

An additional limitation is that ESG scores could not be obtained for the corporate bond, thus allowing for the possibility of the counter-intuitive logic that poor ESG companies can still be invested in through the corporate bond, while not affecting the allocation's ESG score. Moreover, ESG portfolios are formed using 6045 stocks. While this can provide highly diversified portfolios, it still leads to the impracticality of managing such portfolios, such as transaction costs, information overload and liquidity issues. However, this could be fixed using shrinkage methods, such as shrinking the variance-covariance matrix as in Ledoit & Wolf (2003) or introducing a bias to the mean return vector (James & Stein, 1992). Lastly, parameter uncertainty is not accounted for in the VAR(1) model. A possible solution to this is to implement a VAR(1) model which accounts for parameter uncertainty, with or without learning depending on whether time-varying dynamics are to be accounted for (Brandt et al., 2005).

Besides the limitations, this paper nevertheless provides a good foundation for further research into incorporating climate risk factors or creating more insights into the optimal portfolio choice models. Firstly, the CRRA preferences of the investor are assumed to be $\gamma = 5$. It would thus be interesting to investigate the effect on the asset allocation for investors with different risk-return preferences, by altering γ. While the approximate analytical and the numerical solutions are established frameworks to use for long-term asset allocations, reinforcement learning models such as the Proximal Policy Optimization (Schulman et al., 2017) or the Deep Deterministic Policy Gradient (Lillicrap et al., 2015) could also be possibly employed and benchmarked against the analytical and numerical methods, as well as observe how such models handle ESG restrictions. Lastly, the used ESG-integrated portfolio choice models can be further extended to incorporate climate-related risk factors. Possible approaches could be the method of Shen et al. (2019), by including a temperature beta in the optimal portfolio weight solution or to include additional climate-related variables into the VAR(1) framework, such as Carbon prices, precipitation levels or extreme weather events. Another approach could be to incorporate optimistic or pessimistic scenarios to the VAR(1) simulation process, through introducing a bias to the coefficients of climate-related variables.

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Appendix A

A.1: Derivation of the Variance-Covariance Matrix of the Complete VAR(1) Model

To fully understand the derivation of the variance-covariance matrix of the complete VAR(1) model, the model is re-introduced here. The below VAR(1) framework is employed to simulate return dynamics over 10000 scenarios:

$$Y_t = \Phi_0 + \Phi_1 Y_{t-1} + u_t \tag{36}$$

where,

$$Y_t = \begin{bmatrix} Y_{1,t} \\ Y_{2,t} \end{bmatrix}, \Phi_0 = \begin{bmatrix} \mu \\ \alpha + B_0\mu \end{bmatrix}, \Phi_1 = \begin{bmatrix} A_1 & 0 \\ B_0A_1 + B_1 & H_1 \end{bmatrix}, u_t = \begin{bmatrix} \epsilon_{1,t} \\ B_0\epsilon_{1,t} + \epsilon_{2,t} \end{bmatrix}.$$

As noted in the methodology, the innovation of this VAR(1) model follows the bivariate Gaussian distribution $u_t \sim \mathcal{N}(0, \mathbf{\Sigma})$, where the variance-covariance matrix $\mathbf{\Sigma}$ is defined as below:

$$\mathbf{\Sigma} = \begin{bmatrix} Cov(\epsilon_{1,t}, \epsilon_{1,t}) & Cov(\epsilon_{1,t}, (B_0\epsilon_{1,t} + \epsilon_{2,t})) \\ Cov((B_0\epsilon_{1,t} + \epsilon_{2,t}), \epsilon_{1,t}) & Cov((B_0\epsilon_{1,t} + \epsilon_{2,t}), (B_0\epsilon_{1,t} + \epsilon_{2,t})) \end{bmatrix}, \tag{37}$$

where $\epsilon_{1,t}$ and $\epsilon_{2,t}$ are the residuals of the unrestricted and restricted VAR(1) models at time t respectively and B_0 is the coefficient matrix which estimates the instantaneous effect of the longer existing assets and state variables $Y_{1,t}$ onto the shorter sampled variables $Y_{2,t}$. By further writing out the (co-) variances as in (37) and using the assumption that the residuals are independent from each other, $E(\epsilon_{1,t}\epsilon_{2,t}) = 0$ for $\forall\, t$, this leads to the results as presented below:

- $Cov(\epsilon_{1,t}, \epsilon_{1,t}) = Var(\epsilon_{1,t}) = \Sigma_{\epsilon_1}$
- $Cov(\epsilon_{1,t}, (B_0\epsilon_{1,t} + \epsilon_{2,t})) = E(\epsilon_{1,t}(B_0\epsilon_{1,t} + \epsilon_{2,t})') = E(\epsilon_{1,t}\epsilon'_{1,t}B'_0) + E(\epsilon_{1,t}\epsilon'_{2,t}) = Var(\epsilon_{1,t})B'_0$ $= \Sigma_{\epsilon_1}B'_0$
- $Cov((B_0\epsilon_{1,t} + \epsilon_{2,t}), \epsilon_{1,t}) = B_0\Sigma_{\epsilon_1}$
- $Cov(B_0\epsilon_{1,t}+\epsilon_{2,t}, B_0\epsilon_{1,t}+\epsilon_{2,t}) = E((B_0\epsilon_{1,t}+\epsilon_{2,t})(B_0\epsilon_{1,t}+\epsilon_{2,t})') = E((B_0\epsilon_{1,t}\epsilon'_{1,t}B'_0+\epsilon_{2,t}\epsilon'_{2,t}+$ $2B_0\epsilon_{1,t}\epsilon'_{2,t})) = B_0Var(\epsilon_{1,t})B'_0 + Var(\epsilon_{2,t}) = B_0\Sigma_{\epsilon_1}B'_0 + \Sigma_{\epsilon_2}$

A.2: Derivation of the Constant Relative Risk Aversion γ

For the derivation of the risk-return preference parameter γ of the power utility function, the power utility function is re-introduced here:

$$U_\gamma(W_t) = \begin{cases} \log(W_t), \text{if } \gamma = 1, \\ \frac{W_t^{1-\gamma}}{1-\gamma}, \text{otherwise,} \end{cases} \tag{38}$$

where for a certain γ, the utility for a certain level of wealth W_t can be obtained. Given that returns are uncertain in the future, the portfolio choice models make use of a return scenario set

S of size 10000. For a specific scenario s at time t, a certain portfolio return $R^p_{s,t}$ is obtained, which can be used to collect the accumulated wealth of an investor for a specific scenario s at time t, represented by W_t. For the exact derivation of $W_{s,t}$ using the portfolio return, please refer to section 3.3.1 of this paper.

Having $W_{s,t}$ for all scenarios in S, the expected utility (EU) is obtained by using $W_{s,t}$ as input to the power utility function in (38) and taking the average utility across all the scenarios. By inverting the power utility function, the Certainty Equivalent Rate (CER) can be obtained, as outlined in more detail in (33). Besides this approach, the CER can also be derived through using a low-order Taylor approximation:

$$CER \approx E(W_T) + \frac{1}{2}\frac{U''(E[W_T])}{U'(E[W_T])}V(W_T), \tag{39}$$

where W_T is the vector of accumulated wealth at terminal time T. When viewing the CER relative to $E[W_T]$, the relative risk aversion is equivalent to $-E[W_T]\frac{U''(E[W_T])}{U'(E[W_T])}$. By deriving the first and second derivatives of the power utility function of (38), it becomes equivalent to the constant γ and thus can be defined as a constant relative risk aversion parameter.

A.3: Derivation of the Reduced Maximisation Problem For The Approximate Analytical Portfolio Choice Model

As discussed in the methodology, the optimal asset allocation from the approximate analytical framework of Viceira (2001) is obtained by essentially optimising the utility that can be gained from the accumulated wealth at the end of the investment horizon τ. This is formalised using the lognormal-power utility framework as follows:

$$\max_{\{\alpha_z\}_{z=t}^{t+\tau-1}} \mathbb{E}_t\Big[U(W_{t+\tau})\Big] = \max_{\alpha_{t+\tau}^{(\tau)}} \mathbb{E}_t\Big[r_{p,t+\tau}^{(\tau)}\Big] + \frac{1}{2}(1-\gamma)Var_t\Big[r_{p,t+\tau}^{(\tau)}\Big], \tag{40}$$

where, $W_{t+\tau}$ is the terminal wealth, U(.) represents the power utility function as seen in (38). As the wealth is dependent on the cumulative portfolio returns over time, the maximisation problem transforms into a problem where the mean τ-period portfolio return $\mathbb{E}_t\Big[r_{p,t+\tau}^{(\tau)}\Big]$ (following (41)) is maximised, while simultaneously keeping the volatility of the portfolio return $Var_t\Big[r_{p,t+\tau}^{(\tau)}\Big]$ (following (42)) at the lowest possible level for that specific portfolio. This is because for $\gamma > 1$, $(1-\gamma)$ becomes negative and thus a higher portfolio return volatility would penalise the utility gained from the terminal wealth, given a specific portfolio return. Therefore, the maximisation problem tries to find the optimal asset allocation through optimising the risk-return trade-off among the included assets.

$$\mathbb{E}_t\Big[r_{p,t+\tau}^{(\tau)}\Big] = \tau\Big(\mu_{f,t+\tau}^{(\tau)} + \alpha_t^{(\tau)\prime}(\mu_t^{(\tau)} + \frac{1}{2}\sigma^2) - \frac{1}{2}\alpha_t^{(\tau)\prime}\Sigma\alpha_t^{(\tau)}\Big), \tag{41}$$

$$Var_t\Big[r_{p,t+\tau}^{(\tau)}\Big] = \tau\Big(\sigma_f^{(\tau)2} + 2\alpha_t^{(\tau)\prime}\sigma_{A,f}^{(\tau)} + \alpha_t^{(\tau)\prime}\Sigma^{(\tau)}\alpha_t^{(\tau)}\Big), \tag{42}$$

Given that the approximate analytical solution can only find a solution when i.i.d assumptions are enforced, it is assumed that the solution provides a buy&hold allocation, where the asset allocation remains constant across the investment horizon. In the maximisation problem, the only unknown parameter becomes this buy&hold vector of portfolio weights, $\alpha_t^{(\tau)}$. As is provided in the methodology in (23), the optimal asset allocation for the analytical solution is:

$$\alpha_t^{(\tau)} = \frac{1}{\gamma}\Big(\Big(1-\frac{1}{\gamma}\Big)\Sigma^{(\tau)} + \frac{1}{\gamma}\Sigma\Big)^{-1}\Big(\mu_{A,t}^{(\tau)} + \frac{1}{2}\sigma^2 + \Big(1-\gamma\Big)\sigma_{A,f}^{(\tau)}\Big). \tag{43}$$

To obtain the above expression, the partial derivative of (40) with respect to $\alpha_t^{(\tau)}$ needs to be taken and set equal to zero, as in (44). By further working out this expression, it can be shown that it becomes equal to (43), as desired.

$$\begin{aligned}
\frac{\partial F}{\partial \alpha_t^{(\tau)\prime}} &= \tau\Big(\mu_t^{(\tau)} + \frac{1}{2}\sigma^2 - \Sigma\alpha_t^{(\tau)}\Big) + \frac{1}{2}(1-\gamma)\tau\Big(2\sigma_{A,f}^{(\tau)} + 2\Sigma^{(\tau)}\alpha_t^{(\tau)}\Big) = 0 \qquad (44)\\
&\Rightarrow \mu_t^{(\tau)} + \frac{1}{2}\sigma^2 - \Sigma\alpha_t^{(\tau)} + (1-\gamma)\sigma_{A,f}^{(\tau)} + (1-\gamma)\Sigma^{(\tau)}\alpha_t^{(\tau)} = 0\\
&\Rightarrow -\gamma\Big(-(\frac{1-\gamma}{\gamma})\Sigma^{(\tau)} + \frac{1}{\gamma}\Sigma\Big)\alpha_t^{(\tau)} = -\Big(\mu_t^{(\tau)} + \frac{1}{2}\sigma^2 + (1-\gamma)\sigma_{A,f}^{(\tau)}\Big)\\
&\Rightarrow \Big((1-\frac{1}{\gamma})\Sigma^{(\tau)} + \frac{1}{\gamma}\Sigma\Big)\alpha_t^{(\tau)} = \frac{1}{\gamma}\Big(\mu_t^{(\tau)} + \frac{1}{2}\sigma^2 + (1-\gamma)\sigma_{A,f}^{(\tau)}\Big)\\
&\Rightarrow \alpha_t^{(\tau)} = \frac{1}{\gamma}\Big((1-\frac{1}{\gamma})\Sigma^{(\tau)} + \frac{1}{\gamma}\Sigma\Big)^{-1}\Big(\mu_t^{(\tau)} + \frac{1}{2}\sigma^2 + (1-\gamma)\sigma_{A,f}^{(\tau)}\Big),\ \square.
\end{aligned}$$

A.4: Derivation of the Optimal ESG-integrated Portfolio Weight Using the Lagrangian Function

As explained in the methodology, the goal of the ESG-integrated approximate analytical solution is to restrict the maximisation problem, as defined for the return-only solution in (40), with a constraint that enforces a minimum ESG threshold score $\overline{ESG}$ for the portfolio to be satisfied throughout the investment horizon, based on the ESG (component) scores of the 8 ESG portfolios. This results in the following modified maximisation problem:

$$\begin{aligned}
\max_{\{\alpha_z\}_{z=t}^{t+\tau-1}} \quad & \mathbb{E}_t\Big[U(W_{t+\tau})\Big] = \max_{\alpha_{t+\tau}^{(\tau)}} \mathbb{E}_t\Big[r_{p,t+\tau}^{(\tau)}\Big] + \frac{1}{2}(1-\gamma)Var_t\Big[r_{p,t+\tau}^{(\tau)}\Big],\\
\text{s.t.} \quad & \sum_{j\in\{E,S,G\}} w_j\alpha_t^{(\tau)\prime} ESG_{j,T} \geq \overline{ESG}, \text{for } \forall\, t,
\end{aligned} \tag{45}$$

where w_j is the weight of ESG component j, where $j \in \{E,S,G\}$, $ESG_{j,T}$ is the vector of ESG component j's score for all the eight ESG portfolios at time T, where T is the last recorded observation of the ESG scores. Note that $\mathbb{E}_t\Big[r_{p,t+\tau}^{(\tau)}\Big]$ and $Var_t\Big[r_{p,t+\tau}^{(\tau)}\Big]$ are still defined as in (41) and (42), respectively. To make the notation of the constraint more convenient, the left-hand-side (LHS) of the constraint is rewritten as $\sum_{j\in\{E,S,G\}} w_j\alpha_t^{(\tau)\prime}ESG_{j,T} = \alpha_t^{(\tau)\prime}\sum_{j\in\{E,S,G\}} w_j ESG_{j,T} = \alpha_t^{(\tau)\prime}ESG_T$. Using the method of Lagrange multipliers, the Lagrangian function $\Lambda(\alpha_t^{(\tau)}, \lambda)$ is a translation of the incorporation of the ESG constraint into the maximisation problem, which is expressed as follows:

$$\max_{\alpha_{esg,t}^{(\tau)}} \Lambda(\alpha_t^{(\tau)}, \lambda) = \max_{\alpha_{esg,t}^{(\tau)}} \mathbb{E}_t\Big[r_{p,t+\tau}^{(\tau)}\Big] + \frac{1}{2}(1-\gamma)Var_t\Big[r_{p,t+\tau}^{(\tau)}\Big] - \lambda\Big(\alpha_{esg,t}^{(\tau)}{}'ESG_T - \overline{ESG}\Big), \tag{46}$$

where λ is defined as the lagrange multiplier factor of the ESG constraint. λ has the interpretation of an ESG shadow price on the optimal asset allocation: depending on the magnitude and direction of λ, it can reward or penalise the utility gained from a certain asset allocation, depending on whether or not $\alpha_{esg,t}^{(\tau)}{}'ESG_T$ deviates from $\overline{ESG}$ for the specific allocation.

Unlike in the return-only model, the Lagrangian function now has two unknowns for which it needs to be optimised: $\alpha_t^{(\tau)}$ and λ. To derive the optimal solution for both parameters, the partial derivatives of $\Lambda(\alpha_t^{(\tau)}, \lambda)$ with respect to both parameters is now taken and set to zero, resulting in the following two equations:

$$\frac{\partial \Lambda(\alpha_t^{(\tau)}, \lambda)}{\partial \alpha_t^{(\tau)}{}'} = 0 \Rightarrow \alpha_{esg,t}^{(\tau)} = \frac{1}{\gamma}\Big(\Big(1-\frac{1}{\gamma}\Big)\Sigma^{(\tau)} + \frac{1}{\gamma}\Sigma\Big)^{-1}\Big(\mu_{A,t}^{(\tau)} + \frac{1}{2}\sigma^2 + \Big(1-\gamma\Big)\sigma_{A,f}^{(\tau)} - \lambda(ESG_T)\Big), \tag{47}$$

$$\frac{\partial \Lambda(\alpha_t^{(\tau)}, \lambda)}{\partial \lambda} = -\Big(\alpha_{esg,t}^{(\tau)}{}'ESG_T - \overline{ESG}\Big) = 0 \tag{48}$$

$$\Rightarrow \alpha_{esg,t}^{(\tau)}{}'ESG_T = \overline{ESG}$$

where $\alpha_{esg,t}^{(\tau)}$'s expression is derived in a similar manner as its return-only counterpart, as seen in (44), except that now there is an additional $-\lambda(ESG_T)$ term. The partial derivative with respect to λ essentially states that the total portfolio's ESG should be equal to the threshold $\overline{ESG}$. (48) can be rewritten as in (49), by transposing the whole equation and knowing that $\overline{ESG}$ is a constant, it implies that $\overline{ESG}' = \overline{ESG}$.

$$ESG_T'\alpha_{esg,t}^{(\tau)} = \overline{ESG}, \tag{49}$$

Since the optimal $\alpha_{esg,t}^{(\tau)}$ expression in (47) depends on the unknown λ in the last term on the RHS, by multiplying both sides of (47) from the left with ESG_T' and substituting (49) into this new expression, the new expression becomes (50). By further expanding this expression as seen below, the optimal λ^* can be obtained, which can then be plugged back into (47), such that the optimal $\alpha_{esg,t}^{(\tau)*}$ can also be obtained. This gives us the desired result and in the end the optimal asset allocation under the ESG restriction. Note that when completely worked out, both the nominator and the denominator are constant, and thus making such a division for λ^* possible.

$$ESG_T'\alpha_{esg,t}^{(\tau)} = ESG_T'\Big[\frac{1}{\gamma}\Big(\Big(1-\frac{1}{\gamma}\Big)\Sigma^{(\tau)} + \frac{1}{\gamma}\Sigma\Big)^{-1}\Big(\mu_{A,t}^{(\tau)} + \frac{1}{2}\sigma^2 + \Big(1-\gamma\Big)\sigma_{A,f}^{(\tau)} - \lambda(ESG_T)\Big)\Big]. \tag{50}$$

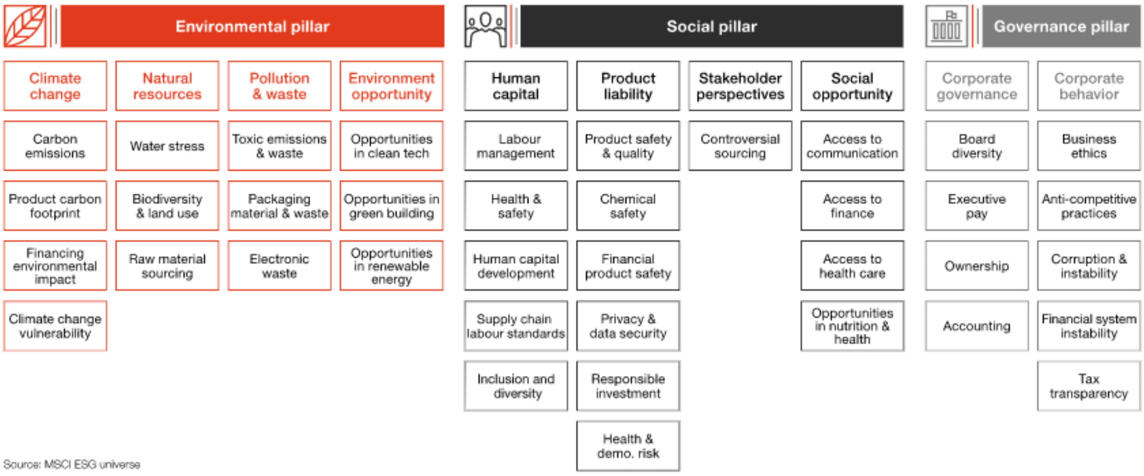
$$\Rightarrow \overline{ESG} = ESG_T'\Big[\frac{1}{\gamma}\Big(\Big(1-\frac{1}{\gamma}\Big)\Sigma^{(\tau)} + \frac{1}{\gamma}\Sigma\Big)^{-1}\Big(\mu_{A,t}^{(\tau)} + \frac{1}{2}\sigma^2 + \Big(1-\gamma\Big)\sigma_{A,f}^{(\tau)} - \lambda(ESG_T)\Big)\Big]$$

$$\Rightarrow \gamma\overline{ESG} = ESG_T'\Big[\Big(\Big(1-\frac{1}{\gamma}\Big)\Sigma^{(\tau)} + \frac{1}{\gamma}\Sigma\Big)^{-1}\Big(\mu_{A,t}^{(\tau)} + \frac{1}{2}\sigma^2 + \Big(1-\gamma\Big)\sigma_{A,f}^{(\tau)}\Big)\Big] - ESG_T'\Big(\Big(1-\frac{1}{\gamma}\Big)\Sigma^{(\tau)} + \frac{1}{\gamma}\Sigma\Big)^{-1}\lambda ESG_T$$

$$\Rightarrow ESG_T'\Big(\Big(1-\frac{1}{\gamma}\Big)\Sigma^{(\tau)} + \frac{1}{\gamma}\Sigma\Big)^{-1}ESG_T\lambda = ESG_T'\Big[\Big(\Big(1-\frac{1}{\gamma}\Big)\Sigma^{(\tau)} + \frac{1}{\gamma}\Sigma\Big)^{-1}\Big(\mu_{A,t}^{(\tau)} + \frac{1}{2}\sigma^2 + \Big(1-\gamma\Big)\sigma_{A,f}^{(\tau)}\Big)\Big] - \gamma\overline{ESG}$$

$$\Rightarrow \lambda^* = \frac{ESG_T'\Big(\frac{1}{\gamma}\Sigma + (1-\frac{1}{\gamma})\Sigma^{(\tau)}\Big)^{-1}\Big(\mu_{A,t+\tau}^{(\tau)} + \frac{1}{2}\sigma^2 + (1-\gamma)\sigma_{A,f}^{(\tau)}\Big) - \gamma\overline{ESG}}{ESG_T'\Big(\frac{1}{\gamma}\Sigma + (1-\frac{1}{\gamma})\Sigma^{(\tau)}\Big)^{-1}ESG_T}, \square.$$

Appendix B

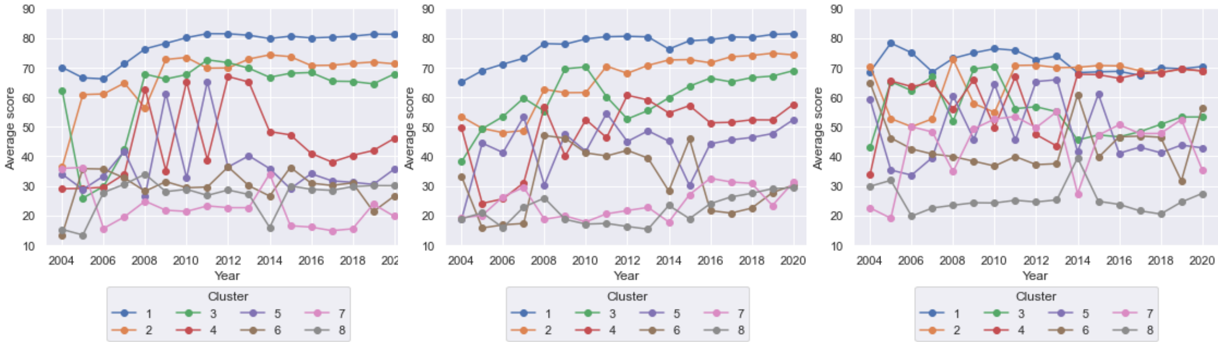


Source: MSCI ESG universe

Figure B.1: The topics within the Environment, Social and Governance components of ESG, from the Thomson Reuters database.

Table B.2: The number of companies within a specific sector per major region

	Region		
Sector	America	Asia	Europe
Basic Materials	156	199	83
Consumer Discretionary	403	303	215
Consumer Staples	106	145	93
Energy	123	72	55
Financials	500	235	265
Health Care	502	160	116
Industrials	382	328	311
Real Estate	155	158	95
Technology	266	186	80
Telecommunications	66	64	36
Utilities	71	76	40



(a) Environment (b) Social (c) Governance

Figure B.3: The average scores of the Environment, Social and Governance components of ESG across the 8 ESG portfolios, under the K-means sorting procedure.

(a) Environment (b) Social (c) Governance

Figure B.4: The average scores of the Environment, Social and Governance components of ESG across the 8 ESG portfolios, under the simple sorting procedure.

Table B.5: The number of companies within a specific sector within each of the 8 ESG portfolios at the terminal period of 2021, with the K-means sorting procedure.

	Cluster							
Industry	1	2	3	4	5	6	7	8
Basic Materials	26	46	48	59	39	110	56	54
Consumer Discretionary	36	57	92	125	148	205	101	157
Consumer Staples	25	32	35	41	41	74	39	57
Energy	17	23	30	28	24	52	40	36
Financials	33	47	46	169	183	237	121	164
Health Care	16	25	42	63	230	150	62	190
Industrials	43	76	136	146	132	219	106	163
Real Estate	18	15	54	70	93	54	45	59
Technology	18	42	33	68	114	97	46	114
Telecommunications	15	20	14	24	15	32	14	32
Utilities	15	25	13	31	15	44	20	24

Table B.6: The number of companies within a specific sector within each of the 8 ESG portfolios at the terminal period of 2021, with the simple sorting procedure.

	Cluster							
Industry	1	2	3	4	5	6	7	8
	1	2	3	4	5	6	7	8
Basic Materials	145	51	26	15	58	14	43	86
Consumer Discretionary	222	125	60	47	120	65	71	211
Consumer Staples	114	49	18	8	35	9	38	73
Energy	73	39	18	5	35	7	15	58
Financials	192	69	28	94	189	79	62	287
Health Care	118	78	22	69	101	114	33	243
Industrials	278	146	77	54	141	45	85	195
Real Estate	123	66	26	31	36	17	54	55
Technology	129	69	32	37	42	45	106	72
Telecommunications	61	13	6	8	27	5	8	38
Utilities	64	26	15	4	20	1	24	33

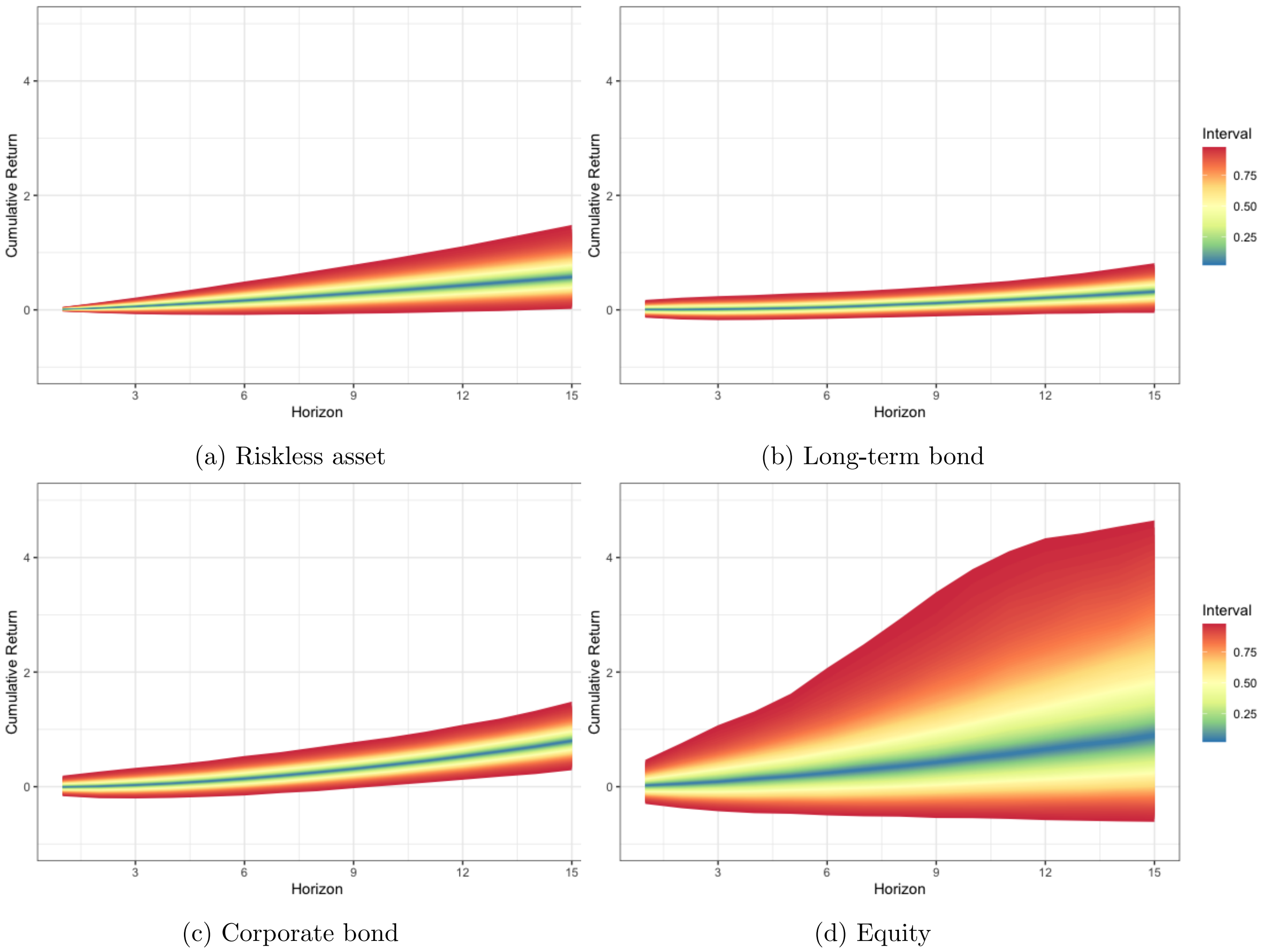
Appendix C

Table C.1: The correlation matrix for excess log returns of the ESG stock portfolios, the remaining asset classes and the state variables, under the simple sorting procedure

	$r_{c1,t}$	$r_{c2,t}$	$r_{c3,t}$	$r_{c4,t}$	$r_{c5,t}$	$r_{c6,t}$	$r_{c7,t}$	$r_{c8,t}$	$r_{f,t}$	$r_{lb,t}$	$r_{cor,t}$	rr_t	YS_t	CS_t	PE_t	ΔT_t	π_t
$r_{c1,t}$	1.00																
$r_{c2,t}$	0.68	1.00															
$r_{c3,t}$	0.59	0.53	1.00														
$r_{c4,t}$	0.54	0.59	0.53	1.00													
$r_{c5,t}$	0.57	0.55	0.56	0.61	1.00												
$r_{c6,t}$	0.65	0.64	0.52	0.48	0.55	1.00											
$r_{c7,t}$	0.63	0.62	0.57	0.53	0.60	0.58	1.00										
$r_{c8,t}$	0.60	0.66	0.58	0.66	0.57	0.54	0.59	1.00									
$r_{f,t}$	-0.05	-0.13	-0.10	-0.07	-0.10	-0.06	-0.08	-0.11	1.00								
$r_{lb,t}$	-0.24	-0.34	-0.27	-0.34	-0.23	-0.23	-0.26	-0.31	0.06	1.00							
$r_{cor,t}$	0.24	0.13	0.09	0.08	0.19	0.14	0.27	0.17	-0.14	0.37	1.00						
rr_t	0.04	-0.04	-0.04	0.00	-0.03	0.02	0.03	0.00	0.95	0.01	-0.08	1.00					
YS_t	-0.02	0.03	0.02	0.03	0.03	0.03	-0.02	-0.01	-0.64	-0.09	0.00	-0.66	1.00				
CS_t	-0.16	-0.18	-0.07	-0.14	-0.04	-0.06	-0.13	-0.16	-0.43	0.22	0.09	-0.48	0.45	1.00			
PE_t	0.17	0.06	0.09	0.11	0.09	0.07	0.14	0.10	0.35	-0.12	-0.03	0.35	-0.25	-0.31	1.00		
ΔT_t	0.10	0.09	-0.01	-0.05	0.07	-0.03	0.10	0.04	0.03	0.02	0.07	0.06	-0.04	0.01	-0.05	1.00	
π_t	-0.06	-0.03	-0.04	-0.02	-0.05	-0.10	-0.10	-0.06	0.08	-0.07	-0.14	-0.04	-0.07	-0.32	0.15	-0.09	1.00

Table C.2: The first-order autocorrelation matrix for excess log returns of the ESG stock portfolios, the remaining asset classes and the state variables, under the simple sorting procedure

	$r_{c1,t}$	$r_{c2,t}$	$r_{c3,t}$	$r_{c4,t}$	$r_{c5,t}$	$r_{c6,t}$	$r_{c7,t}$	$r_{c8,t}$	$r_{f,t}$	$r_{lb,t}$	$r_{cor,t}$	rr_t	YS_t	CS_t	PE_t	ΔT_t	π_t
$r_{c1,t-1}$	0.08	0.11	0.05	0.10	0.05	0.01	0.14	0.11	-0.01	-0.19	0.20	0.05	-0.02	-0.28	0.16	0.07	0.05
$r_{c2,t-1}$	0.17	0.15	0.16	0.16	0.12	0.07	0.21	0.20	-0.09	-0.14	0.19	-0.02	0.02	-0.28	0.07	0.05	-0.01
$r_{c3,t-1}$	0.10	0.13	0.09	0.17	0.07	0.07	0.15	0.20	-0.07	-0.18	0.23	-0.02	0.03	-0.20	0.10	0.01	-0.03
$r_{c4,t-1}$	0.08	0.05	0.10	0.05	0.05	0.04	0.11	0.07	-0.02	-0.19	0.09	0.01	0.05	-0.23	0.09	-0.09	0.02
$r_{c5,t-1}$	0.04	0.01	0.06	0.10	0.02	-0.05	0.12	0.11	-0.08	-0.19	0.17	-0.03	0.04	-0.15	0.08	0.02	0.03
$r_{c6,t-1}$	0.11	0.14	0.10	0.16	0.12	0.04	0.17	0.14	-0.04	-0.18	0.19	0.04	0.03	-0.18	0.09	0.04	-0.06
$r_{c7,t-1}$	0.12	0.11	0.06	0.14	0.03	0.03	0.09	0.13	-0.01	-0.22	0.16	0.03	0.00	-0.25	0.14	0.07	0.01
$r_{c8,t-1}$	0.14	0.11	0.09	0.14	0.12	0.05	0.14	0.12	-0.04	-0.18	0.21	0.01	-0.01	-0.27	0.10	-0.05	0.03
$r_{f,t-1}$	-0.01	-0.08	-0.05	-0.02	-0.05	-0.03	0.00	-0.04	0.95	0.02	-0.14	0.94	-0.63	-0.39	0.35	0.04	0.06
$r_{lb,t-1}$	0.00	-0.06	0.01	-0.01	0.03	0.08	0.06	-0.04	0.03	0.05	0.23	0.02	-0.08	0.18	-0.11	0.12	-0.24
$r_{cor,t-1}$	0.05	-0.02	-0.03	-0.02	-0.03	-0.01	0.04	-0.01	-0.11	-0.12	0.22	-0.11	0.02	-0.01	-0.02	0.11	0.07
rr_{t-1}	0.00	-0.08	-0.07	-0.03	-0.06	-0.03	-0.01	-0.05	0.97	0.01	-0.10	0.99	-0.66	-0.46	0.35	0.06	0.03
YS_{t-1}	-0.02	0.01	-0.01	0.00	0.02	0.02	-0.02	-0.02	-0.67	0.08	0.10	-0.65	0.97	0.45	-0.27	-0.03	-0.07
CS_{t-1}	-0.02	-0.02	0.04	-0.01	0.08	0.06	0.03	0.00	-0.48	-0.01	0.22	-0.50	0.49	0.94	-0.28	0.03	-0.33
PE_{t-1}	0.02	-0.08	-0.03	0.01	-0.03	-0.07	0.00	-0.02	0.36	-0.09	-0.07	0.36	-0.24	-0.32	0.97	-0.06	0.18
ΔT_{t-1}	0.02	0.02	0.01	-0.05	-0.06	-0.05	-0.01	0.03	0.04	0.11	0.10	0.05	-0.06	0.02	-0.04	0.34	-0.06
π_{t-1}	-0.02	-0.03	0.06	0.05	-0.01	0.03	-0.02	0.00	0.13	0.07	-0.09	0.04	-0.07	-0.26	0.14	-0.19	0.46



(a) Riskless asset (b) Long-term bond

(c) Corporate bond (d) Equity

Figure C.3: The cumulative excess return fan plots across the four main asset classes, under the K-means sorting procedure.

Table C.4: The full descriptive statistics of the simulated training set's annual excess log returns of the K-means clustered ESG stock portfolios, the other asset classes and the state variables

	Mean	**Std. dev.**	**SR**	**Skew**	**Kurt**	**Min**	**Max**
K-means ESG portfolios							
Cluster 1 ($r_{c1,t}$)	0.043	0.059	0.735	0.159	3.064	-0.182	0.284
Cluster 2 ($r_{c2,t}$)	0.079	0.048	1.656	0.093	3.019	-0.090	0.270
Cluster 3 ($r_{c3,t}$)	0.077	0.049	1.554	0.114	3.094	-0.103	0.268
Cluster 4 ($r_{c4,t}$)	0.070	0.047	1.474	0.105	3.066	-0.127	0.286
Cluster 5 ($r_{c5,t}$)	0.052	0.051	1.027	0.180	3.037	-0.103	0.245
Cluster 6 ($r_{c6,t}$)	0.002	0.061	0.026	0.153	3.068	-0.220	0.258
Cluster 7 ($r_{c7,t}$)	0.033	0.098	0.342	0.279	3.292	-0.285	0.536
Cluster 8 ($r_{c8,t}$)	0.020	0.058	0.355	0.206	3.070	-0.185	0.275
Return of other asset classes							
Riskless asset ($r_{f,t}$)	0.031	0.013	0.000	0.003	3.018	-0.017	0.083
Long-term bond ($r_{lb,t}$)	0.009	0.012	0.724	0.020	3.036	-0.033	0.057
Corporate bond ($r_{cor,t}$)	0.031	0.017	1.814	0.022	3.065	-0.038	0.096
State variables							
real rate (rr_t)	0.161	0.186	-	0.509	3.517	-0.368	1.208
yield spread (YS_t)	0.014	0.004	-	-0.047	3.018	-0.004	0.030
credit spread (CS_t)	0.022	0.003	-	0.024	3.082	0.011	0.033
PE ratio (PE_t)	21.677	1.946	-	0.022	2.980	14.841	29.277
temperature change (ΔT_t)	0.040	0.050	-	0.148	3.135	-0.144	0.252
log inflation rate (π_t)	0.024	0.006	-	-0.012	2.978	0.001	0.045

Table C.5: The full descriptive statistics of the simulated training set's annual excess log returns of the simple clustered ESG stock portfolios, the other asset classes and the state variables

	Mean	**Std. dev.**	**SR**	**Skew**	**Kurt**	**Min**	**Max**
Simple sorting ESG portfolios							
Cluster 1 ($r_{c1,t}$)	0.086	0.069	1.253	0.163	3.039	-0.123	0.399
Cluster 2 ($r_{c2,t}$)	0.062	0.066	0.938	0.189	3.143	-0.173	0.330
Cluster 3 ($r_{c3,t}$)	0.120	0.053	2.262	0.077	3.079	-0.086	0.368
Cluster 4 ($r_{c4,t}$)	0.055	0.061	0.898	0.201	3.077	-0.153	0.295
Cluster 5 ($r_{c5,t}$)	0.031	0.049	0.633	0.103	3.025	-0.143	0.224
Cluster 6 ($r_{c6,t}$)	0.004	0.070	0.064	0.201	3.125	-0.262	0.365
Cluster 7 ($r_{c7,t}$)	0.054	0.064	0.854	0.139	2.955	-0.148	0.316
Cluster 8 ($r_{c8,t}$)	0.042	0.049	0.870	0.150	3.103	-0.128	0.283
Return of other asset classes							
Riskless asset ($r_{f,t}$)	0.031	0.013	0.000	0.032	3.054	-0.022	0.081
Long-term bond ($r_{lb,t}$)	0.009	0.012	0.714	0.056	3.042	-0.049	0.055
Corporate bond ($r_{cor,t}$)	0.031	0.017	1.806	0.064	2.987	-0.040	0.101
State variables							
real rate (rr_t)	0.165	0.187	-	0.493	3.515	-0.396	1.233
yield spread (YS_t)	0.014	0.005	-	-0.060	2.986	-0.003	0.030
credit spread (CS_t)	0.022	0.003	-	0.022	2.952	0.010	0.034
PE ratio (PE_t)	21.703	1.952	-	-0.001	2.992	14.716	29.295
temperature change (ΔT_t)	0.038	0.050	-	0.121	3.096	-0.147	0.262
log inflation rate (π_t)	0.024	0.006	-	-0.005	2.996	-0.001	0.045

Appendix D

Table D.1: Test-statistics of the augmented Dickey-Fuller test for stationarity of the return and state variables

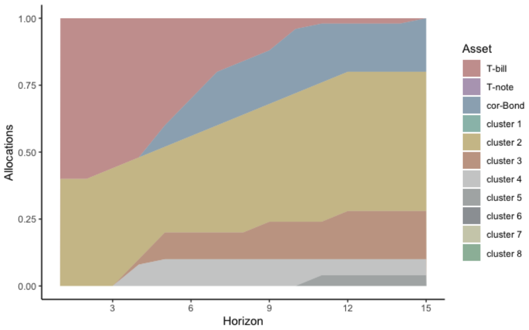
Variables	**Test statistic**	**p-value**
$r_{c1,t}$	-6.976***	0.000
$r_{c2,t}$	-16.643***	0.000
$r_{c3,t}$	-13.772***	0.000
$r_{c4,t}$	-14.585***	0.000
$r_{c5,t}$	-15.325***	0.000
$r_{c6,t}$	-18.266***	0.000
$r_{c7,t}$	-16.347***	0.000
$r_{c8,t}$	-15.304***	0.000
$r_{f,t}$	-1.626	0.470
$r_{lb,t}$	-11.549***	0.000
$r_{cor,t}$	-14.393***	0.000
rr_t	-1.484	0.541
YS_t	-3.955***	0.002
CS_t	-4.174***	0.001
PE_t	-3.048**	0.031
ΔT_t	-6.857***	0.000
π_t	-3.796***	0.003

Note. * $= p < 0.1$, ** $= p < 0.05$, *** $= p < 0.01$.

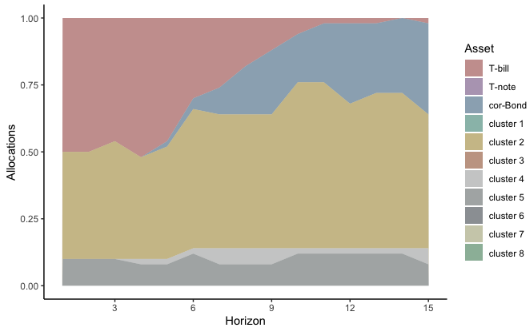
Table D.2: The more detailed restricted and unrestricted VAR(1) and AR(1) model coefficients and significance

				Restricted VAR(1) and riskless asset's AR(1) models					
	rr_t	YS_t	CS_t	PE_t	ΔT_t	π_t	$r_{lb,t}$	$r_{cor,t}$	$r_{f,t}$
μ	0.00(0.00)	0.00(0.00)***	0.00(0.00)**	0.33(0.40)	0.02(0.01)	0.00(0.00)**	0.01(0.01)	-0.01(0.01)*	0.00(0.00)***
rr_{t-1}	0.98(0.01)***	0.01(0.01)**	0.00(0.01)	-0.55(2.17)	-0.05(0.07)	0.01(0.00)	0.09(0.05)**	0.11(0.04)***	-
YS_{t-1}	0.03(0.02)	0.95(0.01)***	0.01(0.01)	-10.37(4.74)**	-0.05(0.16)	0.00(0.01)	0.25(0.1)**	0.18(0.09)**	-
CS_{t-1}	-0.05(0.03)	0.11(0.03)***	0.94(0.02)***	16.73(8.90)*	-0.28(0.30)	-0.05(0.02)***	-0.11(0.19)	0.52(0.16)***	-
PE_{t-1}	0.00(0.00)	0.00(0.00)**	0.00(0.00)*	0.97(0.01)***	0.00(0.00)	0.00(0.00)	0.00(0.00)*	0.00(0.00)	-
ΔT_{t-1}	0.00(0.00)	-0.01(0.00)**	0.00(0.00)	0.71(1.27)	0.34(0.04)***	0(0)	0.06(0.03)**	0.04(0.02)*	-
π_{t-1}	0.35(0.06)***	0.13(0.06)**	0.08(0.05)*	1.46(19.33)	-1.80(0.65)***	0.42(0.04)***	0.78(0.40)*	0.21(0.35)	-
$r_{lb,t-1}$	0.02(0.01)**	-0.01(0.01)	-0.01(0.01)**	-0.63(2.52)	0.06(0.08)	-0.03(0.01)***	0.14(0.05)***	0.29(0.05)***	-
$r_{cor,t-1}$	-0.02(0.01)**	0.01(0.01)	-0.02(0.01)***	3.08(2.51)	0.00(0.08)	0.02(0.01)***	-0.09(0.05)*	0.13(0.05)***	-
$r_{f,t-1}$	-	-	-	-	-	-	-	-	0.94(0.02)***
				Unrestricted VAR(1) model					
	$r_{c1,t}$	$r_{c2,t}$	$r_{c3,t}$	$r_{c4,t}$	$r_{c5,t}$	$r_{c6,t}$	$r_{c7,t}$	$r_{c8,t}$	
α	0.03(0.02)	0.03(0.02)	0.03(0.02)	0.02(0.02)	0.01(0.02)	0.03(0.03)	0.06(0.03)**	-0.02(0.02)	
				Lagged Y_2 coefficients: H_1					
$r_{c1,t-1}$	0.01(0.09)	-0.09(0.08)	-0.02(0.08)	0.08(0.08)	-0.28(0.09)	0.09(0.11)	-0.24(0.11)	-0.03(0.09)	
$r_{c2,t-1}$	0.02(0.08)	-0.16(0.07)	0.01(0.07)	-0.15(0.07)	0.09(0.08)	0.15(0.1)	0.02(0.1)	0.09(0.08)	
$r_{c3,t-1}$	-0.14(0.08)	-0.01(0.08)	-0.12(0.08)	-0.05(0.07)	-0.04(0.08)	-0.01(0.11)	-0.08(0.1)	-0.13(0.08)	
$r_{c4,t-1}$	-0.06(0.08)	0.07(0.08)	0.06(0.08)	0.01(0.07)	0.09(0.08)	-0.04(0.11)	0.27(0.1)***	-0.01(0.08)	
$r_{c5,t-1}$	0(0.07)	0.05(0.07)	0.05(0.07)	0.07(0.06)	0.05(0.07)	-0.08(0.09)	0.05(0.09)	0.06(0.07)	
$r_{c6,t-1}$	0.09(0.05)*	0.05(0.05)	0.05(0.05)	0.03(0.04)	0.05(0.05)	-0.2(0.06)	0.02(0.06)	0.05(0.05)	
$r_{c7,t-1}$	0.03(0.06)	-0.04(0.05)	-0.04(0.05)	0(0.05)	-0.03(0.06)	-0.04(0.07)	-0.08(0.07)	0.02(0.06)	
$r_{c8,t-1}$	0.01(0.07)	0.04(0.06)	-0.02(0.06)	0.04(0.06)	0.08(0.07)	-0.03(0.09)	0.02(0.08)	0.02(0.07)	
				Contemporaneous Y_1 coefficients: B_0					
rr_t	-48.14(22.38)	-50.59(20.79)	-62.08(20.7)	-8.59(19.85)	-21.35(22.02)	-70.76(28.47)	-32.57(26.79)	-48.69(22.79)	
YS_t	-55.42(22.77)	-56.7(21.16)	-68.87(21.07)	-16.75(20.2)	-21.14(22.41)	-77.76(28.97)	-36.32(27.26)	-57.68(23.19)	
CS_t	-3.76(13.63)	15.74(12.66)	5.7(12.61)	3.06(12.09)	-30.07(13.41)	15.39(17.34)	-8.45(16.31)	-8.31(13.88)	
PE_t	0.03(0)***	0.03(0)***	0.03(0)***	0.02(0)***	0.03(0)***	0.03(0)***	0.02(0)***	0.03(0)***	
ΔT_t	0.06(0.08)	0.1(0.08)	-0.04(0.08)	0.03(0.07)	0.06(0.08)	0.04(0.1)	0.05(0.1)	0.07(0.08)	
π_t	-49.06(22.35)	-51.68(20.77)	-64.12(20.68)	-10.47(19.83)	-21.45(22)	-73.62(28.44)	-34.66(26.76)	-50.93(22.77)	
$r_{lb,t}$	-6.99(2.57)	-9.4(2.39)	-9.64(2.38)	-3.11(2.28)	0.02(2.53)	-11.86(3.27)	-4.34(3.08)	-6.83(2.62)	
$r_{cor,t}$	0.67(1.13)	1.66(1.05)	1.11(1.05)	0.77(1)	-2.06(1.11)	2(1.44)	0.04(1.36)	0.01(1.15)	
				Lagged Y_1 coefficients: B_1					
rr_{t-1}	48.06(22.36)**	50.36(20.78)**	61.98(20.68)***	8.57(19.83)	21.31(22.01)	70.47(28.45)**	32.76(26.77)	48.7(22.77)**	
YS_{t-1}	55.7(22.95)**	57.14(21.33)***	69.76(21.24)***	17.23(20.36)	21.25(22.59)	79.05(29.21)***	36.87(27.48)	58.31(23.38)**	
CS_{t-1}	3.02(13.68)	-16.66(12.71)	-6.55(12.66)	-3.39(12.14)	29.88(13.47)**	-16.94(17.41)	7.05(16.38)	8.2(13.94)	
PE_{t-1}	-0.03(0)	-0.03(0)	-0.03(0)	-0.02(0)	-0.03(0)	-0.03(0)	-0.02(0)	-0.03(0)	
ΔT_{t-1}	-0.1(0.08)	0.07(0.08)	-0.1(0.08)	0(0.07)	0.02(0.08)	-0.03(0.1)	-0.02(0.1)	-0.09(0.08)	
π_{t-1}	50.06(22.41)**	52.75(20.83)**	64.05(20.73)***	10.58(19.88)	23.57(22.06)	73.53(28.51)**	35.51(26.83)	51.18(22.83)**	
$r_{lb,t-1}$	0.03(0.19)	-0.15(0.17)	0.2(0.17)	0.13(0.17)	0.08(0.18)	-0.16(0.24)	-0.06(0.22)	0.32(0.19)*	
$r_{cor,t-1}$	-0.3(0.16)	-0.17(0.15)	-0.24(0.15)	-0.23(0.14)	-0.2(0.15)	-0.24(0.2)	-0.18(0.19)	-0.3(0.16)	
				Contemporaneous and lagged effect of $Y_{1,t}$ on $Y_{2,t}$: $B_0A_1 + B_1$					
rr_{t-1}	0.04(26.53)	-0.21(24.45)	-0.06(23.84)	0.00(1075.17)	-0.07(58.23)	0.49(31.52)	0.25(49.48)	0.02(42.70)	
YS_{t-1}	-0.12(26.49)	-0.30(24.41)	-0.40(23.80)	0.00(1073.69)	0.09(58.15)	1.03(31.47)	-0.21(49.40)	0.07(42.64)	
CS_{t-1}	0.01(15.79)	0.25(14.55)	-0.22(14.18)	0.00(642.08)	-0.12(34.72)	0.68(18.75)	0.31(29.48)	0.01(25.44)	
PE_{t-1}	-0.05(0.00)	0.27(0.00)***	-0.26(0.00)	0.00(0.13)	0.02(0.01)***	-0.66(0.00)	0.16(0.01)***	-0.05(0.01)	
ΔT_{t-1}	0.12(0.10)	-0.38(0.09)	0.75(0.09)***	0.00(3.88)	0.01(0.21)	1.77(0.11)***	0.28(0.18)	0.05(0.15)	
π_{t-1}	-0.14(26.58)	0.52(24.49)	-0.73(23.88)	0.00(1073.92)	-0.02(58.24)	1.22(31.58)	-0.06(49.51)	0.03(42.73)	
$r_{lb,t-1}$	0.31(0.58)	0.08(0.52)	-0.59(0.46)	0.00(119.04)	-0.01(4.19)	1.28(0.49)***	0.13(2.70)	0.06(2.36)	
$r_{cor,t-1}$	0.05(0.33)	0.12(0.30)	0.32(0.27)	0.00(52.49)	-0.07(1.92)	-0.04(0.31)	0.50(1.28)	-0.03(1.12)	

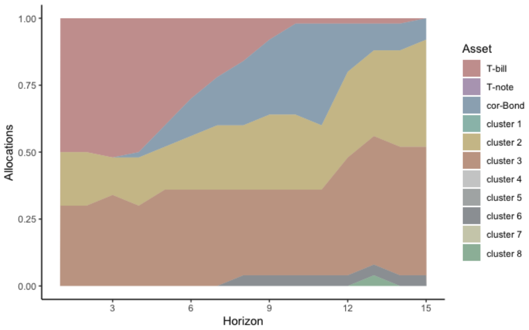
Note. * = $p < 0.1$, ** = $p < 0.05$, *** = $p < 0.01$.



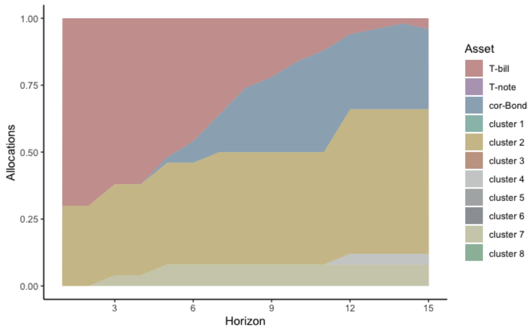
(a) Average Investor



(b) Investor 1



(c) Investor 2



(d) Investor 3

Figure D.3: The optimal asset allocations across the remaining ESG investors.