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Downside risk hedging of spot and future prices in large portfolios with (Hierarchical) Archimedean Copulas and Vine Copulas

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Abstract

Effectively hedging spot and future prices decreases the risk of a portfolio. These optimal hedging ratios depend on the dependence structure of the portfolio, which can effectively be determined by the help of copulas. The financial timeseries which are modelled by copula can be used to determine a hedge ratio based on a certain objective. Copulas that are considered in this paper are the Clayton copula, C-vine, D-vine and Hierarchical Archimedean copula. The main finding of this paper is that by using copula the downside risk is reduced significantly, especially when the Expected Shortfall is used . The Hierarchical Archimedean Copula performs best overall and the Clayton copula performs consistently well and has the shortest computation time. In addition copulas can be applied to large portfolios and cross hedging.

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1 Introduction

Two well-known ways of reducing risk is diversifying and hedging. Focusing on the latter, one can hedge the movement of a derivative by hedging it with the underlying stock. For example when going long in a future it can be hedged with going short in the corresponding spot price. This raises the question of how this hedge ratio should be determined. In addition to that many portfolios are highly dimensional, which complicates determining the hedge ratios. Therefore it is paramount to model the joint downside risk of spot and futures prices. One tool to model this dependence are Copulas. Copulas need less assumptions than other modelling techniques which results in a better empirical fit.

Reducing risk in the framework of this paper means that there is a certain objective function that is minimized. Value-at-Risk (VaR) and Expected Shortfall (ES) are two objectives that are frequently applied. Many companies use models that require one of these risk measures. Therefore the effect on downside risk reduction can be determined by using these measures as objective.

The main goal is to have a model that reduces risk for many different portfolios. Obtaining a more flexible hedging method is relevant for many different parties, but especially parties that hold large portfolios. For example hedge funds and asset management companies. This brings up the research question of this paper. *Are models that apply copulas able to reduce downside risk by hedging spot and future prices in large portfolios?*. To answer this research question the following subquestions are addressed. First of all we need to address different copulas to model the timeseries. In addition, we need to find how an optimal hedging ratio is determined with the modelled timeseries. Finally a certain performance measure should be determined, that assesses the strength of the models in out of sample hedging .

To answer the main question and subquestions parts of the methodology are based on the paper of Sukcharoen and Leatham (2017). Their goal is similar, they determine an optimal hedge ratio with vine copula. This method is extended by different copulas and by using different data. The data from this paper are spot and future prices of different assets and commodities. The data includes weekly Wednesday spot and future prices and spans from December 31, 1999 to December 31, 2020.

With the obtained data four different copulas are estimated. The four copulas used are the Clayton copula, C-vine copula, D-vine copula and Hierarchical Archimedean Copula (HAC). With all the mentioned copulas, the optimal hedging ratio is determined while minimizing VaR 99% and ES 97.5% as objective. These hedge ratios are then used to determine the hedging performance out of sample, which makes it possible to compare the hedging power of the different methods.

The main findings of this paper are that all models consistently yield a positive result in downside risk reduction when the Expected Shortfall is used a objective. However, when the VaR is considered the models performs significantly different.The HAC model yields the best results over all. The Clayton copula is a good contender, since it yields good results and is relatively easy to compute. The D-vine increases the risk in some cases and is also the most volatile method. Furthermore the vine copula are not feasible in all settings and have a relatively long running time. Therefore from a risk management perspective the Hierarchical Archimedean Copula and the multivariate Clayton copula are the best choices based on these findings. In addition to that, determining an optimal hedge ratio based on the Expected Shortfall results in better risk reduction than using the Value-at-Risk as objective function.

The contribution to current literature is that this paper hedges in larger portfolios. Research often focuses on the bivariate case which is not always realistic. Another contribution is that this paper uses a method that is applicable to many different portfolio scenarios. Lastly some results motivate that not all models work well in high dimensions, for example the vine copula performs worse than expected in this setting.

First this paper provides a literature in Section 2, which gives background on previous copulas and hedging models. This is followed by the method for obtaining the data in Section 3. After that, the methodology is illustrated in Section 4. In Section 5 the in sample hedging ratios and out of sample downside risk reduction for all models are presented. Finally in Section 6 the conclusion is drawn from the results and there are some suggestions for further research.

2 Literature Review

Reducing risk is a major part of multiple aspects in the financial industry. One way to reduce risk is hedging a current position with the use of derivatives. An issue with hedging involves determining the optimal hedge ratio. The optimal ratio relies on a certain objective function. By optimizing this function the optimal hedge ratio can be derived. This leads to another issue: determining the objective function. There are many different ways to obtain this objective function and the corresponding hedge ratio. One method relies on minimizing the variance of the hedged portfolio. (Ghosh (1993)) A disadvantage of this method that it does not take expected return into account. Another method that does take that into account is using the utility function as the objective, for example as demonstrated by Cecchetti et al. (1988). However this requires using specific utility function and return distributions. Another approach is by using ordinary least squares (OLS) as objective. This involves regressing the change in future prices on the change in spot prices. To ensure this method is valid and efficient the OLS assumptions must be satisfied. In reality we find that these assumptions often do not hold.

To improve upon the previously mentioned models different GARCH models have been implemented. This technique allows for a dynamic hedging ratio. The GARCH models can then be used to estimate a minimum variance hedging ratio. Hsu et al. (2008a) implement a copula GARCH model for dynamic hedging. Furthermore they look at financial indices and their future contracts. The hedge ratio in their research depends on the variance and covariance between the spot price and the future price. For their empirical study they mostly focus on improving the hedge ratio by means of in sample variance reduction. In the case of out of sample performance one period forecasts are also assessed by means of variance reduction. In addition to that they demonstrate the results for cross hedging, in their case hedging the MSCI-SWI index with the Swiss Frank future. They conclude that for all their methods the copula-GARCH models outperform the other GARCH models.

In addition, the GARCH model is used in more complex frameworks. For example Lee (2009) develop a regime-switching Gumbel–Clayton (RSGC) copula GARCH model which they apply to optimal futures hedging. This method allows for switching between different copulas over time. Their research only considers single product hedging.

Obtaining a solution for the minimum downside hedge ratios calls for estimating the full joint distribution of the future and spot price movements. For single product hedging one of the standard methods is non parametric, for example the historical simulation method. However this often turns out to be inaccurate since the method depends on historical data. Barbi and Romagnoli (2014) show that introducing bivariate Archimedean copulas greatly reduce the risk in the case of single product hedging. However it is not realistic to stay in a single product setting. To extend the dimension size Sukcharoen and Leatham (2017) suggest using vine copula. This model was initially introduced by Joe (1996). The vine copula is a class of multivariate copula and is able to account for fat-tailedness and skewness.The vine copula allows for separating the modelling of the marginal distribution unlike other multivariate copula.

As mentioned by Aas et al. (2009) the vine copula model is extremely flexible compared to other methods. Furthermore special types of vines, like C-, D- or R-vines, help with reducing the number of possible alternative models. However the issue of a large amount of potential candidate models remains. Okhrin et al. (2013) provide a solution for this with the Hierarchical Archimedean copula (HAC). In their research HAC is not applied to future hedging but to determine the Value-at-Risk of a certain portfolio. They found that HAC did not beat the vine models in their research. However, there is not much literature where both the vine copula and HAC are applied to future hedging.

3 Data

The kinds of assets and commodities that are usually traded using future contracts are grains, livestock, currencies, financials, softs, metals and energies. For example Sukcharoen and Leatham (2017) focus on the US crude oil futures and spot prices while Hsu et al. (2008b) focus on indices and currencies. As a starting point two popular and highly traded index futures are considered. These are the S&P 500 and the AEX future contracts. These indices contain the most traded companies in the United States stock exchange and Amsterdam stock exchange respectively. As mentioned by Hsu et al. (2008b) index futures are more popular, which is one of the aspects where this research differs from Sukcharoen and Leatham (2017).

The first extension is including the U.S. Dollar index future and spot prices (USDX). As explained by Clyman et al. (1997) the USDX futures contract serves as a benchmark for the value of the US dollar in an international context. The USDX enables monitoring the movement of the US dollar relative to other world currencies. This implies that this index has some relation to the S&P 500 and AEX future contracts. To increase the dimensions, future and spot prices from other industries are added. By adding CBT corn composition and 100 oz Gold Futures the dimension is increased to ten timeseries. Finally ECBOT-soybeans, ECBOT-wheat and

ICE natural gas are added, which leaves us with sixteen timeseries. Furthermore cross hedging is considered, this means that the future prices and spot prices are from different financial instruments.

Since future contracts expire on a a certain date the methodology of Sukcharoen and Leatham (2017) is used to construct the timeseries of future prices. The cleaned data consists of weekly Wednesday closing spot and futures prices. Tuesday prices are taken when it occurs that Wednesday prices are missing. All the spot and future prices are obtained from the Thomas Reuters Datastream database. The prices span from December 31, 1999 to December 31, 2020. In this timeframe there are many different future contract with different maturities. From all this information a timeseries of weekly changes in spot and futures prices is constructed. For calculating the changes in future prices the closing prices for the nearest to expiration futures are taken. The rollover to the next futures contract occurs on a Wednesday the week before a contract expires. This means that a week before the contract expires, prices of the next nearest to expiration future contract are taken. Consequently a lot of data cleaning needs to be done. For example for the AEX there are more than 400 dead future contracts. This means that the raw data includes around 73000 data points. After the pre-described data cleaning this is cut down to 1096 weekly Wednesday future prices. Table 3.2 shows the summary statistics of the spot and future prices. Figure A.3 shows some of the spot and future prices over time. The plots of the other spot and future prices can be found in Figure A.1 and A.2

	S&P 500		AEX		USDX		Corn		Gold	
	S	F	S	F	S	F	S	F	S	F
μ	8.94e-04	8.80e-04	-1.47e-05	-1.16e-05	-1.03e-04	-1.02e-04	8.42e-04	5.61e-04	0.00171	0.00172
σ	0.0239	0.0241	0.0313	0.0316	0.0108	0.0109	0.0419	0.0365	0.0236	0.0248
Skew	-0.884	-0.868	-0.580	-0.664	-0.232	-0.0011	-0.137	0.210	-0.528	-0.385
Kur	8.91	9.02	9.73	10.0	6.64	4.80	5.51	11.4	5.91	6.89

Table 3.1: Summary statistics for weekly log returns of S&P 500, AEX and USDX spot and future prices. The weekly prices consist of all the Wednesday closing prices for the sample period 31/12/1999 - 31/12/2020. Where S are the spot prices, F the future prices, μ the std, σ the standard deviation, skew the skewness and kur the kurtosis

	Soy		Gas		Wheat	
	S	F	S	F	S	F
μ	0.0025	7.10e-04	0.0014	0.0013	9.77e-04	6.69e-04
σ	0.1335	0.0329	0.0785	0.0807	0.0527	0.048
Skew	-0.539	1.23	0.939	2.22	0.171	0.687
Kur	55.1	35.7	7.36	19.0	6.67	6.05

Table 3.2: Summary statistics for weekly log returns of S&P 500, AEX and USDX spot and future prices. The weekly prices consist of all the Wednesday closing prices for the sample period 31/12/1999 - 31/12/2020. Where S are the spot prices, F the future prices, μ the std, σ the standard deviation, skew the skewness and kur the kurtosis

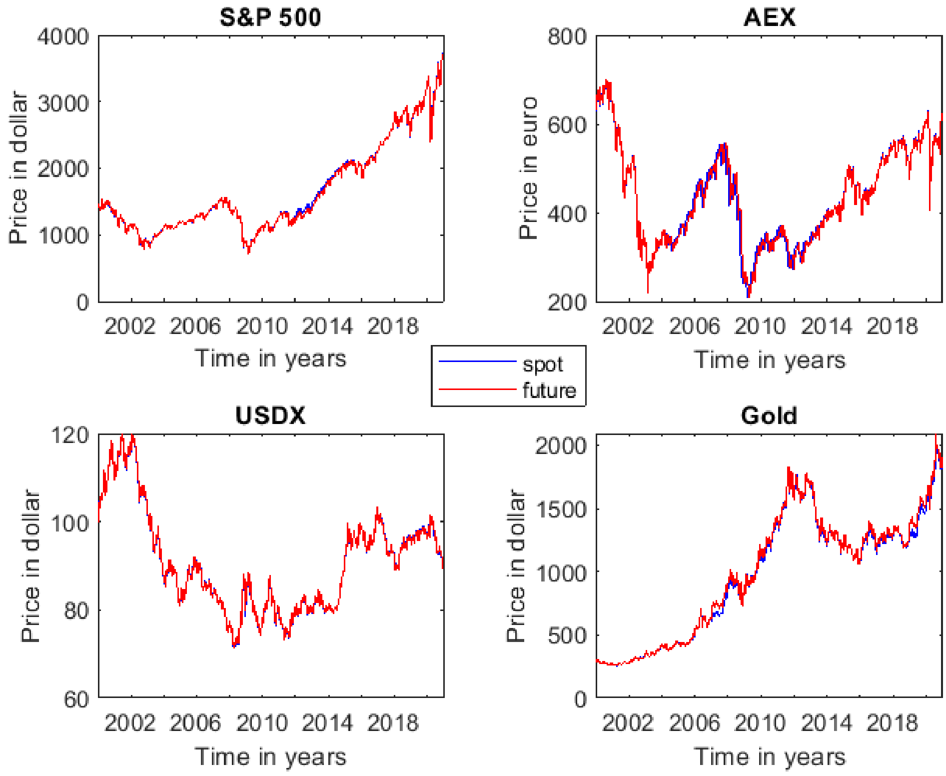

Figure 3.1: Plot of weekly spot and future prices of the S&P 500, AEX, USDX and US gold prices. The weekly prices consist of all the Wednesday closing prices for the sample period 31/12/1999 - 31/12/2020

The summary statistics show that the spot and future log returns display excess kurtosis, which implies the data has heavy tails. In addition the data is negatively skewed, which implies that there is a fatter tail on the left side of the distribution. This indicates that the log returns are not normally distributed. In addition, the spot and future prices are very similar. This is because the future prices that are closest to expiration are taken. Close to expiration future prices converge upon their spot price.

4 Methodology

This section describes the different hedging objectives and corresponding copula methods. The benchmark copula model will be a simple multidimensional Clayton copula approach which will firstly be extended to Sukcharoen and Leatham (2017) and their vine copula approach. This model will then be extended by using Hierarchical Archimedean copulas (HAC) and lastly by penalized HAC. In addition to changing the copula the dimensions are increased. The methodology section first addresses the objective hedging function, then the different copula methods and finally how the performance will be measured. The methods are implemented with MATLAB (2022)

4.1 Hedging Problem

Hedging in its essence is to combine investments from the spot market and the derivatives market into one portfolio that reduces the fluctuation in the portfolio value. Here the derivative market of interest is the futures market.

Following Chen et al. (2003) the initial assumption is a long spot position and a short future position. In addition, it is assumed that all portfolios are equally weighted portfolios. This means that when the portfolio is extended each stock will receive the same weight. This is a 'naive' approach and very likely to not be the optimal stock allocation. However DeMiguel et al. (2009) show that this allocation remains hard to beat. In addition the equally weighted portfolio is computationally easy and therefore a suitable choice. Lastly the goal is to determine an optimal hedge ratio and not optimal equity weights. This results in the following profit on the hedged ratio.

$$y_t(b) = \Delta S_t - B\Delta F_t \tag{1}$$

With hedge ratio(s) B, which is a vector of all the individual hedge ratios. This means that the amount of stocks bought will be the same, but the amount of futures sold will change according to the hedge ratio. Following Sukcharoen and Leatham (2017), the assumption is that futures positions are taken in period t-1 and liquidated in period t. The goal is to select the optimal hedge ratios e^* that minimize the downside risk of the hedged Prices and Losses (P&L) Mathematically,

$$e^* = \arg\,\min Risk(y_t(e)) \tag{2}$$

where $Risk(yt(e))$ is the measure of downside risk which is defined on objective $yt(e)$. The risk measures that are considered can be found in Section 4.2.3. However, to solve for a certain risk measure copula are needed to model the joint distribution of random variables.

4.2 Copula

Merging marginal distributions into joint distributions is done by using Copula. copulas measure dependence between different variables and are a form of linking mechanisms. A copula has uniform marginals on the interval [0,1] and is a k-dimensional distribution function $C : [0,1]^m \rightarrow [0,1]$ .The multivariate distributions can be modelled conforming to Sklar's theorem.

Theorem 4.1 (Sklar's theorem) *If F is a k-dimensional distribution function with marginal cdf $F_{X_1}, \ldots, F_{X_k}$. Then a copula $C \; \forall \; x_1, \ldots, x_k \in \mathbb{R}^k$ exists satisfying*

$$F(x_1, \ldots, x_k = C\{F_{X_1}(x_1), \ldots, F_{X_k}(x_k)\} \tag{3}$$

$F_{X_1}, \ldots, F_{X_k}$ are continuous if C is unique . If $F_{X_1}, \ldots, F_{X_k}$ are distribution functions and C is a copula, then the function $F(x_1, \ldots, x_k)$ is a joint distribution function with the marginal cdf $F_{X_1}, \ldots, F_{X_k}$.

A base point could be any standard Archimedean copula, since they allow for capturing asymmetric and nonlinear dependence between random variables. However these copulas might not be adequate to capture the dependence when the dimension increases, as then it is assumed that all the variables have the same degree of co-movement. To check if this is actually inadequate as a base case the starting point is the multidimensional Clayton Copula. The Clayton copula captures lower tail dependence. The derivation of the multivariate log likelihood can be found in section A.1 in the Appendix.

4.2.1 Vine Copula

One way to go beyond the standard multivariate copulas is by utilizing a vine copula approach. This approach allows for different pairs of variables to have varying dependence patterns. Via bivariate copulas and their marginal distribution function a multivariate copula can be obtained. This copula is a vine copula. Each copula allows to distinguish the dependence pattern of each pair. In essence, the joint density function can be expressed in term of the individual marginal distributions and a group of the bivariate copulas. The copulas do not have to originate from the same copula family, which provides a lot of flexibility. However, because of the many possible combinations the amount of parameters increases when the number of dimensions increase. Since the methodology partly follows Sukcharoen and Leatham (2017), two vine copula methods are implemented. This methodology focusses on six dimensions, which can be decreased or increased and on two vine copulas, the C-vine and D-vine. First the C-vine copula is considered. In every node of the C-vine copula one variable is connected to all the other variables. In a D-vine copula each variable is at most connected by two other variables. Both copula are given by Sukcharoen and Leatham (2017) using the followoing notation.:

$$f(x_1,x_2,x_3,x_4,x_5,x_6) = f_1 f_2 f_3 f_4 f_5 f_6 c_{1,2} c_{1,3} c_{1,4} c_{1,5} c_{1,6} c_{2,3|1} c_{2,4|1} c_{2,5|1} c_{2,6|1} c_{3,4|1,2} c_{3,5|1,2} c_{3,6|1,2} c_{4,5|1,2,3} c_{4,6|1,2,3} c_{5,6|1,2,3,4} \quad (4)$$

$$f(x_1,x_2,x_3,x_4,x_5,x_6) = f_1 f_2 f_3 f_4 f_5 f_6 c_{1,2} c_{2,3} c_{3,4} c_{4,5} c_{5,6} c_{1,3|2} c_{2,4|3} c_{3,5|4} c_{4,6|5} c_{1,4|2,3} c_{2,5|3,4} c_{3,6|4,5} c_{1,5|2,3,4} c_{2,6|3,4,5} c_{1,6|2,3,4,5} \quad (5)$$

Where Equation (4) and Equation (5) are the six-dimensional C-vine and D-vine respectively. $c_{i,j}$ is a copula function linking x_i and x_j. These equations already show that when dimensions increase vine copulas soon become infeasible.

Fitting a joint distribution function utilizing the C- or D-vine copula can be described in four steps. First the marginal distributions are modelled. For each timeseries the marginal distribution is estimated by using an empirical distribution function. Consequently the timeseries are transformed into standard uniform variables.

The second step entails selecting an order of the variables for the C- or D-vine copula. This order is paramount for the structure of the vine. Considering the C-vine, the variable with the highest degree of association is selected. (Czado et al. (2012))

To determine the degree of association the absolute values of pairwise Kendall's tau coefficients are summed.

$$\Theta_{\tau}^{i} = \sum_{j=1}^{n} i \neq j|\tau_{i,j}| \tag{6}$$

The second variable is the variable that has the highest degree of association with the other variables. This process is reiterated till all the variables are part of the structure.

For the D-vine copula structure the variables are ordered in such a way that the sum of the absolute values of pairwise Kendall's tau coefficients

$$\Theta_{\tau}^{i} = \sum_{j=1}^{n-1} |\tau_{i,i+1}| \tag{7}$$

is maximized(Dißmann et al. (2013)).

The third step is to choose a bivariate copula for all pair copulas. As in Sukcharoen and Leatham (2017) a sequential estimation approach proposed by Aas et al. (2009) is used. This approach uses different copula families. A copula family is selected by using the Akaike Information Criterion (AIC). The fourth and final step is to estimate all copula parameters. The estimation is done consecutively. This means that the estimation starts from the first vine tree with the maximum pseudo likelihood method (Genest et al. (1995)).

4.2.2 HAC

As mentioned before, a multivariate multivariate Archimedean copula implies that all the variables have the same degree of co-movement. Since this is rarely feasible this was solved by using the vine copula. However the vine copula has a lot of parameters that need to be estimated. An alternative to this are hierarchical Archimedean copulas (HACs).

Unlike the Archimedean copulas, HACs define the dependence structure recursively. At the first level the dependence between two variables is modelled by a certain copula function. Then at the second level another copula is used to model the previous level with the new variable. This stds we get a nested copula. HACs rely on generator function $\phi \in \mathcal{L} = \{\phi : [0;\infty) \rightarrow [0,1] | \phi(0) = 1, \phi(\infty) = 0; (-1)^k \phi^{(k)} \geq 0; k \in \mathbb{N}\}$ and the non decreasing inverse $(-1)^k \phi^{(k)}(x) x \in [0,\infty)$. The copula function for fully nested copulas is given by the following equation (Okhrin et al. (2013))

$$C(u_1,\ldots,u_k) = \phi_{k-1}(\phi_{k-1}^{-1} \circ \{\phi_{k-2}[\ldots(\phi_2^{-1} \circ \phi_1[\phi_1^{-1}(u_1) + \phi_1^{-1}(u_2)] + \phi_2^{-1}(u_3)) + \cdots + \phi_{k-2}^{-1}(u_{k-1})]\} + \phi_{k-1}^{-1}(u_k) \tag{8}$$

The generator for the copula can come from the same family or from different generator families. The difference between HAC and vine copulas is that for the HAC the density needs to be derived from the copula function. This is done by taking derivatives with respect to the generator functions. Usually the generator functions are not that difficult and it is possible to compute numeric derivatives, even when the dimensions increase.

As the name already implies the structure of the copula is hierarchical. At each node of the tree there is a marginal distribution which is also given by a HAC. In addition, the recursive structure, HAC is helpful in modelling time dependence. There exist many possible structures of HAC, so an algorithm to determine the structure is necessary. Let $M = \{\phi_j\}, j = 1, \ldots, M$ be the set of familiar generator functions. Each function ϕ_j depends on a vector of parameters θ_j. Let $(X_1, \ldots X_k)$ denote the vector of independent random variables and $X = x_{j,i}$ are the samples for $i = 1, \ldots, n$ and $j = 1, \ldots, k$. As in Okhrin et al. (2013) the notation s is introduced, which denotes the structure of the HAC. This variable notes how the variables are joined at a single node. The steps for determining the structure are follow below:

1. Estimate the parameters θ_j using the maximum likelihood estimator for each set $X_i, i \in I_{kj}, j = 1, \ldots, 2, k-1$ and each $\phi \in M$.

2. Determine the optimal subset of variables I_1 to be grouped by the given copula. Then introduce a pseudo-variable defined by $Z_1 = \hat{C}(I^1)$. After that consider the set $Z_1 \cap \{X_j\}_{j \in \{1 \ldots k\}}$ with size $k_2 = k - dim(I^1) + 1$

3. With the new obtained set, repeat step 1.

4. Let the obtained grouping be denoted as I^2. Then repeat step 2.

5. Keep repeating all steps until $k_j = 1$

Now the issue remains to decide which grouping is the best. This can be done according to multiple methods. One of the methods Okhrin et al. (2013) use is grouping based on goodness of fit. The goodness of fit test assesses how close the estimated structure is to the real data. Since there are many possible HAC structures this test reduces computation time to select the optimal structure. This test is given by the following equation as mentioned by Genest et al. (2009):

$$S_n = \int_{[0,1]^d} C_n(u)^2 dC_n(u) \tag{9}$$

With $C_n = \frac{1}{n} \sum_{i=1}^{n} 1(U_{i1} \leq u_1, \ldots, U_{id} \leq u_d)$, also called 'empirical copula'. A lower test statistic suggest that the distance between the estimation and true value is smaller. The goodness of fit test is applied after the entire HAC structure is estimated. From all the HAC structures the one with the lowest goodness of fit test statistic is chosen.

4.2.3 Performance

To evaluate the heging effectiveness of the different models the percentage of downside risk reduction of the hedged P&Ls compared to that of the unhedged P&Ls is computed

Sukcharoen and Leatham (2017) uses four different hedging objectives. This paper uses two of these: Value-at-Risk (VaR) and Expected Shortfall (ES). As mentioned by Yamai and Yoshiba (2005) VaR is the standard risk measure because of its conceptual simplicity. However VaR has a few problems like disregarding the loss beyond the VaR level. Furthermore, VaR is not a coherent risk measure. ES is the conditional expectation of losses beyond the VaR level and is a coherent risk measure.

In this paper the VaR covers the largest potential loss for a time period of one week for a confidence level (p). More universally, VaR at a confidence level p is given by:

$$VaR_p = -F^{-1}(1-p) \tag{10}$$

where F is the cdf evaluated at a certain confidence interval. The other risk measure, ES, is given as:

$$ES_p = -E[y_t | y_t \leq -VaR_p] \tag{11}$$

Which computes the expected loss given that the loss exceeds the VaR at confidence level p.

The two downside risk measures are computed by using a Monte Carlo simulation. This is applied to all models.

With the estimated copula draws, standard uniform variables are generated. These standard uniform variables are then converted to the joint distribution by using the inverse cdf.

As presented in the Data section the financial timeseries do not seem to be normally distributed. On accounts of that, both the normal distribution and the t-distribution are taken into consideration to convert the draws. The simulated spot and future price changes are then used to calculate the hedged prices and losses. After that, the optimal hedge ratios are derived by solving the minimization problem in Equation (1).

To asses out of sample performance a moving window of 5 years (260 weeks) is used. With these five years the optimal hedging ratio is determined which is then used to construct the hedged P&L for the next 130 weeks. The window is moved forward by one week. This results in many optimal hedge ratios, which are used to compute all the different P&Ls.

Finally the hedging effectiveness will be measures as a percentage reduction in the downside risk compared to the unhedged P&Ls. The mean of the different hedged P&Ls are calculated across the different test windows.

$$HE = (1 - \frac{Risk(y_t(e*))}{Risk(y_t(0))}) * 100\% \tag{12}$$

Where $y_t(0)$ the unhedged P&L and $y_t(e*)$ is the hedged P&L.

5 Results

5.1 In sample results

This sections presents the optimal hedge ratios which are obtained in sample. First the bivariate hedge ratios are estimated with the Clayton copula. After that the multivariate hedge ratios are estimated with the Clayton, C-vine, D-vine, HAC. The risk objectives are to minimize the VaR at a 99% significance level and ES at a 97.5 % significance level. This is based on the risk measures suggested by Basel III (Chang et al. (2019)). Basel III risk measures are part of an European law which aims to secure the financial stability of banks.

Table 5.1 displays the optimal hedge ratios for the bivariate case.This means that the Clayton copula is used to hedge between the future contracts and its corresponding spot price. Both the normal distribution and the student-t distribution with $n-1$ degrees of freedom are used to estimate the optimal hedge ratios. The first 261 observations are lost because a moving window of 261 weeks is used.

	S&P	AEX	USDX	Corn	Gold
Normal distribution					
optimal hedge ratio VaR 99%	0.997(0.0168)	0.947(0.00732)	0.991(0.0015)	0.968(0.0058)	0.985(0.0032)
optimal hedge ratio ES 97.5%	0.998(0.00851)	0.948(0.0101)	0.991(0.0014)	0.966(0.0064)	0.985 (0.0024)
Student-t distribution					
optimal hedge ratio VaR 99%	0.981(0.00811)	0.926(0.0177)	0.991(0.0016)	0.968(0.0059)	0.986(0.0033)
optimal hedge ratio ES 97.5%	0.986(0.00482)	0.927(0.0210)	0.991(0.0014)	0.967(0.0064)	0.985 (0.0027)

Table 5.1: Mean and standard deviation of optimal hedge ratios for S&P, AEX, USDX, Corn and Gold spot and future prices for 12/31/2004 until 12/31/2020. Both the hedge ratio's for Value-at-Risk and expected shortfall are given.

It appears that it makes no significant difference if the normal distribution or the student t-distribution is used to transform the joint distribution. This could be explained by the central limit theorem, the sample is large enough to converge to the normal distribution (Heyde (2006)). Therefore the normal distribution is used in the remaining estimations. Table 5.2 shows the other bivariate hedge ratios.

	Gas	Soy	Wheat
Normal distribution			
optimal hedge ratio VaR 99%	0.972 (0.0047)	0.867 (0.0324)	0.937(0.0149)
optimal hedge ratio ES 97.5%	0.972 (0.0049)	0.870 (0.0388)	0.935(0.0161)

Table 5.2: Mean and standard deviation of optimal hedge ratios for Gas, Soy, Oil and Wheat spot and future prices for 12/31/2004 until 12/31/2020. Both the hedge ratio's for Value-at-Risk and expected shortfall are given.

Figure 5.2 shows all the hedge ratios that correspond to Table 5.1 and Table 5.2

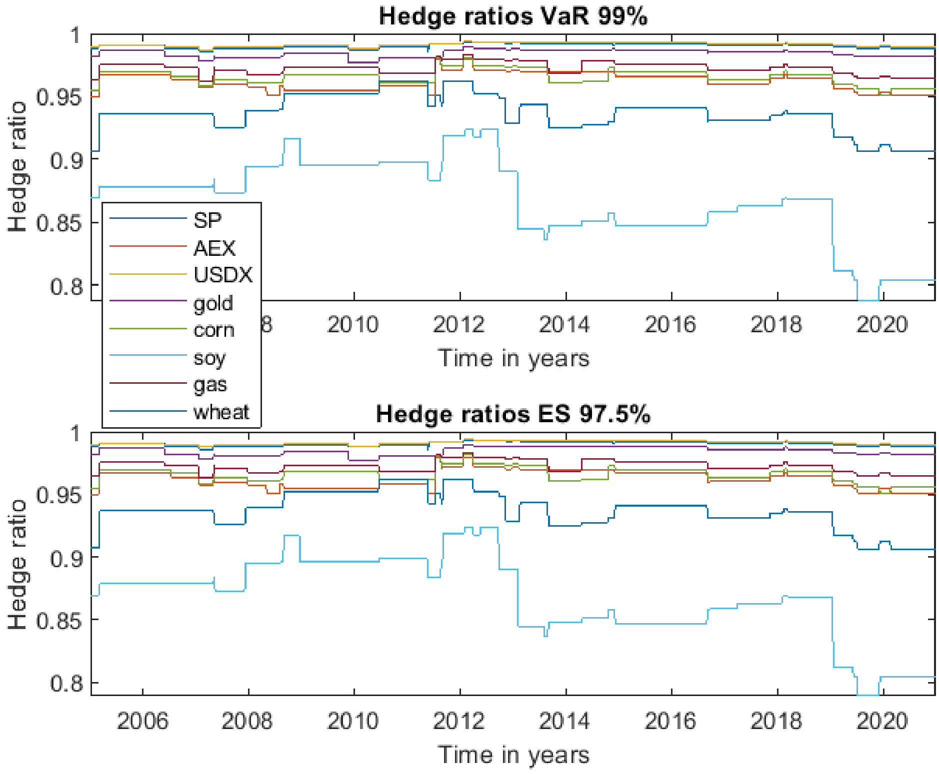

Figure 5.1: Optimal VaR 99% and ES 97.5% hedge ratios for S&P, AEX, USDX, corn, gold, gas, soy and wheat spot and future prices for 12/31/2004 until 12/31/2020

As expected from the future and spot prices as given in figure A.3 the hedge ratios stay close to one when only one future contract with the corresponding spot price is included. This is because the spot prices and future prices move closely together and the main objective is to reduce risk. For the expected shortfall the optimal hedge ratio is slightly higher than for the Value-at-Risk. However the difference is extremely small, which can be explained by the fact that a VaR of 99% is similar to an ES of 97.5%. The hedge ratios in the bivariate case are quite similar, therefore the remainder of the results consider the multivariate cases. The multivariate Clayton copula assumes that all timeseries can be explained by one dependence structure.

In the case of the vine copulas a bivariate copula family is chosen at each node of the vine. Table 5.3 and 5.4 present which family's are chosen in which node of the vine. This is given for both the C-vine and D-vine. The starting point is four dimensions, where we only look at the S&P and AEX. The copula families in the vine copula are selected based on the AIC as described in the methodology. For this estimation the MATLAB (2022) Vine Package by Coblenz (2021) is employed. This package can choose between 14 different bivariate copulas.

	Family		
Tree 1	Joe	Plackett	Plackett
Tree 2	t	t	
Tree 3	t		

Table 5.3: Copula family's selected for the C-vine at each node

	Family		
Tree 1	Joe	Plackett	Plackett
Tree 2	t	t	
Tree 3	Frank		

Table 5.4: Copula family's selected for the D-vine at each node

As described in the methodology in the C-vine one variable is connected with all variables. In the D-vine one variable is at most connected with two variables. For the C-vine the first tree gives the family's of $c_{1,2}, c_{1,3}$ and $c_{1,4}$. The second tree for $c_{2,3|1}$, $c_{2,4|1}$. The last tree gives the family for $c_{3,4|1,2}$. The D-vine is structured in a way that the first tree shows the family's for $c_{1,2}, c_{2,3}$ and $c_{3,4}$. The second tree consists of $c_{1,3|2}$, $c_{2,4|3}$. The third tree presents the family for $c_{1,4|2,3}$. Therefore both methods select different copula families. However the families that are selected are quite similar, only one family differs.

This estimation method can be extended to more dimensions. Table 5.5 and Table 5.6 presents the families that are chosen when US dollar spot and future prices are added to the portfolio.

	Family				
Tree 1	Plackett	Plackett	Joe	Plackett	Plackett
Tree 2	t	t	t	t	
Tree 3	t	Gaussian	t		
Tree 4	independent	t			
Tree 5	Clayton				

Table 5.5: Copula family's selected for the C-vine at each node

	Family				
Tree 1	Plackett	t	Plackett	Plackett	Plackett
Tree 2	t	t	t	t	
Tree 3	t	t	t		
Tree 4	Plackett	t			
Tree 5	t				

Table 5.6: Copula family's selected for the D-vine at each node

Again different families are chosen for the different structures. It appear that the t family and Plackett family are chosen the most frequently. The same method is also applied to higher dimensions. These Tables can be found in the Appendix section A.4.

Increasing the dimensions has two disadvantages. First of all the computation time increases. This can cause problems for larger portfolios, but stays within reason for sixteen timeseries. Another problem is that because of the many combinations (close to) singular matrices occur. These matrices do not have full rank and are therefore not invertible matrices occur. This especially poses a problem for estimating the t-copula. The consequence is that this is not a flexible method, some asset combinations will cause errors.

To estimate the HAC copula the MATLAB (2022) HACopula Toolbox by Gorecki et al. (2018) is used. This toolbox provides nine families of generators, which are given in Table 5.7. For more detailed explanation see Gorecki et al. (2018).

Family	Generator
Ali-Mikhail-Haq	$\frac{1-\theta}{e^t-\theta}$
Clayton	$(1+t)^{1/\theta}$
Frank	$-log(1-(1-e^{-\theta})exp(-t))/\theta$
Gumbel	$exp(-t^{1/\theta})$
Joe	$1-(1-exp(-t))^{1/\theta}$
BB1 (12)	$(1+t^{1/\theta})^{-1}$
BB1 (14)	$(1+t^{1/\theta})^{-\theta}$
BB1 (19)	$\theta/log(t+e^{\theta})$
BB1 (20)	$log^{-1/\theta}(t+e)$

Table 5.7: The nine copula families and corresponding generators implemented in toolbox.

This gives many option for choosing the structure. The toolbox also provides a goodness of fit test. As mentioned by Gorecki et al. (2018) homogeneous and some heterogeneous structures are possible. Table 5.8 presents the optimal structure for the HACs when homogeneous structures and heterogeneous structures are considered.

Data	Family (homogeneous)	GoF	Family (heterogeneous)	GoF
S&P 500 and AEX	BB1 (12)	0.364	BB1(12)	0.364
S&P 500, AEX and USDX	Gumbel	0.194	Ali-Mikhail-Haq, Clayton, BB1(19)	0.238
S&P 500, AEX, USDX, Corn and Gold	Joe	0.148	Ali-Mikhail-Haq, Clayton, BB1(19)	0.0748
S&P 500, AEX, USDX, Corn, Gold, Soy, Gas, Wheat	Gumbel	0.0126	Ali-Mikhail-Haq, Clayton, BB1(19)	0.0347
Cross hedge 1	Gumbel	0.103	Clayton, BB1(14)	0.0507
Cross hedge 1	Gumbel	0.0301	ALi-Mikhail-Haq, Clayton	0.0693

Table 5.8: Optimal HAC structures in the homogeneous setting vs the heterogeneous setting. The cross hedge considers the SP and AEX spot prices with the Corn and Gold future prices

It differs per set of variables if the homogeneous HAC or heterogeneous HAC is chosen. The model with the lowest goodness of fit for each different model is chosen.

Table 5.9 presents the optimal hedge ratios in the multivariate case. In addition to only the Clayton copula the optimal hedge ratios obtained in more dimensions with the C-vine, D-vine and HAC copula are given. The Table show the mean and standard deviation of the hedge ratios up to the ten dimensional case.

Model	Hedging objective									
	VaR (99%)					ES (97.5%)				
	$b_{S\&P}$	b_{AEX}	$b_{\$}$	b_{gold}	b_{corn}	$b_{S\&P}$	b_{AEX}	$b_{\$}$	b_{gold}	b_{corn}
Clay	1.06	0.851	-	-	-	1.23	0.684	-	-	-
	(0.122)	(0.121)	-	-	-	(0.198)	(0.194)	-	-	-
C-vine	0.891	1.17	-	-	-	0.002	1.97	-	-	-
	(0.440)	(0.422)	-	-	-	(-0.557)	(0.544)	-	-	-
D-vine	1.07	1.00	-	-	-	1.11	0.929	-	-	-
	(0.276)	(0.197)	-	-	-	(0.283)	(0.203)	-	-	-
HAC	1.31	0.367	-	-	-	1.35	0.318	-	-	-
	(0.106)	(0.103)	-	-	-	(0.0589)	(0.0533)	-	-	-
Clay	1.23	0.690	0.979	-	-	1.55	0.388	0.966	-	-
	(0.43)	(0.519)	(0.441)	-	-	(0.305)	(0.533)	(0.574)	-	-
C-vine	1.41	1.64	-0.008	-	-	0.139	2.61	-0.246	-	-
	(0.412)	(0.502)	(0.288)	-	-	(0.156)	(0.720)	(0.702)	-	-
D-vine	-0.930	-0.145	-1.88	-	-	-0.686	0.0432	-2.33	-	-
	(1.28)	(1.82)	(1.92)	-	-	(1.05)	(1.07)	(1.07)	-	-
HAC	1.66	1.15	0.273	-	-	1.94	0.952	-0.0053	-	-
	(0.489)	(0.500)	(0.416)	-	-	(0.263)	(0.291)	(0.329)	-	-
Clay	0.938	1.02	0.803	0.927	1.11	1.10	1.52	1.21	0.802	0.134
	(0.335)	(0.317)	(0.281)	(0.360)	(0.514)	(0.631)	(0.927)	(0.752)	(0.588)	(0.943)
C-vine	1.02	1.16	1.06	1.33	0.626	0.0343	1.49	0.899	2.08	0.389
	(0.614)	(0.588)	(0.402)	(0.589)	(0.330)	(0.305)	(0.473)	(0.450)	(0.643)	(0.330)
D-vine	0.500	0.597	0.743	1.22	1.57	0.165	0.248	1.427	1.538	1.24
	(0.589)	(0.451)	(0.488)	(0.417)	(0.384)	(0.480)	(0.444)	(0.537)	(0.419)	(0.551)
HAC	1.29	0.557	0.984	0.829	0.667	1.32	0.407	0.927	0.729	0.596
	(0.176)	(0.245)	(0.119)	(0.175)	(0.146)	(0.142)	(0.113)	(0.0829)	(0.0638)	(0.0839)

Table 5.9: std and standard of optimal hedge ratios for SP, AEX, USDX, gold and corn spot and future prices for 12/31/2004 until 12/31/2020 based on the normal distribution. Both the hedge ratio's for Value-at-Risk and expected shortfall are given.

From the Table it is apparent that the different methods results in very different hedge ratios. In addition the HAC method seems the least volatile. This becomes more clear when for example the hedge ratios for the S&P, AEX and USDX are plotted in Figure 5.2 and 5.3

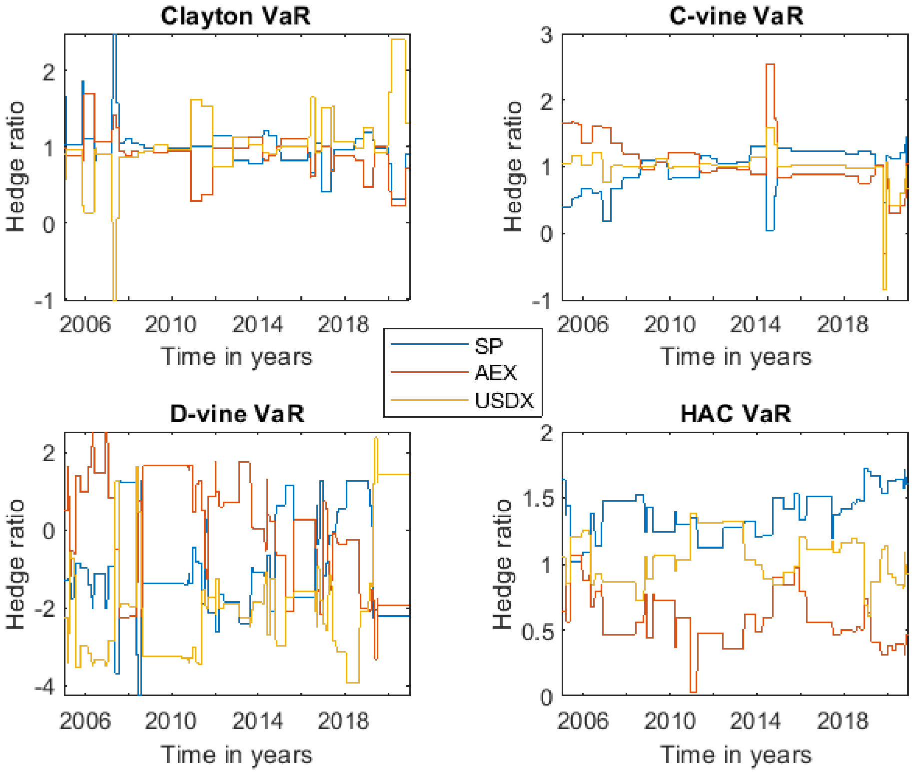

Figure 5.2: Optimal VaR 99% hedge ratios for S&P, AEX and US dollar spot and future prices for 12/31/2004 until 12/31/2020

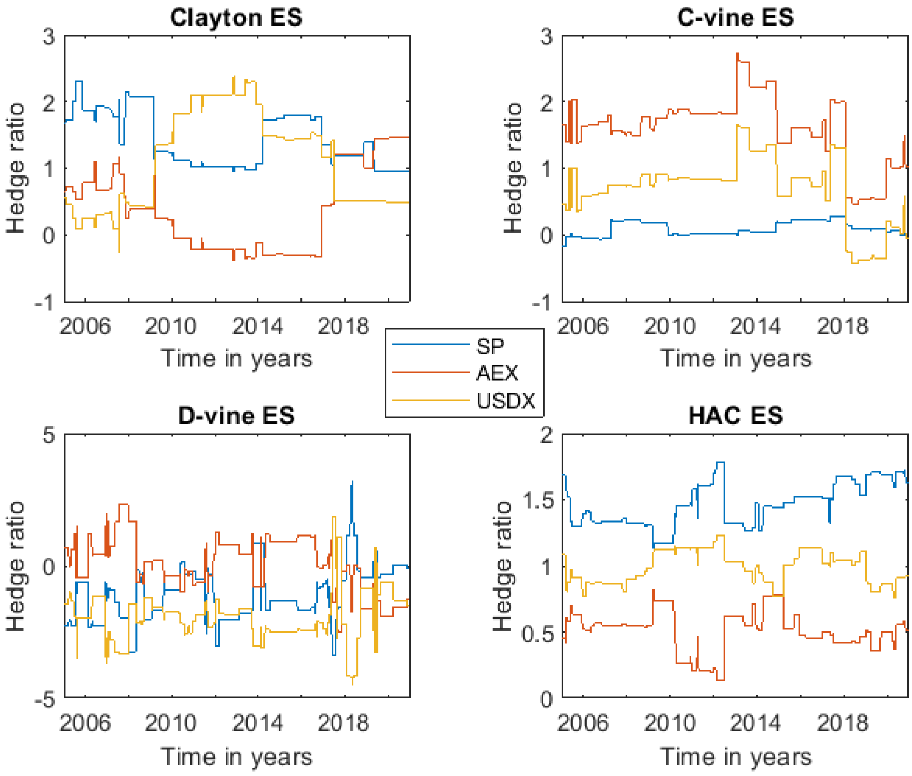

Figure 5.3: Optimal ES 97.5% hedge ratios for S&P, AEX and US dollar spot and future prices for 12/31/2004 until 12/31/2020

The D-vine takes on the most extreme values and is the most volatile. Some other observations are that the C-vine is gives pretty stable estimates but has a few outliers. Furthermore the HAC never takes a negative hedge ratio, this means that the position in the future contract is always a short position.

In addition to hedging a future contract with the corresponding spot price cross hedging is also a possibility. This means that a future contract is hedged with a different spot price. Table 5.10 presents the optimal hedge ratios when cross hedging is considered in the case of four financial instruments. In addition to that cross hedging is applied in larger portfolio's which is described in more detail in Section 5.2.

Model	Hedging objective			
	VaR (99%)		ES (97.5%)	
	$b_{S\&P/gold}$	$b_{AEX/corn}$	$b_{S\&P/gold}$	$b_{AEX/corn}$
Clay	1.12	0.748	0.961	0.899
	(0.490)	(0.500)	(0.289)	(0.289)
C-vine	1.01	1.00	1.16	0.877
	(0.399)	(0.385)	(0.513)	(0.489)
D-vine	1.63	0.534	1.81	0.310
	(0.566)	(0.595)	(0.555)	(0.565)
HAC	0.381	0.522	0.458	0.268
	(0.335)	(0.310)	(0.189)	(0.252)

Table 5.10: std and standard of optimal cross hedge ratios for SP, AEX, gold and corn spot and future prices for 12/31/2004 until 12/31/2020 based on the normal distribution. Where in $b_{i/j}$ i is the spot price and j the future price. Both the hedge ratio's for Value-at-Risk and expected shortfall are given.

Figure 5.4 and 5.5 show the hedge ratios over time.

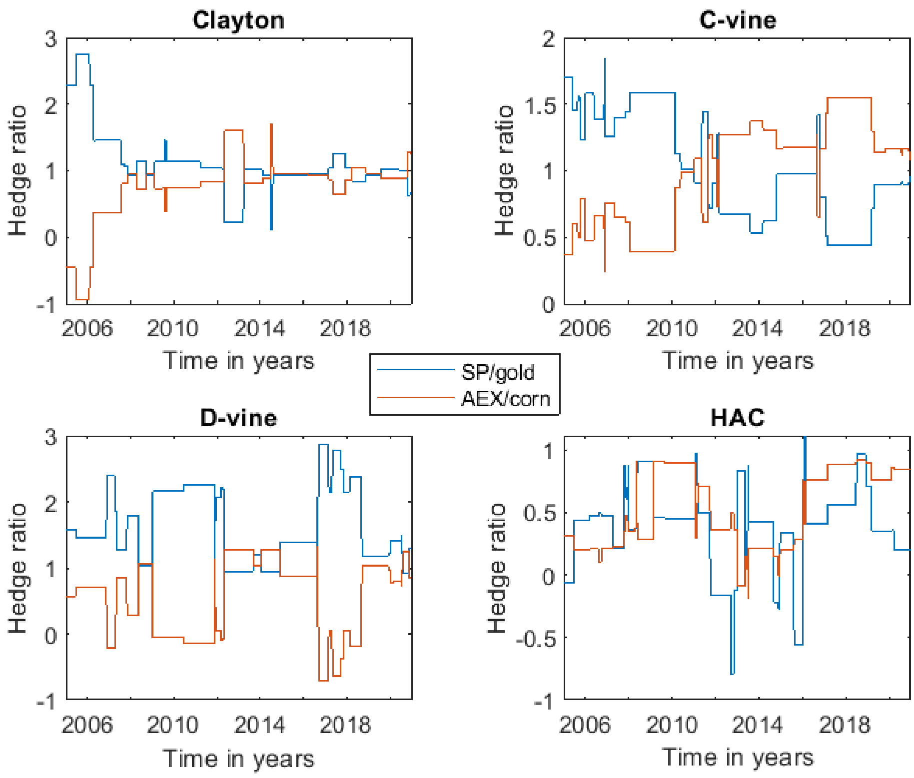

Figure 5.4: Optimal VaR 99% cross hedge ratios for S&P, gold, AEX and corn spot and future prices for 12/31/2004 until 12/31/2020

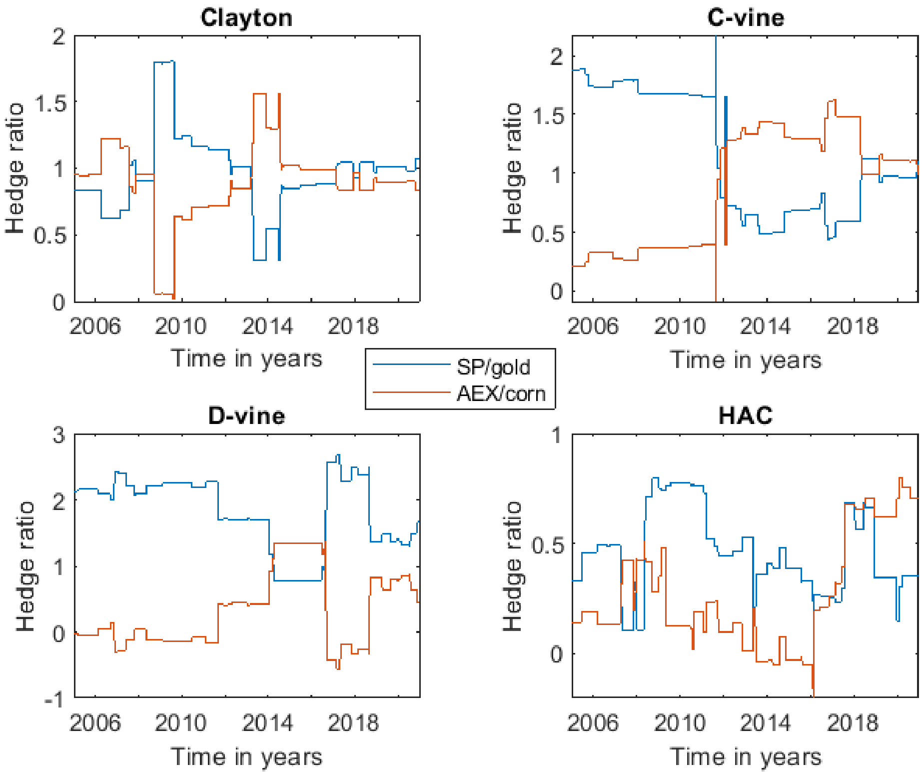

Figure 5.5: Optimal ES 97.5% cross hedge ratios for S&P, gold, AEX and corn spot and future prices for 12/31/2004 until 12/31/2020

One striking result is that the Clayton, C-vine and D-vine hedge ratios seem to be mirrored, both for the VaR and ES. The only method that does not result in such a pattern is HAC. Also the estimated hedge ratio's differ in magnitude. For example around 2018 the C-vine and D-vine suggest large opposite positions while the Clayton copula suggest a similar position close to one. This also holds both for VaR and ES. Finally, the optimal hedge ratios for the VaR are negative more often than for the ES.

5.2 out of sample results

This section presents how the obtained hedge ratios perform out of sample as described in the methodology. Since this means that another 130 weeks are lost in the estimation we end up with 705 values for the hedging effectiveness. Table 5.11 displays the reduction in downside risk in percent for each method. The downside risk reduction is only given for two bivariate cases, the S&P 500 and the AEX. This is because the main goal is to reduce risk in larger portfolios. In addition to reducing risk in portfolio that include the future with the corresponding spot price there are also two portfolio's that include cross hedging. The first portfolio includes two stocks and two futures and the second portfolio four stocks and four futures. The motivation for this is that an optimal model should be flexible and therefore successful in different situations.

Model	Risk reduction	
	VaR (99%)	ES (97.5%)
	S&P	
Clayton	87.2(4.33)	95.2(1.95)
	AEX	
Clayton	10.7(41.7)	66.2(16.1)
	S&P and AEX	
Clayton	29.3(32.5)	73.5(17.9)
C-vine	39.69(29.8)	62.4(13.8)
D-vine	**60.2(13.6)**	**85.5(4.93)**
HAC	56.8 (7.11)	84.3(4.19)
	S&P, AEX and USDX	
Clayton	42.3 (23.0)	83.7 (6.95)
C-vine	43.9 (22.2)	64.4 (14.9)
D-vine	-58.7 (46.3)	40.7 (19.0)
HAC	**57.6(14.4)**	**85.6(4.53)**
	S&P, AEX, USDX, gold and corn	
Clayton	46.9 (14.3)	64.4 (14.5)
C-vine	29.6 (20.0)	59.1 (13.8)
D-vine	30.6 (19.3)	62.2 (10.8)
HAC	**51.1(10.8)**	**73.3 (6.81)**
	S&P, AEX, USDX, gold, corn, gas, soy, wheat	
Clayton	31.4(25.8)	69.4(8.98)
C-vine	**34.5(18.7)**	63.4(18.5)
D-vine	17.9(35.1)	58.6(15.0)
HAC	31.6(27.5)	**70.4(13.0)**
Cross hedge	*S&P + AEX spot and gold + corn future*	
Clayton	-27.4 (21.6)	52.8 (14.2)
C-vine	-31.4 (33.2)	51.1 (17.4)
D-vine	-41.4 (34.6)	46.6 (16.3)
HAC	**-3.87(18.7)**	**59.7(4.21)**
Cross hedge	*corn+ gas+soy+wheat spot and S&P + AEX +USDX +gold future*	
Clayton	**6.14 (7.11)**	**60.7 (4.43)**
C-vine	-1.72 (15.4)	53.4(10.9)
D-vine	4.16 (9.75)	59.3(8.80)
HAC	2.05 (7.48)	59.2 (5.06)

Table 5.11: mean and standard deviation of percentage reduction of downside risk for 27/06/2007 until 31/12/2020. The hedging effectiveness is given for all models. Both the hedging effectiveness for VaR 99% and ES 97.5% are given.

The results show that all methods substantially reduce the downside risk in all cases for the ES. However this is not always the case for VaR. Figure 5.6 shows an example of how the returns of the hedged and unhedged Prices and Losses look like.

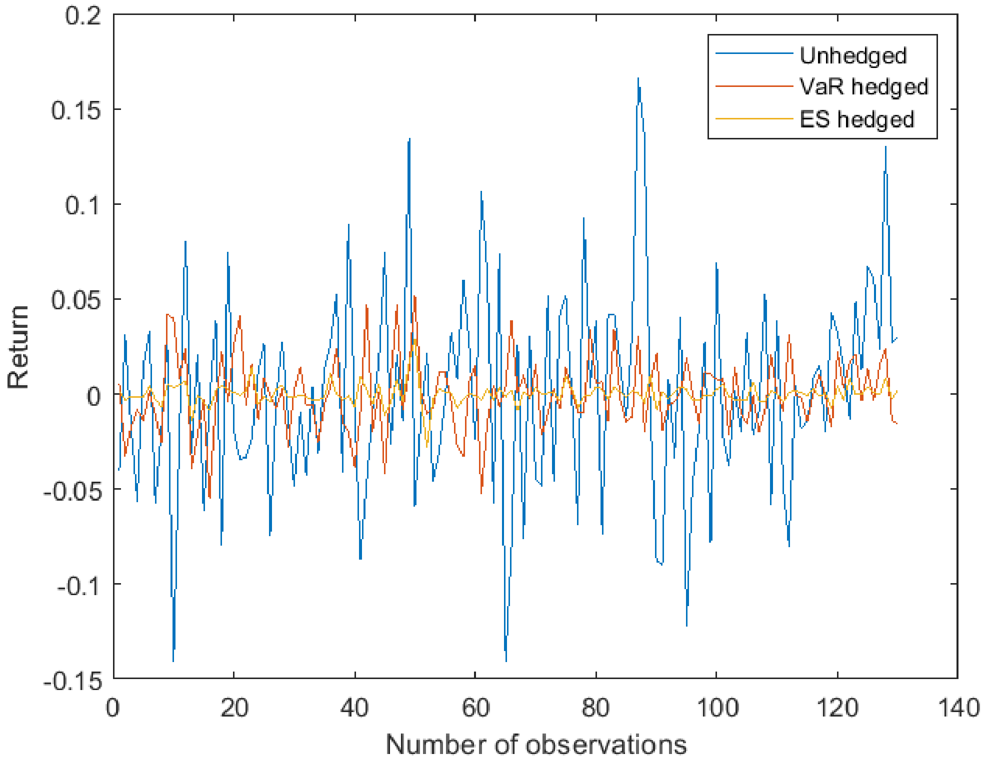

Figure 5.6: Returns for the out of sample hedged an unhedged Prices and Losses for one window of observations in the six dimensional situation (S&P, AEX and USDX spot and future returns)

The Figure shows that the returns determined with ES hedge ratios are less volatile. This happens in most cases and therefore we see that the ES reduction is larger than the VaR reduction. When the portfolio size increases this effect becomes more apparent. In the bivariate case the risk reduction for VaR and ES is quite similar. However, ES performs well consistently, also in larger portfolios. Therefore the objective of minimizing ES seems to improve the optimal hedge ratios for reducing risk. A reason for this could be that ES is a coherent risk measure and VaR is not.

When we consider the different models, the HAC has the largest mean risk reduction in the six dimensional, ten dimensional and four dimensional cross hedge case. HAC does not always outperform the other models, but performs well in all cases. In addition, the estimation of the HAC goes quite fast compared to for example the vine copula.

Another interesting result is that the multivariate Clayton copula performs consistently well. The Clayton copula reduces the risk substantially in all cases, except cross hedging in the four dimensional case for VaR. However, in this scenario all models performs poorly when reducing VaR is considered. On the contrary the vine copula give mixed results. Especially the D-vine performs poorly in the six dimensional case. Here the D-vine model actually increases the risk. The D-vine yields very different results in the different cases. The C-vine is more consistent but does not often outperform the other models.

Figure 5.7 and 5.8 show the reduction in downside risk over time in the case when the D-vine underperforms. These plots display that the values of the D-vine are more volatile than the ones estimated by the other models. This coincides with the hedging ratio's over time, these are also more volatile for the D-vine.

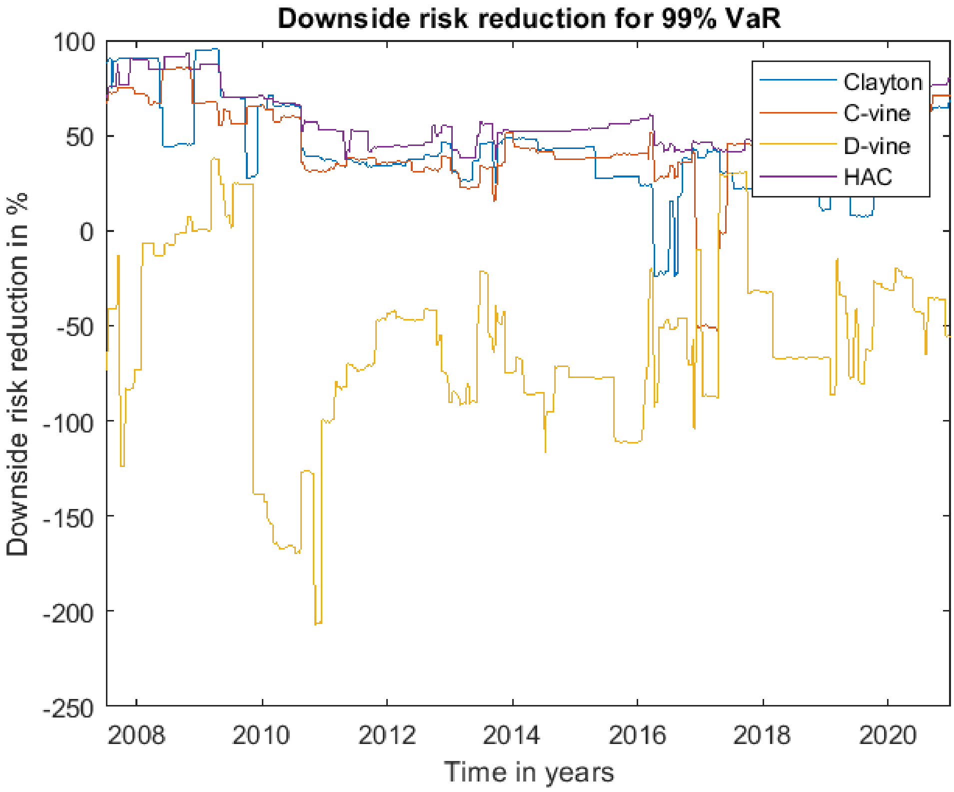

Figure 5.7: Percentage reduction of downside risk for 27/06/2007 until 31/12/2020. The hedging effectiveness is given for all models for VaR 99% and ES 97.5% in the six dimensional case. This includes S&P 500, AEX and USDX spot and future prices.

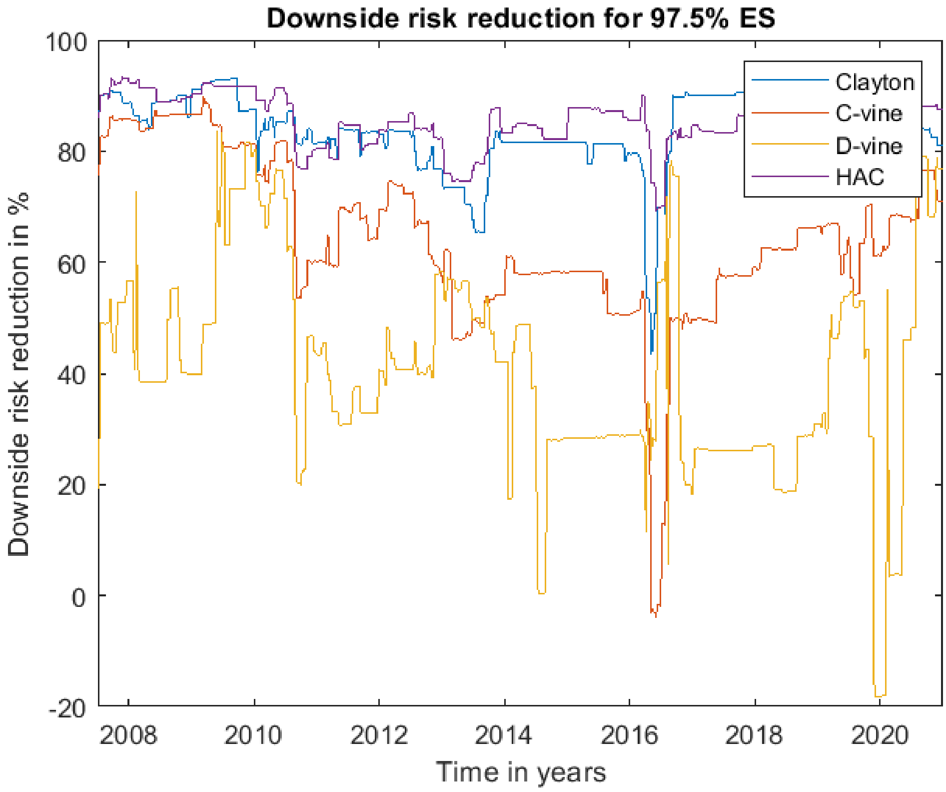

Figure 5.8: Percentage reduction of downside risk for 27/06/2007 until 31/12/2020. The hedging effectiveness is given for all models for VaR 99% and ES 97.5% in the six dimensional case. This includes S&P 500, AEX and USDX spot and future prices.

When the cross hedging is considered the all models have a bad performance when the VaR is considered as risk objective. When looking at the ES risk reduction there is no clear winner or loser. 5.10. Figure 5.9 shows the reduction in downside risk over time when four financial instruments are included. HAC performs a lot better than the other models for VaR reduction in the four dimensional case.

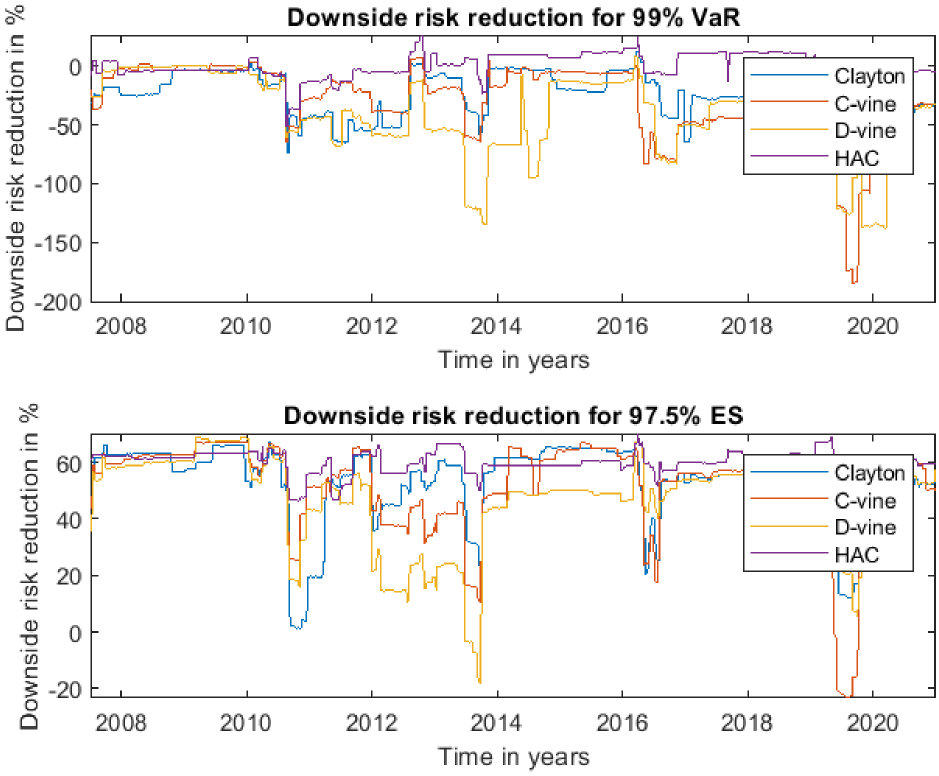

Figure 5.9: Percentage reduction of downside risk for 27/06/2007 until 31/12/2020. The hedging effectiveness is given for all models for the cross hedge. This includes S&P 500 and AEX spot prices with gold and corn future prices.

A possible reason why HAC outperforms the other model is that the HAC model has a different pattern than the other models. The other three models move in a quite similar. When the larger cross hedging portfolio is considered all models reduce risk a lot better when the VaR is considered. The difference is less extreme for the ES. In general the risk reduction for the ES is more stable than the risk reduction for VaR in all scenarios. The ES risk reduction does not change substantially when financial instruments are added or when cross hedging is considered. This is not the case for the VaR reduction.

6 Conclusion

In conclusion using copulas to hedge spot and future prices has a positive effect in risk reduction, especially when Expected Shortfall is considered. Many different copulas are applied to different portfolios and in almost all cases this reduces the risk with as objective minimizing the Value-at-Risk and Expected Shortfall. First, an interesting result is that using Expected Shortfall as hedging objectives yields better results than using VaR. The hedge ratios that are obtained with Expected Shortfall result in less volatile portfolio returns. In addition to that, the results are quite stable for all porftolios. This is not the case when Value-at-Risk is considered as risk objective. Another significant result is that the multivariate Clayton copula yields promising results in all portfolios. The multivariate Clayton copula reduces the risk in all cases and sometimes outperforms the other models. One advantage of the Clayton copula model is that the computation time is low and can be used for large data sets. Similar to the Clayton copula the C-vine performs well and manages to reduce the risk in most cases. The model that shows mixed results is the D-vine copula model. The D-vine sometimes outperforms all the other models, but in other cases performs poorly. The hedge ratio's and hedging effectiveness over time show that the D-vine is the most volatile. In one case this model even increases the risk. Therefore when reducing risk is the main goal this is not a suitable model. Lastly the HAC model ouperforms the other models in most scenarios. This could mean that the HAC is the most flexible and captures the dependence structures the best. Based on these results there is no clear best model. The Clayton copula model and HAC appear to be the best choice. However there are some limitations that should be considered. First of all only risk reduction with certain objectives are taken into account. It could be valuable to look at other objectives, for example taking return into account. Another improvement could be to look at even larger portfolio's. This research already increased the size of the portfolio's compared to other papers that discuss hedging with copula. However larger steps could be made to check if it is then still feasible to reduce risk with these models.

References

Aas, K., Czado, C., F., and A., Bakken, H. (2009). Pair-copula constructions of multiple dependence. *Insurance: Mathematics and Economics*, 44(2):182–198.

Barbi, M. and Romagnoli, S. (2014). A copula-based quantile risk measure approach to estimate the optimal hedge ratio. *Journal of Futures Markets*, 34(7):658–675.

Cecchetti, S. G., Cumby, R. E., and Figlewski, S. (1988). Estimation of the optimal futures hedge. *The Review of Economics and Statistics*, 70(4):623–630.

Chang, C.-L., Jimenez-Martin, J.-A., Maasoumi, E., McAleer, M., and Pérez-Amaral, T. (2019). Choosing expected shortfall over var in basel iii using stochastic dominance. *International Review of Economics Finance*, 60:95–113.

Chen, S.-S., few Lee, C., and Shrestha, K. (2003). Futures hedge ratios: a review. *The Quarterly Review of Economics and Finance*, 43(3):433–465.

Clyman, D. R., Allen, C. S., and Jaycobs, R. (1997). Liquidity without volume: The case of finex, dublin. *Journal of Futures Markets*, 17(3):247–277.

Coblenz, M. (2021). Matvines: A vine copula package for matlab. *SoftwareX*, 14:100–700.

Czado, C., Schepsmeier, U., and Min, A. (2012). Maximum likelihood estimation of mixed cvines with application to exchange rates. *Statistic Modelling*, 12(3):229–255.

DeMiguel, V., Garlappi, L., and Uppal, R. (2009). Optimal versus naive diversification: How inefficient is the 1/n portfolio strategy? *The review of Financial studies*, 22(5):1915–1953.

Diks, C., Panchenko, V., and van Dijk, D. (2010). Out-of-sample comparison of copula specifications in multivariate density forecasts. *Journal of Economic Dynamics Control*, 34:1596–1609.

Dißmann, J., Brechmann, E., Czado, C., and Kurowicka, D. (2013). Selecting and estimating regular vine copulae and application to financial returns. *Computational Statistics Data Analysis*, 59:52–69.

Genest, C., Ghoudi, K., and Rivest, L.-P. (1995). A semiparametric estimation procedure of dependence parameters in multivariate families of distributions. *Biometrika*, 82(3):543–552.

Genest, C., Rémillard, B., and Beaudoin, D. (2009). Goodness-of-fit tests for copulas: A review and a power study. *Insurance: Mathematics and Economics*, 44(2):199–213.

Ghosh, A. (1993). Hedging with stock index futures: Estimation and forecasting with error correction model. *Journal of Futures Markets*, 13(7):743–752.

Gorecki, J., Hofert, M., and Holena, M. (2018). *Hierarchical Archimedean Copulas for MATLAB and Octave: The HACopula Toolbox.*

Heyde, C. (2006). *Central Limit Theorem.* John Wiley Sons, Ltd.

Hsu, C.-C., Tseng, C.-P., and Wang, Y.-H. (2008a). Dynamic hedging with futures: A copula-based garch model. *Journal of Futures Markets*, 28(11):1095–1116.

Hsu, C.-C., Tseng, C.-P., and Wang, Y.-H. (2008b). Dynamic hedging with futures: A copula-based garch model. *Journal of Futures Markets*, 28(11):1095–1116.

Joe, H. (1996). Families of m-variate distributions with given margins and m(m-1)/2 bivariate dependence parameters. *Lecture Notes-Monograph Series*, 28:120–141.

Lee, H.-T. (2009). A copula-based regime-switching garch model for optimal futures hedging. *Journal of Futures Markets*, 29(10):946–972.

MATLAB (2022). *9.8.0.1323502 (R2022a).* The MathWorks Inc., Natick, Massachusetts.

Okhrin, O., Okhrin, Y., and Schmid, W. (2013). On the structure and estimation of hierarchical archimedean copulas. *Journal of Econometrics*, 173(2):189–204.

Sukcharoen, K. and Leatham, D. (2017). Hedging downside risk of oil refineries: A vine copula approach. *Energy Economics*, 66:493–507.

Yamai, Y. and Yoshiba, T. (2005). Value-at-risk versus expected shortfall: A practical perspective. *Journal of Banking Finance*, 29(4):997–1015.

A Appendix

A.1 Pseudo-loglikelihood Clayton Copula

The multivariate Clayton copula pdf has the following density

$$c = \{\prod_{i=1}^{k}(1+(i-1)\theta)\}\{\prod_{i=1}^{k} v_i^{-(\theta+1)}\}\{\sum_{i=1}^{k} v_i^{-\theta} - k + 1\}^{-(\theta^{-1}+k)}$$

and is obtained from Diks et al. (2010). The variable k the number of dimensions and v_i is the empirical cdf. From the pdf the pseudo-loglikehood can be derived. With this expression maximum likelihood estimation can be executed.

$$L(\hat{\theta}) = \sum_{t=1}^{T}\{\sum_{i=1}^{k}(log(1+(i-1)\hat{\theta}) - \sum_{i=1}^{k}(\hat{\theta}+1)log(v_i) - (\hat{\theta}^{-1}+k)log(\sum_{i=1}^{k} v_i^{-\hat{\theta}} - k + 1)\}$$

Where $L(\hat{\theta})$ is maximised by the maximum likelihood estimator $\hat{\theta}$.

A.2 Generating uniform variables from Clayton Copula

By following this algorithm uniform values for a given copula can be simulated.

- Draw $V' = (V_1,' \ldots, V_d') \sim V(0,1)^{k-1}$
- $V_1 = C^{-1}(V_1'|v_d)$
- $V_2 = C^{-1}(V_2'|v_1, v_d)$
- $\vdots$
- $V_I = C^{-1}(V_I'|v_1, v_2, \ldots, v_d)$

When in particular this algorithm is applied to the Clayton copula it results in the following expression

$$C^{(-1)(\theta)}(v_i'|v_1,\ldots,v_{i-1},v_k) = (1 + (1-(i-1) + \sum_{d=1}^{j-1} v_d^{-\theta})(v_i^{\prime - \frac{\theta}{1+(i-1)\theta}} - 1))^{-\frac{1}{\theta}}$$

A.3 Spot and Future prices plots

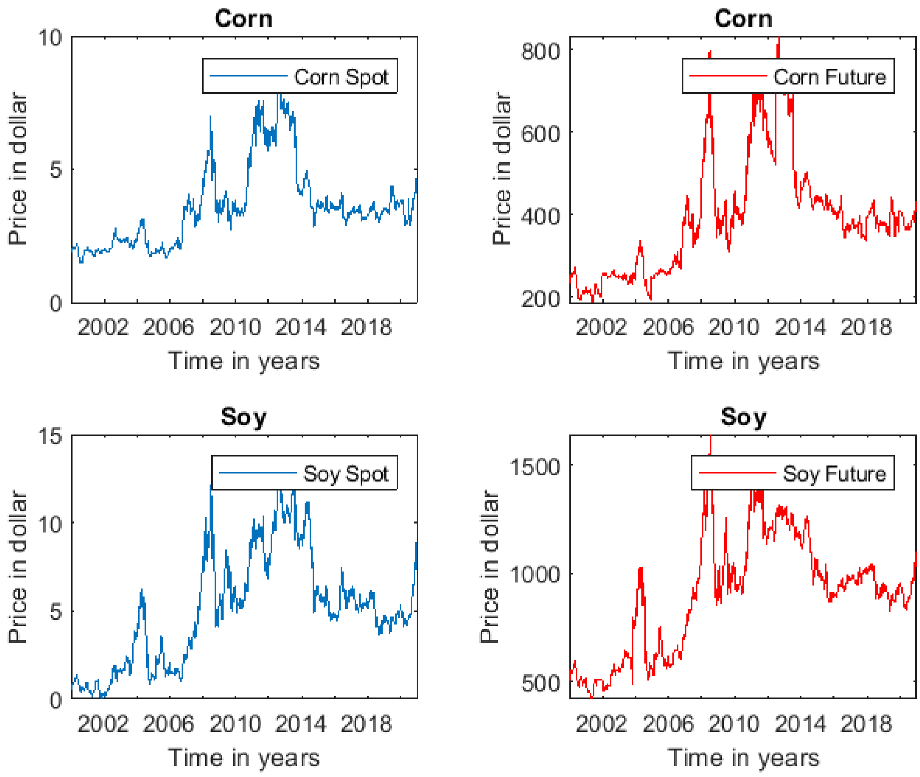

Figure A.1: Plot of weekly spot and future prices of Corn and Soy prices. The weekly prices consist of all the Wednesday closing prices for the sample period 31/12/1999 - 31/12/2020

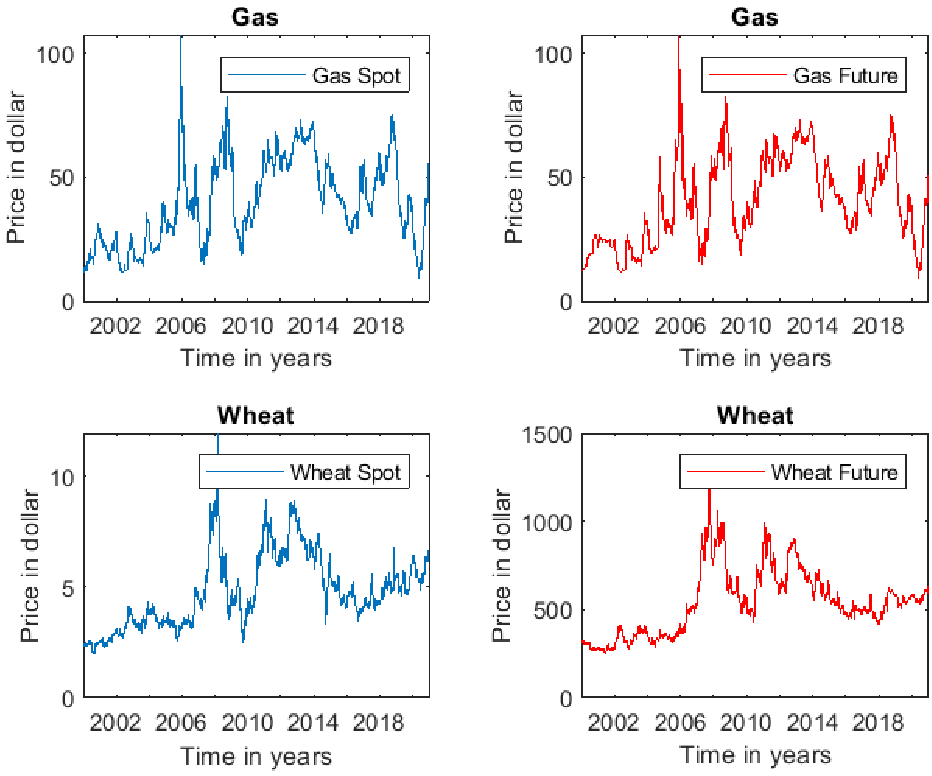

Figure A.2: Plot of weekly spot and future prices of Natural Gas and Wheat prices. The weekly prices consist of all the Wednesday closing prices for the sample period 31/12/1999 - 31/12/2020

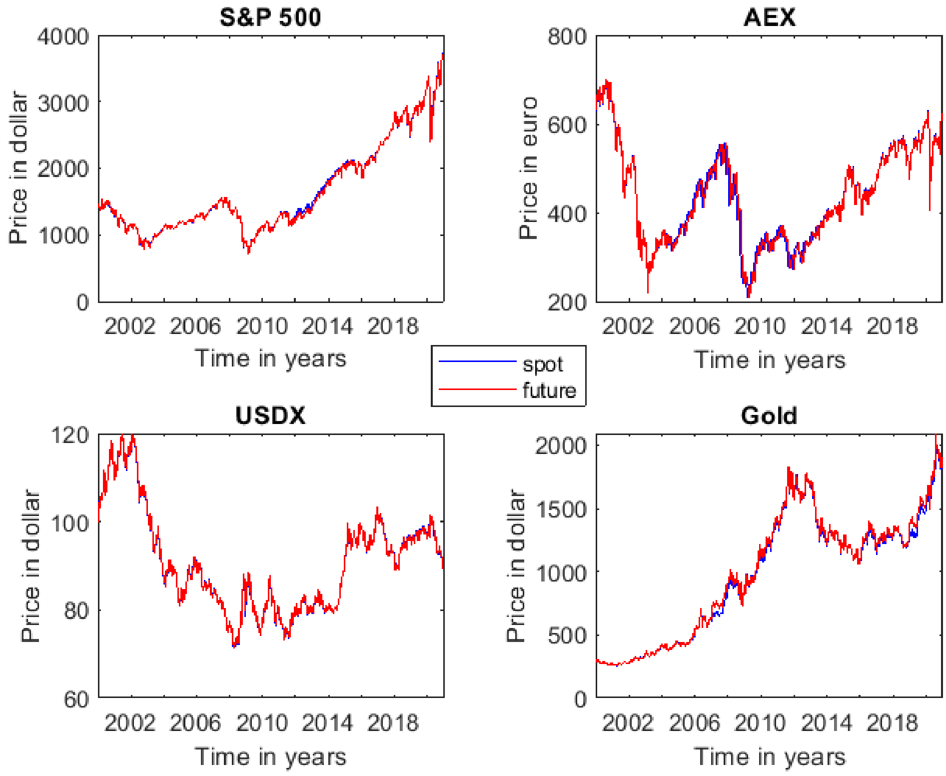

Figure A.3: Plot of weekly spot and future prices of the S&P 500, AEX, USDX and US gold prices. The weekly prices consist of all the Wednesday closing prices for the sample period 31/12/1999 - 31/12/2020

A.4 Families selected for vine copula

	Family								
Tree 1	Plackett	Plackett	Joe	Joe	Joe	Plackett	Plackett	Joe	Frank
Tree 2	t	t	t	t	t	t	t	independent	
Tree 3	t	t	t	Gaussian	t	t	Plackett		
Tree 4	Gaussian	independent	t	independent	t	fgm			
Tree 5	independent	Surjoe	independent	Plackett	fgm				
Tree 6	independent	t	t	independent					
Tree 7	Clayton	t	Surjoe						
Tree 8	Palckett	t							
Tree 9	independent								

Table A.1: Copula family's selected for the C-vine at each node for SP, AEX, USDX, gold and corn spot and future prices.

	Family								
Tree 1	Plackett	t	Plackett	Joe	Joe	Plackett	Plackett	Plackett	Plackett
Tree 2	t	t	t	t	t	t	t	t	
Tree 3	amhaq	surgumbel	t	t	t	t	t		
Tree 4	independent	t	t	t	surgumbel	t			
Tree 5	t	t	t	t	t				
Tree 6	Plackett	t	t	Plackett					
Tree 7	t	Clayton	Surgumbel						
Tree 8	t	fgm							
Tree 9	t								

Table A.2: Copula family's selected for the D-vine at each node for SP, AEX, USDX, gold and corn spot and future prices.

B Optimal hedge ratio

	Family													
Tree 1	'plackett'	'joe'	'joe'	'plackett'	't'	'frank'	'joe'	'plackett'	'plackett'	'joe'	'frank'	'plackett'	'plackett'	'plackett'
Tree 2	't'	't'	'ind'	't'	'joe'	't'	't'	't'	't'	'ind'	't'	'ind'	't'	-
Tree 3	't'	'plackett'	't'	'plackett'	't'	'gauss'	't'	't'	'plackett'	't'	't'	'plackett'	-	-
Tree 4	'fgm'	'amhaq'	'fgm'	'ind'	't'	'ind'	't'	'fgm'	'gauss'	'ind'	'clayton'	-	-	-
Tree 5	'ind'	'fgm'	'ind'	'surjoe'	'ind'	'plackett'	'fgm'	'plackett'	'ind'	'plackett'	-	-	-	-
Tree 6	'ind'	'plackett'	'ind'	'amhaq'	'gauss'	'ind'	'plackett'	'ind'	'fgm'	-	-	-	-	-
Tree 7	'ind'	'tawn'	'frank'	'ind'	'ind'	'ind'	'plackett'	'ind'	-	-	-	-	-	-
Tree 8	'surjoe'	't'	'ind'	't'	'ind'	't'	'amhaq'	-	-	-	-	-	-	-
Tree 9	't'	't'	'joe'	'ind'	't'	'plackett'	-	-	-	-	-	-	-	-
Tree 10	't'	't'	't'	'ind'	't'	-	-	-	-	-	-	-	-	-
Tree 11	't'	'ind'	'gauss'	'ind'	-	-	-	-	-	-	-	-	-	-
Tree 12	'surclayton'	'ind'	'ind'	-	-	-	-	-	-	-	-	-	-	-
Tree 13	'fgm'	'gauss'	-	-	-	-	-	-	-	-	-	-	-	-
Tree 14	'plackett'	-	-	-	-	-	-	-	-	-	-	-	-	-
Tree 15	-	-	-	-	-	-	-	-	-	-	-	-	-	-

Table A.3: Copula family's selected for the C-vine at each node for SP, AEX, USDX, gold, corn, soy, gas and wheat spot and future prices.

	Family														
Tree 1	’plackett’	’t’	’plackett’	’joe’	’plackett’	’t’	’t’	’frank’	’plackett’	’plackett’	’plackett’	’plackett’	’plackett’	’t’	’t’
Tree 2	’t’	’t’	’t’	’surgumbel’	’t’	’t’	’surclayton’	’ind’	’t’	’t’	’t’	’t’	’t’	’t’	-
Tree 3	’amhaq’	’surgumbel’	’ind’	’plackett’	’t’	’surjoe’	’surgumbel’	’amhaq’	’t’	’t’	’plackett’	’plackett’	’t’	-	-
Tree 4	’ind’	’gauss’	’surgumbel’	’t’	’ind’	’amhaq’	’fgm’	’amhaq’	’t’	’t’	’surgumbel’	’plackett’	-	-	-
Tree 5	’clayton’	’surgumbel’	’t’	’amhaq’	’ind’	’t’	’t’	’ind’	’t’	’fgm’	’plackett’	-	-	-	-
Tree 6	’ind’	’tawn’	’frank’	’t’	’ind’	’gauss’	’plackett’	’fgm’	’ind’	’plackett’	-	-	-	-	-
Tree 7	’fgm’	’amhaq’	’t’	’gauss’	’ind’	’frank’	’ind’	’fgm’	’t’	-	-	-	-	-	-
Tree 8	’fgm’	’t’	’surgumbel’	’t’	’t’	’t’	’t’	’fgm’	-	-	-	-	-	-	-
Tree 9	’plackett’	’t’	’t’	’plackett’	’plackett’	’plackett’	’plackett’	-	-	-	-	-	-	-	-
Tree 10	’t’	’surjoe’	’surgumbel’	’gauss’	’plackett’	’gauss’	-	-	-	-	-	-	-	-	-
Tree 11	’t’	’clayton’	’clayton’	’plackett’	’tawn’	-	-	-	-	-	-	-	-	-	-
Tree 12	’t’	’fgm’	’plackett’	’gauss’	-	-	-	-	-	-	-	-	-	-	-
Tree 13	’ind’	’gauss’	’gauss’	-	-	-	-	-	-	-	-	-	-	-	-
Tree 14	’t’	’ind’	-	-	-	-	-	-	-	-	-	-	-	-	-
Tree 15	’surjoe’	-	-	-	-	-	-	-	-	-	-	-	-	-	-

Table A.4: Copula family’s selected for the D-vine at each node for SP, AEX, USDX, gold, corn, soy, gas and wheat spot and future prices.