ERASMUS UNIVERSITY ROTTERDAM
ERASMUS SCHOOL OF ECONOMICS
Bachelor Thesis Economics & Business

Cryptocurrency and portfolio diversification

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Finish date: 19/06/2023

ABSTRACT

The emergence of cryptocurrencies has disrupted traditional financial systems, presenting novel investment opportunities. This study aims to explore the diversification prospects of cryptocurrencies, specifically Bitcoin, Ethereum, and Binance Coin, when combined with traditional asset classes such as Forex Markets, Gold, Corporate Bonds, Real Estate, S&P 500, and Mid Equity Market. The study employs a rigorous approach, utilizing the mean-variance optimization technique within the framework of Markowitz (1952). By constructing various portfolio types, including equally weighted constraint, unconstraint, and long-only portfolios, the study investigates the impact of incorporating these cryptocurrencies on portfolio performance. The research evaluates the potential of cryptocurrencies in enhancing portfolio diversification by analyzing factors such as expected returns, correlations, portfolio variance, and downsize risk as Value-at-Risk at the 95% confidence level. This study contributes to the existing literature by providing a comprehensive assessment of the diversification benefits that prominent cryptocurrencies can offer within a portfolio context. The findings shed light on the role of cryptocurrencies in portfolio optimization strategies and their ability to provide risk-adjusted returns, offering insights for investors seeking effective diversification strategies in the evolving financial landscape.

Keywords: cryptocurrencies, portfolio diversification, risk, correlation, traditional assets, optimization.

JEL codes: look up the fitting JEL codes: http://www.aeaweb.org/journal/jel_class_system.html

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CHAPTER 1 Introduction

The rise of cryptocurrencies has significantly impacted traditional financial systems, opening new investment opportunities for individuals and institutions (Mendoza Tello, 2019). While some people see cryptocurrencies as a potential threat to financial stability, others view them as a gateway to extraordinary returns and success stories Claus DierksMeier, (2016). This study aims to explore the potential benefits of diversifying portfolios with cryptocurrencies and evaluate the additional risks of their evolving market.

Portfolio diversification involves strategically spreading investments across various assets, including traditional ones and cryptocurrencies. The goal is to reduce risk and optimize returns by selecting assets with different risk and return characteristics. In this study, I consider six traditional asset classes – Forex Markets, Gold, Corporate Bonds, Real estate index (specifically referring to the Vanguard Real Estate Index), S&P 500 and Mid Equity Market – alongside three prominent cryptocurrencies: Ethereum, Bitcoin, and Binance Coin.

The research question is: "What is the impact of including cryptocurrencies in a diversified portfolio of traditional assets?"

To address this question, I examine various portfolio strategies such as equally weighted (naïve), constraint-based with the possibility of short selling and weights distribution under 25%, unconstrained with the possibility of short selling, and long-only without the possibility of short-selling strategies. To construct efficient portfolios in all scenarios except for the naive approach, this study adopts a variation of Markowitz's (1952) optimization theory. An efficient portfolio is defined as one that achieves the maximum expected return for a desired level of risk. The constrained portfolio allows allocating up to 25 percent weight to each asset, regardless of whether it is a long or short position. On the other hand, the unconstrained portfolio permits shorting and has no limitations on individual asset weights within the budget constraint. In a long-only portfolio, individual asset weights are effectively limited to 100 percent of the portfolio, with no shorting allowed. The study creates traditional portfolios and crypto-inclusive portfolios for four different scenarios, incorporating three prominent cryptocurrencies (Ethereum, Bitcoin, and Binance Coin) alongside traditional assets.

Additionally, I employ the sophisticated Filtration Historical Simulation (FHS) method to estimate Value-at-Risk (VaR) and evaluate portfolio risk effectively. This approach offers a comprehensive perspective on the diversified portfolio's performance and aligns with the research's focus on enhancing risk assessment methodologies. The data spans from January 1, 2018, to January 1, 2023, and is collected from financeyahoo.com, enabling a robust analysis of the chosen portfolios.

This study stands out as one of the few that explores how diversification potential can be achieved through cryptocurrency using a portfolio optimization approach. Previous studies in this field have mainly concentrated on general portfolio optimization (Ehrgott et al., 2004; Krokhmal et al., 2002; Cai et al., 2000; DeMiguel et al., 2009; Gaivoronski & Pflug, 2005) or explored cryptocurrencies and digital currencies without conducting extensive diversification analysis (Colombo et al., 2021; Ma et al., 2020). Some research has assessed the diversification potential of Bitcoin using limited indices, assets, or variables (Platanakis & Urquhart, 2020; Garcia et al., 2020; Pal and Mitra, 2019; Eisl et al., 2015). Wang and Ngene (2020) emphasized the possibility of significantly improving portfolio performance by including different cryptocurrencies in a well-diversified portfolio using various optimization strategies. They found that Bitcoin continues to maintain its dominance in cryptocurrency portfolios. However, Ma et al. (2020) argued that incorporating multiple cryptocurrencies could enhance portfolio performance further, with Ethereum demonstrating better diversification potential than Bitcoin. Nevertheless, most of these studies have primarily focused on examining the diversification potential of Bitcoin rather than investigating multiple cryptocurrencies extensively.

Surprisingly, there have yet to be any articles that specifically explore the combination of Bitcoin, Ethereum, and Binance Coin using a portfolio optimization approach. This study adds to the existing literature by using various variables and recent data to provide new evidence on how these three prominent cryptocurrencies can diversify a portfolio. The findings from this study could also offer valuable insights into hedging strategies for Bitcoin or Binance Coin, which may help investors mitigate risks. However, it's important to note that during the COVID-19 pandemic period, when policy uncertainty was high, the hedging abilities of Bitcoin and Binance Coin decreased significantly (Santosha et al., 2020). My study uses data from January 1, 2018, to January 1, 2023, which covers the pandemic period and can contribute additional insights to existing literature.

The results obtained from this study shed light on the key findings. Traditional portfolios had lower variance and lower returns compared to portfolios that included cryptocurrencies. This suggests that while crypto-inclusive portfolios generated higher returns, they also carried higher risks. These results challenge the idea of cryptocurrencies as safe-haven assets or reliable diversifiers in more stable economic conditions or under potential market regulations. Furthermore, the portfolio that only invests in assets for the long term showed better performance when compared to the equally weighted portfolio. It generated higher returns for both traditional and crypto-inclusive assets. One interesting observation is how Binance Coin behaved differently. It had a higher average return and a relatively lower correlation with traditional assets.

In conclusion, this study has explored the integration of cryptocurrencies into diversified portfolios, shedding light on the potential benefits and risks associated with these emerging digital assets. The findings emphasize the complex relationship between cryptocurrencies and traditional assets, indicating that cryptocurrencies can offer higher returns but also introduce elevated risks. The diversification strategies examined, ranging from naive allocations to constraint-based and long-only approaches, underscore the importance of portfolio construction in managing risk and optimizing returns. Moving forward, the remainder of this thesis will delve into the detailed analysis of each portfolio strategy, highlighting the strengths and limitations of each approach in the context of a rapidly evolving market. By investigating the performance of these strategies during various market conditions and considering the implications of policy changes and global events, this thesis aims to provide a comprehensive understanding of the role of cryptocurrencies in modern portfolio management. This exploration will gain a deeper insight into the dynamics of cryptocurrency investments, contributing to the broader discourse on the intersection of digital assets and traditional financial systems.

CHAPTER 2 Theoretical Framework

2.1 Cryptocurrencies

The world of cryptocurrency remains highly unpredictable but also draws the attention of investors with its potential. Over the past decade, cryptocurrencies have matured significantly, transitioning from an immature market to a more developed one. Additionally, these digital assets operate independently from traditional financial instruments (Corbet, Meegan, et al., 2018). The growth of new trading platforms and exchanges, along with increased trading volumes and frequency (Watorek et al., 2021), has played a role in this evolution. Other studies, such as Cheah and Fry (2015) and Katsiampa (2018), highlight that market sentiments can affect Bitcoin's volatility, with significant price shocks playing a semi-important role in determining its value. Some investors may hesitate to venture into cryptocurrencies due to perceived risks and a lack of regulation (Arias Oliva et al., 2019). Private investors have been the primary players in cryptocurrency trading, while professional investors have only recently begun to participate actively rather than relying on algorithms (Dyhrberg et al., 2018). This suggests that cryptocurrencies, including Bitcoin specifically, are considered investable assets by Phillip (Chan & Peiris (2018).

However, unlike stocks, cryptocurrencies have no inherent value (Baur et al., 2018; Cheah & Fry, 2015; Baek & Elbeck, 2015), making them susceptible to forming bubbles. Moreover, younger individuals with lower income and education levels have displayed higher levels of optimism regarding the future value of cryptocurrencies. This optimism even extends to latecomers who enter the market at a later stage (Matteo Benetton, 2021). Such belief stems from the allure of easy money and the potential to attain a dream life without exerting much effort.

As the cryptocurrency market witnessed a significant downturn in the first quarter of 2018, there was a noticeable shift in investor pools toward buy-and-hold miners, indicating changes in market dynamics (Corbet & Gurdgiev, 2018; Wilson, 2018; Celeste et al., 2020). This raises questions about how independent cryptocurrencies are from macroeconomic factors and their effectiveness as portfolio diversifiers. During the peak period of crypto market valuations from late 2017 to early 2018, prominent media outlets extensively covered stories about early investors amassing extraordinary profits. These narratives sparked an influx of new investors and speculators enticed by the potential for substantial gains (Bishop, 2017; Kharpal, 2018). In the year 2021, we saw a significant impact from institutional involvement. Hedge funds and corporations, like MicroStrategy and MassMutual, made their way into the crypto market, bringing in substantial capital and adding credibility to the industry. Notable companies such as Tesla and PayPal also entered the crypto space, drawing attention and providing further legitimacy to cryptocurrencies, Daniel Roberts, (2021). This shift towards institutional adoption marked a noticeable departure from the retail-driven frenzy we witnessed in 2017 (Shu & Song, 2021).

2.2. Mean-variance portfolios

Mean-variance portfolios have been extensively studied to assess their effectiveness in achieving optimal diversification compared to simpler strategies. Despite criticism for focusing solely on mean and variance rather than considering the complete distribution of financial returns, the mean-variance methodology remains popular due to its simplicity and strong theoretical foundation.

While widely recognized, the mean-variance framework faces challenges related to accurate parameter estimations, as highlighted by research conducted by Black and Litterman (1992) and Kan and Zhou (2007). Additionally, DeMiguel et al. (2009) demonstrated that using estimated means and variances may result in inferior outcomes compared to a basic equal weighting strategy (1/N). Platanakis et al. (2018) explored the impact of cryptocurrencies such as Bitcoin, Dash, Litecoin, and Ripple on portfolio selection and found that they might not significantly outperform the 1/n portfolio strategy. These findings align with the conclusions drawn by DeMiguel, Garlappi, and Uppal (2009). However, Ackermann et al. (2017) provided evidence suggesting that mean-variance analysis can outperform the naïve 1/N strategy in currency markets by leveraging predictive interest rates for future returns.

2.2.2 Mean variance crypto portfolio applications

These findings highlight the importance of using the mean-variance approach when discussing portfolio optimization, even in relation to modern assets like cryptocurrencies. Majdoub et al. (2021) emphasized the exploration of hedging possibilities for Bitcoin and other cryptocurrencies with different assets. Similarly, Kinkyo (2020) and Jareño et al. (2021) stressed the significance of studying how cryptocurrencies hedge against other markets, especially during times of economic uncertainty. Ongoing research on the diversification and safe haven properties of cryptocurrencies further supports the investigation into their potential impact on other cryptocurrencies. Based on this, there are two hypotheses that I would like to test in this study.

2.3 Cryptocurrency Portfolios

When it comes to cryptocurrency portfolios, various studies have presented diverse findings and approaches. Florin Aliu (2021) conducted an analysis using Markowitz's adjusted risk model and Markowitz Cornish Fisher's model. The results showed that including cryptocurrencies did not significantly improve the efficiency of portfolio strategies. On the other hand, Liu (2019) focused solely on cryptocurrencies in his analysis and found that employing portfolio selection methodologies could enhance investment outcomes in this market. However, despite their complexity, sophisticated models failed to outperform a simple rule called the "1/N rule" when considering the Sharpe ratio criterion (Sharpe, 1963; Sharpe, 1964). Another commonly used approach involves creating a minimum variance portfolio (Clarke et al., 2011; Markowitz, 1952).

The concept of this portfolio originates from the mean-variance optimization paradigm (Jagannathan & Ma, 2003). What sets it apart is that it does not rely on expected return data, making it computationally simple and widely popular among financial market practitioners. Numerous studies have demonstrated the superiority of the minimum variance portfolio compared to other asset allocation approaches. Some researchers have explored alternative risk-return measures like the Omega ratio (Wu and Pandey, (2014) and found that holding a small number of bitcoins in a diverse portfolio can benefit individual investors. Additionally, studies have utilized value at risk (VaR) and conditional value at risk (CVaR) (Eisl et al., 2015; Aggarwal et al., 2018; Selmi et al., 2018) to assess how Bitcoin can enhance portfolio diversification through optimization. These investigations suggest that Bitcoin can positively impact the risk-return tradeoffs of optimal portfolios. In a more recent study by Kajtazi and Moro (2019), they analyzed the effect of Bitcoin on portfolio optimization and diversification for US, European, and Chinese investors across four different portfolio scenarios: naïve, long only, unconstrained, and semi-constrained. Their findings indicate that while Bitcoin improved returns, it also increased the overall riskiness of the portfolios. In this research, I try to analyze the diversification potential of cryptocurrencies by implementing both in-sample and out-of-sample performance evaluations.

Hypothesis 1: *The inclusion of Bitcoin, Ethereum, and Binance Coin affects the asset allocation of a well-diversified portfolio and improves its risk-return profile.*

CHAPTER 3 Data

Table 1 comprehensively depicts the descriptive statistics for the chosen assets in the analysis. The monthly frequency data utilized in this study was obtained from financeyahoo.com, covering the period from January 1, 2018, to January 1, 2023. Subsequently, each asset was imported into Excel to compute descriptive statistics and the correlation matrix. Each asset downloaded using a specific ticker: Binance (BNB), Bitcoin (BTC), Forex Markets, Gold (GLD), Corporate Bonds, a real estate index (referring to the Vanguard Real Estate Index), S&P 500, and Mid Equity Market. Table 1 presents descriptive statistics, including measures such as mean, median, minimum, maximum, standard deviation, variation, Sharpe ratio, and number of observations for this dataset.

Bitcoin and Ethereum have considerably larger market capitalizations compared to Binance Coin. As of July 2023, Bitcoin's market capitalization stood at an astonishing $569 trillion, while Ethereum closely followed with $225 trillion. In contrast, Binance Coin lags with a market capitalization of $37 trillion (Coin Market Cap). However, it is essential to note that Binance Coin still holds a prominent position in the cryptocurrency market as it ranks fourth in market capitalization after Bitcoin, Ethereum, and Tether. Binance Coin is known for its essential role in supporting the operations of different projects associated with the Binance exchange. With the significant growth of the Binance exchange expected in 2023, it becomes interesting to analyze how Binance Coin performs as part of a diverse investment portfolio —considering its importance within the Binance ecosystem and market trends.

Table 1: Descriptive statistics of portfolios variables

	Binance Coin	**Bitcoin**	**Dollar Forex**	**Ethereum**	**Gold**	**Corp. Bonds**	**Real Estate**	**S&P 500**	**Small Cap**
Mean	0.054	0.02	0.002	0.008	0.006	0.00	0.004	0.006	0.002
Median	0.03	-0.03	0.003	0.03	-0.004	0.001	0.015	0.02	0.01
Minimum	-0.61	-0.47	-0.05	-0.77	0.07	-0.1	0.22	-0.13	-0.25
Maximum	0.04	0.47	0.05	0.58	0.1	0.09	0.12	0.12	0.17
Standard D.	0.32	0.22	0.02	0.3	0.04	0.04	0.06	0.05	0.07
Variance	0.1	0.05	0.0003	0.09	0.002	0.002	0.004	0.003	0.005
Skewness	1.6	-0.15	-0.35	-0.2	0.44	-0.25	-0.92	-0.49	-0.7
Kurtosis	7.1	-0.29	0.51	-0.17	-0.4	0.33	2.14	-0.09	1.94
Observations	62	62	62	62	62	62	62	62	62

Note: Data is monthly and provided in percentages.

Data from Table 1 will be used to construct correlation matrix, traditional and crypto-inclusive portfolios. Mean returns showcase the average monthly performance of each asset, with Binance Coin and Bitcoin displaying comparatively higher returns at 5.4% and 2%, respectively. Median returns provide insight into the central tendency of the data, with Dollar Forex, Ethereum, and Gold exhibiting more diverse median values. Minimum and maximum values indicate the range of returns over the study period, with Dollar Forex showing the lowest minimum return of -5% and Ethereum displaying the highest maximum return of 58%. The standard deviation and variance values quantify the dispersion of returns, with Binance Coin and Bitcoin exhibiting the highest standard deviation and variance among the assets.

Most of variables have positive skewness. Binance Coin has the highest positive skewness (1.6). The kurtosis values are generally greater than 3 for most of the variables, which suggests that the distributions have heavy tails and are more peaked than a normal distribution. Binance Coin has positive kurtosis value that is greater than 3, indicating that distribution has more extreme values in the tails compared to a normal distribution. Bitcoin, Ethereum, Gold and S&P 500 have negative kurtosis values, indicating that their distributions are less extreme in the tails and flatter than a normal distribution. Overall, the skewness and kurtosis values suggest that variables may not follow a normal distribution and have characteristics that deviate from the assumptions of a standard normal distribution.

These descriptive statistics lay the groundwork for understanding the individual asset behaviors that contribute to the broader portfolio performance. The data illustrates the diversity in asset returns and their associated risks, which play a crucial role in constructing optimal portfolios that balance risk and reward effectively.

5.1 Correlation matrix

Correlation matrix highlights the complex relationships between cryptocurrencies and traditional assets.

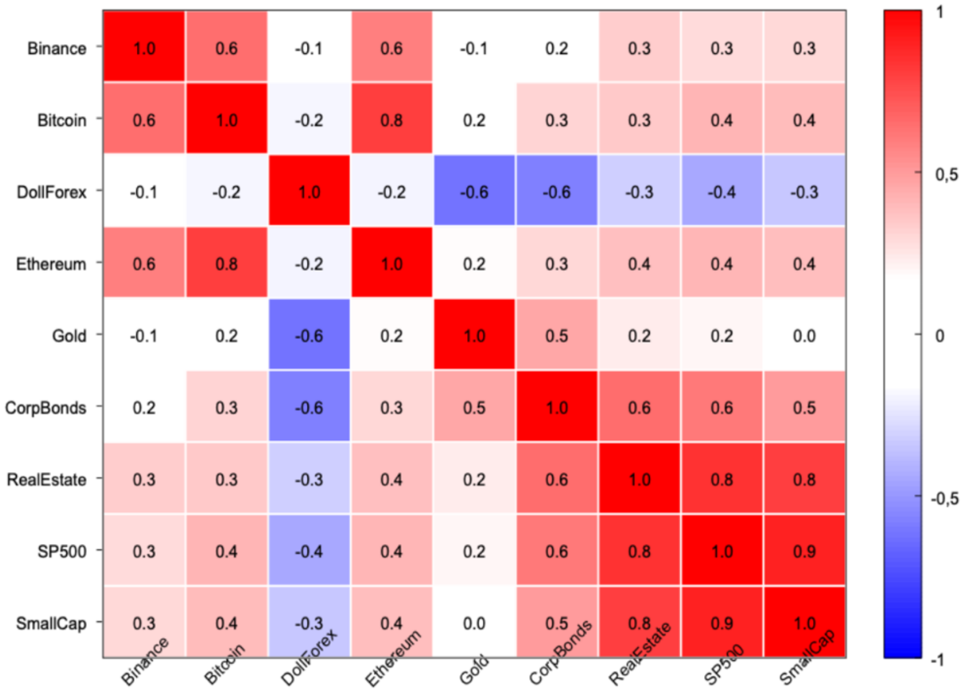

Figure 1 Correlation matrix for monthly growth rates.

Note: Each color identifies the extent of the correlation. Pole on the right side of the table shows the variation between color and correlation sign. Red colors show that assets are positively correlated, and blue colors show that assets are negatively correlated with each other. Data is shown in percentages.

Based on the results from Figure 1, it is evident that Ethereum (ETH), Bitcoin (BTC), and Binance Coin (BNB) exhibit positive correlations with most traditional assets. The strongest positive correlation for Bitcoin is observed with the Russel 2000 index (40%), SP500 (40%), The Vanguard Real Estate Index (30%) and the iShares Long-Term Corporate Bond ETF (30%). Ethereum has the highest positive correlation with Russel 2000 index (40%), SP500 (40%), The Vanguard Real Estate Index (40%). Binance Coin with Russel 2000 index (30%), SP500 (30%), The Vanguard Real Estate Index (30%). However, there are some noteworthy differences in their correlation patterns. BNB stands out as having a smaller positive correlation with traditional assets compared to ETH and BTC. It also exhibits a small negative correlation with the US Dollar Index and Gold, while BTC and ETH show negative correlations only with the US Dollar Index.

These results lead us to interesting conclusions. Bitcoin has positive correlation with Gold (0.2), which align with Article Bouoiyour and Selmi (2017) argues that Bitcoin exhibits the properties of weak safe haven for traditional assets in the short and long terms. On the other hand, Ethereum displays the highest positive correlation with the Russel 2000 index, SP500, The Vanguard Real Estate Index, The iShares Long-Term Corporate Bond ETF, and Bitcoin during the given period.

The findings of my study align with the research conducted by the T Iyer, (2022). The study reveals that the correlation coefficient between Bitcoin price and the S&P 500 has shown a notable increase from 0.01 in the period of 2016-2017 to 0.36 in the period of 2020-2021. These results reinforce the notion that both asset classes exhibit a growing tendency to rise and fall together.

However, there are limitations regarding to correlation matrix, because it does not always provide accurate results due to the small number of observations, especially with volatile price movements. Financial markets may also experience non-stationary periods that are not captured by correlation matrix. Monthly data may miss intramonth movements that also affects reliability of results.

CHAPTER 4 Method

4.1 Mean-Variance Optimization

Markowitz (1952, 1958) introduced a classical approach to portfolio optimization that aims to strike a balance between maximizing the expected return and minimizing portfolio risk, as represented by variance. This approach, known as the Markowitz covariance model or mean-variance approach (MV), involves calculating the expected portfolio return *R* as a weighted average of *n* constituent asset returns $\{\mu_i\}_{i=1}^{n}$ and subtracting portfolio risk through variance ($w^T \sum w$), which is the covariances between the returns of assets. The vector $\boldsymbol{w} \in R^n$ represents the set of weights for each asset. The unconstrained Markowitz MV model can be described by the following model:

$$max \sum_{i=1}^{i=n} \mu_i w_i - w^T \sum w \quad (1)$$

Subject to:

$$\sum_{i=1}^{n} w_i = 1 \quad (2)$$

$$-1 \leq w_i \leq 1 \qquad \forall i \in \{1, \dots, n\} \quad (3)$$

Where Equality 2 represents the budget constraint. Constraint 3 ensures the feasibility of both long and short positions. Equation 1 shows that the objective function consists of a term that maximizes the expected portfolio return, and a second term penalizes the portfolio for higher volatility by incorporating the variance-covariance matrix. Thus, the objective function for MV optimization problem maximizes the portfolio's expected return subject to a given level of risk (variance). We will also evaluate the performance of two variants of the MV model.

The first variant is the “Long Only” MV portfolio. This variant can be obtained by replacing constraint 3 by the following constraint:

$$w_i \geq 0 \qquad \forall i \in \{1, \dots, n\} \quad (4)$$

Where the non-negativity of weights imposed by Constraint 4 makes all strategies with short positions infeasible. The second variant is the “Constrained” MV model. We call this model “constrained” as we limit the allocation to any asset of the total available capital to be at most 25%. This variant can be obtained by replacing constraint 3 by the following constraint:

$$-0.25 \leq w_i \leq 0.25 \qquad \forall i \in \{1, \dots, n\} \quad (5)$$

4.2 Holding and Formation Periods

To gain insights into the portfolios' performance over time, we establish a consistent approach with the literature. Specifically, we employ a fixed size moving window of 36 months for the formation period, aligned with the methodology utilized in the research by Pavan Kumar et al. (2022). To enhance the robustness of our findings, we also adopt the rolling window technique, consistent with study like Bessler and Wolff (2015). The selection of a 36-month moving window is rooted in the requirements of the Mean-Variance (MV) approach, which necessitates the estimation of the covariance matrix. Given the cryptocurrency-inclusive MV portfolios involve the estimation of 45 parameters and traditional MV portfolios require 21 parameters, the 36-month window size strikes a balance. It considers potential inaccuracies arising from a small window size and the need for sufficient out-of-sample observations for portfolio performance testing. This choice is particularly crucial since the forecasted parameters are subject to estimation errors in real-world scenarios, as highlighted in works such as Welch and Goyal (2008) and other related studies. Consequently, our study accommodates both in-sample settings, which implicitly assume perfect forecasts, and out-of-sample performance assessments, offering a comprehensive evaluation of portfolio dynamics and risk-return trade-offs.

4.3 Portfolio Assessment Metrics

We will estimate both promised and realized portfolio returns and variances for each portfolio. Besides these metrics, we will present other performance metrics that shed light on riskiness and other aspects of these portfolios. This is done for both in-sample and out-of-sample performance evaluations.

4.3.1. Value at Risk

The MV approach has a notable drawback as it oversimplifies investors' risk-preferences. While variance considers both upside and downside volatility, rational investors in the real-world typically view only the downside component as undesirable. To address this limitation, alternative risk measures, such as Value-at-Risk (VaR), have been proposed in the literature Jorion (1996) and Camplbell et al. (2005).

We proceed to explain the VaR metric. This is a risk measure that quantifies the maximum potential loss for a given probability and time horizon, focusing solely on downside risk. It is commonly applied by practitioners for risk management purposes and has an extensive literature (Rijba et al., 2015). For a given confidence level α it is formally defined as the $(1-\alpha)\%$ percentile of portfolio returns:

$$VaR_t = \varphi^{-1}(1-\alpha) \tag{6}$$

Where $\varphi^{-1}(x)$ is the inverse cumulative distribution function of portfolio returns R_t. There is an extensive literature of methods co estimate the VaR of a portfolio. An example is the Bootstrap Historical Simulation technique (BHS), which produces an estimate by computing the average $(1-\alpha)\%$ percentile of portfolio returns based on bootstrapped samples from historical data (Rijba et al., 2015). A weakness of BHS is its underperformance in the presence of dependence in the sample (Hall, 1985). We proceed with the Filtration Historical Simulation (FHS) method proposed by Barone-Adesi et al. (1999). This method provides a model that is a better representation of asset return data. This is since it combines the BHS method with volatility modelling. Consequently, it can handle excess kurtosis and clustering of volatility (Barone-Adesi et al., 1999). Rijba et al. (2015) study the performance of several VaR estimation techniques and confirm that FHS outperforms BHS for financial data with the use of a GARCH (1,1) volatility model. We proceed to predict VaR based on the FHS GARCH (1,1) model (See Figure 2). Note that similar to Rijba et al. (2015) we generate 1000 bootstrap samples for the model. Furthermore, we set the size of each bootstrap sample equal to 300.

Algorithm of FHS GARCH (1,1) predictive model

1. Given optimal weights and estimation sample, generate sample portfolio returns
2. Fit a GARCH (1,1) model with zero mean based on normally distributed innovations is fitted to the sample portfolio returns via maximum likelihood
3. Compute standardized portfolio returns $Z_t = R_t/\sigma_t$
4. Estimate σ_{N+1} with the aid of the fitted GARCH (1,1) model
5. Generate a sample of N standardized portfolio returns via bootstrapping
6. Determine the $(1-\alpha)\%$ percentile Z^* of the sample of standardized portfolio returns in step 4.
7. Generate one estimate of the FHS VaR: $\widehat{VaR}_{N+1,i} = \sigma_{N+1} Z^*$
8. Repeat steps 5 to 7 for K times to obtain a sample of estimates for FHS VaR
9. $VaR_{N+1} \approx \frac{1}{K} \sum_{i=1}^{K} \widehat{VaR}_{N+1,i}$

Figure 2: **Algorithm of FHS GARCH (1,1) predictive model**

To evaluate the out-of-sample performance of our VaR estimates for different types of portfolios, we use the "VaR Violation" metric proposed by Christoffersen (1998):

$$I_t(\alpha) = \begin{cases} 1 & if \quad R_t < VaR_t \\ 0 & otherwise \end{cases} \tag{7}$$

By computing the sample mean over the out-of-sample predictions, I can verify whether the promised VaR holds for the given portfolio strategy and asset composition.

4.3.2. Sharpe Ratio and Jobson Korkie Test

The Sharpe ratio measures the risk-adjusted return of a portfolio, indicating how much excess return a portfolio earns per unit of risk taken. The Sharpe ratio is calculated as follows:

$$SHARP \; = \frac{E(\mu)-Rf}{\sigma} \tag{8}$$

I will compute this metric for both promised and realized returns. The methodology employed in this study does come with certain limitations. One noteworthy constraint pertains to the definition of the risk-free rate. Given the dissimilarity between traditional assets and cryptocurrencies, employing a standard measure like the Treasury bond rate may not accurately reflect the true risk-free rate. Considering this challenge, a simplification was made by considering the risk-free rate as 0%. Nonetheless, it's important to acknowledge that this assumption could potentially impact the precision of the results obtained.

The realized Sharpe ratio will be used to assess the relative performance of portfolios with and without cryptocurrency restrictions. For each pair of portfolios differing only by the cryptocurrency choice restriction, I compare their realized Sharpe ratios based on the corrected the corrected version of the Jobson-Korkie Test (Jobson & Korkie, 1981) proposed by Memmel (2003):

$$Z = \frac{SHARP_{high} - SHARP_{low}}{\sqrt{\hat{\theta}}} \tag{9}$$

Where $SHARP_{high}$ represents the portfolio with the larger Sharpe ratio and $SHARP_{low}$ the other one. The parameter $\hat{\theta}$ is defines as follows:

$$\hat{\theta} = \frac{1}{N}(2 - 2\rho_{high,low} + \frac{1}{2}(SHARP_{high}^2 SHARP_{low}^2 - 2SHARP_{high}SHARP_{low}\rho_{high,low}^2)\,) \tag{10}$$

The parameter N represents the amount of portfolio returns, $\rho_{high,low}$ represents the correlation of the returns of both portfolios. The hypotheses are:

$$H0: SHARP_{high} = SHARP_{low}$$

$$H1: SHARP_{high} > SHARP_{low}$$

Under the null hypothesis, Z follows a standard normal distribution. I use a significance level of 5% and thus use the the critical value of 1.645. With the aid of this test, we can test whether the portfolios differ in realized performance.

4.2 Portfolios

The study constructs 8 portfolios, 4 traditional and 4 crypto-inclusive ones. There are equally weighted, unconstraint, long only and constraint portfolios, which I will describe further in details. Traditional portfolios comprise a diversified mix of assets, including equity, forex, mid equity, real estate markets, corporate bonds, and gold. On the other hand, the crypto-inclusive portfolios go beyond the traditional assets and incorporates three prominent cryptocurrencies: Ethereum, Bitcoin, and Binance Coin. This diversified approach allows for a comprehensive analysis of how the inclusion of cryptocurrencies impacts portfolio performance.

Portfolio 1: Equally weighted Portfolio ($\omega_i = \frac{1}{N}\ \forall i$)

The naïve portfolio is constructed so that all assets are allocated equally irrespective of potential effects on the risk-return ratio. Each traditional asset is allocated a weight of 16.7% in the portfolio, ensuring an equal distribution of weights among the assets. In the crypto-inclusive portfolio each asset allocated a weight of 11.1%. The study utilizes the same monthly dataset as for correlation matrix, obtained from Yahoo Finance to calculate expected returns for each asset based on monthly returns. This step enables a comparison of the performance and risk characteristics of the traditional and crypto-inclusive portfolios, shedding light on the potential benefits and drawbacks of including cryptocurrencies in a diversified investment strategy.

4.2.1 Mean-variance portfolios

Portfolio 2: Unconstrained Portfolio ($\omega_i \in R: -1 \leq \omega_i \leq 1; \sum \omega_i = 1$)

The unconstrained portfolio, with its absence of limitations on asset weights, allowed for 100% shorting. Theoretically, this framework aimed to achieve the highest risk-adjusted return among all portfolio frameworks. By permitting such flexibility, it frequently tested the theoretical boundaries for the potential benefits of including a particular asset in a well-diversified portfolio.

Portfolio 3: Long Only Portfolio ($\omega_i \in R: \omega_i \geq 0; \sum \omega_i = 1$)

This approach allowed for asset weights to be fully allocated, with no option for shorting. It offered a more feasible scenario for investors, especially when dealing with assets where shorting was not a viable option due to market limitations. The primary objective of this optimization framework was to maximize the Sharpe ratio while adhering to the no short-selling constraint, making it a pragmatic and realistic approach to portfolio diversification in real-world investment situations.

Portfolio 4: Constraint portfolio ($\omega_i \in R: -0.25 \leq \omega_i \leq 0.25; \sum \omega_i = 1$)

The approach aimed to optimize the risk-adjusted return of the portfolio by introducing constraints that could be adjusted both ways. This approach was valuable for finding the theoretical solution to an optimization challenge, while also accommodating the possibility of allocating up to 25 percent of the portfolio's weight to each asset, whether in long or short positions.

The methodology employed in this study does come with certain limitations. One noteworthy constraint pertains to the definition of the risk-free rate for Sharpe Ratio calculations. Given the dissimilarity between traditional assets and cryptocurrencies, employing a standard measure like the Treasury bond rate may not accurately reflect the true risk-free rate. Considering this challenge, a simplification was made by considering the risk-free rate as 0%. Nonetheless, it's important to acknowledge that this assumption could potentially impact the precision of the results obtained.

CHAPTER 5 Results & Discussion

5.1 Results

In this section, I present the outcomes of our analysis. This encompasses the determined optimal weights for both Traditional and Crypto-inclusive portfolios. Also, I evaluate the portfolios' performance using in-sample and out-of-sample results, and explore the insights derived from the Jobson-Korkie Test. These analyses are performed using Python, utilizing libraries like pandas, numpy, cvxpy, arch, and math.

Table 2 shows the distribution of mean optimal weights for crypto-inclusive portfolios. These weights are calculated for different portfolio types: "Long Only," "Naive," "Unconstrained," and "Constraint."

Table 2: Crypto-Inclusive weights

Weights	Long Only	Naive	Unconstrained	Constraint
Binance	40.89%	11.11%	76.57%	25.00%
Bicoin	0.00%	11.11%	5.92%	25.00%
Doll Forex	0.00%	11.11%	14.56%	25.00%
Ethereum	0.00%	11.11%	-39.47%	-23.35%
Gold	46.37%	11.11%	100.00%	25.00%
Corp. Bonds	0.00%	11.11%	-100.00%	-22.67%
Real Estate	0.00%	11.11%	-11.94%	25.00%
S&P 500	12.7%	11.11%	100.00%	25.00%
Small Cap	0.00%	11.11%	-45.74%	-3.98%

Note: All values are shown in percentages.

The "Unconstrained" portfolio demonstrates a significant allocation of 76.57% to Binance Coin, suggesting a strong bullish stance on its potential. Interestingly, while Bitcoin holds no allocation in the "Long Only" portfolio, it does appear in other portfolios, 5.92% allocation in “Unconstraint”. Additionally, allocations to Ethereum vary, with negative allocations in the "Unconstrained" and "Constraint" portfolios, hinting at an attempt to short Ethereum, a strategy that might face practical limitations. Gold receives considerable attention, particularly in the "Unconstrained" portfolio where a leverage of 100% is employed. The allocations to other assets, including Doll Forex, Corporate Bonds, Real Estate, S&P 500, and Small Cap, vary across portfolios, reflecting diverse risk-taking strategies.

Table 3 presents the in-sample results for the different crypto-inclusive portfolio strategies, shedding light on their performance characteristics.

Table 3: In-Sample results for Crypto-Inclusive Portfolios

Portfolio	Long Only	Naive	Unconstrained	Constraint
Average Return	2.56%	1.10%	4.97%	2.01%
Return Variance	1.10%	0.77%	2.00%	0.46%
Standard Dev.	10.51%	8.77%	14.16%	6.75%
Mean Sharpe Ratio	24.32%	12.51%	35.11%	29.80%
Mean VaR (95%)	-2.80%	-4.80%	-3.60%	-2.34%

Note: All values are based on monthly return data and are shown in percentages.

The "Unconstrained" portfolio demonstrated the highest average return of 4.97%, showcasing its potential for generating substantial returns even within a risk-sensitive context. In terms of return variance, the "Unconstrained" strategy again led the pack with a variance of 2.00%, while the "Constraint" portfolio displayed the lowest variance at 0.46%, highlighting its risk-mitigation attributes. Assessing risk-adjusted performance, the "Unconstrained" portfolio achieved a remarkable mean Sharpe ratio of 35.11%, indicating superior returns per unit of risk. Mean Sharpe ratio is computed using the mean promised return and standard deviation of the mean promised variance. In contrast, the "Naive" strategy, with its mean Sharpe ratio of 12.51%, appeared to trade higher risk for comparatively modest returns. Additionally, the "Mean VaR (95%)" metric demonstrated the portfolio's resilience to extreme events, with the "Unconstrained" strategy registering a lower VaR of -3.60% than its counterparts, further attesting to its potential risk management capacity.

Table 4 shows the out-of-sample results for the crypto-inclusive portfolios.

Table 4: Out-Of-Sample results for Crypto-Inclusive Portfolios

Portfolio	Long Only	Naive	Unconstrained	Constraint
Average Return	3.06%	0.90%	7.13%	2.17%
Return Variance	1.75%	0.96%	5.19%	1.11%
Standard Dev.	13.22%	9.80%	22.78%	10.56%
Sharpe Ratio	23.11%	9.17%	31.32%	20.60%
Mean Violation	34.62%	34.62%	23.08%	30.77%

Note: All values are based on monthly return data and are shown in percentages.

The "Long Only" approach displayed increased potential, yielding an average return of 3.06%, outperforming its in-sample performance. The "Unconstrained" strategy demonstrated consistent excellence, boasting an average return of 7.13%, while the "Constraint" portfolio maintained its stability with an average return of 2.17%. Return variance across strategies continued to align with the in-sample pattern, underscoring the "Unconstrained" portfolio's higher risk profile.

A significant finding is the dynamic performance of the Sharpe ratio, where the "Unconstrained" strategy maintained its leadership with a robust ratio of 31.32%. The "Long Only" portfolio also remained competitive, recording a Sharpe ratio of 23.11%, emphasizing its risk-adjusted performance capacity. However, the "Naive" strategy's Sharpe ratio decreased to 9.17%, highlighting its sensitivity to changing market conditions. Mean Sharpe ratio is computed using the mean promised return and standard deviation of the mean promised variance. The "Mean Violation" metric indicated that both in-sample and out-of-sample periods shared consistent risk levels, further emphasizing the value of managing risk exposure. These results highlight the potential benefits and complexities of integrating cryptocurrencies into diversified portfolios. The "Unconstrained" strategy consistently demonstrated higher average returns and Sharpe ratios but also exhibited greater volatility and mean violations. On the other hand, the "Long Only" and "Constraint" portfolios achieved competitive performance while imposing constraints on asset weights. The differences in these metrics emphasize the trade-offs between risk and return associated with various portfolio strategies when considering both traditional and crypto assets.

Comparing the results presented in Tables 3 and 4 offers a comprehensive view of the performance dynamics of the crypto-inclusive portfolio strategies across both in-sample and out-of-sample periods. Notably, the "Unconstrained" portfolio consistently demonstrated strong potential in both scenarios, showcasing higher average returns, albeit with increased volatility. The "Long Only" approach also displayed resilience, maintaining competitive average returns and exhibiting a relatively stable performance trajectory. In contrast, the "Naive" strategy appeared sensitive to market changes, displaying a marked drop in average return and Sharpe ratio in the out-of-sample period. The "Constraint" portfolio, with its focus on risk mitigation, demonstrated stable performance across both sample periods, showcasing its capacity to navigate market uncertainties.

The return variance and standard deviation exhibited consistent patterns across in-sample and out-of-sample periods, suggesting the presence of systematic factors influencing portfolio volatility. The superior risk-adjusted performance of the "Unconstrained" portfolio in both samples underscores its appeal to investors seeking higher returns despite taking on higher risk. Meanwhile, the "Long Only" strategy presented a balanced risk-reward profile, suitable for risk-sensitive investors looking to capitalize on market opportunities without venturing into short positions. On the other hand, the "Constraint" approach maintained its commitment to risk reduction, with relatively lower returns but also lower volatility.

This comparative analysis reveals that the portfolio strategies' underlying dynamics remain stable across different timeframes, providing valuable insights for investors. The observed trends highlight the potential benefits of diversification and risk management, while also emphasizing the importance of tailoring portfolio strategies to individual risk preferences and market conditions.

Table 5 displays the mean optimal weights for traditional portfolios composed of DollForex, Gold, Corporate Bonds, Real Estate, S&P500, and Small Cap. These weights are calculated for different portfolio types: "Long Only," "Naive," "Unconstrained," and "Constraint."

Table 5: Traditional weights

Weights	Naive	Long Only	Unconstrained	Constraint
Doll Forex	16.67%	0.00%	19.97%	25.00%
Gold	16.67%	66.45%	100.00%	25.00%
Corp. Bonds	16.67%	0.00%	-100.00%	3.29%
Real Estate	16.67%	0.00%	72.71%	25.00%
S&P 500	16.67%	33.55%	100.00%	25.00%
Small Cap	16.67%	0.00%	-92.68%	-3.29%

Note: All values are shown in percentages.

Turning to Table 5, it outlines the optimal asset allocations for traditional portfolios without cryptocurrencies. Among these assets, Gold and S&P 500 are prominent choices with substantial allocations across all portfolios, especially in the "Long Only" and "Unconstrained" strategies. Conversely, Corporate Bonds do not receive allocation only in the "Long Only" portfolio, while the "Unconstrained" portfolio opt to short it entirely. Real Estate finds allocations in the "Long Only" and "Unconstrained" strategies. Small Cap is shorted in the "Unconstrained" and "Constraint" strategies.

Table 6 presents the in-sample results for the traditional portfolios.

Table 6: In-Sample results for Traditional Portfolios

Portfolio	Long Only	Naive	Unconstrained	Constraint
Average Return	0.59%	0.34%	1.27%	0.44%
Return Variance	0.10%	0.11%	0.22%	0.07%
Standard Dev.	3.24%	3.31%	4.72%	2.63%
Sharpe Ratio	18.23%	10.22%	26.98%	16.73%
VaR (95%)	-0.81%	-1.03%	-1.90%	-0.78%

Note: All values are based on monthly return data and are shown in percentages.

The "Unconstrained" portfolio showed the highest average return at 1.27%, followed by the "Constraint" portfolio with an average return of 0.44%. Regarding return variance, the

"Unconstrained" portfolio displayed the highest value at 0.22%, while the "Constraint" strategy exhibited the lowest return variance at 0.07%. The "Unconstrained" portfolio also exhibited the highest standard deviation at 4.72%, while the "Constraint" approach had the lowest standard deviation at 2.63%. Analyzing the mean Sharpe ratio, the "Unconstrained" portfolio outperformed the others with 26.98%, while the "Naive" strategy had the lowest ratio (10.22%). Mean Sharpe ratio is computed using the mean promised return and standard deviation of the mean promised variance. The mean Value-at-Risk (VaR) at the 95% confidence level was most favorable for the "Constraint" portfolio at -0.78%, while the "Unconstrained" approach had the highest VaR at -1.90%.

Table 7 provides the out-of-sample results for the traditional portfolios.

Table 7: Out-Of-Sample results for Traditional Portfolios

Portfolio	Long Only	Naive	Unconstrained	Constraint
Average Return	0.22%	-0.03%	2.46%	0.30%
Return Variance	0.14%	0.12%	0.27%	0.10%
Standard Dev.	3.68%	3.44%	5.20%	3.10%
Sharpe Ratio	5.85%	-0.76%	47.30%	9.67%
Mean Violation	38.46%	30.77%	23.08%	34.62%

Note: All values are based on monthly return data and are shown in percentages.

The "Unconstrained" portfolio demonstrated the highest average return of 2.46%, followed by the "Constraint" portfolio at 0.30%. Return variance mirrored the pattern observed in the in-sample results, with the "Unconstrained" portfolio having the highest value at 0.27%, while the "Constraint" strategy had the lowest return variance at 0.10%. Examining the standard deviation of returns, the "Unconstrained" portfolio also showcased the highest value at 5.20%, while the "Constraint" approach displayed a lower standard deviation at 3.10%. The Sharpe ratio indicated remarkable performance for the "Unconstrained" portfolio at 47.30%, followed by the "Constraint" portfolio at 9.67%. However, it's important to note that the "Naive" strategy showed a negative Sharpe ratio in the out-of-sample period. The "Long Only" approach displayed a modest Sharpe ratio at 5.85%.

Comparing the in-sample and out-of-sample results reveals insightful dynamics in the traditional portfolio strategies. The "Unconstrained" portfolio consistently showcased strong potential across both scenarios, indicating a capacity to navigate market fluctuations. Interestingly, the "Constraint" portfolio exhibited stability, demonstrating a reliable performance trajectory in both sample periods. On the other hand, the "Naive" strategy appeared sensitive to market changes, displaying negative returns and a negative Sharpe ratio in the out-of-sample period.

Table 8 presents the outcomes of the Jobson-Korkie Test, which examines the significance of differences in out-of-sample Sharpe ratios between traditional and crypto-inclusive portfolios.

Table 8: Results of Jobson-Korkie Test between Traditional-Crypto Portfolio Pairs

Portfolio	**Long Only**	**Naive**	**Unconstrained**	**Constraint**
Z-Score	0.679	0.458	0.579	0.158

Note: All values are based on monthly return data and are shown in percentages.

While various portfolio pairs were examined, the absence of statistical significance in Z-Scores suggests that the observed differences in Sharpe ratios between traditional and crypto-inclusive portfolios do not hold at the 5% confidence level.

The results obtained from the analysis of both crypto-inclusive and traditional portfolios provide valuable insights regarding hypothesis 1, which asserts that the inclusion of Bitcoin, Ethereum, and Binance Coin can impact the asset allocation of a diversified portfolio and enhance its risk-return profile.

In the in-sample analysis, the crypto-inclusive portfolios exhibit promising characteristics. The "Unconstrained" crypto-inclusive portfolio boasts a higher average return of 4.97%, outperforming the "Unconstrained" traditional portfolio, which stands at 1.27%. Examining the risk metrics, the crypto-inclusive portfolios demonstrate mixed results. The "Unconstrained" crypto-inclusive portfolio has a higher return variance of 2.00%, compared to the "Unconstrained" traditional portfolio's 0.22%. This divergence indicates that while the crypto-inclusive portfolio offers potentially higher returns, it also exhibits higher volatility. The Sharpe ratios further highlight the divergence in risk-adjusted returns. The "Unconstrained" crypto-inclusive portfolio displays a significantly higher Sharpe ratio of 35.11%, while the "Unconstrained" traditional portfolio has a Sharpe ratio of 26.98%. This suggests that the inclusion of cryptocurrencies has the potential to improve the risk-adjusted returns of a portfolio.

When comparing the traditional and crypto-inclusive portfolios' out-of-sample results, similar trends emerge. The "Unconstrained" crypto-inclusive portfolio maintains a higher average return of 7.13%, surpassing the "Unconstrained" traditional portfolio's 2.46%. This confirms that the presence of cryptocurrencies can contribute to better performance in real-world scenarios. In terms of risk metrics, the "Unconstrained" crypto-inclusive portfolio maintains higher return variance and standard deviation compared to its traditional counterpart. Despite this, the crypto-inclusive portfolio's Sharpe ratio remains remarkably higher at 31.32%, while the traditional portfolio's ratio stands at 47.30%. This result implies that while the crypto-inclusive portfolio might exhibit higher volatility, its risk-adjusted returns remain competitive.

In summary, the comparison between the results of traditional and crypto-inclusive portfolios underscores the potential benefits of incorporating Bitcoin, Ethereum, and Binance Coin. Despite the higher volatility associated with cryptocurrencies, the analysis reveals that their inclusion can enhance both average returns and risk-adjusted performance, affirming the hypothesis that these cryptocurrencies can positively impact portfolio asset allocation and risk-return profiles.

5.2 Limitations

Firstly, in both the crypto-inclusive and traditional portfolios, the calculated mean violations are substantially above the expected threshold of 5%. This outcome indicates that the FHS VaR method may not be functioning effectively in accurately capturing the risk levels. This limitation pertains to the performance of the GARCH (1,1) model in forecasting volatility. This implies that the GARCH (1,1) model, which is commonly used to estimate volatility in financial time series data, may not be well-suited for capturing the complex dynamics of cryptocurrency markets. These larger-than-expected violations suggest that the model might not adequately capture extreme events or tail risk, which are more pronounced in the cryptocurrency market due to its inherent volatility. This limitation highlights the need for alternative volatility modeling approaches that can better account for the unique characteristics of cryptocurrency data.

The second limitation concerns the findings of the Jobson-Korkie test conducted to assess the performance of portfolios with cryptocurrencies compared to those without cryptocurrencies. Surprisingly, the test results indicated that there was no statistically significant difference in out-of-sample Sharpe ratios between portfolios with cryptocurrencies and those without cryptocurrencies across the four different models considered. This is unexpected because, in-sample, the portfolios with cryptocurrencies consistently exhibited higher Sharpe ratios compared to traditional portfolios in all but the unconstrained model. This discrepancy between in-sample and out-of-sample results suggests that the superior risk-adjusted returns observed for crypto portfolios during the estimation period may not necessarily persist in future market conditions. It underscores the importance of considering the stability and robustness of portfolio strategies when incorporating cryptocurrencies, as their performance can vary substantially over time and under different market conditions.

5.3 Discussion

The findings of this study align with the conclusions drawn from prior research regarding the potential advantages of integrating cryptocurrencies into diversified portfolios. Notably, studies by Boiko et al. (2021) and Wang and Ngene (2020) also emphasized the substantial improvement in portfolio performance achievable by including different cryptocurrencies using diverse optimization strategies. This alignment arises due to the inherent nature of cryptocurrencies, particularly their unique market behavior characterized by high volatility and the potential for high returns. The increased returns observed in crypto-inclusive portfolios can be attributed to the significant price fluctuations in the cryptocurrency market, which provide opportunities for outsized gains.

Furthermore, Ma et al. (2020) supported that incorporating multiple cryptocurrencies could enhance portfolio performance, particularly highlighting Ethereum's potential for better diversification than Bitcoin. This alignment can be understood through Ethereum's distinctive characteristics and role as a platform for decentralized applications, offering additional utility beyond Bitcoin. As a result, Ethereum's inclusion can provide broader exposure to the blockchain ecosystem, potentially leading to enhanced diversification benefits.

However, while this study corroborates these consistent findings, it's essential to recognize the nuances and potential variations that can arise due to the dynamic nature of cryptocurrency markets. The challenges tied to accurately estimating parameters within the mean-variance framework, as highlighted by Black and Litterman (1992) and Kan and Zhou (2007), can impact the outcomes. Cryptocurrencies' complex and rapidly evolving nature can lead to significant variations in expected returns and risk, affecting the optimization outcomes. The alignment with DeMiguel et al. (2009) also reinforces that estimated means and variances might not always lead to superior results compared to simpler strategies like equal weighting (1/N). This complexity is further echoed by the alignment with Platanakis et al. (2018), indicating that cryptocurrencies can influence portfolio performance. However, their effect may differ depending on market conditions, asset correlations, and investor risk preferences.

In summary, while similarities between this study's findings and previous research showcase a consistent trend of enhanced performance with cryptocurrency inclusion, the unique nature of the cryptocurrency market and the intricacies of portfolio optimization underscore the importance of careful consideration and tailored strategies to account for potential variations and risks.

CHAPTER 6 Conclusion

As I navigate the intricacies of modern investment strategy, a central inquiry emerges: "What is the impact of including cryptocurrencies in a diversified portfolio of traditional assets?" This fundamental question has steered our exploration, driving us to investigate the intricate interplay between cryptocurrencies and traditional assets within investment portfolios.

The meticulous analysis of diverse crypto-inclusive and traditional portfolios yields invaluable insights into the role of cryptocurrencies in optimizing portfolios. The outcomes illuminate the potential advantages and complexities of integrating cryptocurrencies—especially Bitcoin, Ethereum, and Binance Coin—into diversified investment strategies. These findings directly address the research question, illuminating how the inclusion of these prominent cryptocurrencies influences the asset allocation of well-diversified portfolios and enhances risk-return profiles. The distribution of mean optimal weights across portfolios prominently showcases the impact of these cryptocurrencies on portfolio composition, underscoring their potential as diversification instruments. The significant allocation of the "Unconstrained" portfolio to Binance Coin and the varying allocations of Ethereum underscore their pivotal role in shaping portfolio performance. However, the heightened risk and volatility that come with cryptocurrency inclusion also underscore the need for vigilant risk management and alignment with investor risk preferences.

Nevertheless, it's essential to acknowledge the study's limitations. The assumption of independent asset returns and the use of VaR, better suited for normally distributed data, might affect the generalizability of our findings. Additionally, the identified mean violations within the FHS VaR method accentuate the necessity for more sophisticated risk assessment models capable of addressing the intricate interdependencies within the dataset.

The findings of this study have important implications for investors, financial institutions, and regulators. Firstly, investors should carefully assess cryptocurrencies' risks and potential benefits before incorporating them into their portfolios. They should consider the specific risk appetite, investment objectives, and time horizon when making investment decisions involving cryptocurrencies. Financial institutions should enhance their risk management frameworks and regulatory oversight to address the unique risks posed by cryptocurrencies. Additionally, regulators should continue to monitor and adapt regulations to ensure investor protection, market stability, and integrity. Further research in this area should explore alternative diversification methods, consider a broader range of cryptocurrencies, and examine the potential long-term impact of decentralized finance on traditional financial systems.

Overall, this study contributes to the existing body of knowledge by providing insights into the impact of cryptocurrencies on traditional portfolios. It emphasizes the need for a comprehensive understanding of the risks and limitations associated with cryptocurrencies while acknowledging the sustained enthusiasm of investors toward this emerging asset class.

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APPENDIX A Historical Prices

Figure 1 Bitcoin Historical Price

Note: Data is collected from Yahoo Finance from 2018 January 1 to 2023 January 1. The graph provides information about historical value of Bitcoin. Prices are measured in USD.

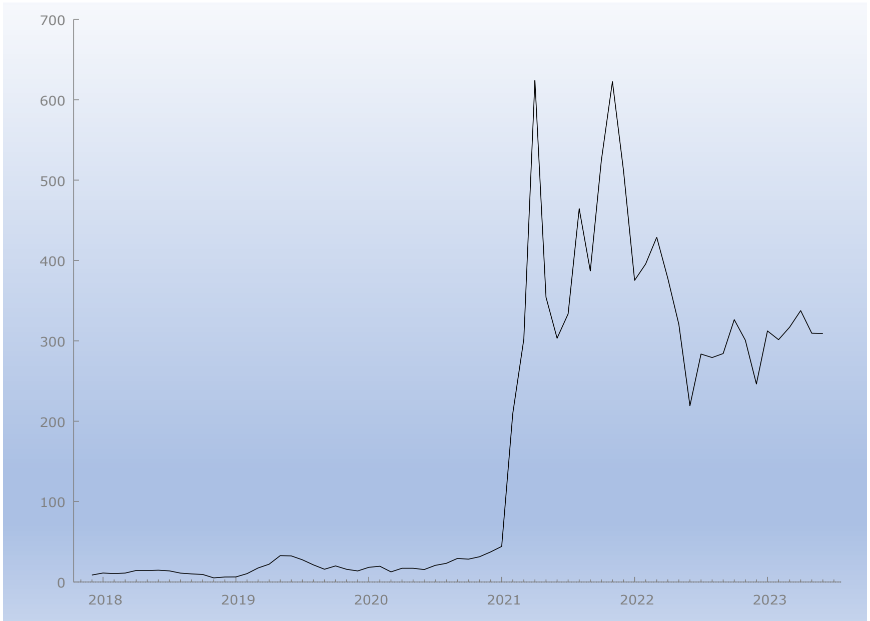

Figure 2 Binance Coin Historical Price

Note: Data is collected from Yahoo Finance from 2018 January 1 to 2023 January 1. The graph provides information about historical value of Binance Coin. Prices are measured in USD.

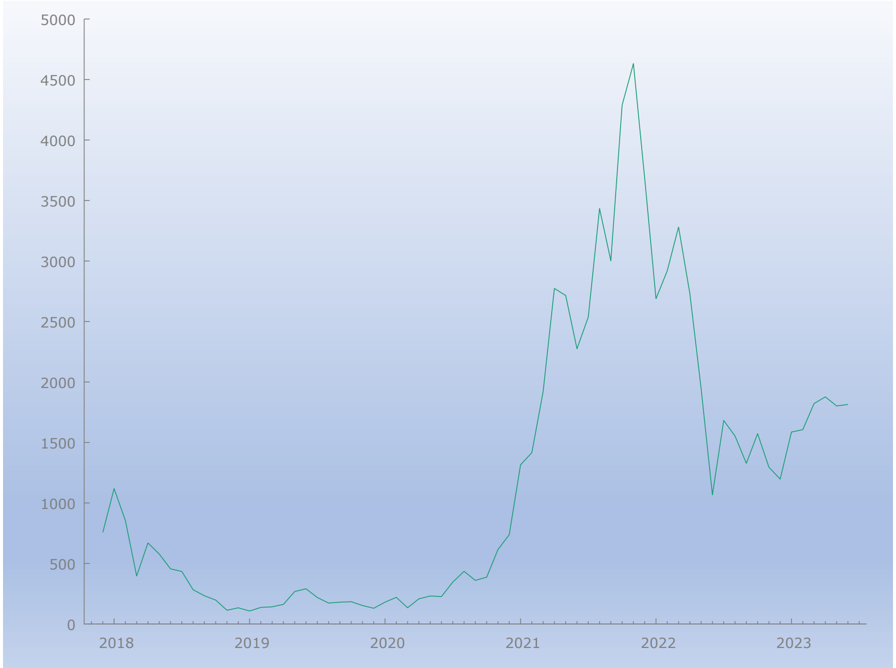

Figure 3 Ethereum Historical Price

Note: Data is collected from Yahoo Finance from 2018 January 1 to 2023 January 1. The graph provides information about historical value of Ethereum. Prices are measured in USD.

APPENDIX B Python Calculations

```
import pandas as pd

import numpy as np

import math


from portfolioExecution import portExecute


#this function provides the Jobson Korkie statistic  based on the Memmel correction

def Jobson_Korkie(returnsA, returnsB):

	#assume 0% rf

	sharpA = np.mean(returnsA) / np.std(returnsA)

	sharpB = np.mean(returnsB) / np.std(returnsB)


	rho = np.corrcoef(returnsA,returnsB)[0][1]

	theta = (1/len(returnsA))*( 2 - 2 * rho + (1/2) * ( (sharpA**2) * (sharpB**2) - 2 * sharpA * sharpB * (rho**2) ) )

	print(theta)

	print(np.abs(sharpA - sharpB))

	return np.abs(sharpA - sharpB) / math.sqrt(theta)




returnsDF = pd.read_csv('C:/Users/Maria/Desktop/monthlyDataThesis.csv')

windowSize = 36

port1 = portExecute(returnsDF,windowSize)
```

#asset = "Traditional" or whatever else(everything else leads to including crypto)

#strategy = "Mean-Variance-Efficient" or whatever else(everything else leads to Naive Diversification Strategy)

#constraint = "Long-Only", "Constrained", everything else leads to unconstrained opt(note that each weight will have an absolute size of at most 1)

```
asset="Traditional"

strategy="Mean-Variance-Efficient"

#strategy="Naive"

constraint="Constrained"

dictionaryTest = port1.executeStrategy(asset, strategy, constraint)

print(dictionaryTest)

#define arrays to be iterated over

asset = ["Traditional", "Crypto"]

strategy = ["Mean-Variance-Efficient", "Naive"]

constraint = ["Long-Only", "Constrained", "Unconstrained"]

outputDictionary={}

for a in asset:

    #You chose an asset

    for s in strategy:

        #You chose a strategy

        for c in constraint:
```

```
                    #You chose your constraint setting

                    outputDictionary["{}-{}-{}".format(a,s,c)]                    =
port1.executeStrategy(a,s,c)
#print(outputDictionary)
"""
#Printing of Crypto inclusive weight choices
firstRowWeights = ["Weights","Long-Only","Constrained", "Unconstrained", "Naive"]
#This order of assets is maintained overall in the codes thus no problems with indexation
#firstColWeights = {"Binance":0 ,"Bitcoin":1, "Doll Forex":2, "Ethereum":3,
"Gold":4,"Corp. Bonds":5, "Real Estate":6, "S&P 500":7,"Small Cap":8}
firstColWeights = ["Binance" ,"Bitcoin", "Doll Forex", "Ethereum", "Gold","Corp. Bonds",
"Real Estate", "S&P 500","Small Cap"]
for s in strategy:
        if s=="Mean-Variance-Efficient":
                #f
                columnsWeight = []
                for c in constraint:
                        #c
                        print(outputDictionary["Crypto-Mean-Variance-Efficient-
{}".format(c)]["Optimal Weights"])
                        columnsWeight.append(outputDictionary["Crypto-Mean-Variance-
Efficient-{}".format(c)]["Optimal Weights"][0])
        else:
                #constraint does not matter
```

columnsWeight.append(outputDictionary["Crypto-Naive-Constrained"]["Optimal Weights"][0])

print("{}, {}, {}, {}, {}".format(firstRowWeights[0], firstRowWeights[1], firstRowWeights[2], firstRowWeights[3], firstRowWeights[4]))

for row in range(len(columnsWeight[0])):

*print("{}, {:.2f}%, {:.2f}%, {:.2f}%, {:.2f}%".format(firstColWeights[row],100*columnsWeight[0][row], 100*columnsWeight[1][row], 100*columnsWeight[2][row], 100*columnsWeight[3][row]))*

OUTPUT:

Weights, Long-Only, Constrained, Unconstrained, Naive

Binance, 40.89%, 25.00%, 76.57%, 11.11%

Bitcoin, -0.00%, 25.00%, 5.92%, 11.11%

Doll Forex, 0.00%, 25.00%, 14.65%, 11.11%

Ethereum, -0.00%, -23.35%, -39.47%, 11.11%

Gold, 46.37%, 25.00%, 100.00%, 11.11%

Corp. Bonds, -0.00%, -22.67%, -100.00%, 11.11%

Real Estate, 0.00%, 25.00%, -11.94%, 11.11%

S&P 500, 12.74%, 25.00%, 100.00%, 11.11%

Small Cap, -0.00%, -3.98%, -45.74%, 11.11%"""

#printing of tables in CSV Format

#Printing of Traditional portfolios inclusive weight choices

firstRowWeights = ["Weights","Long-Only","Constrained", "Unconstrained", "Naive"]

#This order of assets is maintained overall in the codes thus no problems with indexation

```
#firstColWeights = {"Binance":0 ,"Bitcoin":1, "Doll Forex":2, "Ethereum":3, "Gold":4,"Corp. Bonds":5, "Real Estate":6, "S&P 500":7,"Small Cap":8}

firstColWeights = ["Doll Forex", "Gold","Corp. Bonds", "Real Estate", "S&P 500","Small Cap"]

"""


for s in strategy:

    if s=="Mean-Variance-Efficient":

        #f

        columnsWeight = []

        for c in constraint:

            #c

            print(outputDictionary["Traditional-Mean-Variance-Efficient-{}".format(c)]["Optimal Weights"])

            columnsWeight.append(outputDictionary["Traditional-Mean-Variance-Efficient-{}".format(c)]["Optimal Weights"][0])

    else:

        #constraint does not matter

        columnsWeight.append(outputDictionary["Traditional-Naive-Constrained"]["Optimal Weights"][0])


print("{}, {}, {}, {}, {}".format(firstRowWeights[0], firstRowWeights[1], firstRowWeights[2], firstRowWeights[3], firstRowWeights[4]))

for row in range(len(columnsWeight[0])):

    print("{}, {:.2f}%, {:.2f}%, {:.2f}%, {:.2f}%".format(firstColWeights[row],100*columnsWeight[0][row], 100*columnsWeight[1][row], 100*columnsWeight[2][row], 100*columnsWeight[3][row]))
```

```
#now we make csv table for crypto portfolio
#Portfolio     Long Only     Naive   Unconstrained          Constraint
firstRowInsample = ["Portfolio","Long-Only","Constrained", "Unconstrained", "Naive"]
firstColInsample = ["Average Return", "Average Return Variance", "Standard Dev.", "Mean Sharpe Ratio", "Mean VaR (95%)"]

for s in strategy:
    if s=="Mean-Variance-Efficient":
        #f
        columnsIn = []
        for c in constraint:
            #c
            print(outputDictionary["Crypto-Mean-Variance-Efficient-{}".format(c)]["Promised Returns"])
            meanPromise_Returns = np.mean(outputDictionary["Crypto-Mean-Variance-Efficient-{}".format(c)]["Promised Returns"][0])
            meanPromise_Variance = np.mean(outputDictionary["Crypto-Mean-Variance-Efficient-{}".format(c)]["Promised Variances"][0])
            col = [meanPromise_Returns, meanPromise_Variance , math.sqrt(meanPromise_Variance), meanPromise_Returns/math.sqrt(meanPromise_Variance), np.mean(outputDictionary["Crypto-Mean-Variance-Efficient-{}".format(c)]["Mean Predicted Var"])]

            columnsIn.append(col)
```

```
    else:
        #constraint does not matter
        meanPromise_Returns = np.mean(outputDictionary["Crypto-Naive-Constrained"]["Promised Returns"][0])
        meanPromise_Variance = np.mean(outputDictionary["Crypto-Naive-Constrained"]["Promised Variances"][0])
        col = [meanPromise_Returns, meanPromise_Variance , math.sqrt(meanPromise_Variance), meanPromise_Returns/math.sqrt(meanPromise_Variance), np.mean(outputDictionary["Crypto-Naive-Constrained"]["Mean Predicted Var"])]

        columnsIn.append(col)

print("{}, {}, {}, {}, {}".format(firstRowInsample[0], firstRowInsample[1], firstRowInsample[2], firstRowInsample[3], firstRowInsample[4]))
for row in range(len(columnsIn[0])):
    print("{}, {:.2f}%, {:.2f}%, {:.2f}%, {:.2f}%".format(firstColInsample[row],100*columnsIn[0][row], 100*columnsIn[1][row], 100*columnsIn[2][row], 100*columnsIn[3][row]))
print("Traditional")
#now we make csv table for traditional portfolio
#Portfolio	Long Only	Naive	Unconstrained		Constraint
firstRowInsample = ["Portfolio","Long-Only","Constrained", "Unconstrained", "Naive"]
firstColInsample = ["Average Return", "Average Return Variance", "Standard Dev.", "Mean Sharpe Ratio", "Mean VaR (95%)"]

for s in strategy:
```

if s=="Mean-Variance-Efficient":

#f

columnsIn = []

for c in constraint:

#c

print(outputDictionary["Traditional-Mean-Variance-Efficient-{}".format(c)]["Promised Returns"])

meanPromise_Returns = np.mean(outputDictionary["Traditional-Mean-Variance-Efficient-{}".format(c)]["Promised Returns"][0])

meanPromise_Variance = np.mean(outputDictionary["Traditional-Mean-Variance-Efficient-{}".format(c)]["Promised Variances"][0])

col = [meanPromise_Returns, meanPromise_Variance , math.sqrt(meanPromise_Variance), meanPromise_Returns/math.sqrt(meanPromise_Variance), np.mean(outputDictionary["Traditional-Mean-Variance-Efficient-{}".format(c)]["Mean Predicted Var"])]

columnsIn.append(col)

else:

#constraint does not matter

meanPromise_Returns = np.mean(outputDictionary["Traditional-Naive-Constrained"]["Promised Returns"][0])

meanPromise_Variance = np.mean(outputDictionary["Traditional-Naive-Constrained"]["Promised Variances"][0])

col = [meanPromise_Returns, meanPromise_Variance , math.sqrt(meanPromise_Variance), meanPromise_Returns/math.sqrt(meanPromise_Variance), np.mean(outputDictionary["Traditional-Naive-Constrained"]["Mean Predicted Var"])]

```
        columnsIn.append(col)

print("{}, {}, {}, {}, {}".format(firstRowInsample[0], firstRowInsample[1], firstRowInsample[2], firstRowInsample[3], firstRowInsample[4]))

for row in range(len(columnsIn[0])):

    print("{}, {:.2f}%, {:.2f}%, {:.2f}%, {:.2f}%".format(firstColInsample[row],100*columnsIn[0][row], 100*columnsIn[1][row], 100*columnsIn[2][row], 100*columnsIn[3][row]))
```

Portfolio, Long-Only, Constrained, Unconstrained, Naive

Average Return, 2.56%, 2.01%, 4.97%, 1.10%

Average Return Variance, 1.10%, 0.46%, 2.00%, 0.77%

Standard Dev., 10.51%, 6.75%, 14.16%, 8.77%

Mean Sharpe Ratio, 24.32%, 29.80%, 35.11%, 12.51%

Mean VaR (95%), -2.80%, -2.34%, -3.60%, -4.80%

Traditional

Portfolio, Long-Only, Constrained, Unconstrained, Naive

Average Return, 0.59%, 0.44%, 1.27%, 0.34%

Average Return Variance, 0.10%, 0.07%, 0.22%, 0.11%

Standard Dev., 3.24%, 2.63%, 4.72%, 3.31%

Mean Sharpe Ratio, 18.23%, 16.73%, 26.98%, 10.22%

Mean VaR (95%), -0.81%, -0.78%, -1.90%, -1.03% """

```
realizedReturnsCrypto = {}

realizedReturnsTrad = {}

#now we make csv table for crypto portfolio

#Portfolio	Long Only	Naive	Unconstrained		Constraint

firstRowOutsample = ["Portfolio","Long-Only","Constrained", "Unconstrained", "Naive"]

firstColOutsample = ["Average Return", "Average Return Variance", "Standard Dev.", "Mean Sharpe Ratio", "Mean Violation"]

for s in strategy:

	if s=="Mean-Variance-Efficient":

		#f

		columnsOut = []

		for c in constraint:

			#c

			#print(outputDictionary["Crypto-Mean-Variance-Efficient-{}".format(c)]["Promised Returns"])

			print(outputDictionary["Crypto-Mean-Variance-Efficient-{}".format(c)]["Actual Returns"])

			realizedReturnsCrypto["{}-{}".format(s,c)] = outputDictionary["Crypto-Mean-Variance-Efficient-{}".format(c)]["Actual Returns"]

			actualReturns = np.mean(outputDictionary["Crypto-Mean-Variance-Efficient-{}".format(c)]["Actual Returns"])

			actual_Variance = np.var(outputDictionary["Crypto-Mean-Variance-Efficient-{}".format(c)]["Actual Returns"])

			col = [actualReturns, actual_Variance , math.sqrt(actual_Variance), actualReturns/math.sqrt(actual_Variance), np.mean(outputDictionary["Crypto-Mean-Variance-Efficient-{}".format(c)]["Mean Violation"])]
```

```
                columnsOut.append(col)
        else:
            #constraint does not matter

            actualReturns = np.mean(outputDictionary["Crypto-Naive-Constrained"]["Actual Returns"])
            realizedReturnsCrypto["Naive-Constrained"] = outputDictionary["Crypto-Naive-Constrained"]["Actual Returns"]
            actual_Variance = np.var(outputDictionary["Crypto-Naive-Constrained"]["Actual Returns"])
            col = [actualReturns, actual_Variance , math.sqrt(actual_Variance), actualReturns/math.sqrt(actual_Variance), np.mean(outputDictionary["Crypto-Naive-Constrained"]["Mean Violation"])]

            columnsOut.append(col)

print("{}, {}, {}, {}, {}".format(firstRowOutsample[0], firstRowOutsample[1], firstRowOutsample[2], firstRowOutsample[3], firstRowOutsample[4]))
for row in range(len(columnsOut[0])):
    print("{}, {:.2f}%, {:.2f}%, {:.2f}%, {:.2f}%".format(firstColOutsample[row],100*columnsOut[0][row], 100*columnsOut[1][row], 100*columnsOut[2][row], 100*columnsOut[3][row]))
print("Traditional")
#Traditional
for s in strategy:
    if s=="Mean-Variance-Efficient":
        #f
```

```
columnsOut = []
for c in constraint:
    #c
    #print(outputDictionary["Crypto-Mean-Variance-Efficient-{}".format(c)]["Promised Returns"])
    #print(outputDictionary["Traditional-Mean-Variance-Efficient-{}".format(c)]["Actual Returns"])
    realizedReturnsTrad["{}-{}".format(s,c)] = outputDictionary["Traditional-Mean-Variance-Efficient-{}".format(c)]["Actual Returns"]
    actualReturns = np.mean(outputDictionary["Traditional-Mean-Variance-Efficient-{}".format(c)]["Actual Returns"])
    actual_Variance = np.var(outputDictionary["Traditional-Mean-Variance-Efficient-{}".format(c)]["Actual Returns"])
    col = [actualReturns, actual_Variance , math.sqrt(actual_Variance), actualReturns/math.sqrt(actual_Variance), np.mean(outputDictionary["Traditional-Mean-Variance-Efficient-{}".format(c)]["Mean Violation"])]

    columnsOut.append(col)
else:
    #constraint does not matter

    actualReturns = np.mean(outputDictionary["Traditional-Naive-Constrained"]["Actual Returns"])
    realizedReturnsTrad["Naive-Constrained"] = outputDictionary["Traditional-Naive-Constrained"]["Actual Returns"]
    actual_Variance = np.var(outputDictionary["Traditional-Naive-Constrained"]["Actual Returns"])
```

col = [actualReturns, actual_Variance , math.sqrt(actual_Variance), actualReturns/math.sqrt(actual_Variance), np.mean(outputDictionary["Traditional-Naive-Constrained"]["Mean Violation"])]

columnsOut.append(col)

print("{}, {}, {}, {}, {}".format(firstRowOutsample[0], firstRowOutsample[1], firstRowOutsample[2], firstRowOutsample[3], firstRowOutsample[4]))

for row in range(len(columnsOut[0])):

*print("{}, {:.2f}%, {:.2f}%, {:.2f}%, {:.2f}%".format(firstColOutsample[row],100*columnsOut[0][row], 100*columnsOut[1][row], 100*columnsOut[2][row], 100*columnsOut[3][row]))"""*

Portfolio, Long-Only, Constrained, Unconstrained, Naive

Average Return, 3.06%, 2.17%, 7.13%, 0.90%

Average Return Variance, 1.75%, 1.11%, 5.19%, 0.96%

Standard Dev., 13.22%, 10.56%, 22.78%, 9.80%

Mean Sharpe Ratio, 23.11%, 20.60%, 31.32%, 9.17%

Mean Violation, 34.62%, 30.77%, 23.08%, 34.62%

Traditional

Portfolio, Long-Only, Constrained, Unconstrained, Naive

Average Return, 0.22%, 0.30%, 2.46%, -0.03%

Average Return Variance, 0.14%, 0.10%, 0.27%, 0.12%

Standard Dev., 3.68%, 3.10%, 5.20%, 3.44%

Mean Sharpe Ratio, 5.85%, 9.67%, 47.30%, -0.76%

Mean Violation, 38.46%, 34.62%, 23.08%, 30.77%

"""

firstRowTest = ["Portfolio","Long-Only","Constrained", "Unconstrained", "Naive"]

print("{}, {}, {}, {}, {}".format(firstRowTest[0], firstRowTest[1], firstRowTest[2], firstRowTest[3], firstRowTest[4]))

long_test = Jobson_Korkie(realizedReturnsTrad["Mean-Variance-Efficient-Long-Only"], realizedReturnsCrypto["Mean-Variance-Efficient-Long-Only"])

uncon_test = Jobson_Korkie(realizedReturnsTrad["Mean-Variance-Efficient-Unconstrained"], realizedReturnsCrypto["Mean-Variance-Efficient-Unconstrained"])

constr_test = Jobson_Korkie(realizedReturnsTrad["Mean-Variance-Efficient-Constrained"], realizedReturnsCrypto["Mean-Variance-Efficient-Constrained"])

naive_test = Jobson_Korkie(realizedReturnsTrad["Mean-Variance-Efficient-Long-Only"], realizedReturnsCrypto["Naive-Constrained"])

print("Z-Score, {:.3f}, {:.3f}, {:.3f}, {:.3f}".format(long_test, constr_test, uncon_test, naive_test))

"""

Portfolio, Long-Only, Constrained, Unconstrained, Naive

Z-Score, 0.679, 0.458, 0.579, 0.158"""

Portfolio Execution

```
import pandas as pd
import numpy as np
import cvxpy as cp
from arch import arch_model
import random
```

```
import math



from sklearn.covariance import EmpiricalCovariance
class portExecute():
	def __init__(self, returnsDF, windowSize):
		self.returnsDF = returnsDF
		self.windowSize = windowSize

		self.assetReturn = []
		random.seed(0)



		for col in  self.returnsDF.columns:
			if col != 'Date':
				oneAsset = self.returnsDF[col].tolist()
				self.assetReturn.append(oneAsset)

		self.assetNames = np.array(returnsDF.columns)[1:]
		self.traditional_Assets = [2,4,5,6,7,8]

		#convention index 0 = Binance ..... index

		self.returnsArray = np.array(self.assetReturn)
		self.n_assets = len(self.returnsArray)
		self.n_periods = len(self.returnsArray[0])



	#This function gives the monthly variance of a portfolio based on given cov matrix
and chosen weights
	def getPortfolio_variance(self, weights, covParameter):
		return np.dot(weights, np.dot(covParameter, weights))
```

```
#This function gives a prediction for VaR based on the FHS GARCH(1,1) algorithm presented in my thesis
def fhsVaR_estimation(self, weights, estimationSample, nSamples, confAlpha):
    #Step 1: Generate the portfolio estimation sample portfolio returns
    trainingReturns = []
    for t in range(self.windowSize):
        monthReturns = [estimationSample[k][t]*weights[k] for k in range(len(weights))]
        trainingReturns.append(np.sum(monthReturns))

    #Step 2: Fit a GARCH(1,1) model to the training portfolio returns
    model = arch_model(trainingReturns, mean='Zero', vol='GARCH', p=1, q=1)
    # fit the  model
    model_fit = model.fit()
    print(model_fit.summary())
    #print(dir(model_fit))
    #print(model_fit.conditional_volatility)
    #print(model_fit._params)
    #Through visual inspection of the three lines above, we establish that model_fit._params = [omega, alpha, beta]

    fittedVariances = model_fit.conditional_volatility



    #Step 3: Standardize returns
    standardReturns_Training = [ trainingReturns[t]/math.sqrt(fittedVariances[t]) for t in range(self.windowSize)]

    #Step 4: Predict volatility of period right outside training window
    sigmaSpecial = math.sqrt(model_fit.forecast(horizon=1, reindex=False).variance.values[0][0])



    estimatesVaR = []
```

```
        #set the seed for reproducibility
        #Step 8
        for k in range(nSamples):
            #Step 5: sample through bootstrapping
            bootSample = random.choices(standardReturns_Training, k=300)

            #Step 6 find (1-condAlpha)% percentile return of boothstrap
            zSpecial = np.percentile(bootSample,100-confAlpha , method =
"closest_observation")

            #step 7 store new VaR estimate
            estimatesVaR.append(sigmaSpecial*zSpecial)
        return np.mean(estimatesVaR)






    def executeStrategy(self, asset, strategy, constraint):
        allWeights = []
        #actual return and variances
        allPort_Returns = []
        #allPort_Variances = []

        allPort_PromisedReturns = []
        allPort_PromisedVariances = []

        #95% VaR
        allPredicted_VaR = []

        allViolations = []
```

```
		#determine number of portfolios over the sample
		#we subtract the windowSize as the first months are used to estimate the first
portfolio
		nrPortfolios = self.n_periods - self.windowSize


		for i in range(nrPortfolios):
			print("Portfolio")
			print(i)
			#first define the observations for the estimation sample

			#first index of rolling windon INCLUSIVE
			startInd = i

			lastInd = startInd + (self.windowSize-1)


			#Define Holding period actual returns, one month!!, right after end of
moving window
			holdingPeriod = lastInd + 1


			holdingStart = lastInd + 1
			#print("holding start")
			#print(holdingStart)
			holdingEnd = holdingStart + self.windowSize - 1
			#print("holding end")
			#print(holdingEnd)



			#Then define the observations for the execution of the strategy

			#Asset Class
```

```
                    if asset=="Traditional":
                        #restrict focus to traditional assets
                        self.assets_Available = [self.returnsArray[i] for i in
self.traditional_Assets]
                    else:
                        #all assets are available
                        self.assets_Available = [self.returnsArray[i] for i in
range(len(self.returnsArray))]



                    self.estimationWindow_Assets = [[self.assets_Available[i][k] for k in
range(startInd,lastInd + 1)] for i in range(len(self.assets_Available)) ]

                    estimated_MeanReturns =  np.array([np.mean(self.assets_Available[i])
for i in range(len(self.assets_Available))])

                    #we take the transpose of the estimationwindow_assets because
empiricalcovarivance takes data in the format nObs x nFeatures
                    #np.array(a).T.tolist()
                    estimated_Covariance = EmpiricalCovariance(store_precision=True,
assume_centered=False).fit(np.array(self.estimationWindow_Assets).T.tolist() ).covariance_
                    #print("estimated Covariance")
                    #print(estimated_Covariance)

                    self.holdingActual_Returns = [self.assets_Available[i][holdingPeriod]
for i in range(len(self.assets_Available)) ]
                    #Determine the number of available assets and use this to generate the
number of decision variables
                    weights = cp.Variable(len(self.assets_Available))

                    if strategy == "Mean-Variance-Efficient":
                        #mean efficient optimization
                        if constraint == "Long-Only":
                            #long only constraint
```

```
                    constraints = [cp.sum(weights) == 1, weights >= 0]
                elif constraint == "Constrained":
                    #Constrained
                    constraints = [cp.sum(weights) == 1, weights >= -0.25,
weights <= 0.25]
                else:
                    #unconstrained
                    constraints = [cp.sum(weights) == 1,  weights >= -1,
weights <= 1]
                objective =
cp.Maximize(cp.sum(weights*estimated_MeanReturns) - weights @ estimated_Covariance
@ weights)
            else:
                #naive diversification
                naiveWeight = 1/len(self.assets_Available)
                constraints = [cp.sum(weights) == 1, weights == naiveWeight]
                #objective in case of naive diversification will automatically
determined, but we add the following code to make it consistent
                #with the rest of the code
                objective = cp.Maximize(cp.sum(weights
*estimated_MeanReturns))

            problem = cp.Problem(objective, constraints)

            #computing the actual performance of our strategy in the current
holding period

            problem.solve()
            optimal_weights = weights.value
            allWeights.append(optimal_weights)
            allPort_Returns.append(np.dot(optimal_weights,
self.holdingActual_Returns))
            #holdingCovariance = EmpiricalCovariance(store_precision=True,
assume_centered=False).fit(np.array(self.holdingActual_Returns).T.tolist() ).covariance_
```

```
                #allPort_Variances.append(self.getPortfolio_variance(optimal_weights,
holdingCovariance))

                #print(len(self.holdingActual_Returns))
                #print(self.holdingActual_Returns)

                #print(optimal_weights)
                #print(estimated_MeanReturns)

    allPort_PromisedReturns.append(np.dot(optimal_weights,estimated_MeanReturns))

    allPort_PromisedVariances.append(np.dot(optimal_weights,np.dot(estimated_Covaria
nce,optimal_weights)))

                predictionVaR = self.fhsVaR_estimation(optimal_weights,
self.estimationWindow_Assets, 1000, 95)

                allPredicted_VaR.append(predictionVaR)

                violationNow = 0
                actualReturn = np.dot(optimal_weights, self.holdingActual_Returns)
                if actualReturn < predictionVaR:
                    violationNow = 1
                allViolations.append(violationNow)


            """
            print("All weights")
            print(len(allWeights))
            print(allWeights)

            print("All Actual Returns")
            print(len(allPort_Returns))
            print(allPort_Returns)
```

```
        print("All Promised Returns")
        print(allPort_PromisedReturns)

        print("All Promised Variances")
        print(allPort_PromisedVariances)
        """


        resultDictionary = {"Assets": asset , "Strategy":strategy,
"Constraint":constraint, "Optimal Weights": allWeights, "Actual Returns": allPort_Returns,
"Promised Returns": allPort_PromisedReturns, "Promised Variances":
allPort_PromisedVariances, "Mean Predicted Var": np.mean(allPredicted_VaR), "Mean
Violation":np.mean(allViolations)}
        print("violation mean")
        print(np.mean(allViolations))
        return resultDictionary
```