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Portfolio diversification effect of Bitcoin before and during the COVID-19 pandemic

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ABSTRACT

In this thesis, I study whether Bitcoin provides diversification benefits to a well-diversified portfolio of the American Investor. This study has been done in two scenarios: before and during the Covid-19 pandemic. I use six assets, including Bitcoin, and compare two portfolios in each scenario: one portfolio without Bitcoin and one including Bitcoin. Within each scenario, two portfolios are made: equally weighted and optimally risky. I use the Mean-Variance and Conditional Value-at-Risk approach to check both up- and downside potential of Bitcoin. I find that the expected return and Sharpe ratio of the Bitcoin included portfolios is higher than the Bitcoin excluded portfolio. The Conditional Value-at-Risk only improves with an optimally risky portfolio during the pandemic. Although the risk characteristics do not improve with Bitcoin, the overall risk-return characteristics improve with the inclusion of Bitcoin.

Keywords: Bitcoin, Portfolio Diversifcation, Mean-Variance, Conditional Value-at-Risk, COVID-19 pandemic

JEL codes: G01, G11, G14 http://www.aeaweb.org/journal/jel_class_system.html

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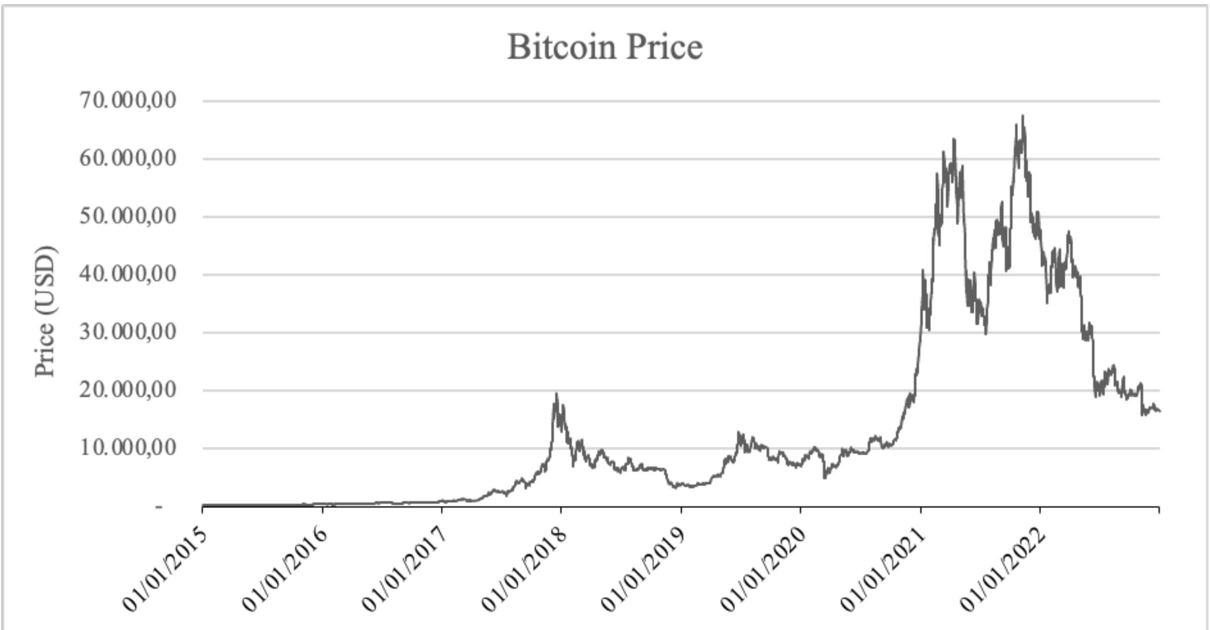
CHAPTER 1 Introduction

Whereas putting multiple different assets in a portfolio is a staple in contemporary investors, this idea is not even eighty years old. With Markowitz (1952) introducing his Modern Portfolio Theory, an emphasis is put on putting many different assets into a portfolio to maximize their potential return while mitigating risk. Institutional and private investors are using every method and asset they find to make their portfolio as profitable as they can, while minimizing risk. The traditional portfolio diversifies with traditional assets such as stocks, bonds, gold, and real estate. With the introduction of the COVID-19 virus, a pandemic was born, and the economy contracted worldwide, putting many countries into recessions and bankrupting companies. Stock prices plummeted but the cryptocurrency market went the opposite way, conveying a new pathway to diversification. The most influential coin of them all, Bitcoin, hits an all-time high price of almost $69.000, almost doubling since the start of the COVID-19 outbreak. Despite Bitcoin's current price of $27200, ten years ago it had a price of $20. In Figure 1, the price development of Bitcoin is shown, from 2015 to 2022.

Figure 1

Bitcoin Price in US Dollars



Note. Price of Bitcoin has been taken over the period January 1, 2015, to December 31, 2022.

This sharp increase, also during the pandemic instigated much attention from the institutional and private investors. Bitcoin, published by Satoshi Nakamoto (2008), is meant to be a decentralized digital currency via the Blockchain that can be globally used. Mid-2010s, the thought of Bitcoin rising to all-time highs was a narrative that could not be silenced. Whereas cryptocurrencies were unheard of twenty years ago, it has become a debated asset for the modern portfolio regarding its diversification role and ability to endure market stress. Current literature has not come to a definitive answer to this debate, whether Bitcoin provides diversification effects to the average portfolio. Adding a period of market stress, the pandemic, into this matter has institutional and private investors interested about Bitcoin as a diversification tool. Therefore, the main research question is:

"Does Bitcoin contain diversification effects for the average U.S. portfolio before and during the COVID-19 pandemic?"

Briere et al. (2013) provided a first provisory look on the economics of Bitcoin. The correlations found put Bitcoin into a safe-haven category. Adding to that, they found fiercely improved risk-return characteristics of a well-diversified portfolio by adding a small proportion of Bitcoin. However, this was data from early-stage behavior and does not provide a reliable view for future fulfilment. Eisl et al. (2015) examined the impact of Bitcoin on a well-diversified investment portfolio, using a Conditional Value-at-Risk framework. They found that Bitcoin is included even in well-diversified portfolios. The inclusion of Bitcoin increases the risk-return characteristics. Corbet et al. (2018) explored the relationship between cryptocurrencies and other financial assets. They found a role for cryptocurrencies in an investment portfolio due to low correlation with the main financial assets. However, they point out a difficult hedging position for the cryptocurrency market due to idiosyncratic risk. Klein et al. (2018) where they made a comparison of properties between a traditional asset in an investment portfolio, gold, and Bitcoin. This paper found a positive correlation between Bitcoin and downward markets. Adding to that, no evidence is found for hedging properties for Bitcoin. A limitation which many studies stumbled upon is the immature market in which Bitcoin finds itself. Corbet et al. (2019) analyzed cryptocurrencies as a financial asset. In this research, a list is identified multiple gaps in the literature regarding cryptocurrencies. Among those points is 'evaluate the benefits of cryptocurrencies as an asset class part of a diversified portfolio.' This research adds to current literature by evaluating the effect of Bitcoin on both sides of risk, up- and downside risk. The mean-variance approach measures the risk-return characteristics and focuses mainly on the optimization of total risk-return. Adding the Sharpe and Sortino ratio, volatility is measured and compared in both total volatility and downside volatility. To supplement this global view of risk, a CVaR approach checks for the downside risk of the portfolio. Implementing both methods into the paper creates a comprehensive view on Bitcoin its diversification effect. Furthermore, this research supplements literature on Bitcoin's characteristics during periods of market stress. By studying the characteristics of a Bitcoin included portfolio within different time periods, an in-depth view of its diversification properties.

This paper uses major asset classes from the United States. These asset classes are Bitcoin, equities, bonds, gold and silver, real estate, and oil. The analysis of the United States is chosen because they have the highest trading volume on online exchanges (Countries with most Bitcoin | Statista, 2023). Adding to that, involving multiple countries makes this research too broad. For all asset classes above, data is found on Yahoo Finance. The data that will be used from these indices is the adjusted close. Weekly data will be taken over the period January 1, 2018, to March 1, 2023. Weekly data is chosen due to availability. Daily data is available for cryptocurrencies, whereas the other markets are closed in the weekends. The methodology will start with a correlation analysis to calculate the correlation between the cryptocurrencies and the other assets used in the dataset. This analysis will be done before and during the COVID-19 pandemic. A high correlation suggests there are little diversification benefits. Next, this paper uses mean-variance optimization to maximize the portfolio its risk-adjusted returns or to minimize the overall risk. First, the equally weighted portfolio will be constructed and analyzed, in which each asset has the same weight. Secondly, Bitcoin is added to a new portfolio and new weights will be determined. This paper compares the characteristics of the two portfolios and analyzes the overall risk-return trade-off change in portfolios. To check for the statistical significance, this research performs a t-test on the difference in mean returns. This determines if the addition of cryptocurrency is statistically significant. Adding to that, evaluating measures as the Sharpe ratio, and comparing them for both portfolios can also support Bitcoin. An increase in Sharpe ratio suggests an improvement in the risk-adjusted performance of the portfolio with Bitcoin added. Adding to the element of both sides of risk, this research wants to study both the up- and downside potential of Bitcoin. This paper studies the downside potential by using a Conditional Value-at-Risk (CVaR) approach. Firstly, CVaR is chosen before VaR due to the non-normal distribution of returns for Bitcoin. Secondly, this research considers the COVID-19 pandemic which is an extreme event. VaR cannot fully capture the tail risk associated with a volatile asset, like Bitcoin. The CVaR approach is done four times, twice for each period. The result from the average portfolio is compared to the Bitcoin included portfolio. If the value of the CVaR decreases with the addition of Bitcoin, it suggests a decrease in downside risk and thus potential diversification benefits.

This paper predicts that Bitcoin will have a low correlation with other assets in both periods. However, due to the market stress of the pandemic, the correlation in this period rises. Furthermore, Bitcoin will have an improved effect on the risk-return characteristics of the average portfolio, using the mean-variance approach. The Sharpe ratio improves for the Bitcoin included portfolios. For the CVaR approach, there is also an improved effect expected within the portfolio with Bitcoin, regarding the downside potential. These predictions hold for both periods, before and during the COVID-19 pandemic.

Chapter two elaborates literature regarding this subject; chapter three; chapter four explains the methodology this paper uses; chapter five interprets the results; chapter six concludes this paper.

CHAPTER 2 Theoretical Framework

2.1 Portfolio management

The article of Harry Markowitz (1952) forms the foundation for portfolio management, referred to as Modern Portfolio Theory. Although investors were known with the benefits of diversifying in assets, Markowitz developed a model that quantitatively analysed these benefits (Witt & Dobbins, 1979). Diversification is dividing your investment into multiple assets, instead of throwing all your eggs into one basket. Every asset has a trade-off between return and risk. The idea is to maximize the returns, for a given level of risk, while minimizing risk, for a given level of return. Multiple portfolios will be created with the highest expected return for a certain amount of risk. All these optimal portfolios together form the efficient frontier. When a portfolio finds itself below this efficient frontier, its risk-return trade-off is non-optimal. Portfolios above this line are not achievable. Risk in this model is expressed as the portfolio standard deviation. When picking assets, they must be not perfectly correlated to be considered for the portfolio. The lower the correlation between two assets, the less they move in the same direction. After Markowitz's Modern Portfolio Theory other theories are developed, such as CAPM by Treynor (1962), Sharpe (1964), Lintner (1965) and Mossin (1966) separately. Even though this model is developed in the 1950s, this model is the base of portfolio management and is still being used in recent articles and books, such as Avellaneda and Lee (2010) and Fabozzi et al. (2006). Another measure for risk management is the Value-at-Risk approach which became popular during the 1990's. Value at Risk (VaR) is a statistical measurement of portfolio losses that could happen, expressed in a single, summary number (Linsmeier & Pearson, 1996). Precisely, it measures the losses during "normal" market conditions over a certain period with an accompanying confidence interval. Whereas this measure has been adopted worldwide into the portfolio management scene, there are disadvantages to this method. These disadvantages are (1) its non-convexity, (2) the lack of sub-additivity, and (3) the focus on a single quantile. Dealing with these disadvantages of VaR, a solution is found in the Conditional Value at Risk (CVaR). Rockafellar & Uryasev (2000) invented this measurement and is considered more suitable as a measure of risk. This paper uses both the risk-return trade-off analysis and the CVaR discussed above. Whereas the risk-return trade-off analysis provides a view on the upside potential, the CVaR pictures the downside risk of the portfolio. Examining both will provide a holistic view on the portfolio performance. Both these methods will be further explained in the Methodology section.

2.2 Bitcoin

Bitcoin is a decentralized digital coin for electronic transactions, without a trusted third party to process the payment. These transactions are publicly announced and made into a chain of timestamps. This chain of so called 'nodes' is the Blockchain (Nakamoto, 2008). It was designed to be independent of banks, government, and other central institutions. This is a solution to the double-spending problem using a peer-to-peer network. The transactions are recorded into an ever-going chain of transactions, making a format that cannot be directly changed. All these transactions are available to the public. The network that Bitcoin functions on is simply structured and the nodes work with little coordination. Bitcoin is a virtual currency and should not be treated the same as 'electronic money' that sits on your bank account. Yermack (2015) discusses the properties of Bitcoin and if it is a real currency. Bitcoins faces challenges in the three properties of a currency: (1) a medium of exchange, (2) unit of account and (3) a store of value. In terms of being a medium of exchange, only a small proportion of consumers use it to purchase products. Therefore, Bitcoin transactions appear to be very rare, and its usage is dominated by speculative investors. An obstacle of this point is the limited number of Bitcoins and its low liquidity with big bid-ask spreads. In terms of the unit of account, Bitcoin its volatility makes it nearly impossible to compare products on a day-to-day basis. Stores must recalculate prices daily to sell the same product for the same price, which is costly to the store. Adding to that, multiple exchanges trade Bitcoin on a different price, which violated the law of one price. Maybe the most overlooked argument, according to Yermack, is the relatively high cost of Bitcoin compared to other currencies. This causes products to have multiple decimal points and makes it difficult for accounting software to accommodate. Regarding the store of value, customers must make costs to evaluate the security plus hold and secure it carefully. Furthermore, Bitcoin its volatility does not compare with other currencies to let it function as a store of value. The volatile price of Bitcoin classifies it more as a speculative asset than a real currency. When the prices become more stable, Bitcoin will be more reliable for property 2 and 3. Baur et al. (2018) analyses if Bitcoin's functions as a medium of exchange or just a speculative asset. They conclude that Bitcoin is used for investment purposes rather than being used for transactions. Adding to that, Bitcoin's size is relatively small compared to other assets and is not a direct risk for current financial stability. However, a global acceptance of Bitcoin could affect the role and value of fiat currencies and twist the function of monetary policies, due to Bitcoin's decentralized nature. Dyhrberg (2016) examines Bitcoin against gold and the dollar via a GARCH volatility analysis and compares their financial capabilities. Bitcoin is like the dollar and gold regarding hedging characteristics and benefits as a medium of exchange. Dyhrberg claims that Bitcoin finds itself between a currency and a commodity. In a more recent paper, Corbet, Meegan, et al. (2018) analyse the relationship between cryptocurrencies, not just Bitcoin, and other financial assets. An isolation from other assets has been found and is interesting for investors with short term goals for their portfolio due to its diversification effects. Above papers point out that Bitcoin's role tends to follow the speculative route and not the role as a stable currency, despite its advantages as a medium of exchange. However, it has interesting characteristics for the investor and the immaturity as an 'asset' gives Bitcoin potential in the world of asset classes.

2.3 Portfolio management with Bitcoin

Dyhrberg (2016) states that Bitcoin is interesting to risk-averse investors awaiting bad news due to its diversification effects. It combines benefits from both commodities and currencies that makes this a convenient tool for portfolio management. Brière et al. (2013) analyse Bitcoin from the view of an American investor with a diversified portfolio over a three-year period, 2010-2013. Bitcoin shows noticeable features, with its extremely high return and volatility. An addition of Bitcoin with minimal weight into the already well-diversified portfolio improves the risk-return characteristics exceptionally. Gasser et al. (2015) investigate the effect of Bitcoin on an already well-diversified portfolio, using a Conditional Value-at-Risk framework. This paper states that the addition of Bitcoin in these portfolios is a good inclusion. The CVaR itself increases but the returns offset this and bettered the risk-return ratio. Carpenter (2016) uses a mean-variance methodology to investigate Bitcoin its diversification effect in an efficient portfolio. He has shown that Bitcoin seems an attractive investment, but this view may be hindered by a speculative bubble that happened within the used period. Aggarwal et al. (2018) studied Bitcoin in a mean-CVaR context on the Indian market and found that portfolios containing Bitcoin have better risk-adjusted return. However, this paper points out that Bitcoin may not be suited for risk-averse investors due to its high volatility.

2.4 Portfolio management during the COVID-19 pandemic

Chkili et al. (2021) researches the hedging properties of Bitcoin to Islamic stock markets pre-and during the COVID-19 pandemic. Bitcoin is suggested as a safe haven due to its low, and usually negative, dynamic correlation. In addition, Bitcoin its diversification effects are consistent and are enhanced during unstable periods. However, this paper uses a period till May 2020 and only covered the start of the pandemic. X. Huang et al. (2022) examine diversification effects of multiple cryptocurrencies, pre and post Covid-19. They find the same diversification effects in both stable and unstable environments. Combined with that, a lower risk aversion from the investor leads to more benefits from cryptocurrencies as a diversifier. Y. Huang et al. (2021) investigate the potential for Bitcoin to function as a safe haven against negative moments in stock and bond prices in multiple economies during the COVID-19 bear market. Apart from the United States, Bitcoin contributes to diversification benefits both within and across borders. Conlon and McGee (2020) analyze the safe haven characteristics of Bitcoin during the Covid-19 bear market. Bitcoin does not act as a safe haven. When compared to the S&P 500, distributing a small weight to Bitcoin increases the portfolio downside risk greatly. Current literature clashes with their outcomes on Bitcoin’s role as a diversifier in times of crisis. This paper expands the knowledge about this discussion.

2.5 Hypotheses

The following hypotheses will be formulated to support our main research question:

(1) H0: Bitcoin provides no diversification effects looking at the upside potential in the portfolio without and with Bitcoin of the average American investor before and during the Covid-19 pandemic.

HA: There are diversification effects looking at the upside potential in the portfolio without and with Bitcoin of the average American investor before and during the Covid-19 pandemic.

Regarding the Conditional Value-at-Risk approach, the following hypotheses are formulated to support the research question:

(2) H0: Bitcoin does not improve downside risk in the average portfolio of the American investor before and during the Covid-19 pandemic.

HA: Bitcoin improves downside risk in in the average portfolio of the American investor before and during the Covid-19 pandemic.

CHAPTER 3 Data

This paper uses six major asset classes from the United States. These asset classes are cryptocurrencies (Bitcoin), equities, bonds, gold and silver, real estate, and oil. Bitcoin (BTC-USD)[1] is picked as a representative for the cryptocurrency market because it has the highest market capitalization of the currencies: Bitcoin leads with a market capitalization of 520 billion dollars, with Ethereum following Bitcoin's footsteps with a market capitalization of 205 billion dollars. Representing the equities market is the Standard & Poor's 500 Index (^GSPC). This index is considered as one of the best indicators of important American equities' performance and for the stock market generally. Bonds are considered in the portfolio due to its reduction of volatility compared to other asset classes. The iShares Core U.S. Aggregate Bond ETF (AGG) is the index chosen for the bond market. This index is used as a benchmark for the bond market by bond traders and mutual funds managers. A more alternative asset class in this paper are the costly metals gold and silver. Gold and silver are used in this portfolio because of its low correlation with other major asset classes (Morgan Stanley, 2022). Moreover, current literature has made many comparisons with Bitcoin and gold. The index used in this paper is the PHLX GOLD and SILVER SECTOR I (^XAU). Another alternative asset class used in the portfolio is real estate. Reasons for investing in real estate are diversification and a regular recurring income for the investor. The representative for this asset class is Vanguard Real Estate Index Fund (VNQ). The last asset in our portfolio is oil. Oil can also be seen as a portfolio diversifier. In addition to this, oil can be used as a hedge against inflation (Arouri & Nguyen, 2010). The index that stands for the American oil market is the United States Oil Fund, LP (USO). The analysis of the United States is chosen because this country contains the highest trading volume of Bitcoin on online exchanges (Countries with most Bitcoin | Statista, 2023). Furthermore, involving multiple countries makes this research too broad. This paper wants to research a specific market and therefore only American indices are examined. Weekly data will be taken over the period January 1, 2015, to December 29, 2022. Weekly data is chosen due to availability. Daily data is available for cryptocurrencies, whereas the other markets are closed in the weekends. The year 2015 was chosen as a starting point because this period depicts a good length for the development of Bitcoin with its significant movements in price. Adding to that, the pioneer of the cryptocurrencies experienced a crash in 2011 and 2014-2015 before the current starting point (Baur & Dimpfl, 2017). We want to examine the effect of COVID-19 on the performance, whereas picking a starting point further back in time contains multiple crashes. The usage of multiple crashes in the dataset could provide a skewed view on its performance. The third reason for this starting point is the availability of data, mainly Bitcoin's data. The accessibility of data from 2015 onwards is the main reason. The data were obtained from the adjusted close price, found on Yahoo Finance. This price is generally used when examining the returns or performance. In

[1] The words in brackets after the asset classes are the tickers found on Yahoo Finance.

this paper, we separate our dataset into two periods: before and during the Covid-19 pandemic. Before is from January 1, 2015, to 30 January, 2020; during is from 30 January, 2020, to 29 December, 2022. This specific date is chosen because the World Health Organization (WHO) declared a Public Health Emergency of International Concern (PHEIC). SARS-CoV-2 is officially named as a pandemic on 11 March 2020, but the economy is already disrupted by this point. This first signal of the WHO is therefore more relevant to this research. In Table 1 can we analyse the descriptive statistics over the complete period.

Table 1

Descriptive Statistics of the Six Assets Before and During the Covid-19 Pandemic.

Variable	Mean	Median	St. Dev.	Kurtosis	Skewness	Minimum	Maximum
Before the Covid-19 pandemic							
Bitcoin	0,0189	0,0101	0,1080	2,7557	0,3659	-0,3949	0,4452
Stocks	0,0020	0,0040	0,0164	2,3198	-0,9569	-0,0669	0,0405
Bonds	0,0006	0,0006	0,0044	0,0400	-0,2020	-0,0124	0,0124
Gold/Silver	0,0025	0,0044	0,0505	1,0765	0,2516	-0,1538	0,1876
Real Estate	0,0014	0,0027	0,0204	1,6485	-0,5460	-0,0725	0,0655
Oil	-0,0009	0,0005	0,0453	1,0470	0,2245	-0,1478	0,1817
During the Covid-19 pandemic							
Bitcoin	0,0090	-0,0003	0,1029	1,6187	0,1320	-0,3379	0,3319
Stocks	0,0015	0,0055	0,0314	3,6576	-0,9882	-0,1252	0,1131
Bonds	-0,0006	-0,0002	0,0108	28,6708	0,7114	-0,0710	0,0766
Gold/Silver	0,0027	0,0064	0,0576	1,0218	-0,1174	-0,1622	0,2094
Real Estate	0,0006	0,0044	0,0393	7,9670	-1,3918	-0,2245	0,1450
Oil	0,0018	0,0103	0,0774	9,5028	-1,7091	-0,4334	0,2301
Total dataset							
Bitcoin	0,0153	0,0076	0,1062	2,3780	0,2930	-0,3949	0,4452
Stocks	0,0018	0,0042	0,0230	6,1363	-1,1416	-0,1252	0,1131
Bonds	0,0002	0,0004	0,0074	47,2199	0,5610	-0,0710	0,0766
Gold/Silver	0,0026	0,0052	0,0532	1,0845	0,0827	-0,1622	0,2094
Real Estate	0,0011	0,0029	0,0287	11,5033	-1,4372	-0,2245	0,1450
Oil	0,0001	0,0035	0,0590	10,9079	-1,2945	-0,4334	0,2301

Note. This table displays the descriptive statistics over the complete dataset. The total period is from January 1, 2015, to December 29, 2022. The period before the Covid-19 pandemic is from. January 1, 2015, to January 30, 2020. The period during the Covid-19 pandemic is from January 30, 2020, to December 29, 2022.

CHAPTER 4 Method

4.1 Markowitz's Portfolio Optimization (Markowitz Model)

Before Markowitz (1952), investors made decisions based on their belief system. Markowitz his main contribution to this problem is diversifying a portfolio to decrease the standard deviation of the portfolio. The methodology and calculations of this optimization problem are inspired by Romeo (2019). The mean μ, also known as the expected value, refers to the returns within our optimization problem. The American investor can buy a certain amount of assets, six in our case, with a corresponding expected return and variance. To calculate the expected return of the portfolio, we use the following formula:

$$E(r_p) = w_1 E(r_1) + w_2 E(r_2) + w_3 E(r_3) + w_4 E(r_4) + w_5 E(r_5) + w_6 E(r_6) = \sum_{j=1}^{6} w_j E(r_j)$$

In which r is the return and w is the weight of the asset. This $E(r_p)$ equals the weighted average of the expected returns on the assets. If the portfolio holds more than two assets, a matrix multiplication to determine the portfolio its optimal asset weights. This leads to the following formula:

$$E(r_p) = W^T R = [w_1 \quad \dots \quad w_j] \begin{bmatrix} E(r_1) \\ \vdots \\ E(r_j) \end{bmatrix}$$

Where W is the vector of weights of the individual assets found in the portfolio, from 1 to j, and R is the vector of expected returns of the individual assets found in the portfolio, from 1 to j. The variance expressed as σ^2 indicates the risk in the mean-variance problem. The higher the variance, the higher the risk or volatility is corresponding to that asset. A higher variance thus relates to possibly bigger fluctuations in the returns. The formula for the variance with two assets is defined as

$$\sigma_P^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2 w_1 w_2 \, Cov\,(r_x, r_y).$$

where σ_P^2 denotes the variance of the returns for portfolio P, σ^2 is the variance for a certain asset, w is the weight allocated within the portfolio to an asset, and $Cov\,(r_x, r_y)$ represents the covariance between the returns of two assets, X and Y. This research works with six different indices for asset classes. To accommodate for the extension of assets, a new generalized equation for the variance is made:

$$\sigma_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \; Cov(r_x, r_y)$$

In the variance formula, the covariance is considered. The covariance studies the relationship of the returns between asset X and Y. This can vary from positive, $+\infty$, to negative, $-\infty$, values. When this covariance holds a positive value, the variables move together with the same sign. With a negative value, the variables tend to move in the opposite direction of each other. When a covariance is close to zero, the relationship between the two returns is neutral and has little to no relationship. The covariance indicates the type of relationship the variables have.

$$Cov(r_1, r_2) = \frac{1}{N} \sum_{i=1}^{N} (r_{1i} - \mu_1) * (r_{2i} - \mu_2)$$

where μ represents the expected (average) returns of the assets. Whereas the covariance is an indicator for the type of relationship that two variables have, it does not examine the magnitude of this relationship. Both the variance and covariance can be hard to interpret. Thus, another measurement can be used, derived from the variance and covariance, to interpret the relationship easier: the correlation coefficient. This number shows how strongly two variables move together. The formula for the correlation coefficient between asset 1 and 2 is defined as

$$\rho 12 = Cov\ (X1, X2) \sqrt{Var\ (X1)} \sqrt{Var\ (X2)}$$

where ρ has a value between -1 and 1. The higher the number, the stronger the relationship between the two variables. The variance can also be denoted as $\sigma = \sqrt{Var(X)}$ to interpret the results easier. In our dataset, no asset class is perfectly correlated with one another. Table 2 displays the correlation over the total period, before, and during the Covid-19 pandemic. During the Covid-19 pandemic, the correlations rise, compared to before the pandemic This aligns with research of Sandoval and De Paula Franca (2012) who point out that the higher volatility in the market is directly linked with a rise in correlation between the two variables. These different markets move as one market during the great crashes they use in their research. The same difference can be seen in Table 2; every correlation rises during the Covid-19 pandemic.

Table 1

Correlation analysis between the six asset classes for the total period, before the Covid-19 pandemic, and during the Covid-19 pandemic.

	Bitcoin	S&P 500	Bond	Gold/Silver	Real estate	Oil
			Total Period			
Bitcoin	1					
S&P 500	0,278	1				
Bond	0,160	0,253	1			
Gold/Silver	0,167	0,386	0,379	1		
Real estate	0,212	0,741	0,444	0,407	1	
Oil	0,165	0,438	0,100	0,258	0,345	1
			Before the Covid-19 pandemic			
Bitcoin	1					
S&P 500	0,067	1				
Bond	-0,026	-0,173	1			
Gold/Silver	0,056	0,251	0,353	1		
Real estate	0,027	0,480	0,315	0,349	1	
Oil	0,037	0,368	-0,206	0,216	0,133	1
			During the Covid-19 pandemic			
Bitcoin	1					
S&P 500	0,512	1				
Bond	0,325	0,413	1			
Gold/Silver	0,347	0,517	0,441	1		
Real estate	0,411	0,864	0,495	0,477	1	
Oil	0,318	0,476	0,232	0,306	0,458	1

Note. The total period is from January 1, 2015, to December 29, 2022. Before the Covid-19 pandemic is from January 1, 2015, to 30 January, 2020. During the Covid-19 pandemic is from 30 January, to 29 December, 2022.

To find the optimal weights w_j that minimalize our volatility of the portfolio, given a certain return, we use the following constraints.

$$min_w \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j Cov(r_i r_j)$$

$$\sum_{j=1}^{n} w_j r_j = z$$

$$\sum_{j=1}^{n} w_j = 1$$

$$w_j \geq 0$$

(1) Objective is to minimize the portfolio variance.

(2) The program runs continuously till every return has been, from $z = r_{min}\ to\ z = r_{max}$.

(3) The sum of the weights needs to be one.

(4) We want to compute the basic version of the Mean-Variance framework which does not include short selling, risk-lending, and borrowing opportunities.

We are looking at two kinds of portfolio in this part: (1) equally weighted portfolio and (2) Optimally risky portfolio. The equally weighted portfolio is a portfolio where each asset has the same weight, 1/N where N is the amount of assets. Thus, if we make the portfolio without Bitcoin with five assets, each asset has a weight of 1/5 inside of the portfolio. With Bitcoin added, each asset has a weight of 1/6. With the optimally risky portfolio, we maximize our Sharpe Ratio by changing these weights. After calculating the returns, we create a variance-covariance matrix of the weekly returns. Weekly data is used which must be made into annual returns. With the matrix, we calculate the expected return and standard deviation for the equally weighted portfolio. To check for its statistical significance, we test the difference in means between the portfolios with and without Bitcoin, for both periods. We use a t-test with a significance level set at 0,05. If the obtained p-value falls below the 0,05 threshold, we reject our null hypothesis of no difference in means. This leads to concluding a significant difference in means. To supplement our significance test on the difference in means, another statistic will be tested. With these data, the Sharpe ratio can be calculated, which is a measurement for the portfolio performance. When the weights are optimal in the portfolio, the value of the Sharpe Ratio maximizes. The formula for the Sharpe ratio is

$$S_p = \frac{E(r_p) - r_f}{\sigma_p}$$

where r_f the risk-free rate is and σ_p the standard deviation of the portfolio. The risk-free rate is obtained from the Economic Research Federal Reserve Bank of St. Louis. We calculated a risk-free rate before and during the Covid-19 pandemic by taking the average over the corresponding period, 2,26% and 1,76% respectively. After the equally weighted portfolio, an optimally risky portfolio is calculated to maximize this Sharpe ratio. Within this optimally risky portfolio, the weights might change compared to the equally weighted portfolio. Firstly, these portfolios will be created before and during the Covid-19 pandemic, with and without Bitcoin. The results will be compared, and the significance of the difference will also be looked at for the optimally risky portfolio. We can also test the significance of the Sharpe ratios in both periods.

4.2 Conditional Value-at-Risk

Whereas the Markowitz Model made the foundation of portfolio management, this model also has its disadvantages. There are two main disadvantages that occur in our research. Firstly, the mean-variance approach assumes normally distributed returns. Bitcoin has non-normally distributed returns (Gasser et al., 2015). Secondly, Mean-variance optimization uses variance, which does not control for tail risk. Therefore, we use a risk measure approach to solve these problems. This measure reflects the downside risk and, together with the mean-variance approach, will give us a holistic view of Bitcoin its portfolio diversification effect. Rockafellar and Uryasev (2000) invented the Conditional Value-at-Risk (CVaR) approach as a risk measure. The Value-at-Risk is a loss in the worst scenario with a corresponding probability and time horizon. The CVaR is a percentage that shows the expected value of losses beyond the Value-at-Risk level. It essentially targets the part beyond the worst-case threshold. Its focus lies in capturing potential downside risk. The reason CVaR is used in this paper is because of its coherence: all four conditions are met for a risk measure. These four properties are (1) monoticity, (2) translation invariance, (3) homogeneity, and (4) sub-additivity. The fourth property does not hold for Value-at-Risk. There are four main advantages of the CVaR: (1) Convexity, (2) Coherence, (3) severity of losses and (4). Firstly, the CVaR accounts for the non-normal distribution of loss and provides a completer view of the tail losses beyond the capacity of VaR. Secondly, it is more robust than the VaR and behaves more consistent with respect to risk aggregation, diversification, and other mathematical properties (Sarykalin, 2008). Thirdly, CVaR considers beyond the threshold level of expected losses that VaR assumes; it gives a more holistic view of the downside risk of the portfolio. The fourth, and last, advantage is the sub-additivity of CVaR. This supports the diversification effects found in a portfolio and provides a more comprehensive picture on the risk of the portfolio. The value of the Conditional Value-at-Risk with a probability α is

$$CVaR_a(X) = E[X \mid X \leq VaR_a(X)]$$

where $E[X \mid X \leq VaR_a(X)]$ shows the expected returns of the portfolio given the returns are less or equal to the VaR at probability, or confidence level, α. First or all, the returns are calculated. Following, the scenarios are numbered, and the assets are firstly put together in a portfolio that is equally weighted. If we take the before Covid-19 scenario, we create an equally weighted portfolio with and without Bitcoin and do the following for both. We calculate the returns for each asset and take the average of these returns to figure out the return on the portfolio for that specific scenario. We give a number to each scenario, each week. This method is called historical simulation. We number the returns from worst to best. With these scenarios, we use a Value-at-Risk of 95%. This states that there is a 0,05 probability that the portfolio will lose a specific amount. Via statistical software in Excel, the value of VaR is calculated. However, we work with CVaR and this measurement takes the average of the values beyond the the VaR threshold. Especially, a portfolio with Bitcoin included contains more volatility than the ones without Bitcoin. We want to account for those extreme losses. The CVaR will be calculated by

To test the significance of the difference in CVaR with and without Bitcoin, a t-test is made by comparing means of the CVaR values. The p-value obtained examines the probability of observing the null hypothesis of no difference in these CVaR values. We set our significance level at 0,05. Thus, when a p-value falls below the 0,05-significance level, we reject our null hypothesis of no difference and state that there is a significant better value of CVaR in the portfolio with Bitcoin added. This will be done for both periods, before and during the Covid-19 pandemic.

CHAPTER 5 Results & Discussion

5.1 Mean-Variance approach

5.1.1 MV approach before Covid-19 pandemic: Equally Weighted portfolio

Firstly, we calculated the returns and variances of the assets used in our dataset. Table 3 displays the results. We find relatively high returns for Bitcoin, S&P 500, and Gold/Silver whereas Oil obtains a negative return. With this high return, Bitcoin and Gold/Silver are paired with a relatively high variance. S&P 500 showcases a low variance, compared to the other two. Bond is spotted with the lowest variance.

Table 3

Returns and Variances from the six assets before and during and during the Covid-19 pandemic.

	Bitcoin	S&P 500	Bond	Gold/Silver	Real Estate	Oil
Before the Covid-19 pandemic						
Average Weekly Return	1,890%	0,195%	0,061%	0,249%	0,144%	-0,087%
Weekly Variance	1,167%	0,027%	0,002%	0,255%	0,042%	0,205%
Average Annual Return	98,276%	10,163%	3,160%	12,949%	7,472%	-4,506%
Annual Variance	60,694%	1,400%	0,101%	13,281%	2,159%	10,667%
During the Covid-19 pandemic						
Average Weekly Return	0,904%	0,154%	-0,061%	0,270%	0,057%	0,179%
Weekly Variance	1,059%	0,098%	0,012%	0,332%	0,154%	0,599%
Average Annual Return	47,020%	8,019%	-3,177%	14,041%	2,963%	9,311%
Annual Variance	55,055%	5,112%	0,601%	17,266%	8,014%	31,136%

We make a variance-covariance matrix to help determining the risk level and diversification benefits of a portfolio. In Appendix A, Table A1 showcases the variance-covariance matrix before the pandemic. In the diagonal, we find the variances of the assets. The variance tells us how data points differ from the corresponding mean. A high variance thus means that the data points surrounding the mean are very far apart. For example, Bitcoin has the highest variance of the used assets, followed by S&P 500 and Gold/Silver. The non-diagonal values are the covariances. These values tell us the sign of the relationship; positive values resemble a positive relationship and vice versa.

For the calculation of the risk-free rate, we take the average of the market yield on U.S. treasury securities at 10-year constant maturity. Before Covid-19, we look at the same period as in our dataset. This results in an average of 2,263%. During Covid-19, the risk-free drops to 1,7585%. The results are found in Table 4.[2]

Table 4

Results from Portfolio 1 and 2, regarding the Equally Weighted and Optimally Risky Portfolio. The Expected Return is the average expected gain or loss from the investment, the Standard Deviation displays the volatility of the portfolio, and the Sharpe evaluates the risk-adjusted performance of the portfolio.

	Portfolio 1		**Portfolio 2**	
	Equally Weighted	**Optimally Risky**	**Equally Weighted**	**Optimally Risky**
Expected Return	5,85%	10,16%	21,25%	12,15%
Standard Deviation	7,41%	5,27%	17,66%	6,90%
Sharpe Ratio	0,484	1,500	1,075	1,43

After calculating the results for Portfolio 1, we incorporate Bitcoin into a new portfolio: Portfolio 2. The same steps as for Portfolio 1 are taken and the results are also found in Table 4.[3] We obtain a strong rise in expected return, from 5,85% to 21,25%, and Sharpe Ratio, from 0,484 to 1,075, in the equally weighted portfolio. Examining the equally weighted portfolios, we want to check if the observed expected return in the equally weighted portfolios is higher in Portfolio 2.[4] The results imply that the expected returns of the Bitcoin included portfolio is bigger than the portfolio without Bitcoin, in the scenario of an equally weighted portfolio. We know that the returns are better for Portfolio 2, but we also want to check the risk-adjusted performance of the portfolio. We observe that the Sharpe ratio more than doubles with the addition of Bitcoin. This is due to the expected return being higher than the standard deviation, whereas in Portfolio 1 the standard deviation is higher. All in all, the expected return as well as the risk-adjusted performance improves for the Bitcoin included portfolio when we observe equal weights. There are diversification gains for a Bitcoin included portfolio before the Covid-19 pandemic.

[2] The corresponding weights for the four portfolios in the Mean-Variance approach are found in Appendix A: Tables

[3] The corresponding weights are found in Appendix A: Tables

[4] The hypothesis and t-statistic are in Appendix B: Hypothesis Tests.

5.1.2 MV approach before Covid-19 pandemic: Optimally Risky Portfolio

After calculating and testing the equally weighted portfolios before the Covid-19 pandemic, we want to check the optimally risky portfolios in this period. The expected return increases from 10,16% to 12,15% with the inclusion of Bitcoin, in the optimally risky portfolios. The expected return of Portfolio 2 with optimally risky weights is significantly bigger than the expected return of Portfolio 1. Whereas the expected return increases by adding Bitcoin, maximizing the Sharpe Ratios works out better for Portfolio 1. This is caused by the rise in standard deviation for the Bitcoin included portfolio, from 5,27% to 6,90%. Adding to that, Portfolio 1 forms a corner portfolio when it is optimally risky. Baxter (2018) points out that the error maximizing nature is a shortcoming of traditional portfolio optimization and can lead to this 'corner solution' problem, where large weights are assigned to few assets. In our scenario, all weight is put into the S&P 500 which does not make it favorable for risk-averse investors, because of high idiosyncratic risk. All in all, expected return improves with Bitcoin included but the risk-adjusted performance does not necessarily improve for the Bitcoin included portfolio compared to a portfolio without Bitcoin.

5.1.3 MV Approach during Covid-19 pandemic: Equally Weighted Portfolio

In Table 3 can we find the returns and variances during the pandemic. We observe a decrease in almost every average annual return except with the S&P 500 and Oil. Adding to that, almost every annual variance rises except for Bitcoin its variance. This annual variance drops from 60,694% to 55,055% in a time of financial turmoil. In Appendix A, we find the Variance-Covariance matrix of the period during Covid-19. Again, we see a rise in most variances and covariances due to the financial turmoil that the pandemic brings with it. We display the results of Portfolio 3 and 4 in Table 5.

Table 5

Results from Portfolio 3 and 4, regarding the Equally Weighted and Optimally Risky Portfolio. The Expected Return is the average expected gain or loss from the investment, the Standard Deviation displays the volatility of the portfolio, and the Sharpe evaluates the risk-adjusted performance of the portfolio.

	Portfolio 3		**Portfolio 4**	
	Equally Weighted	**Optimally Risky**	**Equally Weighted**	**Optimally Risky**
Expected Return	6,23%	10,43%	13,03%	39,52%
Standard Deviation	23,86%	26,26%	28,01%	61,04%
Sharpe Ratio	0,187	0,330	0,402	0,619

The expected return in the equally weighted portfolios rises from 6,23% to 13,03% with the inclusion of Bitcoin. This rise in expected return looks favourable for investors in Bitcoin during the pandemic. This rise in expected return makes the expected return for the Bitcoin included portfolio significantly higher than the portfolio without Bitcoin.[5] We can also check the rise in the Sharpe ratio, from 0,187 to 0,402. This is more than a doubling in the Sharpe ratio due to the sharp increase in expected return. During the pandemic, both the expected return and the risk-adjusted performance of the Bitcoin included portfolio is higher than a Bitcoin excluded portfolio.

5.1.4 MV approach during Covid-19 pandemic: Optimally Risky Portfolio

With an improvement in both the expected return and Sharpe ratio in the equally weighted portfolio, we also want to check the optimally risky portfolio. Observing the expected returns in Table 5, a rise from 10,43% to 39,52% takes place. The results imply that the expected returns for the Bitcoin included portfolio are significantly bigger during the Covid-19 pandemic. Adding to the expected return, we also want to check the improvement in Sharpe Ratio. This measurement increases from 0,330 to 0,619. Whereas the standard deviation doubles, the expected return almost increases by four times. Whereas Bitcoin only has 7,5% before the pandemic, this weight rises to 77,2%.[6] Bitcoin plays a big part in this optimally risky portfolio. All in all, both the expected return and the risk-adjusted performance of the Bitcoin included portfolio is better than a Bitcoin excluded portfolio.

5.2 Conditional Value-at-Risk approach

5.2.1 CVaR approach before Covid-19 pandemic: Equally Weighted

Whereas the focus of 5.1 lies in the upside potential, we want to check the downside risk as well. The method used in the CVaR framework is the historical method, which orders the returns from biggest losses to biggest wins. We start with the equally weighted portfolios before the Covid-19 pandemic. Looking at Table 6, the results from the historical method are displayed.

Table 6

Results from the Equally Weighted Portfolios before and during the Covid-19 pandemic. Alpha is the threshold of the X% worst case scenarios, Total Scenarios is the amount of weekly data points, Position is where the threshold position where X% of the Total Scenarios is found, Value-at-Risk is the loss of the dataset, ranked from worst to best, found in position X, Conditional Value-at-Risk is the average of the points found beyond the VaR threshold, and Target Return is a requirement return to compare the portfolios properly.

[5] Results of the hypothesis test are available in Appendix B: Hypothesis Tests.

[6] These weights are displayed in Appendix A: Weights

	Portfolio 1	Portfolio 2	Portfolio 3	Portfolio 4
Alpha	5%	5%	5%	5%
Total Scenarios	264	264	153	153
Position	13	13	8	8
Value-at-Risk	-3,065%	-3,406%	-5,366%	-5,061%
Conditional Value-at-Risk	4,242%	5,079%	8,896%	9,583%
Portfolio Return	0,112%	0,409%	0,120%	0,251%

We use an alpha of 5%. We find a higher CVaR at the Bitcoin included portfolio, with a value of 5,079% compared to 4,242%. This higher CVaR suggests a bigger magnitude of possible losses beyond the VaR threshold. However, this comes with almost four times the portfolio return compared to portfolio 1. Though, this is interesting when we want to control for the return aspect of the CVaR framework, we are focussing on the risk potential of the portfolios in this section. Portfolio 1 is better, based on that condition.

5.2.2 CVaR approach during Covid-19 pandemic: Equally Weighted

We follow the procedure of 5.2.1 but use the dataset corresponding to the Covid-19 pandemic. Table 6 displays the results for the equally weighted portfolios. We observe the same problem as in 5.2.1. Although the Bitcoin included portfolio has a portfolio return twice as high, the accessory CVaR value is higher than that of the Bitcoin excluded portfolio. With the focus on risk management, Portfolio 2 does not improve on Portfolio 1.

5.2.3 CVaR approach before Covid-19 pandemic: Optimal weights

We have now seen in the results of 5.2.1 the results of an equally weighted portfolio. With the optimal weights we minimize our CVaR while setting the target return at a certain level.[7] This specific target return obtained in Portfolio 1 will be used as the target return for minimizing CVaR in Portfolio 2. This way, we can check what happens to the CVaR in both situations. In the before as well as during Covid-19 scenario, the same target returns of 0,2% is used. This target return is achievable for every portfolio. When keeping the target return constant, we can focus on the change in CVaR. We observe the results from the minimization of CVaR in Table 7.

Table 7

Results from the minimization of CVaR before and during the Covid-19 pandemic. Alpha is the threshold of the X% worst case scenarios, Total Scenarios is the amount of weekly data points,

[7] The exact weights of the minimization of CVaR for both scenarios can be found in Appendix A.

Position is where the threshold position where X% of the Total Scenarios is found, Value-at-Risk is the loss of the dataset, ranked from worst to best, found in position X, Conditional Value-at-Risk is the average of the points found beyond the VaR threshold, and Target Return is a requirement return to compare the portfolios properly.

	Portfolio 1	Portfolio 2	Portfolio 3	Portfolio 4
Alpha	5%	5%	5%	5%
Total Scenarios	264	264	153	153
Position	13	13	8	8
Value-at-Risk	-2,611%	-3,406%	-6,217%	-3,438%
Conditional Value-at-Risk	4,293%	5,079%	9,503%	6,573%
Target Return	0,200%	0,200%	0,200%	0,200%

S&P 500 has the biggest role in Portfolio 1 when optimizing weights. A strange phenomenon is the high weight of oil in the Bitcoin included portfolio, with 79,59%; Oil has no role in Portfolio 1. Bitcoin follows oil and is the second biggest asset in Portfolio 2. With both portfolios maintaining the same target return, Portfolio 1 has a lower CVaR and makes it more suitable for minimizing the risk of significant losses. Portfolio 2 is more interesting for achieving a higher target return. However, the CVaR-approach in our context puts the focus on the risk side. Using optimal weights, the Bitcoin included portfolio does not improve the risk characteristics when comparing it to a Bitcoin excluded portfolio. Bitcoin does not improve downside risk in the average portfolio of the American investor before the Covid-19 pandemic. In both equally weighted and optimal weighted, the Bitcoin excluded portfolio holds better risk management characteristics. However, the Bitcoin included portfolio outperforms in terms of expected return.

5.2.4 CVaR approach during Covid-19 pandemic: Optimal Weights

We follow the procedure of 5.2.2 and obtain the result, displayed in Table 7. In a time of financial instability, Gold/Silver plays a big role in Portfolio 1. Gold is known as a safe haven in times of financial crisis (Arouri et al., 2015). This safe haven characteristic is also captured in Portfolio 3, in Appendix A6. Adding to that, S&P 500 has the biggest weight in this minimization portfolio. In Portfolio 1, before the Covid-19 pandemic, S&P 500 contributed the biggest weight to the portfolio. When looking at Portfolio 4, bonds make up more than half of the portfolio. This stable asset can have such a big part due to the big role that Bitcoin has in this portfolio, amounting for almost a quarter of the portfolio. Bitcoin its big variance is offset by the low variance of bonds.[8] Again, Gold/Silver has a weight due to its safe haven characteristics in times of financial turmoil. The other assets hold no weight, including the S&P 500. When observing the CVaR in Table 7, Portfolio 4 has a lower CVaR

[8] This can be observed in Appendix A: Tables – A6

than Portfolio 3 when maintaining the same target return. This is interesting for our risk management context. Especially, when Bitcoin has a weight of 22,51 percent in the portfolio. The results imply that Portfolio 4 shows better risk management characteristics when the CVaR is minimized, and optimal weights are obtained.[9] To conclude the CVaR approach, Bitcoin does generally not improve the risk management characteristics: only in one situation did the CVaR improve by including Bitcoin. Bitcoin has a high volatility and offers mainly diversification gains from the high return perspective. This risk characteristic can be offset by the higher expected returns found in the Bitcoin included portfolios.

[9] The results of the test are found in Appendix B: Hypothesis Tests.

CHAPTER 6 Discussion

The findings in this paper are in line with other existing papers regarding this subject. Markowitz (1952) and his mean-variance approach states that assets with low correlations can be combined into portfolios that cause better risk-adjusted returns, compared to investments in assets, on its own. Burniske and White (2016) discuss Bitcoin and see it as a unique asset class. Bitcoin has a high average return and volatility, plus low correlations with other assets before the Covid-19 pandemic, such as found in the research of Brière et al. (2015). These findings are in line with this paper. However, the inclusion of Bitcoin increases the CVaR of the portfolio in three of the four cases. This downside risk does not improve with Bitcoin. Although this measurement rises, the Bitcoin included portfolio improves massively on the return side. All in all, the risk-return characteristics improve with the inclusion of Bitcoin. Gasser et al. (2015) studies Bitcoin within a Conditional Value-at-Risk framework and suggests an inclusion of Bitcoin into an already well-diversified portfolio. The CVaR increases with the addition of Bitcoin but the high returns are so favorable that the risk-return characteristics improve. This finding of Gasser et al. (2015) is like the findings in this paper. We experience a rise in the CVaR but the returns are higher with the Bitcoin included portfolio. Both, Gasser et al. (2015) and Brière et al. (2015) examine the viewpoint of an American investor which supports our paper even more. Aggarwal et al. (2018) uses the mean-CVaR approach on an already well-diversified portfolio with the same asset classes, but on the Indian market. This paper found that Bitcoin offers diversification gains for equally weighted portfolios. Moreover, it shows a rise in CVaR with the inclusion of Bitcoin, but that this is offset by the high returns that Bitcoin offers. This finding is like this paper but studied in a different context. When we compare the results regarding the mean-variance approach, we also find similarities with current literature. Carpenter (2016) studies Bitcoin with an adjusted mean-variance model and finds a big increase in the risk-return ratio of a diversified portfolio. He claims that Bitcoin is still in an early stage with extreme volatility and returns. Within our paper, Bitcoin matured, and we find similar results. These similarities show that if a small proportion of Bitcoin is added to a well-diversified portfolio, the investor experiences diversification gains in terms of the upside potential. A different result is found, compared to Conlon and Mcgee (2018). They found that a small proportion of Bitcoin greatly increases downside portfolio risk. This research is done with the start of the Covid-19 pandemic. However, they only use till March 20th, 2020, whereas this paper continues till December 29th, 2022. In this first period of the pandemic, assets experienced more market stress and a higher volatility. With this longer period of the Covid-19 pandemic, we find a decrease in the portfolio downside risk by including Bitcoin. This weight is not even small; Bitcoin holds a weight of 22,51% of the portfolio.

CHAPTER 7 Conclusion

In this thesis I have looked at the diversification effect of Bitcoin on the average portfolio of an American Investor. This is done before and during the Covid-19 pandemic. Previous research shows that Bitcoin provides diversification effects. However, this proof is limited, and Bitcoin is relatively young. Also, the Covid-19 pandemic offers a new dimension in which Bitcoin can be observed as a portfolio diversifier. Therefore, the main question in this thesis was: "Does Bitcoin contain diversification effects for the average U.S. portfolio before and during the COVID-19 pandemic?"
To answer this research question, prices of six different assets, including Bitcoin, are studied within a portfolio. One portfolio is made up of five assets excluding Bitcoin and the other portfolio is complete with all assets. These portfolios are compared with one another with the Mean-Variance method and the Conditional Value-at-Risk approach. Both portfolios are examined with equal weights within the portfolio and optimally risky weights. We look at the expected return, the Sharpe Ratio, and the Condtional Value-at-Risk to obtain a holistic view of Bitcoin its diversification benefits. These comparisons are made before and during the Covid-19 pandemic. The mean-variance method found diversification effects for every Bitcoin included, except with optimally risky weights before the pandemic. This was due to a lower Sharpe ratio from the Bitcoin included portfolio. However, this other portfolio had a corner solution and is not practical due to the lack of diversification. The expected return rises in every Bitcoin included portfolio. The Conditional Value-at-Risk only improves with minimization during the Covid-19 pandemic. Bitcoin has high volatility and does not help minimizing downside risk. But the higher CVaR in the other situations is paired with a higher expected return. This is in line with previous literature.
This study therefore shows that Bitcoin provides many diversification effects in terms of the low correlation with other assets and a high expected return. However, this comes paired with a high volatility and does not improve the portfolio risk characteristics. Nonetheless, this risk gets outweighed by its return characteristics. All in all, this paper suggests that the risk-return characteristics of the portfolio generally improve when a weight is given to Bitcoin.

CHAPTER 8 Limitations

The first limitation of this research is the data. We look at six asset classes, but the results can change if we take more assets into account. Adding to this, combining Bitcoin with other cryptocurrencies in a portfolio give a more holistic view on the cryptocurrency market overall. Andrianto (2017) used Bitcoin, Ripple, and Litecoin in already well-formed portfolios and improves the effectiveness of the portfolio by decreasing the risk and to give an investor more choices to pick from. Adding to the data, Bitcoin its 'closing price' is available from 2015. It would be interesting to take the early-stage price development of Bitcoin and see if the found benefits change with this extra data. Secondly, this paper simplifies assumptions made during the analysis. We use a Conditional Value-at-Risk due to VaR its coherence. However, adding multiple assumptions into this research provides more accurate results. Examples of these assumptions could be involving transaction costs, taxes, and other investment constraints. Natarajan et al. (2010) use the expected utility of an investor to optimize their portfolio. Accounting for risk tolerance and the utility changes the outcome of the optimally risky portfolio. Where Portfolio 1 is 100% made up of S&P 500, a risk-averse investor lowers this weight and thereby changes the dynamics of the portfolio. Even involving the risk-free rate as an investment alternative is an option. I encourage future researchers to continue studying Bitcoin as a portfolio diversifier. By applying the points above, our knowledge of Bitcoin its diversification effect will be more comprehensive.

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APPENDIX A: Tables

Table A1

Variance-Covariance Matrix of assets before Covid-19 pandemic

	Bitcoin	**S&P 500**	**Bond**	**Gold/Silver**	**Real Estate**	**Oil**
Bitcoin	0,6046	0,0061	-0,0006	0,0158	0,0030	0,0094
S&P 500	0,0061	0,0139	-0,0006	0,0108	0,0083	0,0141
Bond	-0,0006	-0,0006	0,0010	0,0041	0,0015	-0,0021
Gold/Silver	0,0158	0,0108	0,0041	0,1323	0,0186	0,0256
Real estate	0,0030	0,0083	0,0015	0,0215	0,0215	0,0064
Oil	0,0094	0,0141	-0,0021	0,0256	0,0064	0,1063

Table A2

Variance-Covariance Matrix of assets during the Covid-19 pandemic

	Bitcoin	**S&P 500**	**Bond**	**Gold/Silver**	**Real Estate**	**Oil**
Bitcoin	0,5470	0,0853	0,0185	0,1064	0,0859	0,1308
S&P 500	0,0853	0,0508	0,0072	0,0483	0,0549	0,0597
Bond	0,0185	0,0072	0,0060	0,0141	0,0108	0,0100
Gold/Silver	0,1064	0,0483	0,0141	0,1715	0,0557	0,0706
Real estate	0,0859	0,0549	0,0108	0,0557	0,0796	0,0718
Oil	0,1308	0,0597	0,0100	0,0706	0,0718	0,3093

Table A3

Mean-Variance Approach: Weights for Portfolio 1 and 2.

	Portfolio 1		**Portfolio 2**	
	Equally Weighted	**Optimally Risky**	**Equally weighted**	**Optimally Risky**
Bitcoin	-	-	16,7%	7,5%
S&P 500	20%	100%	16,7%	27,1%
Bond	20%	0,00%	16,7%	65,4%
Gold/Silver	20%	0,00%	16,7%	0,00%
Real estate	20%	0,00%	16,7%	0,00%
Oil	20%	0,00%	16,7%	0,00%
Sum	100%	100%	100%	100%

Note. The weights used in the equally weighted portfolio are 1/6 for each asset. Making this fraction into a percentage round to one number behind the comma makes it 16,7%.

Table A4

Mean-Variance Approach: Weights for Portfolio 3 and 4.

	Equally Weighted	Optimally Risky	Equally Weighted	Optimally Risky
			16,7%	77,2%
S&P 500	20%	59,9%	16,7%	0
Bond	20%	0,00%	16,7%	0
Gold/Silver	20%	40,1%	16,7%	22,8%
Real estate	20%	0,00%	16,7%	0
Oil	20%	0,00%	16,7%	0
Sum	100%	100%	100%	100%

Note. The weights used in the equally weighted portfolio are 1/6 for each asset. Making this fraction into a decimal number leads to rounding it to 0,167.

Table A5

Weights from the minimization of CVaR before the Covid-19 pandemic

	Portfolio 1	Portfolio 2
Bitcoin	-	13,58%
S&P 500	85,35%	1,94%
Bond	1,75%	1,01%
Gold/Silver	12,90%	2,31%
Real Estate	0,00%	1,58%
Oil	0,00%	79,59%
Sum	100%	100%

Note. We use two decimal points behind the comma to display the weights more accurately.

Table A6

Weights from the minimization of CVaR during the Covid-19 pandemic

	Portfolio 3	Portfolio 4
Bitcoin	-	22,51%
S&P 500	60,46%	0,00%
Bond	0%	64,26%
Gold/Silver	39,54%	13,23%
Real Estate	0,00%	0,00%

Oil	0,00%	0,00%
Sum	100%	100%

APPENDIX B: Hypothesis Tests

To test if the difference in expected returns is significant, we make the following hypotheses to support our test:

H0: The expected return of both portfolios are equal.
Ha: The expected return portfolio of the Bitcoin included portfolio is higher than the expected return of Bitcoin excluded portfolio.

We use the Welch test which contains unequal variances due to the high volatility of Bitcoin. For the equally weighted portfolios before the pandemic, we obtain a t-statistic of 13,065 when filling in all the accompanying data points. With a critical t-value of approximately 1,65, our t-statistic is bigger, and we reject our null hypothesis. For the optimally risky portfolios before the pandemic, we attain a t-statistic of 3,724. With a critical value of approximately 1,65, we reject our null hypothesis. For the equally weighted portfolios during the pandemic, we obtain a t-statistic of 2,286. Using a critical t-value of approximately 1,65. we reject our null hypothesis that the expected returns are equal and accept our alternative hypothesis. For the optimally risky portfolios during the pandemic, the corresponding t-statistic is 7,116 and is bigger than the critical value of t with approximately 1,65. Again, the null hypothesis is rejected. Concluding, every expected return of a Bitcoin included portfolio is higher than those portfolios without Bitcoin.
To check if the difference in Conditional Value-at-Risk is significant, we use a paired t-test with the following hypothesis:

H0: The CVaR values of Portfolios 3 and 4 are equal.
Ha: The CVaR of Portfolio is smaller than the CVaR of Portfolio 3.

Filling in the required data, we obtain a t-statistic of -13,890. Using a one-sided test with a confidence level of 95% and a sample of 153 observations, the critical t-value is approximately -1,65. Our t-statistic is lower, and we reject our null hypothesis. We accept our alternative hypothesis of a lower CVaR in Portfolio 4 compared to Portfolio 3.