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Time-varying maintenance cost optimization for two components

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Contents

Abstract

Offshore wind turbines are often hard to reach and as their components need to be replaced once in a while, it is of great importance that one knows when such a component needs to be replaced. Sometimes combining these maintenance jobs for several components is cost-efficient. The objective of this paper is to find the most optimal policy for two-component maintenance when minimizing costs. We look at three policies including age replacement, block replacement, and modified block replacement taking into account time-varying costs with different levels of cost differences throughout the year. We model the cost function with a cosine function and adapted scale and shift to fit the weather of the seasons. For the lifetime failure probabilities, we assume the component follows a known lifetime distribution. To compare the results to single-component maintenance, we replicate the paper of T. N. Schouten et al. (2022), which optimizes the maintenance cost of the same policies of the single component. We find that the age replacement policy is the most cost-efficient policy to maintain. Using this policy for the two-component model is more efficient than the single-component model. Also, this optimum is independent of the indicated cost fluctuations.

1 Introduction

Generating sustainable energy through wind turbines, particularly offshore, is effective. The Netherlands, The UK, and Germany currently produce 12GW of power from offshore wind turbines and plan to triple this by 2030. At the moment maintenance costs amount to around 25-30% of the total life costs of a wind turbine. With high expectations of growth in both the number of wind turbines and the size, an increase in costs is unavoidable. Both preventive and corrective maintenance is necessary to keep wind turbines operational. Preventive maintenance can be conditioned-based or time-based, or in other words, age-based. However, planning maintenance can be challenging due to unpredictable weather during both the seasons and within the seasons. The main factor influencing this is wind speed fluctuations throughout the year.

Research has explored maintenance optimization, but few studies consider time-varying maintenance costs. One of those papers includes T. N. Schouten et al. (2022). The paper of T. N. Schouten et al. (2022) presents a maintenance planning problem for a single component with predictable time-dependent maintenance costs, suggesting a periodically-based age replacement policy, block replacement policy, and modified block replacement policy to optimize maintenance costs. Results show substantial savings can be obtained by scheduling maintenance during periods with lower cost rates. In practice, there are multiple wind turbines that need to be maintained. Also, a wind turbine has multiple components. The first step to solve this problem is to maintain two components, this arises the following research question:

How can we optimize time-varying maintenance costs for two components?

This paper presents a maintenance planning problem for 2 components with predictable time-dependent maintenance costs. These 2 components can either be identical, for example when we take the same component of two different wind turbines, or they are not identical, for example when you take two different components within a wind turbine. As the paper of I. T. N. Schouten et al. (2019), we suggest a periodically-based age, block, and modified block policy. The age policy replaces a component when it fails, the block replacement policy replaces a component after a fixed interval in time depending on favorable period-related costs. The modified block policy is an in-between strategy in which the component is replaced after a fixed period of time like the block policy. However, when the component fails before that time, the component is replaced like the age policy and the next preventive maintenance block will be skipped when this moment is planned too soon. It is interesting to see to what extent two-component maintenance benefits over single-component maintenance. Therefore, another question to be discussed is:

Is there a benefit of maintaining two components in comparison with one component?

This question can be answered by performing a replication of I. T. N. Schouten et al. (2019). This paper uses the same policies to optimize maintenance time-varying costs for the single component. The paper provides mathematical programming models to optimize policy parameters using CPLEX (version 22.1.0.0). Results show that we save costs by scheduling replacements for two components instead of one. Also, the ARP policy is the most efficient policy to maintain costs, independent of the number of components to maintain (single or two components) and independent of any cost fluctuation.

In the remainder of the paper, we find ideas for the two-component maintenance optimization in section 2, Literature Review. We describe the problem and our corresponding approach in sections 4 and 5. In the latter, Methodology, we discuss our replication for single-component optimization as well. Then, we discuss our analysis in section 6 and end our paper with section 7, the conclusion.

2 Literature Review

The literature focusing on maintenance problems is a broad study. The most important topics to be discussed are maintenance optimization models, maintenance and production planning decisions, and wind park maintenance. We are mainly interested in extending a single component into more components. What is the current research in multiple components? And do we experience a cost reduction when we bundle components and repair them at the same time?

2.1 Policies to tackle component replacement problems

Various maintenance policies have been discussed in the literature, the most relevant are block and age replacement policies. Here, block replacement represents a certain moment in time when a component must be replaced. Age replacement is the replacement of a component, which is done when a component reaches a certain threshold. A third strategy for replacing components is to construct a combination policy of the latter two strategies. The replacement is done every block unless a certain threshold is reached. We replace instantly when the threshold is reached and skip the replacement of the block if no minimum age is reached before the block replacement is scheduled. We find this, and more links to such deepening literature in T. N. Schouten et al. (2022). More literature can be found in this paper. First of all, in the literature on maintenance planning, the relationship between job scheduling and opportunity maintenance is recognized and must be scheduled well, since this maintenance opportunity arrives often randomly.

2.2 Multiple components

Studies on wind park maintenance are also reviewed in the literature of T. N. Schouten et al. (2022), especially regarding the optimization of the wind park design, infrastructure, operations, and logistics. Also, one can find some of the maintenance strategies for wind turbines to include time-varying costs, as production rates and downtime costs are mainly dependent on wind speed. Literature on condition-based costs is given, this is achieved with the help of sensors. In our paper, we want to address the challenging feature of time-dependent cost rates in double-component maintenance optimization. The paper that extends the standard age, block, and modified block replacement policy is the paper of I. T. N. Schouten et al. (2019) which deals with multiple-component models for all replacement policies.

We want to extend our model from a single-component maintenance model to a two-component maintenance model, as we focus on the maintenance of offshore wind turbines. Byon and Ding (2010) show potential failures of different modes when extending the state space by adding states into a multiple-component state space. Also, critical factors are taken into account when looking at the feasibility of maintenance regarding the weather. The Markov process becomes more complex with the components added. Although we keep it linear, the state space grows exponentially. The paper describes the deterioration of components in the transition probability matrix. With the help of dynamic programming, the minimum cost policy is found, adapting to operating conditions. Especially compared to scheduled maintenance, dynamic maintenance can achieve improvements in both reliability and costs.

We can encounter difficulties when moving from single-component to multi-component methods. As a result, multiple methods should be explored to make this transition. Several solutions are dividing the work and taking every component separately, we can group them as a whole, or everything in between. Hameed and

Vatn (2012) investigates the grouping of different activities and corresponding policies to optimize the reliability and maintenance of wind turbines. Tasks such as inspection, and preventive replacement at the wind farm, taking into account important factors such as travel time, and costs. To tackle this problem, a hybrid approach of using the block replacement policy and condition-based model is used. This approach finds the optimum in grouping different activities. Other factors that are considered are access issues, logistics, transportation, and weather. The paper aims to optimize the frequency of visits to the wind farm.

2.3 Methods of prediction component failure

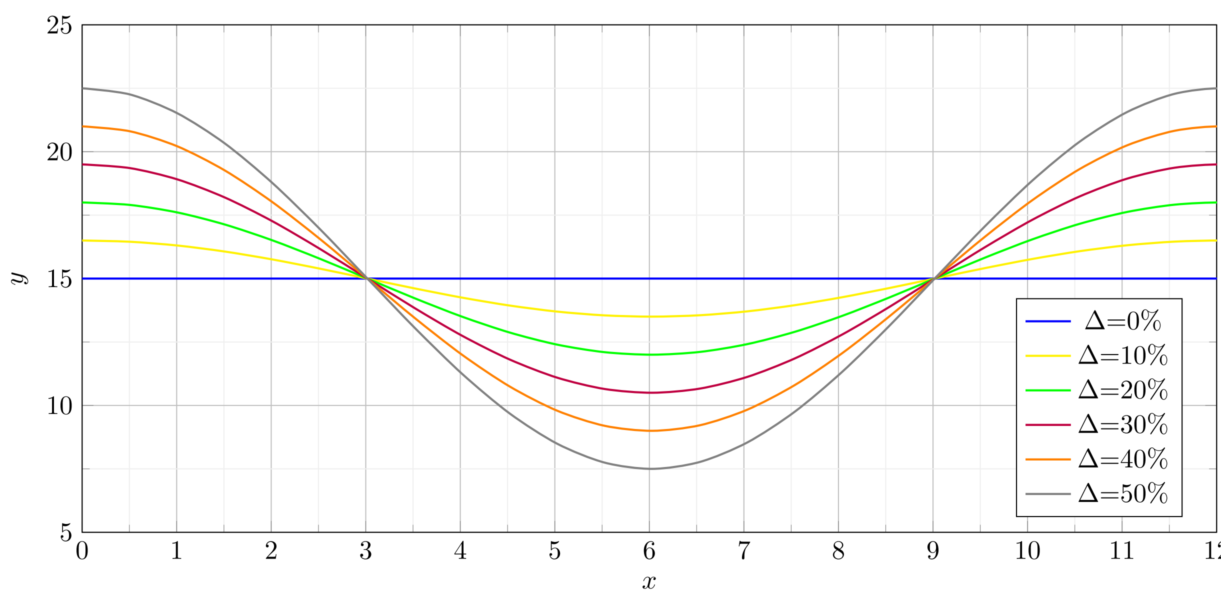
Tian et al. (2011) uses an artificial neural network to replicate the deterioration of components. They determine the deterioration level of each component so they can find the optimal moment when maintenance must be performed. In their example, they look at 5 wind turbines, each containing 4 components including a gearbox, generator, rotor, and main bearing. The maintenance costs used are computed by Tijms (2003). The approach to this problem is measurement-based. We do not assume that we have sensors in the wind turbines to measure the state of the components. Therefore, we predict failures based on components that follow a certain distribution. With the help of this knowledge, we predict failures and schedule maintenance.

Shafiee et al. (2015) investigates optimal condition-based maintenance for an offshore wind turbine, taking environmental factors into account such as corrosion cracking and environmental shocks. The type of maintenance used is an age-based group maintenance policy, in which the costs of maintenance are assumed to be constant over the year because the weather is assumed to be constant over the year. This is a significant simplification so we do not take this assumption in this paper. Seasonality in wind patterns influences both the corrective and preventive costs. In the next section, we describe what data we use and its interpretation. We describe in other sections the problem and solution-wise models of multi-component optimization using time-varying costs.

3 Data

Many studies on the maintenance optimization of wind turbine components do the study with sensor data. With the help of sensors, one can extract information regarding the state of the component. Maintenance is dependent on these data Ciang et al. (2008). However, this is not always possible to obtain. In this paper, we do not consider condition-based replacement, but time-varying-based replacement. The cost data are obtained by the paper Fingersh et al. (2006), which takes a 2,0 MW wind turbine into consideration.

We take the lifetime distribution of a 2.0 MW offshore wind turbine, which is derived by Tian et al. (2011). For a component, the time to failure can be modeled by a distribution, in which the failure rate increases. This distribution is dependent on the lifetime of the component. Note that for 2, not identical components, different parameters and maybe different distributions need to be derived. When using identical components, one can use the same input and the same distribution. This implies an average time to failure of 10.6 months. From the derived distribution of Tian et al. (2011), we can develop a probability matrix. Maintenance costs consist of manpower, material, and lost production costs due to preventive or corrective maintenance downtime. These lost production costs are dependent on the weather and season. Weather changes and so do the production costs. Therefore, to calculate the periodically varying costs, we use a cosine function with a shift such that the costs fit the seasons. Kittler and Darula 2013 mentions that the path of the sun which can be described by a cosine, which in its way influences the weather, which means that we can use the cosine function to replicate the weather roughly. However, we can not foresee weather shocks or temporary deviated weather conditions. This does not only influence the costs but also the capacity to perform maintenance. Some weather conditions do not allow maintenance at all, one should consider for other conditions whether the scheduled time is the optimum time to perform maintenance regarding weather-influenced costs. These costs consist of preventive and corrective costs, both computed separately with the cosine function. These costs are dependent on the delta (Δ), which is the relative difference in costs per period in time.



The higher the Δ, the higher the cost difference per period. The figure below shows the corrective cost function for $\Delta = 0\%, 10\%, 20\%, 30\%, 40\%, 50\%$. We have a corrective maintenance mean of 15 and the function deviates more when the Δ increases. We can conclude that the tendency of replacing in summer a component in summer increases with the Δ. Furthermore, we have the set-up costs, we assume these costs to be constant throughout the paper. Further description can be found in the problem description.

4 Problem description

Offshore wind turbines are far away from the coast and are often hard to reach. Sometimes specific components within a wind turbine fail to work and need to be repaired. Then, workers with their equipment and materials need to drive to the coast, set sail on a boat and get on the wind turbine to replace the component with an as-good-as-new component. This time-consuming and often costly process is preferably done the least number of times possible. We use three different policies, age-replacement, block replacement, and modified block replacement, that minimize costs while choosing the best moment to replace the component. In 4.3, we go more in-depth about these policies. Furthermore, offshore wind turbines are often grouped with multiple wind turbines. To anticipate this, we extend the formulations and test the three policies regarding 2 components.

4.1 Markov decision process

To implement these policies, we need to create a probability matrix indicating what time of the year it is and what the current age of the component is. Also, the action of whether the replacement of the component is done must be mentioned. The first two factors are complex as time is continuous. However, when we discretize time into months we can make use of a Markov process. This Markov process describes the deterioration of components in multiple states. When a component is added to the process, the domain of non-zero probabilities increases rapidly. From having only 4 different kinds of non-zero states for one component to 16 for two components, this shows an exponential increase. This probability matrix is used to determine the deterioration of components and find the optimal deterioration level to apply maintenance and minimize costs.

4.2 Distribution-dependent costs

We found three policies to minimize costs, which include the age replacement policy(ARP), the block replacement policy(BRP), and the modified block replacement policy(MBRP). The policies are used under the assumption that we know the lifetime distribution of components and the corresponding costs of replacement. As explained in section 3, the distribution used is Weibull and the corrective and preventive costs are time-varying. These are individually computed using a cosine function replicating the seasons' costs. The three policies are implemented and adjusted for one and two components.

4.3 Policies

The ARP is a maintenance strategy that handles replacing the component when the component fails to work. Block-based maintenance is a maintenance strategy in which preventive maintenance is planned beforehand and we know in advance when the operations need to take place. The maintenance operations therefore can be planned periodically, which is convenient for the maintenance workers. We optimize the length and time of interval when this needs to take place. Having a fixed moment of preventive maintenance has the downside that it is independent of corrective maintenance. It is inefficient when corrective maintenance happens to be performed just before preventive maintenance. Then, it would be better to wait for the preventive maintenance and apply it in the next scheduled period. This is what is done in the last policy, the modified block replacement policy. If a component has not reached the age at which preventive maintenance must be done and the component still fails, preventive maintenance is scheduled out and corrective maintenance is applied. The block replacement is skipped and scheduled for the next block.

In the next section, we describe the model that we use to solve the problem of time-varying costs for two components and how the probability matrix of the states is built. Furthermore, we show and explain the policies in detail and give mathematical formulations.

5 Methodology

In this section, the research design and implementation show how we want to approach and answer our research question. Then, we do the replication and thus we perform different policies to optimize maintenance costs for a single component in an off-shore wind turbine. Afterward, we extend the paper with a formulation containing two not identical components in an off-shore wind turbine, or even two offshore wind turbines, having each component in one offshore wind turbine.

5.1 Research Design and implementation

This paper will investigate whether there is a cost reduction when we model two components in a windmill, contrary to a single-component model. Hence, our research questions are: How can we optimize time-varying maintenance costs for two components? And is there a benefit of maintaining two components in comparison with one component? First, we reproduce the paper of T. N. Schouten et al. (2022) about the maintenance optimization of a single gearbox in an off-shore windmill. Secondly, we read academic papers to broaden our knowledge about component maintenance and use ideas from this for the extension of multiple components. Then, we extend the three different policies for two components, these policies include age replacement, block replacement, and modified block replacement. Each policy optimizes the maintenance costs using time-varying costs. To ensure this, we introduce another state in the Markov chain and adapt the transition probabilities and costs accordingly. We solve this new Markov decision chain in the same way as the single component. Next, we do a numerical study and compare the results for the two-component case with those of the single-component case.

5.1.1 Implementation

Before we program the three formulations in javascript and solve them with the solver CPLEX (version 22.1.0.0), we start softcoding the multi-dimensional parameters, this includes the probability matrix and the costs. We describe the relevant classes. For the costs, we introduce two separate classes to create the preventive and corrective costs, named *preventiveCosts* and *correctiveCosts* respectively, these are generated by a cosine function mentioned in section 5. The preventive and corrective costs are just part of the bigger picture. When performing an action, we need to add set-up costs and sometimes use combinations or multiples of preventive and/or corrective costs. Therefore, we introduce another class, *costs*, that can call the separate methods and create the corresponding variables' costs.

Also, the probability matrix is developed in 2 classes. One, named *probMatrix*, is able to compute the probability given the input: difference in costs (Δ), the number of periods, and the current period in the year. Using this input, it generates the probability with the help of the Weibull distribution. The main requirement for this distribution is that the failure rate should show an increase in failure rate over time. Then, it is up to the component what kind of input and distribution must be used. This requirement is satisfied using the Weibull distribution. The *probMatrix* calls the class *Weibull*, and assigns the right state variables to the right probabilities. This class processes all necessary input, including the variables indicating the moment transition in the year, the age transition of both components, the action taken whether to maintain or not for component 1 and 2, the number of periods, and the age for which preventive maintenance is done. All these factors are considered using the input: $i_0, i_1, i_2, j_0, j_1, j_2, a_0, a_1, N, M$ respectively. Furthermore, this probability matrix ensures that for any component k taking no action (action = 0), any age equal to

M (the age preventive maintenance is performed) or 0, will set the probability equal to zero. Moving on, we can develop the three policy formulations for two components, program them, and analyze the results. We discuss the results comparing the single-component to the two-component minimized cost objectives and their computation times. We look at the rate of decrease in costs regarding differences in weather influenced by the seasons.

5.2 Replication

We replicate the p-ARP, p-BRP, and p-MBRP formulations by implementing the LP and MIP formulation in Java, in IntelliJ. We compute the costs with the solver CPLEX (version 22.1.0.0) We start by computing the preventive and corrective costs, which depend on the number of periods N in a year. Our cost function is a function of $i_0 \in I_0$ with $I_0 = \{1, 2, .., 12\}$, which indicates a certain period of time within the year, in this case, we use 12 for months. $\bar{c}_p$ is the average preventive maintenance costs and Δp is its corresponding difference in costs throughout the periods. The higher the delta in percentage, the higher the difference in costs. This paper uses the delta's 0%, 10%, 20%, 30%, 40%, and 50%. We make use of a cosine function because the cost function is repetitive with the years. Furthermore, a shift $-\frac{2\pi}{12}$ is inserted so the cost equals the cost average in the first period. The corrective costs are computed likewise.

$$c_p(i_0) = \bar{c}_p + \Delta\bar{c}_p \cos\left(2\pi\frac{i_0}{N} - \frac{2\pi}{12}\right) \tag{1}$$

$$c_f(i_0) = \bar{c}_f + \Delta\bar{c}_f \cos\left(2\pi\frac{i_0}{N} - \frac{2\pi}{12}\right) \tag{2}$$

Furthermore, we develop a method that generates state probabilities which indicate the probability going from state i_0, i_1 to state j_0, j_1. Here, $i_1 \in I_1$ with $I_1 = \{0, 2, .., M\}$ representing the set of ages of a component can have. If $i_1 \in \{0, M\}$ the component is broken or reached its preventive maintenance age respectively.

$$\pi(i_0, i_1)(j_0, j_1)(0) = \begin{cases} 1 - p_{i_1} & \text{for } j_0 = i_0 + 1 \pmod N, j_1 = i_1 + 1, i_1 \notin \{0, M\} \\ p_{i_1} & \text{for } j_0 = i_0 + 1 \pmod N, j_1 = 0, i_1 \notin \{0, M\} \\ 0 & \text{else,} \end{cases} \tag{3}$$

$$\pi(i_0, i_1)(j_0, j_1)(1) = \begin{cases} 1 - p_1 & \text{for } j_0 = i_0 + 1 \pmod N, \quad j_1 = 1 \\ p_1 & \text{for } j_0 = i_0 + 1 \pmod N, \quad j_1 = 0 \\ 0 & \text{else.} \end{cases} \tag{4}$$

These probabilities are based on the Weibull distribution, given the softcoded scale parameter α and shape parameter β, we compute p_{i_1} and p_1. We illustrate this with an example. The lifetime of the component is followed by a Weibull distribution with scale parameter α and shape parameter β. The expected time to failure can be described as:

$$\mathbb{E}(X) = \alpha\Gamma(1 + \frac{1}{\beta}) \tag{5}$$

For example, with an input of $\alpha = 12$ ($= 1$ year) and $\beta = 2$. The expected time to failure of the component is 10.63 so approximately 10 months and 2,5 weeks. For the probability of failure in the first period, we use $\alpha = 1$ year and $\beta = 2$, to derive the following equation:

$$p_1 = \mathbb{P}(X \leq 1) \tag{6}$$

using

$$\mathbb{P}(X \leq i_1) = F(i_1) - F(i_1 - 1) \tag{7}$$

with

$$F(i_1) = 1 - \exp(-\frac{i_1}{\alpha})^{\beta} \tag{8}$$

Furthermore, using the same distribution, we can calculate probability p_{i_1} for all discretized ages expressed in months.

$$\mathbb{P}(X = i_1 | X \geq i_1) = \frac{\mathbb{P}(X = i_1) \wedge \mathbb{P}(X \geq i_1)}{\mathbb{P}(X \geq i_1)} \tag{9}$$

Now we derived the parameters p_{i_1} and p_1. With these values, we derive the transition probability matrix. Now, we program the p-ARP, p-BRP, and p-MBRP formulation.

5.3 Extension

Table 1: Nomenclature

N	Number of periods within a year
N^+	Set of positive integers, 1, 2, ...
I_0	Set of periods within a year
I_k	Set of component ages of component k
$c^p(i_0)$	PM cost in period $i_0 \in I_0$.
$c^f(i_0)$	CM cost in period $i_0 \in I_0$.
$\bar{c}_p$	Average PM cost
$\bar{c}_f$	Average CM cost
X	Random lifetime of a component
$E(X) = \mu$	Average lifetime of a component
A(a_1, a_2)	The set of possible actions in Markov decision process for components 1 and 2.
M	A large number representing the maximum age of the component

5.3.1 Time-varying expected maintenance costs

When repairing wind turbines, we want to minimize costs. For each component we add to our model, the number of actions doubles since we can maintain or not maintain the component. This means that we have five possibilities with a double-component model. The state space becomes 3-dimensional, $I = I_0 \text{x} I_1 \text{x} I_2$. We see that the number of decision variables increases exponentially in the number of components, which makes finding the optimal age-based maintenance policy for a double component more time-consuming than a single component.

Throughout the research, we want to choose the optimal policy while minimizing the costs, the costs that need to be minimized consist of:

- set-up costs (c_s)
- preventive maintenance costs (c_p)
- corrective maintenance costs (c_f)

For all formulations, we make the set-up costs constant, equal to a constant value. The preventive and corrective maintenance costs (PM and CM) are however a function of the period in the year indicated as i_0. As CM or PM increases proportionally for the repair of the components of two wind turbines, the costs simply double compared to one component. However, it matters for the set-up costs when one repairs two wind turbines at two different times, instead of both at the same time. The set-up costs stay constant regardless of the number of components that are necessary to be repaired at the same time. In the model, we split the set-up costs from the CM and PM costs. We make the assumption that the set-up costs are constant over time, and CM and PM costs are periodically different for different times of the year. We compute these values in the same way as we did in the replication. For multiple components, the CM and PM costs are not computed differently because they are determined by the period in time and not the age of the component. For component k, with k equal to component 1 or 2, we use the following to compute the costs.

$$c(i_0, i_k)(a) = \begin{Bmatrix} 0 & \text{if } a = 0 \\ c_p(i_0) & \text{if } a = 1,\ i_k \neq 0 \\ c_f(i_0) & \text{if } a = 1,\ i_k = 0 \end{Bmatrix} \tag{10}$$

Whenever a component fails, we directly apply corrective maintenance and replace the component with an as-good-as-new component. Also, when the other component is scheduled to be replaced in this period, preventive maintenance is applied to this as well. Then, the CM cost turns into CM plus PM costs. The set-up costs stay the same. However, there is one exception in which we need to pay the set-up costs twice, this is when one wind turbine fails to work and corrective maintenance is applied. If the other wind turbine fails as well within the same time period, the corrective maintenance needs to be applied again. We discretize the time in a solvable number of periods which is the number of months, 12. The possible actions we can take for the 2 components are as follows.

$$a = \begin{Bmatrix} 0,0 & \text{if we do not maintain} \\ 1,0 & \text{if we maintain component 1 only} \\ 0,1 & \text{if we maintain component 2 only} \\ 1,1 & \text{if we maintain both components} \end{Bmatrix} \tag{11}$$

To be able to come up with a linear program, we need a similar formulation to the approach for the one-dimensional setting. For the linear formulation, we need a finite state space. Assume that we save set-up costs whenever PM joins a CM action. We distinguish costs for components one and two, so the cost function is applicable if the components are not identical. When two identical components are used, one can set them equal for both corrective and preventive maintenance costs. The specified cost function becomes:

$$c_{i,a} = \begin{cases} 0, & \text{if } a = \{0,0\} \\ c_s + c^{f1}(i_0), & \text{if } a = \{1,0\}, i_1 = 0 \\ c_s + c^{f2}(i_0), & \text{if } a = \{0,1\}, i_2 = 0 \\ c_s + c^{p1}(i_0), & \text{if } a = \{1,0\}, i_1 \neq 0 \\ c_s + c^{p2}(i_0), & \text{if } a = \{0,1\}, i_2 \neq 0 \\ 2c_s + c^{f1}(i_0) + c^{f2}(i_0), & \text{if } a = \{1,1\}, i_1 = i_2 = 0 \\ c_s + c^{f1}(i_0) + c^{p2}(i_0), & \text{if } a = \{1,1\}, i_1 = 0, i_2 \neq 0 \\ c_s + c^{p1}(i_0) + c^{f2}(i_0), & \text{if } a = \{1,1\}, i_1 \neq 0, i_2 = 0 \\ c_s + c^{p1}(i_0) + c^{p2}(i_0), & \text{if } a = \{1,1\}, i_1 \neq 0, i_2 \neq 0 \end{cases} \tag{12}$$

Using this function we can save set-up costs. Note that the only moment that we pay twice the CM costs in one period is when both components shut down and corrective maintenance needs to be performed directly. In other words, if one component shuts down, it needs to be repaired directly. Afterward, when in the same period the other component shuts down, and the set-up costs need to be paid again because the maintenance can not be combined for both components. Furthermore, if a component fails before the other component's planned PM, the maintenance actions can be combined and set-up costs are saved.

5.4 Parameter specification and p-age replacement policy

Both components fail according to a distribution which gives the transition probability of failure p_x^k, for age x and component k. If we investigate the identical components, the distribution is the same. Hence, $p_x^1 = p_x^2 = p_x$. In the computation below, we determine the transition probabilities for non-identical components with a probability of $\pi_{ij}(a)$ transition state i_0, i_1, i_2 to j_0, j_1, j_2, and action a_0, a_1, one for each component. For all these probabilities that contain action $a_k = 0$ for component k, we implement an additional restriction in the probability matrix:

$$x_{i,a}^k = 0 \quad \forall k \leq n, i \in I, a \in A : i_k \in 0, M, a_k = 0 \tag{13}$$

We implement this by adding:

$$\textbf{if } a_k = 0, \quad \textbf{then } i_k \notin \{0, M\} \tag{14}$$

For the sake of order and cleanliness, we do not show this restriction in the probability matrix models below. $\pi_{(i_0,i_1,i_2),(j_0,j_1,j_2)}(a_1, a_2)$ gives the following output.

$$\pi_{ij}(0,0) = \begin{cases} (1 - p_{j_1}^1)(1 - p_{j_2}^2) & \text{for } j_0 = i_0 + 1, , j_1 = i_1 + 1, , j_2 = i_2 + 1, \\ (1 - p_{j_1}^1)p_{j_2}^2 & \text{for } j_0 = i_0 + 1, , j_1 = i_1 + 1, , j_2 = 0, \\ p_{j_1}^1(1 - p_{j_2}^2) & \text{for } j_0 = i_0 + 1, , j_1 = 0, , j_2 = i_2 + 1, \\ p_{j_1}^1 p_{j_2}^2 & \text{for } j_0 = i_0 + 1, , j_1 = 0, , j_2 = 0, \\ 0 & \text{else.} \end{cases} \tag{15}$$

$$\pi_{ij}(1,0) = \begin{cases} (1-p_1^1)(1-p_{j_2}^2) & \text{for } j_0 = i_0+1, j_1 = 1, j_2 = i_2+1, \\ (1-p_1^1)p_{j_2}^2 & \text{for } j_0 = i_0+1, j_1 = 1, j_2 = 0, \\ p_1^1(1-p_{j_2}^2) & \text{for } j_0 = i_0+1, j_1 = 0, j_2 = i_2+1, \\ p_1^1 p_{j_2}^2 & \text{for } j_0 = i_0+1, j_1 = 0, j_2 = 0, \\ 0 & \text{else.} \end{cases} \tag{16}$$

$$\pi_{ij}(0,1) = \begin{cases} (1-p_{j_1}^1)(1-p_1^2) & \text{for } j_0 = i_0+1, j_1 = i_1+1, j_2 = 1, \\ (1-p_{j_1}^1)p_1^2 & \text{for } j_0 = i_0+1, j_1 = i_1+1, j_2 = 0, \\ p_{j_1}^1(1-p_1^2) & \text{for } j_0 = i_0+1, j_1 = 0, j_2 = 1, \\ p_{j_1}^1 p_1^2 & \text{for } j_0 = i_0+1, j_1 = 0, j_2 = 0, \\ 0 & \text{else.} \end{cases} \tag{17}$$

$$\pi_{ij}(1,1) = \begin{cases} (1-p_1^1)(1-p_1^2) & \text{for } j_0 = i_0+1, j_1 = 1, j_2 = 1, \\ (1-p_1^1)p_1^2 & \text{for } j_0 = i_0+1, j_1 = 1, j_2 = 0, \\ p_1^1(1-p_1^2) & \text{for } j_0 = i_0+1, j_1 = 0, j_2 = 1, \\ p_1^1 p_1^2 & \text{for } j_0 = i_0+1, j_1 = 0, j_2 = 0, \\ 0 & \text{else.} \end{cases} \tag{18}$$

Keep in mind $i_0 \in I_0$. For the 2 components, we can now define the following linear program to find the optimal cost policy.

$$\min \sum_{(i_0,i_1,i_2)\in I} \sum_{a\in A} c_{i,a} x_{i,a} \tag{19}$$

$$\sum_{a\in A(i)} x_{i,a} - \sum_{j\in I} \sum_{a\in A(j)} \pi_{j,i}(a) x_{j,a} = 0, \quad \forall i = (i_0,i_1,i_2) \in I \tag{20}$$

$$\sum_{\forall i_2\in I_2} \sum_{\forall i_1\in I_1} \sum_{\forall a\in A(i_1,i_2} x_{i_0,i_1,i_2,a} = 1/N \quad \forall i_0 \in I_0 \tag{21}$$

$$x_{i,a} \geq 0, \quad \forall i = (i_0,i_1,i_2) \in I, \forall a \in A(i) \tag{22}$$

In objective 19 we minimize the total costs, which includes the total preventive and corrective costs. Constraint 20 ensures that the sum of the long-run probabilities of the system if it is in state i = (i_0, i_1, i_2) should be equal to the sum of, the probability of the transition to another state times the long-term probabilities of the next state j = j_0, j_1, j_2. Constraint 21 represents the long-run presence of components 1 and 2 in each period i_0, summed over all ages and actions. This should be equal to $1/N$, in other words, one over the number of periods. This implies that you spent each period the same time in the states. The long-run probability that the system is in state i = $(i_0, i_1, i_2 \in I)$ at the beginning of the period, and the decision $a \in A(i_1, i_2)$ is chosen. We implement this LP formulation using CPLEX in Java.

5.5 P-block replacement policy

In this section we add extra constraints, to make the LP into a MIP formulation. We end up with a periodically based blocked replacement policy (p-BRP). We add variable y_i that helps to decide when to scheme out preventive maintenance.

$$y_i^k = \begin{cases} 1 & \text{if we maintain component k preventively in period } i_0 \in I, \\ 0 & \text{else.} \end{cases} \tag{23}$$

with $k \in K$ and $K = \{1, 2\}$. The constraints that make sure that we do this, are equations 24 and 25.

$$x_{i,a} \leq 1 - y_{i_0}^k \qquad \forall i = (i_0, i_k) \in I : i_k > 0, \quad a_k = 0, \quad k \in K \tag{24}$$

$$x_{i,a} \leq y_{i_0}^k \qquad \forall i = (i_0, i_k) \in I : i_k > 0, \quad a_k = 1, \quad k \in K \tag{25}$$

If we maintain component k, from 24, $x_{i,a}$ must be zero for no action taken place, but we are allowed to have $x_{i,a}$ to be zero or non-zero for action $a_k = 1$ for this component(see 25). In short, maintaining a component k results in taking action for that component. After constructing the new variable and adding the latter equations to the ARP, we can define the new MIP formulation.

$$\min \sum_{(i_0,i_1,i_2) \in I} \sum_{a \in A} c_{i,a} x_{i,a} \tag{26}$$

$$\sum_{a \in A(i)} x_{i,a} - \sum_{j \in I} \sum_{a \in A(j)} \pi_{j,i}(a) x_{j,a} = 0, \quad \forall i = (i_0, i_1, i_2) \in I \tag{27}$$

$$\sum_{i \in I} \sum_{a \in A(i)} x_{i,a} = 1 \tag{28}$$

$$x_{i,a} + y_{i_0}^k \leq 1 \qquad \forall i = (i_0, i_k) \in I : i_k > 0, \quad a_k = 0, \quad k \in K \tag{29}$$

$$x_{i,a} - y_{i_0}^k \leq 0 \qquad \forall i = (i_0, i_k) \in I : i_k > 0, \quad a_k = 1, \quad k \in K \tag{30}$$

$$x_{i,a} \geq 0, \quad \forall i = (i_0, i_1, i_2) \in I, \quad \forall a \in A(i) \tag{31}$$

$$y_{i_0}^k \in \{0, 1\} \qquad \forall i_0 \in I_0, \quad k \in K \tag{32}$$

The objective 26 and the first constraints 27 and 28 are the same as the ones from the p-ARP model. In constraints 29 and 30 we ensure that if we perform an action in a certain period, we must perform preventive maintenance for any age i_1 or i_2 bigger than 0 and vice versa. Also, constraint 31 represents the long-run presence of components 1 and 2 in each period i_0, summed over all ages and actions, this should be equal to

$1/mN$, in other words, one over the total number of periods (number of years times periods in a year). This formulation for 2 components can be used to schedule the maintenance optimally. Since we have 2 identical components, we know that these are scheduled at the same time in the optimal solution. This ensures that the maintenance actions are grouped and can be forced by the constraint 33.

$$y_{i_0}^{k_1} = y_{i_0}^{k_2} \quad \forall i_0 \in I_0,\ k_1 \leq n,\ k_2 \leq n \tag{33}$$

5.6 P-Modified block-replacement policy

In the literature of T. N. Schouten et al. (2022), constraints were added to the LP of the single component age-base maintenance model to obtain a MIP for modified maintenance policies. A similar approach can be taken for two components. In this section, we use the same formulation as the p-ARP formulation, but we introduce extra constraints to end up with a modified block-based maintenance policy (p-MBRP).

Below we introduce an extra variable:

$$z_{i_0,i_k}^k = \begin{cases} 1 & \text{if component } k \text{ is maintained preventively in period } i_0 \text{ at age } i_k \\ 0 & \text{otherwise} \end{cases} \tag{34}$$

z indicates the period in which a component needs to be maintained when it reaches a certain age. This implies that if we are in the indicated period in time and if the component has reached this age or is older, then we must maintain as well. We construct the following constraint to ensure this:

$$z_{i_0,i_k} \leq z_{i_1,j_k} \qquad \forall i \in I,\ \forall j_k \in Ik : i_k < j_k, \tag{35}$$

Also, when we set $z_{i_0,i_k} = 1$ for a certain k, we must set y_{i_0} =1. This does not go vice versa, When we set y_{i_0} =1, $z_{i_0,i_k} = 1$ can be either 1 for component one or two. Hence, we end up with the next formula.

$$z_{i_1,i_2} - y_{i_1} \leq 0 \quad \forall i_0 \in I_0, \forall i_k \in I_k, \quad \forall k \in K \tag{36}$$

When the corrective maintenance is done right before the scheduled preventive maintenance, we must skip the preventive maintenance. To ensure this, we must add a minimum age to the component k at which the component is maintained in period i_0.

$$t_{i_0}^k \leq N + i_0 - j_0 y_{j_0} - N y_{j_0} \qquad \forall i_0,\ j_0 \in I_0 :\ j_0 < i_0 \tag{37}$$

$$t_{i_0}^k \leq N + i_0 - j_0 y_{j_0} \qquad \forall i_k, j_k \in I_k : j_k > i_k \tag{38}$$

When $y_{i_0} = 0$, $t_{i_0}^k$ can take up any value. After creating a minimum age, we need to connect the z and y variables to this minimum age so we do not maintain for ages under $t_{i_0}^k$. We introduce a big M that supports the either or formulation. Constraints41 and 40 to ensure the following restriction:

$$z_{i_1,i_2} = \begin{Bmatrix} y_{i_1}, & \text{if } i_2 \geq t_{i_1} \\ 0, & \text{else} \end{Bmatrix} \tag{39}$$

$$My_{i_0} - Mz_{i_0,i_k} - t^k_{i_0} + 1 + i_2 \leq M \quad \forall i_0 \in I_0, i_k \in I_k \tag{40}$$

$$Mz_{i_0,i_1} + t^k_{i_0} - i_k \leq M \quad \forall i_0 \in I_0, i_k \in I_k \tag{41}$$

Resulting in the complete formulation shown below:

$$\min \sum_{(i_0,i_1,i_2)\in I} \sum_{a\in A} c_{i,a}x_{i,a} \tag{42}$$

$$\sum_{a\in A(i)} x_{i,a} - \sum_{j\in I} \sum_{a\in A(j)} \pi_{j,i}(a)x_{j,a} = 0, \quad \forall i = (i_0, i_1, i_2) \in I \tag{43}$$

$$x_{i,a} + z^k_{i_0,i_k} \leq 1 \qquad \forall i = (i_0, i_k) \in I : i_k > 0, a_k = 0, k \in K \tag{44}$$

$$x_{i,a} - z^k_{i_0,i_k} \leq 0 \qquad \forall i = (i_0, i_k) \in I : i_k > 0, a_k = 1, k \in K \tag{45}$$

$$\sum_{i\in I} \sum_{a\in A(i)} x_{i,a} = 1 \tag{46}$$

$$z_{i_1,i_2} - y_{i_1} \leq 0 \quad \forall i_0 \in I_0, \forall i_k \in I_k, \quad \forall k \in K \tag{47}$$

$$z_{i_0,i_k} - z_{i_1,j_k} \leq 0 \forall i \in I, \quad \forall j_k \in Ik : i_k < j_k, \tag{48}$$

$$t^k_{i_0} + j_0 y_{j_0} + N y_{j_0} \leq N + i_0 \ \forall i_0, \quad j_0 \in I_0 : \ j_0 < i_0 \tag{49}$$

$$t^k_{i_0} + j_0 y_{j_0} \leq N + i_0 \ \ \forall i_k, j_k \in I_k : j_k > i_k \tag{50}$$

$$My_{i_0} - Mz_{i_0,i_k} - t^k_{i_0} \leq M - 1 - i_2 \quad \forall i_0 \in I_0, i_k \in I_k \tag{51}$$

$$Mz_{i_0,i_1} + t^k_{i_0} \leq M + i_k \quad \forall i_0 \in I_0, i_k \in I_k \tag{52}$$

$$x_{i,a} \geq 0 \quad \forall i \in I, a \in A \tag{53}$$

$$z_{i_0,i_k} \in \{0,1\} \quad \forall i0 \in I0, i_k \in IK \tag{54}$$

$$y_{i0} \in \{0,1\} \quad \forall i_0 \in I_0 \tag{55}$$

$$t_{i_0} \in \mathbb{N} \ \ \forall i_0 \in I_0 \tag{56}$$

The objective 42 and the first constraints 43 and 46 are the same as the one from the p-BRP model. If we perform an action in a certain period, then we must perform preventive maintenance in that period for any age i_1 or i_2 bigger than 0 and vice versa, this is ensured by constraints 47 and 48. The long-run presence of components 1 and 2 summed over all ages and actions, should be equal to 1. To ensure that the critical age is satisfied, we implement constraints 49 and 50.

6 Results

This section presents the results of the introduced policies ARP, BRP, and MBRP for time-varying costs. We define the input for the method used, analyze the output, and mention the most important results.

6.1 Two component model for identical components

Suppose that we have two identical components, this implies that we use the same distribution for the components. Also, we use the same scale and shape parameters within the same distribution. Not only do we use the same lifetime distribution, but also do we take the same cost function. Then we solve the LP formulation for the ARP and the MIP formulation for BRP and MBRP. Throughout all results we consider the number of periods equal to 12, indicating the months in a year. The preventive maintenance age is 12 as well.

Table 2: Cost minimalization result, for a single component, using policies ARP, BRP, and MBRP, and using the Weibull distribution with $\alpha = 1$ year and $\beta = 2$. We set $c_s = 5$, $c_p = 10$, and $c_f = 50$ using deltas 0%, 10%, 20%, 30%, 40%, 50% representing cost difference over the year.

	pARP 2		**pBRP 2**		**pMBRP 2**	
Δ	costs	savings	costs	savings	costs	savings
0%	91.39		96.29		94.01	
10%	91.28	0.12%	96.23	0.06%	93.99	0.02%
20%	90.80	0.65%	95.25	1.08%	93.17	0.89%
30%	89.94	1.59%	94.28	2.09%	92.32	1.80%
40%	88.71	2.93%	92.54	3.89%	90.75	3.47%
50%	87.24	4.73%	90.70	5.81%	89.19	5.13%
Running time	1 s		18 s		212 s	

From the table above, we find that the ARP is the optimal maintenance policy from the two-component formulations for any Δ. Also, MBRP is more cost-efficient than BRP. However, the computation time of BRP, which is 18 seconds, is much shorter than the one from MBRP. Still, this difference is too small to prefer the BRP over MBRP. Furthermore, an increase in the difference in costs over the year (Δ), results in a decrease in cost for each policy. Besides that, savings are increasingly rising with the difference in costs for any policy.

6.2 Single-component versus two-components

Both single-component and two-component have their best model as ARP. Increasing the number of components gives bigger differences in costs between the models. So the two-component models decrease faster in costs. As a result, the savings of the corresponding costs decrease faster as well. In addition, the ARP of the 2-component model is the lowest in cost. The biggest savings for both are the BRP with a cost fluctuation of 50%. Those savings are 9.92% and 5.81% for the single and two-component respectively. We see a significant increase in computation time for the BRP and MBRP when increasing the number of components. For the ARP, the computation time stays the same.

Table 3: Cost minimalization result, for a single component, using policies ARP, BRP, and MBRP, and using the Weibull distribution with $\alpha = 1$ year and $\beta = 2$. We set $c_s = 5$, $c_p = 10$, and $c_f = 50$ using deltas 0%, 10%, 20%, 30%, 40%, 50% representing cost difference over the year. We doubled the costs to make it comparable to the two-component analysis.

	pARP		**pBRP**		**pMBRP**	
Δ	costs	savings	costs	savings	costs	savings
0%	98.79		106.25		101.06	
10%	98.62	0.17%	105.70	0.52%	100.97	0.09%
20%	97.97	0.83%	103.20	2.87%	100.19	0.86%
30%	96.83	1.98%	100.70	5.22%	99.17	1.87%
40%	95.14	3.69%	98.21	7.57%	97.63	3.39%
50%	93.18	5.68%	95.71	9.92%	95.71	5.29%
Running time	1 s		3 s		1 s	

In our first figure, we plot the actions of components one and two, the axes of the components indicate the age. With an age equal to 0 when the component is broken and must be repaired. We use the following colors to indicate for which component we need to repair in which period of time and for which the probability of taking this action is non-zero. For example, a cell, which is based on the age of each component, containing the color dark orange ($a(0, 1)$), has a $\mathbb{P}(a(0, 1)) > 0$ for which we perform maintenance for component two and do not perform maintenance for component one.

- dark red, $\mathbb{P}(a(0, 0)) > 0$ for every month
- dark orange, $\mathbb{P}(a(0, 1)) > 0$ for every month
- dark yellow, $\mathbb{P}(a(1, 0)) > 0$ for every month
- dark green, $\mathbb{P}(a(1, 1) > 0$ for every month

The graph can be described as follows:

- When both components fail, both need to be maintained, this is shown by the dark green square at the bottom left.
- When one of the components fails and the age of the other component has not reached 3 months, only that component is repaired, shown by the dark orange and dark yellow bars.
- When one component fails and the age of the other component has reached 3 months, both components are repaired, shown by the dark green bars.
- When both components do not fail, preventive maintenance is performed for both once one of the components reaches the age of 12.
- There is a zero probability to reach a white cell.
- We observe complete symmetry because we work with components following an identical distribution.
- There is no cost difference, hence no preference for the time when performing maintenance.

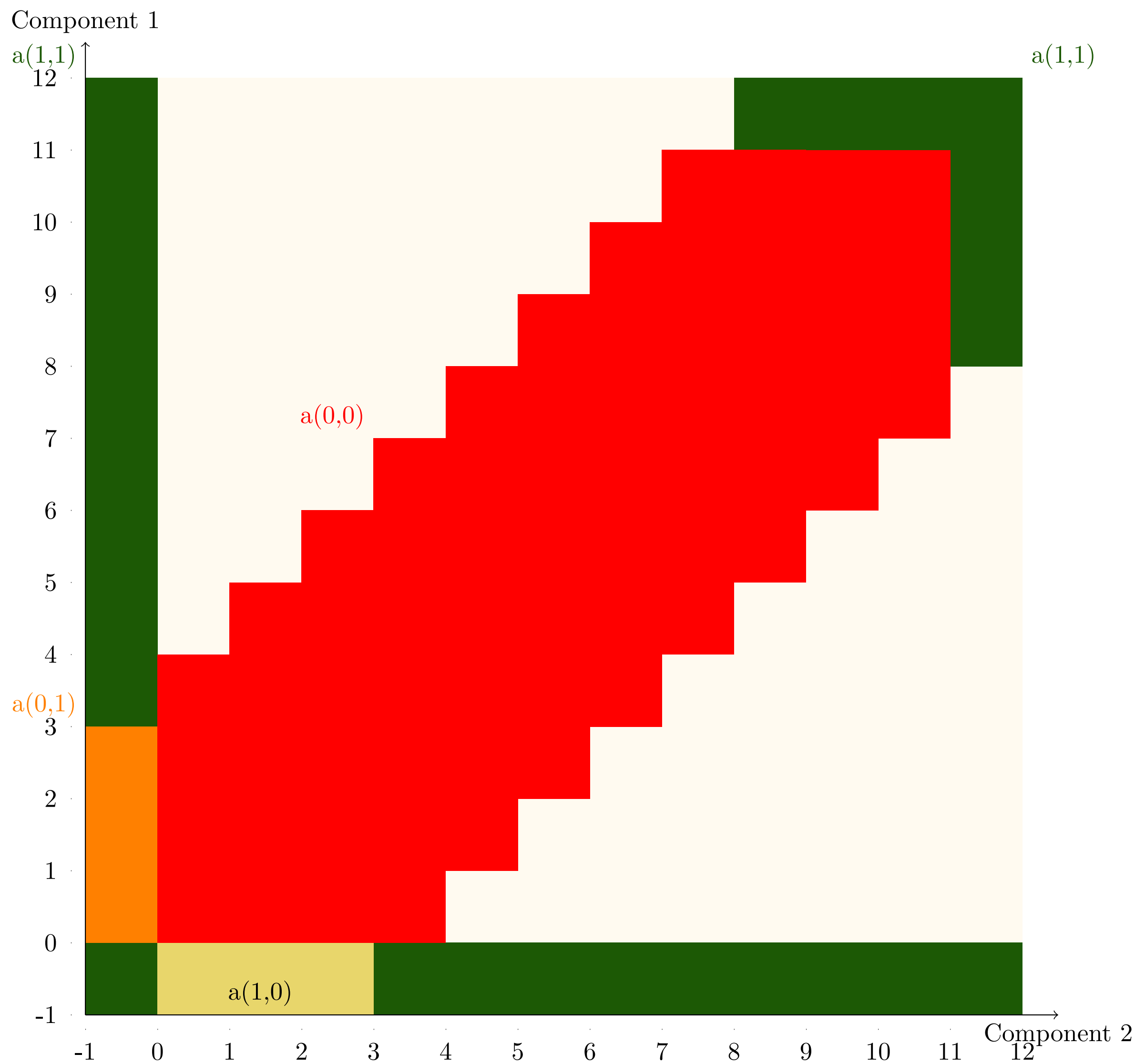


Figure 1: With the lifetime of the two components with each indicating its age on an axis, the action to maintain for which the probability is non-zero is plotted for $\delta = 0$. Because there is no cost difference, there is no preference to maintain in a certain period. We set $\alpha = 1$ year, $\beta = 2$, $c_S = 5$, $c_p = 5$ and $c_f = 15$

It is more interesting to look at the graph with the cost difference. For example, if we take $\Delta = 20$ we get the graph below. For this graph, we need to add some colors indicating the following:

- dark red, $\mathbb{P}(a(0,0)) > 0$ for every month
- dark orange, $\mathbb{P}(a(0,1)) > 0$ for every month
- dark yellow, $\mathbb{P}(a(1,0)) > 0$ for every month
- dark green, $\mathbb{P}(a(1,1) > 0$ for every month
- light red, $\mathbb{P}(a(0,0)) > 0$ approximately for 6 months of the year. In the other months we observe $\mathbb{P}(a(0,0)) = 0$

- light orange/yellow, $\mathbb{P}(a(0,1)) > 0$/ $\mathbb{P}(a(1,0)) > 0$ approximately for 5 months of the year. In the other months we observe $\mathbb{P}(a(1,1) > 0$
- green, $\mathbb{P}(a(1,1) > 0$ for at least 6 months of the year. For the other months we observe $\mathbb{P}(a(0,0)) > 0$.
- light green, $\mathbb{P}(a(1,1) > 0$ for less than 6 months of the year. Again, for the other months, we observe $\mathbb{P}(a(0,0)) > 0$.

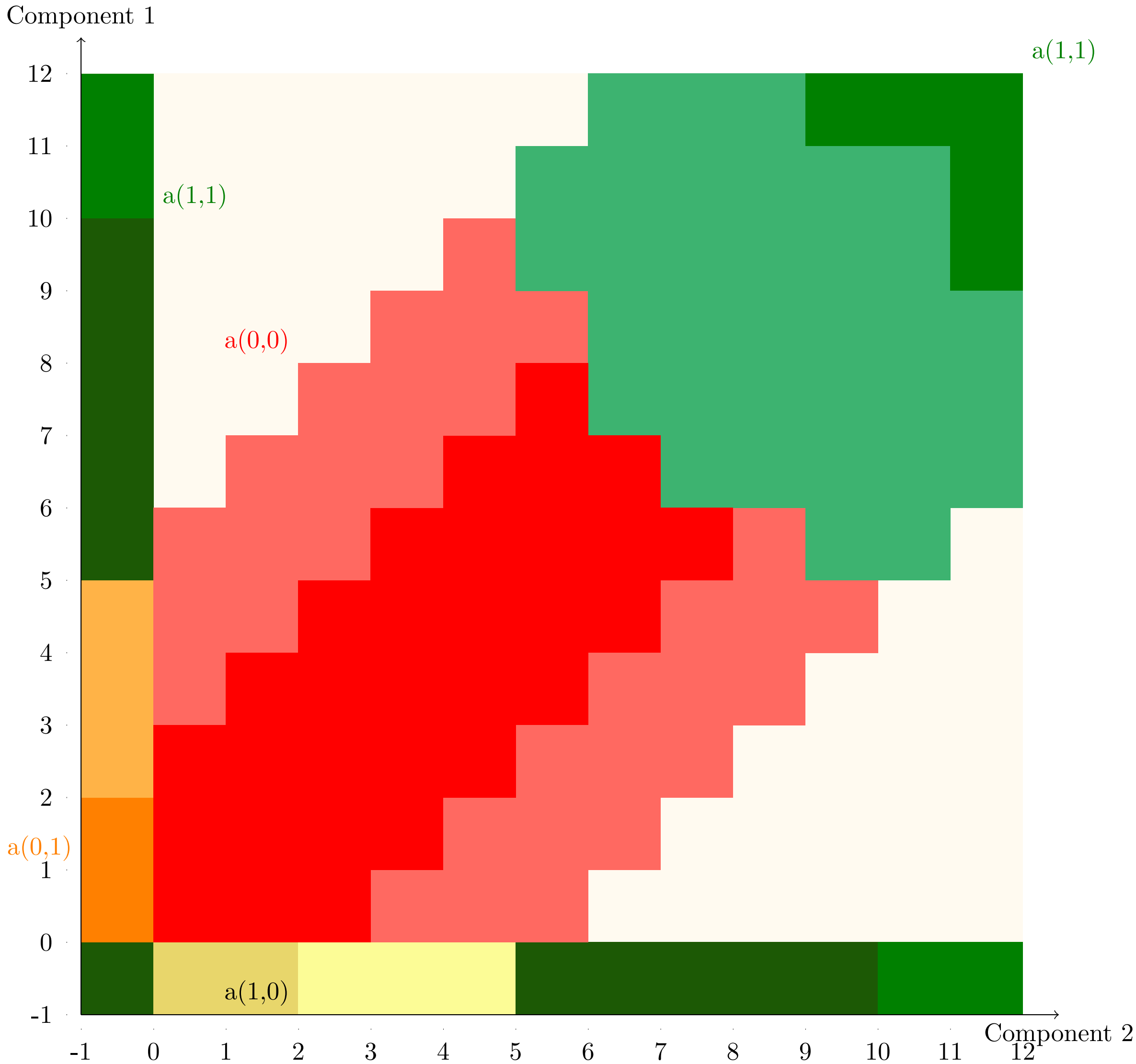


Figure 2: With the lifetime of the two components with each indicating its age on an axis, the action to maintain for which the probability is non-zero is plotted for $\delta = 20$. Because there is no cost difference, there is no preference to maintain in a certain period. We set $alpha = 1$ year, $beta = 2$, $c_S = 5$, $c_p = 5$ and $c_f = 15$

Interesting results from this graph are first of all, the state space for taking certain actions in every month, decreases. Whereas the total state space is larger than when there is no cost difference and the action taken often depends on the current month. Also, the graphs are roughly alike, but the second graph shows more of a gradual change between the different actions taken. Furthermore, the graph can be understood intuitively like the graph of no cost difference.

6.2.1 Minimum age for maintenance MBRP

The minimum age for the two-component MBRP policy is for most periods in the year 0. For the months of February and August, there is a minimum age of 4 months for both components. In these months we perform preventive maintenance as well. In other words, preventive maintenance is skipped for component k if component k has failed at least once in the last four months. For the single component, we have the months August and September with minimum ages of 11 months and 1 month respectively. Here, we perform preventive maintenance in September only.

6.3 Conclusions for two-component setting

The maintenance costs decrease for increasing cost differences due to seasonality. These costs decrease at a faster rate than linear, and so do the savings. We see these results for all policies ARP, BRP, and MBRP. Looking at the single component, we can conclude the same. The ARP policy is the cheapest policy to implement. The MBRP can be preferred over BRP which is logical as the MBRP has more flexibility in its policy. Furthermore, we prefer a maintenance plan for two components than one for one component.

7 Conclusion

The objective of this paper is to answer our two research questions:

How can we optimize time-varying maintenance costs for two components?

Is there a benefit of maintaining two components in comparison with one component?

The age replacement policy turns out to be the optimal model to optimize time-varying maintenance costs for both single and two components independently to what extent the difference in costs over the year was taken. The lowest costs can be obtained if we optimize for two components, taking the ARP policy with a relative cost difference of 50%. To answer the second question: for any policy of the two-component models, computing the costs turns out to result in lower costs than using that same policy for the single-component model. The age replacement policy is the optimum policy when minimizing costs and is thus the most efficient. Knowing that the increase of a single-component to two components is efficient arises the question if expanding to even more components becomes more efficient. This is mainly interesting regarding the computation time as the ones from the BRP and MBRP policies increased a lot for the two-component in comparison with the single component.

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