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Optimizing Mobile Dentistry Services in Rural Areas: Exploring the Impact of Working Week Structures and Tour Designs

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Abstract

This research explores the potential implementation of mobile dentistry services in rural areas using the Mobile Dentistry Network Design Problem (MDNDP), a mixed-integer optimization model. By using different balances between patient-treated maximization, net revenue maximization, and fair patient allocation, the model generates solutions containing only back-and-forth trips to a central hub. A Montana case study demonstrates that a 5-day working week in the MDNDP leads to a fairer patient allocation (minimum disparity of 17%) compared to a 4-day working week (minimum disparity level of 31%) but at the expense of significantly decreased net revenues. The inclusion of tours in the Mobile Dentistry Network With Tours Design Problem (MDNWTDP) demonstrates favorable impacts on patient-treated-maximizing solutions but does not yield improvements when maximizing net revenues. These findings are consistently supported by a simulation study. However, the incorporation of tours does result in increased computational time. To address this challenge, a heuristic approach is introduced, which offers relatively good solutions with a maximum net revenue gap of 14,2% in the simulation study. Nonetheless, the heuristic does not surpass the performance of the MDNDP model.

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1 Introduction

Regular attendance to dental care is essential for maintaining optimal oral health (Thomson et al., 2010), which, in turn, plays a critical role in supporting overall health and well-being (Davis et al., 2010). For instance, oral infections have been associated with an increased risk of developing diabetes, heart diseases, and stroke (U.S. Department of Health and Human Services, 2000). While it is known that inequalities in the availability of oral health care give rise to various conditions such as malnutrition, childhood speech problems, infections, diabetes, heart disease, and premature births (National Research Council, 2012), access to dental care in rural areas is still marked by significant disparities (U.S. Department of Health and Human Services, 2004). Therefore, an imperative exists to identify a solution.

Crall (2006) describes how financial barriers create disparities in dental care access. A public insurance program called "Medicaid" (Medicaid.gov, 2016) in the United States provides coverage for dental services, however, patients may face challenges in finding dental providers who accept this insurance. Non-profit Community Health Centers (CHCs) are often important access points for dental care, but especially in rural areas, there can still be a large distance from a person's home to the nearest CHC. Mobile dentistry has emerged as a viable solution to address the challenges of limited access to dental care. This approach entails dentists, dental hygienists, and dental assistants traveling in vans or trailers with onboard dental equipment.

One of the states in the United States that is known for its rural areas is Montana. The reference paper by Thorsen and McGarvey (2018) describes a study on the Mobile Dentistry Network Design Problem (MDNDP) to assess the potential of a mobile dentistry service in Montana's rural regions. The MDNDP involves the operational framework of the mobile dentistry service, which is initially based at a central hub. From this hub, the mobile dentistry unit is permitted to travel to other CHCs. To optimize efficiency, the mobile dentistry unit has the flexibility to stay overnight at other CHCs than the hub, thus reducing travel time. However, it is important to note that the MDNDP prohibits direct travel between non-hub CHCs as Thorsen and McGarvey (2018) argue against incorporating tours due to the time required for setup and breakdown. Additionally, it is required to return to the central hub for the weekend. The MDNDP is solved using an algorithm that utilizes a Mixed Integer Linear Program (MILP) formulation.

Thorsen and McGarvey (2018) implement a 4-day working week, during which the dental care team operates for 10 hours each day. The study indicates that serving six locations yields a minimum disparity level of 19%. In this context, "disparity" refers to the relative difference between the actual number of patients served at a particular location and the number of patients that should be served proportionally to the demand at that location and other locations. The inclusion of three additional locations results in an increased minimum disparity level of 31%, and thus has a negative impact on fair patient allocation.

This research aims to explore potential enhancements to the MDNDP. Specifically, two improvements are considered: transitioning from a 4-day to a 5-day working week and incorporating tours into the model. The introduction of a 5-day working week allows for increased planning flexibility and has the potential to generate solutions for lower disparity levels. We find that employing a 5-day working week while serving nine locations leads to a significant reduction in the minimum disparity level, reaching as low as 17%. However, this transition necessitates a reduction in daily working hours from 10 to 8, maintaining a total of 40 working hours per week. As traveling becomes a relatively larger part of the working day, less time is left to treat patients, resulting in a decline in net revenues. The most profitable solution achieved when employing a 4-day working week generates \$36,910, while the most profitable solution when using a 5-day working week generates \$26,370, indicating a significant decrease in profitability.

To evaluate the potential benefits of incorporating tours into the MDNDP, we introduce the Mobile Dentistry Network With Tours Design Problem (MDNWTDP). This modification allows for travel between locations through overnight stays. This entails the mobile dentistry staff staying overnight at a non-hub CHC, and the following morning, they travel to another non-hub CHC. Such tours are limited to once per week to reduce the maximal total travel time in one week and thereby accommodate the mobile dentistry staff's comfort and well-being (Wheatley & Bickerton, 2016). We apply the MDNWTDP to both the original 4-day working week and the extended 5-day working week. We see that the MDNWTDP outperforms the MDNDP in both scenarios in terms of patient-treated maximized solutions, however, no significant improvement is made for net revenue maximized solutions.

As the MDNWTDP shows improvements in the Montana case study of Thorsen and McGarvey (2018), its potential value in other contexts merits examination. Consequently, we conduct a simulation involving locations characterized by relatively high-demand locations, situated distant from the hub, with smaller demand locations located close to the high-demand location. In this simulation, the MDNWTDP exhibited superiority over the MDNDP when optimizing for total patients, presenting solutions that treat a greater number of patients at the selected disparity levels. However, the improvements in profits for maximized net revenue solutions remain minimal, with a maximum improvement of only 0.5% compared to the MDNDP. Notably, all solutions generated by the MDNDP are encompassed within the set of solutions obtained by the MDNWTDP, ensuring that the MDNWTDP never produces inferior solutions. However, it is important to consider that the computational time required to solve the MDNWTDP is significantly longer than that of the MDNDP.

To address the computational challenge, we introduce a heuristic. This heuristic rapidly generates solutions that are relatively close in quality to those produced by the exact models, with net revenues approaching the levels achieved by the exact models. In the simulation study, the gaps between the solutions obtained by the heuristic and the profit-maximizing solutions range from a minimum of 6.4% to a maximum of 14.2%. However, it is important to note that the heuristic never surpasses the performance of the MDNDP model.

To provide a concise overview, our investigation reveals that, a 5-day working week leads to a decrease in the minimum disparity level compared to a 4-day working week. However, it also results in a reduction in net revenues. In terms of generating solutions that maximize the number of patients treated, the MDNWTDP model outperforms the MDNDP model, however, the MDNWTDP model is not superior in providing solutions that maximize profits.

The subsequent sections aim to present additional literature related to topics similar to the Mobile Dentistry problem. Afterward, we formally introduce our methods and provide a detailed description of the data employed to derive the eventual results. Finally, we provide a conclusion based on the findings obtained.

2 Literature Review

Thorsen and McGarvey (2018) conducted a thorough investigation on the feasibility of providing mobile dentistry services in Montana. The introduced MILP formulation to solve the MDNDP includes constraints to account for overnight stays, the number of treated patients, and net revenues to obtain optimal solutions. Additionally, disparity constraints are incorporated to ensure a fair distribution of patients across different locations. An algorithm is proposed in which the MILP is solved two times in order to find the efficient frontiers per disparity level in terms of treated patients and annual net revenue. A negative relationship is identified between the disparity of patient distribution and net revenue, whereby an increase in the number of patients served from remote areas resulted in a decrease in net revenue due to travel time and overnight expenses. However, tours are not used in this formulation as explained in Section 1, but the inclusion of tours in similar cases has been investigated in other studies. In the next subsection, I elaborate on these studies.

2.1 Tour-based facility problems: challenges and approaches

Facility problems involving the selection of the most optimal routes can be effectively addressed through the utilization of a set covering approach. Nevertheless, Halper and Raghavan (2011) assert that these problems present computational challenges due to their NP-hard nature. While an algorithm has been introduced to tackle the set covering problem within a polynomial time frame, it solely focuses on maximizing the fulfillment of demand and does not incorporate revenue optimization. Furthermore, the algorithm does not consider any disparity constraints. Given these limitations, the heuristic is deemed inapplicable to our specific problem. Consequently, an alternative approach is pursued, considering the computational complexity inherent in the set covering methodology. Another study on mobile healthcare facilities was conducted by Doerner, Focke and Gutjahr (2007), who proposed three algorithms to approximate tours for mobile healthcare in Senegal. However, it should be noted that the problem described in the paper did not include the constraint of healthcare workers being available at their homes during weekends. Consequently, directly applying the algorithms proposed in that study becomes challenging in our context, where adherence to such constraints is essential. To optimize the route and schedule of a fleet of mobile facilities that serve customers with uncertain demand, Lei, Lin and Miao (2014) proposed a two-stage stochastic formulation to minimize the total cost incurred during the planning horizon. Lei, Lin and Miao (2016) further developed a two-stage robust optimization approach for mobile facility fleet sizing and routing under demand uncertainty using a network flow formulation. These approaches only consider stochastic demand and do not account for setup and breakdown times. In contrast, the variant of the vehicle routing problem proposed by Doerner et al. (2007) considers setup and breakdown times, with a setup time of 10 minutes. However, Thorsen and McGarvey (2018) reported that other mobile dentistry operations in Montana required setup times of around 30-45 minutes, with similar breakdown times. In contrast to Lei et al. (2016), we do not have to optimize for fleet size, as a fixed fleet size is chosen by Thorsen and McGarvey (2018). Berman, Krass and Drezner (2003) consider the maximum cover location problem (MCLP) with partial coverage defined by two coverage radii S_1 and S_2 ($S_1 < S_2$). It is fully covered if a demand point can be covered by its closest facility in less than distance S_1. If the distance is between S_1 and S_2, the demand point is partially covered. If the distance is more than S_2, the demand point is not covered. The objective of MCLP is to establish a set of m facilities that maximize the total weight of "covered" customers, where a customer is considered covered if he is located within a specified distance away from the closest facility. However, it's crucial to consider the implications of following MDNDP's maximum disparity levels. Opening all facilities becomes necessary to meet these requirements, making the Maximum Covering Location Problem (MCLP) approach inadequate for our mobile dentistry challenges. Current and Schilling (1994) developed the median tour problem (MTP) and maximal covering tour problem (MCTP). The objective of the MTP is to reduce the overall travel distance between each demand node and its closest node on the tour, while also minimizing the length of the tour. On the other hand, the MCTP aims to maximize the total demand within a predetermined distance from a stop on the tour. The study by Ozbaygin, Yaman and Karasan (2016) aims to maximize demand coverage subject to a distance limit similar to the MCTP. However, their coverage function differed in that it fully covered the demands of vertices on the tour, while only a certain percentage of the demand of a vertex was covered if it was not visited but within a specified distance of a tour stop. However, the MTP, MCTP and the variant proposed by Ozbaygin et al. (2016) are not suitable for solving the MDNWTDP due to the need to optimize net revenue and distribute patients proportionally across locations. We see that the problem at hand presents a distinctive set of challenges that distinguish it from the issues discussed in the aforementioned literature. Specifically, our situation necessitates adherence to disparity constraints, thus rendering full coverage of demand at a single location unattainable. Furthermore, it is essential to acknowledge the inherent limitations of mobile dentistry, where only a limited number of patients can be treated per hour and the mobile dentistry workers must be at home on weekends. Consequently, the combination of disparity constraints with optimization techniques encompassing patient allocation and net revenue accentuates the uniqueness of the mobile dentistry problem, calling for the development of a tailored approach.

Section 3 provides a comprehensive elucidation of the methods employed to address these challenges and solve the MDNWTDP effectively.

2.2 Heuristic approaches

Heuristics serve as valuable tools for reducing computational time, albeit at the expense of obtaining sub-optimal solutions. Among these heuristics, two prominent types are greedy and local search algorithms. Merz and Freisleben (2002) describe how greedy algorithms are intuitive strategies that make greedy choices to attain a specific objective. On the other hand, local search algorithms, such as hill-climbing, are improvement heuristics that explore the neighborhood of the current solution in search of better alternatives until no further enhancements can be made. A hybrid algorithm combining both greedy and local search techniques, referred to as a greedy local search, applies a hill-climbing approach by selecting the local move that yields the greatest improvement in the objective function. Although this method terminates at a local optimum, it often serves as a reasonable approximation of the global optimum (Selman & Gomes, 2006).

3 Methodology

In this section, we formulate the MDNDP and MDNWTDP as MILPs and introduce an algorithm that is used to compute the efficient frontiers in terms of the number of patients treated and net revenues. Furthermore, we formally introduce the heuristic that is used to approach the solutions of the MDNDP and MDNWTDP.

3.1 Model for the MDNDP

First, we introduce the notation that is used in the MDNDP model. The notation is contained in Table 1.

Index sets	i	$i \in$ location set I
Data variables	a_i	the total time required for a round trip, including travel time and setup/breakdown times, to visit location i from the hub
	b_i	the maximum number of patients that can be treated per day at location i
	c	revenue per patient
	e	total fixed expense of the operation
	f	total patients who can be treated per hour
	g	the overall count of working days within the horizon, accounting for two weeks of non-operation within a 26-week period
	p_i	percentage of total under-served patient population living within a prescribed radius of location i
	α	lever to adjust the sensitivity of total net revenue
	β	lever to adjust the sensitivity of service disparity across locations
	γ	lever to adjust the sensitivity of total patients treated
	O	overnight stay cost per night (based on a three-night trip)
	O'	marginal additional cost for a two-night overnight trip
	$\hat{O}$	marginal additional cost for a one-night overnight trip
	Q_i	the increase in the number of patients treated when staying overnight at location i, taking into account the elimination of round trip travel time on the additional day
Continuous variables	v	total net revenue
Integer variables	z	total patients treated
	y_i	total patients to be served at location i
	x_i	number of days that location i is visited
	w_i	total number of overnight stays at location i
	u_i	number of three-night overnight stays at location i
Binary variables	u'_i	number of two-night overnight stays at location i
	$\hat{u}_i$	number of one-night overnight stays at location i

Table 1: List of the variables used in the MDNDP model

Using the introduced variables, we formulate the following MILP model to solve the MDNDP:

$$\text{Maximize: } z = \sum_{i \in I} y_i \tag{1}$$

$$\text{Subject to: } \sum_{i \in I} x_i = g \tag{2}$$

$$y_i = b_i x_i + Q_i w_i \qquad \forall i \in I \tag{3}$$

$$y_i \leq d_i \qquad \forall i \in I \tag{4}$$

$$v = c \sum_{i \in I} y_i - \sum_{i \in I} (O w_i + O' u'_i + \hat{O} \hat{u}_i) - e \tag{5}$$

$$v \geq \alpha \tag{6}$$

$$z \geq \gamma \tag{7}$$

$$y_i \leq p_i (1 + \beta) z \qquad \forall i \in I \tag{8}$$

$$y_i \geq p_i (1 - \beta) z \qquad \forall i \in I \tag{9}$$

$$w_i \leq 0,75 * x_i \qquad \forall i \in I \tag{10}$$

$$3u_i + 2u'_i + \hat{u}_i = w_i \qquad \forall i \in I \tag{11}$$

$$u'_i,\, \hat{u}_i \in \mathbb{B} \qquad \forall i \in I \tag{12}$$

$$y_i,\, x_i,\, w_i \in \mathbb{N} \qquad \forall i \in I. \tag{13}$$

The objective function states that we want to maximize the number of patients served. Constraint (2) ensures that we visit exactly one location each day over our horizon. Constraints (3) identifies the number of patients that are served per location based on how many times the dental team visits and overnights at a location, and Constraints (4) enforces that the number of patients served at a location is not higher than the number of underserved patients at that location. Constraint (5) computes the net revenue. Constraints (6) and (7) are used in the sensitivity analysis. Using these constraints, we can demonstrate and evaluate the trade-off between net revenue and patients served. We can do another sensitivity analysis regarding the service disparity using Constraints (8) and (9). When we set β equal to 0, the total patients served at location i has to be equal to the percentage of the total under-served population residing within a prescribed radius of location i. Constraints (10) limit the number of overnights at each location to at most three nights per round-trip, as initially a 4-day working week is used. In Section 4.2 we show which model is used for the 5-day working week. Finally, Constraints (11) count the total number of overnights per location. One-night or two-night trips are more expensive per night than a three-night trip, which will be explained in Section 4. As the formulation attempts to maximize net revenue, it will prefer to assign as many three-night overnight stays as possible, and would never perform more than a single one-night or two-night overnight stay, which allows us to define u'_i and $\hat{u}_i$ as binary variables, which is denoted in Constraints (12). Constraints (13) denote the domain of the other decision variables.

3.2 The MDNDP model using a 5-day working week

A 5-day working week presents an opportunity to incorporate four overnight stays, necessitating modifications to the model proposed in the previous subsection. To accommodate this, we introduce an additional variable, $\tilde{O}$, which represents the marginal cost associated with planning a trip involving three overnight stays. The variable u_i denotes the number of four nights used at location i in this formulation. The binary variable $\tilde{u}_i$ is used to indicate whether a three-night stay at location i is included in the planning. Consequently, Constraint (5) is adjusted as follows:

$$v = c\sum_{i\in I} y_i - \sum_{i\in I}(Ow_i + O'u'_i + \hat{O}\hat{u}_i + \tilde{O}\tilde{u}) - e. \tag{14}$$

Furthermore, we change Constraints (10) and (11) into the following constraints:

$$w_i \leq 0.8 * x_i \qquad \forall i \in I \tag{15}$$

$$4u_i + 3\tilde{u} + 2u'_i + \hat{u}_i = w_i \qquad \forall i \in I. \tag{16}$$

3.3 Algorithm for sensitivity analysis

We propose Algorithm 1 to conduct the sensitivity analysis. Define the optimization model in 3.1 as MILP1. Furthermore, a second optimization model (MILP2) is defined where Constraint (5) is set as the objective value, and Equation (1) is added as a new constraint. Thus in this formulation, we maximize net revenue. We explore various values of β in the set K to construct efficient frontiers. The iteration process involves maximizing the number of patients treated and subsequently adjusting α slightly higher than the profits obtained for this iteration. This iterative approach aims to achieve higher net revenues in subsequent solutions. As a result, the final solution obtained for each disparity level represents the one that maximizes net revenue.

Algorithm 1. Generation of efficient frontiers

Step 0: Set $\beta = 0$
Step 1: Loop over the elements $k \in K$
  Step 1.1: Set $\alpha = -\inf$
  Step 1.2: While MILP1 and MILP2 are feasible for α and β
    Step 1.2.1: Set $\gamma = 0$
    Step 1.2.2: Solve MILP1, recording optimal objective function as z*
    Step 1.2.3: Set $\gamma = z^*$
    Step 1.2.4: Solve MILP2, recording optimal objective function value as v^*
    Step 1.2.5: Record v^* and z^* as optimal solutions for the current iterates (j, k)
    Step 1.2.6: Set $\alpha = 1.000001 * v^*$ and return to 1.2.1
  Step 1.3: Set $\beta = \beta + 0.01$ and return to 1.1

3.4 Model for the mobile dentistry network with tours design problem

In the MDNWTDP we use a similar notation as the MDNDP. However, the differences and additions are stated in Table 2.

Index sets	i	$i \in$ location set I
	t	$t \in$ working day set T
	$\hat{t}$	$\hat{t} \in$ working week set $\hat{T}$
Data variables	m_{ik}	indicator if travel is possible between location i and location k
	q_i	extra patients treated at location i when travel is started from location i
	$\hat{q}_{ik}$	extra patients at location k when travel is used from location i to location k
	h_1	the costs of one overnight stays
	h_2	the costs of two overnight stays
	h_3	the costs of three overnight stays
	h_4	the costs of four overnight stays
	M	large number
	L	length of one working week
Integer variables	$A_{i\hat{t}}$	number of nights at location i in week $\hat{t}$
	t_{ij}	travel time between location i and j in minutes
Binary variables	x_{it}	denotes whether location i is visited at day t
	s_{ikt}	denotes whether a tour between location i and k is used between day t and $t+1$
	w_{it}	denotes whether a overnight to stay at location i is used at location i on day t
	$a^n_{i\hat{t}}$	equal to zero if $A_{i\hat{t}}$ is smaller than n ($n \in$\{1, 2, 3, 4\})
	$b^n_{i\hat{t}}$	equal to zero if $A_{i\hat{t}}$ is greater than n ($n \in$\{1, 2, 3, 4\})
	$o^n_{i\hat{t}}$	equal to 1 if $A_{i\hat{t}}$ is equal to n ($n \in$\{1, 2, 3, 4\})

Table 2: List of the added or changed variables used in the MDNWTDP model

The variables described in Table 2 are used to define the model that solves the MDNWTDP. We add several routing and overnight constraints compared to the model that solves the MDNDP. Moreover, a time dimension is added. Day $t = 0$ is defined as the first day.

$$\text{Maximize: } z = \sum_{i \in I} y_i \tag{17}$$

$$\text{Subject to: } \sum_{i \in I} x_{it} = 1 \qquad \forall t \in T \tag{18}$$

$$y_i = \sum_{t \in T} b_i x_{it} + \sum_{t \in T} \sum_{k \in I} q_i s_{ikt} + \sum_{t \in T} \sum_{k \in I} \hat{q}_i s_{kit} + \sum_{t \in T} Q_i w_{it} \qquad \forall i \in I \tag{19}$$

$$y_i \leq d_i \qquad \forall i \in I \tag{20}$$

$$s_{ikt} \leq m_{ik} \qquad \forall i, k \in I \tag{21}$$

$$w_{it} \leq x_{it} \qquad \forall i \in I, t \in T \tag{22}$$

$$w_{i,t-1} \leq x_{it} \qquad \forall i \in I, t \in T \tag{23}$$

$$s_{ikt} \leq x_{it} \qquad \forall i, k \in I, t \in T \tag{24}$$

$$s_{ik,t-1} \leq x_{kt} \qquad \forall i, k \in I, t \in T \tag{25}$$

$$\sum_{t=\hat{t}}^{\hat{t}+5} \sum_{i \in I} \sum_{k \in I} s_{ikt} \leq 1 \qquad \forall \hat{t} \in \hat{T} \tag{26}$$

$$A_{i\hat{t}} = \sum_{t=\hat{t}}^{\hat{t}+L-1} w_{it} + \sum_{t=\hat{t}}^{\hat{t}+L-1} \sum_{k \in I} s_{ikt} \qquad \forall i \in I, \hat{t} \in \hat{T} \tag{27}$$

$$A_{i\hat{t}} \leq L - 1 \qquad \forall i \in I, \hat{t} \in \hat{T} \tag{28}$$

$$\sum_{i \in I} w_{it} + \sum_{i \in I} \sum_{k \in I} s_{ikt} \leq 1 \qquad \forall t \in T \tag{29}$$

$$\sum_{i \in I} w_{it} + \sum_{i \in I} \sum_{k \in I} s_{ikt} = 0 \qquad \forall t \in \{L-1, 2L-1, 3L-1, ..., g-1\} \tag{30}$$

$$A_{i\hat{t}} \geq 4.1 - M(1 - a^4_{i\hat{t}}) \qquad \forall i \in I, \hat{t} \in \hat{T} \tag{31}$$

$$A_{i\hat{t}} \leq 3.9 - M(1 - b^4_{i\hat{t}}) \qquad \forall i \in I, \hat{t} \in \hat{T} \tag{32}$$

$$1 - o^4_{i\hat{t}} \leq a^4_{i\hat{t}} + b^4_{i\hat{t}} \qquad \forall i \in I, \hat{t} \in \hat{T} \tag{33}$$

$$A_{i\hat{t}} \geq 3.1 - M(1 - a^3_{i\hat{t}}) \qquad \forall i \in I, \hat{t} \in \hat{T} \tag{34}$$

$$A_{i\hat{t}} \leq 2.9 - M(1 - b^3_{i\hat{t}}) \qquad \forall i \in I, \hat{t} \in \hat{T} \tag{35}$$

$$1 - o^3_{i\hat{t}} \leq a^3_{i\hat{t}} + b^3_{i\hat{t}} \qquad \forall i \in I, \hat{t} \in \hat{T} \tag{36}$$

$$A_{i\hat{t}} \geq 2.1 - M(1 - a^2_{i\hat{t}}) \qquad \forall i \in I, \hat{t} \in \hat{T} \tag{37}$$

$$A_{i\hat{t}} \leq 1.9 - M(1 - b^2_{i\hat{t}}) \qquad \forall i \in I, \hat{t} \in \hat{T} \tag{38}$$

$$1 - o^2_{i\hat{t}} \leq a^2_{i\hat{t}} + b^2_{i\hat{t}} \qquad \forall i \in I, \hat{t} \in \hat{T} \tag{39}$$

$$A_{i\hat{t}} \geq 1.1 - M(1 - a^1_{i\hat{t}}) \qquad \forall i \in I, \hat{t} \in \hat{T} \tag{40}$$

$$A_{i\hat{t}} \leq 0.9 - M(1 - b^1_{i\hat{t}}) \qquad \forall i \in I, \hat{t} \in \hat{T} \tag{41}$$

$$1 - o^1_{i\hat{t}} \leq a^1_{i\hat{t}} + b^1_{i\hat{t}} \qquad \forall i \in I, \hat{t} \in \hat{T} \tag{42}$$

$$v = c \sum_{i \in I} y_i - \sum_{i \in I} \sum_{\hat{t} \in \hat{T}} \sum_{l=1}^{L-1} (h_l o^l_{i\hat{t}}) - e \tag{43}$$

$$v \geq \alpha \tag{44}$$

$$z \geq \gamma \tag{45}$$

$$y_i \leq p_i(1 + \beta)z \qquad \forall i \in I \tag{46}$$

$$y_i \geq p_i(1 - \beta)z \qquad \forall i \in I \tag{47}$$

In this linear model, we maximize the number of patients treated. Furthermore, Constraints (18) ensure that one location is visited each day, whereas Constraints (19) compute the number of patients per location. Due to the Constraints (21), only tours between locations are utilized, when this is possible. If an overnight is planned at location i at time t in order to operate the next day at location i as well, it necessitates visiting location i on both day t and $t + 1$. Similarly, if a tour is scheduled from location i to k at time t, it requires visiting location i at time t and location k at time $t + 1$. Constraints (22), (23), (24), and (25) ensure that the visits to a location align with the planned overnight stays. Because of Constraints (26), only one overnight is combined with a tour per week. Constraints (27) tallies the number of overnight stays at location i in week $\hat{t}$. Constraint (28) ensures that the total number of overnight stays within a single working week does not exceed the available number of nights in that week. Only one overnight can be used per night, due to Constraints (29). Overnights on the last day of the working week are prohibited, as indicated by Constraints (30). Constraints (31) sets $a^4_{i\hat{t}}$ to zero if $A_{i\hat{t}}$ is less than 4.1. Likewise, Constraints (32) sets $b^4_{i\hat{t}}$ to zero if $A_{i\hat{t}}$ exceeds 3.9. Since $A_{i\hat{t}}$ is an integer, both $a^4_{i\hat{t}}$ and $b^4_{i\hat{t}}$ are simultaneously zero only when $A_{i\hat{t}}$ equals 4. Consequently, the value of $o^4_{i\hat{t}}$ must be 1 when $A_{i\hat{t}} = 4$ due to Constraints (33). However, we have no constraints that set $o^4_{i\hat{t}} = 0$ when $A_{it} \neq 4$. For this problem, we define the model described in Equations (17) to (47) as MILPT1. Additionally, we define another model as MILPT2. In the MILPT2, Constraint (43) represents the objective function. Equation (17) is included as a constraint. Notably, MILPT2 aims to maximize net revenues, and as setting $o^4_{i\hat{t}} = 1$ negatively affects net revenue, it is set to zero whenever feasible during the solution process of MILPT2. Note that Constraints (34) to (42) follow a similar structure and operate in the same manner. As the MDNWTDP model contains more decision variables, it is likely to have a larger running time than the MDNDP model (Watson & Woodruff, 2011). Therefore, computing all efficient frontiers via Algorithm 1 can be computationally challenging. However, we can compute the patient-treated-maximizing solutions by solving MILPT1, setting $\gamma = z$, and then solving MILPT2. For net-revenue-maximizing solutions, we can solve MILPT2 first, set $\alpha = v$, and then solve MILPT1. Note that when using a 4-day working week, Constraints (31), (32), and (33) can be left out of the model. Constraints (20), (44), (45), (46), and (47) are explained in Section 3.1.

3.5 Heuristic for the MDNWTDP

As mentioned in the previous subsection, the generation of MDNWTDP solutions can potentially pose computational challenges. To mitigate this, a greedy local search algorithm is proposed to obtain feasible solutions for the MDNWTDP. The concept is to identify a route with the maximum net revenue per day that serves the location that serves the relatively least patients in the solution of the previous iteration. Subsequently, a search is performed to find a combination of routes that are included in the selected routes, collectively spanning a duration equal to the route that will be added, serving the location that serves the relatively most patients and generating the least profits per day. In cases where no route is found to accommodate the relatively least serving location, the search proceeds to consider the second relatively least serving location, followed by the third, and so forth. This iterative process continues until a feasible route is identified. Similarly, if no routes are found that can be removed that serve locations serving the relatively most patients, the search expands to explore routes that can be removed that serve locations serving the second relatively most patients, then the third, and so on, until a suitable solution is obtained. Following this, an assessment is made to determine if the maximum disparity has decreased. If not, the originally added route is removed from the pool of potential routes. Additionally, the instance variables are reverted to their values from the previous iteration. This is also done if the addition and removal of routes in that iteration ensure that patients treated at a location exceed the demand at that location. This assessment is only made when $\beta < 1$ as we start from an infeasible solution due to this demand constraint, and we are only interested in solutions below a disparity level of 100%. The *Route* object encompasses several instance variables, such as *revenue*, *patientsServed*, and *days*. These instance variables can be accessed using the functions $getRevenue()$, $getPatients()$, and $getDays()$. Furthermore, the heuristic utilizes the functions $changeAddition(Route\ j)$ and $changeRemoval(Route\ j)$ to update the number of patients served per location, the number of visits per location, and the number of overnight stays per location. We introduce a variable ϵ, which denotes the maximum disparity level we want to achieve. It should be noted that when a too-low value of ϵ is selected, the heuristic fails to terminate. The heuristic is described in Algorithm 2.

Algorithm 2.

---

0. Initialize $x_h = 120$, ϵ = desired disparity level, generate routes and store in $listOfRoutes$, fill *selectedRoutes* with 120 one-day-visits to hub, $\beta = 10000$

while $\beta > \epsilon$ do

  1.1 Compute $disp_i = \frac{p_i * z - y_i}{p_i * z} \quad \forall i \in I$

  1.2 Order locations i on basis of $disp_i$ in list *listLocs*, $\beta = max_{i \in I}|disp_i|$

  while

    1.3 Find route $r \in listOfRoutes$ which has the highest net revenue per day of all routes that serve the first location in *listLocs*

    1.4 If no route r is found; do find route $r \in listOfRoutes$ which has the highest net revenue per day of all routes that serve the location that serves the next location in *listLocs*, until a route is found

    1.5 $changeAddition(r)$

    1.6 Initialize $daysToFill = r.getDays()$, initialize *combinationToDelete*

    1.7 reverse *listLocs*

    Loop iteratively over locations i in *listLocs* until $daysToFill = 0$:

      1.8.1 Find the route $w \in selectedRoutes$ that serve i with lowest net revenue and add to *combinationToDelete*

      1.8.2 $daysToFill = daysToFill - j.getDays()$

    Loop over routes $j \in combinationToDelete$

      1.9.1 Remove j from *selectedRoutes*

      1.9.2 $changeRemoval(j)$

    1.10 Add route r to *selectedRoutes*

    1.11 Compute $newDisp_i = \frac{p_i * z - y_i}{p_i * z}$ $\forall i \in I$, $\beta_{test} = max_{i \in I}|newDisp_i|$

    If $\beta_{test} > \beta$ or $\exists\ i \in I$ for which $y_i > d_i$ when $\beta < 1$ do

      2.1 remove r from selected routes, $changeRemoval(r)$

      2.2 Add every route $j \in combinationToDelete$ to *selectedroutes*, call $changeAddition(j)$ $\forall j \in combinationToDelete$

      2.3 remove r from $listOfRoutes$ and return to 1.3

    else return to 1.1

$totalRevenue = \sum_{j \in selectedRoutes} j.getRevenue()$

$totalPatients = \sum_{j \in selectedRoutes} j.getPatients()$

---

4 Data

In this section, we discuss the data utilized in the Montana case study in Section 4.1, followed by an examination of the data employed in the simulation study in Section 4.2. In both studies, we adopt a planning horizon of six months for a single dentistry clinic as Patel et al. (2010) state that a six-month recall interval is associated with more restored teeth but less active caries. Furthermore, Thorsen and McGarvey (2018) estimate that the mobile dentistry service can treat an average of 2.75 patients per hour (f), with an average patient revenue (c) of \$85. Additionally, the costs associated with one-night, two-night, and three-night trips for the mobile dentistry staff (comprising four individuals) are estimated at \$670, \$1,238, and \$1,806, respectively. Therefore, we can set $O = \$602$, $O' = \$34$ and $\hat{O} = \$68$. The estimation of overnight stay costs is based on the per diem rates provided by the US General Services Administration (US GSA). These rates include a lodging fee (for Montana, the rate is \$91 per night) and a meals and incidental expenses (M&IE) rate (for Montana, the rate is \$51 per day). However, the M&IE rate is reduced by 25% for the first and last day of a trip. Additionally, fixed costs are estimated to be \$152,225. For the sake of comparability, we also use these facts for the simulation study. In the case of the 4-day working week, it is assumed that there are 96 working days within a span of 26 weeks. Each working day consists of 10 hours, which includes travel times, therefore we set patients treated per day b_i equal to $\lfloor f(10 - a_i) \rfloor$ and additional patients treated per overnight Q_i equal to $\lfloor f * 9 \rfloor - b_i$.

When transitioning to a 5-day working week, the possibility of a four-night stay arises. Utilizing the same calculations as those employed for one-night, two-night, and three-night stays, it is estimated that a four-night stay incurs a cost of \$2,374. This means that $O = \$593.5$, $O' = \$76.5$, $\hat{O} = \$51$ and $\tilde{O} = \$25.5$.

When considering a tour followed by another overnight stay, the M&IE is increased by 50% for the day of traveling due to the risk of higher incidental expenses that come when overnighting at multiple locations after each other. This is included in the model, as it computes overnight costs per week per location. Consider the scenario where the mobile dentistry staff stays overnight at two different locations within a week. In this case, 75% of the M&IE costs are calculated for the first location visited on the day of departure to the second location. However, 75% of the M&IE costs are calculated for the day before overnighting at the second location. As this is the same day, this results in a total of 150% estimated M&IE costs for that particular day. However, no increase in M&IE is applied when planning a tour without an overnight stay at the second location, as the M&IE is not augmented on days when the mobile dentistry team travels without scheduling an overnight stay. Furthermore, we reformulate b_i to $\lfloor f(8 - a_i) \rfloor$ and Q_i to $\lfloor f * 7 \rfloor - b_i$, as a working day lasts 8 hours in a 5-day working week, to maintain a total of 40 working hours per week. We set extra patients treated at location i when utilizing a tour from location i (q_i) equal to $\lfloor \frac{2.75 * t_{Ai}}{60} \rfloor$ and set extra patients treated at location k when utilizing a tour from i to k ($\hat{q}_{ik}$) equal to $\lfloor \frac{2.75 * (t_{Ak} - t_{ik})}{60} \rfloor$, with index A coinciding with the hub that is used. We set m_{ik} equal to 1 if $t_{Ai} + t_{Ak} - t_{ik} \geq 60$, else it is set equal to zero.

4.1 Montana case study

The data utilized for addressing the Montana problem was collected by Thorsen and McGarvey (2018), who provide a comprehensive explanation of the data collection process. The demand per location is as follows: Big Sky (412), Clyde Park (297), Emigrant (524), King Arthur Park (12,323), Livingston (2846), and Wilsall (194). Big Timber (663), Townsend (951), and White Sulphur Springs (503) denote the three additional locations used in the nine-location problem. The round-trip travel times include 60 minutes per day for setup and breakdown (30 minutes each), based on expert stakeholder estimates. The round-trip travel times and setup hours for each location, with Livingston as the hub, are as follows: King Arthur Park (138 minutes), Big Sky (226 minutes), Clyde Park (108 minutes), Emigrant (112 minutes), Wilsall (124 minutes), Townsend (220 minutes), White Sulphur Springs (202 minutes), and Big Timber (126 minutes). Overnight stays are limited to locations where the round-trip travel time exceeds two hours from the hub in Livingston.

The travel times utilized for constructing the tours are obtained using Google Maps and can be found in Appendix A. To ensure comparability between the solutions of the MDNDP and the MDNWTDP, the same travel times for round-trip journeys to Livingston as used by Thorsen and McGarvey (2018) are employed. This choice aims to ensure that the solutions remain consistent when tours are not utilized.

4.2 Simulation study

In the simulation, the locations are labeled as A through I. The demand at location A is 2500, at location B it is 4000, at location C it is 600, at location D it is 350, at location E it is 400, at location F it is 40, at location G it is 60, at location H it is 100, and at location I it is 70.

Table 9 in Appendix B displays the travel times between locations. These travel times conform to the triangle inequality and exhibit symmetry. The simulation characterizes by medium to high-demand locations located distant from the hub but surrounded by other small-demand locations in closer proximity.

5 Results

In this section, we elaborate on the results obtained from our models. First, we explain the results using six locations in the Montana case, whereafter we discuss the implications of adding the three other locations and shifting to a 5-day working week. Furthermore, we show the performance of the MDNWTDP and the heuristic in the Montana case and the simulation.

5.1 Initial problem: serving six locations in Montana

Utilizing a 4-day working week, the minimum disparity level that results in a feasible solution is 19%. In this solution, 2067 patients are treated, and a net annual revenue of $45,600 is obtained. When increasing the disparity level to 100%, we obtain a profit-maximizing solution that generates $55,100, with 2115 patients treated. Figure 1 demonstrates that lower disparity levels are linked to reduced net revenues and a smaller patient count. Conversely, permitting higher disparities can result in increased revenues, even with a constant number of treated patients. An example can be seen in the solutions that treat 2080 patients for disparity levels of 30% and 50%. The 30% disparity solution generates a profit of \$46,674, while the 50% disparity solution generates \$49,150. A more precise examination of these solutions and their distribution of patients across the locations is given in Appendix C.1. Besides that, we can conclude a trade-off in patients treated and net revenue for each disparity level. We obtain the same results as Thorsen and McGarvey (2018), and a further sensitivity analysis is made in Appendix C.2.

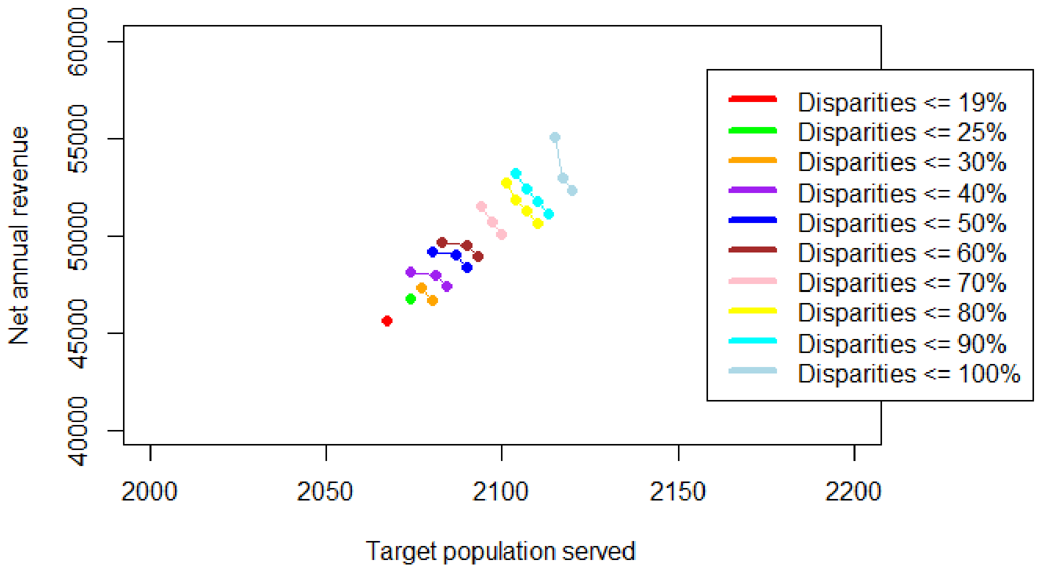

Figure 1: Trade off between net annual revenue and patients served per disparity level

5.2 The expanded case: serving nine locations in Montana

When we expand the set of locations with three additional locations, the lowest disparity level that results in a feasible solution is 31%. This solution generates an annual net revenue of \$36,910 and treats 2008 patients. When comparing the solution for $\beta \leq 31\%$ in Figure 2 with the $\beta \leq 30\%$ curve in Figure 1, we observe a notable shift downwards and leftwards, suggesting a reduction in both the number of patients treated and net revenue. For all other β values, the expansion of service has minimal impact on the total number of treated patients. However, it leads to a significant decrease in net annual revenue. This decline is primarily attributed to the utilization of overnight stays necessary to achieve the required number of treated patients in Townsend and White Sulphur Springs. Once again, we observe a trade-off between the number of patients treated and net revenues, where higher disparity levels result in more treated patients and higher revenues. A comparison similar to the one in the previous subsection reveals that a 35% disparity solution, treating 2079 patients, generates \$41,552 in net revenues, while a 70% disparity solution, treating a comparable 2080 patients, generates \$47,810. More detailed information on the allocation of patients in these solutions can be found in Appendix C.1. The results in the nine-location problem correspond to those found by Thorsen and McGarvey (2018).

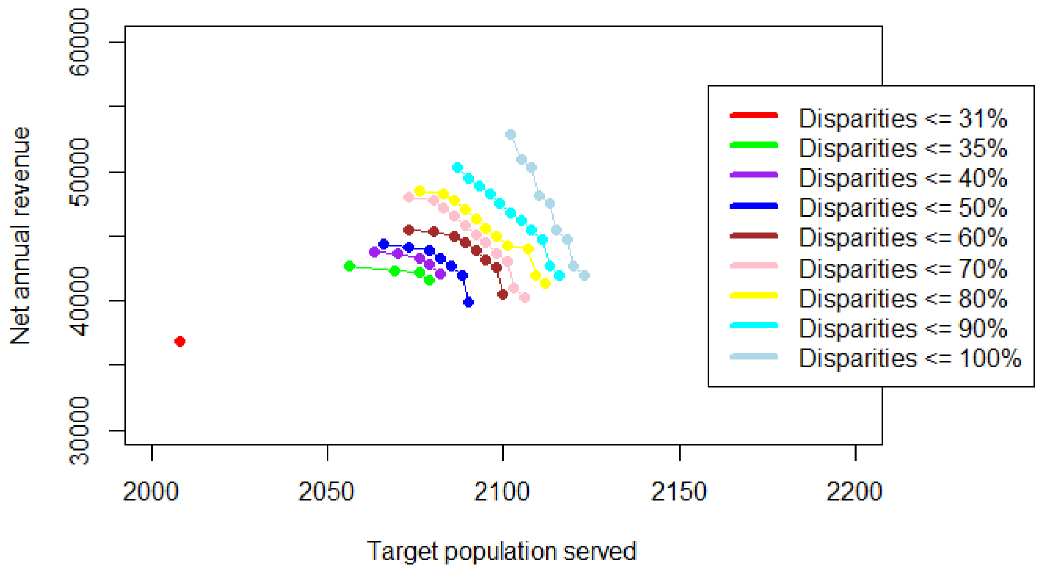

Figure 2: Trade off between net annual revenue and patients served per disparity level when serving to nine locations

5.3 The effects of introducing tours in a 4-day working week

The MDNWTDP's performance in the 4-day working week was deemed to be unprofitable in a study conducted by Thorsen and McGarvey (2018). However, an analysis of the results presented in Table 3 demonstrates that incorporating a tour from Big Sky (BS) to King Arthur Park (KAP) proves advantageous in terms of maximizing the number of treated patients for most disparity levels. By utilizing the tour, an additional 3 patients can be treated, albeit at a cost per patient extra (CPPE) of $209. It is worth noting that the running times (RT) for the MDNWTDP are longer compared to the MDNDP, but they remain within acceptable limits.

	MDNDP			MDNWTDP				
β	patients	net rev	RT	patients	net rev	RT	tours	CPPE
$\leq$ %31	2008	$36,910	0,1s	2008	$36,910	3,5s	-	-
$\leq$ %40	2082	$42,062	0.2s	2085	$41,436	3.5s	BS to KAP	$209
$\leq$ %50	2090	$39,810	0.1s	2093	$39,184	8,7s	BS to KAP	$209
$\leq$ %60	2100	$40,374	0.2s	2103	$39,748	1,6s	BS to KAP	$209
$\leq$ %70	2106	$40,258	0,2s	2109	$39,632	1,6s	BS to KAP	$209
$\leq$ %80	2112	$41,728	0.1	2115	$40,652	1.6s	BS to KAP	$209
$\leq$ %90	2116	$41,958	0.1s	2219	$41,332	1,5s	BS to KAP	$209
$\leq$ %100	2123	$42,012	0.2s	2123	$42,012	3,0s	-	-

Table 3: Comparison between outcomes that patients-treated per disparity level in MDNDP and MDNWTDP in the Montana case study for a 4-day working week

The inclusion of a tour from Big Sky to King Arthur Park appears to be a comprehensive option, considering the travel times involved. It takes 86 minutes to drive from Big Sky to Livingston via King Arthur Park, whereas direct travel from Big Sky to Livingston takes 83 minutes. Therefore, opting to drive via King Arthur Park proves advantageous in terms of time savings compared to the scenario where the mobile dentistry has to return to Livingston from Big Sky and then travel to King Arthur Park on a separate day.

When comparing the performance of the MDNDP and the MDNWTDP in terms of profit-maximizing solutions, it is evident that the MDNWTDP does not surpass the MDNDP except for the disparity level of 60%. In this solution, a tour from Townsend to White Sulphur Springs is incorporated, resulting in an additional patient. Consequently, the MDNWTDP generates $170 more in revenue than the MDNDP model for the disparity level of 60%. However, this is only a 0.4% increase compared to the solution generated by the MDNDP. A tour between Townsend and White Sulphur Springs appears rational due to their distance from Livingston (80 minutes and 71 minutes, respectively) and their relatively close proximity to each other (43 minutes). Utilizing a tour between these locations can result in significant time savings. However, when considering the costs associated with overnighting, it is observed that these costs do not outweigh the benefits of treating additional patients in general. This is evident from Table 4, where only one tour is incorporated, and from Table 3, where all tours come with extra costs.

	MDNDP			MDNWTDP				
β	patients	net rev	RT	patients	net rev	RT	tours	RPPE
$\leq$ %31	2008	$36,910	0,1s	2008	$36,910	1,6s	-	-
$\leq$ %40	2063	$43,784	0.8s	2063	$43,784	49,1s	-	-
$\leq$ %50	2066	$44,294	0.4s	2066	$44,294	231,4s	-	-
$\leq$ %60	2073	$45,484	1,2s	2074	$45,654	1872,2s	T to WSS	$170
$\leq$ %70	2073	$47,960	1,3s	2073	$47,960	25,7	-	-
$\leq$ %80	2076	$48,470	1,5s	2076	$48,470	26,8s	-	-
$\leq$ %90	2087	$50,340	1,6s	2087	$50,340	17,9s	-	-
$\leq$ %100	2102	$52,890	0.2s	2102	$52,890	5,0s	-	-

Table 4: Comparison between outcomes that maximize net revenue per disparity level in MDNDP and MDNWTDP in the Montana case study for a 4-day working week

5.4 Difference in working days and its impact on revenue and patient treatment in the MDNDP

In Figure 3, we observe that the introduction of a 5-day working week leads to a substantial reduction in net revenue. The solution yielding the highest profitability corresponds to net revenue of $26,370, accommodating the treatment of 1946 patients with a disparity level of 100%. In contrast, the efficient frontier solution generating the lowest profits within a 4-day working week yields, $36,910 while treating 2008 patients for a disparity level of 31%. The difference in net revenue arises due to the fact that a working day in a 4-day week spans 2 hours longer than a working day in a 5-day week. Consequently, in a 5-day working week, travel constitutes a relatively larger proportion of the working day in comparison to a working day in a 4-day working week. Considering a patient treatment rate of 2.75 patients per hour, an opportunity cost of 5 patients per day arises. However, the introduction of the 5-day working week does lead to a reduction in the minimum disparity level to 17%. For this disparity level, we obtain two solutions. When prioritizing patients mobile dentistry generates $6,628 while treating 1859 patients. The other solution treats 3 patients less and generates $7,458. Notably, in the original scenario with nine locations, the MDNDP achieved a minimum disparity level of 31%, indicating a significant improvement in achieving a fairer distribution of patients across the locations.

Figure 3: Trade off between net annual revenue and patients served per disparity level when introducing a 5-day working week

5.5 The effects of introducing tours in a 5-day working week

A comparison is drawn between patient-treated-maximizing solutions of the MDNDP model and the MDNWTDP model in Table 5. In the "tours" column, locations are abbreviated by their initials. It can be observed that solutions generated by the MDNWTDP model often involve a tour between Big Sky and King Arthur Park, a comprehensive result explained in Section 5.3. In these solutions (except for the $\beta \leq 17\%$ solution), the cost per patient extra (CPPE) is $209. In the patient-treated-maximizing solution at a disparity level of 30%, the CPPE reaches a minimum of $158. On the other hand, the maximum CPPE occurs for a disparity level of 60%, as a tour from Townsend to White Sulphur Springs results in an additional patient at the expense of $966 fewer profits. Although the running times (RT) for the patient-treated-maximizing solutions in the MDNWTDP are deemed acceptable (with a maximum of 34.4 seconds for $\beta \leq 50\%$), computing solutions that maximize net revenue is computationally challenging. Nevertheless, by solely solving the MILPT2, we can determine the maximum net revenue achievable in the MDNWTDP. Although the number of patients treated is not optimized, this approach provides insight into the potential of utilizing the MDNWTDP to enhance profits. However, we observe that none of the net revenues obtained surpass those generated by the MDNDP. Consequently, it can be inferred that while the MDNWTDP model excels in generating patient-treated-maximizing solutions, it cannot surpass the net revenues attained by the MDNDP model in the Montana case study. Exact maximized net revenues per disparity level and running times for the MDNDP and MDNWTDP are presented in Appendix C.5.

The heuristic approach may be employed to address the computationally challenging solutions. However, the lowest disparity level for which the heuristic terminates is 93%, generating annual revenues of $13,506 by treating 1962 patients. This approach utilizes a tour from Big Sky to King Arthur Park and another from King Arthur Park to Big Sky. In contrast, the least-generating solution for the MDNDP yields $15,484 per year by treating 1987 patients for a disparity level of 93%. When the disparity level is set to 100%, the annual net revenue of the solution generated by the heuristic amounts to $16,546 by treating 1972 patients, which is relatively close to the patient-treated-maximizing solutions in terms of net revenue (0.00007%), but significantly lower than the net-revenue-maximizing solution of the MDNDP, which generates $26,370 (a percentage difference of 37.3%). Although the heuristic approach approximates the net revenues achieved by the patient-prioritizing solutions, it is unable to improve upon the solutions generated by the MDNDP.

	MDNDP			MDNWTDP				
β	patients	net rev	RT	patients	net rev	RT	tours	CPPE
$\leq$ %17	1859	$6,628	0.3s	1859	$6,628	15.4s	BS to KAP	-
$\leq$ %20	1907	$10,040	0.2s	1907	$10,040	18.3s	-	-
$\leq$ %30	1911	$10,720	0.3s	1922	$8,978	3.8s	T to WSS and BS to KAP	$158
$\leq$ %40	1927	$9,828	0.2s	1930	$9202	3.7s	BS to KAP	$209
$\leq$ %50	1937	$11,528	0.2s	1940	$10,902	34.4s	BS to KAP	$209
$\leq$ %60	1954	$12,146	0.2s	1955	$11,180	4.0s	T to WSS	$966
$\leq$ %70	1965	$12,880	0,2s	1968	$12,254	3.2s	BS to KAP	$209
$\leq$ %80	1978	$13,954	0.1	1981	$13,328	2.6s	BS to KAP	$209
$\leq$ %90	1987	$15,484	0.1s	1990	$14,858	2.2s	BS to KAP	$209
$\leq$ %100	2000	$16,558	0.3s	2000	$16,558	7,3s	-	-

Table 5: Comparison between outcomes that prioritize patients treated in MDNDP and MDNWTDP in the Montana case study

5.6 Maximizing patients treated in the simulation study

The MDNWTDP's benefits in maximizing patient treatment warrant further exploration in other scenarios. In Table 6 we compare the solutions of the simulation study that maximize patients treated for the MDNDP and the MDNWTDP. It is observed that the MDNWTDP outperforms the MDNDP on nearly all disparity levels in terms of patients treated. Computational times are on an acceptable level, with the solution of the MDNWTDP as the only outlier with a computational time of 682,6 seconds. Furthermore, it can be concluded that trips to high-demand location B are favored, as well as tours from medium-demand location E. This can be explained by the fact that these are both locations that are relatively distant from the hub, but surrounded closely by other low-demand locations such as G, F, and I, which makes it profitable to incorporate a tour. We see that the CPPE is minimal for the solution that maximizes patients for a disparity level of 40% and that it is maximal for a disparity level of 100%.

	MDNDP			MDNWTDP				
β	patients	net rev	RT	patients	net rev	RT	tours	CPPE
$\leq$ %29	2058	$45,410	0.7s	2058	$45,410	4.1s	-	-
$\leq$ %40	2231	$67,596	1.8s	2256	$66,894	53.1s	E to B, E to F , G to B, I to B	$28
$\leq$ %50	2262	$71,730	1.0s	2279	$69,464	58.2s	2x E to B, E to F , G to B, I to B	$133
$\leq$ %60	2290	$76,490	1.1s	2307	$74,224	682,6s	2x E to B, E to F , G to B, I to B	$133
$\leq$ %70	2324	$79,794	0.6s	2340	$77,358	80,7s	2x E to B, E to F, G to B, 2x I to B	$152
$\leq$ %80	2351	$84,384	1.7s	2371	$81,696	66,8s	3x E to B, F to G , I to B	$134
$\leq$ %90	2368	$87,654	1.3s	2411	$86,020	115,6s	3x E to B, F to G , G to B, I to B	$38
$\leq$ %100	2421	$95,148	1.5s	2440	$91,154	126,8s	3x E to B, G to B , I to B	$210

Table 6: Comparison between outcomes that prioritize patients treated in MDNDP and MDNWTDP in the simulation study

5.7 Comparison between maximizing net revenue solutions of MDNDP and MDNWDP and the heuristic in the simulation study

As shown in Table 7, the MDNWTDP solutions for net revenue maximization achieve slightly higher revenues at disparity levels of 60% and 70%. However, The improvements are minimal, with a 0.5% increase in net revenue for the 60% disparity level compared to the MDNDP model. This is the result of two tours added, namely G to F and I to B. Similarly, the MDNWTDP solution at the 70% disparity level sees a 0.07% net revenue increase, due to the addition of the two times the tour E to F and one time from I to B. In contrast to the Montana case study, computing the optimal solution in terms of patients for net revenue maximization is possible, although longer running times were observed for higher disparity levels. The heuristic approach addresses disparity levels above 38%. While its performance is within 6.4% (smallest percentage margin) of the maximized net revenue at a 40% disparity level, it falls short by 14.2% (largest percentage margin) compared to the maximized net revenue at a 100% disparity level.

	MDNDP			MDNWTDP			Heuristic		
β	patients	net rev	RT	patients	net rev	RT	patients	net rev	RT
$\leq$ %29	2058	$45,410	0.9s	2058	$45,410	3.8s	-	-	-
$\leq$ %40	2210	$71,250	0.1s	2210	$71,250	98.9s	2207	$66,720	0,5s
$\leq$ %50	2239	$76,180	0.7s	2239	$76,180	79.83s	2225	$69,780	0.4s
$\leq$ %60	2262	$80,090	0.7s	2280	$80,470	80.7s	2249	$73,860	0.6s
$\leq$ %70	2289	$84,680	0.7s	2313	$84,740	77.5s	2270	$77,430	0.6s
$\leq$ %80	2318	$89,610	1.6s	2318	$89,610	134.3s	2286	$80,150	0.8s
$\leq$ %90	2349	$94,880	2.0s	2349	$94,880	5754,5s	2311	$84400	0.4s
$\leq$ %100	2394	$102,530	1.5s	2394	$102,530	53,4s	2332	$87,970	0.5s

Table 7: Comparison between the maximized revenues solutions of MDNDP and MDNWTDP and solutions of heuristic

6 Conclusion

In this paper, we explored the potential enhancements of the MDNDP introduced by Thorsen and McGarvey (2018). Firstly, we examined the improvements a 5-day working could make over a 4-day working week. The introduction of a 5-day working week improved the minimal disparity level from 31% to 17%. However, the number of patients-treated and net revenues dropped significantly. Moreover, we introduced the MDNWTDP, which incorporates tours in the MDNDP. The inclusion of tours proved to be beneficial for both the 4-day working week and the 5-day working week when maximizing the number of patients treated. Nevertheless, the MDNWTDP does not surpass the MDNDP in terms of net revenue maximization. Furthermore, the MDNWTDP proves to have significantly longer running times than the MDNDP. Therefore a heuristic is introduced. However, the heuristic only solved for high disparity levels, with net revenues comparable to those of the patient-treated maximizing solutions.

A simulation study is conducted where high-demand locations were situated at a distance from the hub while being surrounded by small-demand cities in close proximity. Once again, the MDNWTDP proves superior in patient-treated maximization but generates comparable solutions as the MDNDP when maximizing net revenues. Compared to the solutions that maximize profits in the MDNWTDP, the heuristic performances within 6,4% (smallest margin) and 14,2% (largest margin) for the selected disparity levels. It should be noted that the heuristic never outperforms any of the solutions in the MDNDP. In conclusion, for organizations seeking to implement a mobile dentistry service and prioritize maximizing the number of patients treated, the MDNWTDP model is recommended, particularly in situations with small-demand locations near large-demand locations located far from the hub. However, if the main goal is to maximize net revenue, the MDNDP provides comparable solutions with lower computational costs.

6.1 Limitations and further research

A major limitation of the MDNDP is its financial sustainability, and while the MDNWTDP does not demonstrate significant improvements in profit-maximizing solutions, financial sustainability remains a challenge. In order to enhance the net revenues of the MDNWTDP, a strategy could be to increase the number of patients treated per hour. By utilizing the time saved through the inclusion of tours for treating patients, tours become a more profitable option. Therefore it is interesting to explore the implications of improved productivity in mobile dentistry services including tours.

Furthermore, the heuristic only provides solutions for high disparity levels in the Montana case, hence it is worth considering an alternative algorithm that allows for iterations with increasing maximum disparity levels. This approach can help escape local optima and potentially lead to more beneficial outcomes. As the heuristic provides relatively good solutions in the simulation study, it can be useful to explore its applicability on larger problems, and if it can outperform the MDNDP in terms of patient allocation, net revenues, and running times.

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Appendix

A Travel times between locations in Montana

In Table 8, the locations are abbreviated using their initials. These times are obtained from Google Maps.

	L	KAP	BS	CP	E	W	T	WSS	BT
L	0	39	83	24	26	32	80	71	33
KAP	39	0	47	55	57	61	61	93	66
BS	83	47	0	101	102	105	105	140	115
CP	24	55	101	0	43	8	71	47	39
E	26	57	102	43	0	50	97	86	53
W	32	61	105	8	50	0	58	44	47
T	80	61	105	71	97	58	0	43	115
WSS	71	93	140	47	86	44	43	0	86
T	33	66	115	39	53	47	115	86	0

Table 8: Travel times between locations in Montana

B Travel times between locations in simulation study

Table 9 contains the travel times between locations in the simulation study.

	A	B	C	D	E	F	G	H	I
A	0	60	30	50	70	100	90	40	90
B	60	0	30	100	60	90	80	100	30
C	30	30	0	30	90	120	110	70	60
D	50	100	30	0	100	130	120	30	110
E	70	60	90	100	0	30	20	100	70
F	100	90	120	130	30	0	20	130	110
G	90	80	110	120	20	20	0	100	110
H	40	100	70	30	100	130	100	0	120
I	90	30	60	110	70	110	110	120	0

Table 9: Travel times between simulated location

C Sensitivity analysis

In this section, we dive more deeply into the solutions generated by the MDNDP and MDN-WTDP models.

C.1 Comparing solutions that treat equal patients for different disparity levels

The solution serving the most patients (2080) with $\beta = 0.30$ generates a \$46,674 profit. However, if we set $\beta = 0.50$, the solution that serves 2080 patients generates \$49,150. This is entirely the result of allocating the patients in a different manner, which is illustrated in Figure 4. The solution with a disparity level of 30% utilizes a two-night trip to Big Sky, whereas no overnight is used in the solution for a disparity level of 50%. The requirement for overnight stays to ensure compliance with disparity constraints has a direct impact on reducing net revenues. Additionally, we observe that as the disparity level increases, more patients in Livingston receive treatment, resulting in an increased number of planned visits to Livingston. This is a comprehensive result since Livingston serves as the hub, and during a visit to Livingston, more patients can be treated since no travel time is incurred, and thus more revenue will be generated, compared to a visit to another location.

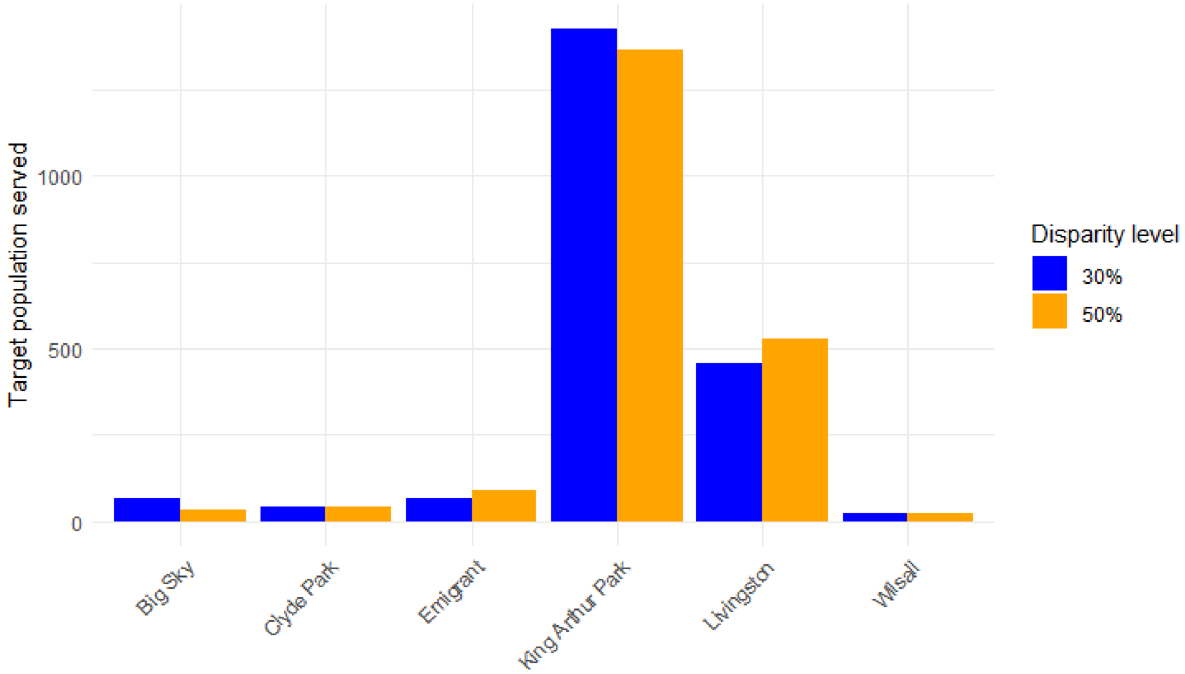

Figure 4: Allocations of feasible solutions serving 2080 patients, using different disparity levels

A similar comparison can be made in the situation where we serve nine locations. In Figure 2, a $\beta \leq 0.35$ solution demonstrates 2079 treated patients and a net revenue of \$41,552, while a $\beta \leq 0.7$ solution treats 2080 patients while generating a net revenue of \$47,810. Once again, reducing the number of visits to more distant locations significantly increases the net revenue, while maintaining a nearly equal total number of treated patients. The distribution of patients of these solutions can be seen in Figure 5. Once again, it is evident that in the higher disparity solution, a greater number of patients in Livingston are treated. This outcome can be attributed to the fact that visits to Livingston yield higher profitability compared to other locations due to the fact that no travel time is incurred.

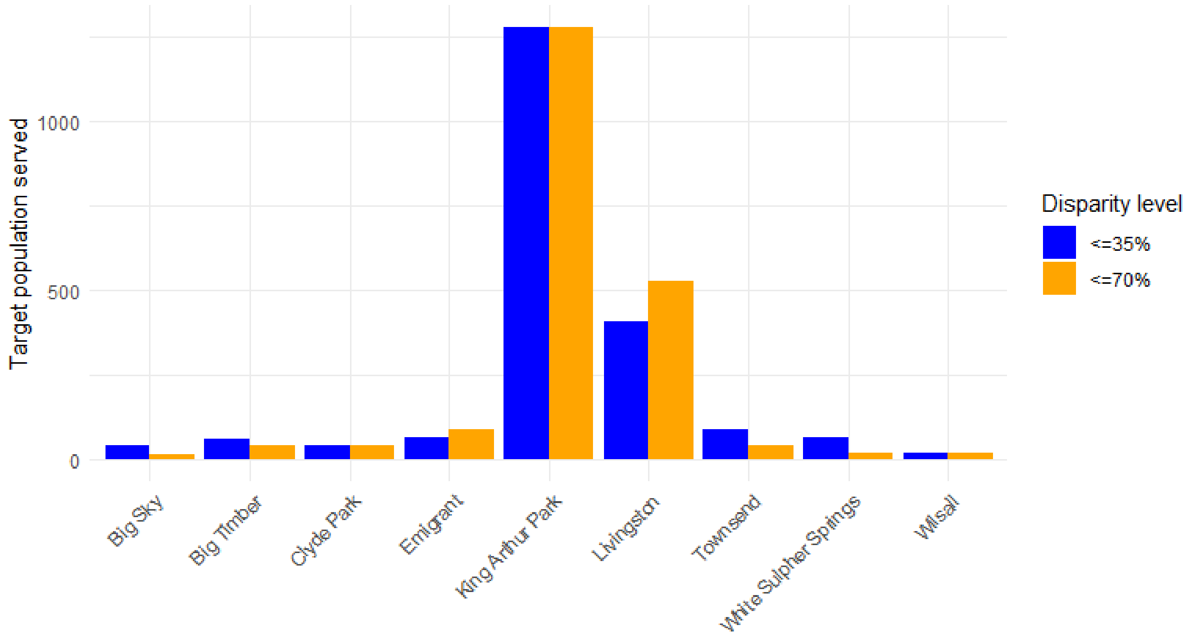

Figure 5: Allocations of feasible solutions serving around 2080 patients, using different disparity levels

C.2 Impact of varying the average reimbursement rate

The average reimbursement rate to community health centers, after removing variable costs, is estimated to be $137,75 (Thorsen & McGarvey, 2018). Using this revenue rate per patient, instead of the $85 we used in the previous sections, the solutions on the efficient frontiers for the original and extended case do not differ, besides a few exemptions. The annual net revenue has risen to respectively $278,232.50 and $274,651 for the original and extended cases. If we set the average reimbursement rates equal to $102.58, we obtain maximums of $129,463.40 and $126,796.32 for the baseline and extended location set. Average reimbursements rates of $120.16 result in maximum net revenues of $203,826.80 and $200,702.64 for the baseline and extended locations set. All solutions correspond to the solutions presented by Thorsen and McGarvey (2018), besides the solution that generates a net revenue of $274,651 for the nine-location problem and uses an average reimbursement rate of $102.58. Thorsen and McGarvey (2018) determine a net revenue of $274,200 for this instance.

C.3 Mixed fixed-site (at hub) and traveling operations

Another option is to split up the 26-week period into two parts of three months. In the first three months, we only treat patients at the hub Livingston. This results in $48 * \lfloor 2.75(10) \rfloor = 1296$ patients treated in three months in Livingston. The disparity constraint does not apply for the Livingston location, and thus only for the eight locations receiving traveling service. The solution with the highest revenue for $\beta = 0.75$ treats 2296 patients in total. Plots for the feasible solutions for $\beta = 0.75$ and $\beta = 1$ are given in Figure 6. If we compare the solution that serves 2080 ($\beta \leq 0.7$) patients when traveling full-time, we see that we treat much fewer (528) patients in Livingston than when we travel only half of the time (1296 patients). However, patients treated in the other locations drops drastically. If we accumulate all patients treated in the eight other locations when we travel full-time, we obtain 1552 patients. In the solution where we only travel half of the time, this number is reduced to 1000. The annual net revenues rise significantly. We generate $85,870 when traveling half of the time, compared to the $ 47,810 annual net revenue in the solution where we travel full-time. The allocations of patients can be found in Figure 7. Our solutions fully correspond to those of Thorsen and McGarvey (2018).

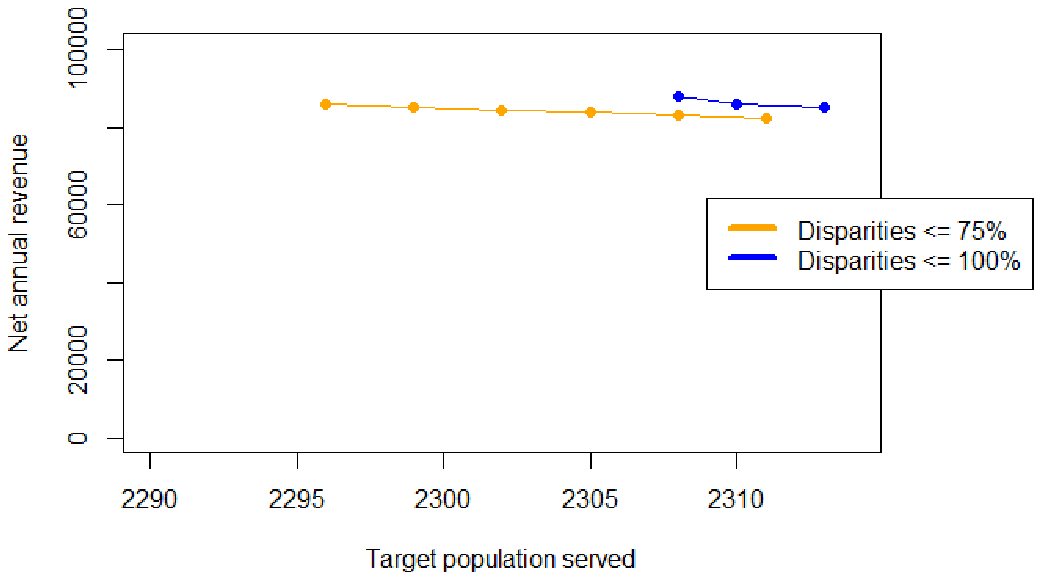

Figure 6: Trade off between net annual revenue and patients served per disparity level for the mixed service

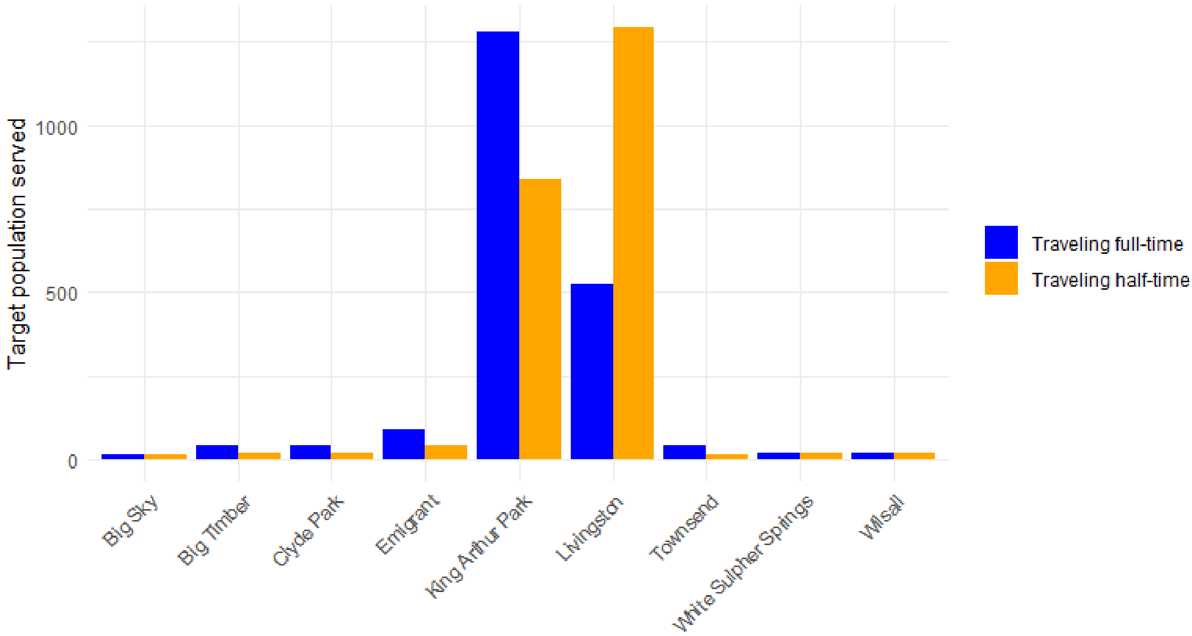

Figure 7: Patient allocation when traveling full-time vs traveling half-time

C.4 Minimum patients treated per location

A notable finding observed consistently in our study pertains to the significantly elevated visitation rate in Livingston. This occurrence can be attributed to the model's inclination to prioritize larger populations located in close proximity to the central hub (Livingston) in order to minimize travel time. To address this particular issue, a supplementary analysis is undertaken. This analysis involves resolving the complete set of efficient frontiers while imposing the constraint that a minimum of 100 patients are served in each of the nine locations (as presented in Table 3). It is necessary to set $\beta = 361\%$ to achieve a feasible solution. This solution yields an annual net revenue of $58,544, which is also the solution that Thorsen and McGarvey (2018) found.

Location	Number of patients served	Number of visits per six months
Livingston	1344	56
King Arthur Park	105	5
Big Sky	106	5 (3 overnights)
Clyde Park	110	5
Emigrant	110	5
Wilsall	105	5
Townsend	106	5 (3 overnights)
White Sulphur Springs	108	5 (3 overnights)
Big Timber	105	5

Table 10: Solution for $\beta = 3.61$ which treats at least 100 patients at every location

C.5 Comparing net-revenue-maximizing solutions of the MDNDP and MDNWTDP for the 5-day working week

As described in Section 5.5 it is computationally challenging to optimize the net-revenue-maximizing solutions of the MDNWTDP for patients treated. The solutions presented in the MDNWTDP column are generated by solely solving MILPT2, and hence not optimized for patients. However, we see that the MDNWTDP does not surpass the MDNDP in revenues for any of the selected disparity levels.

	MDNDP		MDNWTDP	
β	net rev	RT	net rev	RT
$\leq$ %17	\$7,458	0.9s	\$7,458	165.7s
$\leq$ %20	\$10,040	0.2s	\$10,040	18.3s
$\leq$ %30	\$11,728	0.1	\$11,728	98,9s
$\leq$ %40	\$13,448	0.5	\$13,448	53.3s
$\leq$ %50	\$15,148	0.5s	\$15,148	50,3s
$\leq$ %60	\$16,434	0.1s	\$16,434	189,7s
$\leq$ %70	\$18,930	0.4s	\$18,930	77,5s
$\leq$ %80	\$20,950	1.0s	\$20,950	18,8s
$\leq$ %90	\$23,140	0,4s	\$23,140	20,7s
$\leq$ %100	\$26,370	0,4s	\$26,370	46,0s

Table 11: Comparison between outcomes that maximize net revenue in MDNDP and MDNWTDP in the Montana case study by solving MILPT1 for the MDNWTDP

D Programming code

In this section, we explain the code that is used to solve the MDNDP model, the MDNWTDP model, and the heuristic. First, we elaborate on the MDNDP model.

D.1 Code MDNDP model

The classes $MobileDentistry$, $EightHoursMobileDentistry$, and $SimNormalMobileDentistry$ all have the same layout but differ in the fact that $MobileDentisty$ is used as the code to replicate the results of Thorsen and McGarvey (2018), $EightHourMobileDentistry$ implements a 4-day working week and $SimNormalMobileDentistry$ generates results for the MDNDP simulation study. Furthermore, these classes basically do the same, and therefore we explain the code of these classes together. At the beginning of the code of the MDNDP model, all instance variables and fixed constants are initialized. These can be modified to change easily from a 4-day working week to a 5-day working week or to switch from the 6-location problem to the 9-location problem. We implement two methods that generate results: $testBeta()$ and $Algorithm1$. The method $testBeta()$ is used to obtain results for specific values of α and β. After we set values for α and β, we run the method $MILP1$ and $MILP2$ to compute the efficient solution. We write the important results in a text file, to be able to make graphs for example. Some valuable information is directly printed as well, as it delivers the user a quick overview of the most important results. $Algorithm1$ implements the algorithm proposed in Section 3.3. It starts from

$\beta = 0$ and iterates over the potential values of β with steps of 0.01. If a solution is found via the usage of $MILP1$ and $MILP2$, we update α to have higher revenue in the next iteration, while still adhering to the same disparity level. We continue looping until we cannot find a higher revenue for the disparity level. In that case, we increment the β with 0.01 and do the same process for the new disparity level. For each disparity level, we write the important features of all solutions in a separate text file, so we can use the information to draw for example graphs For each solution, we record the running time using the $System.currentTimeMillis()$ call when we start computing the solution, and after a solution is found. When we subtract these times from each other, we find the running time. Dividing the running time by 1000 gives us the running time in seconds. The methods $MILP1$ and $MILP2$ implement the MILPs described in subsection 3.1. The implementation makes use of CPLEX to solve the MILPs. The reasoning behind the constraints is extensively described in subsection 3.1, and comments are placed in the code to see which lines of codes create which constraints. In the $MILP1$ we maximize the value of $totalPatients$, whereas we maximize the value of $netRevenue$. Furthermore, we used text files to incorporate the data from the Excel file and created methods that were able to store the data of the text files in vectors. These methods are called $readVector$. When changing from a 4-day working week to a 5-day working week or vice versa, it is important to call the right data files when calling the $readVector$ function.

The class $MixedMobileDentistry$ implements mixed mobile dentistry as proposed in subsection 7.3.1. The class has a similar structure as the previous ones mentioned in this subsection, but we added constraints to equal the visits of Livingston to 48 days and equal the treated patients to 1296. Furthermore, we did not include the disparity constraints for the Livingston location and used the total amount of patients treated without the location Livingston to form the disparity constraints for all other locations.

D.2 Code for the MDNWTDP model

The MDNWTDP model is implemented in the following classes: $ExtensionMobileDentistry$ and $MobileDentistrySim$. The first implements the MDNWTDP model for the Montana case study, and the other for the simulation study. Both are nearly identical, except for the data that is used to run them. These classes have similar structures as the MDNDP models, as the $testBeta$ and the $Algorithm1$ perform in the same manner. However, some instance variables and constants are changed or added to accommodate the class to the MDNWTDP model. In the $MILP1$ and $MILP2$, we implement the models described in subsection 3.4. The constraints are explained in this section as well, and in the classes itself is described which lines of code implement certain constraints. by comments. We use the same methods as in the MDNDP model to transform the Excel data to vectors, however, due to the addition of tours, we also add a method that can read text files with two-dimensional data, and convert it to a two-dimensional matrix. The instance variables introduced at the beginning of the code can be used to change from a 4-day working week to a 5-day working week. However, it is needed to change the text files containing the data when a transition in the working week structure is done.

D.3 Code for the heuristic

The code that implements the heuristic is less alike than the two models described in the previous subsections, as it is structured differently. This implementation uses the methods $readVector$ and $readMatrix$ as well, to transform text file data into one- or two-dimensional vectors. The method that actually implements the heuristic is called *heuristic* and starts by transforming all data it needs from text files into vectors. Then it calls the *makeRoutes* method. This method generates all possible routes, back-and-forth trips to the hub without overnighting, back-and-forth trips to the hub with overnighting, and tours that visit 2 locations before returning to the hub or routes that use combinations of these trips. Only tours using one overnight to travel are generated, as we only considered those routes to accommodate the mobile dentistry staff's comfort. Then we implement the heuristic as it is described in subsection 3.5. In the implementation, we make use of the following methods: *changeAddition*, *changeRemoval*, *selectRoutes*, and *removalRoutes*. The method *changeAddition* has a route that is added to the selected Routes as an argument and decides whether this route employs overnights, tours, or none of them and updates the instance variables accordingly. The method *changeRemoval* does the opposite of *changeAddition* and updates the instance variables if the route is removed from the selected. routes. The method *selectRoutes* takes a location as an argument and finds the route that has the highest revenue per day while treating the location it has as the argument. The method *removalRoutes* takes a location as an argument as well. However, it returns a combination of tours that serve the location it has as input, and generate the least profits per day. The tours together span the number of days which was given as input. When the new routes are added to the selected routes, and the routes returned by *removalRoutes* are removed, the new maximum disparity level is computed. If it is higher than the previous disparity level, we remove the route that was initially added from the list of potential routes to add and change the instance variables back to their values of the previous iteration. If the new disparity level is lower, we loop again in order to search for another swap of tours to repress the maximal disparity level. If a too-low value of ϵ is chosen, the code will fail to terminate. When the maximum disparity level is beneath ϵ we print the most important information and calculate total revenue and the number of patients treated. An important note is that we only check if the patients treated per location do not exceed the demand per location if a disparity level beneath 100% is encountered. As we start with a solution that is infeasible for this constraint, the heuristic will fail to work otherwise. Not checking the demand constraint for disparity levels above 100% is not necessarily a problem since we are mainly interested in disparity levels beneath 100% in this paper.