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Combining Naive with Strategic: A Comparative Analysis of Portfolio Selection Strategies

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Abstract

Kirby and Ostdiek (2012) investigate the performance of many different mean-variance based strategies of portfolio selection. A drawback of these strategies in comparison to naive diversification is the presence of estimation risk. We attempt to reduce this risk by combining the sophisticated strategies with the 1/N portfolio with the goal to improve on the strategies of Kirby and Ostdiek (2012). We examine three different methods of combining portfolio selection strategies. Our results show that combinations based on maximum utility perform the best. In many cases, these combination strategies show similar or improved performance in comparison to the individual mean-variance based strategies. This advantage is most pronounced for combinations with strategies that suffer from large estimation risk.

1 Introduction

Portfolio management has always been one of the most significant research topics in finance as it presents practical information and tools to investors in order to optimize their economic gains. A large part of this field of study is to find and compare useful portfolio selection strategies. The equally weighted portfolio, which places a weight of 1/N on each of N assets, is a classical and intuitively appealing approach. It might seem naive, as it does not utilize any information on the history of assets and therefore seems sub-optimal. However, studies like DeMiguel, Garlappi and Uppal (2009) and Bloomfield, Leftwich and Long Jr (1977) show that in many cases more sophisticated methods perform worse than the 1/N portfolio. These methods are mostly variants on the Markowitz (1952) mean-variance optimization theory. They point to the high estimation error in estimating expected returns and variances as the main cause of the diminishing performance of the mean-variance strategies out of sample.

Kirby and Ostdiek (2012) respond to DeMiguel et al. (2009) in stating that their research design limits the performance of mean-variance strategies. This is because the mean-variance portfolios implicitly target extremely high expected returns, resulting in high estimation risk and turnover. They then show that constraining the model to have the same expected return as the 1/N portfolio, performance is greatly improved and now competes with the naive strategy. Finally, they find two strategies that significantly outperform the 1/N portfolio in most cases. The first is a strategy based on volatility timing, and the second on reward-to-risk timing.

So forms of mean-variance based portfolio selection strategies can be implemented in order to improve over the traditional equally weighted portfolio. But that does not mean that we have to throw it out altogether. As Kirby and Ostdiek (2012) point out, the timing strategies can face challenges depending on the data set. For example, risk-to-reward timing does not outperform the 1/N portfolio for the industry sorted dataset. This is because the cross-sectional variation in the expected returns is relatively low in this dataset. The result of this is that the estimates for these returns do not contain enough useful information in order to offset the decrease in performance caused by the estimation risk of the timing strategies. Moreover, Kan and Zhou (2007) find that combining different portfolio strategies is essential in the presence of estimation risk. This stems from their result that combining the standard tangency portfolio with the more robust global minimum variance portfolio greatly improves performance.

These observations leave the question: *How do linear combinations in weights of naive and mean-variance portfolio selection strategies compare to the individual strategies?* We perform research on this topic by replicating and extending the paper of Kirby and Ostdiek (2012). We combine their methods of volatility timing and reward-to-risk timing, as well as some mean-variance efficient strategies, with the equally weighted portfolio to examine if the 1/N portfolio can still provide value to strategies that have been shown to outperform it.

We analyze three different methods of combining the different strategies. The first is an intuitively appealing approach where we combine strategies in a way that maximizes utility. Secondly, we examine equally weighted combinations. Finally, Guo, Boyle, Weng and Wirjanto (2019) have examined the market characteristics that indicate how difficult it is to beat the 1/N portfolio. We investigate the practical implications of these findings. We do this by examining to what effect these market characteristics can be used to dynamically change the weights given to each strategy in the portfolio combination.

We find that the method based on maximum utility gives the best performance. Combinations using this method often show similar or improved performance in comparison to the individual strategies that comprise those combinations. This performance improvement is greatest for combinations with strategies that suffer from large estimation risk. The reason this works is that periods with high estimation risk often follow each other, which means they can be predicted. In these periods more weight is then allocated to the 1/N portfolio, which is better suited for periods with high estimation risk, as it requires no estimation.

The paper proceeds as follows. In Section 2, we give an overview of the existing literature that forms the basis for our research. In Section 3, we describe the data that we use to perform our analysis. Afterwards, in Section 4, we discuss the individual portfolio selection strategies and the ways that we combine those strategies. Then we discuss our empirical findings in Section 5. Finally, we conclude the paper in Section 6.

2 Literature Review

Tu and Zhou (2011) analyze the performance of combinations of the 1/N portfolio with four more sophisticated, mean-variance based methods. They find that these combinations outperform the individual mean-variance methods in terms of utility. It should, however, be noted that these methods perform quite badly on their own in comparison to the equally weighted portfolio, with some even achieving negative risk-adjusted utilities. It should therefore not be a surprise that combining with the better-performing 1/N strategy, results in better performance overall. The portfolio combinations, however, struggle to outperform the 1/N portfolio in many cases. Only in the case of low risk aversion do the combinations consistently compete with it.

In contrast to their sophisticated methods, Kirby and Ostdiek (2012) have examined two mean-variance timing strategies which outperformed naive diversification, even when accounting for transaction costs. Thus our first contribution to this literature is to analyze the potential of combinations consisting of naive and strategic approaches in a different environment. We try to investigate whether linearly combining the weights of the mean-variance timing strategies with the 1/N strategy can result in better performance in comparison with the individual performance of the mean-variance and 1/N strategies.

Our second contribution relates to a novel approach exploiting market characteristics found by Guo et al. (2019) that indicate how difficult it is to beat the 1/N strategy. They start their investigation by finding the theoretical market condition in which the equally weighted portfolio is equal to the Sharpe ratio maximizing portfolio. This turns out to be when the expected return of each asset is proportional to the sum of its covariances. They use this to construct a measure of how favourable a market is to the performance of the 1/N portfolio, which they call the 1/N favorability index. This measure reflects the angle between the vector of expected return and the vector of aggregate covariances. If the angle is small, the market is close to optimal conditions for the 1/N strategy, corresponding to a 1/N favorability index close to 1. Following this, it intuitively makes sense to let this index control the weight of our portfolio combination. if the market is favourable to an equally weighted portfolio, we let the weight for this strategy increase, and in the opposite case, we put more emphasis on the mean-variance timing strategy.

The methodology of portfolio combinations is quite similar to that of forecast combinations. And as the existing literature on portfolio combinations is not very extensive, it is useful to discuss findings in this related field. Clemen (1989) and Stock and Watson (1998) find that combining different forecasts leads to an increase in forecast accuracy compared to the individual forecasts. They explain that this is because different models or methods may capture different aspects of the underlying data-generating process, and combining their predictions can mitigate the weaknesses of individual models. This finding is a justification for our approach of combining mean-variance based strategies with the 1/N portfolio. This is the case because the performance of mean-variance strategies is negatively impacted by estimation error. Therefore combining these strategies with the 1/N strategy, which requires no estimation, might help mitigate this weakness. Moreover, Smith and Wallis (2009) and Claeskens, Magnus, Vasnev and Wang (2016) point out an interesting phenomenon in forecast combinations commonly referred to as the forecast combination puzzle. This is the finding that simple forecast combinations often outperform more sophisticated combination weighting methods. They point to the estimation error of the combining weight as the root cause of this discrepancy in performance. We investigate if a parallel can be drawn from these findings to the field of portfolio combinations.

Finally, we discuss a few papers that have commented on Kirby and Ostdiek (2012). For example, Zakamulin (2017) is quite critical of their research. They state that the superior performance of the novel timing strategies of Kirby and Ostdiek (2012) appears as a result of some known market anomalies. Firstly they point out that these strategies inherently make use of the low-volatility anomaly documented by Blitz and Van Vliet (2007). This anomaly refers to the finding that low-volatility stocks often receive higher returns than those with higher volatility. This is also the reason why the minimum variance portfolio often performs well as found by Clarke, De Silva and Thorley (2011). Secondly, they comment on certain datasets used by Kirby and Ostdiek (2012) that are exposed to established factor premiums, like book-to-market ratio and momentum. mean-variance based strategies will improve performance over the 1/N strategy by profiting from these factor premiums. They find that it is not clear that the superior performance provided by portfolio optimization is still present when controlling for these anomalies. Similarly, Lo and MacKinlay (1990) have established that sorting stocks into portfolios by some empirically motivated characteristics of those stocks introduces a statistical bias. Specifically, this means that portfolio sorts based on important asset-pricing factors can lead to an increased likelihood of obtaining statistically significant results. Hsu, Han, Wu and

Cao (2018) also find that very few strategies significantly outperform the 1/N strategy when accounting for these biases.

3 Data

In order to replicate and extend the methods of Kirby and Ostdiek (2012), we use two of the datasets that they also examine, namely the 10 Industry and the 10 Momentum datasets. The sample period is July 1963 to December 2008 (546 monthly observations). Like Kirby and Ostdiek (2012), we draw these datasets from the Ken French data library.

Figure 1: Reward and Risk Characteristics of the Datasets

This figure gives a picture of the sample reward and risk characteristics of the 10 Industry and the 10 Momentum datasets. The graphs on the left show the cross-section of annualized return for the 10 portfolios and the graphs on the right show the cross-section of the annualized standard deviation. The reported statistics correspond to the period that is used to evaluate the out-of-sample performance of the portfolio strategies, which is from July 1973 until December 2008.

The 10 Industry dataset contains the returns on 10 different industries sorted by their SIC code. In the analysis of Kirby and Ostdiek (2012), this dataset posed the biggest challenge to their mean-variance methods in comparison to naive diversification. Thus for our research, we expect that we find the largest benefit of combining mean-variance strategies with naive diversification using this dataset.

The 10 Momentum dataset contains the returns of 10 portfolios that are sorted according to their recent price trend. As shown by Carhart (1997), momentum is an important factor in explaining excess returns. Consequently, sorting firms according to momentum promotes significant cross-sectional spread in returns. This, in turn, makes this dataset favorable to mean-variance timing strategies, as it causes estimators of the conditional expected return to carry more useful information. Figure 1 provides more detail on the reward and risk characteristics of both datasets.

We also use one dataset that contains the returns of the factors of the Fama and French (1992) three-factor model as well as the risk-free rate. Finally, we use a dataset that contains the returns of the momentum factor. These datasets are also drawn from the Ken French data library. We use the risk-free rate to be able to calculate the excess returns of the 10 Industry and 10 Momentum datasets. And we need the factor returns to calculate the betas for the Carhart (1997) four-factor model. It will become clear in the methodology section why we need these betas.

4 Methodology

The methodology for volatility and reward-to-risk timing strategies is taken from Kirby and Ostdiek (2012). The methodology we use for combining portfolio strategies is based on several previous studies like Kan and Zhou (2007), DeMiguel et al. (2009) and Tu and Zhou (2011). These have all analyzed portfolio combinations.

4.1 Portfolio Selection Strategies

We start by discussing the varying portfolio selection strategies that were examined in Kirby and Ostdiek (2012). First is the naive portfolio, where we place equal weights on each asset. Next there are four strategies based on the classic mean-variance framework, namely: the tangency portfolio, the minimum variance portfolio, an optimal constrained portfolio (OC), which targets the expected return of the 1/N strategy, and finally OC with short selling constraints.

Aside from these approaches, Kirby and Ostdiek (2012) propose several timing strategies. These strategies do not necessarily result in theoretically optimal portfolios, however they use practical rules to exploit sample information on the assets while limiting estimation error. One method, which they call 'volatility timing', solely uses information on the individual asset variances. For the second method, they also incorporate sample information on the expected returns. They call this 'reward-to-risk timing'.

4.1.1 Tangency Portfolio

The most traditional approach according to mean-variance theory is to choose the portfolio weights $\omega_{p,t}$ that maximize the quadratic utility function of an investor:

$$Q(\omega_{p,t}) = \omega'_{p,t}\mu_t - \frac{\gamma}{2}\omega'_{p,t}\Sigma_t\omega_{p,t}, \tag{1}$$

where $\mu_t = E_t(r_{t+1})$ is the conditional expected excess return of period $t+1$ using the information up until period t. similarly, $\Sigma_t = E_t(r_{t+1}r'_{t+1}) - E_t(r_{t+1})E_t(r_{t+1})'$ denotes the conditional expected covariance matrix. γ is the investors' relative risk aversion. We follow Kirby and

Ostdiek (2012) in using a simple rolling window estimator to estimate both the conditional return and covariance. These are calculated as $\hat{\mu}_t = (1/L)\sum_{i=0}^{L-1} r_{t-i}$ and $\hat{\Sigma}_t = (1/L)\sum_{i=0}^{L-1}(r_{t-i} - \hat{\mu}_t)(r_{t-i}-\hat{\mu}_t)'$. Here L denotes the window size which we set to be 120 months. Let $r_t = R_t - \iota R_{f,t}$ be the vector of excess returns of N assets at period t (R_t is the vector of regular returns and $R_{f,t}$ is the risk-free rate). For each strategy, we only consider weight allocation to the risky assets and not to the risk-free asset. The reason is that our objective is to compare strategies directly with each other and it is more difficult to inspect differences in those strategies if they can be caused by different allocations to the risk-free asset. From now on in the text we will use returns instead of excess returns for simplicity.

The analytical solution to the maximization problem where the investor only allocates his wealth to the risky assets is

$$\omega_{TP,t} = \frac{\Sigma_t^{-1}\mu_t}{|\iota'\Sigma_t^{-1}\mu_t|}. \tag{2}$$

This equation gives the weights of the tangency portfolio (TP). We take the absolute value of the denominator because the tangency portfolio would otherwise be conditionally inefficient if $\iota'\Sigma_t^{-1}\mu_t < 0$ (see Kirby and Ostdiek (2012)).

The tangency portfolio forms a foundation for mean-variance optimization theory. In the history of portfolio management, it is seen as the basic framework that is used to create many kinds of different asset allocation strategies. Even though it has been very significant throughout finance, the literature shows that the tangency portfolio exhibits very significant practical limitations which makes it unviable as an investing strategy. For example, Michaud and Michaud (2008) state that it is extremely sensitive to small changes in input parameters. Consequently, the risk of estimation error is magnified greatly. DeMiguel et al. (2009) also point to these reasons to explain why the 1/N strategy often outperforms strategies based on mean-variance optimization. Kirby and Ostdiek (2012) show another shortcoming of the tangency portfolio: turnover. the tangency portfolio often gives extreme weights to assets. And because of the input sensitivity, these extreme weight allocations also change drastically over time. Therefore the TP is often characterized by very high turnover and thus large transaction costs, which impedes the performance of this method even further.

4.1.2 Minimum Variance

The minimum variance portfolio (MV) is constructed by minimizing the variance $\omega_{p,t}'\Sigma_t\omega_{p,t}$ subject to $\omega_{p,t}'\iota = 1$. Like the tangency portfolio, it has a straightforward analytical solution:

$$\omega_{MV,t} = \frac{\Sigma_t^{-1}\iota}{\iota'\Sigma_t^{-1}\iota}. \tag{3}$$

Despite its simplicity, the minimum variance portfolio often performs quite well. This strategy is stable and as expected delivers low volatility. Surprisingly, this strategy also generally achieves high returns, which can be attributed to the low-volatility anomaly, as discussed in Section 2. Additionally, a big advantage of minimum variance over the tangency portfolio lies in the estimation risk. As conditional expected returns are not involved in the estimation process, the estimation error is greatly diminished. To add to this, Merton (1980) state that covariances of returns can be estimated more accurately than the mean returns. These characteristics of the minimum variance portfolio make it attractive as an asset allocation strategy.

Although to a much lesser extent than the TP, a downside of the minimum variance portfolio is its turnover. As can be seen in our replication of the results of Kirby and Ostdiek (2012), the turnover of the MV portfolio often greatly exceeds the turnover of the 1/N and mean-variance timing strategies. This aspect of this strategy leaves room for improvement.

4.1.3 Optimal Constrained Portfolio

The mean-variance optimization that results in the tangency portfolio is a very aggressive strategy that implicitly targets extreme returns. DeMiguel et al. (2009) find that the TP has conditional expected returns that often exceed 100% per year. It is unreasonable to expect such a strategy to perform well, which is why we look at a constrained version of the tangency portfolio which Kirby and Ostdiek (2012) calls the optimal constrained (OC) portfolio. The constraint that is imposed is that the OC portfolio targets conditional expected returns that are equal to the expected returns of the 1/N strategy (i.e. $\hat{\mu}_t{}'\iota/N$).

The weights of the OC strategy are calculated as:

$$\omega_{p,t} = \left(\frac{\mu_{e,t} - \mu_{MV,t}}{\mu_{TP,t} - \mu_{MV,t}}\right)\left(\frac{\Sigma_t^{-1}\mu_t}{\iota'\Sigma_t^{-1}\mu_t}\right) + \left(1 - \frac{\mu_{e,t} - \mu_{MV,t}}{\mu_{TP,t} - \mu_{MV,t}}\right)\left(\frac{\Sigma_t^{-1}\iota}{\iota'\Sigma_t^{-1}\iota}\right), \tag{4}$$

where $\mu_{e,t}$, $\mu_{MV,t}$ and $\mu_{TP,t}$ denote the conditional expected returns of the 1/N, MV and tangency portfolios. Note that this constrained version of the TP is a linear combination of the weights of the tangency portfolio and the minimum variance portfolio. OC mimics the aggressiveness of the 1/N portfolio which makes it a lot less volatile than the TP. Finally, Kirby and Ostdiek (2012) also examine a version of the optimal constrained strategy which prohibits short sales. This is mainly done to reduce turnover. they call this the OC+ strategy.

4.1.4 Volatility Timing

Next, we discuss the mean-variance timing strategies introduced by Kirby and Ostdiek (2012), starting with volatility timing (VT). This strategy can be seen as a constrained version of the minimum variance approach. The constraint is that it imposes all non-diagonal elements of the covariance matrix to be zero. Their reasoning behind this choice is that, firstly, it reduces the estimation risk of MV even further. This is because we now do not have to estimate any sample covariances and we do not need to invert the covariance matrix. Secondly, this volatility timing strategy is designed to counteract the substantial turnover that is normally present in the minimum variance portfolio. This is a result of the fact that setting covariances to zero prevents negative weights. The weight of asset i in period t for this constrained MV approach is given by

$$\hat{\omega}_{it} = \frac{(1/\hat{\sigma}_{it}^2)}{\sum_{i=1}^{N}(1/\hat{\sigma}_{it}^2)}, \quad i = 1, 2, ..., N, \tag{5}$$

where $\hat{\sigma}_{it}$ is the estimated conditional volatility of the return on asset i. This weight allocation strategy turns out to be quite simple. The weights are directly proportional to the assets' inverse volatility. This means that if an asset has low variance it will receive a large weight and vice versa.

One final adjustment is made to this strategy to allow the investor some control over the aggressiveness of the strategy, which then automatically also controls the turnover. This is done by introducing a tuning parameter η in equation 5 as follows:

$$\hat{\omega_{it}} = \frac{(1/\hat{\sigma_{it}}^2)^\eta}{\sum_{i=1}^{N}(1/\hat{\sigma_{it}}^2)\eta}, \quad i = 1, 2, ..., N, \tag{6}$$

where $\eta \leq 0$. η is the sensitivity to changes in the volatilities of the assets. If η is set to zero, the weights are completely insensitive to changes in volatility, in which case we get the 1/N portfolio. If $\eta = 1$, we retrieve the strategy in equation 5. The larger η becomes, the quicker the weights will change as a result of a volatility shift. Thus, a larger η results in higher turnover. We refer to the class of strategies that use equation 6 to find the weights as $VT(\eta)$.

4.1.5 Reward-to-risk Timing

The volatility timing strategies only use information on the assets' variances. This, however, ignores information on the returns. Therefore we also examine a strategy that uses conditional expected returns, namely reward-to-risk timing. The reward-to-risk timing strategy is designed similarly to the $VT(\eta)$ strategy in the sense that it uses a diagonal covariance matrix. But instead of taking the approach of a minimum variance strategy, we look at the sample tangency portfolio. The weights for this constrained version of the sample tangency portfolio are given by

$$\hat{\omega}_{it} = \frac{(\hat{\mu}_{it}/\hat{\sigma}_{it}^2)}{\sum_{i=1}^{N}(\hat{\mu}_{it}/\hat{\sigma}_{it}^2)}, \quad i = 1, 2, ..., N, \tag{7}$$

where $\hat{\mu}_{it}$ is the estimated conditional mean of the return of asset i. A result of incorporating conditional expected returns into the strategy is of course increased estimation risk, but the hope is that they contain enough information to mitigate this extra risk. Unlike with VT, the version of reward-to-risk timing in equation 7 does not come with the inherent property of prohibiting short sales. This could still allow extreme weights, which is the main thing we wish to avoid. For this reason all negative $\hat{\mu}_{it}$ are set to zero. Secondly, like for the volatility timing, we introduce tuning parameter η to control the aggressiveness of the strategy. This gives the reward-to-risk weights as

$$\hat{\omega}_{it} = \frac{(\hat{\mu}_{it}^+/\hat{\sigma}_{it}^2)^\eta}{\sum_{i=1}^{N}(\hat{\mu}_{it}^+/\hat{\sigma}_{it}^2)^\eta}, \quad i = 1, 2, ..., N, \tag{8}$$

where $\hat{\mu}_{it}^+ = \max(\hat{\mu}_{it}, 0)$. Kirby and Ostdiek (2012) abbreviate this class of strategies as RRT(μ^+, η).

Kirby and Ostdiek (2012), however, point out that the rolling sample estimates of the conditional expected returns will likely not be very accurate. Therefore they propose an alternative estimator of these returns, which is based on the capital asset pricing model (CAPM). If CAPM holds, all of the systemic variation between the returns of different assets is explained by their betas. Using betas instead of expected returns will give us a more robust picture of the assets' relative expected returns. We follow Kirby and Ostdiek (2012) in using the Carhart (1997) four-factor model to estimate the betas, after which we use the average of these four betas. This results in the RRT(β^+, η) strategy, which is given by

$$\omega_{it} = \frac{(\overline{\beta}_{it}^{+}/\sigma_{it}^{2})^{\eta}}{\sum_{i=1}^{N}(\overline{\beta}_{it}^{+}/\sigma_{it}^{2})^{\eta}}, \quad i = 1, 2, ..., N, \tag{9}$$

where $\overline{\beta}_{it}^{+} = \max(\overline{\beta}_{it}, 0)$ and $\overline{\beta}_{it} = (1/K)\sum_{j=1}^{K}\beta_{ijt}$ is the average conditional beta of asset i with respect to K factors.

4.2 Portfolio Combinations

We continue ahead to the discussion of how these strategies are combined with naive diversification. The basic setup is to linearly combine the portfolio weights of the 1/N and mean-variance timing strategies. The weights of the combination strategy at time t are then given by

$$\omega_{c,t} = \delta_t \omega_e + (1 - \delta_t)\omega_{p,t}, \tag{10}$$

where ω_e is the vector of weights for the equally weighted portfolio (i.e. $\omega_e = \iota/N$). $\omega_{p,t}$ is the vector of weights for the mean-variance timing strategy, and $0 \leq \delta_t \leq 1$ is the combining weight at time t. This is all simple enough, but the real challenge lies in the way we determine this combining weight. We take three different approaches to do this. Firstly, we estimate the δ_t using utility optimization. Secondly, we examine the equally weighted combination strategy. Finally, we use the 1/N favourability index proposed by Guo et al. (2019) to determine the combining weight.

4.2.1 Utility Optimization

We want to optimally choose δ_t by maximizing the expected utility of a mean-variance investor with quadratic utility:

$$Q(\omega_{c,t}) = \omega_{c,t}'\mu_t - \frac{\gamma}{2}\omega_{c,t}'\Sigma_t\omega_{c,t}. \tag{11}$$

Here γ represents the investors' relative risk-aversion. We examine the standard case where $\gamma = 1$. The obvious upside of this method is that, in theory, the combination of strategies dominates the performance of both naive diversification and mean-variance timing. This is because both of these strategies are included in the possibilities of the portfolio optimization, specifically when $\delta = 1$ and $\delta = 0$. This method is designed to put more emphasis on a particular strategy in the combination if it is expected to perform well. For example, in a period where there is high estimation error, the naive diversification strategy would likely be preferred over mean-variance strategies. On the other hand, if, for example, the cross-sectional variation in returns happens to be high, more weight will be put on mean-variance strategies such as RRT(μ^+, η). It is an attempt to get the best of both worlds.

It is important to note that optimizing utility in-sample would not be effective. This happens because the mean-variance strategies are designed to use the sample information on returns and covariances to construct their weights. If we then try to optimize δ using the same information, the mean-variance based strategies will always be preferred over 1/N. In this case, δ_t will always be equal to zero, which would destroy the potential usefulness of combining these strategies.

The way to combat this issue is to introduce a validation period into the estimation process. This is a period that lies between the training sample, which is used to estimate the expected returns and covariance, and the out-of-sample period. The validation sample is then used to estimate the combining weight on unseen data without compromising the integrity of the test sample. This method creates an even playing field and thus allows the strengths of the 1/N strategy, diversification and no estimation error, to play a role. Using a validation sample is the most popular way to tune hyperparameters in the literature and is used successfully by several big papers like Gu, Kelly and Xiu (2020) and Chen, Pelger and Zhu (2023).

A choice has to be made on the length of the validation period. Too short a period will not give reliable estimates for δ as the sample size will be too small. If the period is too long it means that the size of the training sample is shorter. As a result, the estimates for the expected returns and covariance would become less accurate. Through experimentation we find that a validation period of three years strikes a good balance as it gives the best performance. Consequently, the training sample is seven years to ensure that our model uses the same information as the individual weight allocation strategies. This way we can sensibly compare the performance of this combining strategy with the results of Kirby and Ostdiek (2012). So to find δ_t, we use the rolling window estimators for the expected returns and covariances with $L = 84$ to find the weights for the specified mean-variance strategy for each date in the validation sample (i.e. the three years prior to period t). We then find the δ_t that maximizes the sum of the utilities over the entire validation sample. For the covariance matrix in the utility function in equation 11 we use the sample covariance matrix of the three-year validation period. Finally, we find the weights of the combination strategy using equation 10. Note that we still use the 10-year rolling window estimates to determine $w_{p,t}$.

4.2.2 Equally Weighted Combination

It, however, remains to be seen if the utility optimization process translates to better performance out-of-sample. To avoid estimation risk, it is worth looking at a method that does not require any optimization. After all, these characteristics are exactly what has made the 1/N portfolio so historically successful. The most straightforward approach is to simply give equal weights to the two strategies that we would like to combine. This means we set $\delta_t = \frac{1}{2}$ for every period t. The evidence from forecast combinations supports this approach. Recall that Smith and Wallis (2009) and Claeskens et al. (2016) find that simple combinations of forecasts often outperform more sophisticated combination weighting methods.

4.2.3 1/N Favorability Index

Our final portfolio combination method relates to the measure proposed by Guo et al. (2019) to describe how difficult it is to beat the 1/N portfolio. The goal of their research was to find conditions of a market in which the naive diversification strategy has good performance. They did not use empirical analysis to reach this goal, but they looked at this problem from a theoretical point of view. They analyzed the condition in which the 1/N portfolio would be the optimal choice for the utility-optimizing investor. This is the same as determining when the 1/N portfolio is equal to the tangency portfolio. This turns out to be when the expected return of each asset is proportional to the sum of its covariances. More formally, the condition

$$\mu = c\Sigma\iota \tag{12}$$

needs to hold for some $c > 0$. Conversely, if this condition holds for some $c < 0$, the 1/N portfolio would be the Sharpe ratio minimizing portfolio. This means that the 1/N portfolio would be outperformed by every other portfolio where the sum of the weights is equal to one. Therefore a sensible measure of the performance strength of the naive diversification strategy would reflect the proportionality of the returns of assets with their aggregate covariances. With this rationale they developed the 1/N favorability index (we abbreviate with FI), which is given by

$$FI_t = \cos(\mu_t, \Sigma_t\iota) = \frac{\mu_t'\Sigma_t\iota}{||\mu_t||\,||\Sigma_t\iota||}, \tag{13}$$

where $||\cdot||$ represents the Euclidian norm of a vector. Importantly, Guo et al. (2019) show that the rolling window estimator of FI_t (i.e. $\cos(\hat{\mu}_t, \hat{\Sigma}_t\iota)$) is consistent. We put the research of Guo et al. (2019) to practical use in hopes of improving the performance of the combined portfolio. We do this by using the FI as the combining weight δ_t. With this combining weight we can then calculate the weights of the combining strategy using equation 10. When the market is optimal for the 1/N portfolio, $FI_t = 1$ and we put all of the weight on the 1/N strategy. If there is no proportionality of the returns with their aggregate covariances, $FI_t = 0$. In this case we put 100% of the weight on the mean-variance based strategy. If it happens that $FI_t < 0$, the 1/N strategy theoretically performs very poorly which means we should short this strategy. In the worst case for the 1/N portfolio (i.e. $FI = -1$), we short 100% of the 1/N portfolio and invest 200% in the mean-variance based strategy.

4.3 Performance Evaluation

In this section we describe how we evaluate the performance of the different weight allocation strategies. We do this in the same way as Kirby and Ostdiek (2012). We use two different measures for the out-of-sample performance of a strategy. The first is the Sharpe ratio, $\lambda_p = \mu_p/\sigma_p$. This ratio is estimated using the sample mean and variance of the out-of-sample returns, thus $\hat{\mu}_p = (1/T)\sum_{t=h+1}^{T+h} r_{pt}$ and $\hat{\sigma}_p^2 = (1/T)\sum_{t=h+1}^{T+h}(r_{pt} - \hat{\mu}_p)^2$. The reported values of $\hat{\lambda}_p$ are based on the annualized values of $\hat{\mu}_p$ and $\hat{\sigma}_p$ for each portfolio strategy. The second measure of performance is one with an economic interpretation. It is a performance fee that signifies the maximum fee an investor would be willing to pay to switch from one strategy to another. If we compare strategies i and j, and such a fee Δ_γ is imposed on strategy j, we then must have that the expected utilities of strategy i and j are equal, i.e. $E[U(R_{p_i,t})] = E[U(R_{p_j,t} - \Delta_\gamma)]$. We consider two levels of risk aversion (namely $\gamma = 1$ and $\gamma = 5$), and we report the sample maximum annualized fee in basis points that an investor would be willing to pay to switch from the 1/N strategy to each mean-variance based strategy that we consider.

As turnover is an important factor in determining whether a strategy is viable, we also report the performance measures of each strategy when accounting for the performance decrease caused by transaction costs. Kirby and Ostdiek (2012) define turnover as the fraction of invested wealth traded in a given period to rebalance the portfolio. Before we can calculate this, it is important to note that the weights of assets change according to the size of their returns. This happens prior to rebalancing towards the desired weights of the next period. Thus we first calculate the weights of each asset i before the portfolio is rebalanced at time t:

$$\tilde{\omega}_{i,t} = \frac{\omega_{i,t-1}(1 + R_{i,t})}{\sum_{i=1}^{N} \omega_{i,t-1}(1 + R_{i,t})}. \tag{14}$$

Then the turnover at time t is calculated as follows:

$$\tau_{p,t} = \sum_{i=1}^{N} |\omega_{i,t} - \tilde{\omega}_{i,t}| + \left| \sum_{i=1}^{N} (\omega_{i,t} - \tilde{\omega_{i,t}}) \right|, \tag{15}$$

where $\omega_{i,t}$ is the desired weight in asset i at time t. Then the portfolio return net of transaction costs for period t (i.e. $\tilde{R}_{p,t}$) is calculated as

$$\tilde{R}_{p,t} = (1 + R_{p,t})(1 - c\tau_{p,t}) - 1, \tag{16}$$

where c is the level of proportional costs per transaction. We follow Kirby and Ostdiek (2012) in setting transaction costs to 50 basis points of the total turnover. For the statistical inference for the differences in Sharpe ratios between strategies as well as for the performance fees we use GMM estimation as described in Kirby and Ostdiek (2012).

5 Results

In this section we first revisit the results of Kirby and Ostdiek (2012) for the 10 industry and 10 momentum datasets. This provides context so we can adequately discuss and compare the results of the three different portfolio combining strategies. They examine the performance of all strategies discussed in the methodology. For the timing strategies they investigate three different cases, namely $\eta = 1$, $\eta = 2$ and $\eta = 4$.

5.1 The Kirby and Ostdiek (2012) Results Revisited

In general, the results of our replication are very similar to the results of Kirby and Ostdiek (2012). The minor differences that are present are likely caused by changes in the data from the Ken French data library. The only case where these changes in the data have a significant impact is for the tangency portfolio. This only corroborates the arguments of Michaud and Michaud (2008), Kirby and Ostdiek (2012), and many other papers that state that the tangency portfolio is extremely sensitive to changes in the input data.

We report the results of our replications of the findings for the 10 industry dataset in Table 1. Kirby and Ostdiek (2012) find that this dataset provides the biggest challenge for the timing and mean-variance efficient strategies to outperform the 1/N portfolio. The 1/N strategy performs quite well, with the annualized mean and standard deviation of the excess returns being 5.85% and 15.01% respectively. This yields a Sharpe ratio of 0.390 without transaction costs. As expected, this strategy has a low turnover at only 2.4%. This only decreases the Sharpe ratio to 0.380.

The volatility timing strategies show a lot of promise for this dataset. As for $\eta = 1, \eta = 2$ and $\eta = 4$ they have both a higher $\hat{\mu}_p$ and a lower $\hat{\sigma}_p$ than 1/N. Thus the Sharpe ratios also exceed that of the 1/N strategy. VT greatly reaps the benefits of not estimating the covariances between returns. This is reflected in its low turnover, which ranges from 2.5% to 3.5%, only slightly higher than that of the 1/N strategy. Therefore these strategies remain better after the inclusion of turnover.

In contrast to VT, RRT(μ^+, η) strategies do not perform very well on this dataset in terms of Sharpe ratio. $\hat{\lambda}_p$ is 0.334 for $\eta = 1$, 0.328 for $\eta = 2$ and 0.319 for $\eta = 4$. They also underperform in terms of turnover, which ranges from 7.5% for $\eta = 1$ to 11.7% for $\eta = 4$. These strategies suffer from the low cross-sectional variation in the returns of this dataset. As a consequence, There is not much useful information to be gained by including the expected conditional returns in the weight allocation process, but the estimation risk of estimating the expected returns is still present. This estimation risk, however, can be greatly diminished by using the alternative estimator of the conditional expected returns, namely the average betas of the Carhart (1997) four-factor model. The Sharpe ratios for the RRT(β^+, η) strategies are 0.419 for $\eta = 1$, 0.430 for $\eta = 2$ and 0.433 for $\eta = 4$. After transaction costs the Sharpe ratios become 0.407, 0.413 and 0.406 respectively.

So both timing strategies are able to outperform the 1/N strategy, even with a challenging dataset. Now we turn to the mean-variance efficient (MVE) strategies. Firstly, as expected, the tangency portfolio performs horribly. It achieves a negative mean return, and thus a negative Sharpe ratio with a value of -0.276. Turnover is extremely high, exceeding 500% of wealth invested, which deteriorates its performance even more. The OC strategy performs considerably better with a Sharpe ratio of 0.449. It does suffer from high turnover which is 28.9%. With the inclusion of transaction costs its Sharpe decreases to 0.318. OC+ outshines OC in terms of turnover, which becomes 13.1% after imposing short selling constraints. It has a turnover adjusted Sharpe of 0.405. The best performer among the MVE strategies is the minimum variance strategy. MV has the lowest standard deviation and the highest mean return of all strategies we examine. This results in a high Sharpe ratio of 0.515 without transaction costs. As for the other MVE strategies, however, its turnover is quite high at 16.1%. With transaction costs its performance drops as the Sharpe ratio becomes 0.440.

Now we shift our focus to the replication of the results for the 10 Momentum dataset. These results can be found in Table 2. This dataset provides a lot more opportunity for sophisticated weight allocation strategies to perform well relative to the 1/N strategy. This stems from the fact that the cross-sectional variation in the returns is a lot higher than for the 10 industry dataset.

The values of $\hat{\mu}_p$, $\hat{\sigma}_p$ and $\hat{\lambda}_p$ for the 1/N strategy are 4.68%, 16.67% and 0.281. Again transaction costs are low at 1.8%. All three VT strategies now significantly outperform 1/N at the 1% level, with Sharpe ratios 0.329 for $\eta = 1$, 0.355 for $\eta = 2$ and 0.375 for $\eta = 4$.

RRT(μ^+, η) strategies benefit the most from the large cross-sectional variation in returns. Their Sharpe ratios are 0.454 for $\eta = 1$, 0.475 for $\eta = 2$ and 0.481 for $\eta = 4$. The differences with

1/N are all significant at the 1% level. This is true even in the presence of transaction costs. The turnover adjusted Sharpe ratios range from 0.433 to 0.470. In contrast to the 10 industry dataset, using the alternative estimator for μ_t does not make much of a difference. The Sharpe ratios then become 0.397, 0.448 and 0.481 respectively.

For the MVE strategies, MV is still a strong performer, with a Sharpe ratio of 0.491, which is significantly better than the 1/N strategy at the 5% level. This is still greatly decreased after accounting for transaction costs, with an adjusted Sharpe of 0.378. Additionally, the OC strategy emerges as one of the best performers without transaction costs. It has a Sharpe ratio of 0.579 without, and 0.445 with transaction costs. This is significant at 1% without transaction costs and at 10% with transaction costs.

5.2 Results of the Combination Strategies

We can now discuss the results of the three methods of combining the more sophisticated strategies with the 1/N strategy. We start with the maximum utility combinations, where the combining weight is determined by optimizing the total utility in the validation sample.

5.2.1 Maximum Utility Combination

The results of the maximum utility combinations for the 10 industry dataset are presented in Table 3. We compare these values to the values in Table 1, which allows us to see the impact of combining portfolio selection strategies relative to the individual strategies that comprise that combination. It becomes apparent that combining strategies with the 1/N strategy impacts performance differently depending on the mean-variance based strategy used.

For the VT strategies, the combinations showed similar or slightly improved performance in comparison to those strategies individually. Notably, the results for the VT(1) combination stayed exactly the same. This happens because the individual VT(1) strategy consistently dominates the 1/N strategy in the validation sample, which causes the combining weight to be equal to zero for almost every period. There are a few periods around 1996 where this is not the case, but even then the δ_t were mostly small. For $\eta = 2$ there is a slight improvement compared to the individual VT(2) strategy, as its Sharpe ratio increased from 0.459 to 0.464. The Sharpe ratio of the combination of the VT(4) strategy improved with a larger margin from 0.467 to 0.487. It seems that the more aggressive a strategy is, the more benefit combining with 1/N has to its performance. The explanation for this is that an aggressive strategy more sharply changes the weights when its input parameters change. If there happens to be large estimation error in estimating these inputs, performance then deteriorates more quickly. In these cases, the aggressive strategy will likely be outperformed by the 1/N portfolio in the validation sample. This allows the 1/N strategy to 'help' the aggressive strategy in periods of high estimation risk. The fact that this method of combining strategies improves performance means that these periods of high estimation risk are predictable to some extent.

Combining with the 1/N portfolio gives large benefits for the RRT(μ^+, η) strategies. The Sharpe ratios for the individual RRT ranged from 0.319 to 0.334, and they become 0.392 for $\eta = 1$, 0.389 for $\eta = 2$ and 0.396 for $\eta = 4$. So the RRT strategies that performed poorly individually for this dataset, are now up to par with the 1/N strategy. And again the most aggressive strategy receives the greatest increase in performance when combined with 1/N. A problem for the original RRT(μ^+, η) strategies is their turnover, but this does not change much after combining strategies. This might be unexpected, considering that one of the strong points of the 1/N strategy is its low turnover. However, the benefit of combining with a low turnover gets offset by the added turnover that is caused by shifts in the combining weight δ_t.

If we shift our focus to the RRT strategies using the alternative estimator of μ_t, the combinations do not perform very differently from the individual RRT(β^+, η) strategies. Individually, their Sharpe ratios ranged from 0.419 to 0.433. After combining these become 0.413 for $\eta = 1$, 0.418 for $\eta = 2$ and 0.429 for $\eta = 4$. The main cause that combining with 1/N does not make much difference for this estimator is that there is less estimation error for the individual RRT(β^+, η) strategies. Therefore there are fewer periods where the 1/N strategy is needed to counteract the estimation risk.

As combining with the 1/N portfolio seems to have a more positive effect in the presence of high estimation risk, it should be especially helpful for the mean-variance efficient strategies, which is indeed the case. The tangency portfolio performed atrociously individually, achieving a negative Sharpe ratio of -0.276. After combining with 1/N, its Sharpe ratio skyrockets to 0.414, which means that it now outperforms the individual 1/N strategy in the absence of transaction costs. An investor would be willing to pay a maximum fee of 38 basis points for $\gamma = 1$ and 35 for $\gamma = 5$ to switch from the 1/N strategy to the combination with the tangency portfolio. Its turnover decreased significantly after combining with 1/N, from over 500% to 13.8%. This is much more manageable, but still deteriorates its performance quite a bit, as its turnover adjusted Sharpe is 0.359. The performance of the OC and OC+ strategies also gain some ground. Their Sharpe ratios were 0.449 and 0.466 respectively, and they increase to 0.492 and 0.490 after combining with 1/N. The turnover for the OC strategy decreases from 28.9% to 23.4% after the combinations. When accounting for transaction costs the Sharpe then becomes 0.386, which now slightly outperforms the 1/N strategy. The combination with OC+, with its lower turnover, achieves an even higher turnover adjusted Sharpe of 0.430. The MV strategy sees a similar increase to the VT(η) strategies. Its Sharpe ratio increases from 0.515 to 0.521 after combining with 1/N, while the transaction costs remain roughly the same. Without transaction costs this combination strategy is statistically significant at the 10% level.

In general for this dataset, combining a strategy with the 1/N portfolio achieves similar or improved results no matter the strategy. This is interesting, as weight allocation strategies based on in-sample utility optimization (e.g. the tangency portfolio) often yield poor results. This is the case because of their high estimation risk. In contrast, utility optimization can be effective when done in the validation sample. In this case, utility optimization works by finding periods where mean-variance based strategies are likely to perform poorly in comparison to 1/N. By doing this it makes predictions on the level of estimation risk in the next period.

A practical implication of this combination method is that can be employed to mitigate the downside risk of a mean-variance based strategy. Because, if an individual strategy performs worse than the 1/N strategy, combining the two will give you performance that is at least on par with the 1/N strategy itself. And if the individual strategy performs better than 1/N, the corresponding combination strategy will not harm the results much, and might even slightly improve over the individual strategy.

It is, however, important to understand that the success of combining with the 1/N portfolio is a bit dependent on the dataset that is used. At least for the timing strategies, it seems a requirement that the performance of the 1/N strategy comes somewhat close to that of the mean-variance based strategies. Otherwise, the 1/N portfolio does not provide enough value to mitigate the estimation risk of estimating the combining weight δ. This can be seen in the results for the 10 Momentum dataset, which is shown in Table 4. The 1/N for this dataset performed quite poorly, achieving a Sharpe ratio of only 0.281. This poor performance can be explained by the observation that the 1/N strategy is quite dubious for a momentum sorted dataset. As discussed in the data section, momentum is an important factor in explaining returns. Therefore the portfolios with low momentum generally achieve lower returns, while the high momentum portfolios exhibit higher returns. Knowing this information, it then makes less sense to equally distribute wealth among these momentum portfolios. Nevertheless, we shall see that combining strategies with 1/N can still prove quite useful for certain mean-variance based strategies.

Of the timing strategies, VT showed the least difference in performance after combining with the 1/N strategy. The Sharpe ratios for the individual VT strategies were 0.329, 0.355 and 0.375 in ascending order of η. The VT combinations have Sharpe ratios of 0.328, 0.350 and 0.366.

The decrease in performance of the RRT(β^+, η) strategies after combining with 1/N is a bit more drastic. A reason for this, as discussed, is the poor relative performance of the 1/N portfolio compared to RRT. Specifically, the individual RRT strategies have Sharpe ratios of 0.454, 0.475 and 0.493, which are all more than 50% higher than that of the 1/N strategy. We see this in the results of the combination strategies, which have lower Sharpe ratios of 0.431 for $\eta = 1$, 0.447 for $\eta = 2$ and 0.463 for $\eta = 4$. Although these Sharpe ratios are still significantly better than that of the individual 1/N strategy at the 1% level, they give little reason to use the combinations over the individual RRT strategies. The same holds true if we use the alternative estimator for the conditional expected returns. The Sharpes of the combination strategy with RRT(β^+, η) are 0.385, 0.430 and 0.459 compared to 0.397, 0.448 and 0.481 of the individual RRT strategies.

Combining strategies with 1/N starts to shine again when it comes to the MVE strategies. For these, the estimation risk is so high that even a poorly performing 1/N portfolio is quite useful. This can most saliently be seen in the combination with the tangency portfolio. The individual TP was extremely aggressive for this dataset, with a $\hat{\mu}_p$ and $\hat{\sigma}_p$ of 507 and 1702 respectively. Therefore the inclusion of the 1/N was needed to create a more balanced strategy. The combination achieved an annualized mean return of 7.93 and a standard deviation of 16.88, resulting in a healthy Sharpe ratio of 0.470. Its turnover shot down from over 100000% to a more modest 19.7%. When accounted for transaction costs its Sharpe ratio becomes 0.400, which is significantly better than that of the 1/N portfolio at the 5% level.

The combination of 1/N with the OC strategy proves especially successful. Without transaction costs, the OC strategy by itself already performed very well with a Sharpe ratio of 0.579. The large turnover of 34,7%, however, decreases it to 0.445 after accounting for transaction costs. This is still quite good, but now it is outperformed by several RRT strategies. Combining OC with 1/N improves this performance. Without transaction costs, the Sharpe of the combination becomes 0.590. Importantly, the turnover drops to 28,0%, which gives an adjusted Sharpe ratio of 0.478. This means that the OC combination has the best performance of all individual strategies, even when accounting for transaction costs. This really shows the potential of combining weight allocation strategies.

For the MV strategy combining with the 1/N portfolio does not have a lot of impact. Without transaction costs the Sharpe ratio drops from 0.491 for the individual MV to 0.479 for the combination. This decrease in performance is largely offset in the presence of transaction costs, due to the lower turnover of the MV combination.

5.2.2 Equally Weighted Combination

We have seen that utility optimization to obtain the δ_t proves useful in a lot of cases, especially when the mean-variance based strategy in the combination suffers from high estimation risk. Now let us see what happens if we do not use optimization to find the combining weight, and instead let $\delta_t = \frac{1}{2}$ for every period t. The results for the equally weighted combinations can be seen in Table 5 for the 10 Industry dataset and Table 6 for the 10 Momentum dataset.

When looking at the results for both datasets it becomes quite apparent that they do not show many abnormal improvements over the individual strategies that comprise the combinations. The performance of the combinations in terms of Sharpe ratio mostly lies around the average of the Sharpes of their components. This can be illustrated with an example from the 10 Industry dataset: here, the Sharpe ratio of the 1/N portfolio is 0.390, and that of VT(1) is 0.431, which gives an average of 0.4105. Meanwhile the Sharpe ratio of the equally weighted combination of these strategies is 0.410. This suggests that a constant combining weight δ_t leads to a combined performance with the same weight. This is evidence that it is not very straightforward to draw a parallel between portfolio combinations and findings in the field of forecast combinations. Recall that simple combinations of different forecasts often outperformed more sophisticated methods of determining the combining weight (Smith and Wallis (2009), Claeskens et al. (2016)). In contrast, in our research the maximum utility combinations almost always outperform the equally weighted combinations. This also confirms our conclusion that the potential of combining portfolio strategies with 1/N lies mainly in using the 1/N portfolio only when it performs relatively better than the other component strategy (which occurs due to periods with high estimation risk). This is because we find that the combinations perform worse if the 1/N strategy is included with the same combining weight for every period.

Apart from these general findings, there are a few minor things to note about the results of the equally weighted combinations. These relate to the combinations of the MVE strategies. Firstly for the 10 Industry dataset, the MV combination now significantly outperforms the individual 1/N strategy at the 5% level in terms of Sharpe ratio. This is not the case for the MV strategy by itself, even though it achieves a higher Sharpe than the MV combination. This happens because the MV combination is more highly correlated with 1/N, which results in a lower standard error of the difference in Sharpe ratio. Consequently, less of a difference in performance is needed to make that difference significant. Secondly, we find that the equal combinations can sometimes outperform their component strategies if those strategies suffer from high turnover. For example for the 10 Industry dataset, the individual OC strategy has a high turnover of 23.4% and a Sharpe of 0.318 when adjusted for transaction costs. Yet the combination with 1/N has an average monthly turnover of 14.0% and an adjusted Sharpe ratio of 0.384. Thus this combination outperforms both 1/N and the individual OC strategy after accounting for transaction costs.

5.2.3 Favorability Index Combination

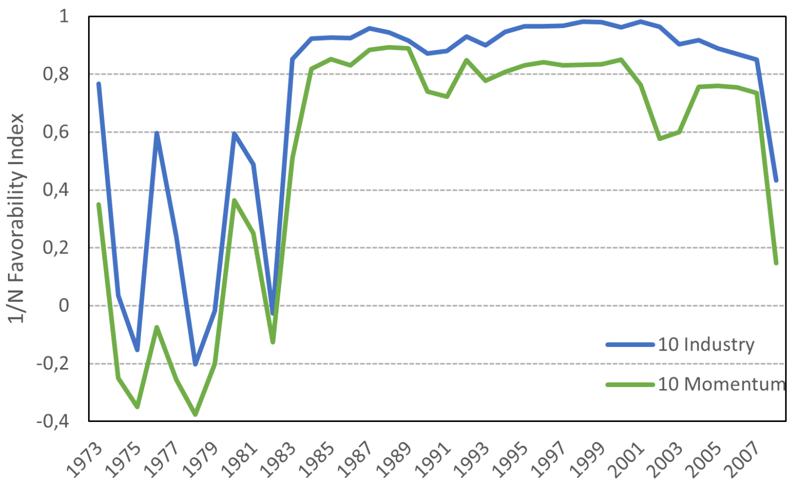
We now turn to the results of the final method of making portfolio combinations, which is done by letting the 1/N favorability index control the combining weight δ_t. The results for the 10 Industry dataset are shown in Table 7 and for the 10 Momentum dataset in Table 8.

With a quick glance at these results, it can be gathered that this method of combining different portfolio strategies does not perform very well. This is true even when comparing them to the results of the equally weighted combinations. For the 10 Industry dataset, every combination strategy using the favorability index has a worse Sharpe ratio than that of its equally weighted counterpart, with or without accounting for transaction costs. For the 10 Momentum dataset, only the FI RRT combinations outperform the equally weighted RRT strategies, but they are still outclassed by the maximum utility combinations. We investigate what causes the favorability index to fail as a method of determining the combining weight δ_t.

The most prominent reason is that this method puts too much emphasis on the 1/N strategy. This can be seen from the values of δ_t for both datasets. The mean combining weight for the 10 Industry dataset is 0.72, which means that on average 72% of the weights of the combination strategy are determined by the 1/N rule. For the 10 Momentum dataset this value is 0.53. This is a problem as most of the timing strategies outperform the 1/N portfolio. Therefore putting much emphasis on the 1/N portfolio is most definitely suboptimal. In contrast, for the maximum utility combinations, if an individual mean-variance based strategy outperformed the 1/N portfolio, we often saw that the δ_t of their combinations were close to zero for many periods. In Figure 2 we see the progression of the FI for both datasets over time. The FI is quite volatile in the first decade of the out-of-sample period. After that it remains consistently very high almost until the end of the out-of-sample period. The fact that the δ_t are so high for the FI method reflects a fundamental flaw that prevents us from using the index successfully. That flaw is that the FI does not use any information on the performance of the mean-variance based strategies. Therefore it fails to predict periods of high estimation risk, in which the 1/N strategy is likely to outperform the mean-variance strategies.

It should be noted, however, that the favorability index does contain some useful information on the performance of the 1/N portfolio. Looking at Figure 2, we see that the FI for the 10 Industry dataset is always at least as high as the FI for the 10 Momentum dataset. And as we know, the 1/N portfolio has much better performance for the 10 Industry dataset. So although the FI does not contain the right information to facilitate high performing portfolio combinations, it can be used to quickly compare the potential performance of the 1/N portfolio for different datasets.

Figure 2: Plot of the time series of the 1/N favorability index



6 Conclusion

Kirby and Ostdiek (2012) examined the performance of many different mean-variance based strategies of portfolio selection. This includes two novel strategies, namely volatility timing and reward-to-risk timing, which both showed an ability to outperform the historically well-performing 1/N portfolio. We provide an analysis of combinations of these strategies with the 1/N strategy by linearly combining their weights. We do this to investigate the potential of combining different portfolio selection strategies by comparing performance to the mean-variance based strategies examined by Kirby and Ostdiek (2012).

We examined three different methods to combine the mean-variance based strategies with the 1/N strategy. Firstly we choose the combination that maximizes the investors' utility. Secondly, we examine equally weighted combinations of the strategies. Finally, we determine the composition of the combinations using the 1/N favourability index proposed by Guo et al. (2019).

We find that the maximum utility method of creating portfolio combinations is the best performing of the three. In many cases, these combination strategies show similar or improved performance in comparison to the individual mean-variance based strategies. This method works, because it makes predictions on the level of estimation risk in the next period. Thus in periods with high estimation risk more emphasis is put on the 1/N strategy, as this strategy requires no estimation. This way the 1/N strategy can help mitigate a large weakness of mean-variance based strategies. This research gives a reason for investors to consider adopting combinations of asset allocation strategies to achieve greater economic gains and limit downside risk.

For future research in this area, one could investigate combinations of strategies that do not involve the 1/N strategy. One could also investigate the performance of three or more different strategies put together. Finally, as our methods for estimating the conditional returns vector and covariance matrix are quite simple, it would be interesting to see if our findings are robust to different estimation methods.

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Table 1: Replication Results 10 Industry

This table reports our replication of the results of Kirby and Ostdiek (2012) for the 10 Industry dataset. It shows the performance of the 1/N strategy, three VT(η) strategies, three RRT(μ^+, η) strategies, three RRT(β^+, η) strategies, and four mean variance efficient strategies. It reports the following sample statistics of the time series of returns that is generated by each strategy: the annualized mean ($\hat{\mu}_p$), the annualized standard deviation ($\hat{\sigma}_p$), the annualized Sharpe ratio ($\hat{\lambda}_p$), the average monthly turnover ($\hat{\tau}_p$), the fee in basis points that an investor would be willing to pay to switch from the 1/N strategy another particular strategy and p-values corresponding to the difference in Sharpe ratios with the 1/N strategy as well as the p-values for the performance fees. A '-' for the TP strategy signifies that it was not possible to calculate this value, as there was no real value for a fee to make the investor indifferent between the TP and 1/N strategies. The sample period is July 1963 - December 2008 (546 observations) and the reported values are calculated using the period July 1973-December 2008 (426 observations), as the first 120 observations are held out to initialize the rolling window estimates for the conditional mean vector and the conditional covariance matrix.

	No Transaction Costs								Transaction Costs						
				vs. 1/N							vs. 1/N				
Strategy	$\hat{\mu}_p$	$\hat{\sigma}_p$	$\hat{\lambda}_p$	p-val	$\hat{\Delta}_1$	p-val	$\hat{\Delta}_5$	p-val	$\hat{\tau}_p$	$\hat{\lambda}_p$	p-val	$\hat{\Delta}_1$	p-val	$\hat{\Delta}_5$	p-val
1/N	5.85	15.01	0.390						0.024	0.380					
Panel A. Volatility Timing Strategies															
VT(1)	6.10	14.15	0.431	0.018	38	0.127	89	0.007	0.025	0.421	0.021	37	0.131	89	0.007
VT(2)	6.17	13.44	0.459	0.048	54	0.203	146	0.020	0.028	0.447	0.055	52	0.215	143	0.022
VT(4)	5.97	12.77	0.467	0.181	43	0.365	171	0.102	0.035	0.451	0.203	36	0.387	163	0.112
Panel B. Reward-to-Risk Timing Strategies															
RRT($\mu^+, 1$)	5.08	15.23	0.334	0.859	-80	0.879	-94	0.881	0.075	0.304	0.930	-111	0.945	-125	0.938
RRT($\mu^+, 2$)	5.09	15.54	0.328	0.813	-84	0.806	-117	0.861	0.087	0.294	0.893	-122	0.894	-155	0.924
RRT($\mu^+, 4$)	5.17	16.20	0.319	0.782	-86	0.742	-163	0.870	0.117	0.276	0.877	-142	0.858	-219	0.936
RRT($\beta^+, 1$)	6.12	14.62	0.419	0.106	33	0.173	57	0.063	0.030	0.407	0.130	30	0.202	54	0.076
RRT($\beta^+, 2$)	6.19	14.39	0.430	0.122	43	0.203	81	0.073	0.040	0.413	0.170	33	0.262	71	0.101
RRT($\beta^+, 4$)	6.11	14.10	0.433	0.194	39	0.301	93	0.128	0.062	0.406	0.300	15	0.418	70	0.195
Panel C. Mean-Variance Efficient Strategies															
MV	6.58	12.76	0.515	0.135	104	0.267	232	0.100	0.161	0.440	0.300	21	0.450	149	0.203
TP	-22.48	81.36	-0.276	0.976	-6083	0.952	-	-	5.287	-0.604	0.994	-5162	0.997	-12087	1.000
OC	5.96	13.28	0.449	0.307	36	0.418	136	0.232	0.289	0.318	0.700	-124	0.759	-24	0.552
OC+	6.00	12.89	0.466	0.182	45	0.360	166	0.109	0.131	0.405	0.386	-20	0.563	101	0.226

Table 2: Replication Results 10 Momentum

This table reports our replication of the results of Kirby and Ostdiek for the 10 Momentum dataset. It shows the performance of the 1/N strategy, three VT(η) strategies, three RRT(μ^+, η) strategies, three RRT(β^+, η) strategies, and four mean variance efficient strategies. It reports the following sample statistics of the time series of returns that is generated by each strategy: the annualized mean ($\hat{\mu}_p$), the annualized standard deviation ($\hat{\sigma}_p$), the annualized Sharpe ratio ($\hat{\lambda}_p$), the average monthly turnover ($\hat{\tau}_p$), the fee in basis points that an investor would be willing to pay to switch from the 1/N strategy another particular strategy and p-values corresponding to the difference in Sharpe ratios with the 1/N strategy as well as the p-values for the performance fees. A '-' for the TP portfolio signifies that it was not possible to calculate this value, as there was no real value for a fee to make the investor indifferent between the TP and 1/N strategies. The sample period is July 1963 - December 2008 (546 observations) and the reported values are calculated using the period July 1973-December 2008 (426 observations), as the first 120 observations are held out to initialize the rolling window estimates for the conditional mean vector and the conditional covariance matrix.

	No Transaction Costs								Transaction Costs						
				vs. 1/N							vs. 1/N				
Strategy	$\hat{\mu}_p$	$\hat{\sigma}_p$	$\hat{\lambda}_p$	p-val	$\hat{\Delta}_1$	p-val	$\hat{\Delta}_5$	p-val	$\hat{\tau}_p$	$\hat{\lambda}_p$	p-val	$\hat{\Delta}_1$	p-val	$\hat{\Delta}_5$	p-val
1/N	4.68	16.67	0.281						0.018	0.274					
Panel A. Volatility Timing Strategies															
VT(1)	5.25	15.96	0.329	0.001	69	0.017	116	0.002	0.017	0.323	0.001	70	0.016	117	0.002
VT(2)	5.54	15.62	0.355	0.002	103	0.018	172	0.002	0.018	0.348	0.002	103	0.019	172	0.002
VT(4)	5.76	15.37	0.375	0.003	129	0.022	214	0.002	0.026	0.365	0.004	124	0.027	209	0.003
Panel B. Reward-to-Risk Timing Strategies															
RRT(μ^+, 1)	7.61	16.77	0.454	0.001	291	0.000	285	0.001	0.058	0.433	0.002	267	0.001	261	0.002
RRT(μ^+, 2)	8.13	17.12	0.475	0.002	338	0.000	307	0.002	0.061	0.454	0.004	312	0.001	281	0.004
RRT(μ^+, 4)	8.61	17.44	0.493	0.003	379	0.001	326	0.005	0.068	0.470	0.006	349	0.001	296	0.009
RRT(β^+, 1)	6.40	16.09	0.397	0.000	181	0.000	220	0.000	0.019	0.390	0.000	180	0.000	219	0.000
RRT(β^+, 2)	7.26	16.20	0.448	0.000	266	0.000	298	0.000	0.024	0.440	0.000	263	0.000	294	0.000
RRT(β^+, 4)	7.94	16.49	0.481	0.001	329	0.000	341	0.001	0.034	0.469	0.001	319	0.001	331	0.001
Panel C. Mean-Variance Efficient Strategies															
MV	7.38	15.04	0.491	0.034	296	0.061	401	0.027	0.281	0.378	0.181	137	0.238	243	0.122
TP	507.07	1701.65	0.298	0.481	-	-	-	-	124.797	-0.248	0.910	-	-	-	-
OC	9.01	15.56	0.579	0.008	451	0.013	525	0.009	0.347	0.445	0.080	253	0.106	327	0.067
OC+	5.38	15.21	0.354	0.059	93	0.143	188	0.031	0.123	0.305	0.258	30	0.368	125	0.107

Table 3: Results Maximum Utility Combinations 10 Industry

This table reports the results of the maximum utility combining strategies for the 10 Industry dataset. It shows the performance of strategies where the 1/N portfolio is combined with several strategies using the combining weight based on utility optimization. These strategies are: three VT(η) strategies, three RRT(μ^+, η) strategies, three RRT(β^+, η) strategies, and four mean variance efficient strategies. Finally the performance of the individual 1/N strategy is shown as a reference for the other strategies. It reports the following sample statistics of the time series of returns that is generated by each strategy: the annualized mean ($\hat{\mu}_p$), the annualized standard deviation ($\hat{\sigma}_p$), the annualized Sharpe ratio ($\hat{\lambda}_p$), the average monthly turnover ($\hat{\tau}_p$), the fee in basis points that an investor would be willing to pay to switch from the 1/N strategy another particular strategy and p-values corresponding to the difference in Sharpe ratios with the 1/N strategy as well as the p-values for the performance fees. A '-' for the TP portfolio signifies that it was not possible to calculate this value, as there was no real value for a fee to make the investor indifferent between the TP and 1/N strategies. The sample period is July 1963 - December 2008 (546 observations) and the reported values are calculated using the period July 1973-December 2008 (426 observations), as the first 120 observations are held out to initialize the rolling window estimates for the conditional mean vector and the conditional covariance matrix. The combining weight δ_t is calculated using a training sample of 84 observations and a validation sample of 36 observations.

	No Transaction Costs								Transaction Costs						
				vs. 1/N							vs. 1/N				
Strategy combined with 1/N	$\hat{\mu}_p$	$\hat{\sigma}_p$	$\hat{\lambda}_p$	p-val	$\hat{\Delta}_1$	p-val	$\hat{\Delta}_5$	p-val	$\hat{\tau}_p$	$\hat{\lambda}_p$	p-val	$\hat{\Delta}_1$	p-val	$\hat{\Delta}_5$	p-val
1/N (for reference)	5.85	15.01	0.390						0.024	0.380					
Panel A. Volatility Timing Strategies															
VT(1)	6.10	14.15	0.431	0.018	38	0.124	89	0.007	0.025	0.421	0.020	37	0.131	88	0.007
VT(2)	6.24	13.46	0.464	0.035	62	0.171	152	0.016	0.030	0.451	0.043	58	0.186	149	0.018
VT(4)	6.26	12.87	0.487	0.109	71	0.269	193	0.059	0.044	0.466	0.138	59	0.306	181	0.071
Panel B. Reward-to-Risk Timing Strategies															
RRT(μ^+, 1)	5.86	14.93	0.392	0.472	2	0.484	7	0.442	0.076	0.362	0.696	-29	0.734	-23	0.685
RRT(μ^+, 2)	5.93	15.24	0.389	0.502	5	0.472	-9	0.553	0.089	0.354	0.692	-34	0.691	-48	0.755
RRT(μ^+, 4)	6.17	15.58	0.396	0.465	23	0.402	-13	0.557	0.104	0.356	0.638	-25	0.607	-60	0.752
RRT(β^+, 1)	6.06	14.66	0.413	0.131	26	0.196	47	0.065	0.036	0.399	0.191	19	0.271	40	0.099
RRT(β^+, 2)	6.03	14.45	0.418	0.173	27	0.269	61	0.089	0.047	0.398	0.275	12	0.388	47	0.149
RRT(β^+, 4)	6.07	14.16	0.429	0.168	35	0.281	86	0.091	0.065	0.401	0.303	10	0.436	61	0.170
Panel C. Mean-Variance Efficient Strategies															
MV	6.65	12.77	0.521	0.083	112	0.214	239	0.057	0.162	0.445	0.247	28	0.421	156	0.150
TP	6.24	15.06	0.414	0.274	38	0.256	35	0.270	0.138	0.359	0.702	-31	0.704	-34	0.720
OC	6.53	13.25	0.492	0.120	93	0.239	194	0.081	0.234	0.386	0.473	-34	0.603	68	0.313
OC+	6.37	13.01	0.490	0.099	80	0.246	195	0.060	0.129	0.430	0.260	17	0.443	132	0.146

Table 4: Results Maximum Utility Combinations 10 Momentum

This table reports the results of the maximum utility combining strategies for the 10 Momentum dataset. It shows the performance of strategies where the 1/N portfolio is combined with several strategies using the combining weight based on utility optimization. These strategies are: three VT(η) strategies, three RRT(μ^+, η) strategies, three RRT(β^+, η) strategies, and four mean variance efficient strategies. Finally the performance of the individual 1/N strategy is shown as a reference for the other strategies. It reports the following sample statistics of the time series of returns that is generated by each strategy: the annualized mean ($\hat{\mu}_p$), the annualized standard deviation ($\hat{\sigma}_p$), the annualized Sharpe ratio ($\hat{\lambda}_p$), the average monthly turnover ($\hat{\tau}_p$), the fee in basis points that an investor would be willing to pay to switch from the 1/N strategy another particular strategy and p-values corresponding to the difference in Sharpe ratios with the 1/N strategy as well as the p-values for the performance fees. A '-' for the TP portfolio signifies that it was not possible to calculate this value, as there was no real value for a fee to make the investor indifferent between the TP and 1/N strategies. The sample period is July 1963 - December 2008 (546 observations) and the reported values are calculated using the period July 1973-December 2008 (426 observations), as the first 120 observations are held out to initialize the rolling window estimates for the conditional mean vector and the conditional covariance matrix. The combining weight δ_t is calculated using a training sample of 84 observations and a validation sample of 36 observations.

	No Transaction Costs									Transaction Costs					
				vs. 1/N							vs. 1/N				
Strategy combined with 1/N	$\hat{\mu}_p$	$\hat{\sigma}_p$	$\hat{\lambda}_p$	p-val	$\hat{\Delta}_1$	p-val	$\hat{\Delta}_5$	p-val	$\hat{\tau}_p$	$\hat{\lambda}_p$	p-val	$\hat{\Delta}_1$	p-val	$\hat{\Delta}_5$	p-val
1/N (for reference)	4.68	16.67	0.281						0.018	0.274					
Panel A. Volatility Timing Strategies															
VT(1)	5.23	15.95	0.328	0.001	66	0.020	115	0.002	0.019	0.320	0.001	66	0.021	114	0.002
VT(2)	5.46	15.61	0.350	0.002	95	0.023	165	0.002	0.024	0.341	0.002	92	0.027	162	0.002
VT(4)	5.61	15.33	0.366	0.003	114	0.028	202	0.002	0.036	0.352	0.006	103	0.043	191	0.003
Panel B. Reward-to-Risk Timing Strategies															
RRT($\mu^+, 1$)	6.89	15.99	0.431	0.001	232	0.001	278	0.000	0.058	0.409	0.002	208	0.001	254	0.000
RRT($\mu^+, 2$)	7.18	16.07	0.447	0.001	260	0.001	300	0.000	0.065	0.422	0.003	231	0.003	272	0.001
RRT($\mu^+, 4$)	7.48	16.16	0.463	0.002	288	0.001	323	0.001	0.079	0.434	0.005	252	0.004	287	0.003
RRT($\beta^+, 1$)	6.08	15.80	0.385	0.000	154	0.001	212	0.000	0.026	0.375	0.000	150	0.001	207	0.000
RRT($\beta^+, 2$)	6.75	15.67	0.430	0.000	223	0.001	289	0.000	0.034	0.417	0.000	213	0.001	279	0.000
RRT($\beta^+, 4$)	7.21	15.73	0.459	0.000	269	0.001	331	0.000	0.048	0.440	0.001	251	0.002	314	0.000
Panel C. Mean-Variance Efficient Strategies															
MV	7.08	14.78	0.479	0.011	270	0.031	391	0.006	0.253	0.376	0.115	128	0.187	249	0.054
TP	7.93	16.88	0.470	0.003	321	0.001	307	0.001	0.197	0.400	0.020	213	0.014	204	0.013
OC	8.86	15.02	0.590	0.000	445	0.001	551	0.000	0.280	0.478	0.010	286	0.024	394	0.006
OC+	5.51	15.18	0.363	0.027	107	0.088	204	0.012	0.123	0.314	0.175	43	0.292	140	0.058

Table 5: Results Equally Weighted Combinations 10 Industry

This table reports the results of the equally weighted combination strategies for the 10 Industry dataset. It shows the performance of strategies where the 1/N portfolio is equally combined with: three VT(η) strategies, three RRT(μ^+, η) strategies, three RRT(β^+, η) strategies, and four mean variance efficient strategies. Finally the performance of the individual 1/N strategy is shown as a reference for the other strategies. It reports the following sample statistics of the time series of returns that is generated by each strategy: the annualized mean ($\hat{\mu}_p$), the annualized standard deviation ($\hat{\sigma}_p$), the annualized Sharpe ratio ($\hat{\lambda}_p$), the average monthly turnover ($\hat{\tau}_p$), the fee in basis points that an investor would be willing to pay to switch from the 1/N strategy another particular strategy and p-values corresponding to the difference in Sharpe ratios with the 1/N strategy as well as the p-values for the performance fees. A '-' for the TP portfolio signifies that it was not possible to calculate this value, as there was no real value for a fee to make the investor indifferent between the TP and 1/N strategies. The sample period is July 1963 - December 2008 (546 observations) and the reported values are calculated using the period July 1973-December 2008 (426 observations), as the first 120 observations are held out to initialize the rolling window estimates for the conditional mean vector and the conditional covariance matrix.

	No Transaction Costs								Transaction Costs						
				vs. 1/N							vs. 1/N				
Strategy combined with 1/N	$\hat{\mu}_p$	$\hat{\sigma}_p$	$\hat{\lambda}_p$	p-val	$\hat{\Delta}_1$	p-val	$\hat{\Delta}_5$	p-val	$\hat{\tau}_p$	$\hat{\lambda}_p$	p-val	$\hat{\Delta}_1$	p-val	$\hat{\Delta}_5$	p-val
1/N (for reference)	5.85	15.01	0.390						0.024	0.380					
Panel A. Volatility Timing Strategies															
VT(1)	5.97	14.56	0.410	0.016	19	0.121	47	0.005	0.024	0.401	0.017	19	0.124	47	0.005
VT(2)	6.01	14.12	0.426	0.035	29	0.189	82	0.011	0.025	0.415	0.039	28	0.196	81	0.012
VT(4)	5.91	13.46	0.439	0.107	28	0.327	119	0.040	0.028	0.426	0.121	25	0.343	116	0.044
Panel B. Reward-to-Risk Timing Strategies															
RRT($\mu^+, 1$)	5.47	14.87	0.368	0.801	-36	0.857	-28	0.778	0.041	0.351	0.868	-47	0.912	-38	0.849
RRT($\mu^+, 2$)	5.47	14.86	0.368	0.730	-36	0.769	-27	0.699	0.044	0.350	0.802	-48	0.837	-39	0.776
RRT($\mu^+, 4$)	5.51	14.97	0.368	0.675	-33	0.691	-30	0.669	0.060	0.344	0.778	-55	0.795	-52	0.774
RRT($\beta^+, 1$)	5.99	14.78	0.405	0.093	17	0.166	31	0.048	0.025	0.395	0.105	16	0.179	30	0.053
RRT($\beta^+, 2$)	6.02	14.63	0.411	0.101	23	0.191	46	0.049	0.029	0.400	0.128	19	0.226	43	0.061
RRT($\beta^+, 4$)	5.98	14.39	0.415	0.149	22	0.278	59	0.075	0.039	0.399	0.220	13	0.366	50	0.110
Panel C. Mean-Variance Efficient Strategies															
MV	6.21	13.00	0.478	0.047	65	0.220	180	0.025	0.081	0.440	0.124	30	0.361	146	0.057
TP	-8.32	41.95	-0.198	0.979	-2189	0.952	-5739	0.964	8.776	-0.534	0.971	-8759	0.977	-	-
OC	5.91	13.19	0.448	0.150	31	0.359	137	0.073	0.140	0.384	0.474	-39	0.669	67	0.240
OC+	5.93	13.54	0.438	0.109	29	0.323	115	0.045	0.072	0.406	0.256	-1	0.503	86	0.103

Table 6: Results Equally Weighted Combinations 10 Momentum

This table reports the results of the equally weighted combination strategies for the 10 Momentum dataset. It shows the performance of strategies where the 1/N portfolio is equally combined with: three VT(η) strategies, three RRT(μ^+, η) strategies, three RRT(β^+, η) strategies, and four mean variance efficient strategies. Finally the performance of the individual 1/N strategy is shown as a reference for the other strategies. It reports the following sample statistics of the time series of returns that is generated by each strategy: the annualized mean ($\hat{\mu}_p$), the annualized standard deviation ($\hat{\sigma}_p$), the annualized Sharpe ratio ($\hat{\lambda}_p$), the average monthly turnover ($\hat{\tau}_p$), the fee in basis points that an investor would be willing to pay to switch from the 1/N strategy another particular strategy and p-values corresponding to the difference in Sharpe ratios with the 1/N strategy as well as the p-values for the performance fees. A '-' for the TP portfolio signifies that it was not possible to calculate this value, as there was no real value for a fee to make the investor indifferent between the TP and 1/N strategies. The sample period is July 1963 - December 2008 (546 observations) and the reported values are calculated using the period July 1973-December 2008 (426 observations), as the first 120 observations are held out to initialize the rolling window estimates for the conditional mean vector and the conditional covariance matrix.

	No Transaction Costs								Transaction Costs						
				vs. 1/N							vs. 1/N				
Strategy combined with 1/N	$\hat{\mu}_p$	$\hat{\sigma}_p$	$\hat{\lambda}_p$	p-val	$\hat{\Delta}_1$	p-val	$\hat{\Delta}_5$	p-val	$\hat{\tau}_p$	$\hat{\lambda}_p$	p-val	$\hat{\Delta}_1$	p-val	$\hat{\Delta}_5$	p-val
1/N (for reference)	4.68	16.67	0.281						0.018	0.274					
Panel A. Volatility Timing Strategies															
VT(1)	4.97	16.30	0.305	0.001	35	0.016	60	0.001	0.017	0.298	0.001	35	0.015	60	0.001
VT(2)	5.11	16.11	0.317	0.001	52	0.017	90	0.001	0.017	0.311	0.001	53	0.017	91	0.001
VT(4)	5.22	15.94	0.327	0.002	66	0.020	115	0.001	0.020	0.320	0.002	65	0.022	113	0.002
Panel B. Reward-to-Risk Timing Strategies															
RRT(μ^+, 1)	6.15	16.53	0.372	0.001	149	0.000	159	0.000	0.033	0.360	0.001	140	0.000	150	0.000
RRT(μ^+, 2)	6.41	16.60	0.386	0.001	174	0.000	179	0.000	0.034	0.374	0.002	164	0.001	169	0.001
RRT(μ^+, 4)	6.64	16.66	0.399	0.001	196	0.000	197	0.001	0.038	0.385	0.002	184	0.001	185	0.001
RRT(β^+, 1)	5.54	16.31	0.340	0.000	92	0.000	116	0.000	0.018	0.333	0.000	92	0.000	116	0.000
RRT(β^+, 2)	5.97	16.28	0.367	0.000	136	0.000	162	0.000	0.019	0.360	0.000	135	0.000	161	0.000
RRT(β^+, 4)	6.31	16.33	0.386	0.000	168	0.000	192	0.000	0.024	0.377	0.001	165	0.000	188	0.000
Panel C. Mean-Variance Efficient Strategies															
MV	6.03	15.09	0.400	0.013	160	0.047	263	0.006	0.134	0.346	0.088	90	0.174	193	0.033
TP	255.87	850.77	0.301	0.478	-	-	-	-	110.196	-0.263	0.914	-	-	-	-
OC	6.85	15.21	0.450	0.002	240	0.009	335	0.001	0.166	0.385	0.029	151	0.068	246	0.013
OC+	5.03	15.81	0.318	0.045	49	0.132	106	0.019	0.068	0.292	0.209	19	0.335	76	0.068

Table 7: Results Favorability Index Combinations 10 Industry

This table reports the results of the favorability index combining strategies for the 10 Industry dataset. It shows the performance of strategies where the 1/N portfolio is combined with several strategies based on the combining weight resulting from the 1/N favorability index. These strategies are: three VT(η) strategies, three RRT(μ^{+}, η) strategies, three RRT(β^{+}, η) strategies, and four mean variance efficient strategies. Finally the performance of the individual 1/N strategy is shown as a reference for the other strategies. It reports the following sample statistics of the time series of returns that is generated by each strategy: the annualized mean ($\hat{\mu}_p$), the annualized standard deviation ($\hat{\sigma}_p$), the annualized Sharpe ratio ($\hat{\lambda}_p$), the average monthly turnover ($\hat{\tau}_p$), the fee in basis points that an investor would be willing to pay to switch from the 1/N strategy another particular strategy and p-values corresponding to the difference in Sharpe ratios with the 1/N strategy as well as the p-values for the performance fees. A '-' for the TP portfolio signifies that it was not possible to calculate this value, as there was no real value for a fee to make the investor indifferent between the TP and 1/N strategies. The sample period is July 1963 - December 2008 (546 observations) and the reported values are calculated using the period July 1973-December 2008 (426 observations), as the first 120 observations are held out to initialize the rolling window estimates for the conditional mean vector and the conditional covariance matrix.

	No Transaction Costs									Transaction Costs					
				vs. 1/N							vs. 1/N				
Strategy combined with 1/N	$\hat{\mu}_p$	$\hat{\sigma}_p$	$\hat{\lambda}_p$	p-val	$\hat{\Delta}_1$	p-val	$\hat{\Delta}_5$	p-val	$\hat{\tau}_p$	$\hat{\lambda}_p$	p-val	$\hat{\Delta}_1$	p-val	$\hat{\Delta}_5$	p-val
1/N (for reference)	5.85	15.01	0.390						0.024	0.380					
Panel A. Volatility Timing Strategies															
VT(1)	5.92	14.74	0.401	0.028	11	0.168	27	0.022	0.029	0.390	0.067	8	0.252	24	0.036
VT(2)	5.93	14.47	0.410	0.063	16	0.243	49	0.035	0.040	0.393	0.167	6	0.392	39	0.071
VT(4)	5.83	14.02	0.416	0.179	12	0.396	72	0.089	0.064	0.389	0.389	-12	0.596	47	0.184
Panel B. Reward-to-Risk Timing Strategies															
RRT(μ^{+}, 1)	4.91	16.39	0.300	0.932	-116	0.960	-205	0.968	0.106	0.261	0.979	-165	0.990	-253	0.986
RRT(μ^{+}, 2)	4.83	16.53	0.292	0.936	-126	0.954	-224	0.971	0.105	0.254	0.977	-175	0.987	-273	0.987
RRT(μ^{+}, 4)	4.94	16.65	0.296	0.913	-118	0.916	-225	0.962	0.111	0.256	0.967	-170	0.973	-277	0.984
RRT(β^{+}, 1)	5.84	14.94	0.391	0.410	0	0.493	5	0.278	0.033	0.378	0.679	-6	0.761	-1	0.563
RRT(β^{+}, 2)	5.82	14.87	0.392	0.404	-1	0.520	8	0.229	0.041	0.375	0.742	-11	0.839	-3	0.600
RRT(β^{+}, 4)	5.75	14.79	0.389	0.530	-7	0.649	7	0.351	0.054	0.367	0.841	-25	0.910	-12	0.743
Panel C. Mean-Variance Efficient Strategies															
MV	5.82	13.80	0.422	0.285	15	0.433	86	0.172	0.168	0.348	0.705	-72	0.793	-2	0.508
TP	-26.76	92.50	-0.289	0.971	-7513	0.935	-	-	4.409	-0.549	0.989	-4412	0.993	-9919	0.997
OC	5.16	13.98	0.369	0.614	-54	0.691	7	0.476	0.242	0.264	0.920	-186	0.935	-128	0.844
OC+	6.20	14.03	0.442	0.077	49	0.200	108	0.047	0.097	0.400	0.294	5	0.464	63	0.158

Table 8: Results Favorability Index Combinations 10 Momentum

This table reports the results of the favorability index combining strategies for the 10 Momentum dataset. It shows the performance of strategies where the 1/N portfolio is combined with several strategies based on the combining weight resulting from the 1/N favorability index. These strategies are: three VT(η) strategies, three RRT(μ^+, η) strategies, three RRT(β^+, η) strategies, and four mean variance efficient strategies. Finally the performance of the individual 1/N strategy is shown as a reference for the other strategies. It reports the following sample statistics of the time series of returns that is generated by each strategy: the annualized mean ($\hat{\mu}_p$), the annualized standard deviation ($\hat{\sigma}_p$), the annualized Sharpe ratio ($\hat{\lambda}_p$), the average monthly turnover ($\hat{\tau}_p$), the fee in basis points that an investor would be willing to pay to switch from the 1/N strategy another particular strategy and p-values corresponding to the difference in Sharpe ratios with the 1/N strategy as well as the p-values for the performance fees. A '-' for the TP portfolio signifies that it was not possible to calculate this value, as there was no real value for a fee to make the investor indifferent between the TP and 1/N strategies. The sample period is July 1963 - December 2008 (546 observations) and the reported values are calculated using the period July 1973-December 2008 (426 observations), as the first 120 observations are held out to initialize the rolling window estimates for the conditional mean vector and the conditional covariance matrix.

	No Transaction Costs									Transaction Costs					
				vs. 1/N							vs. 1/N				
Strategy combined with 1/N	$\hat{\mu}_p$	$\hat{\sigma}_p$	$\hat{\lambda}_p$	p-val	$\hat{\Delta}_1$	p-val	$\hat{\Delta}_5$	p-val	$\hat{\tau}_p$	$\hat{\lambda}_p$	p-val	$\hat{\Delta}_1$	p-val	$\hat{\Delta}_5$	p-val
1/N (for reference)	4.68	16.67	0.281						0.018	0.274					
Panel A. Volatility Timing Strategies															
VT(1)	4.98	16.27	0.306	0.010	37	0.048	64	0.011	0.021	0.298	0.013	35	0.054	62	0.012
VT(2)	5.11	16.08	0.318	0.016	53	0.059	93	0.012	0.028	0.307	0.026	46	0.079	86	0.015
VT(4)	5.14	15.92	0.323	0.037	58	0.091	109	0.018	0.040	0.308	0.074	45	0.145	95	0.028
Panel B. Reward-to-Risk Timing Strategies															
RRT(μ^+, 1)	7.28	17.34	0.420	0.012	249	0.002	203	0.010	0.086	0.391	0.023	208	0.006	163	0.027
RRT(μ^+, 2)	7.70	17.69	0.435	0.015	285	0.002	213	0.017	0.084	0.407	0.025	245	0.006	174	0.039
RRT(μ^+, 4)	8.05	17.94	0.449	0.015	315	0.002	225	0.023	0.085	0.420	0.025	274	0.005	185	0.047
RRT(β^+, 1)	5.73	16.37	0.350	0.003	110	0.004	131	0.003	0.029	0.339	0.005	103	0.005	124	0.004
RRT(β^+, 2)	6.31	16.46	0.383	0.006	166	0.003	181	0.004	0.040	0.369	0.008	153	0.006	167	0.006
RRT(β^+, 4)	6.83	16.66	0.410	0.008	215	0.003	216	0.006	0.053	0.391	0.013	194	0.006	195	0.011
Panel C. Mean-Variance Efficient Strategies															
MV	5.27	15.29	0.345	0.209	81	0.277	172	0.122	0.217	0.260	0.573	-38	0.610	52	0.360
TP	616.42	2023.44	0.305	0.475	-	-	-	-	138.077	-0.270	0.915	-	-	-	-
OC	6.11	15.52	0.393	0.087	161	0.128	237	0.062	0.248	0.298	0.387	23	0.435	100	0.252
OC+	4.43	15.76	0.281	0.492	-10	0.562	51	0.223	0.102	0.242	0.807	-60	0.832	0	0.502