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Comparative Analysis of Portfolio Strategies for Commodity Futures

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Abstract

This thesis compares the performance of mean-variance and active portfolio selection strategies to naïve diversification for commodity futures. Commodity futures offer diversification benefits and act as inflation hedges, making them an attractive asset class. Analyzing monthly returns of commodity futures from 1987 to 2008, we find that active timing strategies outperform mean-variance strategies for commodity futures. Risk-to-reward timing strategies even outperform naïve diversification, particularly for investors with a low level of relative risk aversion. However, short sales introduce volatility, limiting the superiority of risk-to-reward timing strategies to investors with a low level of relative risk aversion. Overall, risk-to-reward timing strategies are more effective for commodity futures, while volatility timing and mean-variance strategies underperform. These findings provide insights for constructing optimal portfolios in the commodity futures market.

1 Introduction

In the world of portfolio optimization, two fundamental approaches get a significant amount of attention. These approaches are mean-variance optimization, introduced by Markowitz (1952) and naïve diversification. Mean-variance optimization is a technique that considers the expected asset returns and variances of asset returns. This technique aims to optimize portfolio weights to maximize expected return for a given level of risk. Naïve diversification often called the 1/N strategy, distributes capital equally across all assets. The 1/N strategy does not consider expected returns or asset variances.

In the paper "It's All in the Timing: Simple Active Portfolio Strategies that Outperform Naive Diversification", Kirby & Ostdiek (2012) compare these strategies using various data sets. However, their study mainly focused on equity assets and did not dive into commodity futures. As an extension to their work, this thesis aims to perform a similar comparison for commodity features, a relatively unexplored asset class in most of the current academic literature.

Several factors motivate an extension of this research with commodity futures. Firstly, commodity futures offer diversification benefits because of their low correlation with traditional and well-studied asset classes such as equities and bonds (Gorton & Rouwenhorst, 2006). Besides this, commodities provide a hedge against inflation because they are physical assets. This provides a layer of protection during volatile economic periods (Bodie & Rosansky, 1980). Lastly, the market for commodity futures has grown in recent years (Büyükşahin & Robe, 2014), making it attractive to include commodity futures in investment portfolios for institutions and individual investors.

Besides these motivating factors, it is worth noting that the mean-variance portfolio selection has received a lot of criticism because of the sensitivity to input assumptions (Michaud, 1989). Therefore, testing the performance of mean-variance portfolio selection in the context of commodity futures is worth testing because of their differing return characteristics compared to traditional asset classes like equities and bonds. Because commodity futures contracts require no initial investment, their risk premia differ from traditional assets like stocks. Generally, risk premia for stocks are expected to be positive in the long term. However, risk premia for commodity futures contracts can be positive and negative. Additionally, taking short and long positions in commodity futures is equally easy. Short-selling stocks usually require borrowing shares and complying with regulations and market rules. The relative ease of taking short positions makes short-selling restrictions less reasonable for the portfolio selection of commodity futures.

Despite the significant growth of the commodity futures market that sparked the increased interest of institutional investors and individual investors (Büyükşahin & Robe, 2014), the literature on portfolio selection strategies for commodity futures is relatively scarce. The unique characteristics of commodity futures motivate the investigation of the best-performing portfolio strategies for this asset class. This thesis aims to contribute to filling this gap in the literature by comparing the performance of naïve diversification and a combination of mean-variance optimization strategies and active portfolio selection strategies for commodity futures portfolios.

Therefore the research question of this thesis is: *"What is the performance of mean-variance and active portfolio selection for commodity futures, and how does it compare to naïve diversi-*

fication?".

This thesis aims to replicate and extend the research of Kirby & Ostdiek (2012) to address the main research question. Specifically, this thesis considers the asset class of commodity futures, while the original paper considers equities. This thesis investigates whether the same mean-variance and active portfolio selection strategies that outperform naïve diversification in the original paper outperform naïve diversification for commodity futures. The results of this analysis can be helpful for investors and institutions in constructing more effective and diversified portfolios.

This thesis finds that active timing strategies perform better for commodity futures than mean-variance strategies. The risk-to-reward timing strategies even outperform naïve diversification for commodity futures for investors with a low level of relative risk aversion. The additional volatility that comes with allowing short-sales, results in the risk-to-reward timing strategies only outperforming naïve diversification for investors with an even lower level of relative risk aversion.

In Section 2, the literature is given. Next, in Section 3, the data is explained. After this, in Section 4, the investment strategies are described, and an explanation is given for the performance measures of these strategies. Section 5 contains the results. Lastly, Section 6 contains concluding remarks and suggestions for future research.

2 Literature

Markowitz (1952) introduced the theory behind portfolio optimization. His work significantly impacted how investors view risk and returns of investments. He presented how investments contribute to the overall portfolio's risk and return instead of viewing risk and return characteristics per asset. Markowitz (1952) laid the foundation for current portfolio theory, which aims to maximize the risk-adjusted returns for a portfolio.

Building onto Markowitz's theory, Sharpe (1964) introduced the Capital Asset Pricing Model (CAPM). This model simplified the portfolio optimization process by introducing the market portfolio idea. This theory states that every investor's optimal portfolio combines the market portfolio and the risk-free asset.

Fama & French (1993) extended the CAPM model with the three-factor model. This model increased the explanatory power of expected returns by adding two factors. These two additional factors are the size (SMB) and book-to-market (HML) factors. These factors are used to capture the different risk premia associated with a stock's size and value.

Naïve diversification, also known as the 1/N strategy, is a strategy where capital is allocated equally across all available assets. The 1/N strategy is a simplistic portfolio that still has a competitive performance compared to more complex portfolio selection strategies. DeMiguel et al. (2009) found that the 1/N strategy often outperforms MV strategies due to the estimation error of these MV strategies, despite the simplicity of the 1/N strategy.

Kirby & Ostdiek (2012) found that the application of timing rules to active portfolio selection strategies improve their performance and even outperform the 1/N strategy for various datasets, which accentuates the potential benefits of active portfolio selection. However their research is limited to traditional assets like stocks and they do not consider other asset classes like commodity futures.

Commodity futures contracts are agreements to buy or sell a commodity at a date in the future for a price that is agreed upon today (Gorton & Rouwenhorst, 2006). The types of commodities typically include metals, softs (grown products like cocoa, sugar and coffee), grains, energy, and meat.

Commodity futures have received a lot of attention as an asset class due to their benefits which include diversification, hedging against inflation, and providing returns that are uncorrelated with traditional asset classes (Erb & Harvey, 2006). Besides this, commodity futures have other unique characteristics regarding different maturities. When it comes to commodity futures, investors are able to choose futures contracts with different expiration dates. The maturity represents the time horizon for the delivery of settlement of the underlying commodity.

Erb & Harvey (2006) found that a diversified portfolio of commodity futures seems to be an excellent diversifier of a traditional stock and bond portfolio. These diversification benefits are due to their low correlation with equities and bonds (Gorton & Rouwenhorst, 2006). Adding to this, You & Daigler (2013) show that there are benefits to adding commodity futures to traditional portfolios such as stocks and bonds. They also show that adding commodity futures by applying mean-variance optimization provides benefits over naïve optimization. However, their study is limited because it does not consider portfolios that solely consist of commodity futures but rather a combination of commodity futures and traditional assets. Besides this, their research only investigates mean-variance optimization but not active portfolio selection strategies like volatility timing or risk-to-reward timing.

Vrugt et al. (2004) investigate timing strategies for commodities futures and find that certain timing strategies deliver superior portfolio returns. However, the timing strategies applied in their research are related to business cycles, monetary environment, and market sentiment. Therefore, this thesis investigates to which extent the latter conclusions hold regarding the performance of mean-variance, volatility timing, and risk-to-reward timing strategies compared to naïve for a portfolio that solely consists of commodity futures.

3 Data

3.1 Replication

For the replication part of this thesis, ten industry portfolios and the market portfolio are used, and the size, value, and market portfolio are from July 1963 until November 2004 to replicate the characteristics of the 1/N and MVE strategies. For the replication of the out of sample performance of the different strategies, the 10 industry, 10 momentum and market portfolios are used from July 1963 until December 2008.

All the data that is used for the replication part of this thesis is monthly data. All the data is publicly available and is obtained from Ken French's website. The data does not need any processing and does not have any missing observations. Table 1 gives an overview of all the used data for the replication part of this thesis. N is the number of risky assets in each data set, and Obs is the number of observations in each data set.

Figures 1 to 6 contain reward and risk characteristics for the 10 Industry, 3 Factor, and 10 Momentum data sets, namely the cross-section of annualized mean returns and the cross-section

Table 1: Overview of data used for replication

Dataset	**N**	**Obs**	**Time Period**	**Abbreviation**
Ten industry portfolios & market portfolio	10 + 1	546	06/1963 - 12/2008	10 FF Industry
Size, Value, and the market porfolio	2 + 1	497	06/1963 - 11/2004	MKT/SMB/HML
Ten portfolios formed on Momentum	10	546	06/1963 - 12/2008	10 MOM

of annualized return standard deviations.

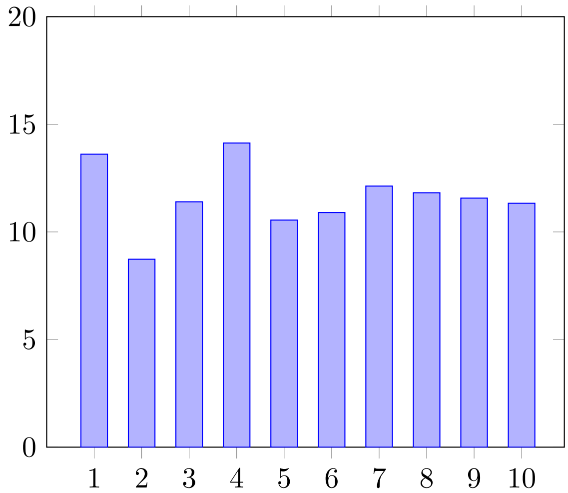

Figure 1: Mean Return FF10

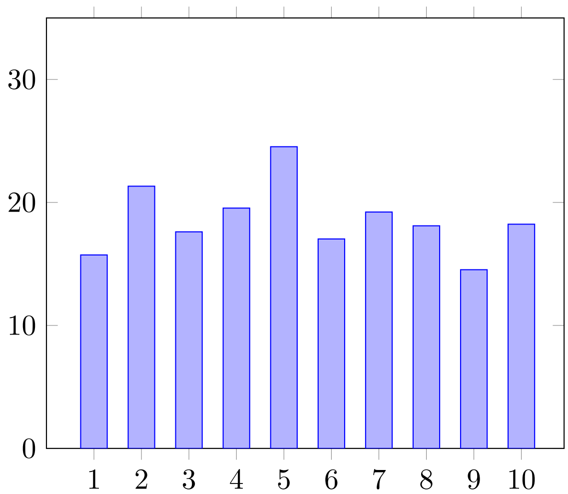

Figure 2: Volatility FF10

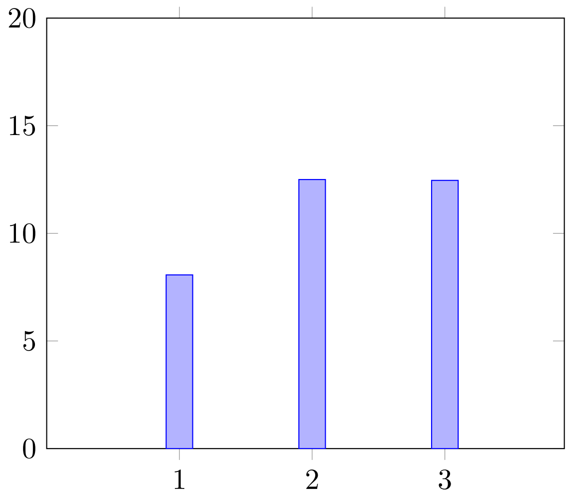

Figure 3: Mean Return FF3

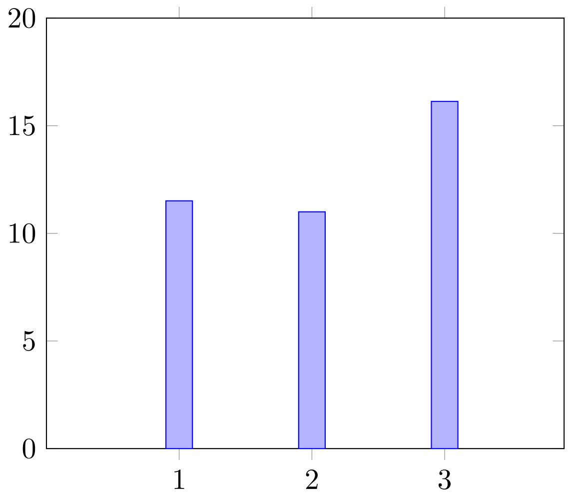

Figure 4: Volatility FF3

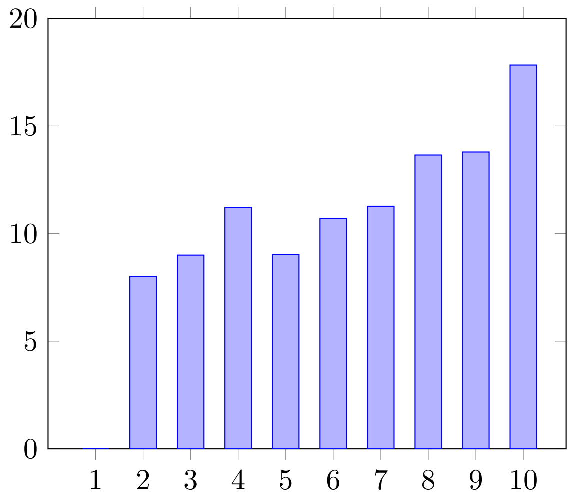

Figure 5: Mean Return MOM10

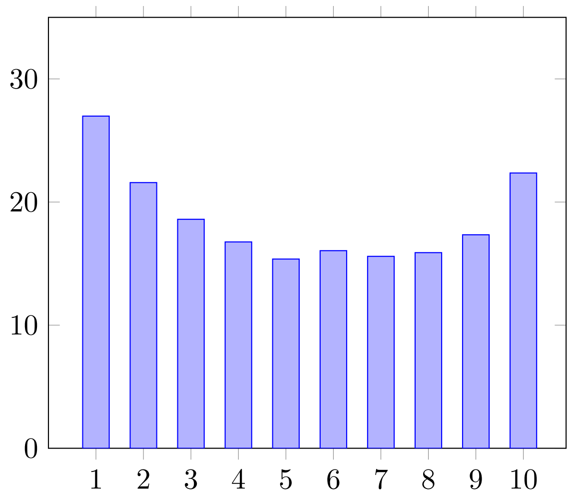

Figure 6: Volatility MOM10

3.2 Extension

For the extension part of this thesis, five data sets will be added to the original paper. All five data sets include monthly data for commodity futures. The futures contracts that are used for this thesis consist of gold, gas oil, sugar, wheat, and feeder cattle. These commodities each represent a different commodity type (metal, energy, softs, grains, and livestock). The maturity of these commodity futures is one month. The Goldman Sachs Commodity Index can obtain a proxy for the market portfolio for commodity futures. This index includes various commodity futures contracts across the different commodity sectors. An overview of the used data for the extension can be found in Table 2, containing the data set, number of observations, and period of the data set. The five commodity futures and the SPGSCI will be combined with the risk-free rate to create the CC5 data set.

Table 2: Overview of data used for extension

Dataset	**Obs**	**Time Period**	**Abbreviation**
Gold	253	12/1987 - 12/2008	AU
London Gas Oil	253	12/1987 - 12/2008	LGA
London Sugar	253	12/1987 - 12/2008	LS
London Wheat	253	12/1987 - 12/2008	LW
Feeder Cattle	253	12/1987 - 12/2008	FC
Goldman Sachs Commodity Index	253	12/1987 - 12/2008	SPGSCI

Figures 5 and 6 contain reward and risk characteristics for the 5 Commodity Futures data sets, namely the cross-section of annualized mean returns and the cross-section of annualized return standard deviations.

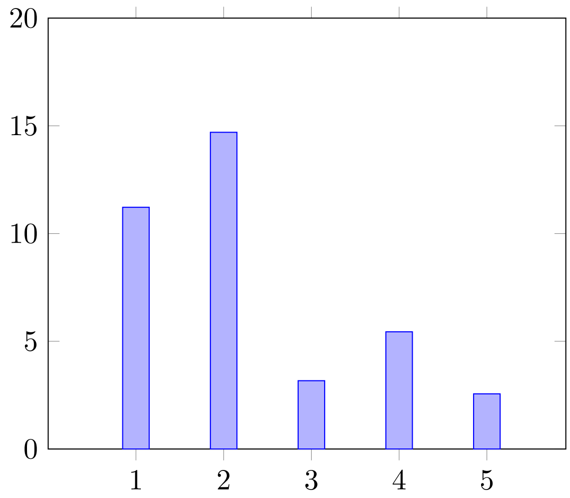

Figure 7: Mean Return CC5

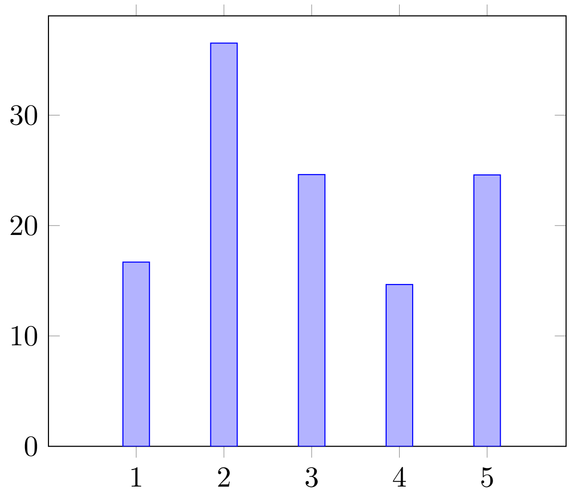

Figure 8: Volatility CC5

To compare the results from the extension to the original paper, the data must be adjusted for factors that could affect the comparability to the US stock data from the original paper. Hence, all the extension data has to be adjusted for sample periods. The data for the commodity futures and the SPGSCI is obtained from the investing.com website.

4 Methodology

This section describes all the methods that will be used to replicate specific results from the paper from Kirby & Ostdiek (2012) and the methods that will be used to perform the extension of the paper. All the calculations will be performed in Python.

4.1 1/N and Mean-Variance Efficient Strategies

The 1/N portfolio strategy consists of holding a portfolio of N assets where the same weight is assigned to each asset. The vector of weights for the naïve portfolio ($1/N$) is constructed by dividing one by the number of assets as depicted in the following formula.

$$\omega_{1/N,t} = 1/N \tag{1}$$

Here N is the number of assets and r_i is the return of asset i.

The tangency portfolio (TP) is the portfolio that lies on the efficient frontier. The first order condition of this problem is depicted as:

$$\mu_t + \lambda_t \iota - \gamma \Sigma_t \omega_{pt} = 0, \tag{2}$$

where λ_t is the Lagrange multiplier of the constraint $\omega'_{pt}\iota = 1$.

The vector of weights for the tangency portfolio at time t is calculated using the following formula.

$$\omega_{TP,t} = \frac{\Sigma_t^{-1} * \mu_t}{\iota' * \Sigma_t^{-1} * \mu_t} \tag{3}$$

Here Σ_t^{-1} is the inverse of the covariance matrix of asset returns at time t and μ_t is the mean of the asset return at time t and ι is the vector of ones. These weights are then normalized such that they sum up to 1.

The minimum-variance portfolio (MV) is the portfolio that is obtained by minimizing $\omega_t' \sum_t \omega_t$ subject to $\omega_t'\iota = 1$. The vector of weights for the minimum-variance portfolio at time t is calculated using the following formula.

$$\omega_{MV,t} = \Sigma_t^{-1} * \iota * (\iota' * \Sigma_t^{-1} * \iota)^{-1} \tag{4}$$

Here Σ_t^{-1} is the inverse of the covariance matrix of asset returns at time t and ι is the vector of ones.

The optimized unconstrained portfolio (OU) is a portfolio where the investor divides his wealth between the risk-free asset and the tangency portfolio. The investor chooses the portfolio in period t by minimizing the conditional risk of the portfolio for a specified value of the conditional expected excess return. The optimized constrained portfolio (OC) is similar to the optimized unconstrained portfolio (OU), except that the weight in the risk-free asset has been transferred to the minimum-variance portfolio. The OC portfolio also targets the conditional expected return of the 1/N portfolio. The vector of weights of the OC portfolio can be calculated using the following formula.

$$\omega_{OC,t} = (\frac{\mu_{pt} - \mu_{MV,t}}{\mu_{TP,t} - \mu_{MV,t}}) * (\frac{\Sigma_t^{-1}\mu_t}{\iota'\Sigma_t^{-1}\mu_t}) + (1 - \frac{\mu_{pt} - \mu_{MV,t}}{\mu_{TP,t} - \mu_{MV,t}}) * (\frac{\Sigma_t^{-1}\iota}{\iota'\Sigma_t^{-1}\iota}) \tag{5}$$

Here μ_{pt} is the conditional mean vector of the excess portfolio returns, and Σ_t is the conditional covariance matrix of the excess portfolio returns. $\mu_{TP,t}$ and $\mu_{MV,t}$ denote the conditional expected excess returns for the tangency and MV portfolios

Next, the annualized mean excess return is obtained by multiplying the mean of the excess returns by 12, and the annualized excess return standard deviation are obtained by multiplying the standard deviation of the excess returns by the square root of 12. Then the Sharpe Ratio is calculated by the following formula.

$$SR = \frac{\hat{\mu}}{\hat{\sigma}} \tag{6}$$

Here $\hat{\mu}$ and $\hat{\sigma}$ are the annualized mean excess return and the annualized excess return standard deviation respectively.

The turnover at time t is calculated with the following formula.

$$\tau_t = \sum_{i=1}^{N} |\omega_{it} - \tilde{\omega_{it}}| + |\sum_{i=1}^{N}(\omega_{it} - \tilde{\omega_{it}})| \tag{7}$$

Where ω_{it} is the desired weight in asset i at time t and $\tilde{\omega_{it}}$ is the weight in asset i before the portfolio is rebalanced at time t and it can be calculated using the following formula.

$$\tilde{\omega_{it}} = \frac{\omega_{i,t-1}(1 + R_{it})}{\sum_{i=1}^{N} \omega_{i,t-1}(1 + R_{it}) + (1 - \sum_{i=1}^{N} \omega_{i,t-1})(1 + R_{ft})} \tag{8}$$

Here R_{it} is the return of asset i at time t, and R_{ft} is the risk-free rate at time t. The turnover in the results is expressed as a fraction of wealth invested ($\hat{\tau}$)

A rolling estimator is used with a 120-month window length, and transaction costs are assumed to be 0. Panel B contains the minimum, median, and maximum values of each strategy's estimated conditional expected return over the months in the out-of-sample period.

4.2 Volatility Timing and Risk-To-Reward Timing Strategies

The analysis is extended by adding volatility timing (VT) and reward-to-risk strategies. The volatility timing strategies have four key characteristics: they do not require optimization, they do not require co-variance matrix inversion, they do not generate negative weights, and they allow the sensitivity of the weights to volatility changes to be adjusted via a tuning parameter. The weights of the VT strategy can be calculated using the following formula.

$$\hat{\omega}_{it} = \frac{(1/\hat{\sigma}_{it}^2)^\eta}{\sum_{i=1}^N (1/\hat{\sigma}_{it}^2)^\eta} \tag{9}$$

Here η is the tuning parameter, and it is a measure of timing aggressiveness. As η moves to 0, the naive portfolio is recovered, and as η moves to ∞, the weight on the asset with the lowest volatility approaches 1. For the VT strategy, we assume a diagonal covariance matrix (i.e., no correlations). The RRT strategy also considers the estimated conditional mean for asset i. The weights for the RRT strategy can be calculated by using the following formula.

$$\hat{\omega}_{it} = \frac{(\hat{\mu}_{it}^+/\hat{\sigma}_{it}^2)^\eta}{\sum_{i=1}^N (\hat{\mu}_{it}^+/\hat{\sigma}_{it}^2)^\eta} \tag{10}$$

Here $\hat{\mu}_{it}^+ = \max(\hat{\mu}_{it}, 0)$ and η is once again the tuning parameter. As η approaches 0, the strategy approaches naive diversification. As η approaches ∞, a weight of 1 is put on the asset with the maximum estimated reward-to-risk ratio.

An alternative estimator of conditional expected returns is also used. The following formula calculates the weights for this RRT strategy.

$$\omega_{it} = \frac{(\bar{\beta}_{it}^+/\sigma_{it}^2)^\eta}{\sum_{i=1}^N (\bar{\beta}_{it}^+/\sigma_{it}^2)^\eta} \tag{11}$$

Here $\bar{\beta}_{it}^+ = max(\bar{\beta}_{it}, 0)$ and $\bar{\beta}_{it} = (1/K)\sum_{j=1}^K \beta_{ij,t}$ is the average conditional beta of asset i with respect to the K factors.

The portfolio returns at time t are constructed by taking the dot product of the weights vector and the returns vector as depicted in the following formula.

$$R_{i,t} = \omega_{i,t} \cdot r_t \tag{12}$$

Here $\omega_{i,t}$ is the weight of strategy i at time t, and r_t is the asset return vector at time t. $R_{i,t}$ is the return of portfolio i at time t. The excess returns are then calculated by subtracting the risk-free rate from each individual portfolio return.

4.3 Risk-To-Reward Timing Strategies With Short-Selling

The RRT strategies in Kirby & Ostdiek (2012) are constructed in a way that they do not generate negative weights. However, in the context of commodity futures, short-selling is relatively easy. To construct the RRT strategies while allowing for short-selling, the tuning parameter η is removed from the weights equations and the assumptions that an investor has a strong prior belief that $\mu_{it} \geq 0$ and $\bar{\beta}_{it} \geq 0$ are also removed from the RRT strategy construction. Therefore the new weights formulas for the RRT strategies that allow short-selling are:

$$\hat{\omega}_{it} = \frac{(\hat{\mu}_{it}/\hat{\sigma}^2_{it})}{\sum_{i=1}^{N}(\hat{\mu}_{it}/\hat{\sigma}^2_{it})} \tag{13}$$

for the RRT(μ_t) strategy, and the weights formula for the RRT($\bar{\beta}_t$) strategy becomes:

$$\omega_{it} = \frac{(\bar{\beta}_{it}/\sigma^2_{it})}{\sum_{i=1}^{N}(\bar{\beta}_{it}/\sigma^2_{it})} \tag{14}$$

4.4 Performance Measures

Additionally, to evaluate portfolio performance, the maximum per period fee an investor would pay to switch from strategy i to strategy j is calculated, which is denoted by $\hat{\Delta}_\gamma$ and γ is the individual investors coefficient of relative risk aversion. When γ is high, it indicates that an investor is more sensitive to changes in risk. The formula for this metric for different levels of risk aversion is shown in the following formula:

$$\hat{\Delta}_\gamma = -\frac{1}{\gamma}(1 - \gamma E[R_{j,t+1}]) + \frac{1}{\gamma}\sqrt{(1 - \gamma E[R_{j,t+1}])^2 - 2\gamma E[R_{i,t+1} - R_{j,t+1} - \frac{\gamma}{2}(R^2_{i,t+1} - R^2_{j,t+1})]} \tag{15}$$

Here $R_{i,t+1}$ is the return of portfolio i for the period $t+1$, and γ is the investor's level of risk aversion. All the abovementioned strategies are compared to the 1/N portfolio.

To understand each strategy's relative performance, the p-values of the $\hat{\Delta}_\gamma$ are calculated for each strategy. The t-statistics are constructed using the generalized method of moments (GMM). Suppose there is a $J \times 1$ vector of random variables Y_t and a $J \times 1$ vector of disturbances. This disturbance vector e is shown as:

$$e(Y_t, \theta) = \begin{pmatrix} r_{p_i t} - \theta_1\theta_3 \\ r_{p_j t} - \theta_2\theta_4 \\ (r_{p_i t} - \theta_1\theta_3)^2 - \theta_3^2 \\ (r_{p_j t} - \theta_2\theta_4)^2 - \theta_4^2 \end{pmatrix} \tag{16}$$

The GMM estimator $\hat{\theta}$ is the value of θ for which the average value of the disturbance vector is zero. The limiting distribution of $\hat{\theta}$ is:

$$\sqrt{T}(\hat{\theta} - \theta) \rightarrow N(0, V) \tag{17}$$

Here $V = D^{-1}SD^{-1}$ with $D = E(\partial e(Y_t, \theta)/\partial\theta')$ and $S = \sum_{j=-\infty}^{\infty} E(e(Y_t, \theta)e(Y_{t-j}, \theta)')$

The distribution of the maximum per period fee an investor is willing to pay to switch from the 1/N strategy to another strategy for a given level of risk aversion is:

$$\sqrt{T}(\hat{\Delta}_\gamma - \Delta_\gamma) \sim N(0, d_\Delta V_\Delta d'_\Delta) \tag{18}$$

Here, $d_\Delta = \partial\Delta_\gamma/\partial\theta'$ and V_Δ is the asymptotic covariance matrix implied by the disturbance vector e. To identify strategies that outperform naïve diversification, the 1/N strategy is specified as strategy i and the p-values are reported for $H_0 : \Delta_\gamma \leq 0$ based on the following t-statistic:

$$\sqrt{T}(\frac{\hat{\Delta}_\gamma}{(\hat{d}_\Delta \hat{V}_\Delta \hat{d}'_\Delta)^{1/2}} \sim N(0, 1) \tag{19}$$

The p-values are determined from 10,000 trials of a stationary block bootstrap with an expected block length of 10, as seen in Kirby & Ostdiek (2012). All performance measures are reported assuming no transaction costs and proportional transaction costs of 50 basis points. For the comparison considering 50 basis points of transactions, the portfolio returns are adjusted by the following formula:

$$\tilde{R}_t = (1 + R_t)(1 - \tau_t c) - 1 \tag{20}$$

Here $\tilde{R}_t$ is the net portfolio return at time t after accounting for transaction costs, R_t is the portfolio return at time t, τ_t is the estimated expected turnover at time t, and c is the level of proportional transaction costs. In this case, c is set to 0.005 to assume 50 basis points of transaction costs.

The data that is used for the characteristics of the 1/N and MVE strategies is the 10 Industry and 10 Momentum data set from July 1963 until December 2004. The data that is used for the out-of-sample performance comparison of the 1/N, MVE, and active timing strategies are the 10 Industry, 10 Momentum, and 5 Commodity Futures data sets from July 1963 until December 2008 for the industry and momentum data set and December 1987 until December 2008 for the commodity futures data set.

5 Results

5.1 Replication

This part of the replication section contains all the tables and figures that are replicated from Kirby & Ostdiek (2012). Some values might not match exactly due to using a different data set. Kirby & Ostdiek (2012) use a different data set than the one publicly available on the Kenneth R. French library. For most estimated values for each portfolio, these differences are minor. However, the tangency portfolio, for example, is very sensitive to small changes in the data. Therefore the results for the tangency portfolio differ more than expected from the results from Kirby & Ostdiek (2012).

5.1.1 Characteristics of the 1/N and MVE Strategies

One thing that stands out in Table 3 is that for the 10 industry portfolios, the 1/N strategy beats the TP strategy in terms of the Sharpe Ratio. The MV and OC portfolios outperform the 1/N portfolio in terms of Sharpe ratio with 0.54 and 0.49, respectively, but they also come with a significantly higher turnover due to the rebalancing. Therefore it becomes a trade-off between turnover and risk-adjusted performance. With no transaction costs, the MV portfolio is the best-performing portfolio due to its high Sharpe Ratio.

For the 3-factor data set, once again, the 1/N strategy outperforms the TP strategy and, in this case, also the OC portfolio in terms of Sharpe Ratios. Once again, the MV portfolio is the best-performing portfolio due to its high Sharpe Ratio and also low turnover compared to the other portfolios.

Table 3: Characteristics of the 1/N and MVE Strategies

	FF 10 Industry				Mkt/SMB/HML			
	Panel A. Summary Statistics							
Strategy	$\hat{\mu}_p$	$\hat{\sigma}_p$	$\hat{\lambda}_p$	$\hat{\tau}_p$	$\hat{\mu}_p$	$\hat{\sigma}_p$	$\hat{\lambda}_p$	$\hat{\tau}_p$
TP	108	457	0.24	515	5.66	7.47	0.76	0.06
1/N	7.14	15.21	0.47	0.02	4.93	6.35	0.78	0.02
MV	7.07	13.11	0.54	0.46	4.80	5.55	0.86	0.02
OC	6.54	13.27	0.49	0.64	4.77	6.25	0.76	0.06
	Panel B. Estimated Conditional Expected Returns							
Strategy	Min.	Med.	Max.		Min.	Med.	Max.	
TP	21.9	47.4	12.216		3.4	6.7	15.5	
1/N	-3.2	7.9	14.1		0.3	1.0	2.3	
MV	-1.7	4.2	12.2		1.8	4.3	8.6	
OC	-0.3	8.4	15.8		2.6	5.0	8.6	

Table 3 compares the TP, 1/N, MV, and OC strategies. It reports the annualized mean, standard deviation, Sharpe Ratio, and average monthly turnover for the FF10 Industry and FF3 data set. Additionally, it also displays the minimum, medium, and maximum of the estimated conditional expected returns for each strategy.

5.1.2 Out of Sample Performance Comparison of Strategies

For the 10 Industry data set considering no transaction costs, one thing to note is that the fee an investor is willing to pay to switch from the 1/N strategy is positive for all strategies except for three of the RRT strategies and the TP strategy.

For the second part of out-of-sample performance comparison, 50 basis points of transaction costs are assumed. This does not impact the maximum fees that investors would be willing to pay to switch from the 1/N strategy because all fees are still positive except for the OC portfolio. The introduction of transaction costs makes investors not want to switch from the 1/N to the OC portfolio. The reason for the relatively small impact imposing transaction costs has is due to the low turnover for every strategy.

Table 4: Comparison of the strategies for the 10 industry data set without transaction costs

				Versus 1/N				
Strategy	$\hat{\mu}_p$	$\hat{\sigma}_p$	$\hat{\lambda}_p$	$\hat{\Delta}_1$	p-val	$\hat{\Delta}_5$	p-val	$\hat{\tau}_p$
1/N	5.78	15.07	0.383	-	-	-	-	0.022
Panel A. Volatility Timing Strategies								
VT(1)	5.98	14.29	0.419	32	0.108	79	0.020	0.023
VT(2)	6.03	13.62	0.443	46	0.158	132	0.026	0.027
VT(4)	5.82	12.85	0.453	35	0.258	162	0.047	0.036
Panel B. Risk-to-Reward Timing Strategies								
RRT(μ_t^+,1)	4.85	15.70	0.309	-102	0.000	-142	0.000	0.078
RRT(μ_t^+,2)	4.77	16.36	0.291	-121	0.000	-205	0.000	0.088
RRT(μ_t^+,4)	4.89	17.48	0.280	-128	0.000	-290	0.000	0.114
RRT($\bar{\beta}_t^+$,1)	5.92	15.08	0.393	14	0.000	14	0.000	0.027
RRT($\bar{\beta}_t^+$,2)	5.95	15.21	0.0.391	15	0.203	6	0.239	0.035
RRT($\bar{\beta}_t^+$,4)	5.89	15.64	0.376	2	0.297	-34	0.400	0.050
Panel C. Mean Variance Efficient Strategies								
MV	6.54	12.54	0.522	122	0.211	255	0.112	0.421
OC	6.09	12.79	0.480	64	0.255	194	0.151	0.682
TP	-5.90	69.95	-0.084	-3536	0.852	-	-	41.14

Table 4 contains the out-of-sample performance comparison of the 1/N, 3 VT, 6 RRT, and 3 MVE strategies without transaction costs for the 10 Industry data sets. The relative performance of each strategy is compared to the 1/N strategy through the maximum per period fee an investor would pay to switch from the 1/N strategy to another strategy for different levels of risk aversion.

Table 6 contains the same comparison as Table 4 but for the 10 Momentum data set. What stands out in Table 6 is that the 3 RRT strategies that did not yield a higher utility than the 1/N strategy for the 10 Industry data set do yield a higher utility than the 1/N strategy for the 10 Momentum data set for both levels of risk aversion. For most RRT strategies, the differences to the 1/N strategy are statistically significant for the 1% level. All RRT strategies have a $\hat{\Delta}_\gamma$ larger than 100 for both a risk aversion level of 1 and 5. This means that investors would be willing to pay a maximum per-period fee of over 100 basis points to switch from the 1/N strategy to any RRT strategy.

When it comes to the VT and the MVE strategies, once again, the maximum per period fee an investor would be willing to pay to switch from the 1/N strategy to any of the VT strategies is positive for both levels of risk aversion. Hence, for the 10 Momentum data set, the 1/N strategy seems to perform the worst among all strategies.

The results do not change much for the same results assuming 50 basis points of transaction costs. The maximum per period fee an investor would be willing to pay to switch from the 1/N strategy to another is still significantly positive for all strategies and both levels of risk aversion. The small impact of imposing transaction costs is that all strategies have a relatively low turnover.

Table 5: Comparison of the strategies for the 10 industry data set with transaction costs

		Versus 1/N			
Strategy	$\hat{\lambda}_p$	$\hat{\Delta}_1$	p-val	$\hat{\Delta}_5$	p-val
1/N	0.375	-	-	-	-
Panel A. Volatility Timing Strategies					
VT(1)	0.409	18	0.201	66	0.024
VT(2)	0.431	30	0.224	116	0.033
VT(4)	0.436	13	0.334	140	0.065
Panel B. Risk-to-Reward Timing Strategies					
RRT(μ_t^+,1)	0.280	-149	0.000	-189	0.000
RRT(μ_t^+,2)	0.259	-174	0.000	-258	0.000
RRT(μ_t^+,4)	0.241	-197	0.000	-359	0.000
RRT($\bar{\beta}_t^+$,1)	0.382	-2	0.000	-3	0.000
RRT($\bar{\beta}_t^+$,2)	0.377	-6	0.300	-15	0.339
RRT($\bar{\beta}_t^+$,4)	0.357	-28	0.381	-64	0.464
Panel C. Mean Variance Efficient Strategies					
MV	0.319	-143	0.472	-1	0.341
OC	0.154	-349	0.536	-223	0.504
TP	-	-	-	-	-

Table 5 contains the out-of-sample performance comparison of the 1/N, 3 VT, 6 RRT, and 3 MVE strategies assuming transaction costs of 50 basis points for the 10 Industry data sets. The relative performance of each strategy is compared to the 1/N strategy through the maximum per period fee an investor would pay to switch from the 1/N strategy to another strategy for different levels of risk aversion.

5.2 Extension

A few results that stand out are that for the VT strategies, the maximum fee an investor would pay to switch from the 1/N strategy to the VT strategies is negative for a risk aversion level of 1 but positive for a risk aversion level of 5 for all values of eta. So the VT strategies yield higher utility for investors that are more risk averse than the 1/N strategy. All of the VT strategies also yield a higher Sharpe ratio than the 1/N strategy for the commodity futures data set.

Contrary to this, for the RRT strategies, the maximum per period fee an investor would pay to switch from the 1/N strategy to the RRT strategies is positive for a risk aversion level of 1 and negative for a risk aversion level of 5 for all values of eta. This is due to the high volatility that comes with commodity futures, making this asset class less attractive to investors that are more risk-averse. This means that the RRT strategies yield higher utility for investors that are less risk averse than the 1/N strategy. All of the RRT strategies also yield a higher Sharpe ratio than the 1/N strategy for the commodity futures data set.

For the MVE strategies, we see different results for each strategy. For the MV strategy, we get the same result as with the VT strategies, namely that the maximum per period fee an

Table 6: Comparison of the strategies for the 10 momentum data set without transaction costs

				Versus 1/N				
Strategy	$\hat{\mu}_p$	$\hat{\sigma}_p$	$\hat{\lambda}_p$	$\hat{\Delta}_1$	p-val	$\hat{\Delta}_5$	p-val	$\hat{\tau}_p$
1/N	4.71	16.57	0.284	-	-	-	-	0.017
Panel A. Volatility Timing Strategies								
VT(1)	5.23	15.91	0.328	62	0.167	106	0.120	0.016
VT(2)	5.48	15.60	0.351	92	0.166	156	0.112	0.017
VT(4)	5.66	15.36	0.369	115	0.173	194	0.124	0.024
Panel B. Risk-to-Reward Timing Strategies								
RRT(μ_t^+,1)	7.63	16.93	0.450	285	0.000	260	0.000	0.057
RRT(μ_t^+,2)	8.27	17.49	0.473	340	0.000	276	0.000	0.056
RRT(μ_t^+,4)	8.89	18.12	0.490	390	0.000	280	0.000	0.063
RRT($\bar{\beta}_t^+$,1)	6.21	16.25	0.382	155	0.084	177	0.050	0.018
RRT($\bar{\beta}_t^+$,2)	7.28	16.52	0.441	258	0.028	262	0.000	0.022
RRT($\bar{\beta}_t^+$,4)	8.59	17.25	0.498	376	0.000	329	0.000	0.029
Panel C. Mean Variance Efficient Strategies								
MV	6.20	14.62	0.424	180	0.217	304	0.169	0.402
OC	7.66	15.00	0.510	320	0.152	422	0.128	0.477
TP	131.23	210.93	0.622	-	-	-	-	180.57

Table 6 contains the out-of-sample performance comparison of the 1/N, 3 VT, 6 RRT, and 3 MVE strategies without transaction costs for the 10 Momentum data set. The relative performance of each strategy is compared to the 1/N strategy through the maximum per period fee an investor would pay to switch from the 1/N strategy to another strategy for different levels of risk aversion.

investor would be willing to pay to switch from the 1/N strategy is negative for low levels of risk aversion and positive for higher levels of risk aversion. The Sharpe ratio for the MV strategy is higher than the Sharpe ratio of the 1/N. For the OC and TP strategies, investors would not switch from the 1/N strategy for both levels of risk aversion, and the Sharpe ratio for these strategies is also lower than the 1/N strategy.

Table 9 shows the out-of-sample comparison of the unrestricted RRT strategies (i.e. allowing short-sales). When looking at the unrestricted version of the RRT(μ_t) strategy, the Sharpe ratio increases but the maximum fee an investor is willing to pay to switch from the 1/N strategy to this RRT strategy is significantly negative. This is because there seems to be a large increase in the annualized return standard deviation to 104.62. Because of this increase even investors with a level of relative risk aversion at 1 are not willing to switch away from the 1/N strategy. Because of this, $\hat{\Delta}_\gamma$ is also calculated for a relative risk aversion level of 0.5. For this level of relative risk aversion we actually see a significant increase and investors with this level of risk aversion would have positive utility by switching from the 1/N strategy to this RRT strategy that allows short-sales.

For the RRT($\bar{\beta}_t$) we see once again that the annualized return standard deviation increased by a large amount but now also the annualized return mean is negative, resulting in a negative sharp ratio. Because of this the utility gain from switching from the 1/N strategy to this RRT

Table 7: Comparison of the strategies for the 10 momentum data set with transaction costs

		Versus 1/N			
Strategy	$\hat{\lambda}_p$	$\hat{\Delta}_1$	p-val	$\hat{\Delta}_5$	p-val
1/N	0.278	-	-	-	-
Panel A. Volatility Timing Strategies					
VT(1)	0.323	53	0.182	97	0.125
VT(2)	0.344	82	0.182	146	0.128
VT(4)	0.359	100	0.191	179	0.128
Panel B. Risk-to-Reward Timing Strategies					
RRT(μ_t^+,1)	0.430	251	0.000	226	0.000
RRT(μ_t^+,2)	0.454	306	0.000	242	0.000
RRT(μ_t^+,4)	0.469	352	0.000	242	0.000
RRT($\bar{\beta}_t^+$,1)	0.375	144	0.086	166	0.060
RRT($\bar{\beta}_t^+$,2)	0.433	245	0.033	249	0.000
RRT($\bar{\beta}_t^+$,4)	0.488	359	0.000	312	0.000
Panel C. Mean Variance Efficient Strategies					
MV	0.258	63	0.343	61	0.293
OC	0.320	33	0.279	136	0.233
TP	-	-	-	-	-

Table 7 contains the out-of-sample performance comparison of the 1/N, 3 VT, 6 RRT, and 3 MVE strategies assuming transaction costs of 50 basis points for the 10 Momentum data set. The relative performance of each strategy is compared to the 1/N strategy through the maximum per period fee an investor would pay to switch from the 1/N strategy to another strategy for different levels of risk aversion.

strategy is negative for both levels of relative risk aversion however, the utility decrease is even worse for an investor with a relative risk aversion of 1. For both strategies there is a substantial increase in average monthly turnovers, and because the utility gains are already mostly negative, there is no motivation to compare these results with the results of a 50 basis points increase in transaction costs because the utility gains from that comparison will be even worse than the ones that are obtained now.

Table 8: Comparison of the strategies for the 5 commodity data set without transaction costs

Strategy	$\hat{\mu}_p$	$\hat{\sigma}_p$	$\hat{\lambda}_p$	$\hat{\Delta}_1$	p-val	$\hat{\Delta}_5$	p-val	$\hat{\tau}_p$
				Versus 1/N				
1/N	4.29	13.57	0.316	-	-	-	-	0.044
Panel A. Volatility Timing Strategies								
VT(1)	3.76	10.63	0.354	-17	0.481	127	0.174	0.040
VT(2)	3.63	9.93	0.365	-23	0.448	151	0.215	0.041
VT(4)	3.29	10.32	0.319	-61	0.455	97	0.280	0.050
Panel B. Risk-to-Reward Timing Strategies								
RRT(μ_t^+,1)	10.52	26.81	0.392	354	0.11	-779	0.907	0.320
RRT(μ_t^+,2)	11.11	28.22	0.394	375	0.104	-931	0.928	0.314
RRT(μ_t^+,4)	11.66	29.70	0.393	387	0.096	-1111	0.943	0.313
RRT($\bar{\beta}_t^+$,1)	6.80	18.60	0.366	170	0.140	-164	0.682	0.092
RRT($\bar{\beta}_t^+$,2)	7.64	21.64	0.353	193	0.188	-400	0.805	0.109
RRT($\bar{\beta}_t^+$,4)	8.14	24.79	0.328	169	0.249	-737	0.908	0.120
Panel C. Mean Variance Efficient Strategies								
MV	3.37	9.66	0.349	-46	0.497	138	0.226	0.050
OC	1.87	9.52	0.196	-195	0.622	-5	0.452	0.133
TP	-19.27	42.33	-0.455	-3166	0.000	-6883	0.000	6.51

Table 8 contains the out-of-sample performance comparison of the 1/N, 3 VT, 6 RRT, and 3 MVE strategies without transaction costs for the 5 Commodity Futures data set. The relative performance of each strategy is compared to the 1/N strategy through the maximum per period fee an investor would pay to switch from the 1/N strategy to another strategy for different levels of risk aversion.

Table 9: Comparison of the RRT strategies allowing short sales to the 1/N strategy

Strategy	$\hat{\mu}_p$	$\hat{\sigma}_p$	$\hat{\lambda}_p$	$\hat{\Delta}_{0.5}$	p-val	$\hat{\Delta}_1$	p-val	$\hat{\tau}_p$
				Versus 1/N				
RRT(μ_t)	43.87	104.62	0.419	1248	0.000	-1569	0.000	1.375
RRT($\bar{\beta}_t$)	-30.90	198.63	-0.156	-13564	0.902	-25187	0.881	4.435

Table 9 contains the out-of-sample performance comparison of the 2 RRT strategies without transaction costs for the 5 Commodity Futures data set. The relative performance of each strategy is compared to the 1/N strategy through the maximum per period fee an investor would pay to switch from the 1/N strategy to another strategy for different levels of risk aversion.

6 Conclusion

In this thesis, we consider to which extent mean-variance and active portfolio selection strategies perform compared to naïve diversification for commodity futures. To get an answer to this problem, we analyze monthly returns on five commodity futures from December 1987 until December 2008 with a maturity of one month. To get an answer to this problem we construct the 3 VT, 6 RRT, and 3 MV portfolios and compare their performance to naïve diversification through the maximum fee investors are willing to pay to switch from the 1/N strategy to the mean-variance or active timing strategies for different levels of an investor relative risk aversion.

Adding to this research, we also construct two more risk-to-reward timing strategies that allow for the possibility of short sales and report the same values.

When comparing the performance of mean-variance and active portfolio selection strategies for traditional assets like stocks we see that investors are not willing to switch from the 1/N strategy to the RRT(μ_t^+, η) and TP strategy for the 10 industry portfolios for both levels of relative risk aversion. For all other strategies and data sets, investors are willing to pay a fee per period to switch from the naïve strategy to any of the mean-variance and active strategies. For the commodity futures data set investors are not willing to pay a fee to switch from the 1/N strategy to the mean-variance efficient strategy. When we consider the active timing strategies, investors with a low level of relative risk aversion are willing to switch from the 1/N strategy to the RRT strategies but investors with a high level of relative risk aversion will not switch from the 1/N strategy to the RRT strategies. This is due to the high volatility that comes with commodity futures investments. For the volatility timing strategies, the opposite is true. Because volatility timing strategies take into account the volatility, investors with a low level of relative risk aversion are not willing to switch from the 1/N strategy to the VT strategies, but investors with a high level of risk aversion are willing to switch because of the low volatility. The Sharpe ratios are also higher for almost all of the RRT strategies when considering commodity futures portfolios than for traditional asset portfolios, indicating that RRT strategies perform better in terms of risk adjusted returns for commodity futures than for traditional assets. The opposite is true for the VT and MVE strategies.

Because of the interesting results of the RRT strategies for commodity futures, further investigation into these strategies is performed by allowing short sales for these strategies. The restriction of short sales is removed because of the relative ease of short-selling commodity futures contracts. When allowing short sales, the utility gains for investors with a low level of relative risk aversion are negative, resulting in investors not being willing to switch from the 1/N strategy to the RRT strategies. A possible explanation for this could be that the possibility of short-selling increases volatility which is in line with the findings of Duong et al. (2023) for bond prices. When looking at investors with an even lower level of relative risk aversion we find that they are only willing to switch from the 1/N strategy to the RRT(μ_t).

To conclude, it appears that the performance of risk-to-reward timing strategies performs well when considering commodity futures compared to traditional assets, while volatility timing and mean-variance efficient strategies perform worse for commodity futures portfolios compared to traditional assets. When comparing risk-to-reward timing strategies to naïve diversification, only investors with a low level of relative risk aversion are willing to switch from naïve diversification to risk-to-reward timing strategies due to the high volatility that comes with this portfolio selection strategy. These results are even stronger when allowing for short-sales since the volatility increases even more and the levels of relative risk aversion need to be even lower for investors to gain utility from switching from the naïve strategy to the risk-to-reward timing strategy.

An interesting avenue for future research might be to take the maturities of the commodity futures contracts into consideration, because considering different maturities in commodity futures is particularly relevant in the context of asset pricing. In addition to this, it might be interesting to also take into account the varying risk premia in commodity futures. Szymanowska et al. (2014) finds that risk premia for commodity futures are not constant over time, but that they show signs of a term structure. Further investigation into these risk premia across different maturities can help make more informed decisions about the construction of optimal portfolios and also the evaluation of mean-variance efficient strategies compared to naïve diversification in the commodity futures market.

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