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Examining the Indirect Influence of Relative Attendance on Seasonal Home Advantage: The Mediating Role of Referee Home Bias in English Professional Football

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Abstract

Peeters and van Ours (2021) allege that pressure exerted on referees by larger home crowds may be a key driver of seasonal home advantage in English professional football. In our research, using match data from the four English professional football divisions from 2000 to 2018, we address the correctness of this claim by means of the mediation analysis framework of Baron and Kenny (1986). To proxy for referee home bias, we construct the so-called referee home bias index as the weighted sum of three home bias measures in terms of disciplinary sanctions, where the weights are computed from a robust Shapley values approach. Using a bootstrap algorithm to ensure the reliability of the coefficient estimates of our mediation analysis, we find compelling evidence that referee home bias completely mediates the effect of relative attendance on relative seasonal home advantage, and verify the statistical significance of this mediation effect by means of the percentile, bias-corrected and reduced bias-corrected bootstrap confidence intervals, such that our results strongly support the proposition made in Peeters and van Ours (2021). Our findings advocate for referee training programs focused on mitigating home bias among arbiters, a more optimised referee assignment process in which the referees that are least prone to home bias are appointed to those matches in which the influence of the crowd is expected to be highest, and the preservation of the use of the Video Assistant Referee for crucial decisions in professional football.

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1 Introduction

The existence of home advantage in professional football is a well-established phenomenon. Recent research by Peeters and van Ours (2021) suggests that pressure exerted on referees by larger home crowds may be a key driver of seasonal home advantage in English professional football. However, a concise motivation behind this claim of the existence of an indirect effect of relative attendance on home advantage via the referee is missing from their paper, such that the question regarding the true nature of home advantage in football remains. Disentangling the effect of relative attendance into direct and indirect components is highly relevant in addressing this query, and may have rather important implications for taking policy decisions to ensure the fairness and integrity of football competitions. For this reason, in this paper, we investigate the correctness of the proposition made in Peeters and van Ours (2021) regarding the existence of an indirect effect of relative attendance via the referee on seasonal home advantage in English professional football. The research question is formulated as follows: *To what extent is the effect of relative home attendance on relative seasonal home advantage in English professional football mediated by referee home bias?*

For this, we consider the mediation analysis procedure proposed by Baron and Kenny (1986) using match data from the four English professional football divisions from 2000 to 2018. In particular, we utilise the mathematical framework of Clarke and Norman (1995) to compute home advantage and the eight regression models proposed by Peeters and van Ours (2021) to assess its determinants. Furthermore, we design three referee home bias measures, being the difference in the number of fouls, the number of yellow cards, and the number of red cards given to the home and away team, respectively. From this, we construct the so-called referee home bias index as the weighted sum of these measures, in which the weights reflect the relative importance of each measure with respect to relative seasonal home advantage and are determined using a robust Shapley values approach to account for the correlation between these metrics.

We observe that the difference in the number of red cards awarded to the home and away team is the major component in this index, which makes sense from the fact that among the three referee bias measures, the red card differential is most influential on match outcomes. To ensure the accuracy and reliability of coefficient estimates in our mediation analysis, we consider a bootstrap algorithm which mitigates the influence of potential outliers in the data. From this, we find compelling evidence that referee home bias completely mediates the effect of relative attendance on relative seasonal home advantage. In addition, we verify the statistical significance of this mediation effect by means of the percentile, bias-corrected, and reduced bias-corrected bootstrap confidence interval methods. Hence, we conclude that our results strongly support the proposition made in Peeters and van Ours (2021). That is, the effect of relative attendance on relative seasonal home advantage in English professional football is likely to be mediated completely through referee home bias.

The main practical implications of this finding are the need for referee training programs focused on mitigating home bias among arbiters, and a more optimised referee assignment process in which the referees that are least prone to home bias are appointed to those matches in which the influence of the crowd is expected to be highest. In addition, our findings strongly advocate for the preservation of the use of the Video Assistant Referee (VAR) for crucial, potentially game-changing decisions.

We proceed as follows. In Section 2, we review past literature regarding the determinants of home advantage in professional football and the econometric models and methods we consider in our research. Next, in Section 3, we demonstrate the characteristics of our data. Then, in Section 4, we describe the considered framework of computing home advantage, the utilised models and their variables, the used measures of referee home bias and the method of constructing the referee home bias index, and the mediation analysis procedure. In Section 5, we present the results of our analysis and explain their interpretations in detail. Finally, in Section 6, we formulate an answer to our research question and suggest how future research can build upon on our conclusions.

2 Literature Review

Courneya and Carron (1992) define home advantage in team sports as "the consistent finding that home teams in sports competitions win over 50% of the games played under a balanced home and away schedule". Evidence of the existence of such advantage in various sports abounds[1], and the quest for the true underlying factors of home advantage has hence been ever existent since. In particular, extensive research aiming to unveil the determinants of home advantage in professional football has been conducted, resulting in numerous potential contributing factors to this phenomenon. For instance, Goumas (2013) finds the psychological effect of crowd support to be positively associated with home advantage in international club competitions, whereas Armatas and Pollard (2014) and Dohmen (2008) find increased crowd proximity due to the absence of a running track around the pitch to be positively related to home advantage in Greek and German professional football, respectively. Moreover, Krumer and Lechner (2018) conclude that the home advantage German Bundesliga clubs enjoy during the weekends vanishes when playing mid-week matches, due to lower home attendance and the perception of players that these matches are relatively less relevant. This psychological effect related to the day of play is confirmed by Goller and Krumer (2020) for various European football leagues, whereas Krumer (2020) finds the kick-off times of UEFA Europa League group stage games to affect the extent of enjoyed home advantage, motivated by the effect kick-off times have on the size of the present stadium crowd.

In addition to these mental facets, there exist various physiological factors affecting home advantage in professional football as well. Pollard, Silva and Medeiros (2008) show that differences in climatic conditions and fatigue resulting from longer travel distances could explain the inferior away team performance relative to that of the home team in Brazil. The importance of travel distance has been further emphasised by demonstrating a decline in home advantage in games between teams from the same city in English (Clarke & Norman, 1995), Turkish (Seckin

[1] Schwartz and Barsky (1977), Varca (1980), Snyder and Purdy (1985), Agnew and Carron (1994), Adams and Kupper (1994), Moore and Brylinsky (1995), A. M. Nevill, Newell and Gale (1996), A. M. Nevill and Holder (1999), Pollard and Pollard (2005)

& Pollard, 2008), and French, Italian, Portuguese, and Spanish professional football (Pollard & Gómez, 2009). Besides changes in climatic conditions, McSharry et al. (2007) find altitude increases to negatively impact the physiological performance of away teams in South America, which magnifies the home advantage of such higher altitude teams. Moreover, Van Damme and Baert (2019) support the relevance of altitude in the magnitude of home advantage in European international football as well. Furthermore, Goumas (2014) emphasises the influence of the number of time zones crossed by the away team on their physiological performance, which significantly affects the level of home advantage in Australia. Finally, the importance of familiarity with local conditions in the context of home advantage has been further substantiated by Barnett and Hilditch (1993), finding a substantial positive additional home advantage of playing on an artificial pitch.

Besides the aforementioned factors influencing home advantage via players, numerous studies investigate the influence of referees on home advantage. In particular, Dawson, Dobson, Goddard and Wilson (2007) conclude there exists a referee bias in terms of issuing disciplinary sanctions in favour of home teams in the English Premier League, argued to be due to pressure that the home crowd exerts on referees. Ponzo and Scoppa (2018) corroborate the influence of crowd support on the referee by analysing same-stadium derbies across Europe, hence controlling for the aforementioned physiological factors such as travel distance or familiarity with home conditions. Similarly, A. M. Nevill, Balmer and Williams (2002) show the degree of home crowd noise to be of significant influence on the extent to which referees make biased decisions in favour of the home team regarding fouls in the English Premier League, which is in accordance with earlier obtained evidence by A. Nevill, Balmer and Williams (1999) regarding the ability of the crowd to influence officiating. Furthermore, Erikstad and Johansen (2020) conclude for the Norwegian Premier League that successful teams are more likely to receive an incorrect penalty compared to their opponents on the one hand and less likely to be denied a penalty they should have been awarded on the other, indicating that referees' decisions may be biased by a team's success. Boyko, Boyko and Boyko (2007) find compliant results regarding referee bias favouring the home team in terms of yellow cards and penalties in English Premiership football. Finally, Garicano, Palacios-Huerta and Prendergast (2005) and Sutter and Kocher (2004) respectively show that Spanish and German referees systematically shorten close games in which the home team is leading on the one hand, and lengthen close games in which the home team is trailing on the other, with the level of bias increasing with the merits for the home team of winning the game, deemed to be due to the referees' desire to satisfy home crowds.

Peeters and van Ours (2021) acknowledge the abundance of evidence of the existence of home advantage in professional football, but emphasise that the relative importance of the found determinants of this phenomenon in the aforementioned studies remains unclear. Consequently, the question of whether and why certain clubs enjoy a relatively larger home advantage than others is left unsolved, such that the authors ask to what extent seasonal home advantage is a relevant concern from a competitive standpoint. Namely, if the home advantage is equally distributed over the teams in a competition, it should cancel out at the end of classic double round-robin competitions, whereas if some teams do enjoy a larger home advantage than others due to certain determinants, this may introduce an element of unfairness and the need for adequate policy decisions to mitigate this issue. Their results indicate a strong positive correlation of both the nature of the pitch and relative home attendance with relative seasonal home advantage, the latter of which they argue to be due to the ability of a relatively larger home crowd to exert pressure on referees, causing bias in their decision-making favouring the home team.

To investigate the correctness of this claim, we endeavor to disentangle the effects of relative home attendance and referee bias on relative seasonal home advantage by means of the mediation analysis model proposed by Baron and Kenny (1986). When taking statistical inference on the magnitude of the mediation effect, a plethora of past evidence[2] shows that we should account for the non-normal sampling distribution of the point estimate of this indirect effect. For this, Efron (1979) proposes a so-called bootstrap algorithm, which encompasses a re-sampling technique used to estimate the empirical distribution of a statistic based on which statistical analysis can be applied. In particular, among others, Hayes and Scharkow (2013) recommend the use of confidence intervals rather than significance tests in the context of mediation analysis to ensure power. To construct such intervals, Efron and Tibshirani (1994) propose to apply the bootstrap method described above to obtain an empirical bootstrap sampling distribution by repeatedly re-sampling the original data set with replacement and estimating the mediation effect for each bootstrap sample.

To determine the limits of this confidence interval, Efron and Tibshirani (1994) suggest the so-called percentile bootstrap confidence interval (PBCI), where the $\left(\frac{\alpha}{2} \times 100\right)$th and $\left(1 - \frac{\alpha}{2} \times 100\right)$th percentiles of the ordered observed bootstrap distribution form the lower and upper limits of the $(1-\alpha) \times 100\%$ confidence interval, respectively. Inherently, the PBCI assumes the bootstrapped indirect effect to be an unbiased estimator of the true indirect effect. However, a myriad of evidence[3] shows that bootstrap sampling distributions of mediation effects tend to be substantially skewed, which could invoke inaccurate and unreliable confidence intervals when using the PBCI approach due to its underlying assumptions. For this reason, the so-called bias-corrected bootstrap confidence interval (BCBI) introduced by Efron (1987) improves on the PBCI by adjusting for possible estimation bias arising from the median of the empirical distribution of the bootstrap estimates not being the true parameter value, which obtains more accurate confidence intervals in case the mediation effect is non-zero as shown by MacKinnon, Lockwood and Williams (2004). When comparing the accuracy of the PBCI and the BCBI, Tibbe and Montoya (2022) show that the BCBCI has the highest power and highest type I error rate, whereas the PBCI has the lowest power and lowest type I error rate, emphasizing the existence of a certain trade-off between the two methods in terms of significance levels and power. Their novel proposed reduced bias-corrected bootstrap confidence interval (rBCBI) turns out to be the golden middle way, providing the most optimal balance between low significance levels and high power. Compared to the BCBI, the rBCBI approach applies the bias correction only once rather than twice, namely merely at the bootstrap indirect effect level and not at the sample indirect effect level.

[2] Craig (1936), Aroian (1947), Lomnicki (1967), Aroian, Taneja and Cornwell (1978)

[3] Stone and Sobel (1990), MacKinnon and Dwyer (1993), Shrout and Bolger (2002), Mallinckrodt, Abraham, Wei and Russell (2006), MacKinnon, Fritz, Williams and Lockwood (2007)

3 Data

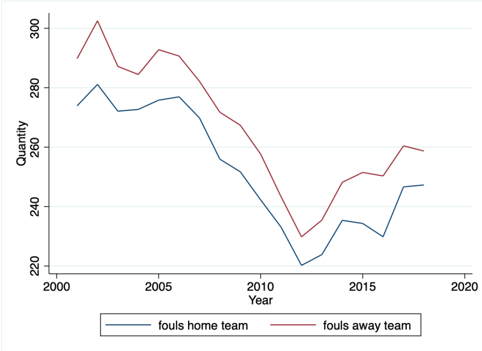
In our research, we analyse a large dataset containing information on various aspects of all professional football matches played in England's highest four divisions from August 12, 2000 to May 13, 2018, amounting to 36646 matches in total. We acquire the primary part of this game-level dataset from Peeters and van Ours (2021) and augment it with information regarding shots, shots on target, corners, fouls, yellow cards, and red cards obtained from www.football-data.co.uk. Table 1 below contains the descriptive statistics of the aforementioned measures.

Table 1: Descriptive statistics game-level data English professional football

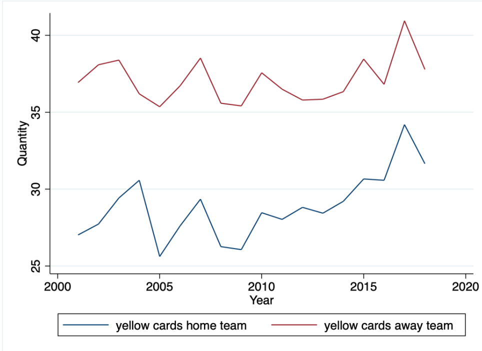
Variable	Description	Mean	SD
win	1 if home team wins, 0 otherwise	0.441	0.497
draw	1 if home team draws, 0 otherwise	0.270	0.444
loss	1 if home team loses, 0 otherwise	0.289	0.453
points	points obtained by home team	1.593	1.304
sc	goals scored by home team	1.464	1.228
op_sc	goals scored by away team	1.133	1.076
shots	shots by home team	12.282	4.656
op_shots	shots by away team	9.824	4.123
corners	corners by home team	6.049	2.959
op_corners	corners by away team	4.900	2.658
fouls	fouls committed by home team	11.406	3.827
op_fouls	fouls committed by away team	12.062	3.962
yellow	yellow cards issued to home team	1.305	1.141
op_yellow	yellow cards issued to away team	1.675	1.273
red	red cards issued to home team	0.071	0.269
op_red	red cards issued to away team	0.104	0.326

From this table, we observe that there exists a clear home advantage at game level, as the home team has a systematically higher chance of winning than the visiting opponent and, logically, obtains more points from their home game than the away team as a consequence. In accordance with this, the home team scores more goals than the away team on average, emphasizing the existence of a clear home advantage at game level. Furthermore, based on the statistics on the number of shots and corners, we observe that the home team presumably plays more offensive football than its opponent. Moreover, we conclude from the information regarding disciplinary sanctions that the referee issues fewer fouls, yellow cards, and red cards to the home team compared to the away team, which could indicate the existence of a certain referee home bias.

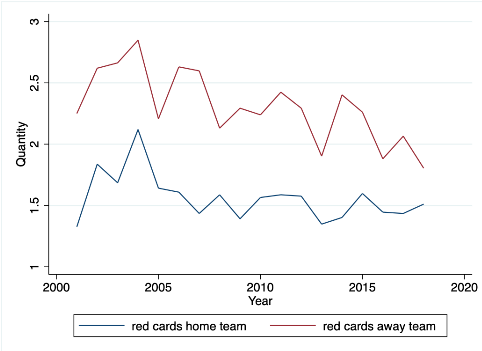
Figure 1 below shows the development of the differences in disciplinary sanctions issued to the home and away team. In particular, Figures 1a 1b and 1c respectively display the development of the average seasonal number of fouls, number of yellow cards and number of red cards over time, from which we observe that there indeed exists a home advantage in terms of disciplinary sanctions in favour of the home team, assuming both teams play equally fair. Furthermore, Figure 1d depicts the development of the foul, yellow card, and red card differentials over time, which are computed by subtracting the seasonal average number of fouls, number of yellow cards, and number of red cards issued to the home team from the seasonal averages corresponding to the away team. From this, we observe that the three measures are presumably rather correlated.



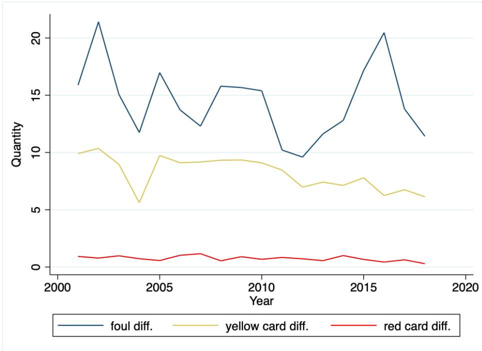
(a) Seasonal number of fouls home and away team



(b) Seasonal number of yellow cards home and away team



(c) Seasonal number of red cards home and away team



(d) Seasonal foul, yellow card and red card differential

Figure 1: Development of differences in disciplinary sanctions; 2000-2018

To asses the correlation structure of these measures to a deeper level, we show their correlation matrix in Figure 2 below, from which we conclude that the different measures are moderately correlated, which we should take into account when constructing the referee home bias index.

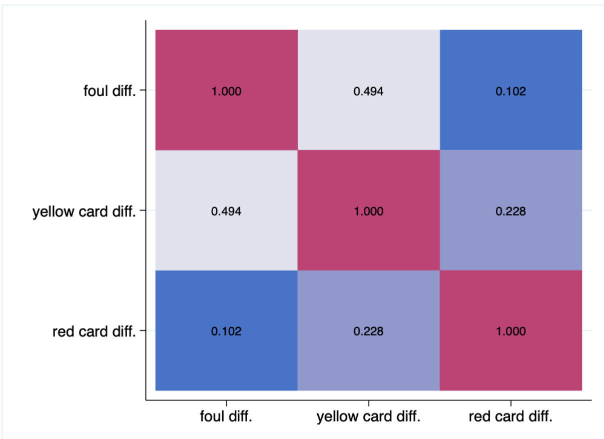


Figure 2: Correlation matrix disciplinary sanctions differentials

In order to analyse seasonal home advantage, we aggregate the extensive game-level dataset described above to season level, in which each observation contains information on the seasonal performance of the respective club in the respective season, coming down to 1656 observations. The methods we use to perform this analysis are presented in the next section.

4 Methodology

First, Section 4.1 shows the formulas used to compute seasonal home advantage. Then, Section 4.2 describes our baseline quantitative analysis, highlighting the considered regression models and the definitions of the incorporated independent variables. Next, Section 4.3 shows the formulations of the different referee home bias measures and the method of constructing the referee home bias index from these. Finally, Section 4.4 describes the mediation analysis model and the procedures used to calculate the different considered bootstrap confidence intervals.

4.1 Calculating Seasonal Home Advantage

Similar to Peeters and van Ours (2021), we use the method proposed by Clarke and Norman (1995) to decompose the performance of team c in season t into team quality and seasonal home advantage, with the latter either being measured in terms of home goal difference or home point difference. Below, we provide a brief overview of the process of calculating seasonal home advantage in terms of home goal difference and home point difference, respectively.

4.1.1 Seasonal Home Advantage: Home Goal Difference

First, we define an indicator function to model when team c plays at home:

$$\mathrm{Ih}_{c,k,t} = \begin{cases} 1, & \text{if team } c \text{ is playing at home in game } k \text{ in season } t, \\ 0, & \text{otherwise} \end{cases}. \tag{1}$$

Then, defining $N_{j,t}$ as the total number of games each team in division j played in season t, and letting v be the index for the opponent team that team c faces in game k, we formulate the seasonal home goal difference for team c in season t as follows:

$$HGD_{c,t} = \sum_{k=1}^{N_{j,t}} \sum_{v=1}^{V} \left(HGpro_{c,k,t} - HGag_{v,k,t}\right) \mathrm{Ih}_{c,k,t}, \tag{2}$$

with $HGpro_{c,k,t}$ being the number of goals pro team c when playing at home in game k in season t, and $HGag_{v,k,t}$ the number of goals against team c when playing at home in game k in season t. Similarly, we define the seasonal away goal difference for team c in season t as follows:

$$AGD_{c,t} = \sum_{k=1}^{N_{j,t}} \sum_{v=1}^{V} \left(AGpro_{c,k,t} - AGag_{v,k,t}\right) \left(1 - \mathrm{Ih}_{c,k,t}\right), \tag{3}$$

with $AGpro_{c,k,t}$ being the number of goals pro team c when playing away in game k in season t, and $AGag_{v,k,t}$ the number of goals against team c when playing away in game k in season t. Then, we calculate the average seasonal home goal difference in division j in season t:

$$MHGD_{j,t} = \frac{\sum_{f=1}^{C_{j,t}} HGD_{f,t}}{C_{j,t} - 1}, \tag{4}$$

with $C_{j,t}$ being the number of teams in division j in season t. From this, we calculate the seasonal home advantage of team c in season t in terms of goal difference (GD) as follows [4]:

$$SHA_{c,t}^{GD} = \frac{(HGD_{c,t} - AGD_{c,t} - MHGD_{j,t})}{C_{j,t} - 2}, \tag{5}$$

4.1.2 Seasonal Home Advantage: Home Point Difference

In addition to the notation introduced in the previous section, we define the following indicator function to model the home point difference:

$$Ip_{c,v,k,t} = \begin{cases} 3, & \text{if team } c \text{ wins game } k \text{ against team } v \text{ in season } t, \\ 0, & \text{if teams } c \text{ and } v \text{ draw in game } k \text{ in season } t, \\ -3, & \text{if team } c \text{ loses game } k \text{ against team } v \text{ in season } t, \end{cases} \tag{6}$$

Then, we formulate the seasonal home point difference for team c in season t as follows:

$$HPD_{c,t} = \sum_{k=1}^{N_{j,t}} \sum_{v=1}^{V} \mathrm{Ip}_{c,v,k,t} \mathrm{Ih}_{c,k,t}. \tag{7}$$

Similarly, we define the seasonal away point difference for team c in season t as follows:

$$APD_{c,t} = \sum_{k=1}^{N_{j,t}} \sum_{v=1}^{V} \mathrm{Ip}_{c,v,k,t} \left(1 - \mathrm{Ih}_{c,k,t}\right). \tag{8}$$

From this, we calculate the average seasonal home point difference in division j in season t:

$$MHPD_{j,t} = \frac{\sum_{f=1}^{C_{j,t}} HPD_{f,t}}{C_{j,t} - 1}, \tag{9}$$

with, as before, $C_{j,t}$ being the number of teams in division j in season t. Finally, we calculate the seasonal home advantage of team c in season t in terms of point difference (PD) as follows[4]:

$$SHA_{c,t}^{PD} = \frac{(HPD_{c,t} - APD_{c,t} - MHPD_{j,t})}{C_{j,t} - 2}. \tag{10}$$

4.2 Baseline Quantitative Analysis

In our analysis, we focus on relative seasonal home advantage such that calendar year fixed effects are removed from the analysis:

$$RSHA_{c,t}^{z} = SHA_{c,t}^{z} - \frac{1}{C_{j,t}} \sum_{f=1}^{C_{j,t}} SHA_{f,t}^{z}, \tag{11}$$

for $z = GD, PD$, corresponding to measuring seasonal home advantage either in terms of home goal difference (GD) or home point difference (PD), respectively. Below, we first specify our independent variables, after which we describe the used regression model estimation methods.

[4] We refer the reader to Appendices B and C of Clarke and Norman (1995) for the derivation of (5) and (10).

4.2.1 Independent Variables

To start with, relative attendance could be an important driver of relative seasonal home advantage. For instance, higher home attendance relative to the division average might benefit the home team by motivating the home players more on the one hand and by putting more pressure on the away team players on the other. In addition, higher home attendance may exert pressure on the referee to favour the home team, as argued by Peeters and van Ours (2021) based on Garicano et al. (2005). Defining the average home attendance for team c in season t as:

$$AvAtt_{c,t} = \frac{2}{N_{j,t}} \sum_{k=1}^{N_{j,t}} Att_{k,t} \mathrm{Ih}_{c,k,t}, \tag{12}$$

with $Att_{k,t}$ the home attendance at game k in season t, we formulate the relative home attendance for club c in division j in season t as follows:

$$RelAtt_{c,t} = \log \left(\frac{AvAtt_{c,t}}{\frac{1}{C_{j,t}} \sum_{f=1}^{C_{j,t}} AvAtt_{f,t}} \right). \tag{13}$$

Furthermore, a team being newly promoted or relegated to a new division may influence the club's seasonal home advantage, as visiting teams are likely to be less familiar with their grounds compared to other grounds in the respective division. Therefore, we define dummy variable $Pr_{c,t}$, which takes value one if team c is promoted in season $t-1$, and is zero otherwise. Similarly, we define dummy variable $Rg_{c,t}$, which takes value one if team c is relegated in season $t-1$, and is zero otherwise.

Finally, we consider relative wage as a possible underlying factor of relative seasonal home advantage. Namely, as better players generally require higher wages, a higher wage sum relative to the division average presumably indicates having the availability of such better players, who can potentially exploit the absolute home advantage relatively more, for instance by instilling timidity to punish them in the referee with their status. We define relative wage for team c in division j in season t as follows:

$$RelW_{c,t} = \log \left(\frac{Wage_{c,t}}{\frac{1}{C_{j,t}} \sum_{f=1}^{C_{j,t}} Wage_{f,t}} \right), \tag{14}$$

with $Wage_{c,t}$ the total wage cost of team c in season t. Having defined these independent variables, we now formulate our considered panel data regression models.

4.2.2 Regression Models

Using the variables defined above, the baseline panel regression model for club c in season t is formulated as follows:

$$RSHA^z_{c,t} = \alpha_c + \gamma_t + \beta_1 RelAtt_{c,t} + \beta_2 Pr_{c,t} + \beta_3 Rg_{c,t} + \beta_4 RelW_{c,t} + \sum_{j=1}^{4} \beta_{j+4} D_j + \epsilon_{c,t}, \tag{15}$$

for $z = GD, PD$, with α_c representing club-specific fixed effects, γ_t encompassing time-specific fixed effects, and with D_j being divisional dummies indicating in which division j team c plays in season t, with $j = 1, 2, 3, 4$. To enhance the robustness of our research, we consider (15) excluding relative wage from the model as well, similar to Peeters and van Ours (2021).

To estimate the model, we make use of the two panel regression model estimation methods employed in Peeters and van Ours (2021) for panel data, being Pooled Ordinary Least Squares (POLS) and Fixed-Effects (FE) panel regression in particular. Starting with POLS, this estimation method can be described as simply performing OLS on panel data, relying on the assumption of an equal relationship between the dependent and independent variables for all clubs across all seasons. Namely, as the name suggests, all observations from different clubs over different time periods are pooled together, treating them as one cross-sectional dataset in which club- and time-specific effects are not accounted for. That is, POLS sets $\alpha_c = \alpha \ \forall c$ and $\gamma_t = 0 \ \forall t$ when estimating (15). In contrast to POLS, FE panel regression does allow for the inclusion of club- and time-specific fixed effects, such that the regression controls for time-invariant club-specific heterogeneity and for club-constant season-dependent heterogeneity. For both estimation methods, to account for the potential presence of heteroscedasticity, we compute the standard errors corresponding to the coefficients in (15) by means of the Huber-White sandwich estimator. For a detailed explanation and motivation for the use of this method, we refer the reader to Heij et al. (2004, Chapter 5.4.2).

In conclusion, in order to ensure the robustness of our research, we differentiate between the used measure of home advantage, the inclusion or exclusion of relative wage in the model equation, and the regression model estimation method used. Hence, similar to Peeters and van Ours (2021), we consider eight regression models in our analysis in total.

4.3 Referee Home Bias Measures and Index

To quantify the extent to which the referee favours the home team over the away team, we consider several measures of this bias. In particular, we introduce the foul differential, yellow card differential and red card differential, which represent the seasonal difference in the number of fouls, number of yellow cards, and number of red cards issued to the away and home team, respectively. For the sake of conciseness, we formulate the general form of a disciplinary sanctions bias measure, based on which the foul, yellow, and red card differentials can be computed by means of substitution. That is, the disciplinary sanctions differential for team c in season t is defined as:

$$dsD_{c,t} = \sum_{k=1}^{N_{j,t}} \sum_{v=1}^{V} \left(dsV_{v,k,t} - dsH_{c,k,t}\right) \mathrm{Ih}_{c,k,t}, \tag{16}$$

with ds being the disciplinary sanctions type, i.e. $ds = F, Y, R$, corresponding to fouls, yellow cards, and red cards, respectively. Furthermore, $dsV_{v,k,t}$ is the number of disciplinary sanctions of type ds called against visiting team v in game k in season t, and $dsH_{c,k,t}$ the number of disciplinary sanctions of type ds called against home team c in game k in season t, such that a positive value of $dsD_{c,t}$ indicates a possible referee bias in terms of sanction type ds in favour of team c when playing at home in season t, again for $ds = F, Y, R$. From this general measure, the foul differential, yellow card differential, and red card differential can be obtained by replacing ds in (16) with F, Y, R, respectively.

Based on this, we construct the so-called referee home bias index, which is a weighted sum of the different measures of referee home bias. We motivate the use of this index by noting that neither of the incorporated measures is likely to be able to fully encompass referee bias, such that taking a weighted combination of these might better gauge the level of referee partiality. Defining $FD_{c,t}, YD_{c,t}, RDc, t$ as the foul differential, yellow card differential and red card differential of team c in season t, we formulate the referee home bias index for team c in season t as:

$$RHI_{c,t} = w_1 FD_{c,t} + w_2 YD_{c,t} + w_3 RD_{c,t}, \tag{17}$$

with w_i representing the relative importance of the respective measure in contribution to relative seasonal home advantage, for $i = 1, 2, 3$. As noted in Section 3, we should account for the existing correlation between the metrics in determining these weights from statistical analysis. In such situations, Lipovetsky and Conklin (2001) suggests the use of the Shapley values algorithm, which consistently estimates the relative importance of independent variables on the dependent variable in the presence of multicollinearity. For that reason, we use the above referee home bias index computed from this Shapley values algorithm as our general proxy for referee home bias. In Appendix Section B, we perform sensitivity analysis with respect to this decision by considering the first component resulting from applying scaled principal component analysis on the three measures for this matter, from which we conclude that our research is robust to the used proxy for referee bias.

In short[5], the Shapley values algorithm determines the average marginal contribution of each referee home bias measure in explaining relative seasonal home advantage by considering all possible combinations of these measures. Below, we provide a brief overview of the main formulas encompassing the Shapley values algorithm in our case of three independent variables, and refer the reader to Franses (2021) for the formulations for the general case of K regressors. Consider the following regression:

$$RSHA^z_{c,t} = \omega_c + \chi_t + \zeta_1 FD_{c,t} + \zeta_2 YD_{c,t} + \zeta_3 RD_{c,t} + \nu_{c,t}, \tag{18}$$

for $z = GD, PD$, corresponding to the case of measuring seasonal home advantage either in terms of home goal difference (GD) or home point difference (PD), respectively, with ω_c representing club-specific fixed effects and χ_t comprising time-specific fixed effects. As before, we estimate this regression model by means of both POLS and FE. Then, ignoring indices specifying the measure of home advantage and estimation method used in (18) for the sake of clarity, we define the contribution of regressor i to relative seasonal home advantage as:

$$SH_i = \sum_{S \subseteq N, J \in S} \frac{(s-j)!(n-s)!}{n!} \left[R^2_S - R^2_{S_J} \right], \tag{19}$$

for $i = 1, 2, 3$ corresponding to the three regressors in (18), with N being the set containing all

[5]We refer the reader to Chantreuil and Trannoy (2013) for a more detailed explanation of this concept.

possible combinations of these regressors, s the number of regressors in set $S \in N$, j the number of regressors in set $J \in S$ and n the total number of regressors, i.e. $n = 3$ in our case. Moreover, R^2_S is the R^2 of the model that includes the regressors from set $S \subseteq N$, and $R^2_{S_J}$ the R^2 of the model that includes the regressors from set $J \in S$. Then, denoting R^2_{123} as the R^2 of the model that includes all three regressors, the Shapley weight of the ith regressor is defined as:

$$s_i = \frac{SH_i}{R^2_{123}}, \tag{20}$$

again for $i = 1, 2, 3$. We refer the reader to Appendix Section C for the manual working out of the above algorithm. By differentiating both the used measure of home advantage and the considered model estimation method for (18) in the Shapley values algorithm, we obtain four different instances of the referee home bias index.

4.4 Mediation Analysis

We break this section down into three parts. First, we show the mediation analysis model and describe how we estimate the mediation effect. Then, we explain the bootstrap algorithm we use to obtain the empirical distribution of the estimated mediation effect. Finally, we describe the three bootstrap confidence interval methods we consider to assess the significance of the estimated mediation effect.

4.4.1 Mediation Analysis Model

We investigate the correctness of the conclusion in Peeters and van Ours (2021) regarding the existence of an indirect effect of relative attendance on relative seasonal home advantage via referee home bias using the mediation analysis proposed by Baron and Kenny (1986), which, in our case, is expressed in its simple form[6] by the following three equations:

$$RSHA^z_{c,t} = \xi_{1;c} + \varrho_{1;t} + \psi RelAtt_{c,t} + \eta_{1;c,t}, \tag{21}$$

$$RHI^z_{c,t} = \xi_{2;c} + \varrho_{2;t} + \phi RelAtt_{c,t} + \eta_{2;c,t}, \tag{22}$$

$$RSHA^z_{c,t} = \xi_{3;c} + \varrho_{3;t} + \lambda RelAtt_{c,t} + \theta RHI_{c,t} + \eta_{3;c,t}, \tag{23}$$

again for $z = GD, PD$, with $\xi_{e;c}$ representing club-specific fixed effects and $\gamma_{e;t}$ comprising time-specific fixed effects in equation $e = 1, 2, 3$. Again, as before, we estimate these equations by means of POLS and FE. In this model, $RSHA^z_{c,t}$ is the dependent, $RelAtt_{c,t}$ the independent and $RHI_{c,t}$ the mediating variable. Furthermore, in the first equation, ψ quantifies the effect of $RelAtt_{c,t}$ on $RSHA^z_{c,t}$, whereas ϕ measures the effect of $RelAtt_{c,t}$ on $RHI_{c,t}$ in the second. Moreover, in the third equation, λ calibrates the effect of $RelAtt_{c,t}$ on $RSHA^z_{c,t}$ after adjusting for the effect of the mediating variable, and θ gauges the effect of $RHI_{c,t}$ on $RSHA^z_{c,t}$ after adjusting for the effect of the independent variable. Then, as showed by MacKinnon, Warsi and Dwyer (1995), the mediating effect is defined as $\phi\theta$, which is estimated by the product of the

[6]We show the simple form for the sake of clarity. However, we add the regressors from (15) as control variables in each equation of this simple model when estimating the mediation model.

Ordinary Least Squares (OLS) estimates $\hat{\phi}$ and $\hat{\theta}$ in (22) and (23), respectively. As mentioned in Section 2, past literature shows that the sampling distribution of $\hat{\phi}\hat{\theta}$ is non-normal, which should be accounted for when constructing a confidence interval around this point estimate to assess the magnitude of the mediation effect properly.

4.4.2 Bootstrap Algorithm

To obtain this empirical distribution, we consider a bootstrapping algorithm, which relies on generating random bootstrap samples from our original seasonal dataset with replacement. For panel data, there exist several methods for this re-sampling process. For example, one could re-sample either at the individual or time level, which preserves the covariance structure either in the time or individual dimension, respectively, which could be beneficial for the accuracy and reliability of the results if there exists a pronounced covariance structure in either dimension. If this is not the case, one could re-sample at the joint level, which allows for examining the overall sensitivity of the results to different combinations of individuals and time periods. In our case, as dependencies seem to exist neither between variables observed for each team across seasons nor between variables observed for each season across teams, we decide to re-sample at the joint level in our bootstrap algorithm. Unreported results from applied sensitivity analysis with respect to this choice show that the used re-sampling technique does not influence our results and their interpretations, which further amplifies the robustness of our research[7].

To create one such bootstrap sample, we randomly select team-year combinations from the original dataset until we have the same number of observations as in this dataset. In particular, each time after drawing a team-year combination from the original dataset, we place this observation back in the pool of observations to draw from, such that the eventual bootstrap sample may contain the same team-year combination multiple times. We obtain one bootstrap sample in each iteration of our bootstrap algorithm and use this sample to perform our mediation analysis on. By estimating the mediation effect in each bootstrap iteration, we eventually obtain the empirical distribution of the mediation effect, based on which we construct confidence intervals to assess its significance. In particular, to ensure robustness, we consider three confidence interval variations, being the percentile bootstrap confidence interval (Efron & Tibshirani, 1994), the bias-corrected bootstrap confidence interval (Efron, 1987), and the reduced bias-corrected confidence interval (Tibbe & Montoya, 2022), which' procedures are described briefly below.

4.4.3 Bootstrap Confidence Interval Methods

Starting with the percentile bootstrap confidence interval (PBCI), first draw P bootstrap samples with replacement from the original sample as described above. Then, let $\hat{\phi}_p$ and $\hat{\theta}_p$ be the estimates of ϕ and θ in the pth bootstrap sample, respectively, with $p = 1, ..., P$, and obtain $\hat{\phi}_p\hat{\theta}_p$ as the estimated mediation effect in the pth bootstrap sample. Order the obtained mediation effects from smallest to largest to retrieve the observed bootstrap sampling distribution, such that the $\left(\frac{\alpha}{2} \times 100\right)$th and $\left(1 - \frac{\alpha}{2} \times 100\right)$th percentiles of this distribution are the lower and upper limits of the $(1 - \alpha) \times 100\%$ PBCI, respectively.

[7]Results are available upon request.

Then, regarding the bias-corrected bootstrap confidence interval (BCBI), repeat the first steps of the PBCI to obtain $\hat{\phi}_p\hat{\theta}_p$ for each bootstrap sample p, and calculate the z-score corresponding to the proportion of bootstrap estimates that are below the original sample estimate $\hat{\phi}\hat{\theta}$ using the following formula:

$$\hat{z}_{adj} = \Phi^{-1}\left(\frac{1}{P}\sum_{p=1}^{P} \mathrm{I}\left[\hat{\phi}_p\hat{\theta}_p < \hat{\phi}\hat{\theta}\right]\right), \tag{24}$$

with $\mathrm{I}[A] = 1$ if A occurs and 0 otherwise, and Φ^{-1} being the inverse normal cumulative distribution function, such that $\hat{z}_{adj}$ can be interpreted as the z-score of the percentile of the observed sample mediation effect. Then, the $\left[\Phi\left(2\hat{z}_{adj} + z_{\alpha/2}\right) \times 100\right]$th and $\left[\Phi\left(2\hat{z}_{adj} + z_{1-\alpha/2}\right) \times 100\right]$th percentiles of the ordered observed bootstrap sampling distribution are the lower and upper limits of the $(1 - \alpha) \times 100\%$ BCBI, respectively, with Φ being the normal cumulative distribution function, α the significance level, $z_{\alpha/2}$ the z-score corresponding to the $\left(\frac{\alpha}{2} \times 100\right)$th percentile of the standard normal distribution, and $z_{1-\alpha/2}$ the z-score corresponding to the $\left(1 - \frac{\alpha}{2} \times 100\right)$th percentile of the standard normal distribution.

Finally, compared to the BCBI, the reduced bias-corrected bootstrap confidence interval (rBCBI) is obtained by adding the bias correction coefficient $\hat{z}_{adj}$ only once to the z-scores corresponding to the percentiles of the lower and upper confidence limits of the PBCI instead of twice, such that the $\left[\Phi\left(\hat{z}_{adj} + z_{\alpha/2}\right) \times 100\right]$th and $\left[\Phi\left(\hat{z}_{adj} + z_{1-\alpha/2}\right) \times 100\right]$th percentiles of the ordered observed bootstrap sampling distribution are the lower and upper limits of the $(1 - \alpha) \times 100\%$ rBCBI, respectively. The next section discusses the results from applying the presented methodology.

5 Results

In this section, we present our obtained results. We conduct our entire analysis in STATA 18. First, Section 5.1 shows the results of our season-level baseline quantitative analysis of seasonal home advantage. Then, Section 5.2 highlights the composition of the referee home bias index and the results from our mediation analysis. In all tables, we use an asterisk ($*$), dagger (†) and double dagger (‡) to indicate regression coefficient significance at 10%, 5% and 1%, respectively.

5.1 Results Year-Level Baseline Quantitative Analysis

We break our baseline quantitative analysis of relative seasonal home advantage down into three parts. In this section, we show the results of our season-level baseline analysis of relative seasonal home advantage, in which we investigate the determinants of home advantage for all teams in all four divisions in each year of our sample period. We refer the reader to Appendix Section A for the results of our analysis of home advantage at club and division level, in which we examine the average level of home advantage enjoyed by individual clubs and the average level of home advantage present in each division over our sample period, respectively.

As explained in Section 4.2, we investigate the determinants of relative seasonal home advantage by estimating the regression model shown in (15) using our sample from 2000 to 2018, from which the obtained results are shown in Table 2 below.

Table 2: Coefficient estimates of regression model (15); 2000-2018

	Relative Home Advantage GD				Relative Home Advantage PD			
Variables	POLS	FE	POLS	FE	POLS	FE	POLS	FE
Relative Attendance	0.051^{*} (0.027)	$0.140^{\dagger}$ (0.066)	$0.120^{\dagger}$ (0.053)	0.062 (0.099)	0.027 (0.042)	0.134 (0.097)	0.138^{*} (0.079)	0.057 (0.145)
Promoted	0.061^{*} (0.032)	0.058^{*} (0.034)	0.033 (0.040)	0.050 (0.042)	0.069 (0.047)	0.069 (0.049)	0.041 (0.058)	0.066 (0.062)
Relegated	-0.023 (0.043)	-0.020 (0.035)	0.014 (0.055)	-0.001 (0.049)	-0.012 (0.066)	-0.003 (0.061)	0.019 (0.084)	-0.008 (0.088)
Relative Wage			-0.065 (0.048)	-0.002 (0.064)			-0.132^{*} (0.069)	-0.043 (0.102)
R^2	0.004	0.006	0.006	0.003	0.002	0.003	0.004	0.002

From this table, we observe that relative attendance has a significant positive influence on relative seasonal home advantage in terms of goal difference in three out of the four considered cases, implying that a larger home crowd significantly benefits the home team with regards to scoring goals. Contrarily, relative attendance only significantly positively affects relative seasonal home advantage in terms of points in one of the four considered cases, suggesting that a larger home crowd does not necessarily increase the number of points the home team gains from their home match. Hence, the influence of relative attendance seasonal home advantage seems to depend on the respective utilised measure, which is in contrast to the harmonious results of Peeters and van Ours (2021), showing a significant positive effect of relative attendance on seasonal home advantage in terms of both measures in general over their considered sample. In addition, the aforementioned paper finds the state of being relegated to significantly negatively impact seasonal home advantage, which does not hold in any of the eight considered cases over our sample. Furthermore, as opposed to the aforementioned paper, we do observe rather stark differences in parameter estimates between the two used regression model specifications, from which we conclude that the inclusion of club fixed effects does matter for the magnitude of the coefficients of the considered variables. Despite these contrasts, the infrequencies of the significance of being promoted and of relative wage are in fact in accordance with the results of Peeters and van Ours (2021), and imply that neither newly promoted nor richer clubs necessarily exploit a higher degree of home advantage. Finally, the values of R^2 of the different models are all extremely low, suggesting that the included variables explain only a minor portion of the total variance in home advantage, indicating the need for model and estimation improvement.

5.2 Results Referee Home Bias Index and Mediation Analysis

As explained in Sections 4.3 and 4.4, we examine the correctness of the claim by Peeters and van Ours (2021) of the existence of an indirect effect of relative attendance on relative seasonal home advantage via referee home bias by means of a mediation analysis. In particular, we use a referee home bias index as mediating variable, which we obtain from applying the Shapley values algorithm to (18). In this regression model, we measure relative seasonal home advantage either in terms of home goal difference (GD) or home point difference (PD) and estimate the model either using POLS or FE, such that we obtain four different instances of the referee home bias index as a consequence. Table 3 summarises their compositions in terms of the absolute and relative weights awarded to each referee home bias measure, indicating each instance in the top row with the respective measure of home advantage and estimation method used in (18).

Table 3: Composition of referee home bias index in four different instances

	GD, POLS		GD, FE		PD, POLS		PD, FE	
	Abs. wght.	Rel. wght.	Abs. wght.	Rel. wght.	Abs. wght.	Rel. wght.	Abs. wght.	Rel. wght.
$FD_{c,t}$	0.0006	0.0333	0.0005	0.0273	0.0004	0.0306	0.0002	0.0134
$YD_{c,t}$	0.0015	0.0866	0.0011	0.0671	.0005	0.0365	0.0004	0.0292
$RD_{c,t}$	0.0157	0.8800	0.0152	0.9057	.0116	0.9329	0.0123	0.9574

In this table, we first observe that each referee home bias measure gets awarded a positive weight, suggesting that all three measures contribute positively to relative seasonal home advantage. In particular, we observe that the red card differential weighs substantially heavier in each of the four instances of the index, with a relative weight ranging between 0.88 and 0.96. Consequently, for all four instances, the foul and yellow card differential are both assigned far lower weights, of which the latter is slightly more prominent than the former. These findings make sense from the fact that a difference in awarded red cards is most influential on game outcomes as one of the teams simply has fewer players on the pitch than the other, making it harder to defend, score goals and win the game, granting the team with more players a substantial advantage as a consequence. The effect of a difference in awarded yellow cards is more subtle, as it does not immediately result in a game-changing advantage with one team outnumbering the other, but it could still very well be of influence on match outcomes. For example, a defender already on a yellow card must be more cautious in defending the opposing striker to avoid receiving a second yellow card leading to a dismissal, which limits his ability to stop the opponent from scoring goals. Finally, the difference in fouls is expected to be least influential on match outcomes, as more fouls called against one of the two teams generally does not lead to a substantial advantage for the other, as was the case for yellow and especially red cards. However, the foul differential could still be of influence on match outcomes through arising chances from free-kicks consequencing from a committed foul in potentially dangerous spots around the opposing goal, possibly leading to more goals for the team that gets more fouls called in favour of them.

Then, as explained in Section 4.4, in order to construct proper confidence intervals around the point estimate of the mediation effect, we consider a bootstrap algorithm to obtain an observed bootstrap sampling distribution of the indirect effect. To ensure statistical validity and reliability of regression coefficient estimates, we use the bootstrap algorithm to conduct the entire mediation analysis, consisting of three main steps. First, we establish the relationship between the dependent variable and the independent variable by regressing relative seasonal home advantage on relative attendance and a set of control variables, the results of which are shown in Table 4 below. This table can be interpreted as a bootstrapped version of Table 2.

With respect to Table 2, apart from the change in the significance of relative attendance in the second model, the results and their interpretations are similar to those of the aforementioned baseline table. However, we do observe substantial increases in the R^2 of each considered model, which implies that our initial results may have been heavily affected by the presence of potential outliers in the model, as bootstrapping is a re-sampling technique known for mitigating the

Table 4: Results of the first step of the mediation analysis procedure; 10000 bootstrap iterations

	Relative Home Advantage GD				Relative Home Advantage PD			
Variables	POLS	FE	POLS	FE	POLS	FE	POLS	FE
Relative Attendance	0.051^{*} (0.027)	0.143 (0.092)	$0.120^{\dagger}$ (0.053)	0.062 (0.135)	0.027 (0.042)	0.138 (0.141)	0.138^{*} (0.079)	0.057 (0.203)
Promoted	0.061^{*} (0.032)	0.060 (0.047)	0.034 (0.040)	0.052 (0.058)	0.070 (0.047)	0.071 (0.069)	0.042 (0.058)	0.067 (0.087)
Relegated	−0.023 (0.043)	−0.021 (0.055)	0.014 (0.055)	0.000 (0.073)	−0.012 (0.066)	−0.006 (0.091)	0.019 (0.084)	−0.005 (0.125)
Relative Wage			−0.065 (0.047)	−0.001 (0.088)			-0.132^{*} (0.069)	−0.043 (0.138)
R^2	0.008	0.010	0.012	0.010	0.005	0.007	0.011	0.009

impact of such influential points, stabilizing the model and providing more reliable estimates as a consequence. Most importantly, from Table 4, we conclude that in three of the eight considered cases, relative attendance has a significant effect on relative seasonal home advantage, which, according to Baron and Kenny (1986), provides profound impetus to explore the potential mediating effect of referee bias in these cases. Furthermore, Shrout and Bolger (2002) argue that even if the relationship is not significant but there does exist a compelling theoretical background regarding the relation between the dependent and independent variable, we can still proceed to the second step of the mediation analysis. Therefore, as Peeters and van Ours (2021) provide sufficient theoretical and empirical evidence of the existence of a significant relation between relative seasonal home advantage and relative attendance, we advance with the second step of our mediation analysis for all considered cases.

In the second step, we assess the extent to which the dependent variable influences the mediating variable by regressing the referee home bias index on relative attendance and the same control variables as in step 1, the results of which are shown in Table 5 below. We specify the used measure of relative seasonal home advantage in (22) in the top row, with GD referring to home goal difference and PD to home point difference, respectively.

Table 5: Results of the second step of the mediation analysis procedure; 10000 bootstrap iterations

	Referee Home Bias, GD				Refeee Home Bias, PD			
Variables	POLS	FE	POLS	FE	POLS	FE	POLS	FE
Relative Attendance	$1.163^{\ddagger}$ (0.151)	$1.466^{\dagger}$ (0.600)	$0.785^{\ddagger}$ (0.278)	1.142 (0.812)	$0.921^{\ddagger}$ (0.138)	$1.034^{\dagger}$ (0.505)	$0.641^{\dagger}$ (0.252)	0.835 (0.687)
Promoted	$0.469^{\ddagger}$ (0.175)	0.375 (0.239)	0.320 (0.216)	0.234 (0.297)	$0.423^{\ddagger}$ (0.157)	0.345^{*} (0.208)	0.303 (0.195)	0.245 (0.259)
Relegated	−0.296 (0.229)	−0.292 (0.297)	−0.323 (0.293)	−0.180 (0.378)	−0.169 (0.211)	−0.135 (0.267)	−0.189 (0.272)	−0.062 (0.343)
Relative Wage			0.335 (0.258)	−0.285 (0.436)			0.265 (0.235)	−0.255 (0.372)
R^2	0.045	0.041	0.046	0.040	0.037	0.032	0.039	0.034

From this table, we conclude that there exists a strongly significant relation between referee home bias and relative attendance in general. In particular, except for the fourth and eighth model, the coefficient corresponding to relative attendance is significant for all considered cases at 5% at least. From this, we conclude that there exists a potential for mediation, which further consolidates the need for further exploration.

In the third step, we aim to determine the extent to which the effect of the independent variable on the dependent variable flows through the mediating variable. For this, we start by regressing relative seasonal home advantage on both relative attendance and the referee home bias index, in addition to the conventional control variables.

Table 6: Results of the third step of the mediation analysis procedure; 10000 bootstrap iterations

	Relative Home Advantage GD				Relative Home Advantage PD			
Variables	POLS	FE	POLS	FE	POLS	FE	POLS	FE
Referee Home Bias	$0.016^{\ddagger}$	$0.019^{\ddagger}$	$0.014^{\dagger}$	0.015^{*}	$0.025^{\ddagger}$	$0.037^{\ddagger}$	$0.025^{\ddagger}$	$0.035^{\dagger}$
	(0.005)	(0.007)	(0.006)	(0.009)	(0.008)	(0.013)	(0.009)	(0.016)
Relative Attendance	0.033	0.115	$0.109^{\dagger}$	0.044	0.004	0.100	0.122	0.028
	(0.028)	(0.095)	(0.053)	(0.138)	(0.043)	(0.143)	(0.080)	(0.206)
Promoted	0.054^{*}	0.053	0.029	0.049	0.060	0.058	0.035	0.059
	(0.032)	(0.047)	(0.040)	(0.058)	(0.047)	(0.069)	(0.058)	(0.086)
Relegated	−0.019	−0.016	0.019	0.002	−0.008	−0.001	0.024	−0.003
	(0.043)	(0.054)	(0.055)	(0.073)	(0.065)	(0.089)	(0.084)	(0.123)
Relative Wage			−0.070	0.004			$-0.139^{\dagger}$	−0.034
			(0.048)	(0.089)			(0.069)	(0.139)
R^2	0.016	0.019	0.018	0.016	0.012	0.018	0.018	0.019

The results of this regression displayed in Table 6 are rather convincing. Namely, first, the coefficient of referee home bias index is significant at 10% in one case, at 5% in two cases, and at 1% in all other considered cases. Concurrently, the coefficient of relative attendance is significant at 5% in the third case, but insignificant in all other models. From this, we first conclude that there is partial mediation in the third model with relative attendance still being significant, implying that the referee home bias serves as an intermediate variable that only partially explains the influence of relative attendance on relative seasonal home advantage. For all other models, however, the effect of relative attendance disappears when including referee home bias in the model, such that there appears to be complete mediation in these models, meaning that relative attendance influences relative seasonal home advantage solely through the mediating referee home bias. Apart from relative attendance no longer being significant in most models, the coefficients of the control variables after adding referee home bias to the models stay fairly similar to those excluding this variable as displayed in Table 4, such that their interpretations again remain as described below baseline Table 2. However, with respect to Table 4, we do observe substantial increases in the R^2 of each considered case, with percentage growth ranging from 50% to 157% , from which we conclude that referee home bias is an essential determinant of relative seasonal home advantage to incorporate in the model.

Then, as explained in Section 4.4, to assess the significance of the mediating effect that referee bias has on relative seasonal home advantage, we construct three bootstrap confidence intervals around the estimated effect in each of the eight considered models, which' results are shown in Table 7 below. In this table, ME AP shows the mediation effect obtained from taking the average of the mediation effects computed in each bootstrap iteration, whereas ME PA reflects the mediation effect obtained from multiplying the bootstrap average estimates of ϕ and θ, which are the coefficients of relative attendance and referee home bias in Tables 5 and 6, respectively. The used measure of seasonal home advantage (GD or PD), estimation method (POLS or FE), and exclusion or inclusion of relative wage in the model (RW) are indicated in the top row.

Table 7: Magnitude of mediation effects ($\phi\theta$) and their statistical significance

	GD, POLS	GD, FE	GD, POLS, RW	GD, FE, RW
ME AP[1]	0.018	0.028	0.011	0.018
ME PA[2]	1.163×0.016	1.466×0.019	0.785×0.014	1.142×0.015
PBCI	[0.007; 0.031]	[0.010; 0.052]	[0.001; 0.025]	[0.000;0.045]
BCBI	[0.008; 0.031]	[0.012; 0.055]	[0.002; 0.027]	[0.001; 0.048]
rBCBI	[0.008; 0.031]	[0.011; 0.054]	[0.002; 0.026]	[0.001; 0.046]
	PD, POLS	PD, FE	PD, POLS, RW	PD, FE, RW
ME AP[1]	0.023	0.038	0.016	0.029
ME PA[2]	0.921×0.025	1.034×0.037	0.641×0.025	0.835×0.035
PBCI	[0.008; 0.040]	[0.012; 0.073]	[0.002; 0.037]	[-0.001; 0.075]
BCBI	[0.009; 0.041]	[0.014; 0.076]	[0.003; 0.040]	[-0.000; 0.077]
rBCBI	[0.009; 0.041]	[0.013; 0.075]	[0.002; 0.038]	[-0.001; 0.076]

[1] Mediation effect from taking average of products: $\frac{1}{P}\sum_{p=1}^{P}\hat{\phi}_p\hat{\theta}_p$.

[2] Mediation effect from taking product of averages: $\left(\frac{1}{P}\sum_{p=1}^{P}\hat{\phi}_p\right) \times \left(\frac{1}{P}\sum_{p=1}^{P}\hat{\theta}_p\right)$.

Comparing the magnitudes of the mediation effect across the eight models, we observe fairly similar values ranging from 0.011 to 0.029. Furthermore, we conclude that, apart from the eighth model, the mediation effect of referee home bias is significantly greater than zero for all three confidence interval methods for each considered model at 5% significance level. From this, we conclude that our results strongly support the proposition made in Peeters and van Ours (2021). That is, the effect of relative attendance on relative seasonal home advantage in the professional divisions of English football is likely to be mediated completely through referee home bias.

6 Conclusion

In this paper, in order to assess the correctness of the proposition made in Peeters and van Ours (2021), we investigate the extent to which the effect of relative home attendance on relative seasonal home advantage in English professional football is mediated by referee home bias using the mediation analysis procedure proposed by Baron and Kenny (1986), based on match data from the four English professional football divisions from 2000 to 2018. To proxy for referee home bias, we construct the so-called referee home bias index as the weighted sum of three referee home bias measures, being the difference in the number of fouls, yellow cards, and red cards awarded to the home and away team, respectively. The weights in this index reflect the relative importance of each measure with respect to seasonal home advantage, and are determined based on a robust Shapley values approach to account for the correlation between these metrics. From this, we observe that the difference in the number of red cards awarded to the home and away team is the major component in the referee home bias index, which makes sense from the fact that the red card differential is most influential on match outcomes among the three referee home bias measures. In order to ensure the accuracy and reliability of the regression coefficient estimates in our mediation analysis, we consider a bootstrap algorithm which mitigates the influence of potential outliers in the data. From this, most importantly, we find compelling evidence that referee home bias completely mediates the effect of relative attendance on relative seasonal home advantage. We verify the statistical significance of this mediation effect by means of the percentile, bias-corrected, and reduced bias-corrected bootstrap confidence interval methods, from which we conclude that our results strongly support the proposition made in Peeters and van Ours (2021). That is, the effect of relative attendance on relative seasonal home advantage in English professional football is likely to be mediated completely through referee home bias.

Our findings have several important practical implications. To start with, the substantial influence referee bias appears to have on relative seasonal home advantage highlights the need for referee training programs that focus solely on mitigating such biases, such that consistent decision-making is enhanced and the impact of referee home bias on match outcomes is minimised. Furthermore, football associations could attempt to ensure a more level playing field for both the home and away team in all matches by assigning referees that are least prone to home bias to those matches in which home bias is expected to be the highest, which could be computed based on past literature regarding, for example, the influence of the place and status of the home team in the league, the number of home fans expected to attend the game, and the noise the home crowd is expected to produce. Finally, given the recent discussion regarding the use of Video Assistant Referee (VAR) in professional football, our research findings convincingly advocate for the current application of VAR. Namely, among other things, the VAR currently monitors decision-making with regards to red cards, and as our findings demonstrate that referee home bias appears to be mainly evident in the difference in the number of red cards awarded to the home and away team, the VAR is expected to mitigate referee home bias substantially. In fact, one could argue that the use of VAR should be extended to even more referee decisions, for example regarding fouls and yellow cards, which could further enhance the fairness of the sport.

Due to limited data availability, we are confined to the three aforementioned measures of referee home bias with respect to disciplinary sanctions. However, a fruitful avenue for future research could be to design other measures, for example in terms of the difference in the number of given penalties to the home and away team, or the difference in awarded extra time in close games in which the home team is leading on the one hand and trailing on the other. Furthermore, exploring which referee characteristics can potentially predict the level of referee home bias proneness and what match conditions or team behaviours contribute to this could help football associations further optimise their referee assignment procedures. In addition, one could even attempt to predict the outcome or statistics of certain past matches based on these factors by means of scenario analysis, and possibly extend this model to predict future matches and their statistics. Moreover, by clustering referees based on such characteristics and their match decision-making, we are interested to assess the extent to which home advantage is related to the type of referee home teams get assigned more often, and to examine which teams have enjoyed the most home advantage over the years as a consequence. Finally, improving on the linear regression model used in Peeters and van Ours (2021) may allow future research to investigate the relations between relative attendance, referee home bias and relative seasonal home advantage on an even deeper level. For instance, one could test for non-linearities and design their models based on those results, for example creating a non-linear regression model or a threshold regression model, which might better explain the nature of seasonal home advantage.

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A Division- and Club-Level Analysis

In this appendix section, we present the results of our analysis regarding division- and club-level seasonal home advantage, performed in a similar fashion as Peeters and van Ours (2021). Table 8 below shows the magnitude of average absolute seasonal home advantage per division over the observation period in terms of both measures of home advantage, from which we conclude that the level of average divisional home advantage is positively related with the rank of the respective division. Although the pattern in the magnitudes of average divisional home advantage is in concordance with Peeters and van Ours (2021), the magnitudes are substantially lower for all divisions for both measures. As we only consider the last eighteen years of the full sample used in Peeters and van Ours (2021), which runs from 1973 to 2018, this observation suggests that average divisional seasonal home advantage is declining over time. In addition to the aggregated values presented in the table above, Figure 3 below shows the development of average absolute seasonal home advantage per division over our sample period.

Table 8: Average absolute seasonal home advantage per division; 2000-2018

Average Home Advantage	Premier League	Championship	League One	League Two
Home Point Difference	0.554	0.464	0.444	0.396
Home Goal Difference	0.397	0.332	0.317	0.300

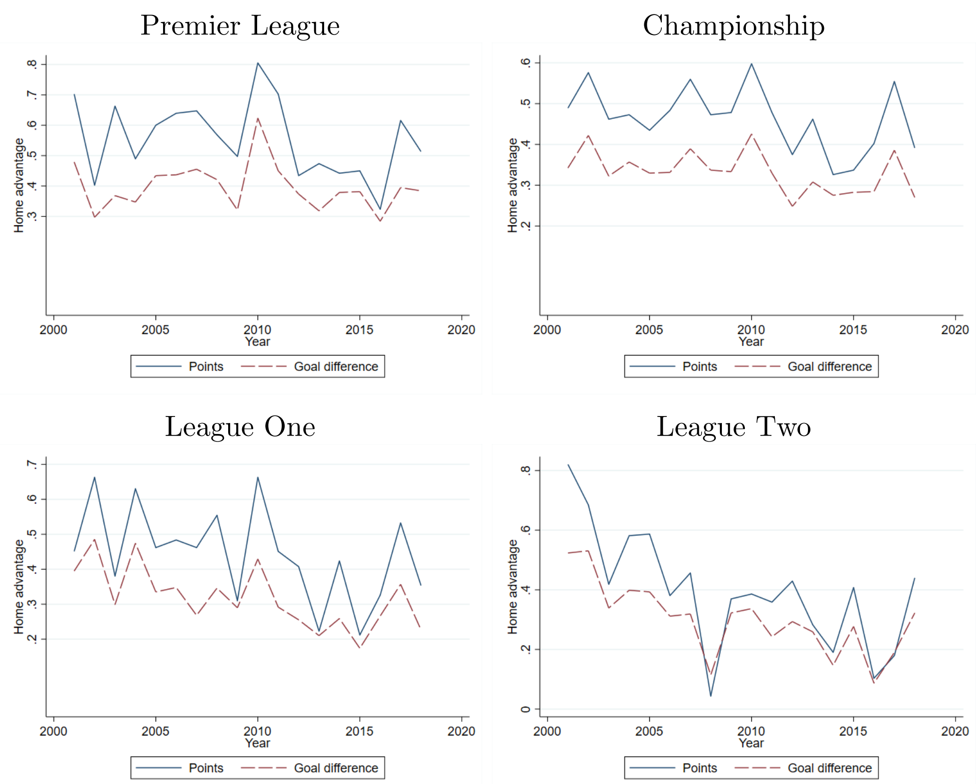

Figure 3: Development of average absolute seasonal home advantage per division over time; 2000-2018

From this, we observe that there exists a clear positive average seasonal home advantage in each division, strongly fluctuating from year to year and slightly declining over time. In general, the absolute size of home advantage seems to be larger when measured in points rather than goals, but this is simply due to the nature of the respective numbers, making such differences to be expected.

In addition to the division-level analysis, Table 9 shown on the next page contains an overview of the magnitude of average absolute seasonal home advantage on club level measured in terms of home point (PD) and goal difference (GD), respectively, along with the number of seasons each club played in each division, for the 68 English professional football clubs that appeared in one of the top four divisions every season over the observation period. From this, we observe that each club has had a positive average absolute seasonal home advantage over these 18 years for both measures, with the exception of Wigan Athletic FC having a slight average home disadvantage in terms of goal difference. Furthermore, the degree of advantage differs substantially amongst clubs, as for example Norwich City and Gillingham FC enjoy large home point and goal advantages of over 0.9 and 0.6, respectively, whereas Wigan Athletic FC and Wycombe Wanderers attain home point and goal advantages of below 0.02 and 0.05.

With respect to the correlation between the two used measures of home advantage, Figure 4 below shows the spread in the two metrics, from which we conclude a strong but non-perfect correlation.

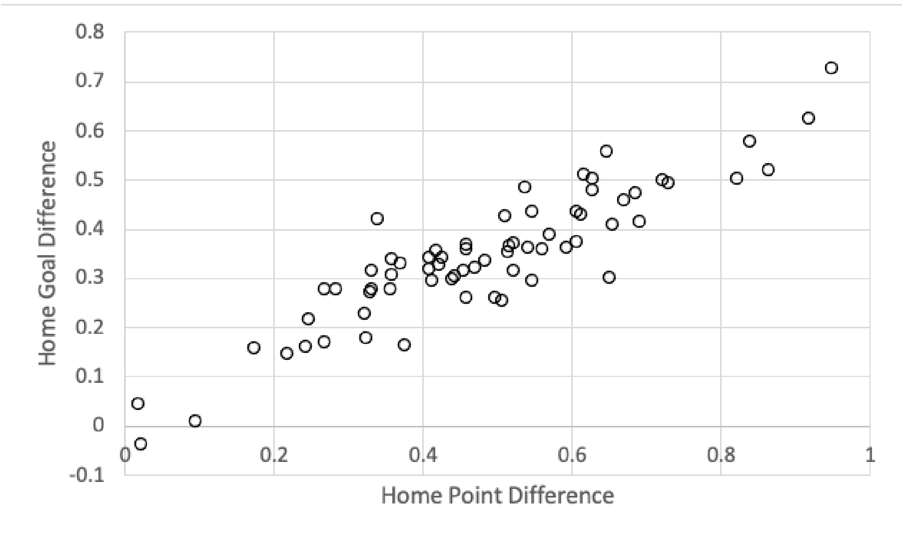

Figure 4: Scatter plot of home point and home goal difference; 2000-2018

Table 9: Overview average absolute seasonal home advantage per club; 2000-2018

Club	Division				Home Advantage		Club	Division				Home Advantage	
	1	2	3	4	PD	GD		1	2	3	4	PD	GD
Arsenal	18	0	0	0	0.625	0.503	Middlesbrough	10	8	0	0	0.650	0.301
Aston Villa	16	2	0	0	0.330	0.281	Millwall	0	11	7	0	0.441	0.306
Barnsley	0	12	6	0	0.591	0.363	Newcastle United	16	2	0	0	0.838	0.579
Birmingham City	7	11	0	0	0.456	0.262	Northampton Town	0	0	8	10	0.437	0.301
Blackburn Rovers	11	6	1	0	0.425	0.345	Norwich City	5	12	1	0	0.947	0.729
Blackpool FC	1	7	8	2	0.417	0.358	Nottingham Forest	0	15	3	0	0.654	0.411
Bolton Wanderers	11	6	1	0	0.605	0.375	Notts County	0	0	9	9	0.369	0.331
Bournemouth AFC	3	2	10	3	0.495	0.262	Oldham Athletic	0	0	18	0	0.266	0.280
Bradford City	1	3	8	6	0.267	0.170	Peterborough United	0	3	12	3	0.355	0.281
Brentford	0	4	12	2	0.512	0.356	Plymouth Argyle	0	6	4	8	0.481	0.336
Brighton and Hove Albion	1	9	7	1	0.330	0.317	Port Vale	0	0	12	6	0.668	0.460
Bristol City	0	9	9	0	0.568	0.389	Portsmouth FC	7	5	2	4	0.646	0.558
Burnley	4	14	0	0	0.610	0.430	Preston North End	0	14	4	0	0.720	0.502
Bury FC	0	0	7	11	0.242	0.163	Queens Park Rangers	3	12	3	0	0.863	0.522
Cardiff City	1	14	2	1	0.546	0.296	Reading	3	13	2	0	0.458	0.360
Charlton Athletic	7	6	5	0	0.216	0.147	Rochdale AFC	0	0	6	12	0.357	0.309
Chelsea	18	0	0	0	0.357	0.340	Rotherham United	0	7	5	6	0.407	0.344
Chesterfield	0	0	10	8	0.458	0.369	Scunthorpe United	0	3	9	6	0.514	0.367
Colchester United	0	2	14	2	0.410	0.295	Sheffield United FC	1	11	6	0	0.452	0.317
Coventry City	1	11	5	1	0.506	0.256	Sheffield Wednesday FC	0	14	4	0	0.172	0.160
Crewe Alexandra	0	5	8	5	0.469	0.324	Southampton	11	5	2	0	0.559	0.362
Crystal Palace FC	6	12	0	0	0.246	0.219	Southend United	0	1	7	10	0.406	0.320
Derby County	3	15	0	0	0.615	0.512	Stoke City	10	6	2	0	0.685	0.473
Everton FC	18	0	0	0	0.727	0.494	Sunderland AFC	14	4	0	0	0.320	0.228
Fulham	13	5	0	0	0.691	0.417	Swansea City	7	3	4	4	0.606	0.437
Gillingham FC	0	5	9	4	0.916	0.627	Swindon Town	0	0	15	3	0.540	0.362
Huddersfield Town	1	6	10	1	0.509	0.427	Tottenham Hotspur	18	0	0	0	0.820	0.503
Hull City	5	8	1	4	0.545	0.438	Walsall FC	0	3	14	1	0.375	0.165
Ipswich Town	2	16	0	0	0.520	0.371	Watford	4	14	0	0	0.327	0.273
Leeds United	4	11	3	0	0.323	0.179	West Bromwich Albion	12	6	0	0	0.282	0.278
Leicester City	7	10	1	0	0.520	0.316	West Ham United	15	3	0	0	0.419	0.329
Liverpool	18	0	0	0	0.625	0.481	Wigan Athletic	8	5	5	0	0.021	-0.035
Manchester City	17	1	0	0	0.536	0.487	Wolverhampton Wanderers	4	13	1	0	0.093	0.012
Manchester United	18	0	0	0	0.338	0.423	Wycombe Wanderers FC	0	0	6	12	0.015	0.046

B Sensitivity Analysis on Used Proxy for Referee Home Bias

In this section, we investigate the sensitivity of our results with respect to the method of constructing the proxy for referee home bias. In particular, as an alternative to the referee home bias index, which, as explained in Section 4.3, we establish by means of the Shapley values algorithm, we investigate the extent to which our results change if we approximate referee home bias with the first principal component resulting from applying scaled principal component analysis (sPCA) to the three measures of referee home bias. We start by motivating the use of sPCA, after which we provide the main formulas used to perform this method. Finally, we show that the results and their implications are not affected by the used proxy for referee home bias, which adds to the robustness of our research.

Huang, Jiang, Li, Tong and Zhou (2022) propose sPCA as an improved version of principal component analysis (PCA), which is a well-known dimension reduction technique introduced by Pearson (1901). Primarily, PCA is useful in preventing in-sample overfitting and addressing multicollinearity, by reducing a large number of regressors into so-called principal components, being orthogonal linear combinations of the original variables that capture the maximum variance in the data. However, this dimension reduction method completely ignores the explanatory power of the regressors with regards to the dependent variable in constructing these components, such that the principal components may not be optimal in providing insights into the relationships with the target variable as a consequence. To tackle this issue, in essence, sPCA scales each independent variable with the absolute value of its estimated effect on the dependent variable, such that higher weights are assigned to those regressors with stronger explanatory power.

In our case, as shown in Section 3, the foul and yellow card differential are moderately correlated, whereas the correlations of these measures with the red card differential are substantially lower, such that the first principal component is likely to mainly reflect the foul and yellow card differentials as a consequence of this correlation structure. However, unreported results[8] show that these have substantially lower explanatory power with respect to relative seasonal home advantage than the red card differential, such that the first principal component does not optimally reflect the relation between referee home bias and relative seasonal home advantage. As explained above, scaled principal component analysis addresses this deficiency of PCA by letting the dependent variable guide the direction of dimension reduction, which motivates considering this method as a reasonable alternative to the Shapley values approach to proxy for referee home bias. We explain the procedure of sPCA in more detail below.

First, we define the standardized disciplinary sanctions differential for team c in season t as:

$$dsD_{c,t}^{std} = \frac{dsD_{c,t} - \mu_{ds}}{\sigma_{ds}}, \tag{25}$$

for disciplinary sanctions type $ds = F, Y, R$, corresponding to fouls, yellow cards and red cards, respectively. Then, for team c in season t, we regress relative seasonal home advantage on each

[8] Results are available on request.

standardized disciplinary sanctions differential:

$$RSHA^{z}_{c,t} = o_c + \iota_t + \kappa_{ds} dsD^{std}_{c,t} + \upsilon_{c,t}, \tag{26}$$

for $z = GD, PD$ corresponding to the case of measuring seasonal home advantage either in terms of home goal difference (GD) or home point difference (PD), respectively, again with $ds = F, Y, R$ corresponding to fouls, yellow cards and red cards, respectively, and with o_c representing club-specific fixed effects and ι_t comprising time-specific fixed effects. As before, we estimate this regression model by means of POLS and FE. We then scale each disciplinary sanctions differential by the absolute value of κ_{ds} and obtain the scaled referee home bias measures, which' covariance matrix we then use to apply PCA on. As explaining principal component analysis is not the aim of this section, we refer the reader to Huang et al. (2022) for a general description of this procedure. Below, we report the results regarding sPCA below.

First, in Table 10 below, we show the composition of the first principal component that results from applying normal PCA to the three standardized measures of referee home bias.

Table 10: Composition of first principal component

	Absolute weight	Relative weight
Foul Diff.	0.634	0.376
Yellow Card Diff.	0.677	0.402
Red Card Diff.	0.373	0.221

From this, we observe that PCA indeed assigns higher weights to the foul and yellow card differential with the aim of maximizing the proportion of variance explained, resulting in a proportion of total variance explained of 52.9%. However, at the same time, we observe that PCA attributes a substantially lower weight to the red card differential, which shows the method's ignorance regarding the substantial explanatory power of this measure with respect to relative seasonal home advantage. Below, Tables 11 and 12 respectively summarise the compositions of and the proportions of variance explained by the first principal components that follow from applying sPCA. The used measure of relative seasonal home advantage in and the considered estimation method of (26) are specified in the top row of both tables.

Table 11: Compositions of first scaled principal component

	GD, POLS		GD, FE		PD, POLS		PD, FE	
	Abs. w.	Rel. w.	Abs. w.	Rel. w.	Abs. w.	Rel. w.	Abs. w.	Rel. w.
Foul Diff.	0.006	0.005	0.008	0.007	0.009	0.008	0.005	0.005
Yellow Card Diff.	0.091	0.083	0.079	0.073	0.056	0.053	0.053	0.050
Red Card Diff.	0.996	0.911	0.997	0.920	0.998	0.939	0.999	0.945

Table 12: Proportions of variance explained by first scaled principal component

	GD, POLS	GD, FE	PD, POLS	PD, FE
Proportion Var. Expl.	0.894	0.911	0.946	0.954

From these, we observe that sPCA performs exactly as portrayed in Huang et al. (2022). Namely, compared to normal PCA, sPCA assigns a far higher weight to the red card differential compared to normal PCA, appreciating the substantial explanatory power this measure has with regards to relative seasonal home advantage. Furthermore, we observe a stark improvement in the proportion of variance explained by the first scaled principal, attaining values of over 89% for all four instances.

Comparing the relative weights of the first scaled principal component with those of the referee home bias index as shown in Table 3, we observe great harmony between the two proxies for referee home bias with regards to the order and level of importance of the three referee home bias measures. Furthermore, regarding the three steps of the mediation analysis, unreported results[9] show that using the first scaled principal component as proxy for referee home bias produces identical outcomes in terms of coefficient significance and interpretation as those shown in Tables 4, 5 and 6 , which use the referee home bias index as proxy. In Table 13, we report the estimated magnitudes of the mediating effects and the confidence intervals around them for all eight models when using the first scaled principal component as proxy for referee bias. The used measure of seasonal home advantage (GD or PD), estimation method (POLS or FE), and exclusion or inclusion of relative wage in the model (RW) are indicated in the top row.

Table 13: Magnitude of mediation effects ($\phi\theta$) and their statistical significance

	GD, POLS	GD, FE	GD, POLS, RW	GD, FE, RW
ME AP[1]	0.021	0.020	0.019	0.018
ME PA[2]	0.023×0.960	0.021×0.965	0.019×0.997	0.019×0.964
PBCI	[0.012; 0.034]	[0.003; 0.042]	[0.005; 0.037]	[-0.006;0.049]
BCBI	[0.012; 0.035]	[0.004; 0.044]	[0.006; 0.038]	[-0.006; 0.049]
rBCBI	[0.012; 0.034]	[0.004; 0.043]	[0.006; 0.037]	[-0.006; 0.049]
	PD, POLS	**PD, FE**	**PD, POLS, RW**	**PD, FE, RW**
ME AP[1]	0.029	0.028	0.026	0.025
ME PA[2]	0.029×0.991	0.028×0.986	0.025×1.035	0.025×0.979
PBCI	[0.014; 0.046]	[0.004; 0.060]	[0.007; 0.051]	[-0.009; 0.070]
BCBI	[0.015; 0.047]	[0.006; 0.063]	[0.008; 0.054]	[-0.008; 0.071]
rBCBI	[0.014; 0.047]	[0.005; 0.061]	[0.007; 0.052]	[-0.009; 0.070]

[1] Mediation effect from taking average of products: $\frac{1}{P}\sum_{p=1}^{P}\hat{\phi}_p\hat{\theta}_p$.

[2] Mediation effect from taking product of averages: $\left(\frac{1}{P}\sum_{p=1}^{P}\hat{\phi}_p\right) \times \left(\frac{1}{P}\sum_{p=1}^{P}\hat{\theta}_p\right)$.

Again, we observe great harmony with respect to the results in Table 7 that follow from performing the mediation analysis with the referee home bias index as proxy for referee home bias. In particular, although the coefficient estimates of relative attendance ($\frac{1}{P}\sum_{p=1}^{P}\hat{\phi}_p$) and the proxy for referee home bias ($\frac{1}{P}\sum_{p=1}^{P}\hat{\theta}_p$) differ across the two tables due to the scaling applied in sPCA, the resulting magnitudes of the mediation effect computed as the product of these estimated coefficients are rather alike for both tables for all considered cases. In addition, both Table 7 and 13 show that, apart from the eighth model, all three bootstrap confidence interval methods demonstrate the significance at 5% of the mediation effect for each considered model. Hence, our conclusion regarding the effect of relative attendance on relative seasonal home advantage

[9]Results are available upon request.

being likely to be mediated completely through referee home bias is robust to the used measure of referee home bias. That is, irrespective or the used proxy for referee home bias, our results strongly support the proposition made in Peeters and van Ours (2021) of the existence of an indirect effect of relative attendance on relative seasonal home advantage via referee home bias in English professional football.

C Manual Working Out Shapley Values Algorithm

To clarify the Shapley values algorithm explained in Section 4.3, we show how to manually work out (19) and (20) in this section. For this, we first define the set N which contains all possible combinations of the three regressors we consider. That is, $N = \{1, 2, 3, 12, 13, 23, 123\}$, with each element representing which referee home bias measures are included as regressors in the regression model shown in (18), with 1 corresponding to the foul differential $FD_{c,t}$, 2 to the yellow card differential $YD_{c,t}$, and 3 to the red card differential $RD_{c,t}$, respectively. Then, again ignoring indices specifying the measure of home advantage and estimation method used in (18) for the sake of clarity, the contribution of each regressor to relative seasonal home advantage are computed as follows:

$$SH_1 = \frac{1}{3}R_1^2 + \frac{1}{6}(R_{12}^2 - R_2^2) + \frac{1}{6}(R_{13}^2 - R_3^2) + \frac{1}{3}(R_{123}^2 - R_{23}^2), \tag{27}$$

$$SH_2 = \frac{1}{3}R_2^2 + \frac{1}{6}(R_{12}^2 - R_1^2) + \frac{1}{6}(R_{23}^2 - R_3^2) + \frac{1}{3}(R_{123}^2 - R_{13}^2), \tag{28}$$

$$SH_3 = \frac{1}{3}R_3^2 + \frac{1}{6}(R_{13}^2 - R_1^2) + \frac{1}{6}(R_{23}^2 - R_2^2) + \frac{1}{3}(R_{123}^2 - R_{12}^2), \tag{29}$$

with 1 corresponding to the foul differential $FD_{c,t}$, 2 to the yellow card differential $YD_{c,t}$, and 3 to the red card differential $RD_{c,t}$, respectively. From these, the Shapley weight of each regressor can be computed as:

$$s_1 = \frac{SH_1}{R_{123}^2}, \tag{30}$$

$$s_2 = \frac{SH_2}{R_{123}^2}, \tag{31}$$

$$s_3 = \frac{SH_3}{R_{123}^2}, \tag{32}$$

again with 1 corresponding to the foul differential $FD_{c,t}$, 2 to the yellow card differential $YD_{c,t}$, and 3 to the red card differential $RD_{c,t}$, respectively. As mentioned in Section 4.3, we differentiate the used measure of seasonal home advantage, i.e. either GD or PD, and the employed model estimation method, i.e. POLS or FE, in (18), such that we obtain four instances of each equation above. Hence, we also obtain four instances of the referee home bias index from the Shapley values algorithm as a consequence.