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Comparing Portfolio Performance: Active strategies versus Naive Diversification in Normal and Large-Dimensional Settings with Factor Models and DCC-NL Estimation

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Abstract

Building upon influential research by Kirby & Ostdiek (2012), this paper explores the mean-variance optimization, timing strategies and naive diversification in portfolio management. Our analysis suggests incorporating timing strategies, especially reward-to-risk timing (RRT) strategies with factor models, can greatly improve the returns compared to naive diversification, while also accounting for transaction costs. Based on the empirical data, mean-variance optimization performs better than naive diversification in the absence of transaction costs. However, for reducing turnover and improving performance, volatility timing (VT) and RRT strategies show promise, particularly with specific datasets. Additionally, this paper investigates the use of a dynamic conditional correlation model with non-linear shrinkage (DCC-NL) and an approximate factor model (AFM) for covariance estimation. Our analysis demonstrates that incorporating these techniques improves the performance of the minimum variance strategy, outperforming naive diversification. Furthermore, it highlights potential future research directions, such as exploring additional datasets, alternative risk factors or models, tuning parameters, conducting robustness tests, and evaluating covariance estimation techniques in various market contexts.

1 Introduction

Modern portfolio theory has relied heavily on mean-variance optimization as a fundamental building block, making it a crucial aspect of academic research in recent years. However, a classic literature DeMiguel et al. (2009) raises doubts about the effectiveness of mean-variance optimization when compared to a simple naive diversification strategy. The authors explore various variants of the standard mean-variance model across multiple datasets and conclude that none of the models consistently outperforms a 1/N portfolio in terms of Sharpe ratio or CEQ return. Their findings challenge the notion that mean-variance optimization is superior to the straightforward equal-weighting approach. Nonetheless, several subsequent studies have refuted this viewpoint, and Kirby & Ostdiek (2012) is one of the seminal papers among them. In this study, the authors show that the results of DeMiguel et al. (2009) are primarily due to their research design. Regardless of the individual risk or return characteristics of each asset in a portfolio, the 1/N portfolio strategy involves allocating an equal amount of capital to each one that is available. While seeking the most effective trade-off between risk and return based on the mean-variance framework, MVE (Mean Variance Efficient) strategies seek to optimize the portfolio allocation by incorporating the expected returns and volatilities of assets. MVE strategies take into account the benefits of diversification and permit different asset weights, which could result in better risk-adjusted returns than the 1/N approach. They compare 13 actively managed MVE strategies to the naive strategy and conclude that MVE portfolios are superior to 1/N portfolio, and even high transaction costs can not prevent the two timing strategies from outperforming the 1/N strategy.

The proposed strategies not only serve as mean-variance timing rules but also draw upon extensive literature on asset allocation considering estimation error and portfolio holding constraints. This study addresses several aspects explored in previous research (e.g., Kirby & Ostdiek (2012); Tu & Zhou (2011)), focusing on investors who acknowledge the changing nature of conditional means and variances of asset returns over time. Consequently, a new group of active portfolio strategies is introduced to capitalize on sample information concerning volatility dynamics, effectively reducing the impact of estimation risk. These methods, known as volatility timing (VT), are rebalanced on a monthly basis entirely based on swings in predicted conditional volatilities. The degree to which portfolio weights are sensitive to these variations is controlled by a tuning parameter that represents timing aggressiveness. This method ensures that the recommended solutions keep competitive turnover levels comparable to naive diversification. Furthermore, a broader range of timing strategies is proposed, including the utilization of sample information on the dynamics of conditional expected returns. These reward-to-risk timing (RRT) methods are rebalanced on a monthly basis solely based on changes in anticipated reward-to-risk ratios. Two estimators of conditional expected returns are employed: a straightforward rolling estimator devoid of parametric assumptions and an estimator designed to reduce estimation risk by incorporating predictions derived from asset pricing theories.

While Kirby & Ostdiek (2012) is a notable study that employed robust methodologies and yielded fruitful results, it is essential to acknowledge potential areas for improvement. One significant concern is the use of a simplified estimated diagonal covariance matrix in estimating the weights for the two timing strategies. This approach, which involved simple rolling estimation, may overlook valuable information embedded in the correlations between assets and potentially introduce bias. To address this limitation, a more sophisticated approach proposed by De Nard et al. (2019) and R. F. Engle et al. (2019) can be employed. This approach involves estimating the covariance matrix using a dynamic factor model, dynamic conditional correlation model (DCC) and non-linear shrinkage method (NL) to extract additional information from the asset correlations when the dimension of the assets is large.

Factor models have a longstanding history in finance and are widely employed in portfolio construction (Chincarini et al., 2006; Meucci, 2005). For instance, the Capital Asset Pricing Model (CAPM) introduced by Sharpe (1964) is a prominent example of a factor model that relates an asset's expected return to its systematic risk and the risk-free rate. We choose to incorporate approximate factor models (AFM) instead of exact factor models (EFM) due to the overly strict assumptions associated with EFM in practical applications. EFM assumes a precise and predetermined set of factors that completely capture the variation in asset returns, which is often unrealistic. In contrast, AFM relaxes these strict assumptions by allowing for a more flexible and adaptive representation of factors that can better capture the complexities of real-world financial markets, leading to improved modeling accuracy and performance. (De Nard et al., 2019; Fan et al., 2013). Then we aim to apply the DCC-NL method to model the time-varying conditional correlations among the residuals from factor models. DCC-NL extends the traditional DCC model by allowing for nonlinear dependencies in the conditional correlations (R. Engle, 2002), which consists of two main steps: univariate volatility estimation and correlation estimation R. F. Engle et al. (2019). We use GARCH (1,1) model to estimate the univariate estimation. To estimate the parameters of the model and the correlations, we utilize two innovative techniques, namely the composite likelihood method (R. Engle et al., 2008) and the NL shrinkage method proposed by R. F. Engle et al. (2019). The method can help in capturing the dynamic dependencies and tail behavior in the covariance matrix estimation, especially in a large-dimensional setting.

The paper's results section examines the out-of-sample performances of the MVE, VT, and RRT strategies in comparison to the 1/N strategy across various datasets. The implementation approach utilized for all strategies aligns with the method employed by Kirby & Ostdiek (2012) in their study. The empirical findings support mean-variance optimization outperforming 1/N strategy in the 10 Industry dataset, in the absence of transaction costs. The timing strategies (VT and RRT) exhibit promise in effectively managing turnover and enhancing performance, particularly in the 10 Momentum dataset. For instance, the VT strategies demonstrate higher estimated Sharpe ratios (range from 0.48 to 0.52) compared to the 1/N strategy (0.43) when transaction costs are not considered. In the 10 Industry dataset, the incorporation of a 4-factor risk model improves RRT strategies by reducing estimation risk. Moreover, transaction costs have minimal impact on the effectiveness of both VT and RRT strategies in both datasets. For instance, the Sharpe ratios of RRT strategies with a factor model in the 10 Industry dataset (after accounting for 50 bp transaction costs) range from 0.56 to 0.60, which closely aligns with the range without transaction costs (0.56 to 0.61). Overall, dynamic timing strategies, especially RRT with a factor model, enhance risk-adjusted returns compared to naive diversification. Furthermore, AFM-DCC-NL estimation is able to improve the performance of MV portfolio and help it outperform 1/N strategy.

Future research should examine active portfolio strategies using additional datasets, examine the effects of alternative risk factors or models on timing strategies, and assess the best tuning parameters and robustness tests, among other things. It is also advised to evaluate the AFM-DCC-NL estimation method's applicability in various market contexts and contrast it with alternative covariance estimation methods.

For the remaining of this paper, we first introduce the central research question and relevant sub-questions in Section 2, then we provide an overview of the variables and data-sets in Section 3. Next, in Section 4, we analyze seven different portfolio strategies by using various estimators and measures, and give an extension to estimate covariance matrix under large-dimensional condition. Essentially, we present important results and conclusion in Section 5 and Section 6, respectively.

2 Theory

The main research question of this study is whether active portfolio strategies that integrate volatility timing and reward-to-risk timing can outperform traditional mean-variance optimization and naive diversification. Additionally, we aim to determine if the performance of the active strategy can be sustained when employing factor models and the DCC-NL method, in cases where the number of assets is comparable to the number of observations.

To address this research question, several sub-questions will be explored. First, we will examine whether traditional mean-variance efficient (MVE) portfolios and actively managed portfolios constructed using volatility timing and conditional beta estimation, as proposed by Kirby & Ostdiek (2012), outperform the naive diversification strategy. The second objective of this study is to explore methods for reducing the dimension of the covariance matrix when constructing portfolios that include a large number of individual assets. Additionally, we aim to investigate whether active portfolio strategies maintain their superior performance compared to naive diversification in such high-dimensional settings. The methodology we used and the corresponding background are elaborated further in Section 4.

3 Data

The data we used for this paper contains monthly excess returns on U.S. equity portfolios for the period from July 1963 to December 2022, providing a total of 714 monthly observations for the empirical analysis, with a 120 month window length. The data consists of three datasets used in Kirby & Ostdiek (2012) and DeMiguel et al. (2009), including 10 Industry portfolios, 10 Momentum portfolios and 25 Factor portfolios. These datasets are drawn from the data library maintained by Kenneth R. French.

The industry portfolios (10 Industry) are constructed based on the Standard Industrial Classification (SIC) system and include Consumer Discretionary, Consumer Staples, Energy, Financials, Health Care, Industrials, Information Technology, Materials, Telecommunication Services, and Utilities. Next, we consider the momentum portfolios (10 Momentum) that are formed based on the 12-2 momentum strategy of Jegadeesh & Titman (1993) and sorted into deciles based on their past 12-month returns excluding the most recent month.

In Section 4.2.1, we use an alternative estimator of conditional expected returns. Furthermore, we employ a different method to estimate the expected covariance matrix. To accounts for correlation information, we utilize a special shrinkage estimator that can capture the correlation structure under large-dimensional setting, as proposed by De Nard et al. (2019). These methods involves incorporating factor returns, including market, size, momentum and book-to-market factors, which are used to estimate the loadings for the factor models. The data for these factor returns, as well as risk-free returns, is also obtained from the same data library. Furthermore, the 500 individual assets returns data used in the Extension Section 4.5 is obtained from the Center for Research in Security Prices (CRSP).

4 Methodology

In order to assess the performance of various strategies, we commence the portfolio formation process by implementing the strategies outlined in Section 4.1. Additionally, we discuss several approaches to estimate the conditional moments of returns in this section, namely the fixed-window rolling estimator, an alternative estimator of the conditional expected returns, and AFM-DCC-NL scheme (De Nard et al., 2019). Furthermore, we introduce two criteria, namely the Sharpe ratio and the alteration fee between two strategies, to evaluate the performance of different strategies (Kirby & Ostdiek, 2012).

4.1 Portfolio Strategies

In this section, we outline the various strategies employed in our analysis. Firstly, we consider the naive diversification strategy across the risky assets, commonly known as the 1/N portfolio (1/N), which involves allocating an equal amount to each asset in the portfolio. The other strategies are classified into two main categories: optimal strategies under quadratic loss, including the Minimum-Variance strategy (MV), Tangency Portfolio strategy (TP), and Optimal Constrained strategy (OC), and strategies that utilize sample information about conditional means and variances to reduce estimation risk, including the Volatility-Timing strategy (VT) and Reward-to-Risk Timing strategy (RRT). It is important to note that throughout this paper, we assume that there are N risky assets and a single risk-free asset. The selection of strategies is based on Kirby & Ostdiek (2012).

4.1.1 Optimal strategies under quadratic loss

In order to achieve the optimal strategies under quadratic loss, we need to solve the maximization quadratic objective function by choosing the $N \times 1$ vector of risky assets weights w_{pt} (the weight of the risk-free asset is determined by $1 - w_{pt}^{'}\iota$. First, we reduce the risk free returns from the risky-asset returns to obtain the excess return $r_t = R_t - \iota R_{ft}$, where $\mathbf{R_t}$ is an $N \times 1$ vector of risky assets, R_{ft} is the risk-free rate, and ι is an $N \times 1$ vector of ones. Next, we use the unconstrained optimal function as follows to form the optimal unconstrained portfolio (OU):

$$Q(w_{pt}) = w_{pt}^{'}\mu_t - \frac{\gamma}{2} w_{pt}^{'}\Sigma_t w_{pt}, \tag{1}$$

where μ_t is the the conditional mean vector of the excess returns of risky assets, $E_t(r_{t+1})$. The conditional covariance matrix of the excess returns of risky assets Σ_t is equal to $E_t(r_{t+1}r_{t+1}^{'}) - E_t(r_{t+1})E_t(r_{t+1})^{'}$. Here γ indicates the investor's preference, which is the coefficient of relative risk aversion. The well-known solution for this objective function is $w_{pt} = \frac{\Sigma_t^{-1}\mu_t}{\gamma}$, which indicates a TP of only risky assets with weights:

$$w_{TP,t} = \frac{\Sigma_t^{-1}\mu_t}{\iota'\Sigma_t^{-1}\mu_t}. \tag{2}$$

The fraction on the investor's wealth allocated to the TP is $x_{TP,t} = \frac{\iota'\Sigma_t^{-1}\mu_t}{\gamma}$, and the fraction that allocated to the risk-free asset is $1 - x_{TP,t}$.

To obtain the vector of weights for the MV portfolio without risk-free asset, we minimizing the objective function $Var = w_{pt}^{'}\Sigma_t w_{pt}$ subject to $w_{pt}^{'}\iota = 1$. The classic solution for this function is as follows:

$$w_{MV,t} = \frac{\Sigma_t^{-1}\iota}{\iota'\Sigma_t^{-1}\iota}. \tag{3}$$

Next, we aim to study the mean-variance efficient strategy with only risky assets to form the OC portfolio, which refer to solve the function (1) subject to $w_{pt}^{'}\iota = 1$. We solve it by taking the first order condition with Lagrange multiplier constraint:

$$\mu_t + \delta_t\iota - \gamma\Sigma_t w_{pt} = 0, \tag{4}$$

then the following optimal weights vector is

$$w_{pt} = \frac{\Sigma_t^{-1}\mu_t}{\gamma} + \frac{\delta_t}{\gamma}\Sigma_t^{-1}\iota. \tag{5}$$

It is obvious that the first term on the right-hand side of Equation (5) is proportional to $w_{TP,t}$, and the second term is proportional to $w_{MV,t}$. Furthermore, we solve for δ_t and obtain

$$w_{pt} = x_{TP,t} \frac{\Sigma_t^{-1}\mu_t}{\iota'\Sigma_t^{-1}\mu_t} + (1 - x_{TP,t}) \frac{\Sigma_t^{-1}\iota}{\iota'\Sigma_t^{-1}\iota}. \quad (6)$$

The OC portfolio is similar to the OU portfolio, except that the allocation in the risk-free asset has been shifted to the MV portfolio. Equation (8) implies that

$$\mu_{pt} = x_{TP,t}\mu_{TP,t} + (1 - x_{TP,t})\mu_{MV,t}, \quad (7)$$

where $\mu_{TP,t}$ and $\mu_{MV,t}$ represent the conditional expected excess returns for the TP and MV portfolios, and can be described as follows:

$$w_{pt} = (\frac{\mu_{pt} - \mu_{MV,t}}{\mu_{TP,t} - \mu_{MV,t}}) \frac{\Sigma_t^{-1}\mu_t}{\iota'\Sigma_t^{-1}\mu_t} + (1 - \frac{\mu_{pt} - \mu_{MV,t}}{\mu_{TP,t} - \mu_{MV,t}}) \frac{\Sigma_t^{-1}\iota}{\iota'\Sigma_t^{-1}\iota}. \quad (8)$$

In addition, we present a special version of the OC portfolio, referred to as OC+, which prohibits short sales. Following Kirby & Ostdiek (2012), We only consider the OC and OC+ strategy that targets the estimated expected return of the 1/N portfolio, in the other words, $\hat{\mu}_{pt} = \hat{\mu}_t'\iota/N$.

4.1.2 Volatility-Timing strategy

Although the OC strategy outperforms the TP strategy according to Kirby & Ostdiek (2012), but it may not consistently beat the 1/N strategy due to transaction costs. Turnover is viewed as the primary obstacle to taking advantage of mean-variance optimization, as it can be costly. Instead of focusing solely on portfolio optimization, we seek to develop portfolio selection methods that enhance performance by exploiting sample information, while keeping the desirable features of naive diversification. Thus, we consider incorporating several techniques to reduce the turnover and begin with volatility timing. By using volatility-timing strategy, we are allowed to rebalance the weight monthly based on changes in the estimated conditional covariance matrix of returns. Fleming et al. (2003) found that VT strategies significantly outperform unconditionally MVE portfolio strategies and used future contracts in their study. However, we aim to use a different approach that is able to avoid short sales and keep turnover as low as possible, with the setting that all the estimated correlations between the excess risky asset returns are zero ($\hat{\Sigma}_t$ is a diagonal matrix). The weights are as follows, $\hat{w}_{it} = \frac{1/\hat{\sigma}_{it}^2}{\sum_{i=1}^N (1/\hat{\sigma}_{it}^2)}$, where σ_{it}^2 is the diagonal element of the expected covariance matrix, which is the estimated conditional volatility of the excess return of the ith asset. While the weights do not offer any flexibility to adjust to changes in volatility, it is part of the broader category of VT strategies, which have weights

$$\hat{w}_{it} = \frac{(1/\hat{\sigma}_{it}^2)^\eta}{\sum_{i=1}^N (1/\hat{\sigma}_{it}^2)^\eta}, \quad (9)$$

where η (≥ 0), the tuning parameter, measures how aggressive the investor adjust the weights based on volatility changes.

4.1.3 Reward-to-Risk Timing strategy

Given the VT strategies do not incorporate information about conditional expected returns, we introduce the Reward-to-risk timing strategies (RRT), the weights of the simple RRT strategy are calculated by $\hat{w}_{it} = \frac{\hat{\mu}_{it}/\hat{\sigma}^2_{it}}{\sum_{i=1}^{N}(\hat{\mu}_{it}/\hat{\sigma}^2_{it})}$, where $\hat{\mu}_{it}$ is the estimated conditional mean for the excess return of the ith asset. Similarly, this equation can also be regarded as an example of the broader category of RRT strategies, which have weights

$$\hat{w}_{it} = \frac{(\hat{\mu}^+_{it}/\hat{\sigma}^2_{it})^\eta}{\sum_{i=1}^{N}(\hat{\mu}^+_{it}/\hat{\sigma}^2_{it})^\eta}, \tag{10}$$

where $\eta \geq 0$, and $\hat{\mu}^+_{it} = max(\hat{\mu}_{it}, 0)$. We refer to the portfolios with weights formatting in Equation (10) as the RRT(μ^+_t, η) portfolio.

4.2 Rolling Estimators

We aim to follow Kirby & Ostdiek (2012) to use a rolling window to estimate μ_t and Σ_t. This approach is designed to balance the trade-off between using more observations, which can lead to efficiency gains and using less timely observations, which can lead to a loss in forecast precision. To estimate the expected returns, we use the following formula:

$$\hat{\mu}_t = (1/L)\sum_{l=0}^{L-1} r_{t-l}, \tag{11}$$

where r_t is the return of the asset at time t and L is the length of the rolling window. We then use a similar formula to estimate the expected covariance matrix,

$$\hat{\Sigma}_t = (1/L)\sum_{l=0}^{L-1}(r_{t-l} - \hat{\mu}_t)(r_{t-l} - \hat{\mu}_t)'. \tag{12}$$

Regarding to the choice of window length, we follow Kirby & Ostdiek (2012) and only consider $L = 120$ in this paper. While rolling estimators of conditional expected excess returns offer simplicity, their use in portfolio optimization introduces a significant level of estimation risk. It is widely acknowledged that a substantial amount of return data is required to accurately estimate μ_t (Merton, 1980). Consequently, we explore an alternative estimator for expected return in order to mitigate estimation risk when implementing RRT strategies under specific conditions.

4.2.1 Alternative estimator of conditional expected returns used in RRT strategy

In order to exploit the relationship between the first and second moments of excess returns, we consider an alternative method to estimate μ_t, which is to estimate conditional betas instead. First, we assume the conditional CAPM model holds, which implies that the cross-sectional variation in conditional expected excess return is due to cross-sectional variation in conditional betas. Then we calculate the weights as follows:

$$w_{it} = \frac{(\beta_{it}^{+}/\sigma_{it}^{2})^{\eta}}{\sum_{i=1}^{N}(\beta_{it}^{+}/\sigma_{it}^{2})^{\eta}}, \tag{13}$$

where β_{it}^{+} denotes the conditional market beta of asset i at period t, and $\beta_{it}^{+} = max(\beta_{it}, 0)$. Furthermore, we employ multi-factors model to extend the method to allow for more factors. The weights with multiple betas for the RRT strategy become:

$$w_{it} = \frac{(\bar{\beta}_{it}^{+}/\sigma_{it}^{2})^{\eta}}{\sum_{i=1}^{N}(\bar{\beta}_{it}^{+}/\sigma_{it}^{2})^{\eta}}, \tag{14}$$

Similarly, $\bar{\beta}_{it}^{+} = max(\bar{\beta}_{it}, 0)$, and $\bar{\beta}_{it}$ is equal to $1/K\sum_{j=1}^{K}\beta_{ijt}$. In this case, β_{ijt} represents the conditional beta of the ith asset with respect to the jth factor at period t. Therefore, the $\bar{\beta}_{it}$ is the average conditional beta of asset i regarding to the K factors at period t.

We use a rolling estimator of the factor returns and conditional covariance matrix to estimate conditional betas as well, with the same window length ($L = 120$). In addition, we aim to follow Kirby & Ostdiek (2012) to use the Carhart (1997) 4-factor model as the multi-factors model, which is an extension of the Fama & French (1993) 3-factor model. We refer to this portfolio as the RRT$(\bar{\beta}_{t}^{+}, \eta)$ portfolio.

4.3 Performance measures: Sharpe ratio and Quadratic Utility

Following Kirby & Ostdiek (2012), we introduce two criteria to evaluate the performance of the strategies. Assuming a dataset with $T + L$ observations, where T represents the out-of-sample period, the performance of the strategies is evaluated based on two criteria after computing the sequence $\{r_{pt}\}_{t=L+1}^{T+L}$ of out-of-sample excess returns for each strategy. The first criteria is the Sharpe ratio,

$$\lambda_p = \mu_p/\sigma_p, \tag{15}$$

where μ_p and σ_p are the mean and standard deviation of the excess return. In order to estimate the ratio, we calculate the sample mean and variance by:

$$\hat{\mu}_p = (1/T)\sum_{t=L+1}^{T+L} r_{pt}, \tag{16}$$

$$\hat{\sigma}_p^2 = (1/T)\sum_{t=L+1}^{T+L} (r_{pt} - \hat{\mu}_p)^2, \tag{17}$$

which are reported annualized. Thus, we are able to obtain the annualized estimator, $\hat{\lambda}_p$. The difference in the estimated Sharpe ratio for 2 strategies is the first measure of relative performance.

The second criteria is based on quadratic utility (Fleming et al., 2003), which is a second-order approximation of the investor's real utility function, can be calculated as follows:

$$U(R_{p,t+1}) = W_t(1 + R_{p,t+1}) - \frac{1}{2}\alpha W_t^2(1 + R_{p,t+1})^2, \tag{18}$$

where α is the level of absolute risk aversion, W_t is wealth at time t, $R_{p,t+1}$ is the portfolio return at $t+1$.In order to keep αW_t constant, we introduce its equivalent condition, which set the relative risk aversion $\gamma_t = \alpha W_t/(1-\alpha W_t)$ to a specified γ. The second relative measure, a Δ_γ fee, is obtained by equating the expected utilities generated by 2 strategies ($E[U(R_{p_i t})] = E[U(R_{p_j t} - \Delta_\gamma)]$), which means the investor with relative risk aversion at the level of γ would be indifferent between 2 strategies after imposing the maximum Δ_γ fee. The quadratic formula is as follows:

$$\Delta_\gamma = -\gamma^{-1}(1-\gamma E[R_{p_i t}]) + \gamma^{-1}((1-\gamma E[R_{p_i t}])^2 - 2\gamma E[U(R_{p_i t}) - U(R_{p_j t})])^{1/2}. \quad (19)$$

In order to estimate the delta for the out-of-sample period, the estimation was reconstructed using a method. Additionally, the details of the estimation can be found in Appendix **??** of this paper. We assess the performance of various strategies (denoted as j) with respect to two levels of relative risk aversion ($\gamma = 1$ and $\gamma = 5$), using naive diversification as strategy i. The annualized basis point value of Δ_γ is estimated as $\hat{\Delta}_\gamma$ for each active strategy j.

4.4 Transaction costs and Turnover

Strategies that have high turnover rates are more affected by transaction costs. To illustrate this, we aim to present a second set of results that show returns after deducting transaction costs. According to Kirby & Ostdiek (2012), we make the assumption that transaction costs remain consistent across assets throughout the entire sample period, and assign a value of 50 basis points ($c = 50\ bp$) for the level of proportional costs per transaction. We estimate the expected turnover $\hat{\tau}_p$ for each strategy as follows:

$$\hat{\tau}_p = (1/T)\sum_{t=L+1}^{T+L} \hat{\tau}_{pt}. \quad (20)$$

Here, $\hat{\tau}_{pt}$ is the turnover at time t, which is defined as the sum of the absolute value of the weight differences across N assets: $\hat{\tau}_{pt} = \sum_{i=1}^{N}(|\hat{w}_{i,t+1} - \hat{w}_{i,t^*}|)$. $\hat{w}_{i,t+1}$ denotes the desired weight of asset i at time $t+1$ after rebalancing, while $\hat{w}_{i,t^*}$ represents the weight before rebalancing at $t+1$. Therefore, the effect of implementing transaction costs can be determined by subtracting $\hat{\tau}_p c$ from the sample mean of $R_{p,t}$.

4.5 Extension: Alternative estimator of expected covariance matrix – AFM1-DCC-NL of De Nard et al. (2019)

Kirby & Ostdiek (2012) used a different covariance matrix for the volatility timing and reward-to-risk strategies than for the four MVE strategies. The reason for this difference is that the MVE strategies are based on the assumption of mean-variance efficiency, which implies that investors care only about the expected return and variance of their portfolio. the VT and RRT strategies, on the other hand, are intended to catch higher moments of the return distribution. By using a diagonal covariance matrix, the authors of Kirby & Ostdiek (2012) assume that the estimated pair-wise correlations between the returns are all 0. This allows the authors to separate the effects of VT and RRT strategies from the effects of mean-variance optimization.

Therefore, in order to improve the estimation of the covariance matrix for portfolio selection in high-dimensional settings, we employ the one-factor AFM-DCC-NL method proposed by De Nard et al. (2019). The AFM-DCC-NL method is a two-step approach used to estimate large-dimensional covariance matrices using factor models. This method combines the Asymmetric Factor Model (AFM) and the Dynamic Conditional Correlation (DCC) model to incorporate time-varying conditional heteroskedasticity and capture the correlation dynamics between assets. In the first step, the AFM is employed to estimate the factor model, allowing for the asymmetric responses of asset returns to market-wide risk factors (Connor & Korajczyk, 1986; Bai & Ng, 2002; Fan et al., 2008). The DCC model is employed to represent the varying correlation between assets in the portfolio. This process allows for capturing the evolving dependencies and correlations among the assets as time progresses. In the subsequent stage, the NL shrinkage estimator suggested by R. F. Engle et al. (2019) is utilized to enhance the precision of the estimated factor loadings matrix. This shrinkage estimator helps to reduce noise in the estimated factor loadings by pulling them towards a common value, thereby enhancing the precision and reliability of the estimation. Ultimately, this can lead to improved performance when applying the estimated covariance matrix in out-of-sample scenarios.

By incorporating factor modeling techniques, it captures the common factors driving asset returns and allows for time-varying covariances (Meucci, 2005; Chincarini et al., 2006). To minimize additional estimation uncertainty, we employ a single factor model instead of a multiple factors model from the dynamic approximate factor models based on the DCC-NL estimation scheme, as demonstrated in the study by De Nard et al. (2019). we aim to evaluate the out-of-sample performance of the dynamic one-factor AFM-DCC-NL using the Sharpe ratio as our performance measure.

To use one-factor approximate factor model (AFM1), we first introduce the unconditional single market factor dynamic model as follows:

$$r_{i,t} = \alpha_i + \beta_i' f + u_{i,t}, \tag{21}$$

where $r_{i,t}$ is the return of asset i at time t, f_t is the return for market factor at t, and $u_{i,t}$ represents the residual. The intercept α_i and the factor loading β_i are both time-invariant under the assumptions of unconditional dynamic models (Avramov & Chordia, 2006; Ang & Kristensen, 2012; R. F. Engle, 2016).

Secondly, we introduce the estimator of the time-varying covariance matrix of r_t according to De Nard et al. (2019), which is as follows:

$$\hat{\Sigma}_{r,t} = \hat{B}' \hat{\Sigma}_f \hat{B} + \hat{\Sigma}_{u,t}, \tag{22}$$

where B is an $1 \times N$ matrix with β_i as ith column, the covariance matrix of factor, $\hat{\Sigma}_f$ is assumed to be time-invariant to make the dynamic component solely due to the error terms. Therefore, the first component of the right side of Equation (22) is time-invariant, while the second component, $\hat{\Sigma}_{u,t}$ is time-varying, which is the estimator of covariance matrix of the residuals $\{u_t\}$, the starting point is the sample covariance matrix $S_{\hat{u}}$. When the number of assets is of the similar magnitude as the number of observations, e.g., $N = 500$, we need to use

DCC model (R. F. Engle, 2000; R. F. Engle et al., 2019; R. Engle, 2002) to achieve an accurately estimated covariance matrix, and they employ the NL-shrinkage method proposed by Ledoit & Wolf (2011) for the estimator of the DCC model (Ledoit & Wolf, 2015). The AFM1-DCC-NL method makes the estimation of the covariance matrix with large dimensions feasible.

We use the method proposed by R. F. Engle et al. (2019) to apply NL shrinkage in the DCC model to handle covariance matrix with large dimensions, using two important tools: composite likelihood method (R. Engle et al., 2008) and non-linear shrinkage method (Ledoit & Wolf, 2011). First, for each asset, we use the fitted model univariate GARCH(1,1) to estimate the conditional variances of each residual series,

$$d_{i,t}^2 = w_i + a_i u_{i,t-1}^2 + b_i d_{i,t-1}^2, \tag{23}$$

where $u_{i,t}$ is the residuals for asset i at time t, $d_{i,t}^2$ denotes the conditional variance of $u_{i,t}$, and (w_i, a_i, b_i) are the parameters of the model. Thus, we are able to devolatilize the residual series as follows,

$$s_{i,t} = u_{i,t}/d_{i,t}, \tag{24}$$

where $s_{i,t}$ denotes the devolatilized residual series. The conditional covariance matrix of devolatilized series Q_t can be calculated as follows:

$$Q_t = (1-\alpha-\beta)C + \alpha s_{t-1}s_{t-1}^{'} + \beta Q_{t-1}, \tag{25}$$

where C is the unconditional correlation matrix of the u_t, which we will explain in the following step, and (α, β) are the DCC parameters in correlation space, which we will address in the final step. Then we are able to obtain the conditional correlation matrix as:

$$R_t = Diag(Q_t)^{-1/2} Q_t Diag(Q_t)^{-1/2}. \tag{26}$$

Accordingly, the time-varying $\hat{\Sigma}_{u,t}$ can be represented by $D_t R_t D_t$, where D_t is the diagonal matrix with $d_{i,t}$ as the ith diagonal element.

Secondly, we need to employ the nonlinear function of Ledoit & Wolf (2011) to estimate the correlation targeting matrix in Equation (25) to replace the sample correlation matrix C, it allows we have a Σ_u estimator with better out-of-sample performance when the dimension of assets N is large. As we know, $C = Corr(u_t) = cov(s_t)$, thus C can also be regarded as the covariance matrix of devolatilized residuals $S = [s_{i,t}]$, then the sample covariance matrix $\hat{C}$ is equal to $\frac{1}{T}SS^{'}$. We begin with decomposing it into a set of descending order eigenvalues $(\lambda_1, \lambda_2..., \lambda_N)$ and corresponding eigenvectors $(v_1, v_2..., v_N)$, then we are able to calculate $\hat{C}$ by $\sum_{i=1}^{N} \lambda_i v_i v_i^{'}$. To offer a consistent estimate of out-of-sample variance under large-dimensional asymptotic conditions, we then use the nonrandom multivariate function called the Quantized Eigenvalues Sampling Transform (QuEST) proposed by Ledoit & Wolf (2015) and asymptotic optimal nonlinear shrinkage formula of Ledoit & Péché (2011).[1] This function is a multivariate deterministic function that maps the interval $[0, \infty)$ to itself for a given dimension N. It takes a

[1]As the shrinkage formula and QuEST function are both complex, we will not repeat them here, they are clearly stated in Ledoit & Péché (2011) and Ledoit & Wolf (2015) respectively. The relevant codes are included in the DCC-NL code package published on Michael Wolf's personal website.

set of population eigenvalues $(p_1, p_2, ..., p_N)$ as input and returns a deterministic equivalent of the sample eigenvalues, denoted as $Q_{N,T}(t) = (q^1_{N,T}(t), q^2_{N,T}(t), ..., q^N_{N,T}(t))$. The QuEST function is used to estimate population eigenvalues by numerically inverting the QuEST function. This means that given a set of sample eigenvalues, we are able to use the function to obtain estimates of the corresponding population eigenvalues:

$$\tilde{g} = argmin_{t \in [0,\infty)^N} \frac{1}{N} \sum_{i=1}^{N} [q^i_{N,T}(t) - \lambda_i]^2, \tag{27}$$

and then receive the shrunk eigenvalues $\tilde{\lambda}(\tilde{g})$ by using the formula of Ledoit & Péché (2011). Thus, the covariance matrix shrinkage estimator can be represented by

$$\tilde{C} = \sum_{i=1}^{N} \tilde{\lambda}_i(\tilde{g}) v_i v_i^{'}. \tag{28}$$

Finally, we must estimate the DCC-NL model parameters in order to generate the time-varying conditional covariance matrix of the residuals. This can be done by using the composite likelihood technique (R. Engle et al., 2008). The idea is to simplify the estimation technique by focusing on pairwise residual relationships rather than the full likelihood function, which can be computationally demanding in high-dimensional scenarios. The goal is to convert the total joint likelihood of the data into a product of conditional likelihoods. Given the calculated volatilities, each conditional likelihood corresponds to the conditional distribution of a certain pair of residuals. In addition, to ensure that the diagonal element of the estimation of C equals zero, we renormalized it by dividing each column and row by the square root of the corresponding diagonal element, as cited in R. F. Engle et al. (2019).

Next, our objective is to utilize the AFM1-DCC-NL estimation scheme to address the challenge of estimating the $1/N$ portfolio and global minimum variance (GMV) portfolio when short-sales constraints are not present and there is no transaction costs. The GMV portfolio aims to minimize the portfolio's risk, the problem is defined as: $\min_{w} w^{'} \sum_{r,t} w$, subject to $w^{'}\iota = 1$. In this optimization problem, $\Sigma_{r,t}$ represents the time-variant covariance matrix of asset returns, w denotes the vector of portfolio weights. Thus, the estimator of weights for GMV portfolio at time t can be described as follows,

$$\hat{w}_t = \frac{\hat{\Sigma}^{-1}_{r,t}\iota}{\iota^{'}\hat{\Sigma}^{-1}_{r,t}\iota}. \tag{29}$$

We specifically choose the GMV portfolio as it demonstrates attractive out-of-sample properties, not solely in terms of risk reduction but also in terms of the reward-to-risk trade-off. We use a commonly used risk-adjusted performance measures to analyze and compare the performance of the naive diversification approach and the GMV strategy: the Information Ratio. The indicator provides useful insights into the strategies' risk-adjusted returns and enable for a full assessment of their effectiveness.

5 Results

To make a direct comparison, we begin by revisiting and replicating the results of DeMiguel et al. (2009) according to Kirby & Ostdiek (2012).[2] In the first panel of Table 1, we present the summary statistics for the 1/N strategy and three MVE strategies applied to 10 Industry portfolios and Mkt/SMB/HML portfolios. These statistics encompass the annualized mean excess return, annualized excess return standard deviation, annualized Sharpe ratio, and the average monthly turnover.

Table 1: Characteristics of the 1/N and MVE strategies

The table presents key sample characteristics of four portfolio strategies: 1/N, MV, OC, and Tangency Portfolio (TP), for two of the five datasets from Kirby & Ostdiek (2012). The first dataset is the "FF 10 Industry," and the second dataset is "Mkt/SMB/HML". Panel A of the table reports performance without transaction costs. These metrics include the mean excess return ($\hat{u}_p$), excess return standard deviation ($\hat{\sigma}_p$), Sharpe ratio ($\hat{\lambda}_p$), and average monthly turnover as a fraction of invested wealth ($\hat{\tau}_p$). Panel B presents the minimum, median, and maximum estimates of the annualized time-series of the conditional expected excess return for each strategy. The MV, OC, and TP strategies employ a 120-month rolling estimator to estimate the conditional mean vector and covariance matrix of excess returns. Additionally, the OC strategy targets the estimated conditional expected excess return of the 1/N strategy. The sample period spans from July 1963 to November 2004, comprising 497 monthly observations.

(a) Summary Statistics

	FF 10 Industry				**Mkt/SMB/HML**			
Strategy	$\hat{u}_p$	$\hat{\sigma}_p$	$\hat{\lambda}_p$	$\hat{\tau}_p$	$\hat{u}_p$	$\hat{\sigma}_p$	$\hat{\lambda}_p$	$\hat{\tau}_p$
TP	108	457	0.24	473	5.66	7.47	0.76	0.06
1/N	7.15	15.23	0.47	0.02	4.92	6.36	0.77	0.02
MV	7.07	13.13	0.54	0.43	4.80	5.56	0.86	0.02
OC	6.55	13.30	0.49	0.65	4.77	6.26	0.76	0.06

(b) Estimated conditional expected returns ($\hat{u}_{pt}$)

	FF 10 Industry			**Mkt/SMB/HML**		
Strategy	Min.	Med.	Max.	Min.	Med.	**Max.**
TP	22.7	47.4	12,216	3.4	6.7	15.5
1/N	-3.2	7.9	14.1	-1.3	3.17	8.5
MV	-1.7	4.2	12.2	1.8	4.3	8.6
OC	-0.3	8.4	15.8	2.6	5.0	8.6

To construct portfolios based on various strategies, we utilize rolling estimators with a window length of 120, assuming the absence of transaction costs. This approach generates monthly

[2]We would like to thank the authors of DeMiguel et al. (2009) generously sharing their data and Matlab programming codes. With their valuable resources, we were able to replicate their results. For the purpose of this paper, we rescaled all the monthly results to annualized results.

return series for each strategy. Subsequently, we convert the monthly returns to annualized returns by multiplying by 12. Additionally, the annualized standard deviation is computed by scaling the monthly standard deviation by $\sqrt{12}$, and the annualized Sharpe ratio is obtained using the annualized return and standard deviation.

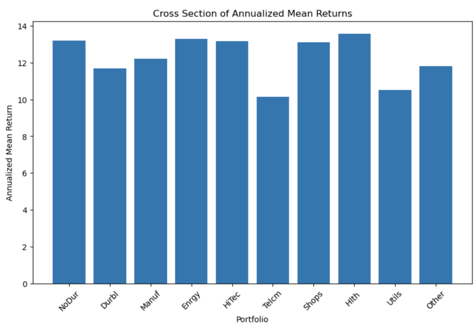
The results obtained match those reported by DeMiguel et al. (2009) and Kirby & Ostdiek (2012). Pane A of Table 1 presents compelling evidence regarding the distinct reward and risk characteristics of the TP strategy in comparison to other strategies. Within the 10 Industry dataset, TP's excess returns exhibit remarkable estimates for both standard deviation and mean, surpassing 100% per year, with the lowest Sharpe ratio and an extremely high turnover. On the other hand, when considering the Mkt/SMB/HML dataset, the turnover of the TP portfolio aligns with that of the OC strategy (0.06), and the estimated mean excess return ($\hat{\mu}_{TP}$) for this dataset is 5.66%, only slightly higher than the mean for other strategies.

Figure 1: Reward and Risk Characteristics of 2 Portfolios

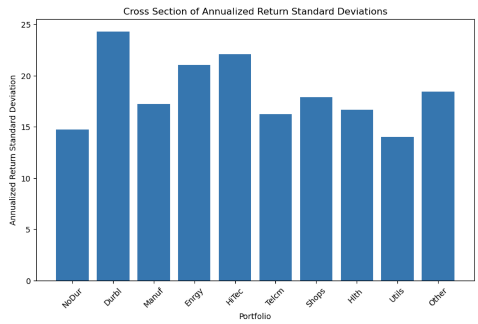
The figure presents a summary of the sample reward and risk characteristics for two datasets: the 10 Industry dataset (a and b) and the 10 Volatility dataset (c and d). Each graph panel consists of two graphs. The first graph in each panel displays the cross section of annualized mean returns, while the second graph shows the cross section of annualized return standard deviations. The sample period spans from July 1963 to December 2022, totaling 714 monthly observations. However, the reported statistics pertain specifically to the subset of observations 121 to 714, which were used to evaluate the out-of-sample performance of the portfolio strategies.

10 Industry Portfolios

(a) Mean Return

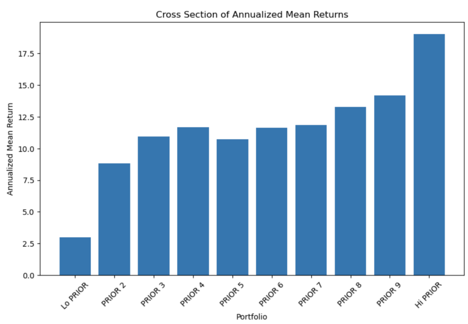


(b) Volatility

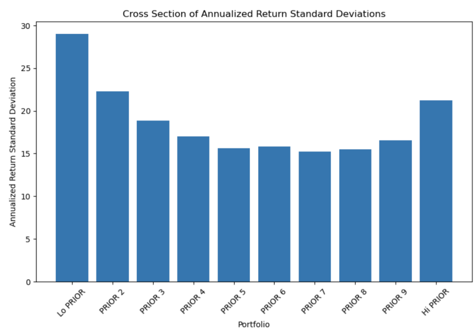


10 Momentum Portfolios

(c) Mean Return



(d) Volatility



The subsequent analysis in Panel B of Table 1 sheds light on the underlying reasons for these observations. It is revealed that the median of $\hat{\mu}_{TP,t}$ for the Mkt/SMB/HML dataset stands at a mere 6.7%, significantly lower than the median for the 10 Industry dataset (47.4%). Such exceptionally high estimated expected returns in the latter dataset result in extreme turnover.

Consequently, when excluding the TP strategy, the results derived from the 10 Industry dataset provide stronger empirical support for mean-variance optimization when compared to the 1/N strategy. Regarding the outcomes of the Mkt/SMB/HML dataset, it is difficult to conclude that the MVE strategies outperform the 1/N strategy.

5.1 Results for the 10 Industry Portfolios

In Table 2 and Table 3, specific entries are marked as ”-” for certain strategies, indicating that the corresponding sample statistic cannot be computed or lacks practical significance. This circumstance mainly arises due to two reasons. Firstly, it occurs when an actual performance fee value is unavailable, resulting in an investor being indifferent between the strategy and the 1/N strategy. Secondly, this situation may arise when the turnover for the TP strategy exceeds 20,000% in one or more months, leading to a wealth of zero when factoring in the assumed transaction costs.

Table 2: Results for the 10 Industry Portfolios

The table presents the out-of-sample performance of 14 portfolio strategies for the 10 Industry portfolios. The strategies include the 1/N strategy, three volatility timing strategies (Panel A), six reward-to-risk timing strategies (Panel B), and four Minimum Variance Efficient (MVE) strategies (Panel C). For each strategy, the table reports various sample statistics based on the monthly excess returns time series. These statistics include the annualized mean ($\hat{u}_p$), annualized standard deviation ($\hat{\sigma}_p$), annualized Sharpe ratio ($\hat{\lambda}_p$), average monthly turnover as a fraction of invested wealth ($\hat{\tau}_p$), annualized basis point fee for switching from the 1/N strategy to the timing or MVE strategy ($\hat{\Delta}_\lambda$), p-value for the difference in Sharpe ratio between the timing or MVE strategy and the 1/N strategy (”vs. 1/N p-val”), and p-values for the basis point fees. The timing and MVE strategies utilize a 120-month rolling estimator to estimate the conditional mean vector and covariance matrix of excess returns. The OC and OC^+ strategies target the estimated conditional expected excess return of the 1/N strategy. The performance measures are reported under two scenarios: assuming no transaction costs and assuming proportional transaction costs of 50 basis points. The p-values are obtained from 10,000 trials of a stationary block bootstrap method with an expected block length of 10. Certain entries in the table are marked as ”-” for several strategies. This indicates that the corresponding sample statistic cannot be computed. This could be due to the absence of a real performance fee value that makes the investor indifferent between the strategy and the 1/N strategy, or if the turnover for the TP strategy exceeds 20,000% in one or more months, leading to zero wealth considering the assumed transaction costs. The sample period for the analysis is from July 1963 to December 2022, comprising 714 monthly observations. The first 120 observations are excluded to initialize the rolling estimators. Further details on each strategy can be found in the accompanying text.

	No Transaction Costs									**Transaction Costs = 50 bp**						
				vs.1/N							**vs.1/N**					
Strategy	$\hat{u}_p$	$\hat{\sigma}_p$	$\hat{\lambda}_p$	$p-val$	$\hat{\Delta}_1$	$p-val$	$\hat{\Delta}_5$	$p-val$	$\hat{\tau}_p$	$\hat{\lambda}_p$	$p-val$	$\hat{\Delta}_1$	$p-val$	$\hat{\Delta}_5$	$p-val$
1/N	7.81	15.32	0.51						0.02	0.50					
A: Volatility Timing Strategies															
VT(1)	7.92	14.15	0.56	0.02	44	0.12	82	0.01	0.01	0.56	0.06	43	0.14	81	0.01
VT(2)	7.85	13.33	0.59	0.04	32	0.18	113	0.02	0.02	0.58	0.12	29	0.22	110	0.02
VT(4)	7.63	12.44	0.61	0.10	-13	0.53	86	0.21	0.03	0.60	0.16	-15	0.56	84	0.21
B: Reward-to-Risk Timing Strategies															
RRT(u_t^+,1)	7.22	14.90	0.51	0.09	-43	0.78	-	-	0.07	0.48	0.63	-69	0.84	-	-
RRT(u_t^+,2)	7.58	13.67	0.55	0.12	-21	0.72	52	0.19	0.09	0.52	0.17	47	0.20	126	0.17
RRT(u_t^+,4)	8.15	13.48	0.60	0.22	37	0.17	108	0.07	0.12	0.57	0.27	33	0.32	104	0.15
RRT($\bar{\beta}_t^+$,1)	8.15	14.05	0.55	0.12	44	0.17	142	0.08	0.02	0.54	0.19	39	0.18	137	0.10
RRT($\bar{\beta}_t^+$,2)	8.21	14.64	0.56	0.13	50	0.19	176	0.09	0.03	0.56	0.23	43	0.25	170	0.15
RRT($\bar{\beta}_t^+$,4)	8.27	14.28	0.58	0.18	62	0.25	192	0.12	0.06	0.58	0.26	54	0.32	184	0.16
C: Mean-Variance Efficient Strategies															
MV	5.77	10.63	0.54	0.21	-110	0.91	52	0.10	0.21	0.42	0.87	-	-	-	-
OC	7.81	12.88	0.61	0.15	-11	0.08	62	0.12	0.36	0.44	0.72	-	-	-	-
OC^+	6.58	13.01	0.62	0.14	-31	0.86	102	0.32	0.08	0.47	0.68	-	-	-	-
TP	63.01	134.47	0.47	0.36	-	-	-	-	32.75	-	-	-	-	-	-

Table 2 provides a comprehensive analysis of the out-of-sample performance of 1/N, MVE, VT, and RRT strategies for the 10 Industry dataset. Firstly, when transaction costs are imposed, the 1/N strategy exhibits only a minor impact on performance, resulting in a slight decrease in estimated performance. For instance, the revised Sharpe ratio is 0.50 compared to the original 0.51. Moving on to the VT strategies, they demonstrate low estimated expected turnover similar to the 1/N strategy. However, they outperform the 1/N strategy in terms of estimated Sharpe ratios, which are statistically significant at the 10% level. Moreover, the VT strategies exhibit superior estimated means for $\eta = 1$ and $\eta = 2$, 7.92% and 7.85%, respectively.

The results of the RRT strategies in Panel B indicate less favorable performance compared to the VT strategies for $\eta = 1$ and $\eta = 2$. These RRT strategies exhibit relatively higher turnover, negative utility fees, and lower $\hat{\lambda}_p$ with p-values exceeding 10%. With transaction costs of 50 bp, the $\hat{\lambda}_p$ decreases to 0.48 for $\eta = 1$ and 0.52 for $\eta = 2$. Figure 1 presents the evidence of the dataset characteristics through graphs. The plot on the left shows a narrow range for the annualized sample mean of excess industry portfolio returns, with most values between 10% and 13%. In contrast, the plot on the right displays a larger dispersion in sample volatilities (14.0%-24.3%). This suggests that VT strategies can outperform RRT strategies for this data set. Furthermore, employing a 4-factor risk model enhances the performance of the RRT strategies and reduces estimation risk in comparison to the standard rolling estimator. Notably, the expected turnover estimates for the RRT strategies using alternative estimators are significantly lower, ranging from 2% for $\eta = 1$ to 6% for $\eta = 4$. The estimates of Δ_γ range from 44 bp to 62 bp for $\gamma = 1$ and from 142 bp to 192 bp for $\gamma = 5$, before taking transaction costs into account. Two out of the three values of $\hat{\Delta}_\gamma$ for $\gamma = 5$ demonstrate statistical significance at the 10% level, indicating that RRT strategies employing alternative.

Moving to Panel C, the results of the 4 MVE strategies display mixed performance in terms of Sharpe ratio. The TP strategy performs the worst, with a $\hat{\lambda}_p$ of 0.47, which is smaller than the value of 1/N (0.51). Transaction costs significantly impact the performance of MVE strategies, leading to lower Sharpe ratios and non-significant performance gains. Specifically, the value of $\hat{\lambda}_p$ decreases to 0.42 for MV, 0.44 for OC, and 0.47 for OC+. Prohibiting short sales significantly reduces turnover by 8% and 36%, respectively, and improves the Sharpe ratio for the OC+ strategy compared to the OC strategy. Overall, these results provide a comprehensive assessment of the performance of various strategies in the 10 Industry dataset, shedding light on their effectiveness and sensitivity to transaction costs.

5.2 Results for the 10 Momentum Portfolios

The results from the 10 Industry dataset highlight the significance of turnover in our investigation. Almost all MVE strategies show superior performance compared to naive diversification when transaction costs are not considered. Although statistical insignificance limits conclusive analysis, the initial evidence generally supports mean-variance optimization. However, the presence of transaction costs diminishes the advantage of MVE strategies due to high turnover. Thus, effective control of turnover is crucial for improving mean-variance strategies. The timing strategies (VT and RRT) demonstrate promising results in this regard for the 10 Industry dataset, but further evidence is necessary for definitive conclusions.

Table 3: Results for the 10 Momentum Portfolios

The table presents the out-of-sample performance of 14 portfolio strategies for the 10 Momentum portfolios. The strategies include the 1/N strategy, three volatility timing strategies (Panel A), six reward-to-risk timing strategies (Panel B), and four Minimum Variance Efficient (MVE) strategies (Panel C). For each strategy, the table reports various sample statistics based on the monthly excess returns time series. These statistics include the annualized mean ($\hat{u}_p$), annualized standard deviation ($\hat{\sigma}_p$), annualized Sharpe ratio ($\hat{\lambda}_p$), average monthly turnover as a fraction of invested wealth ($\hat{\tau}_p$), annualized basis point fee for switching from the 1/N strategy to the timing or MVE strategy ($\hat{\Delta}_\lambda$), p-value for the difference in Sharpe ratio between the timing or MVE strategy and the 1/N strategy ("vs. 1/N p-val"), and p-values for the basis point fees. The timing and MVE strategies utilize a 120-month rolling estimator to estimate the conditional mean vector and covariance matrix of excess returns. The OC and OC$^+$ strategies target the estimated conditional expected excess return of the 1/N strategy. The performance measures are reported under two scenarios: assuming no transaction costs and assuming proportional transaction costs of 50 basis points. The p-values are obtained from 10,000 trials of a stationary block bootstrap method with an expected block length of 10. Certain entries in the table are marked as "-" for several strategies. This indicates that the corresponding sample statistic cannot be computed. This could be due to the absence of a real performance fee value that makes the investor indifferent between the strategy and the 1/N strategy, or if the turnover for the TP strategy exceeds 20,000% in one or more months, leading to zero wealth considering the assumed transaction costs. The sample period for the analysis is from July 1963 to December 2022, comprising 714 monthly observations. The first 120 observations are excluded to initialize the rolling estimators. Further details can be found in the accompanying text.

	No Transaction Costs								**Transaction Costs = 50 bp**							
				vs.1/N							**vs.1/N**					
Strategy	$\hat{u}_p$	$\hat{\sigma}_p$	$\hat{\lambda}_p$	$p-val$	$\hat{\Delta}_1$	$p-val$	$\hat{\Delta}_5$	$p-val$	$\hat{\tau}_p$	$\hat{\lambda}_p$	$p-val$	$\hat{\Delta}_1$	$p-val$	$\hat{\Delta}_5$	$p-val$
1/N	7.37	17.27	0.43						0.02	0.42					
A. Volatility Timing Strategy															
VT(1)	7.73	16.13	0.48	0.00	47	0.01	91	0.01	0.01	0.48	0.00	46	0.01	90	0.00
VT(2)	7.86	15.65	0.50	0.01	68	0.01	132	0.00	0.01	0.50	0.00	66	0.01	131	0.00
VT(4)	7.91	15.26	0.52	0.01	98	0.01	174	0.00	0.02	0.51	0.01	96	0.01	172	0.00
B. Reward-to-Risk Timing Strategies															
RRT(u_t^+,1)	9.62	16.42	0.59	0.00	176	0.00	171	0.00	0.06	0.58	0.00	157	0.00	153	0.00
RRT(u_t^+,2)	10.12	16.57	0.61	0.00	208	0.00	192	0.00	0.07	0.60	0.00	181	0.00	166	0.00
RRT(u_t^+,4)	10.57	16.75	0.63	0.00	296	0.00	274	0.00	0.09	0.62	0.00	266	0.00	245	0.00
RRT($\bar{\beta}_t^+$,1)	8.56	16.07	0.53	0.00	75	0.02	108	0.02	0.01	0.52	0.01	72	0.00	105	0.00
RRT($\bar{\beta}_t^+$,2)	9.13	15.97	0.57	0.00	96	0.01	138	0.00	0.02	0.56	0.00	93	0.00	132	0.00
RRT($\bar{\beta}_t^+$,4)	9.60	16.09	0.60	0.00	109	0.00	176	0.00	0.03	0.59	0.00	108	0.00	175	0.00
C. Mean-Variance Efficient Strategies															
MV	7.22	12.98	0.56	0.14	-11	0.52	96	0.21	0.44	0.34	0.94	-	-	-	-
OC	6.63	14.63	0.45	0.07	-82	0.82	40	0.01	0.19	0.38	0.86	-	-	-	-
OC$^+$	6.24	15.00	0.42	0.56	-94	0.89	-102	0.62	0.09	0.38	0.82	-	-	-	-
TP	-143.21	793.02	-0.18	-	-	-	-	-	74.09	-0.74	-	-	-	-	-

To examine the generalizability of these findings, we analyze a 10 Momentum dataset, where firms are sorted into portfolios based on a momentum measure. This sorting scheme is specifically designed to distribute conditional expected returns. Table 3 presents that the $\hat{\lambda}_p$ of 1/N strategy is 0.43, Imposing transaction costs has little effect on it, the revised ratio is 0.42. Panel A shows the results of VT strategies, $\hat{\lambda}_p$ values range from 0.48 to 0.52. Estimated Sharpe ratio increase compared to naive diversification is statistically significant at the 1% level. Estimated performance fees range from 47 bp to 98 bp for $\gamma = 1$ and 91 bp to 174 bp for $\gamma = 5$, all statistically significant at the 1% level. Imposing transaction costs has little effect on the results, the revised $\hat{\lambda}_p$ values range from 0.48 to 0.51, significantly larger than 0.42 of 1/N portfolio.

Regarding the RRT strategies in Panel B of Table 3, their performance is even more impressive. Without transaction costs, the standard rolling estimator of $\hat{\mu}_p$ yields $\hat{\lambda}_p$ values of 0.59 for $\eta = 1$, 0.61 for $\eta = 2$, and 0.63 for $\eta = 4$. The differences compared to naive diversification are statistically significant at the 1% level. The estimated performance fees range from 176 bp to 296 bp, which are also statistically significant. Expected turnover estimates (6%, 7%, and 9%) are higher than those of the 1/N strategy, but the differences are not substantial. Consequently, imposing transaction costs does not affect our conclusions. Similar results are obtained when using the estimator of $\hat{\mu}_p$ derived from the 4-factor risk model. Expected turnover estimates are 3 to 6 times lower than those produced by the rolling estimator of $\hat{\mu}_p$, consistent with the 10 Industry dataset.

In Panel C, the MVE strategies exhibit less competitive performance in this dataset. The TP strategy shows extreme results, while the MV and OC strategies have estimated Sharpe ratios of 0.56 and 0.45, respectively, lower than the $\hat{\lambda}_p$ of 1/N.

5.3 Empirical Analysis and results for the Extension

To initiate the empirical analysis of Section 4.5, we download US daily individual asset returns data set and also daily market factor (Fama & French, 2015) returns over the period from January 16, 1978 to December 31, 2017, achieved from the Center of Research in Security Prices through Wharton Research Data Services. In this analysis, we choose a daily time frequency to better align the strategy with real-world portfolio management methods. However, in order to reduce turnover and transaction costs, we update the portfolio periodically. Instead of altering portfolio weights, this strategy entails holding a fixed number of shares each month. Our decision is consistent with the findings of De Nard et al. (2019), and it aids in striking a balance between collecting daily market changes and effectively managing trading expenses.

To maintain simplicity and consistency, we adhere to the commonly used convention that considers 21 consecutive trading days as one month. The out-of-sample analysis spans 480 months or 10,080 days, from January 16, 1978 to December 31, 2017. All portfolios, identified by investment dates $h = 1, ..., 480$, are updated monthly. We estimate the covariance matrix at each investment date h using the most recent 1260 daily returns, which roughly equates to a five-year historical data window, align with De Nard et al. (2019) and different from the length used elsewhere in this paper. This method ensures that we capture essential historical information while keeping the analysis small and meaningful for daily data set.

We obtain the annualized average return, standard deviation and information ratio of the GMV portfolio and the naive diversification strategy respectively by following the method described in Section 4.5 and De Nard et al. (2019); R. F. Engle et al. (2019).[3] The results are reported annualized as follows, the average return of the 1/N strategy is 13.9, larger then the average return of the GMV portfolio (12.7). However, the standard deviation of the AFM1-DCC-NL-estimated GMV portfolio is much smaller (8.1) than that of the $1/N$ approach (16.8). This decrease in volatility adds to the allure of the GMV method. The Information Ratio for the naive diversification ($1/N$) strategy is 0.8, while the GMV strategy employing the dynamic AFM1-DCC-NL estimate method obtains an excellent Information Ratio of 1.6. This indicates that the GMV approach outperform in terms of risk-adjusted performance. In conclusion, the

[3]We would like to thank Michael Wolf generously sharing their data and Matlab programming codes on his personal website. With his valuable resources, we were able to initiate the analysis faster and understand the method correctly.

AFM1-DCC-NL technique greatly improves the GMV portfolio's performance, exceeding the naive diversification strategy by a wide margin.

6 Conclusion

In conclusion, Our findings demonstrate the promising results of timing strategies (VT and RRT) in controlling turnover and improving performance specifically in the 10 Momentum dataset. our analysis favors mean-variance optimization over the 1/N strategy without transaction costs in the 10 Industry dataset, while MVE strategies in the Mkt/SMB/HML dataset remain inconclusive. We emphasize the crucial role of turnover control in strategy performance and stress the need for effective turnover management to enhance mean-variance strategies.

Moreover, we observe that the use of a 4-factor risk model enhances the performance of RRT strategies and reduces estimation risk in 10 Industry dataset. Additionally, the imposition of transaction costs has minimal impact on the effectiveness of VT and RRT strategies in both datasets, further supporting their efficacy. Overall, our results suggest that the incorporation of dynamic timing portfolio allocation strategies, particularly RRT with a factor model, can significantly enhance risk-adjusted returns compared to naive diversification. Successful implementation requires careful consideration of transaction costs and turnover control. Furthermore, our analysis employing the dynamic AFM1-DCC-NL estimate method reveals that the minimum variance portfolio can outperform the naive diversification strategy in terms of risk-adjusted performance, as evidenced by a higher Information Ratio. This emphasizes the importance of utilizing advanced estimation techniques to improve portfolio performance.

In summary, our study provides valuable insights into the effectiveness of mean-variance optimization, timing strategies, turnover control, and risk modeling. It highlights the potential for enhanced risk-adjusted returns through dynamic timing portfolio allocation strategies and emphasizes the significance of transaction cost management and turnover control for successful implementation. Additionally, the application of advanced estimation methods, such as the dynamic AFM1-DCC-NL estimate, demonstrates the superior performance of the minimum variance portfolio compared to naive diversification in terms of risk-adjusted metrics. These findings contribute to the understanding of portfolio management practices and offer valuable implications for practitioners and researchers in the field.

This study suggests several avenues for future research. Firstly, it is recommended to explore the performance of active portfolio strategies, including volatility timing and reward-to-risk timing, using additional datasets beyond those analyzed in this study. This would provide a broader understanding of the strategies' effectiveness across different market conditions and asset classes. Additionally, future research could investigate the impact of incorporating other risk factors or alternative risk models on the performance of the timing strategies. Furthermore, exploring the optimal tuning parameters for the timing strategies and conducting robustness tests would contribute to a deeper understanding of their potential. Lastly, it is advisable to investigate the applicability of the AFM-DCC-NL estimation method in different market environments and strategies, and evaluate its performance relative to alternative covariance estimation techniques.

References

Ang, A. & Kristensen, D. (2012). Testing conditional factor models. *Journal of Financial Economics*, *106*(1), 132-156. Retrieved from https://www.sciencedirect.com/science/article/pii/S0304405X12000694 doi: https://doi.org/10.1016/j.jfineco.2012.04.008

Avramov, D. & Chordia, T. (2006, 02). Asset Pricing Models and Financial Market Anomalies. *The Review of Financial Studies*, *19*(3), 1001-1040. Retrieved from https://doi.org/10.1093/rfs/hhj025 doi: 10.1093/rfs/hhj025

Bai, J. & Ng, S. (2002). Determining the number of factors in approximate factor models. *Econometrica*, *70*(1), 191-221. Retrieved from https://onlinelibrary.wiley.com/doi/abs/10.1111/1468-0262.00273 doi: https://doi.org/10.1111/1468-0262.00273

Carhart, M. M. (1997). On persistence in mutual fund performance. *The Journal of Finance*, *52*(1), 57–82. Retrieved 2023-05-07, from http://www.jstor.org/stable/2329556

Chincarini, L. B., Kim, D. & Daehwan, K. (2006). *Quantitative equity portfolio management: An active approach to portfolio construction and management*. McGraw-Hill.

Connor, G. & Korajczyk, R. A. (1986). Performance measurement with the arbitrage pricing theory: A new framework for analysis. *Journal of Financial Economics*, *15*(3), 373-394. Retrieved from https://www.sciencedirect.com/science/article/pii/0304405X86900279 doi: https://doi.org/10.1016/0304-405X(86)90027-9

DeMiguel, V., Garlappi, L. & Uppal, R. (2009). Optimal versus naive diversification: How inefficient is the 1-n portfolio strategy? *Review of Financial Studies*, *22*(5), 1915-1953. Retrieved from https://dx.doi.org/hhm075

De Nard, G., Ledoit, O. & Wolf, M. (2019, 01). Factor Models for Portfolio Selection in Large Dimensions: The Good, the Better and the Ugly. *Journal of Financial Econometrics*, *19*(2), 236-257. Retrieved from https://doi.org/10.1093/jjfinec/nby033 doi: 10.1093/jjfinec/nby033

Engle, R. (2002). Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models. *Journal of Business Economic Statistics*, *20*(3), 339–350. Retrieved 2023-06-30, from http://www.jstor.org/stable/1392121

Engle, R., Shephard, N. & Sheppard, K. (2008, 02). Fitting vast dimensional time-varying covariance models. *Journal of Business Economic Statistics*, *39*. doi: 10.1080/07350015.2020.1713795

Engle, R. F. (2000, May). Dynamic conditional correlation - a simple class of multivariate garch models. Retrieved from https://ssrn.com/abstract=236998 (UCSD Economics Discussion Paper No. 2000-09) doi: 10.2139/ssrn.236998

Engle, R. F. (2016, 08). Dynamic Conditional Beta. *Journal of Financial Econometrics*, *14*(4), 643-667. Retrieved from https://doi.org/10.1093/jjfinec/nbw006 doi: 10.1093/jjfinec/nbw006

Engle, R. F., Ledoit, O. & Wolf, M. (2019). Large dynamic covariance matrices. *Journal of Business & Economic Statistics*, *37*(2), 363-375. Retrieved from https://doi.org/10.1080/07350015.2017.1345683 doi: 10.1080/07350015.2017.1345683

Fama, E. F. & French, K. R. (1993). Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics*, *33*(1), 3-56. Retrieved from https://www.sciencedirect.com/science/article/pii/0304405X93900235 doi: https://doi.org/10.1016/0304-405X(93)90023-5

Fama, E. F. & French, K. R. (2015). A five-factor asset pricing model. *Journal of Financial Economics*, *116*(1), 1-22. Retrieved from https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2287202

Fan, J., Fan, Y. & Lv, J. (2008). High dimensional covariance matrix estimation using a factor model. *Journal of Econometrics*, *147*(1), 186-197. Retrieved from https://www.sciencedirect.com/science/article/pii/S0304407608001346 (Econometric modelling in finance and risk management: An overview) doi: https://doi.org/10.1016/j.jeconom.2008.09.017

Fan, J., Liao, Y. & Mincheva, M. (2013). Large covariance estimation by thresholding principal orthogonal complements. *Journal of the Royal Statistical Society. Series B (Statistical Methodology)*, *75*(4), 603–680. Retrieved 2023-06-18, from http://www.jstor.org/stable/24772450

Fleming, J., Kirby, C. & Ostdiek, B. (2003). The economic value of volatility timing using "realized" volatility. *Journal of Financial Economics*, *67*(3), 473-509. Retrieved from https://www.sciencedirect.com/science/article/pii/S0304405X02002593 doi: https://doi.org/10.1016/S0304-405X(02)00259-3

Jegadeesh, N. & Titman, S. (1993). Returns to buying winners and selling losers: Implications for stock market efficiency. *The Journal of Finance*, *48*(1), 65–91. Retrieved from https://doi.org/10.2307/2328882 doi: 10.2307/2328882

Kirby, C. & Ostdiek, B. (2012). It's all in the timing: Simple active portfolio strategies that outperform naïve diversification. *The Journal of Financial and Quantitative Analysis*, *47*(2), 437–467. Retrieved from http://www.jstor.org/stable/41499477

Ledoit, O. & Péché, S. (2011). Eigenvectors of some large sample covariance matrix ensembles. *Probability Theory and Related Fields*, *151*(1-2), 233–264. Retrieved from https://doi.org/10.1007/s00440-010-0298-3 doi: 10.1007/s00440-010-0298-3

Ledoit, O. & Wolf, M. (2011). *Nonlinear shrinkage estimation of large-dimensional covariance matrices* (IEW - Working Papers No. 515). Institute for Empirical Research in Economics - University of Zurich. doi: 10.1214/12-AOS989

Ledoit, O. & Wolf, M. (2015). Spectrum estimation: A unified framework for covariance matrix estimation and pca in large dimensions. *Journal of Multivariate Analysis*, *139*(C), 360-384. Retrieved from https://www.econ.uzh.ch/apps/workingpapers/wp/iewwp515.pdf

Merton, R. C. (1980). On estimating the expected return on the market: An exploratory investigation. *Journal of Financial Economics*, *8*(4), 323-361. Retrieved from https://www.sciencedirect.com/science/article/pii/0304405X80900070 doi: https://doi.org/10.1016/0304-405X(80)90007-0

Meucci, A. (2005). *Risk and asset allocation*. Springer. doi: 10.1007/978-3-540-27904-4

Sharpe, W. F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk*. *The Journal of Finance*, *19*(3), 425-442. Retrieved from https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1540-6261.1964.tb02865.x doi: https://doi.org/10.1111/j.1540-6261.1964.tb02865.x

Tu, J. & Zhou, G. (2011). Markowitz meets talmud: A combination of sophisticated and naive diversification strategies. *Journal of Financial Economics*, *99*(1), 204-215. Retrieved from https://www.sciencedirect.com/science/article/pii/S0304405X10001893 doi: https://doi.org/10.1016/j.jfineco.2010.08.013

A Appendix A: Additional Figures

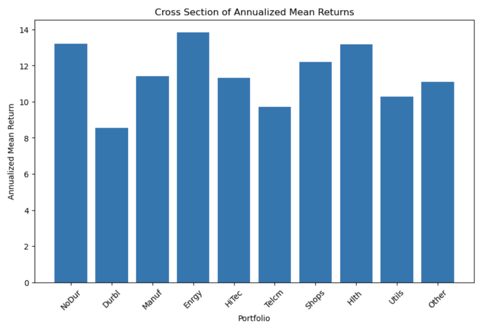
Figure 2 presents the mean return graphs and volatility graphs for 2 datasets we used in this paper, using the samples from July 1963 to December 2008, matching those reported by Kirby & Ostdiek (2012).

Figure 2: Reward and Risk Characteristics of 2 Portfolios

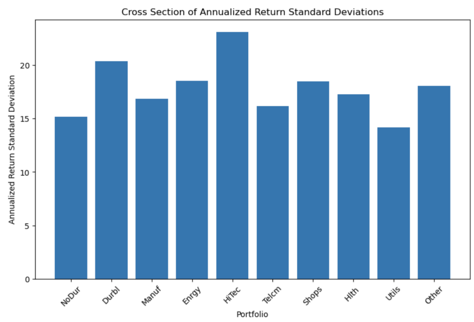
The figure presents a summary of the sample reward and risk characteristics for two datasets: the 10 Industry dataset (a and b) and the 10 Volatility dataset (c and d). Each graph panel consists of two graphs. The first graph in each panel displays the cross section of annualized mean returns, while the second graph shows the cross section of annualized return standard deviations. The sample period spans from July 1963 to December 2008, totaling 546 monthly observations. However, the reported statistics pertain specifically to the subset of observations 121 to 546, which were used to evaluate the out-of-sample performance of the portfolio strategies.

10 Industry Portfolios

(a) Mean Return

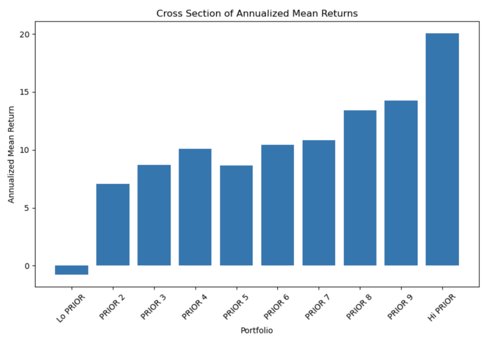


(b) Volatility

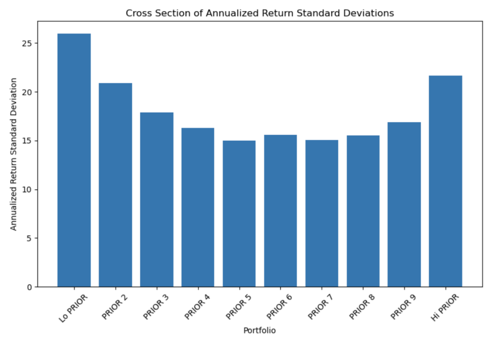


10 Momentum Portfolios

(c) Mean Return



(d) Volatility



B Appendix B: Brief description for programming codes used in this paper

In this paper, several programming codes were utilized to obtain the results presented in the tables and figures. The Matlab codes package provided by the authors of DeMiguel et al. (2009) was employed to calculate the summary statistics and estimate the conditional expected returns for three mean-variance efficient portfolios and the 1/N portfolio, as shown in Table 1. Detailed descriptions of these codes can be found in their respective documentation.

Additionally, the figures displayed in Figure 1 and Figure 2 were generated using a specific code snippet in the Python code file. The code comments provide further information on locating the relevant snippet for reproducing these figures.

Furthermore, various programming codes were employed to derive the results presented in Table 2 and Table 3. These codes encompassed a combination of the Matlab codes package from DeMiguel et al. (2009), Python codes, and R Studio codes. By examining the code comments, researchers can locate the specific code snippets used to calculate the annualized return mean, standard deviation, Sharpe ratio, average monthly turnover, p-value and utility fees.

Lastly, the results discussed in Section 5.3 were obtained using the Matlab codes package provided by Michael Wolf, one of the authors of De Nard et al. (2019). These codes come with detailed descriptions, facilitating the replication of the findings.

Overall, the employed programming codes, including those from published sources and custom-developed codes, played a crucial role in obtaining the empirical results presented throughout this study.