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Equal Weights, but not quite.

Partially Egalitarian Portfolio Selection in equity ETFs

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Abstract

In this paper I apply the Partially Egalitarian Portfolio Selection (PEPS) framework to equity Exchange Traded funds (ETFs). PEPS adds two penalty terms to Mean-Variance optimization that select and shrink asset weights to equality. Previous research finds PEPS is an improvement to both Equal Weights and Mean-Variance portfolios. The drivers behind this outperformance and whether the results hold up for asset classes other than stocks have not been researched yet. I fill this gap in the literature by applying PEPS to equity ETFs and analyzing PEPS' returns using the Fama French factors and portfolio distance to Equal Weights. In line with previous research, I find PEPS outperforms Equal Weights and Mean-Variance Weights. PEPS, however, underperforms the market. I find PEPS selects high-Market-beta ETFs exposed to the Momentum factor. Furthermore, I emphasize the importance of covariance shrinkage for PEPS and find that relatively stricter asset selection occurs in larger PEPS portfolios.

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1.0 Introduction

In this paper, I apply the Partially Egalitarian Portfolio selection (PEPS) to equity ETFs. PEPS is a novel way of selecting assets in a portfolio that outperforms the Equal Weights Portfolio and the Mean-Variance Portfolio (Peng & Linestky, 2022). I apply PEPS to Equity Traded Funds (ETFs) since previously it has only been applied to stocks. The main research question of this paper is whether and how PEPS' outperformance relative to the Mean-Variance and Equal Weights portfolios holds up for ETFs.

The PEPS framework is a model that aims to combine Markowitz's Mean-Variance model and Naive Diversification. The Mean-Variance (MV) model revolutionized the finance literature landscape as investors could model the tradeoff between risk and return in portfolio optimization. Out-of-sample performance of the MV model is poor however, due to the model's sensitivity to estimation error (Chopra & Ziemba, 1993, Britten-Jones, 1999). Naive Diversification generally outperforms MV portfolios and doesn't suffer from estimation error since each asset is given the same weight.

Partially Egalitarian Portfolio Selection is introduced by Peng & Linetsky (2022) and boils down to a Mean-Variance model with two regularization terms. These two terms select assets and then shrink surviving weights towards equality. The regularization terms of PEPS serve as a hedge for the estimation error often found in models such as Mean-Variance. The authors find higher Sharpe Ratios for PEPS compared to the Mean-Variance portfolio and the Equal Weights portfolio. Larger PEPS portfolios also perform better. The results of Peng & Linetsky (2022) are promising, however, the authors do not apply the model to other assets or determine what type of assets are selected.

I apply PEPS to liquid equity ETFs in the period 2007 to 2022. I analyze both the performance of PEPS as well as its selection of ETFs. I take a simplified approach to PEPS since I set the second penalty term of PEPS to an extreme value, ensuring surviving weights are set to equal weights. I use the Equal Weights portfolio as the benchmark since PEPS and Equal Weighting only differ in selection. Selection is proxied using the distance in portfolio weights between two portfolios. I use the CAPM, Fama French 3, 5, and 6-factor regressions to determine what drives PEPS returns and compare the coefficients to the benchmark portfolio. To investigate the importance of covariance shrinkage to PEPS, I differentiate PEPS portfolios between portfolios using an estimated covariance matrix and the Ledoit-Wolf (2004) covariance matrix.

In line with Peng & Linetsky (2022), I find that PEPS' Sharpe Ratios are higher than those of the Equal Weighted portfolio and the Mean-Variance portfolio. I find PEPS' outperformance is the result of selecting high-market-beta ETFs that have previously performed well and continue to do so. The coefficient for the Market factor is significantly positive in all regressions. Coefficient estimates for PEPS exceed 1 whilst for

Equal Weights these fall below 1. The Momentum factor is significantly positive for all but the smallest PEPS portfolio whilst Equal Weights has no significant Momentum exposure.

Furthermore, I find that all PEPS portfolios have negative Fama French alphas. These alphas indicate that PEPS does not reap enough returns given its risk exposures. The alphas are likely caused by underperformance to the market and the Fama French factors capturing less variation in excess returns for larger PEPS portfolios. Lastly, I emphasize the use of a shrunk covariance matrix since the estimated covariance matrix results in inefficient PEPS portfolios with few assets. PEPS portfolios with shrunk covariance matrix select relatively fewer assets as portfolio size increases. This phenomenon falls in line with the idea that as portfolio size increases, the number of redundant assets rises.

Finance literature benefits from this paper as a relatively novel method of portfolio selection is being analyzed leading to new insights. Previous research has extensively analyzed Equal Weights portfolios and Mean-Variance portfolios. Combining the strengths of both models whilst attempting to mitigate the weaknesses is why the PEPS model is so relevant. The analysis of a new model such as PEPS sparks the creation of future research.

The relevancy for investors lies in the possibility of PEPS' applicability in the real world. Investors want to stay ahead and continue outperforming other investors. Staying up-to-date with the newest models can help give these investors an edge. Furthermore, investors can benefit from the fact that this paper applies PEPS to ETFs. ETFs are a relatively new type of asset that has grown exponentially in popularity in the past decade. ETFs allow investors to cheaply diversify portfolios in one trade. Here investors can also gain an edge compared to other investors.

2.0 Literature Review

2.1 Mean-Variance & Equal Weights

One of the largest breakthroughs in finance literature is the Markowitz Mean-variance model. The model introduced the idea of how investors not only care about the expected returns of their portfolio but also the variance of these returns. This leads to a tradeoff between the portfolio's return and its risk. The Markowitz model can be summarized by the following equations:

$$\max_{w} w'\mu - \frac{y}{2} w'\Sigma w \quad (1)$$

$$\text{s.t.} \quad w'1 = 1 \quad (2)$$

Here '*w'* is a vector of asset weights within the portfolio, μ is a vector of expected returns. By taking the product of these two variables one obtains the portfolio's expected return. У is the investor's risk-aversion (assumed to be >0) and ∑ is the covariance matrix of asset returns in the portfolio. This matrix needs to be estimated since the true covariance matrix is unknown. The budget constraint (2) states the weights must add up to 1.

Naïve Diversification remains popular despite the presence of the Mean-Variance portfolio due to its superior performance out of sample (DeMiguel et al., 2009). Naïve Diversification gives each asset an equal weight. In contrast to more complex investment strategies such as the Mean-Variance model, Equal Weighting is a strategy requiring no extensive analysis or data collection. Complex strategies such as the Mean-Variance portfolio suffer from a bottleneck where an investor may not be able to acquire the necessary data to properly estimate a model's parameters.

Mean-Variance portfolios are vulnerable to estimation error (Chopra & Ziemba, 1993, Britten-Jones, 1999). This vulnerability is caused by the model's parameters being sensitive to changes in sample means (Best & Gauer 1991). DeMiguel et al. (2009) investigate the Mean-Variance's underperformance and treat Equal Weights as a benchmark. In-sample performance of complex strategies is often superior to the Equally Weighted portfolio, as per design. The main problem here is that out-of-sample, the sophisticated portfolio strategies tend to underperform the 1/N strategy. In line with previous research, estimation error is the cause. DeMiguel et al. (2009) proxy estimation error by comparing a strategy's in-sample Sharpe Ratio to its out-of-sample Sharpe Ratio. The estimation error is so big that "the gains that Mean-Variance strategies seem to make in-sample, are offset by estimation errors in the out-of-sample" (DeMiguel et al., 2009). The estimation window would need to be at least (an infeasible) 3000 months for the sample-based Mean-Variance strategy to outperform the 1/N portfolio based on certainty equivalent returns.

Kirby & Ostdiek disagree with DeMiguel et al. (2009) and argue that MV portfolios can outperform Equal Weights. The paper by DeMiguel et al. (2009) is generally seen as a critique of Mean-Variance optimization. Kirby & Ostdiek (2012 argue the methods of DeMiguel et al. (2009) favor the 1/N portfolio. The Mean-Variance portfolio's conditional expected returns are larger than that of the naive portfolio which, according to the authors "magnifies estimation risk and leads to excessive turnover" (Kirby & Ostdiek, 2012). A fair comparison between the strategies can only be done when conditional expected returns are equal. The results indicate (general) outperformance of the MV portfolio compared to the 1/N portfolio in the absence of transaction costs.

The idea that the 1/N portfolio outperforms more sophisticated portfolios is not a given fact as the portfolio has periods of underperformance (Kirby & Ostiek, 2012. Taljaard & Mare (2021) present an overview of the likely causes for this short-term underperformance. The first source given relates to the monthly rebalancing. Since markets tend to be subject to momentum, monthly rebalancing causes the Equal Weights portfolio to sell outperforming stocks that keep outperforming and buy underperforming stocks that keep underperforming. The second possible source occurs when stock volatilities are low or are heavily correlated with one another. Rebalancing highly correlated stocks to equal weighting doesn't result in true equal weighting in risk due to these correlations (Taljaard & Maré, 2019).

One way of solving the Mean-Variance's vulnerability to estimation error is applying shrinkage to the covariance matrix. Research has shown that the sample covariance matrix is a relatively poor estimator of the population covariance matrix in situations where the dimensions of the sample covariance matrix are large compared to the sample size (Ledoit & Wolf, 2012). Ledoit & Wolf (2003) applied shrinkage to the sample covariance matrix. This method shifts the most extreme values within the matrix towards a more centrally located value. Given that these extreme values tend to be the values that are most affected by estimation error, this shrinkage "systematically [reduces] estimation error where it matters most" (Ledoit & Wolf, 2003).

2.2 PeLASSO

PEPS draws its inspiration from Diebold & Shin's (2019) paper on forecast combinations. The Equal Weight puzzle is also present in forecast combination literature (Aruoba et al., 2019, Smith & Wallis, 2009) and is therefore relevant. Diebold & Shin's (2019) methodology later inspired Peng & Linetsky (2022) to apply the framework to portfolio weights.

Diebold & Shin (2019) argue Equal Weights useful direction for weight shrinkage due to its relatively good out-of-sample performance. Equal Weights, however, allow for redundant forecasts to enter the 'optimal' forecast combination. The authors argue that it is better to leave certain forecasts out of the combination since the information they hold is already present in either the other individual forecasts or the combination of forecasts. To achieve this selection, a filter needs to be added setting the weights of these forecasts to 0 whilst shifting the remaining weights to the equal weights that perform so well.

Equation (3) displays a forecast optimization problem that aims to minimize the sum of squared errors of the forecast combination (Diebold & Shin, 2019). β_i is the weight given to forecast f_{it} at time t. y_t is the observed value at time t. A penalty term is introduced in equation (4), which penalizes the value of weights in the optimization. The effect of the penalty depends mainly on q; When q = 2 one obtains a Ridge penalty that only shrinks weights. When q=0 the penalty term selects and sets other weights to 0. Lastly when q=1 one obtains the LASSO (Least Absolute Shrinkage and Selection Operator) term which achieves shrinkage as well as selection (Tibshirani, 1996). The LASSO penalty is the first of two terms used in the PEPS framework.

$$\hat{\beta}_{\text{Penalized}} = \arg \min_{\beta} \sum_{t=1}^{T} \left(y_t - \sum_{i=1}^{K} \beta_{i} f_{it}\right)^2 \tag{3}$$

$$\sum_{i=1}^{K} |\beta_i|^q \leq c. \tag{4}$$

$$\hat{\beta}_{\text{Penalized}} = \arg \min_{\beta} \left(\sum_{t=1}^{T} \left(y_t - \sum_{i=1}^{K} \beta f_{it}\right)^2 + \lambda \sum_{i=1}^{K} |\beta_i|^q\right) \tag{5}$$

To obtain a LASSO model that selects and shifts the weights towards equal weights an extra penalty term must be added. As can be seen in equation (6), the newly added term penalizes deviations of surviving weights from equal weighting (Equal weights depicted as $\frac{1}{p(\beta)}$), and as a result, shrinks the weights towards equal weighting. Diebold & Shin (2019) name this penalized LASSO regression the partially-egalitarian LASSO (peLASSO) regression. The authors find evidence that peLASSO outperforms equal weighting and performs similarly to the best out-of-sample forecaster. Lastly, the authors advise that in the peLASSO regression, the penalty term for shrinkage should be extreme and the penalty term for selection should be high. These penalties result in relatively few forecasts surviving, avoiding redundant forecasts, and ensuring equal weights of surviving forecasts.

$$\hat{\beta}_{\text{peLASSO}} = \arg \min_{\beta} \left(\sum_{t=1}^{T} \left(y_t - \sum_{i=1}^{K} \beta_i f_{it} \right)^2 + \lambda_1 \sum_{i=1}^{K} |\beta_i| + \lambda_2 \sum_{i=1}^{K} \left| \beta_i - \frac{1}{p(\beta)} \right| \right) \tag{6}$$

2.3 PEPS

Peng & Linetsky (2022) build upon the peLASSO methodology by applying it to stocks and portfolio formation. Their formulation of the peLASSO is given (in vector notation) in equation (7). The first two terms of the equation present the Markowitz mean-variance model from equation (1). Peng & Linetsky (2022) use this model as their base and add the two penalty terms that select and shrink to equal weights with the budget constraint adding weights up to 1 (Equation 8)

$$\min_{w} \quad \frac{\gamma}{2} w' \Sigma w - w' \mu + \lambda_1 \|w\|_1 + \lambda_2 \left\| w - \|w\|_0^{-1} 1 \right\|_1 \tag{7}$$

$$\text{s.t.} \quad w' 1 = 1 \tag{8}$$

$$w_i \geq 0, i = 1, \dots, N \tag{9}$$

The first penalty term, $\lambda_1 \parallel w \parallel_1$, is the LASSO term applied to portfolio weights. However, when setting a short-sale constraint (Equation 9) this penalty becomes redundant (Peng & Linetsky, 2022) and can be left out.

The second penalty term's formulation, $\lambda_2 \parallel w - \parallel w \|_0^{-1} 1 \|_1$, is different from the Diebold & Shin (2019) paper but works the same way. The term calculates the amount of surviving weights and then computes what the equal weights should be for these surviving stock weights. Then the deviations from these equal weights are penalized by the λ_2 term. A higher λ_2 term therefore shifts the surviving weights toward this equality of weights whilst a lower λ_2 term makes the surviving weights shift more toward Mean-Variance weights.

The PEPS framework outperforms the 1/N portfolio and the Mean-Variance portfolio. Using historical means for the expected return, Sharpe Ratios of PEPS are always higher than MV portfolios. The same goes for the 1/N portfolio but here the differences are marginal. Peng & Linetsky (2022) present two ways the PEPS model can be implemented, a 2-step approach and a 1-step approach. The usage of the 1-step approach resulted in superior Sharpe Ratios. Lastly, the authors conclude that inserting a predictive model using momentum and a reversal factor rather than using historical mean into the expected return variable, significantly improves the PEPS returns.

2.4 ETFs

An exchange-traded fund (ETF) is a fund that is traded on exchanges. ETFs consist of a collection of assets that can be bought and sold similarly to regular stocks. ETFs are either active or passive. The passive ETF attempts to replicate an index, whilst the active ETF has a manager picking assets to include in the portfolio (Simpson, 2022). There is also a variety of types of ETFs including Bond/Fixed-income ETFs, Commodity ETFs, Currency ETFs, and more. Equity ETFs, baskets of stocks, are the most common type of ETF. ETFs allow investors to diversify into hundreds or even thousands of companies with a single trade.

ETFs have known an exponential increase in popularity. In 1990 the first ETF was introduced in Canada. Three years later the US followed. The number of ETFs has grown rapidly along with the total value of ETF assets: In 2009 the total amount of ETF assets stood at $1 Trillion, in 2018 this number reached $6.5 trillion. The ETF has become so popular because it allows investors to cheaply diversify their portfolios (History of ETFs | Vanguard Canada, z.d.).

Angel et al. (2016) find that ETFs are not always as cheap to trade as they may seem. The authors note that since ETFs are a collection of assets, the price of ETFs is bound by what assets they hold. The price of an ETF is therefore determined by the market. In the scenario that the price of the ETF is below the value of its assets, investors arbitrage the difference away. Da & Shive (2018) however, finds that an ETF can increase comovement among the assets it holds, to the point where non-fundamental shocks can be sent through the ETF to the stocks. ETFs are therefore not always bound to the assets. Angel et al. (2016) do note that ETF prices are not exact representations of their assets since arbitrage is not always possible. This can happen when ETFs hold foreign assets that are not traded at the same time as the ETF. This can also occur when transaction costs make arbitrage unworthwhile. For this reason, Angel et al. (2016) use the difference between the ETF price and the value of the underlying (measured as Net Asset Value) as a proxy for transaction costs. When deviations of this proxy exceed the Bid-Ask spreads of ETFs, investors pay higher transaction costs and are likely unaware. This is indeed found for smaller and less-liquid ETFs. These ETFs are less easily arbitraged. Lachance (2022) also linked trading costs to ETF liquidity as the author illustrates how older ETFs tend to have more developed secondary markets and are (usually) cheaper to trade than the underlying assets. The opposite tends to be true for younger ETFs.

3.0 Data

This section will describe the data selection in this paper. This includes which ETFs are chosen, acquiring Fama French Factors, and data patterns.

3.1 CRSP

Data on the ETFs is collected between 31-01-2000 and 31-12-2022 from CRSP's stock file. All securities with Share Code 73 are classified as an ETF. CRSP notes that American Trust Components could also be included in this database. Lipper classes were used to verify that all assets within the database were ETFs. To avoid ETFs that do not track equities, this paper will use the same ETF classification method Crego et al. (2022) used. Only Domestic Equity ETFs will be considered in the PEPS framework. A description of which Lipper classes belong to the Domestic Equity classification can be found in Table (1). A total of 622 unique ETFs belong to one of these classes in the sample.

Table 1: Included Lipper Classes

Lipper Class	Abbreviation	Lipper Class	Abbreviation
Absolute-Return Funds	ABR	Industrials Funds	ID
Precious Metals Equity Funds	AU	Long/Short Equity Funds	LSE
Basic Materials Funds	BM	Mid-Cap Funds	MC
Capital Appreciation Funds	CA	Micro-Cap Funds	MR
Consumer Goods Funds	CG	Natural Resources Funds	NR
Commodities Funds	CMD	Real Estate Funds	RE
Consumer Services Funds	CS	Small-Cap Funds	SG
Equity Leverage Funds	DL	S&P 500 Index Objective Funds	SP
Dedicated Short Bias Funds	DSB	Science & Technology Funds	TK
Equity Income Funds	EI	Telecommunication Funds	TL
Growth Funds	G	Utility Funds	UT
Growth and Income Funds	GI	Equity Market Neutral Funds	EMN
Health/Biotechnology Funds	H	Financial Services Funds	FS

Note: This table presents the Lipper Classes and their abbreviations of ETFs used in the sample.

CRSP provides monthly data on adjusted holding returns, share volume traded, and prices. The database uses the CUSIP-8 identifier for funds. CRSP calculates the price of an ETF by taking the closing price on the final trading day of the month. If no value is obtainable for this final day of the month, the price is set to the Bid/Ask Average. CRSP indicates the Bid/Ask Average as a negative value. CRSP then calculates the return by taking the value of a stock/ETF at time *t* and dividing it by the value of the stock/ETF at t-1. The database adjusts the return for cash dividends and potential splits/buybacks. In the case of a delisting, there is a missing value for the return variable on that month. The CRSP database provides a delisting return which only has a valid value on the month of delisting. This paper will not dig deeper into what the reason for delisting is. ETF liquidity is proxied by dollar trading volume.

3.2 Fama French Factors

Data on the Fama French factors is obtained from the Data Library at Dartmouth College Tuck School of Business for the period 2011-12 to 2022-12. The first factor, MKT-Rf, measures the difference between the return on the market portfolio and the risk-free rate. Here the risk-free rate is the 1-month T-bill rate from Ibbotson and Associates Inc. The Small-minus-Big (SMB) factor indicates the difference in returns between small and big-cap companies. The High Minus Low (HML) factor similarly represents the difference in returns between firms with a high book-to-market ratio and firms with a low book-to-market ratio (Fama & French, 1993). The Investment factor (CMA) presents the difference in returns between aggressive and conservative investment firms. The Robust-minus-Weak (RMW) factor indicates the difference in returns between firms that have robust profitability and weak profitability (Fama & French, 2015). Lastly, this paper will also include the Momentum factor (MOM). Momentum captures the difference in returns of buying assets that have positive historical returns, the winners, whilst shorting assets that have negative historical returns, the losers.

Table 2 presents the correlations of the Fama French factors over the sample. Most notable is that the Momentum factor is negatively correlated with all other factors including the Market. Furthermore, the CMA and HML are positively correlated (0.632).

Table 2: Fama French Factors Correlations

Factor	Mkt-RF	SMB	HML	RMW	CMA	MOM
Mkt-RF	1					
SMB	0.335	1				
HML	0.052	0.255	1			
RMW	0.012	-0.396	0.136	1		
CMA	-0.214	-0.017	0.632	0.180	1	
MOM	-0.387	-0.253	-0.333	-0.088	-0.036	1

Note: This table presents correlations of monthly Fama French factors between 2011-12 and 2022-12.

3.3 Data Patterns

Table (3) confirms that the number of ETFs in the sample increases over time. The sample sees its largest jump in 2007, when the number of unique ETFs reaches 100. From 2007 onwards there is generally an increase in unique ETFs. The highest count of unique ETFs is in 2017 reaching 378. After 2017 the number of ETFs declines. This drop is caused by the sampling in this paper as firstly, the total amount of ETFs is far larger than 378, and secondly, the count of total ETFs hasn't stopped increasing.

Nearly all the largest ETF losses occur in March 2020 due to the covid-19 crash (Mazur et al., 2021). Most ETFs that lost significantly rebounded in the following months. Most notable is Direxion Daily S&P Oil & Gas Exp. & Prod. Bull 2X Shares (GUSH). In March 2020 this ETF lost 96%. A month later the same ETF gained 162%. Despite this extreme rebound, this ETF has not reached pre-COVID price levels yet. ProShares Ultra VIX Short Term Futures ETF did the opposite, in March 2020 this ETF gained 155% during the COVID crash in March (and later lost its gains). This paper will not leave out these returns since COVID-19 is a market-wide event. Comparing portfolio performance is not an issue given that the event affects each portfolio similarly. Leaving these returns out of the sample could potentially even bias the results. Additionally, despite the crash, mean returns are positive in 2020. This is a contrast compared to 2008 where the returns are less extreme but mean returns are strongly negative.

2016 is a volatile year for ETFs also. More specifically almost all top gainers in 2016 are ETFs by Direxion that seek to follow either gas and oil drilling exploration firms or gold and silver mining exploration firms. The gains are possibly related to the gold and oil prices rising sharply in the first half of 2016. These gains are generally lost in the following years.

Table 3: Summary Statistics Sample

Year	Nr. ETFs	Mean Return	σ	Max. Return	Min. Return
2000	9	0.20	7.42	18.24	-20.51
2001	9	-0.46	7.18	19.12	-24.91
2002	9	-1.53	6.72	24.77	-16.69
2003	9	2.16	4.30	13.09	-6.69
2004	19	1.15	3.30	9.74	-8.48
2005	20	0.71	4.03	16.78	-12.60
2006	22	1.24	4.18	26.50	-11.79
2007	113	0.53	5.86	20.68	-18.70
2008	165	-3.08	13.27	51.90	-58.96
2009	199	0.49	13.74	60.74	-66.59
2010	245	0.48	11.28	44.27	-34.79
2011	280	-0.81	11.34	92.77	-53.88
2012	291	0.43	8.89	81.82	-50.91
2013	279	0.61	8.07	60.75	-37.79
2014	287	-0.09	9.19	95.80	-66.67
2015	340	-0.60	10.28	87.29	-59.97
2016	378	0.74	12.49	143.12	-69.33
2017	375	0.97	6.47	60.04	-47.91
2018	378	-0.61	9.73	92.90	-89.65
2019	349	1.54	9.48	65.04	-45.70
2020	361	1.12	15.77	162.77	-97.41
2021	311	1.14	8.06	124.72	-40.78
2022	289	-1.68	11.74	72.56	-74.19

Note: This table presents summary statistics for the entire sample. The mean return is the geometric mean in %, σ is the standard deviation of returns of all ETFs within a given year in %.

4.0 Methodology

This section will present the methodology employed in this paper. It should be noted that to obtain the PEPS portfolio weights, a large part of the methodology will follow that of Peng & Linetsky (2022). Nonetheless, every part will be described.

4.1 Asset Selection

To minimize the potential effects of transaction costs, this paper will use the most liquid ETFs in the PEPS model. An ETF's liquidity is determined by a rolling average liquidity variable. This variable is calculated by taking the product of the *Volume* and the *Price or Bid/Ask* variables and then on a given month *t*, taking the previous 12 values to compute the average for month *t*. The same method is used for the calculation of the historical mean and covariance matrices. The historical mean and covariance matrix use the previous 24 and 30 returns respectively.

To generate out-of-sample returns, the values at month *t* are skipped for these calculations. Skipping the current month essentially means that the PEPS weights will be calculated on the last day of the previous month. The ETFs are then held on the first trading day of the month until the last, leading to out-of-sample returns. ETFs that do not have the required 30 consecutive values of returns or the 12 consecutive values for both the Volume or Price or Bid/Ask variables on a given month, will not be considered in the PEPS framework for that month. A dummy is then created which takes on the value 1 if the rolling liquidity variable falls in the top N ETFs. Here N is the number of ETFs that will be considered in the PEPS framework, the trading universe. In this paper, N will be 20, 40, 60, 80 and 100 ETFs.

4.2 Equal Weights & Mean-Variance

The Equal Weights portfolio and Mean-Variance portfolios consist of the same ETFs as the PEPS portfolios. This means that the ETFs that will be included in the Equal Weighted portfolio require 30 months of consecutive returns data even though the covariance matrix is not a requirement here. All portfolios having the same trading universe means a fair comparison can be made.

The Equal Weighted portfolio can be formally modeled as such:

$$R_t = \sum_i^N (w_i R_{i,t}) \qquad i = 1, \ldots, N \tag{10}$$

$$\text{s.t.} \qquad w_i = \frac{1}{N}, \qquad i = 1, \ldots, N \tag{11}$$

Equation (11) already implies the weights add up to 1 to comply with a portfolio budget constraint.

I apply a maximum weights constraint to the Mean-Variance portfolio to avoid extreme weights. The MV portfolio (Equations 12, 13, and 14) is nearly the same as equation (1). Constraint (14) is added to prevent weights exceeding -1 and 1. The Mean-Variance model becomes more realistic as the portfolio avoids extreme concentrations in one ETF. This in turn improves diversification. The Mean-Variance model that will be used can be formally described as follows:

$$\max_w \qquad w'\mu - \frac{\gamma}{2} w'\Sigma w \tag{12}$$

$$\text{s.t.} \qquad \sum_i^N w_i = 1, \qquad i = 1, \ldots, N \tag{13}$$

$$-1 \leq w_i \leq 1, \qquad i = 1, \ldots, N \tag{14}$$

4.3 PEPS Framework

This paper will employ the same 1 step formulation that Peng & Linetsky (2022) used. Additionally, this paper will use the same mixed integer solver Gurobi (version 10). The model can be written as follows:

$$\min_{w,\, y, u,\, z,\, s,\, t,\, v} \frac{\gamma}{2} w'\Sigma w - w'\mu + \lambda_1 s + \lambda_2 t \tag{15}$$

s.t.

$$-Mz_i \leq w_i \leq Mz_i, \qquad i = 1, \ldots, N \tag{16}$$

$$\sum_{i=1}^N y_i z_i = 1, \qquad i = 1, \ldots, N \tag{17}$$

$$y_i = G, \qquad i = 1, \ldots, N \tag{18}$$

$$-u_i \leq w_i \leq u_i, \qquad i = 1, \ldots N \tag{19}$$

$$\sum_1^N u_i = s, \qquad i = 1, \ldots, N \tag{20}$$

$$-v_i \leq w_i - y_i * z_i \leq v_i, \qquad i = 1, \ldots N \tag{21}$$

$\sum_i^N v_i = t, \quad i = 1, \ldots, N$ (22)

$v_i \geq 0, \quad i = 1, \ldots, N$ (23)

$u_i \geq 0, \quad i = 1, \ldots, N$ (24)

$z_i \in \{0,1\}, \quad i = 1, \ldots, N$ (25)

$\sum_i^N w_i = 1, \quad i = 1, \ldots, N$ (26)

$w \geq 0, \quad i = 1, \ldots, N$ (27)

In this formulation w[i] is the weight of ETF i in the portfolio of month T. The variable M dictates the upper and lower bounds of the weight of each ETF. Values of M larger than 1 would indicate that weights over 1 and below -1 are possible. This paper avoids extreme weights by setting M to 1, similar to Peng & Linetsky (2022). Z[i] is a binary variable that indicates whether an ETF[i] will have a non-zero weight (takes on value 1) or a zero-weight (takes on value 0) Constraint (16) combines both variables to set w[i] to 0 when Z[i] equals 0.

4.3.1 The λ_1 Term

The λ_1 (Lasso) penalty term is implemented using constraints 19, 20, and 24. Constraint (19) sets the values for *u[i]*, which are summed up and ultimately penalized in variable *s* (Constraint 20). Since the model aims to minimize the penalty of *s*, values of *u* are set (close) to the values of *w*. Constraint (19) is essentially the same as setting $w_i = u_i$, which is the same as what is penalized in the Lasso penalty. Using the variable *u* alongside constraint (19) is more efficiently solved by Gurobi's mixed integer solver (Peng & Linetsky, 2022) than simply penalizing the sum of weights. Implementation of the short-sale constraint in equation (27) makes the λ_1 redundant and can therefore be left out when solving the model.

4.3.2 The λ_2 Term

The λ_2 penalty is implemented using constraints 17, 18, 21, and 22. Constraint (17) calculates the weight that every ETF with z = 1 would have under Equal Weights. For practical reasons, these weights are placed in variable y[i]. Variable y[i] does not guarantee that all y[i] are equal. Constraint (18) is introduced to enforce equality by giving all y[i] the same value *G*. Constraint (21) then calculates deviations of ETF weights to G and penalizes these deviations in equation (22).

It is crucial in constraint (21) to multiply each y[i] with z[i] as 0 weights mustn't be penalized for their deviations to the Equal Weights of surviving ETFs. To illustrate, out of 10 ETFS, 8 have a non-zero weight (z =1). ETF [1] has a weight set to 0, however, the model will set y[1] to ⅛. Multiplying y[1] with z[1] sets the penalty of ETF[1] to 0 and only allows for ETFs with a non-zero weight to be penalized for deviations from Equal Weights (v[i]) among these surviving ETFs.

Constraint (22) lastly sums deviations from Equal Weights in variable t which are then penalized by the $\lambda 2$ term. Not multiplying y[1] with z[1] would cause the penalty to increase by 1/8, its deviation from what the surviving ETFs' weights would be. The $\lambda 1$ term's selection would be (partially) offset as selecting ETFs would naturally increase the penalty of $\lambda 2$. An alternative approach would be to set a lower bound of 0 on v (Equation 23).

4.4 Covariance Shrinkage

This methodology will use the sample covariance matrix and the Ledoit-Wolf (2004) covariance matrix. The shrinkage proposed by Ledoit-Wolf (2004) boils down to finding a target covariance matrix and a sample covariance matrix. The target covariance matrix' mean squared error is fully biased but represents no variance. Conversely, the mean squared error of the sample covariance matrix has no bias but represents all variance (Ledoit-Wolf, 2004). Ledoit-Wolf shrinkage aims to find the weights these two covariance matrices need to have (adding up to 1) to best represent the true (unobservable) covariance matrix. This is therefore a tradeoff between bias and variance. This can be formally written as equation (28) where F is the target covariance matrix, S is the sample covariance matrix, and $\hat{\delta}^*$ is the optimal shrinkage weight.

$$\hat{\Sigma}_{\text{Shrink}} = \hat{\delta}^* F + \left(1 - \hat{\delta}^*\right) S \qquad (28)$$

Ledoit-Wolf (2004) use the identity matrix as the target matrix and subsequently estimate $\hat{\delta}^*$ using a quadratic loss function. The optimal shrinkage intensity is therefore a tradeoff between the quadratic loss of the sample covariance matrix and the shrunken covariance matrix. The minimization problem is displayed in Equations (29) and (30), where $\sum^*$ is the shrunken covariance matrix, $\sum$ is the true covariance matrix and v*I is the target matrix with weight ρ. Software packages such as Python's scikit-learn can efficiently solve this optimization problem.

$$\min_{\rho,\mathrm{v}} \mathrm{E}\,[\| \textstyle\sum^* - \sum \|^2] \quad (29)$$

$$s.t.\ \textstyle\sum^* = \rho v I + (1-\rho) S \quad (30)$$

4.5 Hyperparameter Tuning

For the PEPS framework, 3 hyperparameters can be optimized. The *λ1*, *λ2*, and the risk aversion term y. I follow Peng & Linestky's (2022) lead in treating the risk-aversion term as a constant and setting it to 5. I also follow Diebold & Shin's (2019) advice of setting the shrinkage term *λ2* to an extreme value, 9.

Giving the *λ2* an extreme value sets surviving weights to equal weighting. The *λ1* is then the only hyperparameter that needs to be tuned. For this hyperparameter, a larger variety of values can be explored whilst keeping the computation time to a feasible level. This is crucial for larger trading universes as the Gurobi Mixed Integer solver takes up considerable time to find solutions for large N for each month.

Hyperparameter tuning is done using rolling window cross-validation. The rolling window consists of a training window, a validation window, and a test date. The windows have fixed sizes. The training window consists of 38 months, the validation window consists of 19 months and the test date consists of one month. The Test month is the month for which the portfolio is held. For each iteration, all windows shift one month into the future. Trends that occurred far in the past will not affect the model's solutions. Additionally, trends unique to the training window have a lesser influence since the validation window filters them out.

For each combination of hyperparameters, the PEPS model is run for each date in the training window. Subsequently, the returns and the Sharpe Ratio of the portfolio are calculated for this training period. The hyperparameter combinations that give the 20 highest Sharpe Ratios are then applied to the validation set. The Sharpe-Ratio is calculated as follows:

$$Sharpe\ Ratio = \left(\frac{R_p - R_f}{\sigma_p}\right) \quad (31)$$

Where R_f is the mean risk-free rate, R_p is the mean return on the portfolio and the σ_p is the standard deviation of the portfolio's excess returns. It is a popular measure used in portfolio optimization to describe risk-adjusted returns (Sharpe, 1998) The higher the Sharpe ratio the better the risk-adjusted performance of the portfolio.

I derive Sharpe Ratio 95% confidence intervals using a stationary bootstrap for the validation window. The validation window is significantly shorter than the training window. As Peng & Linetsky (2022) note, calculating the Sharpe Ratio for such a small window introduces random noise. In their paper, they opted for a bootstrap using 12-month blocks within their validation window. Due to the smaller validation window in this paper, I use the stationary bootstrap (without blocks).

For a given combination of hyperparameters, the bootstrap is performed as follows: In the validation window, the excess returns are computed for each month using the PEPS model. This set of returns is the sample for the bootstrapping. Then for 300 iterations one of the returns is replaced randomly by a randomly chosen return from the bootstrap sample. Each time the Sharpe Ratio is computed. This leads to a distribution of 300 Sharpe Ratios. From here on out I follow Peng & Linetsky's (2022) method by choosing the hyperparameter combination that gives the highest lower bound of the 95% confidence interval of these Sharpe Ratios. The combination of hyperparameters that is obtained is then used in the model for the test month.

The first holding month of PEPS is December 2011. The final month the PEPS portfolio is held is in December 2022. December 2011 is used due to the requirement of having 100 ETFs that have 30 returns observations to calculate the sample covariance matrix. In this paper's sample, the 100-ETF mark is passed in 2007, adding thirty months lead to the first train portfolio being held in late 2009. Adding a further 57 months (training and validation window) would lead to the first test portfolio to be held in 2014. I am aiming for 10 years of returns data to analyze. I deemed 8 years of returns too little. For this reason, I choose to omit the training window for the first 2 years of the model. The first two years of returns are therefore calculated using all hyperparameter combinations in the validation window.

Table 4: List of strategies

No.	Description	Abbreviations
1	Equal weighting portfolio	1/N
2	Short-sale constrained Mean-Variance model with Ledoit-Wolf covariance matrix	MV(Constrained + Ledoit)
3	Short-sale constrained Mean-Variance model	MV(Constrained)
4	Mean-Variance model with Ledoit-Wolf covariance matrix	MV(Ledoit)
5	Mean-Variance model	MV()
6	Short-sale constrained PEPS model with Ledoit-Wolf covariance matrix	PEPS(Constrained + Ledoit)
7	Short-sale-constrained PEPS model	PEPS(Constrained)
8	PEPS model with Ledoit-Wolf covariance matrix	PEPS(Ledoit)
9	PEPS model	PEPS()

Note: This table presents each abbreviation of models used in this paper.

4.5 Portfolio Evaluation

This subsection will present the metrics that will be used to evaluate the PEPS portfolio. Besides the following metrics, the earlier presented Sharpe Ratio is also used to evaluate PEPS' performance.

4.5.1 Distance

How asset selection of PEPS portfolios differs from the Equal Weights portfolios can be measured using portfolio *Distance* (Equation 32) used in the paper by Plyakha et al. (2012). *T* is the number of dates, w_n^i and w_n^j are the respective portfolio weights to asset *n* of portfolio *i* and *j* at time *t*.

$$Distance_{i,j} = \frac{1}{T} \sum_{t=1}^{T} \sqrt{\sum_{n=1}^{N} \left(w_n^i - w_n^j\right)^2} \qquad (32)$$

Distance is calculated by taking the root of the sum of squared differences in portfolio weights for each ETF. The metric is then averaged over the evaluation period. It is a measure that indicates alikeness between two portfolios. Distances close to 0 indicate the weights are very alike, alternatively higher distances indicate big differences. Since the PEPS portfolio is advised to have strong penalties that, rather than shrink, set the surviving weights to equal weighting (Diebold & Shin 2019), the distances to the 1/N portfolio are likely to be of small sizes. Differences in Distances between portfolios are then ultimately determined by the selection of ETFs.

4.5.2 Fama-French Regressions

The return evaluation includes running 4 regressions: The CAPM, the Fama French 3 Factor, 5 Factor, and 6 Factor model. The regressions are presented respectively in Equations 33, 34, 35, and 36. With $R_t - R_{ft}$ being the excess return of the portfolio at time t.

$$R_t - R_{ft} = \alpha + \beta(R_{Mt} - R_{ft}) + \epsilon_t \tag{33}$$

$$R_t - R_{ft} = \alpha + \beta_1(R_{Mt} - R_{ft}) + \beta_2 SMB_t + \beta_3 HML_t + \epsilon_t \tag{34}$$

$$R_t - R_{ft} = \alpha + \beta_1(R_{Mt} - R_{ft}) + \beta_2 SMB_t + \beta_3 HML_t + \beta_4 CMA_t + \beta_4 RMW_t + \epsilon_t \tag{35}$$

$$R_t - R_{ft} = \alpha + \beta_1(R_{Mt} - R_{ft}) + \beta_2 SMB_t + \beta_3 HML_t + \beta_4 CMA_t + \beta_4 RMW_t + \beta_5 MOM_t + \epsilon_t \tag{36}$$

The CAPM model is used to check whether a portfolio out/underperforms a benchmark, usually the market. This out/underperformance is presented by the alpha of the regression: A significantly positive alpha would indicate a portfolio's outperformance whilst a significantly negative alpha indicates underperformance.

The results of the Fama French regressions display which factors the PEPS portfolio is exposed to. A portfolio's exposure to a risk factor can be seen as the weighted average exposure for each asset to said risk factor. Each ETF's exposure is in turn caused by the exposures of the equities the ETF holds. A significant coefficient for the SMB factor would indicate that the PEPS' excess returns have significant exposure to the difference in returns between Small and Big-cap companies. The alphas in the Fama French Regressions portray the portion of returns that is not attributable to the risk factors, the abnormal return.

4.5.3 Sortino Ratio

The Sortino Ratio is a metric that measures the portfolio's returns vs. downside risk (Sortino & Van Der Meer, 1991). The formula is nearly identical to the Sharpe ratio except for the denominator. Rather than taking the standard deviation of excess returns, the standard deviation of the portfolio's negative excess returns ($\sigma_{pdownside}$) is used. This changes the Sharpe Ratio from a metric that measures a portfolio's risk-adjusted return to a metric that measures a portfolio's downside-risk-adjusted return. Similarly to the Sharpe Ratio, higher positive values indicate better performance. A loss-averse investor who cares about negative returns tails is more likely to use the Sortino ratio because it holds more information on these negative tails.

$$Sortino\ Ratio = \left(\frac{R_p - R_f}{\sigma_{pdownside}}\right) \tag{37}$$

5.0 Results

Table 5 presents the performance metrics of each investment strategy except MV. Table 15 depicts the Mean-Variance portfolios' performance. Since the MV portfolios are not the focus of this paper, the analysis is presented in the Appendix. The main takeaway from the MV portfolios is its poor risk-adjusted performance depicted by near-0 and/or negative Sharpe Ratios.

5.1 Equal Weights Performance

Equal Weights underperforms the market. The Equally Weighted 20 ETF portfolio outperforms its larger variants. This is displayed firstly in this portfolio having the largest mean monthly return, standing at 0.33%, and the highest annualized Sharpe Ratio among all 1/N portfolios (0.22). Increasing the number of ETFs to 40 or 60 lowers the average monthly return to 0.04% and 0.07% respectively. This drop in returns drives the Sharpe Ratios down to 0.03 and 0.06 respectively. All Sharpe Ratios fall below that of the market, 0.83.

ETFs introduced in the 21st through 60th most liquid category perform worse than the most liquid 20 ETFs. The relatively poor returns of the 40 and 60 ETF portfolios indicate that the 21st through the 60th most liquid ETFs for each month happen to consistently perform poorly, driving performance down compared to the most liquid 20 ETFs. Further increasing the number of ETFs in the portfolio improves both the Sharpe Ratio and the mean return, yet these remain below the level of the 20 ETF portfolio. The Sharpe Ratio of the 80 and 100 ETF portfolios are respectively 0.15 and 0.13.

Larger Equal Weights portfolios generally have lower risk. Increasing the number of ETFs has, apart from the 60 ETF portfolio, a negative effect on portfolio std. deviations. This trend is most notable when moving from the 20 ETF portfolio (5.23%) to the 40 ETF portfolio (4.93%). The remaining portfolio sizes have std. deviations around 4.3%. This trend is evidence that including more ETFs in the Equal Weights portfolio improves diversification. Diversification benefits stagnate when moving to portfolios larger than 60 ETFs as the std. deviations remain around 4.3%.

Table 5: Portfolio Performance

N	Portfolio	Sharpe Ratio	Sortino Ratio	Mean Return	σ	Max.	Min.	Kurtosis	Skewness
20	PEPS()	0.33	0.20	1.13	11.93	92.82	-25.76	27.18	3.71
40	PEPS()	0.23	0.14	0.80	12.12	92.82	-21.20	25.62	3.63
60	PEPS()	0.23	0.15	0.82	12.41	92.82	-21.20	23.11	3.39
80	PEPS()	0.21	0.13	0.73	12.33	92.82	-21.99	23.84	3.44
100	PEPS()	0.19	0.12	0.69	12.32	92.82	-25.43	22.93	3.17
20	PEPS(Constrained)	0.26	0.12	0.72	9.38	46.68	-25.76	5.39	0.99
40	PEPS(Constrained)	0.14	0.07	0.38	9.6	46.68	-21.2	4.49	1.05
60	PEPS(Constrained)	0.14	0.07	0.40	9.98	46.68	-21.2	3.83	0.99
80	PEPS(Constrained)	0.14	0.07	0.39	9.91	46.68	-21.99	3.92	1.00
100	PEPS(Constrained)	0.11	0.05	0.29	9.72	46.68	-25.43	1.49	0.42
20	PEPS(Ledoit)	0.42	0.16	0.68	5.72	22.59	-21.33	2.43	-0.38
40	PEPS(Ledoit)	0.44	0.17	0.75	5.91	22.31	-18.62	1.82	-0.3
60	PEPS(Ledoit)	0.32	0.13	0.54	5.72	20.09	-14.81	1.23	-0.2
80	PEPS(Ledoit)	0.30	0.13	0.52	5.98	21.83	-15.82	1.21	-0.1
100	PEPS(Ledoit)	0.37	0.17	0.73	6.79	28.65	-15.58	2.72	0.49
20	PEPS(Constrained + Ledoit)	0.41	0.16	0.67	5.73	22.59	-21.33	2.39	-0.38
40	PEPS(Constrained + Ledoit)	0.46	0.18	0.77	5.81	22.43	-17.12	1.74	-0.23
60	PEPS(Constrained + Ledoit)	0.30	0.12	0.49	5.66	18.42	-14.81	1.03	-0.25
80	PEPS(Constrained + Ledoit)	0.31	0.13	0.53	5.94	21.83	-15.98	1.29	-0.10
100	PEPS(Constrained + Ledoit)	0.34	0.16	0.67	6.75	28.65	-15.85	2.88	0.51
20	1/N	0.22	0.08	0.34	5.23	14.55	-18.18	1.84	-0.52
40	1/N	0.03	0.01	0.04	4.93	14.13	-21.43	3.43	-0.71
60	1/N	0.06	0.02	0.07	4.26	13.1	-17.93	3.52	-0.58
80	1/N	0.15	0.05	0.18	4.34	13.92	-16.86	2.88	-0.44
100	1/N	0.13	0.05	0.17	4.31	13.75	-16.64	2.97	-0.50
-	Market	0.83	0.37	1.03	4.31	13.65	-13.39	1.26	-0.47

Note: This table presents portfolio metrics over the period 12-2011 to 12-2022 for Equal Weights and covariance-shrunk PEPS portfolios. N is the trading universe. Mean Return is the geometric mean of excess returns in %. σ is the standard deviation of excess returns in %. The Sharpe Ratio is annualized. Max. and Min. are respectively the highest and lowest monthly returns over the entire sample. Metrics on the Market are derived from the Fama French MKT-RF factor.

5.2 PEPS and sample covariance matrix

PEPS portfolios using the sample covariance matrix select ETFs too harshly. The 100 PEPS portfolio selects at most 13 ETFs in a single month, whilst the constrained 100 ETF PEPS portfolio selects at most 11 ETFs. Weights of 1, 0.5, and 0.33 are the most common in all portfolios, regardless of the size of the trading universe. The latter is confirmed by the Distances to the Equal Weights in Table 8. All Distances are approximately 0.6.

PEPS is vulnerable to estimation error. The high concentration of portfolio weights means that portfolio diversification depends on the diversification of the ETFs. This is not inherently a disadvantage if these ETFs fully diversify risk. Nonetheless, an extreme portfolio weight of 1 implies that this asset provides the best return-risk tradeoff, better than any combination of assets. Extreme weights occur similarly in Mean-Variance portfolios that perform poorly out-of-sample due to estimation errors in the covariance matrix. The PEPS model using the same estimated covariance matrix and suffering from highly concentrated weights can therefore be interpreted as PEPS' vulnerability to estimation error.

The PEPS() and PEPS(Constrained) portfolio metrics in Table 5 are inflated by the COVID-19 rebound. The unconstrained PEPS portfolios all gain +92.8 % on 04-2020 by setting a portfolio weight of 1 in Direxion Daily Junior Gold Miners Idx Bull 2X Shs (JNUG), whilst the unconstrained portfolios assign a lower weight to this ETF (0.33). Due to the unprecedented nature of COVID-19, I can't ascribe the extreme returns post-covid to PEPS. For the analysis on PEPS() and PEPS(Constrained) portfolios, I omit the returns in February, March, and April 2020 in Table 6 to avoid the influence of the extreme rebound. The Covid-19 rebound continued in the following months. Comparison of metrics is only possible between PEPS() and PEPS(Constrained) portfolios here.

Larger PEPS portfolios with estimated covariance matrix have worse risk-adjusted performance. The Sharpe Ratios display a negative relation with portfolio size, caused by drops in mean returns. The 20 ETF PEPS portfolio has an annualized Sharpe Ratio of 0.23, moving to 60 ETFs already brings this ratio down to 0.11. The 100 ETF portfolio's Sharpe Ratio is 0.02. The same pattern occurs in the short-constrained portfolios. The drop in Sharpe Ratios is attributable to a similar decline in mean excess returns. The 20 ETF PEPS() portfolio's mean return is 0.58%, dropping step by step down to 0.06% for the 100 ETF PEPS() portfolio.

PEPS and short-sale constrained PEPS using the sample covariance matrix have identical performance. The omission of COVID-19 returns in Table 6 displays the similar nature of the PEPS() and PEPS(Constrained) portfolios. The constrained PEPS portfolio is either equally as good or better than its unconstrained counterpart. The differences in Sharpe Ratios are, however, marginal. The 20 PEPS(Constrained) portfolio's Sharpe Ratio is only 0.004 higher. For the 100 PEPS(Constrained) portfolio this difference is relatively larger (0.01).

Table 6: PEPS Excluding Feb., Mar. & Apr. 2020

N	Portfolio	Sharpe Ratio	Mean Return	σ
20	PEPS()	0.23	0.58	8.86
40	PEPS()	0.13	0.35	8.95
60	PEPS()	0.11	0.30	9.46
80	PEPS()	0.06	0.17	9.37
100	PEPS()	0.02	0.06	9.27
20	PEPS(Constrained)	0.23	0.59	8.85
40	PEPS(Constrained)	0.14	0.36	8.95
60	PEPS(Constrained)	0.11	0.30	9.46
80	PEPS(Constrained)	0.09	0.26	9.41
100	PEPS(Constrained)	0.03	0.08	9.12

Note: This table presents the Mean excess return, std. deviation (**σ**) of excess returns and the Annualized Sharpe Ratio for all months except February, March, and April 2020.

5.3 PEPS and Ledoit-Wolf covariance matrix

Applying Ledoit-Wolf shrinkage to the covariance matrix leads to less concentrated ETF selection. The main weakness of the previously discussed PEPS models is the extreme portfolio concentrations. Shrinking the covariance matrix causes the 100 ETF (Ledoit) portfolio to have months where it selects 55 out of the 100 ETFs. Months where the model selects 4 ETFs do still occur, but far less frequently. Mean distances range from 0.07 to 0.22 for PEPS portfolios with Ledoit-Wolf covariance matrix vs. 0.56 to 0.65 for PEPS portfolios with sample covariance matrix.

PEPS with Ledoit-Wolf shrinkage has better risk-adjusted performance than Equal Weights. Table 5 depicts how each PEPS(Ledoit) portfolio has a higher Sharpe Ratio than its equally weighted counterpart. The 40 PEPS(Ledoit) portfolio is the most efficient, having the highest Sharpe Ratio (0.44). This portfolio also improves the most compared to its 1/N counterpart, which has the lowest Sharpe Ratio among all 1/N portfolios (0.03). The Sortino Ratios tell the same story (0.01 to 0.17). Given that the ETFs that are introduced when moving from a 20ETF to a 40ETF 1/N portfolio perform relatively poorly. The 40 ETF PEPS(Ledoit) portfolio benefits the most by selecting well-performing ETFs from this group.

PEPS outperformance to Equal weights is driven by returns increasing more than risk. All PEPS portfolios have higher standard deviations of excess returns compared to Equal Weights. Equal Weights std. deviations range from 4.26% to 5.23% PEPS std. deviations range from 5.72% to 6.79%. The smallest relative jump in mean returns is that of the 20 ETF portfolio. This portfolio has double the mean excess return than that of Equal Weights (0.68% vs. 0.34%).

Larger PEPS portfolios select relatively fewer ETFs. Table 8 provides the means of Distances to Equal Weighting for PEPS portfolios. The Table displays how increasing the number of ETFs increases the average distance. This implies that the 20 PEPS(Ledoit) portfolio is more similar to the 20 ETF Equal Weights portfolio (0.068 average distance) than the 100 PEPS(Ledoit) portfolio is to the 100 ETF Equal Weights portfolio (0.216 average distance). Since differences in Distance are caused only by selection, the larger Distances for bigger PEPS sizes indicate harsher selection. Harsher selection can be attributed to the omission of redundant ETFs. Like forecast combinations, increasing the total number of forecasts increases the number of redundant forecasts. The 20 PEPS(Ledoit) has relatively few ETFs that do not benefit the risk vs. return tradeoff, the redundant ETFs. The lack of redundant ETFs is why the 20 ETF PEPS(Ledoit) is so similar to the 1/N portfolio. The 100 PEPS(Ledoit) portfolio has more redundant ETFs to omit, making it less alike to the 1/N portfolio. Figure 1 shows the bigger distances of larger PEPS portfolios persist over the entire sample. Figure 1 also shows how the 20 ETF PEPS(Ledoit) portfolio has months where it becomes the Equal Weights portfolio.

PEPS and short-sale-constrained PEPS are nearly identical portfolios. The short-sale constraint on PEPS(Ledoit) portfolios has no clear effect on performance. Distances to Equal Weighting are but for one instance all equal both the short-constraint and unconstrained PEPS portfolios. Sharpe Ratios differ only marginally. The 100 ETF short-constrained portfolio's Sharpe is 0.06 lower than the unconstrained portfolio. All other Sharpe Ratios differ less than this 0.06. The 40 (+0.02) and 80 (+0.01) ETF constrained portfolios have marginally higher Sharpe Ratios. The constrained PEPS and unconstrained PEPS do differ in asset selection at times. The 20 PEPS(Constrained+ Ledoit) has a different weight for an ETF 23% of the time. The 100 ETF portfolio differs 14.8% of the time. Differences in weights change portfolio performance marginally, therefore differences in these weights are likely small also. In a month where 60 ETFs have a non-zero weight, adding the 61st non-zero weight changes 61 weights that month. At the same time, the return of that month might only change by 0.001. The lack of differences between the portfolios is most likely caused by setting the λ_2 term to an extreme value, essentially preventing negative portfolio weights. The only difference then is which ETFs end up in the portfolio.

Larger PEPS portfolios do not always outperform smaller PEPS portfolios. Among the PEPS(Ledoit) portfolios, the Sharpe Ratios of the 20 (0.42) and 40 (0.44) ETF portfolios are the highest. The 60 and 80 ETF portfolios' annualized Sharpe Ratios are lower, respectively 0.32 and 0.30. The 100 ETF portfolio has a higher Sharpe (0.37) but remains below the smallest two portfolios. The Sortino Ratios follow the same pattern, indicating there is a U-shaped pattern in performance vs. portfolio size. The 100 ETF portfolios have approximately the same Sortino Ratio (0.17) as the 20 (0.16) and 40 (0.17) ETF portfolios but the 60 (0.13) and 80 (0.13) ETF portfolios are the lowest.

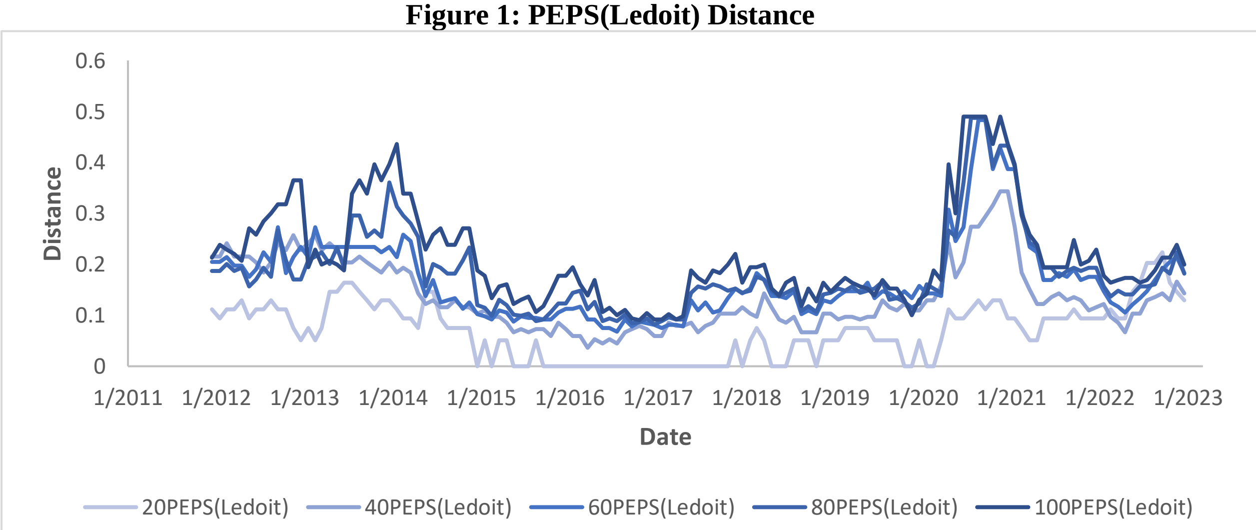
PEPS is vulnerable to market shocks. The most concentrated portfolio weights occur between 2020-04 and 2021-06, visible by the spike for all portfolios in Figure 1. These high weights are likely caused by the market distortion after COVID-19. After approximately 14 months the higher weights disappear. The spike after covid could have happened through three channels: The mean returns, the covariance matrix, and the selection of liquid ETFs.

Table 8: PEPS Distance to Equal Weights

N	Portfolio	Mean	σ
20	PEPS()	0.650	0.209
40	PEPS()	0.568	0.167
60	PEPS()	0.573	0.180
80	PEPS()	0.559	0.191
100	PEPS()	0.595	0.212
20	PEPS(Constrained)	0.642	0.207
40	PEPS(Constrained)	0.564	0.163
60	PEPS(Constrained)	0.569	0.176
80	PEPS(Constrained)	0.561	0.189
100	PEPS(Constrained)	0.590	0.207
20	PEPS(Constrained + Ledoit)	0.069	0.055
40	PEPS(Constrained + Ledoit)	0.138	0.068
60	PEPS(Constrained + Ledoit)	0.169	0.080
80	PEPS(Constrained + Ledoit)	0.183	0.085
100	PEPS(Constrained + Ledoit)	0.215	0.096
20	PEPS(Ledoit)	0.068	0.055
40	PEPS(Ledoit)	0.138	0.068
60	PEPS(Ledoit)	0.170	0.081
80	PEPS(Ledoit)	0.183	0.085
100	PEPS(Ledoit)	0.216	0.098

Note: This table presents the mean distance of each PEPS portfolio to Equal Weights. This means that the 20 ETF PEPS() portfolio's distance is computed compared to the 20 ETF 1/N portfolio, the 40 ETF PEPS compared to 40 ETF 1/N etc., with N being the number of ETFs considered. 'σ' is the standard deviation of distances over the entire sample.

Figure 1: PEPS(Ledoit) Distance



Note: This figure depicts the distance between PEPS(Ledoit) portfolios to Equal Weights. The numbers in front of PEPS(Ledoit) indicate the number of ETFs in the trading universe: as such 20PEPS(Ledoit) means the PEPS(Ledoit) portfolio with 20 ETFs.

5.4 Fama French Results

This section will present the results of performing Fama French Regressions on portfolio excess returns. Due to the inefficient nature of the MV portfolios, these will be left out. The same goes for PEPS portfolios without a shrunk covariance matrix. I shall occasionally refer to PEPS(Constrained+Ledoit) and PEPS(Ledoit) by simply PEPS in this section since the portfolios are alike. Apart from the Fama French 6-factor regression in Table 12, the results of the regressions are presented in the Appendix.

Table 12: Fama French 6 Full Sample

N	Portfolio	Constant	Mkt-RF	SMB	HML	RMW	CMA	MOM	R-squared
20	PEPS(Ledoit)	-0.6695**	1.2214***	0.1200	-0.1205	0.2446	0.2259	0.0197	0.8474
		(0.2266)	(0.0779)	(0.1145)	(0.1561)	(0.1557)	(0.2009)	(0.1158)	
40	PEPS(Ledoit)	-0.7664***	1.3337***	0.0444	-0.0156	0.0784	0.1059	0.3531**	0.8378
		(0.2324)	(0.0764)	(0.1119)	(0.1273)	(0.1313)	(0.1872)	(0.1245)	
60	PEPS(Ledoit)	-0.8909***	1.2756***	-0.0289	-0.2250*	-0.1717	0.1931	0.4367***	0.7733
		(0.2741)	(0.0888)	(0.1238)	(0.1253)	(0.1780)	(0.2254)	(0.1208)	
80	PEPS(Ledoit)	-0.9789***	1.3298***	-0.0124	-0.2169*	-0.1990	0.1618	0.5345***	0.7709
		(0.2757)	(0.0973)	(0.1314)	(0.1248)	(0.1695)	(0.2143)	(0.1308)	
100	PEPS(Ledoit)	-0.8962***	1.4343***	0.0246	-0.2332	-0.1996	0.1172	0.6507***	0.7076
		(0.3390)	(0.1398)	(0.1693)	(0.1647)	(0.2105)	(0.2739)	(0.1717)	
20	PEPS(Constrained + Ledoit)	-0.6847***	1.2269***	0.0996	-0.1228	0.2340	0.2388	0.0101	0.8471
		(0.2273)	(0.0780)	(0.1154)	(0.1578)	(0.1566)	(0.2029)	(0.1160)	
40	PEPS(Constrained + Ledoit)	-0.7531***	1.3377***	0.0284	-0.0374	0.0741	0.1276	0.3555**	0.8439
		(0.2222)	(0.0741)	(0.1067)	(0.1154)	(0.1261)	(0.1771)	(0.1211)	
60	PEPS(Constrained + Ledoit)	-0.9305***	1.2653***	-0.0272	-0.2408**	-0.1604	0.1846	0.4418***	0.7804
		(0.2664)	(0.0859)	(0.1212)	(0.1219)	(0.1688)	(0.2136)	(0.1194)	
80	PEPS(Constrained + Ledoit)	-0.9431***	1.3188***	-0.0184	-0.2015 *	-0.2072	0.1207	0.5265***	0.7735
		(0.2721)	(0.0976)	(0.1291)	(0.1215)	(0.1690)	(0.2145)	(0.1298)	
100	PEPS(Constrained + Ledoit)	-0.9457***	1.4230***	0.0444	-0.2406	-0.1977	0.1413	0.6449***	0.7053
		(0.3364)	(0.1404)	(0.1710)	(0.1638)	(0.2122)	(0.2723)	(0.1723)	
20	1/N	-0.8765***	1.1327***	0.0651	-0.0927	0.1636	0.1144	-0.0239	0.8871
		(0.1849)	(0.0587)	(0.0879)	(0.0876)	(0.1170)	(0.1404)	(0.0702)	
40	1/N	-1.0309***	1.0200***	0.1183	0.1212	0.0665	-0.0190	0.0142	0.8457
		(0.1998)	(0.0662)	(0.0963)	(0.1137)	(0.1332)	(0.1622)	(0.0651)	
60	1/N	-0.8676***	0.8993***	0.0997	0.0736	0.0533	-0.0322	0.0251	0.8661
		(0.1695)	(0.0554)	(0.0750)	(0.0882)	(0.1055)	(0.1310)	(0.0514)	
80	1/N	-0.7695***	0.9207***	0.1100	0.0458	0.0279	0.0127	-0.0189	0.8915
		(0.1522)	(0.0517)	(0.0699)	(0.0643)	(0.0984)	(0.1062)	(0.0472)	
100	1/N	-0.8003***	0.9262***	0.1112	0.0521	0.0333	-0.0011	0.0145	0.8981
		(0.1499)	(0.0521)	(0.0696)	(0.0603)	(0.0966)	(0.1040)	(0.0457)	

Note: This table presents the results of the portfolio's excess returns on the Fama French 6-factor model (Equation 36). Heteroskedasticity-robust standard errors are placed in parenthesis. Three levels of significance are considered: * : $p < 0.1$, ** : $p < 0.05$, *** : $p<0.01$

5.4.1 Equal Weights Exposures

The Equal Weights portfolios exhibit a neutral exposure towards the SMB, HML, RMW, CMA, and MOM factors. The Fama French 3-factor regression displays significantly positive coefficients for the SMB in the 80 and 100 Equal Weights portfolios. In the Fama French 5-factor regression, only the 80 ETF portfolio's SMB is significant. Lastly adding Momentum leads to no significant coefficients for SMB. The RMW and CMA are never significant. The HML factor is significantly positive in only one instance (40 ETF portfolio) in the Fama French 3-factor regression ($p<0.1$). This significance is not robust to Fama French 5 and 6-factor models. In conclusion, apart from the Market factor, there is no evidence that Equal Weight portfolios have exposure to any other Fama French factor considered.

Larger Equal Weights portfolios have more ETFs with a low market beta. In the Fama French 6-factor regression, the 20 ETF Equal Weights portfolio has an estimated Market coefficient of 1.13 ($p<0.01$). The 40 ETF portfolio has a slightly lower Beta of 1.02. The most important evidence for this claim is the Betas for the 60, 80, and 100 ETF portfolios that are respectively, 0.90, 0.92, and 0.93, all below 1. This pattern holds for all full-sample regressions. The Market coefficient indicates a portfolio's exposure to the market. A portfolio's total exposure is the average of its asset's exposures. Given that the exposure is lower for larger portfolios, this means that the assets added to the larger portfolios lower the average exposure to the market and therefore tend to have a market beta below 1.

All Equal Weights portfolios have negative abnormal returns. The CAPM regression confirms that over the entire sample, all Equal Weights underperform the market. This underperformance is implied by all portfolios having significantly negative alphas ($p<0.01$). The alphas in the Fama French 3, 5, and 6-factor regressions are all significantly below 0. The alpha in the Fama French regressions presents the portion of returns that cannot be attributed to the factors. Given that there is only significant market exposure, the negative alpha suggests that the Equal Weights portfolios have underperformed to what is expected for this market exposure. An investor would be better off passively investing in this Market-Exposure than using the Equal Weights portfolio.

Equal Weight portfolios' neutral exposure to HML and MOM over the entire sample is made up of a negative and a positive exposure that even out. Splitting the sample in half uncovers exposures to HML and Momentum. The HML is significantly negative in the first half of the sample for the 20 (-0.34), 40 (-0.24), 60 (-0.23) and 80 (-0.17) ETF portfolios. Furthermore, the sizes of the exposure estimates become closer to 0 with portfolio size to the point where the 100 ETF portfolio has an insignificant coefficient. In the second half of the sample, we see significantly positive HML exposures for the 40 (0.22), 60 (0.16), 80 (0.11), and 100 (0.09) ETF portfolios. Apart from the 20 ETF portfolio, the exposure estimate becomes closer to 0 with portfolio size. Apart from Momentum becoming significantly positive in the second half of the sample (coefficients become insignificant), the same pattern occurs for Momentum exposure.

5.4.2 PEPS Exposures

PEPS selects high-Market-beta ETFs. From the CAPM regression, Market beta estimates exceed 1 for all PEPS portfolios. In the Fama French 3, 5, and 6-factor the beta estimates are higher than that of the CAPM. In the Fama French 6 regression, a high Market beta indicates that a portfolio is more volatile than the Market. The 20 ETF PEPS(Ledoit) portfolio's estimate is 1.22. Similarly, the 100 ETF PEPS (Ledoit) portfolio's estimate is 1.43. These estimates contrast with the Equal Weights portfolios' beta estimates that tend to be slightly higher than 1 for the 20 and 40 ETF portfolios but fall below 1 for the 60, 80, and 100 ETF portfolios (Fama French 6 results). All Market Betas are higher for PEPS than for 1/N. The high Market Betas are therefore the result of omission of low-beta ETFs and selection of high(er) beta ETFs.

The PEPS model selects ETFs subject to momentum. This selection is evident from the significantly positive ($p<0.05$) Momentum coefficients for all but the 20 ETF PEPS portfolios in the Fama French 6 regression. The Momentum coefficients are 0.35, 0.44, 0.53, and 0.65 for the 40, 60, 80, and 100 ETF PEPS(Ledoit) respectively. The increasing coefficients for larger PEPS portfolios imply Momentum exposure is more present for larger PEPS portfolios. Larger ETF PEPS portfolios have access to the same ETFs as all smaller PEPS portfolios and more. The larger PEPS portfolios therefore have more ETFs with exposure towards the Momentum factor available. These are then ultimately chosen, driving up the estimated Momentum exposure. The positive sign of the exposure can be interpreted as follows: The PEPS portfolio consists of ETFs. These ETFs in turn consist of equities that drive the ETF's returns. The equities that have been indirectly selected by the PEPS portfolio collectively have performed well in the past and continue to do so in the holding period.

PEPS portfolios have negative abnormal returns. An efficient portfolio reaps returns equal to or larger than what is expected given its risk exposures. Fama French alphas are then 0 or positive. All PEPS portfolios display significantly negative alphas for all Fama French regressions. In the Fama French 6-factor regression, alphas range from -0.67 for the 20 ETF PEPS(Ledoit) portfolio to -0.98 for the 80 ETF portfolio. These alphas indicate that given the estimated risk exposures, the PEPS portfolios perform worse than what is expected. Similarly, a fund manager who underperforms its index has a negative alpha.

The negativity of alpha is related to PEPS' underperformance to the market. Market coefficients of PEPS portfolios tend to exceed 1, indicating the portfolio is more volatile than the market. The returns that are supposed to come along with this exposure are lower than what is expected. The CAPM alphas for PEPS portfolios are significantly negative are for all but the largest portfolios. All alphas are significantly negative after controlling for the Fama French. The annualized Sharpe Ratio of the Market factor (0.83) being higher than that of all PEPS portfolios confirms PEPS' underperformance.

PEPS' underperformance to the Market portfolio could be related to the sample drawn. The Equal Weights portfolio presents the returns of all assets in the trading universe. Given that the Equal Weights portfolio underperforms the Market, PEPS selection from a pool of underperforming ETFs would need to *extremely* efficient for the model to outperform the market.

PEPS' negative alphas can be the result of omitted risk factors. The Fama French factors capture PEPS returns less efficiently for larger portfolio sizes. The R^2 of the Fama French regressions decreases when moving from smaller PEPS to larger PEPS portfolios. The Fama French 6-factor regression capture around 85 – 89% of the variance of the Equal Weights portfolios. Meanwhile, the PEPS (Ledoit) portfolio's highest R^2 is 84%, decreasing to 70% for the 100 ETF PEPS(Ledoit) portfolio. This pattern indicates there might be other factors that better explain PEPS' returns, factors inherent to PEPS. These factors could explain a portion of PEPS' return that is attributed to alpha in the Fama French regressions.

I find insufficient evidence PEPS selects ETFs exposed to the HML factor. The regression coefficients for HML in the Fama French 3-factor regression are significantly negative ($p<0.01$) for the 60 (-0.32), 80 (-0.36), and 100 (-0.44) ETF PEPS(Ledoit) portfolios. The coefficients become more negative in the Fama French 5-factor regression, -0.42, -0.46, and -0.53 for the 60, 80, and 100 ETF PEPS(Ledoit) portfolios respectively. In the Fama French 6-factor regression, the 100 ETF PEPS(Ledoit) portfolio's HML coefficient loses all significance ($p = 0.108$) and the 60 and 80 ETF portfolios remain only slightly significant ($p<0.1$). The HML coefficients lose significance because the coefficient estimates decrease after the introduction of Momentum: all three estimates are between 0.21 and 0.24. The std. errors remain of similar sizes for the 60, 80, and 100 ETF PEPS(Ledoit) portfolios: 0.14, 0.14, 0.16 respectively in Fama French 5 vs. 0.13, 0.12, 0.16 respectively in Fama French 6. These results indicate that a large portion of the variance of PEPS excess returns that is attributed to the HML, is better explained by the Momentum factor.

Exposure to HML found over the entire sample can be attributed to exposure in the first half of the sample. The full sample Fama French 6-factor regression displays significantly negative HML coefficients for the 60 and 80 ETF PEPS portfolios. The Fama French 6 regression for the first half of the sample results in significantly negative coefficients for 40 (-0.26) and 100 (-0.42) ETF PEPS(Ledoit) portfolios ($p<0.1$) as well as the 60 (-0.32) and 80 (-0.44) ETF PEPS(Ledoit) portfolios ($p<0.05$). I can not attribute these negative exposures to PEPS' selection since the Equal Weights portfolios also tend to have significantly negative exposure toward HML. In the second half of the sample, HML is never significant for PEPS.

If PEPS systematically selects ETFs with negative HML exposure, larger portfolio sizes would have a similar or increasing exposure to the HML factor. Large portfolios have more ETFs to choose from and thus more ETFs with exposure toward HML. At the same time, larger PEPS portfolios select relatively fewer assets. Systematically selecting HML-exposed ETFs would show up strongest in the largest portfolios. The HML factor being insignificant for the 20 and 40 ETF portfolio, and significantly negative for the 60 and 80 ETF portfolio, falls in line with this argument. However, the 100 ETF PEPS portfolio having an insignificant HML exposure fails to provide evidence that PEPS selects 'low-value' ETFs despite the larger pool of ETFs. Given these results, I refrain from drawing conclusions regarding PEPS' asset selection and HML-exposed ETFs.

6.0 Conclusion

This paper applies the Partially Egalitarian Portfolio Selection (PEPS) model to equity ETFs from 2007 until 2022. The model's goal is to utilize the Equal Weights lack of exposure towards estimation error of model parameters whilst obtaining a higher return. Peng & Linetsky (2022) indeed find PEPS outperforms Equal Weights and MV weights but limit their sample to stocks. The main research question of this paper is whether and how PEPS' outperformance to Equal Weights and Mean-Variance weights holds up for ETFs.

The PEPS framework applied to ETFs has better risk-adjusted performance than the Equal Weights and Mean-Variance portfolios when applying the Ledoit-Wolf covariance matrix. Outperformance is best depicted by PEPS' superior Sharpe Ratios and Sortino Ratios. Not shrinking the covariance matrix leads to PEPS selecting ETFs too aggressively. Portfolio concentration becomes too high, with occasional weights of 100%, leaving the portfolio vulnerable to extreme returns. Additionally, in this paper's methodology, the short-sale constraint only marginally alters the PEPS portfolios. Neither portfolio are capable of outperforming the market-portfolio and have negative abnormal returns, apparent from significantly negative Fama French alphas.

I find evidence that PEPS portfolios select high-Market-beta ETFs that have historically performed well and continue to do so. By using a Fama French 6 Factor regression, I show that the 1/N portfolio has no significant Momentum exposure. The PEPS portfolios have significantly positive Momentum coefficients for all but the smallest PEPS portfolio. Similarly, the Market-Betas for the 1/N portfolios are either slightly above 1 (20 and 40 ETF portfolios), or below 1 (60, 80, and 100 ETF portfolios). The estimated Market-betas of the PEPS portfolios are all larger than the Equal Weights counterpart in each of the CAPM, Fama French 3, 5, and 6-factor regressions. The PEPS model therefore selects high(er)-beta ETFs and/or ETFs that have historically performed well and continue to do so.

The finding that the ETFs in PEPS portfolios are exposed to Momentum aligns with previous research by Apergis et al. (2022). The authors found that Momentum explained returns in factor ETFs. The authors present two possibilities. Momentum exposure could be explained by the fact that ETFs tend to pick equities that have historically performed better than other stocks. An alternative explanation is that ETFs do not diversify momentum exposure when selecting assets. Further research could investigate through which channels PEPS' selection of Momentum ETFs occurs. Using alternative measures for the expected return variable in the model could result in different Momentum exposures than when using historical mean return.

Lastly, I find that the PEPS framework selects relatively harsher when more ETFs are available. I display this phenomenon using the distance of portfolio weights between the PEPS portfolios and Equal Weights portfolios (1/N). The more ETFs are available, the larger the average distance to the 1/N portfolio. This falls in line with Diebold & Shin's (2019) goal for the model. The Equally Weighted portfolio would have redundant assets whose information is already present in other ETFs. When more ETFs enter the trading universe, less new information enters. This makes more ETFs redundant. These are then omitted from the PEPS portfolio. The harshest selection occurs directly after a Market-wide shock (COVID).

The results of this paper imply that when investors are doubting when picking an ETF investment strategy, they are better of picking PEPS over Equal Weights and Mean-Variance weights. The selection of assets by PEPS is efficient enough that it omits ETFs redundant for diversification that are present in Equal Weights portfolios, leading to better risk-adjusted returns. The market portfolio is in this paper superior to PEPS, but this could be related to the sample of assets drawn. Future research should apply PEPS to alternative ETFs samples or combine different type of assets to find outperformance to the market.

The methodology in this paper suffers from several inefficiencies. Firstly, I assume no transaction costs. ETFs are often wrongly assumed to be cheap to trade since trading costs are often larger than they seem (Angel et al., 2016). Due to computational restrictions, I simplify PEPS. I use a simplified grid search (set $\lambda 2$ to 9). I also do not implement Peng & Linetsky's (2022) algorithm that manually sets surviving weights to 0 and then resolves the model several times for each month to find more optimal solutions. These simplifications might have influenced the optimality of Gurobi's solutions, and ultimately have led to lower returns. Despite these inefficiencies, the PEPS model manages to outperform the 1/N portfolio and the Mean-Variance portfolio. Given unrestricted computational power, future research should seek to apply and analyze PEPS in its most optimal form.

7.0 References

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8.0 Appendix

8.1 Mean-Variance Performance

The main takeaway from Table 7 is that all MV portfolio variants perform poorly. This performance indicates the need for regularization. Poor MV performance is portrayed by (nearly) all Sharpe Ratios and Sortino Ratios being below 0. These negative Sharpe Ratios are caused by the negative mean returns. The 20 ETF portfolio having the best MV() Sharpe Ratio is the result of a lower standard deviation (10.6%) of returns accompanied by a higher, yet still negative, mean return (-0.76%).

The Mean-Variance portfolios are characterized by heavy tails in the return's distribution, portrayed by the Kurtosis. Each portfolio displays a Kurtosis exceeding 29. For the 20 ETF portfolio, this Kurtosis is caused by a relatively high maximum positive return (+87.6%). For all other MV portfolios, this Kurtosis is the result of negative returns exceeding -100. In this situation, shorted ETFs gain and long positions lose.

Short-sale-constrained MV portfolios have better negative tails and performance. The Sharpe Ratios are still negative but are closer to 0. Furthermore, all standard deviations of (excess) returns have increased. The mean return of the 20 ETF worsens by -0.18% whilst all other portfolios' improvements range between +2.2 and +4.2%. This has led to the 20 ETF portfolio shifting from having the highest Sharpe Ratio to having the lowest. Kurtoses are still higher than 11. Improved performance is the result of lower Min. returns and a strong COVID-19 rebound that made the portfolio gain more than +100%.

Shrinkage partially reduces risk in MV portfolios. The 40 and 60 ETF MV portfolios' Sharpe Ratios improve but remain highly inefficient (Sharpe Ratios < -0.6). All other portfolios' Sharpe Ratios worsen. The negative Sharpe Ratios are caused by negative mean returns, all smaller than -2% per month. The risk of the MV(Ledoit) portfolios is the lowest among all MV portfolios. This is the result of the 40, 60, 80, and 100 ETF portfolios' reduction in std. dev. and extreme returns (Kurtosis is lower). The negative tails are reduced by half. This does come at the cost of the positive tails also becoming smaller. The 20 ETF portfolio is a unique case in which both the std. dev increases and the mean return decreases when applying Ledoit-Wolf Shrinkage. The 20 ETF portfolio sees only its negative tail improve by 8.6%. This is evidence that there is a different effect of shrinkage for smaller and bigger portfolios.

Shrinkage in MV(Constrained) portfolios improves performance but extreme returns are back. Apart from the 40 ETF MV(Constrained + Ledoit) portfolio, all Sharpe Ratios and Sortino Ratios remain negative but are closer to 0. Mean returns improve across the board at the cost of higher std. deviations. The heavy tails that disappeared in the MV(Ledoit) portfolios are back (Kurtosis >11 for all portfolios & max returns > +100%)

Table 7: Mean-Variance Performance

N	Portfolio	Mean	σ	Sharpe Ratio	Sortino Ratio	Kurtosis	Skewness	Max	Min
20	MV()	-0.76	10.65	-0.25	-0.13	36.49	4.13	87.57	-38.97
40	MV()	-4.34	17.35	-0.87	-0.28	41.75	-2.38	100.92	-144.64
60	MV()	-3.79	14.39	-0.91	-0.29	30.76	-3.21	60.31	-115.76
80	MV()	-2.78	13.42	-0.72	-0.21	36.38	-4.31	46.43	-112.07
100	MV()	-2.98	12.96	-0.80	-0.24	29.82	-3.46	45.12	-104.53
20	MV(Constrained)	-0.94	17.37	-0.19	-0.15	14.78	3.12	106.37	-36.32
40	MV(Constrained)	-0.16	20.49	-0.03	-0.02	11.68	1.87	116.24	-83.41
60	MV(Constrained)	-0.07	18.13	-0.01	-0.01	13.67	2.04	116.24	-70.15
80	MV(Constrained)	-0.58	17.49	-0.12	-0.07	16.06	2.22	116.24	-70.15
100	MV(Constrained)	-0.47	17.22	-0.10	-0.06	16.39	2.49	116.21	-61.95
20	MV(Ledoit)	-2.07	14.36	-0.50	-0.34	13.58	2.74	87.46	-30.40
40	MV(Ledoit)	-2.64	13.26	-0.69	-0.27	6.87	-0.72	38.47	-75.45
60	MV(Ledoit)	-2.52	11.32	-0.77	-0.33	3.21	-0.29	28.06	-54.14
80	MV(Ledoit)	-2.84	10.65	-0.92	-0.38	3.75	-0.39	25.69	-52.79
100	MV(Ledoit)	-2.80	10.18	-0.95	-0.38	4.07	-0.55	25.33	-51.61
20	MV(Constrained + Ledoit)	-1.74	17.38	-0.35	-0.26	15.35	3.05	107.82	-38.78
40	MV(Constrained + Ledoit)	0.02	21.22	0.00	0.00	10.34	1.70	116.24	-85.60
60	MV(Constrained + Ledoit)	-0.14	18.87	-0.03	-0.01	12.97	1.48	116.24	-85.19
80	MV(Constrained + Ledoit)	-0.30	18.33	-0.06	-0.03	14.54	1.62	116.24	-83.94
100	MV(Constrained + Ledoit)	-0.07	18.12	-0.01	-0.01	14.71	1.88	116.21	-79.00

Note: This table presents portfolio metrics over the period 12-2011 to 12-2022 Mean-Variance portfolios. N is the trading universe. Mean Return is the geometric mean of excess returns in %. σ is the standard deviation of excess returns in %. The Sharpe Ratio is annualized. Max. and Min. are respectively the highest and lowest monthly returns over the entire sample.

8.2 Fama French Regressions

Table 9: CAPM Full Sample

N	Portfolio	Constant	Mkt-RF	R-squared
20	PEPS(Ledoit)	-0.5701*** (0.2017)	1.2143*** (0.0696)	0.8374
40	PEPS(Ledoit)	-0.5062** (0.2054)	1.2114*** (0.0927)	0.8002
60	PEPS(Ledoit)	-0.5904** (0.2811)	1.0890*** (0.0990)	0.6727
80	PEPS(Ledoit)	-0.6352** (0.3057)	1.1158*** (0.1016)	0.6459
100	PEPS(Ledoit)	-0.5012 (0.3606)	1.1915*** (0.1476)	0.5710
20	PEPS(Constrained + Ledoit)	-0.5850*** (0.2018)	1.2177*** (0.0696)	0.8372
40	PEPS(Constrained + Ledoit)	-0.4824** (0.2006)	1.2086*** (0.0918)	0.8032
60	PEPS(Constrained + Ledoit)	-0.6274** (0.2837)	1.0779*** (0.0981)	0.6724
80	PEPS(Constrained + Ledoit)	-0.6166** (0.3006)	1.1111*** (0.1007)	0.6501
100	PEPS(Constrained + Ledoit)	-0.5518 (0.3602)	1.1830*** (0.1481)	0.5697
20	1/N	-0.8406*** (0.1631)	1.1412*** (0.0454)	0.8825
40	1/N	-1.0406*** (0.1958)	1.0447*** (0.0608)	0.8346
60	1/N	-0.8731*** (0.1598)	0.9162*** (0.0507)	0.8601
80	1/N	-0.7982*** (0.1421)	0.9482*** (0.0422)	0.8841
100	1/N	-0.8108*** (0.1368)	0.9443*** (0.0414)	0.8922

Note: This table presents the results of the portfolio's excess returns on the CAPM model (Equation 33). Heteroskedasticity-robust standard errors are placed in parenthesis. Three levels of significance are considered: * : $p < 0.1$, ** : $p < 0.05$, *** : $p<0.01$

Table 10: Fama French 3-factors Full Sample

N	Portfolio	Constant	Mkt-RF	SMB	HML	R-squared
20	PEPS(Ledoit)	-0.5707***	1.2144***	-0.0023	0.0066	0.8374
		(0.2086)	(0.0686)	(0.0928)	(0.0742)	
40	PEPS(Ledoit)	-0.5221**	1.2269***	-0.0688	-0.0794	0.8038
		(0.2200)	(0.0864)	(0.1278)	(0.0845)	
60	PEPS(Ledoit)	-0.6037**	1.1132***	-0.0696	-0.3188***	0.7119
		(0.2642)	(0.0901)	(0.1374)	(0.0957)	
80	PEPS(Ledoit)	-0.6455**	1.1396***	-0.0593	-0.3617***	0.6907
		(0.2811)	(0.0945)	(0.1595)	(0.0984)	
100	PEPS(Ledoit)	-0.5062	1.2147***	-0.0413	-0.4368***	0.6200
		(0.3343)	(0.1359)	(0.1990)	(0.1101)	
20	PEPS(Constrained	-0.5895***	1.2205***	-0.0178	0.0114	0.8373
	+ Ledoit)	(0.2089)	(0.0687)	(0.0934)	(0.0747)	
40	PEPS(Constrained	-0.5022**	1.2276***	-0.0857	-0.0934	0.8085
	+ Ledoit)	(0.2131)	(0.0842)	(0.1251)	(0.0793)	
60	PEPS(Constrained	-0.6412**	1.1033***	-0.0726	-0.3383***	0.7173
	+ Ledoit)	(0.2601)	(0.0866)	(0.1360)	(0.0945)	
80	PEPS(Constrained	-0.6261**	1.1345***	-0.0563	-0.3623***	0.6955
	+ Ledoit)	(0.2735)	(0.0933)	(0.1569)	(0.0944)	
100	PEPS(Constrained	-0.5525 *	1.2028***	-0.0235	-0.4317***	0.6173
	+ Ledoit)	(0.3341)	(0.1366)	(0.1993)	(0.1086)	
20	1/N	-0.8418***	1.1426***	-0.0055	-0.0106	0.8826
		(0.1685)	(0.0485)	(0.0706)	(0.0551)	
40	1/N	-1.0197***	1.0237***	0.0911	0.1185*	0.8451
		(0.1887)	(0.0568)	(0.0745)	(0.0696)	
60	1/N	-0.8550***	0.9000***	0.0769	0.0596	0.8653
		(0.1590)	(0.0499)	(0.0541)	(0.0553)	
80	1/N	-0.7741***	0.9274***	0.1016**	0.0620	0.8911
		(0.1410)	(0.0423)	(0.0516)	(0.0440)	
100	1/N	-0.7882***	0.9251***	0.0952*	0.0518	0.8979
		(0.1369)	(0.0423)	(0.0524)	(0.0445)	

Note: This table presents the results of the portfolio's excess returns on the Fama French 3-factor model (Equation 34). Heteroskedasticity-robust standard errors are placed in parenthesis. Three levels of significance are considered: * : $p < 0.1$, ** : $p < 0.05$, *** : $p<0.01$

Table 11: Fama French 5-factors Full Sample

N	Portfolio	Constant	Mkt-RF	SMB	HML	RMW	CMA	R-squared
20	PEPS(Ledoit)	-0.6592***	1.2166***	0.1153	-0.1294	0.2412	0.2323	0.8473
		(0.2030)	(0.0738)	(0.1159)	(0.1316)	(0.1473)	(0.1812)	
40	PEPS(Ledoit)	-0.5811***	1.2476***	-0.0404	-0.1752	0.0173	0.2189	0.8072
		(0.2173)	(0.1008)	(0.1436)	(0.1321)	(0.1323)	(0.2089)	
60	PEPS(Ledoit)	-0.6617* *	1.1692***	-0.1338	-0.4223***	-0.2473	0.3329	0.7246
		(0.2693)	(0.1041)	(0.1504)	(0.1381)	(0.1843)	(0.2526)	
80	PEPS(Ledoit)	-0.6984**	1.1995***	-0.1408	-0.4584***	-0.2916	0.3329	0.7042
		(0.2817)	(0.1094)	(0.1725)	(0.1402)	(0.1880)	(0.2490)	
100	PEPS(Ledoit)	-0.5548*	1.2757***	-0.1317	-0.5271***	-0.3123	0.3255	0.6310
		(0.3339)	(0.1551)	(0.2204)	(0.1590)	(0.2323)	(0.3080)	
20	PEPS(Constrained + Ledoit)	-0.6794***	1.2245***	0.0972	-0.1274	0.2322	0.2420	0.8471
		(0.2039)	(0.0739)	(0.1165)	(0.1321)	(0.1485)	(0.1823)	
40	PEPS(Constrained + Ledoit)	-0.5665***	1.2511***	-0.0570	-0.1980*	0.0126	0.2414	0.8126
		(0.2093)	(0.0987)	(0.1390)	(0.1178)	(0.1279)	(0.1960)	
60	PEPS(Constrained + Ledoit)	-0.6986***	1.1576***	-0.1333	-0.4404***	-0.2369	0.3260	0.7295
		(0.2622)	(0.1010)	(0.1478)	(0.1298)	(0.1760)	(0.2401)	
80	PEPS(Constrained + Ledoit)	-0.6668**	1.1905***	-0.1448	-0.4393***	-0.2984	0.2892	0.7078
		(0.2771)	(0.1086)	(0.1694)	(0.1378)	(0.1861)	(0.2500)	
100	PEPS(Constrained + Ledoit)	-0.6072*	1.2658***	-0.1105	-0.5319***	-0.3094	0.3477	0.6291
		(0.3318)	(0.1553)	(0.2209)	(0.1572)	(0.2313)	(0.3057)	
20	1/N	-0.8891***	1.1385***	0.0708	-0.0819	0.1677	0.1068	0.8869
		(0.1738)	(0.0539)	(0.0858)	(0.0764)	(0.1136)	(0.1356)	
40	1/N	-1.0234***	1.0166***	0.1148	0.1148	0.0640	-0.0144	0.8456
		(0.1902)	(0.0618)	(0.0928)	(0.1015)	(0.1303)	(0.1590)	
60	1/N	-0.8544***	0.8931***	0.0937	0.0623	0.0490	-0.0242	0.8658
		(0.1625)	(0.0524)	(0.0710)	(0.0811)	(0.1030)	(0.1317)	
80	1/N	-0.7794***	0.9253***	0.1145*	0.0544	0.0311	0.0066	0.8913
		(0.1475)	(0.0482)	(0.0668)	(0.0566)	(0.0960)	(0.1056)	
100	1/N	-0.7927***	0.9227***	0.1077	0.0455	0.0308	0.0036	0.8980
		(0.1435)	(0.0490)	(0.0678)	(0.0558)	(0.0953)	(0.1073)	

Note: This table presents the results of the portfolio's excess returns on the Fama French 5-factor model (Equation 36). Heteroskedasticity-robust standard errors are placed in parenthesis. Three levels of significance are considered: * : $p < 0.1$, ** : $p < 0.05$, *** : $p<0.01$

Table 13: Fama French 6-factors first half

N	Portfolio	Constant	Mkt-RF	SMB	HML	RMW	CMA	MOM	R-squared
20	PEPS(Ledoit)	-0.3192	1.0053***	0.2130	-0.3142	0.2676	0.4776	-0.2132	0.6978
		(0.3457)	(0.1019)	(0.1706)	(0.2058)	(0.3235)	(0.3476)	(0.1417)	
40	PEPS(Ledoit)	-0.4313	1.0826***	0.1075	-0.2640*	-0.1145	0.1068	0.1118	0.7203
		(0.3024)	(0.1161)	(0.1358)	(0.1537)	(0.2275)	(0.2681)	(0.0954)	
60	PEPS(Ledoit)	-0.6100**	1.1696***	0.0699	-0.3251**	-0.1930	0.0109	0.2903***	0.7848
		(0.2772)	(0.1019)	(0.1203)	(0.1359)	(0.1976)	(0.2585)	(0.0796)	
80	PEPS(Ledoit)	-0.7400**	1.2906***	0.2464	-0.4416**	0.0065	0.1297	0.3875***	0.7787
		(0.2964)	(0.1090)	(0.1607)	(0.1816)	(0.1741)	(0.2666)	(0.0888)	
100	PEPS(Ledoit)	-0.6905*	1.3827***	0.3360	-0.4218 *	0.0501	-0.1130	0.4755***	0.7165
		(0.3979)	(0.1469)	(0.2096)	(0.2251)	(0.2449)	(0.3339)	(0.1138)	
20	PEPS(Constrained + Ledoit)	-0.3670	1.0174***	0.1764	-0.3073	0.2439	0.4823	-0.2235	0.7021
		(0.3468)	(0.1016)	(0.1721)	(0.2065)	(0.3225)	(0.3492)	(0.1428)	
40	PEPS(Constrained + Ledoit)	-0.4585	1.1008***	0.0792	-0.2363	-0.1255	0.1258	0.1511*	0.7376
		(0.2925)	(0.1114)	(0.1293)	(0.1498)	(0.2251)	(0.2690)	(0.0867)	
60	PEPS(Constrained + Ledoit)	-0.6676**	1.1585***	0.0976	-0.3392**	-0.1533	0.0057	0.3010***	0.7812
		(0.2824)	(0.1073)	(0.1221)	(0.1393)	(0.1943)	(0.2525)	(0.0805)	
80	PEPS(Constrained + Ledoit)	-0.7132**	1.2830***	0.2314	-0.3720**	-0.0007	0.0315	0.3804***	0.7876
		(0.2874)	(0.1080)	(0.1530)	(0.1644)	(0.1726)	(0.2556)	(0.0864)	
100	PEPS(Constrained + Ledoit)	-0.7269*	1.3403***	0.3436*	-0.4782**	-0.0363	-0.0199	0.4592***	0.7105
		(0.3922)	(0.1484)	(0.2084)	(0.2352)	(0.2654)	(0.3290)	(0.1135)	
20	1/N	-0.5318**	0.7625***	0.0180	-0.3355***	-0.0394	0.2447	-0.2744***	0.7593
		(0.2360)	(0.0621)	(0.1161)	(0.1291)	(0.1670)	(0.2174)	(0.0773)	
40	1/N	-0.6150***	0.4792***	0.0371	-0.2365**	-0.1032	0.0094	-0.1684***	0.7014
		(0.1851)	(0.0524)	(0.0930)	(0.1024)	(0.1546)	(0.1740)	(0.0618)	
60	1/N	-0.5565***	0.4757***	0.0473	-0.2265**	-0.1016	0.0934	-0.0743	0.7180
		(0.1800)	(0.0596)	(0.0766)	(0.0912)	(0.1402)	(0.1508)	(0.0561)	
80	1/N	-0.5348***	0.5293***	0.0611	-0.1747*	-0.0939	0.0347	-0.1418***	0.7692
		(0.1760)	(0.0606)	(0.0739)	(0.0908)	(0.1404)	(0.1582)	(0.0538)	
100	1/N	-0.5505***	0.5301***	0.0339	-0.1185	-0.0997	-0.0006	-0.0975**	0.8116
		(0.1544)	(0.0526)	(0.0607)	(0.0744)	(0.1171)	(0.1264)	(0.0442)	

Note: This table presents the results of the portfolio's excess returns on the Fama French 6-factor model (Equation 36) for the period 12-2011 to 06-2017. Heteroskedasticity-robust standard errors are placed in parenthesis. Three levels of significance are considered: * : $p < 0.1$, ** : $p < 0.05$, *** : $p<0.01$

Table 14: Fama French 6 Second Half

N	Portfolio	Constant	Mkt-RF	SMB	HML	RMW	CMA	MOM	R-squared
20	PEPS(Ledoit)	-0.7971**	1.3092***	0.0787	-0.0373	0.1880	0.1289	0.1694	0.9113
		(0.3130)	(0.0973)	(0.1517)	(0.1773)	(0.1838)	(0.2391)	(0.1631)	
40	PEPS(Ledoit)	-0.8816**	1.4164***	0.0277	0.0499	0.1090	0.1039	0.4891**	0.8879
		(0.4182)	(0.0961)	(0.1392)	(0.1574)	(0.2084)	(0.2498)	(0.2193)	
60	PEPS(Ledoit)	-1.1013**	1.3120***	-0.0507	-0.1984	-0.1365	0.2324	0.5198**	0.7775
		(0.5230)	(0.1231)	(0.2142)	(0.1744)	(0.2867)	(0.3071)	(0.2153)	
80	PEPS(Ledoit)	-1.1199**	1.3614***	-0.1721	-0.1090	-0.2953	0.1101	0.5928***	0.7810
		(0.5198)	(0.1345)	(0.2133)	(0.1726)	(0.2792)	(0.2794)	(0.2247)	
100	PEPS(Ledoit)	-1.0072	1.4739***	-0.1575	-0.1287	-0.2984	0.1041	0.7200**	0.7190
		(0.6359)	(0.1936)	(0.2454)	(0.2278)	(0.3363)	(0.3689)	(0.2954)	
20	PEPS(Constrained +	-0.7931**	1.3115***	0.0721	-0.0441	0.1885	0.1441	0.1635	0.9093
	Ledoit)	(0.3173)	(0.0981)	(0.1535)	(0.1804)	(0.1877)	(0.2436)	(0.1647)	
40	PEPS(Constrained +	-0.8432**	1.4146***	0.0127	0.0116	0.1066	0.1321	0.4701**	0.8869
	Ledoit)	(0.4139)	(0.0949)	(0.1350)	(0.1493)	(0.2072)	(0.2449)	(0.2200)	
60	PEPS(Constrained +	-1.1103**	1.3049***	-0.0744	-0.2078	-0.1479	0.2166	0.5166**	0.7879
	Ledoit)	(0.5048)	(0.1186)	(0.2092)	(0.1699)	(0.2761)	(0.2919)	(0.2119)	
80	PEPS(Constrained +	-1.0984**	1.3488***	-0.1609	-0.1118	-0.2837	0.0854	0.5914***	0.7802
	Ledoit)	(0.5150)	(0.1361)	(0.2118)	(0.1675)	(0.2778)	(0.2798)	(0.2217)	
100	PEPS(Constrained +	-1.0367	1.4690***	-0.1393	-0.1282	-0.2832	0.1177	0.7174**	0.7181
	Ledoit)	(0.6382)	(0.1943)	(0.2496)	(0.2277)	(0.3414)	(0.3688)	(0.2965)	
20	1/N	-0.8685***	1.2606***	0.0937	-0.0284	0.1252	0.0971	0.1204	0.9481
		(0.2310)	(0.0632)	(0.1029)	(0.0786)	(0.1428)	(0.1460)	(0.0903)	
40	1/N	-0.8302***	1.2116***	0.0780	0.2165**	-0.1623	0.0298	0.0515	0.9448
		(0.2316)	(0.0580)	(0.0925)	(0.0872)	(0.1401)	(0.1413)	(0.1038)	
60	1/N	-0.6665***	1.0488***	0.0321	0.1557**	-0.1546*	-0.0157	0.0174	0.9557
		(0.1925)	(0.0475)	(0.0685)	(0.0617)	(0.0913)	(0.1089)	(0.0656)	
80	1/N	-0.5593***	1.0611***	0.0684	0.1090**	-0.1431	0.0420	0.0039	0.9659
		(0.1670)	(0.0448)	(0.0589)	(0.0488)	(0.0933)	(0.0916)	(0.0627)	
100	1/N	-0.6170***	1.0666***	0.0950	0.0946**	-0.1210	0.0428	0.0382	0.9667
		(0.1630)	(0.0466)	(0.0637)	(0.0478)	(0.0973)	(0.0910)	(0.0603)	

Note: This table presents the results of the portfolio's excess returns on the Fama French 6-factor model (Equation 36) for the period 07-2017 to 12-2022. Heteroskedasticity-robust standard errors are placed in parenthesis. Three levels of significance are considered: * : p < 0.1, ** : p < 0.05, *** : p<0.01