ERASMUS UNIVERSITY ROTTERDAM

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Thesis MSc Policy Economics

FIRM GROWTH DYNAMICS DURING THE COVID-19 PANDEMIC -

AN UNCONDITIONAL QUANTILE REGRESSION APPROACH

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ABSTRACT

During the COVID-19 crisis, the government provided extensive economic support for firms to prevent job loss and market exit. Due to the size of this subsidisation, one can wonder: how did this affect firm dynamics compared to normal times? We answer this question by applying unconditional quantile regression to a large sample of Dutch firms between 2015-2020. We find that firm growth does not follow the random process, as suggested by Gibrat's Law. Rather, through quantile regression, we find heterogeneity across the growth distribution and between size classes. We find negative autocorrelations for slow-growing firms – past growth discourages future growth – and positive autocorrelation for fast-growing firms – past growth contributes to future growth. Moreover, we find that especially micro-firms (< 10 FTE) follow different growth patterns.

We do not find a major break in the firm growth dynamics in 2020 compared to 2019. Therefore, we turn towards two subsets of the firm population. Firstly, we repeat the estimation for cafés and restaurants – a severely affected sector by the pandemic. We find that the regular firm growth dynamics fall apart in 2020. Secondly, we execute the analysis for unsupported and supported firms. We find that the growth dynamics are similar; except for the first autocorrelation. The fastest growing, supported firms had a positive first autocorrelation, compared to a negative first autocorrelation of their unsupported counterparts. This indicates a possibility that this subset of firms used government support to further their growth. We conclude that the pandemic affected firm growth dynamics of specific subsets of the firm population.

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I. INTRODUCTION

Due to the COVID-19 pandemic, firms had to change their business activity dramatically and many received government support to soften the impact from the crisis by preventing severe economic effects. Perhaps paradoxically, unemployment and bankruptcy numbers reached a historic low in this time of crisis, especially compared to the decline in GDP (CBS, 2023a; CBS, 2023b). This apparent contradiction caused economists to voice concerns regarding the effects of providing such unprecedented widespread support for firms. Over three years, the Dutch government spent almost 80 billion euros on the COVID crisis, with approximately 35 billion euros going towards the two main support measures, and additionally, 20 billion euros in tax was deferred, of which the majority, but not all, will be repaid. (Algemene Rekenkamer, 2023). Though this spending reached its short-term goal of guiding firms through the pandemic and preventing job loss, one can wonder how this level of subsidisation affects the economy.

Regular business dynamism is shaped by the ‘Schumpeterian’ process of creative destruction: the least productive firms will lose out in the market, and eventually exit, whereas the most productive firms are able to profit and grow (Schumpeter, 1939; Schumpeter, 1942). This process can be disrupted, if government support is given to firms based on another factor than productivity, potentially causing a loss in productivity in the medium to long run. Recipients of subsidies can ‘hoard resources’, such as labour and capital, with an opportunity cost, as other firms may use these inputs more efficiently. Thus, following the logic by Schumpeter, market exit or firm shrinkage in a competitive market is required to keep the economy optimally productive. Researchers have already found that bankruptcies have at least been severely limited due to government support, specifically bankruptcies among the least productive firms (for instance Davies et al, 2023). Therefore, we wish to look at broad business dynamism through the lens of firm growth, gaining insights beyond the extensive margin.

Firm growth has been a bit of a theoretical mystery: Gibrat’s Law (1931) states that firm growth is a random process, though empirical analysis suggests that there are patterns in how firms grow, for instance related to the size of the firm and autocorrelation of growth. However, limited research has been done on firm growth dynamics in the Netherlands; to our knowledge none taking into consideration the strong heterogeneity in firm size – which is especially important, given the strong presence of micro-firms[1] in the Netherlands –, and using unconditional quantile regression (UQR) compared to conditional quantile regression (CQR). Quantile regression explains the quantile, rather than the conditional mean in ordinary least squares. Unconditional regression does not condition the quantile on the covariates, focusing on the population distributional properties rather than conditional distributions. This provides additional insights, specifically on what explains the

[1] Firms with less than 10 FTE units.

overall distribution of firm growth rather than the conditional firm growth – i.e., within-group effects. With this approach, we contribute to the existing literature by answering the question: how did the COVID-19 pandemic affect firm growth dynamics, across the distribution of firm growth? Additionally, by answering this question, we look at firm growth dynamics across the growth distribution outside of this exceptional crisis, specifically in firm growth across various size classes.

To analyse our research question, we discuss the literature in Section II about firm growth and the economic effects of the COVID-19 pandemic, explaining the differences with our approach. In Section III, we discuss the data we use – including descriptive statistics –, and our measurement choices. In Section IV, we elaborate on the methods we use to estimate our regression, unconditional quantile regression, and explain our specification. Then we report and explain our findings in Section V, to lastly, draw conclusions based on the aforementioned literature.

II. LITERATURE OVERVIEW

The question in this thesis relates to observing business dynamics and firm growth during the pandemic, drawing from two strains of literature. We will cover first the literature on firm growth during normal times, and then discuss the economic effects of the COVID-19 pandemic.

FIRM GROWTH

The relevant literature regarding firm dynamics aims to understand firm growth. Though many theories have been tested, contradictory evidence has complicated the theoretical frameworks surrounding firm growth.

One of the main theories on firm growth is Gibrat’s Law (1931), which stipulates that the expected proportional firm growth rate is independent of firm size in the examined period. This theory comes from the idea that *proportional* firm growth is a random process, where each firm – regardless of size – has equal opportunity to grow or shrink proportionally. Therefore, the distribution of proportional growth would be the same, regardless of size. In a theoretical model, if firm growth is seen as a shock rather than a process with a systemic pattern, the difference in firm size (i.e., firm growth) would only be explained by the shock. Thus, this theory relies on firm growth being a random process. An important early counterargument came from Mansfield (1962), who showed that smaller firms have relatively high exit rates and therefore, the small *surviving* firms have higher growth rates. Gibrat’s Law provides a simple framework for understanding firm growth, although in its empirical investigation, firm growth does not follow a random process.

Evidence on the relationship with firm size and age

Various surveys on this research find that the results on the effects of firm size on firm growth are not *entirely* consistent, and may depend on for instance the sector, country and subset of firms (Daunfeldt and Elert, 2013; Coad, 2012). Nevertheless, a large set of research seems to indicate that there is a negative relationship between size and growth (most notably, Dunne and Hughes, 1994; Evans, 1987a; Evans, 1987b; Kunmar, 1985). It is worth mentioning exceptions that find no such dependence between firm size and growth, such as Audretsch et al (2004) in the hospitality sector in the Netherlands, or even a positive relation between firm size and growth, such as Bentzen et al (2012) on a sample of Danish firms.

This relationship has been further investigated by the use of (conditional) quantile regression, finding heterogenous effects of firm size on growth across the distribution of growth (Coad and Hölzl, 2009; Distante et al, 2018; Leitão et al, 2010; 2017; Reichtstein et al, 2010). The results are not necessarily consistent: authors find that size can function as a ‘symmetric centripetal force’ across the growth distribution (Distante et al, 2018, p.9), encouraging growth for shrinking firms and limiting growth for fast-growing firms, causing size to moderate the existing firm growth distribution

(Distante et al, 2018; Reichstein et al, 2010). However, some found no significant effect over the growth distribution (Leitão et al, 2010), or the reverse relationship, where size negatively affects growth for the slowest-growing firms and encourages growth for the fastest-growing firms (Coad and Hölzl, 2009). The sign of the heterogenous effect of size on growth across the growth distribution thus differs across the various samples, though most papers seem to at least find heterogeneity in this effect. Coad and Hölzl (2009) find that the relationship between size and growth rates across deciles can even differ between size classes. They find that regardless of size class, firm size has a negative effect on growth *except* for the fastest-growing micro-firms[2], these experience a positive effect from size on their growth. Therefore, they show the importance of exploring heterogeneity in firm growth both along the distribution of growth *and* firm size.

In addition to size, studies have also investigated the relationship between age and firm growth. Most empirical inquiries find a negative relation, meaning younger firms experience higher growth than older firms – specifically among *surviving* young firms (Mansfield, 1962; Dunne and Hughes, 1994; Evans, 1987a; Evans, 1987b). These effects have also been estimated with quantile regression; notably Navaretti et al (2014) find a negative effect of age on growth, specifically for growing firms. Lotti et al (2003) look at entrants in the Italian market, and find that smaller, new firms grow quickly towards a size comparable to their larger counterparts, at which point the differences in growth patterns between larger and smaller firms subside. Thus, age seems to have a discouraging effect on growth; older firms are less likely to be the fastest growings, whereas young firms grow quickly to catch up with the market, or fail and exit.

Autocorrelation of firm growth

Gibrat's Law assumes that firm growth is a random process, implying no autocorrelation between growth rates. If every year a firm's proportional growth rate is random, there would be no significant pattern between the growth rate in period t – 1 and the growth rate in period t. Chesher (1979) pointed out that the estimation of the effect of size will be biased in case there is autocorrelation of growth rates, emphasizing the importance of researching both the flow and the stock. A positive autocorrelation implies that growth in one period is persistent towards the next; whereas a negative autocorrelation implies that growth in one period is followed by shrinkage.

Starkly varying rates of autocorrelation have been found. The initial important works found relatively high, positive autocorrelation, even reaching a third (Ijiri and Simon, 1967; Singh and Whittington 1975). However, on more extensive samples, these rates of serial correlation were more moderate and in itself did not predict a large part of the variance, or the autocorrelation was even found to be negative (Dunne and Hughes, 1994; Kumar, 1985).

[2] Firms with less than 10 FTE units.

In order to gain further insights in these contradictory findings, authors deployed methods to investigate whether the autocorrelation follows heterogenous patterns across the distribution of firm growth. Capasso et al (2009) find that persistently high-growth firms co-exist with firms experiencing a one-time extreme growth event. Specifically, it seems as if smaller firms follow more “lumpy” growth patterns, indicated by the negative autocorrelation, whereas larger firms grow in a smoother pattern, indicated by positive autocorrelation (Coad 2007; Coad and Hölzl, 2009). To our knowledge, El-Dardiry and Vogt (2023) are the only recent study that use quantile regression to further understand firm growth with a sample of firms from the Netherlands. They find that for most firms there is no significant autocorrelation – for two lags – except for the extreme deciles, there they find a negative first autocorrelation. The second autocorrelation is barely significant. Therefore, there is relevant heterogeneity across the firm growth distribution when it comes to autocorrelation of growth rates.

Coad and Hölzl (2009) investigate the growth patterns of various size classes over the distribution of growth, and find severely heterogenous effects. With the total sample, they find insignificant / mildly negative autocorrelation for the lower growth deciles and this negative autocorrelation becomes stronger towards the higher deciles. This suggests, especially for the faster growing firms, that growth discourages future growth. However, due to the many micro-firms present in the Austrian dataset, the initial results may be driven by micro-firms, therefore, they repeat the regression for various size classes. They find that small firms follow unstable growth patterns, where growth is often followed with shrinkage. However, larger firms actually have more persistent growth rates. In terms of coefficient size, the most negative autocorrelation they find is approximately -30% for the fastest-growing mid-size firms, whereas the most positive autocorrelation is approximately +25% for the fastest-growing larger firms. Overall, most effects are found in the extreme deciles, rather than at the median. All in all, Coad and Hölzl (2009) show the importance of heterogeneity in terms of size, but also heterogeneity within the distribution of firm growth.

In sum, the process of firm growth is not yet clearly understood. There is a clear impetus for investigating the heterogeneity of firm growth on a wider set of samples, as earlier studies have shown important aspects of heterogeneity in growth patterns, such as the distribution of firm growth and firm size.

EFFECTS OF COVID-19

The second strain of literature focuses on the effect of the pandemic on business dynamism. The COVID-19 pandemic undoubtedly affected businesses. On the one hand, contact-limiting measures restricted the possible business operations; on the other hand, governments rolled out extensive government support programs.

In the Netherlands, the economic effects of the pandemic are very heterogenous. Though the firm growth distribution has moved to the left, there is still a significant number of firms growing – as usual. The hospitality sector was affected most severely, in terms of sales and liquidity position, whereas sectors such as manufacturing were relatively unaffected on these dimensions (de Winter and Pruijt, 2022; Freeman and Bettendorf, 2023). Interestingly, on a sector level, the pandemic has changed more in the lower ends of the distribution of growth rates than in the upper ends (Freeman and Bettendorf, 2023).

The goal of the extensive government support programs was to dampen the aforementioned effects of the contact-limiting measures, through retaining jobs and preventing bankruptcies; temporarily freezing the economy to guide it through this temporary shock[3]. Freezing regular business dynamics comes with an opportunity cost: in normal economic times, firms rise and fall, forcing the least efficient firms to exit and allowing the most efficient firms to grow. This productivity-enhancing process is distorted with such a subsidy, particularly when the subsidy is allocated to firms that do not require it ('deadweight loss') or are unviable in the first place ('displacement costs') (Freeman et al, 2021).

However, whether these costs/losses have occurred is still ambiguous. International comparisons show that government support programs differ significantly and consequently, affect business dynamics differently (Bighelli et al, 2022). In some countries, government support went to growing / viable firms (Bighelli et al, 2022, Croatia, Finland, Slovakia, Slovenia, the Netherlands; Konings et al, 2022, Flanders). However, in the UK, government support did not align with actual sales' losses, and the biggest effects in terms of extended life span favoured the weakest firms (Belghitar, 2022). In the US, the PPP program did not reach the firms or individuals that required the additional liquidity, leading the authors to conclude it was relatively expensive compared to its effect (Chetty et al, 2022; Cho et al, 2022; Granja, 2022). However, in France, they found that significantly less firms exited the market, especially the least productive firms, though factors predicting bankruptcy did not change between 2019 and 2020, noting that support created a 'partial hibernation rather than zombification' of the economy (Cros et al, 2021). All in all, in many countries, the untargeted nature of support can disrupt / postpone the 'Schumpeterian' process of creative

[3] See Appendix A for a more thorough discussion of the government support programs

destruction. However, it can take different forms, like in France, where the regular explanations for bankruptcies have not changed due to the pandemic.

In the Netherlands, particular attention has therefore been paid towards the targeting of COVID-19 government support and the effects on bankruptcies. In terms of targeting, Freeman et al (2021) found that the least solvent and productive firms were most likely to receive government support. Moreover, Schellekens et al (2021) compare revenue losses across sectors in 'normal times' (2018-2019) and compare those with the revenue losses within a sector during the COVID-19 pandemic. They find that about 30% of government support went to subsidizing revenue losses that can be expected in normal times as well. Within sectors, the rate of exits was significantly lower among supported than unsupported firms (Freeman et al, 2021; Overvest and Fareed, 2021). Moreover, using machine learning models, Davies et al (2023) predicted firm exit and found that although many viable firms received government support, unviable and low-productivity firms benefitted the most from government support. This would be a rough indication – no proof – of both deadweight losses and displacement costs. Creemers et al (2022) investigate the potentially heterogeneous effects of government support on productivity – by using unconditional quantile regression (UQR). Like other research, they find strong selection effects: in each part of the productivity distribution, government support went to the less productive firms – this is especially the case for the most productive firms. They find that government support has mainly affected productivity by keeping non-viable firms alive, that normally would likely have exited the market.

This literature found there are early indications that the normal business dynamics have been distorted. However, there is yet limited evidence gathered on output by firms in this period in relation to COVID-19 support, and especially limited causal evidence. Firms could apply to the NOW (one of three support packages, the largest in terms of subsidy), if they expected a revenue loss of at least 20%. Therefore, the fundamental problem of causal inferences arises; we cannot determine whether it is the support, or the underlying characteristics that predict self-selection into support, that drive the measured effects. Therefore, we caution in the interpretation of our results as causal effects and focus on the observable changes in business dynamism.

We contribute to the aforementioned literature by adding a few perspectives. Firstly, we look specifically into firm growth in the Netherlands. Compared to existing studies (El-Dardiry and Vogt, 2023), we are the first to look at heterogenous firm growth between size classes in the Netherlands. Secondly, we use unconditional rather than conditional quantile regression (see Section IV). To our knowledge, unconditional quantile regression is rarely used in the literature on firm growth. UQR will provide additional insights in firm growth dynamics, by focusing on the effects on the population distribution rather than the conditional distribution, or within-group effects. Thirdly, we contribute to the findings on business dynamism during the COVID-19 pandemic to improve our understanding of the economic consequences of the crisis and the widespread government support. Previous research on the Netherlands has focused on selection into support and the effect on market exits. We add additional insights by looking at firm growth, going further than selection and the extensive margin.

III. DATA

DATA SOURCE

We use administrative data from the Central Bureau of Statistics on all limited liability firms between 2015 and 2020 in the Netherlands registered in the General Company Registry – ABR. This dataset is a combination of data sources on Dutch firms, aggregated at the level of enterprise group; their balance sheets and earning and loss statement (NFO), their value added tax reports (DRT), their records on all tax-paying labour contracts (Spolis) and their take-up of each COVID-19 government support programs.[4] We take the revenue from the DRT, over the NFO; the DRT is based on the value added tax reports, and therefore, more recently updated. To indicate the use of government support, we use a dummy in case a firm has taken-up NOW1-NOW3.1, TVL in 2020 or the tax deferral. Furthermore, we use information on firm exits from the ABR; we label firms that exited the market – excluding mergers and acquisitions – and include their revenue as 0 in the year of exit, thus a -100% revenue growth. We include these firms in our quantile regressions. In the transition matrices, we report these firms separately.

In the table below, we show how many observations we have per year – that have data on their revenue available.

Table 1: Observations per year

	2016	2017	2018	2019	2020
Nr. of observations	165,584	168,797	174,350	178,491	182,814

MEASUREMENT CHOICES

Firm size

The most used measurements are the number of employees and sales (Coad and Hölzl, 2012; Delmar, 1997). On the one hand, sales are a general measurement, as all firms need sales to survive. There are a few considerations though: sales are sensitive to input prices – if inputs become more expensive, sales will have to rise along, which could lead to confusing growth with an increase in input prices – and may be sensitive to manipulation (Coad and Hölzl, 2012). On the other hand, the number of employees is equally often used as sales as a measurement of size. However, employment growth has its own emphasis with potential downsides: firms could outsource labour in initial stages of growth, firms could be less labour-intensive and more capital-intense, and using employment for small firms

[4] Results based on calculations by Centraal Planbureau, using non-public microdata from Statistics Netherlands. Under certain conditions, these microdata are accessible for statistical and scientific research. For further information: microdata@cbs.nl.

can lead to quite rigorous jumps (Davidsson, 2005; Coad and Hölzl, 2012). Moreover, during the pandemic, the change in employees will not give a realistic indication of employees – and therefore, dynamism –, as wage subsidies (NOW) were paid to firms on the condition that employees were not laid off (UWV, 2022). The number of employees will therefore indicate how many contracts in FTE units a firm had, but less which proportion of these employees continued to work. Therefore, we use sales rather than employment.

Firm growth

There are roughly three options to measure firm growth, i.e., the change in firm size: (1) absolute growth, (2) relative growth; (3) log differences. Absolute growth is the difference in firm size between period t and period t-1. This measurement is frequently used in research on the growth of small businesses. However, in broader datasets, this measurement is biased towards larger firms (Coad and Hölzl, 2012). Relative growth is measured by dividing the absolute growth by its size in period t-1. Relative growth potentially overemphasises the growth of smaller firms, as a minor change in absolute firm size can cause a large relative change if the denominator is small, though is widely used in the literature as a growth measure (Coad and Hölzl, 2012). Lastly, log differences can be used, as it is less sensitive to heteroscedasticity or severe outliers. As this analysis uses quantile regression, estimation is less sensitive to severe outliers. Moreover, then we cannot include firms with a revenue of 0, which would remove observations from firms that exit the market and have 0 euros of revenue that year. Therefore, we choose to use relative growth rather than log differences.

DESCRIPTIVE STATISTICS

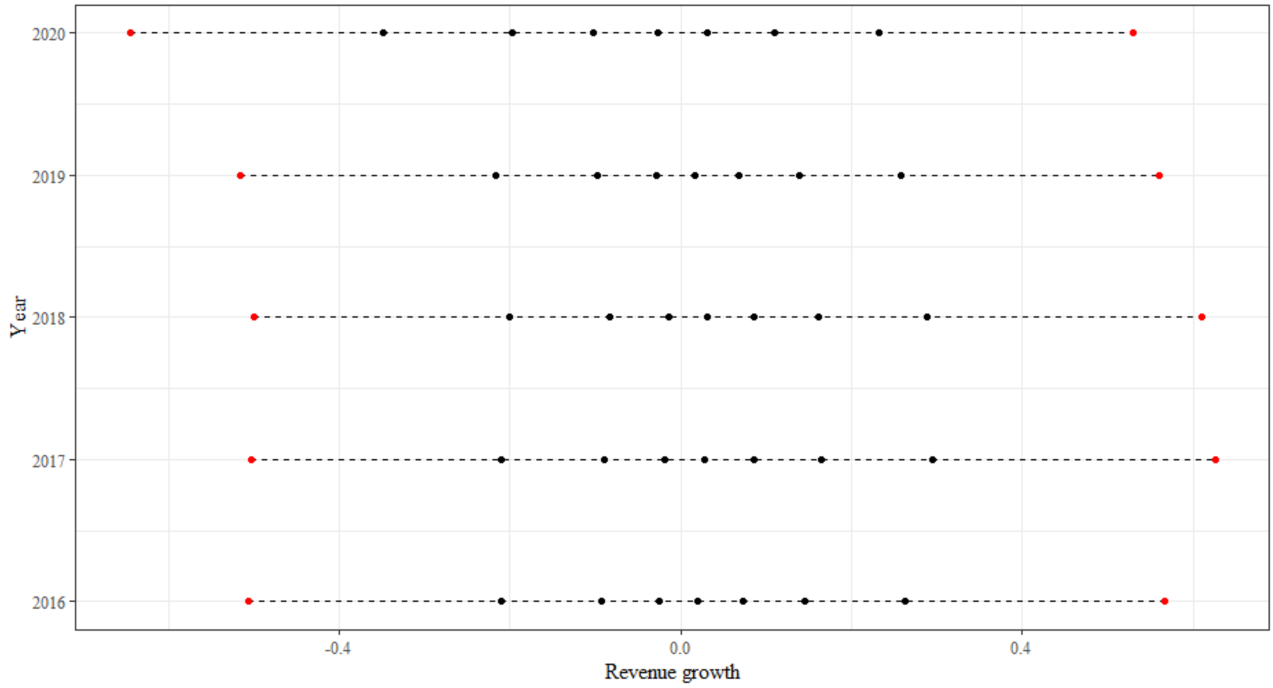
Before we delve into our analysis, we will show some descriptive statistics on firms in the pandemic. In Table 2, one can see that the distribution of revenue growth in 2020 has shifted towards the left compared to previous years. The image is familiar from Freeman and Bettendorf (2023): the right-hand side of the distribution is less severely affected than the left-hand side. This indicates that the main distributional change has been that there are less firms around 0% firm growth, and more firms slightly shrinking. This is further indicated by the fact that the percentile in which 0 lies has moved upwards significantly.

Table 2: quantiles of revenue growth over the years

Year	P10	P20	P30	P40	P50	P60	P70	P80	P90	Percentile of 0
2016	-0.506	-0.210	-0.093	-0.025	0.020	0.073	0.144	0.263	0.566	45
2017	-0.504	-0.210	-0.089	-0.019	0.028	0.086	0.164	0.294	0.626	44
2018	-0.500	-0.200	-0.083	-0.015	0.030	0.086	0.162	0.289	0.610	43
2019	-0.516	-0.216	-0.098	-0.029	0.017	0.069	0.139	0.258	0.560	46
2020	-0.645	-0.348	-0.198	-0.102	-0.028	0.032	0.110	0.233	0.530	54

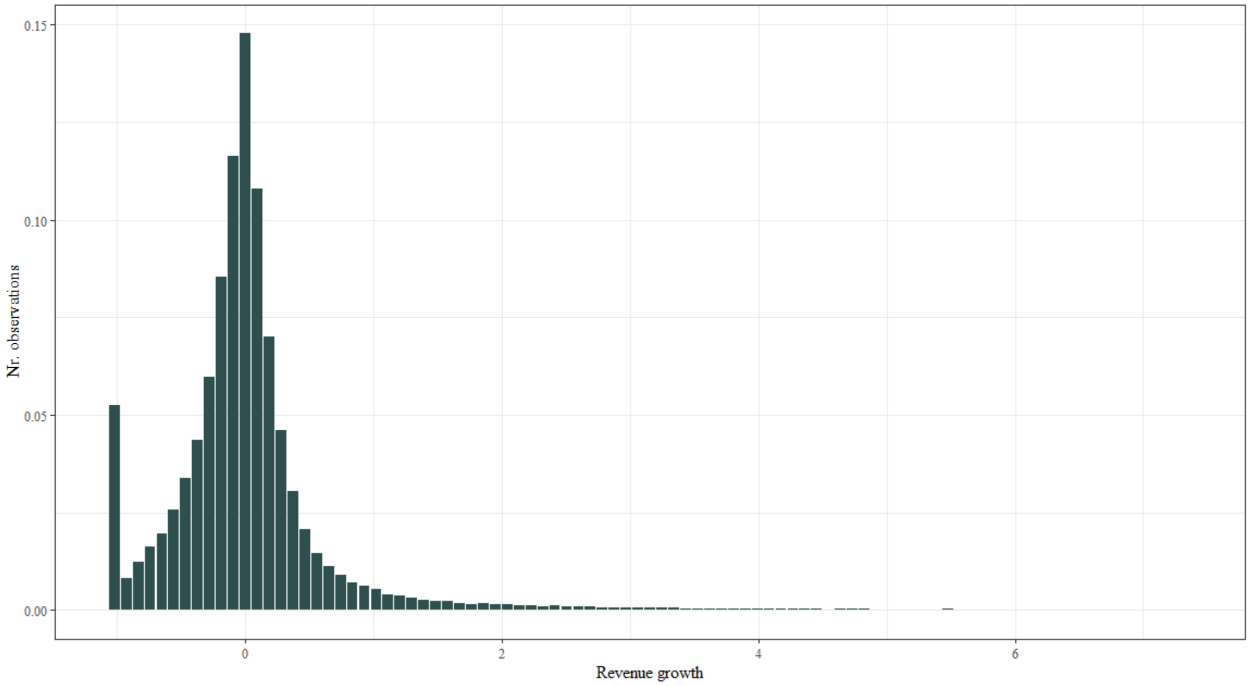
In Figure 1, we also show the table above graphically. The red dots indicate the 10th and 90th percentile; the other dots the deciles in between. This shows the described pattern: around the median and upper end of the distribution there have been minimal changes, though the lower end of the distribution has declined.

Figure 1: Distribution of revenue growth over the years, 2016 – 2020



Below, we show the distribution of revenue growth in 2020 in a histogram. We see a peak at –1, indicating firms that exited the market, and a long right-side tail.

Figure 2: Distribution of revenue growth – 2020



Additionally, we consider the summary statistics of our main covariates. In Table 3 below, we show the distribution for the covariates in 2020. It is particularly worthwhile to point out the difference in the mean and median of the revenue level; the mean is approximately 14 times higher than the median. This suggests severe outliers on the right side of the distribution, which we need to consider when using this variable in our specification.

Table 3: descriptive statistics of main variables in 2020

Variable	25%	Mean	Median	75%	St. dev.
Revenue growth	-0.26	0.03	-0.03	0.16	0.76
Age	4.00	8.52	9.00	15.00	5.28
Revenue level in period t – 1	126,896	6,610,054	464,478	1,773,831	186,304,200

IV. METHODS

QUANTILE REGRESSION

In this analysis, we use quantile regression rather than ordinary least squares (OLS). Ordinary least squares (OLS) explains the conditional mean of the dependent variable. Quantile regression explains not the conditional mean of the dependent variable, but rather the quantiles across the distribution of the dependent variable. This allows us to find the effects of our explanatory variables on the entire distribution of our dependent variable. OLS can answer what the relationship between the covariates and dependent variable is at the conditional mean, quantile regression will estimate this relationship for the distribution, e.g., the 10^{th} percentile, the median, the 90^{th} percentile. Therefore, quantile regression has a few important attributes that help us answer our research question:

(1) From the literature and our data, we gathered that firm growth is a highly heterogenous process. Therefore, regular OLS estimates will give a limited and likely inaccurate idea of the growth patterns of firms. By allowing for heterogeneity along meaningful aspects of firms, we can draw more accurate conclusions on growth patterns of firms.
(2) OLS requires normally distributed error terms; this condition is not met when there are relatively 'fat tails', which is likely the case with firm growth (Coad, 2012). This assumption can be loosened under quantile regression. Quantile regression allows the prediction of the more extreme values, and is therefore less sensitive to these outliers.

CONDITIONAL AND UNCONDITIONAL QUANTILE REGRESSION

Within quantile regression, there are two types of regressions one can use: conditional and unconditional. Conditional quantile regression (CQR) explains the quantile, conditional on the added covariates. With new covariates, the quantiles are conditional on a different set of variables and therefore, the quantiles shift (Koenker and Hallock, 2001). Unconditional quantile regression (UQR), as proposed by Firpo, Fortin and Lemieux (2009), predicts the unconditional quantile. In this sense, UQR will say more about distributional effects on the population distribution.

A clear case to explain the difference in detail would be by introducing sector fixed effects. In UQR, adding these fixed effects has no consequence on our dependent variable – we still estimate the same (unconditional) quantiles, the sector fixed effects control for average effect of a sector on firm growth on the population-wide distribution. In CQR, adding these fixed effects means that the regression now calculates the conditional distribution for each sector and predicts the quantile of a certain sector (*ceteris paribus* implied). For instance, let us say the 90^{th} percentile in the whole sample is 20% revenue growth, but for manufacturing it is 80%; the regressions would predict two fundamentally different things. UQR would predict the 20% growth also for manufacturing firms, and including the sector-fixed effects allows the estimation of the average effect of that sector on the 90^{th} unconditional percentile. However, CQR would predict the 80% growth for manufacturing firms.

Therefore, CQR makes it relatively untransparent – especially with multiple, and continuous covariates – what deciles are estimated. Additionally, in terms of interpretation, CQR looks exclusively at within-group effects in the distribution. Thus, the interpretation is different between the two, as fundamentally different distributions are estimated. In our research, CQR would answer the question: what are firm growth patterns for the distribution of firm growth, conditional on the covariates? On the other hand, UQR would answer the question: what are firm growth patterns for the unconditional distribution of firm growth?

We choose unconditional quantile regression over conditional QR for multiple reasons. Firstly, through UQR, we hope to gain additional perspectives on firm growth, compared to the existing publications using CQR. We are interested in what explains the distribution of growth, and UQR will tell us more about the population effects rather than within-group effects. This is specifically interesting in the pandemic, where most firms were at least to some extent affected. Secondly, through conditioning each quantile on the covariates, CQR requires significant computing power in more extensive specifications. Using CQR would therefore severely limit the amount and types of variables we could use. Thirdly, due to the conditioning of quantiles, the interpretation of CQR coefficients is more difficult and can easily lose meaning, once it is not clear what the quantiles are. With UQR, the estimated quantiles are easier to extract and therefore, precisely interpret. Most of the research on firm growth has used conditional quantile regression. This could be more interesting for those specific research questions, as they aimed at predicting growth patterns for specific (groups of) firms, whereas our question is concerned with changes in the population-wide growth dynamics during the COVID-19 crisis.

UQR weighs observations through calculating the recentered-influence-function (rif), after which we regress outcome of the rif.[5] In this sense, UQR is a transformation of our dependent variable. We follow the notation by Firpo, Fortin and Lemieux (2009). First, the approach starts calculating the influence-function. This is the influence one observation has on that particular quantile. The value of each observation is weighed for the quantile we want to estimate the regression for.

[5] In R, we use the package *dineq* and syntax rif() to calculate the recentered-influence functions for the relevant quantiles, and then regress using regular OLS (Schulenberg, 2018).

$$IF(Y;\ q_t) = \frac{\tau - I(Y\ \le q_t)}{f_Y(q_t)}$$

where:

- q_t: quantile corresponding to the probability τ
  - τ equals $P(Y\ \le q_t)$
- f_Y: the density function of Y
- $I(Y\ \le q_t)$: indicator function that holds value 1 if the condition in the brackets is met

The RIF is then defined as: $RIF(Y;\ q_t) = q_t + IF(Y;\ q_t)$. Some important properties are:

The expectation of the IF is 0.

For a given probability τ, the two elements q_t and $f_Y(q_t)$ are independent from Y and thus, considered a constant.
As τ is the probability corresponding to the quantile, the indicator function will in its expectation equal the probability corresponding to the quantile.

$$E\big(IF(Y;\ q_t)\big) = \frac{\tau - E\big(I(Y\ \le q_t)\big)}{f_Y(q_t)} = \frac{\tau - \tau}{f_Y(q_t)} = 0$$

One may wonder, why recentering is required for estimation – why not use the influence-function. In expectation, the influence function is 0 for *each quantile*. In order to use the influence function in our regression, we recenter it. Then, the expectation of the RIF equals its quantile, which is a very convenient property for interpretation.

$$E\big(RIF(Y;\ q_t)\big) = q_t + E\big(IF(Y;\ q_t)\big) = \ q_t + 0 = q_t$$

Second, the expectation is specified as a linear function of the explanatory variables.

$$E(RIF(Y; q_\tau)) = \beta_0 + \beta_1 x_1 + \beta_2 x_2$$

In essence, UQR is a transformed OLS regression to estimate various quantiles.

REGRESSION SPECIFICATION

Based on Coad and Hölzl (2009), we specify the following regression:

$$RIF\left(G_{i,t};\ q_\tau\right) = \alpha_\tau + \ \beta_{1\tau} * G_{i,\,t-1} + \beta_{2\tau} * G_{i,\,t-2} + \beta_{3\tau} * \log S_{i,\,t-1} + \gamma_\tau * a_{i,\,t} + SBI2_{i,\,t} + \varepsilon_{i,t}$$

where:

G_t: revenue growth in period t

$\log S_{i,\,t-1}$: the log of sales in period t – 1

a_t: age of the firm in period t

q_τ: quantile, given the probability τ (0, 1)

$SBI2_{i,\,t}$: sector fixed-effects at SBI two-digit level

We estimate this regression for t = 2020 and nine values of τ (0.1 to 0.9).

We execute this regression separately for the different size classes. As discussed in Coad and Hölzl (2009), in samples with many micro-firms, regression on the entire sample of firms can lead to results driven by micro-firms, neglecting heterogenous effects for medium to large firms. We follow OECD size classes, based on employment in FTE. We use OECD size classes, except starting the final category of firms at 100 FTE, rather than 250 to keep the group sufficiently large for estimation. In Table 4, we show the number of firms per size class in our dataset for 2020; though micro-firms (less than 10 FTE) only represent 13% of all sales, they represent 81% of the observations.

Table 4: Size class distribution in 2020 in dataset

Size class	**Nr. of observations**	**Proportion of total revenue**	**Proportion of total observations**
1: 0-9 FTE	190,748	0.13	0.81
2: 10-49 FTE	36,174	0.21	0.15
3: 50-99 FTE	4,849	0.10	0.02
4: 100+ FTE	4,058	0.56	0.02

V. RESULTS

TRANSITION-MATRICES

Before we report our regression results, we show transition matrices to show the decile shifts of firms from period t – 1 to period t. The numbers in the cells indicate the proportion of firms, rows adding up to one; decile 1 are firms with the lowest sales growth, and decile 10 with the highest. We consider the entire sample of firms for 2018-2019-2020. We include firms that exited the market as a separate category and calculate the deciles for the remaining firms[6]. There are three main matrices of interest, reported on the following pages, the underlying / additional matrices about government support are reported in Appendix B.

First, we turn towards the matrix for 2019-2020; a few areas draw attention. Foremost, the diagonal – indicating firms that stayed within the same decile between 2019 and 2020 – does not hold a large amount of mass. It is however noticeable that for the middle deciles, approximately half of the firms are situated in deciles around the diagonal. We see relatively less dynamism around the median. – firms that are at the median are quite likely to stay around the median, compared to other deciles. Another observation is that the level of exits is severely higher for the slowest growing firms in the previous year than the exits for the higher deciles – in the extensive margin, proportionally, more firms exit the market when they grew the least (or even shrunk) in the previous period, than if they experienced among the highest growth in the previous period. Additionally, relatively more mass – compared to the other cells in the row – is found in the extreme corners between the first and tenth decile. This indicates that the slowest growing firms have approximately the same probability to be from the lowest or highest decile; and the fastest growing firms have approximately the same probability to be from the lowest or highest decile.

Moreover, there are almost no severe differences between the 2018-2019 and 2019-2020 matrices. This suggests that on these fixed distributions, revenue dynamism between the deciles has not changed severely in the pandemic. To be clear, that does not mean the pandemic had no effects on firms' revenues, merely that the transitions between the annual deciles are unaffected. Perhaps more surprisingly, based on the previous literature on effects of the pandemic on business dynamism in the extensive margin, we also do not find substantial differences in market exit between the years.

To further investigate the effects of the pandemic, we therefore separate the 2019-2020 matrix based on receiving at least one of the government support programs.[7] There are two noticeable differences. Firstly, as a familiar result (e.g., Freeman et al, 2021; Overvest and Fareed, 2021), the

[6] NB: in the regression, firms exiting the market are included in our sample and their revenue growth is -100% (thus, by definition in the lowest decile). In the transition-matrix, we calculate the deciles excluding firms that exit the market. The deciles are slightly different between the transition matrices and the regressions.
[7] NB: We calculate the deciles based on the entire population, label firms based on their place in the distribution, and then separate the matrices. The matrices are thus based on the same distribution.

number of exits is severely lower among supported than non-supported firms – the difference being most severe for firms that grew the slowest in 2019, with 20 percentage points compared to their unsupported counterparts. This result could explain that there is barely any year-on-year change when it comes to market exits: supported firms were significantly less likely to exit the market, whereas unsupported firms were relatively more likely to exit the market than in 2019. Secondly, we see more supported firms on the left-hand side, indicating more supported than non-supported firms were in the lower distribution of growth for 2020 (i.e., the difference is positive). As mentioned before, these results do not indicate a causal effect of government support, as they can also point towards imbalance due to self-selection. In the later sections, we will analyse the regressions, separating unsupported and supported firms, to further inquire the revenue dynamics.

In conclusion, in this simple, unweighted overview, we can draw a few careful, yet important conclusions. The unweighted differences between 2018-2019 and 2019-2020 in terms of transitions between deciles are relatively minor within their respective distributions; both in the extensive and intensive margin. Yet, we see more severe differences between unsupported and supported firms, particularly in the extensive margin, where supported firms are less likely to exit the market. Moreover, based on the transition matrices, we see that the distribution across the row differ a bit: firms that were in the most extreme deciles have a higher probability to stay within the extreme decile or go to the other extreme, whereas firms around the median have a higher likelihood to stay around the median. This result points towards heterogeneity in growth patterns: at first glance, firms in the extreme deciles of firm growth have a different likelihood to be from a certain decile, than firms around the median of firm growth. This motivates the use of quantile regression over ordinary least squares.

Transition-matrix Sales growth: 2018 – 2019

	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Exit
Q1	0.180	0.085	0.051	0.036	0.040	0.034	0.040	0.054	0.096	0.179	0.204
Q2	0.124	0.117	0.097	0.080	0.062	0.074	0.080	0.102	0.125	0.082	0.057
Q3	0.079	0.106	0.118	0.116	0.102	0.106	0.109	0.106	0.086	0.044	0.030
Q4	0.059	0.078	0.107	0.134	0.191	0.121	0.108	0.090	0.064	0.030	0.019
Q5	0.045	0.075	0.108	0.145	0.168	0.147	0.119	0.090	0.058	0.027	0.018
Q6	0.049	0.075	0.109	0.139	0.131	0.153	0.131	0.104	0.068	0.026	0.016
Q7	0.050	0.088	0.117	0.120	0.109	0.138	0.136	0.116	0.078	0.032	0.014
Q8	0.069	0.116	0.121	0.107	0.087	0.106	0.115	0.120	0.100	0.042	0.018
Q9	0.107	0.138	0.107	0.080	0.070	0.078	0.093	0.114	0.119	0.069	0.025
Q10	0.192	0.115	0.072	0.055	0.050	0.056	0.065	0.084	0.125	0.136	0.050

Transition-matrix Sales growth: 2019 – 2020

	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Exit
Q1	0.158	0.093	0.064	0.047	0.041	0.044	0.043	0.054	0.085	0.169	0.202
Q2	0.118	0.110	0.104	0.093	0.079	0.072	0.078	0.092	0.113	0.081	0.061
Q3	0.088	0.106	0.112	0.115	0.107	0.095	0.107	0.106	0.090	0.045	0.030
Q4	0.076	0.097	0.115	0.115	0.126	0.126	0.119	0.098	0.075	0.033	0.020
Q5	0.065	0.079	0.100	0.112	0.126	0.194	0.116	0.092	0.063	0.032	0.019
Q6	0.061	0.082	0.099	0.119	0.141	0.134	0.136	0.110	0.072	0.029	0.016
Q7	0.062	0.083	0.100	0.122	0.129	0.116	0.130	0.118	0.085	0.037	0.018
Q8	0.068	0.097	0.105	0.116	0.109	0.097	0.115	0.123	0.101	0.050	0.019
Q9	0.090	0.116	0.113	0.098	0.092	0.078	0.091	0.103	0.120	0.072	0.027
Q10	0.158	0.123	0.089	0.073	0.060	0.056	0.067	0.083	0.116	0.131	0.044

Transition-matrix Sales growth: 2019 – 2020 (Supported minus non-supported firms)

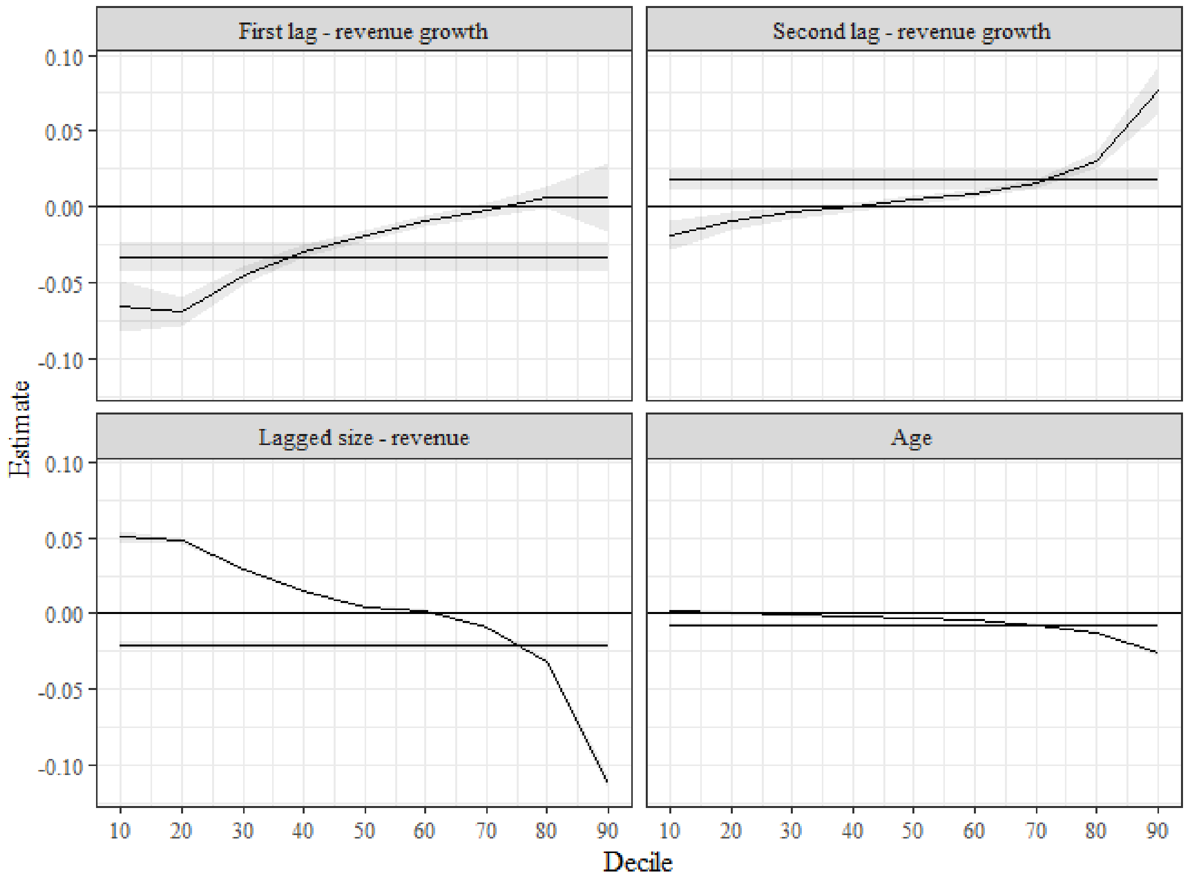
	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Exit
Q1	0.048	0.055	0.034	0.021	0.018	0.007	0.012	0.014	0.019	-0.016	-0.210
Q2	0.033	0.064	0.050	0.038	0.011	-0.010	-0.011	-0.016	-0.050	-0.036	-0.073
Q3	0.044	0.084	0.070	0.045	0.004	-0.030	-0.044	-0.043	-0.063	-0.028	-0.039
Q4	0.053	0.091	0.095	0.044	-0.008	-0.052	-0.061	-0.054	-0.056	-0.024	-0.028
Q5	0.056	0.089	0.096	0.069	0.015	-0.154	-0.050	-0.044	-0.034	-0.020	-0.022
Q6	0.045	0.077	0.073	0.062	0.002	-0.051	-0.060	-0.054	-0.045	-0.024	-0.024
Q7	0.042	0.070	0.071	0.041	0.005	-0.030	-0.044	-0.054	-0.049	-0.027	-0.025
Q8	0.035	0.063	0.054	0.034	0.016	-0.017	-0.032	-0.047	-0.049	-0.031	-0.026
Q9	0.037	0.052	0.040	0.026	0.002	-0.010	-0.007	-0.027	-0.046	-0.036	-0.031
Q10	0.032	0.037	0.024	0.006	0.010	-0.001	0.002	-0.013	-0.024	-0.026	-0.047

UQR RESULTS

WHOLE SAMPLE

In the Figures below, we show the coefficients – including their 95% confidence intervals – from the UQR, across the 9 deciles for which these were estimated. There are two additional horizontal lines: the 0-line and the OLS estimate, including 95% confidence intervals. The complete regression results are presented in Appendix C. For most variables and deciles in Figure 3, we see significantly different results compared to the OLS estimates – as expected.

Figure 3: Coefficients across deciles for the entire sample, all variables & OLS-estimates - 2020



In the coefficient for the first lag, we see a negative effect, except for the higher deciles it trends towards insignificantly different from 0. For the second lag, we see a negative effect for the lower deciles, but a positive autocorrelation for the higher deciles. In our lags, we find support for the following notion: growth for the slowest growing firms is likely to suffer from growth in the past, indicated by the negative first and second autocorrelation, whereas growth for the fastest growing firms is encouraged by previous growth, indicated by the positive first autocorrelation and insignificantly different from zero second autocorrelation. However, the intensity of the autocorrelation is – in line with more recent studies on the autocorrelation – somewhat moderate: for the lowest deciles, if growth was one percentage point higher in period t – 1, this discourages growth with approximately 0.07 percentage points.

For lagged size, by only looking at the OLS estimate, one would see a statistically significant negative effect, though one of little economic significance. One percentage larger firm size in 2019 would lead to an approximately 0.0002 percentage point decrease in revenue growth. We see a severely heterogenous effect across the deciles that switches signs around the median and gains some economic significance. This means that size has a positive effect on the lower deciles in the distribution, whereas size has a negative effect on the higher ends of the distribution. This means that the faster growing firms are experiencing 'mean-reversion', i.e., if a fast-growing firm was larger in period t – 1, this firm is experiencing less (albeit still fast) growth in period t. However, the slowest-growing firms are encouraged in their growth by being larger (in 2019). Although the effect is not highly economically impactful, it indicates that firm size is related to firm growth. We observe a significant effect of age for all deciles. This effect is mildly positive for the lowest deciles, with little economic significance – if a firm is 10 years older, firm growth for the slowest growing firms is encouraged with 0.01 percentage point. The effect of age on firm growth turns negative, and stronger towards the higher deciles. Thus, age has a negative effect for this subset: if a firm is a year older, firm growth for the fastest growing firms is stagnated by 0.026 percentage point. From the literature, this effect is relatively expected: it is young firms that grow the fastest.

Other studies use conditional quantile regression, thus estimate fundamentally different, and consequently not completely comparable effects. We compare to studies, that predict revenue growth, using its lags and firm size. El-Dardiry and Vogt (2023) find a mild inverted-U for the first autocorrelation in the Netherlands – i.e., negative autocorrelation for the lowest and highest deciles. Coad and Hölzl (2009) find with Austrian data no significant AC (for both lags) for the lower deciles, negative AC (for both lags) in the high deciles. Especially for the higher deciles, our results differ from the estimates using CQR. We find that previous growth encourages growth for the fastest-growing firms. For lagged size, Coad and Hölzl (2009) find the reverse effects;[8] These differences can also be explained by differences in sample, measurement, or time-period.

Overall, over the whole sample, we find heterogeneity across the distribution of firm growth in the autocorrelation and the effect of firm size. The lags indicate self-enforcing growth patterns: for the slowest growing firms, additional growth will discourage future growth, whereas for the already fastest growing firms, additional growth will encourage future growth. This effect is moderated by the size of the firm: additional firm size positively affects future growth for the slowest growing firms, and negatively affects growth for the fastest growing firms.

[8] El- Dardiry and Vogt (2023) do not report estimates for the coefficient of firm size on current firm growth.

SIZE CLASSES

We repeat the previous analysis for each separate size class, to see if there is relevant heterogeneity of growth patterns between 4 size groups. In the table below, we report the deciles for the size classes. The complete regression results are presented in Appendix D.

Table 5 shows the distribution of firm growth, between size classes. The distribution of larger firms is narrower than the distribution of smaller firms, especially for micro-firms. Micro-firms compared to large firms have more extreme negative values in the first decile, and more extreme positive values in the ninth decile. This is indicative that firms in different size classes have different growth patterns.

Table 5: Deciles of revenue growth for size classes, 2020

Size	P10	P20	P30	P40	P50	P60	P70	P80	P90
1: 0-9 FTE	-0.516	-0.306	-0.178	-0.087	-0.012	0.048	0.137	0.282	0.658
2: 10-49 FTE	-0.336	-0.196	-0.116	-0.056	-0.004	0.044	0.103	0.182	0.340
3: 50-99 FTE	-0.279	-0.164	-0.096	-0.048	-0.005	0.037	0.088	0.168	0.304
4: 100+ FTE	-0.271	-0.159	-0.096	-0.049	-0.012	0.029	0.075	0.141	0.263

We can see in Figure 4 strong heterogeneity in the effect of the first lag on current revenue growth. In comparing these results to those in Figure 3, we indeed see that the full sample results were driven by micro-firms. For the smallest firms, the effect is consistently negative, albeit minor, whereas for larger firms in the higher deciles, the first lags are positive, and slightly negative / insignificantly different from zero in the lower deciles. Micro-firms experience a consistently negative first autocorrelation; this is indicative of an unstable growth pattern; growth discourages future growth. For larger firms, it is the fastest growing firms that experience the positive autocorrelation. That means that for larger, growing firms, growth stimulates further growth. The first autocorrelation for the fastest growing, larger firms is of economic significance: one percentage point more growth in period t – 1 contributes, all else equal, between 0.2 and 0.4 more percentage points growth in period t.

For the second lag, we see less extreme heterogeneity in the patterns among size groups. For all size classes, the main effect is a positive second autocorrelation for the fastest growing firms. This result in the higher deciles has some economic significance as well: an additional percentage point growth in period t – 2 encourages around 0.1 percentage point growth in period t.

Figure 4: Coefficients across deciles for each size class, First Growth Lag, 2020

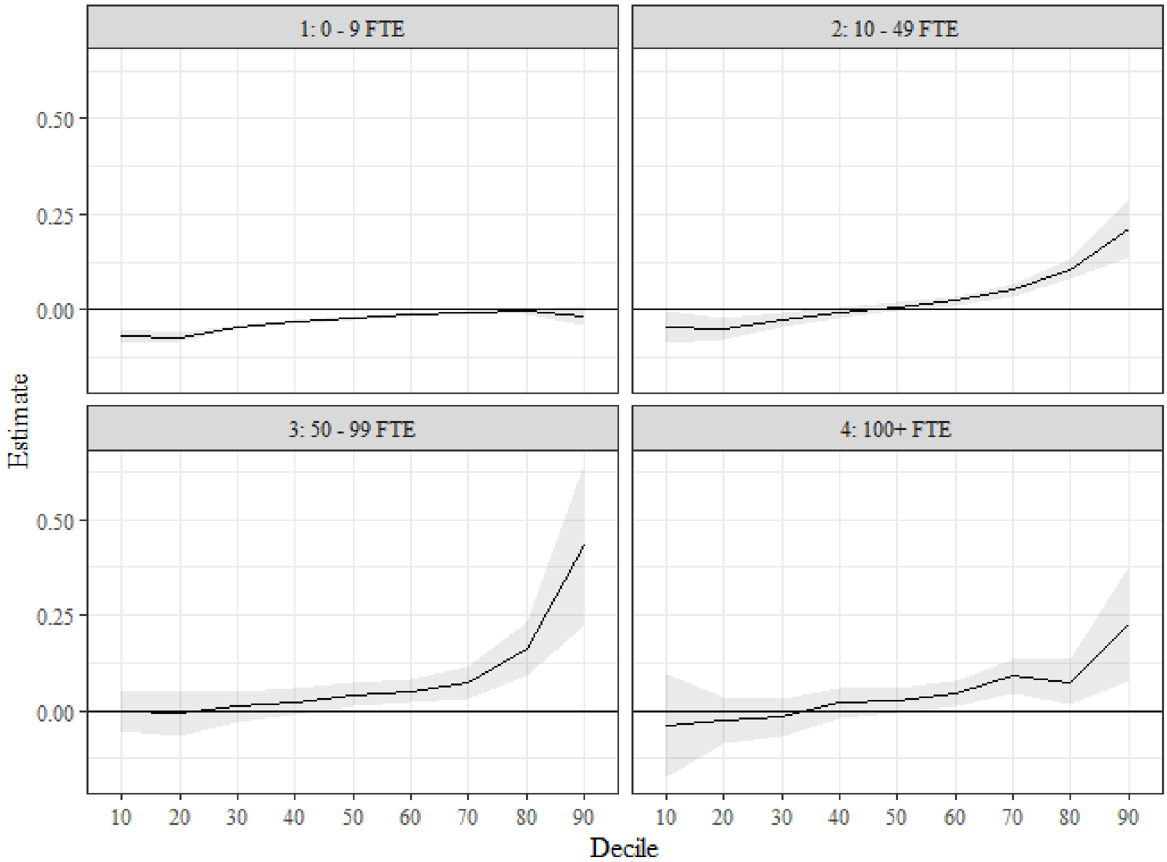


Figure 5: Coefficients across deciles for each size class, Second growth Lag, 2020

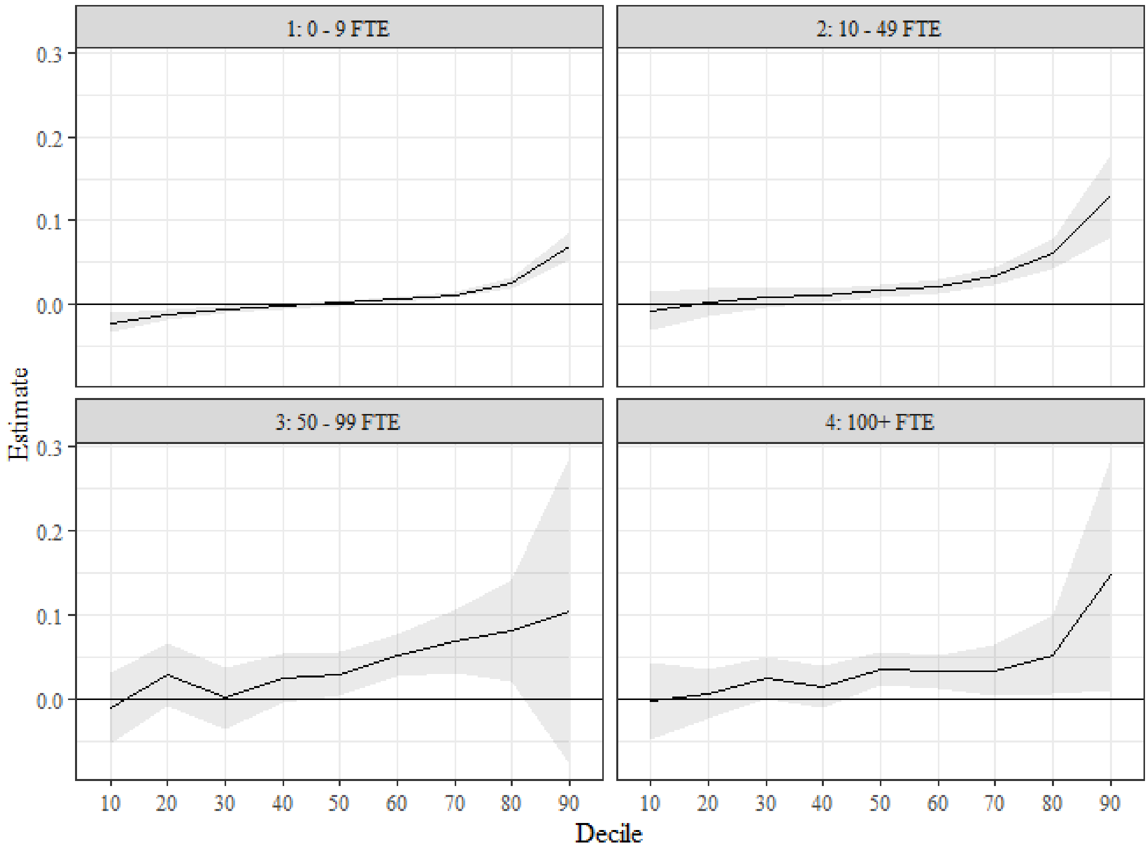


Figure 6: Coefficients across deciles for each size class, Lagged Revenue Levels, 2020

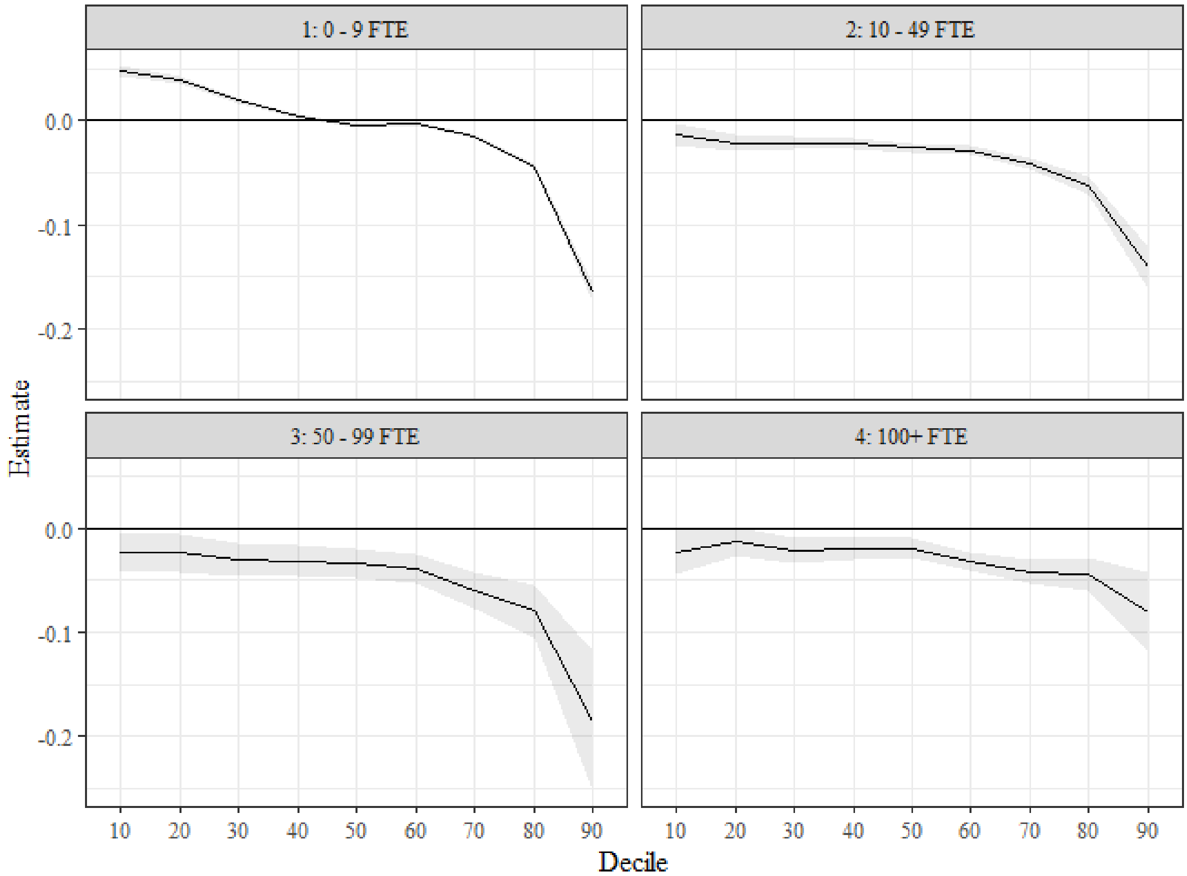
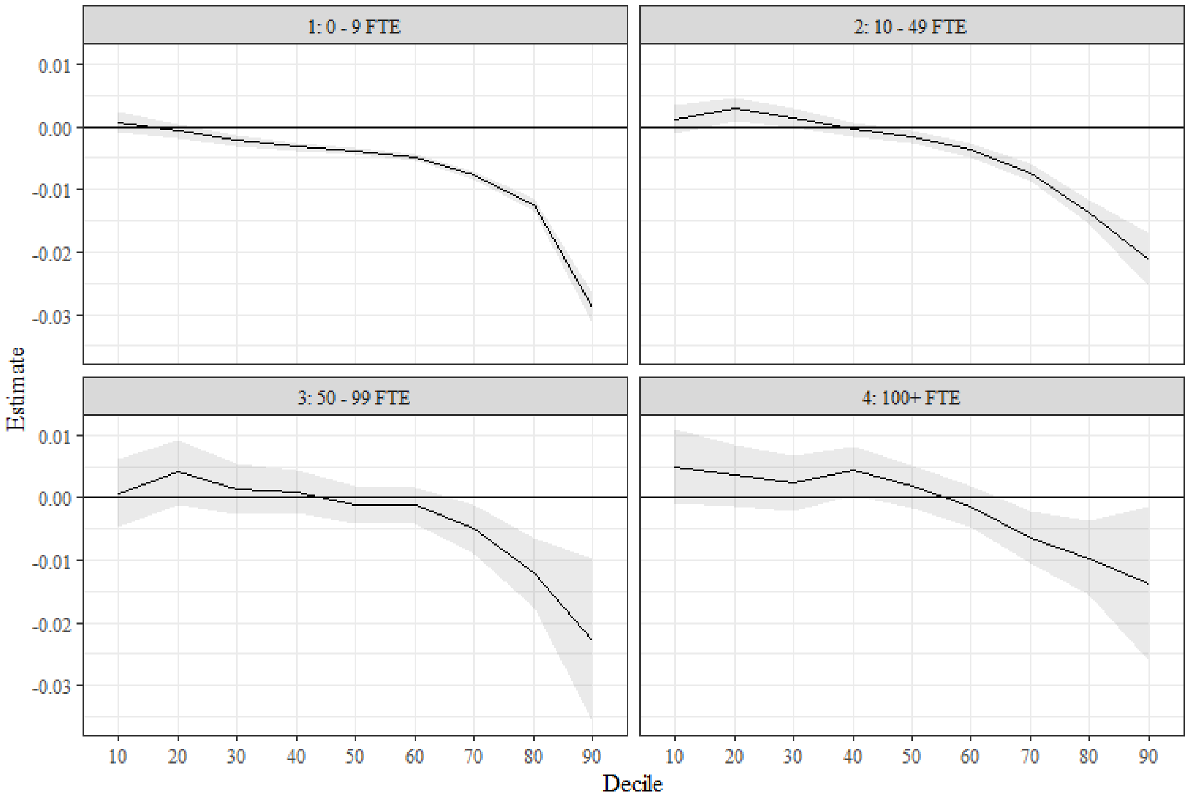


Figure 7: Coefficients across deciles for each size class, Age, 2020



Most types of firms experience a negative effect from lagged firm size on current firm growth, except the slowest growing micro-firms. The initial UQR results were indeed driven by the slow-growing micro-firms: no other size class – decile combination experiences a positive effect of size on growth. The negative effect for the larger firms – ‘mean-reversion’ – means current growth is not independent from size and this confirms earlier findings that the relationship is (largely) negative. The clear exception is the slowest-growing micro-firms; they experience a positive effect from size on growth.

The effect of age on firm growth is relatively consistent across the size classes. In the lowest deciles, we find a mildly positive / statistically insignificant effect. In the highest deciles, the effect has turned stronger and negative. This is a similar result as the previous regression: age has a particularly negative effect for the fastest-growing firms. The biggest difference seems to be that for micro-firms, the effect of age turns negative in the second decile already, whereas for larger firms, the coefficient turns negative around the median.

This section shows the importance of allowing for heterogeneity in researching firm growth, particularly in the first autocorrelation and the effect of firm size. We find that the first autocorrelation term is mildly negative / statistically insignificant for the slowest growing firms, whereas among the fastest growing firms, all firms experience a relatively strong positive autocorrelation (of approximately 0.2 – 0.4), except for the smallest firms, who experience a negative first autocorrelation across the entire growth distribution. Additionally, we find a consistently negative effect of size on firm growth, across the growth distribution and size classes, except for the slowest growing small firms.

COMPARING 2019 AND 2020

To further investigate the effect of the Covid-19 pandemic on business dynamism, we replicate the estimation of UQR for previous years. We estimate the distributions for each year (see table 2) and plot the coefficients for both years, to compare. The complete regression results are presented in Appendix E.

For all covariates, we see no significant deviation in the coefficients for 2020 compared to 2019, except for the first decile. We observe a less negative first autocorrelation, and less positive effect of size on growth in 2020 than in 2019 for the first decile. Thus, for the slowest-growing firms, previous growth discouraged growth less severely, and the moderating effect of size on firm growth was weaker in 2020 than in 2019. There is no break in growth patterns in 2020 compared to 2019; except for the slowest-growing firms, where growth is less related to the first AC and size.

Figure 8: Coefficients across deciles of revenue growth, First Lag, 2019 – 2020

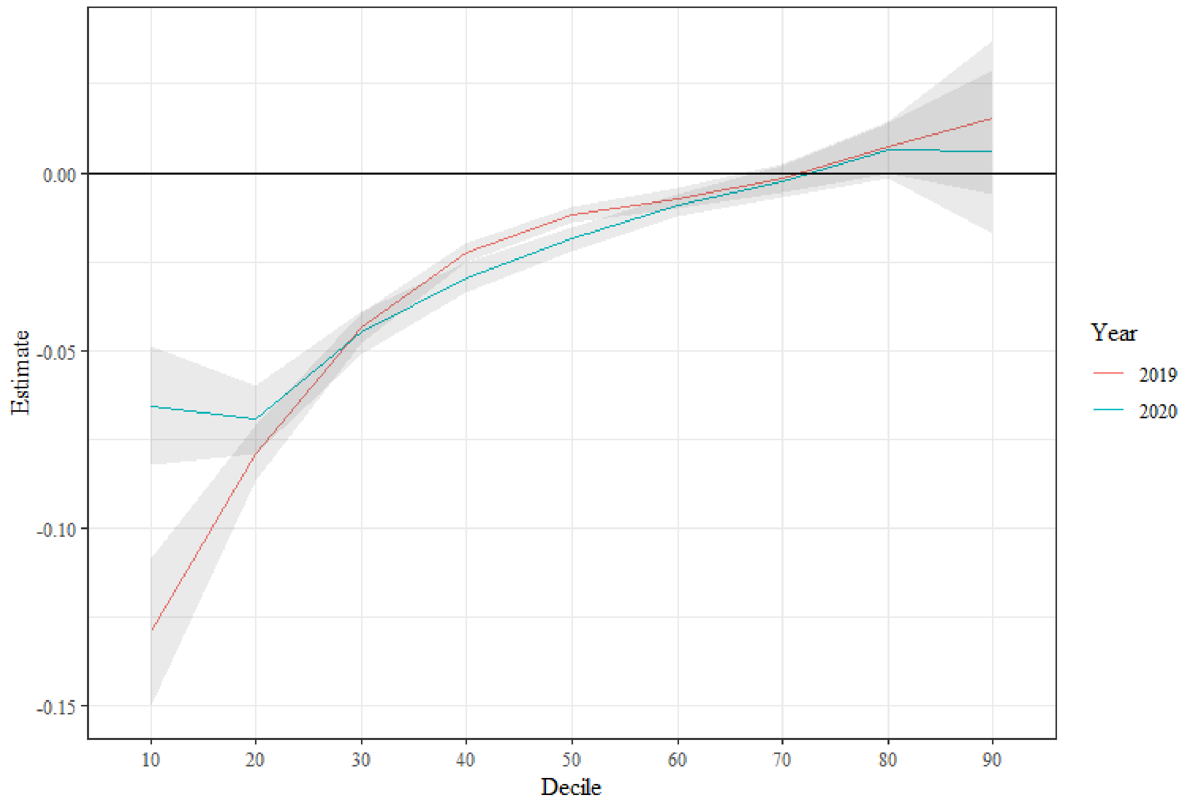


Figure 9: Coefficients across deciles of revenue growth, Second Lag, 2019 – 2020

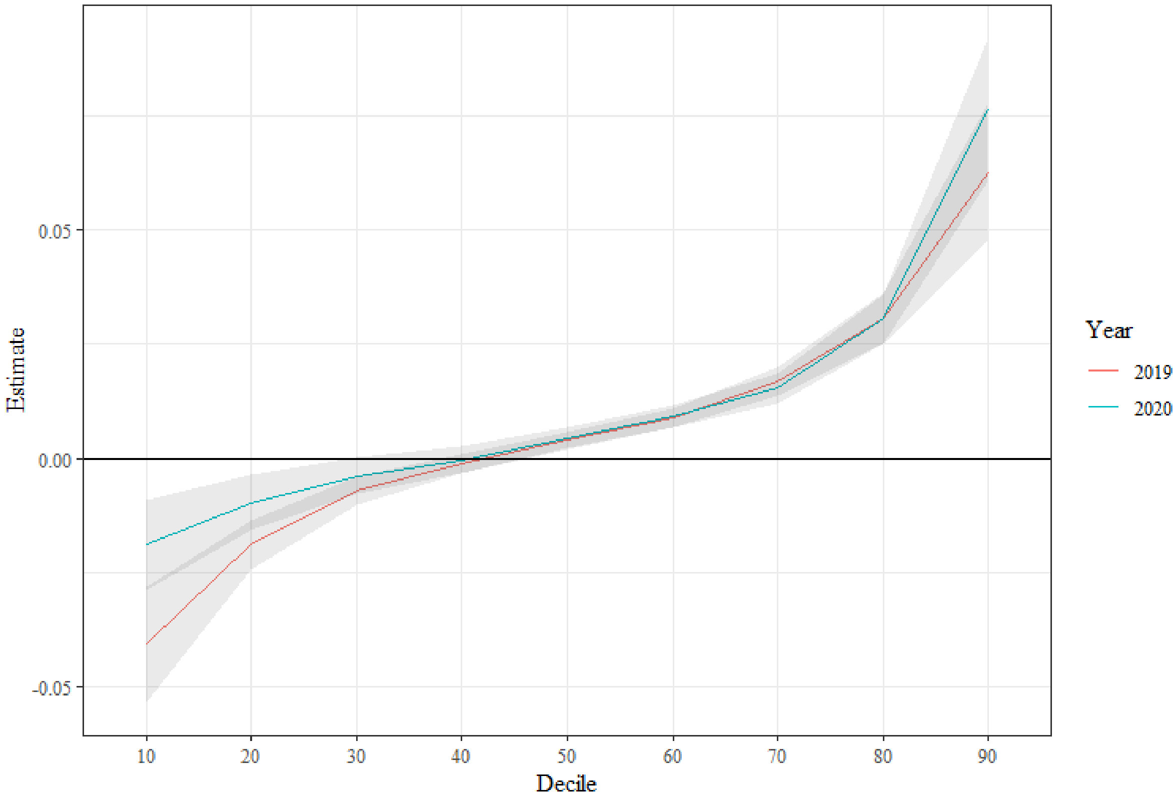


Figure 10: Coefficients across deciles of revenue growth, Lagged Revenue Levels, 2019-2020

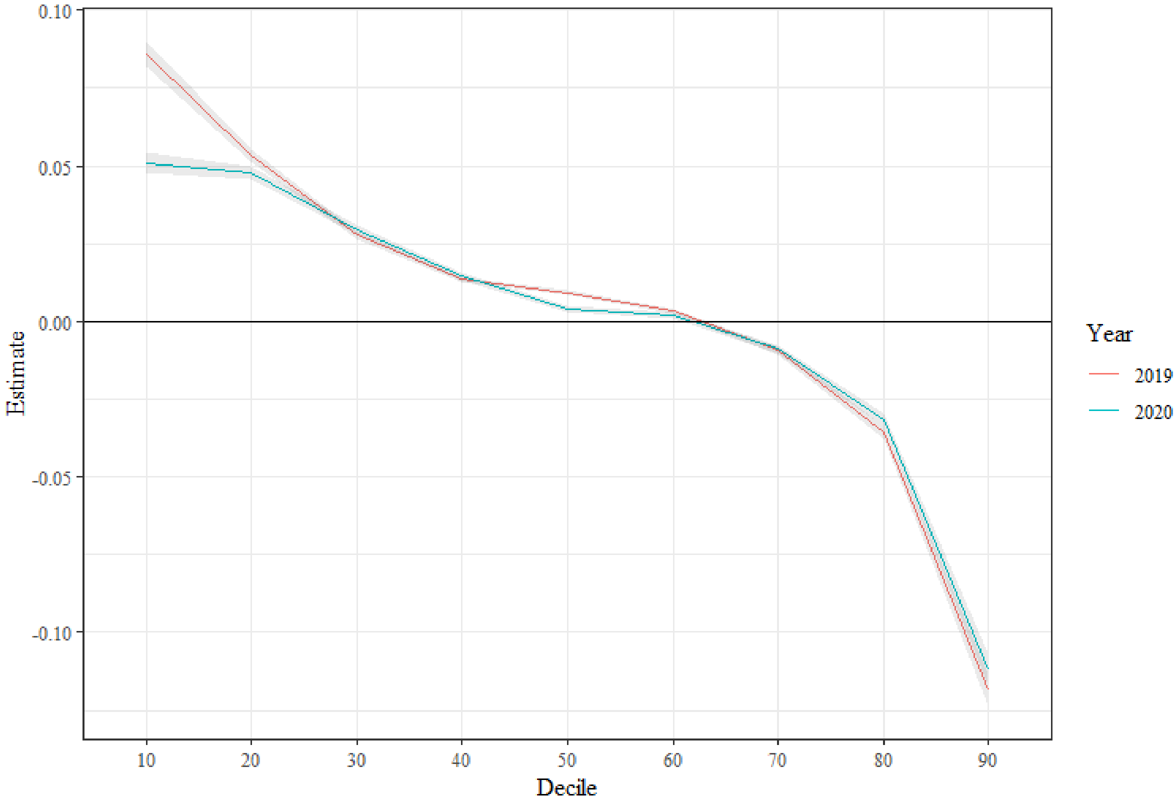
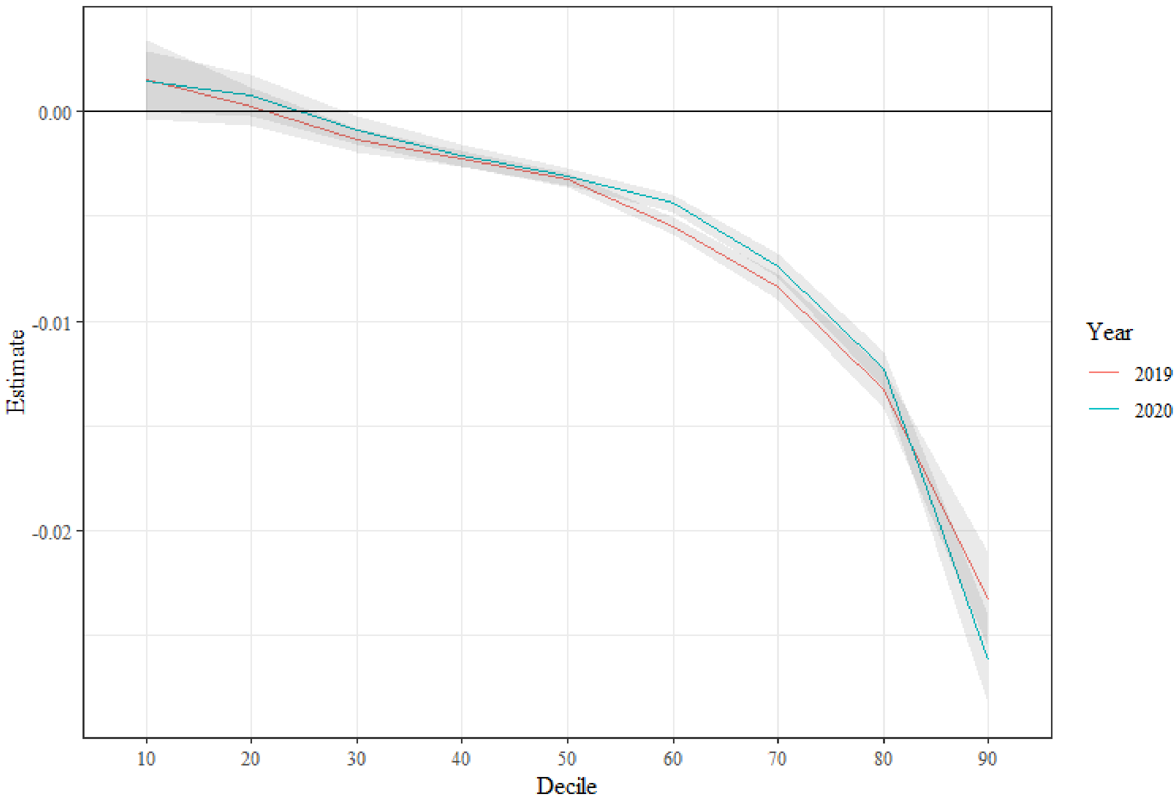


Figure 11: Coefficients across deciles of revenue growth, Age, 2019 – 2020



HOSPITALITY

The relationships between growth and previous growth, size and age follow the same pattern in 2019 as in 2020. Therefore, we turn to more specific areas of the economy where we do expect such a break due to the COVID-19 crisis. We look at one specific sector in our dataset: hospitality, cafés and restaurants specifically. Due to contact-limiting measures, cafés and restaurants could no longer conduct business as usual and lost sales; approximately 60% of firms in the hospitality sector received at least one of the government support packages (CBS, 2022a). Therefore, these businesses are a compelling case for business dynamism disruption.

We estimate the same specification as before; however, we estimate this regression only for this one sector, separately for growth in 2019 and 2020. Below, in table 6, we report the corresponding deciles. In line of expectations, the distribution for cafés and restaurants outlets has shifted towards the left severely. For example, in 2020, the 80th percentile for firm growth has approximately the same value as the 20th percentile in 2019.

The regression results are graphically displayed in Figures 12-15; the full regression results are presented in Appendix F.

Table 6: Deciles of revenue growth for cafés and restaurants – 2019 and 2020

Year	P10	P20	P30	P40	P50	P60	P70	P80	P90
2019	-0.515	-0.165	-0.077	-0.030	0.003	0.034	0.078	0.145	0.356
2020	-0.786	-0.625	-0.532	-0.461	-0.398	-0.330	-0.248	-0.138	0.064

The first lag of revenue growth has a different relationship with current revenue growth in 2019 than 2020 for cafés and restaurants. In 2019, we observe a positive inverted-U, the lowest and highest deciles experience persistence in their growth patterns. That means that the fastest growing cafés and restaurants experience growth, which continues, and the slowest growing (i.e., shrinking) cafés and restaurants experience shrinkages, which continues. In 2020, the significant positive autocorrelations turn into smaller coefficients, not being significantly different from zero.

For the second lag, we see an insignificant relationship in 2019. Interestingly, this relationship changes in 2020 only for the lower deciles. That means that the slowest growing cafés and restaurants suffered from a negative second autocorrelation; growth in 2018 discourages growth in 2020. We know that the lower deciles of revenue growth for cafés and restaurants in 2020 are negative; thus, growth in 2018 discourages growth (i.e., encourages shrinking) for the slowest-growing cafés and restaurants.

For the lagged size of the firm, we see a slightly different relationship between size and firm growth. In 2019, we observe a positive relationship between size and growth for the slowest-growing firms, which turns negative for the highest growing firms. However, in 2020, the relationship between size and growth is negative for all firms, except positive for the slowest-growing firms. The pattern remains that size encourages growth for the slowest-growing firms. The relationship between age and firm growth remains the same in 2019 and 2020.

Overall, for hospitality, we see the existing growth dynamics fall apart in 2020. This is in contrast with the earlier results that year-on-year aggregate dynamics did not change severely. This could mean that the economic effects of the pandemic are sector-specific, rather than affecting the entire population

Figure 12: First Lag coefficient across deciles for cafés and restaurants, 2019 and 2020

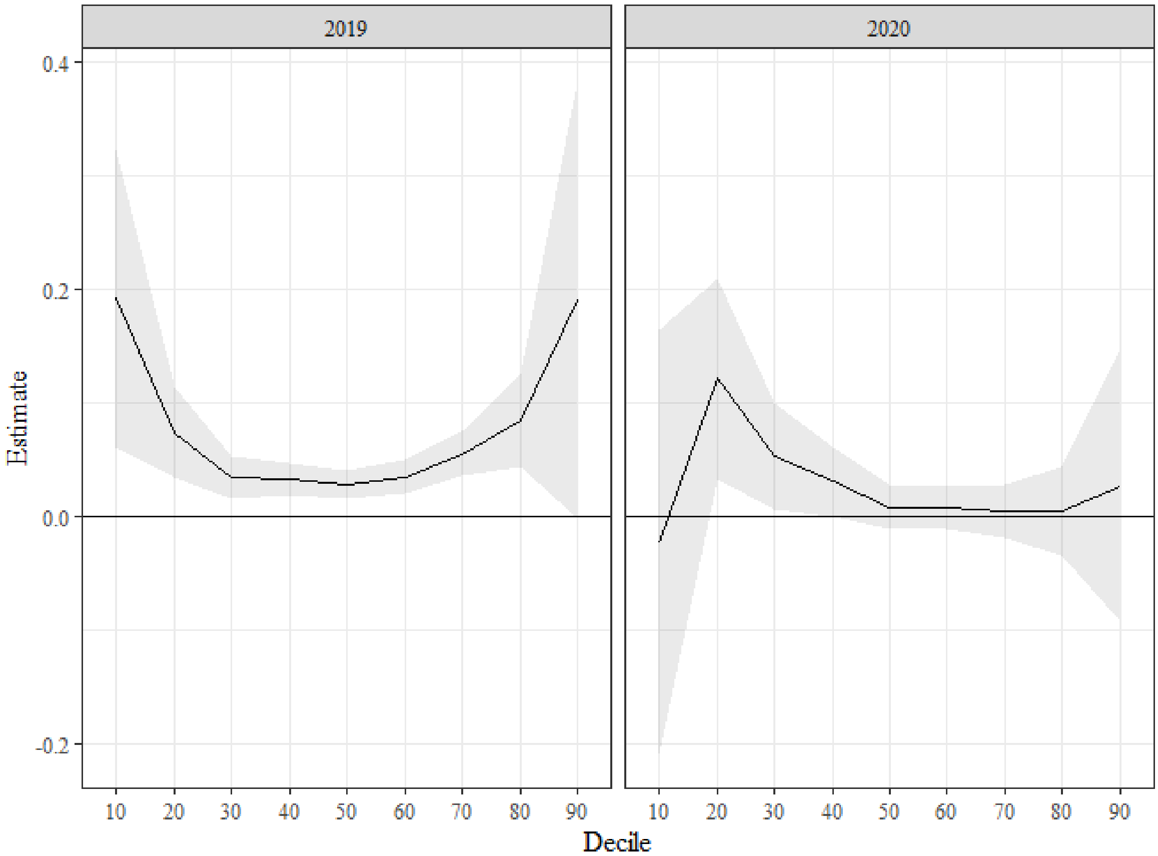


Figure 13: Second Lag coefficient across deciles for cafés and restaurants, 2019 and 2020

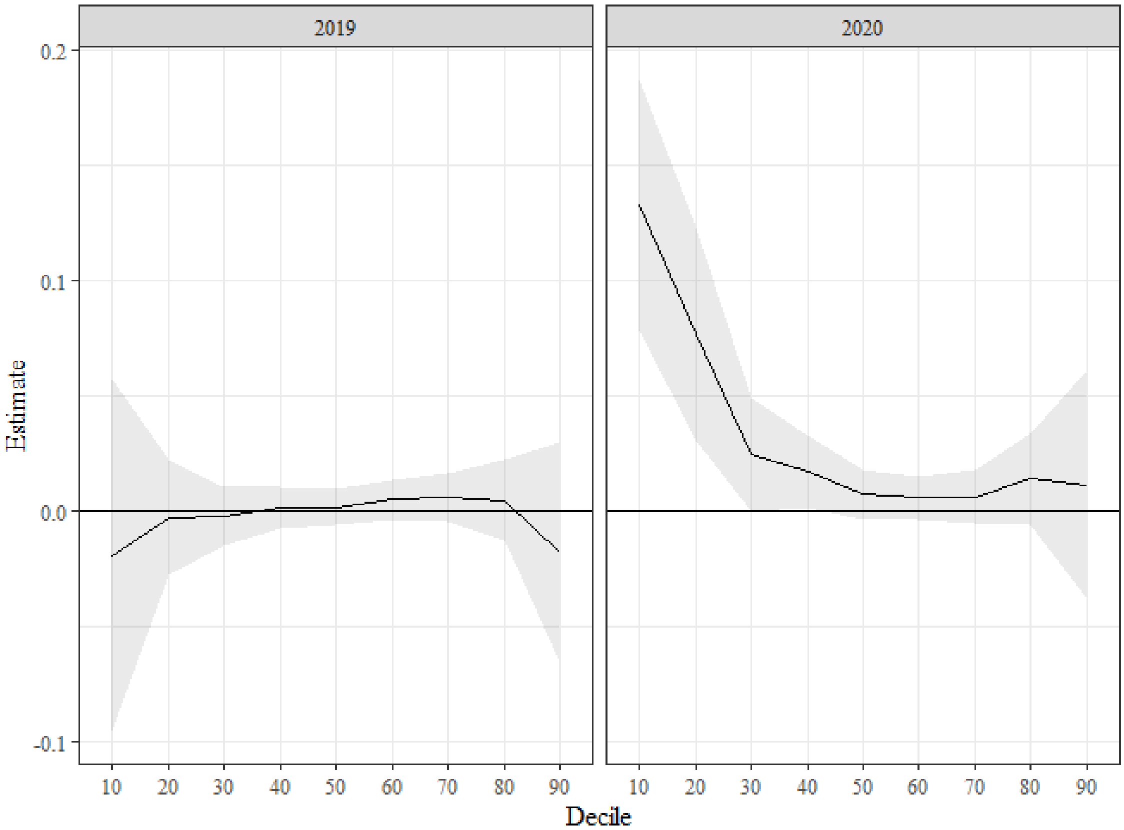


Figure 14: Lagged Revenue Levels coefficients across deciles for cafés and restaurants, 2019 and 2020

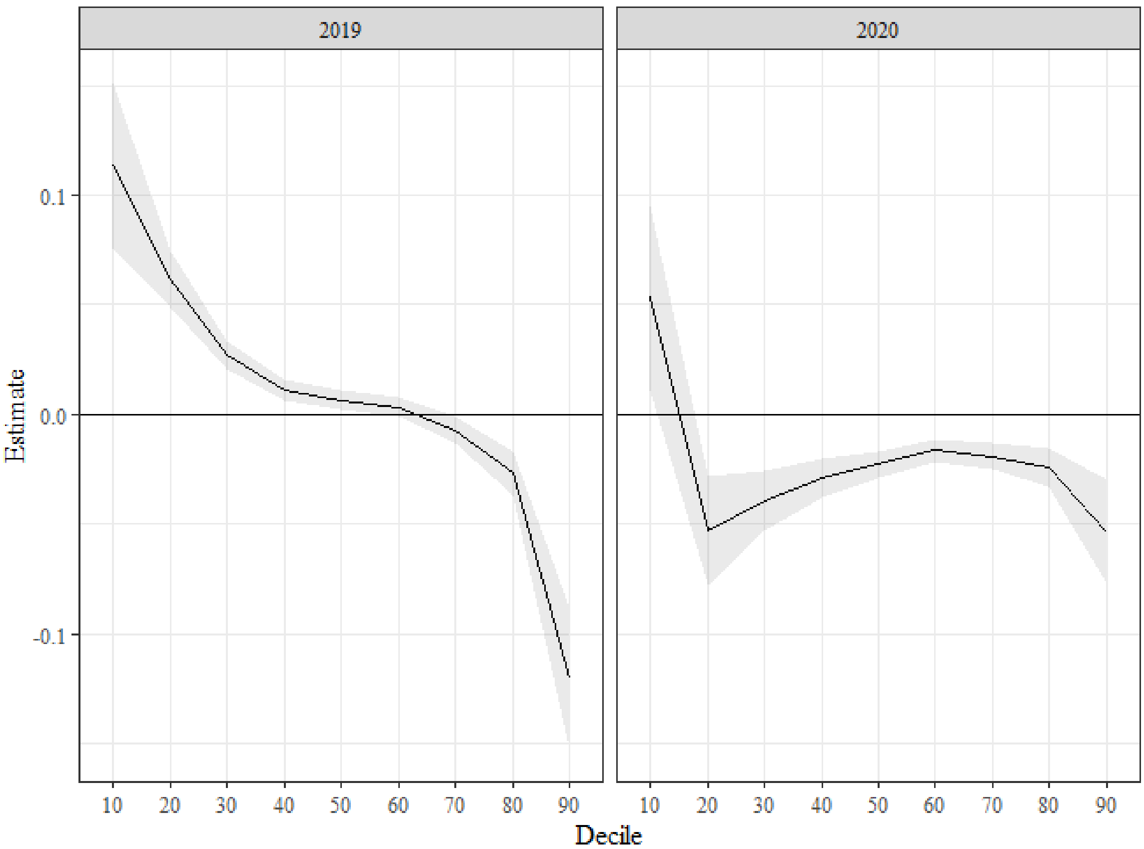
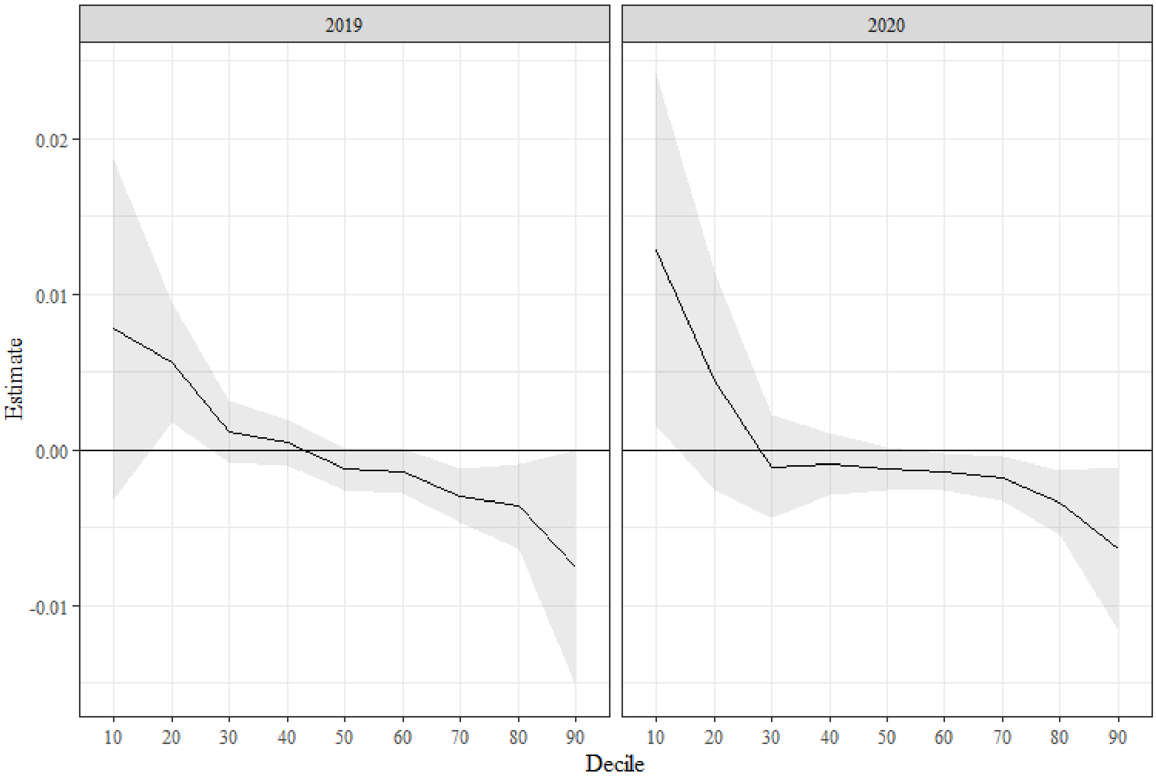


Figure 15: Age coefficients across deciles for cafés and restaurants, 2019 and 2020



RECIPIENTS OF GOVERNMENT SUPPORT

In the previous sections, we have analysed various aspects of firm growth patterns: across size classes, between pre-covid and covid years – 2019 and 2020 –, and in hospitality. From the literature, there has been concerns that the government support may have caused distortions. We do not prove any causal relationship here, but rather show signs of a potential disruption. Due to the endogeneity between government support and revenue growth, we do not include government support in the regressions.

First, we consider descriptive statistics that show the differences between recipient and non-recipient firms.[9] There are a few noticeable differences. Supported firms – on average – experience lower revenue growth, are larger, and older than unsupported firms. Furthermore, the exit rate for supported firms is 6 percentage points lower.

Table 7: Averages of core variables for supported versus unsupported firms, 2020

	Supported	Unsupported
Revenue growth	-0.027	0.074
Revenue levels	8,592,123	5,343,710
Age	9.375	8.022
Market exit	0.012	0.074
Nr. of observations	88346	160405

Additionally, we report the deciles of revenue growth for both supported and unsupported firms.[10]The biggest difference is that unsupported firms have a more extreme distribution of revenue growth. Though the most negative values for revenue growth are lower among unsupported firms, the most positive values of revenue growth are also higher among unsupported firms.

Table 8: Deciles of revenue growth for supported and unsupported firms, 2020

Year	P10	P20	P30	P40	P50	P60	P70	P80	P90
Supported	-0.531	-0.349	-0.235	-0.154	-0.087	-0.023	0.047	0.146	0.364
Unsupported	-0.802	-0.347	-0.159	-0.059	0.003	0.067	0.153	0.289	0.643

We estimate the regressions for 2020 separately for the two populations. In the Figures below, we show the coefficients of the first growth lag for the non-receiving and receiving firms. We see no significant difference in the coefficients for lagged firm size and the second lag of revenue growth; these results are shown in the Appendix G.

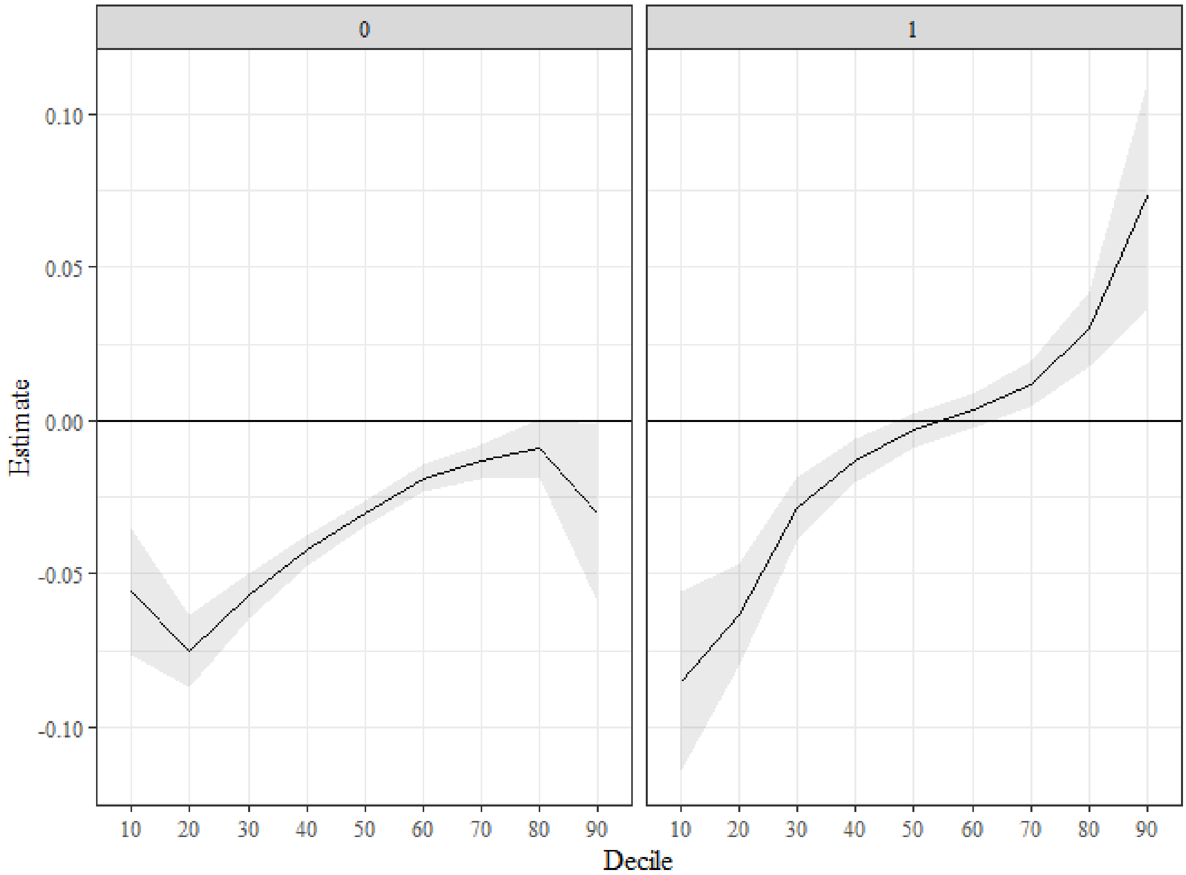
For the first lag, the relationship is relatively similar for the lower deciles, but changes severely for the higher deciles. The fastest growing firms receiving government support experienced a positive effect from their previous growth, compared to a (mildly) negative effect for firms not receiving government support. An explanation is that the fastest-growing firms among the supported

[9] NB: Revenues do not include received government support.
[10] NB: Revenues of firms that exited the market are labelled as 0, and thus, have a -100% revenue growth rate.

firms were able to use government support to further their growth, whereas their unsupported counterpart did not receive any government support to achieve this. Therefore, growth could be particularly persistent for the fastest-growing, supported firms.

However, the eligibility criteria for one of the largest government support packages –NOW, the wage subsidy – is at least 20% *expected* revenue losses. Therefore, there is potentially a problem of selection. It could very well be that it is other underlying characteristics, rather than government support, that causes this difference in the higher deciles.

Figure 16: Coefficients across deciles for unsupported and supported firms, First Lag



VI. CONCLUSIONS

In this thesis, we looked at business dynamism during the COVID-19 pandemic. We discuss with fields of literature: (1) on firm growth, and (2) on the economic effects of the COVID-19 pandemic. With the existing theories in mind, we answer our research question: how did the COVID-19 pandemic affect firm growth dynamics, across the distribution of firm growth?

We make a few contributions to the literature on firm growth. Compared to previous research, we use a different method, unconditional quantile regression instead of conditional quantile regression. By using this method, we can gain additional insights more about population-wide relationships, and the effect of conditioning on the covariates. We find that the results differ, specifically for the higher deciles, with the findings of El-Dardiry and Vogt (2023). They find an inverse-U (negative autocorrelation for the extreme deciles), whereas we find a mild negative effect for the lower deciles, and a positive autocorrelation for the higher deciles. This suggests that the slower growing firms experience negative effects from growth, whereas the fastest growing firms continue to grow further, as their previous growth is persistent. This effect is moderated by (lagged) firm size, for which a negative relationship is found over the deciles. This adds to the notion that firm growth in the Netherlands is at least not a complete random process, in contrast to what Gibrat's Law suggests.

Moreover, we elaborate the analysis of firm growth patterns in the Netherlands by looking at patterns for the different size classes. Here we find first and foremost that the aggregate results were driven by micro-firms, emphasizing the importance of differentiation based on size class if one wants to further understand firm growth. Only for the smaller, slowest-growing firms, we find negative autocorrelation. For larger firms, we find strong persistence in the higher deciles. This suggests that larger, fast-growing firms can continue their growth more easily, whereas smaller, particularly slower-growing firms are unable to profit from their past growth. As well in this differentiation, we see that this effect is moderated by the influence of firm-size: all categories of firms have a (mildly) negative relationship between (lagged) firm size and firm growth, except the slowest-growing micro-firms. Therefore, in both analyses, we find that growth is more persistent for high-growth firms, but size counteracts this effect.

In relation to the COVID-19 pandemic, we find that the pandemic has moved the distribution of revenue growth towards the left. Moreover, we found that market exit of unsupported firms is significantly higher than market exit of supported firms. However, both in the transition-matrices and the comparative regressions between years, the coefficients do not indicate a structural break on average in growth patterns, perhaps surprisingly. Our next question then became: is the disruption completely absent, or are there sectors of the economy where we can observe a disruption? Therefore, we turn to two additional sets of regressions.

Firstly, we inquire growth patterns of hospitality, specifically cafés and restaurants, which were severely affected by the contact-limiting measures in terms of business activities. We find for this sector that the relationships change drastically for the highest and lowest deciles, respectively. This shows that the 'regular' dynamics have been disrupted for hospitality. There is a plethora of explanation for this phenomenon: firstly, government support has disrupted the regular process of creative destruction, giving firms an opportunity through preventing bankruptcies or granting liquidity that otherwise would not be there; secondly, contact-limiting measures have affected firms differently; whereas some firms may have digitalized which allowed for business without contact, the firms that did not may have suffered more (e.g., Freeman and Bettendorf, 2023). Overall, this suggests that the pandemic affected business dynamism among the cafés and restaurants.

Secondly, we further study the effects of government support, by showing the differences in coefficients between non-recipients and recipients of government support during the COVID-19 pandemic. Here, we find that only the first autocorrelation differs significantly. For the highest deciles, we find that while non-recipient firms suffer from a negative first autocorrelation, recipient firms experience a positive autocorrelation. This suggests that among supported firms, the fastest-growing firms were able to make their growth persistent, which unsupported firms were unable to do. For the high-growth recipients of government aid, growth is thus persistent, with one percentage point growth contributing to an additional 0.08 percentage points growth.

We found that the exercise of differentiating (quantile) regressions on Dutch firm growth based on size classes brings about new insights on heterogeneity of firm growth. Furthermore, by using unconditional, rather than conditional regression, we may say more about population-wide effects, rather than the effects on specific conditional quantiles. The pandemic has induced differences in growth patterns, but, mainly for specific groups of firms. Further research could investigate the effects of the pandemic, by including productivity measures, including future years for more medium-term perspectives, and using improved data. Moreover, additional research about Dutch firm growth could expand by estimating the coefficients per sector, allowing for more heterogeneity in the coefficients of each sector, or using a longer timeframe to look at larger time trends.

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VIII. APPENDIX

Appendix A: Government Support Packages During The Covid-19 Pandemic

There are three government support package that we consider in this thesis: (1) Noodmaatregel Overbrugging Werkgelegenheid (NOW), a wage subsidy; (2) Tegemoetkoming Vaste Lasten (TVL), a compensation for the fixed costs of firms in specific sectors; (3) Belastinguitstel, a tax deferral. In this appendix, we briefly cover the main policy aspects of these three main packages of government support.

NOW

The Noodmaatregel Overbrugging Werkgelegenheid (NOW) was a wage subsidy. In this period, the wage subsidy cost 25 billion euros, and was used by 7.3% of all firms (Algemene Rekenkamer, 2023; CBS, 2022b).

Firms could apply to the wage subsidy, if they expected a revenue loss of at least 20% over a specific set of months (often a financial quarter) compared to similar previous months, the set of month differed per NOW package. Based on this expectation, the government subsidized wage costs proportionally, on the condition that firms would retain these jobs (UWV, 2022). The goal was to prevent bankruptcies and prevent unemployment.

TVL

The Tegemoetkoming Vaste Lasten (TVL) was a subsidy for the fixed costs of smaller firms (less than 250 employees) in affected sectors, such as hospitality. In order to be eligible, firms had to have experienced a revenue loss of at least 30%. It costs 10 billion euros, and was used by 12,9% of all firms (Algemene Rekenkamer, 2023; CBS, 2022b).

Tax deferral

All firms could apply to a tax deferral. There were no eligibility criteria. Repayment has started since this year, and the repayment period is 5 years. In total, 20 billion euros has been deferred – measured on June 2022 – by 200.000 firms, estimations are that part of this will not be repaid due to market exit (CBS, 2022b).

Appendix B: Government Support Transition-Matrices

TRANSITION-MATRIX, NON-RECIPIENTS OF GOVERNMENT SUPPORT, 2019-2020

	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Exit
Q1	0.146	0.080	0.056	0.042	0.037	0.042	0.040	0.050	0.081	0.173	0.252
Q2	0.105	0.085	0.085	0.079	0.075	0.076	0.082	0.098	0.132	0.094	0.089
Q3	0.068	0.068	0.081	0.095	0.106	0.109	0.126	0.125	0.117	0.057	0.048
Q4	0.052	0.055	0.071	0.094	0.130	0.150	0.147	0.123	0.101	0.044	0.033
Q5	0.042	0.042	0.060	0.084	0.120	0.258	0.137	0.110	0.078	0.040	0.029
Q6	0.041	0.047	0.066	0.091	0.141	0.157	0.163	0.134	0.093	0.040	0.027
Q7	0.043	0.052	0.069	0.104	0.127	0.129	0.150	0.141	0.106	0.049	0.029
Q8	0.053	0.070	0.082	0.101	0.103	0.105	0.129	0.143	0.122	0.063	0.030
Q9	0.075	0.094	0.097	0.087	0.091	0.082	0.094	0.114	0.139	0.087	0.039
Q10	0.147	0.110	0.081	0.071	0.057	0.056	0.066	0.087	0.125	0.140	0.061

TRANSITION-MATRIX, RECIPIENTS OF GOVERNMENT SUPPORT, 2019-2020

	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Exit
Q1	0.194	0.134	0.090	0.063	0.055	0.050	0.052	0.064	0.100	0.157	0.042
Q2	0.138	0.150	0.134	0.117	0.086	0.066	0.072	0.082	0.081	0.058	0.016
Q3	0.112	0.152	0.151	0.140	0.109	0.079	0.082	0.082	0.055	0.029	0.009
Q4	0.104	0.146	0.166	0.138	0.122	0.098	0.086	0.069	0.045	0.020	0.005
Q5	0.097	0.131	0.156	0.153	0.135	0.104	0.087	0.067	0.044	0.020	0.007
Q6	0.086	0.124	0.138	0.152	0.142	0.106	0.104	0.080	0.048	0.016	0.004
Q7	0.086	0.122	0.140	0.145	0.132	0.099	0.106	0.087	0.057	0.022	0.004
Q8	0.088	0.133	0.136	0.135	0.119	0.087	0.097	0.096	0.073	0.032	0.004
Q9	0.112	0.147	0.137	0.113	0.093	0.072	0.087	0.087	0.093	0.051	0.008
Q10	0.179	0.148	0.105	0.077	0.067	0.055	0.068	0.074	0.100	0.114	0.014

Appendix C: UQR Results, Whole Sample, 2020

	10	20	30	40	50	60	70	80	90
α: constant	-0.972*** (0.030)	-0.728*** (0.022)	-0.398*** (0.017)	-0.137*** (0.014)	0.029** (0.013)	0.094*** (0.013)	0.302*** (0.017)	0.752*** (0.024)	2.121*** (0.058)
β_1: revenue growth, t – 1	-0.065*** (0.009)	-0.069*** (0.005)	-0.045*** (0.003)	-0.029*** (0.002)	-0.019*** (0.002)	-0.009*** (0.002)	-0.002 (0.002)	0.006 (0.004)	0.006 (0.012)
β_2: revenue growth, t – 2	-0.019*** (0.005)	-0.010*** (0.003)	-0.004* (0.002)	0.000 (0.002)	0.004*** (0.001)	0.009*** (0.001)	0.015*** (0.002)	0.031*** (0.003)	0.076*** (0.008)
β_3: firm size, t – 1	0.051*** (0.002)	0.048*** (0.001)	0.029*** (0.001)	0.015*** (0.001)	0.004*** (0.001)	0.002** (0.001)	-0.009*** (0.001)	-0.032*** (0.001)	-0.112*** (0.003)
γ: age	0.001** (0.001)	0.001* (0.000)	-0.001*** (0.000)	-0.002*** (0.000)	-0.003*** (0.000)	-0.004*** (0.000)	-0.007*** (0.000)	-0.012*** (0.000)	-0.026*** (0.001)
R-squared	0.04	0.10	0.09	0.06	0.05	0.03	0.03	0.03	0.04
Nr. observations	140,177	140,177	140,177	140,177	140,177	140,177	140,177	140,177	140,177

Note. Standard errors in parenthesis; p < 0.01 ***, p < 0.05 **, p < 0.10 *

APPENDIX D: UQR Results, Differentiated to Size Class, 2020

Variable	Size class	10	20	30	40	50	60	70	80	90
α: constant	1: 0 - 9 FTE	-0.903*** (0.046)	-0.615*** (0.033)	-0.292*** (0.025)	-0.031 (0.019)	0.120*** (0.017)	0.142*** (0.017)	0.370*** (0.022)	0.944*** (0.034)	2.858*** (0.084)
	2: 10 - 49 FTE	-0.009 (0.083)	0.337*** (0.061)	0.419*** (0.048)	0.467*** (0.042)	0.495*** (0.038)	0.554*** (0.038)	0.787*** (0.050)	1.158*** (0.069)	2.452*** (0.165)
	3: 50 - 99 FTE	-0.002 (0.227)	0.310* (0.181)	0.484** (0.160)	0.556*** (0.140)	0.708*** (0.130)	0.806*** (0.132)	1.161*** (0.174)	1.649*** (0.258)	3.287*** (0.591)
	4: 100+ FTE	0.146 (0.206)	0.224* (0.130)	0.486** (0.113)	0.417*** (0.111)	0.426*** (0.103)	0.755*** (0.108)	0.886*** (0.142)	1.056*** (0.202)	1.810*** (0.493)
β_1: revenue growth, t – 1	1: 0 - 9 FTE	-0.068*** (0.009)	-0.071*** (0.005)	-0.046*** (0.003)	-0.031*** (0.002)	-0.021*** (0.002)	-0.013*** (0.002)	-0.009*** (0.002)	-0.005 (0.004)	-0.015 (0.012)
	2: 10 - 49 FTE	-0.045** (0.021)	-0.049 0.014	-0.028*** (0.009)	-0.008 (0.007)	0.007 (0.006)	0.023*** (0.006)	0.051*** (0.008)	0.107*** (0.014)	0.214*** (0.040)
	3: 50 - 99 FTE	-0.003 (0.026)	-0.006 0.030	0.011*** (0.020)	0.023 (0.018)	0.041*** (0.015)	0.051*** (0.016)	0.074*** (0.021)	0.161*** (0.036)	0.431*** (0.107)
	4: 100+ FTE	-0.039 (0.070)	-0.025 0.031	-0.018*** (0.025)	0.019 (0.020)	0.027 (0.017)	0.043*** (0.017)	0.091*** (0.023)	0.074** (0.030)	0.226*** (0.076)
β_2: revenue growth, t – 2	1: 0 - 9 FTE	-0.021*** (0.006)	-0.012*** (0.003)	-0.006*** (0.002)	-0.002 (0.002)	0.002** (0.001)	0.007*** (0.001)	0.012*** (0.002)	0.026*** (0.003)	0.069*** (0.009)
	2: 10 - 49 FTE	-0.007 (0.011)	0.002 (0.008)	0.009 (0.006)	0.011** (0.005)	0.017*** (0.004)	0.022*** (0.004)	0.034*** (0.005)	0.060*** (0.009)	0.129*** (0.025)
	3: 50 - 99 FTE	-0.011 (0.022)	0.029 (0.019)	0.001 (0.019)	0.025* (0.015)	0.029** (0.013)	0.052*** (0.013)	0.069*** (0.019)	0.080*** (0.031)	0.104 (0.093)
	4: 100+ FTE	-0.003 (0.023)	0.006 (0.015)	0.025** (0.012)	0.014 (0.013)	0.036*** (0.010)	0.032*** (0.010)	0.033** (0.015)	0.052** (0.024)	0.148** (0.070)
β_3: firm size, t – 1	1: 0 - 9 FTE	0.047*** (0.003)	0.039*** (0.002)	0.019*** (0.001)	0.005*** (0.001)	-0.004*** (0.001)	-0.003*** (0.001)	-0.014*** (0.001)	-0.044 (0.001)***	-0.165 (0.004)***
	2: 10 - 49 FTE	-0.013** (0.005)	-0.022*** (0.004)	-0.022*** (0.003)	-0.022*** (0.003)	-0.026*** (0.002)	-0.028*** (0.002)	-0.041*** (0.003)	-0.062 (0.004)***	-0.141 (0.010)***

	3: 50 - 99 FTE	-0.023*** (0.009)	-0.024** (0.010)	-0.030*** (0.008)	-0.031*** (0.008)	-0.034*** (0.007)	-0.039*** (0.007)	-0.060*** (0.009)	-0.080 (0.013)***	-0.185 (0.034)***
	4: 100+ FTE	-0.023** (0.011)	-0.013* (0.007)	-0.022*** (0.006)	-0.020*** (0.005)	-0.019*** (0.005)	-0.033*** (0.005)	-0.042*** (0.006)	-0.045 (0.008)***	-0.081 (0.020)***
γ: age	1: 0 - 9 FTE	0.001 (0.001)	-0.001 (0.001)	-0.002*** (0.000)	-0.003*** (0.000)	-0.004*** (0.000)	-0.005*** (0.000)	-0.008*** (0.000)	-0.012*** (0.000)	-0.029*** (0.001)
	2: 10 - 49 FTE	0.001 (0.001)	0.003** (0.001)	0.001* (0.001)	0.000 (0.001)	-0.002*** (0.001)	-0.004*** (0.001)	-0.007*** (0.001)	-0.014*** (0.001)	-0.021*** (0.002)
	3: 50 - 99 FTE	0.001 (0.003)	0.004 (0.003)	0.001 (0.002)	0.001 (0.002)	-0.001 (0.002)	-0.001 (0.002)	-0.005** (0.002)	-0.012*** (0.003)	-0.023*** (0.007)
	4: 100+ FTE	0.005 (0.003)	0.004 (0.002)	0.002 (0.002)	0.004** (0.002)	0.002 (0.002)	-0.001 (0.002)	-0.006*** (0.002)	-0.010*** (0.003)	-0.014*** (0.006)
R-squared	1: 0 - 9 FTE	0.030	0.065	0.057	0.045	0.036	0.028	0.025	0.030	0.038
	2: 10 - 49 FTE	0.106	0.238	0.208	0.148	0.109	0.081	0.061	0.053	0.039
	3: 50 - 99 FTE	0.239	0.241	0.162	0.123	0.096	0.095	0.081	0.075	0.066
	4: 100+ FTE	0.189	0.237	0.159	0.129	0.118	0.105	0.109	0.091	0.102
Nr. of observations	1: 0 - 9 FTE	104,413								
	2: 10 - 49 FTE	28,671								
	3: 50 - 99 FTE	3,914								
	4: 100+ FTE	3,179								

Note. Standard errors in parenthesis; $p < 0.01$ ***, $p < 0.05$ **, $p < 0.10$ *

APPENDIX E: UQR Results, Per Year 2019 – 2020

Variable	Year	10	20	30	40	50	60	70	80	90
α: constant	2019	-1.355*** (0.045)	-0.849*** (0.024)	-0.429*** (0.015)	-0.193*** (0.011)	-0.091*** (0.009)	0.050*** (0.011)	0.298*** (0.015)	0.780*** (0.023)	2.075*** (0.05)
	2020	-0.972*** (0.03)	-0.728*** (0.022)	-0.398*** (0.017)	-0.137*** (0.014)	0.029** (0.013)	0.094*** (0.013)	0.302*** (0.017)	0.752*** (0.024)	2.121*** (0.058)
β1: revenue growth, t – 1	2019	-0.129*** (0.011)	-0.079*** (0.004)	-0.044*** (0.002)	-0.023*** (0.001)	-0.012*** (0.001)	-0.007*** (0.001)	-0.001 (0.002)	0.007* (0.004)	0.016 (0.011)
	2020	-0.065*** (0.009)	-0.069*** (0.005)	-0.045*** (0.003)	-0.029*** (0.002)	-0.019*** (0.002)	-0.009*** (0.002)	-0.002 (0.002)	0.006 (0.004)	0.006 (0.012)
β2: revenue growth, t – 2	2019	-0.041*** (0.006)	-0.019*** (0.003)	-0.007*** (0.002)	-0.001 (0.001)	0.004*** (0.001)	0.009*** (0.001)	0.017*** (0.002)	0.031*** (0.003)	0.063*** (0.008)
	2020	-0.019*** (0.005)	-0.010*** (0.003)	-0.004* (0.002)	0.000 (0.002)	0.004*** (0.001)	0.009*** (0.001)	0.015*** (0.002)	0.031*** (0.003)	0.076*** (0.008)
β3: firm size, t – 1	2019	0.086*** (0.002)	0.053*** (0.001)	0.028*** (0.001)	0.013*** (0.000)	0.009*** (0.000)	0.003*** (0.000)	-0.009*** (0.001)	-0.036*** (0.001)	-0.119*** (0.003)
	2020	0.051*** (0.002)	0.048*** (0.001)	0.029*** (0.001)	0.015*** (0.001)	0.004*** (0.001)	0.002*** (0.001)	-0.009*** (0.001)	-0.032*** (0.001)	-0.112*** (0.003)
γ : age	2019	0.002 (0.001)	0.000 (0.000)	-0.001*** (0.000)	-0.002*** (0.000)	-0.003*** (0.000)	-0.005*** (0.000)	-0.008*** (0.000)	-0.013*** (0.000)	-0.023*** (0.001)
	2020	0.001 (0.001)*	0.001 (0.000)	-0.001** (0.000)	-0.002*** (0.000)	-0.003*** (0.000)	-0.004*** (0.000)	-0.007*** (0.000)	-0.012*** (0.000)	-0.026*** (0.001)
R-squared	2019	0.025	0.038	0.026	0.013	0.013	0.018	0.027	0.037	0.039
	2020	0.041	0.096	0.087	0.064	0.046	0.035	0.029	0.033	0.036
Nr. of observations	2019	140,177								
	2020	136,738								

Note. Standard errors in parenthesis; p < 0.01 ***, p < 0.05 **, p < 0.10 *

APPENDIX F: UQR Results, Cafés and restaurants, 2019 and 2020

Variable	Year	10	20	30	40	50	60	70	80	90
α: constant	2020	-1.664*** (0.295)	-0.485*** (0.172)	-0.229** (0.093)	-0.086 (0.06)	0.023 (0.042)	0.048 (0.035)	0.15*** (0.045)	0.313*** (0.066)	0.807*** (0.171)
	2019	-1.649*** (0.272)	-0.982*** (0.093)	-0.434*** (0.044)	-0.185*** (0.032)	-0.082*** (0.029)	-0.011 (0.032)	0.172*** (0.04)	0.48*** (0.069)	1.808*** (0.233)
β_1: revenue growth, t – 1	2020	-0.022 (0.095)	0.122*** (0.045)	0.054** (0.024)	0.031* (0.016)	0.009 (0.010)	0.008 (0.009)	0.006 (0.012)	0.004 (0.02)	0.027 (0.061)
	2019	0.193*** (0.067)	0.074*** (0.020)	0.034*** (0.01)	0.033*** (0.007)	0.029*** (0.006)	0.035*** (0.007)	0.056*** (0.01)	0.085*** (0.021)	0.191* (0.098)
β_2: revenue growth, t – 2	2020	0.133*** (0.028)	0.077*** (0.024)	0.025** (0.012)	0.017** (0.008)	0.007 (0.005)	0.006 (0.005)	0.006 (0.006)	0.014 (0.010)	0.011 (0.025)
	2019	-0.019 (0.039)	-0.002 (0.013)	-0.002 (0.006)	0.001 (0.005)	0.002 (0.004)	0.005 (0.004)	0.006 (0.005)	0.005 (0.009)	-0.018 (0.024)
β_3: firm size, t – 1	2020	0.053** (0.021)	-0.053*** (0.013)	-0.039*** (0.007)	-0.029*** (0.004)	-0.023*** (0.003)	-0.016*** (0.003)	-0.019*** (0.003)	-0.024*** (0.005)	-0.053*** (0.012)
	2019	0.114*** (0.019)	0.062*** (0.007)	0.027*** (0.003)	0.011*** (0.002)	0.007*** (0.002)	0.003 (0.002)	-0.007** (0.003)	-0.027*** (0.005)	-0.12*** (0.016)
γ: age	2020	0.013** (0.006)	0.004 (0.004)	-0.001 (0.002)	-0.001 (0.001)	-0.001 (0.001)	-0.001** (0.001)	-0.002** (0.001)	-0.003*** (0.001)	-0.006** (0.003)
	2019	0.008 (0.006)	0.006*** (0.002)	0.001 (0.001)	0 (0.001)	-0.001* (0.001)	-0.001* (0.001)	-0.003*** (0.001)	-0.004*** (0.001)	-0.008* (0.004)
R-squared	2020	0.005	0.008	0.013	0.019	0.024	0.019	0.017	0.017	0.013
	2019	0.015	0.031	0.022	0.010	0.008	0.008	0.014	0.018	0.026
Nr. of observations	2019	4,326								
	2020	4,135								

Note. Standard errors in parenthesis; $p < 0.01$ ***, $p < 0.05$ **, $p < 0.10$

APPENDIX G: UQR Results, Government support, 2020

Variable		**10**	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**
α: constant	Supported	-0.930*** (0.057)	-0.817*** (0.041)	-0.478*** (0.033)	-0.181*** (0.028)	-0.006 (0.024)	0.119*** (0.024)	0.302*** (0.029)	0.663*** (0.04)	1.653*** (0.08)
	Unsupported	-1.055*** (0.037)	-0.906*** (0.026)	-0.624*** (0.020)	-0.375*** (0.016)	-0.177*** (0.015)	-0.099*** (0.016)	0.116*** (0.021)	0.611*** (0.031)	2.194*** (0.077)
β_1: revenue growth, t – 1	Supported	-0.085*** (0.015)	-0.063*** (0.008)	-0.029*** (0.005)	-0.013*** (0.004)	-0.003 (0.003)	0.003 (0.003)	0.012*** (0.004)	0.030*** (0.006)	0.073*** (0.019)
	Unsupported	-0.055*** (0.011)	-0.075*** (0.006)	-0.057*** (0.004)	-0.042*** (0.003)	-0.030*** (0.002)	-0.019*** (0.002)	-0.013*** (0.003)	-0.009* (0.005)	-0.030** (0.015)
β_2: revenue growth, t – 2	Supported	-0.007 (0.008)	0.000 (0.005)	0.002 (0.004)	0.008*** (0.003)	0.011*** (0.002)	0.013*** (0.002)	0.022*** (0.003)	0.034*** (0.004)	0.082*** (0.013)
	Unsupported	-0.026*** (0.006)	-0.016*** (0.004)	-0.008*** (0.002)	-0.006*** (0.002)	-0.001 (0.002)	0.005*** (0.002)	0.010*** (0.002)	0.027*** (0.004)	0.072*** (0.01)
β_3: firm size, t – 1	Supported	0.043*** (0.003)	0.049*** (0.002)	0.031*** (0.001)	0.014*** (0.001)	0.003*** (0.001)	-0.003*** (0.001)	-0.013*** (0.001)	-0.032*** (0.002)	-0.094*** (0.004)
	Unsupported	0.060*** (0.002)	0.064*** (0.001)	0.048*** (0.001)	0.033*** (0.001)	0.021*** (0.001)	0.017*** (0.001)	0.006*** (0.001)	-0.018*** (0.001)	-0.109*** (0.004)
γ: age	Supported	0.005*** (0.001)	0.003*** (0.001)	0.000 (0.001)	-0.002*** (0.000)	-0.003*** (0.000)	-0.004*** (0.000)	-0.007*** (0.000)	-0.010*** (0.001)	-0.019*** (0.001)
	Unsupported	-0.001 (0.001)	-0.001** (0.001)	-0.002*** (0.000)	-0.002*** (0.000)	-0.003*** (0.000)	-0.005*** (0.000)	-0.008*** (0.000)	-0.014*** (0.000)	-0.031*** (0.000)
R-squared	Supported	0.075	0.154	0.129	0.090	0.063	0.049	0.041	0.040	0.37
	Unsupported	0.022	0.046	0.051	0.043	0.029	0.025	0.017	0.021	0.30
Nr. of observations	Supported	59,451								
	Unsupported	80,726								

Note. Standard errors in parenthesis; p < 0.01 ***, p < 0.05 **, p < 0.10 *