ERASMUS UNIVERSITEIT ROTTERDAM

A wind farm case: Optimal maintenance planning for multi-component models considering period-dependent costs

MASTER THESIS IN QUANTITATIVE LOGISTICS

September 4, 2023

Full name: Donna van der Rijst *Student number:* 619034

Supervisor: Prof. Dr. Rommert Dekker
Second assessor: Asst. Prof. Dr. Olga Kuryatnikova

Abstract

This thesis presents the development of a model newly designed to create a planning schedule for multiple components with economic dependence over a finite horizon, specifically adding value to wind farm maintenance planning. The model includes period-dependent maintenance costs, with the option to add maintenance constraints that limit the amount of planned preventive maintenance per period. Markov decision process theory is implemented in the model to shift the individual preventive maintenance with integrity, and optimise over the economic dependence. The model is bench-marked with a two-component infinite Markov Decision Process model and with a two-stage stochastic programming approach model. The limitation of the designed model, which is a mixed-integer program, is that it only works for smaller instances. A future research could be to use this model as a guideline for computing a heuristic that allows larger instances, or using the mixed-integer program on a rolling horizon.

Contents

1 Introduction **1**
1.1 Problem Approach and Sub-questions 2

2 Problem Description **4**

3 Literature Review **7**
3.1 A brief review 7
3.2 Latest research 8

4 Methodology Zhu **10**
4.1 The DEF model 10
4.1.1 Period dependent costs 15
4.2 DEF Heuristic 15
4.2.1 Standard Progressive Hedging Algorithm 16
4.2.2 Heuristic Solving one Scenario 16
4.2.3 Heuristic Solving one Scenario Decomposition 20

5 Methodology Schouten **21**
5.1 Two-Component Markov Decision Process 21
5.2 Two-Component p-ARP policy 23
5.3 Two-Component p-MBRP policy 24
5.4 Retrieving policies 27
5.4.1 Discounted Rewards 27
5.4.2 Markov Manipulation 28
5.5 Costs shift PM 29
5.5.1 Preponing PM 29
5.5.2 Postponing PM 31
5.6 Complete multi-component MIP 33

6 Results **36**
6.1 Extending Schouten to Two-Component Models 36
6.2 Extending Zhu 41
6.3 Multi-Component MIP Model 43

7 Discussion and Conclusions **46**
7.1 MDP approach 46
7.2 Stochastic approach 46
7.3 Multi-Component MIP 47
7.4 Future recommendations 47

References **48**

A Additional Theorem **51**
A.1 Weibull distribution 51

B Tables from other papers **53**

Chapter 1

Introduction

The European climate law sets ambitious goals for reducing greenhouse gas emissions, with a target of at least a 55% reduction by 2030 and climate neutrality by 2050. Achieving these goals requires a significant shift in the way we produce and consume energy, and renewable sources like wind power will play a critical role in this transition. An extra motivation for investing in wind power is the research Way et al. (2022) by Oxford presenting that investing in sustainable energy has a strong economic case. Switching to renewables could save the world $12 trillion, highlighting the potential for the energy transition to be an economic opportunity rather than a burden.

Offshore wind turbines, which face higher wind velocities compared to onshore turbines, are an attractive source of energy. As with all systems, they are prone to failure due to component deterioration. Maples et al. (2013) highlights that operation and maintenance costs account for 12% of total offshore wind farm costs per year. To increase wind farm profitability, minimizing maintenance costs is essential. Preventive and corrective maintenance measures are employed to ensure the turbines continue to operate. Preventive maintenance is carried out before a failure or breakdown occurs, in order to minimize the risk of unplanned downtime. To optimize maintenance schedules, it is important to consider the costs of downtime and since wind velocities are lower in the summer, it is preferable to schedule maintenance during this season. Downtime costs can therefore be seen as period-dependent maintenance costs.

Additionally, it is important to include constraints that may limit the ability to carry out maintenance operations. Offshore Support Vessels and qualified crew are required for offshore maintenance, but scheduling can be challenging due to maintenance constraints such as manpower limitations and weather conditions that may cause high waves and wind velocities that render the Offshore Support Vessels inoperable. A maintenance constraint in the model could be a constraint that limits the number of maintenance jobs done per period.

Besides the challenge of period-dependent costs and maintenance constraints, wind farm maintenance often deals with setup costs such as mobilizing repair crews or transportation to the wind farm. Grouping maintenance for multiple components can lead to shared setup costs and this is called positive economic dependence. This approach can result in significant cost savings when compared to separate maintenance activities.

In current literature, there are multi-component models with economic dependence and maintenance constraints, and there are single-component models with period-dependent costs. However, to the best of our knowledge, there is no multi-component model that deals with economic dependence, maintenance constraints, and period-dependent costs. Developing such a multi-component model can lead to a more precise maintenance schedule, reducing the risk of costly downtime and resulting in more efficient and sustainable offshore wind energy production. Therefore, this thesis aims to answer the following research question:

How can we determine the optimal maintenance schedule for a multi-component model with economic dependence, given maintenance constraints and variable downtime costs?

1.1 Problem Approach and Sub-questions

To answer our research question, we will explore two approaches. A Markov Decision Process (MDP) approach and a two-stage stochastic programming approach. In this section, the thesis approach in chronological order with its corresponding sub-questions can be found. The sub-questions a-e are about extending existing models such that they can be compared. Sub-questions f-g are about extending a model such that we have reached an answer to our research question.

Schouten et al. (2022) uses a MDP approach, and designed several one-component MDP models with period-dependent costs, where the p-ARP and p-MBRP models are the most interesting for our use case. These models have an infinite time horizon. We want to compare these models to a multi-component model and the following sub-question arises:

a. How to transform the p-ARP and p-MBRP models to a two-component model?

The two-stage stochastic programming approach is used in Zhu et al. (2021) and Zhu et al. created amongst others a deterministic extensive form (DEF) of a two-stage stochastic model and a corresponding Progressive-Hedging-Based heuristic (DEF Heuristic). Both are multi-component models with economic dependence for a finite time horizon. This brings the next sub-questions:

b. How to add period-dependent costs to the DEF?
c. How to add period-dependent costs to the DEF Heuristic?

After extending the models of both approaches to a two-component model with economic dependence and period-dependent costs, they can be compared with each other by looking at their policies. The Two-Component p-ARP and p-MBRP models only provide policies for their recurrent states, however, for proper comparison the policies for the transient states are also of interest.

d. For a three-dimensional MDP, how to retrieve policies for transient states?

It is found that the transient states for the Two-Component p-MBRP model can not be retrieved by the method used in this thesis. Additionally, the p-ARP model setting is most similar to the setting of the two-stage stochastic programming approaches and therefore we will continue with the p-ARP model and disregard the p-MBRP model. At this point, the two approaches are extended to multi-component and period-dependent and the next sub-question arises.

e. Should the MDP approach, the two-stage stochastic programming approach, or both approaches be used for the final multi-component model?

For the two-stage stochastic programming approach, it is found that the computation time of DEF is too long to obtain the full policy, and the implementation of the faster DEF Heuristic was not successful enough. The MDP approach is therefore used for the final multi-component model. The final two sub-questions are:

f. How can we create a multi-component model with economic dependence and period-dependent costs using the p-ARP model?
g. How to add the maintenance constraint: 'In period t we only have room for X replacements'?

This thesis is structured as follows. First, we provide a more elaborate description of the problem in Chapter 2. Then, a brief literature review is provided in Chapter 3. We provide our methodology in the succeeding chapters, where Chapter 4 is all about the two-stage stochastic programming approach, and Chapter 5 is about the MDP approach. The results are provided in Chapter 6 and the conclusions, corresponding discussions, and our recommendations for further research are discussed in Chapter 7.

Chapter 2

Problem Description

In this chapter, details are discussed concerning the following aspects. Period-dependent costs, corrective and preventive maintenance, maintenance constraints, economic dependence, repairing method and time horizon. The reasoning behind choices made are from the perspective of offshore wind farm maintenance, however other use cases such as offshore solar panel plants are applicable here too.

Period-dependent costs
Firstly, we will discuss the period-dependent costs. Offshore wind farms' power production is a function of the wind velocity. At a study site north of Spain, the research López et al. (2020) found that wind turbine energy production in winter months is twice as high in summer months, due to the wind velocity. When executing maintenance on a wind turbine, it will not be operating and we call the loss of turnover the downtime costs. Since the wind velocities vary throughout the year, we refer to these downtime costs as period-dependent maintenance costs. We can predict these downtime costs with averaged period-dependent costs.

Corrective and preventive maintenance
For single components, maintenance strategies can be divided into corrective maintenance, preventive maintenance and predictive maintenance. Corrective maintenance is performed after a component fails, preventive maintenance is performed before a component fails, and predictive maintenance uses data from sensors to predict when a component fails and maintains before this time. Due to sensor uncertainties that are involved with monitoring data for which according to Ren et al. (2021) are no studies of effective and robust approaches, we will not consider predictive maintenance. Additionally, Zhao et al. (2022) argues that preventive maintenance is still the preferred maintenance strategy for most enterprises. Therefore, we will look at corrective maintenance and preventive maintenance only. In the case of corrective maintenance, the maintenance is assumed to be done immediately.

Maintenance constraints
Offshore vessels are necessary for performing offshore maintenance and a big offshore vessel with lengthy lead times is necessary for repairing heavy spare parts, such as a rotor blade. In this thesis, we will focus on maintenance for smaller parts which is possible with a more available offshore vessel. Still, executing maintenance needs a qualified crew and limited manpower

could put a limit on the number of maintenance tasks done per month. Additionally, a wind farm might choose to do zero maintenance tasks per month when high waves and wind speeds are expected, as offshore vessels are not allowed to operate in hazardous weather conditions. This all brings us to the maintenance constraints we need in our model. These should constrain the number of planned preventive maintenance tasks per month. The corrective maintenance will be neglected in these constraints.

Economic dependence
Furthermore, the distance to the shore and the distance between turbines cause the components to be economically dependent. The dependency between all components is classified by Thomas (1986) into economic dependence, random dependence, structural dependence and resource dependence. Economic dependence means that the costs of individual maintenance is different to the costs of joint maintenance. The average global distance to shore calculated by Díaz and Soares (2020) in 2020 was 18.8 km and this number is increasing. Combining vessel transport towards the wind farm can reduce costs. Secondly, random dependence means that the failure of one component will affect the performance of the other components. In our wind farm case, a failure of one component may result in the whole corresponding wind turbine being in failure mode, hence we can speak of random dependence. According to Ackermann (2005), the wind farm distances to shore are likely to increase hence one might argue that looking at the economic dependency is more interesting. Therefore in this thesis, only economic positive dependence will be considered. It will be defined as the following. If any maintenance is done, set-up costs d are charged. Performing multiple maintenance activities at the same time will share these constant set-up costs. Here, we only account for the distance to the shore and therefore assume that all components are in the same turbine.

Repairing method
The repairing method used for maintaining a component is that it will be replaced by an as-good-as-new component and the failure distributions of the components are assumed to be independent of the time of the year.

Time horizon
Schleisner (2000) states that the lifetime of a turbine is around 20 years, so for a brand-new wind farm it could be interesting to find a maintenance schedule for 20 years. However, an older wind farm might also be interested in re-planning their maintenance, or a wind farm company wants a one-year free trial of using the planning tool this thesis provides. To reduce the scope, a time horizon between one and three years is chosen.

Summary
To sum up, the following is taken into consideration in this thesis. We estimate the average monthly wind speed and therefore have period-dependent maintenance costs. We only consider corrective and preventive maintenance due to sensor uncertainties. Dependency between components in practice is intricate, since components that are in the same wind turbine have random dependence and economic dependence, and components in difference wind turbines also have economic dependence. Due to the increasing distances to the shore and to manage

the scope of this thesis, we will assume that all components have equal economic dependence which are expressed in the set-up costs d. Furthermore, when maintenance is done it is assumed to be replaced with an as-good-as-new component, the failure distribution of the components are period-independent and a time horizon between one and three years in chosen.

Chapter 3

Literature Review

In this chapter, we begin by providing a brief review of the relevant research and contextualisation of the past decades of both multi-component maintenance in general and offshore maintenance in particular. Next, we look at the most recent research that we could use for our thesis while simultaneously uncovering the newest trends.

3.1 A brief review

Multi-Component Maintenance in General

In 1997, Dekker et al. (1997) conducted a review of models with economic dependence, observing three types of dependency: economic, stochastic and structural. It also distinguished stationary from dynamic models, and within the stationary models, it defined the various options of grouping maintenance activities to grouping corrective maintenance, preventive maintenance, or combining the two. In this thesis, we will only group preventive maintenance. Kobbacy et al. (2008) provides an overview based on dependence, planning aspect, and the optimization approaches used. A conclusion Kobbacy et al. made is that from 1997 already computer intensive approaches are used, like tabu search, genetic algorithms and problem specific heuristics. Wang and Chen (2016) survey all condition based modeling which, like predictive maintenance, needs data collected from monitoring. This is not within the scope of this thesis as we will not be considering sensor-based approaches. Lastly, the review by Zhao et al. (2022) provides us, amongst others, with a clear overview of the different type of maintenance in Figure 3.1.

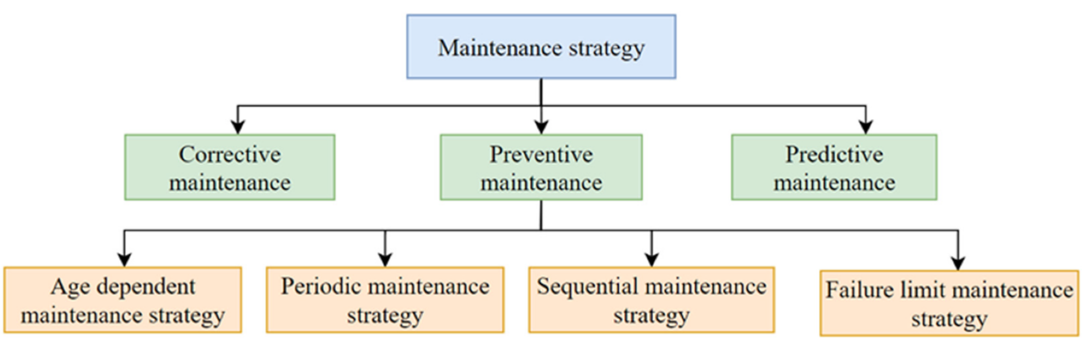


Figure 3.1: Classification of single component maintenance strategies by Zhao et al. (2022).

From Figure 3.1, predictive maintenance is not considered in this thesis. Failure limit maintenance strategy is also not considered, as this is most suitable for components with high reliability requirements which is not necessarily the case in this thesis. Furthermore, Zhao et al. (2022) examined the maintenance strategies employed for industrial equipment, with an additional fourth dependency called resource dependency, meaning that a maintenance operation can only be arranged if the necessary resources are at hand. They also classified the strategies for multi-component models specifically. Possible strategies for our wind farm case out of these classifications are batch grouping, opportunistic grouping, and static preventive grouping. A future challenge noticed by Zhao et al. (2022) is the multi-component maintenance strategies considering maintenance constraints, and highlights its importance as models without constraints are inconsistent with the actual maintenance requirements.

Offshore farm specific
The study of multi-component maintenance optimization models for wind farms began in 1997 by Van Bussel and Schöntag (1997), who was already emphasizing the significance of a maintenance strategy for wind farms. From this point, many articles can be found concerning the operation and maintenance aspects of large offshore wind farms, and the most relevant and recent review is Rinaldi et al. (2021). Ren et al. (2021) gives a clear overview of all the current problems and research done so far in offshore maintenance, and also finds that models that are more complex are often solved by meta-heuristics. It states that major challenges include long distance from shore, weather uncertainty, a lack of information from remote monitoring, unpredicted failures, aging, and subjective factors.

3.2 Latest research

The most recent articles concerning maintenance optimization models using corrective and preventive maintenance, from the year 2021 are provided below. These articles are grouped by the nature of their models which are single-component, machine learning, meta-heuristic and Bayesian.

Single Component with period-dependent costs
Schouten et al. (2022) considers a single-component model, and researches the effect of period-dependent costs where the life of the component follows a discrete-time Markov Decision Process. Schouten et al. (2022) is in line with Figure 3.1 as it provides three models following the age dependent-, period-, and sequential maintenance strategy. The Bachelor's Thesis from Cremers (2022) builds upon Schouten et al. (2022), and formulated three heuristic solutions to construct a least-cost maintenance program for a group of identical components under the constraint of restricted manpower. It calculates the effect on costs of shifting one component to a different maintenance time than its individual optimal maintenance time.

Machine Learning
The machine learning technique that is used the most is deep reinforcement learning, and is used by Nguyen et al. (2022b), Nguyen et al. (2022a), Zhang et al. (2022), Chen and Wang (2023), Zhou et al. (2022), Yousefi et al. (2022), and Pinciroli et al. (2021), where the model of Pinciroli et al. (2021) is wind farm specific and considers multiple crews. Most of them first

formulated the problem as a Markov Decision Process, and then used the deep reinforcement learning model to solve it. As Markov Decision Process suffers from the curse of dimensionality, reinforcement learning is used to also solve the model for larger instances. These reinforcement learning techniques are less accurate, however, have a much broader application. Finally, the model in Zhang et al. (2023) is also wind farm specific and uses a neural network. Its performance is tested on one wind farm and lacks further benchmarks.

Meta-heuristics
Then the meta-heuristics used are genetic algorithm by Maher et al. (2022), Chen and Feng (2022), Bárcena and Castro (2021), particle swarm optimization is used by Dai et al. (2023), and finally a simulated annealing algorithm is used by Tambe (2022), Franciosi et al. (2021). Zhu et al. (2021) shows that with a progressive-hedging-based algorithm, with is a meta-heuristic, solutions for large instances can also be found. This is bench-marked with a smaller instance that is solved with an almost optimal mixed integer program and shows that the objective error compared to the almost optimal mixed integer program , stays under 9.89%. A wind farm specific meta-heuristics is used by Wang and Deng (2022). Wang and Deng (2022) implements maintenance constraints to its multi-component model together with economic dependence. The maintenance constraints are in the form of a time window, in which it is possible to execute maintenance. In the first part of the article, they link the reliability of a component to the two-parameter Weibull distribution and determine what the best values of reliability are to perform corrective, preventive and opportunistic maintenance resulting in a big emphasis on the tuning of parameters.

Bayesian network
Furthermore, Özgür-Ünlüakın et al. (2021) uses a dynamic Bayesian network for a system under corrective maintenance, and Vijayan and Chaturvedi (2021) considers a model with stochastic and economic dependence and uses a Bayesian network to form groups in the system.

Research Gap
To conclude, some trends are seen in these articles. It was remarkable that, of all the pertinent articles discovered, more than half exploited machine learning techniques or a meta-heuristic to solve their model. Additionally, simulation is often used to test the performance of the model and another bench-marking method is often missing. Another observation is that it is difficult to create an accurate model that works for larger instances. It looks like a trade-off between the size of the instance and the accuracy of the solution. To our best knowledge, there is firstly no multi-component model that includes period-dependent costs, and secondly no multi-component model that includes maintenance constraints which is if importance as highlighted by Zhao et al. (2022).

Chapter 4

Methodology Zhu

This chapter describes two methodologies used in Zhu et al. (2021) that have a two-stage stochastic programming programming approach with a finite horizon, and explains how they are extended by adding the period-dependent costs. These two methodologies are both categorised as opportunistic grouping by Zhao et al. (2022). In the paper, first, a multistage stochastic integer program with decision-dependent uncertainty is derived, containing multiple components and economic dependency. The paper of Zhu et al. however concludes that this type of problem is too difficult to solve. Therefore, they provide us with a two-stage model to approximate the exact model and this is the DEF model. The DEF model is found in Section 4.1 together with the period-dependent cost extension. Zhu et al. (2021) also states that for large-size problems the DEF has no moderate CPU time, and therefore the article provides an efficient heuristic to find high-quality solutions which has an objective gap of at most 9.89% from the DEF. This is the Progressive-Hedging-Based Heuristic to which we will refer to as DEF Heuristic, and this model with the period-dependent extension is found in Section 4.2.

Output DEF and DEF Heuristic
Important to note here, is that the DEF and DEF Heuristic produce decision variables for only the starting period. So only the action for the starting period is received as output. By changing the settings of the starting period, a complete policy can be found.

4.1 The DEF model

In this section, we will discuss the DEF model. The objective is to minimize the total costs in a discretized finite planning horizon, where the total costs is the sum of costs made in the first period and the expected maintenance costs in the periods thereafter. The basic idea for these expected maintenance costs is to compute a number of different possible scenarios, of when which component fails, and create the lowest-cost schedule for each scenario. We then calculate the expected maintenance costs of all scenarios. The combination of the known costs in the first period, and the expected costs in the periods thereafter is also why the paper calls the DEF-model a two-stage stochastic programming maintenance model. How this is done in detail will be discussed later. First, we will introduce some important sets, parameters and variables.

Sets and parameters

Zhu et al. (2021) considers a multi-component system that consists of $N = 1, 2, \ldots, n$ components with economic dependence. The length of the planning horizon is T, and the corresponding set is the discrete finite planning horizon denoted as $\mathcal{T} = 0, 1, \ldots, T$. Each component i needs r spares, in case of failure, and the total amount of spares taken into account for component i is q. The corresponding set is $R = 1, 2, \ldots, q$. We indicate an individual spare by I_{ir}, which is the r^{th} replacement of component i. Furthermore, we denote a scenario by ω, and the set of all scenarios is Ω. The probability that a scenario occurs is $p(\omega)$. Each scenario consists of the lifetimes for individuals $I_{ir}, \forall i \in N, \forall r \in R$. We let the lifetime of individual I_{ir} in scenario ω be T_{ir}^{ω}. Finally, Zhu et al. (2021) considers maintenance costs that can vary per component type i and defines the preventive and corrective maintenance costs for type i as $c_{i,PR}$ and $c_{i,CR}$, respectively. We will consider the same maintenance costs for each component type i, so these variables are changed to c_p and c_f for the preventive and corrective maintenance respectively.

Two-stages

The maintenance decision process is divided into two stages. The first stage decision is to decide which individuals $I_{i1}, \forall i \in N$ should be replaced at the first period, and this decision is the same for all scenarios. The first stage decision variables are

$$x_i = \begin{cases} 1, & \text{if an individual of component } i \text{ is replaced at t=0,} \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

At the start at $t = 0$ we also look at all the starting states ξ_i, for all individuals I_{i1}. This is an input variable and by linking ξ_i to x_i in the model, it is possible to make a schedule when not all individuals I_{i1} are functioning. The value of ξ_i is 0 when it is still functioning, and 1 if it starts broken.

The second stage decision is to create the optimal planning schedule for each scenario at time $t \in \mathcal{T} \setminus \{0\}$, after the individual failure states are revealed at $t = 0$ and determine the expected costs over all scenarios. The decision variables for this second stage is defined for all $i \in N, r \in R, t \in \mathcal{T}, \omega \in \Omega$ and is

$$\tilde{x}_{it}^{r\omega} = \begin{cases} 1, & \text{if } I_{ir} \text{ is replaced at or before time } t \text{ in scenario } \omega, \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

The decision variable that keeps track for all $t \in \mathcal{T}, \omega \in \Omega$ whether set-up costs d are incurred is z_t^{ω} and is defined as

$$z_t^{\omega} = \begin{cases} 1, & \text{if any maintenance occurs at time } t \text{ in scenario } \omega, \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

Scenarios size

For n components, q multiples per component and horizon length T we have $(T+1)^{nq}$ scenarios. Computing all these scenarios quickly adds up to memory and therefore a smaller sample size $|\Omega|$ is chosen. Zhu et al. (2021) follows a theorem of Kleywegt et al. (2002) for $|\Omega|$. If we aim the solution to be ϵ-optimal with $(1-\alpha)$ probability, according to Zhu et al. (2021) the sample size

$|\Omega|$ should satisfy

$$|\Omega| \geq \frac{2\sigma^2}{(0.1\sigma - 0.01\sigma)} \ln\left(\frac{n * q * (T+1)}{\alpha}\right) \tag{4}$$

where σ equals $2T\left(\sum_{i \in N} c_p + d\right)$. Following this theorem, we can simply generate scenarios by sampling from the Weibull distributions until we have enough scenarios. Following the paper of Zhu et al., α is chosen to be 0.1, and ϵ is chosen to be 0.1σ. Therefore, the values for $p(\omega)$ all become equal to $(1/|\Omega|)$. Details on the scenarios are computed can be found in Appendix A.1.

Auxiliary variables

Besides x_i, $\tilde{x}_{it}^{r\omega}$, and z_t^{ω}, Zhu et al. introduces the auxiliary variable $Y_i^{r\omega}$ to keep track of preventive and corrective maintenance. The value for this is 1 when individual I_{ir} in scenario ω takes preventive replacement and is 0 when it takes corrective maintenance. In order to compute $Y_i^{r\omega}$, we need two extra auxiliary binaries, $w_{it}^{r\omega}$ and $y_{it}^{r\omega}$. Firstly, $w_{it}^{r\omega}$ takes value 1 if individual I_{ir} is replaced in scenario ω at time t, and value 0 if is is not replaced. Secondly, $y_{it}^{r\omega}$ takes value 1 if $w_{it}^{r\omega} = 1$, and this corresponds to preventive maintenance. It takes a value of 0 if it corresponds to corrective maintenance, or when $w_{it}^{r\omega} = 0$. The DEF can be found below.

$$\text{DEF: min} \quad \sum_{\omega\in\Omega} p(\omega) \left(\sum_{i\in N} \left(c_p \sum_{r\in R} Y_i^{r\omega} + c_p \sum_{r\in R} (\tilde{x}_{iT}^{r\omega} - Y_i^{r\omega}) \right) + \sum_{t\in\mathcal{T}} dz_t^{\omega} \right) \tag{5a}$$

$$\tilde{x}_{it}^{r\omega} \leq \tilde{x}_{i,t+1}^{r\omega} \qquad i \in N, t \in \mathcal{T} \setminus \{T\}, r \in R, \omega \in \Omega \tag{5b}$$

$$\tilde{x}_{i,t+1}^{r+1,\omega} \leq \tilde{x}_{it}^{r\omega} \qquad i \in N, t \in \mathcal{T} \setminus \{T\}, r \in R \setminus \{q\}, \omega \in \Omega \tag{5c}$$

$$\sum_{r\in R} (\tilde{x}_{it}^{r\omega} - \tilde{x}_{i,t-1}^{r\omega}) \leq z_t^{\omega} \qquad i \in N, t \in \mathcal{T} \setminus \{0\}, \omega \in \Omega \tag{5d}$$

$$\tilde{x}_{i,0}^{1,\omega} \leq z_0^{\omega} \qquad i \in N, \omega \in \Omega \tag{5e}$$

$$\tilde{x}_{it}^{r\omega} \leq \tilde{x}_{i,T_{i,r+1}^{\omega}}^{r+1,\omega} \qquad i \in N, t \in \{0, \ldots, T - T_{i,r+1}^{\omega}\}, r \in R \setminus \{q\}, \omega \in \Omega \tag{5f}$$

$$\tilde{x}_{i,T_{i1}^{\omega}}^{1\omega} = 1 \qquad i \in \{j \in N | T_{j1}^{\omega} \leq T\}, \omega \in \Omega \tag{5g}$$

$$\tilde{x}_{i0}^{r\omega} = 0 \qquad i \in N, r \in R \setminus \{1\}, \omega \in \Omega \tag{5h}$$

$$x_i = \tilde{x}_{it}^{1\omega} \qquad i \in N, \omega \in \Omega \tag{5i}$$

$$x_i \geq \xi_i \qquad i \in N \tag{5j}$$

$$w_{it}^{r\omega} = \tilde{x}_{it}^{r\omega} - \tilde{x}_{i,t-1}^{r\omega} \qquad i \in N, r \in R, t \in \mathcal{T} \setminus \{0\}, \omega \in \Omega \tag{5k}$$

$$w_{i0}^{r\omega} = \tilde{x}_{i0}^{r\omega} \qquad i \in N, r \in R, \omega \in \Omega \tag{5l}$$

$$w_{it}^{r\omega} = 0 \qquad i \in N, r \in R, t \in \{T+1, \ldots, T'\}, \omega \in \Omega \tag{5m}$$

$$y_{it}^{r\omega} = w_{it}^{r\omega} - w_{i,t-T_{ir}^{\omega}}^{r-1,\omega} \qquad i \in N, r \in R \setminus \{1\}, t \in \{T_{ir}^{\omega}, T'\}, \omega \in \Omega \tag{5n}$$

$$Y_i^{1\omega} = 1 - w_{i,T_{i1}^{\omega}}^{1\omega} \qquad i \in N, \omega \in \Omega \tag{5o}$$

$$Y_i^{r\omega} = \frac{\sum_{t=T_{ir}^{\omega}}^{T+T_{ir}^{\omega}} \left|y_{it}^{r\omega}\right| + \sum_{t=0}^{T_{ir}^{\omega}-1} w_{it}^{r\omega}}{2} \qquad i \in N, r \in R \setminus \{1\}, \omega \in \Omega \tag{5p}$$

$$\tilde{x}_{it}^{r\omega}, x_i, z_t^{\omega}, w_{it}^{r\omega} \in \{0,1\} \qquad i \in N, r \in R, t \in \mathcal{T}, \omega \in \Omega \tag{5q}$$

$$y_{it}^{r\omega} \in \{-1, 0, 1\} \qquad i \in N, r \in R, t \in \mathcal{T}, \omega \in \Omega \tag{5r}$$

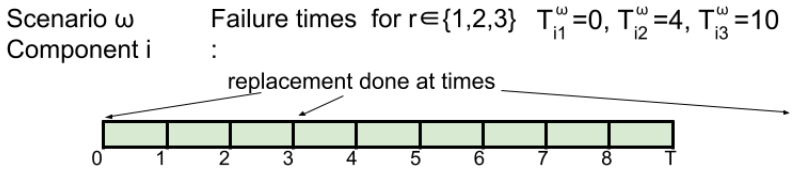
$$Y_i^{r\omega} \in [0,1] \qquad i \in N, r \in R, \omega \in \Omega \tag{5s}$$

Objective
The objective minimizes the total expected maintenance costs over a finite discretized planning horizon, of which the costs made in the first period are known, by weighing each scenario by its probability of occurrence. The first part determines the corrective and preventive maintenance costs, and the second part adds the set-up costs every period any form of maintenance is done.

Constraints
The first constraint (5b) ensures that once an individual I_{ir} in scenario ω is replaced, it remains replaced. Constraints (5c) ensures that for a component type i, the replacement of individual I_{ir} can only take place after the previous individual is replaced. Constraints (5d) and (5e) determine the value of z_t^{ω}, as this is 1 for a replacement at time t and 0 for no replacement in scenario ω. Constraints (5f) and (5g) ensure that individual I_{ir} is replaced before or at the end of its lifetime T_{ir}^{ω}. Constraint (5h) states that for $r > 1$, individuals I_{ir} can not be replaced at $t = 0$, as this is only possible for $r = 1$. Then constraint (5i) is a constraint combining the scenarios

with each other and is called the nonanticipativity constraint. It forces all decisions in the first stage to be the same. Constraint (5j) ensures that when a component starts in failed state at $t = 0$ indicated by ξ_i, it will be replaced. Note that for all scenarios ξ_i will be the same and $T_{i1}^{\omega} > 0$. Why $T_{i1}^{\omega} > 0$ can be found in Appendix A.1.

Auxiliary constraints
Constraints (5k)-(5p) define the variable $Y_i^{r\omega}$ which determines whether the maintenance done was corrective or preventive. These constraints need an extended time horizon length T', where $T' = T + \max\{T_{ir}\}$. Firstly, in constraints (5k), (5l) and (5m) variable $w_{it}^{r\omega}$ is determined. This gets the value of 1 if for individual I_{ir} maintenance is done at time t and scenario ω and value of 0 else. Using $w_{it}^{1\omega}$ the first individuals, the type of maintenance $Y_i^{1\omega}$ can be easily determined by constraint (5o) as we can simply look if maintenance is done at the failure time of the first component. For individuals $r > 1$ this is a bit harder. Here, we use $y_{it}^{r\omega}$ which is defined in constraint (5n). These values remain zero if the maintenance done for I_{ir} was corrective, and can become 1 or -1 in case of preventive maintenance. By using the absolute values and the values of $y_{it}^{r\omega}$ and the values of $w_{it}^{r\omega}$, the final values for $Y_i^{r\omega}$ can be determined in constraint (5p). A visualisation of these values for an example scenario is provided in Figure 4.1.



t	0	1	2	3	4	5	6	7	8	T	10	11	12	13	14	15	T'
x_i					1												
$\tilde{x}_{it}^{1\omega}$	1	1	1	1	1	1	1	1	1	1							
$\tilde{x}_{it}^{2\omega}$	0	0	0	1	1	1	1	1	1	1							
$\tilde{x}_{it}^{3\omega}$	0	0	0	0	0	0	0	0	0	0							
z_t^{ω}	1	0	0	1	0	0	0	0	0	0							
$w_{it}^{1\omega}$	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$w_{it}^{2\omega}$	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
$w_{it}^{3\omega}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$y_{it}^{2\omega}$					-1	0	0	0	0	0	0	0	0	0	0	0	0
$y_{it}^{3\omega}$											0	0	0	-1	0	0	0
$Y_i^{1\omega}$	= (1-1) = 0 → CM																
$Y_i^{2\omega}$	= 0.5 * (|-1| + 1) = 1 → PM																
$Y_i^{3\omega}$	= 0.5 * (|-1| +) = 0.5 → to be corrected for																

Figure 4.1: Visualisation of the decision variables for a component i and a scenario ω. Here, $q = 3$ and the individuals I_{ir} for $r = \{1, 2, 3\}$ have life times 0,4 and 10 respectively.

Correction objective
In Figure 4.1 we see that the value for $Y_i^{r\omega}$ can take the value of 0.5. If this is the case, then the individual $I_{i,r-1}$ is replaced in the horizon, but the individual I_{ir} is not. The value for $\tilde{x}_{iT}^{r\omega}$ is zero. In the objective, however, we have a deviation that causes it to have lower costs than it should have when $c_p < c_p$. A correction is necessary to account for this deviation. Zhu et al., therefore, adds the correction value found in (6) to the objective value. This is not shown in the DEF as this correction value is a constant, however, is important to note in order to get the correct objective value.

$$\sum_{\omega \in \Omega} p(\omega) \sum_{i \in N} \sum_{r \in R} 0.5(c_p - c_p)(\tilde{x}_{iT}^{r-1,\omega} - \tilde{x}_{iT}^{r\omega}) \tag{6}$$

4.1.1 Period dependent costs

The DEF that accounts for period-dependent costs is called the DEF*. In order to add period-dependent costs, we will introduce new binary variables $\tilde{Y}_{it,PR}^{r\omega}$ and $\tilde{Y}_{it,CR}^{r\omega}$ which we can use in the objective, without losing its linearity. They are linked to the decision variables $Y_i^{r\omega}$ and $w_{it}^{r\omega}$ as $Y_i^{r\omega}$ indicates whether we are dealing with corrective or preventive maintenance, and $w_{it}^{r\omega}$ denotes whether at time t we do maintenance or not. Firstly, $\tilde{Y}_{it,PR}^{r\omega}$ has the value 1 when there is preventive replacement at time t in scenario ω for individual I_{ir}, and the value 0 else. Additionally, $\tilde{Y}_{it,CR}^{r\omega}$ has the value 1 when there is corrective replacement at time t in scenario ω for individual I_{ir}, and value 0 else. Furthermore, we extend the cost coefficients from c_p and c_p to $c_{p,t}$ and $c_{f,t}$ respectively. The DEF objective (5a) becomes

$$\sum_{\omega \in \Omega} p(\omega) \sum_{t \in \mathcal{T}} \left(\sum_{i \in N} \sum_{r \in R} (c_{p,t} \tilde{Y}_{it,PR}^{r\omega} + c_{f,t} \tilde{Y}_{it,CR}^{r\omega}) + dz_t^{\omega} \right) \tag{7a}$$

where we do not need the correction from (6). Additionally, we add the constraints (8a)-(8c) to the DEF in order to maintain the correct values for our new auxiliary variables.

$$\tilde{Y}_{it,PR}^{r\omega} + \tilde{Y}_{it,CR}^{r\omega} = w_{it}^{r\omega} \qquad i \in N, r \in R, t \in \mathcal{T}, \omega \in \Omega \tag{8a}$$

$$\tilde{Y}_{it,CR}^{r\omega} \leq 1 - Y_i^{r\omega} \qquad i \in N, r \in R, t \in \mathcal{T}, \omega \in \Omega \tag{8b}$$

$$\tilde{Y}_{it,PR}^{r\omega} \leq Y_i^{r\omega} \qquad i \in N, r \in R, t \in \mathcal{T}, \omega \in \Omega \tag{8c}$$

Constraint (8a) makes sure that when there is no replacement taking place, regardless of the type of replacement, both $\tilde{Y}_{it,PR}^{r\omega}$ and $\tilde{Y}_{it,CR}^{r\omega}$ take the value zero. When there is replacement, either $\tilde{Y}_{it,PR}^{r\omega}$ or $\tilde{Y}_{it,CR}^{r\omega}$ have to take the value of one and which one is determined by constraints (8b) and (8c). If $Y_i^{r\omega}$ is zero, we have corrective maintenance and $\tilde{Y}_{it,CR}^{r\omega}$ takes value of 1. If $Y_i^{r\omega}$ is one, we have preventive maintenance and $\tilde{Y}_{it,PR}^{r\omega}$ takes the value of 1.

4.2 DEF Heuristic

The DEF Heuristic is a faster heuristic to approximate the solution DEF would provide and the details are provided in this section. The idea is to solve all scenarios separately and then put these together in a Standard Progressive Hedging Algorithm to find the best values for $x_i, \forall i \in N$. Not all details of the DEF Heuristic become clear from the paper and a self-invented approach was sometimes necessary. Furthermore, additions are made to implement the period-dependent maintenance costs and this modified model is called the DEF Heuristic*. We will first explain the Standard Progressive Hedging Algorithm in Section 4.2.1, and then we will explain how each scenario is solved independently in Section 4.2.2.

4.2.1 Standard Progressive Hedging Algorithm

This section explains the PHA that is used in Zhu et al. (2021) and will merge all individual solutions into one final solution.

Algorithm 1 The Standard Progressive Hedging Algorithm

1: **Initialization**: Let $v \leftarrow 0, \tilde{\epsilon} \leftarrow 10^{-2}$, $\boldsymbol{x}_\omega^v \leftarrow \text{argmin}_{\boldsymbol{x}}(\boldsymbol{cx} + Q(\boldsymbol{x}, \omega)), \forall \omega \in \Omega$, $\overline{\boldsymbol{x}}^v \leftarrow \sum_{\omega \in \Omega} p(\omega) \boldsymbol{x}_\omega^v$, and $\boldsymbol{W}_\omega^v \leftarrow \rho \left(\boldsymbol{x}_\omega^v - \overline{\boldsymbol{x}}^v \right), \forall \omega \in \Omega$.
2: **Update the iteration counter**: $v \leftarrow v + 1$.
3: **Decomposition**: $\boldsymbol{x}_\omega^v \leftarrow \text{argmin}_{\boldsymbol{x}} \left(\boldsymbol{cx} + \boldsymbol{W}_\omega^{v-1} \boldsymbol{x} + \frac{\rho}{2} \| \boldsymbol{x} - \overline{\boldsymbol{x}}^{v-1} \|^2 + Q(\boldsymbol{x}, \omega) \right), \forall \omega \in \Omega$.
4: **Aggregation**: $\overline{\boldsymbol{x}}^v \leftarrow \sum_{\omega \in \Omega} p(\omega) \boldsymbol{x}_\omega^v$.
5: **Update price**: $\boldsymbol{W}_\omega^v \leftarrow \boldsymbol{W}_\omega^{v-1} + \rho \left(\boldsymbol{x}_\omega^v - \overline{\boldsymbol{x}}^v \right), \forall \omega \in \Omega$
6: **Calculate converge distance**: $g^v \leftarrow \sum_{\omega \in \Omega} p(\omega) \| \boldsymbol{x}_\omega^v - \overline{\boldsymbol{x}}^v \|, \forall \omega \in \Omega$.
7: **Termination**: if $g^v < \tilde{\epsilon}$ **then return** Optimal solution $\overline{\boldsymbol{x}}^v$. **else** Go to step 2. **end if**

New variables

The new variables in the standard PHA are not specifically defined in the paper for the DEF Heuristic, and for this thesis we have chosen them to be the following. Firstly, we choose $\boldsymbol{x}_\omega^v$ to be the array $(x_1 \ x_2 \ \ldots \ x_n)^{\mathrm{T}}$ for iteration v and scenario ω, as in the DEF the output variables are also $x_i \in N$. Secondly, the objective of the DEF is denoted by $(\boldsymbol{cx} + Q(\boldsymbol{x}, \omega))$ where $\boldsymbol{cx}$ are the first-stage costs made at $t = 0$, and $Q(\boldsymbol{x}, \omega))$ the costs made in periods $t > 0$ in scenario ω as a result of choosing $\boldsymbol{x}$. Thirdly, parameter ρ is introduced that represent the Lagrangian penalty for splitting the scenarios and its value represents the trade-off between the solution quality and computational time. As a result, the value for ρ will be chosen iteratively.

Steps explained

In the first initialization step, iteration number v is set to zero, the threshold value $\tilde{\epsilon}$ is chosen, for each scenario ω the array $\boldsymbol{x}_\omega^v$ is determined and the method for this is found in Section 4.2.2, the average values $\overline{\boldsymbol{x}}^v$ are computed and finally the price arrays $\boldsymbol{W}_\omega^v$ are initialized for every ω. The second step is self-evident. The third step is the decomposition step, and nudge the values for x_i towards the average by influencing the calculation of costs when solving for one scenario. This slightly changes the method of finding $\boldsymbol{x}_\omega^v$ and these changes are described in Section 4.2.3. Steps 3-6 are self-evident. Step 7 is the termination step. We terminate if the distance g^v is smaller than $\tilde{\epsilon}$ we return the solution. This solution are the rounded averages in $\overline{x}^v$.

4.2.2 Heuristic Solving one Scenario

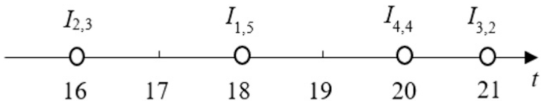
We know $\boldsymbol{x} = (x_1 \ x_2 \ \ldots \ x_n)^{\mathrm{T}}$ will be the output of this heuristic, and how exactly is described in this section. Before we start with the heuristic, it is important to emphasize the non-probabilistic nature of solving for a scenario ω. For a scenario ω the failure times are known, and therefore finding the best solution is all about finding the perfect hindsight replacement times.

Main idea of the heuristic
The main idea of this heuristic is to sort all components I_{ir} according to their optimal replacement times and use various ways of grouping this series. The paper refers to two theorem on which the heuristic is based. The first theorem states that in the optimal grouping, in each group there is at least one component that is replaced at its failure or one time unit before its failure. Since we are adding period-dependent costs, the first theorem does no longer goes since the optimal replacement time is not necessarily one time unit before failure. For a period-dependent model with finite horizon, finding the optimal replacement time is NP-hard and we will use a approximated optimal replacement time. The second theorem states that given a set of operating individuals sorted according to their failure times, there exists an optimal solution for this set such that maintenance activities are executed following the same order. The heuristic provided in the paper however does not always obey this theorem, and therefor the heuristic is slightly adjusted so this theorem is more easily followed. Although this theorem is also not valid in a period-dependent model, we use the idea by ordering the individuals according to their approximated optimal replacement times. We call each cluster of replacements a group, and multiple clusters a grouping. In the heuristic, the goal is to find the grouping that have the lowest maintenance costs. The corresponding $\boldsymbol{x}$ for the first period will be returned.

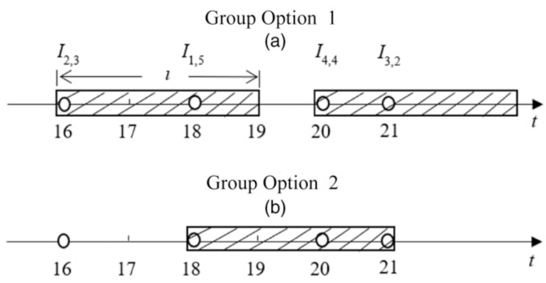
Additional variables
For this heuristic, we introduce some additional variables. For each individual I_{ir} we have β_{ir} which is the approximate optimal replacement time, and β'_{ir} is the newly scheduled replacement time. In the paper, following theorem 1, they chose the value of β_{ir} to be one period before its failure or at its failure and introduce the variable Δ which takes value 0 when the tentative replacement time is at its failure, and 1 for one period before its failure. As we consider period-dependent costs we take a different approach to the value of Δ. We will use Δ to find β_{ir} and allow the component to shift later when the group is formed, over replacement times $\{\beta_{ir}, \ldots, \beta'_{i,r-1} + T_{ir}\}$. The approximated optimal replacement time of individual I_{ir} is the relation

$$\beta_{ir} = t_0 - \operatorname*{arg\,min}_{\Delta \in \{1,2,\ldots,T_{ir}\}} \left[(\Delta - 1) \frac{\overline{c_{p,t}} + d}{E(X_{\alpha_i,\beta_i})} + c_{i,t_0-\Delta,PR} + c_{i,t_0-\Delta+T_{i,r+1}-1} \right]. \tag{9}$$

In this relation two terms can be found. The first term is the failure period of individual, denoted by t_0. The value of t_0 is $\beta'_{i,r-1} + T_{ir}$ for $r > 1$ and $t_0 = T_{ir}$ for r=1. I_{ir}. The second term subtracts a value Δ from this period. To find the value of Δ, we minimize over three terms. The first term is a penalty term of increasing Δ because this causes more individuals to be necessary in the time horizon. The penalty term is dependent on the average costs of doing PM plus the set-up costs, divided by the expectancy of the Weibull distribution denoted as $E(X_{\alpha_i,\beta_i})$. The second term are the PM costs in the new PM costs, and the third term are the difference in PM costs for the successor term. It would be more exact to calculate the PM costs for all successor but for an approximation optimal replacement time we consider this as appropriate. Note Δ can be no higher than the failure time T_{ir}. Note that when $c_{p,t}$ is constant over t, we follow theorem 1 and 2 from Zhu et al. again. Furthermore, we denote $C(G)$ as the costs made for grouping G.



(a) Example from Zhu et al. (2021) showing tentative times of four individuals.



(b) Visualization by Zhu et al. (2021): Options for three values of m.

Figure 4.2: Two figures clarifying the grouping heuristic.

The Heuristic

The heuristic for solving one scenario is provided in Algorithm 2. In Step 1 the set ι is determined. The values in this set will be the allowed sizes of shifting PM. In Step 2 we compute the best grouping and calculate the corresponding costs $C^1(G)$ for each value in ι. In Step 2.1 we initialize by calculating the values $\beta_{i,1}$ for all $i \in N$ with relation (9), putting the corresponding individuals in set K, and creating an empty group G. Some details about K are the following. Set K is a set of all operating individuals that need planning, and K' represent the sorted set of K according to β_{ir}. Zhu et al. (2021) shows an example to demonstrate. In this example, we have $K = \{I_{1,5}, I_{2,3}, I_{3,2}, I_{4,4}\}$ and the corresponding β_{ir} are shown in Figure 4.2a. We see that we get $K' = \{I_{2,3}, I_{1,5}, I_{4,4}, I_{3,2}\}$. Furthermore, we denote the ith position of an individual in our series by $K'[i]$. In Figure 4.2a, $K'[1]$ would return $I_{2,3}$. In Step 2.1 we refer to the Grouping Rule which is described in Algorithm 3. Here, the optimal grouping is determined. From this grouping, we only take the first group g. In Step 2.3, we add all individuals of this group in grouping G, and update set K. Then the β_{ir} for all individuals in K is updated, and if β_{ir} is bigger than the time horizon we remove it again.

Algorithm 2 Heuristic Algorithm for One Scenario

1: **Initialization:** Determine a set of values for ι, $\iota = \{\iota_1, \iota_2, \ldots\}$.
2: For all ι produce groupings and calculate corresponding $C^1(G)$.
  2.1: **Initialization:**
    Assign $\beta_{i,1}$, $\forall i \in N$ with relation (9).
    $K \leftarrow \{I_{1,1}, I_{2,1}, \ldots, I_{n,1}\}$.
    $G \leftarrow \{\}$.
  2.2: Apply **Grouping Rule** for K and denote the first group as g.
  2.3: Update set K.
    $\forall I_{ir} \in g$:
      Add I_{ir} to set G.
      Replace I_{ir} in set K with $I_{i,r+1}$.
    $\forall I_{ir} \in K$:
      Assign $\beta_{i,r}$ with relation (9).
      if $\beta_{ir} > T$, **then** Remove I_{ir} from set K.
      if K is empty, **then** Go to step 2.4. **else** Go to step 2.2. **end if**.
  2.4: Calculate $C^1(G)$.
3: For the group with lowest $C^1(G)$ **return** $\boldsymbol{x}$.

If K is empty, we go to Step 2.4 where we calculate the costs. The costs are calculated the same way as the objective of DEF is calculated and are the total costs with perfect hindsight over the time horizon. For all groupings created for all values of ι, the grouping with the lowest costs is chosen, and the corresponding $\boldsymbol{x}$ is returned.

The Grouping Rule

A set K is grouped according to the Grouping Rule and the heuristic can be found in Algorithm 3. In Step 1, the input set K is sorted in ascending order based on β_{ir}. In Step 2, a number of $|K'| - 1$ different group options are created and the group option with the lowest costs is chosen. We call such a group option m and a visualization by Zhu et al. (2021) is shown in Figure 4.2b.

Algorithm 3 The Grouping Rule

1: Sort K in ascending order based on $\beta_{ir} \Rightarrow$ sorted set K'.
2: Select the option m that has the lowest cost $C^2(G)$.
Group option m : m from 1 to $|K'| - 1$.
2.1: Let $v \leftarrow m$ and $I_{ir} \leftarrow K'[v]$.
2.2: For all $t \in \{\beta_{ir}, \ldots, \beta'_{i,r-1} + T_{ir}\}$ choose t that produce groups with lowest $C'(g)$:
Add individuals to group g in set K' if β_{ir} is before t until $\beta\left(K'\left[v'\right]\right) > t + \iota$, $v' = v+1, v+2, \ldots$.
Calculate $C'(g)$.
2.3: Add best group g to G.
2.4: Let ϑ denote the position of the last individual grouped in Step 2.2, and update actual
replacement times: $\tau'\left(K'\left[v'\right]\right) \leftarrow t, v' = v+1, v+2, \ldots, \vartheta, v \leftarrow \vartheta + 1$.
2.5: **if** $v \geq |K'|$, **then** Go to Step 2.6. **else** Go to Step 2.2. **end if**
2.6: **if** $K'[1] \notin G$ **then** $G \leftarrow G \cup \{(K'[1])\}$. **end if**
2.7: Compute $C^2(G)$.
3: **return** set G.

The procedure is the following. We initialize a group with individual $K'[m]$, in Step 2.1. Then in Step 2.2, we iterate over multiple values of t. For each t we add all individuals to grouping g that have β_{ir} between t and $t + \iota$ and calculate the costs $C'(g)$. The costs are calculated by adding for each individual in g its individual costs $\delta_{c_{ir}}$. The calculation of these individual costs are have the same idea as the calculation of β_{ir} is (9) as are

$$\delta_{c_{ir}} = -d + \left(c_{i,t_0-\Delta_{ir},PR} - c_{i,t_0-1,PR}\right) + \frac{\Delta_{ir} - 1}{E(X_{\alpha_i,\beta_i})}\left(\overline{c_{p,t}} + d\right) + \left(c_{i,t_0-\Delta_{ir}+T_{i,r+1}-1,PR} - c_{i,t_0-1+T_{i,r+1}-1,PR}\right) \tag{10}$$

where $t_0 = \beta'_{i,r-1} + T_{ir}$ if $r > 1$ and $t_0 = T_{ir}$ if r=1. In Step 2.3 we add best group g to G. Then in Step 2.4 we create the starting point of the next group and in Step 2.5 we go to Step 2.2 to create our next group. This is done until all successors from $K'[m]$ are put in a group, as can be read in Step 2.5. The predecessors of $K'[m]$ are put individually in G in Step 2.6 and the costs are calculated in Step 2.7. These costs are the total costs made, including penalty costs for each individual for bringing maintenance forward. The penalty cost is the same as in the paper with a time element added: if individual I_{ir} is grouped with $I_{jr}(\beta_{ir} > \beta_{jr})$, then the penalty cost is $r_i(c_{p,t} + d)$, where r_i is the number of new individuals needed to cover the planning horizon due to this shift and $t = \beta'_{ir} + T_{i,r+1} - 1$. The grouping G with the lowest costs is returned as G in Step 3.

4.2.3 Heuristic Solving one Scenario Decomposition

The calculation of costs $C^1(G)$ in Algorithm 2 is not too complex. We simply add the costs $\boldsymbol{G}_{\omega}^{v-1}\boldsymbol{x} + \frac{\rho}{2}\|\boldsymbol{x} - \overline{\boldsymbol{x}}^{v-1}\|^2$ to the original $C^1(G)$. For the calculation of $C'(G)$ and $C^2(G)$ this is slightly different since not all individuals I_{i1} are in G. Therefore, we calculate the additional costs by

$$\sum_{I_{i1} \in G} \left(W_{\omega,i}^{v-1} x_i + \frac{\rho}{2} \|x_i - \overline{x_i}^{v-1}\|^2 \right). \tag{11}$$

Furthermore, in the decomposition step we not only adjust the calculation of costs, we also allow the values of x_i to be 1 in the Grouping Rule Algorithm 3. By adding

$$\textbf{1a:} \quad \textbf{if} \quad K'[1] = I_{i1}, \quad \forall i \in N \quad \textbf{then} \quad \beta_{i1} \leftarrow 0 \quad \textbf{end if}. \tag{12}$$

to Step 1 the grouping gets the opportunity to also be replaced in the first period.

Chapter 5

Methodology Schouten

This chapter contains all methodologies inspired by the Markov Decision Process (MDP) approach in Schouten et al. (2022). The first four sections are about about creating two-component models and retrieving the policies. Section 5.1 provides a two-component MDP where the components are allowed to have different Weibull parameters. Sections 5.2 and 5.3 provide the Two-Component p-ARP and p-MBRP models that are an extension of the one-component models from Schouten et al. (2022). The model p-ARP is based on the age dependent maintenance strategy from Zhao et al. (2022), where the policy relies on the optimal replacement age for a component. The model p-MBRP is based on the sequential maintenance strategy from Zhao et al. (2022) where the policy looks at the best period to replace a component unless the age is lower than a certain critical age. In Section 5.4, a new method is created to obtain policies of transient states. In the latter part, a multi-component MIP is designed. Section 5.5 explains the calculations of costs when shifting PM which is implemented in the final multi-component MIP found in Section 5.6. The final multi-component MIP is designed in such a manner that maintenance constraints can be added.

5.1 Two-Component Markov Decision Process

In this section, we will transform the single-component MDP from Schouten et al. (2022) to a two-component MDP. The two components are allowed to follow a different distribution.

Set of periods
As stated in the Problem Description, we assume that corrective maintenance is performed directly and that when maintenance is done on a component, it is replaced by an as-good-as-new component. The time is discretized into periods by discretizing one year into N periods. These periods in one year can be extended to m multiple years, if this is the desired cycle length. The set representing these periods is $\mathcal{I}_0$ and we get $\mathcal{I}_0 = \{1, 2, \ldots, mN\}$.

Set of components
The set of components is $k \in K$ and as we have two components we have $K = \{1, 2\}$. A component's lifetime is denoted by X_k and the set of possible ages for k is $\mathcal{I}_k$. In the MDP we identify a maximum age M at which the component should ultimately have preventive replacement. We have $\mathcal{I}_k = \{0, 1, \ldots, M\}$. When a component fails it obtains age 0 and CM is

performed immediately.

Set of states
In our MDP, a state $i = (i_0, i_1, i_2)$ is determined by the period, age of the first component, and age of the second component, respectively. The total state space is defined by $\mathcal{I} = \mathcal{I}_0 \times \mathcal{I}_1 \times \mathcal{I}_2$.

Set of actions
There are four actions a and are represented by the set $a \in \mathcal{A} = \{0, 1, 2, 3\}$. We have $a = 0$ for no action and therefore no replacement, $a = 1$ for only replacing the first component, $a = 2$ for replacing only the second component and $a = 3$ for replacing both components. The state-dependent action space can be expressed as follows as $\mathcal{A}(i_0, i_1, i_2)$, with $i_0 \in \mathcal{I}_0$, $i_1 \in \mathcal{I}_1$ and $i_2 \in \mathcal{I}_2$.

$$\mathcal{A}(i_0, i_1, i_2) = \begin{cases} \{3\} & \text{if } i_1 \in \{0, M\}, i_2 \in \{0, M\}, \\ \{1, 3\} & \text{if } i_1 \in \{0, M\}, i_2 \notin \{0, M\} \\ \{2, 3\} & \text{if } i_1 \notin \{0, M\}, i_2 \in \{0, M\} \\ \{0, 1, 2, 3\} & \text{otherwise,} \end{cases} \tag{13}$$

Transition probabilities
In case of a replacement, there is an instantaneous jump to age 0, after which the component can reach age 1 at the end of the period, or have a failure and end with age 0 again. The failure probability of a component k at age i_k is $p^k_{i_k}$ and is defined by the following relation.

$$p^k_{i_k} = \mathbb{P}(X_k = i_k | X_k \geq i_k) = \frac{\mathbb{P}(X_k = i_k)}{\mathbb{P}(X_k \geq i_k)} \tag{14}$$

Now, let $\pi_{(i_0,i_1,i_2)(j_0,j_1,j_2)}(a)$ be the transition probability from state (i_0, i_1, i_2) to state (j_0, j_1, j_2) under action $a \in \mathcal{A}(i_0, i_1, i_2)$. We find the following values for $\pi_{(i_0,i_1,i_2)(j_0,j_1,j_2)}(a)$. For a shorter notation note that for all cases we have that $\pi_{(i_0,i_1,i_2)(j_0,j_1,j_2)}(a) = 0$ if we do not go to the next period, i.e. $j_0 \neq i_0 + 1 \mod N$.

$$\pi_{(i_0,i_1,i_2)(j_0,j_1,j_2)}(0) = \begin{cases} (1 - p^1_{i_1})(1 - p^2_{i_2}) & \text{for } j_1 = i_1 + 1, j_2 = i_2 + 1 \\ (1 - p^1_{i_1})p^2_{i_2} & \text{for } j_1 = i_1 + 1, j_2 = 0 \\ p^1_{i_1}(1 - p^2_{i_2}) & \text{for } j_1 = 0, j_2 = i_2 + 1 \\ p^1_{i_1} p^2_{i_2} & \text{for } j_1 = 0, j_2 = 0 \\ 0 & \text{else.} \end{cases} \tag{15a}$$

$$\pi_{(i_0,i_1,i_2)(j_0,j_1,j_2)}(1) = \begin{cases} (1 - p^1_0)(1 - p^2_{i_2}) & \text{for } j_1 = 1, j_2 = i_2 + 1 \\ (1 - p^1_0)p^2_{i_2} & \text{for } j_1 = 1, j_2 = 0 \\ p^1_0(1 - p^2_{i_2}) & \text{for } j_1 = 0, j_2 = i_2 + 1 \\ p^1_0 p^2_{i_2} & \text{for } j_1 = 0, j_2 = 0 \\ 0 & \text{else.} \end{cases} \tag{15b}$$

$$\pi_{(i_0,i_1,i_2)(j_0,j_1,j_2)}(2) = \begin{cases} (1-p^1_{i_1})(1-p^2_0) & \text{for } j_1 = i_1+1, j_2 = 1 \\ (1-p^1_{i_1})p^2_0 & \text{for } j_1 = i_1+1, j_2 = 0 \\ p^1_{i_1}(1-p^2_0) & \text{for } j_1 = 0, j_2 = 1 \\ p^1_{i_1}p^2_0 & \text{for } j_1 = 0, j_2 = 0 \\ 0 & \text{else.} \end{cases} \tag{15c}$$

$$\pi_{(i_0,i_1,i_2)(j_0,j_1,j_2)}(3) = \begin{cases} (1-p^1_0)(1-p^2_0) & \text{for } j_1 = 1, j_2 = 1 \\ (1-p^1_0)p^2_0 & \text{for } j_1 = 1, j_2 = 0 \\ p^1_0(1-p^2_0) & \text{for } j_1 = 0, j_2 = 1 \\ p^1_0p^2_0 & \text{for } j_1 = 0, j_2 = 0 \\ 0 & \text{else.} \end{cases} \tag{15d}$$

Cost parameter

The costs $c_{(i_0,i_1,i_2)}(a)$ depend on period i_0, ages i_1, i_2, and action a. It is a function of the period-dependent preventive and corrective maintenance costs $c_p(i_0)$ and $c_f(i_0)$ which are period-dependent. The setup cost for a maintenance task is denoted by d. We have the following costs.

$$c_{(i_0,i_1,i_2)}(a) = \begin{cases} 0 & \text{if } a = 0, \\ c_p(i_0) + d & \text{if } a = 1, i_1 \neq 0, \\ c_f(i_0) + d & \text{if } a = 1, i_1 = 0, \\ c_p(i_0) + d & \text{if } a = 2, i_2 \neq 0, \\ c_f(i_0) + d & \text{if } a = 2, i_2 = 0, \\ 2c_p(i_0) + d & \text{if } a = 3, i_1, i_2 \neq 0, \\ c_p(i_0) + c_f(i_0) + d & \text{if } a = 3, i_1 = 0, i_2 \neq 0, \\ c_p(i_0) + c_f(i_0) + d & \text{if } a = 3, i_1 \neq 0, i_2 = 0, \\ 2c_f(i_1) + d & \text{if } a = 3, i_1 = 0, i_2 = 0 \end{cases} \tag{16}$$

The cost variables $c_p(i_0)$ and $c_f(i_0)$ are assumed to follow a sinus function such that the costs are the lowest in the summer months and highest in the winter months. The average costs for preventive and corrective maintenance throughout the year are $\overline{c_p}$ and $\overline{c_f}$ respectively. A value Δ is used to represent the variation of downtime costs throughout the year. We find the following formula for the time-varying costs. Here, ϕ is chosen such that the lowest maintenance costs are obtained in the month of July.

$$c_p(i_0) = \overline{c_p} + \Delta\overline{c_p}\cos\left(\frac{2\pi i_0}{N} + \phi\right), \qquad c_f(i_0) = \overline{c_f} + \Delta\overline{c_f}\cos\left(\frac{2\pi i_0}{N} + \phi\right) \tag{17}$$

5.2 Two-Component p-ARP policy

In this section, we will introduce the Two-Component p-ARP policy, which has been categorised as an opportunistic grouping by Zhao et al. (2022). This policy is most similar to the policy used in Zhu et al. (2021) as Zhu et al.also only considers age and is therefore chosen to discuss for proper comparison.

Decision variable
The decision variables is $x_{i,a}$ and can be interpreted as the long-run probability that the system is in state $i = (i_0, i_1, i_2) \in \mathcal{I}$ and the decision $a \in \mathcal{A}(i_1, i_2)$ is chosen. For recurrent states, the policy can be found through $x_{i,a}$ by looking for which a it is larger than zero. The values of $x_{i,a}$ for transient states i will always be zero and therefor the policy for these state types can not be obtained by only looking at $x_{i,a}$.

$$\min \quad \sum_{i\in\mathcal{I}} \sum_{a\in\mathcal{A}(i)} x_{i,a} c_{i,a} \tag{18a}$$

$$\text{s.t.} \quad \sum_{a\in\mathcal{A}(i)} x_{i,a} - \sum_{j\in\mathcal{I}} \sum_{a\in\mathcal{A}(j)} \pi_{ji}(a) x_{j,a} = 0 \qquad \forall i \in \mathcal{I} \tag{18b}$$

$$\sum_{i_1\in\mathcal{I}_i} \sum_{i_2\in\mathcal{I}_2} \sum_{a\in\mathcal{A}(i)} x_{i,a} = \frac{1}{mN} \qquad \forall i_0 \in \mathcal{I}_0 \tag{18c}$$

$$x_{i,a} \geq 0 \qquad \forall i \in \mathcal{I}, a \in \mathcal{A}(i) \tag{18d}$$

Objective
The objective (18a) goes through all states and its corresponding set of actions. Here, it sums up all probabilities that we are in state i performing action a multiplied by the corresponding costs. The objective gives the average cost per period. To get the average costs per year the objective value must to be multiplied by N.

Constraints
The first constraint, constraint (18b), puts the transition from one state to all its possible outbound states in equilibrium. Constraint (18c) makes sure that in each period $i_0 \in \mathcal{I}$ equal time is spent.

5.3 Two-Component p-MBRP policy

In this section, we will introduce the Modified Block Replacement Policy (p-MBRP) for the two-component model, which has been categorised as an batch grouping by Zhao et al. (2022). The advantage of block replacement is that it is easier to plan the maintenance ahead and to coordinate maintenance of multiple components.

Critical maintenance age
Firstly, some new variables will be given. As the name says, it is a modification of the original Block Replacement Policy, in which a component is preventively replaced after a fixed amount of time since the previous PM. These times are fixed, regardless of the age of the component at that time. As CM is performed directly, it could be the case that only a month-year-old component is replaced at the planned PM. In the p-MBRP, the same block replacement fixed

moments hold, however, the component is only replaced in period i_0 if a critical maintenance age $t_{(i_0)}$ has been reached. In our two-component model, we let the critical maintenance ages be $t^1_{i_1}$ and $t^2_{i_2}$ for the first and second components respectively.

Other decision variables

Besides $x_{i,a}$ we have two more of decision variables, y_{i_0} and z_i. In block replacement policies, maintenance is performed at periods $T_1, T_2, \ldots, T_n$ where $n \in \mathbb{N}^+$. These policies repeat themselves every m year. The second variable we introduce is $y^k_{i_0}$ for $k \in K$ and indicates in which periods PM is performed.

$$y^k_{i_0} = \begin{cases} 1, & \text{if we maintain component } k \text{ preventively in period } i_0 \in \mathcal{I}_0, \\ 0, & \text{else.} \end{cases} \tag{19}$$

Finally, the decision variables $z^k_{i_0,i_k}$ for $k \in K$ decides for which period and age we maintain the component k.

$$z^k_{i_0,i_k} = \begin{cases} 1, & \text{if we maintain component } k \text{ for age } i_k \in \mathcal{I}_k \text{ in period } i_0 \in \mathcal{I}_0, \\ 0, & \text{else.} \end{cases} \tag{20}$$

MILP for optimal p-MBRP

The following MILP leads to the optimal p-MBRP policy for a two-component model with respect to the long-run average cost criterion.

$$
\begin{aligned}
\min \quad & \sum_{i\in\mathcal{I}}\sum_{a\in\mathcal{A}(i)} x_{i,a}c_{i,a} && && \text{(21a)}\\
\text{s.t.} \quad & \sum_{a\in\mathcal{A}(i)} x_{i,a} - \sum_{j\in\mathcal{I}}\sum_{a\in\mathcal{A}(j)} \pi_{ji}(a)x_{j,a} = 0 && \forall i\in\mathcal{I} && \text{(21b)}\\
& \sum_{i_1\in\mathcal{I}_i}\sum_{i_2\in\mathcal{I}_2}\sum_{a\in\mathcal{A}(i)} x_{i,a} = \frac{1}{mN} && \forall i_0\in\mathcal{I}_0 && \text{(21c)}\\
& x_{i,0} + z^k_{i_0,i_k} \leq 1 && \forall i\in\mathcal{I}, \forall k\in K && \text{(21d)}\\
& x_{i,k} + z^{(3-k)}_{i_0,i_{(3-k)}} \leq 1 && \forall i\in\mathcal{I}, \forall k\in K && \text{(21e)}\\
& x_{i,k} - z^k_{i_0,i_k} \leq 0 && \forall i\in\mathcal{I}, i_k\notin\{0,M\}, \forall k\in K && \text{(21f)}\\
& x_{i,3} - z^k_{i_0,i_k} \leq 0 && \forall i\in\mathcal{I}, i_k\notin\{0,M\}, \forall k\in K && \text{(21g)}\\
& t^k_{i_0} + j_0 y^k_{j_0} + mNy^k_{j_0} \leq mN + i_0 && \forall i_0, j_0\in\mathcal{I}_0, \forall k\in K : j_0 < i_0 && \text{(21h)}\\
& z^k_{i_0,i_k} - y^k_{i_0} \leq 0 && \forall i_0\in\mathcal{I}_0, \forall i_k\in\mathcal{I}_k, \forall k\in K && \text{(21i)}\\
& z^k_{i_0,i_k} - z^k_{i_0,j_k} \leq 0 && \forall i_0\in\mathcal{I}_0, \forall i_k, j_k\in\mathcal{I}_k, \forall k\in K : j_k > i_k && \text{(21j)}\\
& My^k_{i_0} - Mz^k_{i_0,i_k} - t^k_{i_0} \leq M - 1 - i_k && \forall i_0\in\mathcal{I}_0, \forall i_k\in\mathcal{I}_k, \forall k\in K && \text{(21k)}\\
& Mz^k_{i_0,i_k} + t^k_{i_0} \leq M + i_k && \forall i_0\in\mathcal{I}_0, \forall i_k\in\mathcal{I}_k, \forall k\in K && \text{(21l)}\\
& x_{i,a} \geq 0 && \forall i\in\mathcal{I}, \forall a\in\mathcal{A} && \text{(21m)}\\
& z^k_{i_0,i_k} \in \{0,1\} && \forall i_0\in\mathcal{I}_0, \forall i_k\in\mathcal{I}_k, \forall k\in K && \text{(21n)}\\
& y^k_{i_0} \in \{0,1\} && \forall i_0\in\mathcal{I}_0, \forall k\in K && \text{(21o)}\\
& t^k_{i_0} \in \mathbb{N}^+ && \forall i_0\in\mathcal{I}_0, \forall k\in K && \text{(21p)}
\end{aligned}
$$

Similarities with p-ARP

Objective (21a) and constraints (21b), (21c) and (21m) make up the p-ARP model and explanation can be found in the corresponding section 5.2.

Constraints

Then, in order to explain constraints (21d), (21e), (21f) and (21g) a note should be made concerning the p-MBRP model in the paper Schouten et al. (2022). Here, two constraints seem to be forgotten as also noticed by Cremers (2022). These constraints are necessary to relate the decision variable $x_{i,a}$ to the other decision variables and this is done through the variables $z^k_{i_0,i_k}$. Constraints (21d) and (21e) make sure that when maintenance is done for a component k at age

i_k and period i_0, the probability of doing no maintenance for that component k is zero. If $k = 1$ for example, and $z^1_{i_0,i_1} = 1$, we must set $x_{i,0}$ and $x_{i,2}$ to zero as actions 0 and 2 indicate that no maintenance is done for $k = 1$. Constraints (21f) and (21g) constrain from the other side, namely that the value for performing action k for $k \in K$ can only be higher than zero when the value for $z^k_{i_0,i_k}$ allows this. Constraint (21h) ensures the following. When state i has period i_0 and PM can be done it first checks the age of current component k, and if its age is t_{i_0} or higher, a PM can be done. If its age is lower than the critical maintenance age t_{i_0}, no PM can be performed. If there is not even PM possible, $y^k_{j_0}$ is 0 and t_{i_0} becomes unbouded due to the maximum age value M. This might be useful later when we want to add maintenance constraints. Constraint (21i) relates $z^k_{i_0,i_k}$ and $y^k_{i_0}$ with each other, to ensure that maintenance for component k at age i_k can only be done if there is maintenance done for component k in period i_0. In constraint (21j) it says that when we maintain for age i_k in a certain period i_0, we must also maintain for higher ages j_k in this period. Finally, constraints (21k) and (21l) make sure that the value of $z^k_{i_0,i_k}$ is equal to $y^k_{i_0}$ when the age of the component is higher than the critical maintenance age, and that $z^k_{i_0,i_k}$ is zero when the age is lower than the critical maintenance age.

5.4 Retrieving policies

In this section the methodology used to obtain the policies for the transient areas of the Markov Chain is described. As the policies in transient areas are not interesting when wanting to know the optimal policies with corresponding long-term costs, they are not known when solving the Two-Component p-ARP or p-MBRP model. However, looking at the policy in these areas is necessary when comparing Schouten's methods with those of Zhu et al. (2021). Intuitively, when the MDP shows a replacement for a component in state i, one might think that the states after i would also have a replacement. However, due to economic dependence and period-dependent costs, this is not so straightforward and a different procedure is necessary. There are two methods we will discuss here, the Discounted Rewards method by Kallenberg, and the Markov Manipulation heuristic. The Discounted Rewards method is a verified method however is only applicable to retrieving policies for the Two-Component p-ARP model. Therefor, a self-designed heuristic we call the Markov Manipulation heuristic is created to also obtain policies for the Two-Component p-MBRP model. Using the found p-ARP policies, the performance of the Markov Manipulation heuristic can be measured.

5.4.1 Discounted Rewards

Chapter 3 in the book Kallenberg (2011) deals with the total expected discounted reward over an infinite planning horizon. The model is based on the effect of time on the value of money, which overall means that its value decreases. In Chapter 3.5, Kallenberg proves that the following dual linear program returns the optimal policy for the Two-Component p-ARP model with discount factor α. We have no intention of working with a discount factor, and therefor we let $\alpha = 1 - 10^{-10}$. As the alpha is close to one the resulting policy is likely to be also Blackwell optimal and hence also average optimal. The difference however, is that a Blackwell optimal policy also prescribes optimal actions for states that are transient under the average optimal policy.

$$\min \quad \sum_{i \in I} \sum_{a \in \mathcal{A}(i)} c_i(a) x_{i,a} \tag{22a}$$

$$\text{s.t.} \quad \sum_{i \in \mathcal{I}} \sum_{a \in \mathcal{A}(i)} x_{i,a} \left(\delta_{ij} - \alpha \pi_{ij}(a) \right) = \beta_j \qquad \forall j \in \mathcal{I} \tag{22b}$$

$$\beta_j \in \mathbb{R}_{>0} \qquad \forall j \in \mathcal{I} \tag{22c}$$

$$x_{i,a} \geq 0 \qquad \forall i \in \mathcal{I}, \forall a \in \mathcal{A}(i) \tag{22d}$$

Here, the value of β_j can be chosen arbitrarily, as long as it is a positive number. Furthermore, δ_{ij} is the Kronecker delta and only takes value 1 if states i and j are the same. Finally, Kallenberg (2011) proves that for all states i, the linear program produces a positive x_{ia} for some a, which is the optimal policy for state i.

5.4.2 Markov Manipulation

The main idea of the Markov Manipulation heuristic is to do multiple manipulations on the Markov Decision Chain we use in the p-ARP or p-MBRP model such that the actions of replacement are forced to be actions of doing no replacement. This way, the Markov model is forced to be in states that previously were transient and the output of our models will provide policies for these new states. These new policies are then added to the initial policy and another iteration of Markov Decision Chain manipulation can be done. This is done until there are no unknown policies anymore.

Manipulation

When manipulating our Markov Decision Chain we constrain the states of which the action is known. The set of known actions are is denoted by $\mathcal{I}^* \subset \mathcal{I}$. These states are constrained by adding the following constraints.

$$x_{(i_0,i_1,i_2),a} = 0 \qquad \forall i \in \mathcal{I}^*, i_1, i_2 \notin \{0, M\}, \forall a \in \{1,2,3\} \tag{23a}$$

$$x_{(i_0,i_1,i_2),3} = 0 \qquad \forall i \in \mathcal{I}^*, i_1 \in \{0, M\}, i_2 \notin \{0, M\} \tag{23b}$$

$$x_{(i_0,i_1,i_2),3} = 0 \qquad \forall i \in \mathcal{I}^*, i_1 \notin \{0, M\}, i_2 \in \{0, M\} \tag{23c}$$

Here, (23a) makes sure that for all states $i \in \mathcal{I}^*$ that are not along the borders of the MDP have action 0, which is do do no replacement. Constraint (23b) ensures that if for a state i_2 is initially replaced while it is not 0 or M, the state is forced to have action 1 if i_1 is 0 or M. Alongside, constraint (23c) ensures that if for a state i_1 is initially replaced while it is not 0 or M, the state is forced to have action 2 if i_2 is 0 or M. In the Result's section 6.1 a graphical example is shown of how one iteration is done.

5.5 Costs shift PM

In this thesis, we consider a multi-component model with economic dependence. Since for more than two components the MDP will grow unmanageable, we need to think of smarter ways to group our components. We will group our components by first looking at the optimal replacement times per individual component, and then shifting these replacement times so that economic dependence is taken into account, and less start-up costs d needs to be paid. We use the optimal policy R retrieved from the p-ARP model for one component. In this section, we will demonstrate how such a shift looks like and explain how the additional costs of such a shift is calculated.

Shifting PM

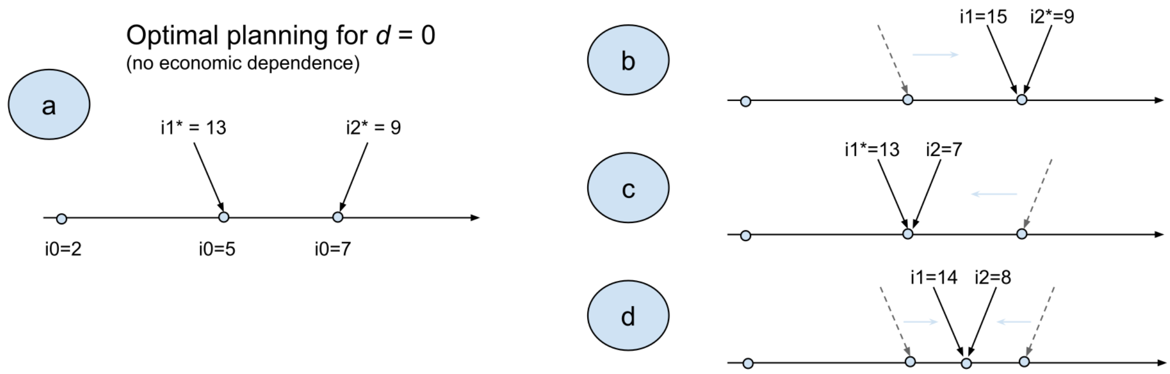


Figure 5.1: In Figure *a* the optimal planning for a situation with no economic dependence is shown, where the first component is replaced at age 13 and the second component at age 9. In Figure *b*,*c*, and *d* three shifting options are shown.

Grouping two components can take many formats as shown in Figure 5.1. Although when shifting these components to a different maintenance time than their individual optimal time, it could be that the costs for these plannings are lower than those in situation when the set-up costs d has a positive value.

5.5.1 Preponing PM

The methodology of calculating the costs of shifting preventive maintenance forward with multiple periods for a single component is based on the theory of Cremers (2022). She based on the theory found in the book Tijms (2003) where it proofs that under mild conditions there exist relative values $v_i(R), \forall i \in \mathcal{I}$, which are defined in such a way that $v_i(R) - v_j(R)$ measures the difference in the total costs when starting in state i rather than starting in state j if we follow policy R. These values for $v_i(R), \forall i \in \mathcal{I}$ are together with the long-term costs $g(R)$ used to calculate the difference in the long-term total costs when starting in state i and first take an action a that is different than the optimal action R_i. This difference in long-term costs is denoted by $\Delta(i, a, R)$. If the optimal action R_i in a state i would be 0 for example, and we choose to do action $a = 1$, the difference in the long-term costs is equal to the costs when we prepone the PM. This is exactly what we need. So, to conclude this alinea, we will use the theory of Tijms (2003) for the relative values $v_i(R)$ and long-term costs $g(R)$ to determine $\Delta(i, a, R)$, and then we will use the theory of Cremers (2022) to calculate the costs of preponing PM.

Calculation $\Delta(i, a, R)$

For an optimal policy R, we can find the values of $v_i(R)$ and $g(R)$ by solving the equations (24a) and (24b). Here, $c_i(R_i)$ is the immediate cost incurred when action R_i is chosen in state i and $\pi_{ij}(R_i)$ is the transition probability from state i to state j under action R_i. Constraint (24b) is added to pin one of the relative values and the value for $q \in \mathcal{I}$ is arbitrarily chosen. Note that the number of constraints of (24a) and (24b) together are exactly the number of unknown variables from $v_i(R) \forall i \in \mathcal{I}$ and $g(R)$.

$$v_i(R) = c_i(R_i) - g(R) + \sum_{j \in \mathcal{I}} \pi_{ij}(R_i) v_j(R), \qquad \forall i \in \mathcal{I} \tag{24a}$$

$$v_q = 0 \tag{24b}$$

When we step beside this optimal policy in state i we find a cost difference from the long-term total costs of

$$\Delta(i, a, R) \approx c_i(a) + \sum_{j \in I} \pi_{ij}(a) v_j(R) - g(R) - v_i(R) \tag{25}$$

where we do action a instead of the optimal action R_i. The approximate sign denotes that this holds under the mild conditions stated by Tijms (2003). So, to conclude, $\Delta(i, a, R)$ is the extra average costs incurred when we choose to take a different action a than the optimal action R_i for state i, for once.

Calculation preponing PM

Here, we use an amended version of the relation found by Roby. The amendment is that we removed the first term. This term is only necessary when working with components that are already shifted and its PM is not scheduled anymore on their optimal PM state.

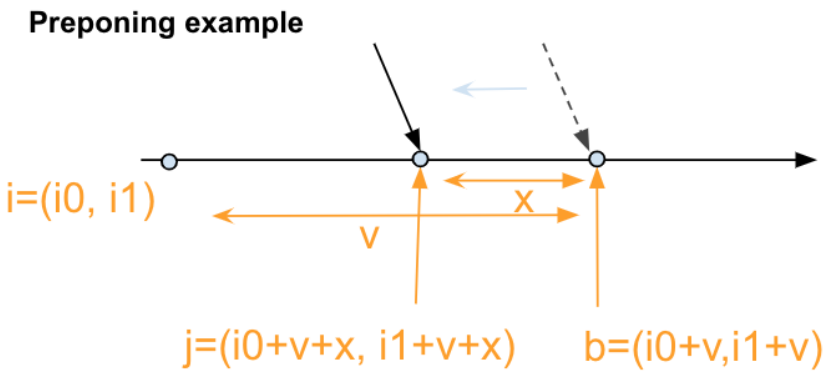


Figure 5.2: Variables used in relation (26) for preponing.

Let's first go through the variables that are also shown in Figure 5.2 for clarity. We are working with one component and therefor the denote the current state we are in by $i = (i_0, i_1)$. Then, the original execution state is $b = (i_0 + v, i_1 + v)$, where v is the length of optimal periods between the current period and the optimal execution date and is found through R. The deviation from the original execution date $i_0 + v$ is denoted by $x \in \{-v, -v+1, \ldots, -1, 1, \ldots, \ell\}$. Here, we only consider $x < 0$. The new state we will plan the PM in is $j = (j_0, j_1) = (i_0 + v + x, i_1 + v + x)$. Finally, the transition probabilities are denoted by $\pi^n_{ij}(R)$ and are the n-step probabilities of going from state i to j in n steps under policy R.

$$\delta(i,x) = \pi_{ij}^{(v+x)}\Delta\left(j,1,R\right) + \pi_{ij}^{(v+x)}(R)\sum_{k=x+1}^{0}\pi_{(i_0+v+x,0),(i_0+v+k,k-x)}^{(k-x)}\Delta\left(\left(i_0+v+k,k-x\right),0,R\right) \tag{26}$$

The fist term, as the extra costs incurred of preponing due to the extra components that are needed to fill up a time horizon, multiplied with the probabilities of reaching state j. The second term are the extra costs in the periods between j and b, due to the forced shifted state. Note that when the actions in the $|x|$ starting periods is the same for all $i_0 \in N$, this second term will always be zero as $\Delta\left(\left(i_0+v+k,k-x\right),0,R\right)$ will become zero. This happens when the costs will not vary too much throughout the year.

5.5.2 Postponing PM

The extra costs incurred when postponing PM for a component is not easy. There are three things that should be taken into account here. Firstly, when postponing PM the probability on corrective maintenance is higher and therefor increase costs. Secondly, postponing PM causes less components to be necessary in the time horizon which will decrease costs. Thirdly, postponing PM increases the probability of not being able to follow the planning as it might fail in between. Especially the last point makes it difficult, because it is hard to assign costs to this schedule insecurity. However, the extra costs incurred for postponing PM will become stable as x goes to infinity because CM is assumed to happen and after CM it will follow its optimal policy again. However, it is not realistic to postpone with $x \to \infty$ when making a planning. In this section, we will first present three methods to calculate the extra costs when postponing PM where we critically look at the method used in Cremers (2022), and at the end the probability of CM is provided to have grip on the credibility of a planning. The same variables are used as in 5.5.1 and for clarity an example for postponing is provided in 5.3.

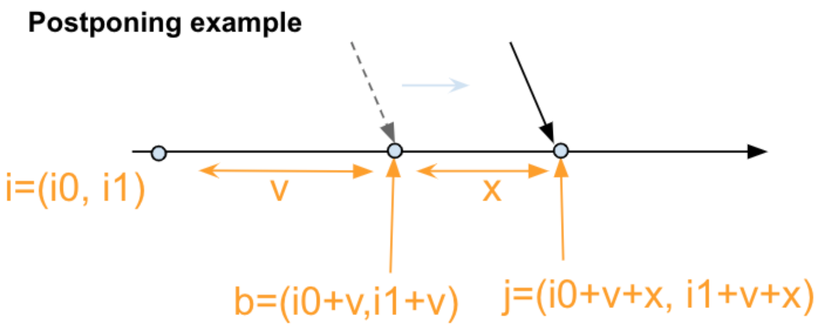


Figure 5.3: Variables used in relation (27) for postponing.

Method 1

We will start with the method used in Cremers (2022). Firstly, she introduces the transition probabilities $\pi_{ij}^{n}(R^0)$ that are the n-step probabilities of going from state i to j in n steps under the null-policy R^0, which means a policy with no PM unless $i_1 = M$. Furthermore, she adapts the policy R to R^{bx} and substitutes this like $\Delta(i,a,R^{bx})$. The new policy R^{bx} is an adjusted policy of R, where the actions between the original and new state $(b_0 + x, b_1 + x)$ are restricted to be 0. The calculation for extra postponing costs are shown in (27). It calculates the extra costs for postponing by one period by calculating the value of Δ and multiplying it by the probability the component reaches the corresponding state. By iteration through b and j, Cremers (2022) states that the postponing costs are found. It is uncertain whether $\Delta(i, a, R)$ is used correctly since R^{bx} is used as input.

$$\delta(i,x) = \sum_{k=0}^{x-1} \pi^{(v+k)}_{i(i_0+v+k,i_2+v+k)}(R^0)\Delta\left((i_0 + v + k, i_2 + v + k), 0, R^{bx}\right) \tag{27}$$

Method 2

As stated in Method 1, it is uncertain whether $\Delta(i, a, R^{bx})$ can be used, and in order to test this $\delta(i, x)$ is also computed when just use R in stead of R^{bx}. This gives us the following relation.

$$\delta(i,x) = \sum_{k=0}^{x-1} \pi^{(v+k)}_{i(i_0+v+k,i_2+v+k)}(R^0)\Delta\left((i_0 + v + k, i_2 + v + k), 0, R\right). \tag{28}$$

Method 3

As an extra method, we will calculate the extra costs incurred when postponing PM by calculating the expected difference in cost by using the theory from Tijms (2003), chapter 6.2. Here, he introduces $V_n(i, R)$ which is the total expected costs over the first n decision epochs when the initial state is i and policy R is used. We find that the expected difference in cost is

$$\delta(i,x) = \pi^{(v)}_{ib}(R)\left(\sum_{k=0}^{x}\sum_{l\in I}\pi^{(k)}_{bl}(R^{bx})c_l\left(R^{bx}_l\right) - \left(\sum_{k=0}^{x}\sum_{l\in I}\pi^{(k)}_{bl}(R)c_l\left(R_l\right)\right)\right) \tag{29}$$

where the first part in the brackets is the total expected costs when postponing the PM by x, and the second part in the brackets is the total expected costs when holding on the the original policy R. This difference is multiplied by the probability of even getting to state b.

Credibility postponing

The probability of failure before the assigned PM state $j = (j_0, j_1)$, when the planning is done in state $i = (i_0, i_1)$ is only dependent on ages i_1 and j_1 since we assume the failure is period-independent. Say X is the lifetime of the component, the probability of failure is

$$p_c(i_1, j_1) = \mathbb{P}\left(X < j_1 | X \geq i_1\right) = \frac{\mathbb{P}\left(i_1 \leq X < j_1\right)}{\mathbb{P}\left(i_1 \leq X\right)}. \tag{30}$$

and can be used to measure the credibility of postponing. If the value is high, planning the postponement might give a distorted planning as it is doomed to fail before.

5.6 Complete multi-component MIP

With knowledge of the policy output of Schouten et al. (2022) and the extra costs incurred when shifting PM from Cremers (2022), a final multi-component MIP is created. This approach is categorised as static preventive grouping by Zhao et al. (2022). The problem with a multi-component MIP for a finite horizon is that the optimal PM time is dependent on the period, and on the PM time of the consecutive component. Therefore, optimising the planning can not be done chronologically, and there is no resetting moment that could divide the optimization problem into multiple stages. As a result, dynamic programming can not be considered and a MIP has been chosen instead. This MIP has a smaller computation time than iteration through all combinations, and through defining the input smartly the MIP's speed can be enhanced even more. We first introduce the sets, parameters and variables used and then provide the MIP to find the optimal solution. The main idea of the optimal solution is that over a time horizon $\mathcal{T}$, the optimal planning is provided considering period-dependent costs and adding a maintenance constraint to constrain the number of maintenance done in month t to be not more than M_t. Every time a component fails and CM must be done, the model can be ran again to obtain the new optimal planning. However, the initial planning can be used as a guidance for a wind-farm company so they can control all operational aspects.

Sets and parameters
In our multi-component model, let I_{kr} be an individual where $k \in K$ denotes the component, and $r \in R$ as the r^{th} replacement. The starting ages of the components are denoted by a_k^r and for $r > 1$ are assumed to be zero. The starting period is t_0. There are two time sets, the main time-set that represent the planning horizon $\mathcal{T} = \{t_0, \ldots, T\}$, and an auxiliary time-set that is slightly bigger as $\mathcal{T}\prime = \{t_0, \ldots, T\prime\}$. Finally, the size of a PM shift for individual I_{kr} is expressed by $x_k^r \in \mathcal{D}_{kt} = \{-v_{kt}, \ldots, v_{kt}\}$ which is dependent on time t which is equivalent to $i0$ in Figure 5.2/ 5.3. Note that if costs are time-independent, v_{kt} would become time-independent too.

Parameters
The parameter v_{kt} is the number of periods until the optimal replacement period. From the p-ARP model, we obtain the optimal policy R^k for each component $k \in K$. For each period $t \in \mathcal{T}$ we can find v_{kt}, by looking at R^k. From equation (26) we can find the extra average costs for shifting PM when $x < 0$, and from equation (27), (28) or (29) the extra average or expected costs for shifting PM when $x > 0$, depending on the method chosen, can be found. Let $\delta^k((i_0, i_1), x)$ be the extra costs for shifting PM by x for component k, when in state (i_0, i_1).

Variables
For every $t \in \mathcal{T}$, we let z_t be the binary variable that states whether there is any PM scheduled in t. For every $t \in \mathcal{T}$ and every individual I_{kr} we have another binary w_{kt}^r that is 1 if maintenance is planned for I_{kr} in t and is 0 else. The non-negative integers b_k^r and j_k^r are the individually optimal replacement time and the new replacement time. Variable $x_k^r \in \mathbb{Z}$ is the number of shifts done for individual I_{kr}. For example, if for individual I_{kr} the optimal individual replacement time is $b_k^r = 9$, but it is replaced at $j_k^r = 7$, we find value $x_k^r = -2$. Finally, the binary y_{kt}^{rx} takes value 1 if

the PM of individual I_{kr} has x shifts and the starting period of the individual is t.

The MIP

$$\min \quad \sum_{r\in R}\sum_{k\in K}\sum_{t\in\mathcal{T}}\sum_{x\in\mathcal{D}_{kt}} y_{kt}^{rx}\cdot\delta^k\left((t,a_k^r),x\right) \quad + \quad \sum_{t\in\mathcal{T}'} dz_t \tag{31a}$$

$$\text{s.t.} \quad z_t \geq w_{kt}^r \qquad \forall t\in\mathcal{T}', \forall r\in R, \forall k\in K \tag{31b}$$

$$w_{kt}^r \geq 1-|t-j_k^r| \qquad \forall t\in\mathcal{T}', \forall r\in R, \forall k\in K \tag{31c}$$

$$b_k^1 = v_{k1}+t_0 \qquad \forall k\in K \tag{31d}$$

$$b_k^r = j_k^{r-1}+\sum_{t\in\mathcal{T}'} w_{kt}^{r-1}\cdot v_{kt}+t_0 \qquad \forall r\in R\setminus\{1\}, \forall k\in K \tag{31e}$$

$$x_k^r = j_k^r - b_k^r \qquad \forall r\in R, \forall k\in K \tag{31f}$$

$$y_{kt_0}^{1x} \geq 1-\left|x_k^1-x\right| \qquad \forall x\in\mathcal{D}_{k1}, \forall t\in\mathcal{T}', \forall k\in K \tag{31g}$$

$$y_{kt}^{rx} \geq 1-\left|t-j_k^{r-1}\right|-|x_k^r-x| \qquad \forall x\in\mathcal{D}_{kt}, \forall t\in\mathcal{T}', \forall r\in R\setminus\{1\}, \forall k\in K \tag{31h}$$

$$x_k^r \in \mathbb{Z} \qquad \forall r\in R, \forall k\in K \tag{31i}$$

$$b_k^r \in \mathbb{N} \qquad \forall r\in R, \forall k\in K \tag{31j}$$

$$j_k^r \in \mathcal{T}' \qquad \forall r\in R, \forall k\in K \tag{31k}$$

$$z_t, w_{kt}^r, y_{kt}^{rx} \in \mathbb{B} \qquad \forall x\in\mathcal{D}_{kt}, \forall t\in\mathcal{T}', \forall r\in R, \forall k\in K \tag{31l}$$

Objective
The objective minimizes over the extra costs incurred a result of shifting and the total set-up costs. The extra costs incurred as a result of shifting is calculation over time horizon $\mathcal{T}$, to not put weight on the choices make after T. Because the value of q is chosen such that there is more than enough individuals, they might accumulate after N causing high negative values for x_k^r. At the same time, we don't want the individuals to have preference for PM after N, and therefor the calculation of the set-up costs is calculation over time horizon $\mathcal{T}'$.

Constraints
Constraint (31b) links the value of z_t to w_{kt}^r to make sure that it takes the value of 1 when maintenance is done in $t\in\mathcal{T}'$. Constraint (31c) makes sure that w_{kt}^r only takes the value of one if individual I_{kr} is replaced at $t=j_k^r$. Constraints (31d) and (31e) define the value of the initial replacement time b_k^r. As b_k^r for $r>1$ is dependent on the replacement times of its predecessor, and the predecessors policies are exactly the same, we use the sum over all values of w_{kt}^{r-1} to keep the MIP linear. The number of shifts x_k^r for individual I_{kr} is defined in constraint (31f). The constraints (31g)-(31h) defines the binary variable y_{kt}^{rx} which is necessary to compute the total shifting costs. The value of y_{kt}^{rx} may only take the value of 1 if both x equals x_k^r and individual I_{kr} is replaced at time t. If they both hold, the right hand side takes value of 1. If only one of

them or none of them hold, the right hand side can take the maximum value of 0. The absolute of $(x - x_k^r)$ is linearized by the auxiliary variables u_{kr} and s_{kr} by the the following constraints.

$$|x - x_k^r| = u_{kr} + s_{kr} \qquad \forall t \in T, \forall x \in \mathcal{D}_{kt}, \forall k \in K, \forall r \in R \tag{31m}$$

$$x - x_k^r = u_{kr} - s_{kr} \qquad \forall t \in T, \forall x \in \mathcal{D}_{kt}, \forall k \in K, \forall r \in R \tag{31n}$$

$$u_{kr}, s_{kr} \in \mathbb{N} \qquad \forall k \in K, \forall r \in R \tag{31o}$$

The same is done for the absolutes of $(t - j_k^{r-1})$ and $(t - j_k^r)$.

Adding maintenance constraint

As stated as one of the goals of this thesis, maintenance limitations should be taken into account. This is done by constraining the amount of PM for each month. This is done by adding the following constraint maintenance

$$\sum_{k \in K} \sum_{r \in R} w_{kt}^r \leqslant M_t \qquad \forall t \in \mathcal{T} \tag{31p}$$

where M_t is the maximum PM that can be scheduled every month.

Chapter 6

Results

In this chapter the results are presented. It consists of three sections. Section 6.1 shows the results about the Two-Component p-ARP and p-MBRP models and policy retrieval methods. Section 6.2 presents the results concerning the extended DEF Model and DEF Heuristic model where period-dependent costs are added. Using the Two-Component p-ARP and DEF models two approaches as a benchmark, the performance of the Multi-Component MIP is measured and provided in Section 6.3. Section 6.3 also evaluates $\delta(i, x)$. We perform our computational study on a computer with Dual-Core Intel Core i5, 1.6 GHz, 4 gigabytes of RAM. The language used is Java, and is used to implement the algorithms with the solver of CPLEX v22.1.0.

6.1 Extending Schouten to Two-Component Models

In this section, the implementation of the original p-ARP and p-MBRP models from Schouten et al. (2022), evaluated, the extensions to two-component models are measured, and the methods concerning transient state policies are tested.
The implementation of the original p-ARP and p-MBRP models has been successful. When using the same settings as used in the paper, the exact same average yearly costs have been found. The values of the paper can be found in Appendix B, and the values of this thesis can be found in Table 6.1, in columns 2 and 6.

Table 6.1: Yearly costs for the original p-ARP and p-MBRP models with $\alpha = 12$ months and $\beta = 2$ in columns 2 and 6. Yearly costs for the extended Two-Component p-ARP and p-MBRP models with $\alpha_1 = 9$ months, $\alpha_2 = 12$ months and $\beta_1 = \beta_2 = 2$ in columns 3-5 and 7-9. Furthermore, we let $\bar{c}_f = 50$, $\bar{c}_p = 10$, $d = 0$, $M = 12$. Results for multiple Δ values are shown.

Δ	p-ARP	Two-Component p-ARP			p-MBRP	Two-Component p-MBRP		
		$k=1$	$k=2$	total		$k=1$	$k=2$	total
0%	40.098	53.160	40.098	93.258	40.311	53.771	40.369	94.140
0.1%	40.035	53.152	40.035	93.187	40.263	53.764	40.263	94.027
0.2%	39.701	53.009	39.701	92.709	39.855	53.717	39.855	93.572
0.3%	39.224	52.686	39.224	91.910	39.338	53.309	39.338	92.647
0.4%	38.461	52.282	38.461	90.743	38.556	52.902	38.556	91.458
0.5%	37.635	51.629	37.635	89.264	37.773	52.005	37.944	89.948
CPU	< 1 s			10 s	3 s			5 min

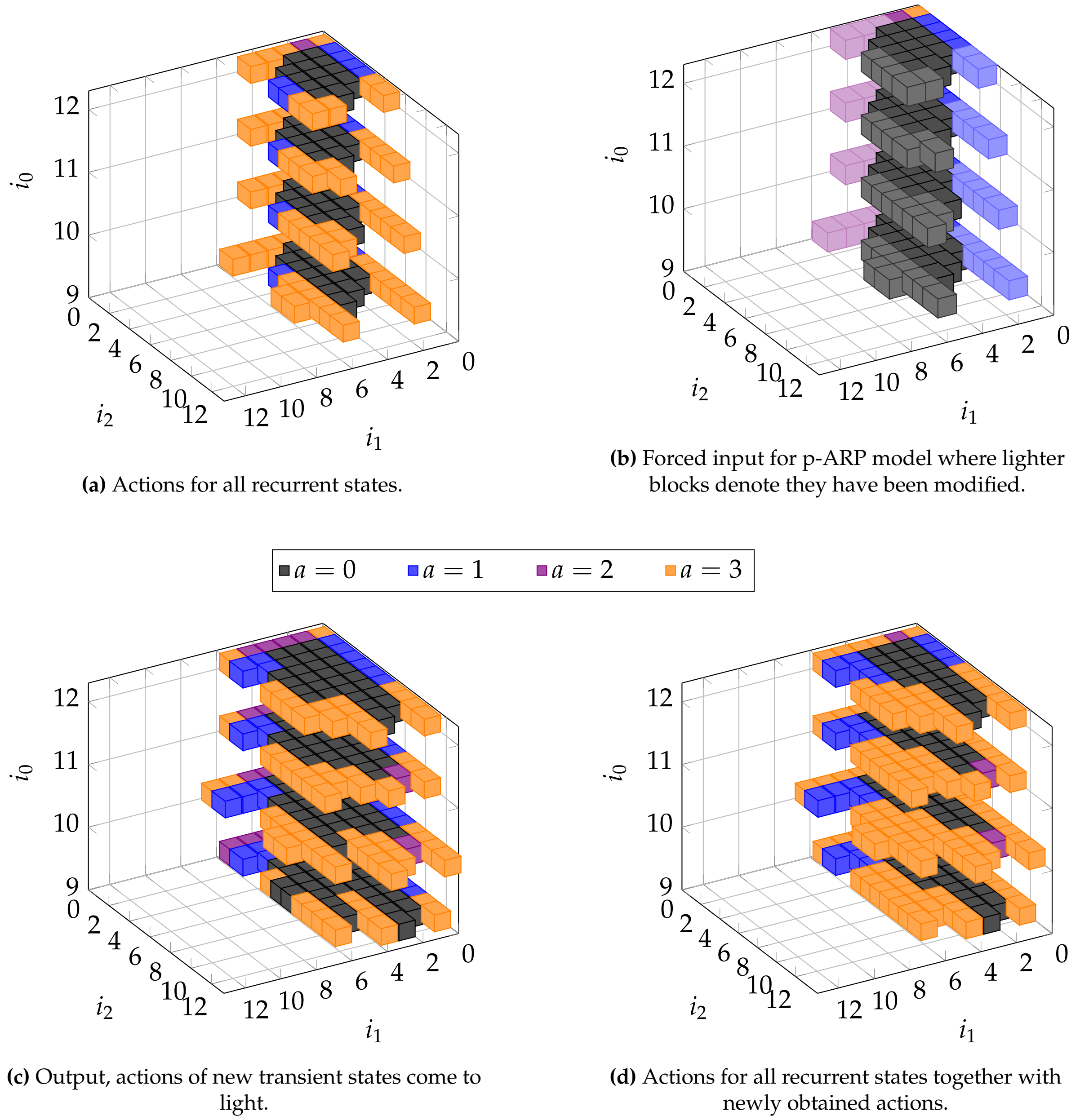
The average yearly costs of the Two-Component p-ARP and p-MBRP models are also found in Table 6.1. The second component used in the two-component models has the same shape and scale parameters as the component in the original p-ARP and p-MBRP models. Since the set-up costs d are set to zero, the costs caused for each component can be calculated independently. You might notice that the column 'total' is the sum of columns $k = 1$ and $k = 2$. It becomes clear that the extension of the p-ARP model is completely successful as all the costs of the second component are the exact same as the costs made in the original model. The p-MBRP extension has also been successful, as the values are almost for every Δ exactly the same. We see that for $\Delta = 0.0$ and $\Delta = 0.5$ there is a small difference that stays below the 0.5%.

Retrieving Policies of Transient States

First, a demonstration of one iteration of the Markov Manipulation method is provided in Figure 6.1. Then the results for finding all the p-MBRP states for two i_0 values are provided. Finally, the Markov Manipulation method is compared to the Discount Reward Model from Kallenberg using the results for the Two-Component p-ARP model.

Markov Manipulation Demonstration

It is chosen to create a three-dimensional plot to demonstrate one iteration of the Markov Manipulation to firstly emphasize the difficulty of finding the actions as we are dealing with three dimensions and economic dependence. Secondly, to clearly show how an iteration is done. Note that states 0 for i_1 and i_2 indicate that they are in failed state and replacement is required.



(a) Actions for all recurrent states.

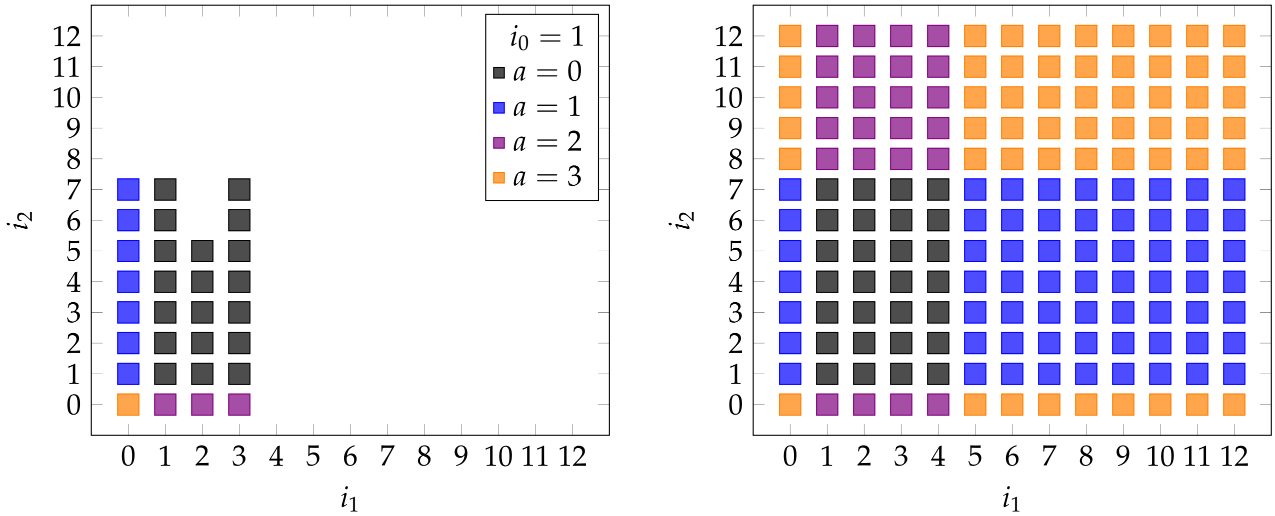
(b) Forced input for p-ARP model where lighter blocks denote they have been modified.

(c) Output, actions of new transient states come to light.

(d) Actions for all recurrent states together with newly obtained actions.

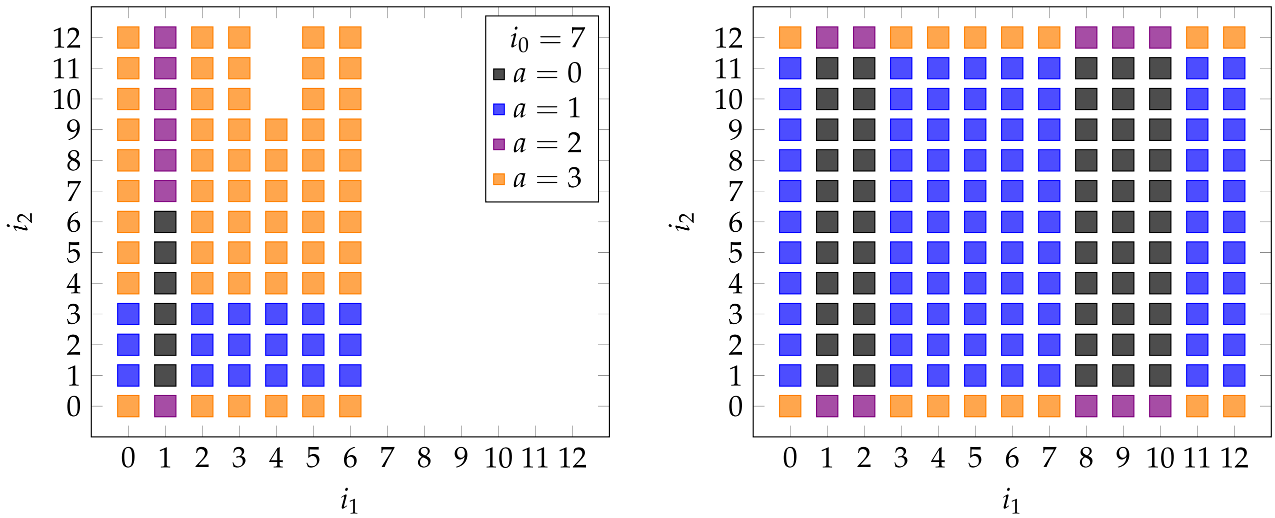
Figure 6.1: Progress of one iteration Markov Chain manipulation in order to retrieve the p-ARP for transient states too. Model used for the time period of one year and we let $d = 5$, $\overline{c_f} = 50$, $\overline{c_p} = 10$ and $\Delta = 0.2$. Components 1 and 2 follow $\alpha_1 = 6$, $\alpha_2 = 12$ and $\beta_1 = \beta_2 = 2$. Policies for $i_0 = \{9, 10, 11, 12\}$ are shown.

Results extending p-MBRP policies with Markov Manipulation
Figure 6.2 shows on the left the known recurrent state policies, and on the right the policies in transient states found with Markov Manipulation for $i_0 = 1$. The Markov Manipulation solutions appear coherent. Note that when the states i_1 and i_2 are 0, this means they are in failed state and is also why the block for (0,0) is has the colour of action 3 which is to replace both components. Figure 6.3 also shows on the left the known recurrent state policies and it is remarkable that so many actions are known. Beacuse $i_0 = 7$ corresponds to July, and is the cheapest month for maintenance, states wait for PM until this month is reached. On the right the found policies are shown. These policies are not coherent and even show different policies for known recurrent actions. We conclude that the p-MBRP policies can not be found with the Markov Manipulation method.



(a) p-MBRP for recurrent states at i_0 =1 **(b)** p-MBRP for all states

Figure 6.2: Plot revealing actions for transient states from p-MBRP model using the Markov Manipulation method. We let $d = 5$, $\overline{c_f} = 50$, $\overline{c_p} = 10$, $M > 12$, and $\Delta = 0.2$. Components 1 and 2 follow $\alpha_1 = 6$, $\alpha_2 = 12$ and $\beta_1 = \beta_2 = 2$.



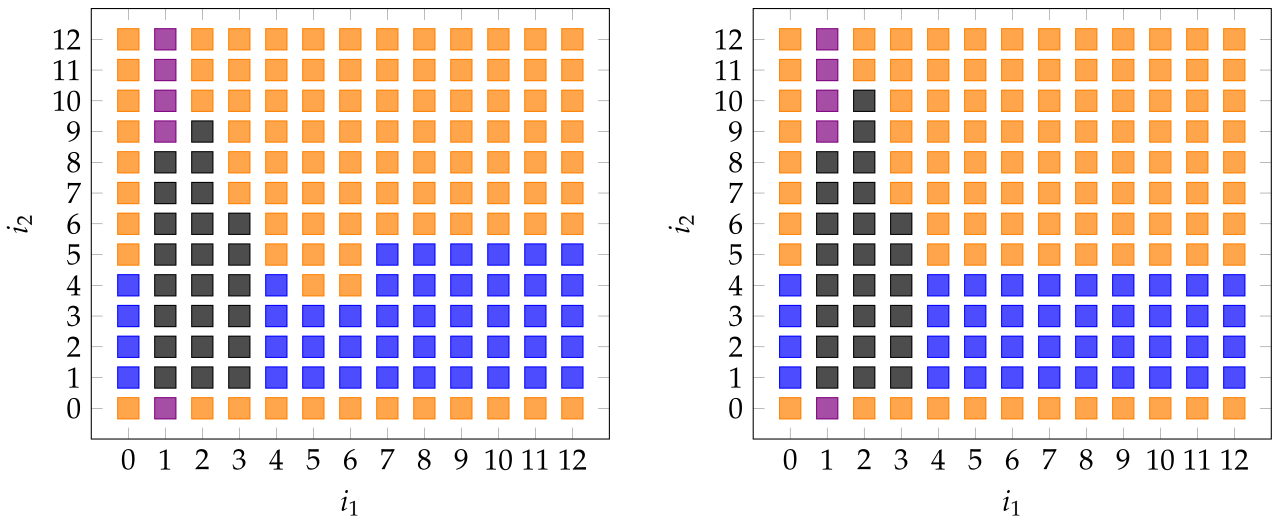
(a) p-MBRP for recurrent states at i_0 =7

(b) p-MBRP for all states

Figure 6.3: Plot revealing actions for transient states from p-MBRP model using the Markov Manipulation method. We let $d = 5$, $\overline{c_f} = 50$, $\overline{c_p} = 10$, $M > 12$, and $\Delta = 0.2$. Components 1 and 2 follow $\alpha_1 = 6$, $\alpha_2 = 12$ and $\beta_1 = \beta_2 = 2$.

Discount reward model Kallenberg

The transient state policies for p-MBRP model can not be found with the Markov Manipulation heuristic. To test whether it is possible for the p-ARP model, we compare the policies found for the p-ARP model with the Markov Manipulation heuristic to the policies found with the acknowledged discount reward theory in Figure 6.4.



(a) p-ARP policies found with the Markov Manipulation heuristic for i_0 =1

(b) p-ARP policies found with the discounted reward theory for i_0 =1

Figure 6.4: Figure revealing actions for transient states from p-ARP model. We let $d = 5$, $\overline{c_f} = 50$, $\overline{c_p} = 10$, $M > 12$, and $\Delta = 0.2$. Components 1 and 2 follow $\alpha_1 = 6$, $\alpha_2 = 12$ and $\beta_1 = \beta_2 = 2$.

Our investigation of the policies for $i_0 = 1$ are not identical. To support this, we looked at the policies for all $i_0 \in \mathcal{I}$, and have found that 8.8% of the found policies were different. We conclude that the Markov Manipulation heuristic is not a proper method to obtain transient state policies.

6.2 Extending Zhu

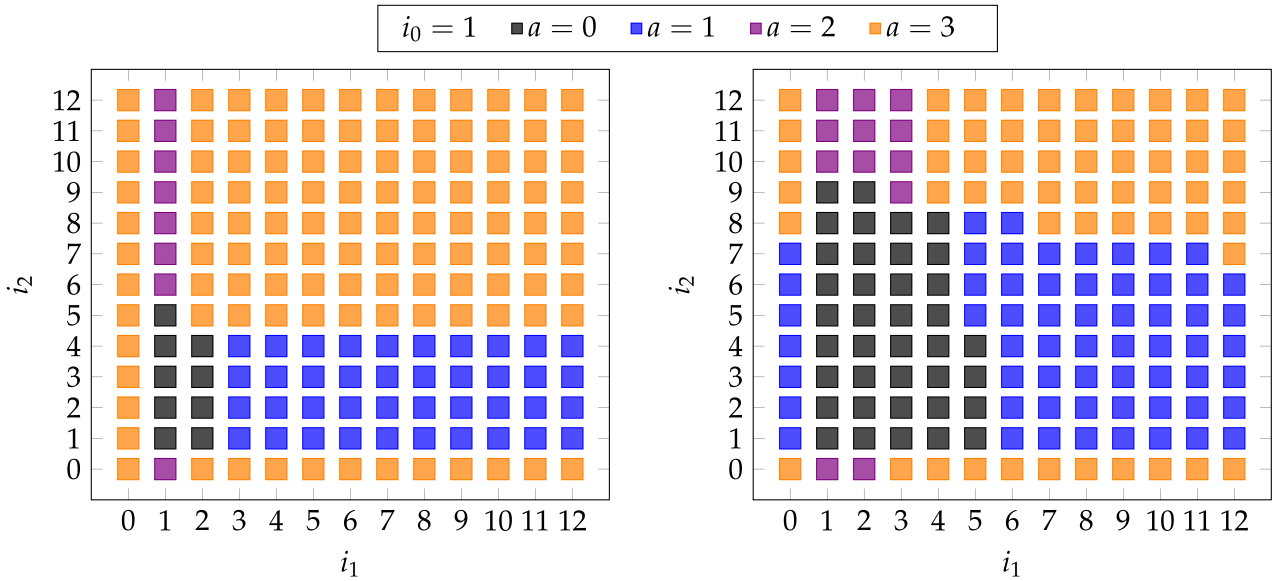
There are two models in the paper Zhu et al. (2021) to which an attempt is done to implement them. These models are already multi-component models, and only the period-dependent costs feature was to be added. The performance of the two original models and their extension are provided in this section. We decide to use the discount reward theory for comparing policies in transient states.

Firstly, the two models DEF and DEF Heuristic are implemented. To check whether this was succesfully done we compare the objective values of the models when using the same setting the paper described they have used. Also, the same scenarios been used. The objective values found by the paper are shown in Appendix B. The objective values found in this thesis are provided in Table 6.2. The implementation of the DEF has been successful, as we find the objective values to have a deviation of no more than 1.1% to the objective values in the paper. A reason for this small deviation could be the choice of costs $c_{f,i}$, as the paper randomly draws these values from a uniform distribution and displays a rounded to one decimal value. We have used these rounded values but perhaps the paper uses slightly different costs. Furthermore, the values for DEF* are the same as those of DEF, however the computation time is longer. This is due to the extra variables and constraints DEF* uses. The results concerning the heuristic tell us a different story. We find different heuristic objectives than the paper finds. They find a maximum objective gap of 9.89%, however our maximum objective gap is 61%. Obviously, some decisions in the implementation are made differently. As we are eventually searching for the policy, we have computed the policy for $t = 1$ for both DEF as DEF Heuristic to see whether there is also a big difference found in the policy at the starting period. These policies can be found in Figure 6.5. Additionally, our heuristic that is adapted to the period-dependent costs show higher objectives than the original DEF Heuristic for $\Delta = 0.0$.

Table 6.2: Results of the objective value from the implemented DEF model, DEF model*, DEF Heuristic and DEF Heuristic*, where the asterisk * implies that it is an adjusted model to period-dependent costs. We let $d = 5$, $\Delta = 0$, first component starts in failure mode, $\xi_1 = 1$, and other components have start age 2. We let $|\Omega| = 1000$, $c_{f,1} = 14.4, c_{f,2} = 11.4, c_{f,3} = 9.4, c_{f,4} = 8.0$. Scenarios are the same as in the paper.

n	T	DEF		DEF*		DEF Heuristic		DEF Heuristic*	
		Obj.	CPU(s)	Obj.	CPU(s)	Obj.	CPU(s)	Obj.	CPU(s)
2	5	25.82	58	25.82	130	29.29	1	29.99	3
	7	28.67	96	28.67	713	42.52	2	44.65	8
	9	33.14	490	N/A	>1 hour	53.49	2	58.66	9
3	5	26.79	79	26.79	310	30.21	2	32.21	9
	7	30.41	265	N/A	>1 hour	46.22	2	49.94	20
4	5	29.97	250	29.83	979	36.47	2	38.88	10

In Figure 6.5 we can find the policies for $t = 1$ computed with DEF and the DEF Heuristic. The policies are not in line with each other. The policies found with DEF are more conservative than the policies found with DEF Heuristic. The computation time for the DEF policies was over 15 hours, and retrieving the policies for all $i_0 \in \mathcal{T}$ would take too longer than a week.



(a) DEF policies for i_0 =1

(b) DEF Heuristic policies for $i_0 = 1$

Figure 6.5: Plot showing difference in policies between DEF and DEF heuristic. We let $d = 5$, $q = 5$, $T = 5$, $n = 2$, $\overline{c_f} = 50$, $\overline{c_p} = 10$ and $\Delta = 0.0$. Components 1 and 2 follow $\alpha_1 = 6$, $\alpha_2 = 12$ and $\beta_1 = \beta_2 = 2$. Computation time for DEF policies was over 15 hours.

Performance DEF and DEF Heuristic**

In order to see the added value of our DEF* and DEF Heuristic*, we compute the objective for different values of Δ. Here, we do not choose N to be the number of months in a year, but we choose the value of 5. That means that the year is divided in 5 periods. As a results from equation (17), the middle period is the least costly. As we are working with expected costs over a time horizon, we use the DEF objective to compare the models with each other. For the DEF this means that the optimal solution is determined when $\Delta = 0.0$, and that the new objective is calculated with costs that obey the new Δ value. For the DEF Heuristic this means that simply the optimal solution is computed, as the varying costs are already used throughout the heuristic. The results can be found in Table 6.3.

Table 6.3: Objective values of the DEF, DEF*, DEF Heuristic and DEF Heuristic* for difference Δ. We let $d = 5$, $T = 5$, $n = 2$, first component starts in failure mode, $\xi_1 = 1$, and other components have start age 2. We let $c_{f,1} = 14.4, c_{p,1} = 1, c_{f,2} = 11.4$, and $c_{p,2} = 1$. Scenarios are the same as in the paper.

Δ	DEF	DEF*	savings	DEF Heuristic	DEF Heuristic*	savings
0.0	25.82	25.82	0.0%	29.3	30.0	-2.7%
0.1	27.29	27.27	0.07%	30.3	31.4	-3.6%
0.2	28.76	28.72	0.14%	31.4	32.7	-4.1%
0.3	30.23	30.18	0.17%	32.44	34.12	-5.2%
0.4	31.70	31.63	0.22%	33.49	35.48	-6.0%
0.5	33.17	33.08	0.27%	34.54	36.86	-6.9%

From Table 6.3 we see that DEF* performs slightly better than DEF and longer computation time pays of in the objective value. We also see that the DEF Heuristic* performs worse than the original model. We will therefor not continue to work with DEF Heuristic*.

6.3 Multi-Component MIP Model

The first part of section will be fully devoted to the characteristics of $\delta(i,x)$ and which of the three postponing methods mentioned in Section 5.5.2 should be chosen to determine the value of $\delta(i,x)$ when x is larger than 0. The second part of this section, measures the performance of the Multi-Component MIP Model when no maintenance constraints and probability constraints are used.

Choice of computation $\delta(i,x)$

Recall, that the computation of $\delta(i,x)$ only looks at one-component p-ARP models. For this $\delta(i,x)$ part of this section, we use a component in state $i = (1,1)$, so period $i_0 = 1$ and age $i_1 = 1$.Additionally, the same variables v, b and j are used as in the example Figures 5.2 and 5.2.

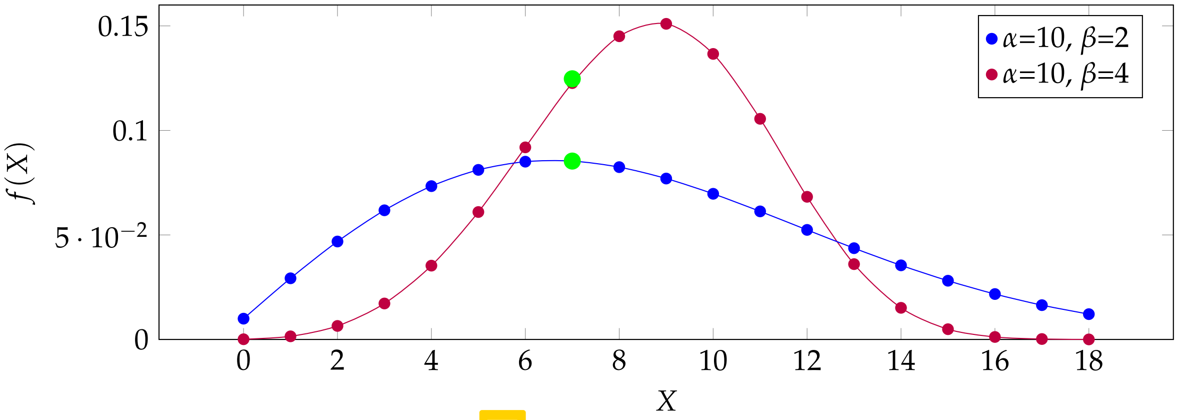


Figure 6.6: Probability distribution function a dicretised Weibull distribution. Scale and shape parameters are denoted by α and β respectively. $X = 0$ means that the component has failed before reaching age 1. The optimal replacement times, b, are indicated with the green dot.

To illustrate $\delta(i,x)$ its characteristics we will work with two Weibull distributions. Both have shape parameter $\alpha = 10$, one has scale parameter $\beta = 2$ and the other has scale parameter $\beta = 4$. Their distributions are shown in Figure 6.6 where X is the lifetime of a component and each value $f(X)$ the probability that the component fails between age X and age $X + 1$. It is clear from the figure that the variance for a larger β is smaller. The green dots in the figure are the optimal replacement times b where they both have age 7. We will use these two distribution to compute the values for $\delta(i,x)$. We denote δ_1, δ_2 and δ_3 as the $\delta(i,x)$ values for a certain x, for which the postponing values are calculated with Method 1, 2 and 3 respectively. The values are shown in the left axis of Figure 6.7.

There are four points to highlight concerning which Figure 6.7. Note that for $x = 0$, the component will be replaced in state b, the green dot in Figure 6.6. Firstly in Figure 6.7 there is no difference between the δ's that use Method 1 or Method 2 for determining the postponing effect, and the suspicion against changing the policy in Method 1 was redundant. Secondly, when going to larger values of x, we find that the δ-values tend to converge to a fixed value. The reason for this is that the probability of reaching the corresponding ages become smaller when x increases. Also, the instant costs of executing PM is multiplied by this probability, so when the probability is low the instant costs of PM is neglected. This is the reason why for δ_1 there is

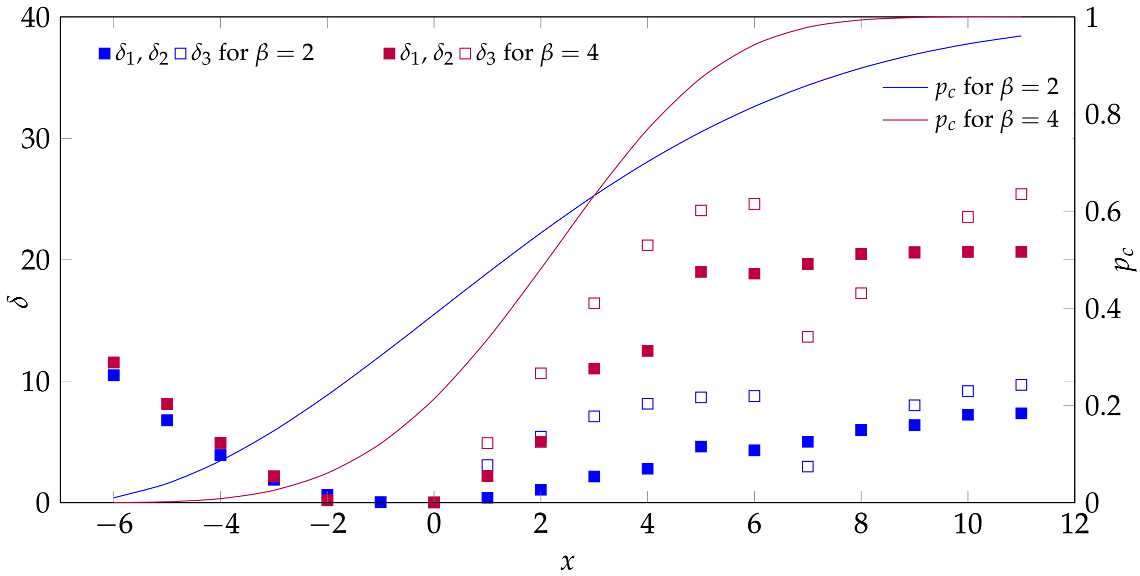


Figure 6.7: On this figures' left y-axis, you can find the values of δ retrieved using Method 1 and Method 2, indicated by δ_1 and δ_2, respectively, as well as the values of δ retrieved with Method 3, indicated by δ_3. These values correspond to the same Weibull distributions shown in Figure 6.6. The corresponding values of p_c are plotted on the right y-axis.

a small decrease for $x = 6$. Thirdly, δ values for the two distributions become constant for at different values. After $x = 0$, the values corresponding to $\beta = 4$ have a steep rise and go toward 20, 25, whereas the values corresponding to $\beta = 2$ go toward 10. As the distribution for $\beta = 4$ has a lower variance, the chances of getting CM are fiercer when planning after $x = 0$. Due to the probabilities multiplied of even getting in a state after v are highest just after $x = 0$ the δ values result having higher values. Higher values than the δ values for a distribution with $\beta = 2$. Fourthly, the sudden drop for δ_3 might have been noticed. As δ_3 calculates the difference in expected costs over a certain time period, the expected costs when following the normal policy has increase of costs at $i_0 = v + 7$. This is the originally planned PM. As these expected costs are subtracted to the expected costs when doing no maintenance, a sudden drop can be found for $x = 7$. The values of δ_1 and δ_2 do not have this drop as they calculate the average expected difference in costs. As δ_1 and δ_2 are more stable it is most appropriate to use these for a more consistent model.

Performance Multi-Component MIP Model

Here, the Two-Component MDP and the DEF model are compared and analyze the differences in policies. We use the equation of Tijms (2003) in Chapter 6.2 to determine the average costs over a finite horizon. The equation is

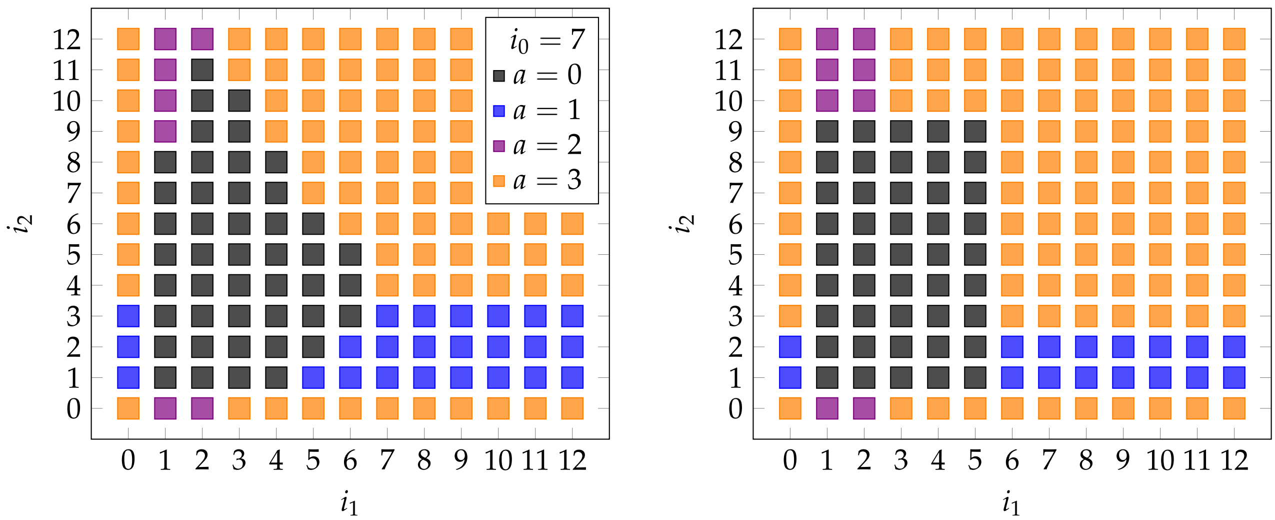
$$V_T(i, R) = \sum_{t=1}^{T} \sum_{j \in \mathcal{I}} \pi_{ij}^{(t)}(R) c_j(R_j) \tag{32}$$

where R is the policy we want to test, T is the length of the time horizon, π the transition matrix, $c_j(R_j)$ the immediate costs when in state j and chosing action R_j, and i is the starting state. A state is $i = (i_0, i_1, i_2)$ for a 2 component model where i_0 is the period and i_1 and i_2 denote the age of the component. When the age is 0, it means that a component is in failure state and it needs to be replaced immediately. We will compare the policies computed by the Two-Component p-ARP model, the multicomponent MIP and the DEF heuristic.

Table 6.4: Average costs for four different models for different starting ages.Setting used are $d = 5$, $\overline{c_f} = 50$, $\overline{c_p} = 10$, $\Delta = 0.2$ and $M = 12$. Components 1 and 2 follow $\alpha_1 = 6$, $\alpha_2 = 12$ and $\beta_1 = \beta_2 = 2$.

Starting ages	Two-Component p-ARP	Multi-component MIP	DEF Heuristic
1, 1	76.2	74.8	138.1
1, 5	99.3	113.6	144.4
1, 10	123.7	87.8	152.6
5, 1	113.6	113.6	141.2
10,1	113.6	113.6	141.2
average	123.45	127.73	146.41

In Table 6.4 you can find the average costs for three models computed by (32) for different starting ages. Also, for all combination of starting states $i_1, i_2 \in M$ the average is computed to represent the difference in performance clearly. The policy differences for the Two-Component p-ARP and Multi-Component MIP is provided for $i_0 = 7$ in Figure 6.8. From this figure, it becomes clear that the Multi-Component MIP model has a stronger bias for replacing the two components at the same time. A reason for this, is that the costs for $\delta(i, x)$ does not completely represent the extra costs of shifting, making the value of d be stronger in the objective than the shifting costs.



(a) Two-Component p-ARP policies for i_0 =7

(b) Multi-Component MIP policies for i_0 =7

Figure 6.8: Figure revealing actions for transient states from p-ARP model. We let $d = 5$, $\overline{c_f} = 50$, $\overline{c_p} = 10$, $M > 12$, and $\Delta = 0.2$. Components 1 and 2 follow $\alpha_1 = 9$, $\alpha_2 = 12$ and $\beta_1 = \beta_2 = 2$.

Chapter 7

Discussion and Conclusions

In this chapter, the conclusions of this thesis together with the discussion points are presented in three sections starting with the MDP approach in Section 7.1, followed by discussing the stochastic approach in Section 7.2 and thirdly the Multi-Component MIP is treated in Section 7.3. Future recommendations are provided in Section 7.4.

7.1 MDP approach

The p-ARP and p-MBRP original model have been implemented such that the results are identical, and we conclude that the paper of Schouten et al. (2022) is clear and concise. Furthermore, the two-component extensions have been rewarding because the extended p-ARP could be used to measure the performance of the designed Multi-Component MIP. The p-MBRP was less appropriate because we could not retrieve the transient state policies, and it is decided to not extend this model because the age replacement principle of the DEF was more in line with the p-ARP model. Additionally, the two component p-MBRP results were almost identical for every Δ, differing slightly by less than 0.5% for two cases. Throughout the computation, we have found the the value of M can impact the output of the p-MBRP model. Unfortunately, the paper of Schouten et al. does not clarify the value of M used in its Table 2 and perhaps this is the reason for the $< 0.5\%$ deviation.

Furthermore, a creative attempt has been done to retrieve policies, but results have shown that this method called Markov Manipulation is not working correctly. Nevertheless, the acknowledged Discounted Reward method by Kallenberg was successfully implemented for the p-ARP policies, yielding satisfactory outcomes. Here, we have made the assumption that by choosing the discount factor α as close to 1 as Java allows, the obtained policy is the actual policy.

7.2 Stochastic approach

The successful implementation of the DEF model and its extended version, DEF*, accounting for period-dependent costs, marks an advancement in this thesis. The extension DEF* performs better when period-dependent costs are present. However, it is worth noting that the integration of period-dependent costs in DEF* comes at the expense of increased computation time due to the additional variables and constraints. In our quest for more efficient computational times, we explored the DEF Heuristic as an alternative. Unfortunately, we encountered challenges in properly implementing the DEF Heuristic, and the DEF Heuristic* did not yield any improvement over the original DEF Heuristic. The DEF Heuristic relies on two theorems that do not hold anymore when taking period-dependent costs in account. Too much needed to be adapted, at the expense of the quality.

While the DEF model holds promise for both short-term planning and situations where long-term costs are of lesser importance, the computation times are too long to obtain the full policy and test its performance next to the two-component p-ARP model and Multi-Component MIP. The exploration of the DEF Heuristic and DEF Heuristic* offers valuable lessons for future research, emphasizing the significance of appropriately adapting heuristic approaches and addressing complex cost structures.

7.3 Multi-Component MIP

The Multi-Component MIP is a model that creates a planning schedule for multiple components that can follow different distributions. The model allows period-dependent costs, and maintenance constraints can be easily added. For wind farm operations, this can be valuable as it enables the careful planning of maintenance activities, along with the allocation of necessary resources and manpower.

For this Multi-Component MIP, first an analysis is done concerning computing $\delta(i,x)$ for $x > 0$. It is found that method 1 and 2 produce identical values and that these values are more consistent than the values produced by method 3, as they compute the expected average costs difference and not the expected costs difference. Consequently, method 1 is chosen for computation δ. Additionally, it is found that the $\delta(i,x)$ value does not represent the chance of getting CM which might influence the performance of the Multi-Component MIP.

Promising results were obtained during testing the Multi-Component MIP with two components, where a comparison against the two-component p-ARP model is done. The Multi-Component is tested for two components and the expected average costs over a time period of 12 months, was 3% below the expected average costs of the two-component p-ARP model. The Multi-Component MIP model can be improved by constraining the value of p_c into the model to discourage the use of high δ values. As the Multi-Component MIP computes a policy for a finite horizon and the two-component p-ARP model computes a policy for an infinite horizon, the objective of the Multi-Component MIP has potential to drop below the objective of the p-ARP model.

7.4 Future recommendations

The Multi-Component MIP can be used as a guideline for designing a heuristic for larger instances with the same use case. Using the Multi-Component MIP, the heuristic can be tested for smaller instances and be bench-marked. Another approach is to use the Multi-Component MIP on a rolling horizon, enabling planning for a longer time horizon

References

Ackermann, T. (2005). Transmission systems for offshore wind farms. *Wind power in power systems*, pages 479–503.

Bárcena, L. B. and Castro, I. T. (2021). Optimizing and evaluating a maintenance strategy for multi-component systems. In *Optimization and Data Science: Trends and Applications: 5th AIROYoung Workshop and AIRO PhD School 2021 Joint Event*, pages 157–168. Springer.

Chen, J. and Wang, Y. (2023). A deep reinforcement learning approach for maintenance planning of multi-component systems with complex structure. *Neural Computing and Applications*, pages 1–14.

Chen, Y. and Feng, H. (2022). Maintenance strategy of multicomponent system based on structure updating and group importance measure. *Communications in Statistics-Theory and Methods*, 51(9):2919–2935.

Cremers, R. (2022). Optimal maintenance planning for offshore wind farms considering time-varying costs and limited manpower. Bachelor's thesis, Erasmus University of Economics, the Netherlands.

Dai, J., Ding, R., Fu, Y., and Qin, Y. (2023). Multi-component system maintenance optimization of rail transit train based on opportunistic correlations. *Transportation Safety and Environment*, page tdad005.

Dekker, R., Wildeman, R. E., and Van der Duyn Schouten, F. A. (1997). A review of multi-component maintenance models with economic dependence. *Mathematical methods of operations research*, 45:411–435.

Díaz, H. and Soares, C. G. (2020). Review of the current status, technology and future trends of offshore wind farms. *Ocean Engineering*, 209:107381.

Franciosi, C., Miranda, S., Veneroso, C. R., and Riemma, S. (2021). A maintenance scheduling optimization model for a multi-component machine in a digitalized manufacturing context. *IFAC-PapersOnLine*, 54(1):1254–1259.

Kallenberg, L. (2011). Markov decision processes. *Lecture Notes. University of Leiden*, 428.

Kleywegt, A. J., Shapiro, A., and Homem-de Mello, T. (2002). The sample average approximation method for stochastic discrete optimization. *SIAM Journal on optimization*, 12(2):479–502.

Kobbacy, K. A., Murthy, D. P., Nicolai, R. P., and Dekker, R. (2008). *Optimal maintenance of multi-component systems: a review*. Springer.

López, M., Rodríguez, N., and Iglesias, G. (2020). Combined floating offshore wind and solar pv. *Journal of Marine Science and Engineering*, 8(8):576.

Maher, H., Aly, M. F., Afefy, I. H., and Abdelmaguid, T. F. (2022). A maintenance optimisation approach based on genetic algorithm for multi-component systems considering the effect of human error. *International Journal of Industrial and Systems Engineering*, 40(1):51–78.

Maples, B., Saur, G., Hand, M., Van De Pietermen, R., and Obdam, T. (2013). Installation, operation, and maintenance strategies to reduce the cost of offshore wind energy. Technical report, National Renewable Energy Lab.(NREL), Golden, CO (United States).

Nguyen, V.-T., Do, P., Voisin, A., and Iung, B. (2022a). Weighted-qmix-based optimization for maintenance decision-making of multi-component systems. In *PHM Society European Conference*, volume 7, pages 360–367.

Nguyen, V.-T., Do, P., Vosin, A., and Iung, B. (2022b). Artificial-intelligence-based maintenance decision-making and optimization for multi-state component systems. *Reliability Engineering & System Safety*, 228:108757.

Özgür-Ünlüakın, D., Türkali, B., and Aksezer, S. Ç. (2021). Cost-effective fault diagnosis of a multi-component dynamic system under corrective maintenance. *Applied Soft Computing*, 102:107092.

Pinciroli, L., Baraldi, P., Ballabio, G., Compare, M., and Zio, E. (2021). Deep reinforcement learning based on proximal policy optimization for the maintenance of a wind farm with multiple crews. *Energies*, 14(20):6743.

Ren, Z., Verma, A. S., Li, Y., Teuwen, J. J., and Jiang, Z. (2021). Offshore wind turbine operations and maintenance: A state-of-the-art review. *Renewable and Sustainable Energy Reviews*, 144:110886.

Rinaldi, G., Thies, P. R., and Johanning, L. (2021). Current status and future trends in the operation and maintenance of offshore wind turbines: A review. *Energies*, 14(9):2484.

Schleisner, L. (2000). Life cycle assessment of a wind farm and related externalities. *Renewable energy*, 20(3):279–288.

Schouten, T. N., Dekker, R., Hekimoğlu, M., and Eruguz, A. S. (2022). Maintenance optimization for a single wind turbine component under time-varying costs. *European Journal of Operational Research*, 300(3):979–991.

Tambe, P. P. (2022). Selective maintenance optimization of a multi-component system based on simulated annealing algorithm. *Procedia Computer Science*, 200:1412–1421.

Thomas, L. (1986). A survey of maintenance and replacement models for maintainability and reliability of multi-item systems. *Reliability Engineering*, 16(4):297–309.

Tijms, H. C. (2003). *A first course in stochastic models*. John Wiley and sons.

Van Bussel, G. and Schöntag, C. (1997). Operation and maintenance aspects of large offshore windfarms. In *EWEC-CONFERENCE-*, pages 272–275. Bookshop for scientific publications.

Vijayan, V. and Chaturvedi, S. K. (2021). Multi-component maintenance grouping optimization based on stochastic dependency. *Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability*, 235(2):293–305.

Wang, R. and Chen, N. (2016). A survey of condition-based maintenance modeling of multi-component systems. In *2016 IEEE International Conference on Industrial Engineering and Engineering Management (IEEM)*, pages 1664–1668. IEEE.

Wang, Y. and Deng, Q. (2022). Optimization of maintenance scheme for offshore wind turbines considering time windows based on hybrid ant colony algorithm. *Ocean Engineering*, 263:112357.

Way, R., Ives, M. C., Mealy, P., and Farmer, J. D. (2022). Empirically grounded technology forecasts and the energy transition. *Joule*, 6(9):2057–2082.

Yousefi, N., Tsianikas, S., and Coit, D. W. (2022). Dynamic maintenance model for a repairable multi-component system using deep reinforcement learning. *Quality Engineering*, 34(1):16–35.

Zhang, C., Li, Y.-F., and Coit, D. W. (2022). Deep reinforcement learning for dynamic opportunistic maintenance of multi-component systems with load sharing. *IEEE Transactions on Reliability*.

Zhang, G., Yin, J., Cheng, C., Li, L., Xiao, Y., and Han, Q. (2023). Optimization of operation and maintenance path of offshore wind turbine based on ics-bp. In *Journal of Physics: Conference Series*, volume 2452, page 012020. IOP Publishing.

Zhao, J., Gao, C., and Tang, T. (2022). A review of sustainable maintenance strategies for single component and multicomponent equipment. *Sustainability*, 14(5):2992.

Zhou, Y., Li, B., and Lin, T. R. (2022). Maintenance optimisation of multicomponent systems using hierarchical coordinated reinforcement learning. *Reliability Engineering & System Safety*, 217:108078.

Zhu, Z., Xiang, Y., and Zeng, B. (2021). Multicomponent maintenance optimization: A stochastic programming approach. *INFORMS Journal on Computing*, 33(3):898–914.

Appendix A

Additional Theorem

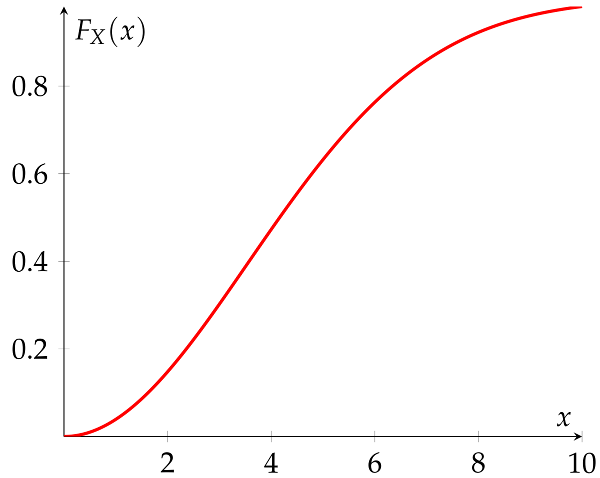
A.1 Weibull distribution

The Weibull distribution is a continuous probability distribution that is often used to model the lifetime or failure times of systems or events. It is characterized by two parameters: the scale parameter (α) which mostly determines the location and the shape parameter (β) which mostly determines the spread.

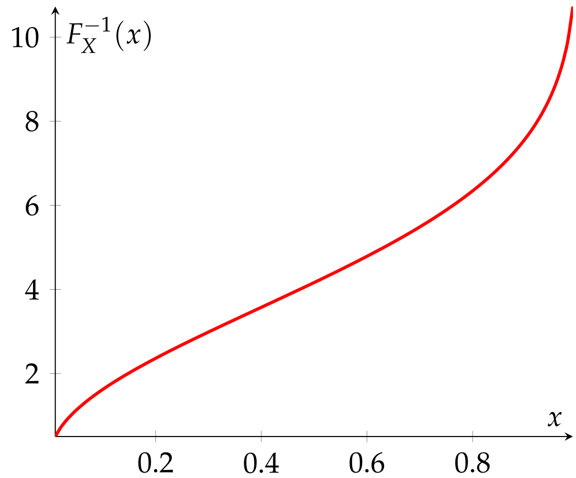
The CDF of the Weibull distribution is given by:

$$P(X \leq x) = F_X(x) = \begin{cases} 1 - e^{-(x/\alpha)^\beta} & x \geq 0 \\ 0 & x < 0 \end{cases} \tag{33}$$

Probabilities Schouten
In Schouten, we use a descretised Weibull distribution to compute $P(X = x)$, the probability that X fails at age x. We use $P(X = x) = F_X(x+1) - F_X(x)$, where $x = 0$ denotes the situation where an individual fails in its first period.



(a) CDF of Weibull distribution, scale and share parameters are $\alpha = 5$ and $\beta = 2$ respectively.



(b) CDF of Inverse Weibull distribution, scale and share parameters are $\alpha = 5$ and $\beta = 2$.

Sampling scenarios
Where in the model of Schouten the age $i_1 = 0$ denotes that an individual fails in its first year,Zhu et al. interprets this as the individual starting off in failure mode and starts computing lifetimes from $i_1 = 1$ by the following methodology:

1. Generate a random number x in (0,1)
2. Round $F_X^{-1}(x)$ to closest integer
3. If the result is 0, return 1.

The inverse Weibull distribution is used to randomly pick the lifetime, and is plotted for $\alpha = 5, \beta = 2$ in A.1.1b. The relation for this distribution is

$$F_X^{-1}(x) = \begin{cases} \alpha\left(-\ln(1-x)\right)^{1/\beta} & 0 \leq x < 1 \\ 0 & \text{else.} \end{cases} \tag{34}$$

To be consistent between the two methods, we will adjust the method mentioned above by replacing 2. and 3. by

1. Return $\lceil F_X^{-1}(x))\rceil$

Important to note here, is that when sampling the scenarios, the lifetimes are rounded to integers, and a lifetime of 0 is not allowed. This might cause the expectancy of the computed lifetimes to deviate slightly from the distributions' original mean.
Often, we deal with individuals that have been used for some periods and therefore has an age g. As we can not have any scenario that has age $i_1 = 0$, we are not allowed to use this age and simply subtract it by one for the next period. In this case, we use the following methodology to generate the lifetime it still has left.

1. Generate a random number x in $(0, 1)$
2. Return $\max\{1, \lceil F_X^{-1}(x))\rceil - g\}$.

Appendix B

Tables from other papers

Table 2
Yearly costs, in thousands of €, savings with respect to constant cost model, and maintenance months for p-ARP, p-BRP and p-MBRP. We let $\bar{c}_f = 50$, $\bar{c}_p = 10$, $\Delta_f = \Delta \cdot \bar{c}_f$ and $\Delta_p = \Delta \cdot \bar{c}_p$ for different Δ. We let $\alpha = 1$ year and $\beta = 2$.

	p-ARP		p-BRP ($m = 1$)			p-MBRP ($m = 1$)			
Δ	costs	savings	costs	savings	mos.	costs	savings	mos.	ages
0%	40.098		41.501			40.311			
10%	40.035	0.16%	41.420	0.20%	6, 11	40.263	0.12%	6, 11	4, 4
20%	39.701	0.99%	40.933	1.37%	6, 11	39.855	1.13%	6, 11	4, 4
30%	39.224	2.18%	40.361	2.75%	6, 10	39.338	2.41%	6, 10	5, 3
40%	38.461	4.08%	39.439	4.97%	6, 10	38.556	4.35%	6, 10	5, 3
50%	37.635	6.14%	38.466	7.31%	7, 10	37.773	6.30%	6, 10	5, 3
CPU	< 1 s		< 1 s			3 s			

Figure B.0.1: Table 2 from Schouten et al. (2022)

Table 1. Algorithm Performance in Solving DEF

			Solver		Algorithm 4		
Case	n	T	CPU time (sec.)	Obj.	CPU time (sec.)	Obj.	Obj. error (%)
1	2	5	103	25.80	17	26.41	2.36%
2		7	202	28.36	14	30.87	7.82%
3		9	503	32.82	25	34.76	5.91%
4	3	5	237	26.92	51	27.78	3.19%
5		7	561	30.14	103	33.12	9.89%
6		9	2,777	34.50	92	37.71	9.30%
7	4	5	472	30.02	145	32.31	7.63%
8		7	N/A		181	38.86	N/A
9		9	N/A		269	44.5	N/A
10	5	5	N/A		324	37.35	N/A
11		7			422	45.59	
12		9			551	53.66	
13	6	5	N/A		623	41.4	N/A
14		7			837	51.28	
15		9			1,117	60.94	
16	7	5	N/A		1,385	48.77	N/A
17		7			2,812	62.13	
18		9			3,398	74.55	

Figure B.0.2: Tabel 1 from Zhu et al. (2021) where they indicate the DEF model with 'Solver' and the DEF Heuristic with 'Algorithm 4'.