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Dutch Annuity and Interest-Only Mortgage Valuation Framework

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Abstract

The present study introduces a framework for valuing Dutch annuity and interest-only retail mortgages. The valuation involves computing the expected values of discounted yearly cash flows over simulated mortgage rates, which are generated via a Hull-White one-factor model. A unique feature of this research is the incorporation of the carry-over option and partial prepayment in the valuation process. The carry-over option allows borrowers to add an additional loan or alter the existing loan when moving houses. This study identifies the specific scenarios in which the carry-over option becomes advantageous. Additionally, a sigmoid regression is employed for partial prepayment. The results of this research have significant implications for the valuation of Dutch retail mortgages and provide valuable insights for lenders and borrowers alike.

Contents

1 Introduction

Dutch retail mortgages are loans secured by collateral in the form of a household's property, providing lenders with recourse to the property in the event of loan default. Fixed-rate mortgages (FRMs) and adjustable-rate mortgages (ARMs) are the two most common mortgage types. FRMs offer a fixed interest rate that remains constant over the loan's life, while ARMs charge an interest rate that varies over time. According to the European Central Bank (Köhler-Ulbrich et al., 2009), FRMs with a fixation period of 5-10 years are the most prevalent interest rate type for Dutch mortgages. There are three main repayment methods for FRMs: annuity, interest-only, and linear. An annuity loan is the most common repayment method, with roughly 75% of mortgages employing it. Under this approach, a fixed monthly amount is paid that includes both interest and principal repayments. Interest-only mortgages, which account for around 22% of mortgages, require only interest payments during the loan term, with the entire principal repaid at the loan's end. Linear mortgages, which account for less than 3% of mortgages, require fixed principal repayments, leading to a reduction in interest payments over time. These percentages are based on data from the Hypotheken Data Netwerk (HDN B.V., 2023).

Mortgage holders/households may choose to prepay their loan, which can be prompted by changes in borrower characteristics, such as increased income, or market circumstances related to the contract, such as a decrease in interest rates. The latter can incentivize prepayment, as investing in mortgages may be more profitable than in interest rates. To mitigate prepayment risk, Dutch lenders may impose penalties in the form of interest rates foregone to offset their losses (Groot and Lejour, 2017 and Mayer et al., 2013). However, there are three major exceptions to this penalty rule for Dutch mortgages. Firstly, the prepayment amount must generally be less than or equal to 10% - 20% of the original loan (depending on the lender). Secondly, if the homeowner sells their property, they are exempt from paying the compensation. Lastly, at the end of the fixation period, the household has the choice to prepay the whole loan or continue with the same lender, before the fixation rate and period are reset.

When a household decides to sell their collateral/house and acquire a new one, they are presented with various options. One possibility is to repay the existing mortgage and obtain a new one from either the same or a different lender. Alternatively, they may opt to retain the current fixed-rate contract with the same lender and obtain an additional loan. This particular option is commonly referred to as the "Carry-Over" agreement and will be one of the primary focus of this research.

Between 2009 and 2019, interest rates exhibited a downward trend, resulting in a decrease in mortgage rates. Consequently, the carry-over agreement option was not commonly utilized since prevailing market rates to establish a new fixation period were lower than those at the time the current fixed rate was set. However, the recent rise in interest rates and corresponding mortgage rates has increased the value of the carry-over agreement option. This research introduces a new approach by including the carry-over agreement option in the valuation of mortgages for both annuity and interest-only principal repayment methods. This allows for a comprehensive analysis of the impact of various factors on mortgage valuations. To achieve this, we will create several scenarios featuring varying loan values, fixed rates, the competitiveness of NN, and time to reset the fixed rates. We aim to provide NN with the value of a loan under each specific scenario, and additionally the relative use of the carry-over option under each scenario. We will assume that the mortgage rates include the household's risk of default.

Research Question:

Can we iteratively value a mortgage loan by calculating the carry-over agreement option of a mortgage with a specific prepayment dynamic and a predefined probability of moving, using the Hull-White short-rate model as a discount factor?

The present paper is organized as follows: Firstly, in Section 2, an overview of the existing literature pertaining to the methods employed in this study will be presented. Secondly, in Section 3, the data utilized in the analysis will be briefly described. Subsequently, Section 4 will provide a detailed exposition of the methodology for valuing loans using partial prepayment and carry-over agreement options. Finally, Section 5 will present the implementation of the aforementioned methodology and the interpretation of the results.

2 Literature review

Prepayment dynamics are the dominant variable to consider when valuing mortgages and mortgage-backed securities. The Constant Prepayment Rate model is the most straightforward method, which assumes that households prepay at a constant rate. However, this might be too simplistic, and therefore Kalotay et al. (2004a) introduces a prepayment model that classifies prepayments as two different types, and models each differently. The first type, turnover, is assumed to be independent of interest rates, and the second, refinancings, is assumed to depend on interest rates.

A similar approach was followed by Azevedo-Pereira et al. (2002) and Hung et al. (2012). Their FRMs valuation framework uses a mean-reverting interest rate model and a log-normal house price diffusion model, where the prepayment is modeled as an American option, as it may be optimal at any time to exercise this option. According to their suggestion, the interest rates are the primary determinants of the prepayment option value when the future house price is larger than 0.95 of the original price. Therefore, our research will solely focus on the interest rate fluctuations and not the housing dynamics when modeling the prepayment.

This research will simulate risk-neutral interest rates that we will transform into market mortgage rates by adding a constant to the short rates. Vasicek (1977) introduced a time-homogeneous one-factor mean-reverting short-rate model. Although the paper started a whole stream of research, the yield curve was not consistent with the market (Nowman, 2010). Cox et al. (1985a) and Cox et al. (1985b) extended the model further which prevents negative interest rates. These models are useful for capturing the yield curve dynamics, however, for derivative pricing their main limitation is that they are time-homogeneous. They are not capturing the market curve exactly, consequently, it induces pricing errors and arbitrage opportunities.

Ho and Lee (1986) overcame this by introducing a no-arbitrage model based on the market interest rates. The limitation is that the model is not mean-reverting, therefore the interest rates can be negative, and the volatility is constant over time. Ergo, Hull and White (1990) created the Hull-White model, which is an extension of the previous model with the mean-reversion and the heteroskedasticity addition.

To calibrate this Hull-White model, Hull and White (2001) describes how a general one-factor model of the short-rate can be implemented as a recombining trinomial tree and calibrated to market prices of actively traded instruments such as caps and swap options. Instead of using a tree, Schlenkrich (2012) describes the efficient non-linear optimization formulation of the problem to calibrate the model and investigate the numerical properties of the iterative solution by means of a Gauss-Newton and an Adjoint Broyden quasi-

Newton method. Gurrieri et al. (2009) propose an efficient approximation formula for the swaption implied volatility which enables to estimate of the mean reversion independently of the volatility. Furthermore, they give the closed forms for exact pricing using explicit integrals of the model parameters and propose parametric forms for the mean-reversion and volatility.

In the field of mortgage valuation, two types of methods can generally be distinguished (Pliska, 2006). The first approach involves creating a stochastic interest rate model and incorporating statistical methods to describe household prepayment dynamics. The resulting cashflows are then discounted to value the mortgage. While this approach can be complex to set up, it can be approximated through a Monte Carlo simulation. Notable examples of this approach include works by Deng (1997), Deng et al. (2000), and Kau et al. (2004).

The second approach to mortgage valuation is an option-based or structural method. One such method, developed by Kalotay et al. (2004b), employs a recursive method that is similar to the binomial option pricing method. At each node, the method compares the difference between refinancing costs and the value of the mortgage. This approach has also been applied to partial prepayment of Dutch mortgages, as shown by Kuijpers and Schotman (2006) and Kuijpers and Schotman (2007). This research will be a follow-up of the second approach.

3 Data

First, in order to calibrate the one-factor Hull-White model for the purposes of this research, it is necessary to obtain relevant data. Given their close association with interest rate dynamics, the Bonds and Swaptions markets are both viable options. However, in light of the Swaptions market's generally higher level of liquidity, we have chosen to utilize prices from this market for our model calibration. Specifically, we will utilize the prices of At-The-Money co-terminal European Euro Swaptions with entry dates ranging from 1-30 years and expiry dates ranging from 1-30 years, as of June 14th. These prices are derived from black-implied volatilities. Additionally, swap rates for the spot rates will also be utilized in the calibration process. Further details regarding the calibration process itself are provided in Section (4.6.2).

During the course of this research, it will be necessary to use the probability of households moving. To this end, data from the Central Agency of Statistics (CBS) will be utilized. However, it should be noted that precise data pertaining to this particular variable is not readily available. As such, an approximation will be employed, whereby the turnover rate of households will be utilized. Specifically, the turnover rate will be calculated as the average number of sold houses divided by the total number of houses in the Netherlands over the years 2012-2022. This calculation yields a value of 4.37%.

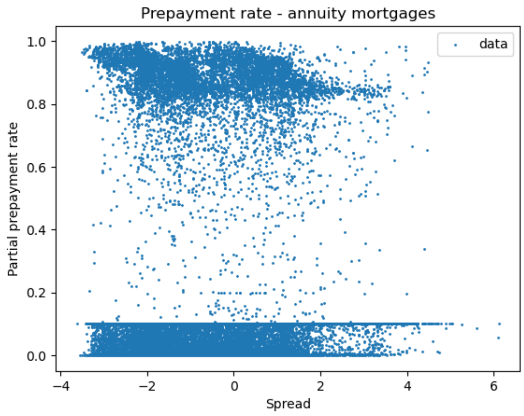
Finally, data pertaining to the partial prepayment of mortgages will be required for modeling purposes. This data was provided by NN and consists of approximately 900,000 distinct mortgages, encompassing six different mortgage types. The data includes information such as the original principal amount of each loan, the fixed rate of the loan, the amount of partial prepayment made by borrowers, the reason for making the prepayment, and the market spread between the prevailing rates and the respective fixed rate. In order to focus solely on the mortgage types relevant to our research, namely interest-only and annuity mortgages, the first step was to filter the data accordingly.

Typically, the principal repayment moments occur on a monthly basis. However, for the purposes of this research, the analysis of repayments will be conducted on a yearly basis. To this end, all the data was condensed to a yearly basis by summing up all the partial prepayments for each specific mortgage. Additionally, the sum of the market spread was weighted to their respective prepayment. Furthermore, we filter out the outliers by setting the lower bound of the original principal to 2000 euros. Finally, to get to the partial prepayment rate, we divide the yearly partial prepayment by the original principal balance. Descriptive statistics pertaining to this filtered data sorted on the two mortgage types can be found in Tables 1-2.

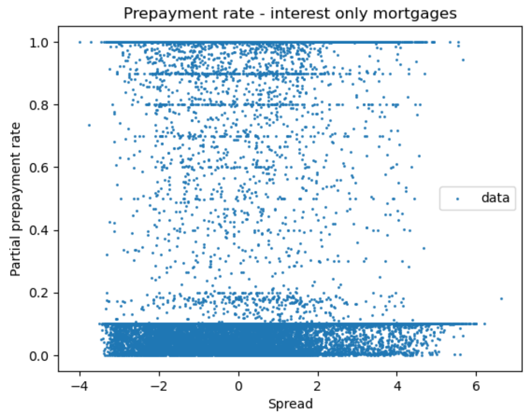
For our partial prepayment model, we will regress the market spreads on the partial prepayment rate of the filtered data. On Figures 1 we can see the plots, of the mortgage spreads on their partial prepayment rates. The first two figures show the full yearly data, and the two last pictures show the filtered yearly data that will be used during the research. As mentioned earlier, the mortgage holders can partially prepay without penalties up to 10% of the original principal per year. This ceiling can be seen on the plots.

Figure 1: Plot of the full and filtered mortgage data for both annuity and interest-only mortgages, along with the partial prepayment rates on the market spreads. The market spreads refer to the difference between the prevailing rates and the fixed rate of the respective loans.

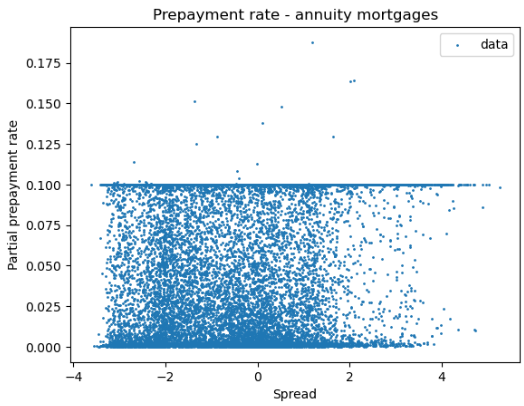
(a) Annuity - Full



(b) Interest Only - Full



(c) Annuity - Filtered



(d) Interest Only - Filtered

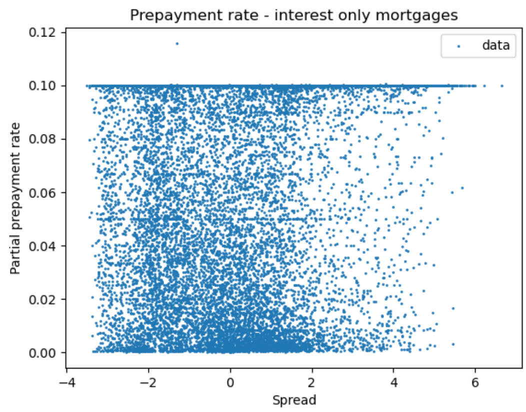


Table 1: Annuity Mortgages

$n = 477113$	Mean	Sd	Median	Min	Max
Original principal	127242	106229.85	103500	2000	1732000
Market Spread (%)	-0.03	0.36	0	-3.62	5.26
Partial prepayment rate (%)	0.23	1.42	0	0	18.76

Table 2: Interest-Only Mortgages

$n = 415893$	Mean	Sd	Median	Min	Max
Original principal	104460	87249.73	88500	2000	4200000
Market Spread (%)	0.01	0.42	0	-3.51	6.64
Partial prepayment rate (%)	0.33	1.68	0	0	11.58

4 Methodology

The carry-over agreement is a financially favorable option at a time t when the expected values of the loans satisfy the inequality:

$$\mathbf{E}\left[V_1(t, T_0, T_1)\right] + \mathbf{E}\left[V_2(t, \hat{T}_0, T_1)\right] \leq \mathbf{E}\left[V_3(t, \hat{T}_0, T_1)\right], \tag{1}$$

where $V_j(t, T_0, T_1)$ represents the value of loan j at time t, with the end of the fixation period at time T_0, and the end of the loan at time T_1. Specifically, L_1 denotes the original mortgage, L_2 represents the additional loan, and L_3 represents the loan offered by the competition (Section 4.1 will expand on this). To calculate the value of each mortgage, it is necessary to compute the cash flows at each time point and discount them using the prevailing market mortgage rates.

We first assume that the final maturities of all the loans are equal to T_1. Hence, after T_1 years, the household will have repaid the whole loan(s). Further, the default fixation period of $V_2(t, \hat{T}_0, T_1)$ and $V_3(t, \hat{T}_0, T_1)$ are set to 10 years, but this can change depending on the lasting $T_1 - t$, as described:

$$\hat{T}_0 = \begin{cases} T_0 & \text{If } T_0 = T_1, \\ T_1 & \text{If } 10 > T_1 - t > T_0 - t, \\ 10 & \text{If } T_1 - t \geq 10. \end{cases} \tag{2}$$

Then, we assume that buying a new house is a long-term, well-thought-through decision. Therefore, we assume that the probability of moving from one house to another is independent of the interest rates. Additionally, we calculate the cash flow yearly to avoid computing the carry-over value each month. Finally, to avoid having a nested problem, we assume that a household can only have a maximum of two loans simultaneously. Consequently, after exercising the carry-over option, you are not able to move from your house and use the carry-over option again.

This section will begin by providing an explanation of the calculation of different loans (Section 4.1). It will then proceed to describe the determination of cash flows for plain vanilla annuity and interest-only repayment methods (Section 4.2), followed by an illustration of how these cash flows change when incorporating prepayment and movement dynamics (Section 4.3). Furthermore, the expected values of the loans of Equation (1) will be calculated and demonstrated in Section 4.4. The section will then move on to describe the modeling of prepayment (Section 4.5) and the computation of market mortgage rates (Section 4.6).

4.1 Different Loans

As previously mentioned, this research will entail the study of three distinct loans. The first loan, denoted by L_1, will be the primary subject of interest, as it will be valued. While determining the price of L_1, we will consider a certain probability p_m of households moving, wherein they may purchase a new house denoted by H_n, which is assumed to be more expensive than their previous abode, H_o. In this scenario, households can either obtain an additional complementary loan at NN or opt for an entirely new loan at NN or from a competing institution.

In this research, we make the assumption of no housing dynamics. Azevedo-Pereira et al. (2002) states that at a loan-to-value (LTV) higher than 95%, the prepayment options' main driver is the interest rates. For this reason and for future simplicity, we also assume an LTV ratio of 1. Thus, at time $t = 0$, we have $L_1 = H_o$, $L_2 = H_n - H_o$ (when $H_n > H_o$), and $L_3 = H_n$. However, as time progresses ($t > 0$), the outstanding principal on L_1 decreases annually due to repayments made by households. The additional loan amount remains thus constant at $L_2 = H_n - H_o$, while the principal of the loan at the competition is $L_3 = L_2 + O_1(t)$, with $O_1(t)$ being the lasting principal of the original loan at time t.

4.2 Principal Repayment Methods

As aforementioned, the household pays a fixed amount yearly in the annuity mortgage repayments. This payment incorporates the interest rates on the payments and the principal payment.

As shown by Azevedo-Pereira et al. (2002), the Yearly Payment (YP) of the annuity loan j at time point t is given as follows:

$$YP_j(t) = \frac{r_j^f(t, T_0) * \left(1 + r_j^f(t, T_0)\right)^{T_0 - t} * O_j(0)}{\left(1 + r_j^f(t, T_0)\right)^{T_0 - t} - 1}, \tag{3}$$

where $r_j^f(t, T_0)$ stands for the fixed interest rate of loan j at time t with end of fixation period at time T_0, and $T_0 - t$ for the loans total number of periods. Currently, we assume that the $T_0 = T_1$. Furthermore, they also showed that the outstanding balance of loan j at time t equals:

$$O_j(t) = \frac{\left(1 + r_j^f(t, T_0)\right)^{T_0 - t} - \left(1 + r_j^f(t, T_0)\right)^{t}}{\left(1 + r_j^f(t, T_0)\right)^{T_0 - t} - 1} * O_j(0). \tag{4}$$

By using Equation (4), we can easily rewrite the $O_j(t-1) - O_j(t)$ to get the Yearly Principal Payment (YPP) of loan j at time t:

$$YPP_j(t) = \frac{\left(1 + r_j^f(t, T_0)\right)^{t} \left[1 - \frac{1}{\left(1 + r_j^f(t, T_0)\right)}\right]}{\left(1 + r_j^f(t, T_0)\right)^{T_0 - t} - 1} * O_j(0). \tag{5}$$

Another alternative to the annuity is the interest-only mortgage repayment method. Under this method, borrowers are only required to make payments towards the interest portion of the loan on an annual basis, with the principal amount remaining outstanding until maturity, at which point it must be repaid in full. Due to the simpler repayment structure, the calculation of the plain vanilla $YP_j(t)$ for an interest-only mortgage is relatively straightforward:

$$YP_j(t) = \begin{cases} r_j^f(t, T_0) * O_j(t-1), & \text{If } t \in \{1, \ldots, T_0 - 1\}, \\ O_j(t) + r_j^f(t, T_0) * O_j(t-1), & \text{If } t = T_0. \end{cases} \tag{6}$$

where $O_j(t)$ equals the lasting principal. Further, the $YPP_j(t) = 0$ and the $O_j(t)$ is constant over time, besides at T_0 where the whole principal is repaid. However, later we will include partial prepayments that will alter the lasting principal.

In both cases, at the end of the fixation period, the loan is repaid. However, as mentioned in Equation (2) the fixation period often does not coincide with the end of the loan. Although, for interest-only mortgages, this does not change the repayment structure, for annuity mortgages it does. When the end of the fixation period is attained at $t = T_0$, we assume that the borrower prepays the whole lasting principal. The next fixed rates are determined based on the prevailing market conditions, which are currently unknown.

4.3 Valuing a Loan

In this Section, we will calculate the value of each loan L_j, denoted as $V_j(t, T_0, T_1)$, with a specific fixed rate $r_j^f(t, T_0)$.

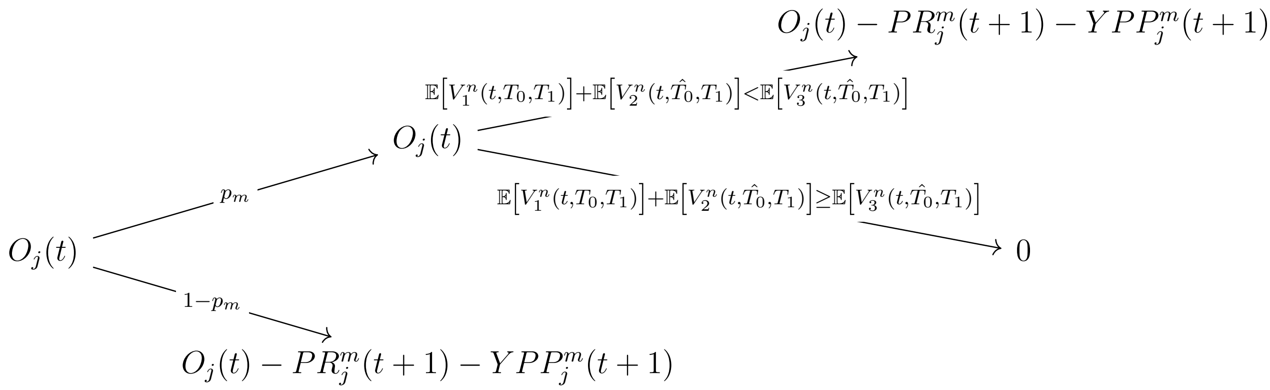
The most straightforward method to value L_j is to discount all the mortgage payments, $YP_j(t+i)$, using their corresponding market mortgage rates, $y^m(t, t+i)$, and then summing up the present values to obtain the value at time t, $V_j(t, T_0, T_1)$:

$$V_j(t, T_0, T_1) = \sum_{i=t}^{T_0} \frac{YP_j(i)}{(1 + y^m(t, i))^{i-t}} \tag{7}$$

An important assumption is that we get the loan principal before t and will for computational proposes denote it as $t - 1$. However, there are no time increments between $t - 1$ and t, hence no interest payments are needed to be paid between those two points.

Households are presented with an annual opportunity to exercise the carry-over agreement while relocating. By forgoing this option, they are required to repay the entire loan at the end of the term without incurring any penalty, resulting in a remaining principal of zero. On the other hand, should they opt to exercise this option, they will repay the loan periodically. Additionally, households can partially prepay their mortgage using a value of $PR_j(t)$, computed using the partial prepayment model detailed in Section 4.5. The outstanding balance at the end of the term is calculated by subtracting both periodic repayments and partial prepayments from the previous outstanding balance. However, partial prepayment may result in a faster repayment of the principal than originally intended. Hence, a boundary must be included when calculating future payments to

Figure 2: The plot illustrates the trajectories of the outstanding principal for loan j from time t to time t+1 is presented. The households relocate with probability p_m. Hence, they must select between utilizing the carry-over option. Whereas with a probability of $1 - p_m$, the household remains in their current residence and repays both the YPP^m and the PR^m.



prevent the repayment of more than the initial loan.

$$YPP_j^m(t) = \begin{cases} \min\left(YPP_j(t), O_j(t-1)\right) & \text{If } t \in \{0, \dots, T_0 - 1\}, \\ O_j(t-1) & \text{If } t = T_0. \end{cases} \tag{8}$$

When the lower bound $O_j(t-1)$ is reached, it means that the predefined Yearly Principal Payment is larger than the outstanding balance. Therefore, the final principal payment will be the $YPP_j^m(t) = O_j(t-1)$. Further, when the $t = T_0$, it means that the fixation period ends, and therefore, we assume that the borrower prepays the whole loan. The same reasoning applies to the prepayment. To avoid paying more than expected, we need to set a lower bound for the prepayment:

$$PR_j^m(t) = \min\left(O_j(t-1) - YPP_j^m(t), PR_j(t)\right), \tag{9}$$

where the first term computes the outstanding balance after the principal repayment, and the second term computes the prepayment term. Hence, by combining these together, we get the following equation for the lasting principal calculation:

$$O_j(t) = O_j(t-1) - YPP_j^m(t) - PR_j^m(t). \tag{10}$$

Now we can calculate $V_j^n(t, T_0, T_1)$ which equals the PV of the loan at time t, with the fixation period ending at time T_0 and end of the loan at time T_1, without assuming the possibility of moving:

$$V_j^n(t, T_0, T_1) = \underbrace{\sum_{i=t}^{T_0} \frac{YPP_j^m(i) + PR_j^m(i)}{(1 + y^m(t,i))^{i-t}}}_{\text{Principal Payment}} + \underbrace{\sum_{i=t}^{T_0} \frac{O_j(i) * r_j^f(t, T_0)}{(1 + y^m(t,i))^{i-t}}}_{\text{Interest}} \tag{11}$$

To add the possibility of moving, we need to add the p_m, which equals the probability of moving. Figure 2 depicts the lasting principal flow. Combining the Eq. (8, 9, 10, 11), we can calculate the value of the loan with the possibility of moving:

$$\begin{aligned} V_j(t, T_0, T_1) = &\sum_{i=t}^{T_0} \frac{(1-p_m)^{i-t} * p_m * \mathbb{I}_{\{V_j^n(i,T_0,T_1), O_j(i)\}}}{(1 + y^m(t,i))^{i-t}} \\ &+ \sum_{i=t}^{T_0} \frac{(1-p_m)^{i-t+1} * \left[YPP_j^m(i) + PR_j^m(i)\right]}{(1 + y^m(t,i))^{i-t}} \\ &+ \sum_{i=t}^{T_0} \frac{(1-p_m)^{i-t+1} * O_j(i) * r_j^f(t, T_0)}{(1 + y^m(t,i+1))^{i-t}}. \end{aligned} \tag{12}$$

The first part of the equation equals the valuation when the households move. The $\mathbb{I}_{\{V_j^n(i,T_0,T_1), O_j(i)\}}$ equals $V_j^n(i, T_0, T_1)$ if the carry-over option is used, and $O_j(i)$ otherwise. When using this carry-over agreement, we assumed that it was not possible to buy again a new house, and therefore we discount at this point the value of the loan computed in Eq. (11). The second part of the equation is the principal payments in case there is no movement, and the last part is the interest payment.

Note that all the equations from this section can be applied to both the annuity mortgage and interest-only repayment method. Although it looks to be more suitable for annuity mortgages, setting the $YPP_j(t)$ equation to the following is adapting the model for the interest-only method:

$$YPP_j^m(t) = \begin{cases} 0 & \text{If } t \in \{0, \ldots, T_0 - 1\}, \\ O_j(t-1) & \text{If } t = T_0. \end{cases} \tag{13}$$

The Equations (9, 10, 11, 12) can stay the same for both repayment methods.

4.4 Carry-Over Agreement

By combining the loan valuation methodology outlined in Section 4.3, the partial prepayment dynamics described in Section 4.5, and the market mortgage rates computed in Section 4.6, we can determine the value of $V_j(0, T_0, T_1)$ for each loan j. It is important to note that this value is highly sensitive to changes in the interest rates, as both the discount factor and the partial prepayment dynamics are dependent on them. To account for this sensitivity, we simulate the mortgage rates 100 times in order to compute the expected value of $V_j(0, T_0, T_1)$. On top of that, in order to value the carry-over option for each loan at each point in time, we need to simulate an additional 100 mortgage rate paths each year, which increases the total number of simulations to 10 (years) * 100 (initial mortgage rates) * 100 (carry-over rates). This enables us to capture the impact of different mortgage rate scenarios on the carry-over option value. Once we have performed these simulations, we can create different loan scenarios and price each loan in each simulated market situation.

We account for the relative differences between L_1 and L_2 in the loan scenarios, whereby we only use a single proportion parameter ι instead of two parameters L_1 and L_2. We fix L_1 and set $L_2 = \iota L_1$, and L_3 will be the sum of the two. The fixed rates $r_j^f(t, T_0)$ have two parameters: the fixed rate itself and the fixation period $T_0 - t$.

When a household moves, the new fixed rate offered by NN is assumed to be the current mortgage rate, $r_2^f(t, \hat{T}_0) = y^m(t, \hat{T}_0)$, where $\hat{T}_0$ is from Eq. (2). When $r_2^f(t, \hat{T}_0) < r_1^f(t, T_0)$, the borrower will never apply the carry-over as he will move to the additional loan rate. Furthermore, $r_3^f(t, \hat{T}_0) = r_2^f(t, \hat{T}_0) + \alpha$, where α represents the competitiveness spread parameter of NN. Afterward, it will be mentioned as the competition spreads. By adjusting the value of α, we can simulate different competitive scenarios, resulting in different mortgage rates of the competition loan. Then, to see whether the carry-over option is valuable at time t, we calculate $V_1^n(t, T_0, T_1)$, $V_2^n(t, \hat{T}_0, T_1)$, and $V_3^n(t, \hat{T}_0, T_1)$ like in Equation (11) for all the simulated future mortgage rates after time t. Then, if the inequality of Equation (1) holds, the carry-over option is valuable.

By varying the aforementioned parameters, we generate various scenarios to simulate the behavior of households with regard to the carry-over option. Subsequently, we can determine the conditional probability of exercising the carry-over option during the fixation period given the spread and the fixation period, denoted by $\mathbb{P}(\text{carry-over}|\alpha, T_0 - t)$.

4.5 Partial Prepayment Model

Partially prepaying on a loan means that you are paying some additional principal on top of the planned $YPP_j(t)$. Consequently, the loan will be repaid faster than expected, and the household will avoid paying further interest on the prepaid principal. For the lender, it means that they lose some interest payments. The Dutch mortgages have a specific characteristic in their prepayment penalty, namely, households are able to prepay up to 10% of the original principal at each payment period without penalty. After that, you will need to pay a penalty equal to the interest foregone, e.i. $r_j^f(t, T_0) - y^m(t, t+i)$, with a small i which represents the short term interest rates. However, prepaying is only advantageous when the mortgage market rates are lower than the fixed rates. This means that your money is better invested in repaying your mortgages.

In the Figures (1c-1d), we can see the partial prepayment rates of the mortgage holders with their specific spreads between their fixed rates and the market rates of their loans. In order to model the partial prepayment rates of a given set of loans, denoted by $p(x_i)$ with $i \in \{1, \ldots, N\}$, where N is the number of loan observations. We propose a regression approach based on the sigmoid function. Specifically, we express $p(x_i)$ as a function of the spread x_i for loan i using the following sigmoid regression model:

$$p(x_i) = a + \frac{b}{1 + e^{-\gamma(x_i - c)}} + \epsilon_i, \tag{14}$$

with a being the convergence in $\lim_{x_i \to -\infty}$, b determines the scale of the transition, therefore convergence when $\lim_{x_i \to +\infty}$ is $a + b$, γ determines the curves smoothness of transition, c determines the threshold or the midpoint of the transition, and the ϵ_i is the error term for loan i's prepayment estimation. We opt for the sigmoid regression over a linear regression model because it allows for low partial prepayment rates when the spreads are low or negative while accommodating large partial prepayment rates when the spreads are high.

4.6 Market Mortgage Rates

This section will begin by explaining the one-factor Hull-White model that will be utilized for simulating the short interest rates. Subsequently, the methodology for calibrating the parameters of this Hull-White model will be introduced. Finally, the approach for transforming the risk-free interest rates to the market mortgage rates will be presented.

4.6.1 Hull-White Model

We will use the Hull-White model to simulate the short interest rates, and then add a constant to incorporate the additional mortgage lending risk. Hull and White (1994) proposes a one-factor short-rate process that evolves under the risk-neutral measure according to the following:

$$dr(t) = [\theta(t) - ar(t)]\,dt + \sigma dW(t), \tag{15}$$

with $dr(t)$ being the change in the interest rates at time t and where the short rates converge to $\frac{\theta(t)}{a}$ with speed $a \in \mathbb{R}^+$ and volatility $\sigma \in \mathbb{R}^+$. Further, $dW(t)$ is a one-dimensional Brownian motion, and $\theta(t)$ is the long-term mean which equals to:

$$\theta(t) = \frac{\delta f^M(0,t)}{\delta T} + af^M(0,t) + \frac{\sigma^2}{2a}\left(1 - e^{-2at}\right), \tag{16}$$

where $\frac{\delta f^M}{\delta T}$ is the partial derivative of $f^M(0,t)$, which is the market instantaneous forward rate is at time 0 for the maturity t, and is equal to:

$$f^M(t,\tau) = -\frac{\delta\tau y(t,\tau)}{\delta\tau}. \tag{17}$$

This research will use the swap rates for the $y(t,\tau)$. We will linearly interpolate the discrete swap rates to get a continuous curve. Then, we can approximate the $f^M(0,t)$ by discretizing the derivative to:

$$f^M(t,\tau) = -\frac{(\tau+\delta)y(t,\tau+\delta) - y(t,\tau)}{\delta}, \tag{18}$$

for small δ. We proceed the same way to compute the $\frac{\delta f^M}{\delta\tau}$. The only two variables in this model are the convergence rate a, and short-rate volatility σ. We calibrate the model to market prices of interest rate derivatives to get these variables.

4.6.2 Calibration

To calibrate the model, we will employ Swaptions, which provide the option to exchange fixed interest rates for floating rates, and vice versa. These financial instruments are typically priced using their implied volatilities. As demonstrated in this section, we will utilize the closed-form Black-Scholes formula to align the theoretical prices with the market prices. By calibrating the Hull-White model using Swaptions, we can perfectly match the current yield curve.

According to Hull and White (1990), the price at time t of a pure discount bond that is paying 1 at time T equals the following:

$$P(t,T) = A(t,T)e^{-B(t,T)r(t)}, \tag{19}$$

where,

$$\begin{aligned} A(t,T) &= \frac{P^M(0,T)}{P^M(0,t)}\exp\left\{B(t,T)f^M(0,t) - \frac{\sigma^2}{4a}\left(1-e^{-2a(T-t)}\right)B(t,T)^2\right\}, \\ B(t,T) &= \frac{1}{a}\left[1-e^{-a(T-t)}\right], \end{aligned} \tag{20}$$

Where the $P^M(0,T)$ equals the market discount rates at time T. Using this, it can be shown that the zero coupon bond call option price at time t with strike X, maturity T, and written on a pure discount bond maturing at time S equals:

$$\begin{aligned} \mathbf{ZBP}(t,T,S,X) &= XP(t,T)\Phi(-h+\sigma_p) - P(t,S)\Phi(-h) \\ \sigma_p &= \sigma\sqrt{\frac{1-e^{-2a(T-t)}}{2a}}B(T,S) \\ h &= \frac{1}{\sigma_p}\ln\frac{P(t,S)}{P(t,T)X} + \frac{\sigma_p}{2} \end{aligned} \tag{21}$$

By utilizing Jamshidian's (1989) decomposition, it is possible to price European swaptions analytically. The swaption price at a given time t can be expressed as $\mathbf{PS}(t,T,\mathcal{T},N,X)$, where X represents the strike rate, T denotes the option maturity, and N represents the nominal value. This financial instrument provides the holder with the right to enter the interest rate swap at time $t_0 = T$, with $n-1$ payments being made at times $\mathcal{T} = t_1, \ldots, t_n$, wherein the holder pays the fixed rate X and receives the short-rate in exchange. The Swaption price can be determined using the following equation:

$$\mathbf{PS}(t,T,\mathcal{T},N,X) = N\sum_{i=1}^{n} c_i\mathbf{ZBP}(t,T,t_i,X_i). \tag{22}$$

with $c_i = \frac{t_i - t_{i-1}}{360}X$ for $i \in \{1,\ldots,n-1\}$ and $c_n = 1 + \frac{t_i - t_{i-1}}{360}X$. Further, the X_i is defined as:

$$X_i := A(T, t_i)e^{-B(T,t_i)r^*} \tag{23}$$

With the r^* as the value of the spot rate at time T satisfying:

$$\sum_{i=1}^{n} c_i A(T, t_i)e^{-B(T,t_i)r^*} = 1 \tag{24}$$

The calibration process involves minimizing the difference between the market value of at-the-money (ATM) swaptions at a given time t with expiration T_i, denoted as $\mathbf{PS}(t, T_i, \mathcal{T}, N, X)^M$, and the theoretical value obtained using Equation (22). This is achieved by minimizing the Root Mean Square Percentage Error (RMSPE), as expressed in the following equation:

$$\min_{a,\sigma} \sqrt{\sum_{i=1}^{N} \left(\frac{\mathbf{PS}(t, T_i, \mathcal{T}, N, X) - \mathbf{PS}(t, T_i, \mathcal{T}, N, X)^M}{\mathbf{PS}(t, T_i, \mathcal{T}, N, X)^M} \right)^2} \tag{25}$$

From this minimization problem, we can get the optimal estimators $\hat{a}$ and $\hat{\sigma}$ and use them for further interest rate simulation.

4.6.3 Market Spot Rates

Having obtained the values of $\hat{a}$ and $\hat{\sigma}$ through the calibration process outlined in Section 4.6.2, it is possible to simulate future paths of short interest rates. Subsequently, as previously mentioned, a constant ζ is added to these rates to arrive at the corresponding mortgage rates:

$$r^m(t) = r(t) + \zeta \tag{26}$$

With the modified short rates in hand, it is possible to reconstruct the term structures for $t \geq 0$. Denoting $\tau_i = T_i - t$ where $i \in 1, \ldots, n$ represents the different time-to-maturities at time t, the following spot rates can be obtained using Eq. (20):

$$y^m(t, t + \tau_i) = -\frac{\ln P(t, t + \tau_i)}{\tau_i} = a(t, \tau_i) + b(\tau_i)r^m(t), \tag{27}$$

with,

$$
\begin{aligned}
a(t,\tau_i) &= f^M(t,t+\tau_i) - b(\tau_i) f^M(0,t) + b(\tau_i)^2 \frac{\tau_i \sigma^2}{4a}\left(1 - e^{-2a(t-c)}\right), \\
b(\tau_i) &= \frac{1 - e^{-a\tau_i}}{a\tau_i}
\end{aligned}
\tag{28}
$$

By rewriting the equation, it becomes evident that the spot rates can be increased by $b(\tau_i) * \zeta$. It is important to note that the additional mortgage risk incorporated in the spot rates decreases over time, as $b(\tau_i) \to 1$ when $\tau_i \to 0$. Thus, $y^m(t, t+\tau_i)$ converges to $r(t) + \zeta$. However, as τ_i increases, $b(\tau_i) \to 0$, hence $b(\tau_i) * \zeta \to 0$ and therefore, $y^m(t, t+\tau_i)$ converges to $y(t, t+\tau_i)$. The model-implied spot rates can be gathered in an n-dimensional vector, which can be reformulated as:

$$
\mathbf{y^m}(t) = \mathbf{a}(t) + \mathbf{b} r^m(t) \tag{29}
$$

with,

$$
\begin{aligned}
\mathbf{y^m}(t) &= \left(y^m(t,t+\tau_1), \ldots, y^m(t,t+\tau_n)\right)^T, \\
\mathbf{a}(t) &= \left(a(t,\tau_1), \ldots, a(t,\tau_n)\right)^T, \\
\mathbf{b} &= \left(b(\tau_1), \ldots, b(\tau_n)\right)^T.
\end{aligned}
\tag{30}
$$

Once we have these spot rates, we can discount each cashflow from time $t+i$ to time t by multiplying it with $(1 + y^m(t,t+i))^{-i}$, with $t \geq 0$ and $i \in \{0, \ldots, T_0\}$ with T_0 being the end of the fixation period.

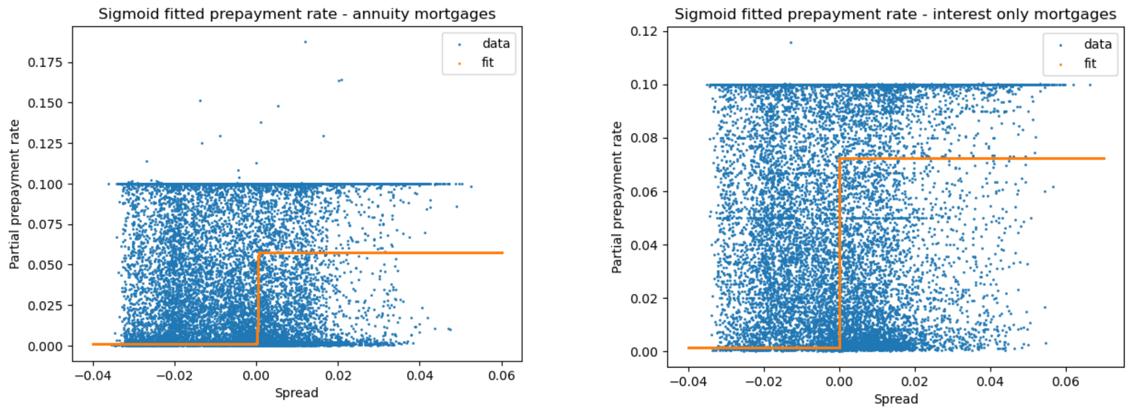
5 Results

5.1 Prepayment model

By utilizing the sigmoid regression method described in Section 4.5 for both annuity and interest-only mortgages separately, we could ascertain the coefficients listed in Table 3. The fitted regressions are shown in Figures 3. It can be observed that both regressions γ exhibit an extreme transition coefficient. With a spread of 0.0398% and 0.0101%, the partial prepayment rates for annuity and interest-only mortgages, respectively, jump from 0.0013 and 0.0015 to 0.0574 and 0.0722. Interest-only mortgages tend to experience higher

Figure 3: Fitted sigmoid curve for the partial prepayment data of annuity and interest-only mortgages.

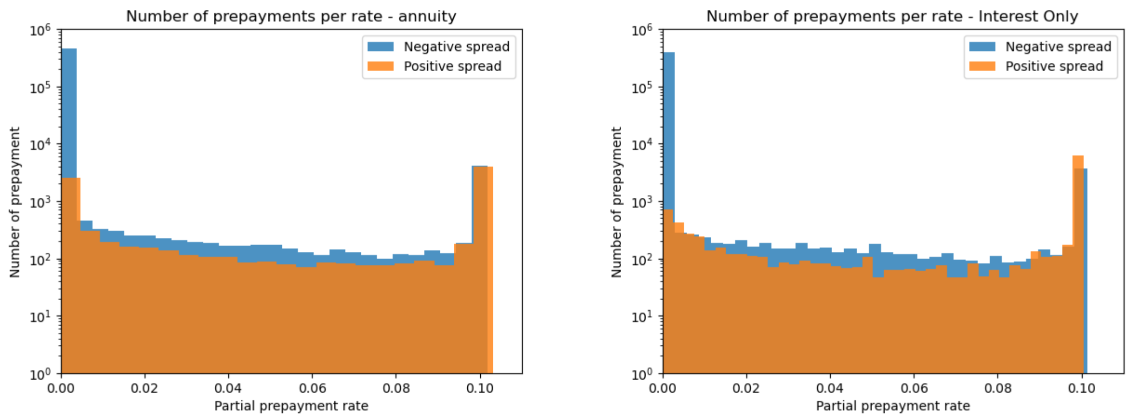
(a) Annuity fitted sigmoid curve (b) Interest-only fitted sigmoid curve



prepayment rates as they do not have a scheduled contractual principal repayment. The Figures 4 illustrate the number of observations per prepayment rates with positive and negative spreads. This can give some more intuition on the extremeness of the transition parameters γ.

Figure 4: Histogram plots of the number of partial prepayment observations per prepayment rates with the data-set split in two: prepayments with positive and negative spreads. Note that the number of observations per prepayment rate is on a log scale.

(a) Annuity histogram plots (b) Interest-only histogram plots



As previously mentioned, prepayment can result in the borrower repaying the loan faster than anticipated, resulting in the lender losing out on interest payments. Figures 5 illustrate the differences in repayments cashflows between plain vanilla repayment loans and their counterparts where prepayment is incorporated. Subfigure 5e displays a simulated market mortgage rate paths using the Hull-White model, with the calibrated results where

Table 3: Coefficients of the sigmoid regression, with significance level of 95%, 97.5%, and 99%, as *, **, and *** respectively.

	Annuity	Interest-only
a	0.0013***	0.0014***
b	0.0561***	0.0707***
c	0.0004	0.0001
γ	22697	123632

$\hat{a} = 0.03356$, $\hat{\sigma} = 0.01071$, and $\zeta = 1.5\%$. The red dots represent the rates used to calculate the market spread. When the dots are below the fixed rate of 4%, annuity, and interest-only mortgages prepay. Note that for these loans, we have $T_0 = T_1 = 10$ and an original principal of 100,000. The Yearly Principal Payment curves for both annuities follow the same trend until the principal is fully repaid for the loan with prepayment at $t = 8$. Although there are several partial prepayment opportunities, the interest-only mortgage does have a large principal amount to pay at the end of the fixation period.

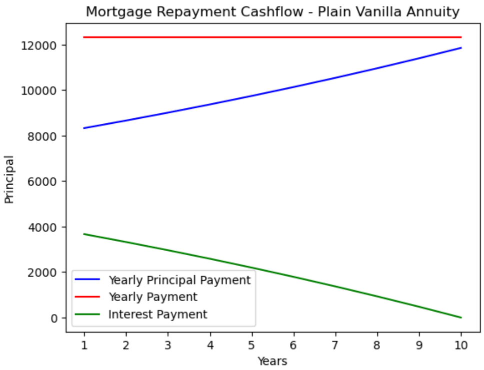
Figure 6 thoroughly explains the impact of the prepayment model on the loan valuation without the carry-over inclusion. We calculated the plain vanilla loan value using Eq. (7). We further simulated 1000 future rates to value the loans of Eq.(11), to get $\mathbb{E}\left[V_j^n(0, T_0, T_0)\right]$, as the prepayments are sensitive to the short rates. At roughly $r^f(0, T_0) = 4\%$, all the loans converge to the original outstanding principal value. In this case, the loan is priced at par. This convergence is due because, under the current market situation, the $y^m(0, T_0) = 4.2\%$, as shown in Figure 7. With lower rates, prices of loans with and without prepayments converge to the same prices for both mortgage types. However, when increasing the fixed rates, we can see that the loans with prepayment have significantly lower values. Further, annuity mortgages are more valuable than interest-only mortgages when the fixed rates are low and inversely when the fixed rates are high.

Borrowers want to minimize the value of their mortgages. Hence, the partial prepayment dynamics should ensure that the values of the loans are minimized over the different fixed rates. By comparing the plain vanilla repayment methods, we can see that annuity and interest-only mortgages have lower or equal loan values over the different fixed rates. This means that prepaying following the model presented in Section 4.5 does incorporate the rational thinking of the borrowers.

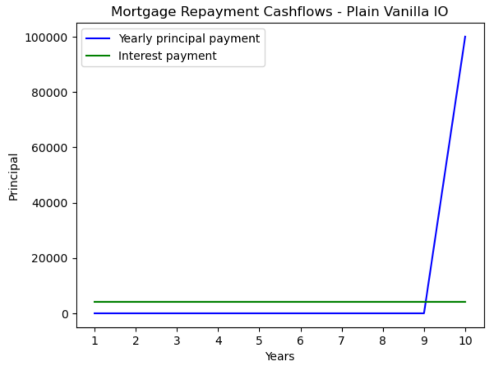
Now that we have analyzed the impact of the partial prepayment model on the plain vanilla valuation model, we can implement the combination of this prepayment model

Figure 5: Plot of the different repayment cashflows of Annuity and Interest-Only mortgage types, with and without prepayment. Note that the loans have a repayment time and a fixation period of 10 years, an original principal of 100,000, and a fixed rate of 4%.

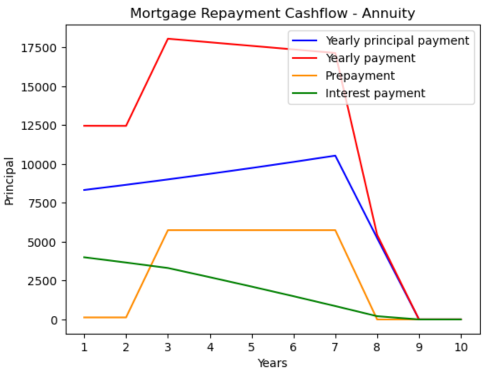
(a) Annuity - Plain Vanilla



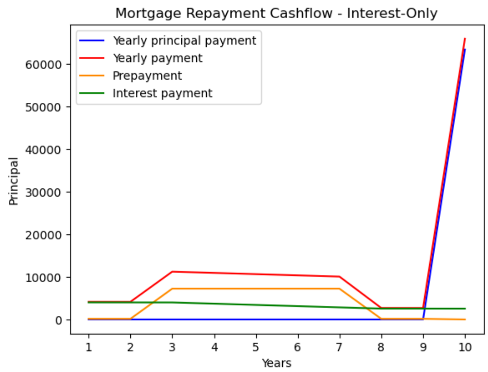
(b) Interest Only - Plain Vanilla



(c) Annuity - With Prepayment



(d) Interest Only - With Prepayment



(e) Path Simulation

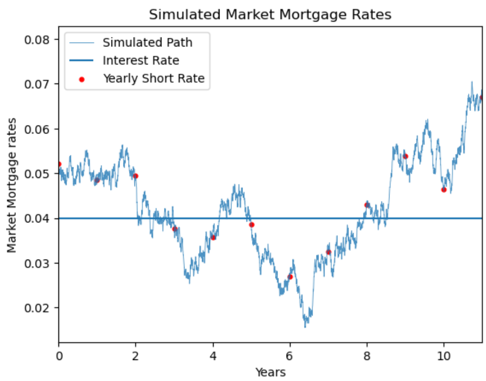


Figure 6: Plot of the average value of the interest-only and annuity mortgage loans with different fixed rates.

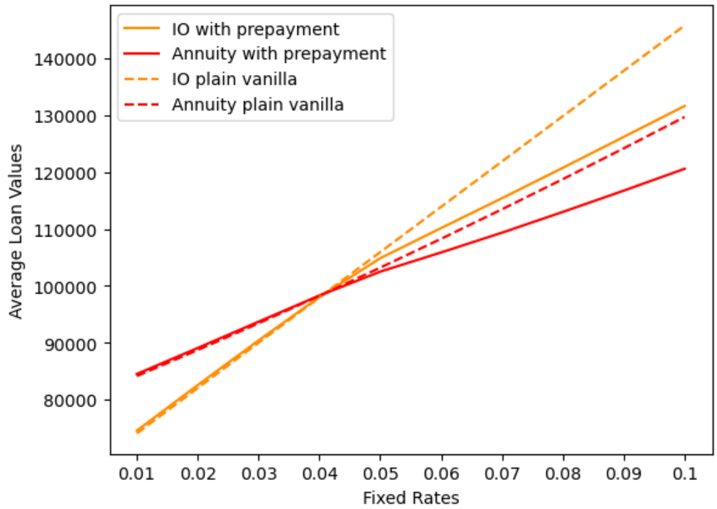
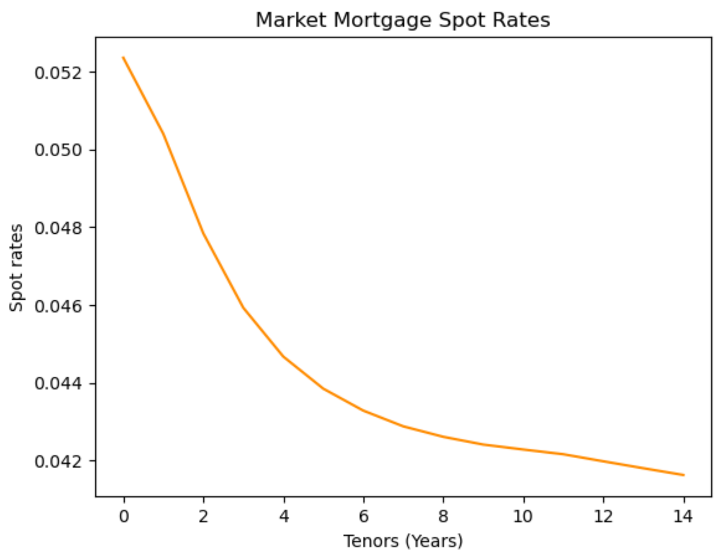


Figure 7: Plot of the market mortgage spot rates.



with the possibility of moving and the use of the carry-over agreement.

5.2 Mortgage Valuation Results

In order to illustrate the practical implementation of the valuation framework as expounded in Section 4, we conducted an exercise to implement it. Specifically, we employed the model to evaluate loans characterized by the original principal of 100, fixed interest rates of $r_1^f(t, T_0) = \{0.5\%, 3\%, 6\%\}$, fixation periods $T_0 - t = \{0, \ldots, 10\}$ years, $\hat{T}_0 = 10$, $T_1 = 30$, and competition spreads that varied between $\alpha \in [-0.02, 0.02]$. It is important to reiterate that the $r_2^f(t, \hat{T}_0) = y^m(t, \hat{T}_0)$, which corresponds to the simulated market mortgage rates at time t. Additionally, we considered various additional loan proportions relative to the original principal, $\iota = \{0.001, 0.5, 1, 1.5\}$. Finally, we assume that $t = 0$, hence the fixation period can be denoted as T_0. An important point to remember is that all the results depend on the current market situation. The current yield curve calculates the market mortgage rates and discount factors.

For annuity mortgages, the conditional probability of the carry-over use is higher when the competition spreads are positive. This means that the competition loans are more expensive and hence less attractive. Further, looking at Figures 8, it is evident that the utilization of carry-over is also impacted by the fixation periods in scenarios involving small or negligible additional proportions for the $r_1^f(0, T_0) = 0.5\%$ and 3%. When $T_0 = 10$, the proportionate use of the carry-over option is distributed across the various α values. Given the small size of the supplementary loan, the competition spread needs to be significantly different from zero to impose a large change in the use of the carry-over agreement. We can see that the carry-over use probability around the $\alpha = 0$ is approximately 0.5. For $r_1^f(0, T_0) = 0.5\%$ only around the spreads $\alpha = \{-2\%, 2\%\}$ can you see probabilities of carry-over use lower or higher than 0.7 and 0.3, respectively. The spread interval to get to these probabilities is slightly lower on the negative side of the spreads for $r_1^f(0, T_0) = 3\%$, as this fixed rate is already less attractive than the previous rate.

By decreasing T_0, we can see through the conditional probabilities that the competition spreads have an increasing influence on the use of the carry-over. When looking at the negative competition spread side of $r_1^f(0, T_0) = 3\%$, the probabilities tend to go close to zero. However, on the positive side, the probabilities do not go to 1 except for small T_0. This can be explained by the additional restriction that we have set on the loans, where when the $r_2^f(t,\hat{T}_0) < r_1^f(t,T_0)$, the borrower will never apply the carry-over because he will move to the additional loans rate instead of keeping this current loan or the competition loan. This additional restriction also explains the almost nonexistent use of the carry-over when $r_1^f(0,T_0) = 6\%$.

When we have $T_0 = 0$, we can see that the use of the carry-over is solely determined by the spreads regardless of the additional loan proportions for the $r_1^f(0,T_0) = \{0.5\%, 3\%\}$. With this fixation period, the original loan will be repaid instantly. Hence, the fixed rate of this loan does not impact the carry-over use. Therefore, the only variable that can affect the carry-over use calculation is the difference between the additional and competition fixed rates. Additionally, we see that the carry-over is used if the competition spreads are larger or equal to zero. The inclusion of the 0 spread in the carry-over use is caused by the inequality of Eq. (1).

The Figures 10, 11 and 12 display the results with increasing ι. We notice that the use of the carry-over is increasingly sensitive to the change in competition spreads. The explanation for this is that the value of the original loan has a smaller influence on the inequality of Eq. (1). Therefore, the differences in the fixed rates of the additional loan and the competition loan have greater importance in the determination of the carry-over use.

Upon examining the carry-over use of interest-only loans displayed in Figures 20, 21, 22 and 23, we observe that the outcomes are highly similar to those of annuity loans. Therefore, the interpretation of the results remains the same.

Having analyzed the carry-over use, we can combine it with the loan valuation process across the various variables. When $T_0 = 10$ and the ι is low, displayed in Figure 9, we observe that the values of the loans across different $r_1^f(0,T_0)$ exhibit a decreasing linear trend with the increase in spreads. Figure 13c shows more in detail the change in the value of the $r_1^f(0,T_0) = 6\%$ loans. This change in valuation is a result of the carry-over use, as the partial prepayments are the same across the different spreads. We can see that when the carry-over option is used, the values of the loans are lower. As mentioned previously, the borrowers want to minimize their loan value, hence, decreasing the loan value when using the carry-over is rationally coherent. When never applying the carry-over, you have a yearly probability p_m of getting all the principal in return earlier.

When decreasing T_0, the lower fixed rate loans increase in value. For these loans, losing a fixed interest payment is offset by the closer discounting of the final principal payment. However, the inverse reasoning holds for the $r_1^f(0, T_0) = 6\%$, as the loan decreases in value while decreasing the fixation period. All the loan values converge to the original principal, 100. When $T_0 = 0$, the loan is immediately repaid. Hence, neither the different spreads nor the different fixed rates alter the value of the loan. When looking at $T_0 = 1$, we can note that the two smaller fixed rates with positive spreads have a sudden drop in the value of their loans. These sudden drops in the values are caused by the combination of the possible carry-over use and the discounting of the first cash flows.

In all cases, Figures 17, 18 and 19 show that increasing the ι enhances the influence of α on the carry-over use, which itself influences the values of the loans. By increasing the ι, the value of the loans becomes increasingly sensitive to the change in competition spreads around $\alpha = 0$. This highlights the impact of the carry-over agreement on the valuation of the loans.

Once again, when applying this to interest-only loans, we observe that the valuation dynamics are similar to those of annuity loans. However, we notice that the values of interest-only mortgages are consistently lower than those of annuity mortgages, which is due to the absence of principal payments planned throughout the life of the loan. Although, Figure 6 shows that the values of the interest-only mortgages become more expensive than the annuity loans at rates higher than $r^f(0, T_0) = 4\%$.

In addition to the valuation of each loan, this framework provides the lender with the ability to understand and anticipate the borrowers' use of the carry-over agreement. The lender aims to maximize the value of the loans, while the borrower aims to minimize it. For instance, when examining the loan with $r_1^f(0, T_0) = 0.5\%$, the values of the loans are consistently below 100. This indicates that the lender incurs a loss on this loan under the current market conditions. We observe that, in this case, having a non-competitive additional loan can minimize the losses of the lender inversely, when the $r_1^f(0, T_0) = 6\%$ all the values are above par. Although the probability of use of the carry-over is very limited, the values of the loans still differ with a change in the competition spreads. Here again, the loans decrease in value with an increase in the competition spread.

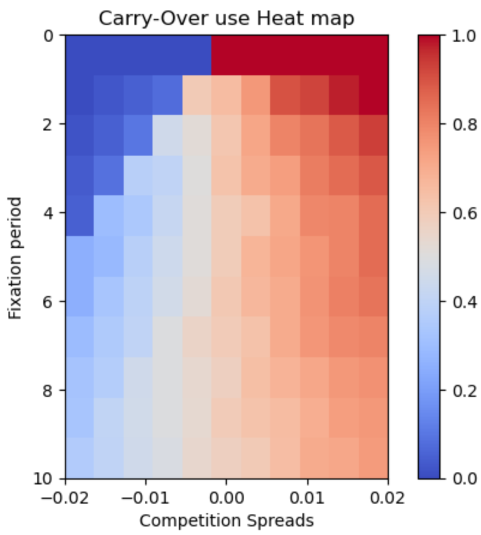
It is noteworthy to mention that in the course of this study, the integration of low rates with wider negative spreads may result in a negative competition rate. However, this aspect has not been factored into the analysis despite its lack of realism.

Over the past few years, interest rates have been low and decreasing, resulting in low fixed rates. Therefore, the carry-over agreement was not frequently used as the spreads were negative. However, in recent times, interest rates have increased significantly, bringing the competition spread to the positive side, and as a result, the carry-over agreement has started to be utilized. The framework captures this real-life dynamic. When examining the low fixed rates of 0.5%, we observe that the carry-over is not frequently used when the spread is negative. However, when the spread is positive, borrowers begin to use the carry-over.

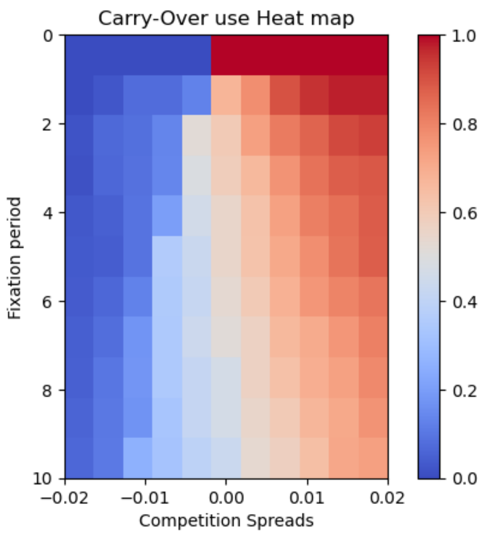
In our case, we used a predefined percentage of moving. By altering this, you can adapt to specific situations. In case of extreme situations, by bringing the $p_m = 0$, the results will be independent of ι and α. Conversely, when you bring the $p_m = 1$, the value of the loan equals the first part of the Equation (12) where it is either equal to the valuation $V_j^n(t, T_0, T_1)$ or $O_j(t)$ depending on whether the carry-over is used.

Figure 8: This figure illustrates the annuity loan heatmap plot of the conditional probability of the carry-over use with a fixed rate of 0.5%, 3% and 6% as a function of the competition spreads and the fixation period, with an additional proportion of 0.001.

(a) Fixed rate of 0.5%.



(b) Fixed rate of 3%.



(c) Fixed rate of 6%.

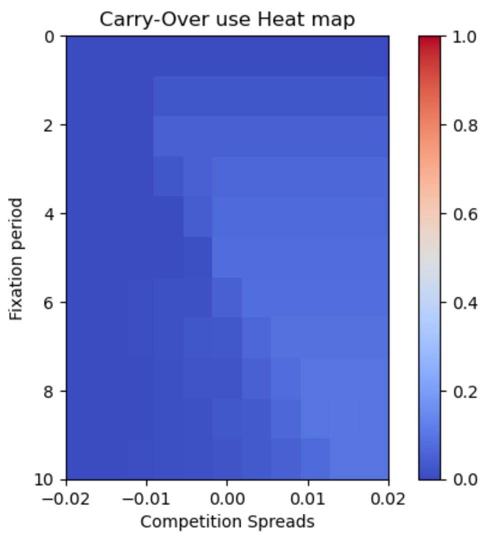
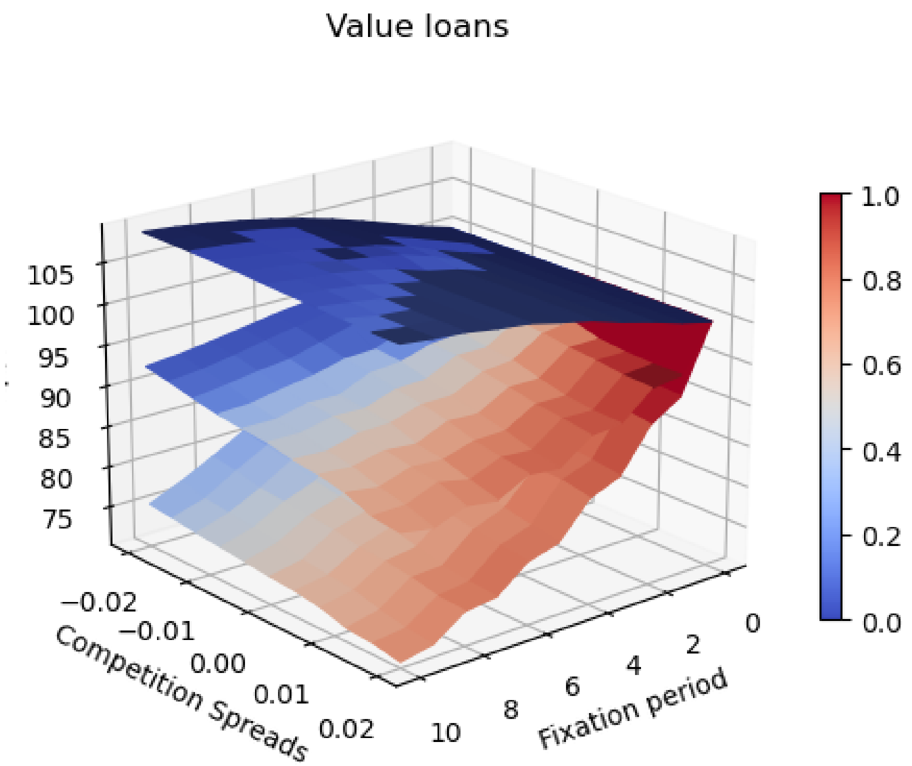


Figure 9: This figure displays the valuation of annuity loans with r_f of 0.5%, 3%, and 6% as a function of lasting fixation periods and competition spreads, with each plot representing the value of loans that are dependent on various additional loans with proportions of 0.001. The planes represent loans with fixed interest rates in descending order of layers, with r_f of 6%, 3%, and 0.5%, respectively. The heatmap plots are used to show the probability of carry-over given the competition spreads and the lasting fixation period.



6 Conclusion

In conclusion, this research presents a framework for valuing loans that includes the possibility of using the carry-over option and making partial prepayments on the loan. The carry-over is calculated by comparing the expected present values of the current loans with the additional loan, and the present value of the competition loan. By employing this framework, it is possible to comprehend the dynamics of the lenders based on variables such as time-to-maturity, lasting fixation period, current fixed rates, fixed rate of the competition, and size of the additional loan. This framework enables more accurate pricing of loans, as it differentiates between partial prepayments and full prepayments in case the carry-over agreement is not utilized. The model was able to capture past market situations.

However, a limitation of this framework is that it is currently limited to specific predefined scenarios. One way to address this limitation is by marginalizing the probabilities of the carry-over use and fitting specific distributions for these probability distributions. This would enable the establishment of a mathematical relationship between the variables, the carry-over use, and the value of the loan. Additionally, this would allow for pricing this specific carry-over agreement option. Further, the framework assumes that the probability of moving is constant over the time of the loan. This might not be very realistic as the households are unlikely to move a year after having bought the house.

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Appendix

Swaption Data

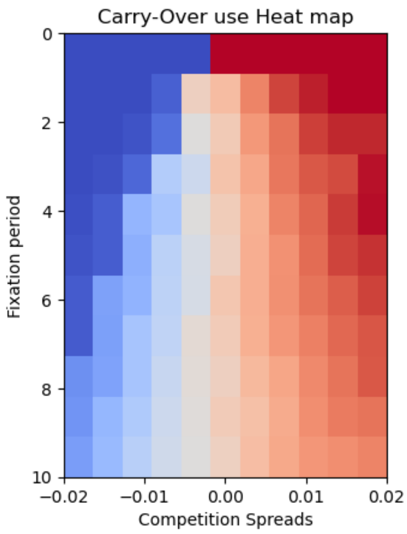
Table 4: Table representing the Swaption market volatilities data for the calibration of the Hull-White model. The first column is the starting period and the first row is the duration of the swaptions. We are only using the swapions where the final end period is smaller than 30 years, as we use a 30-year swap rate curve. Note that these are ATM, co-terminal Euro swaptions, and were collected on the 14th of June 2023 on Bloomberg.

Exp	1	2	3	4	5	7	10	12	15	20	25
1	32.7	36.23	36.99	37.02	36.43	35.37	33.17	32.41	31.78	31.71	32.76
2	38.32	40.02	39.57	38.87	37.9	36.45	34.15	33.21	32.56	32.61	33.61
3	39.94	40.19	39.06	38.05	37.05	35.5	33.35	32.43	32.1	32.31	33.37
4	39.04	38.62	37.39	36.31	35.17	33.7	31.94	31.46	31.09	31.47	32.6
5	36.97	36.57	35.5	34.38	33.38	31.88	30.48	30.47	30.1	30.71	
6	35.27	34.8	33.54	32.48	31.52	30.27	29.5	29.45	29.35	30.07	
7	33.52	32.96	31.68	30.65	29.85	28.83	28.58	28.52	28.7	29.48	
8	31.57	31.21	29.97	29.14	28.6	27.73	27.96	27.85	28.27	29.1	
9	30.04	29.67	28.67	28.18	27.64	27.46	27.52	27.67	27.94	28.85	
10	28.58	28.48	27.9	27.4	26.83	27.34	27.2	27.65	27.75		
12	28.01	28.02	27.3	27.93	28	27.94	28.3	28.47	28.56		
15	32.79	32.72	31.78	30.97	30.04	30.66	30.46	30.56			
20	41.07	41.02	39.64	38.2	36.79	36.52					
25	49.34	48.96	46.99	44.5							

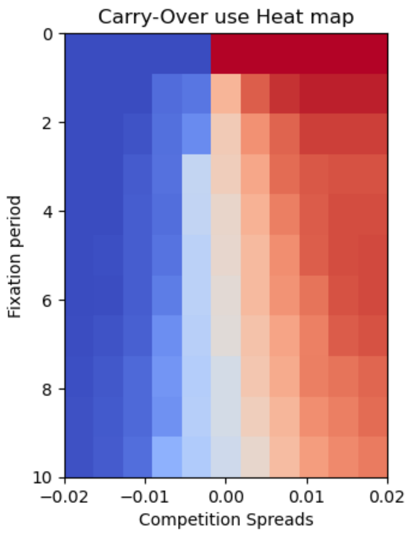
Additional Graphs

Figure 10: This figure illustrates the annuity loan heatmap plot of the conditional probability of the carry-over use with a fixed rate of 0.5%, 3% and 6% as a function of the competition spreads and the fixation period, with an additional proportion of 0.5.

(a) Fixed rate of 0.5%.



(b) Fixed rate of 3%.



(c) Fixed rate of 6%.

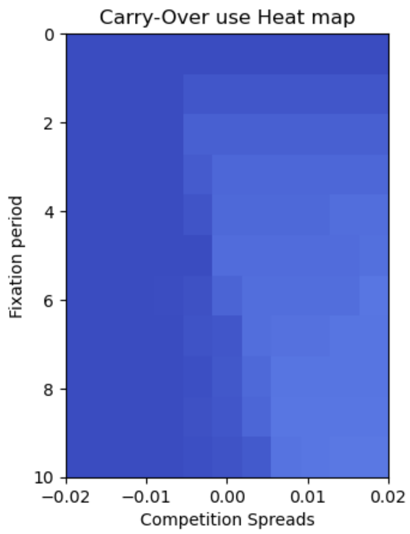
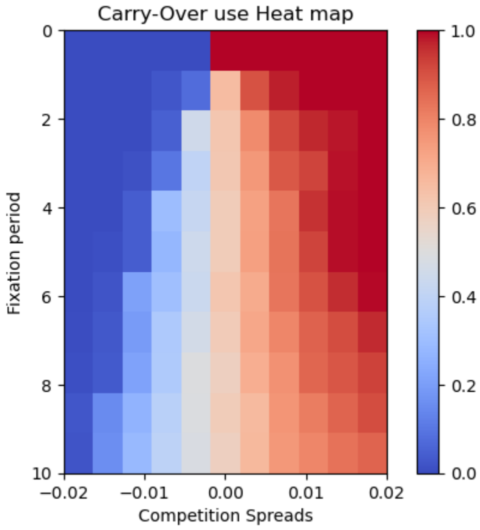
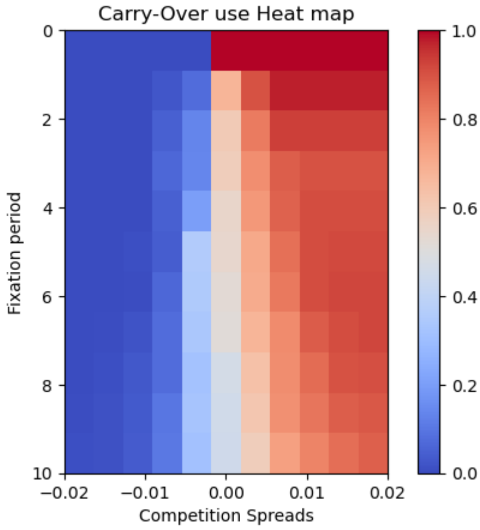


Figure 11: This figure illustrates the annuity loan heatmap plot of the conditional probability of the carry-over use with a fixed rate of 0.5%, 3% and 6% as a function of the competition spreads and the fixation period, with an additional proportion of 1.

(a) Fixed rate of 0.5%.



(b) Fixed rate of 3%.



(c) Fixed rate of 6%.

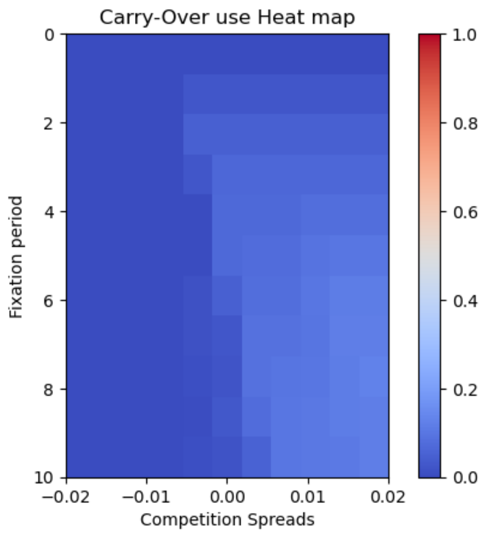
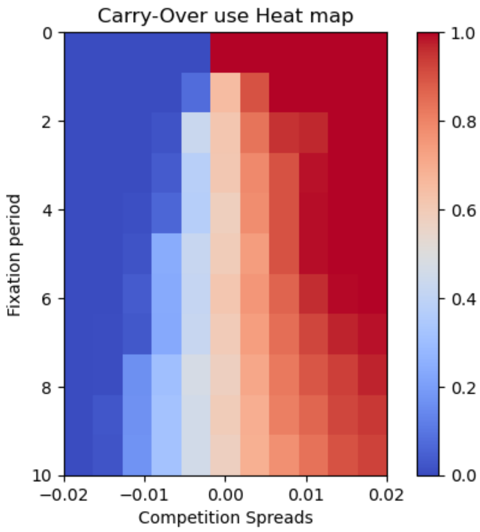
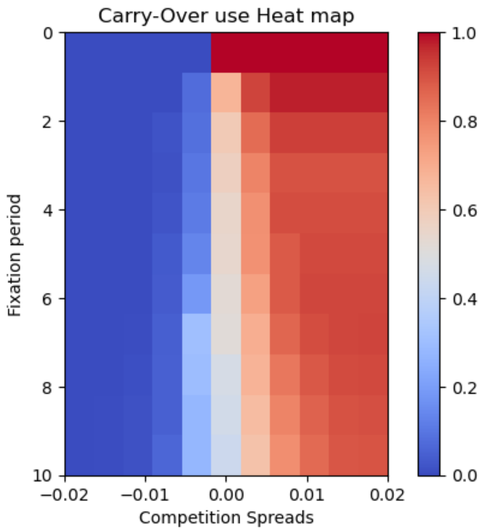


Figure 12: This figure illustrates the annuity loan heatmap plot of the conditional probability of the carry-over use with a fixed rate of 0.5%, 3% and 6% as a function of the competition spreads and the fixation period, with an additional proportion of 1.5.

(a) Fixed rate of 0.5%.



(b) Fixed rate of 3%.



(c) Fixed rate of 6%.

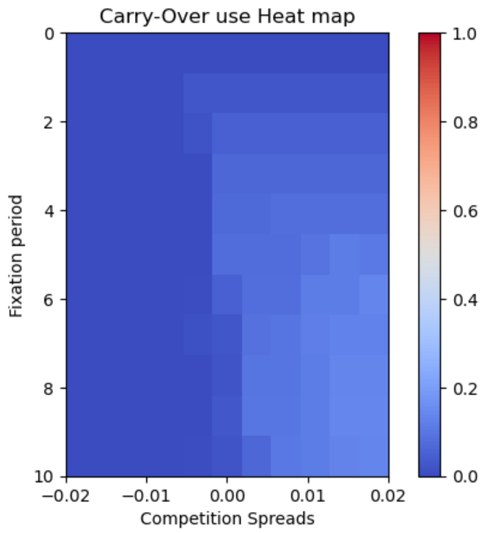
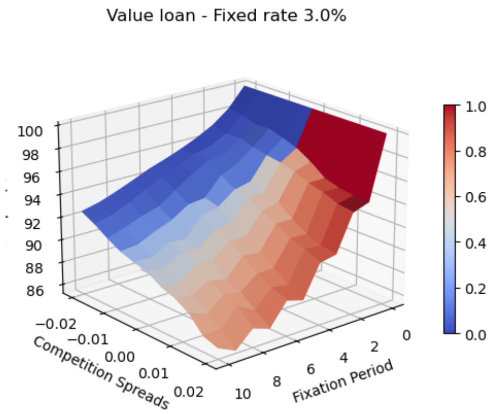


Figure 13: This figure displays the valuation of annuity loans with r_f of 0.5%, 3%, and 6% as a function of lasting fixation periods and competition spreads, with each plot representing the value of loans that are dependent on various additional loans with proportions of 0.001. The heatmap plots are used to show the probability of carry-over given the competition spreads and the lasting fixation period.

(a) Fixed rate of 0.5%.

(b) Fixed rate of 3%.

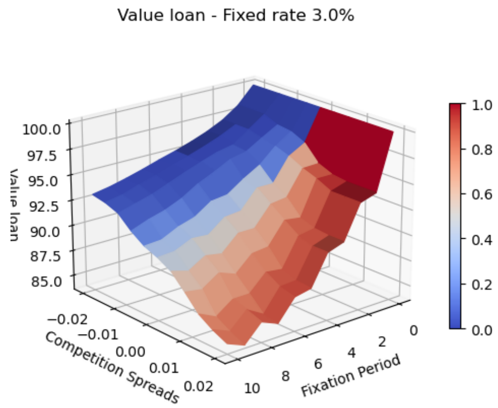


(c) Fixed rate of 6%.

Figure 14: This figure displays the valuation of annuity loans with r_f of 0.5%, 3%, and 6% as a function of lasting fixation periods and competition spreads, with each plot representing the value of loans that are dependent on various additional loans with proportions of 0.5. The heatmap plots are used to show the probability of carry-over given the competition spreads and the lasting fixation period.

(a) Fixed rate of 0.5%.

(b) Fixed rate of 3%.



(c) Fixed rate of 6%.

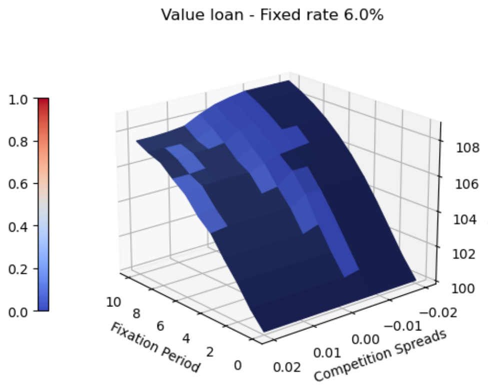
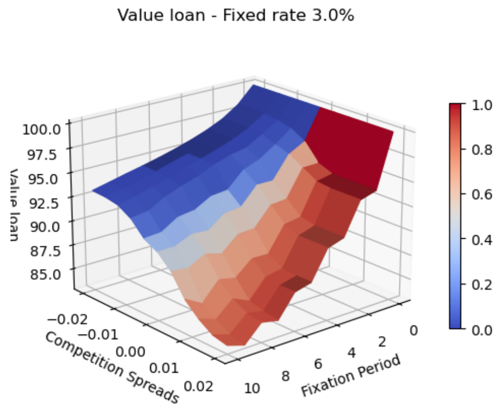


Figure 15: This figure displays the valuation of annuity loans with r_f of 0.5%, 3%, and 6% as a function of lasting fixation periods and competition spreads, with each plot representing the value of loans that are dependent on various additional loans with proportions of 1. The heatmap plots are used to show the probability of carry-over given the competition spreads and the lasting fixation period.

(a) Fixed rate of 0.5%.

(b) Fixed rate of 3%.



(c) Fixed rate of 6%.

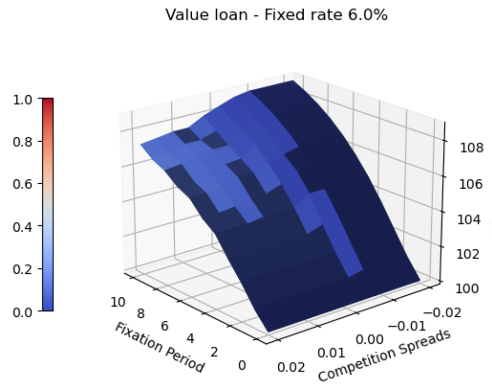


Figure 16: This figure displays the valuation of annuity loans with r_f of 0.5%, 3%, and 6% as a function of lasting fixation periods and competition spreads, with each plot representing the value of loans that are dependent on various additional loans with proportions of 1.5. The heatmap plots are used to show the probability of carry-over given the competition spreads and the lasting fixation period.

(a) Fixed rate of 0.5%.

(b) Fixed rate of 3%.

(c) Fixed rate of 6%.

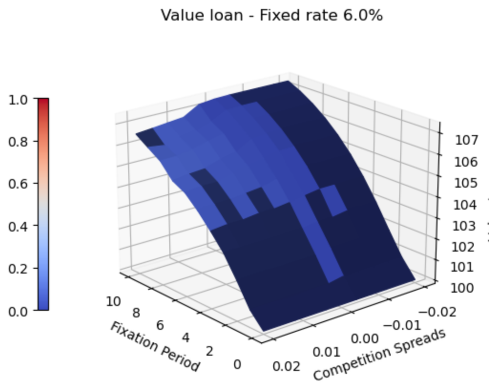


Figure 17: This figure displays the valuation of annuity loans with r_f of 0.5%, 3%, and 6% as a function of lasting fixation periods and competition spreads, with each plot representing the value of loans that are dependent on various additional loans with proportions of 0.5. The planes represent loans with fixed interest rates in descending order of layers, with r_f of 6%, 3%, and 0.5%, respectively. The heatmap plots are used to show the probability of carry-over given the competition spreads and the lasting fixation period.

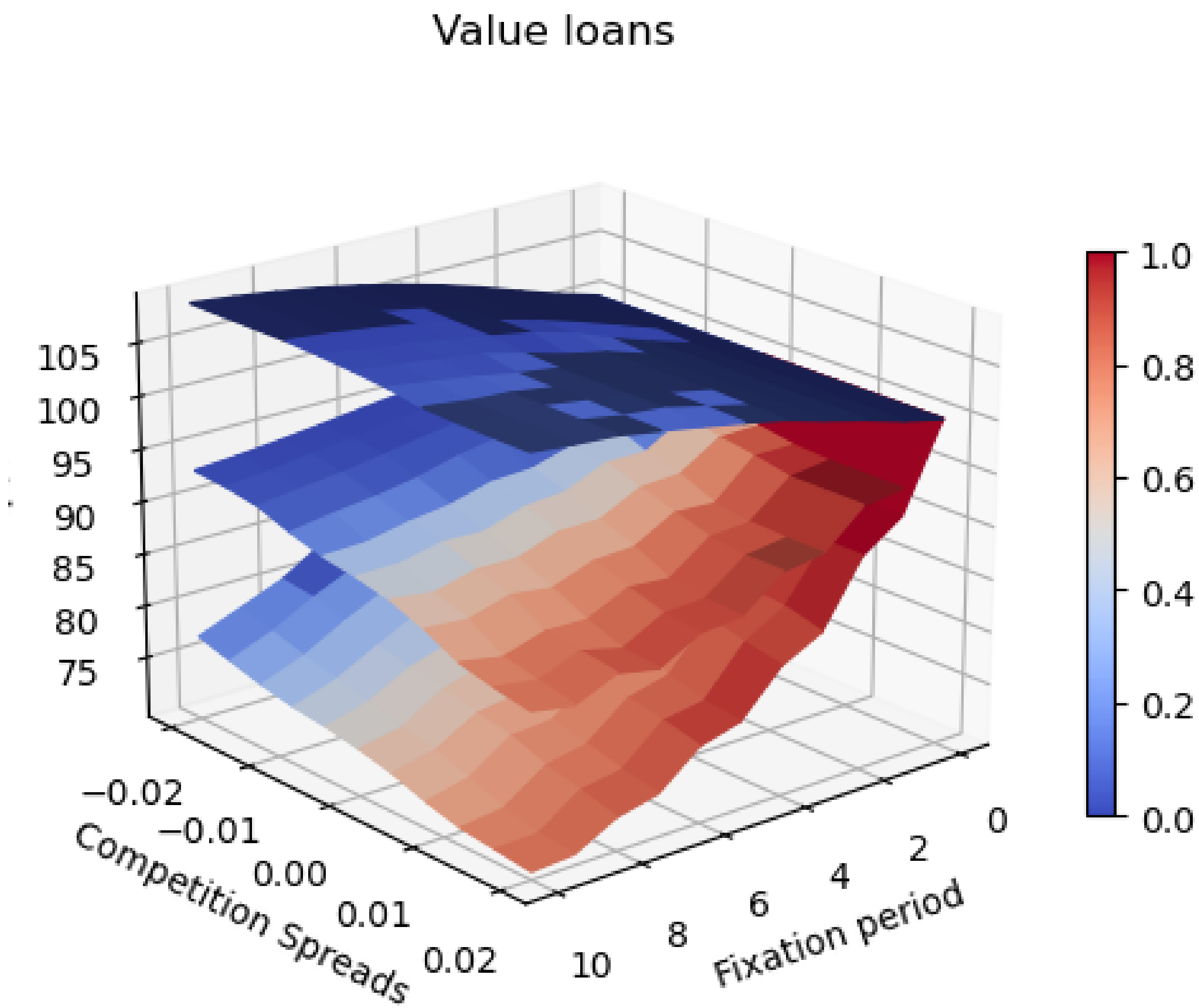


Figure 18: This figure displays the valuation of annuity loans with r_f of 0.5%, 3%, and 6% as a function of lasting fixation periods and competition spreads, with each plot representing the value of loans that are dependent on various additional loans with proportions of 1. The planes represent loans with fixed interest rates in descending order of layers, with r_f of 6%, 3%, and 0.5%, respectively. The heatmap plots are used to show the probability of carry-over given the competition spreads and the lasting fixation period.

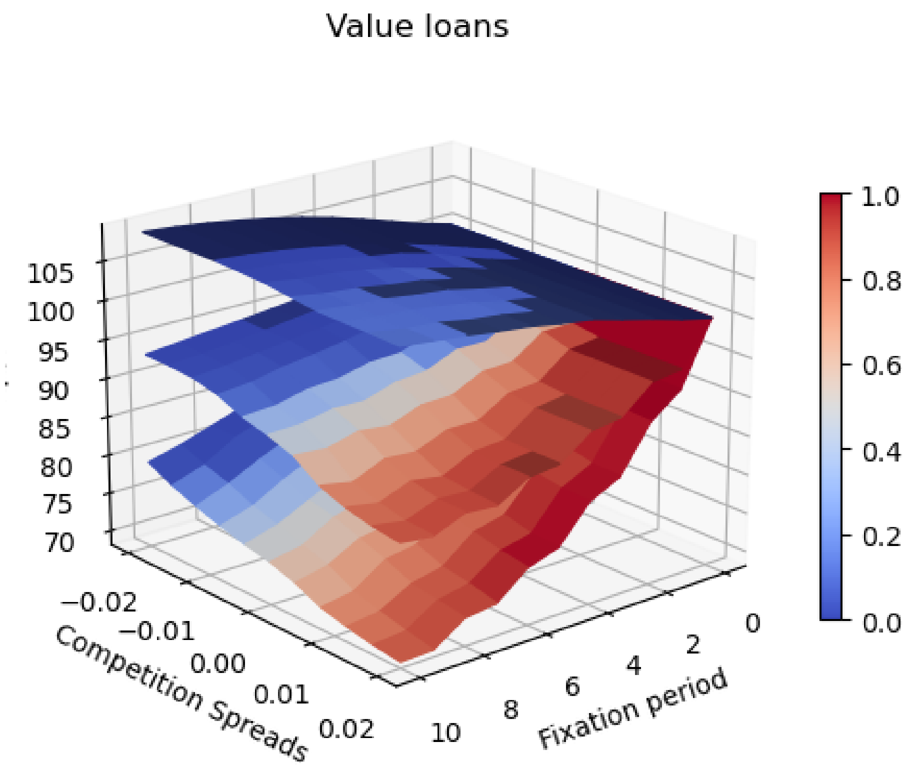


Figure 19: This figure displays the valuation of annuity loans with r_f of 0.5%, 3%, and 6% as a function of lasting fixation periods and competition spreads, with each plot representing the value of loans that are dependent on various additional loans with proportions of 1.5. The planes represent loans with fixed interest rates in descending order of layers, with r_f of 6%, 3%, and 0.5%, respectively. The heatmap plots are used to show the probability of carry-over given the competition spreads and the lasting fixation period.

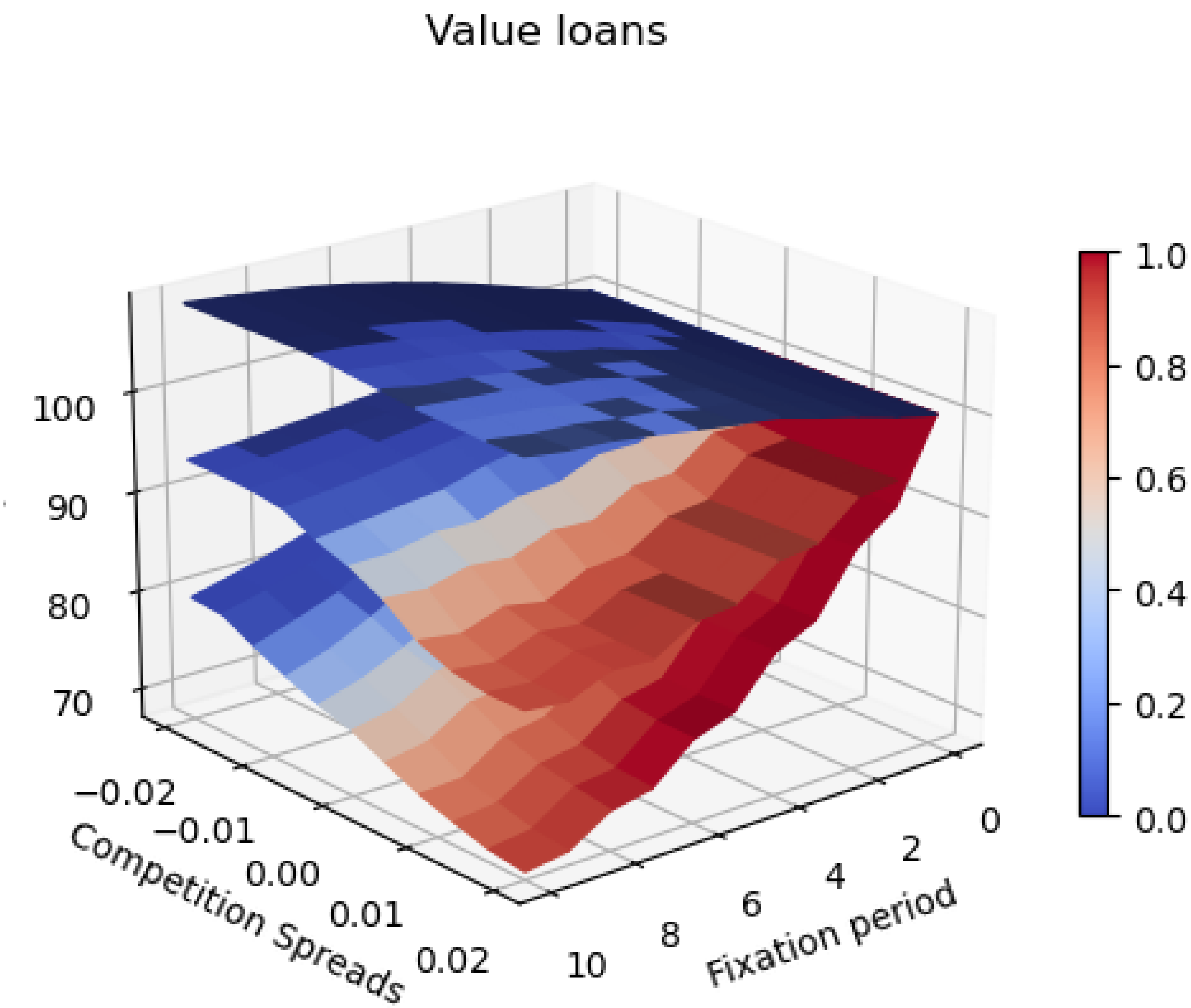
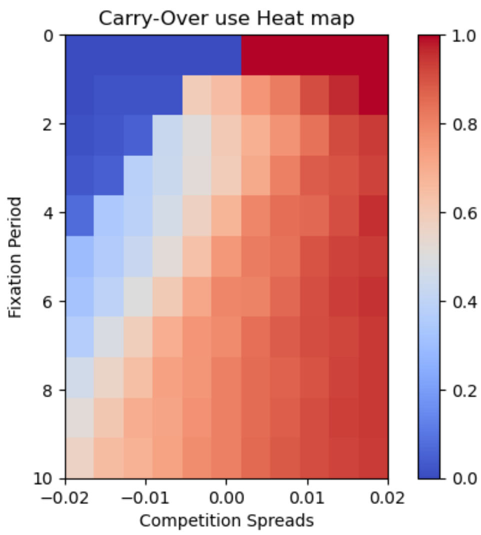
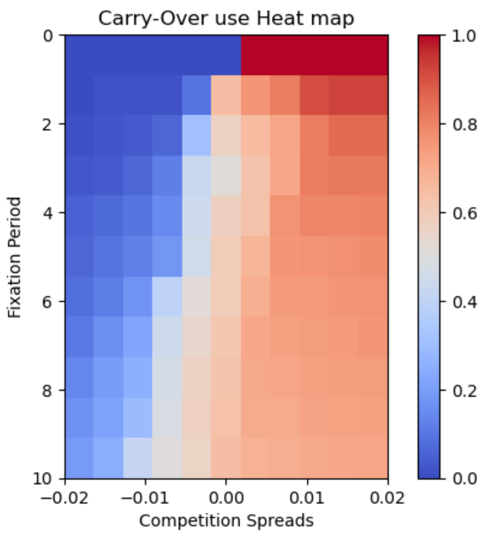


Figure 20: This figure illustrates the interest-only loan heatmap plot of the conditional probability of the carry-over use with a fixed rate of 0.5%, 3% and 6% as a function of the competition spreads and the fixation period, with an additional proportion of 0.001.

(a) Fixed rate of 0.5%.



(b) Fixed rate of 3%.



(c) Fixed rate of 6%.

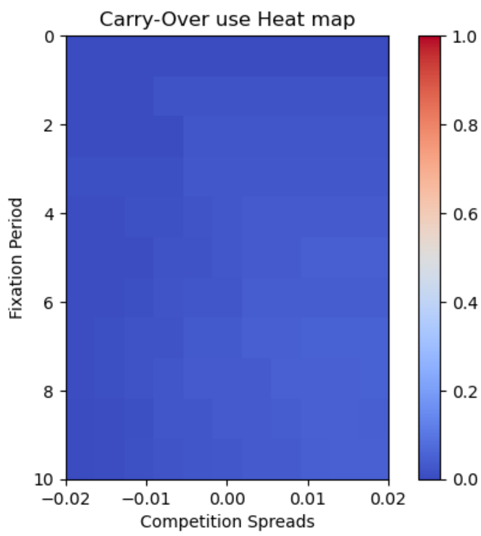
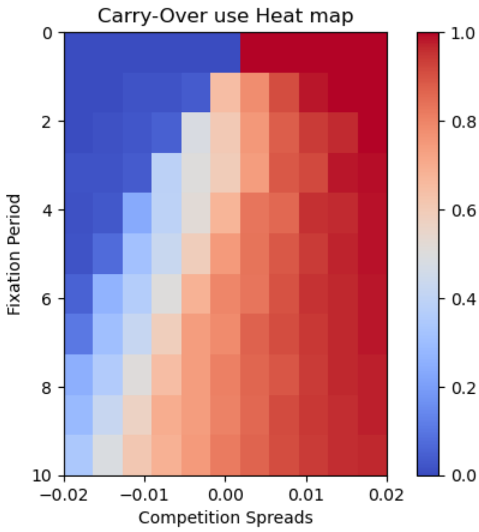
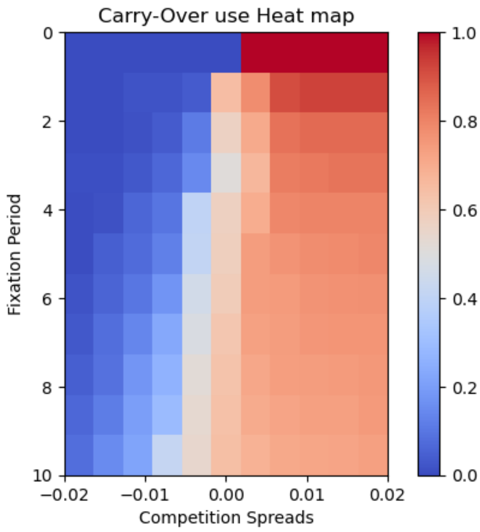


Figure 21: This figure illustrates the interest-only loan heatmap plot of the conditional probability of the carry-over use with a fixed rate of 0.5%, 3% and 6% as a function of the competition spreads and the fixation period, with an additional proportion of 0.5.

(a) Fixed rate of 0.5%.



(b) Fixed rate of 3%.



(c) Fixed rate of 6%.

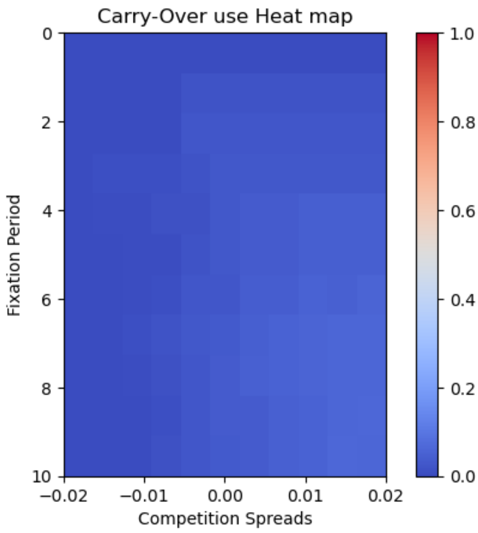
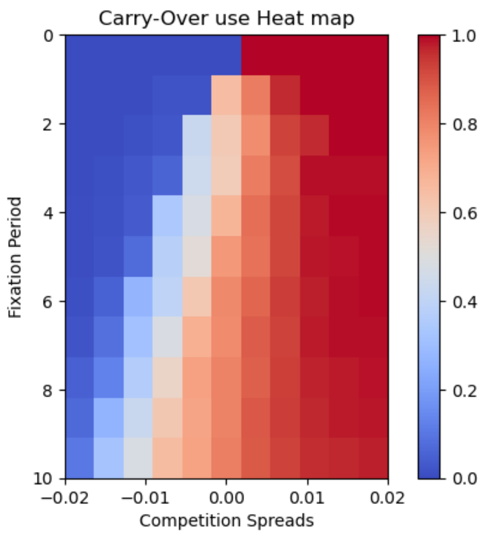
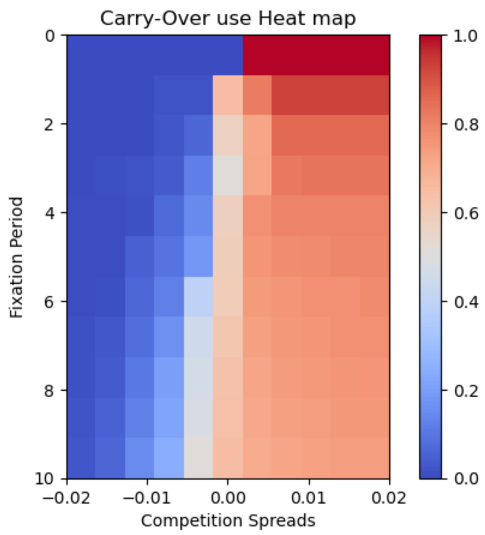


Figure 22: This figure illustrates the interest-only loan heatmap plot of the conditional probability of the carry-over use with a fixed rate of 0.5%, 3% and 6% as a function of the competition spreads and the fixation period, with an additional proportion of 1.

(a) Fixed rate of 0.5%.



(b) Fixed rate of 3%.



(c) Fixed rate of 6%.

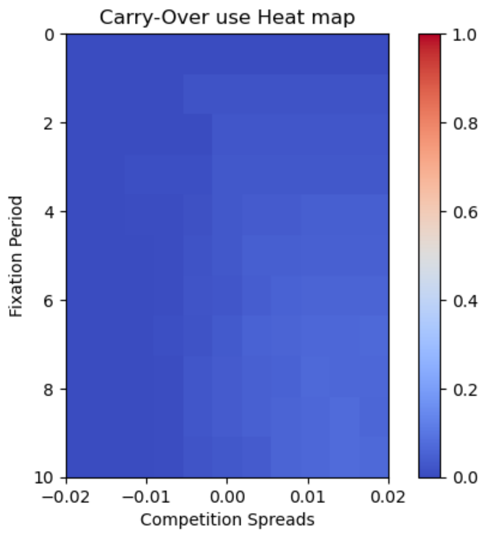
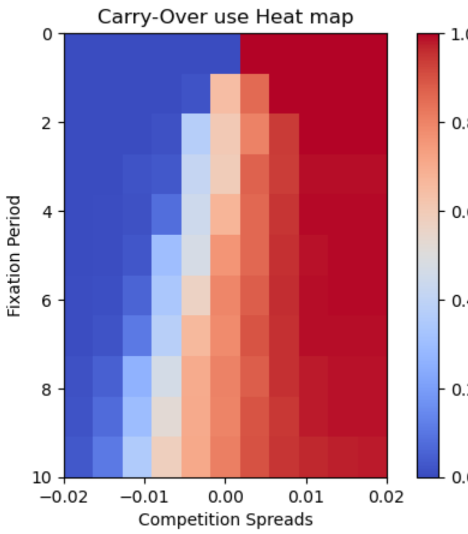
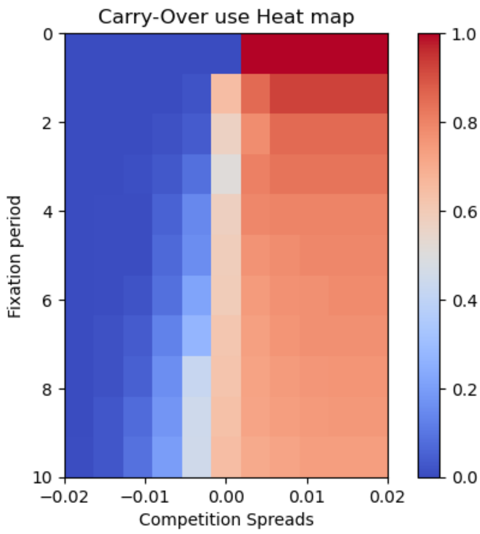


Figure 23: This figure illustrates the interest-only loan heatmap plot of the conditional probability of the carry-over use with a fixed rate of 0.5%, 3% and 6% as a function of the competition spreads and the fixation period, with an additional proportion of 1.5.

(a) Fixed rate of 0.5%.



(b) Fixed rate of 3%.



(c) Fixed rate of 6%.

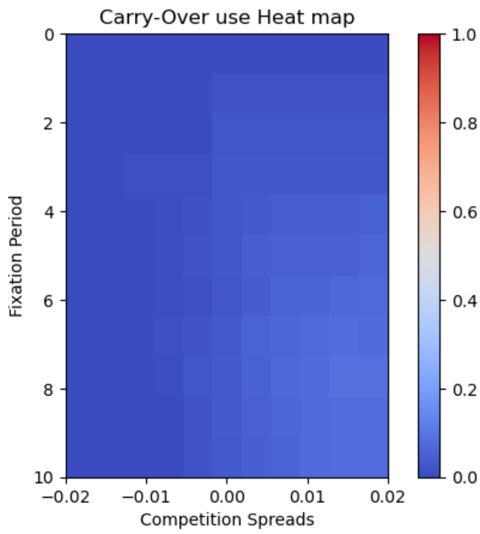
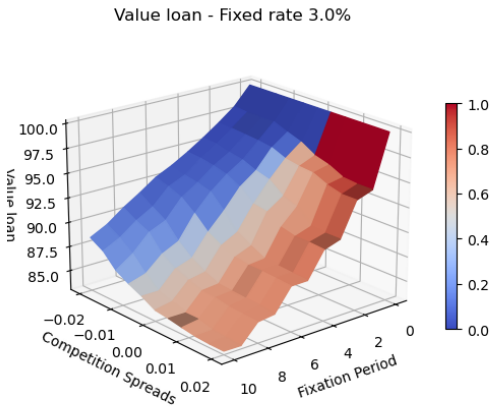


Figure 24: This figure displays the valuation of interest-only loans with r_f of 0.5%, 3%, and 6% as a function of lasting fixation periods and competition spreads, with each plot representing the value of loans that are dependent on various additional loans with proportions of 0.001. The heatmap plots are used to show the probability of carry-over given the competition spreads and the lasting fixation period.

(a) Fixed rate of 0.5%.

(b) Fixed rate of 3%.



(c) Fixed rate of 6%.

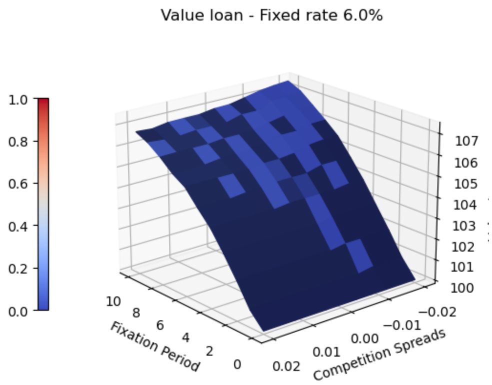
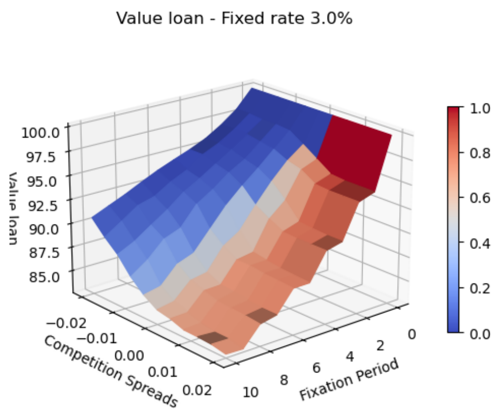


Figure 25: This figure displays the valuation of interest-only loans with r_f of 0.5%, 3%, and 6% as a function of lasting fixation periods and competition spreads, with each plot representing the value of loans that are dependent on various additional loans with proportions of 0.5. The heatmap plots are used to show the probability of carry-over given the competition spreads and the lasting fixation period.

(a) Fixed rate of 0.5%.

(b) Fixed rate of 3%.



(c) Fixed rate of 6%.

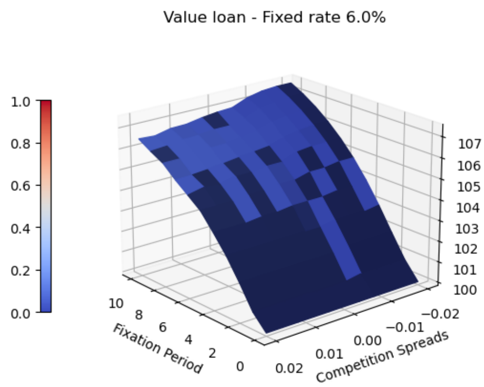
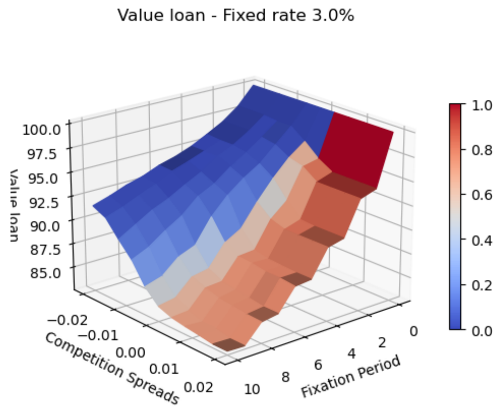


Figure 26: This figure displays the valuation of interest-only loans with r_f of 0.5%, 3%, and 6% as a function of lasting fixation periods and competition spreads, with each plot representing the value of loans that are dependent on various additional loans with proportions of 1. The heatmap plots are used to show the probability of carry-over given the competition spreads and the lasting fixation period.

(a) Fixed rate of 0.5%.

(b) Fixed rate of 3%.



(c) Fixed rate of 6%.

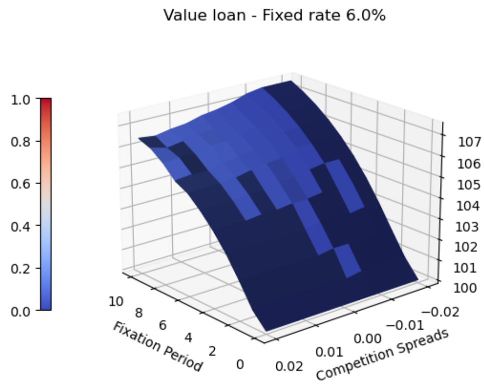


Figure 27: This figure displays the valuation of interest-only loans with r_f of 0.5%, 3%, and 6% as a function of lasting fixation periods and competition spreads, with each plot representing the value of loans that are dependent on various additional loans with proportions of 1.5. The heatmap plots are used to show the probability of carry-over given the competition spreads and the lasting fixation period.

(a) Fixed rate of 0.5%.

(b) Fixed rate of 3%.

(c) Fixed rate of 6%.

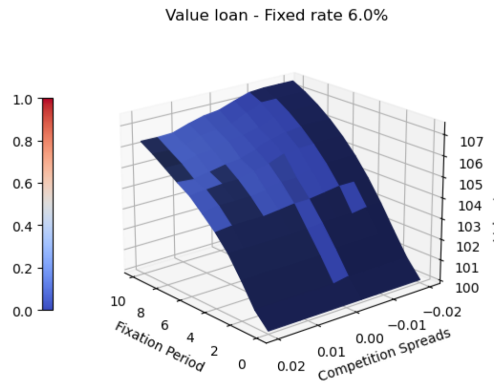


Figure 28: This figure displays the valuation of interest-only loans with r_f of 0.5%, 3%, and 6% as a function of lasting fixation periods and competition spreads, with each plot representing the value of loans that are dependent on various additional loans with proportions of 0.001. The planes represent loans with fixed interest rates in descending order of layers, with r_f of 6%, 3%, and 0.5%, respectively. The heatmap plots are used to show the probability of carry-over given the competition spreads and the lasting fixation period.

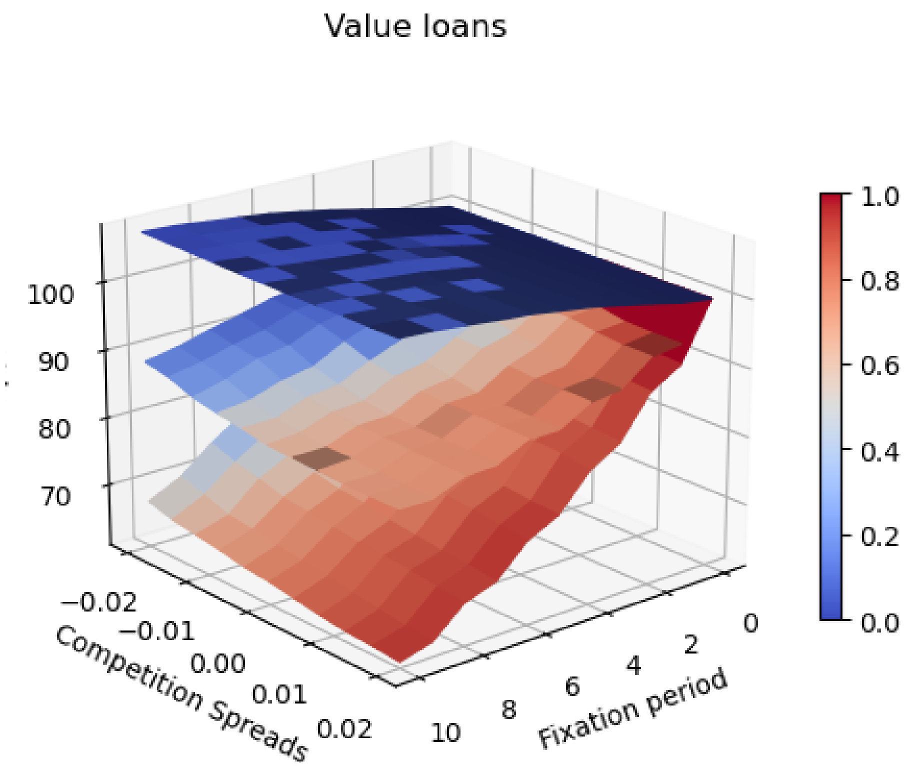


Figure 29: This figure displays the valuation of interest-only loans with r_f of 0.5%, 3%, and 6% as a function of lasting fixation periods and competition spreads, with each plot representing the value of loans that are dependent on various additional loans with proportions of 0.5. The planes represent loans with fixed interest rates in descending order of layers, with r_f of 6%, 3%, and 0.5%, respectively. The heatmap plots are used to show the probability of carry-over given the competition spreads and the lasting fixation period.

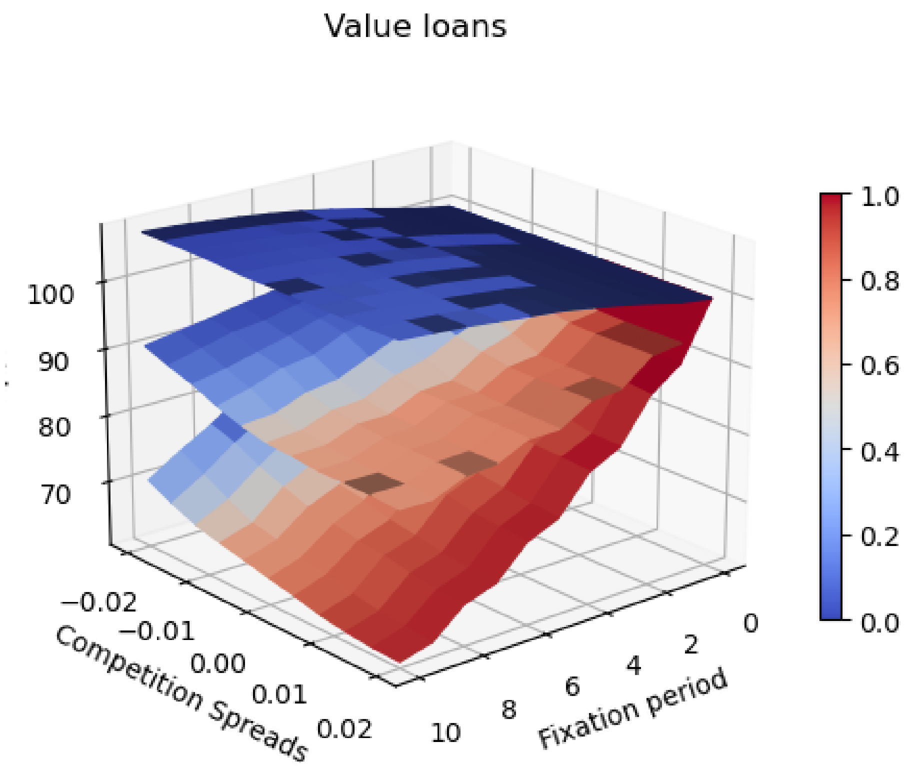


Figure 30: This figure displays the valuation of interest-only loans with r_f of 0.5%, 3%, and 6% as a function of lasting fixation periods and competition spreads, with each plot representing the value of loans that are dependent on various additional loans with proportions of 1. The planes represent loans with fixed interest rates in descending order of layers, with r_f of 6%, 3%, and 0.5%, respectively. The heatmap plots are used to show the probability of carry-over given the competition spreads and the lasting fixation period.

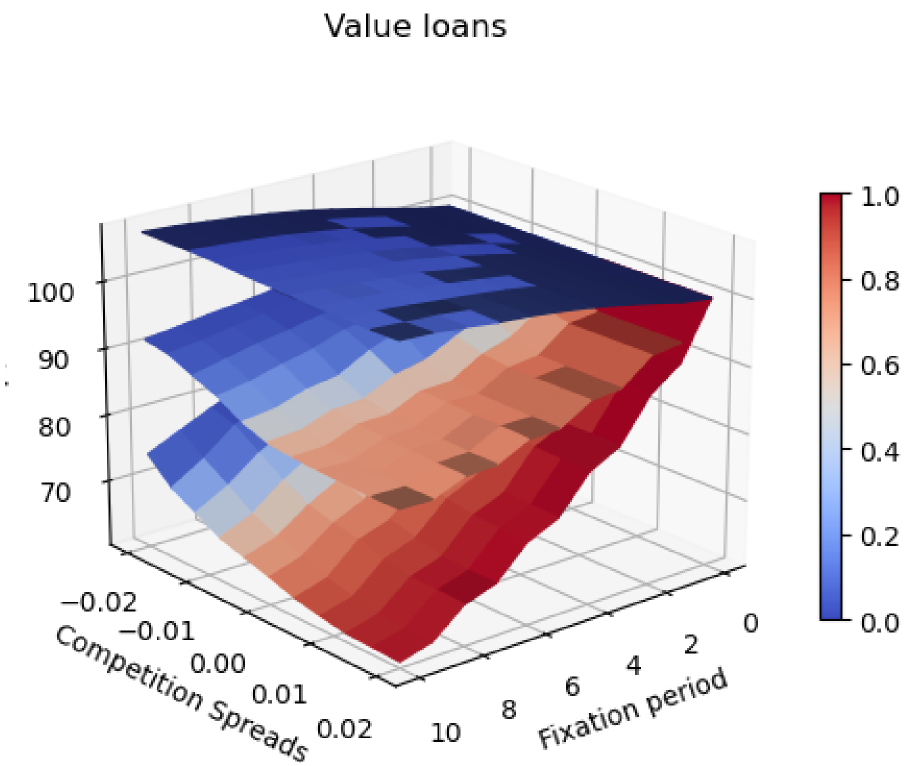


Figure 31: This figure displays the valuation of interest-only loans with r_f of 0.5%, 3%, and 6% as a function of lasting fixation periods and competition spreads, with each plot representing the value of loans that are dependent on various additional loans with proportions of 1.5. The planes represent loans with fixed interest rates in descending order of layers, with r_f of 6%, 3%, and 0.5%, respectively. The heatmap plots are used to show the probability of carry-over given the competition spreads and the lasting fixation period.