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The influence of climate-related and environmental risks on bank loan credit migration

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Abstract

This research analyses the impact of climate-related and environmental risks on credit risk management using a one-parameter and a principal components representation of credit risk and transition matrices. The study finds that incorporating climate risk significantly affects economic capital and credit migration predictions. These effects are relatively higher for portfolios with higher initial credit ratings. The inclusion of a climate risk variable next to an economic risk variable improves the predictive performance of systematic risk in climate risk scenario analysis. In the short-run, the current policies scenario (used as reference) outperforms the disorderly and orderly transition scenarios due to the high costs of the transition towards a low-carbon economy. These costs lead up to approximately 25% more expected yearly losses for the disorderly scenario, and 12.5% for the orderly scenario, highlighting the importance of a quick transition. In the long-run, however, the transition scenarios outperform the reference scenario by effectively mitigating most physical risks after a few years of transitioning, almost reaching a consistent 5% less expected yearly losses. This study recommends integrating climate risk into credit risk management protocols and employing an extension to the one-parameter approach to address potential risks posed by climate change.

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Chapter 1

Introduction

Climate-related and environmental risks (CER) pose a significant threat to households, firms, and the overall financial sector. Within the financial sector, a crucial aspect of managing CER lies in assessing their impact on the credit risk associated with bank loan portfolios. Despite the extensive research and stringent regulatory oversight surrounding credit risk modelling, a significant number of banks are still in the nascent phases of integrating CER into their credit risk models (ECB, 2022). CER encompasses two broad categories: physical risk and transition risk (BCBS, 2021). Physical risk arises from the adverse effects of greenhouse gas (GHG) emissions, resulting in heightened vulnerability to physical damages worldwide. Simultaneously, transition risk emerges from actions taken to mitigate the impacts of global warming. In this context, the objective of this thesis is to assess the influence of CER on the credit risk of bank loan portfolios by conducting a comprehensive evaluation of credit migration through scenario analysis. The integration of CER into a bank's credit risk management framework can have significant implications for the capital required to maintain solvency given the risks it assumes. Hence, the main objective of this study is to answer the following research question: 'What is the effect of CER on the credit migration of bank loan portfolios?'

To address this research question, we will employ two distinct methodologies. The first approach utilises the one-parameter model proposed by Belkin, Suchower and Forest Jr. (1998), which is based on Vasicek (1991)'s asymptotic single-risk factor (ASRF) credit portfolio model. Vasicek's model extends the Merton model for pricing corporate debt (Merton, 1974). This one-parameter threshold model plays a central role in the framework of regulatory capital measurement and standards proposed by the Basel Committee on Banking Supervision (BCBS) in Basel III (BCBS, 2017). Under Basel III guidelines, the Basel internal ratings-based (IRB) ASRF model is used to determine capital requirements for credit risk by estimating the loss conditional to a single systematic economic risk factor (BCBS, 2005). In our second approach, we extend the one-parameter approach by re-estimating the systematic risk in historical transition matrices conditional on the initial rating. Having found multiple systematic risk factors, we apply principal component analysis (PCA) to reduce dimensionality and ensure orthogonality of these factors. Once we establish the two different approaches, we proceed to incorporate CER into our single-factor and multi-factor models. This integration is accomplished through the construction of linear regression models that link economic risk and climate risk to our respective models. Specifically, we utilise GDP growth as a proxy for economic risk (Garnier,

Gaudemet and Gruz (2022) and Gaudemet, Deschamps and Vinciguerra (2022)), and incorporate temperature growth as a proxy for climate risk (Cosemans, Hut & Van Dijk, 2021).

After conducting model estimation, we will evaluate the impact of CER on credit risk through analysis of various scenarios. Our focus is on long-term climate risk scenarios, considering a 30-year time horizon. In line with the 2022 Climate Risk Stress Test methodology of the European Central Bank (ECB) (ECB, 2022), we compare an orderly transition, disorderly transition, and current policies (used as reference) scenario, whose narratives are based on the Network for Greening the Financial System (NGFS) Phase III scenarios (NGFS, 2022). We predict future values of systematic risk, and calculate the corresponding conditional future transition matrices. Lastly we apply these transition matrices to two different portfolios using time steps of one year, and retrieve closed-form expressions for credit migration and the resulting expected losses and economic capital (Vasicek, 2002). This enables us to describe how CER substantiates in changes of credit risk of different bank loan portfolios. Consequently, we construct a model that integrates CER into our existing credit risk framework, serving as a direct extension to the models presently employed by banks for internal capital buffer management (BCBS, 2009).

The proposed methods rely on multiple publicly accessible data sources. To prepare the data, various pre-processing steps are employed. The model estimation makes use of biannual historical corporate transition matrices from S&P Global, covering the period from 2000H1 to 2017H2. These matrices are smoothed to ensure non-zero transition probabilities and adhere to row and column monotonicity. The analysis incorporates historical macro variables, namely GDP and temperature. GDP data is obtained from the Organisation for Economic Co-operation and Development (OECD) and is interpolated to match the credit transition data. The focus of the analysis is the biannual percentage change in GDP. Temperature data is sourced from the National Centers for Environmental Information (NCEI), specifically the monthly temperature anomaly. The anomaly is also interpolated to create biannual data and is transformed into a two-year moving average. The stress scenarios used to evaluate climate risk are based on the NGFS Phase III scenarios. These scenarios encompass both short-term and long-term risks and encompass different policy scenarios, carbon emission pathways, and carbon prices. The scenarios each represent varying levels of physical and transition risk. Two ROBECO portfolios, high-yield and investment-grade, are analysed to assess the impact of climate risk on credit risk. These portfolios differ in their initial credit ratings distribution and level of riskiness.

The paper presents three key findings. Firstly, incorporating temperature anomaly growth significantly enhances the predictive performance of systematic risk when conducting climate risk scenario analysis for credit risk. Neglecting climate risk leads to an underestimation of potential paths of mainly physical risks. Secondly, our analysis demonstrates that the reference scenario produces the most favourable outcomes in terms of expected losses and economic capital in the short-term. Both the disorderly and orderly scenarios experience higher expected losses and economic capital during the early years of the transition, driven by heightened transition risks. These costs lead up to approximately 12.5% more expected yearly losses for the orderly scenario compared to the reference scenario, and 25% for the disorderly scenario, highlighting the importance of a quick transition. However, by effectively mitigating most of the physical risks after a decade of transitioning, the disorderly and orderly transition scenarios consistently

outperform the reference scenario in the long-term, with expected losses reaching almost -5% compared to the reference scenario at the end of the 30-year period. Thirdly, we establish the effectiveness of the widely adopted one-parameter approach, known for its simplicity and robustness, as it outperforms the more comprehensive PCA approach.

The contribution of this thesis is of both a scientific and a practical nature. Firstly, it addresses the challenge of quantifying CER within the context of credit risk modelling, thereby making a significant contribution to the existing body of research. We compare the computation of CER using both a single-factor (one-parameter) and a multi-factor (PCA) approach, and make an assessment on the effects of CER on the credit risk of a bank loan portfolio within these two approaches. Moreover, we consider the effects of climate risk not solely on credit default but also on the wider perspective of credit migration. Secondly, the paper offers a practical application by demonstrating how existing credit risk models can be adapted to incorporate the issue of CER. It outlines the necessary steps to implement different climate policy scenarios and provides guidance on analysing transition pathways, climate impacts, and macro-financial indicators. By offering these tools, the paper enables the management of emerging climate risks by considering it as a driver of existing risks.

The rest of our paper is organised in the following way. The subsequent Chapter 2 presents a comprehensive review of the existing literature on the topic. In Chapter 3, we elaborate on the methodology utilised in our study, providing detailed explanations of the theoretical frameworks and statistical techniques used. Chapter 4 focuses on the data utilised in our research. Chapter 5 delves into the empirical outcomes of our analysis, along with the presentation and discussion of our findings. Chapter 6 presents the conclusions that we draw based on our findings, including the implications of our results and their potential impact. Finally, Chapter 7 provides a discussion, including the research's limitations and suggestions for future research endeavours.

Chapter 2

Literature review

In this section, we conduct a comprehensive literature review on the interplay between credit risk and CER. We begin by examining the current state of the literature pertaining to each topic individually. Firstly, we explore the existing research on CER, considering the various dimensions and implications of CER. Subsequently, we shift our focus to credit risk, analysing the extensive research conducted in this domain. Furthermore, we delve into recent studies that investigate the assessment of climate risk within the framework of credit risk. By reviewing relevant papers in the field and seeking guidance from regulatory authorities and the banking industry itself, we aim to gain insights into the emerging practices and approaches utilised in assessing the impact of climate risk on credit risk.

2.1 Climate-related and environmental risks

The impact of climate change and environmental degradation on economic activity can result in structural changes that affect the financial system and its stability (ECB, 2020). CER refer to the risks of potential effects. These risks arise from the physical impacts of climate change, as well as the transition to a low-carbon economy, and can have far-reaching consequences for businesses, investors, and financial institutions. CER are therefore typically associated with these two primary risk drivers: physical risk and transition risk.

Physical risks are those that arise from the direct impact of climate change on physical assets, such as buildings, infrastructure, and agricultural land. These risks can include damage from extreme weather events, sea level rise, and water scarcity, among other factors. The physical risks can lead to significant financial losses for businesses and investors, particularly those with high exposure to climate-sensitive sectors such as agriculture and real estate. The White House reported in 2021 that in the United States, the economic damages from storms, floods, droughts, and wildfires have already risen to over \$100 billion per year (Boushey, Kaufman & Zhang, 2021). Besides that, Howard and Sterner (2017) point out that the increase in global average temperatures, which could exceed 3 degrees Celsius by the end of this century (IPCC, 2021), could cause annual damages equal to 7 to 11 percent to global GDP.

Transition risks, on the other hand, are those that arise from the process of adjustment to a low-carbon economy. This includes policy and regulatory changes, technological innovation, and changes in consumer behaviour (Bua, Kapp, Ramella & Rognone, 2022). These risks can have a

range of impacts on businesses and investors. An example of transition risk is the introduction of a carbon tax. The United Nations Environment Programme - Financial Initiative (UNEP-FI), for example, indicates that the introduction of a higher tax on GHG emissions implies that a company faces higher operating costs (UNEP-FI, 2018). Higher operating costs due to this tax reduce the company's earnings and therefore its profitability. Expectations of this profitability in turn affect the company's creditworthiness.

Financial institutions, including banks, insurers, and asset managers, are particularly exposed to CER (BCBS, 2021). They may have investments in assets or sectors that are vulnerable to physical or transition risks, or they may be exposed to indirect risks through their lending activities. Failure to manage these risks can lead to financial instability and losses, and may also undermine long-term sustainable economic growth. To address these risks, many financial institutions are adopting climate risk management strategies and integrating environmental, social, and governance (ESG) factors into their investment decision-making. Governments and regulators are also taking action to promote greater transparency and disclosure of CER, as well as to incentivise the transition to a low-carbon economy.

In their report on how CER can impact both banks and the banking system, BCBS (2021) conclude that the impacts of these risk drivers on banks can be observed through traditional risk categories, such as credit, operational, market, and liquidity risk. Of these main risk types, The Sustainable Finance Platform (2022) expects credit risk to be the one that is most affected by CER. The potential effect of CER on credit risk consists of the two risk drivers, physical and transition risk, that are measured through two transmission channels, which we call the income effect and the wealth effect. The income effect is when a borrower's credit risk increases if CER drivers reduce its ability to repay and service its debt (influence on PD). The wealth effect is when a borrower's credit risk increases if CER drivers reduce a banks' ability to fully recover the value of a loan in the event of default (influence on LGD).

2.2 Credit risk

In the realm of quantitative finance, few topics receive as much intensive study as credit risk does today (Lando, 2004). But what is credit risk? Imagine we hold a portfolio of loans. Credit risk is the risk that our portfolio value varies, because of the unexpected changes in credit quality of the assets in our portfolio. It can be divided into two sub-risks. The first part of this risk are losses due to defaults. The second part are losses caused by variations in the credit quality of a counterpart in an external or internal rating system. When dealing with credit risk, one needs to account for both parts. To ensure a comprehensive and accurate approach, it is imperative to first contextualise credit risk within the framework of the Basel accords and common practice within the banking industry (BCBS, 2005).

The BCBS establishes regulatory standards for credit risk modelling in banks. These global accords were implemented as part of a reform package to manage risk in the international banking sector. Although non-binding, member banks are expected to make efforts to incorporate them into their domestic regulations. The accords prescribe certain minimum leverage ratios and reserve capital requirements. The BCBS specifies two approaches for calculating capital requirements for corporate loans: the standardised approach and the IRB approach. If banks

meet some minimum requirements, Basel II allows them to use IRB models (BCBS, 2005). Under the revised and augmented version called Basel III, which was set up to finalise post 2007-2008 financial crisis reforms, banks can still use these models (BCBS, 2017). The accords state that risk measures should capture relevant risks in case of economic downturn. Banks categorise their exposures and estimate risk parameters, such as the probability of default (PD), the loss given default (LGD) and the exposure at default (EAD) using either standard supervisory rules or internal assessments. These parameters are then used to calculate risk-weighted assets (RWA) and regulatory capital for credit risk (BCBS, 2001). These risk parameters are obtained through the application of either standardised supervisory rules (foundation method, or F-IRB) or through internal assessments (advanced methodology, or A-IRB). Under the F-IRB method, banks only compute the PD using a proprietary model. The formulas to compute the other risk parameters are provided by the regulator, according to the standards of Basel II and III. Under the other method, the A-IRB, banks are free to compute all risk parameters using the methods they prefer. The regulator checks the statistical soundness of the proposed methods, and if they meet some other minimum guidelines.

Credit risk modelling approaches play a crucial role in the banking industry as they enable banks to comprehend and make informed decisions about their credit risk. Accurate calibration, stress testing, and sensitivity analysis of dependence parameters are necessary to capture the impact of dependence on credit risk. The choice of modelling approach depends on the specific credit risk factors being considered and the desired level of accuracy. BCBS (2009) emphasises the significance of dependence structure modelling in credit risk modelling, which involves understanding the relationship between credit risk factors. Banks commonly utilise portfolio models based on the condition independence framework, which assumes that the credit risk of individual loans in the portfolio is independent (Bluhm, Overbeck & Wagner, 2003). The ASRF model is widely adopted due to its portfolio invariance property, meaning that changes in the portfolio structure do not affect the model's results (Gordy, 2003). ASRF models rely on the law of large numbers, where a portfolio consisting of numerous small exposures leads to the cancellation of idiosyncratic risk, leaving only systematic risk. In these models, the defaults of individual borrowers depend on a set of common systematic risk factors that describe the state of the economy. The ASRF model is a simplified credit portfolio risk model that underlies the Basel III capital requirements (BCBS, 2017). These systematic risk factors vary over time and are assumed to follow a joint normal distribution. The individual exposures move in a correlated manner but are linked to the systematic risk factors to varying degrees. By modelling these correlations, banks implicitly account for concentration risk.

The discussed portfolio invariance property allows the calculation of credit value-at-risk (VaR) using exposure-specific parameters such as PD, LGD, and the dependence on a common factor. Under the IRB approach, banks employ one of three credit models to estimate PD, often known by their commercial names: Moody's (KMV), CreditMetrics, or CreditRisk+ (BCBS, 2005). While some banks have developed their own internal models, these models align with the structure of one of these types. Researchers have conducted comparative analyses of these methodologies to identify their strengths and weaknesses (a.o., Crouhy, Galai and Mark (2000) and Kollár and Gondžárová (2015)). KMV, building on the asset value model proposed by

Merton (1974), introduced the option pricing approach (Vasicek, 1991). In this model, default occurs when the asset value falls below a certain threshold, and the default process is assumed to be endogenous, related to the firm’s capital structure. CreditMetrics, proposed by JP Morgan, employs a credit migration approach based on the probability of transitioning between credit ratings within a specific time horizon, including default (Belkin et al., 1998). Credit Suisse Financial Products (CSFP) proposes CreditRisk+, an actuarial approach that focuses solely on default and assumes that default for individual bonds or loans follows an exogenous Poisson process (Gundlach & Lehrbass, 2004).

The modelling of portfolio credit risk can be broadly categorised into two primary frameworks: mixture models and threshold models (Bolder, 2022). Mixture models assume that the portfolio comprises sub-portfolios with varying levels of risk, while threshold models consider a specific threshold level of credit risk below which the portfolio is considered safe. In the following section, we will provide a concise overview of both these modelling frameworks and highlight their fundamental concepts.

2.2.1 The mixture model

Incorporating systematic risk into credit risk models is crucial for accurately assessing the likelihood of default events in a portfolio. One approach is the family of mixture models, which combines the binomial structure of the independent-default model with another random variable that influences default probability (Bolder, 2022). This results in a comprehensive depiction of default events that integrates both idiosyncratic and systematic risk factors. The mixture model encompasses the independent-default model as a special case and can be useful for Monte Carlo simulations (Frey & McNeil, 2003).

CreditRisk+ is an influential industry model that uses a Poisson mixture model to compute the (expected) loss distribution of an entire portfolio and enables the computation of VaR for obtaining RWA and capital requirements. The probability distribution for the losses has a fatter right tail under a Poisson distribution, indicating that the probability of large losses is not that small. The Poisson formulation and positivity of factor weights lead to analytical features such as the default distribution of a portfolio being equal to the distribution of a sum of independent negative binomial random variables (Gordy, 2000). However, migration risk is only partially accounted for by stochastic default rates.

Another type of mixture model is the Bernoulli mixture model, which is a type of Generalised Linear Mixed Model (GLMM). Kunene, Mung’atu and Nyarige (2021) use this model as a tool for modelling default dependency. They incorporate rating category (fixed effect) and time (random effect) as the systematic risk of a portfolio. The default dependence is captured with these so-called random effects. They conclude that it would be reasonable to allow for more heterogeneity in the model to incorporate additional random effects, which would only be possible by using more industrial and geographical information of the obligors.

Keijsers, Diris and Kole (2014) give an example on how a set of factors influence credit risk in a mixture model. In the style of Creal, Schwaab, Koopman and Lucas (2014), they are able to demonstrate cyclicity in losses on bank loans by making use of a dynamic factor model. The observations can follow different distributions and depend on a set of latent factors, resolving

issues as the inability to capture time-varying risk and the use of restrictive assumptions about the structure of data. All processes are linked through these latent factors, which include a set of macro factors that capture the business cycle and affect all observed variables, and a set of factors that capture the dynamics of the credit cycle that are unrelated to the business cycle, which are split into loan factors influencing both default and the LGD variables, and a separate set of LGD factors solely affecting the LGD variables. Their model is able to capture the time-varying nature of credit risk and outperforms traditional models in terms of its ability to predict credit events.

2.2.2 The threshold model

The family of threshold models presents an alternative approach for incorporating systematic risk into the independent-default model. Similar to the mixture-model setting, there is a shared component that impacts all obligors, leading to default correlation and systematic risk. In contrast to the mixture-model framework, the class of threshold models adopts a structural approach to the modelling of credit risk. This approach entails endogenising the default event, meaning that the occurrence of default is determined within the model itself rather than being treated as an external event. These models assign a latent state variable to each credit obligor, which is linked to the firm's asset value. This variable is a linear combination of (again) two components: the systematic component and the idiosyncratic component. The default event for each obligor occurs when their state variable falls below a predefined threshold determined by the firm's liabilities. The foundation of these models is laid by Merton (1974), who demonstrated that a firm's default could be modelled by using Black and Scholes (1973)'s option pricing model to value corporate debt. He uses stochastic differential equations to describe changes in a firm's equity value. According to this approach, the stochastic value of a firm's equity corresponds to the payoff of a European call option, with the strike price being equal to the face value of a single debt payment. The PD can be obtained using Merton's framework, expressed in terms of a standard Gaussian cumulative distribution function. An increase in debt and asset volatility raises the PD, while an increase in equity value decreases the PD. The threshold model represents the basis for many industry solutions, such as the KMV and CreditMetrics models.

The KMV model tries to overcome some flaws of the Merton model, such as the Gaussianity assumption, and that default can only happen at maturity (T). In three papers, Vasicek laid the groundwork for the KMV model (Vasicek (1987), Vasicek (1991) and Vasicek (2002)). The model uses an extensive amount of empirical data to create a set of credit quality indicators to calculate the PD. These indicators include factors such as the borrower's credit history, financial position, and other relevant factors. In this way they show that the PD is a function of the borrower's credit quality. Fundamental to this model is the 'Expected Default Frequency' (EDF), which is nothing but the one-year PD according to KMV's methodology (Vasicek, 1987). It uses a structural approach that links a firm's asset values to the EDF. The model assumes that a firm defaults when the value of its assets falls below a certain threshold. The EDF is calculated by making use of a quantity called the DD, which is the distance to the default (DD) threshold, expressed as a number of standard deviations. Companies with the same DD have the same EDF. The standard Gaussian cumulative distribution function (CDF) that is used in the Merton

model, is substituted by an empirical function.

The second industry model, CreditMetrics, differs somewhat from the KMV model. Most importantly, the default threshold is not defined using the liabilities of a company, but through credit ratings (Gupton, Finger & Bhatia, 1997). It is based on credit migrations. These ratings can be calculated internally or externally. In the KMV model, the PD equals the average PD of companies with the same DD, whereas in the CreditMetrics model, the PD equals the average PD of companies with the same credit rating class (Frey & McNeil, 2003). Because thresholds are defined through credit ratings, this approach overcomes the problem related to the correct definition of the liability threshold. Furthermore, the use of credit ratings allows us to compute both the PD and the probability of more subtle creditworthiness deterioration. On the contrary, the model relies heavily on its assumption of normality that may cause an underestimation of tail risks. This can be troublesome when looking at the VaR of extreme events.

In a first extension to the CreditMetrics methodology, Belkin et al. (1998) propose a simplified approach to modelling credit risk and transition matrices, using a single systematic risk factor that drives the correlation between rating migrations of different borrowers, rather than the observed correlations between equity values of the different industries and countries of those borrowers. The one-parameter is estimated using historical transition matrices, taking rating transitions to reflect an underlying continuous credit-change indicator. This indicator is partitioned into a set of disjoint bins that reflect the probability that this indicator falls within a given interval. This corresponds to the historical average transition rate. Broadly speaking, the systematic risk measures the values of default rates and end-of-period credit ratings that were not predicted by applying the historical average transition rates to the initial mix of credit grades. We call this deviation of the historical average the ‘credit cycle’. The one-parameter is assumed to account for the variance of the indicator that can be attributed to systematic risk.

A second extension is made by Li (2000), who uses a copula function approach to default correlation. The normal copula function is essentially already used in Gupton et al. (1997)’s default correlation formula, whilst the use of the copula concept is not stated explicitly. Some standard techniques are used in survival analysis to show that the default correlation between two credit risks is defined as the correlation coefficient of their survival times, indicating that one needs to specify a joint distribution with given marginal distributions when modelling a credit portfolio. Li (2000) argues that the copula function approach provides a convenient way to do so. In the wake of the 2008 financial crisis, however, MacKenzie and Spears (2014) wrote an infamous article titled ‘The formula that killed Wallstreet?’, referring to the Gaussian copula. Their main concern with the Gaussian copula, is that the probability mass of the normal probability function is more concentrated around its mean, causing the Gaussian copula to generate joint symmetric dependence, but no tail dependence. Therefore, by moving far enough into the tail of the loss distribution, default becomes independent in the Gaussian copula case, causing joint defaults to be structurally underestimated in the Gaussian setting. A practical solution to resolve this is to adjust the model to a t -copula. The t -copula can generate joint symmetric tail dependence, hence allowing for fatter joint tails and increasing the probability of joint extreme events.

A third extension to the the Merton-type portfolio models of credit risk, is the multi-factor adjustment of Pykhtin (2004). While these multi-factor Merton-type portfolio models have

become very popular among risk management practitioners, most of them rely on Monte Carlo simulations. Analytical methods are limited to the one-factor case, such as the limiting loss distribution of Vasicek (1991). The multi-factor problem is resolved by replacing the original loss distribution with a loss distribution of an infinitely fine-grained portfolio, thereby providing an analytical method for the multi-factor framework. Essentially, this method is an extension to a simpler granularity adjustment made by Gordy (2003).

2.3 Climate-related and environmental risks within credit risk modelling

In the existing literature, we have examined CER and credit risk separately. Now, we will explore the potential linkage between the two and explore possibilities for assessing the influence of CER on credit risk.

To begin, we turn to the guidelines provided by the ECB to understand how climate risk can be integrated into credit risk modelling (ECB, 2020). The ECB has identified CER as a key focus area for supervision between 2022 and 2024, emphasising the risks posed by climate change and the transition to a net-zero carbon economy for companies, households, and the financial sector¹. Consequently, the ECB expects banks to fully incorporate CER into their internal capital adequacy assessment process (ICAAP) and stress testing by 2024. As part of their annual stress test in 2022, the ECB conducted a climate risk stress test (CRST) on selected significant supervised entities. The purpose of this CRST was to evaluate the effectiveness of these entities' management of climate risk. The findings of the test shed light on how banks are incorporating CER considerations into their credit risk models. Notably, the ECB observed that banks generally preferred to modify their existing credit risk models to capture the impact of CER, rather than developing entirely new models (ECB, 2022).

Green RWA (2020) introduces a mathematical model as an extension to the Basel III regulation, incorporating the two climate factors, physical and transition risk, next to an economic factor. The concepts outlined in this paper are further developed in Garnier et al. (2022). Garnier et al. (2022) employ a structural credit risk model to assess the potential unexpected loss that could impact a bank's credit portfolio. The ASRF model is expanded to a multi-factor framework, presenting an approach for modelling correlated risk factors, known as the Climate-Extended Risk Model (CERM). The CERM estimates CER within a bank's loan portfolio, whilst complying to Basel III. The model incorporates future CER paths using a Gaussian copula model, calibrated with non-stationary macro-correlations. The migration matrices and systematic weights of the model are dynamically updated over time to reflect evolving physical and transition risks.

Gaudemet et al. (2022) propose a calibration method for the model introduced by Garnier et al. (2022). They calibrate the macro-correlations by using a stochastic forward-looking approach. Four fundamental principles form the basis of a model consisting of three interdependent stochastic cumulative growth factors. These equations represent climate-independent economic

¹2022 ECB press release: 'ECB sets deadlines for banks to deal with climate risks'.
<https://www.bankingsupervision.europa.eu/>

growth, cumulative physical damage resulting from climate change, and cumulative economic costs associated with transition efforts.

In a separate study conducted by Cosemans et al. (2021), the authors examine the influence of climate change on the optimal asset allocation for long-term investors. They emphasise that climate risk, encompassing both physical and transition risks, is a significant source of systematic risk over the long term. However, studying climate risk poses challenges due to the scarcity of historical data available on the impact of climate change on long-horizon returns. To address this, the researchers propose a methodology that combines historical data with a theoretical model that elucidates how increasing temperatures affect asset prices, amplifying the likelihood of climate-related disasters that impede economic growth. The authors evaluate the impact of climate change on long-term equity risk and portfolio allocation, considering various perspectives on the pricing of climate risks. While their primary focus lies on physical risks, they acknowledge the potential relevance of transition risks as well. Employing a Bayesian approach, they integrate prior information derived from the theoretical temperature long-run risk model developed by Bansal, Kiku and Ochoa (2019) with the available historical data. To examine the long-term return dynamics, a Vector Auto-regressive (VAR) model is employed, incorporating temperature change as a predictor to proxy for climate risk. The prior information provided by the Bansal et al. (2019) model establishes a structured relationship between returns and temperature change, offering insights into the impact of climate change. Notably, the key finding of the study suggests that investors utilising these informative prior beliefs, perceive returns to be riskier and observe higher long-term return variance compared to investors with uninformative priors.

Chapter 3

Methodology

Our study introduces a comprehensive four-step methodology for assessing the influence of CER on credit risk. The initial two steps establish the foundation of our approach. Firstly, we adopt a generalised credit risk threshold model aligned with the Basel III ASRF framework (BCBS, 2017), building upon the seminal works of Merton (1974) and Vasicek (1991). Our credit risk modelling follows the CreditMetrics approach proposed by Belkin et al. (1998), encompassing both a one-parameter approach and an extension to a multi-factor model, utilising PCA to ensure factor orthogonality and dimensionality reduction. Secondly, we construct a linear regression model that establishes the linkage between the threshold models and a set of explanatory variables, introducing a proxy for climate risk within the credit risk framework.

After establishing the foundation, we progress to the third step, which outlines the methodology for conducting scenario analysis based on various climate pathways. In the fourth and final step, we elucidate how the predicted systematic risk associated with different climate scenarios can be translated into implications for risk management, considering measures such as expected loss and economic capital. This enables us to infer potential downgrade outcomes and compute the total capital impact of incorporating climate risk within the credit risk framework. The subsequent chapter will discuss the results derived from this methodology.

3.1 The threshold model

3.1.1 Credit migration

We initiate our analysis by establishing a threshold model to assess credit risk, using the notations presented in Bolder (2022)'s book on economic capital. The threshold model is built upon the concepts introduced by Merton (1974) and later refined by Vasicek (2002), which propose that default occurs when a firm's equity is completely depleted or, in other words, when the value of its assets falls below its liabilities. A derivation of the basic concepts of the threshold model can be found in Appendix A. In this paper, we adopt the CreditMetrics approach, which incorporates all types of credit migration, recognising that default is just one specific outcome within the broader context of credit migration. The transition from one credit rating to another provides valuable information about the risk level of the portfolio. While modelling default is crucial, it is also essential to model credit improvement or deterioration. In the CreditMet-

rics approach, we assume that rating transitions reflect an underlying (unobserved) continuous indicator of creditworthiness ΔX that follows a standard normal distribution. Following the methodology proposed by Belkin et al. (1998), we consider rating transition matrices as the result of binning the CDF of the latent creditworthiness variable, conditioned on the initial credit rating m (ΔX^m). This enables us to account not only for default but also for gradual deterioration in creditworthiness.

By assuming that creditworthiness follows a discrete-time Markov chain with q distinct discrete credit states, we achieve the desirable property that an obligor's current credit rating is sufficient to determine the credit rating in the subsequent period. In our research, we employ eight credit states: AAA, AA, A, BBB, BB, B, CCC, and D (Default, an absorbing state). We derive the bins for these states from the transition probabilities of moving from credit rating m at time t to credit rating n at time $t + 1$ as

$$\begin{aligned} p^{n,m} &= \mathbb{P}(S_{t+1} = n | S_t = m), \\ &= \Phi(\Delta X^{(n+1),m}) - \Phi(\Delta X^{n,m}), \end{aligned} \tag{3.1}$$

for $n, m = 1, \dots, q$, where S_t denotes the credit state at time t . To construct a transition matrix, we collect all transition probabilities and arrange them in a matrix \mathbf{P} , ensuring that each row sums up to unity. To form our cumulative transition matrix \mathbf{G} (representing the boundaries), we divide the CDF of the creditworthiness index, conditioned on the initial rating m , into a set of disjoint bins $(\Delta X^{n,m}, \Delta X^{(n+1),m}]$. The probability of ΔX^m falling within a specific interval (the transition probability) is denoted as $p^{n,m}$. In our threshold approach, this brings us the cumulative transition matrix

$$G_{\Phi^{-1}} = \begin{bmatrix} \Phi^{-1}(g_{11}) & \Phi^{-1}(g_{21}) & \cdots & \Phi^{-1}(g_{q1}) \\ \Phi^{-1}(g_{12}) & \Phi^{-1}(g_{22}) & \cdots & \Phi^{-1}(g_{q2}) \\ \vdots & \vdots & \ddots & \vdots \\ \Phi^{-1}(g_{1q}) & \Phi^{-1}(g_{2q}) & \cdots & \Phi^{-1}(g_{qq}) \end{bmatrix}, \tag{3.2}$$

where $\Phi^{-1}(g_{n,m})$ is the upper threshold of a transition from rating m to rating n , with

$$g_{n,m} = \sum_{w=n}^q p_{wm}. \tag{3.3}$$

To finalise the boundary transition matrix, we assign an upper threshold of infinity to the highest bin (AAA) and a lower threshold of minus infinity to the lowest bin (default), as outlined by Belkin et al. (1998).

3.1.2 One-parameter

We adopt the perspective of Belkin et al. (1998) regarding systematic risk as a measure of the 'the credit-cycle'. In their one-parameter representation of credit risk and transition matrices, they reduce the systematic component into a single systematic risk factor (ΔZ) and utilise the

generalised version of Equation A.0.7,

$$\Delta X = \alpha \Delta Z + \sqrt{1 - \alpha^2} \Delta w, \quad (3.4)$$

where $\Delta Z \sim \mathcal{N}(0, 1)$ is the systematic risk factor, $\alpha \in (0, 1)$ is the systematic weight, and $\Delta w \sim \mathcal{N}(0, 1)$ the idiosyncratic risk. This equation ensures that the distribution of creditworthiness indicator ΔX is also standard normal.

When considering a specific initial credit rating m in year t , the observed transition matrix may deviate from the historical average transition matrix (\mathbf{P}). According to Yang (2014) and Yang (2017), the historical average transition matrix ($\Delta Z = 0$) can be regarded as a through-the-cycle (TTC) estimate of the transition matrix. Consequently, the credit-cycle parameter (ΔZ_t) represents the disparity between the TTC estimate and a point-in-time (PIT) estimate of the transition matrix (\mathbf{P}_t). The fitted PIT transition matrix is a function of the credit cycle ($\mathbf{P}_t(\Delta Z_t)$), which is the desired outcome. In Subsection 3.2, we will establish a relationship between this credit-cycle parameter and macro variables, with the PIT transition matrix serving as the response variable. By undertaking this process, we are able to investigate the transmission of shocks to macro variables into credit conditions during scenario analysis, thereby elucidating their subsequent impact on our transition matrices.

3.1.3 Principal component analysis

After computing the model using the single systematic risk factor, we proceed to estimate the model with multiple factors. This entails extending the systematic risk factor from the one-parameter approach to a multi-factor approach (Frey and McNeil (2003), and Garnier et al. (2022)). To achieve this, we estimate the systematic risk factor conditioned on the initial credit rating m . This allows us to translate changes in the credit-cycle into effects specific to each initial credit rating.

$$\Delta X_t^m = \alpha_m \Delta Z_t^m + \sqrt{1 - \alpha^2} \Delta w_t \quad (3.5)$$

Consequently, we obtain seven systematic risk factors at each point in time, one for each initial credit rating (AAA, AA, ..., CCC), which we gather in a matrix $\Delta \mathbf{Z}_t$. The corresponding individual systematic weights α_m are placed in vector $\boldsymbol{\alpha}$. Subsequently, we apply PCA to mitigate the impact of the increased number of parameters on our estimation error and to obtain factors that are orthogonal to each other (Stock & Watson, 2002b). By employing PCA, we identify linear combinations of the seven systematic risk factors that are uncorrelated and exhibit maximum variance. Working with orthogonal factors offers several advantages, including the ability to retain a latent form and enhanced convenience (Pykhtin, 2004).

We begin by constructing a factor model using linear combinations of the systematic risk factors;

$$\mathbf{f}_t = \mathbf{V}' \Delta \mathbf{Z}_t, \quad (3.6)$$

where $\mathbf{f}_t = (\mathbf{f}_{1t}, \mathbf{f}_{2t}, \dots, \mathbf{f}_{7t})'$ and \mathbf{V} is an (7×7) matrix with vectors \mathbf{v}_i in its columns for

$i = 1, \dots, 7$. . The first principal component represents the linear combination

$$f_{1t} = \mathbf{v}'_1 \Delta \mathbf{Z}_t, \quad (3.7)$$

that maximises variance and has unit length (Equation 3.8).

$$\text{Var}(f_{1t}) = \mathbf{v}'_1 \boldsymbol{\Sigma} \mathbf{v}_1, \quad \text{s.t. } \mathbf{v}'_1 \mathbf{v}_1 = \sum_{j=1}^N v_{1j}^2 = 1 \quad (3.8)$$

The k -th principal component represents the linear combination

$$f_{kt} = \mathbf{v}'_k \Delta \mathbf{Z}_t, \quad (3.9)$$

again maximising variance and of unit length, but with the additional requirement that it is orthogonal to the other components, i.e.

$$\text{Cov}(f_{kt}, f_{jt}) = \mathbf{v}'_k \boldsymbol{\Sigma} \mathbf{v}_j = 0, \quad (3.10)$$

for $j = 1, \dots, k-1$. We form the Lagrangian to address the maximisation problem. By slightly modifying the resulting conditions, we arrive at

$$\boldsymbol{\Sigma} \mathbf{v}_1 = l \mathbf{v}_1, \quad (3.11)$$

with l the Lagrangian multiplier. The solution \mathbf{v}_1 is an eigenvector of the correlation matrix $\boldsymbol{\Sigma}$ (Stock & Watson, 2002a). We may assume the desirable properties that eigenvectors are normalised and orthogonal to each other. By applying PCA to the covariance matrix of the seven systematic risk factors, we obtain the seven eigenvectors and eigenvalues of the covariance matrix, as:

$$\boldsymbol{\Sigma} \mathbf{v}_i = \lambda_i \mathbf{v}_i, \quad (3.12)$$

for all $1 \leq i \leq 7$, where λ_i denotes the eigenvalue corresponding to eigenvector \mathbf{v}_i . We then arrange the eigenvectors based on their eigenvalues in descending order. By applying these eigenvectors to the correlation matrix of our systematic risk factors, we obtain our principal components. The amount of variance explained by each principal component is determined by dividing each eigenvalue by the sum of all eigenvalues. We select the first r principal components, ensuring that the cumulative explained variance is reasonably large. Finally, we implement the factors in our credit risk model, by changing Equation 3.5 to

$$\Delta X_t = \hat{\boldsymbol{\alpha}}' \hat{\mathbf{f}}_t + \sqrt{1 - \hat{\boldsymbol{\alpha}}' \hat{\boldsymbol{\alpha}}} \Delta w_t, \quad (3.13)$$

where $\hat{\mathbf{f}}_t$ are the first r components, and the systematic weights vector $\hat{\boldsymbol{\alpha}}$ is estimated using least squares.

3.2 Linking macro variables to systematic risk

Our objective is to expand upon the formulated factor approach by incorporating multiple macro variables to explain the systematic risk factor. Building on the works of Cosemans et al. (2021) and Garnier et al. (2022), we aim to extend the analysis of economic explanatory variables by considering climate risk as an additional source of systematic risk. The core idea is to establish a connection between a specified set of variables and the systematic risk factor(s). To model the impact of economic and climate macro variables on the systematic component of the creditworthiness index, we construct a linear regression model. This approach enables us to assess the influence of climate change on credit risk by combining historical data with theoretical data from organisations like NGFS, which describes how rising temperatures affect economic growth as measured by GDP growth.

Utilising a linear regression model offers conceptual and practical advantages when estimating and implementing our model. We will describe credit conditions as a function of an underlying state represented by the macro variables. In this system, we will include proxies for economic risk and climate risk. For instance, following the approach of Gaudemet et al. (2022), we will employ GDP growth as a proxy for economic risk. Moreover, following the work of Bansal et al. (2019) and Cosemans et al. (2021), we will expand the set of macro variables by incorporating temperature anomaly growth as a proxy for climate risk. Our linear regression model is defined, in matrix notation, as

$$\hat{\mathbf{f}}_t = \mathbf{B}'\mathbf{Y}_t + \boldsymbol{\eta}_t, \quad (3.14)$$

with,

$$\mathbf{Y}_t = \begin{pmatrix} 1 \\ \Delta\text{GDP}_t \\ \Delta\text{T}_t \end{pmatrix}, \quad (3.15)$$

with \mathbf{B} a $(k+1) \times r$ matrix, with r the number of principal components and k the number of macro variables (two in our case) extended with a constant, and $\boldsymbol{\eta}_t \sim \mathcal{N}(0, 1)$. The values of this \mathbf{B} parameter are our systematic factor loadings. In the one-parameter case, Equation 3.14 simplifies, by reducing the systematic factor loadings to a $k \times 1$ vector, and replacing $\hat{\mathbf{f}}_t$ with the single parameter ΔZ_t from Equation 3.4.

To summarise, we can now develop a dynamic transition matrix conditioned on macro state variables, aligning with our modelling objective of assessing the impact of CER on credit risk. This enables us to consider the driving macro variables under different scenarios and evaluate their impact on credit migration (Section 3.3). Subsequently, we apply these findings to economic capital calculations (Section 3.4).

3.2.1 Autoregressive and moving average extension

We consider two small extensions to our linear regression model. We extend the linear regression model by adding an autoregressive (AR) term and an moving average (MA) term, *ceteris paribus*.

We make these extensions to consider the ‘smoothing’ effects of making the systematic risk linearly depend on its previous value (AR), and making the error term linearly dependent on its previous value (MA). We first add an AR(1) term to the endogenous variable, thus changing Equation 3.14 to,

$$\hat{\mathbf{f}}_t = \phi \hat{\mathbf{f}}_{t-1} + \mathbf{B}'\mathbf{Y}_t + \boldsymbol{\eta}_t. \quad (3.16)$$

Secondly, we add the MA(1) term, changing Equation 3.14 to,

$$\hat{\mathbf{f}}_t = \mathbf{B}'\mathbf{Y}_t + \boldsymbol{\eta}_t + \theta \boldsymbol{\eta}_{t-1}. \quad (3.17)$$

3.3 Scenario analysis

Having formulated the credit risk model and established its connection to our economic and climate macro variables, we now proceed to the scenario analysis. Once we have estimated the parameters of our linear regression model, we will enter macro state variables and utilise them to predict future values of systematic risk ($\Delta \tilde{\mathbf{Z}}_{t+1}$, or $\tilde{\mathbf{f}}_{t+1}$) under different climate pathway scenarios. Subsequently, we compute the associated predicted transition matrix ($\tilde{\mathbf{P}}_{t+1}(\Delta \tilde{\mathbf{Z}}_{t+1})$, or $\tilde{\mathbf{P}}_{t+1}(\tilde{\mathbf{f}}_{t+1})$), which serves as the foundation of our climate stress testing framework. A further description of the scenarios follows in Subsection 4.3.

3.4 Risk management

After completing the scenario analysis using the different scenarios, our next objective is to evaluate the outcomes. In order to compare the scenarios, we utilise our model to examine the conditional loss distribution of two hypothetical loan portfolios in a PIT framework (Miu and Ozdemir (2006) and Keijsers et al. (2014)). One portfolio is categorised as ‘investment-grade’ (lower risk), while the other is classified as ‘high-yield’ or ‘speculative grade’ (higher risk). A further description of the portfolio can be found in Subsection 4.4. By employing the conditional loss distribution, we derive the expected loss and economic capital for both portfolios. Economic capital is determined as the difference between the 99.9% quantile and the mean of the loss distribution. Following common practice, we use time steps of one year to estimate the expected losses and economic capital (BCBS, 2009). At each point in time, we compute the loss distribution for our portfolios, conditioned on the credit condition, as

$$L(\hat{\mathbf{f}}_t) = \sum_{i=1}^I c_i \gamma_i p_i(\hat{\mathbf{f}}_t), \quad (3.18)$$

with $p_i(\hat{\mathbf{f}}_t)$ the PD conditional on the systematic factors, γ_i the LGD and c_i the EAD of obligor i . To estimate the economic capital, which necessitates the assessment of unexpected loss, we employ our threshold credit risk model in accordance with the framework established by Vasicek

(2002). We calculate the 99.9% quantile of the loss distribution as follows:

$$L(\hat{\mathbf{f}}_t) = \sum_{i=1}^I c_i \gamma_i \Phi \left(\frac{\Phi^{-1}(p_i(\hat{\mathbf{f}}_t)) + \sqrt{\hat{\boldsymbol{\alpha}}' \hat{\boldsymbol{\alpha}}} \Phi^{-1}(0.999)}{\sqrt{1 - \hat{\boldsymbol{\alpha}}' \hat{\boldsymbol{\alpha}}}} \right), \quad (3.19)$$

where Φ denotes the standard normal CDF, Φ^{-1} its inverse, and $\hat{\boldsymbol{\alpha}}$ the systematic weights. In the one-parameter approach, $\hat{\mathbf{f}}_t$ is naturally replaced by ΔZ_t , and $\hat{\boldsymbol{\alpha}}$ by α . Finally, utilising the PIT estimates, we can also derive TTC estimates by averaging the PIT estimates.

3.5 Estimation

To estimate our model, we begin by calculating the historical average transition matrix based on the available data. We apply smoothing techniques to ensure row monotonicity, column monotonicity, non-zero transition probabilities, and that all rows sum to one (Belkin et al. (1998) and Bolder (2022)). Row and column monotonicity are achieved by adjusting the off-diagonal values closest to the diagonal, representing the probabilities of credit ratings remaining the same at the end of the period. We also introduce an initial default state to the transition matrix, creating an absorbing state with a transition probability of 100% from default to default. Once the transition matrix is smoothed, we calculate the corresponding historical average boundary matrix. This matrix is then utilised to estimate the systematic risk variable(s) using historical rating matrices within the sample. Subsequently, we fit a linear regression model to the fitted systematic risk values and make in-sample and out-of-sample predictions. For the multivariate case, we estimate principal components first after finding the systematic risk variables, followed by the linear regression and predictions. Finally, we employ the estimated parameters to conduct scenario analysis using NGFS data.

3.5.1 systematic weights

One-parameter

To estimate the model parameters of our threshold credit risk model, as described in equations 3.4 and 3.5, we begin by determining the relative importance of systematic and idiosyncratic risks (i.e., the systematic weights). Following the procedure outlined by Belkin et al. (1998), we find the systematic risk value that best approximates the observed transition matrix for year t using predefined bins from the smoothed historical average boundary matrix. This value of systematic risk minimises the following weighted mean-squared discrepancies between the historical average and the observed transition probabilities:

$$\min_{\Delta Z_t} \sum_m \sum_n \frac{N_{t,m} (p_t^{n,m} - \Delta(x^{(n+1),m}, X^{n,m}, \Delta Z_t))^2}{\Delta(x^{(n+1),m}, X^{n,m}, \Delta Z_t) (1 - \Delta(x^{(n+1),m}, X^{n,m}, \Delta Z_t))} \quad (3.20)$$

with

$$\Delta(x^{(n+1),m}, X^{n,m}, \Delta Z_t) = \Phi \left(\frac{\Delta X^{(n+1),m} - \alpha \Delta Z_t}{\sqrt{1 - \alpha^2}} \right) - \Phi \left(\frac{\Delta X^{n,m} - \alpha \Delta Z_t}{\sqrt{1 - \alpha^2}} \right). \quad (3.21)$$

N_t^m here represents the number of transitions from initial credit rating m in year t . We label this value as ΔZ_t , which represents the credit-cycle capturing end-of-period credit rating changes not predicted by applying historical average transitions to the initial ratings. Positive values of ΔZ_t indicate good years with fewer defaults or less credit deterioration than the average, while the opposite holds true for bad years.

The systematic weight parameter α governs the correlation between the credit-cycle and the creditworthiness indicator, implying that the systematic risk explains a fraction α of the variance in the creditworthiness index. Or, although fairly close, the actual proportion considered systematic is determined as

$$\frac{\alpha}{\alpha + \sqrt{1 - \alpha^2}}. \quad (3.22)$$

We do not know the value of α a priori. We optimise the least-squares problem from Equation 3.20 for an assumed value of α , and obtain a time series for ΔZ_t conditional on α . By calculating the mean and variance of this systematic risk time series for various α values, we find the value of α that results in a variance of one for ΔZ_t , ensuring ΔX_t remains standard normally distributed¹.

PCA

In estimating systematic risk parameters conditional on the initial rating, we follow the same steps as in the one-parameter case. The main difference lies in the optimisation problem, where we optimise for ΔZ_t^m instead of ΔZ_t to determine the systematic weight α_m instead of α . We rewrite the optimisation problem from Equation 3.20 to

$$\min_{\Delta Z_t^m} \sum_n \frac{N_{t,m} (p_t^{n,m} - \Delta(x^{(n+1),m}, X^{n,m}, \Delta Z_t^m))^2}{\Delta(x^{(n+1),m}, X^{n,m}, \Delta Z_t^m) (1 - \Delta(x^{(n+1),m}, X^{n,m}, \Delta Z_t^m))}, \quad (3.23)$$

with

$$\Delta(x^{(n+1),m}, X^{n,m}, \Delta Z_t^m) = \Phi\left(\frac{\Delta X^{(n+1),m} - \alpha_m \Delta Z_t^m}{\sqrt{1 - \alpha_m^2}}\right) - \Phi\left(\frac{\Delta X^{n,m} - \alpha_m \Delta Z_t^m}{\sqrt{1 - \alpha_m^2}}\right). \quad (3.24)$$

To identify the principal components, we estimate the eigenvalues and eigenvectors of the correlation matrix derived from the seven estimated systematic risk factors. To determine the number of principal components to retain (r), we construct a scree plot. A general rule of thumb is to retain components above the point where the eigenvalues start tapering off gradually (the ‘scree’) (Thorndike (1953) and Walach, Kohls and G uthlin (2010)). Having selected r components, we subsequently solve the following least-squares problem to determine systematic weight $\hat{\alpha}$ from Equation 3.5:

$$\min_{\alpha} \sum_t \sum_m \sum_n \left(p_t^{n,m} - \Delta(x^{(n+1),m}, X^{n,m}, \hat{\mathbf{f}}_t) \right)^2. \quad (3.25)$$

¹If we were to look for some regulatory guidance, we would need to use a value of α in the range of 0.30 to 0.35 (Bolder, 2022). Higher credit-quality firms tend more towards the upper end of this range. We can conclude that by using a higher systematic weight, we would be taking a more conservative approach.

The actual systematic proportion is adjusted accordingly to

$$\frac{\sqrt{\hat{\alpha}'\hat{\alpha}}}{\sqrt{\hat{\alpha}'\hat{\alpha}} + \sqrt{1 - \hat{\alpha}'\hat{\alpha}}}. \quad (3.26)$$

3.5.2 systematic factor loadings

We divide the dataset into a training set and a test set. The in-sample period is used for parameter estimation. The out-of-sample period is used to evaluate forecasting performance of the model. Approximately 80% of the data (2000H1-2014H1) is used for training, while the remaining 20% (2014H2-2017H2) is reserved for testing.

We employ ordinary least squares (OLS) to estimate the linear regression and determine the systematic factor loadings \mathbf{B} from Equation 3.14. We then calculate the in-sample and out-of-sample R^2 to assess the model's performance. After evaluating the results for both the one-factor and the principal components, we estimate \mathbf{B} once again utilising all available data when conducting our scenario analysis.

3.5.3 Portfolio analysis

In our analysis of the capital impact on our two portfolios under different scenarios, we consider each portfolio to consist of 2,000 loans ($I = 2,000$). To focus on credit migration and default risk, we assume that each individual loan has a constant EAD of €1 (c_i), following the approach of Miu and Ozdemir (2006) and Keijsers et al. (2014). Additionally, we set the LGD at 45% for both portfolios. According to BCBS (2017), senior claims on firms, sovereigns, and banks without recognised collateral are subject to a 45% LGD, known as the '45% rule'. It is important to note that the LGD for speculative grade bonds generally exceeds that of investment-grade bonds.

Chapter 4

Data

4.1 Transition matrices

To estimate the model, we utilise biannual historical corporate transition matrices from S&P Global spanning from the first half of the year 2000 (2000H1) to the second half of the year 2017 (2017H2). The dataset consists of an average of 6892 transitions per half-year, with a standard deviation of approximately 833. These matrices are publicly available on the website of the European Securities and Markets Authority (ESMA)¹. We pre-process the 36 individual transition matrices through three simple steps. Initially, due to limited observations in the lowest credit ratings, we consolidate the initial credit ratings CCC, CC, and C from the dataset into a single initial credit state CCC. Similarly, we perform the same consolidation for the end-of-period credit ratings. Finally, we normalise the matrices for withdrawals.

4.2 Historical macro variables

To estimate our systematic factor loadings, we incorporate historical data on GDP and temperature. The historical GDP data is obtained from the Organisation for Economic Co-operation and Development (OECD)². GDP serves as a crucial indicator to capture economic activity, making it suitable for assessing changes in economic risk and its relation to the credit cycle. We utilise yearly OECD world data for our analysis and interpolate the yearly time series to biannual data to align with the credit transition data. We employ cubic spline interpolation for smoothness and higher accuracy in approximating the original function. Following interpolation, we calculate the biannual percentage change in GDP. When plotting the economic risk proxy, we observe that GDP growth is generally positive, with a significant decline during the 2007-2008 financial crisis (Figure 4.1a).

¹ESMA (2023): ‘Central repository (CEREP) for publishing the rating activity statistics and rating performance statistics of credit rating agencies’. <https://www.cerep.esma.europa.eu/>

²OECD: ‘Gross domestic product (GDP)’. <https://data.oecd.org/>

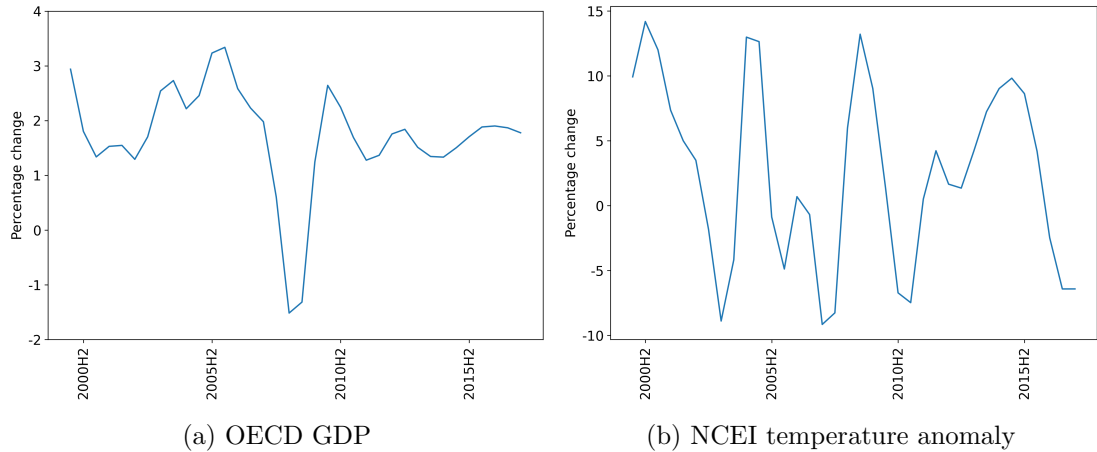


Figure 4.1: Historical macro variables

For temperature, we rely on publicly available data from the National Centers for Environmental Information (NCEI)³. We utilise monthly time series of the global land and ocean temperature anomaly. The anomaly represents the deviation of the average global temperature from a baseline period, calculated relatively to the twentieth-century average (1901-2000). Similarly to GDP, we interpolate the temperature anomaly to biannual data using cubic spline interpolation to match the credit transition data. Subsequently, we calculate the two-year moving average of the temperature, following Cosemans et al. (2021), and calculate the percentage change in this moving average of the temperature anomaly. We make use of the moving average because the temperature anomaly’s growth data reveals cyclical behaviour, as depicted in Figure 4.1b. The use of a two-year moving average differs slightly from the five-year moving average Cosemans et al. (2021) is using. Reason for this, is the small amount of available data. By using the moving average, we account more for the long-run change in temperature, moderating the cyclical behaviour. This is desired when looking at long-run climate change effects.

Fluctuations in the temperature anomaly can be influenced by both natural and human-induced factors affecting Earth’s climate. Human activities, such as GHG emissions that contribute to global warming, and natural phenomena like El Niño and La Niña events⁴, contribute to year-to-year variations in the temperature anomaly. El Niño events, characterised by significant warming of Pacific Ocean surface waters, can lead to increased growth of the global temperature anomaly, while La Niña events have the opposite effect. Figure 4.2⁵ highlights El Niño events (red shaded areas) and La Niña events (blue areas), measured by the Oceanic Niño Index (ONI). The ONI identifies El Niño and La Niña climate patterns based on the running mean of sea surface temperature (SST) anomalies. Both the temperature anomaly percentage change and the ONI exhibit similar cyclical patterns, as observed in figures 4.1b and 4.2.

³NCEI (2023): ‘The Global Anomalies and Index Data’, <https://www.ncei.noaa.gov/>

⁴National Oceanic and Atmospheric Administration (NOAA): ‘What are El Niño and La Niña?’, <https://oceanservice.noaa.gov/>

⁵NCEI: ‘Equatorial Pacific Sea Surface Temperatures (SST)’, <https://www.ncei.noaa.gov/>

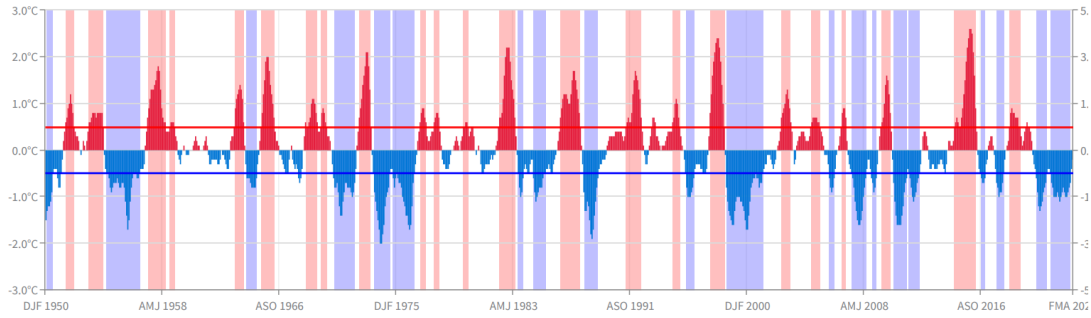


Figure 4.2: 3-month running mean of Niño 3.4 SST Anomalies

4.3 NGFS

The stress scenarios we employ are based on those used by the ECB in their CER stress test conducted in 2022 (ECB, 2022). The ECB’s climate risk stress test in 2022 incorporates a range of short and long-term scenarios to assess transition and physical risks. These scenarios are based on the NGFS Phase II scenarios released in June 2021. While the ECB emphasises that these scenarios should be viewed as hypothetical and not reflective of their own expectations regarding future outcomes, we can consider them as indicative for our climate risk assessment. We focus on three distinct long-term risk scenarios characterised by varying policy scenarios, carbon emission pathways, and prices, resulting in different levels of physical and transition risk. A more rapid transition toward a low-carbon future, accompanied by a higher degree of transition risk, is generally associated with lower physical risk. Conversely, a less stringent carbon policy and lower associated transition risk naturally give rise to higher physical risk (Figure 4.3). This assumption underscores the importance of considering both transmission channels of climate risk within the same model.

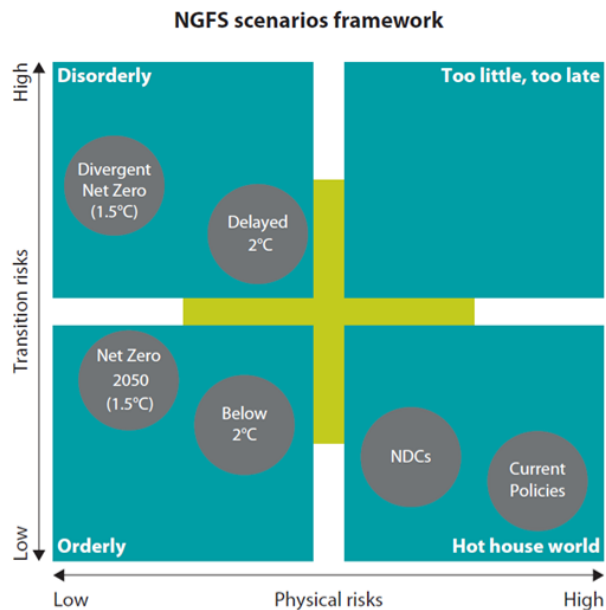


Figure 4.3: The NGFS matrix

The three long-term scenarios developed by the NGFS in their updated 2022 version (NGFS,

2022) that we will employ are as follows:

- **Current Policies Scenario:** This scenario envisions a ‘hot house world’ where some jurisdictions have implemented climate policies, but global efforts remain insufficient to effectively mitigate significant global warming. There is no strengthening of policy ambition, resulting in critical temperature thresholds being surpassed and high physical risks, such as rising sea levels. Global temperature anomaly continues to rise until around 3°C in 2080 due to the lack of reduction in GHG emissions.
- **Delayed Transition Scenario:** In this scenario, there is disorderly transition to a carbon-neutral world, with global annual GHG emissions only starting to decrease from 2030 onwards. Strong policies are required to keep global warming below 2°C due to the temperature increase prior to the delayed transition. The higher temperature anomaly leads to increased carbon prices. Consequently, this scenario exhibits not only higher physical risks but also higher transition risks compared to the previous scenario, the orderly transition.
- **Net Zero 2050 Scenario:** This scenario assumes early introduction and gradual strengthening of climate policies. Due to the proactive approach taken to address climate change, less stringent policies are needed compared to the delayed transition scenario. With net zero CO₂ emissions around 2050, and some more developed countries even reaching net zero for all GHG, the physical risks are relatively subdued. This scenario represents the most ambitious approach, limiting the global temperature anomaly to 1.5°C.

These scenarios have been developed using three integrated assessment models (IAMs). These models incorporate variables that consider global climate mitigation costs, characteristics of the energy system transition, quantification of investments required for transforming the energy system, as well as synergies and trade-offs associated with sustainable development pathways⁶. They have been extensively utilised in numerous peer-reviewed scientific studies on climate change mitigation. Therefore, they possess a well-established track record of providing crucial information to policymakers and decision-makers.

The names of these models are GCAM, REMIND-MAGPIE, and MESSAGEixGLOBIOM. Although they share similarities, each model has its own unique characteristics. From an economic standpoint, the latter two models are general equilibrium models that employ perfect foresight. This means that the equilibria solve intertemporally, enabling them to anticipate changes such as increasing costs of exhaustible resources and decreasing costs of solar and wind technologies. Moreover, perfect foresight allows for endogenous changes in consumption, GDP, and energy demand in response to climate policies. These two models are being utilised by banks to evaluate transition risks (UNEP-FI, 2018). On the other hand, GCAM takes a myopic view and is solved in a recursive dynamic mode.

The models include various climate variables under different policy scenarios, such as temperature. These quantities are used to translate climate variables into a core set of macroeconomic variables, such as GDP. We extract long-run GDP and temperature pathways spanning a 30-year horizon for the three scenarios to serve as input for the macro variables in our stress testing

⁶<https://www.ngfs.net/>

analysis. These variables are utilised to construct future PIT transition matrices, which can then be translated into different TTC estimates to assess long-run average results.

All models are solved by minimising the costs associated with energy and land use systems, while adhering to various constraints such as limiting global warming to below 2°C. These constraints, along with other techno-economic and policy assumptions, vary across scenarios. For our analysis, we utilise the latest version of the MESSAGEix-GLOBIOM model adopted in NGFS Phase III: 1.1-M-R12. This model corresponds to the IIASA (International Institute for Applied Systems Analysis) IAM framework, which combines five complementary models: the energy model MESSAGE, the land use model GLOBIOM, the air pollution and GHG model GAINS, the aggregated macroeconomic model MACRO, and the simple climate model MAGICC (Model for the Assessment of Greenhouse-gas Induced Climate Change). These models interact with each other and exchange input throughout the scenario development process. Figure 4.4⁷ provides a concise overview of the interactions between these different models. The coloured boxes depict specialised disciplinary models integrated to create internally consistent scenarios (Fricko et al., 2017).

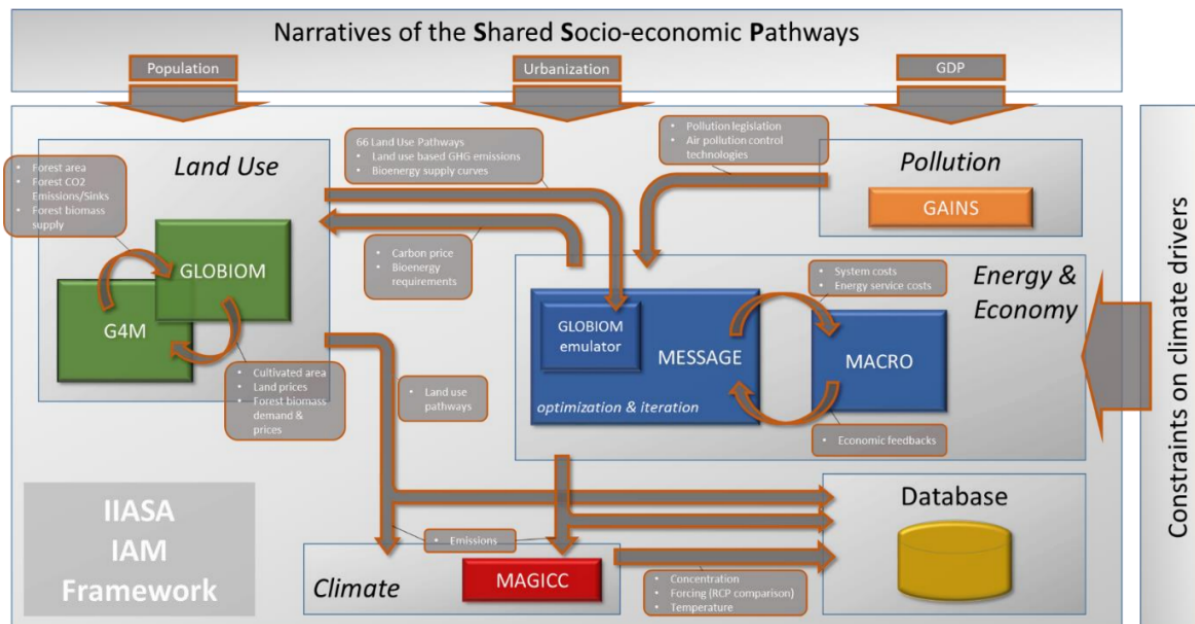


Figure 4.4: Overview of the IIASA IAM framework

In summary, exogenous material demand projections are generated based on GDP projections (Huppmann et al., 2019). From these projections, the model endogenously determines the final energy use for various sectors. The demand for useful energy is adjusted based on demand prices until the models reach equilibrium. This iterative process captures energy efficiency adjustments induced by changes in energy prices. To achieve the desired radiative forcing in each scenario, global GHG emissions are constrained at different levels. These climate constraints are incorporated into the optimisation procedure, and the resulting carbon price is fed back into the iteration. In this way, the carbon price paths serve as key distinguishing features among the scenarios as they drive different levels of transition risk. Finally, the combined emissions

⁷<https://docs.messageix.org/>

results are put into a global carbon-cycle and reduced-complexity climate model, which provides estimates of the climate implications in terms of atmospheric concentrations, radiative forcing, and global-mean temperature.

As mentioned earlier, our scenario analysis will encompass the utilisation of the current policies (referred to as the reference scenario), delayed transition, and Net Zero 2050 scenarios. We will employ median values of GDP and temperature as macro variables across the various climate pathways. The model adopts a base year of 1990 and employs time steps of 5 years until 2050⁸. Once again, we apply cubic interpolation to transform these macro variables into half-yearly data, ensuring consistency with the rest of the research. Subsequently, we calculate the percentage changes of the variables, which are then plotted in 4.5.

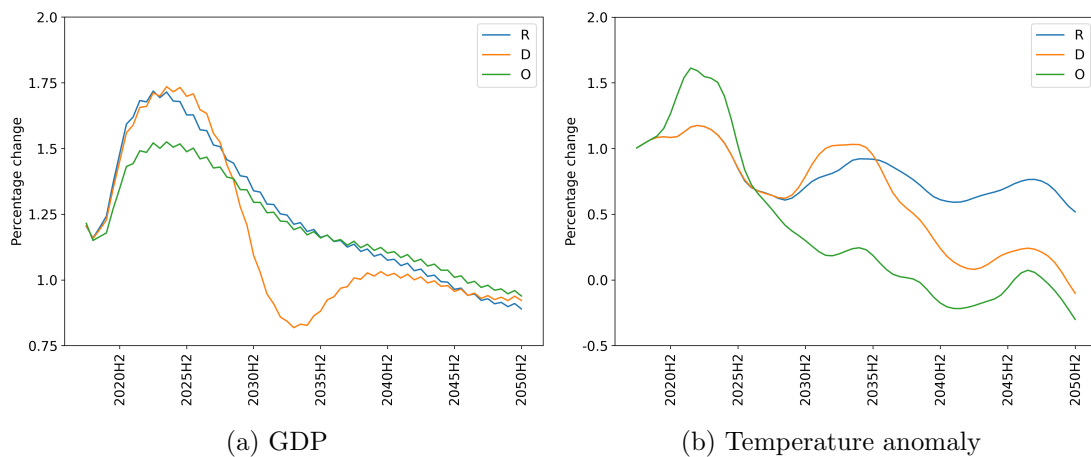


Figure 4.5: Future macro variables in different NGFS scenarios

The orderly scenario, characterised by the financial and economic risks associated with the transition to a low-carbon economy, exhibits an immediate slowdown in GDP growth during the early years, followed by a similar trend in the disorderly scenario (Figure 4.5a). Between 2040 and 2045, the GDP growth in the orderly scenario surpasses that of the reference scenario due to the emergence of substantiating physical damages. Analysing the trajectory of temperature anomaly changes, we observe a pronounced peak in percentage change in the years 2020-2025 for the orderly scenario. One can think of reasons for this more rapid increase, such as that the initial phase of the transition may involve mitigation actions from which the benefits only start to pay off in long term, such as the building of lagging infrastructure for renewable energy installations, and in other ways improving energy efficiency. After the peak, the growth rate gradually decelerates and eventually turns negative between 2035 and 2040. In the disorderly transition scenario, a similar peak occurs but is delayed by 10 years (2030-2040) as expected. In this scenario, growth declines around 2050. Meanwhile, the reference scenario exhibits a continuous increase in the temperature anomaly. Similar to historical temperature data, all scenarios exhibit a cyclical pattern.

⁸More specifically, the full name of the GDP variable is ‘GDP—PPP—including high chronic physical risk damage estimate’, and of the temperature variable is ‘AR6 climate diagnostics—Surface Temperature (GSAT)—MAGICCv7.5.3—50.0th Percentile’.

4.4 Portfolios

We consider two different portfolios to assess effect CER on credit risk with different initial credit mixes (Table 4.1). We consider a high-yield and an investment-grade bond portfolio. The high-yield portfolio is based on the Robeco high-yield Bonds CH EUR, the investment-grade on Robeco investment-grade Corporate Bonds C EUR. These portfolios differ in terms of riskiness. Investment-grade contains mainly A and BBB ratings, while high-yield contains BB, B and CCC.⁹

		IG	HY
Initial rating	AAA	0.4	0.0
	AA	4.5	0.0
	A	35.0	0.0
	BBB	60.1	0.0
	BB	0.0	50.0
	B	0.0	39.7
	CCC	0.0	10.3

Table 4.1: The distribution across credit ratings of the investment-grade (IG) and high-yield (HY) portfolios (in %)

⁹Numbers accessed on 31/03/2023. <https://www.Robeco.com/>

Chapter 5

Results

This section begins with a discussion of the outcomes of the historical systematic risk by utilising our one-parameter and PCA approaches. Afterwards, we delve into the outcomes of our systematic factor loadings utilising the linear regression model, and consider the AR and MA extensions to the model. The estimated parameters empower us to proceed with the scenario analysis, and finally, we draw implications for risk management.

5.1 The threshold model

We start by importing biannual historical S&P transition matrices covering the period between 2000H1 and 2017H2. Using this data, we compute the historical average transition matrix (Table B.2) and apply the in Section 3.5 discussed smoothing techniques to ensure monotonicity, non-zero transition probabilities, and row-sum normalisation (Table 5.1).

		End-of-half-year rating							
		AAA	AA	A	BBB	BB	B	CCC	D
Initial rating	AAA	90.840%	8.663%	0.355%	0.056%	0.056%	0.010%	0.010%	0.010%
	AA	0.166%	94.964%	4.543%	0.190%	0.102%	0.014%	0.010%	0.010%
	A	0.086%	0.976%	96.098%	2.686%	0.107%	0.017%	0.016%	0.014%
	BBB	0.047%	0.051%	1.859%	95.664%	2.093%	0.170%	0.064%	0.051%
	BB	0.028%	0.035%	0.946%	2.592%	91.890%	4.021%	0.336%	0.152%
	B	0.010%	0.014%	0.498%	1.334%	2.949%	90.877%	3.500%	0.819%
	CCC	0.010%	0.010%	0.059%	0.098%	0.137%	10.076%	80.160%	9.450%

Table 5.1: Historical average of the smoothed transition matrix

The highest probabilities are on the diagonal, indicating that the initial ratings have a high probability of staying the same. Default probabilities are high for the initial rating CCC (almost 10%), but quickly decrease for higher initial ratings. The probabilities of moving up or down one rating are generally low, except for the highest and lowest ratings, which are more volatile. Overall, we notice higher probabilities of deterioration than improvement, possibly due to the 2007-2008 financial crisis that is present in the observed time-span.

		End-of-half-year rating							
		AAA	AA	A	BBB	BB	B	CCC	D
Initial rating	AAA	∞	-1.331	-2.578	-2.986	-3.136	-3.432	-3.540	-3.719
	AA	∞	2.936	-1.658	-2.719	-2.996	-3.395	-3.540	-3.719
	A	∞	3.133	2.304	-1.905	-2.959	-3.305	-3.431	-3.628
	BBB	∞	3.306	3.095	2.063	-1.981	-2.764	-3.047	-3.285
	BB	∞	3.447	3.225	2.323	1.799	-1.695	-2.584	-2.963
	B	∞	3.723	3.494	2.561	2.084	1.664	-1.715	-2.400
	CCC	∞	3.719	3.540	3.159	2.916	2.733	1.260	-1.314

Table 5.2: Upper thresholds of the historical average boundary matrix of smoothed transitions

Using the smoothed transition matrix and Equation 3.2, we determine the upper thresholds of the binned CDFs (Table 5.2). As an illustration, we graphically depict the resulting binned standard normal CDF of the creditworthiness indicator for the initial credit rating BBB in Figure 3.4. The dotted line signifies the probability of exactly zero, denoting an average year. The CDF shifts left or right with negative or positive systematic risk, respectively. A shift of the CDF to the left increases the probability of a credit downgrade (bad year). A shift to the right decreases the probability of a credit downgrade (good year).

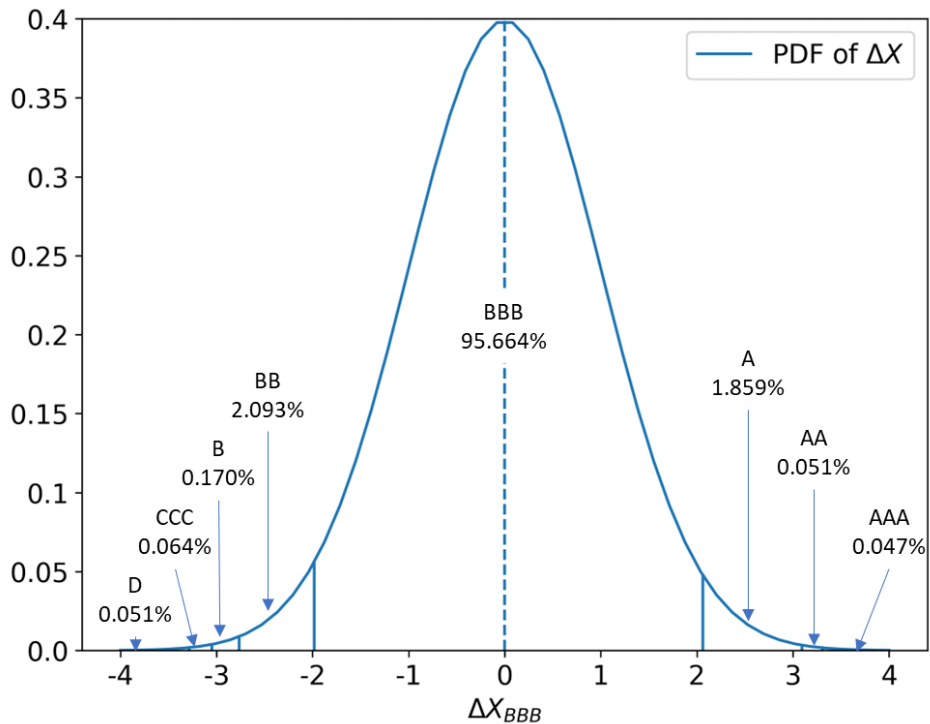


Figure 5.1: Relationship between creditworthiness indicator ΔX_t and rating transitions

5.1.1 One-parameter approach

After obtaining the smoothed historical average transition matrix, we proceed to estimate the systematic risk and weight values for the one-parameter approach by optimising Equation 3.20. The resulting systematic weight value is $\alpha = 0.186$, which yields a ΔZ_t distribution that follows $\sim N(-0.169, 1.000)$. By applying Equation 3.22, we infer that the systematic weight value indicates that 17.1% of the variance in the creditworthiness indicator can be attributed to systematic risk.

5.1.2 PCA approach

After the one-parameter approach, we estimate the systematic risk and weight values conditional on the initial credit rating for the PCA approach, by solving the least squares problem of Equation 3.23. We obtain seven sets of ΔZ_t^m and α_m values, one for each initial credit rating. The resulting mean values for the systematic risk in Table 5.3 appear generally far from zero. This is primarily due to the used numerical search procedure that finds the systematic weight for which the systematic risk factors have a variance of exactly one, resulting in some overfitting. The distributions of the systematic risk factors have mean values that differ the most from zero conditional on the boundary ratings AAA and CCC (BB and B as well).

		α_m	$\mathbb{E}[\Delta Z_t^m]$
Initial rating	AAA	0.953	-0.686
	AA	0.391	0.047
	A	0.211	-0.083
	BBB	0.174	-0.039
	BB	0.159	-0.197
	B	0.225	-0.310
	CCC	0.288	0.223

Table 5.3: Estimates of the systematic part in the multivariate case.

We then apply PCA to the correlation matrix Σ of ΔZ_t and identify seven eigenvalues and their corresponding eigenvectors (principal components) using Equation 3.12. We determine the number of principal components to retain using a scree plot and keep the ones above the scree. We observe that the slope becomes minimal from the fourth principal component onwards in Figure 5.2. Therefore, we select the first three principal components for our research. For more details, please refer to Tables B.3 and B.4 in the appendix.

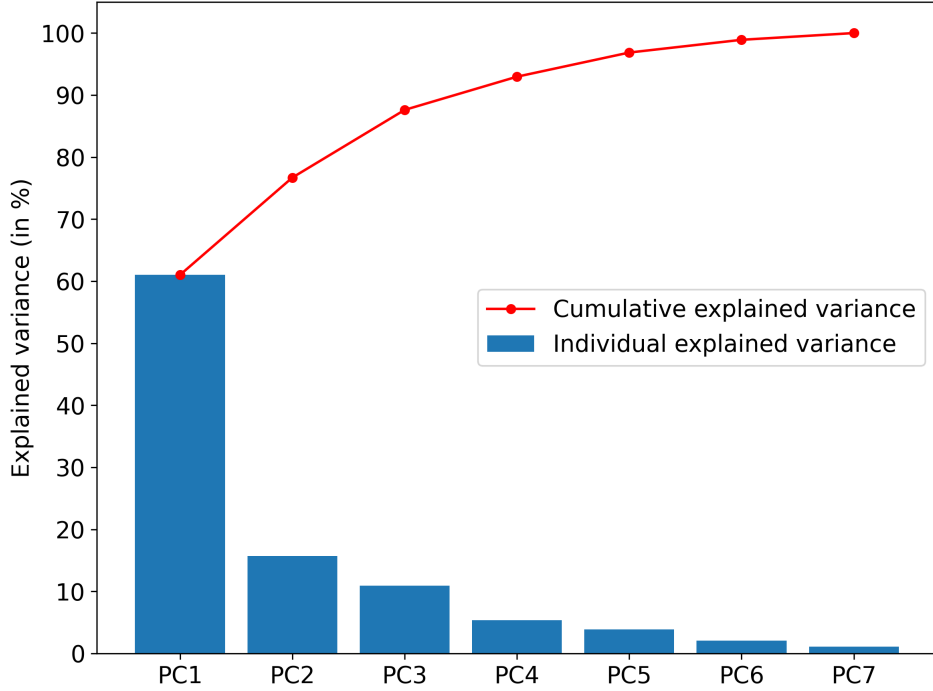


Figure 5.2: Individual and cumulative explained variance of the principal components

We find that the variance explained by the first three components is 61.0%, 15.7% and 10.9% respectively, which accumulates to 87.6% (as seen in Figure 5.2). Table 5.4 displays the principal components associated with these eigenvalues.

		PC ₁	PC ₂	PC ₃
Initial rating	ΔZ^{AAA}	0.117	0.816	0.500
	ΔZ^{AA}	0.363	0.405	-0.439
	ΔZ^A	0.439	0.066	-0.174
	ΔZ^{BBB}	0.427	-0.169	-0.158
	ΔZ^{BB}	0.444	-0.109	-0.007
	ΔZ^B	0.434	-0.106	0.069
	ΔZ^{CCC}	0.309	-0.338	0.705

Table 5.4: The first three principal components

The first component approximately represents level, the second slope, and the third some sort of curvature (Ibsen & de Almeida, 2005). Let us analyse their relationship with the original variables. We find that the first component is composed entirely of positive values, indicating that all original variables are expected to have the same direction. The magnitude of the loadings suggests that the systematic risk associated with the initial ratings that include a larger number of firms in the original dataset is assigned greater weights (Table B.1). The uniform direction of values in the level factor is as anticipated. In a half year characterised by poor credit performance due to negative systematic risk outcomes (which we visualise in Table 5.5), we see that in general, the level of all credit ratings will experience deterioration compared to Table 5.1. This level effect

is caused by the first principal component.

The second component accounts for most of the remaining variance after removing the effect of the first component. It adjusts the level effect, by distinguishing between lower and higher ratings. The slope factor captures contractions or expansions across different credit ratings. It has positive coefficients on the systematic risk variables for the highest-grade initial ratings and negative coefficients on the lowest grades. The magnitude of the loadings is particularly pronounced for the two most extreme grades.

The transition probabilities on the first off-diagonal are smallest for the ratings in the middle. This effect is caught by the third principal component. The third component, the curvature component, demonstrates a positive relationship with the single highest and the two lowest ratings but a negative relationship with those in between. This implies that in a poor credit year, the credit quality of the lowest two ratings and the highest rating deteriorates while the ratings in between show improvement. Similar to the second component, the loadings are strongest for the lowest and highest ratings.

		End-of-half-year rating							
		AAA	AA	A	BBB	BB	B	CCC	D
Initial rating	AAA	87.397%	11.767%	0.581%	0.096%	0.100%	0.019%	0.019%	0.021%
	AA	0.090%	92.859%	6.487%	0.317%	0.181%	0.027%	0.019%	0.020%
	A	0.045%	0.595%	95.081%	4.002%	0.187%	0.032%	0.030%	0.029%
	BBB	0.024%	0.028%	1.176%	95.144%	3.132%	0.285%	0.114%	0.097%
	BB	0.014%	0.018%	0.574%	1.753%	91.074%	5.744%	0.549%	0.274%
	B	0.005%	0.007%	0.290%	0.859%	2.057%	90.472%	4.971%	1.339%
	CCC	0.005%	0.005%	0.031%	0.055%	0.080%	7.243%	79.613%	12.968%

Table 5.5: Calculated transition matrix for bad half year ($\Delta Z_t = -1$)

We proceed to find the systematic weights of these principal components by minimising the least squares problem from Equation 3.25, obtaining systematic weight values of

$$\hat{\alpha} = \begin{pmatrix} 0.080 \\ 0.040 \\ 0.041 \end{pmatrix}. \quad (5.1)$$

The systematic weight is largest for the level factor. Equation 3.26 implies that the first three components account for 9.0% of the total variance in the creditworthiness indicator, which is almost half as much as the 17.1% in the one-parameter approach.

First principal component

In order to perform a more comprehensive comparison between the PCA approach and the one-parameter approach, we conduct computations using solely the first principal component. The level factor is indicative of the optimal linear combination of the estimated seven systematic risk factors that approximates the one-parameter approach. The first principal component

accounts for 61.0% of the overall variance in ΔZ_t . Our findings reveal a systematic weight of $\hat{\alpha} = 0.085$, which corresponds to the first principal component explaining 7.9% of the variance in the creditworthiness indicator.

5.2 Dynamic model

We move on to the systematic factor loadings of the \mathbf{B} parameter from Equation 3.14. We divide the historical data into a training and test set with an 80/20 split, and use the training set to determine the factor loadings. We consider the one-parameter approach first, followed by the PCA approach.

5.2.1 One-parameter approach

The one-parameter approach yields systematic factor loadings and their standard deviations shown in bold in Table 5.6. The coefficient estimates indicate a positive relationship between GDP growth and systematic risk and a negative relationship between temperature anomaly growth and systematic risk. These findings align with our expectations. Positive GDP growth positively influences systematic risk, leading to reduced credit deterioration. Conversely, positive growth of the temperature anomaly signifies heightened climate risk, resulting in more negative systematic risk and increased credit deterioration.

	Intercept	ΔGDP_t	ΔT_t
ΔZ_t	-1.541 *** (0.244)	0.789 *** (0.120)	-0.051 * (0.028)
ΔGDP_t	1.703*** (0.258)	-	0.020 (0.051)
ΔT_t	0.697 (2.406)	0.370 (1.197)	-

Table 5.6: The estimated systematic factor loadings (\mathbf{B} , in bold) using the one-parameter approach, with the standard deviations shown below the coefficients in parentheses (***) indicates significance at a confidence level of 99%, ** at 95%, and * at 90%). Estimates of the macro variables on each other are also included.

A percentage increase of GDP of 2%, which is a reasonable value when looking at the historical data from Figure 4.1a, leads to an increase in systematic risk of around 1.578. Such a value more than offsets the intercept value, which is -1.541. This implies that a biannual increase of approximately 2% is needed to have a ‘credit neutral’ (i.e. systematic risk is equal to 0) period.

Although statistically less significant, the temperature variable is economically meaningful. A percentage increase of 10%, which is a reasonable value when looking at the historical data from Figure 4.1b, leads to a decrease in systematic risk of around 0.51. We saw already in Table 5.5 that such a shift in systematic risk can have big consequences for the transition probabilities.

We compare in-sample and out-of-sample predictions using our model with fitted systematic risk in Figure 5.3, and obtain an in-sample R^2 of 0.634 and an out-of-sample R^2 of 0.371.

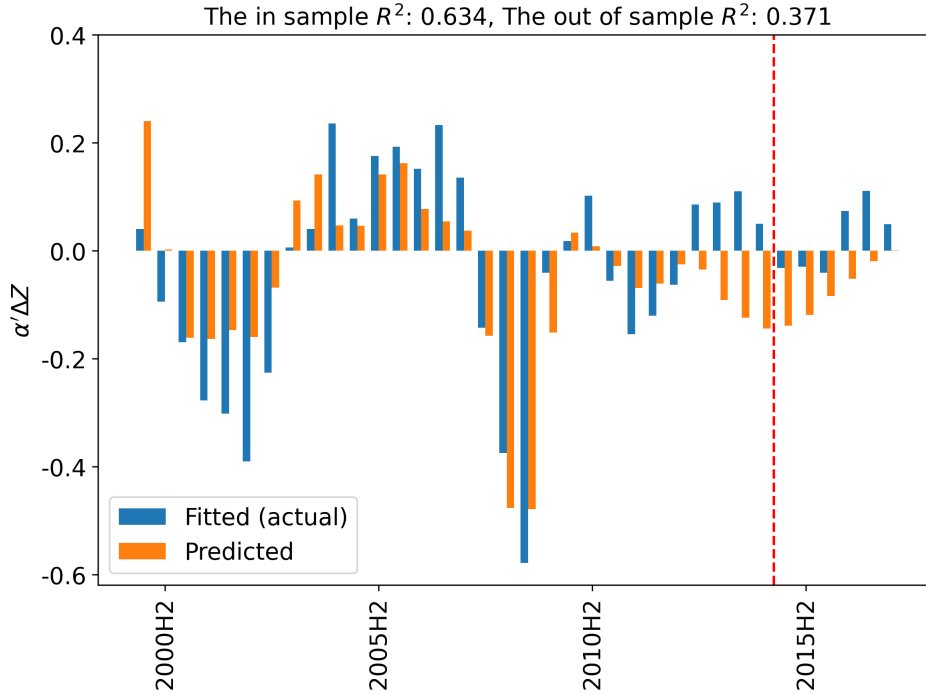


Figure 5.3: In-sample and out-of-sample predictions of the systematic part using the one-parameter approach. The red line indicates the training/test split.

Only GDP growth

If we were to focus exclusively on economic risk, specifically by considering only changes in GDP growth, the estimated systematic factor loadings on the intercept and economic risk would not change much, as is depicted in Table 5.7; both parameter values only become slightly more negative due to the absence of the negative effect that the climate risk variable has on the systematic risk.

	Intercept	ΔGDP_t
ΔZ_t	-1.576***	0.770***
	(0.253)	(0.124)

Table 5.7: The estimated systematic factor loadings (\mathbf{B}) using the one-parameter approach, with only GDP growth as explanatory variable, and the standard deviations shown below the coefficients in parentheses

However, when examining the predicted values against the actual values (Figure 5.4), a noticeable limitation of this model becomes apparent: it performs less well in adjusting for positive and adverse credit cycles, resulting in a forecast that is more concentrated around zero.

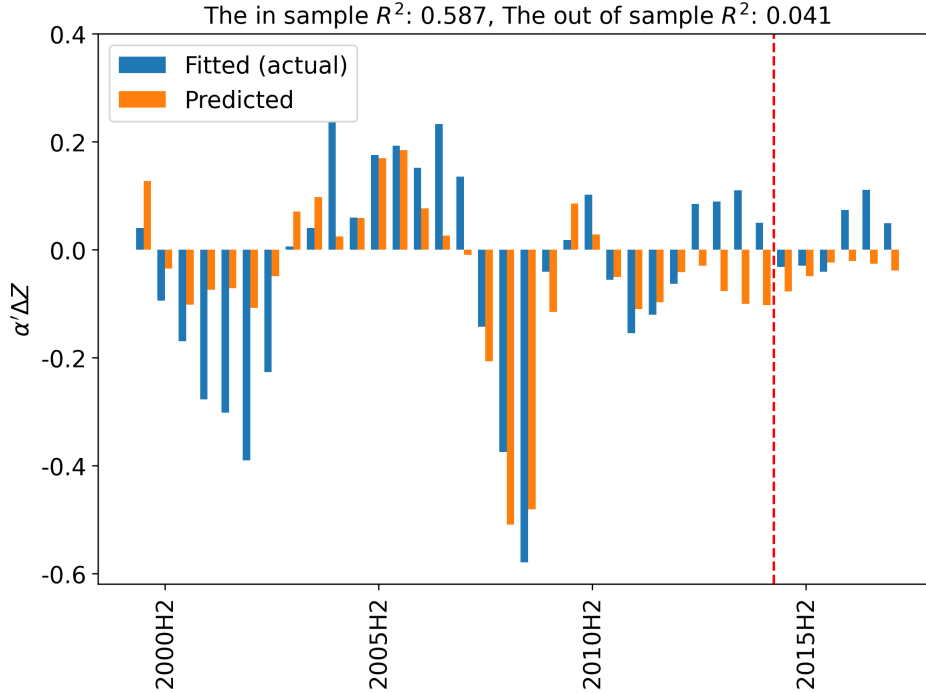


Figure 5.4: In-sample and out-of-sample predictions of the systematic part using the one-parameter approach, with only GDP growth as explanatory macro variable

The in-sample R^2 decreases only slightly for this model, but the out-of-sample R^2 is considerably lower. The inclusion of a separate climate risk variable in the form of temperature change in the model does not change the in-sample fit much, but appears to be more informative for long-horizon forecasts.

Adding an autoregressive or moving average term to the linear regression

In we were to extend Equation 3.14 by adding an AR term (Equation 3.16), the intercept and economic risk variable coefficients do not change much (Table 5.8). The coefficient of the temperature variable however, becomes less high and less significant, indicating some correlation between the temperature variable and the AR term. Adding an MA term (Equation 3.17) is not significant (Table 5.8). The addition of the MA term also has a smaller effect on the rest of the coefficients, including the one for temperature.

	Intercept	ΔZ_{t-1}	ΔGDP_t	ΔT_t	η_{t-1}
ΔZ_t	-1.503*** (0.338)	0.597*** (0.182)	0.758*** (0.143)	-0.022 (0.033)	-
ΔZ_t	-1.514*** (0.293)	-	0.767*** (0.149)	-0.041 (0.028)	0.346 (0.216)

Table 5.8: The estimated systematic factor loadings (\mathbf{B}) using the one-parameter approach, with the AR and MA additions, and the standard deviations shown below the coefficients in parentheses

If we consider the in-sample and out-of-sample predictions of the systematic part (Figure 5.5), we see that the in-sample R^2 improves for the AR(1) extension, and to a lesser extent also for the MA(1) extension. However, both extensions diminish the out-of-sample R^2 to almost zero. They both seem to overestimate the systematic risk negatively. As both extensions do not seem to improve the performance of our model, we do not include them in the scenario analysis and the rest of the research. It is shown, however, that the extension to a VAR model seems not implausible when analysing the dynamics of systematic and climate risks (see for example also the paper of Cosemans et al. (2021)).

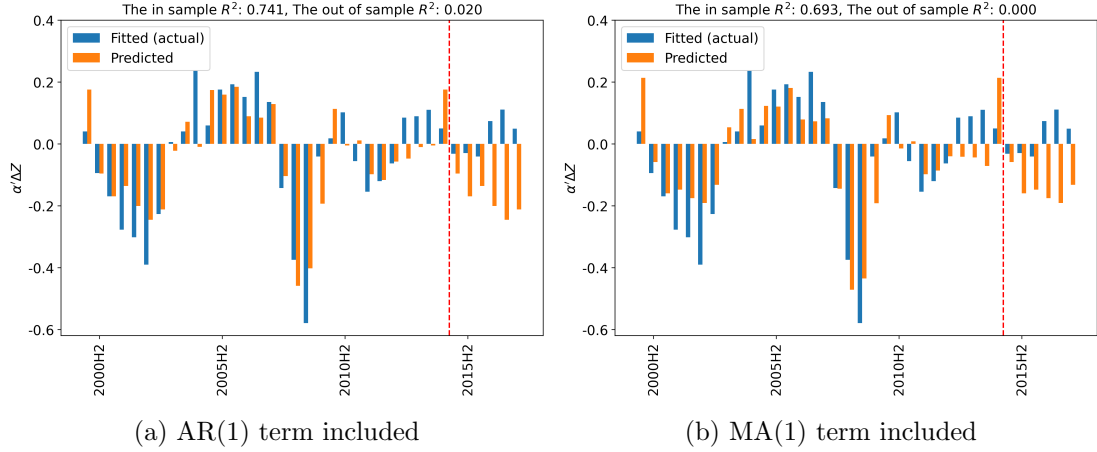


Figure 5.5: In-sample and out-of-sample predictions of the systematic part using the one-parameter approach, with an AR(1) and MA(1) included

5.2.2 PCA approach

First principal component

Our findings show that using only the first principal component to characterise level yields comparable results to the one-parameter case, as demonstrated in Table 5.9. The level factor only contains positive values, resulting in a positive relationship between systematic risk and GDP growth and a negative relationship with temperature anomaly growth. However, the factor loadings' magnitude is more than twice as large as the one-parameter model for both economic and climate risk. Nonetheless, this effect is mostly offset by the systematic weight, which is half as large for the single principal component case as compared to the one-parameter model.

	Intercept	ΔGDP_t	ΔT_t
$\hat{f}_{1,t}$	-2.750***	1.446***	-0.113**
	(0.591)	(0.299)	(0.055)

Table 5.9: The estimated systematic factor loadings (\mathbf{B}) using the first principal component in the PCA approach, with the standard deviations shown below the coefficients in parentheses

The movements of the first PCA approach appear quite similar to the one-parameter case (Figure 5.6), which was expected. The R^2 (both in-sample and out-of-sample) is worse than in

the one-parameter case, though the difference is not big.

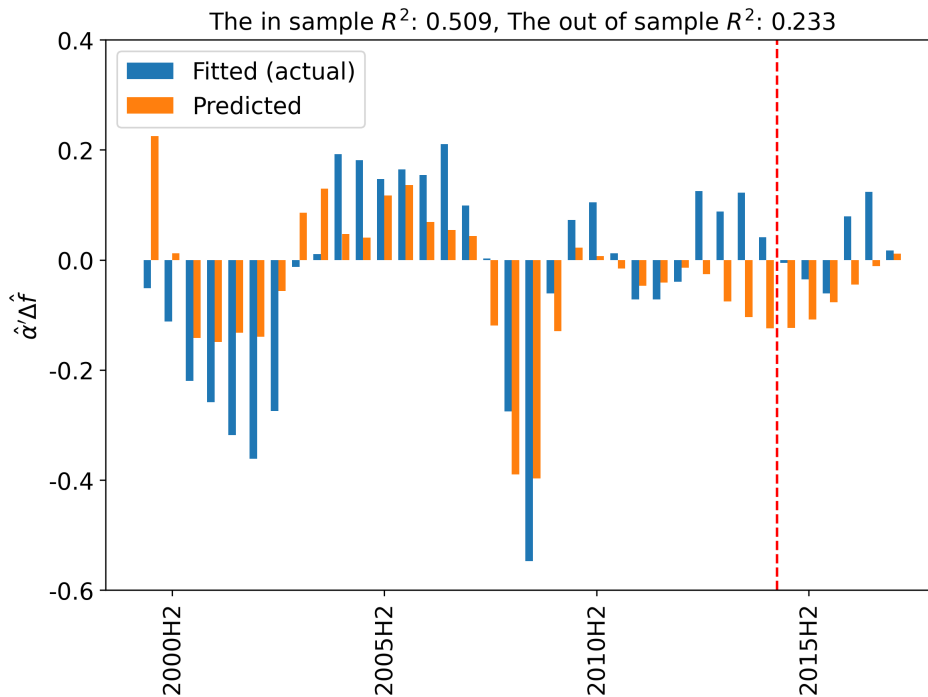


Figure 5.6: In-sample and out-of-sample predictions of the systematic part using the first principal component

Three principal components

When using three principal components, the macro variables' loadings on the level principal component naturally remain the same, since it is the same component. The curvature component has negative relationships with both GDP growth and temperature growth, as shown in Table 5.10. The slope component is also negative for the GDP growth, but is positive for the temperature growth. The parameter estimates for the second and third component are generally not significant.

	Intercept	ΔGDP_t	ΔT_t
$\hat{f}_{1,t}$	-2.750*** (0.583)	1.446*** (0.287)	-0.113* (0.067)
$\hat{f}_{2,t}$	-0.668** (0.320)	-0.081 (0.157)	0.013 (0.037)
$\hat{f}_{3,t}$	-0.018 (0.341)	-0.042 (0.168)	-0.022 (0.039)

Table 5.10: The estimated systematic factor loadings (\mathbf{B}) using the PCA approach, with the standard deviations shown below the coefficients in parentheses

To translate these results back to systematic risk, we examine the effect on the principal components. The slope factor's negative sign implies that positive GDP has a negative relation-

ship with the slope factor, lowering the systematic risk of higher ratings while raising the lower ratings (contraction). The reverse is true for temperature growth due to the positive loading of the slope factor on this variable. The negative relationships with the curvature factor imply that GDP or temperature growth lowers the systematic risk of the highest rating or the bottom two, while raising the ratings in between. The reverse is true for decreases in GDP or temperature growth. In summary, the PCA approach captures more diverse effects.

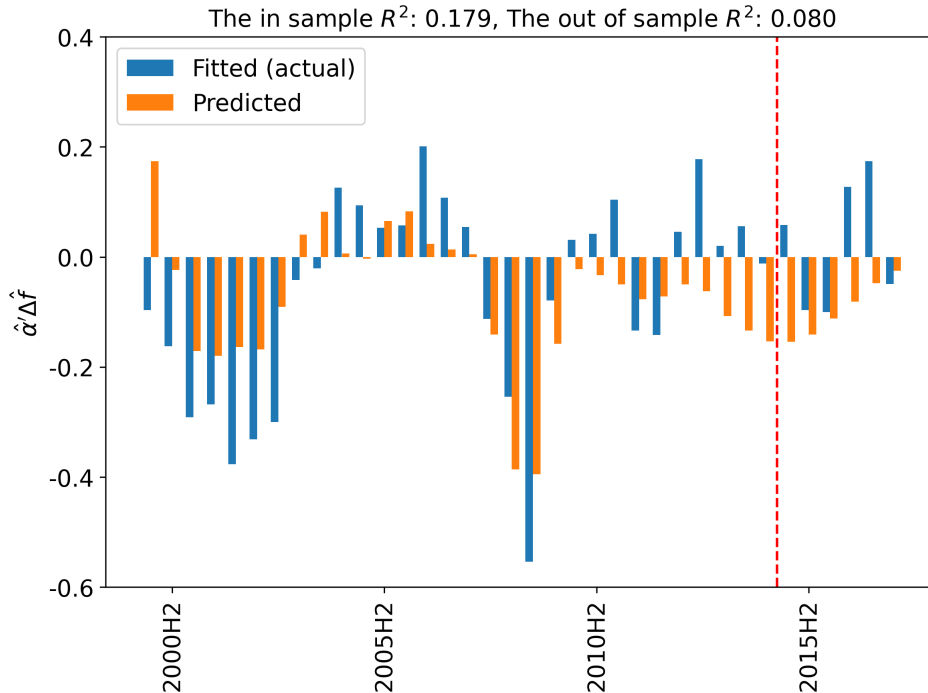


Figure 5.7: In-sample and out-of-sample predictions of the systematic part using the PCA approach

The R^2 values for the principal components are 0.509, 0.014, and 0.015 in-sample and 0.233, 0.003, and 0.004 out-of-sample. The first component fits quite accurately (Figure B.3.1), but the fit of the second and third components are not ideal. The slope factor performs poorly in predicting positive outcomes (Figure B.3.2), and the third factor's magnitude is generally too small (Figure B.3.3). The average R^2 value of this model is 0.179 in-sample and 0.080 out-of-sample. Here the consequences of the overfitting of the systematic part (Table 5.3) become evident. Thus, calculating the systematic risk factor first per initial rating, then using PCA to translate the resulting seven factors back to a three principal components, does not seem to improve performance.

As an example, we add the observed, fitted and predicted transition matrix of example year 2000H2 for both the one-parameter and the PCA approach to the appendix (Tables B.5, B.6, B.7, B.8 and B.9).

5.3 Scenario analysis

After obtaining our systematic weights and factors, we move on to the scenario analysis. To ensure maximum utilisation of available data, we re-estimate the factor loadings \mathbf{B} by employing

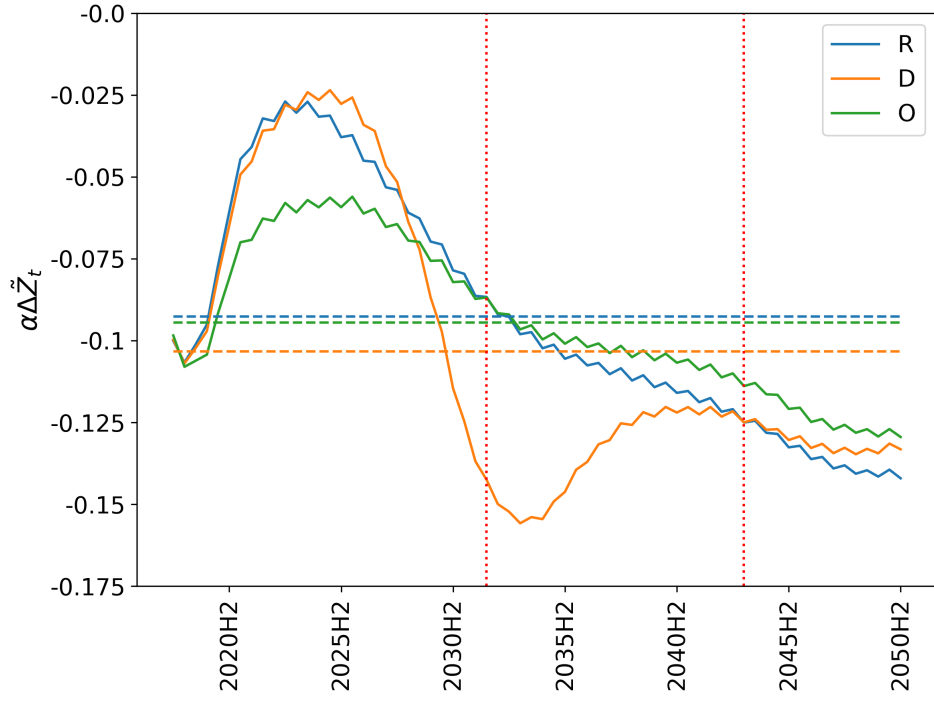
all available data. Table 5.11 displays the parameter values utilised for calibration, which are slightly different from those obtained from the 80/20 split.

	Intercept	ΔGDP_t	ΔT_t
ΔZ_t	-1.437*** (0.229)	0.776 (0.115)	-0.036 (0.024)
$\hat{f}_{1,t}$	-2.557*** (0.536)	1.412*** (0.269)	-0.082 (-0.082)
$\hat{f}_{2,t}$	-0.493 (0.367)	-0.083 (0.184)	0.038 (0.038)
$\hat{f}_{3,t}$	-0.087 (0.307)	-0.038 (0.154)	-0.027 (0.032)

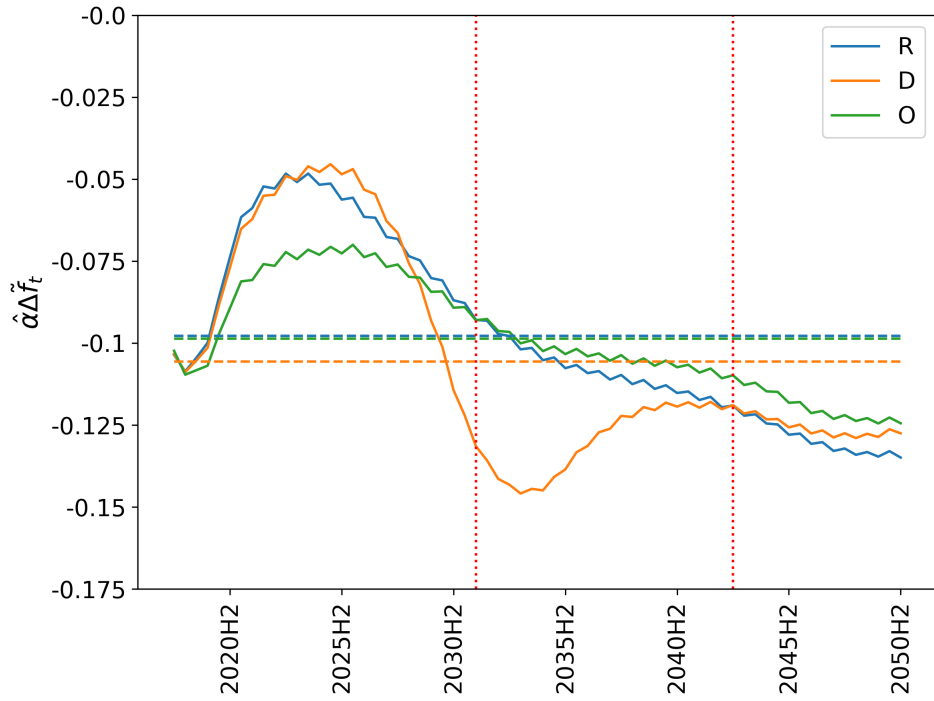
Table 5.11: The estimated systematic factor loadings (\mathbf{B}) using all data in the one-parameter and PCA approaches, with the standard deviations shown below the coefficients in parentheses

In both the one-parameter and PCA scenario, all coefficients and standard deviations bear close resemblance to the 80/20 split. The coefficients and standard deviations have only undergone a slight reduction. As a comparison, we again consider only using GDP growth as an explanatory macro variable. The intercept in this regression changes to -1.490*** (0.230) and the GDP growth coefficient changes to 0.765*** (0.117).

Using the systematic weights and revised factor loadings, we predict the future values for the systematic risk of the creditworthiness index in various scenarios (reference (R), disorderly transition (D), and orderly transition (O)). We begin by examining the results of the one-parameter approach, whereby Figure 5.8a displays the systematic aspect of the creditworthiness indicator for each of the different scenarios. We display the PIT and TTC predictions, where the TTC predictions are calculated as the average of the PIT predictions over time.



(a) One-parameter



(b) PCA

Figure 5.8: PIT (solid lines) and TTC (dotted lines) predictions of future values of systematic risk in different scenarios, with the red dotted line indicating three separate periods

The graphs indicate three distinct periods. The orderly transition towards a carbon-neutral world begins in 2020, leading to a more negative systematic risk in the initial years. However, after 2030, the systematic risk of the orderly scenario becomes more positive than the reference scenario. This marks the point where the benefits of the transition, such as mitigating certain physical risks, start to outweigh the disadvantages, such as regulatory factors (e.g., a carbon

tax) and mitigation costs. In the second period, the orderly scenario experiences the advantages of its early transition and exhibits more positive systematic risk values compared to the other scenarios. The delayed transition, starting ten years later, results in worsened systematic risk once it has begun. It is only beyond 2040 that the disorderly scenario surpasses the reference scenario again. In the last period, the disorderly scenario converges towards the orderly scenario due to the benefits of the in the previous period initiated transition. Meanwhile, the reference scenario experiences further decline due to the materialisation of physical risks.

The PCA approach, where the systematic risk factors were first estimated per initial credit rating, appears to capture different movements compared to the one-parameter approach (Figure 5.8b). Firstly, the orderly and disorderly scenarios surpass the reference scenario earlier. Secondly, the variation in absolute terms is smaller.

When comparing the TTC predictions of the PCA and one-parameter approaches, we observe that the disorderly scenario is the worst one. In the considered time period, the reference scenario remains the most positive, being only slightly better than the orderly scenario. This suggests that, over the 30-year period, the forestalled physical risks do not yet outweigh the transition risks for the orderly and disorderly scenarios in both approaches. The fact that the TTC lines are closer together in the PCA approach can be attributed to the fact that the systematic risk associated with the orderly and disorderly scenarios exceed that of the reference scenario a bit earlier, as compared to the one-parameter approach. However, we expect the TTC to become more positive for the transition scenarios when considering a longer time period, due to the more positive PIT estimates at the end of the considered period and the expectation that physical risks will worsen for the reference scenario.

The TTC systematic risk predictions shift the boundaries of the historical average transition matrix, resulting in different average boundary matrices for each scenario. These predicted average boundary matrices are then translated back to average transition matrices for the respective scenarios (Tables B.10, B.11, B.12, B.13, B.14 and B.15).

When looking at the relative percentage differences, we notice that the transition probabilities primarily change on the off-diagonal elements (Table 5.12). This is off-diagonal effect is caused by the shift in the boundaries of the normal distributions that leads to a relatively greater increase or decrease in the tails of the binned normal distribution. In the normal distributions, there is a large bin at the centre of the distributions representing the probabilities of staying in the same rating end-of-period. Contrarily, the values in the tails of the distribution that represent the probabilities of rating upgrades or downgrades, are relatively small. The further one moves away from the main diagonal, the more one moves into the tail of the normal distribution, the bigger the relative changes are. Overall, the average transition probabilities across the entire transition matrix in Table 5.12 are clearly worse for the orderly scenario, compared to the reference scenario. The fact that the changes occur mainly on the off-diagonal again highlights the importance of considering not only default probabilities but also other forms of credit migration. Such changes can have big consequences for capital requirements, which we will discuss in Section 5.4. Similar observations are made for the disorderly transition scenario (Table B.16) and under the PCA approach (Tables B.18 and B.19).

		End-of-half-year rating							
		AAA	AA	A	BBB	BB	B	CCC	D
Initial rating	AAA	-0.037%	0.294%	0.473%	0.528%	0.565%	0.605%	0.630%	0.691%
	AA	-0.587%	-0.021%	0.342%	0.490%	0.546%	0.601%	0.630%	0.691%
	A	-0.620%	-0.473%	-0.010%	0.383%	0.537%	0.584%	0.611%	0.676%
	BBB	-0.649%	-0.582%	-0.438%	-0.005%	0.387%	0.498%	0.546%	0.618%
	BB	-0.672%	-0.606%	-0.479%	-0.374%	-0.009%	0.342%	0.472%	0.564%
	B	-0.718%	-0.654%	-0.519%	-0.421%	-0.345%	-0.004%	0.337%	0.472%
	CCC	-0.718%	-0.659%	-0.604%	-0.553%	-0.517%	-0.316%	-0.007%	0.304%

Table 5.12: Relative differences of the average transition matrix of the orderly transition scenario compared to the reference scenario, using the one-parameter approach

Only GDP growth

When examining GDP growth as the sole explanatory macro variable, the systematic risk predictions depicted in Figure 5.9 are obtained. Observing the 30-year time-frame, the TTC estimation of the reference scenario exhibits the most positive systematic risk. It is only after 2035 that the PIT systematic risk estimates of the orderly transition scenario becomes more positive than the reference scenario. For the disorderly scenario, this is only after 2045. These points are further ahead in time compared to the scenarios in Figure 5.8, where temperature was also included. The effect of the transition risks is caught by the economic risk variable. The model's limitation seems to be in its failure to properly incorporate physical risks, resulting in a more positive TTC systematic risk estimate for the reference scenario over the 30-year time span.

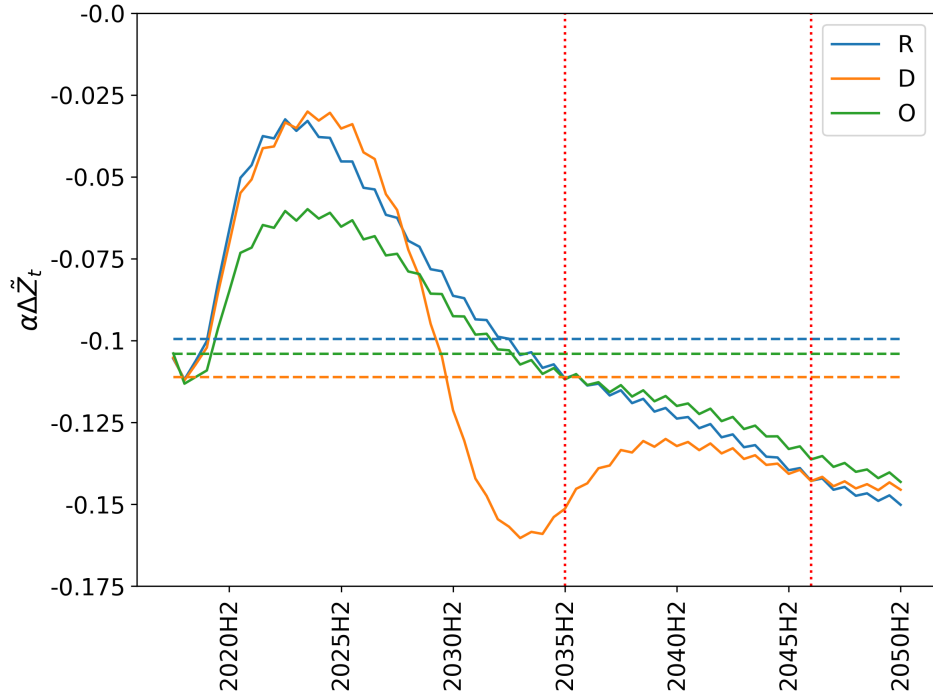


Figure 5.9: PIT (solid lines) and TTC (dotted lines) predictions of future values of systematic part using the one-parameter approach, with only GDP growth as explanatory macro variable

5.4 Implications for risk management

Finally, we turn our attention to the implications for risk management by examining the effects on expected loss and economic capital. We focus on two portfolios characterised by different initial rating compositions. Each portfolio consists of 2,000 loans, with an EAD of exactly €1 per loan and a LGD of 0.45. We make use of time steps of exactly one year. We assume that the portfolios are re-balanced at the start of each time period. Figure 5.10 illustrates the expected losses of the investment-grade portfolio using the two different approaches. Notably, the graphs mirror the movements observed in the systematic risk predictions but in the opposite direction. The transition dips are evidently pointing in the opposite direction compared to Figure 5.8. As expected, a more negative systematic risk translates into a higher probability of default, leading to more expected losses. In absolute terms, the expected losses are small: between approximately €0.80 (0.040%) and €1.30 (0.065%) per year of the total €2,000. This can be mainly attributed to the small systemic weights of 17.1% for the one-parameter approach, and 9.0% for the PCA approach. Furthermore, we observe the same distinct movements in systematic risk that the PCA approach appears to capture; the lines in Figure 5.10b exhibit slightly less volatility.

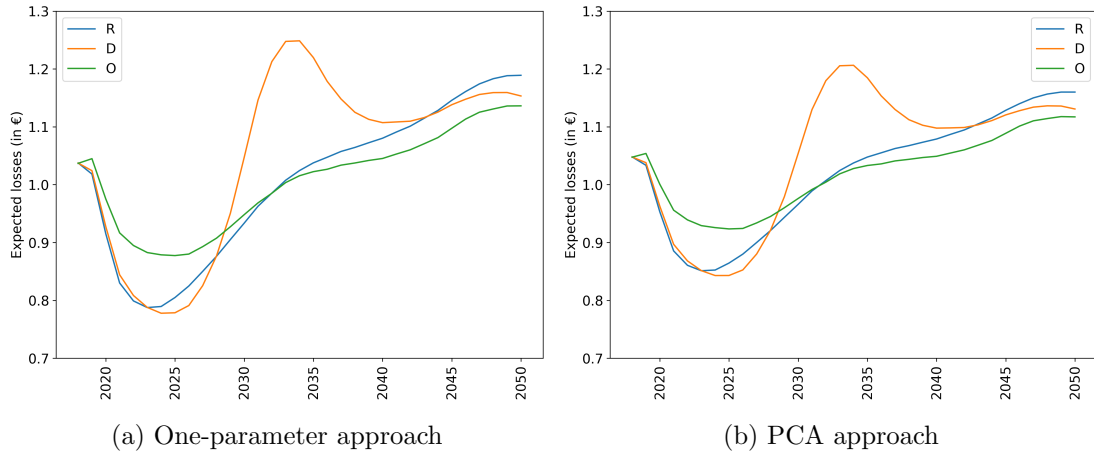


Figure 5.10: Expected losses of the investment-grade portfolio using the two different approaches

Let's consider economic capital. Economic capital is a function of the expected loss, calculated as the difference between the 99.9% quantile and the mean (expected loss) of the loss distribution. The graphs of economic capital exhibit similar patterns (Figure 5.11), which was of course expected. In both approaches, the lines simply shift towards higher values. However, the variability in economic capital values is larger in absolute terms compared to expected losses, as the values are derived from the tail of the loss distribution. Another notable difference is that economic capital is higher in the one-parameter approach compared to the PCA approach, whereas expected losses were higher in the PCA approach. This discrepancy arises from the way that the 99.9% quantile is calculated in Equation 3.19. The systematic weights is smaller in the PCA approach, resulting in a smaller shift when computing economic capital. A higher systematic weight leads to higher estimates of economic capital. Thus, the one-parameter approach can here be considered as the more conservative approach. We observe similar findings for the high-yield portfolio, with the main distinction being that the high-yield portfolio exhibits higher expected losses and economic capital (Figures B.5.1 and B.5.2), as anticipated.

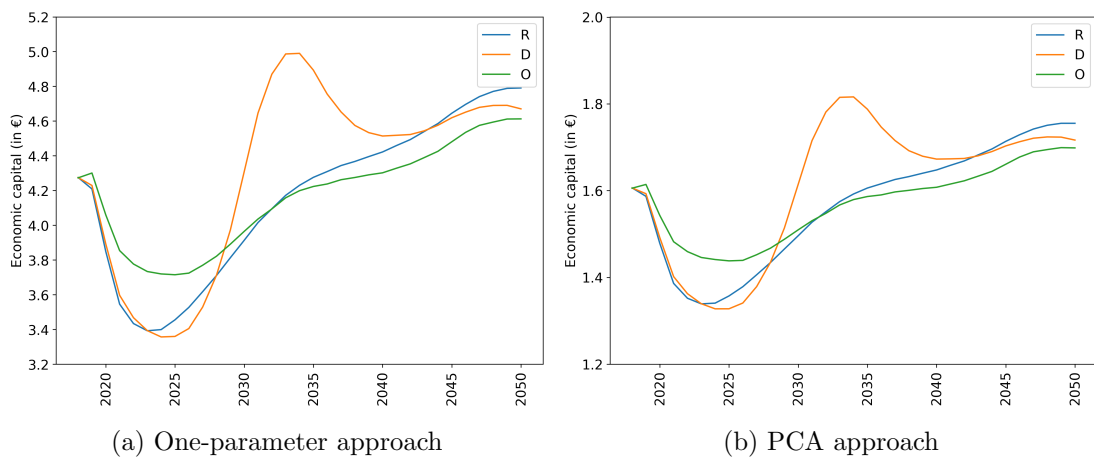


Figure 5.11: Economic capital of the investment-grade portfolio using the two different approaches

Let us examine the scenarios in relative terms (Figure 5.12). When comparing the orderly and disorderly scenarios to the reference scenario, the differences become more evident. The

findings indicate that an investment-grade portfolio experiences a peak of over 12.5% additional expected losses per year in the orderly scenario compared to the reference scenario using the one-parameter approach. A little more than a decade after the start of the transition, the orderly scenario begins to benefit from reduced expected losses, reaching approximately 5% less expected losses per year. These advantages continue to increase over time as physical risks become more apparent. Transition risks in the disorderly scenario are higher, reaching approximately twice as much relative expected losses compared to the orderly scenario. Besides that, the disorderly scenario takes longer to surpass the reference scenario again. This of course highlights the benefits of an early transition towards a carbon neutral world. Using the PCA approach, the relative expected losses of our investment-grade portfolio is lower due to the reduced variability in systematic risk. The peaks of transition risk in expected losses are less pronounced in the PCA approach, and we observe that the orderly and disorderly scenarios cross the reference scenario earlier compared to the one-parameter approach. The disadvantages of the transition are lower in the PCA approach, while the benefits of mitigating physical risks remain the same as in the one-parameter approach. Similar results are observed for the high-yield portfolio, although the differences are smaller (Figure 5.13).

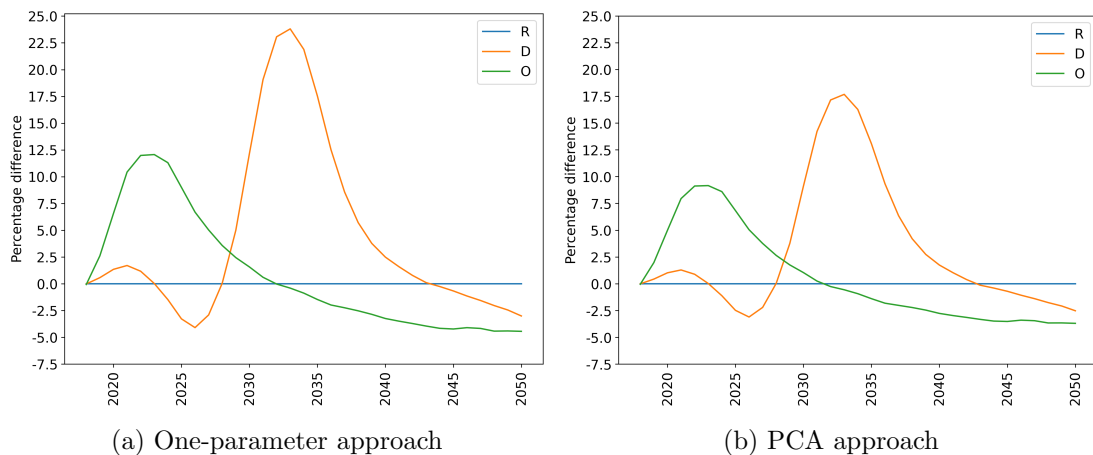


Figure 5.12: Expected losses of the investment-grade portfolio using the two different approaches

Ultimately, both approaches converge to approximately the same values. This suggests that the one-parameter approach estimates higher expected losses and economic capital for transition risks compared to the PCA approach. However, when transition risks become minimal and only physical risks remain in the reference scenario, both models seem to estimate these physical risks similarly.

Analysing the systematic risk weights and loadings, we find that the temperature effect is quite similar in both approaches, while they differ in the loadings on GDP growth. Examining the input NGFS macro variables (Figure 4.5a), we observe a decline in GDP growth towards the end of the 30-year period. This implies that in our research, transition risks mainly emerge in GDP growth, while increased temperature growth primarily manifests itself in physical risks.

Turning to the relative differences of expected losses in high-yield portfolios (Figure 5.13), we notice that they are lower compared to those of investment-grade portfolios. This discrepancy arises because the absolute expected losses of high-yield portfolios are already considerably larger

than those of investment-grade portfolios, resulting in a lower relative difference. The relative differences in economic capital follow a similar pattern for the same reason (Figures B.5.3 and B.5.4).

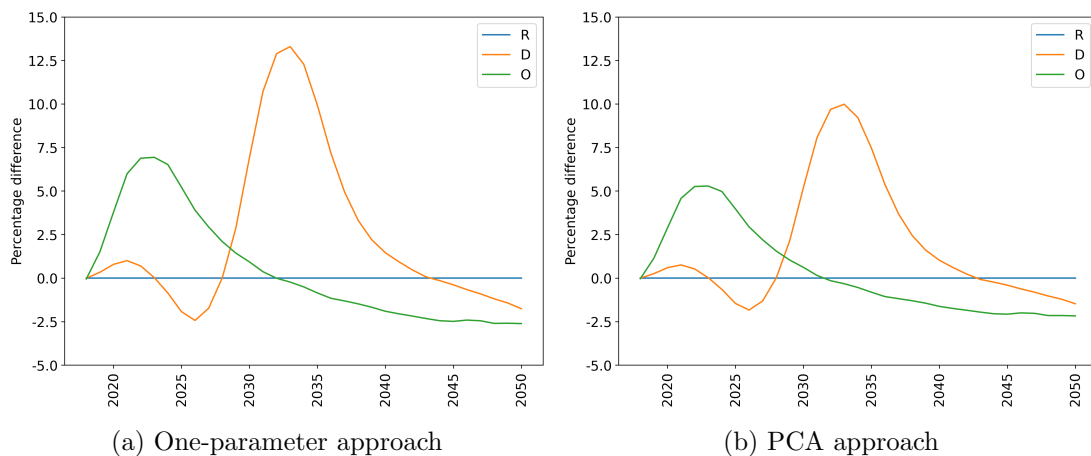


Figure 5.13: Expected losses of the high-yield portfolio using the two different approaches

We investigate the consequences of the resulting implications of CER for credit risk management a bit more specifically. A bank needs to hold the calculated economic capital as capital requirements. Using the one-parameter approach, the economic capital estimates of the disorderly transition scenario are on average 3.49% higher than those of the reference scenario in the considered time period. For the orderly scenario, we find 0.68%. This implies that the disorderly scenario gives us a relative difference in economic capital that is 2.81% higher than that of the orderly scenario. On every euro a bank needs to hold as a capital requirement, it can not use this euro to invest, and make a potential profit. If one assumes that the bank makes a 10% yearly return on its investments, the returns would be much worse under the disorderly transition, costing the bank money each year. These results would be even be stronger if we were to add migration costs due to the various impairments resulting from changes across the whole transition matrix.

5.5 Robustness

For quality assurance purposes, we conduct a robustness test. We investigate what the consequences are when changing the chosen 80/20 training/test split ratio to 70/30. We find that changing the split for the one-parameter approach slightly improves the in-sample fit, yielding an R^2 of 0.717 compared to 0.634, but reduces the out-of-sample R^2 from 0.371 to 0.034. For the PCA approach, using a 70/30 split decreases the average R^2 values from 0.509 to 0.207 in-sample, and from 0.233 to 0.039 out-of-sample. The effect of altering the training/test split ratio is quite big, mainly for the out-of-sample fit. The main reason for this is the small amount of data used.

Chapter 6

Conclusion

The aim of this research was to evaluate the effect of CER on the credit migration of a bank loan portfolio. We find that including climate risk has a substantial effect on credit migrations and should be taken into account when considering the credit risk of a bank loan portfolio. The main findings of this paper are threefold; findings concerning adding the climate variable temperature as an explanatory macro variable, the implications for risk management, and the relative performance of the PCA approach to the widely adopted one-parameter approach.

Firstly, incorporating temperature anomaly growth as an explanatory macro variable improves the predictive capability of assessing systematic risk during a climate risk scenario analysis of credit risk. Although the climate variable is less significant, the addition of it is economically meaningful. When focusing solely on economic risk represented by GDP growth, our findings indicate that the model underestimates systematic risk. This underestimation can be primarily attributed to the model's failure to adequately account for physical risks.

Secondly, our analysis reveals that in the considered 30-year time-span the reference scenario yields the most favourable outcomes in terms of expected losses and economic capital, both in absolute and relative terms. The reference scenario is more favourable because of the high costs of the transition. However, it is expected that the transition scenarios will become much more advantageous in the long-run, due to the forestalled materialisation of physical risks. In the initial decade of the transition period, the yearly expected losses of the orderly scenario exceed the reference scenario up until almost 12.5% for the investment grade portfolio when employing the one-parameter approach. However, within a decade into the transition, the orderly scenario dives below the reference scenario by exhibiting lower expected losses. This trend persists as physical risks are mitigated, resulting in a gradual reduction of yearly expected losses, reaching almost -5% at the end of the 30-year period. The disorderly scenario experiences a transition peak in yearly expected losses that is approximately twice as high, and takes more time to become more favourable than the reference scenario. These results also apply to economic capital, albeit with smaller differences relatively. The consequences of the higher economic capital charges for the disorderly scenario relative to the orderly scenario underline the importance of an early transition. Furthermore, our analysis highlights that the majority of changes in the transition matrices occur off the diagonal, emphasising the importance of considering not only default but all forms of credit migration. We naturally observe the high yield portfolio to have more expected losses and economic capital than the investment grade portfolio, but the relative difference is

smaller due to the already higher expected defaults. The PCA approach displays less volatility in the risk management results due to the smaller systematic weights, making the one-parameter approach the more conservative one.

Thirdly, our findings suggest that the widely adopted one-parameter approach, although simplistic, appears quite powerful by outperforming the more extensive PCA approach. The superior performance of the one-parameter approach primarily stems from its ability to mitigate overestimation issues when estimating systematic risks based on initial ratings. The level component exhibited poorer performance compared to the one-parameter case, while the slope and curvature components fared even worse. Overall, the one-parameter approach exhibited greater significance and proved to be more suitable for predicting future values of systematic risk, and accurate assessment and modelling of credit risk.

To summarise, our findings indicate that incorporating CER into credit risk management significantly impacts economic capital and overall credit migration predictions using the PCA approach, and even more so with the one-parameter approach. The magnitude of these relative differences is greatest for portfolios with higher initial ratings. Thus, we recommend that (central) banks integrate CER into their credit risk management protocols under the reviewed scenarios to address the potential risks posed by climate change. Additionally, we recommend employing the straightforward yet robust one-parameter approach, preferably incorporating all forms of credit migration rather than solely focusing on default.

Chapter 7

Discussion

Every econometric model, including ours, inevitably falls back on assumptions that simplify the infinitely more complex reality. Our research focuses on the effect of incorporating climate risk into the assessment of credit risk and makes certain assumptions regarding which macro variables to select, what data to use, and on which models to employ. The reader should note that there are different approaches that could have been used to assess the effects of climate risks. In this discussion we will mention the consequences of some assumptions and shortcomings together with suggestions for further research on assessing the effect climate risk has on credit risk.

Let us begin by addressing the data shortage. As expected, the amount of data used to parameterise risk models has a significant impact on the results. In our case, we estimate the model using a relatively small amount of data and then utilise this model to make predictions for a 30-year time span. Over time, the value of information naturally diminishes. Even predicting macro-financial trends for just one year is already quite challenging, and it becomes even more speculative when looking five years ahead. Moreover, using a 30-year time frame is necessary to capture certain climate effects that only manifest themselves far into the future. To build a model around systematic risk, we considered an 18-year historical period. Within this period, we utilised a set of 36 half-yearly transition matrices to estimate the systematic risk factors and weights within our sample. Each transition matrix consists of 7 initial ratings and 8 end-of-period ratings. This implies estimating 56 values when, for example, computing the historical average transition matrix or making predictions for future transition matrices. The data shortage became more apparent when we estimated the systematic part conditional on the initial ratings, as it led to overfitting and negatively impacted the overall performance for the PCA approach. Having a more extensive dataset to calibrate the model could have improved the quality of predictions by allowing for the inclusion of a greater number of observed business cycles. However, two challenges arise in this context. Firstly, compiling such a dataset can be difficult and expensive. Secondly, going too far back in time may introduce non-representative data due to structural shifts in economic relationships.

In addition to the data's length, its nature also influences the research. We specifically chose two macro variables to estimate the systematic risk factors. Bolder (2022) aptly described the process of selecting variables: "Selecting the proper set of systematic risk factors is not unlike putting together a list of invitees for a wedding. The longer the list, the greater the cost. Some

people simply have to be there, some would be nice to have but are unnecessary, while others might actually cause problems.” In our case, we opted to invite only one proxy for economic risk and one for climate risk to our ‘wedding’. We based the selection of these two invitees on previous literature. It could be argued that only having two variables is not extensive enough. The linear regression model could naturally be expanded to include more or different variables, allowing for different model assumptions and interactions between the variables, in order to capture various climate effects. We leave this as a suggestion for future research.

Another point of discussion is our research within the context of the conservatism in which risk management is generally shrouded. The estimated systematic weights that we used to estimate economic capital are lower than the values prescribed by regulatory bodies such as BCBS to estimate regulatory capital. Assigning higher systematic weights results in a more conservative approach, suggesting that our predicted systematic risk values might underestimate regulatory capital requirements. In fact, changing the systematic weight from 17.1% to, for example, 30%, would almost double the estimates of expected losses and economic capital. Furthermore, the use of the CreditMetrics approach, which heavily relies on the Gaussian assumption, underestimates tail risk, as indicated by MacKenzie and Spears (2014). By using higher systematic weights and a t distribution, we could have introduced more conservatism.

One more factor contributing to extra conservatism and higher values of economic capital during economic downturns is the inclusion of estimates for impairments due to credit migration. The economic capital approximations in this paper solely focus on the credit-risk economic capital associated with default events. However, transitioning from one credit state to another has implications for the spread (Bolder, 2022). The aggregate migration effect, which depends on the overall magnitude of credit spreads, is expected to be relatively smaller. There is even the possibility of credit gains that can offset loss outcomes. Although the precise magnitude of the migration effect varies based on the initial portfolio composition, it is generally anticipated to be comparatively smaller than default losses. It would be valuable to include conditional migration losses as well, and we suggest this as an avenue for future research.

A final suggestion for a meaningful addition to this research is the incorporation of the influence of climate risk on the LGD. In this paper, we only consider the income effect, which is the influence of CER on the PD. It would be worthwhile to also examine the wealth effect, which is the influence of CER on LGD. One possible approach is presented by Keijsers et al. (2014), who demonstrate that macroeconomic variables, the PD, and LGD of bank loans share common cyclical components. Their model links macroeconomic variables to the PD and LGD, utilising latent factors that follow autoregressive processes, accommodating both time and cross-sectional variation.

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Appendix A

Derivation basic threshold model

To model the creditworthiness index, we make use of a threshold default model (Merton (1974) and Vasicek (2002)). To substantiate this approach, we start by specifying the asset-return dynamics. We describe the asset values of an obligor as the sum of a systematic component (aZ) and an idiosyncratic component ($b\epsilon$),

$$y = aZ + b\epsilon, \tag{A.0.1}$$

where Z and ϵ are i.i.d. $\mathcal{N} \sim (0, 1)$, and $a, b \in \mathbb{R}$ are selected such that also $y \sim \mathcal{N}(0, 1)$. The systematic risk captures a common effect impacting all obligors. This common effect can be approximated by, for example, a global macroeconomic variable. The parameters a and b determine the relative importance of the systematic and idiosyncratic components. In a fine-grained portfolio, the idiosyncratic risks are diversified away, and the systematic risk drives the variance.

We now have a random characterisation of the firms assets. Because we want to describe default as the event that the value of a firm's assets falls below its liabilities, we define K to represent these liabilities. Using Merton (1974), we can write down the event of default of an obligor as

$$\mathcal{D} \equiv \{y \leq K\}. \tag{A.0.2}$$

Thus, if y falls below K given realisations of Z and ϵ , default occurs. We can summarise this as

$$\mathcal{D} = \begin{cases} 1 & : \text{default occurs when } y \leq K, \\ 0 & : \text{survival if } y > K. \end{cases} \tag{A.0.3}$$

Continuous time Merton

Building on this background, we continue to specify the asset-return dynamics. Using the threshold model, we introduce the latent creditworthiness index that according to Merton (1974) relates to the value of an obligor's assets (or asset returns). He uses a stochastic differential equation (SDE) to specify the creditworthiness process of an obligor's latent creditworthiness.

Following Bolder (2022), we use a slight generalisation of this SDE,

$$d\tilde{X}(t) = \sum_{j=1}^J \tilde{\beta}_j dZ_j(t) + \tilde{\sigma} dW(t), \quad (\text{A.0.4})$$

which is the intertemporal dynamic of an obligor's creditworthiness index. The stochastic element in the first term of the equation is a correlated system of J Brownian motions, that have a correlation matrix $\tilde{\mathbf{S}}$ capturing the dependence between these factors. The individual Z_j each represent a systematic risk factor. The stochastic element in the second term of the equation represents the idiosyncratic risk associated with a single obligor, and is described by a standard scalar Wiener process. Each W is by construction i.i.d. of another and the correlated systematic risk factors. In a fine-grained portfolio, the correlated system of Brownian motions and the systematic $\tilde{\beta}_j$ parameters together are responsible for the portfolio effects. Negative outcomes of the systematic risk push the risk of default upwards, and therefore function as the driver of default dependence.

Discretisation

Bolder (2022) simplified the continuous-time framework by discretising it through the use of the Euler-Maruyama method. According to Bolder, models in risk management are typically implemented in discrete time as they are easier to handle mathematically. To ease the notational burden, Bolder set the time interval Δt to 1, resulting in the following discrete-time expression for Equation A.0.4,

$$\Delta X = \sum_{j=1}^J \beta_j \Delta Z_j + \sigma \Delta w, \quad (\text{A.0.5})$$

where $\Delta \mathbf{Z} \sim \mathcal{N}(0, \mathbf{\Omega})$ and $\Delta w \sim \mathcal{N}(0, 1)$. The instantaneous correlation matrix $\tilde{\mathbf{S}}$ for the correlated Brownian system \mathbf{Z} has been replaced by the non-instantaneous correlation matrix $\mathbf{\Omega}$.

Normalisation

To enhance the practical interpretation of the model structure and its results, we aim to clarify the relative importance of the systematic and idiosyncratic elements and standardise the creditworthiness increments. These adjustments are incorporated into the β and σ parameters. To standardise Equation A.0.5, we divide the systematic component by its standard deviation, which is defined as

$$\sigma(\beta \Delta \mathbf{Z}) = (\beta' \mathbf{\Omega} \beta)^{\frac{1}{2}}. \quad (\text{A.0.6})$$

The above standing description with the standard deviation of the systematic component of the creditworthiness index is a rather complicated function of the β parameters and the covariance

structure of the systematic risk factors $\mathbf{\Omega}$. We therefore rewrite Equation A.0.5 to

$$\Delta X = \alpha \sum_{j=1}^J B_j \Delta Z_j + \sqrt{1 - \alpha^2} \Delta w, \quad (\text{A.0.7})$$

where $\alpha \in (0, 1) \subset \mathbb{R}$ and $\sigma = \sqrt{1 - \alpha^2}$ (Bolder, 2022). We have defined

$$\mathbf{B} \equiv \boldsymbol{\beta} (\boldsymbol{\beta}' \mathbf{\Omega} \boldsymbol{\beta})^{-\frac{1}{2}}. \quad (\text{A.0.8})$$

Besides this more breve notation, we have introduced the systematic weight α , which is a parameter that determines the relative importance of the systematic and idiosyncratic components. We have now derived a discretised and normalised creditworthiness index, $\Delta X \sim \mathcal{N}(0, 1)$.

Appendix B

Graphs and tables

B.1 The threshold model

Initial rating	AAA	104	(70.9)
	AA	791	(358.1)
	A	1812	(66.9)
	BBB	1739	(148.0)
	BB	1050	(127.1)
	B	1234	(362.4)
	CCC	162	(52.9)

Table B.1: Mean of number of firms per credit rating at the beginning of the period, with standard deviations in parentheses

		End-of-half-year rating							
		AAA	AA	A	BBB	BB	B	CCC	D
Initial rating	AAA	90.867%	8.666%	0.355%	0.056%	0.056%	0.000%	0.000%	0.000%
	AA	0.166%	95.077%	4.548%	0.191%	0.004%	0.014%	0.000%	0.000%
	A	0.006%	0.977%	96.180%	2.688%	0.107%	0.017%	0.011%	0.014%
	BBB	0.007%	0.051%	1.860%	95.703%	2.094%	0.170%	0.064%	0.051%
	BB	0.008%	0.027%	0.041%	2.617%	92.757%	4.059%	0.339%	0.154%
	B	0.000%	0.014%	0.033%	0.079%	3.001%	92.479%	3.561%	0.833%
	CCC	0.000%	0.000%	0.059%	0.038%	0.137%	10.084%	80.225%	9.458%
	D	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	100.000%

Table B.2: Historical average transition matrix before smoothing

B.2 Principal component analysis

λ_1	4.271
λ_2	1.098
λ_3	0.764
λ_4	0.375
λ_5	0.273
λ_6	0.144
λ_7	0.076

Table B.3: Eigenvalues of correlation matrix Σ

		PC ₁	PC ₂	PC ₃	PC ₄	PC ₅	PC ₆	PC ₇
Initial rating	ΔZ^{AAA}	0.117	0.816	0.500	-0.115	0.237	-0.035	0.019
	ΔZ^{AA}	0.363	0.405	-0.439	0.253	-0.491	0.365	0.271
	ΔZ^A	0.439	0.066	-0.174	0.491	0.127	-0.515	-0.501
	ΔZ^{BBB}	0.427	-0.169	-0.158	0.039	0.698	0.103	0.515
	ΔZ^{BB}	0.444	-0.109	-0.007	-0.455	0.072	0.505	-0.569
	ΔZ^B	0.434	-0.106	0.069	-0.547	-0.364	-0.540	0.270
	ΔZ^{CCC}	0.309	-0.338	0.705	0.418	-0.250	0.207	0.115

Table B.4: Principal components and their loadings on the systematic risk factors conditional on the initial ratings

B.3 Linear regression model

B.3.1 Predictions of separate components

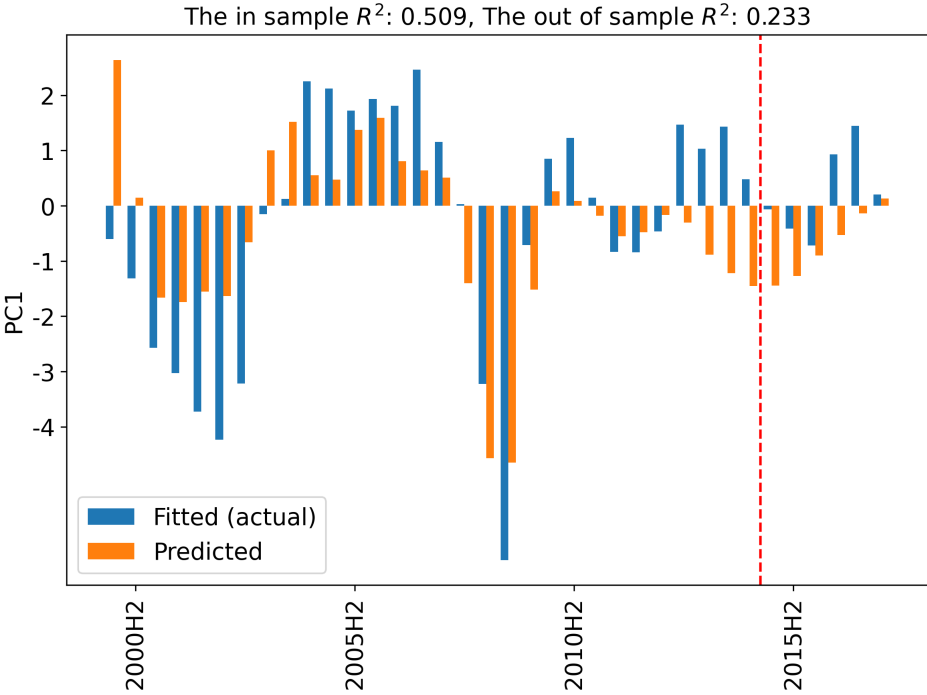


Figure B.3.1: PIT estimates of only the first principal component

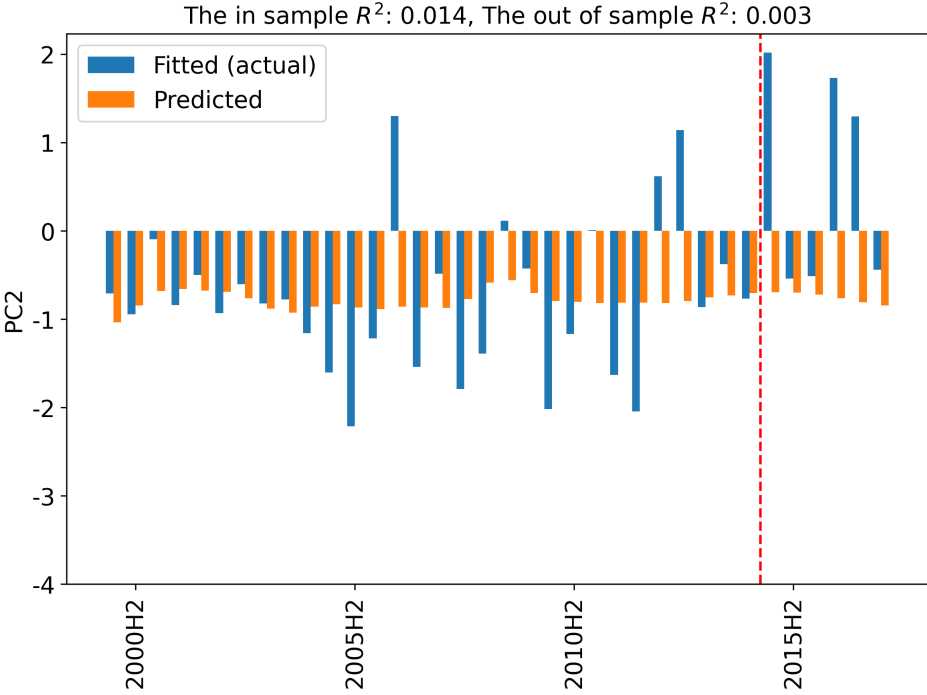


Figure B.3.2: PIT estimates of only the second principal component

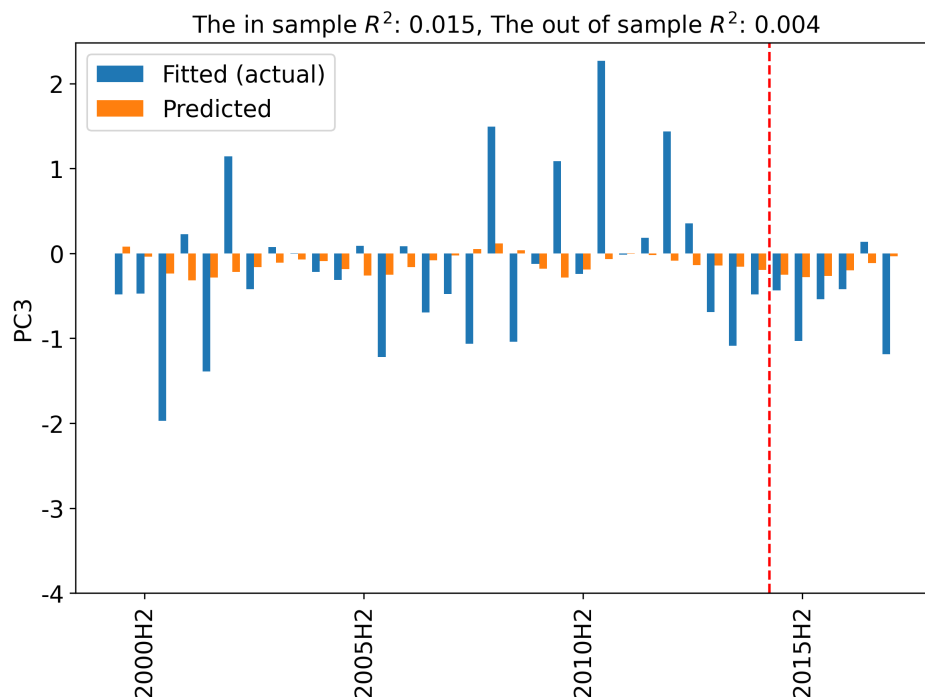


Figure B.3.3: PIT estimates of only the third principal component

B.3.2 Comparison of observed, fitted and predicted transition matrices

		End-of-half-year rating							
		AAA	AA	A	BBB	BB	B	CCC	D
Initial rating	AAA	92.309%	6.833%	0.858%	0.000%	0.000%	0.000%	0.000%	0.000%
	AA	0.568%	95.601%	3.501%	0.341%	0.000%	0.000%	0.000%	0.000%
	A	0.000%	2.068%	93.572%	4.136%	0.174%	0.000%	0.000%	0.061%
	BBB	0.000%	0.164%	1.295%	95.847%	1.799%	0.113%	0.391%	0.391%
	BB	0.000%	0.281%	0.000%	1.842%	92.071%	5.078%	0.552%	0.187%
	B	0.000%	0.000%	0.000%	0.504%	2.754%	90.415%	3.563%	2.754%
	CCC	0.000%	0.000%	0.761%	0.000%	0.758%	5.299%	78.787%	14.395%

Table B.5: Observed transition matrix 2000H2

		End-of-half-year rating							
		AAA	AA	A	BBB	BB	B	CCC	D
Initial rating	AAA	89.187%	10.163%	0.458%	0.074%	0.075%	0.014%	0.014%	0.014%
	AA	0.122%	93.978%	5.467%	0.248%	0.137%	0.020%	0.014%	0.014%
	A	0.062%	0.762%	95.664%	3.303%	0.142%	0.024%	0.022%	0.021%
	BBB	0.034%	0.038%	1.479%	95.490%	2.580%	0.222%	0.086%	0.071%
	BB	0.020%	0.025%	0.737%	2.134%	91.606%	4.840%	0.433%	0.206%
	B	0.007%	0.010%	0.380%	1.071%	2.466%	90.810%	4.201%	1.056%
	CCC	0.007%	0.007%	0.043%	0.073%	0.105%	8.551%	80.074%	11.140%

Table B.6: Fitted transition matrix 2000H2 using one-parameter approach ($\alpha\Delta Z_t = -0.094$)

		End-of-half-year rating							
		AAA	AA	A	BBB	BB	B	CCC	D
Initial rating	AAA	90.877%	8.630%	0.353%	0.055%	0.055%	0.010%	0.010%	0.010%
	AA	0.167%	94.986%	4.522%	0.189%	0.102%	0.014%	0.010%	0.010%
	A	0.087%	0.981%	96.106%	2.672%	0.106%	0.017%	0.016%	0.014%
	BBB	0.048%	0.052%	1.869%	95.666%	2.083%	0.168%	0.064%	0.051%
	BB	0.029%	0.035%	0.952%	2.604%	91.894%	4.003%	0.334%	0.151%
	B	0.010%	0.014%	0.501%	1.341%	2.961%	90.875%	3.484%	0.814%
	CCC	0.010%	0.010%	0.060%	0.099%	0.138%	10.114%	80.157%	9.412%

Table B.7: Predicted transition matrix 2000H2 using one-parameter approach ($\alpha\Delta Z_t = -0.098$)

		End-of-half-year rating							
		AAA	AA	A	BBB	BB	B	CCC	D
Initial rating	AAA	87.870%	11.345%	0.547%	0.090%	0.093%	0.017%	0.018%	0.019%
	AA	0.097%	93.159%	6.215%	0.298%	0.169%	0.025%	0.018%	0.019%
	A	0.049%	0.634%	95.245%	3.814%	0.174%	0.030%	0.028%	0.026%
	BBB	0.026%	0.030%	1.248%	95.249%	2.984%	0.268%	0.106%	0.090%
	BB	0.015%	0.020%	0.612%	1.845%	91.233%	5.503%	0.517%	0.255%
	B	0.005%	0.008%	0.310%	0.909%	2.156%	90.583%	4.767%	1.261%
	CCC	0.005%	0.005%	0.034%	0.059%	0.086%	7.561%	79.765%	12.485%

Table B.8: Fitted transition matrix 2000H2 using PCA approach ($\hat{\alpha}'\hat{f}_t = -0.162$)

		End-of-half-year rating							
		AAA	AA	A	BBB	BB	B	CCC	D
Initial rating	AAA	90.440%	9.028%	0.379%	0.060%	0.060%	0.011%	0.011%	0.011%
	AA	0.154%	94.730%	4.765%	0.204%	0.110%	0.016%	0.011%	0.011%
	A	0.080%	0.917%	96.004%	2.833%	0.115%	0.019%	0.017%	0.016%
	BBB	0.043%	0.048%	1.756%	95.638%	2.209%	0.182%	0.069%	0.056%
	BB	0.026%	0.032%	0.889%	2.469%	91.844%	4.218%	0.358%	0.165%
	B	0.009%	0.013%	0.465%	1.263%	2.820%	90.887%	3.669%	0.874%
	CCC	0.009%	0.009%	0.055%	0.091%	0.128%	9.672%	80.176%	9.860%

Table B.9: Predicted transition matrix 2000H2 using PCA approach ($\hat{\alpha}'\hat{f}_t = -0.024$)

B.4 Scenario analysis

B.4.1 Predicted average transition matrices using the one-parameter approach

		End-of-half-year rating							
		AAA	AA	A	BBB	BB	B	CCC	D
Initial rating	AAA	89.220%	10.134%	0.456%	0.074%	0.075%	0.014%	0.014%	0.014%
	AA	0.123%	93.997%	5.449%	0.247%	0.137%	0.020%	0.014%	0.014%
	A	0.063%	0.765%	95.674%	3.291%	0.142%	0.024%	0.022%	0.020%
	BBB	0.034%	0.038%	1.485%	95.495%	2.570%	0.221%	0.086%	0.071%
	BB	0.020%	0.025%	0.740%	2.142%	91.613%	4.824%	0.431%	0.205%
	B	0.007%	0.010%	0.382%	1.075%	2.474%	90.814%	4.187%	1.051%
	CCC	0.007%	0.007%	0.043%	0.074%	0.105%	8.578%	80.079%	11.107%

Table B.10: Average transition matrix reference scenario using the one-parameter approach

		End-of-half-year rating							
		AAA	AA	A	BBB	BB	B	CCC	D
Initial rating	AAA	89.022%	10.312%	0.469%	0.076%	0.078%	0.014%	0.014%	0.015%
	AA	0.119%	93.876%	5.560%	0.254%	0.141%	0.020%	0.014%	0.015%
	A	0.060%	0.744%	95.615%	3.366%	0.146%	0.024%	0.023%	0.021%
	BBB	0.033%	0.037%	1.447%	95.465%	2.630%	0.227%	0.089%	0.073%
	BB	0.019%	0.024%	0.719%	2.094%	91.565%	4.923%	0.443%	0.212%
	B	0.006%	0.010%	0.370%	1.049%	2.424%	90.789%	4.272%	1.081%
	CCC	0.007%	0.007%	0.042%	0.071%	0.102%	8.417%	80.046%	11.309%

Table B.11: Average transition matrix disorderly transition scenario using the one-parameter approach

		End-of-half-year rating							
		AAA	AA	A	BBB	BB	B	CCC	D
Initial rating	AAA	89.187%	10.163%	0.458%	0.074%	0.075%	0.014%	0.014%	0.014%
	AA	0.122%	93.977%	5.467%	0.248%	0.137%	0.020%	0.014%	0.014%
	A	0.062%	0.762%	95.664%	3.303%	0.143%	0.024%	0.022%	0.021%
	BBB	0.034%	0.038%	1.479%	95.490%	2.580%	0.222%	0.086%	0.071%
	BB	0.020%	0.025%	0.736%	2.134%	91.605%	4.840%	0.433%	0.206%
	B	0.007%	0.010%	0.380%	1.071%	2.466%	90.810%	4.201%	1.056%
	CCC	0.007%	0.007%	0.043%	0.073%	0.105%	8.551%	80.074%	11.141%

Table B.12: Average transition matrix orderly transition scenario using the one-parameter approach

B.4.2 Predicted average transition matrices using the PCA approach

		End-of-half-year rating							
		AAA	AA	A	BBB	BB	B	CCC	D
Initial rating	AAA	89.125%	10.219%	0.462%	0.075%	0.076%	0.014%	0.014%	0.015%
	AA	0.121%	93.940%	5.502%	0.250%	0.139%	0.020%	0.014%	0.015%
	A	0.062%	0.755%	95.646%	3.327%	0.144%	0.024%	0.022%	0.021%
	BBB	0.033%	0.037%	1.467%	95.481%	2.599%	0.224%	0.087%	0.072%
	BB	0.020%	0.025%	0.730%	2.119%	91.591%	4.871%	0.437%	0.208%
	B	0.007%	0.010%	0.376%	1.062%	2.450%	90.803%	4.228%	1.065%
	CCC	0.007%	0.007%	0.043%	0.073%	0.104%	8.500%	80.064%	11.203%

Table B.13: Average transition matrix reference scenario using the PCA approach

		End-of-half-year rating							
		AAA	AA	A	BBB	BB	B	CCC	D
Initial rating	AAA	88.978%	10.352%	0.472%	0.076%	0.078%	0.014%	0.015%	0.015%
	AA	0.118%	93.849%	5.585%	0.256%	0.142%	0.021%	0.015%	0.015%
	A	0.060%	0.739%	95.602%	3.383%	0.147%	0.025%	0.023%	0.021%
	BBB	0.032%	0.036%	1.438%	95.457%	2.643%	0.229%	0.089%	0.074%
	BB	0.019%	0.024%	0.715%	2.084%	91.554%	4.945%	0.446%	0.213%
	B	0.006%	0.009%	0.367%	1.043%	2.413%	90.783%	4.291%	1.088%
	CCC	0.007%	0.007%	0.041%	0.071%	0.101%	8.382%	80.037%	11.354%

Table B.14: Average transition matrix disorderly transition scenario using the PCA approach

		End-of-half-year rating							
		AAA	AA	A	BBB	BB	B	CCC	D
Initial rating	AAA	89.108%	10.234%	0.463%	0.075%	0.076%	0.014%	0.014%	0.015%
	AA	0.120%	93.929%	5.512%	0.251%	0.139%	0.020%	0.014%	0.015%
	A	0.061%	0.753%	95.641%	3.333%	0.144%	0.024%	0.022%	0.021%
	BBB	0.033%	0.037%	1.463%	95.478%	2.604%	0.224%	0.088%	0.072%
	BB	0.020%	0.025%	0.728%	2.115%	91.586%	4.880%	0.438%	0.209%
	B	0.007%	0.010%	0.375%	1.060%	2.446%	90.800%	4.235%	1.068%
	CCC	0.007%	0.007%	0.042%	0.072%	0.103%	8.487%	80.061%	11.221%

Table B.15: Average transition matrix orderly transition scenario using the PCA approach

B.4.3 Percentage differences of the average transition matrices

		End-of-half-year rating							
		AAA	AA	A	BBB	BB	B	CCC	D
Initial rating	AAA	-0.222%	1.760%	2.848%	3.186%	3.411%	3.655%	3.806%	4.185%
	AA	-3.455%	-0.129%	2.051%	2.951%	3.296%	3.632%	3.806%	4.185%
	A	-3.643%	-2.793%	-0.062%	2.299%	3.238%	3.524%	3.694%	4.090%
	BBB	-3.809%	-3.425%	-2.586%	-0.032%	2.324%	3.002%	3.292%	3.735%
	BB	-3.943%	-3.562%	-2.823%	-2.214%	-0.053%	2.054%	2.844%	3.406%
	B	-4.210%	-3.839%	-3.060%	-2.489%	-2.040%	-0.027%	2.021%	2.843%
	CCC	-4.206%	-3.866%	-3.549%	-3.255%	-3.043%	-1.873%	-0.042%	1.820%

Table B.16: Percentage differences of the disorderly transition scenario average transition matrix compared to the reference scenario using the one-parameter approach

		End-of-half-year rating							
		AAA	AA	A	BBB	BB	B	CCC	D
Initial rating	AAA	-0.037%	0.294%	0.473%	0.528%	0.565%	0.605%	0.630%	0.691%
	AA	-0.587%	-0.021%	0.342%	0.490%	0.546%	0.601%	0.630%	0.691%
	A	-0.620%	-0.473%	-0.010%	0.383%	0.537%	0.584%	0.611%	0.676%
	BBB	-0.649%	-0.582%	-0.438%	-0.005%	0.387%	0.498%	0.546%	0.618%
	BB	-0.672%	-0.606%	-0.479%	-0.374%	-0.009%	0.342%	0.472%	0.564%
	B	-0.718%	-0.654%	-0.519%	-0.421%	-0.345%	-0.004%	0.337%	0.472%
	CCC	-0.718%	-0.659%	-0.604%	-0.553%	-0.517%	-0.316%	-0.007%	0.304%

Table B.17: Percentage differences of the orderly transition scenario average transition matrix compared to the reference scenario using the one-parameter approach

		End-of-half-year rating							
		AAA	AA	A	BBB	BB	B	CCC	D
Initial rating	AAA	-0.165%	1.298%	2.100%	2.349%	2.514%	2.693%	2.804%	3.082%
	AA	-2.574%	-0.096%	1.513%	2.176%	2.430%	2.676%	2.804%	3.082%
	A	-2.715%	-2.080%	-0.046%	1.696%	2.387%	2.597%	2.722%	3.012%
	BBB	-2.839%	-2.553%	-1.926%	-0.025%	1.714%	2.214%	2.426%	2.752%
	BB	-2.940%	-2.655%	-2.103%	-1.648%	-0.040%	1.515%	2.097%	2.510%
	B	-3.140%	-2.862%	-2.279%	-1.853%	-1.519%	-0.021%	1.491%	2.097%
	CCC	-3.137%	-2.883%	-2.645%	-2.426%	-2.267%	-1.394%	-0.033%	1.343%

Table B.18: Percentage differences of the disorderly transition scenario average transition matrix compared to the reference scenario using the PCA approach

		End-of-half-year rating							
		AAA	AA	A	BBB	BB	B	CCC	D
Initial rating	AAA	-0.019%	0.148%	0.239%	0.267%	0.286%	0.306%	0.318%	0.349%
	AA	-0.299%	-0.011%	0.173%	0.248%	0.276%	0.304%	0.318%	0.349%
	A	-0.315%	-0.241%	-0.005%	0.193%	0.271%	0.295%	0.309%	0.341%
	BBB	-0.330%	-0.296%	-0.223%	-0.003%	0.196%	0.252%	0.276%	0.312%
	BB	-0.342%	-0.308%	-0.243%	-0.190%	-0.004%	0.173%	0.239%	0.285%
	B	-0.365%	-0.332%	-0.264%	-0.214%	-0.175%	-0.002%	0.170%	0.239%
	CCC	-0.365%	-0.335%	-0.307%	-0.281%	-0.263%	-0.161%	-0.004%	0.153%

Table B.19: Percentage differences of the orderly transition scenario average transition matrix compared to the reference scenario using the PCA approach

B.5 Risk management

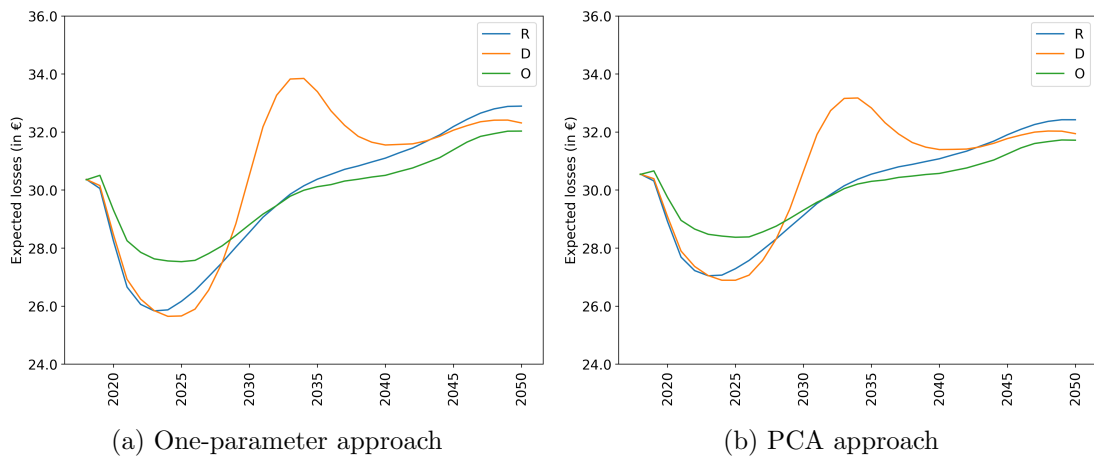


Figure B.5.1: Expected losses for the high-yield portfolio using the two different approaches

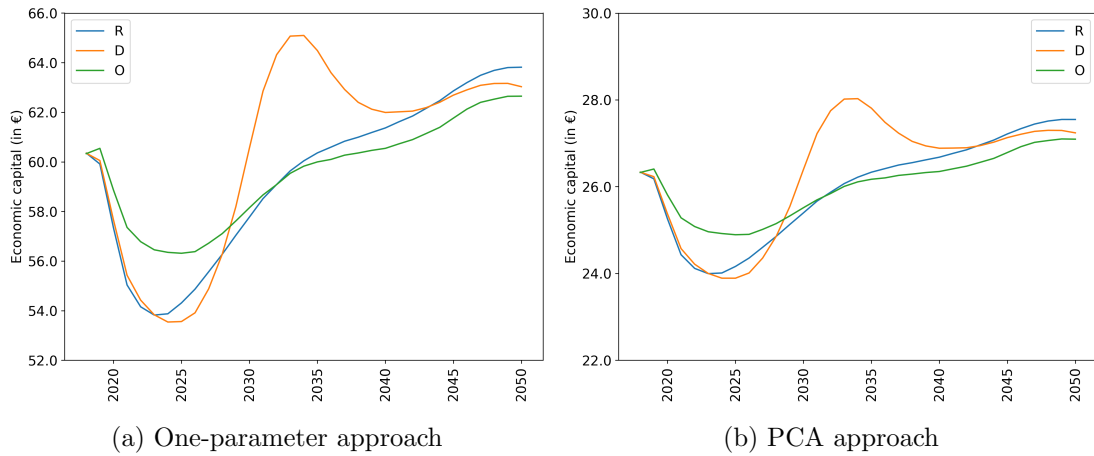


Figure B.5.2: Economic capital for the high-yield portfolio using the two different approaches

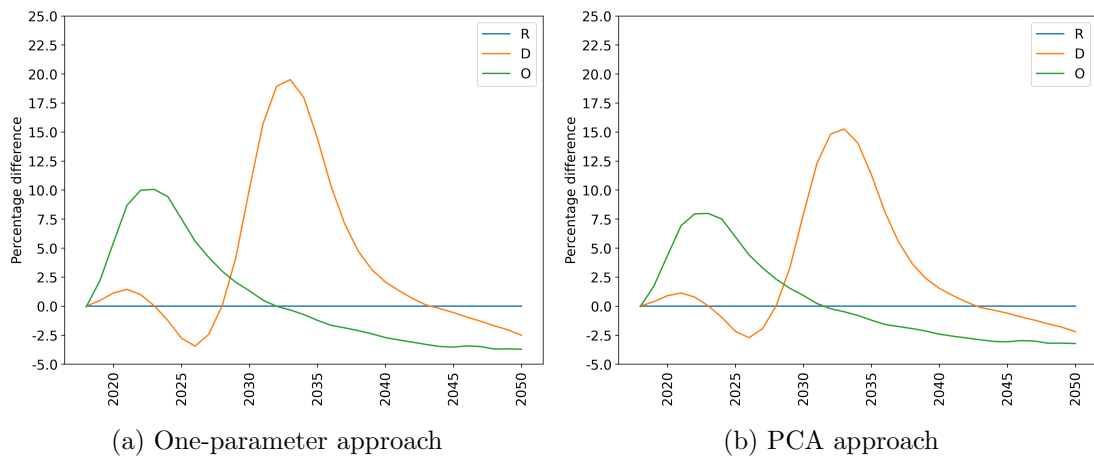


Figure B.5.3: Economic capital relative to the reference scenario for the investment-grade portfolio using the two different approaches

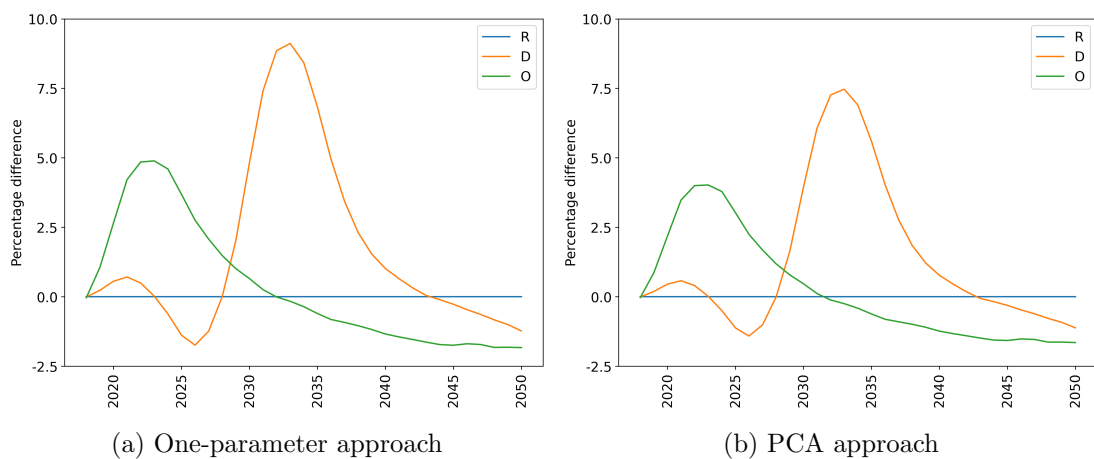


Figure B.5.4: Economic capital relative to the reference scenario for the high-yield portfolio using the two different approaches