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NOT SO NAIVE

An exploration of diversification strategies

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ABSTRACT

How does the choice of a diversification strategy affect portfolio performance? To answer this question, I have used data on excess returns from financial markets, employed five different diversification strategies, and evaluated them with regards to their Sharpe ratio and certainty-equivalent returns. I have found that no sophisticated strategy could consistently outperform the naive approach in terms of the Sharpe ratio, but all of them outperformed the naive approach in terms of certainty-equivalent returns. This finding is (partly) contrary to previous academic literature, showcasing the significance of continued validity ascertainment. Moreover, it presents important takeaways for investors worldwide, striving for a diversified portfolio.

Keywords: portfolio analysis, portfolio performance, diversification

JEL codes: G00, G11

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CHAPTER 1 Introduction

Bernoulli in 1738 wrote: "… it is advisable to divide goods which are exposed to some danger into several portions rather than to risk them all together" (1954, p.30). This is the key to effective diversification. Nevertheless, diversification has been a long-debated topic in the world of finance, but also in the natural world. In fact, diversification is a concept known not only to humans, but animals as a whole. Much like people nowadays diversify their portfolios to minimize risk and achieve highest expected returns, animals millions of years ago have started diversifying to spread their risks and increase their chances of survival in changing environments. Thus, an efficient diversification strategy is necessary in order to thrive. Many such strategies have been created since, but the question remains, which strategy yields greatest results? Since under-diversification can prove quite costly to most (Goetzmann & Kumar, 2008), an answer to this question would be a useful tool for individual and institutional investors alike. Therefore, a deeper understanding of the topic can foster safer and greater returns to all.

Markowitz's (1959) seminal work has become a cornerstone of modern portfolio theory, where he argued that investors should not simply consider the expected return of individual securities, but rather the benefits of diversification across multiple securities. While written primarily with the non-mathematician in mind, Markowitz offers investors optimal balance between risk and return, by selecting a mix of assets that offer the highest expected return for a given level of risk. Other previous research has examined the relationship between portfolio performance and diversification, using various strategies, such as the naive 1/N portfolio, but also more complex methods, including sample-based mean-variance, models considering estimation error or models with certain restrictions or constraints (DeMiguel et al., 2007). Surprisingly, DeMiguel et al. (2007) found that out of all the strategies taken into consideration in the US equity market, none consistently outperform the 1/N rule in terms of Sharpe ratio, certainty-equivalent return, or turnover.

In this study I intend to replicate the findings of DeMiguel et al. (2007), but with the inclusion of more recent data to evaluate the continued validity of their results. In the 16 years that have passed since the publication of their study many things have changed. For one, advancements have been made in the area of diversification and portfolio management as a whole. Furthermore, financial markets are dynamic, meaning significant changes can occur over time, such as change in volatility levels, correlations between assets or overall market performance.

And lastly, the inclusion of new data, including such events as the financial crisis in 2007-2009, the recent Covid pandemic or the Russian invasion on Ukraine could lead to interesting findings with regard to optimal asset allocation. It is possible that these changes would lead to a different conclusion than the one reached by DeMiguel et al. (2007) and perhaps 1/N is not the best strategy anymore. Therefore, I phrase the research question as follows: "How does the choice of a diversification strategy affect portfolio performance?".

To evaluate the performance of these strategies, I will once again follow the footsteps of DeMiguel et al. (2007) and analyse the portfolios in terms of: (1) Sharpe ratio, and (2) the certainty-equivalent (CEQ) return for the expected utility of a mean-variance investor. This should yield numerical data readily available for analysis and comparison of the different strategies. I intend to use exclusively secondary data gathered from the following sources. The data regarding various types of portfolios, i.e., industry portfolios, SMB and HML portfolios, and Fama-French 1-, 3- and 4-factor portfolios will be obtained from Kenneth French's website. Country indices, the world index, and the data regarding sector portfolios of the S&P 500 will be obtained from Yahoo! Finance and GFD (Global Financial Data). The data used will span until 2023, while going back as far as the data is available for each source. I plan to use 90 portfolios in total, akin to the amount used by DeMiguel et al. (2007).

I hypothesize that, regardless of the advancements made and availability of new data, the results will remain constant, meaning the naive approach of the 1/N portfolio will outperform the other strategies. Given the framework described in the previous paragraph, the outperformance of the 1/N portfolio should be apparent in significantly greater results than that of the other portfolios. Since new data will be used, the effectiveness of the outlined strategies will be evaluated to examine whether their use is still warranted and if not, what other strategies could be put into their place. I expect that despite the use of an already well tested method, this will not put an end to the discussion regarding asset allocation. Different investors will seek different returns, risks, industries to invest in or measures of performance; thus, a final answer which strategy is best simply cannot be put forth. Yet, I believe that regardless of these hindrances, the research shall elucidate the advantages and disadvantages of the strategies, offering a better, more well-rounded overview of each.

In the remainder of the paper, I will first discuss relevant literature and previous research in the theoretical framework. Secondly, I will present the data used for the research and the methodology employed in making the necessary calculations. Then, I will showcase and discuss the results of the research. And lastly, I will provide a conclusion of my study, what are the main takeaways and what are its limitations.

CHAPTER 2 Theoretical Framework

2.1 Portfolio analysis: background

Portfolio analysis involves evaluating the performance of an investment portfolio to determine its adequacy for meeting the portfolio's goals and objectives. It often encompasses both the return and risk of a portfolio relative to its benchmark or relative to other portfolios. Its central elements are risk assessment which is essential to understanding a portfolio's potential vulnerabilities, and asset allocation - the distribution of different types of investments in a portfolio, such as equities, bonds, or cash equivalents (Elton et al., 2013).

One cannot talk about portfolio analysis without measuring portfolio performance. Such measures can be split into two categories traditional (unconditional) and conditional. Unconditional measures were mainly influenced by the Capital Asset Pricing Model (CAPM) of Sharpe (Aragon & Ferson, 2006; Marhfor, 2016). Their main shortcoming is the assumption that risk is constant over the entire evaluation period (Marhfor, 2016). Traditional evaluation measures are computed as a ratio, most commonly by dividing the performance (excess returns) by a risk measure. On the other hand, conditional measures allow portfolio risk and market premium to vary over time with the state of the economy.

Lastly, a crucial concept in portfolio analysis is diversification. A diversification strategy in the simplest terms can be viewed as the particular distribution of a firm's resources (Lecraw, 1984). Its aim is to reduce risk generally by spreading investments across different assets, sectors, geographic regions, or other categories. Portfolio diversification can be summarized as a product of four principles, those are: law of large numbers (LLN), correlation, CAPM, and risk contribution (RC) (Koumou, 2020).

LLN as used in portfolio theory, "recommends investing a small fraction of wealth in each of a large number of assets" (Koumou, 2020, p. 301). The principle first phrased by Cardano in the 16^{th} century, but only proved in the 18^{th} century by Bernoulli was confined to the field of statistics and probability. Only later did its application into areas such as insurance, finance, and risk management gain significance. As for correlation, the words of Markowitz (1952) illustrate the point excellently: "A portfolio with sixty different railway securities, for example, would not be as well diversified as the same size portfolio with some railroad, some public utility, mining, various sort of manufacturing, etc. The reason is that it is generally more likely for firms within the same industry to do poorly at the same time than for firms in dissimilar industries." (p. 89). By diversifying across different industries, regions or asset classes one can minimise both probability and severity of loss. The CAPM diversification principle can be split into two components: (1) individual asset diversification and (2) asset class diversification. The former is based on the assumption of CAPM which states that individual asset risk can be decomposed into systematic risk and idiosyncratic risk. The latter is based on the one-fund theorem, as phrased by Tobin (1958), according to which every individual's portfolio can be obtained as a mixture of the risk-free asset and the market portfolio. The final principle is risk contribution or risk parity diversification which serves as an alternative to the market portfolio and the 60/40 rule (60% of the portfolio shall be divided into equity and the remainder into bonds). It is based on risk allocation rather than wealth or capital. Its premise revolves around the idea of equalising assets' absolute risk contributions.

2.2 Empirical studies on naive diversification

Many studies have been conducted in the past regarding what's known as naive portfolio diversification – in other words, the practice of attributing equal weight to all assets in a portfolio – 1/N. But counterintuitively, what might seem like a fool's errand, has historically thrived with great performance, often outperforming more sophisticated diversification strategies.

A seminal work on the topic was conducted by DeMiguel et al. (2007) who investigated the performance of different diversification methods as compared, to what was only supposed to be, a benchmark 1/N strategy. To their surprise, none of these could consistently outperform the "naive" method.

Another paper touching upon the concept of the 1/N heuristic was done by Benartzi and Thaler (2001), but the context of their study relates to defined contribution saving plans. An observation that fascinated them was that many individuals when faced with a choice of a saving plan simply decided to spread their contributions evenly across the funds they were offered. This 1/N heuristic seems to be particularly influenced by the number of options available, underlining the psychological reasons people might default to such a strategy – such as: avoiding perceived decision-making mistakes, familiarity bias, analysis paralysis or avoiding extreme outcomes. All of these reasons point to the simple (for them) choice of the

1/N strategy. A similar conclusion was reach by Statman (2004), in his diversification puzzle, where he dives deep into the cognitive and behavioural underpinnings of investment strategies. He presents evidence where the 1/N strategy may not perform the best in a purely mathematical sense, but rather be born from a place of "bounded rationality" where investors, overwhelmed by choices and information, seek simplicity. The eponymous "puzzle" refers precisely to this gap between classical financial theory and the often irrational behaviours exhibited by real investors, causing them to opt for the simple 1/N. Huberman and Jiang (2006) in a related work, while probing the intricacies of 401(k) retirement plans explore the impact of the number of fund choices on investor behaviour. Their findings are nearly analogous, wherein they find that the sheer volume of options can influence the equity exposure that investors take on, and there's a marked tendency towards naive diversification when faced with a large array of choices.

An earlier work bordering on this topic was done by Fernholz and Shay (1982), who while investigating stochastic portfolio theory, explore how various portfolio strategies behave in probabilistic market conditions. Their theoretical insights offer context for understanding how and why simple strategies like 1/N might perform unexpectedly well in the real-world chaotic markets. They show that when market prices evolve according to certain mathematical models (like geometric Brownian motion), the value-weighted market portfolio outperforms, on average, any other diversified strategy. But, since 1/N can approximate the market portfolio in the absence of significant market concentration, it's inherently a strong strategy in such mathematical contexts. Moreover, in complex and dynamic markets, no single strategy consistently outperforms others due to the ever-changing nature of the market. Given this unpredictability, a simple strategy like 1/N, which doesn't rely on any specific forecasts or assumptions, may fare just as well as more complicated strategies. Throughout the paper, the authors challenge the notion that markets are in equilibrium (as traditionally assumed in economic theory), arguing instead that they are chaotic systems with various forces at play. The absence of equilibrium might make traditional optimization strategies less effective, thereby elevating the efficacy of simpler strategies like 1/N.

2.3 Choice of diversification strategy on portfolio performance

As mentioned before, the first person to consider a mathematical approach of finding an optimal diversifications strategy in terms of portfolio choice was Markowitz (1952). Since investors strive to attain high returns at a low risk, he has constructed an optimization problem relating to maximizing expected returns and minimizing variance. This way the efficient frontier can be created from which an investor can choose their preferred set depending on individual risk aversion. While ground-breaking at the time, it is not used in practice due to huge amounts of data required (given n assets, the model requires $2n + \binom{n}{2}$ parameters) (Marling & Emanuelsson, 2012). As a response to that, Sharpe, in his model, reduced the requirement from the pairwise correlation between the assets to only an estimation of how an asset depends on the market behaviour (which resulted in "only" $3n + 2$ parameters required). Yet, it was clear that more work would be necessary to achieve a superior result.

Two of the people who undertook the challenge were Jagannathan and Ma (2003) who studied the effect of imposing constraints on risk reduction in a portfolio. In the presence of a mean-variance efficient portfolio with extreme negative weights, imposing no-short-sale constraints should have an adverse effect. Yet, the authors find and explain that doing so can reduce risk even in the case that the constraints are wrong. Their model will be later included in the paper that bears the most significance for my work, namely the paper of DeMiguel et al. (2007), due to its high Sharpe ratio.

Finally, DeMiguel et al. (2007) set out to study the performance of the sample-based mean-variance model and its extensions, relative to the so-called "naive" 1/N portfolio which was meant to serve merely as a benchmark. Yet, of the 14 models across seven datasets, none have consistently outperformed the naive portfolio in terms of the Sharpe ratio, CEQ return or turnover. The authors argue that this phenomenon can be explained by the fact that optimal diversification is more than offset by estimation error. But the question remains, how could it be that such a straightforward rule can outperform an array of various complex asset-allocation models?

In a later study, Kirby and Ostdiek (2012) aim to address this issue and create a method that can outperform the naive portfolio. They argue that the poor performance of the mean-variance models used by DeMiguel et al. can be explained solely by the research design which delivers aggressive portfolios where target conditional expected excess returns often exceed 100% per year. This leads to greater estimation error and turnover which can explain the poor out-of-sample performance. They instead propose implementing the mean-variance model such that it targets the conditional expected return of the 1/N portfolio, in which case the resulting portfolios (mostly) do outperform naive diversification by statistically significant margins.

From the aforementioned a simple observation can be made – the world of portfolio theory is deeply convoluted and ever changing. Constantly, new observations are made, and new theories are created, expanding the academic literature, leading to impactful insights. Therefore, with the acquisition of new data, the maintenance of continued validity needs to be ascertained.

Considering the referenced literature, the following hypotheses are created:

H1: The "sophisticated" strategies will not outperform the naive approach in terms of the Sharpe ratio.

H2: The "sophisticated" strategies will not outperform the naive approach in terms of the CEQ returns.

CHAPTER 3 Data

To answer my research question, I collected data necessary to create a set of varied portfolios. A summary of these can be seen in Table 1.

TABLE 1

Empirical datasets

#	Dataset	N	Time period	Abbreviation
1	Sector portfolios of the S&P 500	9+1	22/12/1998-30/06/2023	S&P Sector
2	Industry portfolios	10+1	01/07/1926-30/06/2023	Industry
3	Country indices	9+1	22/12/1998-30/06/2023	International
4	SMB and HML portfolios	2+1	01/07/1926-30/06/2023	MKT/SMB/HML
5	Size- and book-to-market portfolios + MKT	25+1	01/07/1926-30/06/2023	FF-1
6	Size- and book-to-market portfolios + MKT, SMB, HML	25+3	01/07/1926-30/06/2023	FF-3
7	Size- and book-to-market portfolios + MKT, SMB, HML, UMD	25+4	01/07/1926-30/06/2023	FF-4

Note. The table presents all of the different datasets considered along with their abbreviation which will be used in the remainder of the paper. Each dataset considers monthly excess returns over the 90-day nominal US T-bill. N is the number of risky assets considered, where the number following the "+" is the number of factor portfolios available.

The data used for this research will consider seven datasets (Table 1), where each dataset considers monthly excess returns over the 90-day nominal US T-bill (hereafter referred to as *excess returns*). The necessary data will be taken from two different databases. Datasets #2 and #4-7 will be taken from Kenneth French's website[1], offering peer reviewed data used widely in academic research which is continuously updated to offer the latest information. The data for

[1] https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

the 90-day nominal US T-bill will also be taken from this website. Dataset #1 and #3 will be taken primarily from Yahoo! Finance[2], a well-established platform providing comprehensive coverage of real-time market data. One exception is in the International portfolio, where the data for the world index was taken from GFD (Global Financial Data)[3]. Lastly, the data for the 90-day nominal US T-bill was taken from FRED (Federal Reserve Economic Data)[4]. The time period for each dataset can also be viewed – N.B. the values differ, reaching as far back as possible for each dataset, given the availability of data. The specific number of assets used can be seen in the table, totalling 119 assets. The total number of data points amounts to 1,220,647.

Assets in the S&P Sector have been selected so as to imitate the ones used by DeMiguel et al. (2007), but since the authors used a custom dataset provided by Prof. Wessels (whom I contacted, but to no avail, as the dataset was lost to the ages), I have tried to recreate it using select sector SPDR (Standard & Poor's Depository Receipts) ETFs which divide the S&P 500 into 11 sectors: Materials, Real estate, Finance, Utilities, Healthcare, Communication services, Energy, Industrial, Consumer staples, Consumer discretionary and Technology. As Real Estate and Communication Services were created only in 2015 and 2018, respectively, I opted to exclude them from the portfolio (their inclusion would disallow the calculation of the covariance matrix for the full extent of data). This portfolio is not a perfect substitute, but I believe it is similar enough to be used in its stead. Like the authors, I augmented it by adding the excess return on the US equity market portfolio (hereafter, MKT) – defined as the value-weighted return on all NYSE, AMEX, and NASDAQ stocks subtracted by the 1-month T-Bill rate.

The Industry portfolio was created using the exact same dataset as the one used by DeMiguel et al. (2007), only extending the time span, both backwards and forwards. This includes the following industries: Consumer discretionary, Consumer staples, Manufacturing, Energy, High-tech, Telecommunication, Wholesale and Retail, Health, Utilities, and Others.

The International portfolio consists of eight country indices composed of the iShares MSCI ETFs for seven of the eight countries (due to unavailability of data): Canada, France, Germany, Italy, Japan, Switzerland, and the UK. The US was also included by using the SPDR S&P 500

[2] https://finance.yahoo.com/
[3] https://finaeon-globalfinancialdata-com.eur.idm.oclc.org/
[4] https://fred.stlouisfed.org/series/TB3MS

ETF Trust, tracking 500 largest US-based companies. Finally, the world index was included by using the MSCI ACWI (All Country World Index) Price Index (*sic*) which tracks the investment results of large- and mid-cap firms in developed and emerging markets.

MKT/SMB/HML portfolio was created in the same manner as portfolio #2. It includes 3 assets: SMB (small-minus-big), HML (high-minus-low), and MKT.

Portfolios #5-#7 were all created in a similar manner – they all include 25 various size- and book-to-market portfolios (the original research used 20, but the data source only provides an option of 25 portfolios). These were created as intersections of five portfolios formed on size and five portfolios formed on the ratio of book equity to market equity (5x5=25). The only difference between them is the number of factors by which they are augmented – FF-1 was augmented only by MKT, FF-2 by MKT, SMB and HML, and FF-3 by MKT, SMB, HML and UMD (up-minus-down).

Excess returns, as used for my purpose, can be defined as the difference between the actual return of an investment or a portfolio and a benchmark. Therefore, a positive value would indicate that the investment outperformed the benchmark and a negative value that it underperformed. Its unit of measurement is a percentage. As mentioned before, the benchmark used will be the 90-day nominal US T-bill, which can be considered a risk-free asset, widely used for its availability and transparency of data.

In Table 2, summary statistics for the data are given. What's immediately striking is the high standard deviation reaching as high as 1.447, but then looking at minimum and maximum values it is less surprising, seeing as they can reach from -48.460 all the way to 127.940.

Table 2

Summary statistics of excess returns

Variable	Mean	St. Dev.	Min	Max
S&P Sector	0.039	1.456	-20.141	16.475
Industry	0.045	1.227	-20.080	27.060
International	0.029	1.425	-15.645	19.790
MKT/SMB/HML	0.016	0.798	-17.440	15.760
FF-1	0.049	1.447	-48.460	127.940
FF-3	0.047	1.404	-48.460	127.940
FF-4	0.046	1.387	-48.460	127.940
Total	0.044	1.359	-48.460	127.940

Note. The table presents mean, standard deviation, minimum and maximum values for excess returns for each of the datasets considered as well as for all of the data combined.

CHAPTER 4 Method

To answer my research questions, I will investigate five diversification methods, seen below in Table 3.

TABLE 3

Diversification methods

#	Model	Abbreviation
1	1/N portfolio (benchmark)	1/N
2	Sample-based mean-variance portfolio	mv
3	Minimum variance portfolio	min
4	Sample-based mean-variance portfolio with short-sale constraints	mv-c
5	Minimum variance portfolio with short-sale constraints	min-c

The first method used will be the naive diversification strategy and it will serve as a benchmark to which all of the other strategies will be compared. To recapitulate, this strategy divides investment equally among all of the securities within a portfolio. It is the simplest method used, allowing for a clear comparison between the different methods.

The second method employed is the sample-based mean-variance portfolio, first developed by Markowitz (1952). This method utilizes equation (1) below, where at each time t, the decision-maker selects x_t to maximize expected utility[5]. In which γ is the investor's risk aversion. Moreover, μ_t is the N-dimensional vector denotes the expected returns on the risky asset in excess of the risk-free rate, and Σ_t denotes the corresponding N × N variance-covariance matrix of returns ($\hat{\mu}_t$ and $\hat{\Sigma}_t$ are their sample counterparts).

$$\max_{x} x_t^T \hat{\mu}_t - \frac{\gamma}{2} x_t^T \hat{\Sigma}_t x_t$$

[5] It is important to note that the choice of x_t is entirely up to the decision-maker, based on his risk-aversion. The expected return could easily be maximised by simply putting the entire weight of a portfolio into the asset with historically highest returns, but this often leads to very high variance (which is undesirable). Given that my dataset had naturally very high variance, I opted to choose variance such that it fell somewhere in the range between that of min and 1/N. There is not one right choice here, but for further investigation that could be done see the conclusion section.

Under the minimum variance strategy, the decision maker chooses a portfolio of assets, such that, as the name suggests, variance is minimized. This can be seen below in equation (2):

$$\min_{w_t} w_t^T \Sigma_t w_t, \quad \text{s.t.} \quad 1_N^T w_t = 1$$

Where 1 is an N-dimensional vector of ones. This strategy ignores estimates of the expected returns and focuses solely on the estimate of the covariance matrix of asset returns (the sample covariance matrix).

Moreover, I do not present results for mv-c and min-c, as while investigating the best strategies, imposing short-sale constraints seemed to always lead to them being dominated by non-short-sale constrained strategies.

Then, in order to evaluate the effectiveness of these aforementioned methods I will use the following two methods. First of all, I will calculate the Sharpe ratio for each strategy k, using equation (3) below:

$$\widehat{\mathrm{SR}}_{\mathrm{k}} = \frac{\hat{\mu}_{\mathrm{k}}}{\hat{\sigma}_{\mathrm{k}}}$$

Where $\hat{\mu}_k$ is the sample mean of excess returns (over the risk-free asset) or simply put – expected return, and $\hat{\sigma}_k$ is the sample standard deviation. This will provide me with the excess return generated by an investment over and above the risk-free rate. In order to compare the Sharpe ratio between strategies, I will also compute a t-test to gain an insight whether the differences are statistically significant (comparing the Sharpe ratio of the naive strategy to the Sharpe ratios of mv and min). To do so, I will use the approach suggested by Jobson and Korkie (1981), later improved by Memmel (2003).

The second approach I will use is the certainty-equivalent (CEQ) return – the risk-free rate an investor would rather accept than undertake any risky portfolio strategy. To calculate CEQ return, formula (4) can be used, as seen below:

$$\widehat{\mathrm{CEQ}}_{\mathrm{k}} = \hat{\mu}_{\mathrm{k}} - \frac{\gamma}{2}\hat{\sigma}_{\mathrm{k}}^2$$

Where $\hat{\mu}_k$ and $\hat{\sigma}_k^2$ are the mean and variance of excess returns, respectively, and γ is the risk aversion. I use $\gamma = 2$ which is consistent with literature and can be described as moderate risk-seeking. Once again, I will calculate the test statistic based on a paired t-test (comparing the naive strategy CEQ return to mv and min CEQ returns).

CHAPTER 5 Results & Discussion

The results of the Sharpe ratio analysis can be seen in Table 4. Each strategy is presented in a separate row, while the different datasets are presented in columns. Here, a value of e.g., 0.037 (1/N strategy for the S&P Sector) indicates the portfolio's excess return over the risk-free rate is 0.037 units for every unit of risk (as measured by the standard deviation) it has taken on. The values observed are rather low, seeing as a typical Sharpe ratio one would strive for would be 1.0 or greater than 1.0, anything below seen as suboptimal. This can be explained partly by the low expected return of the portfolios, these consist mostly of ETFs after all. And on the second hand, all of the portfolios have high variance which, in turn, could be caused by the sheer length of the data, increasing the chance of encountering outliers in the data structure, capturing different market regimes, or even being influenced by structural changes to the market.

It can be seen that the Sharpe ratio is higher for the more sophisticated diversification methods in most cases, except for three instances: both mv and min underperform 1/N in the MKT/SMB/HML dataset and min underperforms 1/N in the FF-3 dataset. Yet, due to the differences being rather low, the statistical significance is lacking, only mv significantly outperforming 1/N in the FF-4 dataset at the 1% significance level, with a Sharpe ratio of 0.063 as opposed to 0.044.

TABLE 4

Sharpe ratios for empirical data

Strategy	S&P Sector	Industry	International	MKT/SMB/HML	FF-1	FF-3	FF-4
1/N	0.037	0.043	0.027	0.036	0.042	0.043	0.044
mv	0.044 (-0.539)	0.047 (-0.630)	0.043 (-1.11)	0.033 (0.781)	0.050 (-1.152)	0.050 (-1.128)	0.063*** (-2.861)
min	0.043 (-0.477)	0.048 (-0.836)	0.040 (-0.900)	0.030 (0.822)	0.044 (-0.247)	0.039 (0.609)	0.055 (-1.482)

Note. This table presents the Sharpe ratio of each strategy employed as explained in Table 3 and the following text. Results for mv-c and min-c not reported, due to overall worse performance, for the sake of brevity and to better illustrate the point at hand. Test statistic in parentheses, computed according to the methodology explained by Jobson and Korkie (1981) described in Chapter 4. *** denotes p-value < 0.01

Similarly, the CEQ returns can be seen in Table 5, with the strategies presented in rows and datasets in columns. Setting aside for a moment the values found, a CEQ return of e.g., 0.5 would mean that the investor would be indifferent between receiving a guaranteed return of 0.5 (or 50%) and taking on the risk of the particular investment or portfolio under consideration. Going back to the data at hand, it can be immediately noticed that all of the CEQ returns in the table are negative, meaning an investor would prefer to lose an x part of his investment for certain, than to undertake the risky investment. Some of the values reaching even as high as below -1.0, meaning an investor would rather lose his entire investment and then some, than to undertake the particular investment. This can once again be explained by the high variance of the portfolios and, moreover, by the choice of γ which is rather arbitrary but affects the outcome significantly. It is also important to note that the absolute value is not necessarily of importance here, but rather the change when moving from 1/N to mv or min.

As seen in the table, mv and min outperform the naive approach in all cases at the 1% of significance. Using FF-4, min has a CEQ return of -0.078, significantly outperforming the 1/N strategy with a CEQ return of -1.053.

TABLE 5

CEQ returns for empirical data

Strategy	S&P Sector	Industry	International	MKT/SMB/ HML	FF-1	FF-3	FF-4
1/N	-1.101	-1.038	-1.072	-0.192	-1.306	-1.142	-1.053
mv	-0.771*** **(-18.596)**	-0.953*** **(-9.419)**	-0.531*** **(-32.955)**	-0.467*** **(-10.571)**	-1.265*** **(-3.964)**	-0.455*** **(-84.733)**	-0.446*** **(-76.989)**
min	-0.531*** **(-34.281)**	-0.694*** **(-40.754)**	-0.391*** **(-43.409)**	-0.136*** **(-14.802)**	-1.061*** **(-24.876)**	-0.118*** **(-142.330)**	-0.078*** **(-142.572)**

Note. This table presents the CEQ returns of each strategy employed. Test statistic in parentheses, computed using a paired t-test, comparing the benchmark to other strategies. Again, results for mv-c and min-c omitted. *** denotes p-value < 0.01

Taking these results into account, H1 is (mostly) accepted – the sophisticated strategies do not outperform the naive approach in a statistically significant manner in terms of the Sharpe ratio. But H2 is rejected – the sophisticated strategies do outperform the naive approach in a statistically significant manner in terms of CEQ returns.

These findings are both in accordance with and contradictory to those of DeMiguel et al. (2007). Similarly, the Sharpe ratio of the proposed strategies cannot consistently outperform the benchmark with high statistical significance. But, on the contrary, in terms of CEQ returns, as opposed to the findings of the authors, the proposed strategies do consistently outperform the benchmark. The answer to this apparent difference is far from obvious, though. One possible reason could be the fact that correlations between assets change over time – if these decreased since 2007, strategies favouring minimum variance could benefit from it. Another reason is financial innovation including the proliferation of ETFs, advancements in trading technology, and increased algorithmic trading, all of which could influence asset price behaviours and, subsequently, the performance of various portfolio strategies.

Similarly, the findings of Statman (2004), and Benartzi and Thaler (2001) concur, generally speaking, with my own, showing that while 1/N might not have mathematically optimal performance, it is a useful heuristic for when a simpler method is needed, or a more complex method cannot be applied. Opting for the "naive" approach might oftentimes prove much better than failing to account for one's bias and selecting an even worse strategy.

Fernholz and Shay (1982) argued that in complex and dynamic markets no single diversification strategy outperforms any other strategy, and in such a context the naive approach could prove appealing. Given the high standard deviation present in my dataset, it makes sense that 1/N performed quite well achieving similar Sharpe ratios to the sophisticated methods.

Lastly, behavioural factors could play a part as well, one such concept is that of "depression babies", wherein people who have experienced an economic downturn during their lifetime are more risk-averse in their trading behaviour (Malmendier and Nagel, 2011), potentially affecting the efficiency of mv and min strategies.

CHAPTER 6 Conclusion

In this thesis I have aimed to investigate the effectiveness of various diversification strategies, as opposed to a benchmark – the naive (1/N) portfolio. According to previous research, none of the more "sophisticated" strategies could consistently outperform the naive portfolio, thus I have embarked on a journey to test whether this truly is the case and whether the results are upheld today. Due to changing market conditions and the creation of new financial technologies it was possible that strategies once unprofitable could become more suitable nowadays. As more and more people are actively investing, not only those with a financial background, an evaluation of these methods to ascertain continued validity seemed appropriate. To this end I created the following research question: "How does the choice of a diversification strategy affect portfolio performance?"

To answer this research question, I have taken into consideration seven datasets, consisting primarily of ETFs for various market sectors, regions, and types. I have evaluated five strategies, i.e. 1/N, mean-variance (also with short-sale constraints), and minimum variance (also with short-sale constraints), in terms of their Sharpe ratio and certainty-equivalent return. According to my findings, none of the strategies could consistently and significantly outperform 1/N in terms of the Sharpe ratio, but all of them did outperform it in terms of certainty-equivalent returns.

Therefore, it is essential to consider the continued effectiveness of simple strategies, but to realise that different strategies might be better adjusted for different occasions. Another takeaway is that the time period studied matters, as I have shown, thus continued efforts should be made to ascertain that previous research deemed valid before is valid today. Lastly, my research shows a certain divergence in different evaluation strategies, showcasing the importance of using multiple measures of performance, since while one can indicate poor performance, another one might reveal quite the opposite.

A potential limitation of this study is the lack of sensitivity analysis, considering different values of γ to include different levels of risk-aversion, or employing an efficiency frontier for mean-variance analysis to find the optimal balance between variance and expected returns. Its inclusion could produce more robust results, less susceptible to arbitrary choice. Another limitation is the limited availability of data taken from Yahoo! Finance, reaching back only until 1998 – future research could perhaps utilise a different source offering more extensive data, if such a source exists.

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