Forecasting inflation rates and recessions with asset prices: A Copula Approach

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Abstract

In this research we report asymmetry in the dependence structure between the growth rate of the Consumer Price Index (CPI) and the federal fund rate. The same also appears to be true for the dependence structure between the growth rate of the Composite Coincident Index (CCI) of the Conference Board and the yield spread. We investigate the importance of capturing non-linearities in the dependence structure in the context of out-of-sample forecasting by comparing the forecast performance of models that capture non-linearities in the dependence structure and those which do not. For predicting future inflation rates, we find that the out-of-sample forecasts can be improved by capturing the effects of an asymmetric dependence structure. For recession forecasts, we do not see evidence of improved forecasting performance however.

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Chapter 1

Introduction

1.1 Background

The interest of economists at central banks and businesses often has been predicting the future state of the economy, in particular recessions as it can lead to severe economic costs. Often in practice, the binary recession indicator published by the NBER Business Cycle Dating Committee is used for dating US recessions. This indicator is based on the following variables: personal income less transfer payments, employment, industrial production and the volume of sales of the manufacturing and wholesale sector adjust for price changes to determine the months of peaks and troughs. The Conference Board‘s work on the composite coincident indicators for the United States combines these 4 series in their CCI index. Therefore there is a close link between CCI growth and the binary NBER recession indicator.[1]

Not surprisingly, forecasts of inflation rates also receive a lot of attention from policy makers and businesses as actions have to be taken before inflationary or deflationary pressures appear in the economy. These efforts are to combat losses as a result of a decline in purchasing power. For example, fixed income streams such as interest rates from fixed deposits or fixed salary will suffer from a diminished purchasing power over time. This exposure can be hedged, but hedging is not an option for everybody. Also if people have a

[1]The NBER does not define a recession in terms of two consecutive quarters of decline in real GDP. Rather a recession is a recurring period of decline in total output, income, employment and trade, usually lasting from six months to a year, and marked by widespread contractions in many sectors of the economy, http://www.nber.org/cycles.html

salary that increases with inflation, people may shift into a higher income tax bracket as a result of higher inflation rates[2].

Asset prices are forward looking in the sense that they have information embedded in its prices about the future state of the economy, hence it's a very interesting class of variables for predicting inflation rates and recessions. The Euler equation[3] from Asset pricing theory implies that the price of an asset is the expectation of the pay-off's and the pricing kernel. Because the pricing kernel has information embedded about the future state of the economy, assets have information embedded in its prices about the future state of the economy.

The paper of Estrella & Mishkin (1997) investigating the predictive content of asset prices to forecast recessions finds that the yield spread best predicts future recessions. Also the authors report that the yield spread performs better by itself rather than in conjunction with other variables. Their results suggest that including more variables leads to over-fitting which deteriorates the forecasting performances.

The findings of Stock & Watson (2003) suggest that the predictive power of the yield spread to predict the output growth may not be stable over time. This is a theoretical argument to suggest that the predictive content of the yield spread to forecast recessions may also not be stable. It has been suggested that the instability may depend on a non-linear relationship between asset prices and output growth.

Various studies have paid attention to non-linear models to forecast future recessions. Markov Switching (MS) models by Hamilton (1989), Smooth Transition Autoregressive (STAR) model by Teräsvirta (1994) and the Threshold Autoregressive (TAR) model by Potter (1995) have been evaluated as potential non-linear forecast models for recessions.

[2] http://www.reservebank.co.za

[3] According to the book of Cochrane (2005), asset prices adjust themselves to the expected pricing kernel and expected payout which is implied by the expression $p_t = E_t(m_{t+1}x_{t+1})$. In a bad state of the world, the pricing kernel m_{t+1} takes on low values while in a good state of the world the pricing kernel takes on high values. Bad states of the world are often characterized by recessions and/or high inflation and the opposite for good states of the world. So there is a well documented theoretical relationship between asset prices and the expected future output growth and inflation rates.

These non-linear specifications all have found to perform quite poorly in an out of sample setting however[4].

The empirical researches of Minksin (1990a), Minksin (1990b) and Minksin (1991) finds that the yield curve has predictive ability to forecast inflation rates. However the findings of Stock & Watson (2003) suggests that the short rate has more predictive power than the yield spread to forecast inflation rates. These same authors suggested that inflation rates may also not stable over time and suggested that the instability may be attributed to a non-linear relationship between inflation rates and asset prices. However the results of the paper Andrew Ang & Wei (2007) indicate that regime switching models perform poorly relative to linear regression models using term structure data.

So the challenge remains open to find non-linear models that produces good out-of-sample forecasts.

[4]The papers of Estrella & Schich (2003) and Stock & Watson (2003) performed literature studies for recession and output growth forecasts which we refer to for the more interested reader.

1.2 Object and Scope

For inflation rates and recession forecasts, we consider the possibility that the dependence structure with asset prices is asymmetric. The empirical research of Dowd (2008) finds non-zero tail dependence in the joint distribution of interest rates and the general price level for the United Kingdom, so it could be possible that the same is true for US data. For the US market, we have seen extreme events in asset prices such as stock market crashes being linked to extreme events in output growth in the form of recessions, which suggest a non-zero tail dependence in the joint distribution of asset prices and output growth.

The impact of not capturing tail-dependence has been demonstrated by the controversial work of Li (2000). The author did not consider non-zero tail dependence in the joint distribution which lead many of the leading investment banks to underestimate the risk of CDO's crashing at the same which is blamed as the main culprit of the recent financial meltdown. In the case of out-of-sample forecasting, recent empirical research of Patton (2005b) finds that portfolio decisions can be improved by capturing non-zero tail dependence.

So in this research we question if the dependence structure of inflation rates and CCI grow with asset prices is asymmetric. If this is the case, we question if capturing the non-linear dependence structure can improve out-of-sample recession and inflation rate forecasts.

1.3 Approach

In this research, we examine the importance of capturing non-linearities in the dependence structure for recession and inflation forecast using asset prices. The papers of Estrella & Hardouvelis (1991) and Estrella & Mishkin (1997) indicate that the Probit model performs quite good for recession forecasts. For predicting future inflation rates, the linear regression model is quite good as it outperformed the non-linear models in an out-of-sample setting using term structure data in the paper of Andrew Ang & Wei (2007). Hence we use these two models as benchmarks for predicting future recessions and inflation rates.

For the linear regression model, we argue that the assumption of independent residuals and explanatory variable does not hold if the dependence structure is asymmetric, which could deteriorate the out-of-sample forecasting performance. A variation on the linear regression model that we developed in this research, is modelling the conditional mean of the residuals with copula functions because this conditional mean is not zero under these conditions.

The Probit model used by Estrella & Hardouvelis (1991) and Estrella & Mishkin (1997) underestimates the conditional probability of a recession if the lower tail dependence coefficient between CCI growth and the yield spread is non-zero. In this research, we develop an alternative forecasting model which calculates the conditional probability of a recession with copula functions, which we refer to as the Copula probability forecast model. Our approach allows for non-zero tail dependence in the joint distribution of CCI growth and the yield spread.

Using US data, the exceedance correlations show asymmetry in the dependence structure with asset prices for both inflation rates and CCI growth. Overall, the results suggests that the Copula-regression model outperforms the standard linear regression model for predicting future inflation rates, especially for long forecasting horizons. For recession forecasts we do not see evidence of improved out-of-sample forecast performance over the Probit model when capturing the non-linear dependence structure between CCI grwoth and yield spread.

1.4 Thesis Structure

The remainder of this thesis proceeds with chapter 2 where we review the Probit model and the linear regression models used by Estrella & Mishkin (1997) and Andrew Ang & Wei (2007) for predicting recessions and inflation rates with term structure data. We also discuss how the predictive accuracy for these models is affected if the dependence structure is asymmetric. In chapter 3 we review copula's which we use to extend the Probit and linear regression model to capture the effects of an asymmetric dependence structure in chapter 4. The data is analysed and non-linear dependence is investigated in chapter 5. In chapter 6 and 7 we present the empirical results of forecast models that captures the effects of an asymmetric dependence structure and of those which do not. This research concludes with chapter 8.

Chapter 2

Forecasting

2.1 Introduction

In this chapter, we describe the linear regression model for predicting future inflation rates and the Probit model for predicting future recessions which are used as benchmark models. We also perform an analysis of the impact on the predictive accuracy when the dependence structure with asset prices is asymmetric for these two forecast models. Later on in this research, we extend the linear regression and binary response model which capture the effects of an asymmetric dependence structure with copula functions.

2.2 Linear regression model

Let the k-period ahead forecast model be described by

$$y_{t+k} = X_t'\beta + \varepsilon_t \tag{2.1}$$

for which k is the length of the forecast horizon and ε_t a normally distributed error term. The best prediction of y_{t+k} given values of X_t is the conditional expectation $E(y_{t+k}|X_t)$. Hence it follows that the best predictor of y_{t+k} is

$$E[y_{t+k}|X_t] = E[X_t'\beta + \varepsilon_t|X_t] = E[X_t'\beta] + E[\varepsilon_t|X_t] = X_t'\beta + E[\varepsilon_t] = X_t'\beta + 0 = X_t'\beta \tag{2.2}$$

Here we assume that the residuals and input are independent. If this independence doesn't hold, $E[\varepsilon_t|X_t] \neq E[\varepsilon_t]$ which leads to $X_t'\beta$ not being the best predictor of y_{t+k}.

2.3 Binary response model

Assume that Y_{t+k} follows a Bernoulli distribution which is defined as $Y_{t+k} \sim BIN(1, \pi)$. In other words, $P(Y_{t+k} = 1) = \pi$ and $P(Y_{t+k} = 0) = 1 - \pi$. Because π is unknown, π is made dependent on a set of explanatory variable X_t such that $Y_{t+k} \sim BIN(1, F(X_t'\beta)$ where F is a function that lies between 0 and 1. Hence $P(Y_{t+k} = 1|X_t) = F(X_t'\beta)$ and $P(Y_{t+k} = 0|X_t) = 1 - F(X_t'\beta)$.

The binary variable Y_{t+k} is often inferred from other variables which we refer to as latent variables. For example, the binary NBER recession indicator is determined by turning points in personal income less transfer payments, employment, industrial production and the volume of sales of the manufacturing and wholesale sector adjust for price changes. The Conference Board combines these same indicators in the CCI index, hence we can make the latent variable observable by the CCI growth. We assume that the latent variable y_t^* gets mapped to the dependent variable by

$$\begin{aligned} \mathrm{Y}_t = 1 \quad if \quad y_t^* \leq 0 \\ Y_t = 0 \quad if \quad y_t^* > 0 \end{aligned} \tag{2.3}$$

for which the explanatory variables describe the continuous latent variable y_t^* by

$$y_t^* = X_t'\beta + \varepsilon_t \tag{2.4}$$

If F is the CDF of the standard normal distribution, we get the Probit model which is

$$F(X_t'\beta) = \Phi(X_t'\beta) = \int_{-\infty}^{X_t'\beta} \frac{1}{\sqrt{2\pi}} exp(-\frac{z^2}{2} dz) \tag{2.5}$$

and if F is the CDF of a standardized logistic distribution, we get the Logit model which is

$$F(X_t'\beta) = \Lambda(X_t'\beta) = \frac{exp(X_t'\beta)}{1 + exp(X_t'\beta)} \tag{2.6}$$

The function of X_t that best estimates Y_{t+k} is the conditional expectation, which is given by

$$E[Y_{t+k}|X_t] = 0 * P(Y_{t+k} = 0|X_t) + 1 * P(Y_{t+k} = 1|X_t) = P(Y_{t+k} = 1|X_t) = F(X_t'\beta) \tag{2.7}$$

Thus we can generate forecasts by

$$\hat{Y}_{t+k} = P(Y_{t+k} = 1|X_t) = F(X_t'\beta) \tag{2.8}$$

2.4 Pitfalls in forecasting

The empirical research of Longin & Solnik (2001) have reported non-linear dependence between stock returns. Similar finding may be found for the dependence structure of inflation rates and output growth with asset prices. This is because extreme events of inflation rates and output growth have been linked with extreme events of asset prices which implies a non-zero tail dependence coefficient in the joint distribution. In this section we investigate the problems of the forecast models previously discussed if the dependence structure is asymmetric.

In the case of the linear regression model, we assumed that residuals and explanatory variables are independent such that $E[\varepsilon_t|X_t] = 0$ holds. Let's assume that the joint distribution of the dependent and explanatory variable has a non-zero upper tail dependence coefficient. In this setting, the dependence or statistical relationship for large returns is greater than a linear dependence structure would imply. Because the linear regression model assumes a linear dependence structure, the linear regression is thus least accurate for large returns. Thus in the setting of asymmetric dependence, the residuals of a linear regression model and the explanatory variables are not independent. The problem with this implication is that ignoring the term $E[\varepsilon_t|X_t]$ does not make sense any more and not implementing this term could deteriorate the forecasting performance.

For the case of the Probit model, we can see from equation 2.9 that non-normal tail probabilities in the joint distribution of the latent and explanatory variables can lead to higher than normal conditional probabilities of observing an event.

$$P(Y_{t+k} = 1|X_t) = P(y^*_{t+k} < 0|X_t) = \int_{-\infty}^{0} \frac{f(y^*_{t+k}, X_t)}{f(X_t)} \delta y^*_t \tag{2.9}$$

Thus the Probit model lacks accuracy for small returns if the lower tail dependence coefficient is non-zero because the conditional probability of observing an event is systematically underestimated.

Chapter 3

Introduction to Copula

The forecast models we develop in this research are based on copula functions. For predicting inflation rate forecasts, we focus on improving the linear regression model by modelling the conditional mean of the residuals with copula's, while for predicting future recessions we consider calculating the conditional probability of a recession given the data at time t.

Therefore it is important to have a clear understanding what copula functions are and how they can be used. In this chapter we explore the most important parts of the books Alexander (2008) and Nelson (1999).

3.1 Definitions & Properties

In this research we use copula's to model joint distributions. Let's assume that X_1 and X_2 are two random variables with marginal cumulative distribution functions $F_1(X_1) = P(X_1 \leq x_1)$ and $F_2(X_2) = P(X_2 \leq x_2)$. The joint cdf is defined as $H(x_1, x_2) = P(X_1 \leq x_1, X_2 \leq x_2)$ with the range $0, 1$ for $H(x_1, x_2)$, $F_1(X_1)$ and $F_2(X_2)$. There are many cases for which the explicit joint distribution $H(x_1, x_2)$ doesn't exist or is very difficult to obtain even though the marginal distributions $F_1(X_1)$ and $F_2(X_2)$ are easy to describe. For example, the multivariate student t-distribution has only 1 degree of freedom parameter so if the marginal distributions do not have the same fatness in the tails, the joint cdf doesn't exist. A solution to this problem are copula's which can be used to link a set of marginal distributions into a multivariate joint distribution defined over the range $[0, 1]$.

Definition 1 A two-dimensional copula is a function $C : [0, 1]^2 \rightarrow [0, 1]$ with the 4 properties

1. $C(u_1, u_2)$ is increasing in u_1 and u_2

2. $C(0, u_2) = C(u_1, 0) = 0$

3. $C(1, u_2) = u_2$ and $C(u_1, 0) = u_1$

4. $C(v_1, v_2) - C(u_1, v_2) \geq C(v_1, u_2 - C(u_1, u_2)$ for every $u_1, u_2, v_1, v_2[0, 1]$ with $u_1 \leq v_1$ and $u_2 \leq v_2$

Theorem 1 Sklars theorem for continuous distributions.

> Let H be a joint distribution function with marginals F_1 and F_2. Then there exist a copula C such that for all real numbers x_1 and x_2, one has the equality as $H(x_1, x_2) = C(F_1(x_1), F_2(x_2))$. Furthermore if F_1 and F_2 are continuous then C is unique. Conversely if F_1 and F_2 are distributions then the function H is a joint distribution function with margins F_1 and F_2.

An important property of the copula function is that it is defined over uniform distributed variable over $[0, 1]$. According to condition 1 we can set $U_i = F_i(X_i)$. The other 3 conditions specify the copula as a distribution function while the Sklars theorem shows that any multivariate distribution function H can be written in the form of a copula function.

This result can be extended to the multivariate case for which Sklar (1959) tell us that, given a fixed set of continuous marginal distributions, distinct copulas define distinct joint CDF. Thus given any joint CDF $F(x_1, ..., x_n)$ with continuous marginals, we can back out a unique copula function C such that $F(x_1, ..., x_n) = C(F_1(x_1), ..., F_n(x_n)$. If it exist, the associated copula CDF is the function

$$c(F_1(x_1), ..., F_n(x_n) = \frac{\delta^n C(F_1 x_1, ..., F_n(x_n)}{\delta F_1(x_1), ..., F_n(x_n)} \tag{3.1}$$

3.2 Tail dependence & Rotated copulas

In this section we present tail dependence that may arise in the joint distribution. It could be possible that we have to impose lower or upper tail dependence when modelling joint distributions. We also present rotated copula's which allows us to have more copula's to work with.

Often in Finance the density function is higher in the corners than normal, suggesting a non-zero dependence in the tails. Define the i,j-th lower tail dependence coefficient as

$$\lambda_{i,j}^{l} = \lim_{q \downarrow 0} P(X_i < F_i^{-1}(q) | X_j < F_j^{-1}(q)) \tag{3.2}$$

provided that the limit exists. The lower tail dependence coefficient can be interpreted as the conditional probability that one variable takes a value in its lower tail, given that the other variable takes a value in its lower tail. Since the coefficient is a conditional probability, $\lambda_{i,j}^{l} \in [0, 1]$. The copula is said to have lower tail dependence for X_i and X_j when $\lambda_{i,j}^{l} > 0$. Similarly the i,j-th upper tail dependence coefficient is defined by the limit assuming it exist:

$$\lambda_{i,j}^{l} = \lim_{q \uparrow 0} P(X_i > F_i^{-1}(q) | X_j > F_j^{-1}(q)) \tag{3.3}$$

It basically represents the conditional probability that one variable takes a value in tis upper tail, given that the other variable takes a value in its upper tail. Since the coefficient is a conditional probability, $\lambda_{i,j}^{u} \in [0, 1]$. The copula is said to have upper tail dependence for X_i and X_j when $\lambda_{i,j}^{u} > 0$. The higher the value of the dependence coefficient, the stronger the upper tail dependence. A copula has symmetric tail dependence if $\lambda_{i,j}^{l} = \lambda_{i,j}^{u}$ for all i,j and asymmetric tail dependence if the upper or lower tail dependence coefficients are different. Examples of copulas with symmetric tail dependence are the normal and student t copulas and others with asymmetric tail dependence are the Clayton and Gumbel copulas. Since

$$P(X_i < F_i^{-1}(q)|X_j < F_j^{-1}(q)) = \frac{P(X_i < F_i^{-1}(q), X_j < F_j^{-1}(q))}{P(X_j < F_j^{-1}(q))} = \frac{C(q,q)}{q} \tag{3.4}$$

Hence the lower tail dependence coefficient can be expressed as

$$\lambda^l = \lim_{q \downarrow 0} q^{-1} C(q,q) \tag{3.5}$$

The limit must lie in the interval $[0,1]$, and if λ^l is positive, the copula has lower tail dependence. Similarly it can be shown that

$$\lambda^u = \lim_{q \uparrow 1} (1-q)^{-1} C(1-q, 1-q) \tag{3.6}$$

where $C(u_1, u_2) = u_1 + u_2 - 1 + C(1-u_1, 1-u_2)$ which is called the survival or rotated copula associated with $C(u_1, u_2)$ while for densities the rotated or survival copula is $c(1-u_1, 1-u_2)$. The rotated or survival copula is rotated by 180 degrees so it only makes sense for asymmetric copulas such as the Clayton or Gumbel copula. If λ^u is positive, the copula has upper tail dependence which lies in the interval $[0,1]$.

3.3 Maximum Likelihood Estimation

We estimate the copula parameters using maximum likelihood estimation applied to the theoretical joint distribution function. There are three methods to maximizing the likelihood function: canonical maximum likelihood estimation (CML), inference on margins (IMF) and full maximum likelihood estimation (FML). In this research we use the canonical maximum likelihood estimation (CML), but we also give an overview of the other 2 copula calibration methods for the interested reader who wishes to extend this research by investigating how the three methods affect the out-of-sample performance of our forecast models.

In the inference on margins (IMF) approach we specify a functional form for the marginals in contrast to the canonical maximum likelihood (CML) estimation. However both these methods may provide less efficient estimators than full maximum likelihood estimation (FML) which calibrates all parameters of the copula and the marginals at the same time. But the advantage that the inference on margins and canonical maximum likelihood (CML) approach offer, is that they are considerably easier than full maximum likelihood estimation (FML) and they lead to consistent estimators.

The likelihood of observing sample $(x_i, y_i)_{i=1}^{n}$ from the sample (X, Y) with distributions $X \sim F$ and $Y \sim G$ is [1]

$$L(x, y; \theta) = \prod_{i=1}^{n} c(F(x_i), G(y_i)) f(x_i) g(y_i); \theta) \tag{3.7}$$

the maximization of the likelihood function is the same as maximizing the log-likelihood which is

$$\log[L(x, y; \theta)] = \sum_{i=1}^{n} \log[c(F(x_i), G(y_i); \theta] + \sum_{i=1}^{n} \log[f(x_i) g(y_i)] \tag{3.8}$$

In the inference on margins (IMF) approach, a parametric distribution for F,G and copula C is chosen. The term $\sum_{i=1}^{n} \log[f(x_i) g(y_i)]$ of the log-likelihood is maximized first which is then followed by the maximization of the first term described as $\sum_{i=1}^{n} \log[c(F(x_i), G(y_i); \theta]$. The same two step maximization method is performed for the canonical maximum likelihood (CML) approach except that the distribution of F,G are replaced by the corresponding non-parametric empirical distributions.

[1] The sheets of a presentation by Jon de Kort in 2007 regarding copula functions by ABN AMRO at the TU Delt was used in this section

3.4 How to choose the best copula

Recently an out-of sample measure based on the Kullback-Leibler Information Criterion (KLIC) for competing copulas has been proposed by Diks, Panchenko & van Dijk (2008) which measures the distance of the specified copula to the true copula. Based on this selection criterion, the yen-dollar exchange rate returns does not exhibit asymmetric tail dependence which goes against the findings of Patton (2005a). So therefore we are not confident in its ability to select appropriate copulas as of now. Also the test relies on a large sample size for it to be useful which we do not have.

So therefore we stick to the BIC criteria for copulas to determine which symmetric or asymmetric copula provides the best fit to the data, we choose the copula that yields the lowest value of the BIC criteria. The BIC and AIC criteria are defined by

$$BIC = T^{-1}[k \log(T) - 2\log(L)] \tag{3.9}$$

$$AIC = 2k - 2\log(L) \tag{3.10}$$

where T is the number of observations, L the log likelihood function of the copula and k the number of parameters to be estimated. All model selection and estimation will be done with data available prior to the forecast.

Chapter 4

Application of copulas in forecasting

4.1 Introduction

The main lesson from chapter 2 is that the Probit model and linear regression model must be used with caution if the dependence structure is asymmetric when forecasting. In this section we employ copula functions to develop new methods that are more suitable under these conditions.

4.2 Inflation rate forecasts: Copula-regression forecast model

Let y_{t+k} and X_t be random variables that exhibit an asymmetric dependence structure. The residuals of an estimated linear regression model and the explanatory variables are not independent under these conditions. In this case, the forecasts can be improved by capturing the conditional mean of the residuals $E(\varepsilon_t|X_t)$ rather than setting it equal to zero.

The conditional expectation $E[\varepsilon_t|X_t]$ is defined by

$$E[\varepsilon_t|X_t] = \int_{-\infty}^{\infty} \varepsilon_t f(\varepsilon_t|X_t) d\varepsilon_t \tag{4.1}$$

The conditional distributions in the integral can be rewritten into a joint and marginal distribution.This leads to the equation

$$E[\varepsilon_t|X_t] = \int_{-\infty}^{\infty} \varepsilon_t f(\varepsilon_t|X_t) d\varepsilon_t = \int_{-\infty}^{\infty} \varepsilon_t \frac{f(\varepsilon_t, X_t)}{f_X(X_t)} d\varepsilon_t = \int_{-\infty}^{\infty} \varepsilon_t \frac{f(\varepsilon_t, X_t)}{\int_{-\infty}^{\infty} f(\varepsilon_t, X_t) d\varepsilon_t} d\varepsilon_t \quad (4.2)$$

A very simple and convenient approach to specify the joint distributions with given marginals are the copula functions. For given marginal distributions F_ε and F_X, we can estimate the conditional mean of the residuals with copula functions by

$$E[\varepsilon_t|X_t] = \int_{-\infty}^{\infty} \varepsilon_t \frac{c(F_\varepsilon(\varepsilon_t), F_X(X_t))}{\int_{-\infty}^{\infty} c(F_\varepsilon(w), F_X(X_t)) dw} d\varepsilon_t \quad (4.3)$$

Using these results, the **Copula-regression forecast model** is presented which captures the effects of an asymmetric dependence structure by

$$\hat{Y}_{t+k} = X_t\beta + E[\varepsilon_t|X_t] = X_t\beta + \int_{-\infty}^{\infty} \varepsilon_t \frac{c(F_\varepsilon(\varepsilon_t), F_X(X_t))}{\int_{-\infty}^{\infty} c(F_\varepsilon(w), F_X(X_t)) dw} d\varepsilon_t \quad (4.4)$$

4.3 NBER recession forecasts: Copula Probability Forecast Model

While the Probit model used in the papers of Estrella & Hardouvelis (1991) and Estrella & Mishkin (1997) has been successful at forecasting the binary NBER recession variable, in chapter 2 we explained that the forecast performance of the Probit model could be compromised by asymmetry in the dependence structure between the latent and explanatory variable. The assumption of independent residuals and explanatory variables does not hold in this case. In this section we develop new methods to forecast binary response variables that should outperform the Probit model under these conditions.

Let Y_{t+k} be a binary response variable and y^*_{t+k} a latent variable. Also let $Y_{t+k} = 1$ if $y^*_{t+k} < 0$ and 0 otherwise. Then the best forecast of this binary response variable is $P(Y_{t+k} = 1|X_t)$ which we showed in 2.8. Typically $P(Y_{t+k} = 1|X_t)$ is approximated with $F(X_t'\beta)$ for binary response models. However this conditional probability can also be estimated with copula functions. The conditional probability in copula form is given by

$$P(Y_{t+k} = 1|X_t) = P(y^*_{t+k} < 0|X_t) = C(F_{y^*}(y^*_{t+k}) < F_{y^*}(0)|F_X(X_t)) \tag{4.5}$$

where X_t is the set of explanatory variables with marginal distributions F_X and F_{y^*} such that $F_X(X_t)$ and $F_{y^*}(y^*_{t+k})$ are in the interval $[0, 1]$. For convenience, this expression can be rewritten into joint distributions. This leads to

$$C(F_y(y^*_{t+k}) < F_{y^*}(0)|F_X(X_t)) = \int_0^{F_{y^*}(0)} \frac{c(F_{y^*}(y^*_{t+k}), F_X(X_t))}{\int_0^1 c(F_{y^*}(y^*_{t+k}), F_X(X_t))dF_{y^*}(y^*_{t+k})} dF_{y^*}(y^*_{t+k}) \tag{4.6}$$

Then the model to forecast a binary response variable with copula functions is

$$\hat{Y}_{t+k} = P(Y_{t+k} = 1|X_t) = \int_0^{F_{Y^*}(0)} \frac{c(F_{Y^*}(Y^*_{t+k}), F_X(X_t))}{\int_0^1 c(F_{Y^*}(Y^*_{t+k}), F_X(X_t)) dF_{Y^*}(Y^*_{t+k})} dF_{Y^*}(Y^*_{t+k}) \quad (4.7)$$

which is the **Copula Probability Forecast Model** .

Chapter 5

Data

5.1 Data Analysis

We use the annualized h-month growth of the CCI indicator defined by $y_t = (12/h)(z_t - z_{t-h})/z_{t-h}$ where z_t is the original CCI series because the trend appears to be linear which is seen in figure 5.1. The same reasoning applies when transforming CPI inflation which is seen in figure 5.2. The federal fund rate obtained from the Federal Reserve and the yield spread are annualized and are published as percentage change so we don't to transform them. The full sample runs from January 1965 to May 2009, the in-sample period from January 1965 to December 1988 and the out-of-sample period from January 1989 to May 2009.

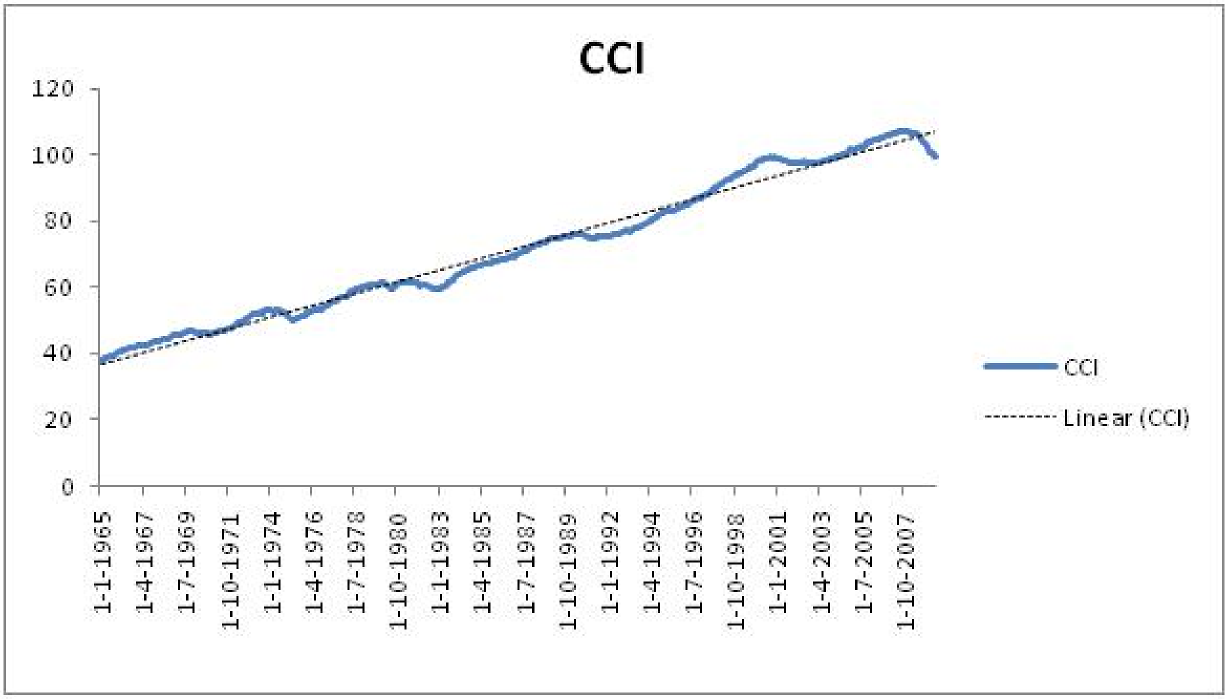

Figure 5.1: Plot of the CCI index over the full-sample period.

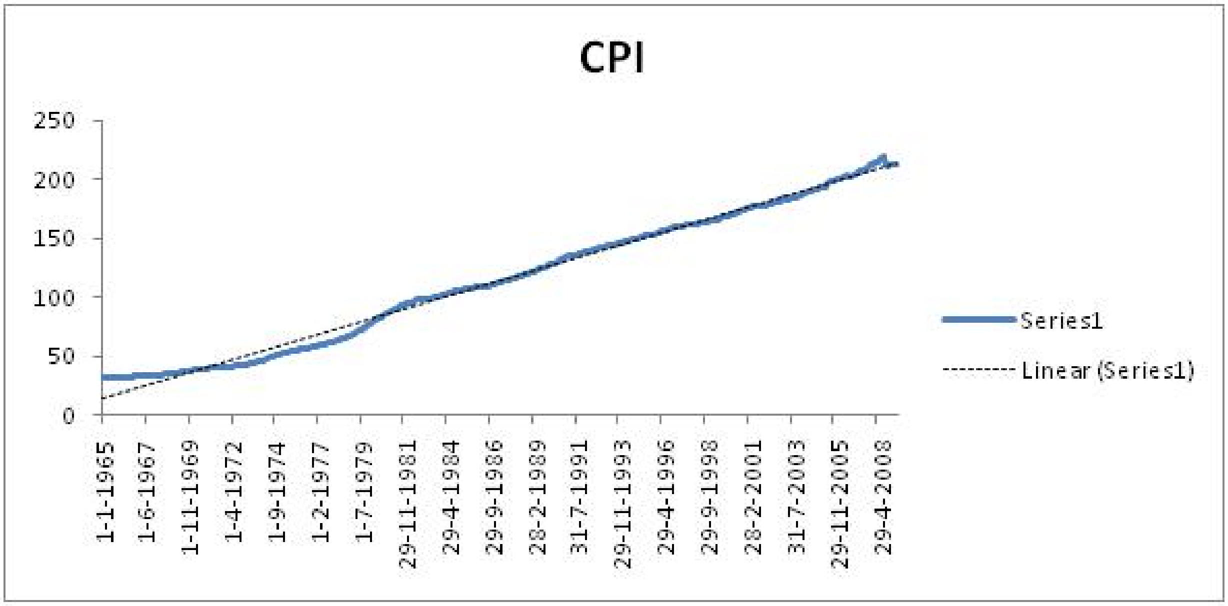

Figure 5.2: Plot of the CPI index over the full-sample period.

In table 5.1 we see evidence of non-normal skewness and excess kurtosis for CCI growth and the term structure spread for the full sample, in-sample and out-of sample period which is supported by the Jarque-Bera test as no significant evidence in favour of normality is found. Similar results are presented in table 5.2 for the federal fund rate and CPI inflation as the Jarque-Bera test finds no significant evidence of normality at a 5% significance level over the 3 sample periods. This is due to asymmetry and fat-tails in the distributions as the skewness parameter is non-zero and the kurtosis higher than 3. The Jung-Box test with 25 as the number of lags, which should be large enough to detect higher order autocorrelations but small enough to retain enough power to detect the presence of autocorrelation, are reported also in the previous tables. There seems to be significant autocorrelation present for return series CCI growth, yield spread, CPI inflation rate and federal fund rate at a significance level of 5% level which is seen in tables 5.1 and 5.2.

Table 5.1: Descriptive statistics of CCI and Yield Spread

	Full Sample		In Sample		Out of Sample	
	CCI	Yield Spread	CCI	Yield Spread	CCI	Yield Spread
Mean	0.0219	0.0085	0.0282	0.0043	0.0143	0.0135
Std. Dev	0.0433	0.0177	0.0471	0.0194	0.0371	0.0139
Skewness	-0.4718	-0.9845	-0.5609	-10,960	-0.6497	0.0533
5% VaR	-0.0515	-0.0250	-0.0523	-0.0330	-0.0478	-0.0070
1% VaR	-0.1037	-0.0476	-0.1037	-0.0563	-0.0943	-0.0116
Kurtosis	45,879	45,386	44,470	40,126	48,273	17,174
Min	-0.1615	-0.0651	-0.1615	-0.0651	-0.1361	-0.0125
Max	0.1661	0.0385	0.1661	0.0328	0.1308	0.0385
Jarque-Bera	75,770	138.66	40,225	69,966	51,114	16,840
p-val	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
Ljung-Box	443.55	3129.5	252.69	1434.6	247.88	1720.8
p-val*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 5.2: Descriptive statistics of CPI Inflation and Federal fund rates

	Full Sample		In Sample		Out of Sample	
	Inflation	Short rate	Inflation	Short rate	Inflation	Short rate
Mean	0.0433	0.0628	0.0565	0.0786	0.03	0.0440
Std. Dev	0.0390	0.0335	0.0393	0.0334	0.03	0.0218
Skewness	0.1685	10,853	0.6726	12,236	-1.4	0.1279
5% VaR	0.0000	0.0126	0.0000	0.0404	-0.01	0.0100
1% VaR	-0.0482	0.0022	-0.0218	0.0372	-0.09	0.0016
Kurtosis	72,942	49,578	39,997	44,069	14.92	27,148
Min	-0.2006	0.0015	-0.0656	0.0329	-0.2	0.0015
Max	0.2172	0.1910	0.2172	0.1910	0.17	0.0985
Jarque-Bera	412.05	189.75	33,709	95,624	1525.7	14,923
p-val	0.0001	0.0001	0.0001	0.0001	0.0000	0.0001
Ljung-Box	1701.7	6742.4	1120.8	2790.9	98,534	2115.9
p-val*	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

One of the objectives of this research is to study the forecast performance of statistical models, hence there is the need to investigate if the selected asset prices have any information about future inflation rates and output growth returns. In table 5.3 we present the correlations for forecasting horizons up to one year. The results indicate that the spread of the term structure is more informative for long-term horizons of CCI growth as the correlations increases with the length of the horizon. The findings for the short rate implies the opposite, the short rate is very informative if the horizon is kept at a short-term as the strength of the correlations decreases beyond the short-term.

Table 5.3: Correlations

Forecasting horizon	CCI & Yield Spread	Inflation & Short rate
0	0.0968	0.5258
1	0.1645	0.5116
2	0.2002	0.4804
3	0.2402	0.4423
4	0.2726	0.4145
5	0.2777	0.3943
6	0.2682	0.3776
7	0.2722	0.3614
8	0.2755	0.3426
9	0.2731	0.3201
10	0.2864	0.3056
11	0.2872	0.2872
12	0.2911	0.2667

5.2 Asymmetric dependence

The paper of Longin & Solnik (2001) have investigated asymmetric dependence between stock returns. To see whether our copula based forecast models are suitable for inflation rate and recession forecast, we first have to establish if the dependence structure between inflation rates and output growth with asset prices is asymmetric.

To see whether the dependence structure is asymmetric, the test for asymmetric correlation assessed by Hong, Tu & Zhou (2007) is employed. In the article of Longin & Solnik (2001) the measure called exceedance correlations is presented which measures the correlation if the variables exceeds an exceedance level c, which can be defined by

$$\rho^{+}(c) = corr(r_{1t}, r_{2t} | r_{1t} > c, r_{2t} > c)$$

$$\rho^{-}(c) = corr(r_{1t}, r_{2t} | r_{1t} < -c, r_{2t} < -c) \tag{5.1}$$

where r_{1t}, r_{2t} are 2 random variables. If the correlation structure is symmetric, the correlations for small returns and large returns should be the same. The null hypotheses of a symmetric correlation structure is given by

$$H_0 : \rho^+(c) = \rho^-(c), \quad \forall \quad c \geq 0. \tag{5.2}$$

So basically we are testing if the correlation between small returns is the same as for large returns. If the null hypothesis is rejected than the correlation structure is asymmetric which is given by

$$H_0 : \rho^+(c) \neq \rho^-(c), \quad \forall \quad c \geq 0. \tag{5.3}$$

The test statistic for testing the null hypothesis is

$$J_\rho = T(\hat{\rho}^+ - \hat{\rho}^-)\hat{\Omega}^{-1}(\hat{\rho}^{-1} - \hat{\rho}^-) \rightarrow^d \chi^2_m \tag{5.4}$$

for which m is number of chosen exceedance levels, $\hat{\rho}^+ - \hat{\rho}^-$ is given by

$$\hat{\rho}^+ - \hat{\rho}^- = [\hat{\rho}^+(c_1 - \hat{\rho}^-(c_1), \, ... \, , \hat{\rho}^+(c_m - \hat{\rho}^-(c_m)]' \tag{5.5}$$

and $\hat{\Omega}$ is an estimator of the variance-covariance matrix[1] of $\sqrt{T}(\hat{\rho}^+ - \hat{\rho}^-)$.

[1]

$$\hat{\Omega} = \sum_{l=1-T}^{T-1} k(l/p)\hat{\gamma}_l \tag{5.6}$$

First, the test is applied for CCI growth and the yield spread over the in-sample period for forecasting horizons up to 1 year. The results of the asymmetric correlation test are presented in table 5.4, which overall do not support the hypothesis that the correlation structure of CCI growth and yield spread is symmetric. When we apply the asymmetric correlation test for CPI inflation rate and the short over the in-sample period, the results appear to support the hypothesis that the correlation structure between CPI inflation rate and the short rate is symmetric.

$$\hat{\gamma}_l(c_i, c_j) = \frac{1}{T} \sum_{t=|l|+1}^{T} \hat{\xi}_t(c_i)\hat{\xi}_{t-|l|}(c_j)$$

$$\hat{\xi}_t(c) = \frac{T}{T_c^+}[\hat{X}_{1t}^+(c)\hat{X}_{2t}^+(c) - \hat{\rho}^+(c)]1(r_{1t} > c, r_{2t} > c)$$

$$-\frac{T}{T_c^-}[\hat{X}_{1t}^-(c)\hat{X}_{2t}^-(c) - \hat{\rho}^-(c)]1(r_{1t} < -c, r_{2t} < -c)$$

$$\hat{X}_{1t}^+(c) = \frac{r_{1t} - \hat{\mu}_1^+(c)}{\hat{\sigma}_1^+(c)}, \quad \hat{X}_{2t}^+(c) = \frac{r_{2t} - \hat{\mu}_1^+(c)}{\hat{\sigma}_2^+(c)}$$

for which $\hat{\mu}(c)$, $\hat{\sigma}(c)$ and T_c are the sample mean, standard deviation and the size of the sample for the observations exceeding the exceedance level c.

Table 5.4: J-test

	CCI & Yield Spread		Inflation & Short rate	
Forecasting horizon	Statistic	p-value	Statistic	p-value
0	278,375	0.1133	216,832	0.3579
1	337,451	0.0279	339,625	0.0264
2	313,241	0.0511	161,202	0.7091
3	423,754	0.0025	159,048	0.7225
4	373,704	0.0106	252,080	0.1936
5	418,313	0.0029	138,766	0.8367
6	438,848	0.0016	183,858	0.5620
7	359,570	0.0156	130,919	0.8734
8	340,209	0.0260	212,282	0.3838
9	305,791	0.0610	105,652	0.9567
10	314,620	0.0494	136,069	0.8499
11	238,299	0.2499	130,855	0.8737
12	196,495	0.4800	192,339	0.5067

Using the J-test from the paper of Hong, Tu & Zhou (2007), the acceptance of a symmetric correlation structure is not solid evidence against the hypothesis of an asymmetric correlation structure. According to simulations of the test statistics performed in the paper of Hong, Tu & Zhou (2007), the test statistic can accept the hypothesis of a symmetric correlation structure when the data has been generated from a copula distribution with an asymmetric dependence structure. Hence the acceptance of a symmetric correlation structure is not solid evidence against the hypothesis of an asymmetric correlation structure.

There is evidence of an asymmetric dependence structure between inflation rates and the short rate when we investigate the exceedance correlations for the qth quantiles which are calculated by

$$\rho(q) = \begin{cases} \text{corr}(X,Y)|X \leq Q_X(q) \bigcap Y \leq Q_Y(q) & \text{if } q \leq 0.5 \\ \text{corr}(X,Y)|X \leq Q_X(q) \bigcap Y > Q_Y(q) & \text{if } q > 0.5 \end{cases}$$

for which $Q_X(q)$ is the qth quantile for random variable X and $Q_Y(q)$ the qth quantile for random variable Y in following of Patton (2005b)

We use canonical maximum likelihood estimation to calibrate symmetric and asymmetric copula's on the inflation rate and short rate. If the data has a linear dependence structure, then the empirical exceedance correlations[2] would be that of the Normal Copula. In figure 5.6 to 5.9 we see for large returns, that the empirical exceedence correlations are stronger than if the data were obtained from the Normal copula. This implies that a linear dependence structure is not suitable for the data. The empirical exceedance correlations shows the same asymmetry as the exceedance correlations from the Gumbell Copula, which suggests that the joint distribution of inflation and the short rate has a non-zero upper tail dependence coefficient. These findings are in contrast with the results of the J-test, which only confirms that the test statistic lacks power for detecting asymmetric correlation structures.

The J-test also appears to make false acceptances of a symmetric correlation structure between CCI growth and yield spread returns. In figure 5.5 and table 5.4, the two results clearly differ. The J-test does not detect significant asymmetry in the correlation structure for a 12 month forecasting horizon, yet the empirical exceedance correlations has the same asymmetry as the exceedance correlation from the Clayton Copula, which suggests that the joint distribution of CCI growth and the yield spread has a non-zero lower tail dependence coefficient.

So the exceedance correlations suggests that the joint distribution of inflation and the short rate has a non-zero upper tail dependence coefficient . Likewise we find evidence of a non-zero lower tail dependence coefficient in the joint distribution of CCI growth and the spread of the term structure.

[2]the empirical exceedance correlations are directly obtained from the data

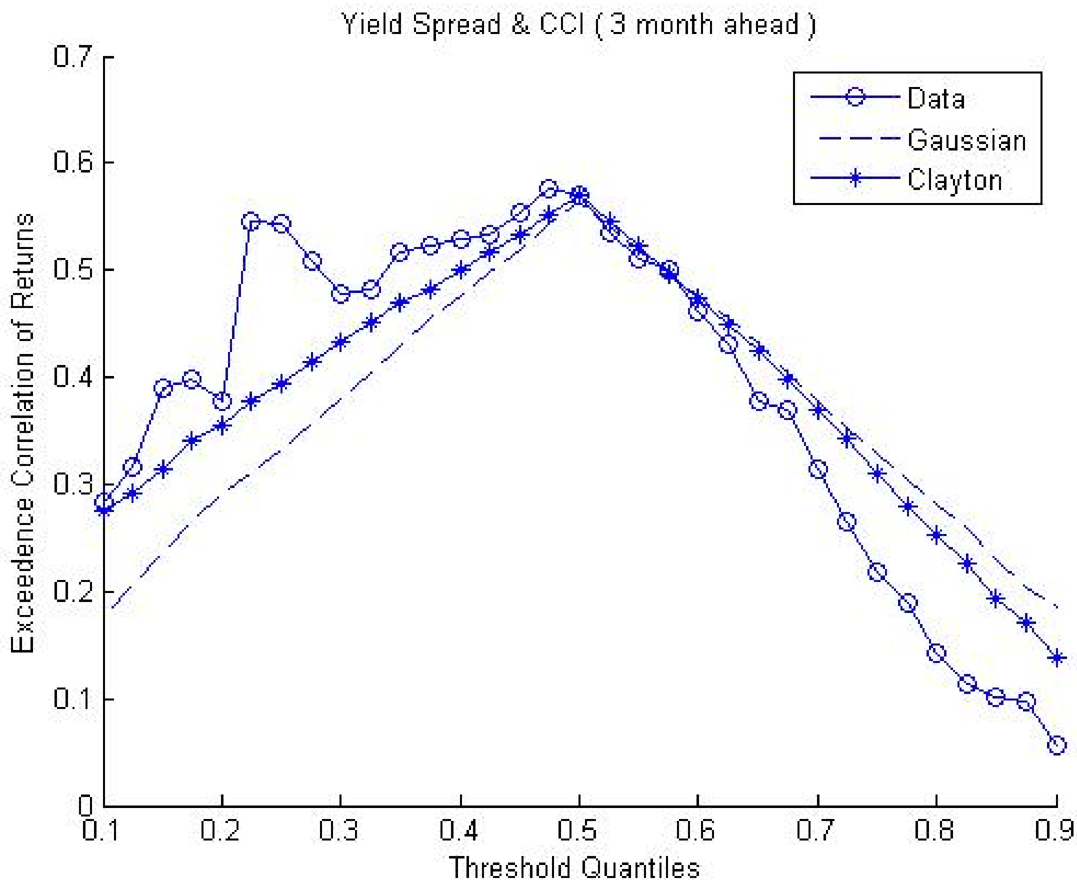

Figure 5.3: Exceedance correlations between transformed return series (U and V) of CCI growth and yield spread using a forecasting horizon of 3 months ($k = 3$) The horizontal axis shows the cutoff quantile, and the vertical axis shows the correlation between the two return series given that both exceed that quantile.

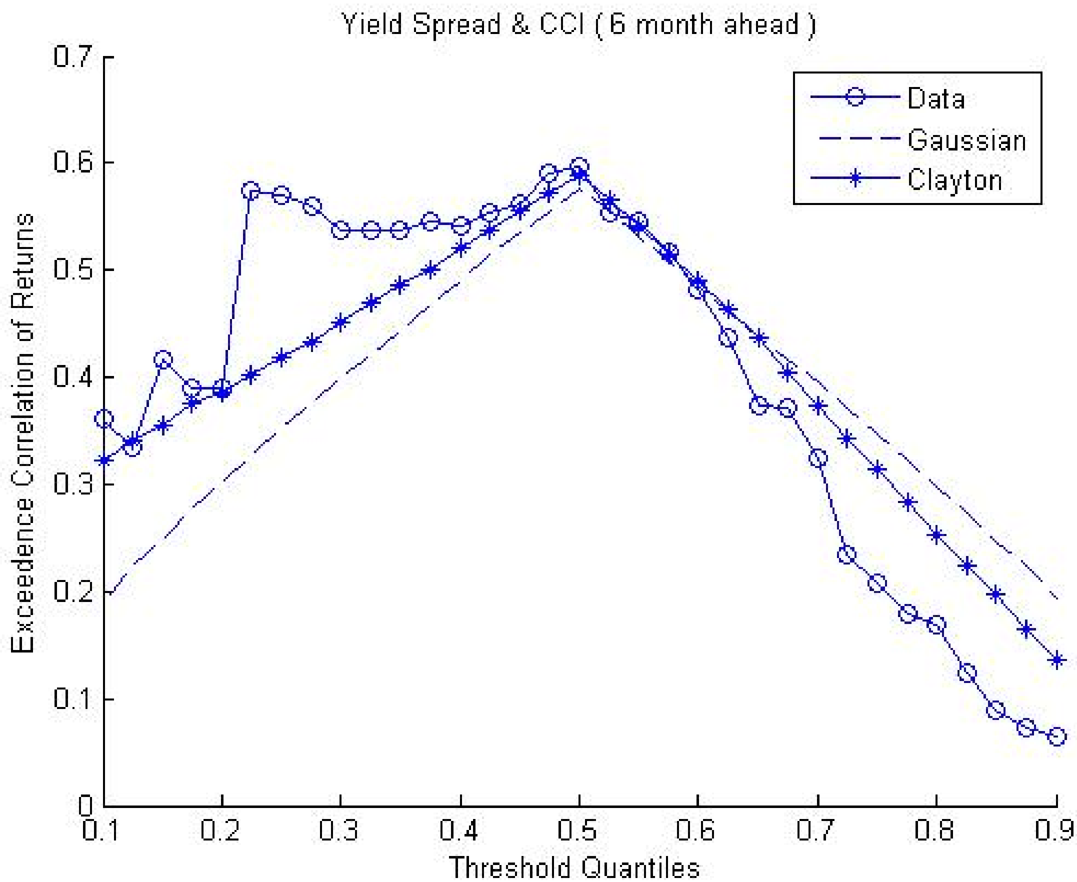

Figure 5.4: Exceedance correlations between transformed return series (U and V) of CCI growth and yield spread using a forecasting horizon of 6 months ($k = 6$) The horizontal axis shows the cutoff quantile, and the vertical axis shows the correlation between the two return series given that both exceed that quantile.

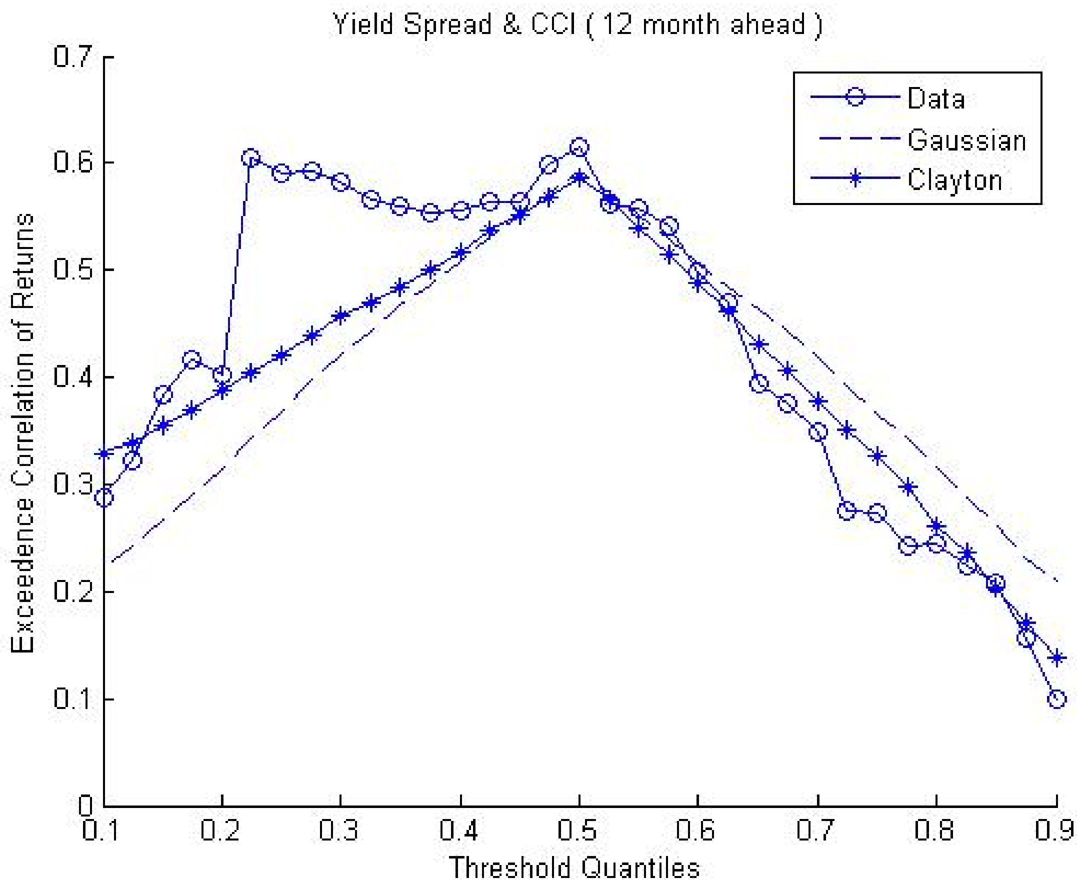

Figure 5.5: Exceedance correlations between transformed return series (U and V) of CCI growth and yield spread using a forecasting horizon of 12 months ($k = 12$) The horizontal axis shows the cutoff quantile, and the vertical axis shows the correlation between the two return series given that both exceed that quantile.

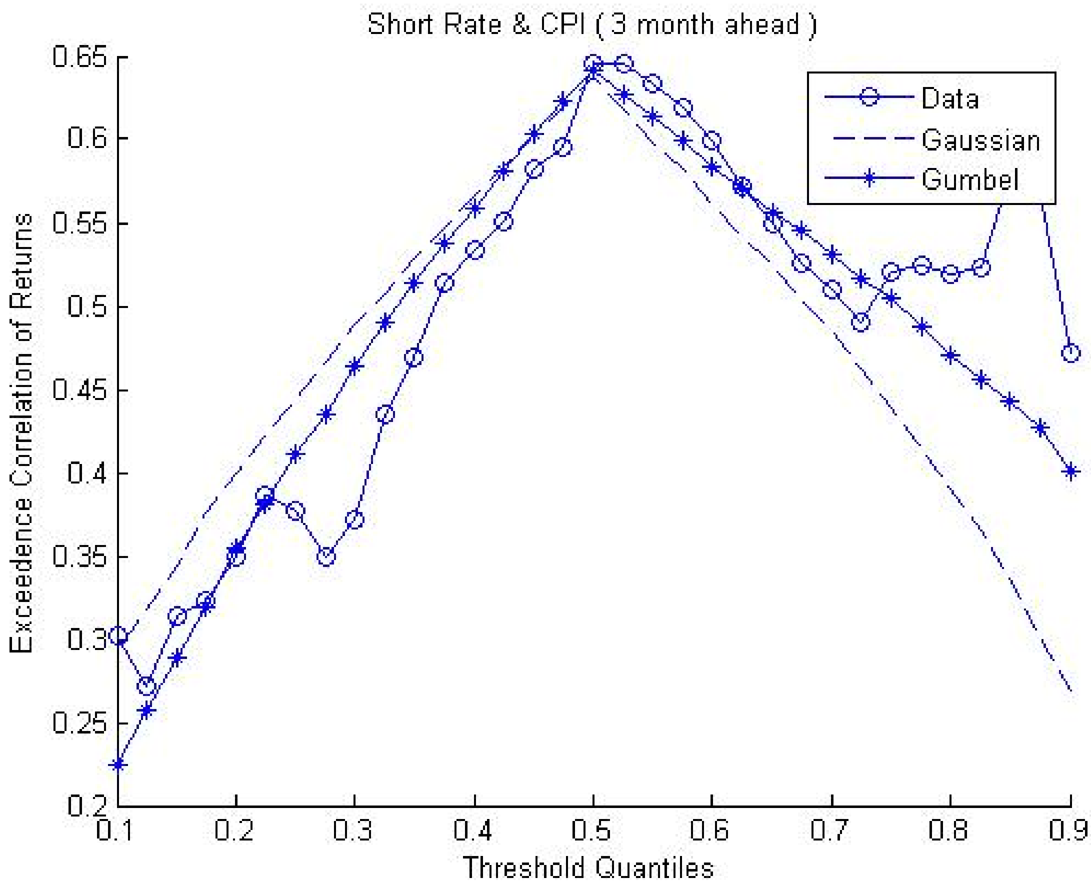

Figure 5.6: Exceedance correlations between transformed return series (U and V) of inflation rate and federal fund rate using a forecasting horizon of 3 months ($k = 3$) The horizontal axis shows the cutoff quantile, and the vertical axis shows the correlation between the two return series given that both exceed that quantile.

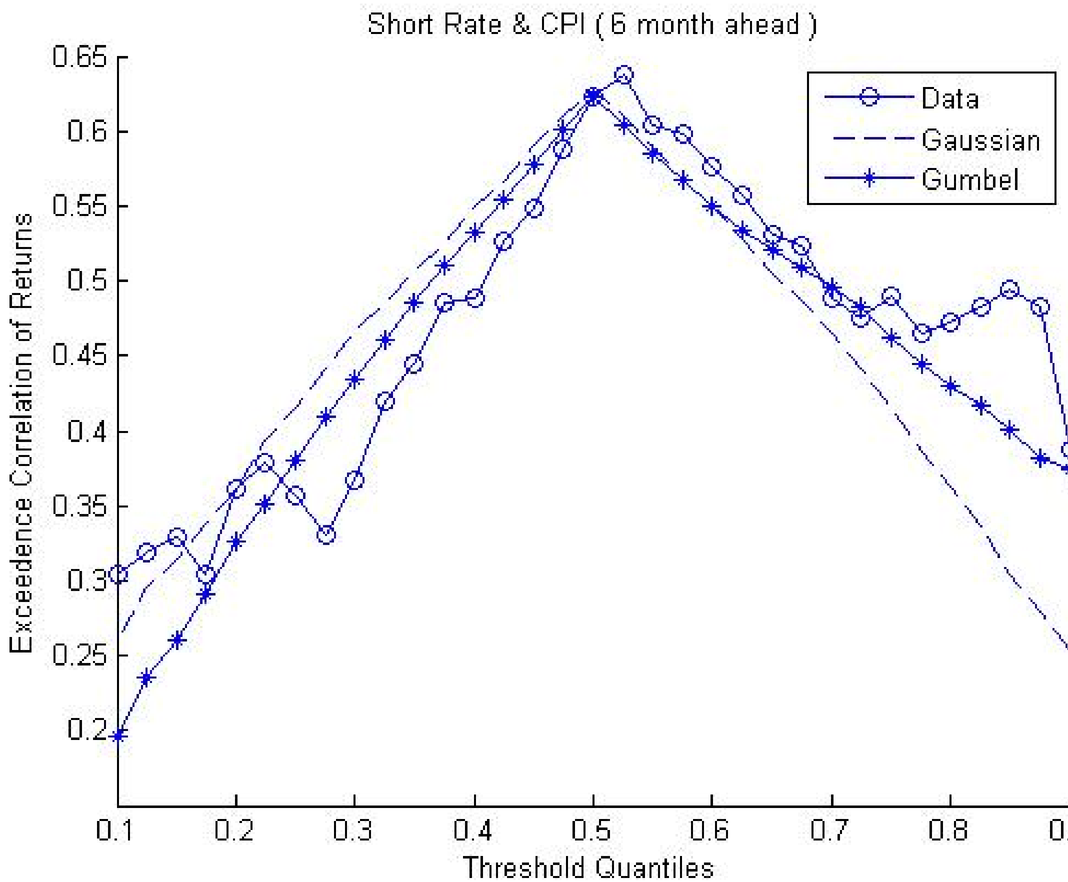

Figure 5.7: Exceedance correlations between transformed return series (U and V) of inflation rate and federal fund rate using a forecasting horizon of 6 months ($k = 6$) The horizontal axis shows the cutoff quantile, and the vertical axis shows the correlation between the two return series given that both exceed that quantile.

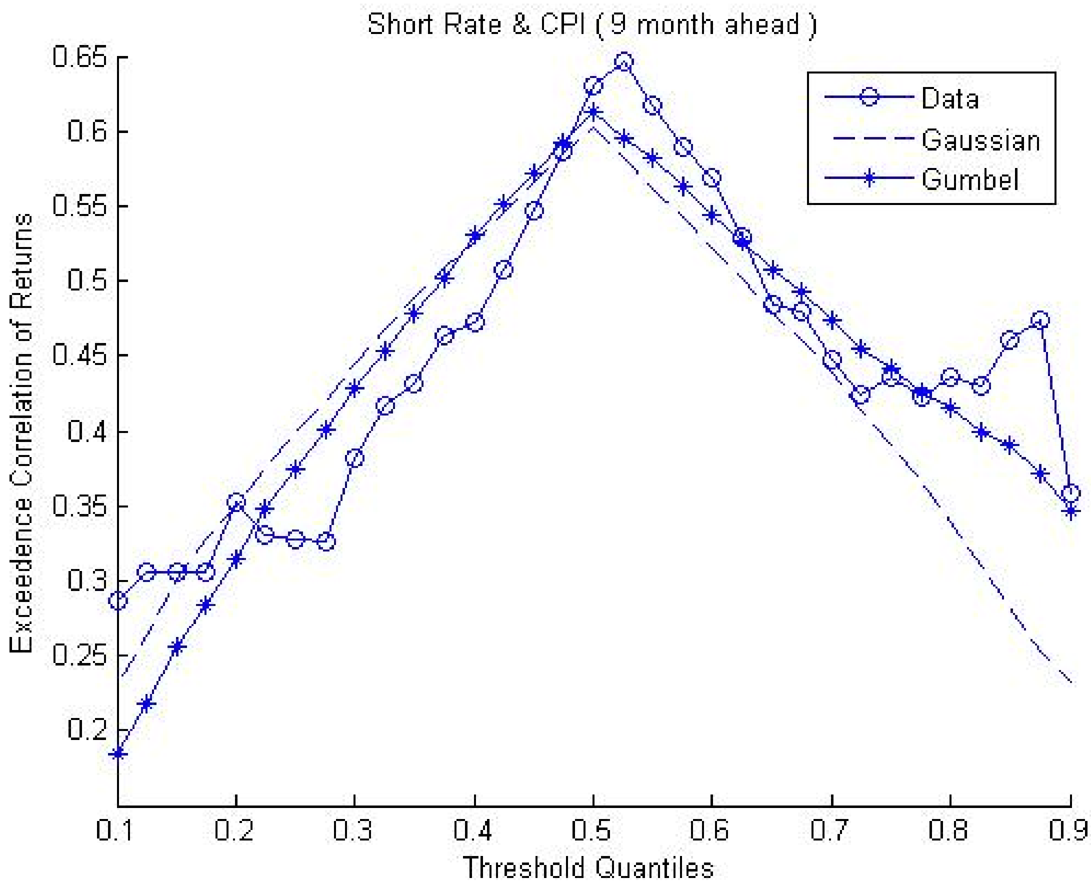

Figure 5.8: Exceedance correlations between transformed return series (U and V) of inflation rate and federal fund rate using a forecasting horizon of 9 months ($k = 9$) The horizontal axis shows the cutoff quantile, and the vertical axis shows the correlation between the two return series given that both exceed that quantile.

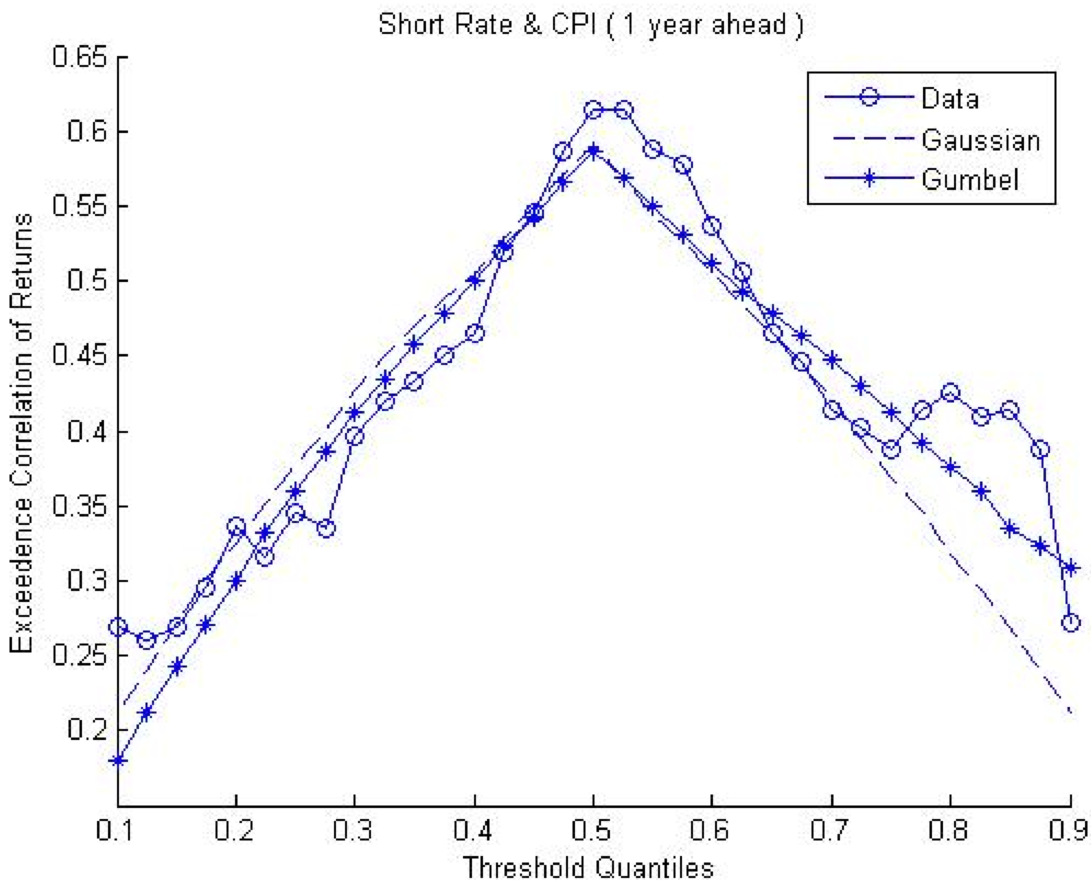

Figure 5.9: Exceedance correlations between transformed return series (U and V) of inflation rate and federal fund rate using a forecasting horizon of 12 months ($k = 12$) The horizontal axis shows the cutoff quantile, and the vertical axis shows the correlation between the two return series given that both exceed that quantile.

Chapter 6

Empirical results: Inflation forecasts

6.1 Introduction

Various studies have found that term structure data contains information for future inflation rates rates. We focus only on the short rate as the results of Stock & Watson (2003) indicate that the short rate is a better predicator than the spread of the term structure. The empirical research of Andrew Ang Wei (2007) performed comparisons of non-linear forecast models against linear forecast models using term structure data. The results indicate that non-linear models do not improve the forecasting performance of linear regression models using term structure data. In this chapter we investigate if the forecasting performance of the linear forecast models using term structure data can be improved by capturing the effects of the non-linear dependence structure with copula functions.

6.2 Linear regression model

The linear regression models defined by equation 6.1 are estimated over the in-sample period using forecasting horizons upto one year because the results in chapter 5 suggests that the statistical relationship between the short rate and future inflation rate is poor for long forecasting horizons.

$$y_{t+k} = \beta_1 + \beta_2 x_{1,t} + \varepsilon_t \tag{6.1}$$

The results of the estimated linear regressions are presented in appendix tables B.4 to B.7. In general, the results indicate that the short rate has a positive effect on inflation rates, so an increase of the short rate will likely result into higher future inflation rates. It also appears that the t-test finds less significant evidence against the null-hypothesis if the forecasting horizon is increased which support the finding of chapter 5. We have identified the same pattern for the R^2 statistic, the R^2 statistic decreases when the length of the forecasting horizon is increased. This means that the explanatory power of the linear regression model declines if the forecasting horizon is increased which supports the previous findings of the t-test and the findings of chapter 5.

So our main conclusion from this section is that linear regression models can be employed to estimate future inflation rates, but caution has to be taken for long forecasting horizons as the explanatory power decreases if the length of the forecasting horizon is increased.

6.3 Copula Approach

The approach to improve forecasts of the linear regression model when the dependence structure is asymmetric, is by modelling the conditional mean of the residuals instead of assuming it's equal to zero. This approach has lead us to the Copula-regression forecast model, which estimates the conditional mean of the residuals with copula functions. The Copula-Regression forecast model can be dividend in two components, the linear regression model forecast $X_t'\beta$ and the conditional mean of the residuals $E[\varepsilon_t|X_t]$. The first component comes from the linear regression, however the second component has to be obtained from copula functions. So in this section we have to investigate which copula functions are useful to estimate the conditional mean of the residuals over the in-sample period.

6.3.1 Finding a copula

The maximum likelihood and information criteria values for each of the copula specifications are presented in table 6.1 to 6.4 and it's seen that the Plackett copula attains the greatest log-likelihood value and the lowest of both information criteria in our in-sample period except for the forecasting horizon $k = 12$. For the forecasting horizon $k = 12$ we find that the Student's t copula has the best fit over the in-sample period. These will be the copula specifications adopted for the respective forecasting horizons.

Table 6.1: Presented here are the nine copula specifications tried for the distribution of the residuals from the linear regression model and the short rate for $k = 3$. The copula likelihood at the optimum is denoted ζ_c . Also presented are the Akaike and Schwarzs bayesian information criteria at the optima.

Model	ζ_c	AIC	BIC
Symmetric Copula			
Normal	15.7778	-31.5486	-31.5358
Students t	16.3621	-32.7102	-32.6847
Plackett	16.4653	-32.9236	-32.9108
Frank	-0.0015	0.0100	0.0227
Asymmetric Copula			
Clayton	-0.0064	0.0198	0.0326
Rotated Clayton	-0.0067	0.0203	0.0331
Joe Clayton	-3.5617	7.1374	7.1629
Gumbel	-13.6465	27.3000	27.3128
Rotated Gumbel	-14.4088	28.8246	28.8374

Table 6.2: Presented here are the nine copula specifications tried for the distribution of the residuals from the linear regression model and the short rate for $k = 6$. The copula likelihood at the optimum is denoted ζ_c . Also presented are the Akaike and Schwarzs bayesian information criteria at the optima.

Model	ζ_c	AIC	BIC
Symmetric Copula			
Normal	26.3777	-52.7483	-52.7355
Students t	26.9816	-53.9490	-53.9232
Plackett	27.7736	-55.5401	-55.5272
Frank	-0.0019	0.0109	0.0237
Asymmetric Copula			
Clayton	-0.0079	0.0229	0.0358
Rotated Clayton	-0.0085	0.0240	0.0369
Joe Clayton	-4.4105	8.8351	8.8608
Gumbel	-16.5836	33.1743	33.1872
Rotated Gumbel	-17.1418	34.2906	34.3035

Table 6.3: Presented here are the nine copula specifications tried for the distribution of the residuals from the linear regression model and the short rate for $k = 9$. The copula likelihood at the optimum is denoted ζ_c . Also presented are the Akaike and Schwarzs bayesian information criteria at the optima.

Model	ζ_c	AIC	BIC
Symmetric Copula			
Normal	32.3395	-64.6719	-64.6589
Students t	33.0100	-66.0057	-65.9797
Plackett	34.1891	-68.3711	-68.3581
Frank	-0.0021	0.0113	0.0243
Asymmetric Copula			
Clayton	-0.0087	0.0244	0.0374
Rotated Clayton	-0.0091	0.0254	0.0384
Joe Clayton	-4.7674	9.5490	9.5750
Gumbel	-17.8785	35.7642	35.7772
Rotated Gumbel	-18.2271	36.4614	36.4744

Table 6.4: Presented here are the nine copula specifications tried for the distribution of the residuals from the linear regression model and the short rate for $k = 12$. The copula likelihood at the optimum is denoted ζ_c . Also presented are the Akaike and Schwarzs bayesian information criteria at the optima.

Model	ζ_c	AIC	BIC
Symmetric Copula			
Normal	0.3435	-0.6797	-0.6666
Students t	4.8495	-9.6846	-9.6585
Plackett	0.0276	-0.0480	-0.0349
Frank	-0.0001	0.0073	0.0204
Asymmetric Copula			
Clayton	0.3287	-0.6503	-0.6372
Rotated Clayton	-0.0023	0.0118	0.0249
Joe Clayton	-0.1771	0.3687	0.3949
Gumbel	-5.4950	10.9972	11.0103
Rotated Gumbel	-0.9600	1.9272	1.9403

6.4 Benchmarking: Out-of-sample forecasting

In this section, our interest is to compare the forecast performances of the linear regression and Copula-regression forecast models over the out-of-sample period. The models are identified on the in-sample period and forecasts are produced over the out-of-sample period. To compare the competing models, we use the MSPE or the mean squared prediction error defined by

$$MSPE = \frac{1}{m}\sum_{h=1}^{m}(y_{n+h} - \hat{y}_{n+h})^2 \tag{6.2}$$

for which m are the amount of observations of the in-sample period and n the amount observations of the out-of-sample period. We can compare 2 competing models by investigating which of the forecast models produces the smallest mean squared prediction error.

To test whether the out-performance is significant, we employ the Diebold-Mariano test with the null hypotheses that the competing models have equal predictive accuracy, given by

$$H_0 : E[d_t] = 0 \tag{6.3}$$

for which $d_t = (y^A_{t+h} - \hat{y}^A_{t+h})^2 - (y^B_{t+h} - \hat{y}^B_{t+h})^2$ in the setting of 2 competing models A and B and a forecasting horizon h. The test statistic of the Diebold-Mariano test is the test statistic from the student's t test which follows a student's t distribution with $n-1$ degrees of freedom.

The results of the MSPE presented in tables 6.5 to 6.7 show that the Copula-linear regression model has smaller MSPE's in comparison to the linear regression model beyond the 6 month forecasting horizon for the full-sample period and when we change the sample period. The Diebold-Mariano test indicate that the out-performances are significant at a significance level of 5%.

There is less evidence to be found for the 3 month forecasting horizon. For the full and 2000-2009 out-of-sample period, we see that the copula-regression forecast model has the same MSPE as the linear regression model, while over the out-of-sample period 1989-1999, the copula-regression forecast model has a significant better MSPE. So it appears that the performance of the Copula-regression forecast model relative to the linear regression model depends on the time period when using a forecast horizon of 3 months. We did not consider the problem of capturing time-varying distributions in this research which could explain why the out-performance using a forecasting horizon of 3 months is time dependent. We could use time-varying copulas that allows the joint distributions to vary over time. The testing and investigation of time-variations in the joint distribution in an out-of-sample forecasting setting is left as a suggestion for future research however.

We find an interesting pattern in the empirical results, the forecast accuracy of the linear regression model are improved the most for longer forecast horizons. The MSPE of the Copula-regression forecasts approximately remains the same if the length of the forecasting horizon is increased, while the forecast performance of the linear regression model deteriorates when the length of the forecasting horizons increases.

Thus the main lesson from this section, is that the Copula-regression model predicts future values of the inflation rate more accurately than the linear regression model beyond the 6 month forecasting horizon regardless of what sample period is used. Also the longer the forecasting horizon, the better are the predictions of the Copula-regression model relative to the linear regression model.

Table 6.5: Mean Squared Prediction Error ($MSPE$)

	linear regression	copula-regression
3 month ahead prediction	0.0012	0.0012
6 month ahead prediction	0.0014	0.0012
9 month ahead prediction	0.0016	0.0013
12 month ahead prediction	0.0018	0.0014

Table 6.6: Mean Squared Prediction Error ($MSPE$), sub-sample 1989-1999

	linear regression	copula-regression
3 month ahead prediction	6.3137e-004	5.8732e-004
6 month ahead prediction	7.6463e-004	6.0346e-004
9 month ahead prediction	9.0250e-004	6.4901e-004
12 month ahead prediction	0.0010	6.9472e-004

Table 6.7: Mean Squared Prediction Error ($MSPE$), sub-sample 2000-2009

	linear regression	copula-regression
3 month ahead prediction	0.0019	0.0019
6 month ahead prediction	0.0021	0.0019
9 month ahead prediction	0.0024	0.0020
12 month ahead prediction	0.0026	0.0021

Table 6.8: p-values of the Diebold-Mariano test

	1989-2009	1989-1999	2000-2009
3 month ahead prediction	2.8057e-004	0.0015	0.0529
6 month ahead prediction	1.8210e-005	4.4126e-007	0.0094
9 month ahead prediction	5.5242e-008	4.3205e-010	9.2471e-004
12 month ahead prediction	4.2290e-011	1.5869e-011	5.2985e-005

Chapter 7

Empirical results: NBER recession forecasts

7.1 Introduction

Various papers have found that the spread of the term structure is a good predictor of real activity. The paper of Estrella & Hardouvelis (1991) reports the same findings for recessions by introducing a Probit model to predict the probability of a recession with the spread of the term structure. This work is extended by Estrella & Mishkin (1997) which compares the spread of the term structure with other asset prices for predicting recessions. The results of the Probit model using the spread of the term structure are quite good, but we believe that the out-of-sample performance can be improved by capturing asymmetric in the dependence structure between CCI growth and the spread of the term structure.

In this section we compare the predictions of the binary response model used by Estrella & Mishkin (1997) and our models that capture the effects of an asymmetric dependence structure between CCI growth and the spread of the term structure. The results of Estrella & Hardouvelis (1991) indicates that the spread of the term structure performs best between 4 and 6 quarters. So we examine the out-of-sample predictive performance for similar forecasting horizons.

7.2 Probit model model

We begin with the Probit model by equation 7.1 with the spread of the term structure as predictor of the NBER recession indicator. The models are estimated by maximum likelihood with forecasting horizons of 2, 4 and 6 quarters over the in-sample period which are presented in figures B.1 to B.3.

$$Y_{t+k} = F(X_t'\beta) + \mu_t \tag{7.1}$$

The results of the Probit models indicate that the spread of the term structure is statistically significant. The corresponding parameter is negative suggesting that the spread of the term structure has a negative effect on the probability of a future recession. This finding suggests that an inverted yield curve increases the probability of a recession.

An interesting observation is that the McFadden R^2 decreases with the length of the forecasting horizon. This is not in line what Estrella & Hardouvelis (1991) reports for the predictive power of the Probit model using the yield spread, as they find that the predictive performance is best for 4 and 6 quarters ahead forecast horizons. This just shows that the model with the strongest explanatory power doesn't guarantee the strongest predictive power.

7.3 Copula Approach

The findings of chapter 5 suggest that the lower tail-dependence coefficient in the joint distribution of CCI growth and the yield spread is non-zero. In chapter 2 we explained that the Probit model systematically underestimates the probability of a recession, because lower tail dependence in the joint distribution result into higher tail probabilities in the conditional distribution which the normal distribution used in the Probit model cannot capture.

In chapter 4 we provided an alternative method to calculate the conditional probability of an event by estimating the conditional probabilities with copula functions, which can accommodate for non-zero tail dependence in the joint distribution of CCI growth and the yield spread. First we have to search for a proper copula specification so that we can calculate the conditional probability of observing a recession k periods into the future.

7.3.1 Finding a copula

Their exist an infinite amount of copula specifications. Some copulas can capture symmetric or asymmetric tail dependence or even both while others can not capture any form of tail dependence. We restrict ourself to a set of copula function most commonly used in practise that can facilitate the previously mentioned dependence structures which are discussed in the Appendix B.

The maximum likelihood and information criteria values for each of the copula specifications are presented in table 7.1 to 7.3 using canonical maximum likelihood estimation to calibrate the copulas. The Joe Clayton copula attains the greatest log-likelihood value and the lowest of both information criteria over the in-sample period for a forecasting horizon of 6 months. The Clayton copula is the best fitting copula over the in-sample period using for the forecasting horizons 12 and 18 months. These are the copula specifications we select for the respective forecasting horizons.

Table 7.1: Presented here are the nine copula specifications tried for the distribution of the CCI growth and the yield spread for $k = 6$. The copula likelihood at the optimum is denoted ζ_c . Also presented are the Akaike and Schwarzs bayesian information criteria at the optima.

Model	ζ_c	AIC	BIC
Symmetric Copula			
Normal	17.3195	-34.6319	-34.6190
Students t	18.5076	-37.0009	-36.9751
Plackett	19.2371	-38.4670	-38.4541
Frank	19.1006	-38.1940	-38.1811
Asymmetric Copula			
Clayton	32.0687	-64.1302	-64.1173
Rotated Clayton	2.5624	-5.1177	-5.1048
Joe Clayton	32.5402	-65.0663	-65.0405
Gumbel	7.5465	-15.0860	-15.0731
Rotated Gumbel	28.0005	-55.9940	-55.9811

Table 7.2: Presented here are the nine copula specifications tried for the distribution of the CCI growth and the yield spread for $k = 12$. The copula likelihood at the optimum is denoted ζ_c . Also presented are the Akaike and Schwarzs bayesian information criteria at the optima.

Model	ζ_c	AIC	BIC
Symmetric Copula			
Normal	15.9377	-31.8678	-31.8543
Students t	16.2374	-32.4597	-32.4326
Plackett	16.6918	-33.3761	-33.3625
Frank	16.5603	-33.1130	-33.0995
Asymmetric Copula			
Clayton	21.5457	-43.0839	-43.0703
Rotated Clayton	5.1159	-10.2242	-10.2107
Joe Clayton	20.6046	-41.1940	-41.1669
Gumbel	9.2191	-18.4306	-18.4171
Rotated Gumbel	19.3724	-38.7372	-38.7237

Table 7.3: Presented here are the nine copula specifications tried for the distribution of the CCI growth and the yield spread for $k = 18$. The copula likelihood at the optimum is denoted ζ_c . Also presented are the Akaike and Schwarzs bayesian information criteria at the optima.

Model	ζ_c	AIC	BIC
Symmetric Copula			
Normal	7.6951	-15.3828	-15.3695
Students t	7.8785	-15.7421	-15.7155
Plackett	8.2258	-16.4442	-16.4309
Frank	8.2175	-16.4276	-16.4143
Asymmetric Copula			
Clayton	10.6263	-21.2452	-21.2319
Rotated Clayton	2.0920	-4.1766	-4.1632
Joe Clayton	9.6021	-19.1894	-19.1628
Gumbel	3.9441	-7.8808	-7.8675
Rotated Gumbel	8.7485	-17.4897	-17.4764

7.4 Benchmarking: Out-of-sample forecasting

Using the results of the previous sections, we benchmark the forecast performances of the Copula probability forecast model against the Probit model using the spread of the term structure as the explanatory variable. We are interested in the fraction of correct predictions by using a binary random variable that is 1 if a prediction is correct and 0 otherwise. Then the hit rate is calculated by the sum of this binary variable divided by the out-of-sample size. The higher the hit rate, the better the model makes predictions.

To transform the predicted conditional probabilities of a recession into binary outcomes $\hat{y}_{t+k}$, we transform the predicted probabilities into binary outcomes by

$$\begin{aligned}
\hat{y}_{t+k} &= 1 \quad \text{if } P(Y_{t+k} = 1|X_t) > c \\
\hat{y}_{t+k} &= 0 \quad \text{if } P(Y_{t+k} = 1|X_t) \leq c
\end{aligned}$$

for which c is the threshold value calculated by the fraction of recessions over the in-sample period.

The results for predicting recessions stated by the NBER are presented in tables 7.3 to 7.6 using the term structure spread. The out-of-sample period is divided in 2 sub-periods from 1989 to 1999 and 2000 to 2009 so that we can investigate if the forecasting performance does not depend on the chosen sample period.

The hit rates are consistently higher for the Probit model. The predictive performance of the Copula Probability model using a forecasting horizon of 18 months is much worse than anticipated, as it falls below the 50% expected hit rates for random predictions over the out-of-sample periods 1989-1999 and 1989-2009.

Recessions are not defined by a 1-month negative CCI growth, but several periods of negative CCI growth which might explain the poor out-of-sample performance of the Copula probability forecast model. So we argue that the performance of the Copula probability forecast model can be improved by calculating the conditional probability of observing several periods of negative CCI growth instead of calculating the probability of a 1-month negative CCI growth which we leave as a remark for future research.

Table 7.4: Hit rate: 1989-2009

	Probit model forecast	Copula forecast
6 month ahead prediction	79.18%	59.18%
12 month ahead prediction	88.98%	60.82%
18 month ahead prediction	83.27%	42.86%

Table 7.5: Hit rate: 1989-1999

	Probit model forecast	Copula forecast
6 month ahead prediction	84.09%	56.06%
12 month ahead prediction	91.67%	55.30%
18 month ahead prediction	90.15%	31.06%

Table 7.6: Hit rate: 2000-2009

	Probit model forecast	Copula forecast
6 month ahead prediction	73.45%	62.83%
12 month ahead prediction	85.84%	67.26%
18 month ahead prediction	75.22%	56.64%

Chapter 8

Conclusion

In this thesis we examined the dependence structure of output growth and inflation rates with asset prices. We have identified lower tail dependence in the joint distribution of CCI growth and the spread of the term structure, a strengthening of the statistical relationship between small returns, while we found upper tail dependence in the joint distribution of CPI inflation rate and the short rate.

The main lesson of chapter 2 is that the linear regression and Probit forecast model must be used with caution, since these forecast models do not capture non-linearities in the dependence structure. If the dependence structure between the dependent and explanatory variable is asymmetric, we discussed for the linear regression model that the residuals are dependent on the explanatory variables. Fortunately, we have copula functions that can model the conditional mean of the residuals rather than assuming it's zero, which leads to the Copula-Regression forecast model. Likewise, we discussed that the Probit model underestimates the conditional probability of an event because the normal distribution used in the Probit model can not accommodate for tail probabilities. We proposed an alternative forecast model which calculates the conditional probability of observing an event with copula functions that can accommodate for tail dependence.

We have seen that the Copula-Regression forecast model consistently outperforms the linear regression model for inflation rate forecasts beyond the 6 month forecasting horizon. Using forecasting horizons shorter than 6 months, we found that the out-performance of the Copula-regression forecast model depends on the time period, which suggests that the joint distribution between the residuals and the explanatory variables might vary over time.

An interest pattern emerged, the longer the forecasting horizon, the greater the Copula-regression forecast model outperforms the linear regression model. So the general conclusion that emerges from our results, is that capturing the effects of an asymmetric dependence structure can improve the forecasting performance for inflation rates, especially for long forecasting horizons.

The results of the Copula probability forecast model were not so great. The Probit model is clearly the better forecasting model for predicting recessions. We argued that the poor performance of the Copula probability forecast model is attributed to an incorrect defined link between CCI growth and the NBER recession indicator. We calculated the conditional probability of a 1-month negative CCI growth, while a recession stated by the NBER is defined by several periods of negative CCI growth in reality.

For future works, it could be interesting to see how the Copula probability forecast model performs relative to the Probit model when calculating the conditional probability of several subsequent periods of negative CCI growth. Also due to the long length of the out-of-sample period (20 years), we cannot rule out the possibility that the joint distributions vary over time, which could be the reason why the out performance of the Copula-regression forecast model changes over time for the forecasting horizon of 3 months.

Furthermore, we restricted ourselves to using only one variable so this research can be extended using multiple asset prices. It might also be interesting to see how full maximum likelihood estimation (FML) and inference on margin (IMF) approach compares to the canonical maximum likelihood approach for prediction future inflation rates and recessions. I'm willing to provide help with research aimed at answering these questions.

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Appendix A

Copula Functional Forms

In this appendix, we describe the copula functions used in this research. This appendix follows the literature of Alexander (2008) and Nelson (1999) which provides further details for the more interested reader.

A.1 Gaussian Copula's

One of the most important implicit copula's from which the dependence part is isolated from is the normal copula which is also called the Gaussian copula. The multivariate normal copula has a correlation matrix Σ for parameters which play a central role in financial analysis. However they are used for convenience rather than accuracy. A normal copula is derived from the n-dimensional multivariate and univariate stand normal distributions functions denoted Φ and Φ respectively. It's then defined as

$$C(u_1, u_2; \Sigma) = \Phi(\Phi^{-1}(u_1), ..., \Phi^{-1}(u_n)) \tag{A.1}$$

Differentiating this leads to the normal or Gaussian density which is given by

$$c(u_1, ..., u_n; \Sigma) = |\Sigma|^{-\frac{1}{2}} exp(-\tfrac{1}{2}\xi'(\Sigma^{-1} - \mathbf{I})\xi)(\text{A.2})$$

For the case n = 2 the normal copula distribution is given by

$$C(u_1, u_2; \varrho) = \Phi(\Phi^{-1}(u_1), \Phi^{-1}(u_2)) \tag{A.3}$$

$$C(u_1, u_2; \varrho) = \int_0^{\Phi^{-1}(u_1)} \int_0^{\Phi^{-1}(u_2)} (2\pi)^{-1}(1-\varrho^2)^{-\frac{1}{2}} exp(-\frac{x_1^2 - 2_1 x_2 + x_2^2}{2(1-\varrho^2)})\delta x_1 \delta x_2 \tag{A.4}$$

The copula distribution cannot be written in a simple closed form. Numerical methods such as the adaptive Simpson quadrature can used to approximate the definite integrands. The bivariate normal copula density is

$$c(u_1, u_2; \varrho) = (1-\varrho^2)^{-\frac{1}{2}} exp(-\frac{\varrho^2\xi_1^2 - 2\varrho\xi_1\xi_2 + \varrho^2\xi_2^2}{2(1-\varrho^2)}) \tag{A.5}$$

where $\xi_1 = \Phi^{-1}(u_1)$ and $\xi_2 = \Phi{-}1(u_2)$ are quantiles of standard normal variables. Since the correlation is the only parameter the bivariate normal copula has its easy to calibrate it.

A.2 Student t copula

The n-dimensional symmetric copula Student t copula is another implicit copula from which the dependence can be isolated from. It is defined by

$$C_\upsilon(u_1, ..., u_n; \Sigma) = t_\upsilon(t_\upsilon^{-1}(u_1), ..., t_\upsilon^{-1}(u_n) \tag{A.6}$$

where t_υ and t_υ are multivariate and univariate Student t distributions with υ degrees of freedom and Σ the correlation matrix. Then the multivariate Student t copula distribution maybe written as

$$C_\upsilon(u_1, ..., u_n; \Sigma) = \int_0^{t_\upsilon^{-1}(u1)} ... \int_0^{t_\upsilon^{-1}(u_n)} k|\Sigma|^{-\frac{1}{2}}(1 + \upsilon^{-1}x'\Sigma^{-1}x)^{-(\upsilon+1)/2}\delta x_1...x_n \tag{A.7}$$

Differentiation of the above expression leads to the student t copula density as

$$c_\upsilon(u_1, ..., u_n); \Sigma = K|\Sigma|^{-\frac{1}{2}}(1 + \upsilon^{-1}\xi'\Sigma^{-1}\xi)^{(\upsilon+2)/2} \prod_{i=1}^{n} 1 + \upsilon^{-1}\xi^{-(+2)/2} \tag{A.8}$$

where by

$$k = \Gamma(\frac{\upsilon}{2})^{(n-1)}\Gamma(\frac{\upsilon + n}{2})(\upsilon\pi)^{-n/2} \tag{A.9}$$

$$K = \Gamma(\frac{\upsilon}{2})^{(n-1)}\Gamma(\frac{\upsilon + 1}{2})^{-n}\Gamma(\frac{\upsilon + n}{2}) \tag{A.10}$$

The tail dependence is given by

$$\lambda_\upsilon(\varrho) = 2t_{\upsilon+1}(\frac{\sqrt{\upsilon + 1}\sqrt{1 - \varrho}}{\sqrt{1 + \varrho}}) \tag{A.11}$$

where $t_{\upsilon+1}$ is the complementary cumulative univariate Students distribution with $\upsilon + 1$ degrees of freedom.

In the case of n=2 we have the symmetric bivariate t copula distribution

$$C(u_1, u_2; \varrho) = \int_0^{\Phi^{-1}(u_1)} \int_0^{\Phi^{-1}(u_2)} (2\pi)^{-1}[1 - \varrho^2)^{-1/2}(1 + \upsilon^{-1}(x_1^2 - 2\varrho x_1 x_2 + x_2^2)]^{-(\upsilon+2)/2}\delta x_1 \delta x_2 \tag{A.12}$$

Like the normal copula, the Student t copula distribution cannot be written in a simple closed form so well have to approximate it with numerical methods.

A.3 Archimedean Copula

An other method for building copulas is based on a generator fuction which will is denoted as u. The corresponding Archimedean copula is defined then as

$$C(u_1, ..., u_n) = \Psi^{-1}(\Psi(u_1, ..., u_n) \tag{A.13}$$

Two simple copulas that are commonly used in market risk analysis are the Clayton and Gumble copulas and the are useful because they can capture and asymmetric tail dependence that we know can be important for modeling many relationships between financial asset returns. The Clayton Copula captures lower tail dependence and the Gumbel copula captures upper tail dependence.

A.3.1 Clayton Copula

Clayton (1978) introduced the following generator function and so is commonly called the Clayton copula

$$\Psi_u = \alpha^{-1}(u^{-\alpha} - 1), \alpha \neq 0 \tag{A.14}$$

So the inverse generator function is

$$\Psi^{-1}(x) = (x + 1)^{1/\alpha} \tag{A.15}$$

The Clayton copula thus has the form of

$$C(u_1, ..., u_n; \alpha) = (u_1^{-\alpha} + ... + u_n^{-\alpha} - n + 1)^{-1/\alpha} \tag{A.16}$$

A Clayton copula has asymmetric tail dependence. In fact it has zero upper tail dependence but a positive lower tail dependence coefficient, when $\alpha > 0$

The Clayton copula density is obtained by differentiating the Clayton copula which yields

$$c(u_1, ..., u_n; \alpha) = (1 - n + \sum_{i=1}^{n} u_i^{-\alpha})^{-n-(1/\alpha)} \prod_{j=1}^{n} u_j^{-\alpha-1}[(j-1)\alpha + 1] \tag{A.17}$$

As the parameter α varies, the Clayton copulas capture a range of dependence with perfect positive dependence as $\Rightarrow \infty$. That is, as α increases the Clayton copulas converge to the Frechet upper bound copula. In the case of n=2, the conditional distributions for Clayton copulas are easy to derive.

A.3.2 Gumbel Copula

Gumbel Copula is an Archimedean copula with generating function

$$\Psi = -(\log[u])^{\delta}. \tag{A.18}$$

Thus the inverse generation function is

$$\Psi^{-1}(x) = exp((-x)^{1/\delta}) \tag{A.19}$$

The Gumbel copula distribution may therefore be written as

$$C(u_1, ..., u_n); \delta) = exp[(-\log[u_1])^{\delta}) + ... + (-\log[u_n])^{\delta}]^{1/\delta} \tag{A.20}$$

Differentiating the Gumbel copula yields the Gumbel copla density which for the bivariate case is

$$c(u_1, u_2; \delta) = (A + \delta - 1)A^{1-2\delta} exp(-A)(u_1 u_2)^{-1}(-\log[u_1])^{\delta-1}(-\log[u_2])^{\delta-1} \quad (A.21)$$

Where $A = ((-\log[u_1])^\delta + (-\log[u_2])^\delta)^{1/\delta}$.

Just like the Clayton copula, the Gumbel copula has asymmetric tail dependence. Unlike the Clayton copula, it has zero lower tail dependence but a positive upper tail dependence coefficient, when $\delta > 1$.

A variation of the Gumbel copula is the rotated Gumbel copula which transform the copula from a upper tail dependence copula into a lower dependence copula.

A.3.3 Frank Copula

The frank copula is a symmetric Archimedean copula with copula distribution

$$C(u_1, u_2; \alpha) = -\frac{1}{\alpha} \log(\frac{((1-e)^{-\alpha} - (1-e)^{-\alpha u_1})(1-e)^{-u_2}}{(1-e)^{-\alpha}}) \quad (A.22)$$

and copula density

$$c(u_1, u_2; \alpha) = \frac{\alpha(1-e)^{-\alpha} e^{-\alpha(u_1+u_2)}}{((1-e)^{-\alpha} - (1-e)^{-\alpha u_1})(1-e)^{-u_2})^2} \quad (A.23)$$

with $\alpha \in (-\infty, \infty) \backslash \{0\}$

A.3.4 Joe-Clayton Copula

This is the modified version of the symmetric Joe-Clayton copula used by Patton (2005a). It can be restricted to the symmetric version by imposing $\lambda^u = \lambda^d$ according to Palaro (2004). The copula distribution of the Joe clayton copula is

$$C(u_1, u_2; \lambda^u, \lambda^l) = 1 - (1 - \{[1 - (1 - u_1)^k]^{-\gamma} + (1 - [1 - u_2]^k)^{-\gamma} - 1\}^{-1/\gamma})^{1/k} \quad \text{(A.24)}$$

$$\text{k}=[\log_2(2 - \lambda^u)]^{-1} \quad \gamma = [-\log_2(\lambda^l)]^{-1} \quad \lambda^u \in (0,1), \lambda^l \in (0,1)$$

for which λ^u and λ^l are the corresponding tail dependence parameters.

A.4 Plackett Copula

The placket copula is a symmetric function with no tail dependence similar to the Gaussian copula. The distinction between the Gaussian and Plackett copula is that the Gaussian copula has greater dependence for large joint observation than the Plackett copula. The distribution of the Plackett copula is

$$C(u, v; \pi) = \frac{1}{2(\pi - 1)}(1 + (\pi - 1)(u + v) - \sqrt{(1 + (\pi - 1)(u + v)^2) - 4\pi(\pi - 1)uv)}) \quad \text{(A.25)}$$

with copula density

$$c(u, v; \pi) = \frac{\pi(1 + (\pi - 1)(u + v - 2uv))}{((1 + (\pi - 1)(u + v)^2) - 4\pi(\pi - 1)uv))^{3/2}} \quad \text{(A.26)}$$

$$\pi \in (0,)\{1\} \quad \text{(A.27)}$$

Appendix B

In-Sample Results

Dependent Variable: NBER				
Method: ML - Binary Probit				
Sample: 7 288				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-1.094691	0.123903	-8.835063	0.0000
YIELDSPREAD(-6)	-57.49061	6.358802	-9.041108	0.0000
S.E of regression	0.275086	McFadden R-squared		0.452028
Log likelihood	-74.67214			

Table B.1: This table presents some statistics of the estimated Probit model over the in-sample period for an forecasting horizon of 6 months ($k = 6$).

Dependent Variable: NBER				
Method: ML - Binary Probit				
Sample: 13 288				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.956104	0.104216	-9.174252	0.0000
YIELDSPREAD(-12)	-40.19603	5.225751	-7.691915	0.0000
S.E of regression	0.341120	McFadden R-squared		0.268545
Log likelihood	-98.75073			

Table B.2: This table presents some statistics of the estimated Probit model over the in-sample period for an forecasting horizon of 12 months ($k = 12$).

Dependent Variable: NBER				
Method: ML - Binary Probit				
Sample: 19 288				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.857676	0.092340	-9.288215	0.0000
YIELDSPREAD(-18)	-23.49448	4.601382	-5.105961	0.0000
S.E of regression	0.379909	McFadden R-squared		0.103862
Log likelihood	-119.8232			

Table B.3: This table presents some statistics of the estimated Probit model over the in-sample period for an forecasting horizon of 18 months ($k = 18$).

Dependent Variable: CPI				
Method: Least Squares				
Sample: 4 288				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.023425	0.005521	4.242883	0.0000
FEDFUND(-3)	0.427471	0.064629	6.614262	0.0000
S.E of regression	0.036574	R-Squared		0.133890

Table B.4: This table presents some statistics of the estimated linear regression model over the in-sample period for an forecasting horizon of 3 months ($k = 3$).

Dependent Variable: CPI				
Method: Least Squares				
Sample: 7 288				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.034015	0.005765	5.900243	0.0000
FEDFUND(-6)	0.295288	0.067440	4.378515	0.0000
S.E of regression	0.038164	R-Squared		0.064082

Table B.5: This table presents some statistics of the estimated linear regression model over the in-sample period for an forecasting horizon of 6 months ($k = 6$).

Dependent Variable: CPI				
Method: Least Squares				
Sample: 10 288				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.042822	0.005848	7.321890	0.0000
FEDFUND(-9)	0.190668	0.068306	2.791383	0.0056
S.E of regression	0.038643	R-Squared		0.027360

Table B.6: This table presents some statistics of the estimated linear regression model over the in-sample period for an forecasting horizon of 9 months ($k = 9$).

Dependent Variable: CPI				
Method: Least Squares				
Sample: 13 288				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.049955	0.005934	8.418349	0.0000
FEDFUND(-12)	0.103761	0.069158	1.500343	0.1347
S.E of regression	0.039099	R-Squared		0.008148

Table B.7: This table presents some statistics of the estimated linear regression model over the in-sample period for an forecasting horizon of 12 months ($k = 12$).

Appendix C

Out-of-sample forecasts: Inflation rate

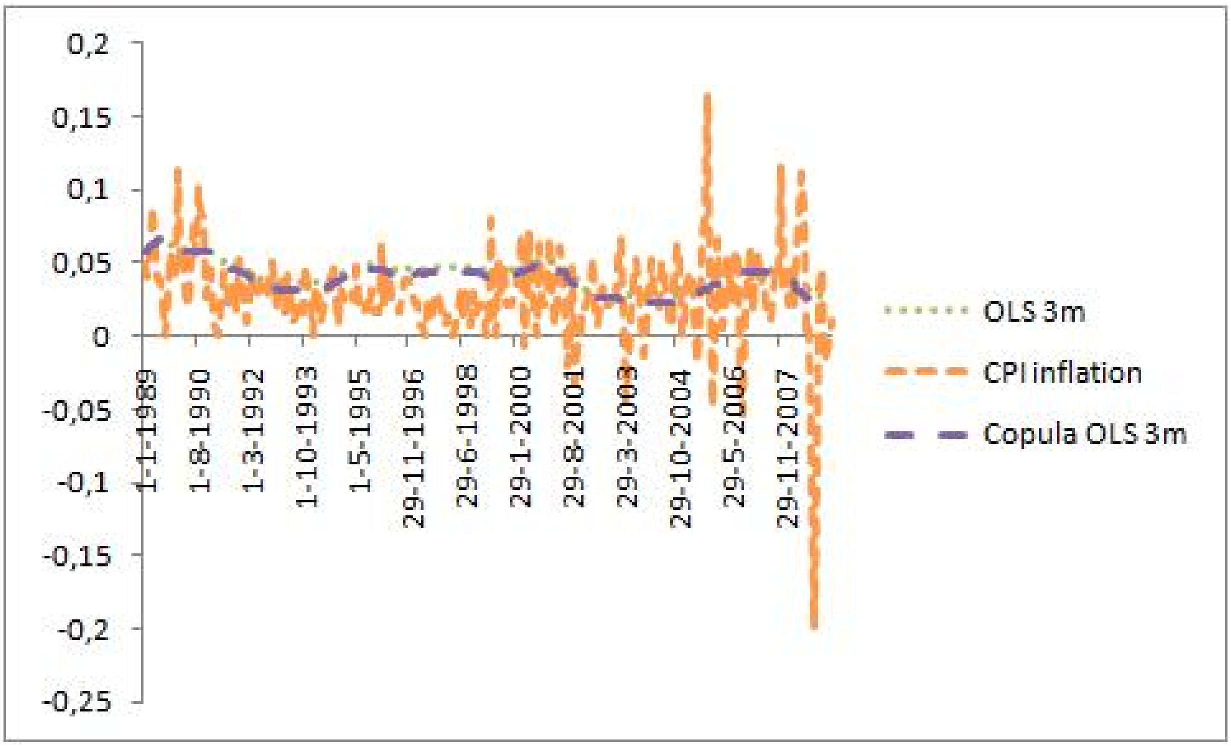

Figure C.1: Out-of-sample forecasts of the linear regression model and the Copula-regression model with the CPI inflation rate over the out-of-sample period (January 1989 to May 2009) using a forecasting horizon of 3 months ($k = 3$). "OLS" stands for the linear regression model, "Copula OLS" stands for the Copula-regression model and "CPI inflation" stands for the CPI inflation rates.

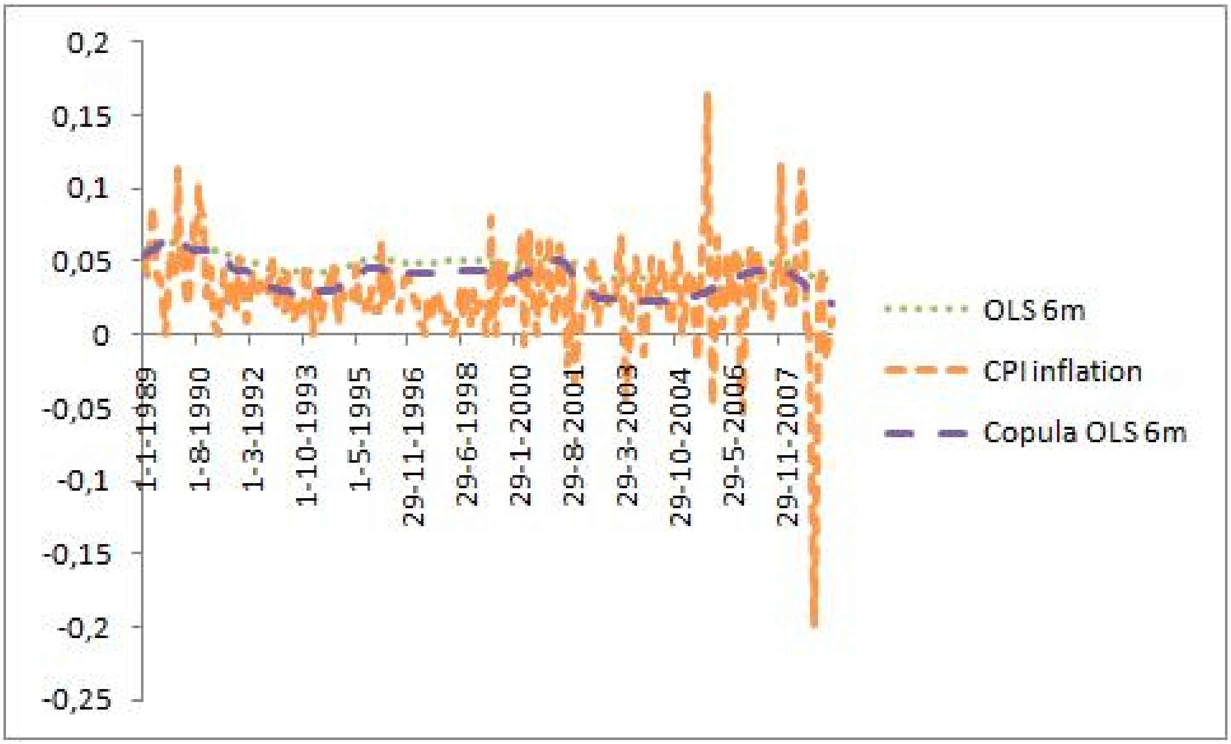

Figure C.2: Out-of-sample forecasts of the linear regression model and the Copula-regression model with the CPI inflation rate over the out-of-sample period (January 1989 to May 2009) using a forecasting horizon of 6 months ($k = 6$). "OLS" stands for the linear regression model, "Copula OLS" stands for the Copula-regression model and "CPI inflation" stands for the CPI inflation rates.

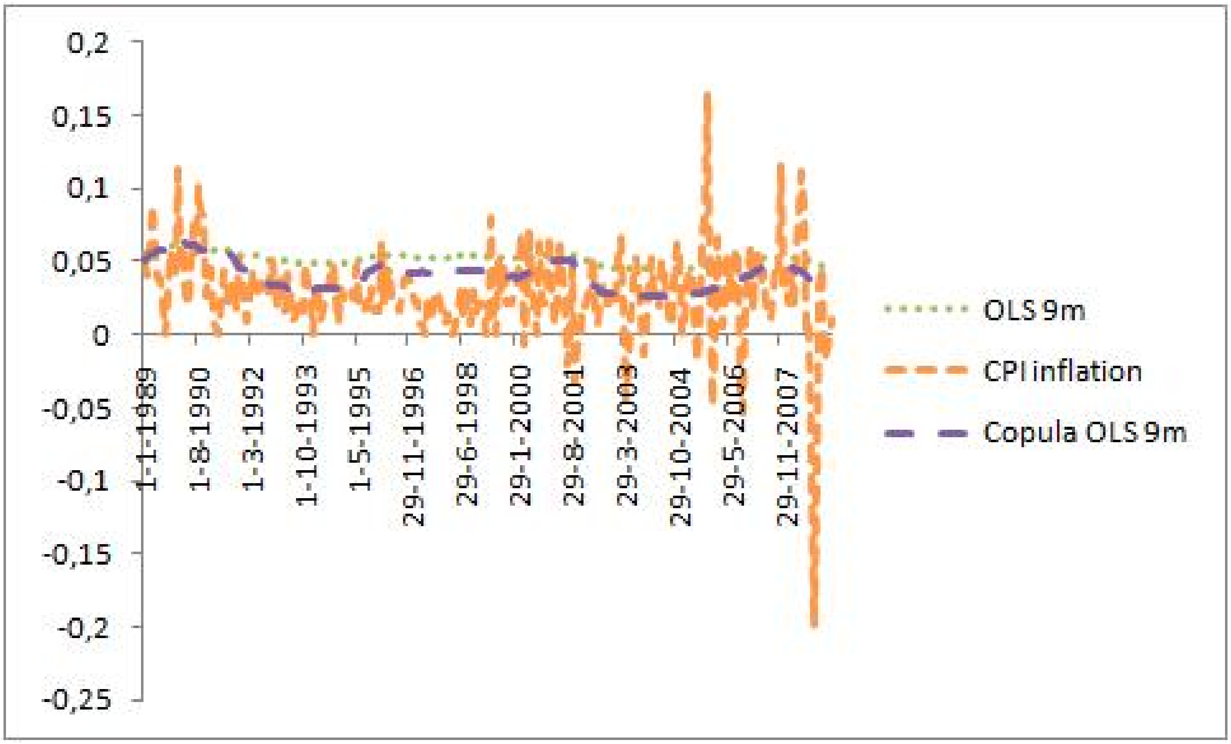

Figure C.3: Out-of-sample forecasts of the linear regression model and the Copula-regression model with the CPI inflation rate over the out-of-sample period (January 1989 to May 2009) using a forecasting horizon of 9 months ($k = 9$). "OLS" stands for the linear regression model, "Copula OLS" stands for the Copula-regression model and "CPI inflation" stands for the CPI inflation rates.

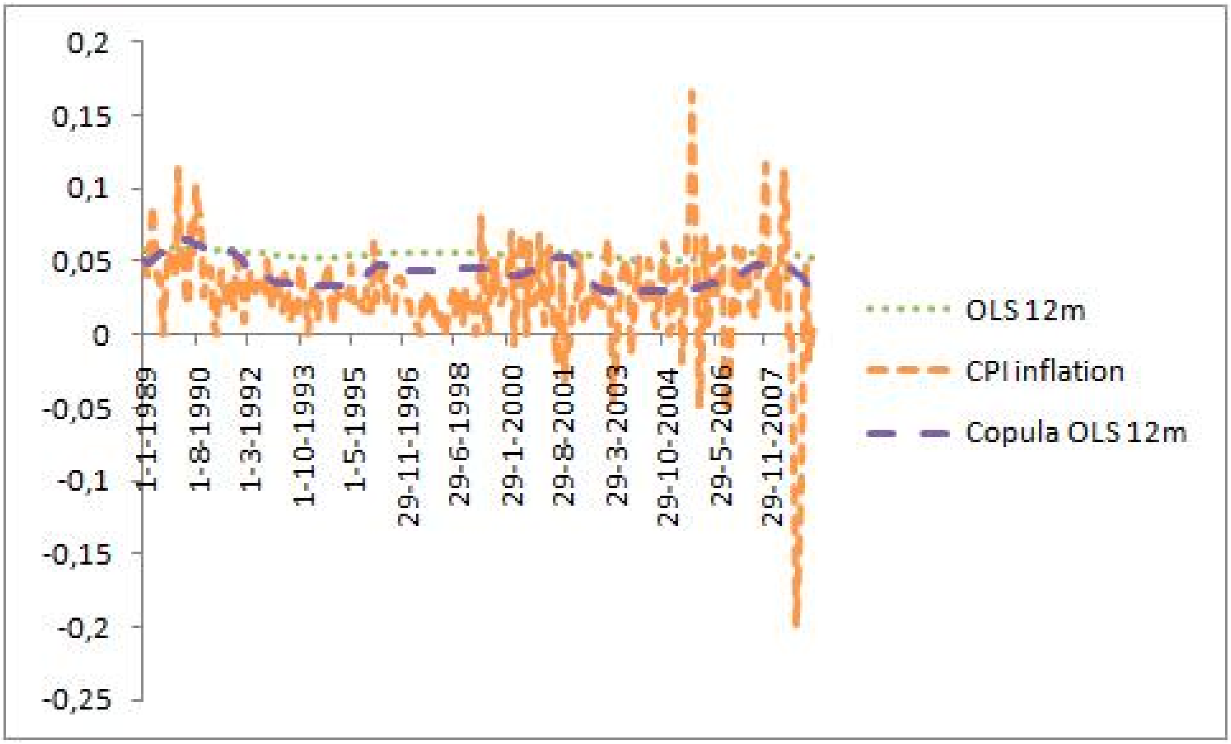

Figure C.4: Out-of-sample forecasts of the linear regression model and the Copula-regression model with the CPI inflation rate over the out-of-sample period (January 1989 to May 2009) using a forecasting horizon of 12 months ($k = 12$). OLS stands for the linear regression model, “Copula OLS“ stands for the Copula-regression model and CPI inflation stands for the CPI inflation rates.

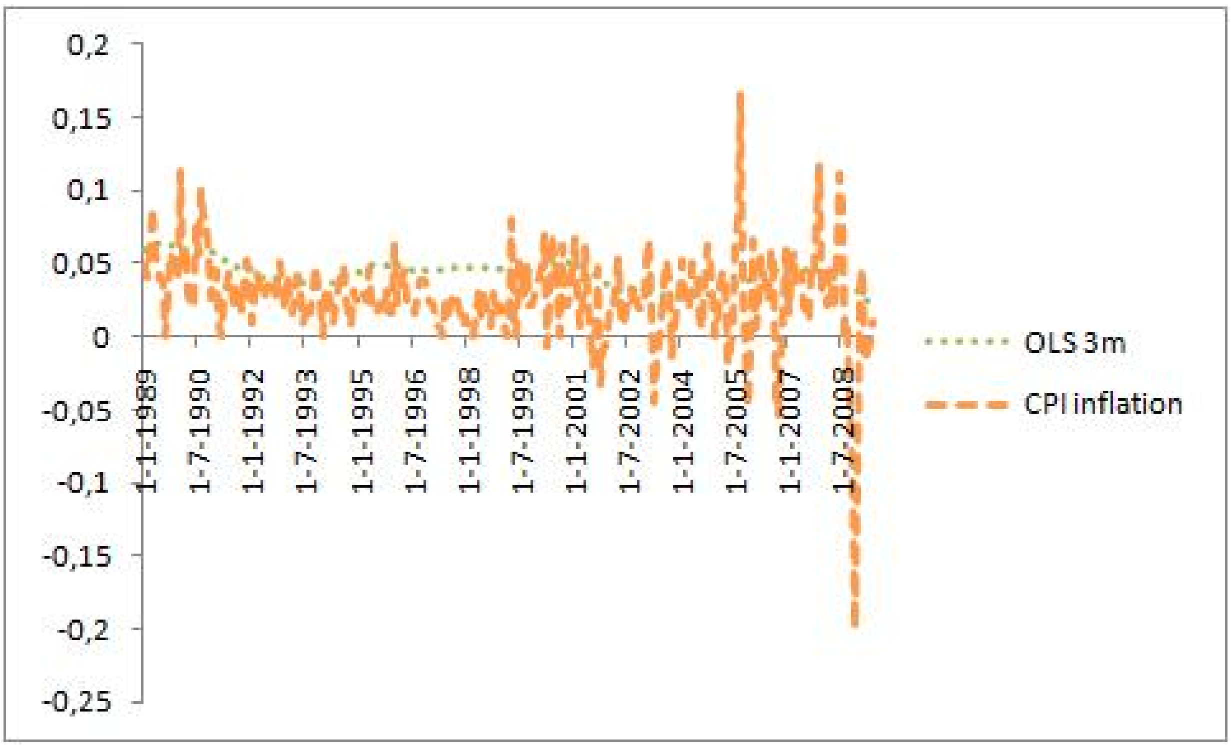

Figure C.5: Out-of-sample forecasts of the linear regression model with the CPI inflation rate over the out-of-sample period (January 1989 to May 2009) using a forecasting horizon of 3 months ($k = 3$). “CPI Inflation“ stands for the inflation rate, “OLS“ stands for the linear regression model.

Figure C.6: Out-of-sample forecasts of the linear regression model with the CPI inflation rate over the out-of-sample period (January 1989 to May 2009) using a forecasting horizon of 6 months ($k = 6$). “CPI Inflation“ stands for the inflation rate, “OLS“ stands for the linear regression model.

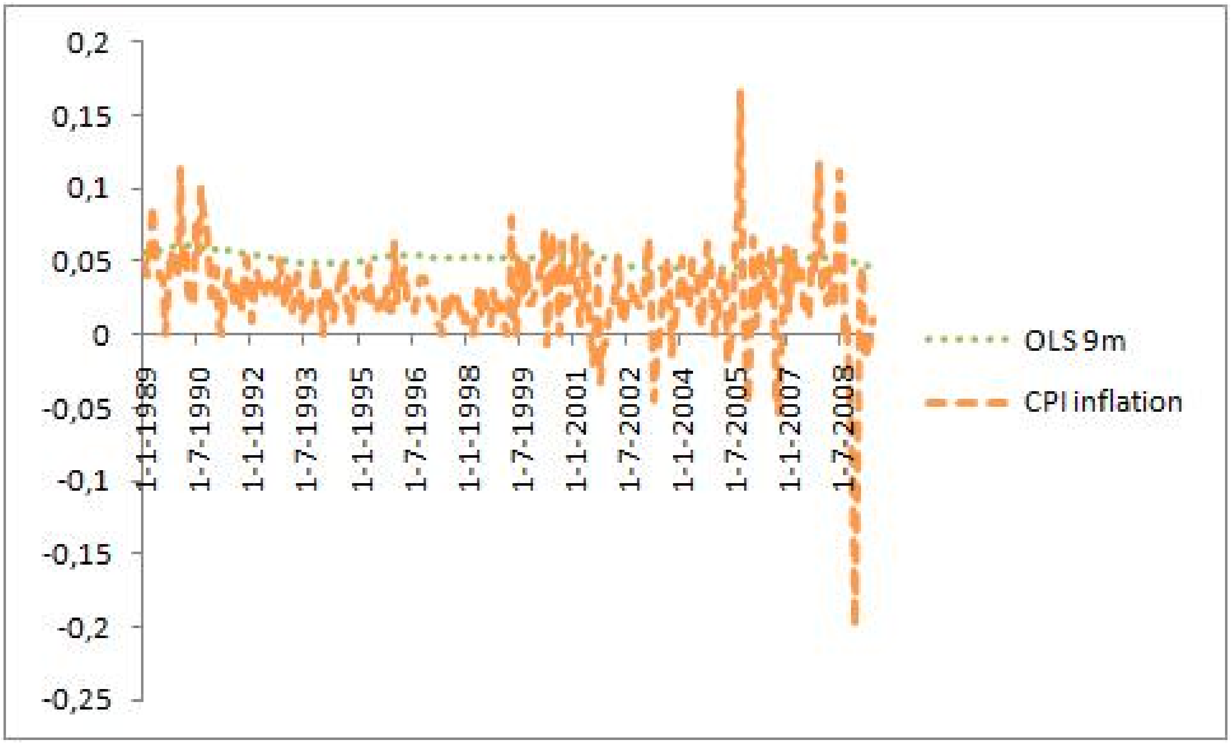

Figure C.7: Out-of-sample forecasts of the linear regression model with the CPI inflation rate over the out-of-sample period (January 1989 to May 2009) using a forecasting horizon of 9 months ($k = 9$). “CPI Inflation“ stands for the inflation rate, “OLS“ stands for the linear regression model.

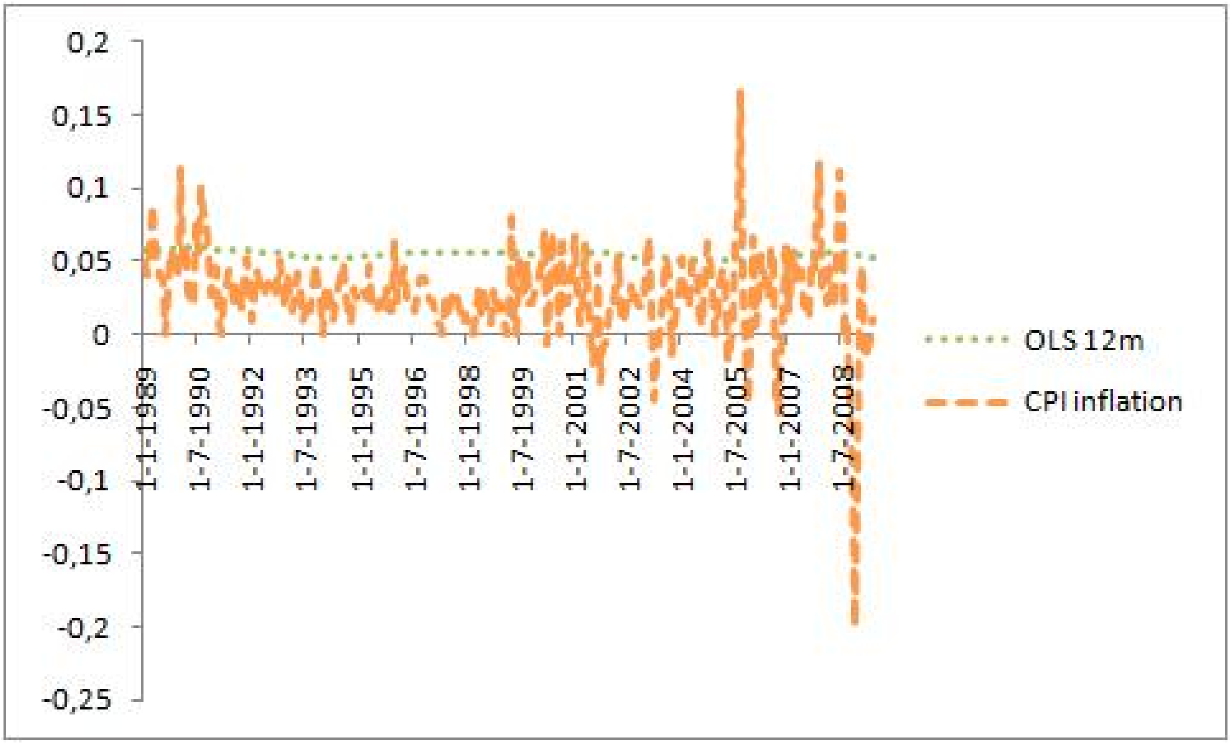

Figure C.8: Out-of-sample forecasts of the linear regression model with the CPI inflation rate over the out-of-sample period (January 1989 to May 2009) using a forecasting horizon of 12 months ($k = 12$). “CPI Inflation“ stands for the inflation rate, “OLS“ stands for the linear regression model.

Appendix D

Out-of-sample forecasts: Recessions

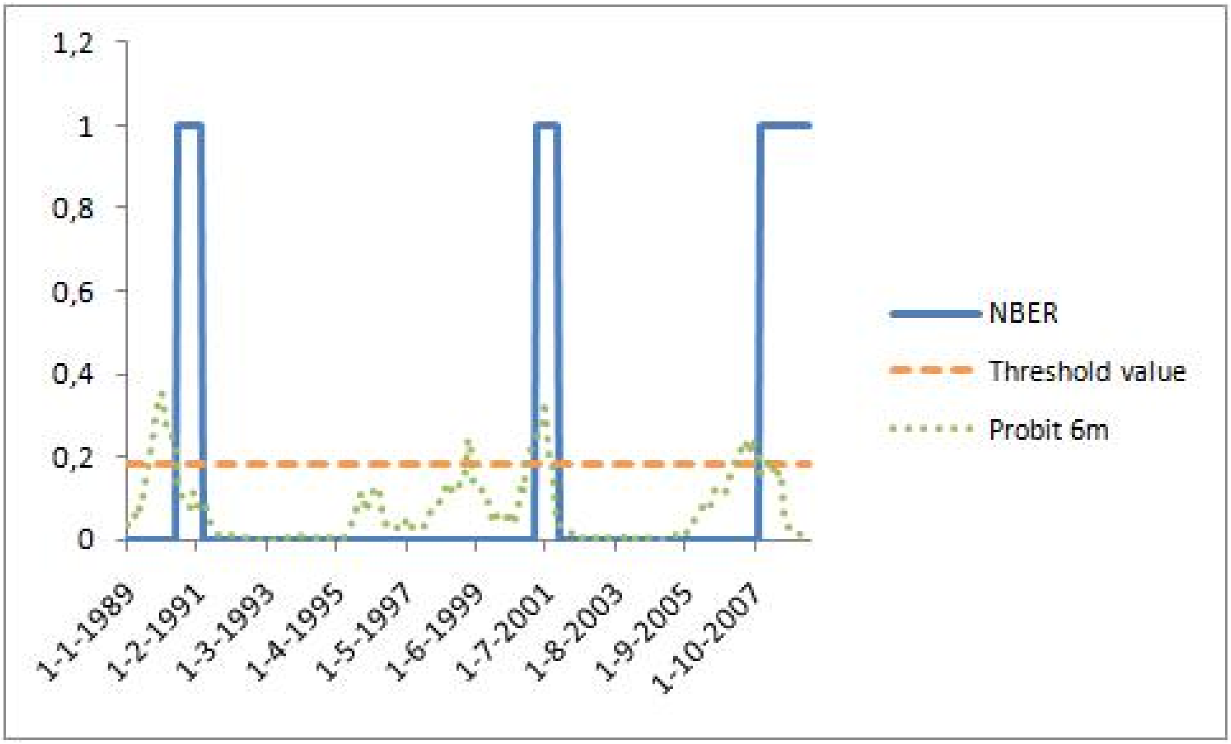

Figure D.1: Out-of-sample forecasts of the Probit model with the NBER recession indicator over the out-of-sample period (January 1989 to May 2009) using a forecasting horizon of 6 months ($k = 6$). “NBER“ stands for the NBER recession indicator, “Probit“ stands for the Probit model and “Threshold“ stands for the cut-off probability.

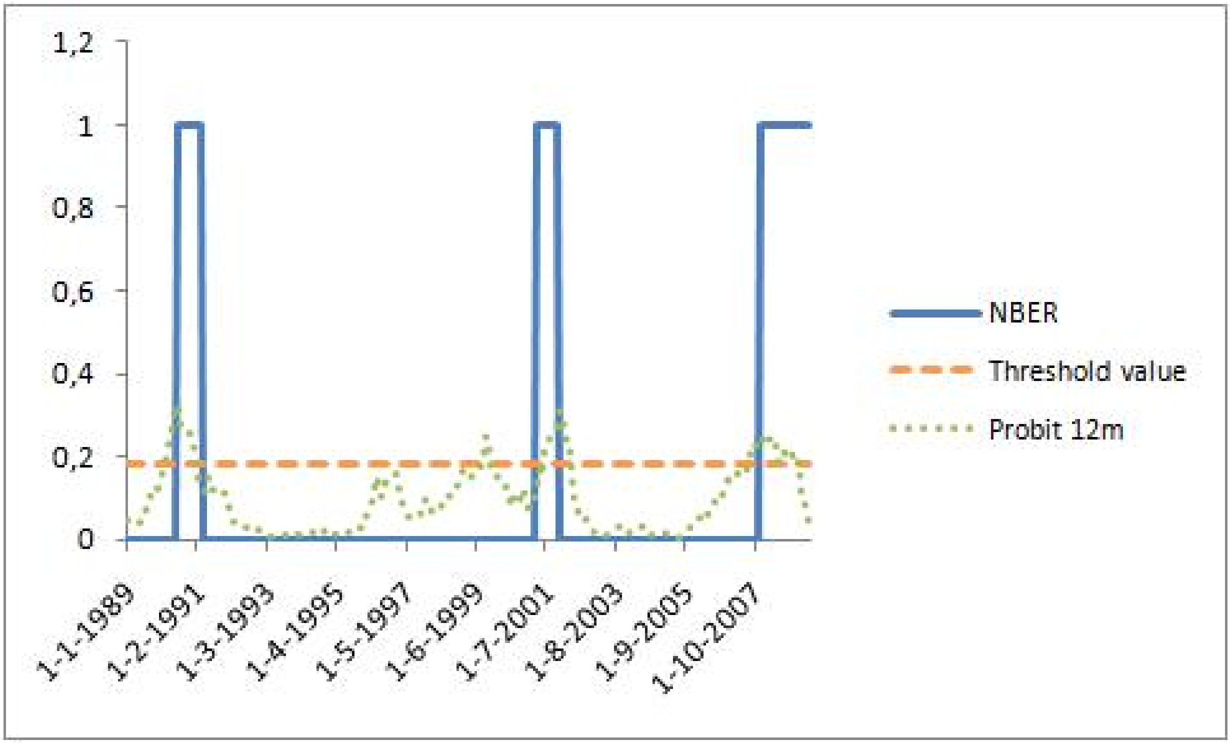

Figure D.2: Out-of-sample forecasts of the Probit model with the NBER recession indicator over the out-of-sample period (January 1989 to May 2009) using a forecasting horizon of 12 months ($k = 12$). "NBER" stands for the NBER recession indicator, "Probit" stands for the Probit model and "Threshold" stands for the cut-off probability.

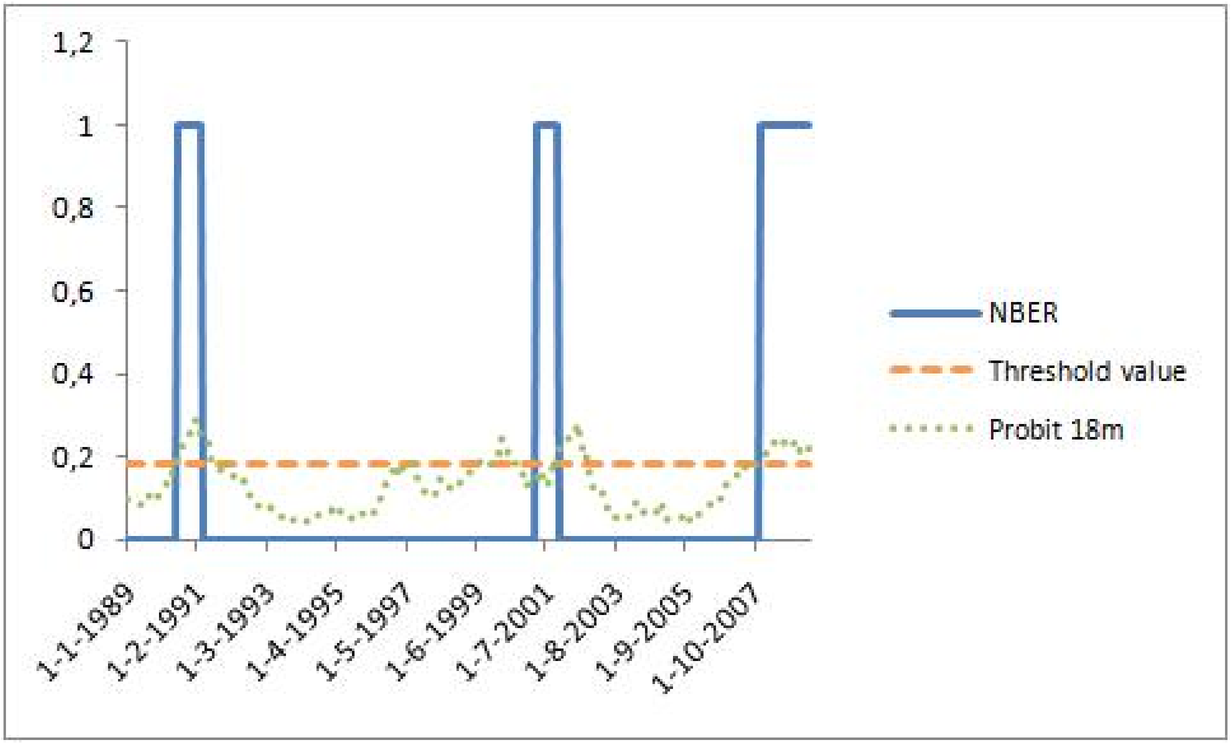

Figure D.3: Out-of-sample forecasts of the Probit model with the NBER recession indicator over the out-of-sample period (January 1989 to May 2009) using a forecasting horizon of 18 months ($k = 18$). “NBER“ stands for the NBER recession indicator, “Probit“ stands for the Probit model and “Threshold“ stands for the cut-off probability.

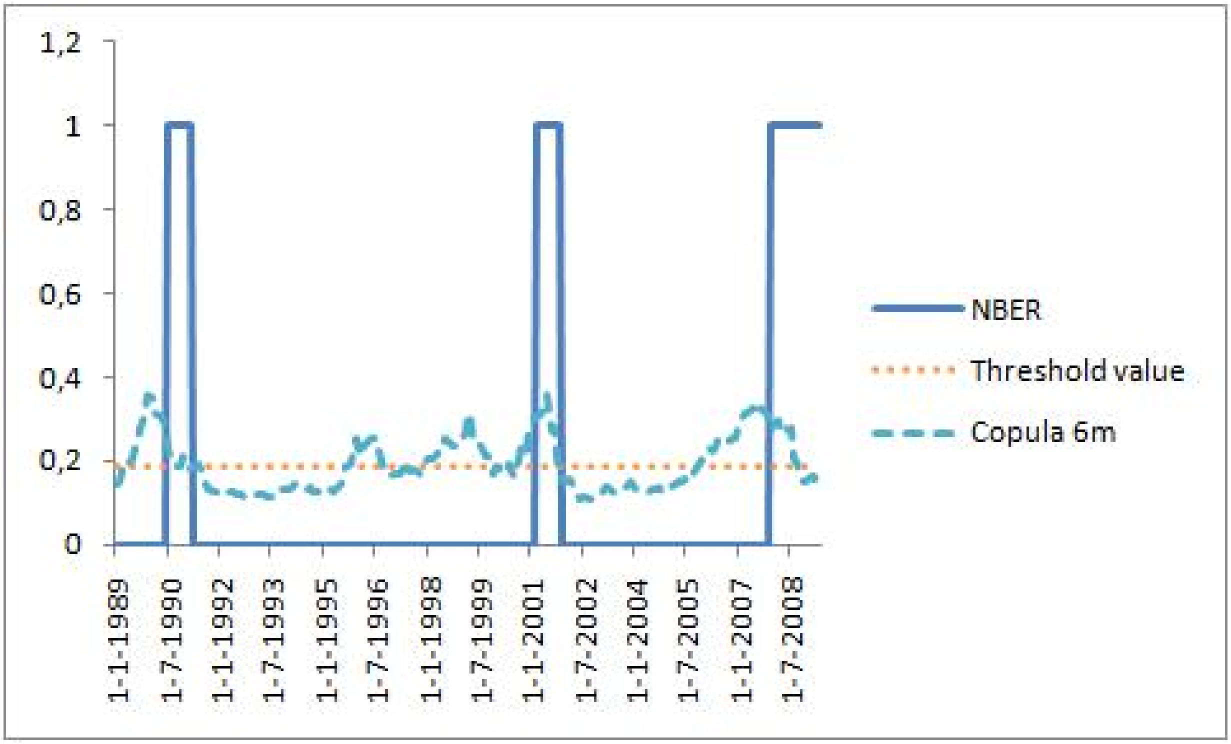

Figure D.4: Out-of-sample forecasts of the Copula probability model with the NBER recession indicator over the out-of-sample period (January 1989 to May 2009) using a forecasting horizon of 6 months ($k = 6$). "NBER" stands for the NBER recession indicator, "Copula" stands for the Copula probability model and "Threshold" stands for the cut-off probability.

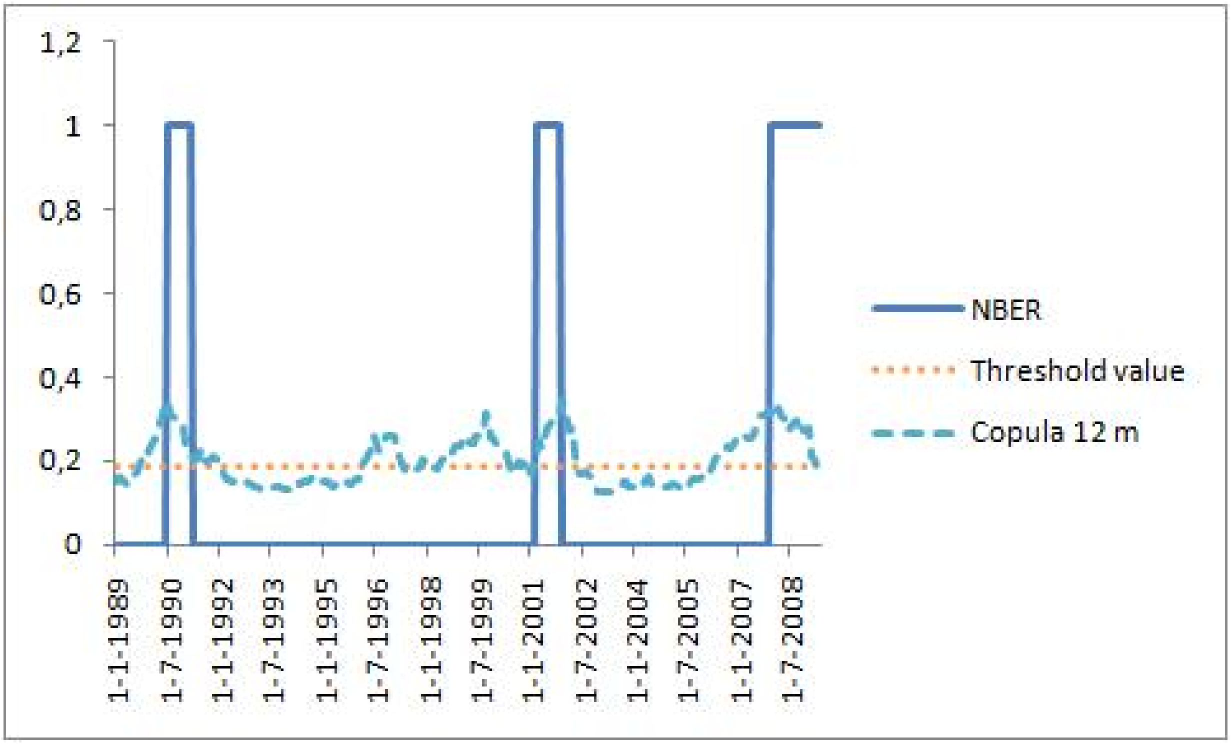

Figure D.5: Out-of-sample forecasts of the Copula probability model with the NBER recession indicator over the out-of-sample period (January 1989 to May 2009) using a forecasting horizon of 12 months ($k = 12$). "NBER" stands for the NBER recession indicator, "Copula" stands for the Copula probability model and "Threshold" stands for the cut-off probability.

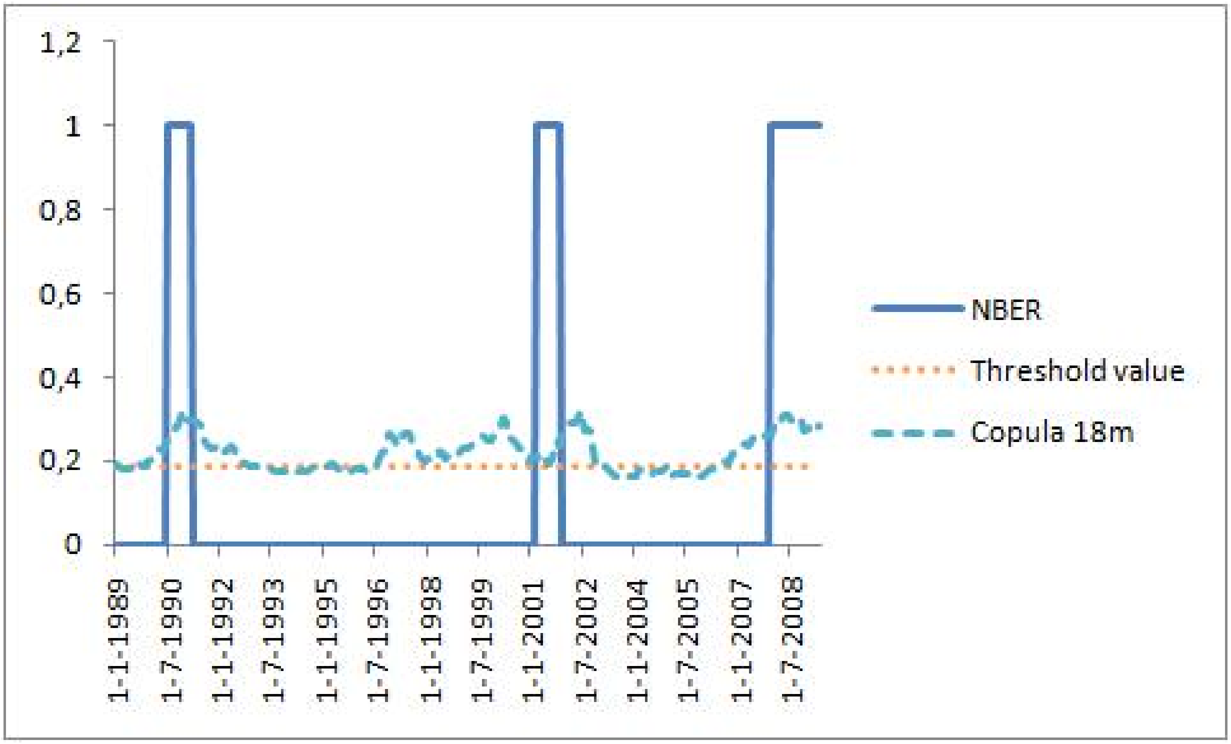

Figure D.6: Out-of-sample forecasts of the Copula probability model with the NBER recession indicator over the out-of-sample period (January 1989 to May 2009) using a forecasting horizon of 18 months ($k = 18$). “NBER“ stands for the NBER recession indicator, “Copula“ stands for the Copula probability model and “Threshold“ stands for the cut-off probability.