The Sensitivity of Infrastructure Investments Towards US Interest Rate Regimes

Econometrics and Management Science: Quantitative Finance
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Abstract

Gaining insights into the ripple effects of U.S. interest rates on infrastructure sectors is crucial in the constantly changing landscape of economic dynamics. Our research employs a nuanced regression model on daily asset returns from 2010 until 2024 and intertwines hidden interest rate regimes with factor analysis. The Hidden Markov Model unveils two distinct regimes which we characterise as stable and volatile. The pivotal discovery is a shift from systematic to idiosyncratic factors during volatile periods, emphasising the dynamic nature of infrastructure sectors. These insights offer significant implications for portfolio management within the infrastructure asset class.

Keywords: Infrastructure, Regime-Switching, Factor Regression, Interest Rates

Contents

1 Introduction

Infrastructure forms the foundation of civilisation and economic growth and requires substantial financing for large-scale projects. In their study, Schwartz et al. (2020) emphasises the substantial financing required for large-scale infrastructure projects. Furthermore, Merna and Njiru (2002) and Grimsey and Lewis (2002) argue that public-private partnerships (PPPs) shift the responsibility and risk of infrastructure delivery to private entities through long-term contracts. However, it is important to note that the success of PPPs relies on a consistent revenue stream, and any disruption or instability in this stream can pose significant financial challenges for private entities involved in the partnership, potentially leading to project delays, cost overruns, or even project abandonment.

Despite the associated risks, Andonov et al. (2021) note that infrastructure investments are gaining traction due to their stable returns, low equity market correlation, and diversification benefits. Folqué et al. (2021) further emphasises the appeal of infrastructure investments, particularly in the energy sector, due to the rise in sustainable investing and Environmental, Social, and Governance (ESG) scores.

Moreover, according to Bird et al. (2014) and Wurstbauer and Schäfers (2015), infrastructure stocks provide long-term cash flow visibility and inflation hedging benefits. Interest rates are crucial in infrastructure investments, as highlighted by Bitner and Newell (2018). Rothballer and Kaserer (2012) explains that lower interest rates can enhance profitability by influencing the cost of debt financing.

Blanc-Brude et al. (2014) note that interest rate fluctuations significantly affect infrastructure investments' value due to their impact on the discount rate used in valuation. The relationship between interest rates and different sectors of the infrastructure asset class depends on various factors, as explained by extensive research (Prud'Homme, 2004; Yeaple & Golub, 2007).

Bamidele Oyedele (2014) finds that infrastructure assets are the main drivers of portfolio stability during times of crisis, suggesting that they may provide a buffer against business cycle fluctuations. However, the rise of financial instruments, funds, and indices has shifted the infrastructure market towards more volatile demand, emphasising the need for a deeper understanding of the dynamics of infrastructure funds within the economy.

Given these considerations, our research takes a unique approach by investigating the sensitivity of infrastructure sectors to interest rates through a regime-switching framework. This methodology allows a more nuanced understanding of infrastructure assets' behaviour under different economic conditions. Ang (2014) recognises that the driving factors can vary depending on the prevailing interest rate regime. These differences can become apparent when considering different exposures to financial markets across various sectors. Consequently, our research question is formulated as follows:

> "*How does the sensitivity towards U.S. interest rate regimes differ between infrastructure sectors?*"

We will employ several factor models to estimate the driving factors in different regimes to answer this. By analysing the factor loadings, we aim to shed light on the influential sector-specific characteristics and their sensitivity to interest rates. Rothballer and Kaserer (2012) explains the increasing variations of infrastructure investments across various sectors, showing the particular relevance of this sectoral perspective.

Interest rates are often subject to regulatory influences, primarily guided by central banks' monetary policy, such as the Federal Reserve in the United States or the European Central Bank. These institutions periodically modify interest rates to manage inflation, ensure economic stability, and fulfil other economic objectives. Therefore, by providing a nuanced understanding of the role of interest rates across different infrastructure sectors, this research aims to bridge the existing knowledge gap, thereby informing more effective decision-making in this vital sector.

In addition to the primary research question, this study will address two key subquestions. The first is "*Which regimes can we identify in the U.S. interest rate between 2010 and 2024?*" This aims to pinpoint the different interest rate regimes during this period. Ang and Bekaert (2004) emphasises identifying these regimes as a crucial first step. This identification allows us to examine the behaviour of infrastructure assets under varied conditions, enriching our understanding of the complexity of the interest rate environment.

The second subquestion, "*Which factors in infrastructure sectors are most susceptible to change when transitioning between regimes?*", delves into the practical implications of interest rate regime changes on infrastructure assets. Answering this subquestion enriches academic understanding and provides a valuable decision-making and risk-management tool for practitioners.

To our knowledge, limited research has been conducted on the driving factors of infrastructure assets under a regime-switching framework. Building upon the work of Ben Ammar and Eling (2015), who investigated the drivers of infrastructure sectors using multiple factor models, our research extends its findings by utilising a regime-switching model. We identify different regimes using a Hidden Markov Model on the U.S. Effective Rate.

By examining different cycles in interest rates, we can identify the most sensitive sector within the asset class. We will analyse the daily returns of twelve infrastructure-specific sectors, including Airport, Communications, Datacenters, Electric Utility, Gas Utility, Multi Utility, Others, Pipelines, Port, Railroads, Toll Roads, and Water Utility, employing three different factor models, similar to the proposed methodology of Ben Ammar and Eling (2015). Here, we analyse a factor model based on daily systematic factors, idiosyncratic factors, and a combined model.

This paper is structured as follows: Section 2 provides a comprehensive overview of the data used and the process of creating the driving factors. Section 3 details the research approach, which includes regime detection and factor analysis. Section 4 presents the findings, and Section 5 draws the research to a close.

2 Data

For our analysis, we delve into the twelve sectors of infrastructure assets, focusing on a fund specialising in infrastructure, particularly the FTSE Core Infrastructure 50/50 index. This index, a global benchmark for many investment managers, provides a comprehensive view of the infrastructure market. We analyse the behaviour using daily data (excluding weekends) from 01-01-2010 until 01-01-2024, resulting in 3650 observations.

2.1 Return data

As mentioned, the infrastructure assets can be categorised into twelve diverse sectors, each with its unique characteristics and market behaviour: Airport, Communications, Datacenters, Electric Utility, Gas Utility, Multi Utility, Others, Pipelines, Port, Railroads, Toll Roads, and Water Utility. Appendix A provides an overview of the different stocks within each sector, offering a glimpse into the rich diversity of the infrastructure market.

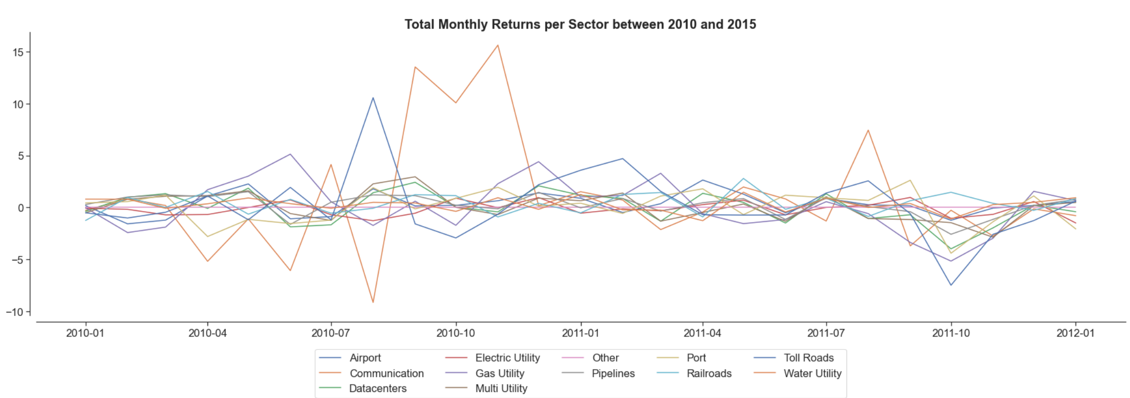

Figure 1: Monthly returns (%) per sector between 01-01-2010 and 01-01-2012

In Figure 1, we see an overview of each sector's daily returns aggregated to monthly observation over two years (2010-2012). There, we see that each sector behaves differently regarding volatility and return direction. We observe, for example, that at the end of 2010, Communications shot up rapidly, while Toll Roads and Gas Utilities showed a substantial drop in returns. This stark contrast in movements and behaviour between the sectors underscores the complexity of our analysis.

Since we aim to utilise a factor model for different regimes, we have also obtained data on the interest rate. As a proxy for interest rates, we use the Federal Funds Effective Rate, a key rate in the U.S. financial market that significantly influences other interest rates. We have sourced this data from FRED Economic Data [1].

Table 1: Overview of the value-weighted daily return per sector of infrastructure funds, and U.S. Federal Fund Effective Rate in percentages between 01-01-2010 and 01-01-2024

	Mean	St. Dev	Min.	Max.	Count
Effective Rate	0.01	0.02	0.00	0.05	-
Airport	0.16	1.43	-10.60	12.36	18
Communication	0.31	2.93	-16.87	24.51	27
Datacenter	0.07	1.28	-11.69	10.99	4
Electric Utility	0.02	1.08	-13.19	6.00	82
Gas Utility	0.07	2.75	-13.14	21.29	35
Multi Utility	0.02	1.03	-10.38	12.05	17
Other	-0.13	1.43	-7.92	9.39	3
Pipelines	0.04	1.26	-16.25	14.49	14
Port	0.09	1.47	-12.28	10.72	18
Railroads	0.03	1.33	-8.98	13.06	18
Toll Roads	0.23	2.53	-10.27	14.10	25
Water Utility	0.04	1.23	-13.49	13.50	22

In Table 1, we see an overview of the summary statistics for each sector, as well as the Federal Funds Effective Rate. From 01-01-2010 until 01-01-2024, the best-performing sector was Communications, with an average daily return of 0.31%. Comparing this with the negative daily return of -0.13% of the Other sector, we see a clear difference. The Other sector is created for companies that do not fall within the remaining sector specifications and, therefore, might not show clear dynamics. It is important to note that the number of stocks that comprise a sector differs, ranging from three for Other and 82 for Electric utility. When we look closer at Table 1 to the differences between sectors, we directly observe that a similar increase in standard deviation accompanies a relatively higher return. This shows

[1] https://fred.stlouisfed.org/series/DFF

that investors require a greater return for taking on more risk. A similar phenomenon is seen in the minimum and maximum gain/loss, as the sectors with higher risk have a more considerable all-time high gain and a more profound all-time loss.

2.2 Factor data

Next, we acquire data to create the factors for our analysis using daily data. The data consists of the Fama and French factors: Market risk, Size, Value, and Momentum (Rm-Rf, SMB, HML, and MOM, respectively). The data for these factors are obtained at Kennedy and French [2]. Next, we must create the lesser-known Fama and French factors for Term and Default premium (Fama & French, 1993). Furthermore, we have created non-return factors for Cash Flow Volatility, Leverage, and Investment Growth based on daily rankings of firm-specific metrics. This offers a more granular analysis, allowing us to focus on specific variables and metrics within each sector and regime. Moreover, by directly examining the changeability of these factors, we can uncover unique insights into the relationship between these characteristics and their sensitivity to different regimes. This approach contributes to a novel perspective on the dynamics of these factors and their impact on stock behaviour.

We gather data on the different stocks from WSJ.com [3] [4] FRED economic research [5] and lastly from Factset [6] where we calculate these idiosyncratic factors as follows:

Term premium - This factor is constructed to find the effect of unexpected movements in the return of government bonds. Using Equation 1, we calculate the term premium factor as the lagged difference between a U.S. 10-year treasury bond and the one-month treasury bill. Similar to Fama and French (1993), this is a proxy for the deviation of long-term bond returns from expected returns due to shifts in interest rates.

$$TERM_t = USTBOND10Y_{t-1} - USTBILL1M_{t-1} \tag{1}$$

Default premium - Secondly, we construct a factor to determine the effect of changes in the probability of default. Since infrastructure projects are prone to high debt ratios, this might result in an increased impact on the default premium. Fama and French (1993) constructs the default premium factor as the difference between a market portfolio of long-term corporate bonds and a long-term government bond. We do the same here, as seen

[2] $https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html$
[3] $https://www.wsj.com/market-data/quotes/bond/BX/TMUBMUSD01M/historical-prices$
[4] $https://www.wsj.com/market-data/quotes/bond/BX/TMUBMUSD10Y$
[5] $https://fred.stlouisfed.org/series/DBAA$
[6] $https://www.factset.com/$

in Equation 2, using Moody's Seasoned Baa Corporate Bond Yields and the U.S. 10-year treasury bond.

$$DEF_t = MoodysBAA_{t-1} - USTBOND10Y_{t-1} \tag{2}$$

Cash flow volatility - This is the only factor not observed daily but monthly. We forward-fill the observations to create a factor with daily values. We are not worried about the repetitive nature of cash flow now because the companies' reporting dates are not aligned. This results in a varying dataset when we aggregate the values to create the final dataset for the factor. This dataset calculates the standard deviation of cash flow divided by sales over a rolling 50-trading-day window. For each date, we rank the measurements for cashflow volatility into the top 30%, the middle 40%, and the bottom 30%. Next, we create the $CFvol_t$ factor by taking the daily average of the top 30% high cash flow values minus the average of the bottom 30% low cash flow volatility.

Investments growth - We incorporate an investment growth factor because we believe that investment will be an essential indicator of profit generation. These profits will explain part of the variation in the equity return of an infrastructure asset or project. Chen et al. (2011) proposes a standard investment growth factor as the change in property, plant and equipment (PPE) with the addition of inventory divided by the lagged value of total assets. However, we devised a different metric since infrastructure stocks usually do not have inventory or changed assets. We divide each company's earnings per share (EPS) by its Capital Expenditure (CAPEX). CAPEX is the funds a company spends to acquire, upgrade, and maintain physical fixed assets, such as property, buildings, and equipment.

$$INVG_{i,t} = \frac{EPS_{i,t-1}}{CAPEX_{i,t-1}} \tag{3}$$

Here, Equation 3 shows how we calculate the metric for investment growth for company i at time t. These values are obtained from Factset, and after ordering the companies for every day, we create the $INVG_t$ factor as the average of the top 30% minus the bottom 30% on day t.

Leverage - We construct a leverage factor based on a high and low debt-to-equity ratio (DER) (Bhandari, 1988). Here, we divide the total debt (difference in book value of total assets and common equity) by the market value of common equity, as seen in Equation 4 below.

$$DER_{i,t} = \frac{BV_{i,t-1}^{TOTASSET} - BV_{i,t-1}^{COMEQUITY}}{MV_{i,t-1}^{COMEQUITY}} \tag{4}$$

Next, we create a daily ranking based on the DER of each company i (top 30%, middle 40%, bottom 30%). The LEV_t factor is the average of the top 30% DER minus the bottom 30% on day t.

Price/Earnings - This factor is relatively straightforward, as the metric is obtained by dividing the price per share by the earnings per share. Here, we create the PE_t factor as the average of the top 30% PE minus the bottom 30% PE stocks on day t.

In Table 2, we see the summary statistics of the factors. Here, we note that not every company has reported all data starting from 2010 or until 2024. Instead of only looking at stocks with data from the beginning until the end, we perform the abovementioned analysis by daily ranking the companies that have available data based on that observation data.

Table 2: Summary statistics on explanatory factors for the period 01-01-2010 until 01-01-2024

	$R_M - R_f$	SMB	HML	MOM	DEF	TERM	CFvol	INVG	LEV	PE
Mean	0.04	-0.01	-0.00	0.02	2.56	1.56	0.10	0.24	3.80	70.75
St. Dev	0.93	0.39	0.50	0.65	0.69	1.22	0.10	0.34	1.21	24.13
Min	-9.62	-5.38	-3.10	-9.40	1.42	-2.06	0.03	0.02	2.16	33.82
Max	8.33	2.05	4.16	3.57	5.97	3.87	0.79	1.13	6.60	121.54

3 Methodology

In this section, we explain the proposed methodology for this research. It consists of two primary analyses combined to analyse the movements of infrastructure assets under changing interest rates. The first part starts with a deep dive into the interest rate dynamics, for which we use the Federal Fund Effective Rate as a proxy. Using a regime-switching model, we determine the different regimes in the data. For each regime, we then estimate specific parameters of the factor model.

3.1 Hidden Markov Model

Regime-switching models are statistical models that allow for changes in a time series' underlying structure or parameters. These models are advantageous in this setting since we analyse data prone to exhibit periods of different behaviours or regimes.

To determine the different regimes, we utilise a Hidden Markov Model (HMM) as done by, for example, Hou (2017), Mor et al. (2021) and Rabiner (1989). Here, we assume that there is a (hidden/unobserved) process that drives the interest rates. Therefore, the implementation of an HMM provides a better understanding of the differences in the process of the driving factor of interest rates.

As stated in the name, a HMM is, in essence, a Markov model where the states are unobserved. With that, a HMM is a stochastic model where states are assigned with a particular transition probability. The probability of being in a specific state depends only on the previous state and the transition probability.

We assume a HMM with an unknown number of regimes K. Also, let $\{S_t\}$ be an observed process, namely the interest rate, and the unobserved state time series is denoted by $\{Y_t\}$. Furthermore, matrix A represents the transition probabilities between states, where a_{ij} is the probability of moving from state i to state j. Here, we must assume that these probabilities are constant over time and constrained such that $\sum_{j=0}^{K} a_{ij} = 1 \quad \forall i$.

The initial state probabilities, denoted by π, represent the probabilities of starting in each state. So, π_i is the probability of starting in state i. Following the same logic as before, we impose the constraint that $\sum_{i=0}^{K} \pi_i = 1$.

The emission probabilities in a Hidden Markov Model (HMM) represent the likelihood of observing a certain value for $S_n = s_n$ given a specific hidden state. We denote the emission probabilities as the matrix B, where $b_{s_i}(k)$ represents the probability of emitting observation s_i from state k. Like the transition probabilities, the emission probabilities are also subject to constraints. Specifically, the sum of probabilities of emitting all possible observations from a particular state should equal 1. This can be expressed as $\sum_{i=0}^{n} b_{y_i}(k) = 1 \quad \forall k$, where n represents the total number of possible observations. These probabilities are visually represented in Figure 2, where we see the different states, probabilities and emitted observations.

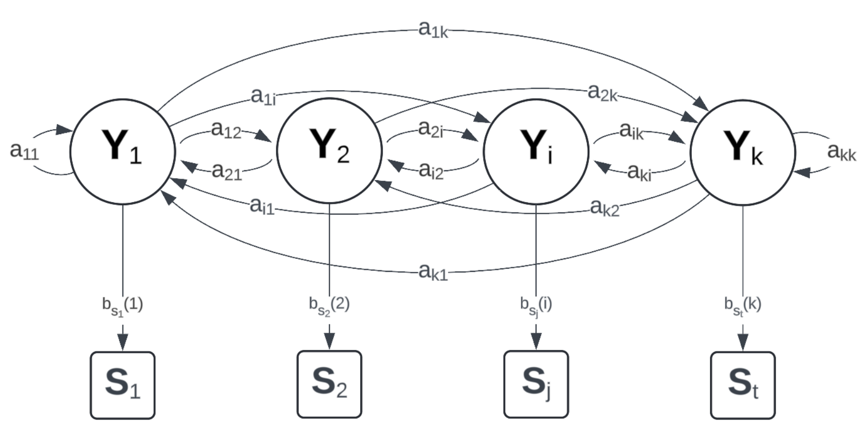

Figure 2: Hidden Markov Model showing the transition and emission probabilities

Estimating the Hidden Markov Model consists of two steps: estimating the Markov parameters (obtaining α_{ij} and $\beta_{s_i}(j)$). We do this through the Baum-Welch algorithm as first proposed by Baum et al. (1970). Secondly, we implement the Viterbi algorithm to find the sequence of regimes that maximises the probabilities of being in state i at time t. The Viterbi algorithm utilises the parameters we estimated with the Baum-Welch algorithm as trade-offs between the probability of being in a specific state and moving to the next state. Both algorithms use the forward-backward equations, a dynamic programming algorithm combining two equations to determine joint probabilities (Ross, 2014).

3.1.1 Forward-Backward algorithm

We begin by explaining the general form of the forward and backward equations. This algorithm calculates the forward probabilities, denoted by $\alpha_t(i) = \mathbb{P}\{S_t = s_t, Y_t = i\}$, which represent the probability of being in state i at time t while having observed $S_{1:t}$. Here we know that, according to the law of conditional probability, $\mathbb{P}\{Y_t = i | S_t = s_t\} = \frac{\alpha_t(i)}{\sum_j \alpha_t(j)}$. The backward probabilities, denoted by $\beta_k(t) = \mathbb{P}\{S_{t+1} = s_{t+1}, \ldots, S_n = s_n | Y_t = k\}$, represent the probability of observing the data $Y_{t+1:T}$ given that the system is in state k at time t.

The forward-backwards algorithm is performed in two steps. First, the forward approach is used to determine $\alpha_t(i)$ according to Formula 5 (full derivation in Appendix A.2)

$$\alpha_t(i) = p(s_t|i) \sum_j \alpha_{t-1}(j) a_{ji}. \tag{5}$$

Starting with $\alpha_1(i) = \mathbb{P}\{Y_1 = i, S_1 = s_1\} = p_i p(s_1|i)$ we use Formula 5 recursively to determine the function up to $\alpha_n(i)$. Next, the backward approach is defined by conditioning on $Y_{t+1} = j$ using the following:

$$\beta_k(t) = \sum_j p(s_{t+1}|j) \beta_j(t+1) a_{jk}. \tag{6}$$

We use Formula 6 starting with $\beta_i(n-1) = \sum_j a_{ji} p(s_n|j)$ to determine $\beta_k(n-2)$ until $\beta_k(1)$. We again refer to Appendix A.2 for the full derivation of the recursion.

To utilise the sequential nature of this approach, we may simultaneously compute the forward and backward equations starting from α_1 and β_{n-1} until we have calculated both $\alpha_k(i)$ and $\beta_k(i)$. The joint probabilities of the observed data and the underlying states, $\mathbb{P}(Y_{1:T}, S_{1:T})$, can be calculated using the forward and backward probabilities as in Formula 7

$$\mathbb{P}\{S_n = s_n, Y_k = i\} = \alpha_k(i) \times \beta_k(i) \tag{7}$$

Ross (2014) explains two schools of thought when analysing the unobserved states. The first one is solely looking at the final observation and maximising the probability $\mathbb{P}(Y_n = i|S_n = s_n)$. However, this would not incorporate the transition probability a_{ij}. For example, the maximum probability at time $t-1$ may be attained in state j and state i for time t. However, that must also mean the a_{ij} should be reasonably high, which is not necessarily the case.

Therefore, analysing the entire sequence in a Hidden Markov Model (HMM) is essential for capturing the temporal dependencies inherent in financial time series data, such as interest rates. A specific form of the forward-backwards algorithm (Viterbi algorithm) leverages the entire sequence to compute the forward and backward probabilities. These probabilities are then combined to yield the conditional probability of being in a specific state at each time point, given all the observed data. This whole sequence analysis enables a nuanced understanding of the regime-switching behaviour in the data, offering insights into the underlying dynamics that single-point predictions may overlook.

3.1.2 Baum-Welch algorithm

In this part, we will explain how we utilised the Baum-Welch algorithm to find the estimated parameters under a fixed number of regimes. Here, the Markov Chain is described by $\theta = (A, B, \pi)$ such that the algorithm searches for the local maxima $\theta^* = \text{argmax}_\theta \mathbb{P}(S|\theta)$. After that, we compare the estimated models across different regimes using the log-likelihood and the AIC and BIC criteria. This will determine the number of regimes we believe are present in the data.

The Baum-Welch algorithm is an Expectation-Maximization (EM) algorithm consisting of two steps. The goal of this algorithm is to find the parameters that maximise $Q(\theta, \theta_k) = \mathbb{E}_{\theta_k}(\log(p_\theta(S, Y))|S = s) = \sum_s \log(p_\theta(s, y))p_{\theta_k}(y|s)$. This leads to the first step:

E-step In this step, we need to take the expectation of the joint distribution of the observed data and the hidden states given the current parameter estimates, represented by θ_k. By taking this expectation, we obtain the following formula:

$$\begin{aligned} Q(\theta, \theta_k) = &\sum_{i=1}^{k} \mathbb{P}_{\theta_k}(Y_1 = i|s)\log(\pi_i) + \sum_{t=2}^{n}\sum_{i=1}^{k}\sum_{j=1}^{k} \mathbb{P}_{\theta_k}(Y_{t-1} = i, Y_t = j)\log(a_{ij}) \\ &+ \sum_{t=1}^{n}\sum_{i=1}^{k} \mathbb{P}_{\theta_k}(Y_t = i|s)\log(f(s_t|\Theta)). \end{aligned} \tag{8}$$

For the Baum-Welch algorithm, we can make Formulas 5 and 6 specific. This results in the forward recursion being: $\alpha_t(i) = b_{s_t}(i)\sum_{j=1}^{k} \alpha_{t-1}(j)a_{ji}$ and the backward recursion is $\beta_t(i) = \sum_{j=1}^{k} \beta_{t+1}(j)a_{ij}b_{s_{t+1}}(i)$. Now, we see that the probabilities in Formula 8 (after using Bayes' Theorem of conditional probability, see Appendix A.2) can be calculated with the forward-backward algorithm such that:

$$\gamma_t(i) = \frac{\alpha_t(i)\beta_t(i)}{\sum_{j=1}^{n} \alpha_t(i)\beta_t(i)} \tag{9}$$

$$\xi_t(ij) = \frac{\alpha_t(i)a_{ij}\beta_{t+1}(j)b_{s_{t+1}}(j)}{\sum_{k=1}^{n}\sum_{w=1}^{n} \alpha_t(k)a_{kw}\beta_{t+1}(w)b_{s_{t+1}}(w)}. \tag{10}$$

M-step Here, we take the expectation we obtained in the previous step and find the value for θ that maximises $Q(\theta, \theta_k)$. This is done by calculating the first-order conditions with respect to the parameters. These derivations are omitted, but we refer to Yang et al. (2017) and Hsiao and Schultz (2011) for the detailed description. Using Formulas 9 and 10 we can update the Markov parameters as:

$$\pi_i^* = \gamma_1(i) \tag{11}$$

$$a_{ij}^* = \frac{\sum_{t=1}^{n-1} \xi_t(ij)}{\sum_{t=1}^{n-1} \gamma t(i)} \tag{12}$$

$$b_{v_w}^* = \frac{\sum_{t=1}^{n} \mathbb{1}(s_t = v_w)\gamma_t(i)}{\sum_{t=1}^{n} \gamma_t(i)}. \tag{13}$$

Formula 12 calculates the number of expected transitions from state i to state j relative to the total number of transitions from state i. Furthermore, Formula 13 calculates the expected number of times the observation is v_k while the system is in state i, relative to the total expected number of times the system is in state i.

After updating the parameters, the algorithm circles back to the E-step and continues the loop until it converges. This convergence is quantified by iterating until the increase in the log-likelihood is below a certain threshold. Depending on the initial parameters, the algorithm may converge to a different solution due to local maxima in the likelihood function. To mitigate this, we rerun the algorithm multiple times with different (random) initial values of θ.

3.1.3 Viterbi algorithm

The Viterbi algorithm is another variant of the forward-backwards algorithm that finds the most probable sequence of states, known as the Viterbi path. In this section, we explain how it works and differs from the forward-backwards algorithm. As mentioned at the beginning of Section 3, our goal here is to predict the sequence of state $Y_{1:T} = \{y_1, \ldots, y_T\}$ based on the observed data $S_{1:T} = \{s_1, \ldots, s_T\}$. Similarly as in Churbanov and Winters-Hilt (2008), we use Formulas 5 and 6 to compute $\mathbb{P}(Y_n = (i_1, \ldots, i_n)|S_{1:T})$ where $Y_n = (i_1, \ldots, i_n)$ is the vector of the first n states. This results in the following formula:

$$\mathbb{P}\{\mathbf{Y}_n = (i_1, \ldots, i_n)|S_n = s_n\} = \frac{\mathbb{P}\{\mathbf{Y}_n = (i_1, \ldots, i_n), S_n = s_n\}}{\mathbb{P}\{S_n = s_n\}}. \tag{14}$$

The objective function maximised by the Viterbi algorithm can be decomposed into a series of sub-problems, each representing the maximum probability of a path leading to a specific state at a specific time. Thus, the most probable sequence of states (or the optimal path) combines the most probable sub-paths. Mathematically the objective function becomes $F(y) = f_1(y_1) + f_2(y_2, y_1) + \cdots + f_n(y_n, y_{n-1})$. The power of the Viterbi algorithm lies in its ability to utilise the Markovian property. If we let $Y^*_{1:T}$ be the optimal sequence and s^*_i is observed in regime k, then we can split the objective function into two parts as,

$$F_{i,k}(y_{1:i-1}) = f_1(y_1) + f_2(y_2, y_1) + \cdots + f_i(k, y_{i-1})$$
$$\bar{F}_{i,k}(y_{i+1,n}) = f_{i+1}(y_{i+1}, k) + \cdots + f_n(y_n, y_{n-1})$$

and maximise them separately. The optimal path from the start to any point in the sequence is part of the overall optimal path. This is a property of dynamic programming problems known as the principle of optimality. Using this logic, we can form the recursion using a special form of the forward-backward algorithm.

The forward part of the Viterbi algorithm calculates a modified version of the forward probabilities, denoted by $V_k(j) = \underset{i_1,\ldots,i_{k-1}}{max} \mathbb{P}\{\mathbf{Y}_{k-1} = (i_1, \ldots, i_{k-1}, Y_k = j, S_k = s_k\}$, which represent the maximum probability of any path that ends at state j at time k.

The calculation of $V_k(j)$ is similar to calculating the forward probabilities in the forward-backward algorithm. Still, instead of summing over the probabilities of each path, it keeps only the maximum probability. This is done by taking the maximum over the previous states. The calculation is as follows: $V_k(j) = p(s_k|j) \max_i V_{k-1}(i)a_{ij}$ or in terms of emission probabilities, $\max_i V_{k-1}(i)a_{ij}b_{s_k}(j)$.

After the $V_n(j_n)$ values have been calculated for each state j_n at each time step n, the Viterbi path can be found by backtracking. Starting from the state at which the maximum $V_n(j_n)$ is obtained at the final time step, the state at each previous time step is chosen to be the state that maximises the product of the V value, the transition probability, and the emission probability at that time step.

This reverse path that is traced out is the Viterbi path, i.e., the most probable sequence of states given the observed data. This backtracking step can be seen as a backward calculation, but it is not the same as the backward equations in the forward-backward algorithm. The main difference is that the backtracking step in the Viterbi algorithm is used to recover the most likely state sequence. In contrast, the backward equations in the forward-backward algorithm are used to compute the backward probabilities and then to calculate the posterior state probabilities.

Using the relation $V_k(j) = \underset{i_1,\ldots,i_{k-1}}{max} \mathbb{P}\{\mathbf{Y}_{k-1} = (i_1, \ldots, i_{k-1}, Y_k = j, S_k = s_k\}$, and seeing that the problem in Formula 14 is maximised through the numerator, we get an iterative algorithm for the whole sequence. The entire derivation is shown in Appendix A.2, where Formula 15 shows the resulting maximisation.

$$\max_{i_1,\ldots,i_n} \mathbb{P}\{\mathbf{Y}_n = (i_1, \ldots, i_n), S_n = s_n\} = p(s_n|j_n) a_{i_{n-1}(j_n),j_n} V_{n-1}(i_{n-1}(j_n)) \tag{15}$$

Here, $i_{n-1}(j_n)$ is the next to last state in the maximising sequence, and we can continue this fashion with $i_{n-2}(i_{n-1}(j_n))$.

3.2 Factor Model

Following the identification of distinct regimes using Hidden Markov Models, we proceed to the second part of the research. Namely, we estimate three factor models for each found regime. For this, we propose linear models to analyse the daily effects of the factors as stated in Section 2. The final layer of the analysis involves fitting the factor model to the aggregated return of different sectors. The parameters of the factor models, including the factor loadings $\beta_{i,X}$ for variable X and the error terms $\epsilon_{i,t}$, are estimated using Generalized Least Squares (GLS).

GLS is a variant of ordinary least squares that allows for certain types of heteroskedasticity and autocorrelation in the error terms. It works by applying a linear transformation to the data to remove the heteroskedasticity and autocorrelation and then performing ordinary least squares on the transformed data. In this context, the objective function for GLS can be written as $\min_{\alpha,\beta} \sum_t (R_t - \alpha - \beta' F_t)' \Omega^{-1} (R_t - \alpha - \beta' F_t)$, where R_t are the returns, F_t are the created factors, α and β are the parameters to be estimated, and Ω is the variance-covariance matrix of the errors. The parameters that minimise this function can be found using numerical optimisation techniques or, in some instances, using algebraic solutions. GLS provides consistent and efficient estimates of the parameters under the model's assumptions. The most critical assumptions under GLS are a linear relation between dependent and independent variables, heteroskedasticity and autocorrelation in the error term, no perfect multicollinearity between independent variables, and normally distributed errors.

3.2.1 Model Estimation

As mentioned, we implement three factor models on the excess return $(R_{i,t} - R_{f,t})$ of each sector i. The first one is constructed using only systematic factors as proposed by Fama and French (1993) with the addition of the momentum term first researched by Carhart (1997) as seen in Formula 16.

$$\begin{aligned} R_{i,t} - R_{f,t} &= \alpha_i + \beta_{i,M}(R_{M,t} - R_{f,t}) + \beta_{i,SMB}SMB_t + \beta_{i,HML}HML_t \\ &+ \beta_{i,MOM}MOM_t + \beta_{i,TERM}TERM_t + \beta_{i,DEF}DEF_t + \epsilon_{i,t} \end{aligned} \tag{16}$$

This six-factor model, which includes systematic factors such as the market premium $(R_{M,t} - R_{f,t})$, size (SMB_t), value (HML_t), momentum (MOM_t), term structure $(TERM_t)$, and default risk (DEF_t), is interesting because it allows us to capture broad market trends and economic conditions that affect all companies to varying degrees. The factors selected are believed to explain a significant portion of the variations in stock returns.

Secondly, we utilise four company-specific characteristics, which are captured in the Cash-flow Volatility, Investment Growth, Leverage and Price-Earnings factors ($CFvol_t$, $INVG_t$, LEV_t and PE_t respectively). This regression is again done for each sector i and is shown in Formula 17.

$$\begin{aligned} R_{i,t} - R_{f,t} &= \alpha_i + \beta_{i,CFvol}CFvol_t + \beta_{i,LEV}LEV_t \\ &+ \beta_{i,INVG}INVGt + \beta_{i,PE}PE_t + \epsilon_{i,t} \end{aligned} \tag{17}$$

This model is interesting because it allows us to capture firm-specific characteristics that may affect the stock's return, independent of macroeconomic conditions. It provides a more detailed look at the factors influencing a particular company's stock performance.

Lastly, we combine both models to create the model of Ben Ammar and Eling (2015), with an additional Price-Earnings factor, which is shown in Formula 18

$$\begin{aligned} R_{i,t} - R_{f,t} &= \alpha_i + \beta_{i,M}(R_{M,t} - R_{f,t}) + \beta_{i,SMB}SMB_t + \beta_{i,HML}HML_t \\ &+ \beta_{i,MOM}MOM_t + \beta_{i,TERM}TERM_t + \beta_{i,DEF}DEF_t \\ &+ \beta_{i,CFvol}CFvol_t + \beta_{i,LEV}LEV_t + \beta_{i,INVG}INVGt + \beta_{i,PE}PE_t + \epsilon_{i,t} \end{aligned} \tag{18}$$

Combining the two previous models, we integrate systematic and idiosyncratic factors. This model is particularly interesting because it provides a more holistic view of the factors that might influence a stock's return. It allows for a comprehensive analysis of the market-wide and firm-specific characteristics that drive stock returns, providing a more complete picture of the dynamics at play.

In these formulas, α_i is said to be the constant factor within sector i, $\beta_{i,X}$ represents the factor loading in sector i on the effect of variable X and $\epsilon_{i,t}$ is the error term of sector i at time t.

3.2.2 Model Comparison

We employ the Wald test to compare the factor loadings between different regimes. The Wald test is a statistical test commonly used to assess the significance of individual coefficients in regression models. In our context, it allows us to determine if there are statistically significant differences in the factor loadings between regimes for each variable.

The test statistic for the Wald test is calculated by taking the difference in factor loadings between the two regimes and dividing it by the square root of the difference between standard errors. The formula for the Wald test statistic is as follows:

$$W = \frac{\hat{\beta}_1 - \hat{\beta}_2}{\sqrt{SE(\hat{\beta}_1)^2 - SE(\hat{\beta}_2)^2}}. \tag{19}$$

Here $\hat{\beta}_1$ and $\hat{\beta}_2$ are the estimated factor loadings for the two regimes being compared, and $SE(\hat{\beta}_1)$ and $SE(\hat{\beta}_2)$ are the corresponding standard errors. This results in a test statistic that follows the Chi-squared distribution.

Furthermore, we also compare the coefficients of a driving variable across the regressions of different sectors. With this comparison, we aim to quantify the differences between industry drivers. The test statistic for the multivariate Wald test is calculated by taking the difference between the coefficient vector and a reference value (in this case, the mean coefficient across different sectors) and multiplying it by the inverse of the joint covariance matrix. The resulting test statistic follows the multivariate chi-squared distribution,

$$\mathbf{W} = \frac{\hat{\boldsymbol{\beta}}_i - \hat{\boldsymbol{\beta}}_\mu}{\hat{\boldsymbol{\Omega}}}. \tag{20}$$

In this formula, $\hat{\beta}_i$ represents the estimated coefficient vector for a specific sector or regime, $\hat{\beta}_\mu$ represents the estimated coefficient vector as the reference value (e.g., the mean coefficient across different sectors), and $\hat{\Omega}$ represents the estimated covariance matrix of the coefficient estimates.

Using the (multivariate) Wald test, we can test the hypothesis of equal coefficients. A significant test result indicates evidence against this hypothesis, suggesting that the coefficients differ between the regimes or sectors being compared.

Performing the Wald test enables us to quantify the differences in factor loadings and coefficients, providing valuable insights into the variations across regimes and sectors.

4 Results

In this section, we will discuss the results of the methodology as explained in Section 3. We first look at the Hidden Markov Model (HMM) and then see the implications of the factor model. We combine the current dataset (2010-2024) with a similar training set from 01-01-2000 until 31-12-2009 to capture the full dynamics in the interest rate data.

4.1 Hidden Markov Model

Since a HMM assumes that the state transition distribution is constant over time, we first test for stationarity by implementing the Augmented-Dickey Fuller test (Dickey & Fuller, 1979). As we will use the full data set to estimate the number of regimes, we will also test the stationarity of this sample. We obtain an ADF test statistic of -1.28 with a P-value of 0.64. This means we reject the null hypothesis of stationarity and transform the data by taking the first difference of the returns.

Next, we determine which HMM suits the data best using valuation criteria. As explained in Section 3.1, we change the number of regimes (1 to 6) and compare the log-likelihood, AIC, and BIC across the different models.

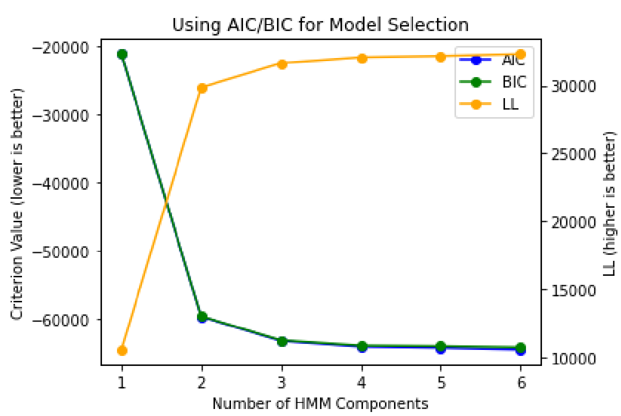

Figure 3: Valuation criteria for different Hidden Markov Model

By looking at Figure 4, we can see that there is a steep drop in the beginning, and after two regimes, it flattens very quickly. Even though there seems to be a slight benefit for three regimes compared to two, the HMM could not converge to an optimal solution for more than two regimes. Therefore, we believe the most robust answers come from a two-regime Markov model, which we will implement from here onwards. With that, we have answered the first subquestion as stated in Section 1.

By selecting the HMM with two regimes to be the best model, we again fit the model to our data, utilising various (random) starting values to decrease the possibility of finding local maxima. After the model converges, we obtain summary statistics to characterise the two regimes.

Table 3: Transition matrix for the two regimes HMM on interest rates between 2010 and 2024

	Regime 1	Regime 2
Regime 1	0.958	0.042
Regime 2	0.766	0.234

Table 4: Statistics on the two regimes in interest rates between 2010 and 2024

	Mean	St. Dev.	Count
Regime 1	-0.00053	0.00014	3420
Regime 2	0.021	0.020	231

The Hidden Markov Model (HMM), as presented in Table 3, provides a dynamic perspective on how interest rate states transition over time. The model reveals a high degree of persistence, particularly for regime 1, which aligns with the expectation that changes in interest rates typically occur slowly and irregularly. Regime 1, characterised by a high self-transition probability, indicates a stable state in the economy. This state captures periods of relative stability when interest rates are maintained at a consistent level. This is a common scenario, as central banks often keep interest rates steady for extended periods to avoid causing abrupt economic disruptions.

The HMM shows that in regime 2, the model is substantially less likely to retain in this state but has a high transition probability from regime 2 to regime 1. Moreover, we note that the model moves from regime 2 to 1 with a higher change than from regime 1 to 2. This suggests that even when the economy experiences changes that push it away from the steady state, there's a strong tendency to revert to stability. Therefore, regime 1 represents the most frequently observed state and serves as an "attractor" that the economy tends to gravitate towards.

Overall, the transition probabilities underscore the central role of regime 1 as the dominant and stabilising force in the dynamics of interest rates. They highlight the economy's propensity to maintain or revert to stability, reflecting the cautious and gradual approach typically taken by central banks in adjusting interest rates.

Table 4 shows the summary statistics for the two different regimes, which are also visualized in Figure 4. Using these statistics, we give a detailed representation of each regime as follows:

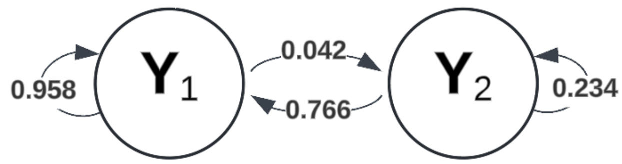

Figure 4: Graphical representation of transition probabilities between the stable (Y_1) and volatile (Y_2) regime

Regime 1 - Stable Regime: regime 1 shows a slight negative mean change in interest rates (-0.00053), suggesting slight decreases or essentially stable rates when the system is in this state. The very low standard deviation (0.00014) further underscores the stability of this regime. As identified earlier, this steady state exhibits a high degree of stickiness, with a high likelihood of remaining in this state once it has entered.

Regime 2 - Volatile Regime: regime 2 is characterised by a mean change in interest rates of 0.021 and a relatively high standard deviation of 0.020. These characteristics suggest a period of active fluctuations in interest rates. While the mean change is positive, indicating an overall upward trend, the high standard deviation underscores the volatility, indicating that rates can rise or fall when the system is in this state. Despite these fluctuations, there's a notable tendency to return to a steady state, reflecting the pull of the stickiness inherent in interest rate dynamics.

These two regimes thus represent distinct states of interest rate changes: regime 1 signifies periods of stability or slight decreases in rates, while regime 2 captures periods of growth with volatility. The transition probabilities between these states further highlight the tendency of the system to revert to the steady state, underscoring the stickiness of interest rate dynamics. In Figure 5, we can see the distinction of the periods in each regime. Here, we note that there are clear clusters of regime 2 followed by a long period of cluster 1. This shows further evidence of the fact that regime 2 represents short but sure interest rate changes. We can see some remarkable historical events within the plot, namely some volatile periods representing the aftermath of the financial crisis. Also, we see the first rate increase since the economic crisis around 2015. Finally, we observe a volatile period during the COVID pandemic in 2020.

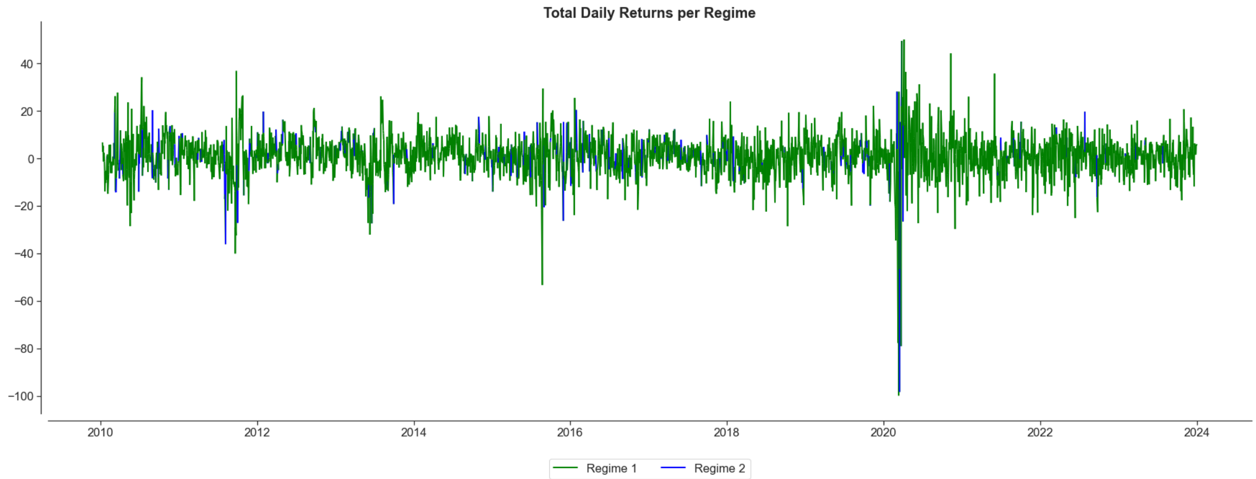

Figure 5: Aggregated daily returns with indication of the different regimes between 2010 and 2024

4.2 Factor model

Here, we explain the results following the factor analysis within different periods and sectors. In Section 4.2.1, we start by analysing all three models on the total aggregated return within each regime. After that, in Section 4.2.2, we delve into the differences between sectors. The tables below provide an overview of the regression results of the three models in both regimes. In every table, the β represents the coefficient value (or constant), SE shows the Standard Error and *pval* is the p-value of a t-test. The last is used to determine the significance level of the parameter, where values are bold if $pval \leq 0.1$.

4.2.1 Regime analysis

As stated above, we first look at the aggregated returns, the sum of the total value-weighted returns of all stocks. We use the Viterbi path, as specified in Section 3.1.3, to separate the return data into observations for two time series. In Table 5, we provide an extensive overview of the three models as can be seen in Formulas 16, 17 and 18 respectively.

We rely on several statistical measures to evaluate the adequacy of our regression models and their fit to the data. Namely, the adjusted R-squared, a commonly used metric that considers the number of predictors in the model, penalises the addition of unnecessary variables that do not improve the model's explanatory power. As Wooldridge (2010) explains, a higher adjusted R-squared indicates a better fit.

The F-statistic, on the other hand, assesses the overall significance of the model by comparing the explained variance to the unexplained variance. A larger F-statistic suggests a more significant model fit, indicating that the included predictors collectively impact the dependent variable Tiku (1967).

Additionally, log-likelihoods measure how well the model fits the observed data based on the probability of observing the data given the estimated model parameters. A higher log-likelihood indicates a better fit. These three tests, adjusted R-squared, F-statistics, and log-likelihoods, play complementary roles in assessing model fit and can help researchers make informed conclusions about their regression models' validity and explanatory power.

In Appendix A.4, we provide Table 24, which gives an overview of the criteria values, as mentioned earlier. Models 1 and 3 almost always form a significant fit, and Model 2 has trouble providing a decisive performance.

Aggregated Returns Table 5 presents the regression results of model 3 on the equally-weighted aggregated returns, with red underlined values indicating significant variables that display the opposite sign when moving from the first to the third model.

Table 5: Factor regression model 3 for two regimes over the aggregated returns from 2010 until 2024

Variable		α	$Mkt-Rf$	SMB	HML	MOM	$TERM$	DEF	$CFvol$	LEV	$INVG$	PE
R1	β	**-4.12**	**5.16**	**2.20**	**2.04**	0.11	**0.47**	**1.28**	0.68	0.05	**1.93**	0.00
	SE	**1.51**	**0.14**	**0.34**	**0.24**	0.19	**0.17**	**0.35**	1.47	0.12	**0.53**	0.01
	Pval	**0.01**	**0.00**	**0.00**	**0.00**	0.57	**0.01**	**0.00**	0.65	0.69	**0.00**	0.60
R2	β	**-11.65**	**5.86**	1.14	0.84	**2.38**	0.58	**3.19**	9.48	**0.83**	1.27	-0.00
	SE	**6.30**	**0.66**	1.56	1.25	**0.87**	0.73	**1.47**	6.56	**0.49**	2.22	0.03
	Pval	**0.07**	**0.00**	0.47	0.50	**0.01**	0.43	**0.03**	0.15	**0.09**	0.57	0.99

* The bold coefficients indicate a value which is significant with at least pval $\leq$ 0.1

** Underlined coefficients represent values that have opposite significance levels between the models.
E.g. if a coefficient is significant in model 3 but not in 1 or 2, it will be underlined, and vice versa.

In regime 1, the positive and significant coefficients for $Mkt-Rf$ (5.16), SMB (2.20), and HML (2.04) in both models indicate that market risk, company size, and value factors are vital in shaping infrastructure returns in this regime. Here, we note that the positive value for SMB indicates that, on average, infrastructure assets behave like small-cap stocks. In contrast, the initial belief is that infrastructure is a large-cap stock. The significant coefficient for DEF (1.28) suggests that default risk plays an important role. Notably, the coefficient for $Mkt - Rf$ is larger than those for SMB and HML, indicating that market risk might be the most influential factor in this regime.

Contrastingly, regime 2 paints a different picture. None of the variables are significant in model 2, implying that these factors may not hold as much relevance in this regime. However, in model 3, leverage (LEV) becomes significant (0.83) at the 10% level, suggesting that the financial structure of firms becomes an essential driver of infrastructure returns in this regime.

Interestingly, the coefficient for PE is significant and positive (0.00) in regime 1 under model 3, suggesting that firms with higher price-earnings ratios tend to perform better in this regime. However, it is worth noting that this relationship is reversed in model 2, highlighting the complexity of the relationship between the price-earnings ratio and infrastructure returns.

Furthermore, while the cash flow volatility ($CFvol$) is significant in model 2 for regime 1, it loses its significance in model 3. This could suggest that while cash flow volatility is essential in a more stable economic environment, its influence may be overshadowed by other factors in different financial conditions.

These results underscore the multifaceted nature of infrastructure returns and the importance of considering different economic regimes when analysing them. The values provided by the regime-switching model offer more nuanced insights, helping investors better understand the dynamics at play and make more informed investment decisions in the infrastructure sector.

4.2.2 Sector analysis

This section will discuss the results of our regressions per sector. By doing this, we better understand the variables actively driving returns within that sector and, hopefully, gain insight into the differences between the regimes.

Airport Here, we will discuss the results of our analysis on the Airport sector, as shown in Table 6.

Table 6: Factor regression model 3 for two regimes over the simple return in the Airport sector from 2010 until 2024

Variable		α	$Mkt-Rf$	SMB	HML	MOM	$TERM$	DEF	$CFvol$	LEV	$INVG$	PE
R1	β	**1.10**	**0.47**	**0.35**	**0.20**	0.01	-0.04	**-0.21**	0.62	-0.03	-0.09	**-0.00**
	SE	**0.29**	**0.03**	**0.07**	**0.05**	0.04	0.03	**0.07**	0.29	0.02	0.10	**0.00**
	Pval	**0.00**	**0.00**	**0.00**	**0.00**	0.77	0.21	**0.00**	0.03	0.22	0.40	**0.00**
R2	β	0.71	**0.34**	0.27	0.26	-0.04	-0.13	-0.17	1.27	0.00	-0.72	0.00
	SE	1.25	**0.13**	0.31	0.25	0.17	0.15	0.29	1.30	0.10	0.44	0.01
	Pval	0.57	**0.01**	0.38	0.30	0.82	0.38	0.56	0.33	0.97	0.11	0.98

* The bold coefficients indicate a value which is significant with at least pval ≤ 0.1
** Underlined coefficients represent values that have opposite significance levels between the models.
E.g. if a coefficient is significant in model 3 but not in 1 or 2, it will be underlined, and vice versa.
*** Variables with an asterisk (*) in the header have a significant difference in loadings between regimes

For the Airport sector, in regime 1, which represents a stable economy with little to no movements in interest rates, the returns are positively influenced by market risk, company size, and value factors, as indicated by the significant and positive coefficients for $Mkt - Rf$ (0.47), SMB (0.35), and HML (0.20). This suggests that larger companies with higher Book-to-Market ratios in the Airport sector tend to offer higher returns, which align with the overall market. This means that airports also show different characteristics for size, as expected. However, the significant and negative coefficient for DEF (-0.21) implies that lower default risk could potentially erode returns. Going against popular beliefs that higher returns should go hand in hand with higher risk. Furthermore, as DEF is underlined and significant, its significance does not hold when considering other models. This could suggest that the impact of default risk on the Airport sector's returns is more nuanced and potentially influenced by other factors captured in different models. This also holds for PE but with a substantially smaller loading.

However, in regime 2, $Mkt - Rf$ (0.34) is the only significant coefficient. This suggests that during periods of interest rate volatility, market risk becomes a critical concern for returns in the Airport sector. The shift in significance from multiple factors in regime 1 to primarily default risk in regime 2 indicates that the sector's returns become less diversified and more sensitive to default risk when interest rates are volatile.

The underlined coefficient for $CFvol$ in both regimes indicates that while cash flow volatility is significant under model 2, its influence diminishes under model 3. This could suggest that the impact of cash flow volatility on the Airport sector's returns may be contingent on the broader economic regime or other factors included in the different models. Also, the PE coefficient in regime 1 is significant and underlined, indicating its significance changes between models. This suggests that the impact of the price-earnings ratio on returns is model-dependent in this regime, highlighting the complexity of this relationship.

Looking at Table 40 in Appendix A.4, we notice no significant difference between the loadings in regime 1 and the same loadings in regime 2. This could indicate that even though there is less evidence in regime 2, the variables still drive as much return as in regime 1, but due to the short sample size, the estimator's precision is lost.

Communication Below in Table 7, we find the results of the factor model on the simple daily returns in the Communications industry.

Table 7: Factor regression model 3 for two regimes over the simple returns in the Communication sector from 2010 until 2024

Variable		α^*	$Mkt-Rf$	SMB	HML	MOM	$TERM$	DEF^*	$CFvol$	LEV	$INVG$	PE
R1	β	0.36	**0.26**	0.20	0.10	**0.17**	-0.07	**0.31**	**-1.57**	-0.04	**-0.40**	-0.00
	SE	0.62	**0.06**	0.14	0.10	**0.08**	0.07	**0.14**	**0.61**	0.05	**0.22**	0.00
	Pval	0.57	**0.00**	0.15	0.31	**0.02**	0.33	**0.03**	**0.01**	0.39	**0.07**	0.17
R2	β	**-6.75**	0.05	0.03	-0.06	0.42	-0.06	**2.39**	-0.51	0.26	0.18	0.01
	SE	**2.48**	0.26	0.62	0.49	0.34	0.29	**0.58**	2.58	0.19	0.87	0.01
	Pval	**0.01**	0.84	0.96	0.90	0.22	0.83	**0.00**	0.84	0.17	0.84	0.60

* The bold coefficients indicate a value which is significant with at least pval ≤ 0.1
** Underlined coefficients represent values that have opposite significance levels between the models.
E.g. if a coefficient is significant in model 3 but not in 1 or 2, it will be underlined, and vice versa.
*** Variables with an asterisk (*) in the header have a significant difference in loadings between regimes

In regime 1, the significant and positive coefficients for $Mkt-Rf$ (0.26), MOM (0.17), and DEF (0.31 suggest that market risk, momentum, and default risk are significant contributors to the returns in the Communication sector during this regime. However, the negative and significant coefficient for $CFvol$ (-1.57) and $INVG$ (-0.40) suggests that higher cash flow volatility and higher investment growth can negatively impact returns in this regime. For cashflow volatility, this is expected, as the inelastic demand should generate higher returns when cashflow is stable. However, for investment growth, we believe that the renewal of physical assets should result in higher returns in the long term.

Contrastingly, in regime 2, we observe a shift in the relation between the factors and return. The coefficient for $Mkt - Rf$ is insignificant, indicating that the sector's returns are less sensitive to overall market movements in this regime. However, α (-6.75) is significant, showing a general negative return unexplained by the present factors. Interestingly, the coefficient for DEF remains significant and positive (2.39), suggesting that default risk plays a vital role in shaping returns in the Communication sector in this regime, potentially even more so than in regime 1. Employing the Wald test, we can conclude that there is a significant increase in the magnitude of the DEF loading, which means that in volatile interest rates, there is a bigger premium for risk.

The significance of these factors changes between the two regimes indicates that the Communication sector's sensitivity to these factors is indeed affected by changes in interest rates. Specifically, when interest rates are stable, the sector's returns appear more sensitive to market risk, momentum, and cash flow volatility. However, there does not appear to be a significant difference in the magnitude of the coefficients between the regimes.

Datacenters Next, we look at the results of our analysis in the Datacenters industry. This is shown in Table 8.

Table 8: Factor regression model 3 for two regimes over the simple returns in the Datacenters sector from 2010 until 2024

Variable		α	$Mkt-Rf$	SMB	HML	MOM	$TERM$	DEF	$CFvol$	LEV	$INVG$	PE
R1	β	-0.07	**0.77**	**-0.45**	**-0.46**	**-0.05**	0.02	-0.00	0.04	0.01	0.07	0.00
	SE	0.21	**0.02**	**0.05**	**0.03**	**0.03**	0.02	0.05	0.20	0.02	0.07	0.00
	Pval	0.74	**0.00**	**0.00**	**0.00**	**0.07**	0.31	0.97	0.86	0.59	0.35	0.83
R2	β	-0.16	**0.82**	-0.13	**-0.32**	0.01	0.11	0.02	0.41	0.04	0.22	-0.00
	SE	0.90	**0.09**	0.22	**0.18**	0.12	0.10	0.21	0.93	0.07	0.32	0.00
	Pval	0.86	**0.00**	0.56	**0.08**	0.92	0.30	0.94	0.66	0.60	0.48	0.59

* The bold coefficients indicate a value which is significant with at least pval ≤ 0.1
** Underlined coefficients represent values that have opposite significance levels between the models.
E.g. if a coefficient is significant in model 3 but not in 1 or 2, it will be underlined, and vice versa.
*** Variables with an asterisk (*) in the header have a significant difference in loadings between regimes

In regime 1, which corresponds to a stable interest rate environment, the significant and positive coefficient for $Mkt-Rf$ (0.77) suggests that market risk is a crucial driver of returns in the Datacenters sector. However, the significant and negative coefficients for SMB (-0.45), HML (-0.46), and MOM (-0.05) indicate that smaller company sizes, lower book-to-market ratios, and negative momentum are associated with higher returns in this sector during this regime. These last observations are interesting, as we believe that large-cap value stocks would outperform small-cap growth stocks.

In contrast, during the more volatile regime 2, the market risk remains a significant driver of returns, as indicated by the significant and positive coefficient for $Mkt-Rf$ (0.82). Similar to regime 1, the significant and negative coefficient for HML (-0.32) suggests that lower book-to-market ratios are again associated with higher returns in this regime.

These shifts in sensitivity between the two regimes suggest that the Datacenters sector's returns are sensitive to changes in interest rates. Specifically, in a stable interest rate environment (regime 1), returns in this sector are influenced by various factors, including market risk, company size, value, and momentum. However, in a volatile interest rate environment (regime 2), market risk and value factors become the primary drivers of returns, with company size and momentum factors losing significance. Despite a shift in significant drivers, the loadings are not significantly different. This indicates that the driver's importance might differ, but the magnitude does not vary as an effect of interest rates.

Electric Utility In Table 9, we find the factor analysis results on the simple daily returns in the Electric Utility industry.

Table 9: Factor regression model 3 for two regimes over the simple returns in the Electric Utility sector from 2010 until 2024

Variable		α	$Mkt-Rf$	SMB	HML	MOM	$TERM$	DEF	$CFvol$	LEV	$INVG$	PE
R1	β	-0.24	**0.35**	**0.32**	**0.18**	0.03	0.01	0.06	**-0.38**	<u>0.01</u>	0.05	0.00
	SE	0.22	**0.02**	**0.05**	**0.04**	0.03	0.03	0.05	**0.22**	0.02	0.08	0.00
	Pval	0.28	**0.00**	**0.00**	**0.00**	0.36	0.81	0.22	**0.08**	0.55	0.54	0.29
R2	β	0.18	**0.50**	0.17	0.08	0.05	-0.05	-0.00	-1.17	-0.01	-0.28	0.00
	SE	0.86	**0.09**	0.21	0.17	0.12	0.10	0.20	0.90	0.07	0.30	0.01
	Pval	0.83	**0.00**	0.43	0.64	0.66	0.60	0.99	0.19	0.88	0.37	0.88

* The bold coefficients indicate a value which is significant with at least pval $\leq$ 0.1
** Underlined coefficients represent values that have opposite significance levels between the models.
E.g. if a coefficient is significant in model 3 but not in 1 or 2, it will be underlined, and vice versa.
*** Variables with an asterisk (*) in the header have a significant difference in loadings between regimes

Beginning with regime 1, we observe the significant and positive coefficients for $Mkt-Rf$ (0.35), SMB (0.32), and HML (0.18), which suggests that market risk, company size, and value factors are key contributors to returns in the Electric Utility sector. Meanwhile, the significant and negative coefficient for $CFvol$ (-0.38) suggests that higher cash flow volatility may negatively impact returns in this sector during this regime. The underlined but non-significant coefficient for LEV (0.01) suggests that the significance of leverage in this sector's returns may vary depending on the model used.

For regime 2, however, only the $Mkt-Rf$ coefficient remains significant (0.50), indicating that market risk continues to be a key driver of returns in the Electric Utility sector. However, all other factors, including SMB, HML, and $CFvol$, lose their significance, suggesting that their influence on returns is diminished in this volatile interest rate environment.

Analysing the shift between the two regimes, we see that the Electric Utility sector's returns are sensitive to changes in interest rates. Specifically, in a stable interest rate environment, returns in this sector are influenced by various factors, including market risk, company size, and value. At the same time, higher cash flow volatility tends to reduce returns. However, in a volatile interest rate environment, market risk emerges as the primary driver of returns, while the influence of other factors seems to vanish.

Again, we do not find a significant drop or increase in the magnitude of the drivers when transitioning to another regime. However, we can predict the loadings more precisely in the stable regime.

Gas Utility Table 10 summarises the results for the Gas Utility sector.

Table 10: Factor regression model 3 for two regimes over the simple returns in the Gas Utility sector from 2010 until 2024

Variable		α^*	$Mkt-Rf^*$	SMB	HML	MOM^*	$TERM$	DEF	$CFvol$	LEV	$INVG$	PE^*
R1	β	0.17	**0.37**	**0.56**	**0.27**	-0.07	0.02	-0.06	0.85	**-0.12**	**0.42**	0.00
	SE	0.57	**0.05**	**0.13**	**0.09**	0.07	0.07	0.13	0.56	**0.05**	**0.20**	0.00
	Pval	0.77	**0.00**	**0.00**	**0.00**	0.33	0.76	0.66	0.13	**0.01**	**0.04**	0.19
R2	β	**7.18**	**0.83**	0.74	-0.32	**0.71**	**-0.57**	**-1.08**	**10.79**	-0.14	-0.86	**-0.06**
	SE	**2.55**	**0.27**	0.63	0.51	**0.35**	**0.30**	**0.60**	**2.65**	0.20	0.90	**0.01**
	Pval	**0.01**	**0.00**	0.24	0.53	**0.04**	**0.06**	**0.07**	**0.00**	0.49	0.34	**0.00**

* The bold coefficients indicate a value which is significant with at least pval ≤ 0.1
** Underlined coefficients represent values that have opposite significance levels between the models.
E.g. if a coefficient is significant in model 3 but not in 1 or 2, it will be underlined, and vice versa.
*** Variables with an asterisk (*) in the header have a significant difference in loadings between regimes

In the first regime, the significant coefficients for $Mkt-Rf$ (0.37), SMB (0.56), HML (0.27) and $INVG$ (0.42) indicate that the market risk, company size, value, cash flow and investment growth factors are significant contributors to returns in this sector. Similar to before, the positive value of the SMB factor indicates that Gas Utility stocks move more like small-caps than large-caps. Also, contradicting financial beliefs is the significant but negative coefficient for LEV (-0.12), which suggests that higher leverage levels may lead to lower returns in the Gas Utility sector during this regime.

In the second regime, the significant positive constant term (α: 7.18) implies that the Electric Utility sector would have a positive return even when all factors are zero. Regarding factors, market risk remains significant ($Mkt-Rf$: 0.83), suggesting its continued influence on returns. The significance of momentum (MOM: 0.71) and cash flow volatility ($CFvol$: 10.79) indicates that these factors become more influential when interest rates are volatile. Here, the positive loading on $CFvol$ shows that there is a premium for more volatile cash flows. Indicating that the sector behaves more like general markets, where stability is not always a requirement to attract investors. On the other hand, term spread ($TERM$: -0.5675) and default risk (DEF: -1.08) have significant negative coefficients, indicating that higher term spread and default risk may negatively impact returns in this regime. This is interesting as we would expect a positive effect on these variables.

These findings demonstrate that the Gas Utility sector's driving factors shift as interest rates move from stable to volatile. Notably, the sector's returns appear to be influenced by a broader range of factors in a volatile interest rate environment, including market risk, momentum, term spread, default risk, cash flow volatility, and the price-earnings ratio. Notice that together with the constant α, the loadings on $Mkr-Rf$, MOM and PE change significantly when transitioning to the other regime. This shows that the sector becomes more influenced by these variables, indicating that past winners are believed to still perform well in a higher volatility regime than the losers. Also, stocks with a high price-earnings ratio in these sectors are expected to perform worse in regime 2. Lastly, the effect of market risk is increased in regime 2 compared to regime 1, which indicates a higher premium for market exposure for gas utility companies.

Multi Utility In this paragraph, we discuss the results for the regressions on simple returns in the Multi Utility sector.

Table 11: Factor regression model 3 for two regimes over the simple returns in the Multi Utility sector from 2010 until 2024

	Variable	α^*	$Mkt-Rf^*$	SMB	HML	MOM	$TERM$	DEF	$CFvol$	LEV	$INVG$	PE^*
R1	β	0.05	**0.59**	**-0.56**	**0.23**	-0.02	0.02	-0.02	-0.17	-0.01	0.00	0.00
	SE	0.16	**0.02**	**0.04**	**0.03**	0.02	0.02	0.04	0.16	0.01	0.06	0.00
	Pval	0.74	**0.00**	**0.00**	**0.00**	0.42	0.39	0.62	0.28	0.55	0.99	0.79
R2	β	**-1.49**	**0.80**	**-0.41**	0.22	-0.04	0.12	0.22	0.12	0.08	0.36	0.01
	SE	**0.71**	**0.07**	**0.18**	0.14	0.10	0.08	0.17	0.74	0.06	0.25	0.00
	Pval	**0.04**	**0.00**	**0.02**	0.12	0.71	0.13	0.18	0.87	0.12	0.15	0.19

* The bold coefficients indicate a value which is significant with at least pval ≤ 0.1
** Underlined coefficients represent values that have opposite significance levels between the models.
E.g. if a coefficient is significant in model 3 but not in 1 or 2, it will be underlined, and vice versa.
*** Variables with an asterisk (*) in the header have a significant difference in loadings between regimes

In the first regime, the returns of the Multi Utility sector are positively associated with market risk and value factors, as indicated by the significant and positive coefficients for $Mkt-Rf$ (0.59) and HML (0.23). However, the significant and negative coefficient for SMB (-0.56) suggests that bigger company sizes may lead to higher returns in this sector during this regime. The non-significant constant term (α: 0.05) indicates that when all explanatory factors are zero, the expected return in the Multi Utility sector is not significantly different from zero.

Transitioning to the second regime, market risk maintains its significance ($Mkt - Rf$: 0.80), implying its continued influence on returns. Contrarily, the constant term becomes significant (α: -1.49), indicating that even if all factors are zero, the 'Multi Utility' sector yields a negative return. Additionally, the significance of the SMB coefficient persists, but its negativity decreases (-0.41), suggesting a reduced impact of company size on returns in this regime.

These findings indicate that the Multi Utility sector's returns exhibit different sensitivities to various factors between the two regimes. In particular, during the first regime, the sector's returns seem to be influenced by broader factors, including market risk, company size, and value factors. However, in the second regime, market risk appears to be the dominant driver of returns, while the influence of company size on returns diminishes.

When transitioning from regime 1 to regime 2, the influence of company size on returns diminishes slightly but not significantly, while market risk continues to play a significantly larger role. This suggests that in a volatile interest rate environment, market risk becomes the dominant driver of returns in the Multi Utility sector while the impact of company size is reduced.

Other Here, we discuss the analysis of sector Other as seen in Table 12.

Table 12: Factor regression model 3 for two regimes over the simple returns in the Other sector from 2010 until 2024

Variable		α^*	$Mkt - Rf$	SMB	HML^*	MOM	$TERM$	DEF^*	$CFvol^*$	LEV	$INVG$	PE
R1	β	**-4.61**	**0.05**	**0.16**	**-0.00**	**-0.06**	**0.27**	**0.98**	**1.94**	0.01	**1.44**	**0.01**
	SE	**0.29**	**0.03**	**0.07**	**0.05**	**0.04**	**0.03**	**0.07**	**0.29**	0.02	**0.10**	**0.00**
	Pval	**0.00**	**0.08**	**0.01**	**0.95**	**0.09**	**0.00**	**0.00**	**0.00**	0.70	**0.00**	**0.00**
R2	β	**-2.17**	**0.11**	**0.36**	**0.39**	-0.12	**0.37**	0.08	-0.58	-0.02	**1.31**	**0.02**
	SE	**1.05**	**0.11**	**0.26**	**0.21**	0.14	**0.12**	0.25	1.10	0.08	**0.37**	**0.01**
	Pval	**0.04**	**0.33**	**0.17**	**0.06**	0.42	**0.00**	0.18	0.31	0.14	**0.07**	**0.00**

* The bold coefficients indicate a value which is significant with at least pval $\leq$ 0.1
** Underlined coefficients represent values that have opposite significance levels between the models.
E.g. if a coefficient is significant in model 3 but not in 1 or 2, it will be underlined, and vice versa.
*** Variables with an asterisk (*) in the header have a significant difference in loadings between regimes

Under regime 1, corresponding to a stable interest rate environment, the significant negative constant term (α: -4.61) suggests that when all explanatory factors are zero, the expected return in the Other sector is negative. Among the factors, market risk ($Mkt - Rf$: 0.05), company size (SMB: 0.16), term spread ($TERM$: 0.27), default risk (DEF: 0.98), cash flow volatility ($CFvol$: 1.94), the level of investment ($INVG$: 1.44), and the price-earnings ratio (PE: 0.01) all have significant coefficients, suggesting that these factors are important drivers of returns in the Other sector during this regime. Remarkably, the signs of most loadings are not directly in line with popular beliefs. The positive values for SMB, $CFvol$, and PE mean this sector has reversed behaviour. Additionally, the negative coefficient for momentum (MOM: -0.06) implies that negative momentum may lead to higher returns in this sector during this regime.

In regime 2, representing a period of volatile interest rates, the significant and negative constant term (α: -2.17) indicates that even when all factors are zero, there would be a negative return in the Other sector. The significant positive coefficients for market risk ($Mkt - Rf$: 0.11), company size (SMB: 0.36), Book-to-Market (HML: 0.39), term spread

($TERM$: 0.37), investment growth ($INVG$: 1.31) and price-earnings (PE: 0.02) suggest that these factors are positively associated with returns in this sector during this regime. Like regime 1, this sector shows more of a growth perspective than a value sector. In contrast to regime 1, cash flow volatility and default risk lose significance, suggesting that these factors are less influential in this volatile interest rate environment.

These shifts in the significant factors between the two regimes suggest that the Other sector's returns are sensitive to changes in interest rates. Specifically, the constant, Book-to-Market, default premium and cash flow volatility change significantly when transitioning between states. Most noticeable is that HML shows a reverse effect in the second regime. In volatile times, the robustness of a growth firm is expected to drive up returns. A final remark is that there are many significant drivers (especially in regime 1), which traces back to the fact that the Other sector contains firms of different characteristics.

Pipelines The results for the sector Pipelines are shown in Table 13 below.

Table 13: Factor regression model 3 for two regimes over the simple returns in the Pipelines sector from 2010 until 2024

Variable		α	$Mkt-Rf$	SMB	HML	MOM	$TERM^*$	DEF	$CFvol$	LEV	$INVG$	PE
R1	β	-0.12	**0.85**	**-0.13**	**0.60**	**-0.07**	0.03	0.04	-0.04	-0.02	**0.12**	0.00
	SE	0.19	**0.02**	**0.04**	**0.03**	**0.02**	0.02	0.04	0.18	0.02	**0.07**	0.00
	Pval	0.54	**0.00**	**0.00**	**0.00**	**0.00**	0.20	0.34	0.83	0.26	**0.08**	0.65
R2	β	-0.19	**0.86**	-0.18	**0.65**	0.07	**0.24**	-0.23	-0.77	0.08	**0.46**	0.00
	SE	0.72	**0.08**	0.18	**0.14**	0.10	**0.08**	0.17	0.75	0.06	**0.25**	0.00
	Pval	0.79	**0.00**	0.31	**0.00**	0.47	**0.00**	0.18	0.31	0.14	**0.07**	0.95

* The bold coefficients indicate a value which is significant with at least pval ≤ 0.1
** Underlined coefficients represent values that have opposite significance levels between the models.
E.g. if a coefficient is significant in model 3 but not in 1 or 2, it will be underlined, and vice versa.
*** Variables with an asterisk (*) in the header have a significant difference in loadings between regimes

In regime 1, corresponding to a stable interest rate environment, the significant and positive coefficients for $Mkt - Rf$ (0.85), HML (0.60), and $INVG$ (0.12) suggest that market risk, Book-to-Market, and the level of investment are major contributors to returns in the Pipelines sector. However, the significant and negative coefficients for SMB (-0.13) and MOM (-0.07) indicate that smaller company sizes and negative momentum may lead to higher returns in this sector during this regime. This implies that stocks outperforming earlier are expected to underperform in the future and vice versa. Also, the loading on SMB suggests that larger firms are believed to perform better than smaller firms, which might indicate the economies of scale in this sector.

In regime 2, representing a period of volatile interest rates, the significant and positive coefficients for $Mkt - Rf$ (0.86) and HML (0.65) suggest that market risk and value factors continue to be key drivers of returns in the Pipelines sector. The significant and positive coefficient for $TERM$ (0.24) indicates that term spread is relevant in influencing returns in this sector during this regime. However, the significant and positive coefficient for $INVG$ (0.46), underlined in both regimes, suggests that the influence of the level of investment on returns may differ depending on the model used.

These shifts in the significant factors between the two regimes suggest that the Pipelines sector's returns are some what sensitive to changes in interest rates. Specifically, returns in this sector are influenced by market risk, company size, value factors, momentum, and the level of investment in a stable interest rate environment (regime 1). However, when interest rates become volatile (regime 2), market risk, value factors, and term spread become the primary drivers of returns. In contrast, the influence of company size and momentum on returns diminishes. However, the magnitude of the loadings is statistically the same except for the term premium. Within the second regime, the precision of the estimates is lost, resulting in larger standard errors and, with that, a non-significant driver.

Port Here, in Table 14, we delve into the factor analysis results for the Port industry.

Table 14: Factor regression model 3 for two regimes over the simple returns in the Port sector from 2010 until 2024

Variable		α	$Mkt-Rf$	SMB	HML	MOM^{*}	$TERM$	DEF	$CFvol$	LEV	$INVG$	PE
R1	β	-0.16	**0.46**	**0.52**	**0.22**	**0.07**	0.05	-0.03	-0.26	**0.06**	-0.03	0.00
	SE	0.30	**0.03**	**0.07**	**0.05**	**0.04**	0.03	0.07	0.29	**0.02**	0.11	0.00
	Pval	0.59	**0.00**	**0.00**	**0.00**	**0.05**	0.18	0.70	0.38	**0.01**	0.74	0.70
R2	β	-1.31	**0.36**	0.24	-0.02	**0.39**	0.13	0.07	0.87	0.12	0.26	0.00
	SE	1.17	**0.12**	0.29	0.23	**0.16**	0.14	0.27	1.22	0.09	0.41	0.01
	Pval	0.26	**0.00**	0.41	0.93	**0.02**	0.34	0.78	0.48	0.17	0.54	0.51

* The bold coefficients indicate a value which is significant with at least pval ≤ 0.1
** Underlined coefficients represent values that have opposite significance levels between the models.
E.g. if a coefficient is significant in model 3 but not in 1 or 2, it will be underlined, and vice versa.
*** Variables with an asterisk (*) in the header have a significant difference in loadings between regimes

Looking at regime 1, we see several factors contributing to the Port sector's returns. Market risk ($Mkt - Rf$: 0.46), company size (SMB: 0.52), Book-to-Market (HML: 0.22), momentum (MOM: 0.07) and (LEV: 0.06) all show significant positive coefficients, indicating that these elements are positively linked with returns in this sector during stable interest rates. Interestingly, the leverage also displays a significant positive coefficient in model 2, suggesting that higher leverage levels might enhance returns in this sector in a stable rate environment.

Transitioning to regime 2, market risk ($Mkt - Rf$: 0.36) and momentum (MOM: 0.39) continue to be strong influencers of returns, as suggested by their significant positive coefficients. This hints that market risk and momentum are pivotal in shaping the Port sector's returns despite interest rate volatility.

These findings demonstrate that the returns in the Port sector exhibit changing loadings for different factors as interest rates shift from stable to volatile. In a stable interest rate environment, returns are influenced by diverse factors, including market risk, company size, value factors, momentum, and leverage. However, during the second regime, the sector's returns appear primarily driven by market risk and momentum, whereas the latter plays a significantly larger role in the more volatile regime.

Railroads Table 15 provides the regression results for the Railroad sector

Table 15: Factor regression model 3 for two regimes over the simple returns in the Railroads sector from 2010 until 2024

Variable		α	$Mkt - Rf$	SMB^*	HML	MOM^*	$TERM$	DEF	$CFvol^*$	LEV	$INVG$	PE^*
R1	β	0.27	**0.39**	**0.80**	**0.34**	-0.01	0.02	-0.08	-0.21	0.02	0.01	-0.00
	SE	0.27	**0.03**	**0.06**	**0.04**	0.03	0.03	0.06	0.26	0.02	0.09	0.00
	Pval	0.32	**0.00**	**0.00**	**0.00**	0.65	0.44	0.18	0.41	0.30	0.87	0.12
R2	β	-1.54	**0.48**	0.22	0.34	0.26	0.18	0.18	-1.18	0.07	0.12	**0.01**
	SE	1.16	**0.12**	0.29	0.23	0.16	0.14	0.27	1.21	0.09	0.41	**0.01**
	Pval	0.19	**0.00**	0.44	0.14	0.10	0.17	0.51	0.33	0.43	0.78	**0.07**

* The bold coefficients indicate a value which is significant with at least pval ≤ 0.1
** Underlined coefficients represent values that have opposite significance levels between the models.
E.g. if a coefficient is significant in model 3 but not in 1 or 2, it will be underlined, and vice versa.
*** Variables with an asterisk (*) in the header have a significant difference in loadings between regimes

In the first regime, the significant coefficients for $Mkt - Rf$ (0.39), SMB (0.80) and HML (0.34) indicate that market risk, company size and Book-to-Market are significant contributors to returns in the Railroads sector during this regime. These findings suggest that smaller or value firms have an advantage over bigger or growth companies, which is interesting as infrastructure is a predominantly large-cap industry.

In the second regime, the coefficient for $Mkt - Rf$ (0.48) remains significant, implying that market risk continues to influence returns in the Railroads sector. Furthermore, for model 3 only, the price-earnings ratio (PE: 0.01) is significant. From this, we can infer that when interest rates change, PE becomes a vital driving power source.

After a transition to regime 2, firm size loses significance with a substantial drop in the loading. However, for HML, an increased standard error results in an insignificant driver. The reverse is seen for PE as in a volatile regime; the price-earnings ratio is significant in predicting the returns and significantly different from the loading in the first regime. It is worth noting that even though MOM is just above the significant threshold, there is a significant difference between the two regimes, hinting at a better performance for past winners in a volatile regime.

Toll Roads Next, we visualise the factor analysis results in the toll roads industry in Table 16.

Table 16: Factor regression model 3 for two regimes over the simple returns in the Toll Roads sector from 2010 until 2024

Variable		α^*	$Mkt-Rf$	SMB	HML	MOM^*	$TERM$	DEF^*	$CFvol$	LEV	$INVG$	PE^*
R1	β	-0.83	**0.28**	**0.43**	0.13	0.07	**0.16**	**0.23**	-0.12	**0.15**	**0.34**	**-0.01**
	SE	0.53	**0.05**	**0.12**	0.08	0.06	**0.06**	**0.12**	0.52	**0.04**	**0.19**	**0.00**
	Pval	0.12	**0.00**	**0.00**	0.12	0.31	**0.01**	**0.06**	0.81	**0.00**	**0.06**	**0.04**
R2	β	**-5.71**	**0.55**	0.10	-0.35	**0.67**	0.33	**1.42**	-0.28	0.26	0.36	0.02
	SE	**2.30**	**0.24**	0.57	0.46	**0.32**	0.27	**0.54**	2.39	0.18	0.81	0.01
	Pval	**0.01**	**0.02**	0.86	0.44	**0.04**	0.22	**0.01**	0.91	0.14	0.66	0.13

* The bold coefficients indicate a value which is significant with at least pval ≤ 0.1

** Underlined coefficients represent values that have opposite significance levels between the models. E.g. if a coefficient is significant in model 3 but not in 1 or 2, it will be underlined, and vice versa.

*** Variables with an asterisk (*) in the header have a significant difference in loadings between regimes

In the first regime, the significant coefficients for $Mkt-Rf$ (0.28), SMB (0.43), $TERM$ (0.16), DEF (0.23), LEV (0.15) and $INVG$ (0.34) indicate that market risk, company size, term spread, default risk, leverage effects and investment growth are major contributors to returns in the Toll Roads sector during this regime. Furthermore, PE (-0.01) shows a significant negative loading, indicating that firms with a lower price-earnings ratio could have a higher return.

In the second regime, the significant negative constant term (α: -5.71) implies that even when all factors are zero, there would be a negative return in the Toll Roads sector. The significant coefficient for $Mkt-Rf$ (0.55) suggests that market risk continues to influence returns in this sector. Additionally, the significant positive coefficient for DEF (1.42) and MOM (0.67) indicates that default risk becomes a driver of returns in the second regime.

The differences in the coefficients between regimes 1 and 2 indicate a potential sensitivity of the Toll Roads sector to changes in interest rates. In regime 1, company size, term spread, leverage, investment growth, and price-earnings have significant coefficients, which are not driving returns in regime 2. This suggests that the sector's characteristics change during a transition of regimes.

However, in regime 2, default risk remains a significant factor and becomes an even bigger driver of returns. Compared to regime 1, we notice that α and MOM now show significant explanatory power. This suggests that during volatile interest rates, market risk, momentum and default risk become the primary drivers of returns in the Toll Roads sector; with these changes in loadings, as well as the varying significance of the drivers, we note that the Toll Roads sector's returns are sensitive to changes in interest rates.

Water Utility Lastly, we discuss the factor analysis results for the water utility sector in Table 17.

Table 17: Factor regression model 3 for two regimes over the simple returns in the Water Utility sector from 2010 until 2024

Variable		α	$Mkt - Rf$	SMB	HML	MOM	$TERM$	DEF	$CFvol$	LEV	$INVG$	PE
R1	β	-0.04	**0.32**	0.01	**0.23**	0.03	-0.02	0.05	-0.02	0.01	-0.00	-0.00
	SE	0.25	**0.02**	0.06	**0.04**	0.03	0.03	0.06	0.25	0.02	0.09	0.00
	Pval	0.89	**0.00**	0.92	**0.00**	0.31	0.49	0.39	0.94	0.80	0.97	0.65
R2	β	<u>-0.41</u>	0.17	-0.28	-0.02	-0.02	-0.10	<u>0.29</u>	0.52	0.06	-0.15	-0.01
	SE	1.00	0.10	0.25	0.20	0.14	0.12	0.23	1.04	0.08	0.35	0.01
	Pval	0.68	0.11	0.27	0.92	0.91	0.40	0.22	0.62	0.45	0.67	0.28

* The bold coefficients indicate a value which is significant with at least pval $\leq$ 0.1
** Underlined coefficients represent values that have opposite significance levels between the models.
E.g. if a coefficient is significant in model 3 but not in 1 or 2, it will be underlined, and vice versa.
*** Variables with an asterisk (*) in the header have a significant difference in loadings between regimes

The analysis of the Water Utility sector in regime 1 reveals that, during a stable economy, coefficients for $Mkt - Rf$ (0.32) and HML (0.23) are significantly driving returns. This indicates that market risk and value factors are significant contributors to returns. These findings suggest that market conditions and the relative value of assets play a crucial role in determining returns during stable interest rates.

However, in the second regime, the previously significant coefficient becomes non-significant, suggesting a reduced impact of market risk on returns. This indicates that the Water Utility sector becomes less sensitive to overall market movements during volatile interest rate periods. It is worth noting that the significance of the α and DEF coefficients in other models should be considered. Although insignificant in the presented model, they are significant in model 1. This suggests that the influence of these variables on returns varies depending on the inclusion of additional factors.

Furthermore, there is no significant difference in the magnitude of the loadings between the regimes. Combined with a low number of significant drivers, this suggests that the sector's predictability is low, and it exhibits minimal sensitivity to fluctuations in interest rates.

If we combine the findings of our regression models across all sectors, a few remarks will be exciting and help us answer our second subquestion. Namely, the constant term α significantly changes in all sectors where it is significant. This indicates that when the interest rates switch regimes, there is a substantial difference in the baseline or average market conditions. Furthermore, this could indicate a change in investors' sentiment regarding their risk premium or an underlying (non-)linear relation that could not be captured. The momentum factor is another variable that changes significantly in all models and drives substantial returns. In the Gas Utility, Port and Toll Roads sector, we see that in actively changing interest rates, the past winners should significantly outperform past losers. Finally, the default premium positively affects the Communications and Toll Roads sectors. Furthermore, its loading increases significantly after transitioning to the second regime. This provides further evidence that these sectors are considerably more risky in changing interest rates, as this has a positive sensitivity to the returns.

4.3 Comparison

This section will discuss the differences between the sectors in each regime. We want to create a clear picture of how each variable affects each sector by determining whether or not a coefficient in a particular sector is statistically different from the mean of the loadings across all sectors.

Table 18: Wald test statistics for the multivariate test of equal coefficients between sectors in regime 1

Sector	α	$Mkt - Rf$	SMB	HML	MOM	$TERM$	DEF	$CFvol$	LEV	$INVG$	PE
Airport	**23.60*** 0.01	4.58* 0.92	1.57* 1.00	0.01* 1.00	1.99 1.00	3.03 0.98	6.99* 0.73	7.03 0.72	2.60 0.99	4.22 0.94	2.31* 0.99
Communications	9.00 0.53	0.58* 1.00	0.29 1.00	0.03 1.00	0.22* 1.00	0.26 1.00	0.30* 1.00	8.20* 0.61	0.17 1.00	2.28* 0.99	0.04 1.00
Datacenters	0.10 1.00	**78.62*** 0.00	**21.09*** 0.02	**22.40*** 0.01	0.02* 1.00	0.38 1.00	0.03 1.00	4.24 0.94	0.17 1.00	0.65 1.00	0.13 1.00
Electric Utility	3.50 0.97	8.60* 0.57	6.70* 0.75	1.48* 1.00	0.14 1.00	0.39 1.00	0.07 1.00	5.34* 0.87	0.34 1.00	0.00 1.00	0.40 1.00
Gas Utility	5.73 0.84	0.33* 1.00	1.73* 1.00	0.04* 1.00	1.37 1.00	0.58 1.00	0.40 1.00	15.69 0.11	1.60* 1.00	1.10* 1.00	0.77 1.00
Multi Utility	1.08 1.00	**61.21*** 0.00	**64.45*** 0.00	7.78* 0.65	0.34 1.00	0.03 1.00	0.56 1.00	1.07 1.00	0.24 1.00	0.06 1.00	0.13 1.00
Other	**1671.90*** 0.00	0.92* 1.00	3.68* 0.96	0.39* 1.00	0.02* 1.00	8.65* 0.57	**90.39*** 0.00	**126.80*** 0.00	0.94 1.00	**139.80*** 0.00	0.55* 1.00
Pipelines	6.37 0.78	**73.09*** 0.00	8.51* 0.58	**31.99*** 0.00	4.71* 0.91	1.05 1.00	0.82 1.00	3.07 0.98	2.46 0.99	0.00* 1.00	1.82 1.00
Port	0.38 1.00	8.09* 0.62	11.03* 0.35	1.10* 1.00	0.00* 1.00	0.09 1.00	1.19 1.00	11.02 0.36	0.04* 1.00	1.06 1.00	0.37 1.00
Railroads	2.55 0.99	4.06* 0.94	**30.01*** 0.00	2.50* 0.99	2.05 1.00	1.28 1.00	4.62 0.92	5.42 0.86	1.31 0.99	1.34 0.99	1.76 0.99
Toll Roads	**31.65** 0.00	0.22* 1.00	1.24* 1.00	0.03 1.00	0.21 1.00	0.00* 1.00	0.32* 1.00	13.63 0.19	0.00* 1.00	1.57* 0.99	0.58* 1.00
Water Utility	0.95 1.00	5.66* 0.84	0.22 1.00	2.51* 0.99	0.06 1.00	0.49 1.00	0.00 1.00	0.00 1.00	0.22 1.00	0.23 1.00	0.28 1.00

* The bold coefficients indicate a value which is significant with at least pval ≤ 0.1

** Coefficients indicated with an asterisk (*) are significant drivers in the third factor model

We start by discussing the results of the Wald test in regime 1, which is visualised in Table 18 below. Here, we see that there is no significant difference between any sector for the variables MOM, $TERM$, LEV or PE. This means that, even though some variables are significant drivers of returns within some sectors, there is no sector where these variables have statistically more explanatory power. Analysing further, we observe that specific sectors exhibit significantly different variable loadings compared to the mean of those variables across all sectors. First, the Airport sector shows significant differences in the coefficient for α, indicating that it may have a distinct effect on returns that is not explained by any factor.

Next, the Datacenter's sector displays significant differences in the coefficients for the $Mkt-Rf$, SMB and HML variables. This indicates that the Datacenters sector may be more sensitive to market and size-related factors in the context of interest rate changes. The distinct variable loadings in this sector may reflect its unique characteristics and dependencies, potentially resulting in differing impacts on stock returns.

The same results are found in the Multi Utility sector (except for HML), where the market risk and size factor ensure that the sector differentiates itself from the rest. This relative outperformance can help decision-making by highlighting the over-sensitivity towards the factors.

Similarly, the Pipeline sector stands out with significant differences in the coefficients for the α, $Mkt-Rf$, SMB, and HML variables. This suggests that the Pipeline sector may be more susceptible to market and size-related factors, potentially making it more vulnerable to interest rate changes than other sectors. These significant differences in variable loadings imply that interest rate fluctuations may have a more pronounced effect on borrowing costs, profitability, and investment decisions within the Pipeline sector.

Interestingly, the Other sector shows most deviation in variable loadings, as α, DEF, $CFvol$ and PE significantly differ from their cross-sector mean. This means this sector can utilise its above-average sensitivity towards factors to make better investment decisions.

Moreover, the Railroads and Toll Roads sectors exhibit significant differences in one variable loading compared to the mean across sectors, as the bold coefficients indicate. These findings suggest that these sectors may have distinct responses towards SMB and α respectively.

On the other hand, sectors such as Communications, Electric Utility, Gas Utility, Port and Water Utility show non-bold coefficients, indicating relatively similar loading compared to the mean across sectors. These sectors may demonstrate a more consistent impact of interest rate fluctuations on their borrowing costs, profitability, and investment decisions. The relatively stable coefficients in these sectors suggest a more predictable response to changes in interest rates, which could be advantageous for investors seeking stability and reduced uncertainty.

Table 19: Wald test statistics for the multivariate test of equal coefficients between sectors in regime 2

Sector	α	$Mkt-Rf$	SMB	HML	MOM	$TERM$	DEF	$CFvol$	LEV	$INVG$	PE
Airport	2.45 0.99	0.33* 1.00	0.18 1.00	0.22 1.00	0.01 1.00	0.03 1.00	0.19 1.00	0.77 1.00	0.00 1.00	2.11 0.99	0.00 1.00
Communications	**27.29*** 0.00	0.16 1.00	0.14 1.00	0.09 1.00	0.54 1.00	0.09 1.00	5.66* 0.84	0.41 1.00	0.32 1.00	0.20 1.00	0.13 1.00
Datacenters	0.03 1.00	3.93* 0.95	0.14 1.00	0.61* 1.00	0.00 1.00	0.08 1.00	0.02 1.00	2.26 0.99	0.00 1.00	0.12 1.00	0.00 1.00
Electric Utility	3.50 0.97	1.79* 1.00	0.26 1.00	0.07 1.00	0.05 1.00	0.01 1.00	0.05 1.00	9.99 0.44	0.00 1.00	0.61 1.00	0.01 1.00
Gas Utility	12.00* 0.29	0.00* 1.00	0.05 1.00	0.95 1.00	0.04* 1.00	1.09* 1.00	1.85* 1.00	12.10* 0.28	0.61 1.00	2.31 0.99	0.63* 1.00
Multi Utility	13.42* 0.20	7.12* 0.71	1.24* 1.00	0.75 1.00	0.01 1.00	0.35 1.00	0.43 1.00	3.27 0.97	0.16 1.00	1.24 1.00	0.04 1.00
Other	14.92* 0.13	0.07* 1.00	0.63* 1.00	0.73* 1.00	0.04 1.00	0.63* 1.00	0.01 1.00	2.19 0.99	0.00 1.00	7.24* 0.70	0.01* 1.00
Pipelines	0.17 1.00	4.89* 0.90	0.88 1.00	2.40* 0.99	0.03 1.00	0.07* 1.00	1.43 1.00	3.05 0.98	0.03 1.00	1.00* 1.00	0.16 1.00
Port	10.81 0.37	0.28* 1.00	0.09 1.00	0.03 1.00	0.32* 1.00	0.02 1.00	0.02 1.00	2.13 0.99	0.01 1.00	0.16 1.00	0.02 1.00
Railroads	1.12 1.00	1.28* 1.00	0.41 1.00	0.76 1.00	0.58 1.00	0.31 1.00	0.04 1.00	14.63 0.15	0.08 1.00	0.04 1.00	0.07* 1.00
Toll Roads	**38.83*** 0.00	0.35* 1.00	0.03 1.00	0.09 1.00	0.46* 1.00	0.12 1.00	2.65* 0.99	4.41 0.93	0.12 1.00	0.36 1.00	0.00 1.00
Water Utility	2.11 0.99	0.13 1.00	0.37 1.00	0.00 1.00	0.00 1.00	0.04 1.00	0.57 1.00	1.83 0.99	0.02 1.00	0.07 1.00	0.00 1.00

* The bold coefficients indicate a value which is significant with at least pval ≤ 0.1

** Coefficients indicated with an asterisk (*) are significant drivers in the third factor model

Next, we discuss the results for the second regime, characterised by higher volatility, as shown in Table 19. In contrast to the substantial differences observed between sectors in the stable regime, we observe fewer significant differences in variable loadings in the second regime. Only the Communications and Toll Roads sectors show deviations in the α variable compared to the average loading.

The reduced number of significant differences in variable loadings in the second regime can be attributed to several factors, including the smaller sample size. The higher volatility during this period introduces challenges in accurately estimating variable loadings. Nevertheless, the deviations observed in the Communications and Toll Roads sectors highlight potential idiosyncratic responses to changes in interest rates within these sectors.

It is important to interpret these results cautiously due to the smaller sample size and the inherent uncertainties associated with periods of higher volatility. However, the deviations in the α variable for the Communications and Toll Roads sectors warrant attention, as they reflect unique characteristics and sensitivities within these sectors.

The most remarkable similarity is that most sectors positively relate to the SMB variable. Even though we believe infrastructure to be a large-cap industry, small-caps can outperform significantly. Furthermore, there are specific quantified differences between sectors when transitioning between regimes. For the sectors: Communications, Datacenters, Electric Utility, Ports and Water Utility, we see that in regime 1, they have around four significant drivers, while in regime 2, they either have zero, one or two. These sectors become more complex to explain, but the value of their loadings is significantly different in the second regime.

Compared to the sectors Other and Toll Roads, almost all factors play a crucial role in estimating their returns in regime 1. However, the factor model can still find many influential loadings for the Other sector in regime 2. Furthermore, four factors are still present for Toll Roads, three of which are significantly different from regime 1. In contrast, Gas Utility has more significant drivers in the second regime, indicating a better fit in volatile times.

5 Conclusion

This paper proposes a regression model that combines hidden interest rate regimes with factor analysis to examine twelve sectors in the infrastructure asset class. We analysed the daily U.S. effective rate by implementing a Hidden Markov Model. We found strong evidence of the presence of two regimes, which we characterise as a stable regime with low volatility and a volatile regime with high volatility. This methodology is aimed to address the main research question: "*How does the sensitivity towards U.S. interest rate regimes differ between infrastructure sectors?*"

After segregating the daily infrastructure returns for each sector into these two regimes, we fitted three different factor models. These models incorporated systematic and idiosyncratic variables, including market risk, size, value, momentum, term premium, default premium, cashflow volatility, leverage, investment growth, and price-earnings ratio.

The findings of our analysis are significant, revealing the sensitivity of infrastructure sectors to U.S. interest rate regimes. Specifically, we observed that the effect of the momentum factor significantly increased in regime 2 compared to regime 1. Furthermore, the default premium significantly increased when its estimated loading in regime 2 was positive. Notably, the default premium loadings in the second regime ultimately diminished for the sectors Airport and Other.

In addition to these sector-specific differences, a noteworthy observation is that systematic variables served as significant drivers for all but two sectors in the first regime. However, their significance diminished substantially in the second regime. This suggests a sensitivity within the infrastructure asset class, wherein as interest rates become more volatile, firm-specific characteristics captured in idiosyncratic factors become more relevant while systematic drivers lose their estimation power.

It is crucial to acknowledge the complexity of this study. Due to the inherent nature of interest rates, the second regime has a relatively small sample size. This could influence the estimation, resulting in insignificant loadings on some variables as the regression could only be done using a few data points. These limitations highlight the intricacies of our research.

While the regime-switching model has provided intriguing insights into the impact of interest rates, it is not the most suitable forecasting method. This opens up avenues for future research to explore more effective forecasting methods. For instance, factor models are often used to estimate parameters that are later used to forecast returns. However, forecasting regimes of interest rates is substantially more challenging, especially when one dataset is relatively small. This may be circumvented by predicting interest rates and implementing the HMM afterwards, offering a potential direction for future studies.

As stated above, effective rates are not known for their volatility. Therefore, the research could also use a different proxy for interest rates, such as short-term bond prices. This could help estimate the hidden states and, in turn, capture more information about the market as bond prices incorporate investors' beliefs about the state of the market. Additionally, when the proxy for interest rates shows more defined characteristics, it gives us a good fit for the regimes. Further research can then be done on implementing forecasting sector returns or on the valuation of non-listed companies.

From a practical standpoint, the findings of this study can be applied in portfolio management settings. The variation in factor loadings across sectors can inform diversification strategies, allowing investment managers to allocate funds to sectors with a higher or lower sensitivity towards specific macroeconomic outcomes or firm-specific characteristics.

In conclusion, this study contributes to understanding how U.S. interest rate regimes affect infrastructure sectors. The findings highlight the importance of integrating regime-specific dynamics and comprehensively analysing systematic and idiosyncratic factors. This will be essential for maximising investment opportunities and ensuring a deepened understanding of characteristics between sectors in the infrastructure asset class.

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A Appendix

A.1 Stock overview

Table 20: Factset ticker names for each stock categorized by sector

Railroads	AZJ-AU	RAIL3-BR	2633-TW	GET-FR	525-HK	66-HK	9020-JP
	9021-JP	9022-JP	CNR-CA	CP-CA	CSX-US	NSC-US	UNP-US
	601333-CN	BTS-TH	601006-CN	FGP-GB			
Communication	SLC-AU	CNU-NZE	DIF-TH	THCOM-TH	ETL-FR	SESG-FR	RWAY-IT
	INW-IT	CLNX-ES	788-HK	UNIT-US	SBAC-US	AMT-US	CCI-US
	IRDM-US	TBIG-ID	TOWR-ID	SATS-US	CCOI-US	CJLU-SG	HTWS-GB
	VTWR-DE	YAHSAT-AE	SITES1A.1-MX	534816-IN	MTEL-ID	LASITEB.1-MX	
Toll Roads	TCL-AU	ECOR3-BR	CCRO3-BR	PINFRA-MX	JSMR-ID	BEM-TH	532754-IN
	531344-IN	ALX-AU	AT-IT	DG-FR	FER-ES	177-HK	200429-CN
	995-HK	107-HK	1052-HK	152-HK	600377-CN	600350-CN	601107-CN
	600012-CN	001965-CN	000828-CN	FGR-FR			
Pipelines	APA-AU	VPK-NL	ENB-CA	TRP-CA	CU-CA	ACO.X-CA	PPL-CA
	WMB-US	OKE-US	KMI-US	LNG-USA	TRGP-US	KEY-CA	ALA-CA
Airport	AIA-NZ	GAPB-MX	OMAB-MX	ASURB-MX	5014-MY	AOT-TH	TAVHL.E-TR
	ADP-FR	FRA-DE	FLU-AT	AENA-ES	FHZN-CH	ENAV-MIL	694-HK
	000089-CN	600008-CN	600009-CN	357-HK			
Multi Utility	VCT-NZ	RENE-PT	ENGI-FR	FTS-CA	ED-US	D-US	CNP-US
	SRE-US	PEG-US	WEC-US	CMS-US	AEE-US	AVA-US	BKH-US
	NWE-US	H-CA	UTL-US				
Electric Utility	ENBR3-BR	EQTL3-BR	ALUP11-BR	TAEE11-BR	TRPL4-BR	ENELAM-CL	ISA-CO
	ENGI11-BR	532779-IN	532898-IN	500400-IN	015760-KR	MER-PH	ENJSA.E-TR
	ELI-BE	ORSTED-DK	ADMIE-GR	PPC-GR	TRN-IT	EVN-AT	RED-ES
	NG-GB	SSE-LON	IBE-MCE	2-HK	6-HK	9502-JP	9504-JP
	9509-JP	9505-JP	9503-JP	9508-JP	9507-JP	9506-JP	9501-JP
	9511-JP	EMA-CA	AEP-US	PNW-US	DUK-US	ETR-US	FE-US
	PCG-US	PPL-US	EIX-US	SO-US	ES-US	POR-US	NEE-US
	XEL-US	AGR-US	LNT-US	MGEE-US	IDA-US	HE-US	EVRG-US
	ALE-US	OGE-US	PNM-US	DTE-US	5110-SA	ENELCHILE-CL	CPFE3-BR
	CESP3-BR	5347-MY	EDPR-PT	2638-HK	RNW-CA	CWEN-US	AQN-US
	ENG-PL	ECL-CL	RWE-DE	ANA-ES	500084-IN	539254-IN	ANE-ES
	AES-US	DEWA-AE	GVOLT-PT	ELE-ES	EXC-US		
Water Utility	CSMG3-BR	SBSP3-BR	IAM-CL	AGUAS.A-CL	MPI-PH	MWC-PH	TTW-TH
	UU-GB	SVT-GB	PNN-LON	270-HK	855-HK	AWK-US	CWT-US
	AWR-US	WTRG-US	SJW-US	601158-CN	000598-CN	MSEX-US	SAPR11-BR
	EYDAP-GR						
Port	STBP3-BR	532921-IN	533248-IN	5246-MY	ICT-PH	NS8U-SG	HHFA-DE
	144-HK	1199-HKG	600018-CN	601880-CN	601000-CN	601326-CN	601298-CN
	601018-CN	001872-CN	000088-CN	ADPORTS-AE			
Gas Utility	532702-IN	539336-IN	532514-IN	PGAS-ID	036460-KR	004690-KR	539957-IN
	IG-IT	SRG-IT	NTGY-ES	ENG-ES	CNA-GB	ENEL-MIL	1193-HK
	2688-HK	3-HK	1038-HK	1083-HK	384-HK	3633-HK	9532-JP
	9531-JP	9534-JP	9536-JP	9533-JP	9543-JP	OGS-US	NI-US
	ATO-US	SR-US	NWN-US	600635-CN	1600-HK	TPE-PL	392-HK
Datacenters	COR-US	DLR-US	CONE-US	EQIX-US	QTS-US		
Other	TFFIF-TH	BIPC-CA	BTSGIF-TH				

A.2 Methodology Algorithms

Forward-equation

$$
\begin{aligned}
\alpha_n(i) &= P\{S^{n-1} = s_{n-1}, S^n = s_n, Y_n = i\} \\
&= \sum_j P\{S^{n-1} = s_{n-1}, Y_{n-1} = j, Y_n = i, S^n = s_n\} \\
&= \sum_j \alpha_{n-1}(j) P\{Y_{n-1} = i, S^n = s_n | S^{n-1} = s_{n-1}, Y_{n-1} = j\} \\
&= \sum_j \alpha_{n-1}(j) P\{Y_{n-1} = i, S^n = s_n | Y_{n-1} = j\} \\
&= \sum_j \alpha_{n-1}(j) a_{ji} p(s_n 1) \\
&= p(s_n|i) \sum_j \alpha_{n-1}(j) a_{ji} \qquad (21)
\end{aligned}
$$

Backward-equation

$$
\begin{aligned}
\beta_k(i) &= \sum_j P\{S^{k+1} = s_{k+1}, \ldots, S^n = s_n | Y_k = i, Y_{k+1} = j\} \times P\{Y_{k+1} = j | Y_k = i\} \\
&= \sum_j P\{S^{k+1} = s_{k+1}, \ldots, S^n = s_n | Y_{k+1} = j\} a_{ij} \\
&= \sum_j P\{S^{k+1} = s_{k+1} | Y_{k+1} = j\} \times P\{S^2 = s_2, \ldots, S^n = s_n | S^{k+1} = s_{k+1}, Y_{k+1} = j\} a_{ij} \\
&= \sum_j p(s_{k+1}|j) P\{S^{k+2} = s_{k+2}, \ldots, S^n = s_n | S^{k+1} = s_{k+1}, Y_{k+1} = j\} a_{ij} \\
&= \sum_j p(s_{k+1}|j) \beta_{k+1}(j) a_{ij} \qquad (22)
\end{aligned}
$$

Forward-Backward algorithm

$$
\begin{aligned}
P\{S^n = s_n, Y_k = i\} &= P\{S^k = s_k, Y_k = i\} \times P\{S_{k+1} = S_{k+1}, \ldots, S_n = s_n | S^k = s_k, Y_k = i\} \\
&= P\{S^k = s_k, Y_k = i\} P\{S_{k+1} = S_{k+1}, \ldots, S_n = s_n | Y_k = i\} \\
&= \alpha_k(i) \times \beta_k(i) \qquad (23)
\end{aligned}
$$

Baum-Welch

$$\begin{aligned}\log(p_\theta(s,y)) &= \log(p_\theta(y_1)) + \sum_{t=2}^{n}\log(p_\theta(y_t|y_{t+1})) + \sum_{t=1}^{n} p_\theta(s_t|y_t)) \\ &= \sum_{i=1}^{k}\mathbb{1}(y_1=i)\log(\pi_i) + \sum_{t=2}^{n}\sum_{i=1}^{k}\sum_{j=1}^{k}\mathbb{1}(y_{t-1}=i, y_t=j)\log(a_{ij}) \\ &+ \sum_{t=1}^{n}\sum_{i=1}^{k}\mathbb{1}(y_t=i)\log(f(s_t|\Theta))\end{aligned}$$

Here $f(s_t|\Theta)$ is the unknown emission distribution of the data, where Θ are distribution variables.

$$\gamma_t(i) = \mathbb{P}(Y_t=i|S,\theta_k) = \frac{\mathbb{P}(Y_t=i,S|\theta_k)}{\mathbb{P}(S|\theta_k)} = \frac{\alpha_t(i)\beta_t(i)}{\sum_{j=1}^{n}\alpha_t(i)\beta_t(i)} \tag{24}$$

$$\begin{aligned}\xi_t(ij) &= \mathbb{P}(Y_t=i,Y_{t+1}=j|S,\theta_k) = \frac{\mathbb{P}(Y_t=i,Y_{t+1}=j,S|\theta_k)}{\mathbb{P}(S|\theta_k)} \\ &= \frac{\alpha_t(i)a_{ij}\beta_{t+1}(j)b_{s_{t+1}}(j)}{\sum_{k=1}^{n}\sum_{w=1}^{n}\alpha_t(k)a_{kw}\beta_{t+1}(w)b_{s_{t+1}}(w)}\end{aligned} \tag{25}$$

Viterbi Using the relation $V_k(j) = \underset{i_1,\ldots,i_{k-1}}{max} P\{\mathbf{Y}_{k-1} = (i_1,\ldots,i_{k-1}, Y_k = j, S^k = s_k\}$, and seeing that the problem in Formula 14 is maximized through the numerator, we get the following iterative algorithm for the full sequence:

$$\begin{aligned}&\max_{i_1,\ldots,i_n} P\{\mathbf{Y}_n = (i_1,\ldots,i_n), S^n = s_n\} \\ &= \max_j V_n(j) \\ &= V_n(j_n) \\ &= \max_{i_1,\ldots,i_{n-1}} P\{\mathbf{Y}_n = (i_1,\ldots,i_{n-1},j_n), S^n = s_n\} \\ &= p(s_n|j_n)\max_i \quad a_{i,j_n}V_{n-1}(i) \\ &= p(s_n|j_n)a_{i_{n-1}(j_n),j_n}V_{n-1}(i_{n-1}(j_n))\end{aligned} \tag{26}$$

Here $i_{n-1}(j_n)$ is the next to last state in the maximizing sequence and we can continue this fashion with $i_{n-2}(i_{n-1}(j_n))$ etcetera.

A.3 Log-Return analysis

Here we provide the first analysis of the three models within two regimes, but we fit the regression model on the simple daily log-returns. We believe that log returns might show better fit due to fact that the normal distribution might fit better. We begin by analysing the first factor model, as specified in Formula 16 above. In Table 21, we see the regression results of this analyses.

Table 21: Factor regression model 1 (GLS) for two regimes over the aggregated log-returns for 2010 until 2024

	Regime 1			Regime 2		
Variable	Coefficient	Std. Error	P-Value	Coefficient	Std. Error	P-Value
α	-0.0004	0.004	0.926	-0.0231	0.017	0.177
$Mkt - RF$	0.0365	0.001	0.000	0.0425	0.004	0.000
SMB	0.0157	0.002	0.000	-0.0044	0.010	0.647
HML	0.0114	0.002	0.000	0.0084	0.008	0.283
MOM	0.0010	0.001	0.449	0.0054	0.005	0.315
$TERM$	0.0014	0.001	0.091	0.0005	0.003	0.848
DEF	0.0001	0.002	0.951	0.0117	0.007	0.119
	Adj R2: 0.304; F: 1.23e-267; LogL: -5448.3			Adj R2: 0.435; F: 4.95e-22; LogL: -348.44		

Secondly, Table 22 gives an overview of the GLS results after fitting the model as determined in Formula 17.

Table 22: Factor regression model 2 (GLS) for two regimes over the log-return for 2010 until 2024

	Regime 1			Regime 2		
Variable	Coefficient	Std. Error	P-Value	Coefficient	Std. Error	P-Value
α	0.0073	0.004	0.059	0.0011	0.014	0.939
$CFvol$	0.0081	0.012	0.508	-0.0279	0.052	0.594
LEV	-0.0004	0.001	0.657	0.0001	0.004	0.975
$INVG$	0.0025	0.003	0.472	-0.0190	0.013	0.154
PE	-5.669e-5	5.22e-5	0.278	0.0002	0.000	0.351
	Adj R2: -0.001; F: 0.744; LogL: 4821.6			Adj R2: -0.008; F: 0.649; LogL: -291.8		

Lastly, we look at the combined model (Formula 18) and compare that to the separate models as given above.

Table 23: Factor regression model 3 (GLS) for two regimes over the log return for 2010 until 2024

	Regime 1			Regime 2		
Variable	Coefficient	Std. Error	P-Value	Coefficient	Std. Error	P-Value
α	-0.0045	0.011	0.673	-0.0705	0.039	0.075
$Mkt - RF$	0.0365	0.001	0.000	0.0423	0.004	0.000
SMB	0.0158	0.002	0.000	-0.0027	0.010	0.786
HML	0.0113	0.002	0.000	0.0071	0.008	0.366
MOM	0.0010	0.001	0.458	0.0064	0.005	0.241
$TERM$	0.0025	0.001	0.038	0.0034	0.005	0.456
DEF	0.0009	0.002	0.725	0.0185	0.009	0.046
$CFvol$	-0.0040	0.010	0.699	-0.0222	0.041	0.590
LEV	3.498e-05	0.001	0.967	0.0042	0.003	0.173
$INVG$	0.0061	0.004	0.098	-0.0040	0.014	0.775
PE	-8.132e-06	5.34e-05	0.879	0.0002	0.000	0.332
	Adj R2: 0.304; F: 8.64e-264; LogL: 5450.2			Adj R2: 0.414; F: 4.63e-20; LogL: 350.27		

A.4 Model evaluation

Table 24: Model evaluation criteria, quantified in the Adjusted R-squared, F-statistic and the Log-Likelihood for the dataset of return between 2010 and 2024

	model 1						model 2						model 3					
	R1			R2			R1			R2			R1			R2		
	R2	F	LL	R2	F	LL	R2	F	LL	R2	F	LL	R2	F	LL	R2	F	LL
Aggregated return	0.309	4.44e-273	-11652	0.333	1.97e-16	-682.59	0.001	0.0796	-12288	-0.011	0.789	-725.86	0.311	2.76e-271	-11645	0.338	4.07e-15	-679.77
Log return	0.304	1.23e-267	5448.3	0.418	4.95e-22	348.44	-0.001	0.744	4821.6	-0.008	0.649	291.78	0.304	8.64e-264	5450.2	0.416	4.63e-20	350.27
Airport	0.083	6.81e-63	-5991.6	0.016	0.161	-354.27	0.000	0.298	-6141.9	0.008	0.234	-356.16	0.085	5.06e-62	-5985.8	0.023	0.147	-351.43
Communications	0.014	6.25e-10	-8607.7	0.066	0.0035	-491.51	0.009	2.07e-7	-8616.6	-0.010	0.728	-500.40	0.017	3.58e-11	-8599.6	0.057	0.0179	-490.31
Datacenters	0.427	0.00	-4793.5	0.318	1.68e-15	-284.24	-0.001	0.940	-5754.9	-0.017	0.968	-325.80	0.426	0.00	-4792.9	0.308	2.03e-13	-283.58
Electric Utility	0.084	3.33e-63	-5022.6	0.131	7.59e-6	-276.68	-0.001	0.715	-5175.0	-0.007	0.641	-292.69	0.084	9.6e-61	-5020.6	0.123	9.51e-5	-275.48
Gas Utility	0.017	2.39e-12	-8319.2	0.029	0.0682	-508.71	0.004	0.001	-8343.2	0.071	0.000963	-505.26	0.020	2.92e-13	-8311.7	0.126	7.55e-5	-495.93
Multi Utility	0.432	0.00	-4004.8	0.483	5.93e-27	-239.22	-0.001	0.980	-4984.1	-0.003	0.497	-307.43	0.432	0.00	-4003.9	0.487	3.02e-25	-236.19
Other	0.019	7.80e-14	-6175.4	0.018	0.147	-325.45	0.044	3.40e-33	-6133.0	0.030	0.0404	-325.22	0.108	2.94e-80	-6010.6	0.079	0.00363	-316.81
Pipelines	0.477	0.00	-4572.5	0.494	6.5e-28	-244.02	0.001	0.521	-5690.2	-0.017	0.950	-315.92	0.477	0.00	-4569.9	0.505	1.26e-26	-239.83
Port	0.079	1.61e-59	-6091.3	0.040	0.029	-339.59	0.001	0.114	-6232.4	-0.005	0.560	-345.28	0.080	7.46e-58	-6087.4	0.038	0.0631	-337.71
Railroads	0.097	2.86e-74	-56766.3	0.082	0.0008	-338.91	0.001	0.554	-5854.0	-0.005	0.563	-349.12	0.098	4.40e-72	-5673.2	0.084	0.0025	-336.61
Toll Roads	0.014	6.43e-10	-8057.2	0.041	0.0271	-477.82	0.006	5.21e-5	-8071.9	-0.015	0.902	-484.58	0.019	5.62e-12	-8047.0	0.047	0.0358	-475.09
Water Utility	0.069	1.86e-51	-5484.3	0.018	0.146	-307.58	-0.001	0.968	-5610.0	-0.013	0.841	-311.75	0.068	2.29e-48	-5484.1	0.006	0.345	-306.69

Aggregated returns

Table 25: Factor regression model (GLS) for two regimes over the aggregated return from 2010 until 2024

	model 1						model 2						model 3					
	R1			R2			R1			R2			R1			R2		
Variable	β	SE	Pval	β	SE	Pval	β	SE	Pval	β	SE	Pval	β	SE	Pval	β	SE	Pval
α	**-1.5040**	**0.632**	**0.017**	-3.2966	2.740	0.230	**1.9408**	**0.550**	**0.000**	1.1059	2.177	0.612	**-4.1220**	**1.506**	**0.006**	**-11.6514**	**6.298**	**0.066**
$Mkt-Rf$	**5.1642**	**0.139**	**0.000**	5.8461	0.653	0.000	x	x	x	x	x	x	**5.1582**	**0.139**	**0.000**	**5.8553**	**0.656**	**0.000**
SMB	**2.1888**	**0.335**	**0.000**	0.7896	1.547	0.610	x	x	x	x	x	x	**2.2048**	**0.335**	**0.000**	1.1419	1.564	0.466
HML	**2.0823**	**0.241**	**0.000**	1.0871	1.251	0.386	x	x	x	x	x	x	**2.0425**	**0.241**	**0.000**	0.8402	1.254	0.504
MOM	0.1100	0.185	0.553	**2.2463**	**0.866**	**0.010**	x	x	x	x	x	x	0.1055	0.185	0.569	**2.3844**	**0.867**	**0.007**
$TERM$	0.0802	0.118	0.497	0.1888	0.434	0.664	x	x	x	x	x	x	**0.4661**	**0.171**	**0.007**	0.5779	0.734	0.432
DEF	**0.8567**	**0.271**	**0.002**	1.7377	1.198	0.148	x	x	x	x	x	x	**1.2780**	**0.347**	**0.000**	**3.1871**	**1.472**	**0.032**
$CFvol$	x	x	x	x	x	x	**3.2331**	**1.737**	**0.063**	7.9677	7.851	0.311	0.6762	1.471	0.646	9.4780	6.557	0.150
LEV	x	x	x	x	x	x	-0.1440	0.136	0.289	0.1401	0.555	0.801	0.0486	0.121	0.688	**0.8259**	**0.489**	**0.093**
$INVG$	x	x	x	x	x	x	**0.8885**	**0.497**	**0.074**	-0.7773	1.999	0.698	**1.9280**	**0.527**	**0.000**	1.2656	2.222	0.570
PE	x	x	x	x	x	x	**-0.0149**	**0.007**	**0.046**	-0.0100	0.032	0.758	0.0040	0.008	0.596	-0.0003	0.033	0.992

* The bold coefficients indicate a value which is significant with at least pval ≤ 0.1

Airport

Table 26: Factor regression model (GLS) for two regimes over the simple return in the Airport sector from 2010 until 2024

	model 1						model 2						model 3					
	R1			R2			R1			R2			R1			R2		
Variable	β	SE	Pval	β	SE	Pval	β	SE	Pval	β	SE	Pval	β	SE	Pval	β	SE	Pval
α	**0.32**	**0.12**	**0.01**	0.4525	0.544	0.406	**0.222**	**0.09**	**0.016**	-0.3282	0.352	0.353	**1.0982**	**0.292**	**0.000**	0.7126	1.250	0.569
$Mkt-Rf$	**0.4663**	**0.027**	**0.000**	**0.3263**	**0.130**	**0.013**	x	x	x	x	x	x	**0.4680**	**0.027**	**0.000**	**0.3356**	**0.130**	**0.011**
SMB	**0.3530**	**0.065**	**0.000**	0.1612	0.307	0.600	x	x	x	x	x	x	**0.3510**	**0.065**	**0.000**	0.2744	0.310	0.378
HML	**0.1960**	**0.047**	**0.000**	0.3011	0.248	0.227	x	x	x	x	x	x	**0.2002**	**0.047**	**0.000**	0.2591	0.249	0.299
MOM	0.0118	0.036	0.742	-0.0650	0.172	0.706	x	x	x	x	x	x	0.0107	0.036	0.765	-0.0385	0.172	0.823
$TERM$	0.0203	0.023	0.375	0.0248	0.086	0.774	x	x	x	x	x	x	-0.0417	0.033	0.209	-0.1275	0.146	0.382
DEF	-0.0826	0.052	0.115	-0.1617	0.238	0.497	x	x	x	x	x	x	**-0.2117**	**0.067**	**0.002**	-0.1720	0.292	0.557
$CFvol$	x	x	x	x	x	x	**0.59**	**0.29**	**0.043**	1.0332	1.271	0.417	0.6248	0.285	0.029	1.2704	1.301	0.330
LEV	x	x	x	x	x	x	-0.01	0.02	0.862	0.0221	0.090	0.806	-0.0288	0.023	0.220	0.0037	0.097	0.970
$INVG$	x	x	x	x	x	x	0.06	0.08	0.492	-0.4546	0.324	0.162	-0.0857	0.102	0.401	-0.7160	0.441	0.106
PE	x	x	x	x	x	x	-0.002	0.001	0.165	0.0051	0.005	0.327	**-0.0043**	**0.001**	**0.004**	0.0001	0.007	0.984

* The bold coefficients indicate a value which is significant with at least pval ≤ 0.1

Communication

Table 27: Factor regression model (GLS) for two regimes over the simple returns in the Communication sector from 2010 until 2024

	model 1						model 2						model 3					
	R1			R2			R1			R2			R1			R2		
Variable	β	SE	Pval	β	SE	Pval	β	SE	Pval	β	SE	Pval	β	SE	Pval	β	SE	Pval
α	**-1.0206**	**0.262**	**0.000**	**-4.0224**	**1.069**	**0.000**	**1.2175**	**0.190**	**0.000**	1.0867	0.717	0.131	0.3579	0.623	0.566	**-6.7455**	**2.477**	**0.007**
$Mkt-Rf$	**0.2478**	**0.057**	**0.000**	0.0579	0.255	0.820	x	x	x	x	x	x	**0.2567**	**0.057**	**0.000**	0.0520	0.258	0.840
SMB	0.1904	0.139	0.170	-0.0317	0.604	0.958	x	x	x	x	x	x	0.2011	0.138	0.146	0.0324	0.615	0.958
HML	0.0845	0.100	0.397	-0.0009	0.488	0.998	x	x	x	x	x	x	0.1010	0.100	0.311	-0.0619	0.493	0.900
MOM	**0.1745**	**0.077**	**0.023**	0.3815	0.338	0.260	x	x	x	x	x	x	**0.1748**	**0.077**	**0.023**	0.4227	0.341	0.217
$TERM$	0.0322	0.049	0.510	-0.2627	0.169	0.122	x	x	x	x	x	x	-0.0695	0.071	0.327	-0.0626	0.288	0.828
DEF	**0.5121**	**0.112**	**0.000**	**1.9854**	**0.467**	**0.000**	x	x	x	x	x	x	**0.3117**	**0.143**	**0.030**	**2.3914**	**0.579**	**0.000**
$CFvol$	x	x	x	x	x	x	**-1.3614**	**0.599**	**0.023**	-1.9429	2.586	0.453	**-1.5714**	**0.609**	**0.010**	-0.5107	2.578	0.843
LEV	x	x	x	x	x	x	-0.0642	0.047	0.170	0.0287	0.183	0.875	-0.0433	0.050	0.388	0.2623	0.192	0.174
$INVG$	x	x	x	x	x	x	**-0.3707**	**0.171**	**0.031**	-0.4421	0.658	0.503	**-0.4000**	**0.218**	**0.066**	0.1765	0.874	0.840
PE	x	x	x	x	x	x	**-0.0061**	**0.003**	**0.017**	-0.0064	0.011	0.550	-0.0043	0.003	0.172	0.0070	0.013	0.595

* The bold coefficients indicate a value which is significant with at least pval ≤ 0.1

Datacenters

Table 28: Factor regression model (GLS) for two regimes over the simple returns in the Datacenters sector from 2010 until 2024

	model 1						model 2						model 3					
	R1			R2			R1			R2			R1			R2		
Variable	β	SE	Pval	β	SE	Pval	β	SE	Pval	β	SE	Pval	β	SE	Pval	β	SE	Pval
α	0.0844	0.087	0.330	0.0142	0.385	0.971	0.0328	0.083	0.692	0.2290	0.303	0.451	-0.0684	0.207	0.741	-0.1607	0.895	0.858
$Mkt-Rf$	**0.7750**	**0.019**	**0.000**	**0.8158**	**0.092**	**0.000**	x	x	x	x	x	x	**0.7745**	**0.019**	**0.000**	**0.8189**	**0.093**	**0.000**
SMB	**-0.4509**	**0.046**	**0.000**	-0.1152	0.217	0.597	x	x	x	x	x	x	**-0.4504**	**0.046**	**0.000**	-0.1297	0.222	0.560
HML	**-0.4610**	**0.033**	**0.000**	**-0.3167**	**0.176**	**0.073**	x	x	x	x	x	x	**-0.4627**	**0.033**	**0.000**	**-0.3151**	**0.178**	**0.078**
MOM	**-0.0455**	**0.025**	**0.073**	0.0172	0.122	0.888	x	x	x	x	x	x	**-0.0457**	**0.025**	**0.072**	0.0128	0.123	0.917
$TERM$	0.0077	0.016	0.633	0.0808	0.061	0.186	x	x	x	x	x	x	0.0239	0.023	0.309	0.1092	0.104	0.296
DEF	-0.0255	0.037	0.491	-0.0181	0.168	0.915	x	x	x	x	x	x	-0.0020	0.048	0.966	0.0154	0.209	0.941
$CFvol$	x	x	x	x	x	x	0.1217	0.261	0.641	0.5563	1.094	0.612	0.0360	0.202	0.858	0.4136	0.931	0.657
LEV	x	x	x	x	x	x	0.0095	0.020	0.641	-0.0188	0.077	0.808	0.0090	0.017	0.587	0.0367	0.069	0.598
$INVG$	x	x	x	x	x	x	0.0466	0.075	0.534	-0.0072	0.279	0.979	0.0678	0.072	0.348	0.2223	0.316	0.482
PE	x	x	x	x	x	x	-0.0004	0.001	0.727	-0.0013	0.004	0.769	0.0002	0.001	0.826	-0.0025	0.005	0.592

* The bold coefficients indicate a value which is significant with at least pval ≤ 0.1

Electric Utility

Table 29: Factor regression model (GLS) for two regimes over the simple returns in the Electric Utility sector from 2010 until 2024

	model 1						model 2						model 3					
	R1			R2			R1			R2			R1			R2		
Variable	β	SE	Pval	β	SE	Pval	β	SE	Pval	β	SE	Pval	β	SE	Pval	β	SE	Pval
α	-0.0087	0.093	0.925	-0.2258	0.371	0.544	-7.777e-05	0.070	0.999	-0.0257	0.258	0.921	-0.2386	0.221	0.280	0.1821	0.860	0.832
$Mkt-Rf$	**0.3470**	**0.020**	**0.000**	**0.4934**	**0.088**	**0.000**	x	x	x	x	x	x	**0.3470**	**0.020**	**0.000**	**0.4963**	**0.090**	**0.000**
SMB	**0.3126**	**0.049**	**0.000**	0.1743	0.210	0.406	x	x	x	x	x	x	**0.3151**	**0.049**	**0.000**	0.1679	0.213	0.432
HML	**0.1818**	**0.035**	**0.000**	0.0740	0.169	0.663	x	x	x	x	x	x	**0.1808**	**0.035**	**0.000**	0.0793	0.171	0.644
MOM	0.0247	0.027	0.363	0.0537	0.117	0.647	x	x	x	x	x	x	0.0250	0.027	0.358	0.0517	0.118	0.663
$TERM$	-0.0180	0.017	0.299	-0.0237	0.059	0.687	x	x	x	x	x	x	0.0061	0.025	0.809	-0.0521	0.100	0.603
DEF	0.0217	0.040	0.584	0.0795	0.162	0.624	x	x	x	x	x	x	0.0621	0.051	0.222	-0.0025	0.201	0.990
$CFvol$	x	x	x	x	x	x	-0.2244	0.221	0.310	-1.3633	0.929	0.144	**-0.3754**	**0.216**	**0.082**	-1.1743	0.895	0.191
LEV	x	x	x	x	x	x	0.0025	0.017	0.886	-0.0275	0.066	0.676	0.0105	0.018	0.554	-0.0101	0.067	0.879
$INVG$	x	x	x	x	x	x	0.0236	0.063	0.708	-0.1929	0.237	0.416	0.0469	0.077	0.544	-0.2756	0.303	0.365
PE	x	x	x	x	x	x	0.0005	0.001	0.583	0.0038	0.004	0.323	0.0012	0.001	0.291	0.0007	0.005	0.877

* The bold coefficients indicate a value which is significant with at least pval ≤ 0.1

Gas Utility

Table 30: Factor regression model (GLS) for two regimes over the simple returns in the Gas Utility sector from 2010 until 2024

	model 1						model 2						model 3					
	R1			R2			R1			R2			R1			R2		
Variable	β	SE	Pval	β	SE	Pval	β	SE	Pval	β	SE	Pval	β	SE	Pval	β	SE	Pval
α	0.3500	0.241	0.146	0.5316	1.163	0.648	0.0261	0.175	0.882	**1.5846**	**0.734**	**0.032**	0.1651	0.573	0.773	**7.1836**	**2.546**	**0.005**
$Mkt-Rf$	**0.3765**	**0.053**	**0.000**	**0.7166**	**0.277**	**0.010**	x	x	x	x	x	x	**0.3739**	**0.053**	**0.000**	**0.8283**	**0.265**	**0.002**
SMB	**0.5620**	**0.127**	**0.000**	0.6436	0.657	0.329	x	x	x	x	x	x	**0.5550**	**0.127**	**0.000**	0.7412	0.632	0.243
HML	**0.2832**	**0.092**	**0.002**	-0.3335	0.531	0.531	x	x	x	x	x	x	**0.2721**	**0.092**	**0.003**	-0.3226	0.507	0.525
MOM	-0.0694	0.071	0.325	**0.7766**	**0.368**	**0.036**	x	x	x	x	x	x	-0.0688	0.070	0.329	**0.7129**	**0.351**	**0.043**
$TERM$	-0.0412	0.045	0.359	0.2736	0.184	0.139	x	x	x	x	x	x	0.0200	0.065	0.759	**-0.5675**	**0.297**	**0.057**
DEF	-0.0962	0.103	0.351	-0.2846	0.509	0.577	x	x	x	x	x	x	-0.0583	0.132	0.659	**-1.0826**	**0.595**	**0.070**
$CFvol$	x	x	x	x	x	x	**0.9678**	**0.554**	**0.081**	**9.7953**	**2.648**	**0.000**	0.8523	0.560	0.128	**10.7867**	**2.651**	**0.000**
LEV	x	x	x	x	x	x	**-0.1198**	**0.043**	**0.006**	-0.0176	0.187	0.925	**-0.1200**	**0.046**	**0.009**	-0.1374	0.198	0.488
$INVG$	x	x	x	x	x	x	**0.4022**	**0.158**	**0.011**	0.7460	0.674	0.270	**0.4172**	**0.200**	**0.037**	-0.8582	0.899	0.341
PE	x	x	x	x	x	x	**0.0041**	**0.002**	**0.087**	**-0.0341**	**0.011**	**0.002**	0.0038	0.003	0.194	**-0.0557**	**0.013**	**0.000**

* The bold coefficients indicate a value which is significant with at least pval ≤ 0.1

Multi Utility

Table 31: Factor regression model (GLS) for two regimes over the simple returns in the Multi Utility sector from 2010 until 2024

	model 1						model 2						model 3					
	R1			R2			R1			R2			R1			R2		
Variable	β	SE	Pval	β	SE	Pval	β	SE	Pval	β	SE	Pval	β	SE	Pval	β	SE	Pval
α	0.0153	0.069	0.824	0.0180	0.308	0.953	0.0466	0.066	0.482	-0.4178	0.277	0.133	0.0543	0.164	0.741	**-1.4919**	**0.708**	**0.036**
$Mkt-Rf$	**0.5918**	**0.015**	**0.000**	**0.8152**	**0.074**	**0.000**	x	x	x	x	x	x	**0.5924**	**0.015**	**0.000**	**0.8033**	**0.074**	**0.000**
SMB	**-0.5563**	**0.036**	**0.000**	**-0.4186**	**0.174**	**0.017**	x	x	x	x	x	x	**-0.5553**	**0.037**	**0.000**	**-0.4066**	**0.176**	**0.022**
HML	**0.2275**	**0.026**	**0.000**	**0.2376**	**0.141**	**0.093**	x	x	x	x	x	x	**0.2279**	**0.026**	**0.000**	0.2180	0.141	0.124
MOM	-0.0166	0.020	0.412	-0.0528	0.097	0.589	x	x	x	x	x	x	-0.0164	0.020	0.417	-0.0369	0.098	0.705
$TERM$	0.0149	0.013	0.247	-0.0126	0.049	0.796	x	x	x	x	x	x	0.0161	0.019	0.388	0.1249	0.082	0.132
DEF	-0.0154	0.030	0.602	-0.0062	0.135	0.963	x	x	x	x	x	x	-0.0186	0.038	0.624	0.2233	0.166	0.179
$CFvol$	x	x	x	x	x	x	-0.0683	0.209	0.744	0.3172	0.999	0.751	-0.1738	0.161	0.279	0.1192	0.737	0.872
LEV	x	x	x	x	x	x	-0.0052	0.016	0.749	0.0216	0.071	0.760	-0.0078	0.013	0.552	0.0848	0.055	0.125
$INVG$	x	x	x	x	x	x	0.0184	0.060	0.758	-0.0603	0.255	0.813	0.0006	0.057	0.992	0.3641	0.250	0.147
PE	x	x	x	x	x	x	-1.322e-05	0.001	0.988	0.0050	0.004	0.220	0.0002	0.001	0.792	0.0050	0.004	0.186

* The bold coefficients indicate a value which is significant with at least pval ≤ 0.1

Other

Table 32: Factor regression model (GLS) for two regimes over the simple returns in the Other sector from 2010 until 2024

	model 1						model 2						model 3					
	R1			R2			R1			R2			R1			R2		
Variable	β	SE	Pval	β	SE	Pval	β	SE	Pval	β	SE	Pval	β	SE	Pval	β	SE	Pval
α	**-1.1156**	**0.129**	**0.000**	0.7430	0.472	0.117	**-0.2708**	**0.092**	**0.003**	**-0.5108**	**0.303**	**0.093**	**-4.6074**	**0.294**	**0.000**	**-2.1723**	**1.054**	**0.041**
$Mkt-Rf$	**0.0632**	**0.028**	**0.026**	0.1600	0.112	0.156	x	x	x	x	x	x	**0.0467**	**0.027**	**0.084**	0.1074	0.110	0.329
SMB	**0.1740**	**0.068**	**0.011**	0.4287	0.266	0.109	x	x	x	x	x	x	**0.1629**	**0.065**	**0.013**	0.3603	0.262	0.170
HML	0.0415	0.049	0.399	**0.3791**	**0.215**	**0.080**	x	x	x	x	x	x	-0.0032	0.047	0.946	**0.3910**	**0.210**	**0.064**
MOM	-0.0609	0.038	0.108	-0.1276	0.149	0.393	x	x	x	x	x	x	**-0.0606**	**0.036**	**0.094**	-0.1180	0.145	0.417
$TERM$	**-0.0810**	**0.024**	**0.001**	-0.0088	0.075	0.907	x	x	x	x	x	x	**0.2730**	**0.033**	**0.000**	**0.3689**	**0.123**	**0.003**
DEF	**0.4318**	**0.055**	**0.000**	-0.3394	0.206	0.101	x	x	x	x	x	x	**0.9796**	**0.068**	**0.000**	0.0804	0.246	0.744
$CFvol$	x	x	x	x	x	x	**2.7962**	**0.292**	**0.000**	0.0834	1.091	0.939	**1.9372**	**0.287**	**0.000**	-0.5831	1.097	0.596
LEV	x	x	x	x	x	x	**-0.1233**	**0.023**	**0.000**	-0.0786	0.077	0.310	0.0092	0.024	0.698	-0.0159	0.082	0.846
$INVG$	x	x	x	x	x	x	**0.6809**	**0.083**	**0.000**	**0.5642**	**0.278**	**0.044**	**1.4445**	**0.103**	**0.000**	**1.3082**	**0.372**	**0.001**
PE	x	x	x	x	x	x	0.0016	0.001	0.204	**0.0080**	**0.004**	**0.075**	**0.0144**	**0.001**	**0.000**	**0.0169**	**0.006**	**0.003**

* The bold coefficients indicate a value which is significant with at least pval ≤ 0.1

Pipelines

Table 33: Factor regression model (GLS) for two regimes over the simple returns in the Pipelines sector from 2010 until 2024

	model 1						model 2						model 3					
	R1			R2			R1			R2			R1			R2		
Variable	β	SE	Pval	β	SE	Pval	β	SE	Pval	β	SE	Pval	β	SE	Pval	β	SE	Pval
α	-0.0739	0.081	0.363	**0.6221**	**0.316**	**0.050**	0.1202	0.081	0.139	-0.1949	0.289	0.501	-0.1182	0.194	0.542	-0.1886	0.721	0.794
$Mkt-Rf$	**0.8449**	**0.018**	**0.000**	**0.8632**	**0.075**	**0.000**	x	x	x	x	x	x	**0.8451**	**0.018**	**0.000**	**0.8599**	**0.075**	**0.000**
SMB	**-0.1345**	**0.043**	**0.002**	-0.1326	0.178	0.458	x	x	x	x	x	x	**-0.1334**	**0.043**	**0.002**	-0.1824	0.179	0.310
HML	**0.6003**	**0.031**	**0.000**	**0.6378**	**0.144**	**0.000**	x	x	x	x	x	x	**0.5984**	**0.031**	**0.000**	**0.6454**	**0.144**	**0.000**
MOM	**-0.0698**	**0.024**	**0.003**	0.0778	0.100	0.436	x	x	x	x	x	x	**-0.0699**	**0.024**	**0.003**	0.0719	0.099	0.470
$TERM$	0.0087	0.015	0.568	**0.1015**	**0.050**	**0.044**	x	x	x	x	x	x	0.0286	0.022	0.195	**0.2359**	**0.084**	**0.005**
DEF	0.0316	0.035	0.364	**-0.3345**	**0.138**	**0.016**	x	x	x	x	x	x	0.0423	0.045	0.343	-0.2280	0.169	0.178
$CFvol$	x	x	x	x	x	x	0.2254	0.257	0.380	-0.1500	1.042	0.886	-0.0416	0.189	0.826	-0.7708	0.751	0.306
LEV	x	x	x	x	x	x	-0.0249	0.020	0.214	0.0483	0.074	0.513	-0.0176	0.016	0.259	0.0838	0.056	0.136
$INVG$	x	x	x	x	x	x	0.1006	0.073	0.170	-0.0189	0.265	0.943	**0.1169**	**0.068**	**0.084**	**0.4641**	**0.254**	**0.070**
PE	x	x	x	x	x	x	-0.0004	0.001	0.702	0.0005	0.004	0.909	0.0004	0.001	0.648	0.0002	0.004	0.952

* The bold coefficients indicate a value which is significant with at least pval ≤ 0.1

Port

Table 34: Factor regression model (GLS) for two regimes over the simple returns in the Port sector from 2010 until 2024

	model 1						model 2						model 3					
	R1			R2			R1			R2			R1			R2		
Variable	β	SE	Pval	β	SE	Pval	β	SE	Pval	β	SE	Pval	β	SE	Pval	β	SE	Pval
α	**0.2734**	**0.126**	**0.030**	0.4034	0.506	0.426	-0.0965	0.095	0.310	-0.5327	0.334	0.112	-0.1614	0.301	0.592	-1.3141	1.168	0.262
$Mkt-Rf$	**0.4584**	**0.028**	**0.000**	**0.3683**	**0.121**	**0.003**	x	x	x	x	x	x	**0.4574**	**0.028**	**0.000**	**0.3609**	**0.122**	**0.003**
SMB	**0.5200**	**0.067**	**0.000**	0.1883	0.286	0.511	x	x	x	x	x	x	**0.5220**	**0.067**	**0.000**	0.2385	0.290	0.412
HML	**0.2212**	**0.048**	**0.000**	0.0186	0.231	0.936	x	x	x	x	x	x	**0.2209**	**0.048**	**0.000**	-0.0200	0.233	0.932
MOM	**0.0731**	**0.037**	**0.048**	**0.3635**	**0.160**	**0.024**	x	x	x	x	x	x	**0.0731**	**0.037**	**0.048**	**0.3895**	**0.161**	**0.016**
$TERM$	0.0276	0.024	0.242	0.0189	0.080	0.814	x	x	x	x	x	x	0.0463	0.034	0.175	0.1311	0.136	0.336
DEF	**-0.0917**	**0.054**	**0.090**	-0.2017	0.221	0.363	x	x	x	x	x	x	-0.0271	0.069	0.695	0.0749	0.273	0.784
$CFvol$	x	x	x	x	x	x	-0.0830	0.300	0.782	0.9460	1.204	0.433	-0.2573	0.294	0.381	0.8666	1.216	0.477
LEV	x	x	x	x	x	x	**0.0582**	**0.023**	**0.013**	0.0734	0.085	0.390	**0.0636**	**0.024**	**0.009**	0.1237	0.091	0.174
$INVG$	x	x	x	x	x	x	-0.1005	0.086	0.242	0.0661	0.307	0.830	-0.0345	0.105	0.743	0.2552	0.412	0.537
PE	x	x	x	x	x	x	0.0001	0.001	0.926	0.0017	0.005	0.739	0.0006	0.002	0.704	0.0040	0.006	0.513

* The bold coefficients indicate a value which is significant with at least pval ≤ 0.1

Railroads

Table 35: Factor regression model (GLS) for two regimes over the simple returns in the Railroads sector from 2010 until 2024

	model 1						model 2						model 3					
	R1			R2			R1			R2			R1			R2		
Variable	β	SE	Pval	β	SE	Pval	β	SE	Pval	β	SE	Pval	β	SE	Pval	β	SE	Pval
α	0.1150	0.112	0.304	0.2442	0.504	0.629	0.0540	0.085	0.526	-0.2449	0.340	0.473	0.2650	0.267	0.320	-1.5437	1.162	0.185
$Mkt - Rf$	**0.3883**	**0.025**	**0.000**	**0.4984**	**0.120**	**0.000**	x	x	x	x	x	x	**0.3900**	**0.025**	**0.000**	**0.4777**	**0.121**	**0.000**
SMB	**0.7937**	**0.059**	**0.000**	0.1960	0.285	0.492	x	x	x	x	x	x	**0.7981**	**0.059**	**0.000**	0.2229	0.288	0.441
HML	**0.3376**	**0.043**	**0.000**	0.3646	0.230	0.115	x	x	x	x	x	x	**0.3396**	**0.043**	**0.000**	0.3387	0.231	0.145
MOM	-0.0139	0.033	0.671	0.2369	0.159	0.138	x	x	x	x	x	x	-0.0148	0.033	0.653	0.2637	0.160	0.101
$TERM$	0.0283	0.021	0.176	0.0257	0.080	0.748	x	x	x	x	x	x	0.0236	0.030	0.437	0.1845	0.135	0.174
DEF	-0.0580	0.048	0.226	-0.0675	0.220	0.760	x	x	x	x	x	x	-0.0822	0.061	0.181	0.1808	0.271	0.506
$CFvol$	x	x	x	x	x	x	-0.0810	0.269	0.763	-1.0003	1.227	0.416	-0.2141	0.260	0.411	-1.1814	1.209	0.330
LEV	x	x	x	x	x	x	0.0232	0.021	0.269	0.0051	0.087	0.953	0.0222	0.021	0.300	0.0711	0.090	0.432
$INVG$	x	x	x	x	x	x	-0.0029	0.077	0.970	-0.2815	0.313	0.369	0.0150	0.093	0.872	0.1151	0.410	0.779
PE	x	x	x	x	x	x	-0.0016	0.001	0.166	0.0077	0.005	0.129	-0.0021	0.001	0.119	**0.0113**	**0.006**	**0.067**

* The bold coefficients indicate a value which is significant with at least pval ≤ 0.1

Toll Roads

Table 36: Factor regression model (GLS) for two regimes over the simple returns in the Toll Roads sector from 2010 until 2024

	model 1						model 2						model 3					
	R1			R2			R1			R2			R1			R2		
Variable	β	SE	Pval	β	SE	Pval	β	SE	Pval	β	SE	Pval	β	SE	Pval	β	SE	Pval
α	-0.3596	0.223	0.107	-1.3186	0.999	0.189	**0.5171**	**0.162**	**0.001**	0.3229	0.663	0.627	-0.8322	0.531	0.117	**-5.7052**	**2.298**	**0.014**
$Mkt - Rf$	**0.2822**	**0.049**	**0.000**	**0.5816**	**0.238**	**0.015**	x	x	x	x	x	x	**0.2834**	**0.049**	**0.000**	**0.5499**	**0.239**	**0.023**
SMB	**0.4196**	**0.118**	**0.000**	-0.0128	0.564	0.982	x	x	x	x	x	x	**0.4330**	**0.118**	**0.000**	0.0992	0.571	0.862
HML	0.1359	0.085	0.110	-0.2644	0.456	0.563	x	x	x	x	x	x	0.1331	0.085	0.117	-0.3527	0.458	0.442
MOM	0.0701	0.065	0.284	**0.5984**	**0.316**	**0.059**	x	x	x	x	x	x	0.0665	0.065	0.308	**0.6682**	**0.316**	**0.036**
$TERM$	**0.0972**	**0.042**	**0.020**	0.0107	0.158	0.946	x	x	x	x	x	x	**0.1595**	**0.060**	**0.008**	0.3317	0.268	0.217
DEF	**0.1704**	**0.096**	**0.075**	**0.7570**	**0.437**	**0.085**	x	x	x	x	x	x	**0.2320**	**0.122**	**0.058**	**1.4173**	**0.537**	**0.009**
$CFvol$	x	x	x	x	x	x	0.2632	0.512	0.607	-0.5326	2.392	0.824	-0.1222	0.518	0.814	-0.2775	2.392	0.908
LEV	x	x	x	x	x	x	**0.1011**	**0.040**	**0.012**	0.0418	0.169	0.805	**0.1464**	**0.043**	**0.001**	0.2646	0.179	0.140
$INVG$	x	x	x	x	x	x	0.0011	0.146	0.994	-0.5909	0.609	0.333	**0.3421**	**0.186**	**0.065**	0.3606	0.811	0.657
PE	x	x	x	x	x	x	**-0.0101**	**0.002**	**0.000**	0.0042	0.010	0.669	**-0.0056**	**0.003**	**0.036**	0.0185	0.012	0.129

* The bold coefficients indicate a value which is significant with at least pval ≤ 0.1

Water Utility

Table 37: Factor regression model (GLS) for two regimes over the simple returns in the Water Utility sector from 2010 until 2024

	model 1						model 2						model 3					
	R1			R2			R1			R2			R1			R2		
Variable	β	SE	Pval	β	SE	Pval	β	SE	Pval	β	SE	Pval	β	SE	Pval	β	SE	Pval
α	-0.0855	0.106	0.419	**-0.7588**	**0.432**	**0.081**	0.0716	0.079	0.366	0.1377	0.283	0.627	-0.0363	0.252	0.886	-0.4076	1.002	0.685
$Mkt-Rf$	**0.3228**	**0.023**	**0.000**	0.1494	0.103	0.148	x	x	x	x	x	x	**0.3232**	**0.023**	**0.000**	0.1651	0.104	0.115
SMB	0.0051	0.056	0.927	-0.2916	0.244	0.233	x	x	x	x	x	x	0.0059	0.056	0.917	-0.2763	0.249	0.268
HML	**0.2338**	**0.040**	**0.000**	-0.0102	0.197	0.959	x	x	x	x	x	x	**0.2343**	**0.040**	**0.000**	-0.0191	0.200	0.924
MOM	0.0320	0.031	0.303	-0.0140	0.136	0.918	x	x	x	x	x	x	0.0318	0.031	0.306	-0.0154	0.138	0.911
$TERM$	-0.0165	0.020	0.403	-0.0395	0.068	0.564	x	x	x	x	x	x	-0.0197	0.029	0.492	-0.0987	0.117	0.399
DEF	0.0585	0.045	0.197	**0.3295**	**0.189**	**0.083**	x	x	x	x	x	x	0.0502	0.058	0.388	0.2886	0.234	0.219
$CFvol$	x	x	x	x	x	x	0.0843	0.251	0.737	0.2253	1.021	0.826	-0.0183	0.247	0.941	0.5193	1.044	0.619
LEV	x	x	x	x	x	x	0.0030	0.020	0.878	0.0417	0.072	0.564	0.0051	0.020	0.803	0.0586	0.078	0.453
$INVG$	x	x	x	x	x	x	0.0316	0.072	0.659	-0.1052	0.260	0.686	-0.0029	0.088	0.974	-0.1505	0.354	0.671
PE	x	x	x	x	x	x	-0.0008	0.001	0.475	-0.0041	0.004	0.324	-0.0006	0.001	0.654	-0.0058	0.005	0.275

* The bold coefficients indicate a value which is significant with at least pval ≤ 0.1

Wald test statistics

Table 38: Wald test statistics for the univariate test of equal coefficients in model 1 between regimes

Sector	α	$Mkt-Rf$	SMB	HML	MOM	$TERM$	DEF
Agg. Return	0.406 0.524	1.044 0.307	0.781 0.377	0.610 0.435	5.825 0.016	0.058 0.809	0.515 0.473
Airport	0.055 0.814	1.119 0.29	0.374 0.541	0.173 0.677	0.192 0.662	0.003 0.96	0.105 0.745
Communications	**7.441** 0.006	0.529 0.467	0.129 0.72	0.029 0.864	0.358 0.55	**2.801** 0.094	**9.397** 0.002
Datacenters	0.032 0.859	0.189 0.663	2.282 0.131	0.651 0.42	0.255 0.614	1.343 0.246	0.002 0.966
Electric Utility	0.322 0.57	2.608 0.106	0.413 0.52	0.388 0.533	0.058 0.81	0.009 0.926	0.12 0.729
Gas Utility	0.023 0.879	1.452 0.228	0.015 0.903	1.308 0.253	**5.111** 0.024	**2.756** 0.097	0.132 0.717
Multi Utility	0.0 0.993	**8.858** 0.003	0.599 0.439	0.005 0.943	0.132 0.716	0.297 0.586	0.004 0.947
Other	**14.441** 0.0	0.698 0.403	0.857 0.354	2.333 0.127	0.188 0.664	0.847 0.358	**13.042** 0.0
Pipelines	**4.554** 0.033	0.056 0.813	0.0 0.992	0.065 0.799	2.071 0.15	**3.154** 0.076	**6.609** 0.01
Port	0.062 0.803	0.531 0.466	1.279 0.258	0.738 0.39	**3.136** 0.077	0.011 0.917	0.234 0.629
Railroads	0.063 0.802	0.807 0.369	**4.227** 0.04	0.013 0.908	2.38 0.123	0.001 0.975	0.002 0.966
Toll Roads	0.877 0.349	1.517 0.218	0.563 0.453	0.743 0.389	2.686 0.101	0.28 0.597	1.721 0.19
Water Utility	2.292 0.13	**2.7** 0.1	1.405 0.236	1.469 0.226	0.108 0.742	0.104 0.747	1.947 0.163

* The bold coefficients indicate a value which is significant with at least pval ≤ 0.1

Table 39: Wald test statistics for the univariate test of equal coefficients in model 2 between regimes

Sector	α	$CFvol$	LEV	$INVG$	PE
Agg. Return	0.012 0.913	1.963 0.161	0.065 0.798	1.913 0.166	0.684 0.408
Airport	1.601 0.206	1.49 0.222	0.008 0.927	**4.023** 0.045	**3.234** 0.072
Communications	0.002 0.969	**2.767** 0.096	0.068 0.794	0.207 0.649	0.221 0.638
Datacenters	0.622 0.43	0.982 0.322	0.272 0.602	0.324 0.569	0.079 0.779
Electric Utility	0.01 0.921	**3.361** 0.067	0.457 0.499	0.852 0.356	0.998 0.318
Gas Utility	**3.753** 0.053	**3.398** 0.065	0.5 0.479	0.069 0.792	**8.009** 0.005
Multi Utility	1.768 0.184	**3.375** 0.066	0.008 0.928	0.83 0.362	**4.0** 0.046
Other	0.453 0.501	1.117 0.291	0.113 0.737	0.186 0.667	0.596 0.44
Pipelines	0.872 0.35	0.69 0.406	0.678 0.41	0.252 0.616	0.121 0.728
Port	1.726 0.189	0.552 0.457	0.063 0.801	0.168 0.682	0.167 0.683
Railroads	0.29 0.59	**5.727** 0.017	0.286 0.593	1.472 0.225	**5.544** 0.019
Toll Roads	0.041 0.839	0.324 0.569	0.162 0.687	1.004 0.316	2.213 0.137
Water Utility	0.062 0.803	0.014 0.907	0.236 0.627	0.364 0.546	0.505 0.478

* The bold coefficients indicate a value which is significant with at least pval ≤ 0.1

Table 40: Wald test statistics for the univariate test of equal coefficients in model 3 between regimes

Sector	α	$Mkt - Rf$	SMB	HML	MOM	$TERM$	DEF	$CFvol$	LEV	$INVG$	PE
Agg. Return	1.089 0.297	1.038 0.297	0.552 0.457	0.804 0.370	6.438 0.011	0.181 0.670	1.039 0.308	0.050 0.824	2.058 0.151	0.282 0.103	0.157 0.692
Airport	0.003 0.955	1.078 0.299	0.113 0.737	0.065 0.799	0.079 0.779	0.189 0.664	0.093 0.76	0.795 0.372	0.004 0.952	2.659 0.103	0.98 0.322
Communications	**7.014** 0.008	0.636 0.425	0.074 0.785	0.104 0.747	0.526 0.468	0.007 0.933	**10.521**| 0.001	0.028 0.867	2.186 0.139	0.256 0.613	1.261 0.261
Datacenters	0.007 0.935	0.164 0.685	1.804 0.179	0.69 0.406	0.226 0.634	0.868 0.351	0.03 0.863	0.219 0.639	0.05 0.822	0.099 0.752	0.129 0.719
Electric Utility	0.762 0.383	2.456 0.117	0.451 0.502	0.379 0.538	0.075 0.784	0.537 0.464	0.483 0.487	1.916 0.166	0.376 0.54	1.529 0.216	0.034 0.854
Gas Utility	**3.631** 0.057	**2.769** 0.096	0.019 0.89	1.017 0.313	**4.128** 0.042	1.486 0.223	1.344 0.246	2.522 0.112	0.093 0.761	2.108 0.146	**11.308** 0.001
Multi Utility	**3.034** 0.082	**7.529** 0.006	0.59 0.443	0.004 0.951	0.034 0.854	2.013 0.156	0.986 0.321	0.823 0.364	1.862 0.172	1.242 0.265	**2.711** 0.1
Other	**9.221** 0.002	0.266 0.606	0.485 0.486	**3.222** 0.073	0.132 0.716	0.198 0.657	**17.368** 0.0	**4.628** 0.031	0.813 0.367	0.233 0.629	0.046 0.83
Pipelines	0.015 0.903	0.031 0.86	0.056 0.813	0.084 0.772	1.996 0.158	**5.086** 0.024	2.564 0.109	0.37 0.543	2.36 0.124	1.641 0.2	0.104 0.747
Port	1.698 0.193	0.561 0.454	0.906 0.341	0.98 0.322	**3.572** 0.059	0.587 0.444	0.51 0.475	1.261 0.261	0.811 0.368	0.65 0.42	0.593 0.441
Railroads	0.668 0.414	0.387 0.534	**4.069** 0.044	0.001 0.976	**3.159** 0.076	1.185 0.276	0.056 0.813	**3.579** 0.059	0.001 0.971	0.011 0.916	**5.336** 0.021
Toll Roads	**4.705** 0.03	1.363 0.243	0.256 0.613	1.109 0.292	**3.416** 0.065	0.343 0.558	**5.191** 0.023	0.274 0.6	0.647 0.421	0.017 0.895	**3.734** 0.053
Water Utility	0.31 0.578	2.096 0.148	1.212 0.271	1.515 0.218	0.125 0.723	0.349 0.555	1.274 0.259	0.371 0.542	0.647 0.421	0.108 0.743	0.864 0.353

* The bold coefficients indicate a value which is significant with at least pval ≤ 0.1

Table 41: Wald test statistics for the multivariate test of equal coefficients in model 1; regime 1

Sector	α	$Mkt-Rf$	SMB	HML	MOM	$TERM$	DEF
Agg. Return	0.406 0.524	1.044 0.307	0.781 0.377	0.610 0.435	5.825 0.016	0.058 0.809	0.515 0.473
Airport	4.989 0.545	**20.922** 0.002	7.51 0.276	0.039 1.0	7.754 0.257	7.007 0.32	**18.613** 0.005
Communications	**63.676** 0.0	2.691 0.847	1.451 0.963	0.161 1.0	1.175 0.978	0.0 1.0	**13.284** 0.039
Datacenters	5.352 0.5	**329.125** 0.0	**99.116** 0.0	**103.76** 0.0	0.441 0.998	0.309 0.999	0.042 1.0
Electric Utility	7.974 0.24	**23.065** 0.001	**16.527** 0.011	1.589 0.953	4.448 0.616	9.14 0.166	4.72 0.58
Gas Utility	1.634 0.95	2.24 0.896	9.162 0.165	0.529 0.997	4.754 0.576	3.793 0.705	5.765 0.45
Multi Utility	0.401 0.999	**255.155** 0.0	**292.478** 0.0	**30.0** 0.0	2.409 0.878	0.417 0.999	2.304 0.89
Other	**253.685** 0.0	4.707 0.582	**14.984** 0.02	3.375 0.76	0.07 1.0	0.002 1.0	**61.295** 0.0
Pipelines	**36.24** 0.0	**270.017** 0.0	**56.269** 0.0	**109.312** 0.0	**35.048** 0.0	**16.024** 0.014	**11.855** 0.065
Port	0.942 0.988	**15.062** 0.02	**23.526** 0.001	0.022 1.0	4.756 0.575	8.393 0.211	**22.786** 0.001
Railroads	3.973 0.68	8.163 0.226	**101.023** 0.0	3.834 0.699	**18.32** 0.005	**12.468** 0.052	**25.62** 0.0
Toll Roads	**17.945** 0.006	2.172 0.903	7.27 0.297	0.03 1.0	0.171 1.0	0.03 1.0	0.229 1.0
Water Utility	9.476 0.149	**20.987** 0.002	1.901 0.929	8.475 0.205	0.765 0.993	3.184 0.785	0.142 1.0

* The bold coefficients indicate a value which is significant with at least pval ≤ 0.1

Table 42: Wald test statistics for the multivariate test of equal coefficients in model 1; regime 2

Sector	α	$Mkt-Rf$	SMB	HML	MOM	$TERM$	DEF
Airport	1.152 0.979	0.394 0.999	0.002 1.0	0.29 1.0	0.568 0.997	0.191 1.0	1.199 0.977
Communications	**45.388** 0.0	0.347 0.999	0.184 1.0	0.234 1.0	1.37 0.968	0.0 1.0	**16.406** 0.012
Datacenters	0.073 1.0	**13.881** 0.031	0.837 0.991	3.682 0.72	0.065 1.0	0.004 1.0	0.185 1.0
Electric Utility	2.658 0.85	4.37 0.627	0.193 1.0	0.006 1.0	0.034 1.0	0.342 0.999	0.003 1.0
Gas Utility	0.108 1.0	0.403 0.999	0.264 1.0	1.205 0.977	0.538 0.997	0.009 1.0	1.034 0.984
Multi Utility	0.163 1.0	**20.757** 0.002	9.739 0.136	0.927 0.988	0.713 0.994	0.354 0.999	0.308 0.999
Other	5.314 0.504	0.004 1.0	1.053 0.984	0.68 0.995	1.53 0.957	0.568 0.997	4.404 0.622
Pipelines	4.782 0.572	**13.339** 0.038	5.755 0.451	5.211 0.517	1.255 0.974	0.954 0.987	**13.148** 0.041
Port	0.814 0.992	0.592 0.997	0.007 1.0	0.311 0.999	0.564 0.997	0.31 0.999	1.943 0.925
Railroads	0.013 1.0	1.173 0.978	0.005 1.0	0.329 0.999	0.008 1.0	0.514 0.998	1.15 0.979
Toll Roads	6.913 0.329	1.041 0.984	0.015 1.0	0.365 0.999	1.109 0.981	0.006 1.0	1.843 0.934
Water Utility	8.809 0.185	1.138 0.98	0.796 0.992	0.128 1.0	0.116 1.0	0.052 1.0	3.487 0.746

* The bold coefficients indicate a value which is significant with at least pval ≤ 0.1

Table 43: Wald test statistics for the multivariate test of equal coefficients in model 2 regime 1

Sector	α	$CFvol$	LEV	$INVG$	PE
Airport	0.0 1.0	**15.786** 0.003	2.018 0.732	1.18 0.881	2.138 0.71
Communications	**23.343** 0.0	**12.721** 0.013	0.054 1.0	1.121 0.891	0.001 1.0
Datacenters	0.286 0.991	1.542 0.819	0.051 1.0	0.106 0.999	0.024 1.0
Electric Utility	0.149 0.997	2.644 0.619	0.089 0.999	0.414 0.981	0.089 0.999
Gas Utility	1.723 0.786	**13.339** 0.01	2.586 0.629	0.177 0.996	1.328 0.857
Multi Utility	0.147 0.997	0.055 1.0	0.033 1.0	0.024 1.0	0.015 1.0
Other	5.708 0.222	2.322 0.677	1.696 0.791	6.445 0.168	0.137 0.998
Pipelines	0.854 0.931	0.55 0.968	0.173 0.996	0.182 0.996	0.037 1.0
Port	0.001 1.0	1.745 0.783	1.023 0.906	0.064 0.999	0.351 0.986
Railroads	0.088 0.999	0.008 1.0	0.002 1.0	0.008 1.0	0.027 1.0
Toll Roads	0.752 0.945	5.001 0.287	0.558 0.968	1.162 0.884	1.633 0.803
Water Utility	0.238 0.993	0.038 1.0	0.02 1.0	0.001 1.0	0.031 1.0

* The bold coefficients indicate a value which is significant with at least pval ≤ 0.1

Table 44: Wald test statistics for the multivariate test of equal coefficients in model 2 regime 2

Sector	α	$CFvol$	LEV	$INVG$	PE
Airport	0.0 1.0	0.37 0.985	0.355 0.986	0.357 0.986	0.345 0.987
Communications	4.041 0.4	**13.409** 0.009	0.526 0.971	0.032 1.0	0.561 0.967
Datacenters	1.22 0.875	4.05 0.399	0.126 0.998	0.014 1.0	0.189 0.996
Electric Utility	1.138 0.888	**11.394** 0.022	0.717 0.949	0.231 0.994	0.964 0.915
Gas Utility	0.004 1.0	**13.113** 0.011	1.599 0.809	1.212 0.876	1.743 0.783
Multi Utility	0.038 1.0	**7.916** 0.095	1.075 0.898	0.279 0.991	1.111 0.892
Other	0.532 0.97	1.697 0.791	0.023 1.0	2.274 0.685	0.14 0.998
Pipelines	0.008 1.0	2.041 0.728	0.309 0.989	0.1 0.999	0.219 0.994
Port	1.315 0.859	1.032 0.905	0.02 1.0	0.0 1.0	0.0 1.0
Railroads	0.86 0.93	**14.216** 0.007	1.337 0.855	0.211 0.995	1.506 0.825
Toll Roads	0.213 0.995	0.065 0.999	0.024 1.0	0.228 0.994	0.014 1.0
Water Utility	0.148 0.997	0.051 1.0	0.015 1.0	0.091 0.999	0.0 1.0

* The bold coefficients indicate a value which is significant with at least pval ≤ 0.1