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Improving the intuitive appeal of optimized portfolios: the incorporation of macroeconomic uncertainty and investor sentiment within the portfolio optimization framework

Author: Adit Chadha
Student number: 590125
Thesis supervisor: Dr. Marshall X. Ma
Second reader: Dr. Jan J.G. Lemmen
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ABSTRACT

This study evaluates the effectiveness of incorporating investor sentiment and macroeconomic uncertainty into the portfolio optimization framework through additional constraints in the optimization problem. A rolling methodology of portfolio construction is employed to assess the out-of-sample performance of several variance-minimizing portfolios. These portfolios, constructed under different combinations of alternative covariance matrix estimators and sentiment and uncertainty constraints, are analyzed using two separate datasets of asset returns from July 2001 to February 2024. The results provide moderately strong evidence in support of incorporating investor sentiment and macroeconomic uncertainty. My findings encourage researchers to assess their proposed optimized portfolios under different datasets of asset returns to note potential data-dependent performance. Furthermore, the effectiveness of my proposed optimized portfolios demonstrates the potential for using optimization constraints to incorporate other variables into the optimization framework.

Keywords: portfolio optimization, investor sentiment, macroeconomic uncertainty

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1 Introduction

While investors may care for more than returns and return variances, the standard portfolio optimization framework fails to account for this. This leads to the construction of intuitively unappealing optimized portfolios (Fisher & Statman, 1997). This study addresses this shortcoming by incorporating investor sentiment (IS) and macroeconomic uncertainty (MU) into the framework. Prior research has documented IS's significant effect on stock returns (Schmeling, 2009) and MU's significant effect on stock return volatility (Iania et al., 2023). Such findings illustrate the potential for integrating IS and MU to develop optimized portfolios with attractive returns and return volatilities. Additionally, incorporating these two factors in a way that allows investors to adjust the extent of their effect on optimized portfolio weights can give rise to more intuitively appealing portfolios by accounting for investor preferences. Behr et al. (2013) show that, against the findings presented in other literature, there are optimization techniques that can consistently outperform the nave, equally weighted portfolio. Accordingly, this study explores the performance of optimized portfolios incorporating IS and MU relative to the naively diversified portfolio.

Researchers have explored several methods of improving upon the standard portfolio optimization framework developed by Markowitz (1952) and addressing its limitations. Behr et al. (2013) develop two portfolio strategies that incorporate endogenous weight constraints that minimize the estimation error present in the sample covariance matrix. Their strategies significantly outperform the naive strategy, in out-of-sample Sharpe ratio, in five of six considered datasets of 45 years of returns on American stocks and portfolios of stocks. On the other hand, Campbell et al. (2001) address the wrongful return normality assumption drawn when using standard deviation as a risk measure, as is done in the standard portfolio optimization framework. They propose Value-at-Risk (VaR) as an alternative risk measure, combined with the assumption of a fatter-tailed student-t distribution. The authors note that by allowing the user to set the confidence level associated with the VaR, the optimized portfolios account for the user’s risk aversion. Banholzer et al. (2019) integrate IS within mean-variance portfolio optimization using a Copula Opinion Pooling approach and consider several stock market indices from 1993 to 2015. Their proposed strategy adjusts portfolio weights to account for price reversals due to initial sentiment-led mispricing, and it outperforms five other benchmark strategies, including the naive strategy, in several performance metrics.

Although Banholzer et al. (2019) demonstrate the effectiveness of incorporating IS into portfolio optimization, their proposed strategy does not allow for the convenient adjustment of the extent to which IS should affect portfolio weights. The importance of convenient adjustment stems from the heterogeneity of investors and their preferences. Campbell et al. (2001) develop a framework that allows for the convenient incorporation of the investor’s risk tolerance but relates to their general risk and not risk tolerance towards the portfolio’s sensitivity to IS and MU. Kirby & Ostdiek (2012) present two active portfolio strategies that incorporate a tuning parameter allowing user-specified control over portfolio turnover. Such user-specified control can increase the strategy's practical applicability by catering to investor-specific preferences. Consequently, this paper explores the incorporation of IS and MU into portfolio optimization in a manner that accounts for the investor’s risk tolerance towards fluctuations in their portfolio value due to changes in IS and MU. The question studied in this paper is the following:

“Can incorporating macroeconomic uncertainty and investor sentiment in the portfolio optimization framework produce portfolios that outperform the equally weighted portfolio in the out-of-sample period?”

The methodology for answering this question follows the standard portfolio construction and out-of-sample testing performed by Behr et al. (2013), with an estimation window of 60 months and a portfolio holding period of 12 months. IS and MU are integrated using additional constraints on the constructed portfolio’s sensitivity to these two factors. Beta values on the IS or MU variable are determined for all assets through standard OLS regressions of asset returns on IS or MU to determine portfolio sensitivity, computed as the weighted average of absolute regression betas of individual assets. The upper limit on the sensitivity constraint is a specified percentile value of the individual asset betas, which can be conveniently adjusted to account for the investor’s risk tolerance to changes in portfolio value due to IS and MU. I analyze various optimization strategies with different combinations of IS and MU constraints and covariance estimators. Finally, I perform significance tests comparing all strategies' mean returns and return variances relative to the naive strategy. My research focuses on the US stock market. I utilize two datasets of asset returns frequently used in portfolio optimization literature: the 17 industry portfolios and the 25 size and book-to-market ratio portfolios from the Kenneth French data library. Macroeconomic variables are all sourced from the Federal Reserve Economic Data, and the proxy for IS is sourced from the American Association of Individual Investors. The proxy for MU is obtained from the proxy’s developers' publishing website.

Based on the evidence of the significant effects of IS on returns and of MU on return volatility, as well as the benefit of implementing IS into portfolio optimization demonstrated by Banholzer et al. (2019), I hypothesized that the IS and MU strategies would significantly outperform the equally weighted benchmark strategy. My obtained results demonstrate moderately strong support for the incorporation of MU and IS when considering overall performance over both datasets; this motivates the incorporation of relevant variables using additional constraints in the portfolio optimization framework.

The remainder of this paper is structured as follows. Chapter 2 expands on academic literature on portfolio optimization, IS, and MU and defines the two hypotheses tested in this study. Chapter 3 discusses the collection and construction of all relevant data. The relevant optimization framework and the significance tests that were performed are explained in Chapter 4. Chapter 5 then illustrates the results of this study, while Chapter 6 discusses the performed robustness tests. Finally, Chapter 7 concludes the results of this study, identifies its limitations, and accordingly poses topics for future research.

2 Theoretical Framework

2.1 Portfolio Optimization

The foundations for modern advancements in portfolio optimization are derived from the framework developed by Markowitz (1952), where efficient portfolios are defined according to two factors: expected returns and variance. As proposed by Markowitz, efficient portfolios are those with the maximum expected portfolio return for a given portfolio variance or the minimum portfolio variance for a given expected portfolio return. This framework aims to narrow the choice of different allocations of invested capital amongst assets to a set of efficient allocations, or portfolios, which the investor can then choose from based on their preferences (p. 91). Additionally, Markowitz segments the portfolio selection problem into two stages. The first stage pertains to forming beliefs over expected asset returns and asset covariances, which are used to determine portfolio variance. In contrast, the second stage pertains to the method of selecting the portfolio based on these beliefs. The focus of Markowitz's paper was on the second stage. Since the publishing of Markowitz (1952), many other papers have expanded on both stages of portfolio selection.

2.1.1 Expansions on stage one of portfolio selection

According to Markowitz, stage one of portfolio selection is forming beliefs on expected asset returns and covariances.

One of the most critical limitations of tangency portfolios, which are portfolios developed from the work of Markowitz (1952) and formed by maximizing Sharpe ratios, is the significant role of estimation error in forming these portfolios. Estimation error in the context of portfolio optimization is the error that arises because historical, sample-based values of all optimization inputs are poor estimators for their true population values (Jagannath & Ma, 2003, p. 1652). This error can be present in the computation of an asset's expected returns, variances, and covariances. Furthermore, portfolio weights are highly sensitive to estimation error, resulting in highly unstable portfolios with poor out-of-sample Sharpe ratios and high turnover when rebalancing the portfolio (Michaud & Michaud, 2007). The shortcoming of estimation errors has incited a notable number of researchers to employ various techniques to overcome this challenge. Some of this work is discussed below.

The application of shrinkage theory to the covariance matrix, an input to the optimization framework, is one mechanism used to address estimation errors. Shrinking the covariance estimator entails reducing the larger estimated covariances amongst assets. Jagannathan & Ma (2003) prove the equivalence of imposing non-negativity constraints on portfolio weights to utilizing a shrunk covariance estimator. When high estimated covariances exist because of upward-biased estimation error, shrinking the covariance estimator through non-negativity constraints can reduce sampling error (p. 1652). Using the 25 Fama-French size and book-to-market sorted portfolios and, separately, a random sample of 500 stocks trading on the NYSE and the AMEX, the authors obtain monthly out-of-sample returns from May 1968 to April 1999 for different optimized portfolios and the naive portfolio. They show that minimum variance portfolios outperform tangency portfolios in out-of-sample Sharpe ratios, arguing that this is due to the reduced impact of estimation errors in minimum-variance portfolios. They also find that minimum variance portfolios constructed under non-negativity constraints perform as well as those constructed under more complicated modifications to the covariance matrix structure, such as covariance estimators derived from assuming specific factor models for returns.

Black & Litterman (1992) provided a seminal contribution to the field of portfolio optimization, significantly increasing the applicability of the mean-variance framework. Their adjustment to this framework allows for incorporating the investor's views, particularly those that differ from the market's. This allows investors to utilize models that can predict returns more accurately than historical estimates and incorporate those predicted returns within the mean-variance optimization framework. The user's confidence in each absolute or relative view is also accounted for. These confidence levels determine the weight to give to the user's view compared to the views held in market equilibrium when computing the new expected returns. The optimized weights are then obtained using these new expected returns in a standard mean-variance optimization framework (refer to section 4.1.2). The authors explain their methodology and its intuition using a dataset on equities, bonds, and currencies of seven different countries from January 1975 to August 1991, standardizing all values to dollar terms.

Banholzer et al. (2019) demonstrate the benefits of introducing new variables in the optimization framework to improve the information held in asset returns and covariances. Upon noting that sentiment can be a signal for medium-term mean reversion of stock prices, they construct a portfolio strategy that incorporates IS, specifically IS-led mispricing, into portfolio optimization. Notably being the first to do so. Their resultant portfolio strategy, formed by maximizing the Sharpe ratio after accounting for sentiment information, entails underweighting assets with high sentiment in the past six months to two years and similarly overweighting assets with low past sentiment. Sentiment in their framework is incorporated using the Copula Opinion Pooling approach, an alternative to the Black-Litterman model that allows for multiple views on each asset and can account for dependencies amongst views. Over an out-of-sample period from August 2004 to September 2015, their proposed strategy outperforms all benchmark strategies, including the standard tangency portfolio and the equally weighted portfolio, in Sharpe ratios and other performance metrics. However, it exhibits considerably higher turnover than most benchmark strategies.

2.1.2 Expansions on stage two of portfolio selection

According to Markowitz, stage two of portfolio selection relates to the method of selecting the optimal portfolio using the beliefs on expected asset returns, covariances, and variances.

In constructing tangency portfolios, the standard deviation is used as a downside risk measure. However, using standard deviation incorrectly assumes that expected returns are normally distributed (Campbell et al., 2001, p. 1790). As concluded by Fama (1965), the normal distribution is an inadequate representation of empirical stock return distributions. Campbell et al. (2001) address this limitation by using the portfolio's VaR, defined as the portfolio's maximum expected loss with a given level of confidence over a specified time horizon (p. 1791), as an alternative risk measure to standard deviation. This allows for the assumption that returns follow other, more appropriate distributions, such as the fatter-tailed student-t distribution. They accordingly adjust Sharpe ratios by utilizing VaR as the appropriate risk measure in the denominator of the Sharpe ratio formula (refer to section 4.3). Tangency portfolios are then created using data from January 1990 until December 1998 on the S&P 500 composite return index, the 10-year US government bond return index, and the 3-month US Treasury Bill rate as the risk-free rate. Their analysis of tangency portfolios created under normal and non-normal distributions leads to the conclusion that standard deviation underestimates the downside risk and allocates too much of a portfolio weight to risky assets. Moreover, the consideration of VaR also allows for the inclusion of the investor's risk aversion in the portfolio optimization framework since the investor sets the confidence level in the VaR (p. 1802).

Green and Hollifield (1992) note the potential introduction of a specification error within the optimization framework when portfolio weight constraints are imposed. In practice, these constraints may be wrong, as relatively high or low weights for certain assets might not result from estimation errors. Resultantly, relevant sample information may be omitted when imposing portfolio weight constraints. Jagannathan and Ma (2003) oppose the certainty of poor out-of-sample performance from introducing minimum weight constraints and provide the argument that such constraints sufficiently reduce the sampling error present in the covariance matrix and lead to improved out-of-sample portfolio performance. This leads to them recognizing the sampling error – specification error trade-off.

Behr et al. (2013) extend the optimization framework by accounting for this trade-off within the optimization problem. Specifically, they propose new strategies that impose endogenous maximum and minimum weight constraints, which are determined by minimizing the estimation error in the covariance matrix estimator (p. 1232). They first obtain a risk function representing the sum of mean squared errors of the covariance matrix entries, a quantification of the estimation error. The risk function is then minimized with respect to the upper limit of the maximum weight constraint and the lower limit of the minimum weight constraint to determine the 'optimal' upper and lower limits. The authors apply their flexible weights mechanism to variance-minimizing portfolios and compare their strategy's performance to naive portfolios and several other minimum-variance portfolios with different structures to the covariance matrix estimator. To analyze the effectiveness of their addition, they compare out-of-sample performance metrics of all portfolio strategies under six different datasets of monthly asset returns from July 1963 to December 2008.

Their proposed strategy, which incorporates flexible maximum and minimum weight constraints, achieves significantly higher out-of-sample Sharpe ratios than the equally weighted portfolio in five out of six datasets and significantly higher out-of-sample variances in all datasets.

Moving beyond the mean-variance and minimum-variance optimization frameworks, researchers have explored alternative frameworks for constructing portfolios. Notably, Kirby & Ostdiek (2012) explore alternative strategies that do not require optimization yet still exploit sample information on mean returns and return variances. The authors construct two simple active portfolio strategies. The first determines portfolio weights based solely on the conditional volatility of individual assets; as an asset's conditional return volatility increases relative to other assets, its weight is decreased. The second strategy functions similarly but considers an asset's relative ratio of conditional expected return to conditional return volatility instead of conditional return volatility alone. These two strategies and other benchmarks are evaluated over four datasets of asset returns from July 1963 to December 2008. Their proposed strategies significantly outperform the equally weighted portfolio in out-of-sample Sharpe ratios, even after accounting for transaction costs. Both strategies also allow for control over portfolio turnover, and thus incurred transaction costs, via a tuning parameter, increasing their practical applicability as they account for preferences over portfolio turnover. Kirby & Ostdiek highlight the importance of using multiple datasets in portfolio literature as their results are shown to depend on the spread of conditional expected returns and conditional return volatilities of assets within a dataset. An increase in the relevant spread improves the relative performance of their proposed strategies.

2.2 Macroeconomic Uncertainty (MU)

A large body of literature on MU focuses on how to measure it, with economic and financial researchers having employed a vast range of methodologies to do so. Jo and Sekkel (2019) use forecast errors in measuring MU, specifically the volatility of forecast errors of several macroeconomic variables from the Survey of Professional Forecasters. On a global level, Van Robays (2016) proxies MU as the volatility of world industrial production growth using a GARCH(1,1) process. The use of forecast errors and volatilities of macroeconomic quantities is one of the more common approaches to measuring MU; however, economic policy uncertainty (EPU) is also often utilized as a proxy for MU. Baker et al. (2016) constructed an EPU index (refer to section 3.2), which serves as a convenient measure of MU in academia. Berger et al. (2017) used this index to confirm the validity of their constructed MU measure's movements in times of uncertainty, and Caldara et al. (2016) used it as one of six measures used to 'infer fluctuations in macroeconomic uncertainty' (p. 188). In essence, this body of research emphasizes the complexity of measuring MU, with no one accepted approach to this measurement.

Resembling approaches used in prior literature, Iania et al. (2023) form their own MU measures for the US based on monthly, one-year-ahead forecasts for ten macroeconomic variables over twenty countries. The authors utilize four different approaches to forming these MU measures. Namely, monthly standard deviations of forecasts, their range and interquartile range, and a measure based on the time-varying monthly variance of forecast errors obtained using an AR-GARCH process. Their analysis uses monthly US stock return data from 1989 to 2019. The considered stocks are identified to belong to 49 different industries, allowing the author to observe inter-industry differences in the effects of MU on returns and return volatility. To identify these differences, they perform four regressions for each industry; a separate regression for each of the four alternative measures of MU. Specifically, they regress (volatility of) stock returns on MU and relevant controls. Their regression results demonstrate clear evidence for a positive effect of MU on return volatility but weak evidence for the observed positive effect of MU on returns. Across the four alternative regressions, 41.7 out of 49 industries, on average, demonstrate significant effects of MU on volatility, while only 13 industries, on average, demonstrate a significant effect on returns.

While MU is typically measured in a manner that treats both positive and negative movements in macroeconomic variables as the same form of MU, Segal et al. (2015) decompose MU into a 'good' and a 'bad' component. The 'good' component of MU is measured by the realized semivariance of positive shocks to macroeconomic variables, such as output and consumption, computed as the variance of positive deviations from the mean values of macroeconomic variables. Contrarily, the 'bad' component is measured by the realized semivariance of negative shocks, computed as the variance of negative deviations from the mean. Regressing aggregate market price variables and, separately, future return volatility on current expected consumption growth, good MU, and bad MU shows that equity price increases with good uncertainty but decreases with bad uncertainty. Furthermore, bad uncertainty is shown to have a larger positive effect on future return volatility. Overall, their findings of the heterogenous effects of good and bad MU motivate asset pricing models to distinguish between the two components.

Corroborating the findings of Segal et al. (2015), Chiu (2020) finds further evidence for differences between good and bad MU. Focusing on the Taiwanese stock market and macroeconomy from January 2000 to December 2017, Chiu finds that stock liquidity decreases with bad uncertainty and increases with good uncertainty. Additionally, the author considers the role of information competition in the MU-liquidity relationship. This is achieved by using interaction effects between both MU components and a proxy for information competition in the regression of liquidity on good MU, bad MU, and relevant control variables. The proxy for information competition is a dummy variable for when the number of informed investors in the market, measured as the number of institutional investors holding the considered stock, exceeds the cross-sectional median. Using separate regressions for each considered stock on the Taiwanese Stock Exchange and Wald's test for significance, they note that when information competition is higher, the positive impact of good uncertainty on stock liquidity increases while the negative impact of bad uncertainty on stock liquidity decreases. Therefore, this study highlights the benefit to all stock market participants of the presence of informed investors in times of good and bad uncertainty.

2.3 Investor Sentiment (IS)

Baker & Wurgler (2007) define IS as ‘a belief about future cash flows and investment risks that is not justified by the facts at hand’ (p. 129), posing an irrational view on sentiment-based trading.

Baker & Wurgler (2006) provided a method for quantifying sentiment using indirect measures to form a sentiment index that is widely utilized in academia (Stambaugh et al., 2012; Yu & Yuan, 2011). Their sentiment index is the first principal component on the current and lagged values of six sentiment proxies, including NYSE share turnover and the annual number of IPOs. Their study then focuses on assessing whether sentiment has larger effects on stocks that have highly subjective valuations and are harder to arbitrage. Subsequently, they consider stocks with these characteristics, such as those that are newly listed, smaller, more volatile, etc. To perform their analysis, the authors looked at monthly stock returns from 1963 to 2001 and formed equal-weighted decile portfolios based on firm characteristics such as the recency of going public, size, return volatility, etc. Then, the decile portfolios are used to form long-short portfolios based on these characteristics and regress their monthly returns on IS. When sentiment is low, being below the sample average, the stocks that are smaller, more recently listed on public exchanges, more volatile, etc., earn higher returns. When sentiment is high, this pattern generally reverses.

Coinciding with the significant, negative IS-return relationship noted by Brown & Cliff (2005), Schmeling (2009) finds that periods of increasing sentiment ‘tend to be followed by lower returns for the aggregate market’ (p. 394). Schmeling then shifts his analysis to explore the interaction effect between sentiment and two factors, the development of national market institutions and cultural proneness to overreaction, when analyzing the impact of sentiment on stock returns. He explored this interaction effect by collecting aggregate market returns and sentiment measure data for 18 industrialized countries from January 1985 to December 2005. The countries are split into two groups based on median values for proxies for the development of national market institutions, such as accounting standards, and proxies for cultural proneness to overreaction, such as a collectivism index. This allows the author to run panel regressions on both datasets of countries for each proxy, revealing that cultural proneness to overreaction strengthens the sentiment-return relation while increased development of national market institutions weakens it.

Literature on the topic of IS is conflicted on the rationality of sentiment-based trading. Some studies, including Schmeling (2009), Black (1986), and Baker & Wurgler (2007), view investors for whom IS is a determinant of trading behavior as irrational, ‘noise’ traders, while others oppose this view. De Long et al. (1990) argue that the presence of sentiment-based, ‘noise’ traders in the market can stimulate professional arbitrageurs to consider sentiment-related information in their strategies (p. 735), implying that incorporating sentiment alone does not categorize an investor as irrational. Brown & Cliff (2004) found that the greatest sentiment-return relationship was the effect of IS on large stock returns. As large stocks are disproportionately traded more often by institutional investors, their findings suggest that rational institutional investors also account for IS information. Verma & Verma (2008) found that institutional IS and individual IS exhibit fundamental differences; the former is more sensitive to changes in relevant risk factors, such as market returns, and thus more rational. Their work signifies that the arguments of both sides of the sentiment rationality conflict can coexist and be treated as credible. The findings of those arguing for irrationality may specifically relate to individual investors.

2.4 Hypotheses

IS is a relevant but inadequately explored variable in portfolio optimization frameworks. Its relevance is first demonstrated by Banholzer et al. (2019), who developed a sentiment reversal-based portfolio optimization strategy that significantly outperforms the equally weighted portfolio in Sharpe ratios. Banholzer et al. conclude their study by motivating the exploration of alternative methods that incorporate 'sentiment as an optimization criterion instead of using it as an input variable' (p. 701). Moreover, sentiment is documented to negatively and significantly impact a portfolio performance metric, specifically mean returns (refer to section 4.3). Building on these findings, I developed a framework that incorporates IS as an optimization criterion and allows the user to decrease the optimized portfolio's sensitivity to IS. I expect the following hypothesis to hold for these optimized portfolios:

Hypothesis 1: ***Optimized portfolios that incorporate investor sentiment as an optimization criterion are able to outperform the equally weighted portfolio in the out-of-sample period.***

Due to MU's significant, positive effect on return volatility observed in the previously discussed literature, limiting an optimized portfolio's sensitivity to MU through appropriate optimization constraints may allow for reduced portfolio volatility. Furthermore, considering the forward-looking nature of MU, its incorporation into portfolio optimization may help mitigate some concerns of estimation error, one of the more significant limitations of portfolio optimization. There is also a notably more prominent absence of MU incorporation within the field of portfolio optimization than there is for IS, further motivating its consideration in this field. This leads to the development of the following hypothesis explored in this study, with a consistent focus on performance relative to the equally weighted portfolio:

Hypothesis 2: ***Optimized portfolios that incorporate macroeconomic uncertainty as an optimization criterion are able to outperform the equally weighted portfolio in the out-of-sample period.***

3 Data

3.1 Asset returns

An optimized portfolio entails the optimal distribution of invested capital across different assets. The assets defined in this study are portfolios of stocks traded on specific US stock exchanges: NYSE, AMEX, and NASDAQ. To perform my analysis, I utilize two separate datasets of portfolios and their average, monthly, value-weighted returns, namely, the 25 size and book-to-market sorted portfolios and the 17 industry-sorted portfolios. These are both collected from the Kenneth R. French Data Library. Despite both datasets consisting of portfolio returns from July 1926 to March 2024, the data on all variables in this study, including portfolio returns, are collected from July 2001 to February 2024, a total of 272 months, as the data on certain variables before July 2001 and after February 2024 are missing. The selection of these datasets was led by their frequent use in portfolio optimization literature (Cai et al., 2024; Behr et al., 2013; Shi et al., 2019), ensuring the validity of the collection and computation of these monthly portfolio returns.

3.1.1 Size & book-to-market portfolios (25SBM)

The 25 portfolios are constructed at the end of June every year, implying potential annual changes to the composition of stocks in each of these 25 portfolios. To construct these annual portfolios, stocks on the NYSE, AMEX, and NASDAQ are first sorted into five portfolios based on size quintiles and independently into five portfolios based on the quintiles of the ratio of book value of equity to market value of equity of the firm (BE/ME). Size quintiles are determined only by the market equity of stocks on the NYSE, while the BE/ME quintiles are determined by the BE/ME ratio of stocks on all three considered US stock exchanges. The 25 portfolios are then formed from the intersection of the five portfolios based on size with the five portfolios based on BE/ME. For example, a stock in the lowest quintile for market equity and the highest quintile for BE/ME would be placed in the 'ME1BM5' portfolio.

Table 1 from Appendix A provides descriptive statistics for all 25 portfolios' monthly returns over the sample period. In determining the effectiveness of optimization strategies, it is insightful to note that, over the entire sample period, the highest average monthly return for a single asset is 1.18%, belonging to the ME1BM5 portfolio, which has a monthly standard deviation of 6.85%, corresponding to a monthly variance of 0.0047. Furthermore, the lowest monthly standard deviation is 4.20%, corresponding to a monthly variance of 0.0018, belonging to the ME5BM2 portfolio, which has an average monthly return of 0.79%.

3.1.2 Industry portfolios (17Ind)

The 17 industry portfolios are also constructed annually, at the end of June, by separating stocks into their respective industries. The separation is based on their four-digit Compustat SIC codes at the end of the previous fiscal year or on their four-digit CRSP SIC codes at the time of portfolio construction. The 17 industries that stocks are categorized into are the following: Food, Mining and Minerals, Oil and Petroleum, Clothes, Consumer Durables, Chemicals, Consumer Products, Construction and Construction Materials, Steel Works, Fabricated Products, Machinery and Business Equipment, Automobiles, Transportation, Utilities, Retail Stores, Financial Services and Others.

Table 2 in Appendix A provides descriptive statistics for all 17 industry portfolios. It is again insightful to note that the Fabricated Products portfolio achieves the highest average monthly return of 1.16% over the entire sample period with a standard deviation of 5.81%, corresponding to a variance of 0.0034. Moreover, the Food portfolio achieves the lowest standard deviation of 3.43%, corresponding to a variance of 0.0012, with an average monthly return of 0.76%.

Note that the statistics in Tables 1 and 2 relate to the entire sample period of 272 months, while the results of optimized portfolios are specific to the out-of-sample period. As explained later in section 4.3, the out-of-sample period pertains to 212 months, with the first month being 60 months ahead of the first month in the overall sample period. However, comparing the lowest variance and highest returns of individual assets from Tables 1 and 2 to those of the optimized portfolios in the relevant dataset can still be insightful. It can help assess the effectiveness of optimization and determine whether optimization was performed correctly.

3.1.3 Risk-free rate

Data on the risk-free rate is collected to perform out-of-sample Sharpe ratio computations for each portfolio strategy. This requires the computation of excess returns, which are calculated as the monthly return minus the monthly risk-free rate. The risk-free rate here is defined as the market yield on US treasury securities at 1-month constant maturity. The data on these yields was collected from the Federal Reserve Economic Data. This definition of the risk-free rate is motivated by the fact that the US Department of the Treasury issues such treasury securities, which are consequently guaranteed by the US government, implying a negligible default rate of such securities. I preprocess the daily annualized yields I collected by dividing by 12 and taking the average in a given month, converting the data to the appropriate monthly frequency.

3.2 Macroeconomic uncertainty (MU)

MU is proxied using the Baker-Bloom-Davis measure of EPU, developed by Baker et al. (2016) and collected from the authors' publications website, where they publish monthly EPU indices for several countries. This choice of proxy follows the decision of Caldara et al. (2016) and Berger et al. (2017), as mentioned in section 2.2. Following macroeconomic shocks, governments tend to implement policies to stabilize the economy, giving rise to uncertainty regarding what policies will be implemented. As such, macroeconomic shocks would generally increase macroeconomic forecast errors, often used to model MU directly (Jo & Sekkel, 2019). Thus, it is intuitive to understand why EPU can proxy MU.

The EPU index is constructed using three main components: newspaper coverage frequency, the number of federal tax code provisions scheduled to expire within ten years, and disagreements in economic forecasts. The latter two components are only used to measure the EPU for the US.

Newspaper coverage frequency is computed using the number of articles published in a month in any of the ten leading newspapers in the US that contain terms relating to all of the following topics: the economy, uncertainty, and policy. Baker et al. also account for variations in the number of articles published in each newspaper over time by appropriately standardizing and normalizing these monthly frequencies, allowing for a more interpretable EPU index across different timeframes (p. 1599). Tax code provisions are laws and regulations specific to calculating and paying taxes for all entities required to pay tax. The expiring tax code provisions account for uncertainty concerning taxation policy in the US. Lastly, the measure of disagreement is specific to that amongst economic forecasters on the future values of the Consumer Price Index (CPI) and federal, state, and local expenditures.

3.2.1 MU regression's control variables

In the regression of returns on MU, proxied by EPU, control variables are chosen by following the methodology of Arouri et al. (2016). Namely, I include the monthly values for inflation rate, change in industrial production, change in unemployment rate, and default spread. The data on all controls is collected from the Federal Reserve Economic Data, and the data frequency for all variables is monthly. The monthly inflation rate is computed as the monthly percentage change in the CPI. The monthly change in industrial production is derived by subtracting the following month's industrial production index (IPI) from that of the current month, as the observations for IPI are collected at the start of each month. Similarly, the monthly change in unemployment rate is computed by subtracting the following month's unemployment rate from that of the current month. Finally, although undefined by Arouri et al. (2016), the default spread is measured as the difference between BAA and AAA corporate bond yields for the given month, as defined by Segal et al. (2015).

3.3 Investor sentiment (IS)

I proxy IS using the American Association of Individual Investors (AAII) index, as is frequently done in financial literature (Chau et al., 2016; Wang et al., 2006). The AAII sends weekly surveys to all members to vote on whether they are bullish, neutral, or bearish on the US stock market in the next six months. Thus, the data on the percentages of individual investors that are bullish, neutral, or bearish is collected from the AAII, and the IS index is formed using these percentages. The consideration of such survey-based IS measures is motivated by the findings of Chau et al. (2016), who noted that survey-based measures, as opposed to indirect measures, play a greater role in influencing the trading behavior of sentiment-driven investors. Similar to Chau et al. (2016) and Wang et al. (2006), I form my index based only on the percentages of bullish and bearish investors at a given point in time, treating neutral investors as insignificant for determining market sentiment. However, instead of computing the IS index as the ratio of bullish percentage to bearish percentage, I compute it in the following manner for improved interpretation as it ranges from 0 to 100:

$$Bullish\_Relative_t = \frac{Bullish\_Percentage_t}{Bullish\_Percentage_t + Bearish\_Percentage_t} \times 100 \quad (1)$$

This alternative index measures the percentage of bullish investors normalized by the sum of percentages of bullish and bearish investors. Multiplying this value by 100 implies that this index takes values between 0 and 100, whereas the ratio-based index would have no clear upper bound and would be undefined when the bearish percentage is 0. As the data published by AAII is in weekly frequency, I transform all variables to monthly frequency by taking the average values of each variable for the month.

Figure 1 in Appendix A illustrates similarities and differences between the IS and MU proxy. In level terms, the standard deviation of the MU proxy is visibly higher than that of the IS proxy, with computed values of 44.8718 and 10.3631, respectively. This may be because the MU proxy does not have a clear upper bound, while the IS proxy ranges from a minimum of 0 to a maximum of 100. Both proxies move as expected during major economic and financial events. For example, MU increased and IS decreased around 2020 when COVID-19 was publicized as a global pandemic. Similarly, MU increased, and IS decreased during late 2008 and 2009, regarded as crucial periods during the Global Financial Crisis (GFC). However, MU in the US continued increasing following the GFC for considerably longer than IS decreased, with IS stabilizing soon after, while the uptrend in MU only reversed around 2012. This suggests that the two factors, IS and MU, can potentially bring forward mutually exclusive information to portfolio optimization, motivating their simultaneous incorporation into the portfolio optimization framework.

3.3.1 IS regression's control variables

The control variables I choose to include in the regression of returns on IS are based on the choice of controls of Schmeling (2009). Specifically, I include the monthly inflation rate, computed as the monthly percentage change in CPI, the industrial production index value, and the term spread, defined as the difference between the yields on the ten-year and two-year treasury notes. A positive term spread represents increasing interest rates, suggesting that future economic growth is expected due to the stabilizing role of interest rates in macroeconomic policy. Following this logic, a negative term spread suggests an expectation of future economic decline. The data on all controls is collected from the Federal Reserve Economic Data at a monthly frequency.

4 Method

4.1 Optimization framework

The proposed methodology to incorporate MU and IS consists of two stages. Stage one requires estimating individual asset returns' sensitivities to MU and, separately, to IS. These sensitivities are proxied by the absolute β values of assets from the regression of asset returns on MU or IS and appropriate controls. Absolute β values are used since sensitivity depends on only the magnitude of the effect and not its direction. Given the documented positive effect of MU on return volatility and the negative effect of IS on returns, the directions of these effects are such that reducing sensitivity to these factors presents the possibility of increasing portfolio returns and decreasing portfolio return volatility. Stage two of the methodology entails incorporating MU and IS into the optimization framework through constraints on the overall portfolio's sensitivity to MU and, separately, to IS. Portfolio sensitivities are computed as follows:

$$\beta_p = \sum_{i=1}^{N}(|\ \beta_i\ | \times w_i) \tag{2}$$

Here β_p is the portfolio's sensitivity to either MU or IS, N represents the total number of assets being considered, β_i is asset i's regression coefficient on either MU or IS and w_i is asset i's weight in the portfolio. Therefore, β_p represents the weighted average of individual assets' absolute β values.

The constraints on portfolio sensitivity are inequalities in the following form:

$$\beta_p \leq x \tag{3}$$

Here, x denotes the x^{th} percentile value of the absolute individual asset β values. I consider the upper limits of the constraints to be percentile values of individual asset sensitivities to ensure the utilized optimization algorithm can reach a solution. Too small of an upper limit may prevent a solution from being found as β_p cannot take such a small value. Conversely, too large of an upper limit may make the constraint meaningless if it exceeds the maximum absolute individual asset β value. The remainder of this chapter expands on the method for determining individual asset β values and the optimization framework that incorporates these constraints.

4.1.1 Determining sensitivities to MU and IS

The sensitivity of individual asset returns to MU is the asset's β_1 value from the following regression:

$$r_t = \beta_0 + \beta_1 MU\ Proxy_t + \beta_2 Unemployment\ Rate\ Change_t + \beta_3 IPI\ Change_t + \beta_4 Inflation\ Rate_t + \beta_5 Default\ Spread_t + \varepsilon_t \tag{4}$$

Here, r_t is monthly asset returns, and MU $Proxy_t$ is the three-component Baker-Bloom-Davis EPU index. The sensitivity of individual asset returns to IS is the asset's β_1 value from this alternative regression:

$$r_t = \beta_0 + \beta_1 IS\ Proxy_t + \beta_2 Term\ Spread_t + \beta_3 IPI_t + \beta_4 Inflation\ Rate_t + \varepsilon_t \quad (5)$$

IS $Proxy_t$ is the sentiment index I created, as described in equation (1). Furthermore, it is important to note that equation (4) considers monthly changes in the IPI value while equation (5) considers monthly IPI values, following the appropriate literature mentioned in sections 3.2.1 and 3.3.1.

Both regressions are simple OLS regressions without adjustments for heteroskedasticity or autocorrelated errors, as these issues are addressed to ensure the use of appropriate standard errors in significance testing. Addressing these issues would not change the coefficient estimates or result in them no longer being unbiased; only correct coefficient values, not standard errors, are required for my proposed methodology.

4.1.2 Mean-variance optimized portfolios

Unconstrained mean-variance optimization, as developed from the work of Markowitz (1952), accounts directly for both portfolio returns through the constraint in equation (7) and variance. It requires finding the column vector of portfolio weights for all assets, w, that solves the following problem:

$$\min\ w'S\,w \quad (6)$$

Subject to the following constraints:

$$E[R_p] = w'\mu \quad (7)$$

$$w'\iota = 1 \quad (8)$$

Equation (6) represents minimizing portfolio variance, $w'S\,w$, with w′ being the transposed, 1 × N row vector of portfolio weights, and S being the N × N sample covariance matrix. $E[R_p]$ is the imposed target portfolio return. The portfolio return is computed by the matrix multiplication of w′ and μ, the N × 1 column vector of expected returns of all assets. Equation (8) is the constraint that the portfolio weights sum to one, with ι being an N × 1 vector of ones.

To obtain the efficient frontier, the efficient set of portfolios according to Markowitz (1952), one can plot the standard deviation of returns of the mean-variance optimized portfolio against the different targeted portfolio returns specified to obtain the optimal weights. To select one optimal portfolio while accounting for both portfolio variance and returns, researchers use tangency portfolios formed by maximizing Sharpe ratios, formulated later in section 4.3.

Jagannathan & Ma (2003, p. 1654) illustrate that tangency portfolios perform worse in the out-of-sample period than minimum-variance portfolios due to greater estimation error prevalence in tangency portfolios. Consequently, this study focuses solely on incorporating MU and IS into minimum-variance portfolios to extend the findings of prior literature.

4.1.3 Minimum-variance portfolios and incorporating MU and IS

Unlike mean-variance optimized portfolios and tangency portfolios, minimum-variance portfolios only indirectly consider expected asset returns through the computation of covariance matrices, used as an input to the optimization framework. Global minimum-variance portfolios are formed by finding the column vector of portfolio weights, w, that solves the following problem:

$$\min \; w'S\,w \tag{9}$$

Subject to the following constraints:

$$w'\iota = 1 \tag{10}$$

The solution to this problem represents the global minimum-variance portfolio, as there are no additional constraints other than the portfolio weights summing to one. In equation (8), S is the N × N sample covariance matrix, which is used instead of Σ, the true covariance matrix, as the latter is unobservable. S is computed as $\frac{1}{T}(r_t - \mu)'(r_t - \mu)$, with μ being the N × 1 column vector of expected asset returns comp and so ($r_t - \mu$) is the N × 1 column vector of demeaned asset returns. The scalar value T represents the number of historical return observations considered in the computation of the covariance matrix of asset returns.

Portfolio variance, computed by the matrix multiplication of w′Sw, has the following non-matrix formula:

$$\sigma_p^2 = \Sigma_{i=1}^{N}\Sigma_{j=1}^{N}(w_i w_j \sigma_{ij}) \tag{11}$$

Here, N is the number of assets in the portfolio, w_i and w_j are the portfolio weights of the considered assets in the outer and inner summation, respectively, and σ_{ij} is the covariance of returns of asset i and asset j.

To form minimum-variance portfolios that incorporate MU, IS, or both, one or both of the following constraints are added to the optimization problem represented by equations (9) and (10):

$$\beta_p^{MU} \leq x_{25}^{MU} \tag{12}$$

$$\beta_p^{IS} \leq x_{25}^{IS} \tag{13}$$

To incorporate only MU, the constraint in equation (12) is added alone, where β_p^{MU} is the portfolio sensitivity to MU, formed according to equation (2), and x_{25}^{MU} is the 25th percentile value of individual assets' β_1 values from regression equation (4). Conversely, to incorporate only IS, the constraint in equation (13) is added alone, where β_p^{IS} is the portfolio sensitivity to IS and x_{25}^{MU} is the 25th percentile value of individual assets' β_1 values from regression equation (5). Both constraints in equations (12) and (13) are added to obtain portfolios incorporating MU and IS. I consider the 25th percentile here since it allows for a tight constraint, ensuring MU and IS are being considered in the optimization to observe their effects. However, this percentile can be readjusted to match the user's preferences on their portfolio's insensitivity to MU and IS; investors with greater preference for this insensitivity would lower the considered percentile.

4.1.4 Non-negativity constraints & the Ledoit-Wolf covariance estimator

In analyzing several minimum-variance portfolios with different constraints and covariance matrix structures, Jagannathan & Ma (2003) find that the non-negativity (NN) constrained portfolio using the sample covariance estimator performs similarly to the other portfolios with more complicated covariance structures. This finding is specific to the use of monthly data. They also found that no portfolio had significantly higher average out-of-sample returns than the NN-constrained portfolio using the sample covariance estimator (p. 1672). Furthermore, the unconstrained portfolios with Ledoit-Wolf (LW) covariance estimators had the lowest out-of-sample variance and were one of the only portfolios with significantly lower out-of-sample variance than the NN-constrained, sample covariance matrix portfolio.

Motivated by these findings, I use either the sample covariance matrix, non-negativity constraints, or the LW covariance estimator to impose structure to the covariance matrix. These alternative covariance estimator structures are combined with constraints that incorporate MU, IS, both MU and IS, or neither when forming minimum variance portfolios. This allows me to observe which combination of constraints and covariance structure performs the best out-of-sample.

NN constraints require each asset's portfolio weight to be greater than or equal to 0. Jagannathan & Ma (2003) prove the equivalency of this constraint to shrinking the covariance matrix, as explained in section 2.1.1. Formally, this constraint can be written into the optimization problem as follows:

$$w_i \geq 0 \quad for\ i = 1, 2, \ldots, \text{N}. \tag{14}$$

The LW covariance matrix used in this study is the shrinkage-theory-based covariance estimator presented in Ledoit & Wolf (2004), where the authors develop a shrunk covariance estimator as the weighted average of the sample covariance estimator and a shrinkage target. Unlike Jagannathan & Ma (2003), this study does not use the estimator presented in Ledoit & Wolf (2003), where the shrinkage target was the covariance matrix derived from assuming stock returns are generated by a one-factor model. Instead, the estimator presented in Ledoit & Wolf (2004) is used, where the shrinkage target is related to the identity matrix, as presented in equation (15). This is because of the lack of easily accessible functions for the former estimator in Python, the software of choice for this study. This should not be a cause for concern as the findings of Ledoit & Wolf (2003) illustrate that the shrunk estimator based on the identity matrix performs similarly to the shrunk estimator based on the one-factor model in out-of-sample portfolio variance.

The LW covariance estimator, S_{LW}, is computed as follows:

$$S_{LW} = \frac{\beta^2}{\delta^2}\mu I + \frac{\alpha^2}{\delta^2}S \tag{15}$$

In this formula $\frac{\beta^2}{\delta^2}$ is interpreted as the weight parameter on the N × N identity matrix, I, and it represents the shrinkage intensity. Conversely, $\frac{\alpha^2}{\delta^2}$ is the weight parameter on the sample covariance matrix, S, since $\alpha^2 = \delta^2 - \beta^2$. Additionally, μ is a scalar value acting as a scaling factor of the identity matrix. These optimal parameters and weights are determined by minimizing a predefined quadratic loss function, the mean squared error of the LW estimator's entries relative to the true covariance estimator's entries.

4.2 Equally weighted portfolio

DeMiguel et al. (2009, p. 1921) emphasize that a portfolio optimization strategy that cannot outperform a basic, unoptimized alternative should not be utilized, as the extra effort of optimization is rendered meaningless. Consequently, equally weighted portfolios are often used in portfolio literature as the basic, unoptimized alternative to compare with the proposed strategies, specifically comparing out-of-sample performance.

Banholzer et al. (2019) note two frequently used equally weighted portfolio strategies: the buy-and-hold, equally weighted portfolio, and the dynamic, equally weighted portfolio. The buy-and-hold portfolio is set to have weights equal to 1/N for all N assets at the beginning of the sample period. These weights are allowed to change since relative returns of assets may differ, thus changing the weight allocation if weights are not actively readjusted, a shift in portfolio weight towards assets with higher relative returns. On the other hand, the dynamic portfolio's weights are also set to 1/N at the start but are readjusted at every considered time period, here every month, to hold the weights fixed at 1/N. The methodology for testing the performance of portfolio strategies, as explained below in section 4.3, is frequently used in portfolio optimization literature and fixes the weights of the considered naive strategy to 1/N for every period. Subsequently, this study considers only the dynamic, equally weighted portfolio strategy as the naive benchmark. To further motivate this choice, it is of interest to note that DeMiguel et al.'s (2009) findings that no optimized portfolio can outperform the equally weighted portfolio consistently, specifically considered the dynamic, equally weighted portfolio.

Henceforth, all considered portfolio strategies will be referred to by their abbreviations, which are defined in Table 3 in Appendix A.

4.3 Out-of-sample testing & performance metrics

Out-of-sample periods are periods in which the corresponding data was not used to determine the optimal portfolio weights but to assess the performance of the constructed portfolios on unseen data.

To obtain out-of-sample returns for all optimization strategies, I employ the rolling methodology used frequently in portfolio optimization literature (Behr et al., 2013; Jagannathan & Ma, 2003). For each month from July 2006 to February 2023, the past 60 months of returns data are used to form the appropriate covariance matrix of returns and, when relevant, the individual asset sensitivities to MU and IS. The covariance matrix and, optionally, the asset sensitivities are used to determine the optimal weights solution to the appropriate variance minimization problem defined in section 4.1. These optimal portfolio weights are held constant for the next year to obtain 12 out-of-sample monthly portfolio returns. The monthly portfolio return for each of these 12 out-of-sample months is computed as $w'r$, where w' is the 1 × N transposed vector of optimal portfolio weights, and r is the N × 1 vector of each asset's returns for that month. The portfolios are readjusted annually; the next starting point for optimization is 12 months from the previous. This rolling methodology gives T – E out-of-sample returns in total, where T is the number of months between and including July 2001 and February 2024, specifically 272 months, and E is the estimation window of 60 months.

For the DEW, portfolio out-of-sample returns are computed for each of the 212 months in the out-of-sample period of July 2006 to February 2024 as follows:

$$r_{DEW,t} = Xr \tag{16}$$

In this equation, $r_{DEW,t}$, is the out-of-sample return for the DEW in month t, X is a 1 × N vector with all entries being 1/N, and r is an N × 1 vector of each asset's returns for that month. For the 25SBM dataset, N is 25, and similarly, for the 17Ind dataset, N is 17.

In order to compare the out-of-sample performance of the portfolio strategies in Table 3, the following performance metrics, as computed by DeMiguel et al. (2009) and Behr et al. (2013, p. 1235), are used:

$$\hat{\sigma}^2 = \frac{1}{T-\tau-1}\Sigma_{t=\tau}^{T-1}(r_{t+1}{w'}_t - \hat{\mu})^2, \qquad with\ \hat{\mu} = \frac{1}{T-\tau}\Sigma_{t=\tau}^{T-1}(r_{t+1}{w'}_t) \tag{17}$$

$$\widehat{SR} = \frac{\hat{\mu}_{ex}}{\hat{\sigma}_{ex}}, \qquad with\ \hat{\mu}_{ex} = \frac{1}{T-\tau}\Sigma_{t=\tau}^{T-1}(r_{t+1}{w'}_t - r_{f,t+1}) \tag{18}$$

$$TRN = \frac{1}{X}\Sigma_{t=0}^{X-1}\Sigma_{j=1}^{N}(|w_{j,t+1} - w_{j,t+}|) \tag{19}$$

In the equations above, $\hat{\sigma}^2$ is the sample variance of the portfolio's out-of-sample returns, r_{t+1} is an N × 1 vector of all asset returns in period t+1, w'_t is a 1 × N vector of optimized weights in period t and $\hat{\mu}$ is the sample mean of out-of-sample portfolio returns, which is also considered as a separate performance metric. Furthermore, T represents the index value of the final month in the out-of-sample period, specifically February 2024, and τ is the estimation window, here 60 months, which reflects the index for the month before the start of the out-of-sample period, June 2006. In equation (18), $\hat{\sigma}_{ex}$ has the alternative definition of being the standard deviation of portfolio out-of-sample excess returns while $\hat{\mu}_{ex}$ is the sample mean of portfolio out-of-sample excess returns; excess returns are measured as the portfolio's out-of-sample return in period t + 1 subtracted by the corresponding risk-free rate for that period, $r_{f,t+1}$. This equation represents the computation of the portfolio's overall out-of-sample Sharpe ratio, which measures risk-adjusted out-of-sample returns.

Equation (19) is the computation for portfolio turnover, reflecting the average percentage of invested funds that are traded every time the portfolio is rebalanced. Accordingly, in equation (19), X is the total number of rebalancing periods, and the outer summation only considers the months when the rebalancing occurs, specifically every month of July in the considered out-of-sample period. Following this logic, t is no longer time in months but in years, as the period between two consecutive rebalancing months is a year. Turnover is based on the sum of absolute differences in portfolio weights of all assets before and after rebalancing, with $w_{j,t+}$ being the weight of asset j at time t+1 before rebalancing and $w_{j,t+1}$ being the weight of asset j at time t + 1 after rebalancing. The weight before rebalancing, $w_{j,t+}$, is computed following the method demonstrated in Figure 2 in Appendix A and $w_{j,t+1}$ is simply the optimized weight for asset j, computed in the subsequent annual rebalancing period**.** Portfolio turnover consideration is crucial; it can indirectly reflect a portfolio strategy's relative mean return and Sharpe ratio performance after accounting for transaction costs. If a portfolio strategy has considerably higher relative turnover, its post-transaction cost Sharpe ratio and mean return decrease relative to other strategies.

I also compute two additional performance metrics for each portfolio strategy: negative return probability (NRP) and median optimized absolute weight (MOAW).

NRP is calculated as the proportion of negative out-of-sample returns for a strategy. Given the large number of total out-of-sample returns, specifically 212, this proportion can proxy the general probability. It is possible that a highly loss-averse investor, one who experiences a much more significant loss in utility from a negative return than a gain from a positive return of equal magnitude, picks the portfolio strategy with a lower Sharpe ratio, for example, if it has considerably lower NRP than its alternatives. The additional insight this performance metric can provide for such investors motivates its consideration.

On the other hand, MOAW is calculated by first storing all optimized weights of a strategy, computed at every rebalancing period, into a single list. These values are then converted into their absolute values, and finally, the median of this list is computed. Absolute values are considered to allow for more intuitive comparisons of this metric between optimization strategies that allow for negative weights and those that do not, such as the DEW and strategies with non-negativity constraints. The MOAW of a portfolio strategy provides insight into the concentration of the portfolios' weights, allowing us to observe whether the strategy allocates negative or positive weight to only a few assets, suggesting poor diversification, or to several assets. The consideration of this metric is motivated by the importance of diversification, as encouraged by Markowitz (1952), which can reduce the portfolio's idiosyncratic risk exposure, the risk specific to a single asset or a small subset of assets. Note that a well-diversified portfolio can be a mixture of both long and short positions, as opposed to just one type of position, thus, considering absolute weights is appropriate.

4.4 Significance tests

To provide statistically validated conclusions to the hypotheses of this study, standard paired t-tests are used to perform a two-tailed test following the methodology of Jagannathan & Ma (2003, p. 1673). The authors use the fact that the variance of a random variable, such as returns, is equal to $E(X^2) - (E(X))^2$, with X representing the random variable, which is out-of-sample portfolio returns in this case, to test for differences in variances. They note that if the tests for differences in mean returns, $(E(X))$, are insignificant, then testing for differences in mean squared returns, $(E(X^2))$, is equivalent to testing for differences in return variances. After observing that the t-tests for differences in mean returns between DEW and all other strategies considered in this study are insignificant, I utilize the t-test for differences in mean-squared returns of a portfolio strategy and the DEW to test the following null hypothesis:

$$H_0\colon\ \hat{\sigma}_i^2 - \ \hat{\sigma}_{DEW}^2 = 0 \tag{20}$$

This t-test comparing mean squared returns and thus return variance is run for all considered optimization strategies, subscript i in equation (20) is replaced by the considered portfolio strategy's abbreviation.

As later discussed in the limitations section, standard t-tests cannot be performed on Sharpe ratio differences since multiple Sharpe ratio observations are required for each portfolio strategy. Additionally, alternative tests, such as the circular block bootstrap test developed by Ledoit & Wolf (2008), are not performed due to their mathematical complexity. Bootstrapping would allow for a distribution of Sharpe ratio values to be obtained, which can then be used to compute appropriate standard errors, test statistics, and p-values.

5 Results & Discussion

5.1 Asset Sensitivities

Appendix A includes figures for the distribution of asset sensitivities in the two considered datasets to potentially draw on to provide reasoning for inter-dataset differences in portfolio performances. Recall that, in this study, asset sensitivity to MU and IS is defined as absolute regression β_1 values from regression equations (4) and (5), respectively. Returns, used as input to out-of-sample performance testing for every portfolio strategy, are converted from percentages to their equivalent decimal values. Therefore, the beta values in the distributions presented in Appendix A, multiplied by 100, represent the percentage change in the asset's returns for a one-unit increase in either the IS or MU proxy.

5.2 Discussion on results of the 25SBM dataset

The results discussed in this section relate to those presented in Table 4, Appendix A. Before discussing the results, I clarify how the assessed performance metrics can be interpreted.

All performance metrics relate to either the out-of-sample returns or the computed weights of a portfolio strategy. It is useful to note that lower values of MOAW indicate worse diversification. Lower values suggest that most assets are assigned very low weights, implying that, at each point of rebalancing, most of the portfolio weight lies in a few assets. The DEW is an exception; it is known that the MOAW will always be 1/N; thus, when N, the number of assets, is high, this value can be low yet still suggest adequate diversification. Lower portfolio turnover, TRN, indicates that we can expect similar out-of-sample performance metric values when accounting for transaction costs as incurred transaction costs increase with portfolio turnover.

The reported mean returns, $\hat{\mu}$, are in decimal format, multiplying them by 100 gives the average monthly out-of-sample returns in percentage format. Similarly, multiplying NRP by 100 expresses the probability as a percentage. Conversely, a portfolio strategy's monthly out-of-sample Sharpe ratio, $\widehat{SR}$, and a strategy's return variance, $\hat{\sigma}^2$, are best interpreted through comparisons with their values for other portfolio strategies. Return variance captures the dispersion of portfolio returns around their mean, while the Sharpe ratio measures the risk-adjusted return.

As noted in Table 4, no optimization strategy significantly increases mean out-of-sample return relative to the DEW, which is especially surprising for the IS-incorporating strategies, given that IS has been shown to have a significant adverse effect on returns (Schmeling, 2009). One potential explanation may be the lack of a significant number of periods with considerably higher IS within the sample period. In such periods of high IS, the returns of non-IS-incorporating strategies would be expected to reduce by a greater degree than those incorporating IS by reducing portfolio sensitivity to IS. However, the insignificance of differences in mean out-of-sample returns for minimum-variance portfolios relative to the DEW aligns with the results of Jagannathan & Ma (2003, p. 1672). This suggests that this observed insignificance may be because minimum-variance portfolios disregard the direct consideration of mean returns within the optimization problem. As stated in section 4.5, this observed insignificance allows for testing the significance of differences in return variances following the approach of Jagannathan & Ma.

The results from Table 4 illustrate that incorporating only MU has the same effect on portfolio return variances as incorporating only IS, as seen by the similar return variances of Min-MU-U and Min-IS-U, which are both insignificantly different from that of the DEW. Furthermore, the simultaneous incorporation of both factors does not lead to the development of portfolios that can significantly outperform the DEW unless combined with non-negativity constraints or the Ledoit-Wolf covariance estimator. This is demonstrated by the return variances of Min-MU-IS-N and Min-MU-IS-L, which are significantly lower than that of the DEW at a 1% significance level.

Similar to the results presented by Jagannathan and Ma (2003), my findings greatly support the benefit of using an LW covariance estimator, as all strategies with the lowest return variance utilize this covariance estimator. Given the equal variances of Min-L, Min-IS-L, Min-MU-L, and Min-IS-MU-L and the significantly lower variance of all four strategies relative to DEW, it can be deduced that MU and IS contributed minimally to decreasing return variance, most of the contribution comes from the use of the LW covariance estimator.

Despite the lack of tests for the significance of Sharpe ratio differences, considering the magnitude of these differences can speak to their economic significance. When considering the magnitude of Sharpe ratio differences, there is strong evidence in favor of optimization; all optimization strategies exhibit considerably higher Sharpe ratios than the DEW, the naive alternative to optimization. Incorporating MU and IS is also shown to be highly beneficial, as the highest Sharpe ratios belong to optimization strategies that incorporate MU, IS, or both. Namely, Min-MU-U, Min-MU-N, and Min-MU-IS-U, which exhibit Sharpe ratios exceeding the DEW's by roughly 36%. The lowest Sharpe ratio belongs to Min-MU-IS-N, exceeding the DEW’s Sharpe ratio by around 10%.

A turnover ten times that of the DEW is considered by Banholzer et al. (2019, p. 690) to be relatively high, while Behr et al. (2013, p. 1239) find a turnover of around five to six times that of the DEW to be comparatively low. Following these bounds, besides the strategies that use an LW covariance estimator, almost all optimization strategies illustrate high turnover relative to the DEW. For these strategies, it is uncertain whether the post-transaction cost Sharpe ratios would exhibit the same considerable outperformance as is observed for pre-transaction cost Sharpe ratios in Table 4. Notably, two optimization strategies demonstrate highly similar turnover to the DEW, specifically Min-N and Min-MU-IS-N. However, both strategies produce highly concentrated portfolios. This high concentration is deduced from their near-zero values for MOAW, which implies that most of the assigned weight is concentrated in a few out-of-sample weights. Additionally, their low turnover values strengthen this deduction, suggesting minimal changes to their highly concentrated portfolio weight distribution over time; most of the portfolio weight is allocated to the same few assets in every rebalancing period.

Incorporating MU and IS does not considerably increase or decrease NRP relative to the DEW. Two potential explanations exist. First, the effective diversification of the DEW limits the prevalence of negative returns due to adverse idiosyncratic shocks, and second, the performed optimization minimizes variance, not NRP. When non-negativity constraints are already imposed, the incorporation of MU and IS can reduce NRP as seen by lower NRP values for Min-MU-N, Min-IS-N, and Min-MU-IS-N in comparison to Min-N; however, these differences are again insubstantial.

5.3 Discussion on results of the 17Ind dataset

The results discussed in this section relate to those presented in Table 5, Appendix A.

Table 5 shows that the out-of-sample mean returns of all optimization strategies from the 17Ind dataset insignificantly differ from the mean returns of the DEW. On the other hand, opposing the findings from the 25SBM dataset, all optimization strategies have significantly lower out-of-sample return variances than the DEW at a 1% significance level, with all variances being around half of the DEW's variance. The significant outperformance of the Min-MU-U coincides with MU's documented significant, positive effect on return volatility (Iania et al., 2023), suggesting that reducing portfolio sensitivity to MU could improve return variances.

To understand why optimization strategies more frequently outperform the DEW in portfolio return variance in the 17Ind dataset, it is essential to recall the findings of Kirby & Ostdiek (2012). One of their proposed strategies considers asset return variances to determine portfolio weights, similar to the minimum-variance strategies employed in this study. The performance of this strategy was shown to depend on the cross-sectional dispersion of return variances in the considered dataset; their strategy performs better when there is a larger range in return volatilities of the assets in the considered dataset. The return volatilities in the 25SBM dataset, as observed in Table 1, range from 0.0420 to 0.0766, a difference of 0.0346, while in the 17Ind dataset, as presented in Table 2, they range from 0.0343 to 0.0947, a larger difference of 0.0604. The larger range of cross-sectional return volatilities in the 17Ind dataset may allow for improved minimum-variance optimization.

Following this logic, I provide another reason for the improved relative performance of the IS and MU incorporating strategies. In the 25SBM dataset, only four out of nine strategies incorporating IS, MU, or both exhibit significantly lower return variance than the DEW, whereas in the 17Ind dataset, all nine strategies exhibit such variance. Notably, strategies incorporating only MU or IS, without alternative covariance structures, also outperform the DEW in return variance. This may be because the range of assets' sensitivities to IS in the 17Ind dataset, as shown in Figure 4, is higher than the range in the 25SBM dataset, as shown in Figure 3. Similarly, the range of sensitivities to MU in the 17Ind dataset, as shown in Figure 6, is higher than the range in the 25SBM dataset, as shown in Figure 5.

When assessing Sharpe ratios computed in the 17Ind dataset, two similarities are observed to those computed under the 25SBM dataset. First, all optimization strategies have a higher Sharpe ratio than the DEW. Second, the Min-MU-IS-N strategy has the lowest Sharpe ratio of all optimization strategies. The latter finding may be because imposing all three constraints, non-negativity, MU sensitivity, and IS sensitivity, leads to excluding assets that cannot fit the constraints but would have still contributed to improving out-of-sample portfolio Sharpe ratios. Furthermore, an increased number of constraints in a model tends to lead to the model overfitting on its trained data, here the 60 months of historical returns, which reduces the generalizability to the untrained data, here the 12-month holding period. In contrast, the Min-MU-IS-LW strategy does not exhibit such relatively poor out-of-sample Sharpe ratios in either dataset. This may be because non-negativity constraints indirectly shrink the covariance estimator to reduce the effect of estimation errors; however, they do so by imposing an additional constraint, resulting in the possible exclusion of relevant assets and overfitting. On the other hand, the LW covariance estimator shrinks the covariance estimator without imposing such an additional constraint.

Another insightful inter-dataset pattern is observed when assessing portfolio turnover, TRN. On average, portfolio turnover is lower in the 17Ind dataset than in the dataset with more assets, 25SBM. This pattern is also observed in the turnover values presented by Cai et al. (2024, p. 22). This follows logically from the formula of portfolio turnover in equation (17), where the turnover value is not normalized by the number of assets being considered. As a result, with more assets, as in the 25SBM dataset, there is a greater likelihood of the occurrence of idiosyncratic events, which are relevant to only a few assets, in the 12-month holding period. Such events may cause shifts in the relative attractiveness of different assets within the appropriate optimization framework, leading to more significant adjustments to portfolio weights in the subsequent rebalancing period and, thus, increased turnover.

The 17Ind dataset provides more substantial evidence than the 25SBM dataset for optimization strategies to preserve their outperformance in Sharpe ratios, relative to the DEW, after accounting for transaction costs. With Behr et al. (2013, p. 1239) finding portfolio turnover of five times that of the DEW to be comparatively low, all optimization strategies have relatively low turnover in the 17Ind dataset, the highest being roughly three times that of the DEW. This implies a negligible effect on Sharpe ratios when accounting for transaction costs can be expected. Therefore, similar, considerable outperformance of optimization strategies relative to the DEW would be observed when assessing post-transaction cost Sharpe ratios, as is noted for pre-transaction cost Sharpe ratios in Table 5.

Aligning with the findings of the 25SBM dataset, Min-N and Min-MU-IS-N exhibit similar turnover to the DEW. Both strategies again produce portfolios with consistently high concentrations of portfolio weights over time due to their low values for MOAW and TRN.

NRP results again do not indicate that incorporating MU and IS has sizable effects on reducing or increasing NRP; all strategies have NRP values within a 10% range from that of the DEW.

5.4 Answers to Hypotheses 1 and 2

Hypothesis 1 states that the IS-incorporating strategies are expected to outperform the DEW in the out-of-sample period. Hypothesis 2 is the expectation of MU-incorporating strategies outperforming the DEW in the out-of-sample period.

As significance tests are not conducted for Sharpe ratios, I will not conclude these hypotheses based on Sharpe ratio differences due to a lack of statistical validity in such conclusions. However, it is worth noting that both datasets illustrate that all strategies that incorporate MU and IS have considerably higher out-of-sample Sharpe ratios than the DEW.

The significance tests for differences in mean returns do not provide evidence in favor of either hypothesis as all optimization strategies have insignificantly different mean returns relative to the DEW. However, the results obtained from the significance of return variance differences between IS and MU incorporating strategies and the DEW provide moderately strong evidence in support of both hypotheses. This is because the significance of these variance differences depends on the assets under consideration, specifically on some cross-sectional characteristics of the assets within the considered dataset. Namely, sufficient cross-sectional dispersion in assets' return variances and sufficient cross-sectional dispersion in asset sensitivities to MU and IS. When these dispersions are low, as in the 25SBM dataset, incorporating only IS or MU requires using an LW covariance estimator to develop minimum-variance portfolios with significantly lower return variances than the DEW. However, when these dispersions are sufficiently high, as in the 17Ind dataset, incorporating IS or MU allows for developing such portfolios without additional considerations. It is uncertain whether the performance of MU and IS incorporating strategies depends on these specific dispersions or other asset characteristics. Nonetheless, the significant, relative outperformance of MU and IS incorporating strategies would still depend on specific characteristics of the assets within the dataset; indicating moderately strong, as opposed to very strong, evidence in favor of the hypotheses.

6 Robustness Tests

Results that are highly dependent on specific parameter values and specifications of considered variables, diminish the validity of any conclusions formed based on those results. To ensure this validity, appropriate robustness checks are performed. The results for all robustness checks are provided in Appendix C.

The first robustness check, encouraged by Banholzer et al. (2019), is changing the estimation window in the rolling methodology for constructing optimized portfolios and testing their out-of-sample performance, as explained in section 4.3. Previously, I used an estimation window of 60 months, which entails that at any point in the out-of-sample period, the prior 60 months of data is used to determine asset sensitivities, covariance matrices, and all other inputs to the optimization framework, which are then used to construct and test the optimized portfolios. To test for robustness to relatively minor changes in the estimation window, Banholzer et al. adjusted their initial window of 120 months down to 108 months and up to 132 months; a 10% decrease and increase in the estimation window. Following these proportions, the results for adjusting my initial window of 60 months down to 54 months are presented in Tables 6 and 7, and the adjustment to 66 months is presented in Tables 8 and 9. The results presented in these tables for both datasets, specifically the nearly identical outcomes of the significance tests on mean return differences and return variance differences, support the same conclusions of relatively strong support for Hypotheses 1 and 2. Consequently, the obtained results are robust to minor changes in the estimation window.

The second robustness check relates to the specification of the IS proxy. As discussed in section 3.3, following Chau et al. (2016) and Wang et al. (2006), I ignore the percentage of neutral investors when forming the IS proxy variable. However, there are instances when this could lead to misrepresenting the stock market as being highly bearish or bullish when it is best described as being neutral. For example, when the percentage of neutral investors is exceptionally high, but the bullish percentage still greatly exceeds the bearish percentage, the previous IS proxy variable misrepresents the market as highly bullish. As a result, I implement an adjusted IS proxy, previously computed using equation (1). The new proxy is the percentage of bullish investors, which accounts for the number of neutral investors in its denominator. Under this new proxy, despite a considerably larger number of bullish than bearish investors, the market would not be incorrectly described as highly bullish when the percentage of neutral investors is exceptionally high. A limitation of this new proxy is that it distinguishes between a bullish and non-bullish market as opposed to a bullish and bearish market since, at lower percentages of bullish investors, it cannot distinguish between whether the market is bearish or neutral. Nonetheless, repeating the analysis under this alternative IS proxy can highlight the robustness of my results to alternative IS proxies. Tables 10 and 11 illustrate that the results obtained are robust to this alternative definition for the IS proxy, as across both datasets, the results for the IS-incorporating strategies are almost identical for both IS proxy definitions. Most importantly, the t-tests for mean and variance differences yield the same results for all IS-incorporating strategies under both IS proxy definitions. Tables 4 and 5 contain the results for the IS-incorporating strategies under the 25SBM and 17Ind datasets, respectively, and the previous IS proxy definition.

The final robustness check involves changing the specification of the regressions for asset sensitivities, specifically regression equations (4) and (5). There may exist alternative regression specifications that allow for better computations of asset sensitivities to MU and IS, thus, it is insightful to observe whether my obtained results highly depend on the exact regression specification I have used. A simple and intuitive way to test for robustness to altered specifications is to add appropriate lagged variables to the regressions. Specifically, I add the first five lags of all control variables, which are listed in sections 3.2.1 and 3.3.1. The new regression equations for asset sensitivity to MU and IS are stated below in equations (21) and (22), respectively.

$$\begin{aligned} r_t = {} & \beta_0 + \beta_1 MU\ Proxy_t + \beta_2 Unemplyment\ Rate\ Change_t + \\ & \beta_3 Unemplyment\ Rate\ Change_{t-1} + \ \dots + \ \beta_7 Unemplyment\ Rate\ Change_{t-5} + \\ & \beta_8 IPI\ Change_t + \ \dots + \ \beta_{13} IPI\ Change_{t-5} + \beta_{14} Inflation\ Rate_t + \ \dots + \\ & \beta_{19} Inflation\ Rate_{t-5} + \beta_{20} Default\ Spread_t + \ \dots + \beta_{25} Default\ Spread_{t-5} + \ \varepsilon_t \end{aligned} \quad (21)$$

$$\begin{aligned} r_t = {} & \beta_0 + \beta_1 IS\ Proxy_t + \beta_2 Term\ Spread_t + \beta_3 Term\ Spread_{t-1} + \ \dots + \\ & \beta_7 Term\ Spread_{t-5} + \beta_8 IPI_t + \ \dots + \beta_{13} IPI_{t-5} + \beta_{14} Inflation\ Rate_t + \ \dots + \\ & \beta_{19} Inflation\ Rate_{t-5} + \ \varepsilon_t \end{aligned} \quad (22)$$

Given the large number of independent variables in the new regression equations, I do not explicitly state the first five lags of all control variables in equations (21) and (22). As monthly data is used in this study, X_{t-n} is the n-month prior value of the variable X.

Lagged variables are included to capture the delayed effects of the control variables on the dependent variable, returns. Therefore, five lags have been chosen as they capture an adequate range of prior months and various delayed effects, including those associated with quarterly cycles in macroeconomic data. In particular, the third lag accounts for the quarterly cycles inherent in the macroeconomic variables used in this study. As I remove observations with missing data and use up to five lags, the sample period is now from December 2001 to February 2024 and the out-of-sample period is from December 2006 to February 2024; both periods start five months ahead of their previously defined range. The results of this robustness check for the 25SBM and 17Ind datasets are presented in Tables 12 and 13, respectively. The results in these tables, specifically the identical significance test results, illustrate that my previously obtained results are robust to alternative specifications for asset return sensitivity regressions.

7 Conclusion

Prior research on portfolio optimization has concentrated on refining the information present in the inputs of the traditional optimization framework, specifically asset returns and covariances. The consideration of other variables within this framework, which can provide comparatively attractive portfolios while addressing the concern that investors value more than returns and variances, has been limited. Given this gap in prior research and evidence provided by relevant literature for the potential benefits of incorporating IS and MU into optimized portfolios, I developed a portfolio optimization framework that can incorporate IS and MU. Moreover, this framework allows investors to conveniently adjust the extent of this incorporation based on their preferences. This addresses the lack of consideration for the factors investors may value beyond returns and variances, such as the extent to which their ideal portfolio should be resilient to changes in IS and MU. Accordingly, this study explores the following question:

"Can incorporating macroeconomic uncertainty and investor sentiment in the portfolio optimization framework produce portfolios that outperform the equally weighted portfolio in the out-of-sample period?"

Two datasets on monthly asset returns, focusing on the US stock market, were used to answer this research question. The proxy for IS was defined using survey data from the AAII, while the proxy for MU was the Baker-Bloom-Davis measure of EPU. MU and IS were incorporated into the optimization framework using constraints on the optimized portfolio's sensitivity to these factors. This study assessed the out-of-sample performance of several minimum variance portfolios, which were constructed under different constraints and covariance structures, relative to a commonly considered benchmark strategy, the DEW. Performing this analysis provides fairly strong evidence supporting the relative outperformance of MU-incorporating and IS-incorporating strategies. This relative outperformance is specifically in terms of significantly lower out-of-sample portfolio variance than the DEW.

The general effectiveness of incorporating MU and IS motivates the use of the proposed methodology to incorporate other intuitive factors, which are documented to have significant effects on returns or return variances, using optimization constraints. Furthermore, my findings strengthen the motivation for portfolio optimization literature to perform their proposed additions under different datasets with different cross-sectional characteristics. This is because the significance of the portfolio variance differences, specifically between MU and IS incorporating strategies and the DEW, is shown to depend on the cross-sectional characteristics of assets specific to a dataset, an insight that would have been unobserved if the analysis had been performed on a single dataset. Consistent with prior literature, my findings demonstrate that significantly outperforming the DEW in mean returns can be challenging for minimum-variance portfolios.

7.1 Limitations & future research

A key limitation of my research is the assumptions drawn in using standard t-tests to evaluate the significance of differences in portfolio variance. As noted by Behr et al. (2013), ‘standard hypothesis tests do not control for time series characteristics in portfolio returns’ (p. 1237). Notably, such tests assume that monthly returns are independent, while they tend to exhibit autocorrelation, and monthly returns are normally distributed, while they have been documented to follow non-normal distributions (Campbell et al. (2001), p. 1796). To account for these shortcomings, Behr et al. (2013) use bootstrapping tests developed by Ledoit & Wolf (2011) to test the significance of portfolio variance differences. However, due to the mathematical complexity of such tests, they were not implemented in this study. Instead, the approach used by Jagannathan & Ma (2003), a highly regarded and frequently cited paper, was employed to test for significant variance differences. Nonetheless, researchers expanding on or replicating my analysis should reconfirm the observed t-test results using the alternative test developed by Ledoit & Wolf (2011).

Another limitation of this study is the absence of testing for the significance of Sharpe ratio differences. Standard hypotheses tests, such as the t-test, are invalid for comparing two Sharpe ratio observations as they require standard errors of Sharpe ratios. While my analysis only provides one Sharpe ratio observation for each portfolio strategy, standard errors are typically determined using standard deviations, which require a series of observations to be computed. Ideally, significance tests that account for the non-normality of returns and return autocorrelation, such as the test developed by Ledoit & Wolf (2008), should be implemented instead of standard hypotheses tests. However, due to their mathematical complexity, they are not implemented in this study. Although there are large, economically significant differences in the Sharpe ratios of my proposed strategies and the DEW, conclusions on relative outperformance cannot be made using Sharpe ratios as they would lack statistical validity. To strengthen the conclusions of this study, formed through significant portfolio variance differences, researchers wishing to expand on or replicate this study should employ the test developed by Ledoit & Wolf (2008) and evaluate the significance of Sharpe ratio differences.

The findings presented in this study on the benefits of incorporating MU into portfolio optimization reveal further points of future research, particularly in combination with the findings of Segal et al. (2015). The authors find that MU can be decomposed into a good and bad component, with equity prices increasing with good uncertainty and decreasing with bad uncertainty. Future research can aim to exploit both components of MU within portfolio optimization by separately considering the opposing effects of good and bad uncertainty on equity prices. For example, this can be achieved by identifying when good or bad MU is predicted to increase and placing greater portfolio weights on assets with higher sensitivity to good MU and lower weights on assets with higher sensitivity to bad MU. The effectiveness of my proposed strategies highlights the potential of such alternative uses of asset sensitivities within the optimization framework.

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APPENDIX A All tables and figures, excluding robustness check results

Table 1. *Descriptive statistics of assets' monthly returns under the 25SBM dataset*

Asset Name	avg.	SD	min.	max.	Asset Name	avg.	SD	min.	max.
ME1BM1	0.0040	0.0766	-0.2385	0.2579	ME3BM4	0.0099	0.0592	-0.2695	0.1666
ME1BM2	0.0081	0.0659	-0.1948	0.2137	ME3BM5	0.0101	0.0685	-0.3125	0.2308
ME1BM3	0.0079	0.0605	-0.2199	0.1930	ME4BM1	0.0096	0.0538	-0.2041	0.1655
ME1BM4	0.0096	0.0592	-0.2651	0.2043	ME4BM2	0.0100	0.0526	-0.2060	0.1578
ME1BM5	0.0118	0.0685	-0.2763	0.4151	ME4BM3	0.0087	0.0538	-0.2488	0.1679
ME2BM1	0.0082	0.0690	-0.2301	0.2123	ME4BM4	0.0090	0.0573	-0.3271	0.1676
ME2BM2	0.0099	0.0609	-0.2329	0.1912	ME4BM5	0.0077	0.0666	-0.3248	0.2043
ME2BM3	0.0100	0.0588	-0.2181	0.1757	ME5BM1	0.0092	0.0452	-0.1486	0.1409
ME2BM4	0.0092	0.0593	-0.2367	0.1867	ME5BM2	0.0079	0.0420	-0.1491	0.1376
ME2BM5	0.0095	0.0710	-0.3206	0.2616	ME5BM3	0.0083	0.0434	-0.1574	0.1426
ME3BM1	0.0082	0.0612	-0.2313	0.2083	ME5BM4	0.0046	0.0543	-0.2748	0.1608
ME3BM2	0.0105	0.0552	-0.1958	0.1912	ME5BM5	0.0073	0.0691	-0.2840	0.2198
ME3BM3	0.0092	0.0540	-0.1794	0.1666					

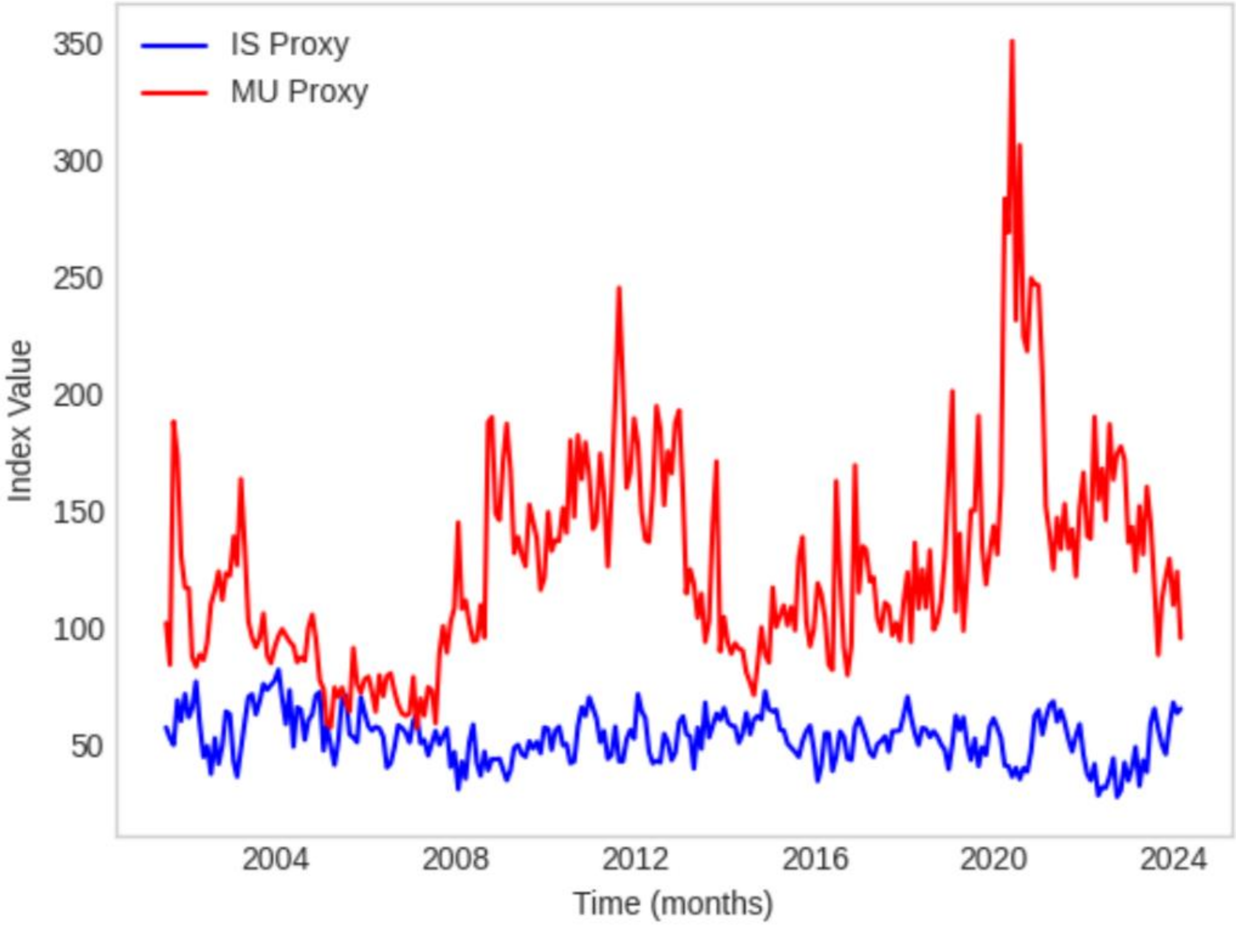
Note. SD is the sample standard deviation, and avg. is the sample average. All values can be converted to percentages by multiplying by 100.

Table 2. *Descriptive statistics of monthly portfolio returns for the 17Ind dataset*

Asset Name	avg.	SD	min	max	Asset Name	avg.	SD	min	max
Food	0.0076	0.0343	-0.1300	0.1032	Fab. Pro.	0.0116	0.0581	-0.2314	0.1894
Mining	0.0104	0.0828	-0.3277	0.2203	Machinery	0.0110	0.0680	-0.2432	0.1942
Oil	0.0089	0.0730	-0.3466	0.3284	Automobiles	0.0111	0.0869	-0.2791	0.3932
Clothes	0.0105	0.0644	-0.2253	0.2393	Transport	0.0098	0.0557	-0.2299	0.1995
Con. Dur.	0.0062	0.0652	-0.2582	0.2963	Utilities	0.0069	0.0410	-0.1294	0.1037
Chemicals	0.0086	0.0633	-0.2200	0.2000	Retail	0.0098	0.0475	-0.1461	0.1828
Con. Pro.	0.0080	0.0361	-0.0977	0.1039	Fin. Ser.	0.0069	0.0573	-0.2123	0.1710
Construct.	0.0114	0.0633	-0.2029	0.1801	Others	0.0082	0.0481	-0.1752	0.1390
Steel	0.0096	0.0947	-0.3241	0.2589					

Note. SD is the sample standard deviation, and avg. is the sample average. All values can be converted to percentages by multiplying by 100.

Figure 1. *Time series plot of MU and IS proxies*



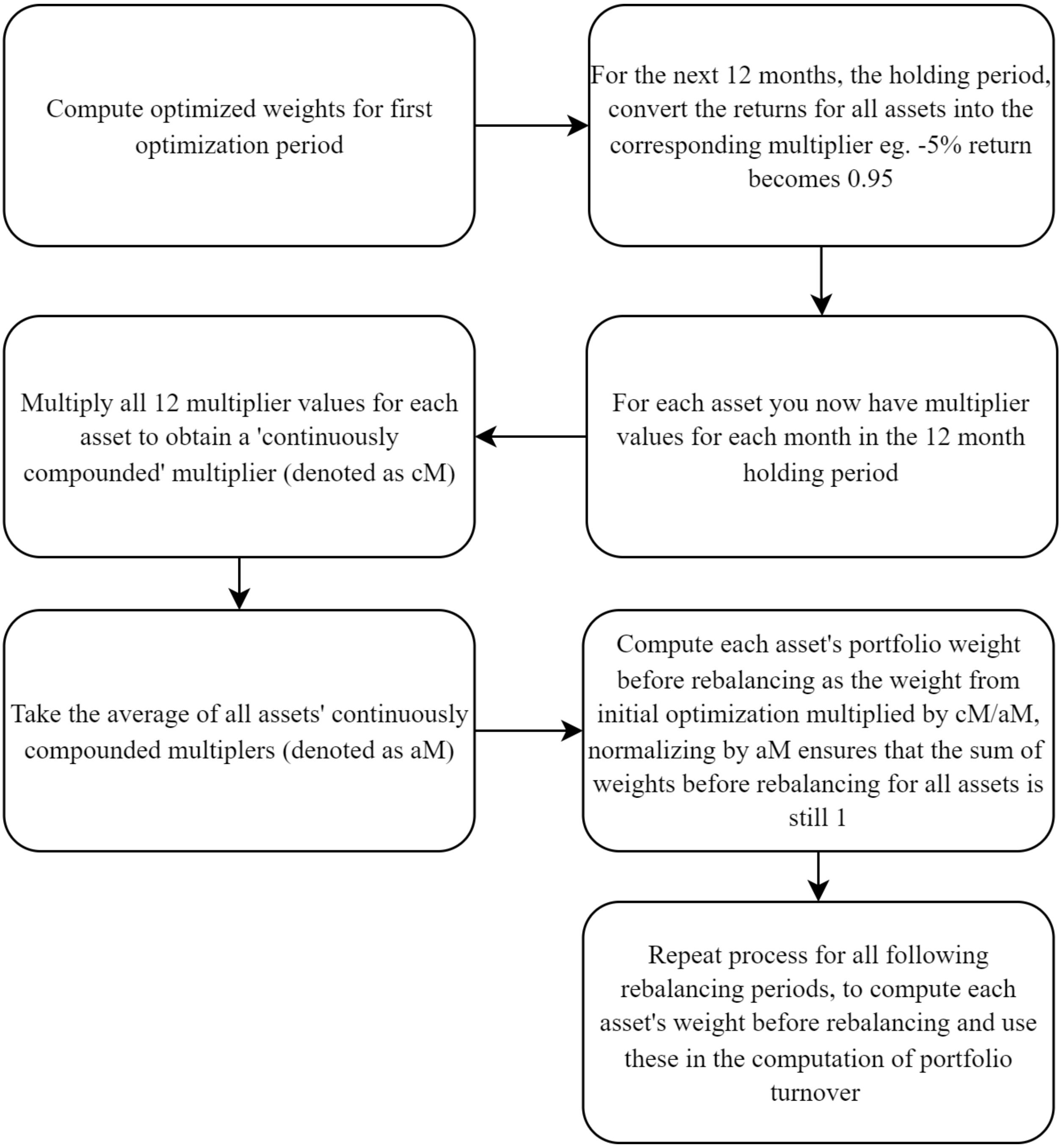
Note. Here, the MU Proxy is the three-component EPU index developed by Baker et al. (2016), and the IS Proxy is Bullish_Relative $_t$, as computed in equation (1). The time horizon plotted is July 2001 to February 2024.

Table 3. *List of all considered portfolio strategies' descriptions and their abbreviations*

#	Portfolio strategy description	Abbreviation
1	Dynamic, equally weighted portfolio	DEW
2	Minimum-variance portfolio without constraints	Min-U
3	Minimum-variance portfolio with NN constraints	Min-N
4	Minimum-variance portfolio with LW covariance estimator	Min-L
5	Minimum-variance portfolio with only MU constraints	Min-MU-U
6	Minimum-variance portfolio with MU and NN constraints	Min-MU-N
7	Minimum-variance portfolio with MU and LW covariance estimator	Min-MU-L
8	Minimum-variance portfolio with only IS constraints	Min-IS-U
9	Minimum-variance portfolio with IS and NN constraints	Min-IS-N
10	Minimum-variance portfolio with IS and LW covariance estimator	Min-IS-L
11	Minimum-variance portfolio with only MU and IS constraints	Min-MU-IS-U
12	Minimum-variance portfolio with MU, IS, and NN constraints	Min-MU-IS-N
13	Minimum-variance portfolio with MU, IS, and LW covariance estimator	Min-MU-IS-L

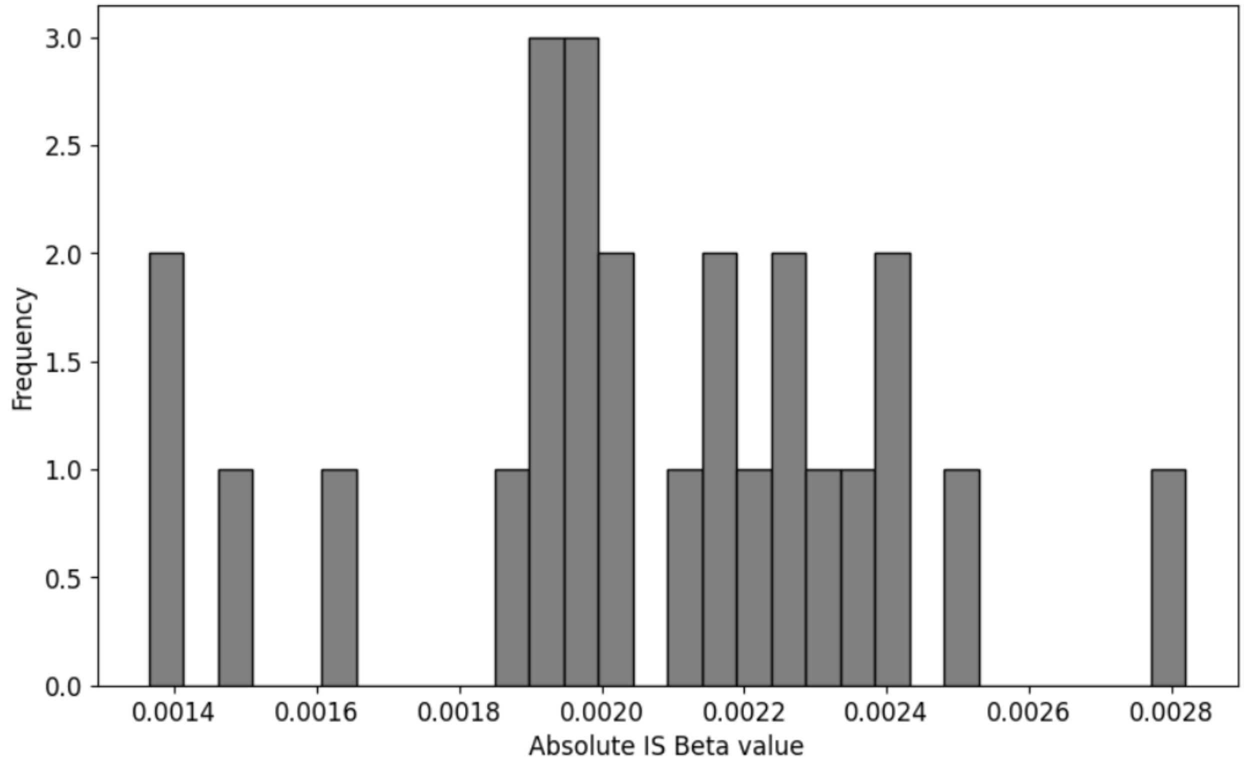
Note. NN stands for non-negativity, LW for Ledoit-Wolf, and 'with MU' means that the MU constraints are used in forming the portfolio, and similarly, 'with IS' means that the IS constraints are used. Portfolio strategies are referred to by their abbreviations in this study.

Figure 2. *Flowchart of methodology for computing weights before rebalancing*



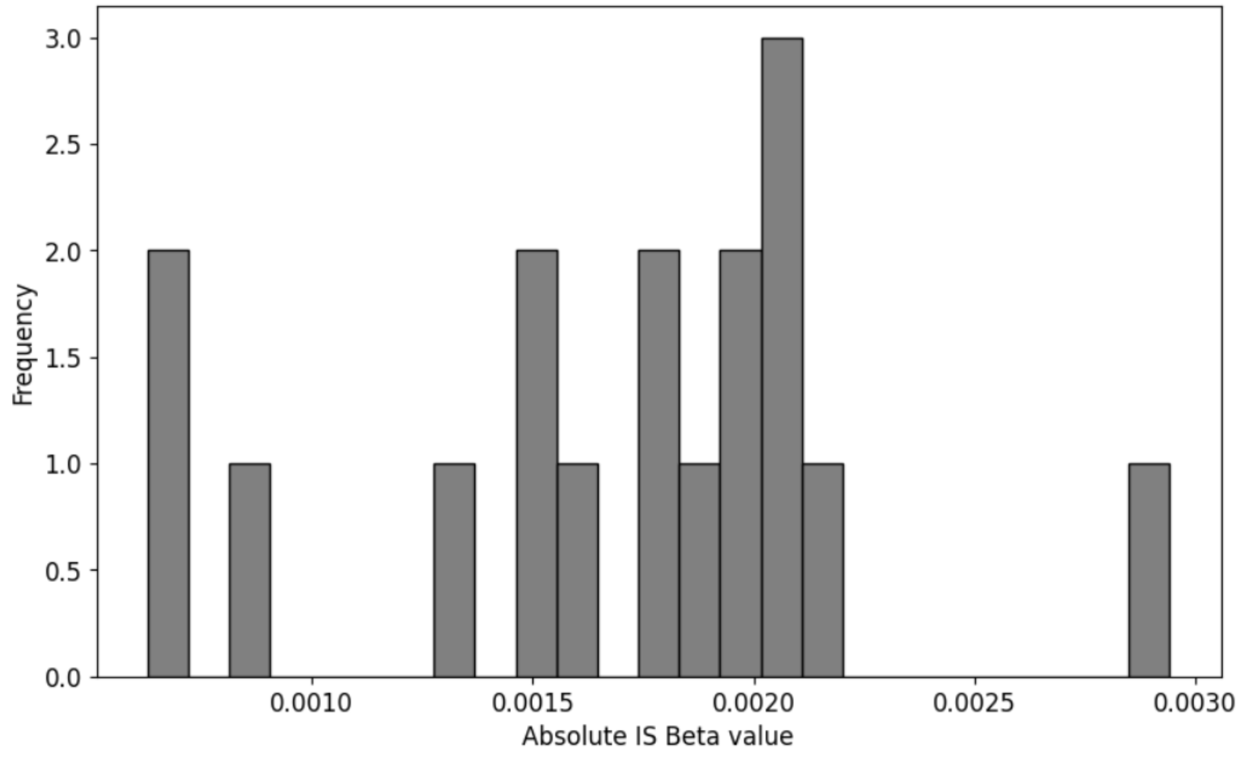
Note. An asset's weight before rebalancing refers to $w_{j,t+}$ from equation (18).

Figure 3. *Histogram of asset sensitivities to investor sentiment under the 25SBM dataset*



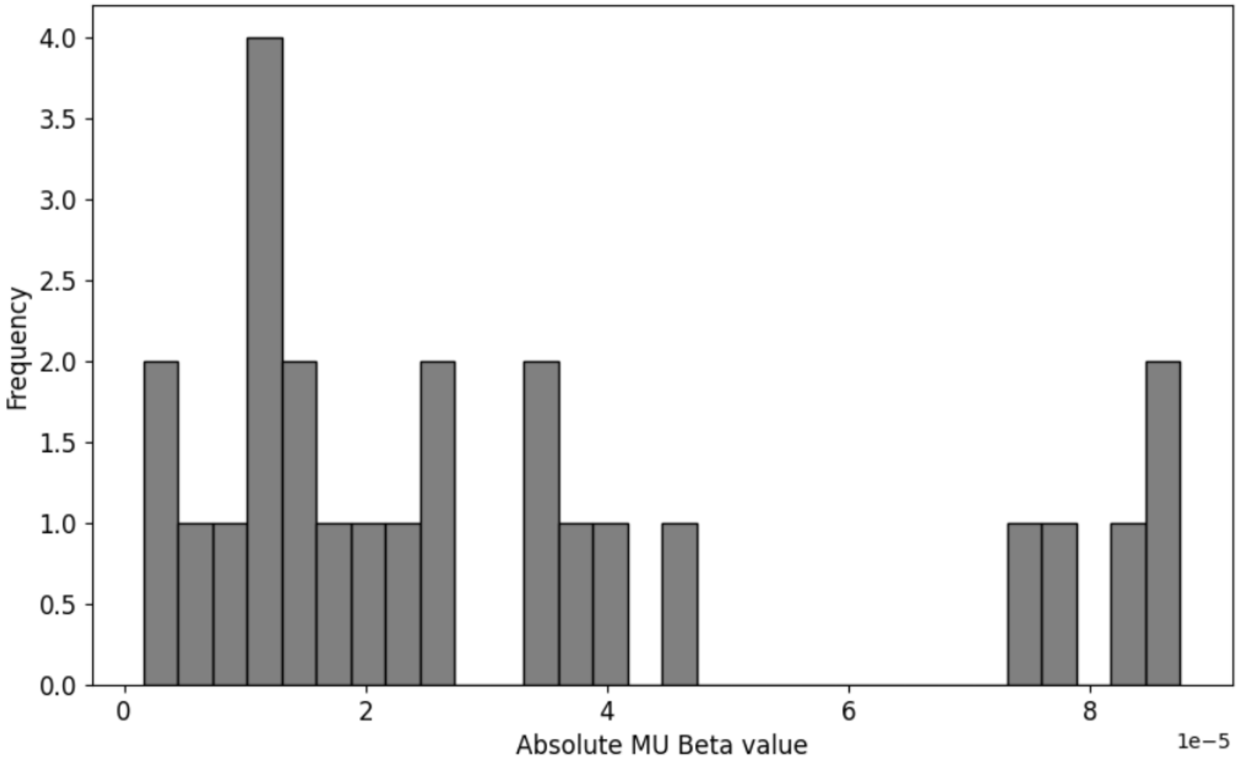
Note. These sensitivities are computed from regression equation (5), using data from the entire sample period from July 2001 to February 2024.

Figure 4. *Histogram of asset sensitivities to investor sentiment under the 17Ind dataset*



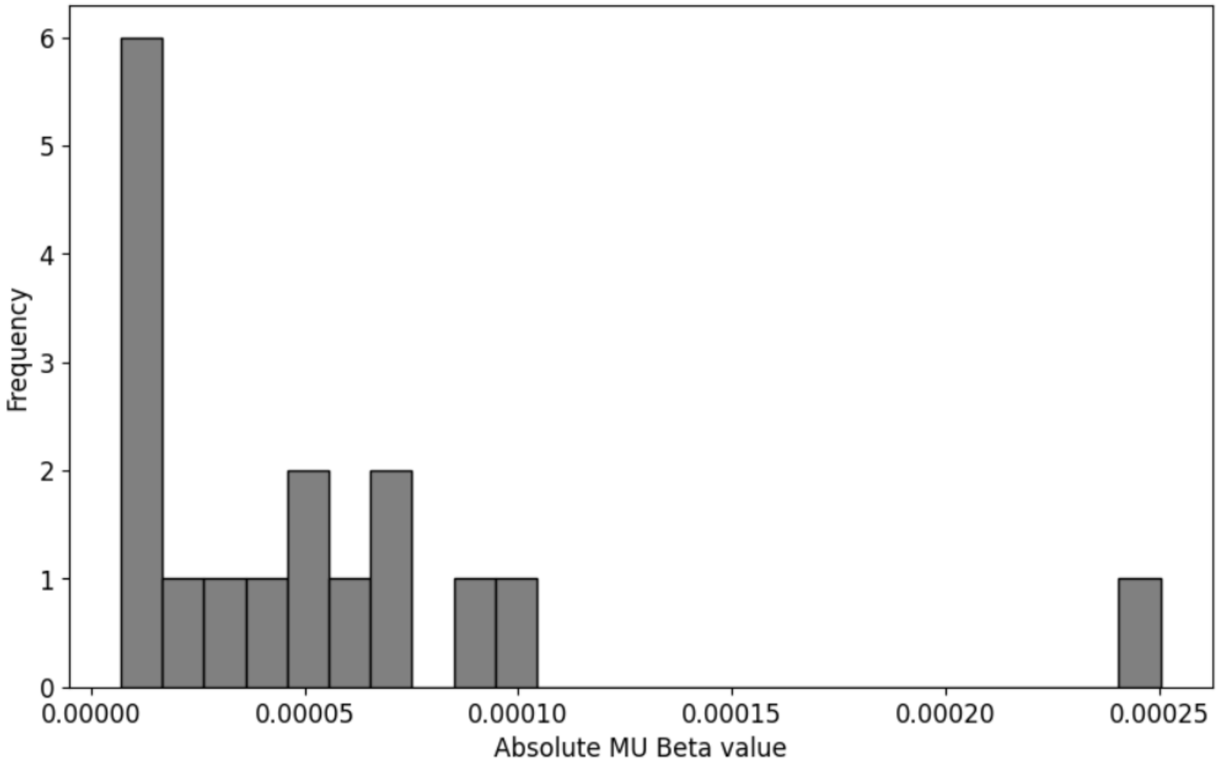
Note. These sensitivities are computed from regression equation (5), using data from the entire sample period from July 2001 to February 2024.

Figure 5. *Histogram of asset sensitivities to macroeconomic uncertainty under the 25SBM dataset*



Note. These sensitivities are computed from regression equation (4), using data from the entire sample period from July 2001 to February 2024. Beta values on the x-axis are multiplied by 10^{-5}.

Figure 6. *Histogram of asset sensitivities to macroeconomic uncertainty under the 17Ind dataset*



Note. These sensitivities are computed from regression equation (4), using data from the entire sample period from July 2001 to February 2024.

Table 4. *Out-of-sample performance metrics for all portfolio strategies under the 25SBM dataset*

Portfolio Abbreviation	$\hat{\mu}$	$\hat{\sigma}^2$	NRP	MOAW	$\widehat{SR}$	TRN
DEW	0.0088	0.0032	0.3585	0.0400	0.1370	0.0510
Min-U	0.0121	0.0036	0.3632	0.3181	0.1838	0.5192
Min-N	0.0083	0.0019***	0.3821	0.0000	0.1675	0.0537
Min-L	0.0084	0.0017***	0.3868	0.1388	0.1791	0.2287
Min-MU-U	0.0121	0.0035	0.3632	0.3117	0.1867	0.5172
Min-MU-N	0.0121	0.0035	0.3632	0.3117	0.1866	0.5172
Min-MU-L	0.0085	0.0017***	0.3868	0.1388	0.1803	0.2287
Min-IS-U	0.0121	0.0036	0.3632	0.3181	0.1838	0.5192
Min-IS-N	0.0121	0.0036	0.3632	0.3181	0.1838	0.5193
Min-IS-L	0.0084	0.0017***	0.3868	0.1388	0.1791	0.2287
Min-MU-IS-U	0.0121	0.0035	0.3632	0.3117	0.1866	0.5172
Min-MU-IS-N	0.0078	0.0020***	0.3726	0.0000	0.1506	0.0489
Min-MU-IS-L	0.0085	0.0017***	0.3868	0.1388	0.1803	0.2287

Note. *p < 0.1, **p < 0.05, ***p < 0.01. Significance tests for $\hat{\mu}$ and $\hat{\sigma}^2$ assess significant differences in mean returns and return variance relative to the DEW. Portfolio abbreviations are explained in Table 1.

Table 5. *Out-of-sample performance metrics for all portfolio strategies under the 17Ind dataset*

Portfolio Abbreviation	$\hat{\mu}$	$\hat{\sigma}^2$	NRP	MOAW	$\widehat{SR}$	TRN
DEW	0.0096	0.0027	0.3679	0.0588	0.1658	0.0818
Min-U	0.0099	0.0014***	0.3821	01469	0.2379	0.2378
Min-N	0.0083	0.0012***	0.3585	0.0000	0.2082	0.0808
Min-L	0.0091	0.0012***	0.3538	0.1105	0.2316	0.1751
Min-MU-U	0.0102	0.0014***	0.3774	0.1502	0.2453	0.2383
Min-MU-N	0.0102	0.0014***	0.3774	0.1502	0.2454	0.2384
Min-MU-L	0.0093	0.0012***	0.3538	0.1122	0.2376	0.1776
Min-IS-U	0.0099	0.0014***	0.3821	0.1469	0.2379	0.2378
Min-IS-N	0.0099	0.0014***	0.3821	0.1469	0.2378	0.2378
Min-IS-L	0.0091	0.0012***	0.3538	0.1105	0.2316	0.1751
Min-MU-IS-U	0.0102	0.0014***	0.3774	0.1502	0.2353	0.2383
Min-MU-IS-N	0.0085	0.0012***	0.3538	0.0000	0.2182	0.0797
Min-MU-IS-L	0.0093	0.0012***	0.3538	0.1122	0.2376	0.1775

Note. *p < 0.1, **p < 0.05, ***p < 0.01. Significance tests for $\hat{\mu}$ and $\hat{\sigma}^2$ assess significant differences in mean returns and return variance relative to the DEW. Portfolio abbreviations are explained in Table 1.

APPENDIX B Optimization in Python

Forming minimum-variance portfolios is a quadratic programming problem. This is because the objective function being minimized is portfolio variance, which is quadratic in nature, as can be inferred from equation (11). The quadratic programming problem here is represented by an objective function to minimize and one or more constraints to satisfy.

Consequently, 'cvxopt', a Python library with optimization capabilities, is used to perform this study's analysis in code. This library contains appropriate modules, specifically 'solvers', and functions, specifically 'solvers.qp', that can solve quadratic programming problems using numerical search algorithms. Such algorithms iteratively search for a solution, converging to a solution over time if one exists.

Here, I present the quadratic programming problem of portfolio variance minimization in the notation that follows the arguments for the solvers.qp function, consistent with the published documentation of the cvxopt library. This can help in understanding the parts of the submitted code that are relevant to this function.

$$\text{minimize} \quad \frac{1}{2} w' P w + q' w \tag{20}$$

$$\textit{Subject to the following constraints}:$$

$$Gw \leq h \tag{21}$$

$$Aw = b \tag{22}$$

As defined in the main section of this study, w is the N × 1 column vector of portfolio weights being solved for, w′ is its transposed, 1 × N row vector, and P is the N × N sample covariance matrix, denoted in the main section as S. Note that $q'w$ reflects the linear term for the objective function; however, since portfolio variance minimization does not have a linear term in the objective function, q' is a 1 × N row vector of zeros. The constraints here are structured as matrix inequality and equality equations. G is an X × N matrix, h is an X × 1 column vector of entries for the inequality constraints' upper bound values, and X is the number of separate constraints in the optimization problem. By way of example, imposing only non-negativity constraints on each asset means that X equals N while imposing non-negativity, IS, and MU constraints means that X equals N + 2. Given that the only equality constraint imposed in every optimization strategy is that the portfolio weights have to sum to one, A is a 1 × N row vector of ones, and b is a 1 × 1 matrix with the only entry being a one.

Appendix C Robustness checks

Table 6. *Out-of-sample performance metrics for all portfolio strategies under the 25SBM dataset and a 54-month estimation window*

Portfolio Abbreviation	$\hat{\mu}$	$\hat{\sigma}^2$	NRP	MOAW	$\widehat{SR}$	TRN
DEW	0.0088	0.0031	0.3624	0.0400	0.1376	0.0534
Min-U	0.0115	0.0026	0.3670	0.3406	0.2041	0.5283
Min-N	0.0078	0.0019***	0.3532	0.0000	0.1546	0.0399
Min-L	0.0083	0.0017***	0.3853	0.1438	0.1738	0.2226
Min-MU-U	0.0114	0.0026	0.3670	0.3472	0.2034	0.5280
Min-MU-N	0.0114	0.0026	0.3670	0.3472	0.2032	0.5279
Min-MU-L	0.0082	0.0017***	0.3853	0.1432	0.1719	0.2228
Min-IS-U	0.0115	0.0026	0.3670	0.3406	0.2041	0.5283
Min-IS-N	0.0115	0.0026	0.3670	0.3406	0.2041	0.5283
Min-IS-L	0.0083	0.0017***	0.3853	0.1438	0.1738	0.2226
Min-MU-IS-U	0.0114	0.0026	0.3670	0.3473	0.2034	0.5280
Min-MU-IS-N	0.0073	0.0020***	0.3532	0.0000	0.1387	0.0399
Min-MU-IS-L	0.0082	0.0017***	0.3853	0.1429	0.1719	0.2228

Note. *p < 0.1, **p < 0.05, ***p < 0.01. Significance tests for $\hat{\mu}$ and $\hat{\sigma}^2$ assess significant differences in mean returns and return variance relative to the DEW. Portfolio abbreviations are explained in Table 1.

Table 7. *Out-of-sample performance metrics for all portfolio strategies under the 17Ind dataset and a 54-month estimation window*

Portfolio Abbreviation	$\hat{\mu}$	$\hat{\sigma}^2$	NRP	MOAW	$\widehat{SR}$	TRN
DEW	0.0096	0.0026	0.3774	0.0588	0.1667	0.0889
Min-U	0.0091	0.0014***	0.3585	01606	0.2126	0.2555
Min-N	0.0084	0.0012***	0.3726	0.0000	0.2090	0.0834
Min-L	0.0090	0.0012***	0.3726	0.1065	0.2274	0.1862
Min-MU-U	0.0091	0.0014***	0.3532	0.1610	0.2141	0.2565
Min-MU-N	0.0092	0.0014***	0.3532	0.1613	0.2142	0.2565
Min-MU-L	0.0091	0.0012***	0.3578	0.1076	0.2287	0.1878
Min-IS-U	0.0093	0.0014***	0.3585	0.1613	0.2184	0.2558
Min-IS-N	0.0093	0.0014***	0.3585	0.1613	0.2184	0.2558
Min-IS-L	0.0091	0.0012***	0.3679	0.1068	0.2299	0.1862
Min-MU-IS-U	0.0095	0.0014***	0.3532	0.1613	0.2206	0.2571
Min-MU-IS-N	0.0085	0.0013***	0.3899	0.0000	0.2067	0.0824
Min-MU-IS-L	0.0092	0.0012***	0.3578	0.1088	0.2320	0.1880

Note. *p < 0.1, **p < 0.05, ***p < 0.01. Significance tests for $\hat{\mu}$ and $\hat{\sigma}^2$ assess significant differences in mean returns and return variance relative to the DEW. Portfolio abbreviations are explained in Table 1.

Table 8. *Out-of-sample performance metrics for all portfolio strategies under the 25SBM dataset and a 66-month estimation window*

Portfolio Abbreviation	$\hat{\mu}$	$\hat{\sigma}^2$	NRP	MOAW	$\widehat{SR}$	TRN
DEW	0.0085	0.0033	0.3641	0.0400	0.1331	0.0517
Min-U	0.0108	0.0025*	0.3835	0.3155	0.1952	0.4771
Min-N	0.0077	0.0019***	0.3835	0.0000	0.1560	0.0346
Min-L	0.0085	0.0018***	0.3835	0.1389	0.1788	0.2259
Min-MU-U	0.0086	0.0023***	0.3932	0.3154	0.1603	0.4772
Min-MU-N	0.0086	0.0023***	0.3932	0.3154	0.1603	0.4771
Min-MU-L	0.0078	0.0018***	0.3835	0.1382	0.1626	0.2262
Min-IS-U	0.0108	0.0025*	0.3835	0.3155	0.1952	0.4771
Min-IS-N	0.0108	0.0025*	0.3835	0.3155	0.1952	0.4771
Min-IS-L	0.0085	0.0018***	0.3835	0.1389	0.1788	0.2259
Min-MU-IS-U	0.0086	0.0023***	0.3932	0.3155	0.1603	0.4772
Min-MU-IS-N	0.0073	0.0022***	0.3738	0.0000	0.1363	0.0413
Min-MU-IS-L	0.0078	0.0018***	0.3835	0.1382	0.1627	0.2263

Note. *p < 0.1, **p < 0.05, ***p < 0.01. Significance tests for $\hat{\mu}$ and $\hat{\sigma}^2$ assess significant differences in mean returns and return variance relative to the DEW. Portfolio abbreviations are explained in Table 1.

Table 9. *Out-of-sample performance metrics for all portfolio strategies under the 17Ind dataset and a 66-month estimation window*

Portfolio Abbreviation	$\hat{\mu}$	$\hat{\sigma}^2$	NRP	MOAW	$\widehat{SR}$	TRN
DEW	0.0094	0.0027	0.3632	0.0588	0.1615	0.0869
Min-U	0.0091	0.0014***	0.3726	01422	0.2209	0.2328
Min-N	0.0083	0.0013***	0.3585	0.0000	0.2065	0.0836
Min-L	0.0088	0.0012***	0.3585	0.1109	0.2249	0.1795
Min-MU-U	0.0096	0.0015***	0.3786	0.1462	0.2202	0.2375
Min-MU-N	0.0096	0.0015***	0.3786	0.1461	0.2203	0.2375
Min-MU-L	0.0091	0.0013***	0.3641	0.1131	0.2244	0.1847
Min-IS-U	0.0091	0.0014***	0.3726	0.1422	0.2209	0.2328
Min-IS-N	0.0091	0.0014***	0.3726	0.1422	0.2209	0.2328
Min-IS-L	0.0088	0.0012***	0.3585	0.1109	0.2249	0.1795
Min-MU-IS-U	0.0096	0.0015***	0.3786	0.1462	0.2202	0.2375
Min-MU-IS-N	0.0085	0.0013***	0.3739	0.0000	0.2138	0.0802
Min-MU-IS-L	0.0091	0.0013***	0.3641	0.1131	0.2244	0.1847

Note. *p < 0.1, **p < 0.05, ***p < 0.01. Significance tests for $\hat{\mu}$ and $\hat{\sigma}^2$ assess significant differences in mean returns and return variance relative to the DEW. Portfolio abbreviations are explained in Table 1.

Table 10. *Out-of-sample performance metrics for IS-incorporating strategies under the 25SBM dataset and the alternative IS proxy*

Portfolio Abbreviation	$\hat{\mu}$	$\hat{\sigma}^2$	NRP	MOAW	$\widehat{SR}$	TRN
Min-IS-U	0.0121	0.0036	0.3632	0.3181	0.1838	0.5192
Min-IS-N	0.0121	0.0036	0.3632	0.3181	0.1838	0.5193
Min-IS-L	0.0084	0.0017***	0.3868	0.1388	0.1791	0.2287
Min-MU-IS-U	0.0121	0.0035	0.3632	0.3117	0.1866	0.5172
Min-MU-IS-N	0.0078	0.0020***	0.3726	0.0000	0.1506	0.0489
Min-MU-IS-L	0.0085	0.0017***	0.3868	0.1388	0.1803	0.2287

Note. *p < 0.1, **p < 0.05, ***p < 0.01. Significance tests for $\hat{\mu}$ and $\hat{\sigma}^2$ assess significant differences in mean returns and return variance relative to the DEW. Portfolio abbreviations are explained in Table 1.

Table 11. *Out-of-sample performance metrics for IS-incorporating strategies under the 17Ind dataset and the alternative IS proxy*

Portfolio Abbreviation	$\hat{\mu}$	$\hat{\sigma}^2$	NRP	MOAW	$\widehat{SR}$	TRN
Min-IS-U	0.0099	0.0014***	0.3821	0.1479	0.2384	0.2377
Min-IS-N	0.0099	0.0014***	0.3821	0.1479	0.2383	0.2376
Min-IS-L	0.0091	0.0012***	0.3538	0.1105	0.2316	0.1751
Min-MU-IS-U	0.0102	0.0014***	0.3774	0.1502	0.2453	0.2383
Min-MU-IS-N	0.0084	0.0012***	0.3585	0.0000	0.2152	0.0792
Min-MU-IS-L	0.0093	0.0012***	0.3538	0.1122	0.2376	0.1775

Note. *p < 0.1, **p < 0.05, ***p < 0.01. Significance tests for $\hat{\mu}$ and $\hat{\sigma}^2$ assess significant differences in mean returns and return variance relative to the DEW. Portfolio abbreviations are explained in Table 1.

Table 12. *Out-of-sample performance metrics for MU-incorporating or IS-incorporating strategies under the 25SBM dataset and the alternative regression specification*

Portfolio Abbreviation	$\hat{\mu}$	$\hat{\sigma}^2$	NRP	MOAW	$\widehat{SR}$	TRN
DEW	0.0085	0.0032	0.3623	0.0400	0.1332	0.0510
Min-MU-U	0.0108	0.0028	0.4010	0.3123	0.1860	0.4944
Min-MU-N	0.0108	0.0028	0.4058	0.3124	0.1860	0.4944
Min-MU-L	0.0087	0.0018***	0.3865	0.1488	0.1810	0.2255
Min-IS-U	0.0105	0.0028	0.4010	0.3123	0.1804	0.4906
Min-IS-N	0.0105	0.0028	0.4010	0.3123	0.1804	0.4906
Min-IS-L	0.0084	0.0018***	0.3913	0.1461	0.1748	0.2232
Min-MU-IS-U	0.0108	0.0028	0.4010	0.3123	0.1860	0.4944
Min-MU-IS-N	0.0082	0.0021***	0.3671	0.0000	0.1555	0.0427
Min-MU-IS-L	0.0087	0.0018***	0.3865	0.1488	0.1810	0.2255

Note. *p < 0.1, **p < 0.05, ***p < 0.01. Significance tests for $\hat{\mu}$ and $\hat{\sigma}^2$ assess significant differences in mean returns and return variance relative to the DEW. Portfolio abbreviations are explained in Table 1. Alternative regression specifications are described in Chapter 6. The results for the DEW in this table relate to the shortened out-of-sample period of 207 months, which is consistent with the other strategies.

Table 13. *Out-of-sample performance metrics for MU-incorporating or IS-incorporating strategies under the 17Ind dataset and the alternative regression specification*

Portfolio Abbreviation	$\hat{\mu}$	$\hat{\sigma}^2$	NRP	MOAW	$\widehat{SR}$	TRN
DEW	0.0094	0.0027	0.3720	0.0588	0.1607	0.0828
Min-MU-U	0.0095	0.0015***	0.3671	0.1579	0.2229	0.2309
Min-MU-N	0.0095	0.0015***	0.3671	0.1579	0.2227	0.2309
Min-MU-L	0.0091	0.0013***	0.3816	0.1091	0.2267	0.1719
Min-IS-U	0.0094	0.0014***	0.3623	0.1571	0.2235	0.2294
Min-IS-N	0.0094	0.0014***	0.3623	0.1572	0.2235	0.2294
Min-IS-L	0.0090	0.0012***	0.3816	0.1078	0.2263	0.1701
Min-MU-IS-U	0.0095	0.0015***	0.3671	0.1579	0.2228	0.2309
Min-MU-IS-N	0.0076	0.0014***	0.3816	0.0000	0.1801	0.0703
Min-MU-IS-L	0.0091	0.0013***	0.3816	0.1091	0.2267	0.1719

Note. *p < 0.1, **p < 0.05, ***p < 0.01. Significance tests for $\hat{\mu}$ and $\hat{\sigma}^2$ assess significant differences in mean returns and return variance relative to the DEW. Portfolio abbreviations are explained in Table 1. Alternative regression specifications are described in Chapter 6. The results for the DEW in this table relate to the shortened out-of-sample period of 207 months, which is consistent with the other strategies.