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Enhanced Beta Estimation through Peer Firm Selection

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ABSTRACT

This thesis examines the use of the SARD method to enhance beta estimation for firms, comparing it to existing models. Two new beta estimators are proposed: one combining peer firm data identified via the SARD method with Bayesian estimators, and another using only peer firm data. Using 2012-2013 data from the S&P Composite 1500, the accuracy of these estimators was assessed through Mean Squared Error (MSE) comparisons. Results show both estimators improve beta accuracy. The Peer beta estimator effectively estimates betas for non-publicly traded firms. This study highlights the potential of the SARD method to enhance beta prediction, providing a versatile tool for financial analysis and research.

Keywords: corporate finance, CAPM, beta, SARD method.

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CHAPTER 1 Introduction

This research will seek to improve beta estimation accuracy by using a new method to find similar firms. Beta[1] is a key input in CAPM and accurate beta estimation is therefore important for capital budgeting decisions of firms and company valuations. Two new beta estimators will be created. In one, information about the peer firms found with the new method will be combined with current Bayesian beta estimators. In the other, only information about the discovered peer firms would be used to create a beta estimator. As both methods are easy to implement, they would enable more precise beta estimation by practitioners if precise. Additionally, as one of the two estimators does not require any stock price history of the firm for which beta is estimated, it would allow for precise beta estimation of firms that are not publicly traded, which is currently not possible.

In firm valuation and capital budgeting processes, CAPM is commonly used. CAPM is a model that enables the calculation of the cost of equity of a firm. The cost of equity is used as the discount rate for cash flows, which is crucial in determining the value of a company or of a project. Beta is one of the key inputs in CAPM. Beta represent the sensitivity of the returns of the stock to the returns of the market, the market being the index to which the firm belongs. Being able to accurately estimate beta is therefore essential in both company valuation and capital budgeting.

The simplest form of measuring beta consists of simply performing a regression of the returns of the firm on the returns of the index to which the firm belongs. However, the true beta of a firm cannot be precisely known through measurement, since there are usually measurement errors or firm-specific events that distort the estimated beta. This means that an estimate must be found that uses as much information as possible to find the true beta. Assuming a normal distribution of betas, this can be done by using the information about the betas of other firms. In his paper 'A NOTE ON USING CROSS-SECTIONAL INFORMATION IN BAYESIAN ESTIMATION OF SECURITY BETAS', Vasicek (1973) therefore proposes a new Bayesian technique for computing beta. Under this method, the average beta of other firms and the variance in these betas are used in addition to the 'simple' beta of the firm, namely by shrinking the measured beta of the individual firm toward the average beta of other firms.[2] The amount of shrinkage depends on the statistical strength of the beta estimate of the individual firm relative to that of the other firms. Vasicek (1973) also states that is it best to use all information on beta available. This also implies that an improved selection of the firms that are used to perform the Bayesian technique, can be relevant.

[1] The sensitivity of a firm's stock returns to the returns of the index to which it belongs.
[2] In practice, the 'other firms' are usually taken as all other firms in an index (e.g. S&P 500) to which the firms belongs.

Karolyi (1992) further expands on the model of Vasicek (1973) by adding two specific factors to the model: firm size and industry. Specifically, he expands the formula used by Vasicek (1973) by dividing all firms into groups (based on industry and size) and shrinking the beta of the individual firm toward the average beta of the groups to which the firms belongs, the amount of shrinkage depending on the statistical strength of the beta estimates of these groups. Thus, simplifying, the beta of the firm will be shrunk to multiple targets. This method yielded an estimation advantage relative to the Vasicek (1973) model.

Lastly, Knudsen et al. (2017) in 'Stick to the Fundamentals and Discover your Peers' proposed the SARD method for selecting firms that are similar ('peer firms'). They applied this method to multiples valuation[3], not to beta estimation. They ranked firms based on four input variables: return on equity, market capitalization, earnings growth estimates and net debt/EBIT. Companies were then grouped based on which companies had the smallest sum of differences in ranks between each other, resulting in peer firms. A key strength of the SARD method is that it allows for a theoretically unlimited amount of input variables to select peer firms from, enabling it to use large amounts of firm information.

Applying the SARD method to beta estimation thus allows for the exploration of a model that can greatly improve the accuracy of the beta estimates by incorporating more relevant information. Two new beta estimators will be computed. The first combines peer firms found through the SARD method with the Vasicek (1973) method, by using the mean beta and standard deviation of the betas of the discovered peer firms in the Vasicek (1973) method. As Vasicek (1973) states that using all relevant information about a firm is necessary and Karolyi (1992) found that incorporating information about industry and market capitalization optimizes beta estimates, using information about firms that are most comparable based on more financial fundamentals can lead to improved estimates.

The second estimator will be computed by taking the mean beta of the peer firms found through the SARD method, without using the measured beta of the individual firm. As this approach would use the beta estimates of highly comparable firms and thus many of the characteristics of the individual firm, this estimator can also lead to accurate beta estimates. The main advantage of this estimator would be that it could be used to compute beta estimates for private firms[4], for which no highly accurate model currently exists.

[3] Valuing firms by determining the market capitalization of similar firms.
[4] I.e. firms that are not publicly traded and thus do not have any stock price history.

Specific characteristics of the SARD model make it suitable to find relevant peer firms for beta estimation. First, the input to the SARD model is not limited to a certain amount of variables. It can accommodate as many variables as desired, which can greatly improve the accuracy of the beta estimates by incorporating more relevant information. Using the input variables as stipulated in the paper of Knudsen et al. (2017) – return on equity, market capitalization, net debt/EBIT, earnings forecasts and industry – can already yield improved forecasting power relative to the input of the Karolyi (1992) paper: size and industry. This flexibility of the SARD method also allows for the exploration of different firm fundamentals as input variables in order to determine the optimal beta estimator.

Second, the SARD method has already proven to be effective at predicting the value of firms in multiples analysis. Thus, it is possible that this predictive power also extends to the prediction of beta. Karolyi (1992) specifically mentions that extending the research by adding more input variables is desirable, indicating that this type of research is useful. Third, the SARD method is not sensitive in outliers in the values of input variables, since peer firms are selected based on rank and not based on absolute values.

Lastly, the SARD method is easy to use and intuitive. When looking at one company specifically, as practitioners do, peer groups can be computed using simple calculations. Additionally, the complexity of the model does not increase if different or more input variables (such as ROE) are used. The Karolyi model, however, is difficult to use relative to the SARD method, with greatly increasing complexity as the number of input variables goes up.

Thus, the research question is:

> To what extent does using prior information about peer firms found with the SARD method enhance beta estimation?

Data will be taken from Compustat, CRSP and Bloomberg. This allows for the usage of data about fundamentals of the firms, which allows for the construction of peer firm groups using the SARD method. Additionally, these sources provide security and index prices, which enables the calculation of betas.

Betas of firms in the dataset will be calculated using the data from 2012 of the S&P Composite 1500. The estimation power will be measured by observing the MSE (mean squared error) of the estimated beta relative to the observed beta in 2013. Daily data will be used to measure beta in 2012, as Levi and Welch (2017) state that this yields the best predictive results. This conclusion can also be drawn from the results of the research by Karolyi (1992). One year of daily data will be used to measure beta, as

Levi and Welch (2017) conclude that using more years of data does not yield significantly greater predictive power, while this would complicate the calculations.

The calculation of the new beta estimators will consist of two main parts. First, peer firm groups will be created for each company. After that, the calculation of the estimated beta will be done using information about those peer firms, after which the estimated betas can be compared to the observed 2013 beta. The MSE will be compared with outcomes of other models (Karolyi, Vasicek market-wide, etc.), which will be calculated from the same data.

By combining the SARD and Vasicek (1973) method, it can be hypothesized that a more predictive model will be found, as more firm information will be included in the model. The crucial question will be whether or not the model will have more predictive power than current models, which this research will test. However, if the model does prove to yield more or similar predictive power, this proves that the application of the SARD model to beta calculations is useful. Additionally, it would provide for an easy-to-use model that takes into account relevant firm information and allows for flexibility in selecting input variables (such as size, industry and growth estimates).

Chapter 1 contained an introduction of the topic. Chapter 2 will give the theoretical framework of this paper, discussing CAPM, several methods for beta estimation and the SARD method.. Chapter 3 discusses the data that is used, while chapter 4 discussed the methodology that is used. Chapter 5 provides the results that were found a discussion of those results, discussing whether or not the proposed method enhances beta estimates. Chapter 6 contains the conclusion to this paper.

CHAPTER 2 Theoretical Framework

2.1 Introduction

In this chapter, the ideas and concepts underlying the research will first be discussed. Two main concepts are relevant: the meaning of beta, specifically in the context of the capital asset pricing model (hereafter referred to as '**CAPM**'), and the concept of Bayesian beta estimation. These two concepts are also crucial to the relationship that this study will examine. This will be explained at the end of this chapter, where this relationship will be specified and hypotheses will be stated.

2.2 Beta

2.2.1 Capital asset pricing model

In order to understand the concept of beta, CAPM must first be discussed. This model seeks to determine the expected return on securities (Lintner, 1965; Mossin, 1966; Sharpe, 1964; Treynor, 1961). The expected return for securities essentially means the return that investors expect from a stock.

In this model, the expected return on a stock is determined in large part by the sensitivity of the return of the stock to the return of the market, a concept that will hereafter be referred to as '**beta**'. Usually, the return on an index to which the stock belongs is taken as the return of the market. The expected return also depends on the contemporary risk-free rate and on the market risk premium, i.e. the expected return on the market portfolio over the risk-free rate. The CAPM formula is:

$$E(R_i) = R_f + \beta_i(E(R_m) - R_f) \tag{1}$$

Where $E(R_i)$ is the expected return on security i, $R_f$is the risk-free rate, β_i measures the sensitivity of the return of the stock to the return of the market and $E(R_m)$ measures the market risk premium.

CAPM has been highly influential and widely used. For example, Sharpe (1965) has been cited 33,471 times according to Google Scholar and a large numbers of CFOs claimed to use this model (Graham & Harvey, 2001). However, its accuracy has been a subject of debate. In their 2004 paper, Fama & French state that most of the present applications of the model are invalid for several reasons.

One problem with CAPM is that its assumptions are unrealistic, such as a lack of transactions costs or taxes, unlimited borrowing and lending and investors possessing homogeneous expectations and being purely rational (Fama & French, 2004). Additionally, during empirical tests, it became clear that the measurements of beta were imprecise, as measurement error leads to relatively imprecise estimations of firm betas (Fama & French, 2004). One solution was the usage of portfolio estimation in order to improve the accuracy of the estimates (Blume, 1970; Friend & Blume, 1970). Another solution will be discussed under '2.3 Bayesian beta estimation'.

Moreover, market capitalization and book-to-market ratios seemed to have an effect on the average returns of stocks, which is contradictory to CAPM (Banz, 1981; Rosenberg et al., 1985). The popular Fama-French three-factor model sought to address this fact by including these two factors into an expanded model (Fama & French, 1992).

Another problem lies in the empirically observed implications of betas. Baker et al. (2016) provide a literature overview that points out that betas possibly do not even have an impact on the expected rate of return, while Frazzini & Pedersen (2014) describe the lack of a consistent positive relationship between betas and average returns. Levi & Welch (2017) find that high beta stocks have historically been outperformed by low-beta stocks. All of these findings contradict CAPM. Lastly, Roll (1977) argues that CAPM cannot be empirically tested, as these tests inevitably use proxies of the true market portfolio, since it is unclear what the true market portfolio truly entails. For example, it is unclear why real estate or other factors should not be included in the definition of the 'market'.

Despite these criticisms, CAPM stills seems to be commonly used by practitioners. Graham & Harvey (2001) for example find that 73.5% of respondent CFOs always or almost always use CAPM. Dessaint et al. (2017) find that the usage of CAPM can be used to explain cumulative abnormal returns in takeovers, indicating a continued usage of CAPM by practitioners. CAPM is used to varying ends, but plays a notable role in determining the cost of equity of companies. Through this function, it plays an essential role in both capital budgeting processes and company valuations. The continued popularity can possibly be explained by the fact that it is easy to use relative to other models.

Because of this continued popularity of CAPM, estimating beta correctly remains an important endeavour. The role that beta also plays in other models, such as the aforementioned Fama-French three-factor model, further signifies the importance of accurate beta estimation. Lastly, the pursuit of other purposes using beta, such as academic ones, also benefits from accurate estimation methods.

2.2.2 Methods of beta estimation

The ‘basic’ method of calculating beta (hereafter referred to as ‘**OLS beta**’) is done using the following formula:

$$\beta_i = \frac{cov(r_i, r_m)}{var(r_m)} \tag{2}$$

Here, β_i is the beta of the individual firm, r_i is the return of the firm’s stock and r_m is the return of the market. The measurement of returns can be based on daily, weekly and monthly stock price history. This is the same as an OLS regression of the return of the individual stock on the return of the market.

However, the beta estimations of individual stocks suffer from measurement error and are therefore unreliable (Fama & French, 2004; Levi & Welch, 2017). Idiosyncratic noise or thin trading bias are some of the reasons that can cause this measurement error (Karolyi, 1992; Roll, 1988). When no stock price history is available and no beta can therefore be calculated, practitioners often use the mean beta of the firm’s industry to estimate the firm’s beta (Levi & Welch, 2017). Additionally, beta suffers from non-stationarity, which further reduces the power of beta estimates (Blume, 1971; Levi & Welch, 2017).

2.3 Bayesian beta estimation

2.3.1 Vasicek (1973)

Vasicek (1973) proposes an alternative method of estimating beta which is designed to diminish the effect of measurement error. He states that OLS regressions do not suffice here, as the true value of beta is assumed to be known in those regressions (OLS calculates coefficients that minimize the difference between the observed outcome and predicted outcome, hence assuming that the observed outcome is the true value). The situation is actually the reverse: because of the presence of measurement error, only the sample coefficient - i.e. not the true but only the measured beta - is known.

Vasicek (1973) therefore develops an estimator, hereafter referred to as ‘**Vasicek beta**’, that takes into account the (assumed to be normal) cross-sectional distribution of betas.[5] If the betas of all other firms are significantly higher than the beta of a single firm, for example, the beta of that firm was most likely underestimated. The beta of the individual firm is therefore shrunk towards the cross-sectional mean of betas. The extent to which beta is shrunk, depends on the statistical ‘strength’ of the

[5] ‘Cross-sectional’ refers to the other firms in the index.

individual firm beta relative to that of the cross-sectional mean, which is measured by the standard error of the beta and the standard deviation of the cross-sectional mean, respectively. The formula of the proposed estimator is:

$$\left(\frac{Stdv(Beta_{market})^2}{Stdv(Beta_{market})^2 + StdError\left(Beta_{firm}\right)^2}\right) * Beta_{market}$$
$$+$$
$$\left(\frac{StdError\left(Beta_{firm}\right)^2}{StdError\left(Beta_{firm}\right)^2 + Stdv(Beta_{market})^2}\right) * Beta_{firm}$$

(3)

Where Stdv($Beta_{market}$) is the standard deviation of the cross-sectional beta estimates, StdError($Beta_{firm}$) is the standard error of the estimated beta of the individual firm, $Beta_{market}$ is the cross-sectional mean beta estimate and $Beta_{firm}$ is the OLS beta of the stock in question.

Vasicek (1973) states that the choice of parameters of prior density depend on the prior information that is available. If one does not possess any information about the stock other than the population it belongs to, the cross-sectional betas of the population will be the prior information. In some cases, more can be known about the stock than the population it belongs to. The Vasicek beta consistently yields better beta estimations than the OLS beta (Karolyi, 1992; Levi & Welch, 2017).

2.3.2 Karolyi (1992)

Karolyi (1992) expands on the earlier work of Vasicek (1973) by incorporating more prior information about the stock into the estimation. Specifically, he utilizes information about firm size (market capitalization) and industry to further enhance beta estimation. Firm size has been shown to influence (Banz, 1981; Chan & Chen, 1988; Reinganum, 1981). The industry in which the firm operates has also been shown to influence beta (Farrell, 1974; King, 1966; Livingston, 1977; Meyers, 1973). Thus, using this information in the Vasicek beta could enhance beta estimation. Therefore, Karolyi (1992) proposes the following estimator that uses three shrinkage targets:

$$\hat{\beta}_{ij} = \frac{\hat{\beta}_i + \sum_j \gamma_{ij} \bar{\beta}_{ij}}{1 + \sum_j \gamma_{ij}}$$

(4)

Where $\hat{\beta}_{ij}$ is the shrinkage estimator that incorporates firm size and industry (hereafter referred to as '**Karolyi beta**'), $\hat{\beta}_i$ is the OLS beta, γ_{ij} is the shrinkage weight ($\gamma_{ij} = \frac{S^2_{\hat{\beta}_i}}{S^2_{ij}}$)and $\bar{\beta}_{ij}$ is the mean beta of group j that the firm belongs to. Group j can denote subscript 0 (market), 1 (industry) or 2 (size). When the betas in the group (market, industry or size) that the firm belongs to have high variance, meaning that belonging to this group has less predictive power, the mean beta of that group will be taken into account less in $\hat{\beta}_{ij}$. Using MSE to verify the forecasting power of the Karolyi beta, (Karolyi, 1992) finds that it yields a 5% reduction in MSE compared to the market-wide technique (Vasicek beta).

2.3.3 Levi & Welch (2017)

In their 2017 study, Levi & Welch provide recommendations for the best practices regarding beta estimation. They recommend taking the Vasicek beta and shrinking this beta by another 20%-30% toward a target that depends on the market capitalization of the firm. This is done to account for aforementioned non-stationarity of the beta and because the study shows that this method results even in better estimations of beta. If beta needs to be estimated for a longer duration, additional shrinkage is recommended, because of the non-stationarity of beta. If beta needs to be estimated for small firms, additional shrinkage is required, as the estimation methods prove less effective for smaller firms. In this study, the practice of taking the mean beta of the industry group of the firm is proven to be ineffective.

2.4 SARD method

The SARD method was developed by Knudsen et al. (2017). The method seeks to enable the selection of relevant peer firm companies for multiples valuation. Thus, the SARD method originally was not intended to play a role in beta estimation. The SARD method entails ranking firms based on proxies for profitability, growth and risk. The paper specifically uses market capitalization, net debt/EBIT, EPS growth estimates and return on equity as these proxies. These will hereafter be referred to as '**input variables**'. After all firms are ranked for these four metrics, peer firm groups are for every individual firm by finding firms with the smallest sum of absolute rank differences relative to the individual firm in question. The method is more powerful when peer firms are only selected from the same industry. The method proves to be highly effective at finding relevant peer firm groups and thus at predicting firm value based on multiples analysis. Additionally, the method enjoys the key advantages of being insensitive to outliers and being able to select based on a theoretically indefinite amount of variables (Knudsen et al., 2017).

Another key advantage of the SARD beta computation is that the method is not sensitive to outliers in one of the input variables, as peer firms will be selected based on multiple input variables and only the rank of the input variables matters, not the absolute value. Thus, any outliers in firm fundamentals should not decrease the forecasting power of this method.

In this paper, the SARD method will also be used to find peer firms. However, instead of using these peer firms in multiples valuation, they will be used to obtain two estimates of beta. One of these estimators, the '**SARD beta**', will use the information about the betas and standard deviation of the betas of the peer firms in formula 3 (instead of information about the entire population of firms). The other estimator, the '**Peer beta**', will take the mean of the betas of the peer firms, without using information about the firm for which the beta is being estimated. The estimation of these betas will be discussed further in the Methodology chapter. As stated before, using the SARD method in beta estimation can lead to improved beta estimates, as more information about the firm is used. Additionally, the Peer beta is purely based on information about other firms, which allows for beta estimation for firms that are not publicly traded. This would provide a key tool for practitioners and further research, as it is currently difficult to accurately estimate beta for private companies.

2.5 Conclusion

As seen, beta is a key concept in CAPM and in some other models and applications. Bayesian estimation can yield better beta estimates (Karolyi, 1992; Levi & Welch, 2017). Karolyi (1992) proves that the choice of prior information is relevant, as including size and industry characteristics of firms in Bayesian estimation improves beta estimates. The SARD method allows for the selection of suitable peer firms group in multiples valuation and thus for more accuracy in multiples valuation, while enjoying the benefit of resilience to outliers and a theoretically indefinite amount of input variables.

As Vasicek (1973) states, all relevant firm information needs to be taken into account. The SARD method can use far more information than other models discussed in this paper, as it is able to incorporate many input variables. Theoretically, it could use an infinite amount of input variables. Additionally, because of its high flexibility, this method can be greatly enhanced in further research by changing the input variables that are used and the relative importance that is assigned to these input variables.

Using the SARD method to select relevant peer firms and using the mean and standard deviation of the betas of these firms in combination with the Vasicek (1973) method can therefore allow for improved beta estimation. Additionally, using the SARD method to compute the Peer beta can allow for a beta estimator that does not require the firm to have stock price history (i.e. to be listed on a stock exchange) but still accurately estimates beta, as the beta estimator is based on a high amount of input variables that contain important information about the firm.

The flexibility of the model can, however, also lead to it being used as justification for the choice of a certain beta, by choosing variables that fit that specific outcome. In order to prevent misuse, it is therefore crucial that research establish clear methodology and suggestions.

The relationship that will be studied in this research will therefore be the relationship between the choice of prior information and the accuracy of (Bayesian) beta estimation. Beta estimation directly influences the effectiveness of CAPM, which influences the accuracy of capital budgeting and the accuracy of company valuation.

One hypothesis is that combining using the SARD method in beta estimation will yield greater beta estimation accuracy than other aforementioned estimators, because it uses more relevant firm characteristics as prior information. Another hypothesis is that the beta estimator that results from the combination will be highly customizable, since input variables can be replaced or added without sacrificing usability or estimating strength. The last hypothesis is that using a beta estimator that is based solely on the mean beta of peer firms found with the SARD method will lead to better estimations than using the industry mean (Levi & Welch, 2017).

CHAPTER 3 Data

3.1 Sample

In order to study the effectiveness of different beta estimators, all firms that were in the S&P Composite 1500 throughout all of 2012 and 2013 will be used. Using the Bloomberg Terminal, 1321 firms were found to have been present in the S&P Composite 1500 throughout both of these years.

Firms were removed if no data or stock price history was found in CRSP using their CUSIP. After, in accordance with Knudsen et al. (2017), firms were removed from the dataset if they were found to have (i) negative earnings before extraordinary items, (ii) negative book value of equity, (iii) negative enterprise value, (iv) negative EBIT, (v) negative net sales, (vi) negative invested capital and (vii) negative market capitalization.

Lastly, firms were removed from the dataset if the Bloomberg Terminal did not have 1 year forward analysts' forecasts or if any of the essential datapoints were missing in the Compustat data. After these steps, 1,039 firms were left in the dataset.

3.2 Variables

The variables that were collected are displayed in Table 1 in Appendix B. 1-year forward EPS estimates were used instead of the EPS estimates used in the original paper on the SARD method due to data availability (Knudsen et al., 2017). However, the metrics are very similar and should both function well as proxies for future growth; The firm fundamental data necessary to find peer firms under the SARD method were figures from the 31st of December, 2012, as this date reflects the date on which the beta is estimated that is compared to the 2013 beta (see 'Chapter 4 – Methodology').

The adjusted daily closing price, hereafter '*Price'*, will be used for beta calculation. Per firm, 224 days of trading were required per year. Thus, 465,472 daily returns were used. Karolyi (1992) and Levi & Welch (2017) find that beta estimations based on daily stock returns provide better estimates. Levi & Welch (2017) also find that 1-year and 3-year stock returns perform relatively similarly; 1 year daily returns will thus be used. The closing price the S&P Composite 1500 will be used to the same end and will hereafter be referred to as the '*Index price*'. The daily market capitalization, hereafter '*Market capitalization*' is used for varying purposes, notably to calculate the Karolyi beta. The SIC industry codes will be used for varying purposes, notably to calculate the Karolyi beta. The 1-year forward Bloomberg estimates, hereafter '*EPS growth*' will be used to compute the SARD beta. All items from Compustat will be used to calculate the SARD beta.

3.3 Summary statistics

In Tables 2, 3, 4 and 5, summary statistics about the firms' characteristics and OLS beta estimates have been given. All three indices contribute approximately one-thirds of the firms in the dataset, meaning that small-cap, mid-cap and large-cap firms are represented relatively evenly. From the statistics on the market capitalization of firms, it appears that the distribution is right-skewed, with outliers with high market capitalization increasing the average relative to the mean.

The Vasicek beta relies on a normal distribution of risk measures, as shrinkage is useful only when the population is distributed around a certain mean. The skewness figures suggest slight violations of the normality assumption. However, Bayesian estimation has proven to yield improved results over other estimation methods in certain settings (Karolyi, 1992; Landsman & Damodaran, 1989; Thisted & Wecker, 1981). Additionally, Levi & Welch (2017) find that the Vasicek beta is a better predictor of beta than the OLS beta. Regardless, this paper seeks to compare the forecasting power of the SARD beta method to the forecasting power of other, already established methods, implying that violations of underlying assumptions of other models are not relevant in the context of this research. As stated above, the SARD method is not sensitive to outliers in firm fundamentals.

Year	Average	Median	Standard deviation	Skewness
2012	1,140	1,074	0,420	0,443
2013	1,109	1,126	0,296	0,530

Table 2: summary statistics OLS beta estimates

S&P	Amount of firms
400	286 (*27,53%*)
500	377 (*36,28%*)
600	376 (*36,19%*)

Table 3: amount of firms per S&P index

GIC Industry Code	Industry	Amount of firms
0100-0999	Agriculture, Forestry and Fishing	3
1000-1499	Mining	27
1500-1799	Construction	18
2000-3999	Manufacturing	398
4000-4999	Transportation, Communications, Electric, Gas and Sanitary service	104
5000-5199	Wholesale Trade	37
5200-5999	Retail Trade	84
6000-6799	Finance, Insurance and Real Estate	202
7000-8999	Services	161
9100-9729	Public Administration	0
9900-9999	Non-classifiable	5

Table 4: amount of firms per GIC industry

	Mean	Standard deviation	Min	Max
Total assets (USD; millions)	26.355,72	138.453,40	73,18	2.359.141
Cash and short term investments (USD; millions)	3.307,03	24.339,82	0,25	471.833
Debt in current liabilities (USD; millions)	2.340,37	24.013,05	0	379.187
Total long-term debt (USD; millions)	3.943,11	17.489,29	0	274.873
Earnings before interest and taxes (EBIT) (USD; millions)	1.422,60	4.406,87	0,91	55.241
Income before extraordinary items (USD; millions)	820,06	2.780,10	0,58	44.880
Total liabilities (USD; millions)	20.199,67	123.201,40	9,17	2.155.072
Net sales (USD; millions)	9.532,30	28.013,60	51,93	467.231
Book value of equity (USD; millions)	6.156,05	19.040,77	14,11	236.956
Market capitalization (USD; millions)	12.204,36	33.218,82	143,09	499.696
Enterprise value (USD; millions)	15.180,81	42.806,79	61,84	554.603,4
Return on equity	4,37%	4,17%	0,01%	57,63%
Estimated 1 yr EPS growth	16,59%	9,22%	5,51%	119,19%
Net debt to EBIT	0,98	16,39%	-487,08	49,91
OLS beta	1,14	0,42	0,17	3,54
Vasicek beta	1,15	0,05	0,96	1,71
Observations	1.039			

Table 5: firm fundamentals

CHAPTER 4 Methodology

Stock price history data from 2012 will be used to estimate firms' betas. Stock price history data from 2013 will be used to determine the accuracy of these estimates. With the 2012 data, six beta estimates will be calculated for each firm for 2012.

First, the OLS beta will be calculated using formula 2.

$$\beta_i = \frac{cov(r_i, r_m)}{var(r_m)} \tag{2}$$

Second, the Vasicek beta will be calculated using formula 3.

$$\left(\frac{Stdv(Beta_{market})^2}{Stdv(Beta_{market})^2 + StdError\left(Beta_{firm}\right)^2}\right) * Beta_{market} + \left(\frac{StdError\left(Beta_{firm}\right)^2}{StdError\left(Beta_{firm}\right)^2 + Stdv(Beta_{market})^2}\right) * Beta_{firm} \tag{3}$$

After, the mean OLS beta of all firms in the same industry group (hereafter referred to as '**Industry beta**') will be calculated by taking the mean OLS beta of all firms in the same Karolyi SIC industry group, as this method is commonly used when firms are not publicly traded and beta therefore cannot be computed using stock returns. The division into Karolyi SIC industry groupings will be based on the groupings as stipulated in Karolyi (1992) and is shown in Table 6.

GIC Industry Code	Industry	Amount of firms
<2000	Agriculture, Forestry and Fishing	48
2000-2399	Food, Tobacco, Textiles	33
2400-2799	Paper, Printing, Lumber	32
2800-3299	Basic	93
3300-3599	Metals	82
3600-3799	Manufacturing	91
3800-4799	Transportation	92
4800-4999	Utilities	79
5000-6000	Wholesale, retail	121
>6000	Finance and Real Estate	368

Table 6: Karolyi (1992) industry classification

Fourth, the Karolyi beta will be calculated using formula 4. Specifically, firms will be divided into industry groups based on Table 6. Additionally, firms will be grouped based on market capitalization into 21 groups of 49 or 50, following the way that firms were divided into groups of 50 in the original study (Karolyi, 1992). Using the variance and mean of the betas of these subgroups, the Karolyi beta will be calculated using formula 4.

$$\hat{\beta}_{ij} = \frac{\hat{\beta}_i + \sum_j \gamma_{ij} \bar{\beta}_{ij}}{1 + \sum_j \gamma_{ij}} \tag{4}$$

Fifth, beta will be estimated by taking the mean betas of peer firm groups and the standard deviation thereof as respectively $Beta_{market}$ and $Stdv(Beta_{market})$ in formula 3, i.e. taking the information about the betas of the peer firm groups as prior information. This beta estimator will hereafter be referred to as the '**SARD beta**'. As mentioned, the peer firm groups will be selected based on the SARD method. This means that all firms are ranked based on (i) market capitalization, (ii) net debt/EBIT, (iii) EPS growth and (iv) return on equity. After, peer firm groups are selected for each firm by taking all firms with the smallest sum of absolute rank differences across these four metrics relative to the firm. Thus, the formula of the SARD beta will be:

$$\left(\frac{Stdv\left(Beta_{peer\ firms}\right)^2}{Stdv\left(Beta_{peer\ firms}\right)^2 + StdError\left(Beta_{firm}\right)^2}\right) * Beta_{peer\ firms}$$
$$+$$
$$\left(\frac{StdError\left(Beta_{firm}\right)^2}{StdError\left(Beta_{firm}\right)^2 + Stdv\left(Beta_{peer\ firms}\right)^2}\right) * Beta_{firm} \tag{5}$$

Where StdError($Beta_{firm}$) is the standard error of the estimated beta of the individual firm, $Beta_{firm}$ is the OLS beta of the stock in question, Stdv($Beta_{peer\ firms}$) is the standard deviation of the OLS beta estimates of the selected peer firms and $Beta_{market}$ is the mean OLS beta estimate of the selected peer firms.

Lastly, the Peer beta by using the SARD method to find peer firms for each individual firm. However, instead of using the information about the peer firms in formula (5), the mean beta of these peer firms will simply be the beta estimate. If this method yields predictive power, this would be a highly useful method for beta estimation for firms that are not publicly traded and thus do not have stock price history.

For both the SARD and Peer beta, different amounts of peer firms can be used in the beta estimator. Therefore, both estimators will be computed using different amount of peer firms. As the amount of peer firms that can be taken from one industry group are limited, both the SARD and Peer beta will also be computed without using industry as an input variable, which allows for a greater amount of peer firms. The MSE of these different versions of the SARD and Peer beta will be compared, which will yield the most effective version of the SARD and Peer beta. These versions of the SARD and Peer beta will then be compared to other beta estimators.

After all betas have been calculated for all individual firms in the dataset for the year 2012, these results will be compared to the observed OLS and Vasicek betas in 2013. This allows the comparison of the 1-year ahead forecasting power of multiple beta estimators, which thus allows for a comparison of the relative precision of the estimators. Both the Vasicek and OLS betas will be used, as Levi & Welch (2017) note that the Vasicek beta is possibly a better measure of the true beta than the OLS beta, as the OLS beta suffers from measurement error. Comparing the precision of all beta estimates to both the observed Vasicek beta and OLS beta will thus grant more insight. This will thus result in two MSE values for all seven beta estimates. The formula for the MSE is:

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(\hat{\beta}_i - \beta_i)^2 \tag{6}$$

Where n is the number of firms, i represents one specific firm, $\hat{\beta}_i$ is the estimated beta, and β_i is the observed beta. Therefore, a lower MSE indicates a lower estimation error.

CHAPTER 5 Results & Discussion

5.1 Interpretation of the results

As discussed in the methodology section, the mean squared error has been used to compare the outcomes of different models. This calculates the sum of squared differences between the predicted and actual outcome, divided by the amount of observations, see formula 6. This gives a reflection of the accuracy of the model, as lower differences between the predicted and actual outcome lead to a lower MSE. Thus, a lower MSE means that the predictor is more accurate.

5.2 SARD and Peer beta results

First, the results of the MSE of different computations of the SARD and Peer betas will be discussed. Different computations of these betas are possible. For example, the amount of used peer firms can be changed. Additionally, it is possible to both include and exclude industry as a selection criterion for peer firms, i.e. changing whether or not peer firms can only be taken from the same industries or from any industry. First, the results of the Peer and SARD beta without industry as a selection criterion will be discussed in order to determine which amount of peer firms is optimal. Then, the results will be discussed for beta estimators for which industry was used as a selection criterion.

5.2.1 SARD and Peer beta without industry

In Table 7, the MSE is given for the Peer and SARD beta with differing amounts of peer firms without using industry as an input variable. MSE has been calculated using both the Vasicek and OLS beta, as stipulated in the methodology section. When the MSE is calculated using the 2013 OLS beta, both the SARD and Peer beta are most accurate when 300 peer firms are used, which is around one third of the dataset. The estimation effectiveness rises with an increasing amount of peer firms, decreasing again when the amount of peer firms approaches the total amount of firms in the dataset. Starting from around fifty peer firms, however, the accuracy does not greatly increase relative to the initial increases. This implies that the optimal amount of peer firms is around one third of the index (market) if the index contains at least 150 firms and industry is not used as a selection criterion.

When the MSE is calculated using the 2013 Vasicek beta, a peculiarity appears. The optimal amount of peer firms is around 900 - 1,000, which is almost the entire dataset. Although not tabulated, taking the average beta of all firms yields a very low MSE relative to the Vasicek beta as well, although slightly higher than using around 900 – 1,000 peer firms. Therefore, incidentally, the data seems to be such that the average beta of all firms is a relatively good predictor for the Vasicek beta of 2013. This does not decrease the power of any model or mean that the data is not valid. Rather, this is a reflection of the fact that the Vasicek beta is shrunk heavily toward the average beta in this specific dataset. If the amount of shrinkage towards the average beta is high, the average beta will be relatively predictive. Additionally, this peculiarity shows why it is relevant to compare results to both the Vasicek and OLS beta.

However, this peculiarity will most likely not be present in other indices or even in the same index (S&P Composite 1500) in another year. Therefore, the earlier conclusion will still be maintained: taking around one third of the dataset but at least 50 firms as peer firms, is the most efficient SARD and Peer beta estimate. Thus, the MSE value for 300 peer firms will be used when comparing the SARD and Peer beta estimates without industry as a selection criterium.

Peer firm group size	Peer beta MSE (OLS)	Peer beta MSE (VCK)	SARD beta MSE (OLS)	SARD beta MSE (VCK)
5	0,118	0,064	0,093	0,060
10	0,100	0,048	0,084	0,046
20	0,095	0,036	0,079	0,035
30	0,091	0,032	0,076	0,030
40	0,090	0,028	0,075	0,028
50	0,088	0,025	0,074	0,025
75	0,087	0,021	0,073	0,020
100	0,086	0,019	0,073	0,018
150	0,086	0,015	0,072	0,015
200	0,085	0,013	0,072	0,012
250	0,084	0,011	0,071	0,011
300	**0,084**	0,009	**0,071**	0,009
400	0,085	0,008	0,072	0,008
500	0,085	0,006	0,072	0,006
600	0,086	0,006	0,073	0,005
700	0,086	0,005	0,073	0,005
800	0,086	0,005	0,073	0,004
900	0,087	**0,005**	0,074	0,004
1000	0,088	0,005	0,075	**0,004**

Table 7: MSE of SARD and Peer beta without industry

5.2.3 SARD and Peer beta with industry

In Table 8, the MSE is given for the Peer and SARD beta with differing amounts of peer firms, with peer firms only from the same industry group. The industry groupings are given in Table 6. Once again, MSE has been calculated using both the Vasicek and OLS beta, as stipulated in the methodology section.

Because the amount of peer firms per industry grouping is relatively limited, the amount of peer firms cannot be as large as when the SARD and Peer beta are calculated without taking into account industry groupings. Thus, three different analyses were made. First, peer firm groups of 5, 10, 15 and 20 firms were made for all firms. Second, peer firm groups of 30, 40 and 50 were made for firms operating in industries with more than 90 firms. Lastly, firms peer firm groups of 75 and 100 were made for firms operating in industries with more than 200 firms. In this case, only one industry contained this amount of firms.

As can be seen in Table 8, the small peer firm groups, which also includes very small industries, is very inaccurate. In fact, the peer firm group including small industries proved to be a worse estimator than simply taking the industry of the firm, based on OLS MSE. This bad performance most likely arises from the fact that taking peer firms from industries with a small amount of firms, renders relatively unsimilar firms, as the choice of firms is small. This is proven by Table 12 This reduces the effectiveness of the SARD method. Additionally, the small amount of peer firms is relatively ineffective in the SARD and Peer betas without industry, giving another reason for the low effectiveness of the estimator. This most likely arises from the fact that peer firm groups with a small amount of peer firms still suffer from measurement error and mean reversal, which will be discussed further under 5.3. The peer firm groups containing companies in industries with over 90 firms, are far more accurate estimators. In all cases, the largest amount of peer firms proved to be the most accurate in this group.

The most accurate SARD and Peer betas, however, were estimated in the largest industry. However, this MSE is based on only one industry. Therefore, it is difficult to state whether or not the higher power arises from enhanced estimation or from an industry characteristic. Additionally, this estimator is relatively useless, as the betas of only about one thirds of the firms in the dataset can be estimated using this method. Thus, the estimator based on a peer firm group of 50 firms will be used to compare the SARD and Peer betas to other beta estimators. In practice, this means that the Peer and Beta estimators using industry can be used when industries have more than 90 firms. 50 peer firms should be used. Otherwise, using SARD and Peer beta without industry yields better results.[6]

[6] The MSE of the SARD beta with small industries was 0.082 (OLS) and 0.71 (VCK). The MSE of the Peer beta with small industries was 0.096 (OLS) and 0.071 (VCK). Since these are significantly higher MSEs than the SARD and Peer beta without industry, see table 7, it is more effective not to take industry into account when the firm is operating in a smaller industry.

Peer firm group size	Observations	Peer beta MSE (OLS)	Peer beta MSE (VCK)	SARD beta MSE (OLS)	SARD beta MSE (VCK)
5	1.039	0,109	0,091	0,093	0,090
10	1.039	0,097	0,076	0,084	0,076
15	1.039	0,096	0,073	**0,082**	0,072
20	1.039	**0,096**	**0,071**	0,082	**0,070**
30	765	0,081	0,028	0,071	0,030
40	765	0,080	0,025	0,070	0,070
50	765	**0,079**	**0,022**	**0,069**	**0,025**
75	368	0,081	0,009	**0,061**	0,009
100	368	**0,075**	**0,006**	0,062	**0,007**

Table 8: MSE of SARD and Peer beta with industry

5.3 Amount of peer firms

As the amount of peer firms that are used is relatively high, the question arises whether or not the enhanced beta forecasts are a result of the SARD method or simply a result of the fact that the average of a large part of the dataset is taken. Although more data is needed to definitely prove this, there is evidence that the methodology of the SARD method itself enhances beta estimates. First, it is important to note that the beta estimation accuracy peaks at 300 peer firms for the OLS MSE, after decreasing again, indicating that it is the selection of relevant peer firms that enhances the beta estimate.[7] It is also important to determine if the similarity between the firm and its peer firm group also holds at a larger amount of peer firms. If the similarity between firms within a peer firm group is very low for peer firm groups of 300, this indicates that it is not the ability of the SARD method to select relevant peer firms that enhances the beta estimates.

In Tables 9 and 10 in Appendix B, the similarities between randomly selected firms and their peer firm groups were shown. For the five firms in Table 9, the average values for peer firm groups of 300 firms without taking into account industry classification were shown. For the other five firms in Table 10, the average values for peer firms groups of 50 firms based on industry classification were shown. This Table gives an insight into the similarities between firms and their peer firms given the optimal amounts of peer firms. The characteristics of firms seem to be reflected well in both peer firm group sizes, with the 300 no industry firm group size reflecting the characteristics more accurately. This follows from the fact that the 300 firms were selected from a larger group, as industry classification was not taken into account.

[7]As the higher dissimilarity between a firm and its peer firm group in larger peer firm groups, results in lower beta estimator accuracy.

However, a more general comparison is necessary to draw conclusions about the effectiveness of the SARD method. For this, a new score was computed. First, the four input variables (market capitalization, EPS growth estimates, net debt/EBIT and return on equity) were normalized using Z-score normalization, which allows for the four proxies to be directly compared. After, for each firm the average difference between all firms in its peer firm group and its own normalized input variable value was calculated for each input variable. The average of these four values was then calculated, which gives a measure of the difference between the firm and its peer firm group on all four metrics. This was done for each firm for every different size of peer firm groups. The average value of these values for each individual peer firm group size is given in Table 11. The table indicates that the selection of relevant peer firms by the SARD method does result in enhanced beta estimates, as the similarity between firms and peer firms is still large for peer firm groups of 300 relative to the similarity that is observed when the full sample is taken as the peer firm group.

Peer firm size	Difference	Standard deviation	Min	Max
5	0,212	0,353	0,027	7,433
10	0,238	0,360	0,035	7,415
20	0,264	0,362	0,049	7,472
30	0,281	0,364	0,062	7,486
40	0,297	0,369	0,071	7,538
50	0,309	0,371	0,080	7,536
60	0,319	0,372	0,085	7,514
70	0,329	0,375	0,093	7,524
80	0,338	0,377	0,097	7,533
90	0,345	0,379	0,101	7,539
100	0,352	0,381	0,106	7,550
150	0,381	0,388	0,122	7,595
200	0,405	0,393	0,138	7,637
300	0,443	0,400	0,161	7,634
400	0,476	0,400	0,184	7,707
500	0,505	0,406	0,217	7,737
600	0,533	0,408	0,247	7,772
700	0,562	0,410	0,280	7,798
800	0,591	0,411	0,319	7,832
900	0,623	0,411	0,365	7,868
1000	0,664	0,412	0,417	7,906
1039	0,686	0,412	0,446	7,923

Table 11: measure of average difference between firm and peer firms per size of peer firm group without taking industry classification into account

The same measure was also constructed for the peer firm groups that take industry classification into account. The results are displayed in Table 12 and also indicate that there is still a relatively high degree of similarity at the 50 peer firm group size. The general degree of similarity is lower for peer firms when industry is taken into account because of the lower amount of firms that can be selected as peer firms as a result of the limited amount of firms per industry classification.

Peer firm size	Observations	Difference	Standard deviation	Min	Max
5	1.039	0,285	0,385	0,027	7,416
10	1.039	0,320	0,380	0,035	7,419
15	1.039	0,344	0,383	0,044	7,438
20	1.039	0,366	0,387	0,050	7,446
30	765	0,390	0,386	0,059	7,526
40	765	0,415	0,388	0,075	7,533
50	765	0,440	0,391	0,088	7,535
75	368	0,446	0,474	0,115	7,570
100	368	0,478	0,480	0,141	7,618

Table 12: measure of average difference between firm and peer firms per size of industry peer firm group

Specifically for the industry peer firm groups, there is further evidence that the SARD method itself results in enhanced beta estimates. The SARD beta and Peer beta estimates for the biggest industry (368 firms) do not increase in precision when measured by MSE relative to the true 2013 OLS beta after respectively 75 and 100 firms.[8] This also indicates that the ability of the SARD method to find comparable peer firms, leads to enhanced beta estimates.

Peer firm group size	Peer beta MSE (OLS)	Peer beta MSE (VCK)	SARD beta MSE (OLS)	SARD beta MSE (VCK)
75	0,081	0,009	**0,061**	0,009
100	**0,075**	0,006	0,062	0,007
150	0,078	0,005	0,064	0,005
200	0,078	0,005	0,064	0,005
250	0,079	0,004	0,064	0,004
300	0,079	0,004	0,064	0,004
367	0,078	**0,004**	0,063	**0,004**

Table 13: MSE for large peer firm groups for firms in the largest industry grouping

[8] Again, when measured relative to the true 2013 Vasicek beta, the highest amount of peer firms is the optimal estimator because of the peculiarity in the data relating to the average beta being a good beta estimate.

There can be several reasons for the relatively high amount of optimal peer firms. First, as mentioned before, there is measurement error in the measurement of betas (Vasicek, 1973). Using more peer firms betas can therefore reduce the effect of this measurement error when using these peer firms betas in the Peer or SARD beta. Second, there is mean reversal in betas (Levi & Welch, 2017). Using larger peer firm groups can reduce this effect as well. Lastly, using larger peer firm groups can be useful if more useful information on the relation between the input variables and beta is taken into account because of these larger peer firm groups.

5.4 Results of all estimators

In Table 9, the MSEs of all estimators have been shown. For the MSE using 2013 OLS beta and 2013 Vasicek beta, the most accurate estimators are shown at the top. The SARD beta is relatively effective at predicting both the OLS and Vasicek betas in 2013. Although not being the most effective beta estimator in either, it ranks second in both cases. The Karolyi beta is better at predicting the 2013 OLS beta. The Vasicek beta is better at predicting the 2013 Vasicek beta.

However, the fact that the SARD beta performs very well in both columns, means that the beta estimator is relatively predictive. As discussed in the theoretical framework, beta estimation can be measured relative to both the OLS and Vasicek beta, as the OLS beta suffers from measurement error. As the SARD beta consistently predicts both betas well, while other estimators (Karolyi and Vasicek beta) only predict one beta well, the model can be said to be a better estimator than others. Additionally, further optimization to the SARD beta estimator is possible, meaning that its accuracy most likely can be improved significantly.

The Peer beta performs relatively well. Although mostly estimating less precisely than the SARD beta, it still ranks above the OLS beta and industry beta. When measuring MSE based on the 2013 OLS beta, the Peer beta (with or without industry) ranks just below the Vasicek beta. When measuring based on 2013 Vasicek beta, it ranks above the Karolyi beta.

As discussed above, this beta does not take into account any beta estimate of the firm in question. This beta can therefore also be computed for firms without any stock return history, i.e. firms that are not publicly traded. As there are currently no straightforward methods to estimate beta for firms that are not publicly traded, this result is of interest.

The research question can be answered using the results. The research question was:

> <u>To what extent does using prior information about peer firms found with the SARD method in the Vasicek calculation enhance beta estimation</u>*?*

Using the SARD method (Knudsen et al., 2017) improves beta estimation accuracy. Although the first implementation of this model does not yet estimate both OLS and Vasicek beta best, the strength of the SARD model lies in the ease with which it can be changed. This means that further improvements can be made. Additionally, using the SARD method allows for accurate beta estimation for firms that are not publicly traded.

Predictor	MSE(OLS)	Predictor	MSE (VCK)
Karolyi beta	0,065	Vasicek beta	0,004
SARD beta - industry	0,069	SARD beta – no industry	0,009
SARD beta – no industry	0,071	Peer beta – no industry	0,009
Vasicek beta	0,075	Peer beta – industry	0,022
Peer beta – industry	0,079	SARD beta – industry	0,025
Peer beta – no industry	0,084	Industry beta	0,051
OLS beta	0,090	Karolyi beta	0,095
Industry beta	0,094	OLS beta	0,152

Table 14: MSE of all beta estimators

Two other observations stand out. First, the Karolyi beta is very effective when estimating the 2013 OLS beta but ineffective when predicting the 2013 Vasicek beta. Karolyi (1992) does not measure the effectiveness of the Karolyi beta when measuring the Vasicek beta. Therefore, it is possible that the Karolyi beta is inaccurate when it comes to predicting the Vasicek beta, which could imply a reduced accuracy of the model. However, it is also possible that this lower accuracy disappears when the dataset is expanded to include more years, which is beyond the scope of this study.

The second observation of note is that the industry beta is somewhat accurate when it comes to predicting the 2013 Vasicek beta. As this contradicts Levi & Welch (2017), who found that the industry beta is wholly ineffective as a beta estimator, it is possible that this finding is not replicated if the dataset is expanded to include more years.

5.5 Implications of beta estimation accuracy

The relevance of accurate beta estimates and the implications of the difference between the MSEs in Table 14 can be shown using an example of a capital budgeting decision. Imagine that the true beta of a firm is 1 and that the firm uses CAPM to obtain its required rate of return. Table 15 gives the betas that would be estimated given the MSE of Table 14, assuming that beta is only overestimated.[9]

The risk-free rate is 2% and the market risk premium 7%. Given formula 1, the true required return on equity is thus 9%. Assume that the project is fully financed with equity. There are four payouts of €800.000: at the end of year 1, 2, 3 and 4. These are discounted at the estimated required return that is found using CAPM. The firm will invest when the discounted total returns are higher than the initial investment. If the initial investment is €2.490.000,-, the true total profit would be approximately €100.000,-. However, the firm would forgo this profit in most cases, because its beta estimates are inaccurate.

MSE	Estimated beta	Required Return	Discounted total returns
0	1	9%	€ 2.591.775,90
0,065	1,255	10,785%	€ 2.493.392,93
0,069	1,263	10,841%	€ 2.490.405,83
0,071	1,266	10,862%	€ 2.489.287,18
0,075	1,274	10,918%	€ 2.486.308,15
0,079	1,281	10,969%	€ 2.483.600,20
0,084	1,291	11,037%	€ 2.479.997,12
0,090	1,3	11,1%	€ 2.476.666,63
0,094	1,307	11,149%	€ 2.474.081,32

Table 15: estimated betas for different MSEs

[9] Note that MSE gives the average error for all firms, thus this overestimation represents an average overestimation. However, in this example there is only one firm. In that case, the given MSE's translate to the betas in Table 15.

5.6 Flexibility of SARD method

A main advantage of the SARD method is its flexibility. More input variables can be added. Additionally, the relative importance of the input variables can be changed by changing the degree to which peer firms are selected using a specific input variable. It can be hypothesized that adding input variables that are relevant to the beta of a firm will increase the beta estimation precision. Additionally, it can be hypothesized that increasing the degree to which more relevant input variables are used in peer firm selection relative to the degree to which less relevant input variables are used, will increase the beta estimation precision.

As an example, two additional input variables were incorporated into the model. The total dividends paid as a percentage of income before extraordinary items and goodwill as a percentage of book value were added. This meant that the altered beta estimator uses six input variables. Peer firm groups of 300 were computed when industry classification was not taken into account and peer firm groups of 50 firms when industry classification was taken into account. The MSEs for the altered beta estimators are shown in Table 16. The MSE for the peer firm groups of 300 without industry classification as an input variable mostly decreased, indicating that the altered estimator is more accurate for peer firm groups of 300. For the peer firms groups of 50 firms, however, the change in the estimator resulted in a very significant decrease in the accuracy of the beta estimates.

The reason for this significant decrease in preciseness of beta estimation, lies in the fact that both the goodwill and dividend variables were 0 for 151 and 293 firms, respectively. This means that a significant part of the datasets had similar ranks for these input variables. In smaller industries, firms were then placed in peer firm groups that were formed to a large extent by these two new input variables, as smaller industries contained fewer firms that were very similar in other input variables. This resulted in significant differences between firms in peer firm groups that were constructed in this way, which decreased the preciseness of the estimator.

This can be proven by again calculating the difference in normalized values measurement as in Table 11 and 12.[10] The 50 peer firm group measurement increased from 0,440 to 0,730 for the reason stated above. This measurement is thus larger than even the average difference when the entire dataset was taken as a peer firm, as shown in Table 11. This explains the significant decrease in the accuracy of the estimator.

[10] For which only the difference between the firm and its peer firm group average for the four original input variables were used in order to facilitate a proper comparison between the altered and earlier beta estimators.

The 300 peer firm group measurement only increased from 0,443 to 0,471. Apparently, the greater dissimilarity in the four original input variables was compensated for by the fact that the two new input variables improved the beta estimates.

This example shows why it is crucial for practitioners to only use input variables that have been proven to yield accurate beta estimates. However, it also shows the flexibility of the SARD model and the fact that beta estimates can be relatively easily improved further by incorporating more input variables.

Peer firm group size	Peer beta MSE (OLS)	Peer beta MSE (VCK)	SARD beta MSE (OLS)	SARD beta MSE (VCK)
50 (industry)	0,413	0,352	0,085	0,090
300 (no industry)	0,082	0,012	0,070	0,012

Table 16: MSE for a different iteration of the SARD and Peer betas

5.6 Discussion

The results of the analyses in this study show that using the SARD method to estimate beta, increases estimation accuracy. Vasicek (1973) proposes a beta estimator that uses cross-sectional betas. Additionally, Karolyi (1992) found that incorporating market capitalization and industry information into the beta estimation, yields more accurate beta estimations. Knudsen et al. (2017) found that the SARD method yields precise value estimates in multiples valuation. Therefore, it could be hypothesized that using the SARD method in beta estimation would lead to improved estimates.

The results of this study seem to confirm this finding. There is evidence to support that the SARD method results in enhanced beta estimates because of its selection of relevant peer firms. The results form a contribution to the current scientific literature, as the flexible nature of the SARD model allows for the usage of different or more input variables, which can further improve the estimation effectiveness of the SARD and Peer betas. The results also form a contribution to the current scientific literature, as the Peer beta allows for a relatively precise estimate of betas of firms that are not publicly traded, for which no straightforward and effective method had thus far been found. This can also help practitioners that need to estimate the beta of firms that are not publicly traded.

CHAPTER 6 Conclusion

This study sought to apply the SARD method to beta estimation. The beta of a firm represents the sensitivity of the firm's share price to the market and is an important input in CAPM, which is commonly used to estimate the cost of equity of firms. Different methods of estimating betas of firms have been uncovered in previous research. Previous research (Knudsen et al., 2017) found that the SARD method proved to be a good estimator of market capitalization when applied to multiples valuation. Therefore, this study sought to discover if applying the SARD method to beta estimation yielded improved beta estimates.

Stock price information of the S&P Composite 1500 throughout 2012 and 2013 was used. Different beta estimators were computed using the 2012 data, after which these beta estimates were compared to the true 2013 betas in order to be able to compare the relative accuracy of these estimators. Two new beta estimators were computed. The SARD beta consisted of the betas of peer firms found with the SARD method in combination with the firm's own beta. The Peer beta consisted of the mean of the betas of the peer firms found with the SARD method. For both beta estimators, the optimal amount of peer firms was found. MSE was used as a measure of accuracy. Both betas were found to yield improved beta estimates relative to most other models. Notably, the Peer beta is a relatively precise method for estimating betas of firms that are not publicly traded.

Therefore, this study shows that using the SARD method in beta estimation can increase the accuracy of beta estimates. Since the SARD method allows for an infinite number of input variables, further research can yield improvements in the accuracy of the SARD beta and Peer beta. If the findings of this study are replicated and substantiated in further research, the new beta estimators will allow for improved beta estimation and for precise beta estimation for firms that are not publicly traded.

6.1 Limitations

Note that improved beta estimates do not address the underlying issues of CAPM.

Further research is required to definitively confirm the findings of this study, as only one year of stock price information was used. Applying the methodology of this study to multiple years of data will allow for further confirmation. Additionally, further research is required to uncover other or more relevant input variables. Another limitation is the fact that the firms in the dataset were limited as described under 'Data'. Lastly, more research is necessary to uncover the reasons behind the precision of the SARD and Peer beta estimators found in this study; this can result from the large amount of peer firms used or from the selection of relevant peer firms, although some evidence suggested the latter.

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APPENDIX A: formula overview

Formula 1: CAPM

$$E(R_i) = R_f + \beta_i(E(R_m) - R_f) \tag{1}$$

Formula 2: OLS beta

$$\beta_i = \frac{cov(r_i, r_m)}{var(r_m)} \tag{2}$$

Formula 3: Vasicek beta

$$\left(\frac{Stdv(Beta_{market})^2}{Stdv(Beta_{market})^2 + StdError\left(Beta_{firm}\right)^2}\right) * Beta_{market}$$
$$+$$
$$\left(\frac{StdError\left(Beta_{firm}\right)^2}{StdError\left(Beta_{firm}\right)^2 + \ Stdv(Beta_{market})^2}\right) * Beta_{firm} \tag{3}$$

Formula 4: Karolyi beta

$$\hat{\beta}_{ij} = \frac{\hat{\beta}_i + \sum_j \gamma_{ij} \bar{\beta}_{ij}}{1 + \ \sum_j \gamma_{ij}} \tag{4}$$

Formula 5: SARD beta

$$\left(\frac{Stdv\left(Beta_{peer\ firms}\right)^2}{Stdv\left(Beta_{peer\ firms}\right)^2 + StdError\left(Beta_{firm}\right)^2}\right) * Beta_{peer\ firms}$$
$$+$$
$$\left(\frac{StdError\left(Beta_{firm}\right)^2}{StdError\left(Beta_{firm}\right)^2 + \ Stdv\left(Beta_{peer\ firms}\right)^2}\right) * Beta_{firm} \tag{5}$$

Formula 6: MSE

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(\hat{\beta}_i - \beta_i)^2 \qquad (6)$$

APPENDIX B: tables and graphs

Variable name	Source	Description	Scale
Firm price	CRSP	Adjusted daily closing price per firm throughout 2012 & 2013.	
Index price	CRSP	Closing price of the S&P Composite 1500 throughout 2012 & 2013.	
Firm market capitalization	CRSP	Daily market capitalization per firm throughout 2012 & 2013.	Thousands of USD
Firm industry	CRSP	SIC industry codes per firm on 31-12-2012.	
EPS estimates	Bloomberg Terminal	1-year forward Bloomberg earnings per share estimates using Bloomberg Terminal on 31-12-2012.	
Assets	Compustat	Assets – Total (at) on 31-12-2012.	Millions of USD
Cash and short-term investments	Compustat	Cash and Short-Term Investments (che) on 31-12-2012.	Millions of USD
Debt in current liabilities	Compustat	Debt in Current Liabilities – Total (dlc) on 31-12-2012.	Millions of USD
Long-term debt	Compustat	Long-Term Debt – Total (dltt) on 31-12-2012.	Millions of USD
EBIT	Compustat	Earnings Before Interest and Taxes (EBIT) on 31-12-2012.	Millions of USD
Income before extraordinary items	Compustat	Income Before Extraordinary Items (ib) on 31-12-2012.	Millions of USD
Liabilities	Compustat	Liabilities – Total (lt) on 31-12-2012.	Millions of USD
Net sales	Compustat	Sales/Turnover (Net) (sale) on 31-12-2012	Millions of USD
Total dividend	Compustat	Total dividends paid in 2012.	Millions of USD
Goodwill	Compustat	Total goodwill on balance on 31-12-2012.	Millions of USD

Table 1: variables

Year	Average	Median	Standard deviation	Skewness
2012	1,140	1,074	0,42	0,443
2013	1,109	1,126	0,296	0,53

Table 2: summary statistics OLS beta estimates

S&P	Amount of firms
400	286 (*27,53%*)
500	377 (*36,28%*)
600	376 (*36,19%*)

Table 3: amount of firms per S&P index

GIC Industry Code	Industry	Amount of firms
0100-0999	Agriculture, Forestry and Fishing	3
1000-1499	Mining	27
1500-1799	Construction	18
2000-3999	Manufacturing	398
4000-4999	Transportation, Communications, Electric, Gas and Sanitary service	104
5000-5199	Wholesale Trade	37
5200-5999	Retail Trade	84
6000-6799	Finance, Insurance and Real Estate	202
7000-8999	Services	161
9100-9729	Public Administration	0
9900-9999	Non-classifiable	5

Table 4: amount of firms per GIC industry

	Mean	Standard deviation	Min	Max
Total assets (USD; millions)	26.355,72	138.453,40	73,18	2.359.141
Cash and short term investments (USD; millions)	3.307,03	24.339,82	0,25	471.833
Debt in current liabilities (USD; millions)	2.340,37	24.013,05	0	379.187
Total long-term debt (USD; millions)	3.943,11	17.489,29	0	274.873
Earnings before interest and taxes (EBIT) (USD; millions)	1.422,60	4.406,87	0,91	55.241
Income before extraordinary items (USD; millions)	820,06	2.780,10	0,58	44.880
Total liabilities (USD; millions)	20.199,67	123.201,40	9,17	2.155.072
Net sales (USD; millions)	9.532,30	28.013,60	51,93	467.231
Book value of equity (USD; millions)	6.156,05	19.040,77	14,11	236.956
Market capitalization (USD; millions)	12.204,36	33.218,82	143,09	499.696
Enterprise value (USD; millions)	15.180,81	42.806,79	61,84	554.603,4
Return on equity	4,37%	4,17%	0,01%	57,63%
Estimated 1 yr EPS growth	16,59%	9,22%	5,51%	119,19%
Net debt to EBIT	0,98	16,39%	-487,08	49,91
OLS beta	1,14	0,42	0,17	3,54
Vasicek beta	1,15	0,05	0,96	1,71
Observations	1.039			

Table 5: firm fundamentals

GIC Industry Code	Industry	Amount of firms
<2000	Agriculture, Forestry and Fishing	48
2000-2399	Food, Tobacco, Textiles	33
2400-2799	Paper, Printing, Lumber	32
2800-3299	Basic	93
3300-3599	Metals	82
3600-3799	Manufacturing	91
3800-4799	Transportation	92
4800-4999	Utilities	79
5000-6000	Wholesale, retail	121
>6000	Finance and Real Estate	368

Table 6: Karolyi (1992) industry classification

Peer firm group size	Peer beta MSE (OLS)	Peer beta MSE (VCK)	SARD beta MSE (OLS)	SARD beta MSE (VCK)
5	0,118	0,064	0,093	0,060
10	0,100	0,048	0,084	0,046
20	0,095	0,036	0,079	0,035
30	0,091	0,032	0,076	0,030
40	0,090	0,028	0,075	0,028
50	0,088	0,025	0,074	0,025
75	0,087	0,021	0,073	0,020
100	0,086	0,019	0,073	0,018
150	0,086	0,015	0,072	0,015
200	0,085	0,013	0,072	0,012
250	0,084	0,011	0,071	0,011
300	**0,084**	0,009	**0,071**	0,009
400	0,085	0,008	0,072	0,008
500	0,085	0,006	0,072	0,006
600	0,086	0,006	0,073	0,005
700	0,086	0,005	0,073	0,005
800	0,086	0,005	0,073	0,004
900	0,087	**0,005**	0,074	0,004
1000	0,088	0,005	0,075	**0,004**

Table 7: MSE of SARD and Peer beta without industry

Peer firm group size	Observations	Peer beta MSE (OLS)	Peer beta MSE (VCK)	SARD beta MSE (OLS)	SARD beta MSE (VCK)
5	1.039	0,109	0,091	0,093	0,090
10	1.039	0,097	0,076	0,084	0,076
15	1.039	0,096	0,073	**0,082**	0,072
20	1.039	**0,096**	**0,071**	0,082	**0,070**
30	765	0,081	0,028	0,071	0,030
40	765	0,080	0,025	0,070	0,070
50	765	**0,079**	**0,022**	**0,069**	**0,025**
75	368	0,081	0,009	**0,061**	0,009
100	368	**0,075**	**0,006**	0,062	**0,007**

Table 8: MSE of SARD and Peer beta with industry

Firm name	Size	Debt	Growth	ROE	Peer firms size	Peer firms debt	Peer firms growth	Peer firms ROE
Conagra Brands Inc.	$11.947.116.500,-	5,8	13,3%	2,2%	$15.579.952.148,44	4,5	14,9%	2,2%
Commerce Bancshares Inc.	$3.209.111.920,-	0,7	12,4%	0,6%	$6.521.804.687,50	0,4	12,3%	2%
O-I Glass Inc.	$3.499.553.100,-	4,4	7,6%	1,2%	$10.407.836.914,06	3,9	11,8%	1,9%
Dine Brands Global Inc.	$1.234.810.000,-	6	15,6%	2,8%	$3.423.374.267,58	4,5	16,7%	2,4%
Hittite Microwave Corp.	$1.234.810.000,-	-3,8	24,5%	11,5%	$6.174.560.546,88	-1,7	19,3%	7,7%

Table 9: average peer firm values for five randomly selected firms; peer firm group size of 300

Firm name	Size	Debt	Growth	ROE	Peer firms size	Peer firms debt	Peer firms growth	Peer firms ROE
Esterline Technologies Corp.	$729.363.522,60	1,7	17,2%	2,1%	$4.714.052.246,09	0,5	18,8%	3,0%
Omnicorm Group Inc.	$4.628.946.000,-	1,2	12,1%	4,7%	$11.488.249.023,44	1,2	16,5%	4,9%
URS Corp.	$4.480.435.470,-	-1	22,6%	10,1%	$13.515.506.835,94	0,1	16,1%	4,6%
CSG Systems Intl. Inc.	$6.342.359.280,-	-0,4	12,6%	3,1%	$13.025.508.789,06	0,9	16%	4,8%
Reinsurance Group Amer Inc.	$336.117.558,70	0,5	14,1%	0,8%	$10.182.756.835,94	1,2	17,1%	5,0%

Table 10: average peer firm values for five randomly selected firms; industry peer firm group size of 50

Peer firm size	Difference	Standard deviation	Min	Max
5	0,212	0,353	0,027	7,433
10	0,238	0,360	0,035	7,415
20	0,264	0,362	0,049	7,472
30	0,281	0,364	0,062	7,486
40	0,297	0,369	0,071	7,538
50	0,309	0,371	0,080	7,536
60	0,319	0,372	0,085	7,514
70	0,329	0,375	0,093	7,524
80	0,338	0,377	0,097	7,533
90	0,345	0,379	0,101	7,539
100	0,352	0,381	0,106	7,550
150	0,381	0,388	0,122	7,595
200	0,405	0,393	0,138	7,637
300	0,443	0,400	0,161	7,634
400	0,476	0,400	0,184	7,707
500	0,505	0,406	0,217	7,737
600	0,533	0,408	0,247	7,772
700	0,562	0,410	0,280	7,798
800	0,591	0,411	0,319	7,832
900	0,623	0,411	0,365	7,868
1000	0,664	0,412	0,417	7,906
1039	0,686	0,412	0,446	7,923

Table 11: measure of average difference between firm and peer firms per size of peer firm group without taking industry classification into account

Peer firm size	Observations	Difference	Standard deviation	Min	Max
5	1.039	0,285	0,385	0,027	7,416
10	1.039	0,320	0,380	0,035	7,419
15	1.039	0,344	0,383	0,044	7,438
20	1.039	0,366	0,387	0,050	7,446
30	765	0,390	0,386	0,059	7,526
40	765	0,415	0,388	0,075	7,533
50	765	0,440	0,391	0,088	7,535
75	368	0,446	0,474	0,115	7,570
100	368	0,478	0,480	0,141	7,618

Table 12: measure of average difference between firm and peer firms per size of industry peer firm group

Peer firm group size	Peer beta MSE (OLS)	Peer beta MSE (VCK)	SARD beta MSE (OLS)	SARD beta MSE (VCK)
75	0,081	0,009	**0,061**	0,009
100	**0,075**	0,006	0,062	0,007
150	0,078	0,005	0,064	0,005
200	0,078	0,005	0,064	0,005
250	0,079	0,004	0,064	0,004
300	0,079	0,004	0,064	0,004
367	0,078	**0,004**	0,063	**0,004**

Table 13: MSE for large peer firm groups for firms in the largest industry grouping

Predictor	MSE(OLS)	Predictor	MSE (VCK)
Karolyi beta	0,065	Vasicek beta	0,004
SARD beta - industry	0,069	SARD beta – no industry	0,009
SARD beta – no industry	0,071	Peer beta – no industry	0,009
Vasicek beta	0,075	Peer beta – industry	0,022
Peer beta – industry	0,079	SARD beta – industry	0,025
Peer beta – no industry	0,084	Industry beta	0,051
OLS beta	0,090	Karolyi beta	0,095
Industry beta	0,094	OLS beta	0,152

Table 14: MSE of all beta estimators

MSE	Estimated beta	Required Return	Discounted total returns
0	1	9%	€ 2.591.775,90
0,065	1,255	10,785%	€ 2.493.392,93
0,069	1,263	10,841%	€ 2.490.405,83
0,071	1,266	10,862%	€ 2.489.287,18
0,075	1,274	10,918%	€ 2.486.308,15
0,079	1,281	10,969%	€ 2.483.600,20
0,084	1,291	11,037%	€ 2.479.997,12
0,090	1,3	11,1%	€ 2.476.666,63
0,094	1,307	11,149%	€ 2.474.081,32

Table 15: estimated betas for different MSEs

Peer firm group size	Peer beta MSE (OLS)	Peer beta MSE (VCK)	SARD beta MSE (OLS)	SARD beta MSE (VCK)
50 (industry)	0,413	0,352	0,085	0,090
300 (no industry)	0,082	0,012	0,070	0,012

Table 16: MSE for a different iteration of the SARD and Peer betas