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An Analysis of the use of Volatility Risk Premium Strategies in the U.S. Options Market.

Can their exploitation lead to positive returns?

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The views stated in this thesis are those of the author and not necessarily those of the supervisor, second reader, Erasmus School of Economics or Erasmus University Rotterdam.

ABSTRACT

This thesis analyses the potential profitability of trading strategies based on the Volatility Risk Premium (VRP) in the US equity options market from 2010 to 2022. Using a considerable daily dataset of more than 3,100,000 observations from IvyDB US of companies quoted in the S&P 500 index, this thesis examines two trading strategies. Namely, the Decile Sorting VRP Portfolio and Delta-Hedged VRP Portfolio. Results from backtesting indicate that both of these strategies yield statistically significant abnormal returns. The Delta-Hedged portfolio delivers a monthly return of 24.5%, with an adjusted alpha of 12.3%, coherent with the previous findings of Goyal and Saretto (2009). Further assessment of our findings using the Fama-French Three-Factor Model confirms that the returns significantly surpass typical market benchmarks, establishing VRP as a valid predictor of abnormal returns and underscoring the efficacy of VRP-based trading strategies in producing consistent returns.

Keywords: options, volatility risk premium, implied volatility, realized volatility

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CHAPTER 1 Introduction

This thesis aims to investigate and optimize the potential profits arising from the volatility risk premium (VRP) in the US equity options market. Historical volatility (HV), also known as realized volatility, reflects the actual past fluctuations in stock prices over a designated period – commonly a year – thus providing a measure of past market volatility. Implied volatility (IV) depicts the expected volatility of the underlying stock, and can be derived from option pricing model, such as the one developed by Black and Scholes (1973). Implied volatility is hence the market expectations regarding the future volatility, and can be observed through the price of the option. The difference between these two components is known as the Volatility Risk Premium (VRP). Throughout this thesis, we refer to the VRP as the individual VRP for each option, per asset basis, and not at an index level. The VRP is particularly relevant as it shows a potential disparity between the market's expectations of future volatility (IV) and the historical volatility (HV). Trading strategies leveraging that misalignment can take advantage of it to potentially generate positive returns. The exploitation of the VRP to generate positive returns is the focus of this thesis.

To develop a comprehensive understanding of the dynamics between implied and historical volatility, particularly concerning their mean-reversion properties which is at the centre of this thesis, it is essential to study academic findings on the topic. A foundational study by Fouque et al. (2000) demonstrates the mean-reverting nature of volatility, which is critical for constructing financial models that treat volatility as a primary variable like ours. In their study, they find that IV generally tends to revert to HV over time, suggesting that significant departures are typically temporary. As a result, this study is especially pertinent to our research as it suggests that large fluctuations in the current implied volatility (IV) levels from the past observed volatility (HV) are not permanent. This reversion suggests that if IV deviates significantly from HV, it is likely to revert back to HV over time, providing a predictable pattern in volatility movements. Building upon this framework, Vasquez (2017) further investigates how deviations from average implied volatility can be leveraged to inform trading decisions. His research sheds light on the practical applications of these deviations, particularly in the equity options market, by demonstrating that they can be exploited to produce profitable trading strategies. These strategies' essence is that when anticipating high IV relative to HV, leading to a large VRP, it supposedly decreases in order to align closer to HV and thus reduces the options' value. This strategic anticipation can be leverage to obtain returns by selling overpriced options to collect premiums or purchasing underpriced options.

These studies collectively form critical support in our understanding of the relevance of the HV-IV gaps, and how their exploitation can lead to positive returns. The profitability of such strategies is particularly relevant during times of market crises. Indeed, their notable use can be observed in the latest crisis, the 2008 financial crisis and the recent market disruptions caused by the COVID-19. These crises saw important spikes in IV, leading to large VRPs, which suggested potential mispricing opportunities that could have been used to obtain good returns. During the 2008 crisis, for example, strategies based on these volatility discrepancies potentially yielded significant profits as traditional investment models underperformed, a situation observed once again during the COVID-19. For example, Goyal and Saretto (2009) demonstrated that a trading approach focused solely on the HV-IV gap could outperform the market, delivering monthly excess returns of up to 20% in U.S. equity options markets under different market conditions. This thesis aims to analyse these findings, offering additional insights for a more effective and optimal use of VRP-based trading strategies.

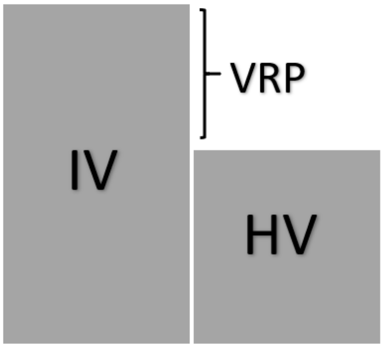
Building upon the foundational research of Goyal and Saretto (2009), which introduced the potential benefits of exploiting large HV-IV gaps in stock options, this thesis proposes an enhanced version of their strategy. Indeed, recognizing from their research that the profitability from these gaps increases with their magnitude, we use diverse ways in order to use the volatility risk premium strategy at its optimal level. This methodological enhancement is expected to enable us to target the most significant and thus profitable VRP more effectively, thereby optimizing the risk-performance balance of a portfolio. Moreover, the extension of the period originally chosen by the authors is beneficial to assess the validity of this trading strategy. Including a diverse economic environment, spanning from 2010 to 2022, with periods of great booms but also of recessions, this these aims at determining the potential of VRP-based trading strategies. Hence, our thesis aims to investigate the following question: **To what extent can a strategy leveraging the volatility risk premium generate positive returns in the US equity options market during the period of 2010 to 2022?** By exploring this question, this thesis seeks to clarify and analyse previous findings, offering an enhanced trading strategy with higher risk-adjusted returns. The primary goal of this thesis is to validate the potential of trading strategies solely based on the VRP, on a longer period that combine both economic downturns and growth. Through that rich period in terms of financial events, from 2010 to 2022, we aim to establish whereas trading strategies on VRP have potentials and are viable under various economic conditions. We expect to find that our VRP-based trading strategies can effectively produce positive returns by throughout the chosen period.

CHAPTER 2 Theoretical Framework

2.1 The Volatility Risk Premium (VRP)

To start this thesis, it is crucial to delve into the theoretical aspects that define the volatility risk premium and its exploitation in the financial markets. As expressed by Feunou et al. (2017), VRP represents the extra yield that traders can potentially obtain by correctly anticipating future volatilities that deviate from those implied by current option prices. As such, it represents the market's misalignment in the valuation of future uncertainty relative to past observed volatility. Specifically, VRP is the gap between IV, representing the market's forecast of future volatility, and HV, recording the actual, realized volatility of the asset. A higher VRP is indicative of a greater aversion to uncertainty, with investors demanding a higher premium to compensate for the possible risk linked to the future volatility. Indeed, they forecast a higher volatility in the future (IV) compared to the actual levels (HV). On the other hand, a lower VRP suggests a higher willingness among investors to engage with uncertainty. This scenario is mostly observable in periods when market participants are comfortable in bearing additional risk, when uncertainty is low, such as in periods of booms. Hence, when IV > HV, this difference is not uncommon but is structural, implemented within the option pricing frameworks. The intuition behind this statement is that the differences in IV-HV does not necessarily implies that options are mispriced. Figure 1 illustrates the concepts of IV, HV and the VRP, and how they relate to each other.

Figure 1
Relationship between IV, HV and VRP.



Note: This is a hypothetical, simplistic figure representing our variables of interest. The VRP can also be null in some settings, or even negative. More information depicting the VRP are presented in Figure 3.

However, even though disparities between IV and HV are normal and should not account as mispricing, as observed by Goyal and Saretto (2009), some high levels in the magnitude of IV-HV differences can consistently predict mispricing opportunities. These differences between implied and historical volatility form the foundation of profitable trading strategies, especially in times of market uncertainty when the VRP tends to widen. Reflecting on the historical context, the 2008 financial crisis and the COVID-19 pandemic provided empirical proof of such scenarios, as noted by Chung (2024), where market volatility was grossly underpredicted by standard financial models. Thus, this underprediction of volatility caused a huge misalignment between IV and HV. This phenomenon is also observed in Endri et al. (2021), they denoted that the COVID had an unpredictable impact on stock prices, and decreased the returns of strategies that were working in the past. As such, it is crucial to develop models that works both in normal market conditions, as well as when uncertainty is dominant within the financial markets.

Moreover, as mentioned earlier, the theoretical implications of volatility's mean-reverting properties, as discussed by Fouque et al. (2000), suggest that significant discrepancies between IV and HV are temporary, thereby offering strategic entry and exit points for trading positions on these options. This theory can be considered as the essence of this thesis, as the assumptions of profits through VRP exploitation relies on this concept. Indeed, the significant results exposed by Fouque et al. (2000) suggest that IV should revert back to HV at some point. Following up on this theory, Vasquez (2017) further explores how these theoretical insights can be practically applied, particularly in constructing models that leverage this mean reversion to obtain positive returns in the options market. His findings reveal that portfolios focusing on options with steeper volatility term structures tend to significantly outperform those with flatter term structures. Thus, his research suggests that the steepness of the term structure can be a good predictor of future returns in the options market. As a steep term structure indicates that market participants expect higher volatility in the future, it directly implies that the exploitation of high IVs can have a predictive power of option returns. Hence, in other words, we suspect that large VRP caused by high IV can also have a predictive power nature. By incorporating this insight from Vasquez's empirical analysis on the predictive power of high implied volatility, our thesis seeks to determine the potential of the VRP-based trading strategies under varied market conditions.

2.1.1 Hypotheses Formulation

Building upon past theoretical insights, our thesis investigates two primary hypotheses in order to analyse the predictability and profitability of VRP-based trading strategies. These hypotheses serve as a roadmap for our result section, and guide us towards our empirical analysis.

2.1.1.1 Hypothesis 1: Market predictability of Volatility

At first, before even investigating the possible returns generated by strategies leveraging VRP, it is crucial to explore if the individual differences between IV and HV are significant and observable. That is, if the VRP is significantly different from zero. The study of these significant differences allows us to establish whether potential mispricing can be exploited, as discussed in the works of Feunou et al. (2017) and further evidenced during historical crises. As a result, we will be at first analysing the capacity of the market to predict future volatility. If in any case, the future volatility (IV) is under or over-estimated, that means VRP being non-zero, this implies a possibility of designing VRP-based trading strategies. Hence, our formulated first hypothesis is the following one:

H_0: There is no significant differences between implied volatility (IV) and historical volatility (HV) for the stocks, the mean VRP is equal to zero. This implies that the market perfectly predicts future volatility.

2.1.1.2 Hypothesis 2: Profitability of VRP-based trading strategies

Secondly, if the observed VRP is significantly different from 0, meaning that we reject our first null hypothesis, H_0, we are able to proceed to the analysis of the efficacy of VRP-based trading strategies in their ability to generate positive returns. This hypothesis also aims to refine the findings exposed by the previous cited papers, and especially the ones from Goyal and Saretto (2009). Therefore, our second hypothesis for this thesis is:

H_0: VRP-based trading strategies do not generate positive return, indicating that exploiting the volatility risk premium does not lead to significant profits.

By using the methods presented in the next subsections, as well as their findings, the aim of this thesis is to, at first, investigate our two hypotheses. Following their analysis, we will be able to answer our research question: can VRP-based trading strategies generate positive return during the 2010 – 2022 period? These diverse methods, especially highlighting the use of HV-IV differences' magnitude to identify trading opportunities and using delta-hedge strategies to profit from these disparities are crucial to solve and investigate our hypotheses, and finally bring an answer to our research question.

2.2 Sources of mispricing opportunities

Mispricing can occur for many reasons, but is the existence of the VRP a reason for mispricing? As previously mentioned, the presence of the VRP is structural in the option pricing frameworks. As such, it does not directly imply a mispricing opportunity. However, as we delve in the topic, significant disparities in implied and historical volatilities, large VRP, can be one source of them. Indeed, this type of anomalies is crucial to explore in the market due to its potential profits' implications, for e.g., arbitrage opportunities. One explicit definition of mispricing would be that *"assets can be mispriced because value-relevant information may not be timely incorporated, thus allowing for the deviations of market prices from the intrinsic values"* (Cao et al., 2024). In addition, as IV is a forward-looking measure implied by the options market, while HV is backward-looking, this timely problem can lead to some mispricing opportunities. The study by Cao et al. (2024) reveals that options markets enhance market efficiency by facilitating the quicker incorporation of value-relevant information into stock prices. These insights are especially relevant for developing VRP-based trading strategies, as they highlight the potential of options markets to signal underlying stock mispricing. On top of this, the study illustrates that the impact of implied volatility on stock mispricing is more pronounced with higher options trading volumes, supporting the notion that greater trading activity can in fact lead to informed trading. This finding can be integrated into our strategy design, suggesting that higher trading volumes may provide more reliable signals for VRP exploitation. This is particularly relevant for our data choice, where we observe a sample of liquid options (picked from the S&P index), in order to properly assess the validity of our VRP strategy.

Alternatively, mispricing can occur due to behavioural factors. Behavioural finance also provides a crucial lens through which to view the anomalies in the options market, particularly the persistent existence of VRP. Traditional financial theories often assume rational markets, but behavioural finance suggests that market anomalies like VRP can arise from systematic biases and irrational behaviour of investors. According to Baker and Wurgler (2007), fear and overreaction to new information can lead to an increase in implied volatility, which directly cause a misalignment with historical volatility. This behavioural factor hence creates the "premia" of the VRP, also known as the IV-IV gap. This psychological aspect of trading can create profitable opportunities for informed traders who can identify and exploit these behavioural biases that causes misprice. Such insights help in understanding the psychological drivers behind the VRP, adding depth and motivation to the strategies that capitalize on these inefficiencies. This suggests that the magnitude of the VRP is particularly relevant is establishing our strategy, as on top of the mean reverting properties of the implied volatility, there is also a behaviour factor concerning the size of the spread.

Finally, noise trading can also lead to mispricing opportunities. In their research, De Long et al. (1990) developed a model that showed a number of financial anomalies. Among these, it demonstrated that anomalies such as the excess volatility of asset prices and the mean reversion of stock returns are persisting. Including this finding in our theoretical framework is critical for our understanding of the VRP. Indeed, their research introduces an influential perspective on the role of noise traders in financial markets, suggesting that their irrational behaviours can significantly influence asset prices and contribute to financial anomalies. As such, noise traders, especially present in highly liquid options, can contribute in the widening of the VRP: excess volatility. This model is particularly pertinent to the study of VRP as it provides a foundational explanation for why implied volatility might often diverge from historical volatility, a core component of this thesis.

2.3 Trading strategies using VRP

Strategies leveraging such financial anomalies have been largely studied due to their high interests. Among these, Della Carte et al. (2016) established a trading strategy that capitalizes on the volatility risk premium (VRP) in foreign exchange markets. The findings from this study are particularly relevant to our case, as they demonstrate that VRP can consistently predict currency returns. This research broadens the theoretical framework of VRP application, suggesting its utility as a predictive tool in other financial classes. It provides a clear example of how VRP can be leveraged in developing trading strategies that are less dependent on traditional risk factors. They show that similar principles applied in equity options markets for capturing the VRP profits can also be effective in currency markets, thus expanding the applicability of VRP-based strategies across different asset classes. Although this thesis is focused on equity options market only, the findings from Della Carte et al. (2016) are still relevant as they show that VRP can be a significant predictor in an asset class, which we are looking to determine in our chosen asset class, the US option market. Such insights are also crucial for a thesis that explores VRP, as they illustrate the broad potential for these strategies to deliver significant returns while also offering diversification benefits.

Following up on this, Bakshi and Kapadia (2003) shed light on the potential of VRP strategies, particularly in delta-hedged option portfolios. A delta-hedged option portfolio aims at neutralizing the directional risk of an option by holding a position in the underlying asset adjusted to match the option's delta. The goal is to offset price changes in the underlying asset, focusing on other profit sources like changes in volatility for our case. Their research reveals important insights on the options market, especially concerning the performance of delta-hedged strategies under varying market conditions. Their findings indicate that delta-hedged portfolios on VRP strategies give positive results, but typically underperform when the VRP is negative. Their study is also particularly relevant to understand the risk profile of VRP strategies under specific market conditions. Indeed, they noted that this underperformance is even more observable during periods of high market volatility, that is typically during crises. This can be attributed to the heightened uncertainty and the increased demand for options as a hedge against anticipated risks, which end up increasing the implied volatility. These observations are crucial for developing a critical thinking of how VRP can be exploited using delta-hedged options portfolios. By demonstrating that the sign and magnitude of VRP are correlated with the mean returns of delta-hedged portfolios, Bakshi and Kapadia (2003) provide a theoretical foundation suggesting that VRP can be a predictive factor in developing effective trading strategies. This relationship is particularly important for our thesis that aims to leverage these insights to develop VRP-based strategies.

Additionally, the methodological approach of Bakshi and Kapadia (2003), using a sample of S&P 500 index options for empirical testing, serves as a model for this thesis' empirical analysis. It underscores the importance of considering the type of options and the market conditions under which these strategies are tested, providing a comprehensive framework within which to evaluate the effectiveness of VRP-based trading strategies. Thus, the study of Bakshi and Kapadia (2003) enriches our understanding of VRP and its application in real-world trading. It also sets the stage for further investigation into how these strategies can be adapted or modified to capture the full potential of VRP under various market scenarios, thereby contributing to more robust and adaptive financial models, which is our exact aim.

Lastly, the paper by Goyal and Saretto (2009), constituting the foundation of this thesis, investigates the cross-section of stock option returns by analysing stocks based on the difference between historical realized volatility (HV) and at-the-money implied volatility (IV). We will aspire from their methodology of sorting stocks based on the VRP magnitude and constructing long-short portfolios for zero-cost trading strategies. Indeed, this can directly apply to this thesis, as this approach helps explore whether similar strategies that identify mispriced options based on VRP can generate positive returns in the US equity options market over our studied period. Moreover, to test the VRP strategies, we replicate their choice of database consisting of selecting only at-the-money (ATM) puts and calls.

Incorporating the methodologies from these three studies using VRP trading strategies into our theoretical framework provides a robust foundation for answering the key question of this thesis: To what extent can a strategy leveraging the Volatility Risk Premium (VRP) generate positive returns in the US equity options market during the period 2010-2022? By examining the findings of VRP strategies in various market conditions and across asset classes, including insights from Della Corte et al. (2016) on foreign exchange markets, Bakshi and Kapadia (2003) and Goyal and Saretto (2009) on stock option returns, this thesis aims to apply these principles to develop and rigorously test a VRP-based trading strategy tailored to the US equity options market. The methodologies from these papers, especially Bakshi and Kapadia's focus on delta-hedged portfolios and Goyal and Saretto's approach to sorting stocks based on the HV-IV differential for constructing long-short portfolios, serve as pivotal models for our empirical testing. These approaches ensure that the strategies are robustly evaluated under the specific market conditions of the period in question. Additionally, these papers provide a strong theoretical ground on the predictive power of VRP, as they demonstrate that such volatility disparities can be effectively leveraged to achieve significant returns.

These methodologies collectively underline the feasibility and potential profitability of VRP-based strategies in various markets. These foundational works collectively form the basis of this thesis, and help us investigate our hypothesis in order to answer to our research question.

CHAPTER 3 Data

The data for this study comes from the IvyDB US and covers the time period from January 2010 to December 2022. The database contains historical volatility, implied volatility, deltas, implied strike and premium, and contains observations on the US markets for 490 optionable securities (all listed in the S&P 500). The list of the chosen company's tickers from our sample are available in Appendix A. The original motivation of picking stocks belonging to the S&P 500 index is to be sure that the liquidity problem is solved, aligned with Bakshi and Kapadia (2003) framework. However, upon retrieving the option data for all the companies in the S&P 500, it appears that some options had very low liquidity throughout our observed period. For some, the reason was that they were not publicly listed in 2010 yet. Hence, from the original 502 companies quoted in the S&P 500, 490 are kept as they aligned with our picking process. The data is daily, and as such for each trading day, we obtain a put and a call (when liquid). Only the at-the-money options (ATM options) are extracted from the data, following Goyal and Saretto (2009) methodology. The identification of such options is based on the delta, keeping deltas of -0.5 and 0.5 for puts and calls respectively. For diverse reason, such as in case the underlying price is not available, or if the implied volatility calculation fails to converge, for example, our database automatically displays "-99.99". As such, the observations containing such values are deleted from our database. We also assume that options with a premium of 0$ immediately implies low liquidity. For that reason, the observations containing a premium of 0$ are deleted as well.

Table 1

Summary Statistics of the entire data.

Statistics	Options	Stocks (S&P 500)[1]	HV	IV	VRP (aggregated for each trading days)
Mean	3.411	82.076	0.272	0.288	0.016
Standard Deviation	6.064	122.199	0.166	0.129	0.060
Min.	0.045	0.204	0.024	0.011	-0.678
Max.	296.831	2690.167	3.750	2.996	0.251
Median	1.862	49.613	0.232	0.259	0.024
Count	490	490	490	490	490

Note. As illustrated in Figure 1, VRP is simply the IV-HV. The VRP measures are averaged for each trading days based on all the company's individual VRPs. The stocks prices' datapoints are duplicated in order to fit our data format: for each date, a call and a put are displayed in 2 separate rows. Hence, duplicating the stock information ensured data completion.

[1] The complete list of the stocks used is available in Appendix A.

Upon cleaning and checking for outliers, the final database is populated with 3,023,554 observations. The summary statistics of our variables of interest are presented in Table 1. Finally, in order to fetch the historical price data on our 490 underlying stock's companies, we use the YahooFinance Python Package for its simplicity. These daily observations of historical adjusted closing prices are then matched to the daily pairs of put and call contracts. In addition to that primary and essential data, we also use the Fama French Three Factor data from Kenneth R. French. On top of this, the benchmark used is the SPDR S&P500 ETF Trust (SPY), obtained through YahooFinance.

CHAPTER 4 Methodology

This chapter describes the methodology employed in this thesis. In Section 4.1, we discuss the pricing of an option under the Black Scholes model and the Cox-Ross-Rubinstein model, particularly relevant to understand for the content of this thesis. In Section 4.2, we explicitly define the concept of implied volatility and historical volatility, and their calculations. In Section 4.3, we describe our portfolio construction and our trading strategies developed around the VRP exploitation. Lastly, in Section 4.4, the backtesting and the methodology concerning our validity assessment of the strategies is detailed.

4.1 Understanding Option Pricing Models

In our thesis, understanding how the valuation of both call and put options works is a fundamental concept. An option is a financial derivative that grants the holder the right, but not the obligation, to buy (call option) or sell (put option) an underlying asset, like a stock, at a predetermined price (known as the strike price, K) by a specified deadline (maturity, T). Options come in various styles: European options can only be exercised on the maturity date T while American options offer more flexibility, allowing exercise at any point from the present time t until the maturity date T.

The Black-Scholes model, from Black and Scholes research in 1973, is a cornerstone of modern financial theory used to price options. This model makes the valuation of options by considering dynamics in the underlying assets, of which are the current price level (S), the strike price (K), the risk-free rate (r), and the volatility of the asset's returns (σ). The latter parameter is especially the focus of our thesis as it is the only parameter unknown by the market. Hence, option prices forecasting techniques typically depends on that volatility parameter.

It calculates the price of a call option as:

$$P_{call} = S \cdot N(d_1) - K \cdot e^{-r\,(T\,-\,t)} \cdot N(d_2)$$

And the one of a put option as:

$$P_{put} = K \cdot e^{-r\,(T\,-\,t)} \cdot N(-d_2) - \; S \cdot N(-d_1)$$

With:

- S : the current price of the underlying stock.
- K : the strike price.
- $T - t$: the time until the option's maturity.
- r : the risk-free rate.
- σ : the volatility of the stock's (log) returns.
- $N(\cdot)$ represents the cumulative distribution function of the standard normal distribution.

The variables d_1 and d_2 are computed as follows:

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + (r + \frac{\sigma^2}{2})(T - t)}{\sigma\sqrt{T - t}}$$

$$d_2 = d_1 - \sigma\sqrt{T - t}$$

The Black-Scholes model not only helps in the pricing of options but also provides a methodological framework for extracting implied volatility from market prices of options. This capability is particularly crucial for our analysis because IV represents the market's expectation of future volatility and plays a central role in the valuation of options, as well as in our thesis. By comparing this implied volatility with the historical volatility, which is the actual volatility that occurs, we can analyse VRP. This premium shows us the difference between what the market expects and what actually happens. Understanding this difference is vital because it reflects the market's expectations and uncertainties. This makes the Black-Scholes model not just a theoretical tool but a practical one that helps us understand and leverage market behaviours in our analysis of volatility.

In addition to the Black-Scholes model, the Cox-Ross-Rubinstein (CRR) binomial pricing model developed by Cox et al. (1979) forms a core part of our methodological framework. This model is particularly relevant for its adaptability in handling American-style options as well as its dynamic approach, where the Black Scholes model might fail. The CRR model uses a binomial tree to simulate multiple scenario and paths an underlying asset's price might take at each step towards the option contract's expiration. At each node of the tree, the model calculates two potential outcomes: the stock price going up or down, thus capturing a range of possible prices at the expiration date. This method involves adjusting the price upward and downward by a factor, respectively u and d. One of the key advantages of using the CRR model in our analysis is its ability to incorporate early exercise features of American options, providing a more flexible and realistic valuation method. This is particularly crucial for our database calculations, which rely on this model to estimate fair values of options in the US equity options market, considering not just the paths of price movements but also the timing of potential option exercises. Hence, the computational methodology used in our IvyDB database is the following to obtain the value of an American call option:

$$C_i = \max\begin{cases} \left[pC_{i+1}^{u} + (1-p)C_{i+1}^{d}\right]\big/R \\ S_i - K \end{cases}$$

With:

- $p = \frac{R-d}{u-d}$: the risk neutral probability.

- $R = \exp([r - q]h)$ with r being the interest rate and q the continuous dividend yield, and where $h = \frac{T}{N}$ is the size of the sub-period.
- C_{i+1}^{u} and C_{i+1}^{d} are the price of the call option at the end of the sub-period, according to if it moved up or down.

The same framework, CRR, is of course also applied to the pricing of puts options in the IvyDB database. For clearance and simplicity, the computational methodology of the puts is not detailed in this thesis.

4.2 Options payoff and profit calculations

In this subsection, we detail briefly the computations of option payoffs and profits. The method for calculating these is straightforward, and similar across all option research papers. The payoff for a bought call option, granting the right to buy the underlying asset at an agreed strike price (K) is given by the following formula, where S_t is the price of the underlying asset at maturity:

$$Payoff_{long\ call} = \max(S_t - K, 0)$$

Conversely, a similar formula stands for the calculation of long put options:

$$Payoff_{long\ put} = \max(K - S_t, 0)$$

In order to derive the profit associated with the purchase of options, the premium, amount paid on the establishment of the option contract, is subtracted from the option payoff. Hence, for both a long put and call, the profits are calculated by:

$$Profits_{long\ call\ (put)} = Payoff_{long\ call\ (put)} - Premium_{long\ call\ (put)}$$

Concerning the payoffs and profits of written puts and call, that is when the options are sold, the following formula applies:

$$Payoff_{call} = -\max(S_t - K, 0)$$

And

$$Payoff_{put} = -\max(K - S_t, 0)$$

Similarly to long options, the profits of written options are then:

$$Profits_{written\ call\ (put)} = Premium_{written\ call\ (put)} - Payoff_{written\ call\ (put)}$$

4.3 Realized VS Implied Volatility

In financial markets, volatility plays a crucial role in pricing and risk management. To understand the dynamics of options trading, particularly when exploiting the volatility premium, it is essential to distinguish between implied volatility and historical volatility.

- **Implied Volatility (IV):** This is a forward-looking measure, representing the market's expectation of the future volatility of the underlying asset. It is derived from the pricing of options; hence, it reflects the sentiment and predictions of traders about future market behaviour. It is not directly observable and is typically calculated using options pricing models where IV is the variable that aligns the model's theoretical price with the observed market price of the option. In our case, our database IvyDB uses a kernel smoothing technique in order to derive the standardized option implied volatilities. From this, it develops a volatility surface, which represent the implied volatility in function of its strike price and it's time to maturity. As our dataset is populated only with maturities of 30 days and deltas of 0.5 and -0.5, the IV contained in our database are just points on the volatility surface. This means that at each point j on the volatility surface, the IV is determined using:

$$\hat{\sigma}_j = \frac{\sum_i V_i \sigma_i \Phi(x_{ij}, y_{ij}, z_{ij})}{\sum_i V_i \Phi(x_{ij}, y_{ij}, z_{ij})}$$

  With:
  - V_i the Vega of the option.
  - σ_i the implied volatility.
  - $\Phi(\cdot)$ the kernel function.

- **Historical Volatility (HV):** In contrast, HV is a backward-looking measure, calculated based on the historical price movements of the underlying asset. According to the method defined by Goyal and Saretto (2009), the annualized historical volatility over an *n*-day period is calculated using the formula:

$$HV = \sqrt{\frac{252}{n} \cdot \sum_{i=1}^{n} (\ln \frac{P_{t-i}}{P_{t-i-1}})}$$

  where P_t represents the price of the asset at time t.

The difference between implied volatility and historical volatility, referred to as the volatility premium, is critical and especially relevant in the case of this thesis. This difference arises because IV often tends to be higher than HV. As mentioned earlier, this is thought to be because option sellers demand more premium for the risk of selling options, or because market participants expect future volatility to be higher than past volatility, hence the name "volatility premium".

4.3.1 The Predictive Power of the Implied and Historical Volatilities

Since IV reflects the expectations and sentiments of market participants, it can serve as a leading indicator of future market volatility. Financial professionals often look at IV to measure market risk and determine potential price instability. Meanwhile, HV provides a historical benchmark, allowing analysts to compare past market behaviours with current expectations, in order to see if they are approximatively aligned. As expressed by Fouque et al. (2000), volatility tends to revert to its mean, suggesting that periods of high volatility are often followed by lower levels, and vice versa.

Understanding the gap between IV and HV offers insights into market conditions—whether the market is uncertain or not—and is vital for managing risks in portfolios. This influences decisions on option pricing and strategic hedging. As mentioned earlier, the VRP is normal to observe. Hence, a strategy based solely on the existence of the VRP, for e.g., selling an option if its individual VRP is positive, would make no sense. Indeed, the findings from Goyal and Saretto (2009) sheds the light on the importance of the magnitude of the VRP. As such, in their study, they separated the different VRPs observed at each period in deciles. Once sorted, they tested two portfolio constructions: delta-hedged and straddles. In both cases, they found that the highest magnitudes had the most significant profits, and had also the most interesting characteristics regarding their risk profiles (higher Sharpe ratios and lower volatility in returns). As such, this study gives us some insight for our portfolio formation, and serves as a starting point.

4.3 Portfolio construction and trading strategies

In this thesis, we construct two trading strategies designed to exploit the VRP in our sample of the U.S. equity option market. The first strategy, the Decile Sorting VRP Portfolio, utilizes a sorting approach where portfolios are constructed based on decile rankings of VRP values, alike Goyal and Saretto (2009) methodology. This strategy sorts the VRP across our sample into deciles, from the lowest (Decile 1) to the highest (Decile 10), and analyzes the performance of each decile to identify optimal trading positions. In addition to this, it allows to identify the optimal magnitude of the VRP, yielding the highest (risk-adjusted) returns. Following the path of Goyal and Saretto, we also perform a Top Minus Bottom portfolio, consisting of going long in the top deciles and short in the lowest deciles' options.

The second strategy, the Delta-Hedged VRP Portfolio, involves the minimization of the directional market risk while still benefiting from the volatility differences. This approach consists of adjusting the position to be delta-neutral position, where the portfolio's sensitivity to price movements in the underlying assets is rebalanced, on a daily basis. This strategy aims to capture the potential VRP profits while avoiding losses due to price movements in the underlying asset. In essence, its aim is to profit from the volatility changes without incurring loss from changes in the underlying stock's price. In theory, the strategy captures the VRP profits by selling options when the VRP is positive, that is selling call options with a -0.5 hedge position (selling half the stock) and selling put options with a 0.5

hedge position (buying half the stock). In the opposite case, when the VRP is negative, the strategy does the inverse way by buying options and hedging at the same time.

The motivation behind that buying and selling of option contracts depending on the sign of the VRP can be explained logically. In simple words, when the VRP is positive, for .e.g, it means that IV > HV, suggesting potential overpriced options. Hence, the process of selling option contracts when the VRP is positive and hedging at the same time to avoid any losses incurred by the underlying stock movements, it technically allows to obtain profits as the supposedly overpriced option will lose value over time.

One important factor to note is that both strategies are constructed without consideration of early exercise options, transaction costs, spread, liquidity issues, or execution delay. Lastly, we attribute an equal weighting across options in our portfolios and assume a risk-free rate of 0%.

4.4 Backtesting and validity assessment

4.4.1. Methodology for backtesting

To validate the effectiveness of our two VRP-based trading strategies, we conduct extensive backtesting using historical data from 2010 to 2022. This backtesting phase aims at recreating a realistic trading environment to simulate the outcomes of our trading strategies if they had been used during the observed period. The programming of the strategies' decision-making process is a hard task as it requires a precise and executable code, where the entry and exit rules, and rebalancing methods need to be clearly specified. Our original goal is to make that backtesting phase as closely realistic as possible. This implies including trading constraints[2] such as a maximal capital, margins rules, drawdown limits, etc. However, it soon becomes apparent that this realistic approach goal is too ambitious for this thesis. Unfortunately, the complexity of modelling and coding such realistic constraints requires resources and computational abilities that extend beyond the scope of this academic project. As such, the backtesting phase only involve the coding of the strategy, along with its daily adjustments for the delta-hedge portfolio, as well as determining the options payoffs.

4.4.2 Validity Assessment

The validity of our trading strategies is supported by robust statistical metrics. Indeed, we evaluate the strategies' robustness and efficacity by calculating key performance metrics such as the Sharpe Ratio, Sortino Ratio, Alpha, Beta, and comparing these to our chosen market benchmarks, namely the SPDR S&P 500 ETF Trust (SPY). The Alpha and Beta is particularly relevant in order to validate our second hypothesis, axed on our trading strategies' ability of generating excess returns. Indeed, the Alpha reflects how much the strategy outperforms (or underperforms, if $\alpha < 0$) our

[2] For example, the ideal backtest would account for fixed and variable transactions costs, lot size constraints, and position limits on trading as well as the parameters cited above. Papers detailing how to integrate them in a backtesting simulation are available, such as the one from Edirisinghe et al. (2009).

benchmark index. On the other hand, Beta gives information about the volatility of the strategies in comparison to the benchmark. For example, a Beta of 1 indicates that our strategies' returns are as volatile as the S&P returns. The Sortino Ratios also help us assess the capacity of our strategy to produce consistent returns, focusing especially on their ability to avoid large losses. Indeed, the Sortino Ratios are computed accordingly to the downward deviation of a strategy. Finally, to further validate and understand the results of our strategies, we also employ the Three-Factor Model by Fama-French. This regression will allow us to explain the returns of the portfolios in terms of three systematic risks factors (namely, Market Factor R_m, Size Factor SMB, and Value Factor HML). These factors are estimated by the following regression:

$$R = \alpha + \beta R_m + \gamma SMB + \vartheta HML + \varepsilon$$

4.4.3 Limitations

Despite the potential promising results, there are several limitations to our thesis that might cause our results to be inflated[3]. First, as mentioned earlier in the methodology of the backtesting (4.4.1), our original goal is to simulate our VRP-based trading strategies in realistic trading conditions, by adding real trading constraints. That approach is unfortunately not honoured, and hence constitutes one substantial limitation. The capital allocated to the backtesting is unlimited, meaning that all the options could be bought on a daily-basis and held until expiration, if that follows the strategy's logic. This also causes our results to be inflated due to the fact that a possible unlimited amount of capital is invested during a period, leading to a supplement limitation of this thesis. Lastly, in addition of being free of real trading constraints, the backtesting phase does not account for transaction costs, spreads, liquidity issues, or trading delays, which can significantly affect real-world trading outcomes, hence constituting a limitation as well. Finally, the assumption of no early exercise of options and equal weighting across options may not reflect actual trading conditions. Overall, even though several limitations are present in our analysis, whom might cause bias, the outcomes of the backtest can still be relevant for future research, and gives a first glance at the potential returns of VRP-based trading strategies

[3] Backtesting strategies is often known to suffer from severe overfitting biases, hence leading to highly significant returns. This subject is deeply discussed in the paper by Novy-Marx (2015).

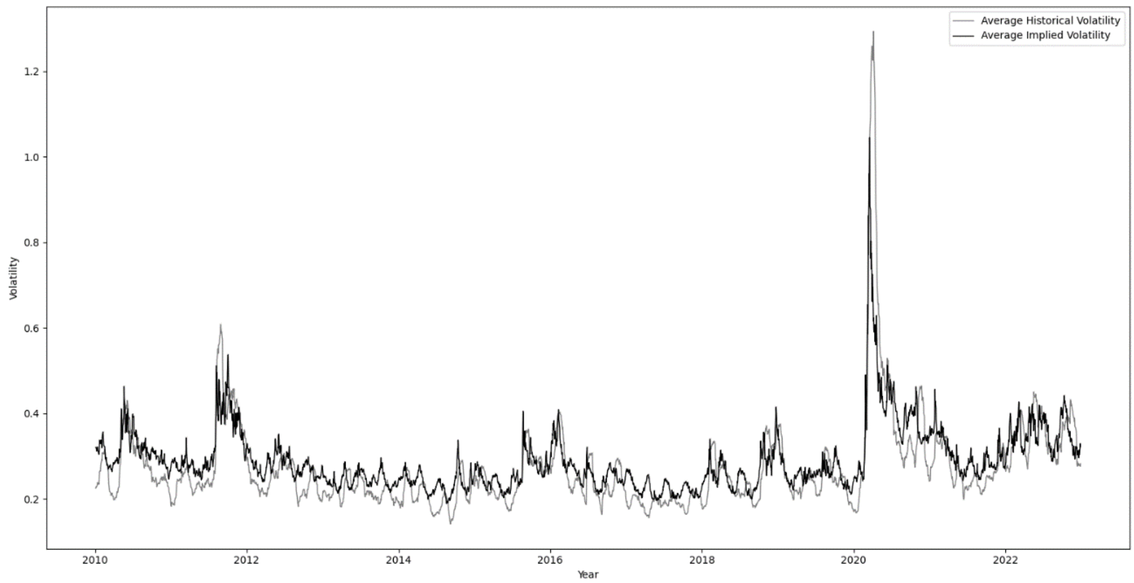
CHAPTER 5 Results & Discussion

5.1 Descriptive Statistics

This section of the thesis examines at first the behaviour of the Volatility Risk Premium (VRP) in the US equity options market over the period from 2010 to 2022. Our first descriptive analysis is underpinned by two key graphical representations: Figure 1 compares average historical volatility with average implied volatility over the sample period, while the second graph, Figure 2, is quite similar and focuses on the VRP itself, calculated as the difference between IV and HV. These two figures give us some interesting insights at first in order to determine whether a strategy based solely on the VRP is feasible or not. That is, if the VRP itself is volatile enough to build strategies depending on it.

Figure 2

Historical VS Implied volatility (2010 – 2022)

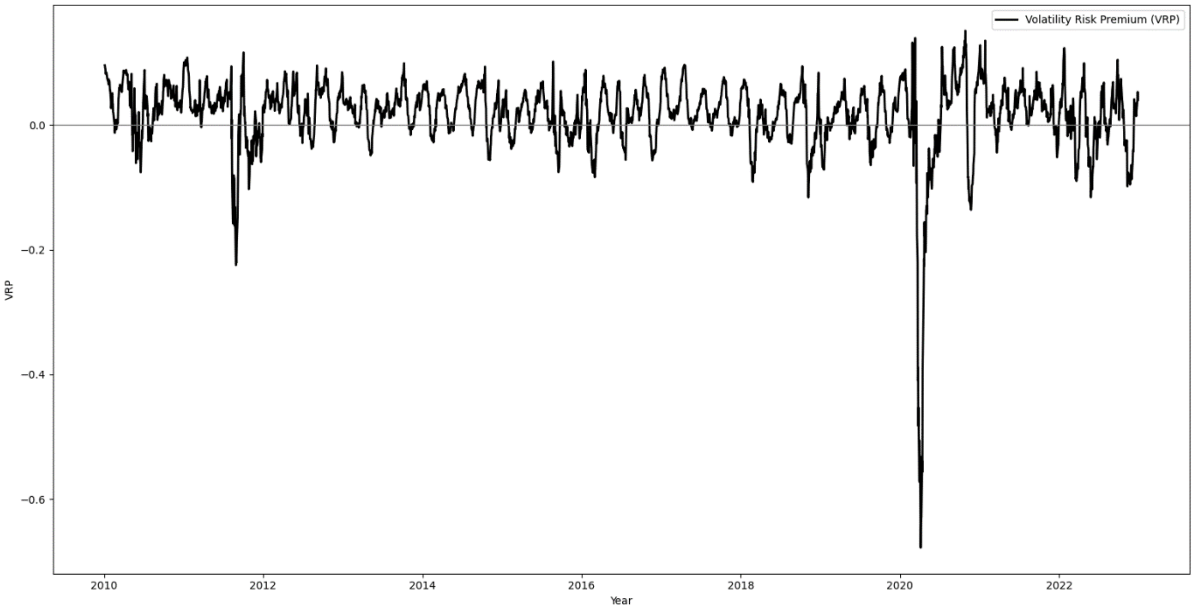


Note. The average HV and IV are computed by aggregating the daily observation of our 490 stocks. Equal weights are attributed.

Figure 1 presents a visual comparison of average historical volatility against average implied volatility over the selected period. The juxtaposition of these two measures provides insights into how market expectations align or diverge from actual market behaviours. Notably, periods of significant divergence between IV and HV suggest opportune moments for VRP exploitation, as these represent periods when the market's expectations are heavily misaligned with actual outcomes, which is the motivation behind the construction of our strategies

Figure 3

Volatility Risk Premium plot over the sample period.



Note. The VRP calculation is simply the difference of the average HV and IV that have been computed by aggregating the daily observation of our 490 stocks. Equal weights have been attributed.

Figure 2 details the Volatility Risk Premium over the same period. VRP is depicted as a continuous measure of the gap between implied and historical volatility. The analysis of VRP trends is crucial for developing effective trading strategies, as it highlights periods of heightened premium, which typically correlate with increased market uncertainty, known as crises. For example, two notable spikes in the VRP can be observed in Figure 2: in late 2011 and early 2020. Concerning the first one, this can possibly be attributed to European Debt Crisis[4] where in late 2011, there were significant concerns over the debt levels of countries such as Greece, Italy, Spain, and Portugal. At the same time, in the US, the Standard & Poor's downgraded the U.S. Credit Rating[5], also participating to the increase in uncertainty. Secondly, in early 2020, a bigger crisis happened: COVID-19. This obviously caused major disruption in the financial markets, and thus increased uncertainty. One explanation for the observation of large negative VRP is that during times of crisis, there tends to be an overreaction with implied volatility spiking more than what is actually observed subsequently. This overestimation of future volatility results from the market's increasing fear and uncertainty during the initial phases of the crisis. Lastly, Figure 2 gives us some interesting insights about the mean reverting characteristic of the VRP, which is further studied in a later part. It also grants us with some insights for our conclusion of our first hypothesis: is

[4] This small crisis emerged by a phenomenon of contagion, caused by the 2008 financial crisis. More information is available on the topic in the paper by Constancio (2012).

[5] On August 5th 2011, the S&P downgraded the US credit rating from AAA to AA+ (in response to the 2008 crisis). This historic decision caused a sudden increased in uncertainty. More information is available in the study of Fadaei (2012).

the VRP significantly different than 0. The implication of that hypothesis is thoroughly analysed in the next section below.

5.2 Statistical Analysis of the VRP

Before establishing potential trading strategies relying solely on the VRP, it is important to understand its behaviour across our sample. To analyse that behaviour, and investigate our first hypothesis, we conduct a statistical test to investigate the VRP in the options market, specifically examining whether the difference between IV and HV significantly deviates from zero across the stocks in our sample. Generally, the VRP is supposed to be positive and statistically different from zero, as IV > HV in most cases. As such, the hypothesis that VRP equals zero would imply that the market perfectly predicts future volatility, which is rarely the case due to the complexity and unpredictability of market dynamics. To assess the VRP, a one-sample T-test is conducted to the differences between implied and historical volatilities for each ticker in the dataset, representing the VRP of each stock present in our sample. The results are presented in the Table 2 below.

Table 2

Summary of T-Test Results for Volatility Risk Premium (VRP) Across Different Significance Levels.

	10 %	5 %	1 %
Number of significant results	442	437	422
Number of insignificant results	48	53	68
Number of results with insufficient data	0	0	0

Note. This table displays the number of significant and insignificant results from one-sample T-tests conducted on the volatility risk premium (VRP) across our tickers. Results are categorized by different significance levels: 10%, 5%, and 1%. The tests assess whether the mean difference between IV and HV significantly deviates from zero.

From Table 2, we can observe that most of the options present in our data (437) have significant results, indicating that with 95% confidence, the VRP differs from zero. This gives a robust indication that IV typically overestimates or underestimates HV. Hence, we can affirm that on all major significance levels, most of the companies present in our data have a VRP that differs significantly from 0. Our first null hypothesis is thus rejected, allowing us to analyse further the potential of VRP-based trading strategies. However, we can observe that some results are insignificant, meaning that these options have a mean VRP of zero at a 90%, 95% and 99% confidence levels. Upon analysing the tickers that have insignificant results, where the mean VRP is zero, we observe that these are mainly large companies. Indeed, the biggest companies from our sample are present, such as Google (GOOG), Meta

(META), Netflix (NFLX), Bank of America (BAC), AbbVie (ABBV), Microsoft (MSFT), and others. A reason for this could be that larger companies tend to be more closely followed by analysts, investors, and the media, leading to a more efficient absorption of information into their stocks. As a result of this efficient pricing, the IV, reflecting the market's expectation of future volatility, is likely to align more closely with the actual volatility (HV) of these stocks. The negative relationship between the market capitalization (size of the company) and the size of the VRP was highlighted by Han and Zhou (2012), where they showed negative significant impact of firm’s size on the average VRP magnitude. This alignment reduces the gap between IV and HV, often leading to a VRP near zero. The implication of this first finding is that we can now suspect a VRP–strategy to use more low capitalization options than large ones. It also induces that VRP-based trading strategies will be effective on most of our sample (more than 400 tickers eligible out of the 490).

5.3 Decile-sorting VRP Portfolio

We are now going to delve in the results of our first strategy, the decile sorting inspired by Goyal and Saretto (2009). In order to do so, the cross-sectional VRPs is initialized and sorted in deciles. Decile 1 represent the 10% lowest observed VRP across the sample period, from 2010 to 2022. The results of this strategy are presented in Table 3, along with the relevant statistical metrics in order to compare the returns between deciles. The difference with Goyal and Saretto (2009) is that they constructed straddle and delta-hedged portfolio around that strategy. Here, no special portfolio strategy is implemented, hence this can explain the different returns observed here compare to Goyal and Sarreto’s 2009 paper.

Table 3
VRP-decile sorting Portfolio

Statistic	1	2	3	4	5	6	7	8	9	10	10 - 1
Mean VRP	-0.197	-0.058	-0.023	0.000	0.017	0.033	0.049	0.068	0.096	0.176	0.372
Mean Return	-0.07	0.256	0.422	0.466	0.491	0.480	0.494	0.399	0.256	-0.027	0.043
Standard Deviation	5.524	7.698	8.551	8.081	8.256	8.107	8.028	7.300	6.192	4.492	5.244
Sharpe Ratio	0.152	0.144	0.149	0.161	0.161	0.162	0.165	0.169	0.175	0.177	0.024
Sortino Ratio	1.074	1.354	1.532	1.576	1.606	1.595	1.610	1.510	1.357	1.057	-0.027
Alpha	-0.08	0.12	0.23	0.24	0.26	0.25	0.26	0.20	0.12	-0.02	0.06
Beta	-0.09	0.973	1.231	1.256	1.379	1.320	1.381	1.189	0.970	-0.08	-0.28

Note. The deciles are determined on the total sample size (from 2010 to 2022). The risk-free rate considered in the computations is 0%. The returns of the options are computed accordingly to the maximization of the payoffs on their expiration date, and are expressed monthly. No options are exercised early. Equal weighting is attributed to all the options. Transaction costs, spread, liquidity issue are not accounted for. The benchmark used to assess the alphas and betas is the SPDR S&P 500 ETF Trust (SPY).

As observed in Table 3, the VRP means of each decile range from -0.19 to 0.17, nearly centred at 0 like a normal distribution. The mean return corresponding to the decile looks also to be normally distributed, with a bell shape, where the returns for the deciles 4-5-6-7 are the highest. This pattern, observable as the middle deciles (4-5-6-7) outperform the extremes, could indicate that moderate levels of under/over-estimation of volatility provide more valid opportunities for VRP-based trading strategy. Indeed, the extreme deciles, 1 and 10, have a relatively high mean VRP compared to the other groups. This can be attributed to the fact that high levels of (absolute value of) VRP are observed in times of crises, or in times of uncertainty. Hence, we suppose that the observations contained in these deciles might be concentrated during such uncertain times, where financial strategies struggle, even the ones based on volatility parameters. This is highlighted in the study by Ge (2016), where he points out that volatility strategies struggled in such periods, although still outperforming traditional models. Those "underperforming" tails (relative to the rest of the deciles), is also highlighted in the study by Alankar et al. (2023) where they evidence that the very left and right tails underperform the middle, even in momentum strategies.

From Table 3 we can also observe that the highest monthly mean return is 49.35% for decile 7. Overall, our analysis of the Decile Sorted portfolio presented in Table 3 suggests evidences that goes against the findings of Goyal and Saretto (2009). Indeed, in their study, they find that the return increased with the VRP magnitude. However here, it appears that the closer to 0 the VRP is, the highest the returns are. Our finding goes also against a similar paper from Bollerslev et al. (2009), where they show that VRP *"with high (low) premia predicts high (low) future returns"*. There is hence a supposedly relationship between the magnitude of VRP sign and its returns developed in the existing literature, but unfortunately this relation is not apparent in our thesis. The elaboration of the possible reasons behind these different findings between our paper and the existing literature will be extensively discussed in Section 5.6.

Another interesting observation is the fact that the alphas of the deciles look mirrored, forming an inversed U-shape, with the highest alphas observed for the middle deciles. This evidence suggests that excess returns diminish as the absolute value of the VRP increases, observation confirmed by the study of Lee et al. (2002). Indeed, they also find that higher excess returns are associated with a decrease in the absolute value of the volatility risk premium. Furthermore, the extreme standard deviation observed for each decile, more than 500% monthly, gives us insight on the high variability and high-risk profile of this strategy. Indeed, high monthly returns combined with a +500% standard deviation suggest an incredibly high-risk and high-reward profile. The large standard deviation suggests that the returns of this strategy of every decile are extremely volatile. Further implications and discussion about this strategy are extensively discussed in Section 5.6. The following part is now Section 5.4, focused on the analysis of the Delta-Hedged VRP Portfolio. This choice is motivated by the aim to hedge the possible downside risk evoked earlier, and to test if positive returns are achievable, while still using the VRP. Indeed, while the decile sorted portfolio offers high returns, the high standard deviation makes this strategy very risky.

5.4 Delta-Hedged VRP Portfolio

The results of our second strategy are presented in this sub-section, the Delta-Hedging Portfolio. At first, it is important to recall the definition of "delta-hedging". It is a strategy used in options trading to manage risk by offsetting changes in the price of the underlying asset. The strategy involves adjusting the position in the underlying so that it remains delta-neutral, which means that the portfolio's value doesn't change with small movements in the underlying asset's price. This approach allows traders to profit from the volatility risk premium (VRP) without worrying about the possible loss incurred by the loss of value of the underlying asset. This portfolio construction seeks to dynamically adjust the hedge, on a daily basis, so that traders can capture the potential volatility disparities while minimizing directional risk. The results of this strategy are presented in Table 4, along with the relevant statistical metrics.

Table 4
Delta - Hedged Strategy

Metrics	Delta - Hedged
Mean	0.245
Standard Dev.	0.183
Sharpe Ratio	1.343
Alpha	0.205
Beta	0.502

Note. The risk-free rate considered in the computations is 0%. The returns of the options are computed accordingly to the maximization of the payoffs on their expiration date, and are expressed monthly. No options were exercised early. Equal weighting is attributed to all the options. Transaction costs, spread, liquidity issue are not accounted for. The benchmark used to assess the alphas and betas is the SPDR S&P 500 ETF Trust (SPY).

As we can observe in Table 4, the Delta-Hedge portfolio produce returns that are more interesting than the Decile Sorting Portfolio. Indeed, with an Alpha of 0.205, the strategy produces a monthly excess return of 20.5%. On top of this, the Beta of 0.502 suggests that the strategy is less sensitive to market movements, thereby offering a good level of market diversification. The SR of 1.34 suggests that there is a strong risk-adjusted performance, highlighting that the strategy provides good returns relative to the risk taken. Finally, and most importantly, the Delta Hedge strategy delivers a mean return of 24.5% per month with a standard deviation of 18.3%, over the observed period. This result is in line with Goyal and Saretto's findings in their 2009 paper. Even though those findings are in line with our expectations, running the Fama French regression to attest their validity is necessary in order to conclude on our second hypothesis.

5.5 Fama-French Three-Factor Model

Following up on the results of our portfolio, we decide to run a Fama French Three - Factor regression in order to get more insights on the returns of our portfolio. Indeed, running this type of regression allows us to decompose the returns into market, size and value factors. This enables us to assess if the returns generated by the strategies are solely due to systematic market risks or if it's generating strong alphas, thereby indicating idiosyncratic (or specific) risks. The results of these regressions are presented in the Table 5 below.

Table 5
Fama-French Three Factor regression

	(1)	(2)
Alpha	0.093*** (3.668)	0.123*** (4.610)
MKT - Rf	0.049 (1.141)	0.005*** (3.163)
SMB	-0.011 (-0.138)	0.0025 (1.035)
HML	0.021 (0.352)	-0.0028* (-1.61)

Note. The first column (1) is the Decile Sorted Portfolio, with a long in decile 7 and a short in decile 2. The second column (2) is the Delta Hedged Portfolio. The returns of those portfolio are regressed on the 3 Fama French Factors, including the Market Risk Premium (MKT – Rf), the Small Minus Big (SMB) and the High Minus Low (HML). The regression equation is $R = \alpha + \beta R_m + \gamma SMB + \vartheta HML + \varepsilon$. The first row gives the coefficients while the second row gives the t-statistics in brackets.

The findings summarized in Table 5 indicate that both VRP strategies are robust in generating abnormal returns, independent of risk factors like market risk premium, size, and book-to-market ratios. The significance of the alpha coefficients allows for the rejection of our null hypothesis, and hence induces that VRP-based trading strategies produce significant and positive returns. In addition to being positive, those returns are abnormal and outperform the market. As we can observe in Table 5, both the alphas of the Decile Sorted and the Delta-Hedged portfolio are positive and significant. Indeed, these coefficients suggest that both our VRP-based strategies outperform the benchmark by 9.3% and 12.3% respectively. This outperformance is statistically significant, and we can thus infer a robust positive and abnormal returns for both our strategies. Although the Delta Hedged strategy yields a higher alpha, the results imply that both strategies are able to produce abnormal returns unexplained by traditional risk factors. Some risk factor's coefficients are still significant for the delta-hedged portfolio, but very small. This aspect may suggest that the hedging method is not perfect and hence require further investigation or adjustment of the hedging mechanism to ensure better market neutrality.

To test the Decile-Sorted Portfolio, we choose to apply the same methodology as Goyal and Saretto (2009). Indeed, after investigating the deciles' returns, they constructed a zero-cost trading strategy, consisting of going long on portfolios with a large positive difference between HV and IV (their decile 10), and short on those with a negative difference (their decile 1). However, as our results diverge slightly from those exposed in their paper, we apply the same methodology but with different deciles. The chosen deciles in our case are the seventh and the second. Indeed, they both have interesting risk and return characteristics compare to the other possible combinations. The obtained results from the Fama-French Three Factor regression consolidate the findings of Goyal and Saretto (2009): Top-Minus-Bottom VRP-based portfolio produce significant positive returns also on our sample's period. Overall, Table 5 attests the validity of the result previously exposed: the exploitation of the VRP is possible and it can indeed produce positive (and abnormal) returns, that traditional risk factors fail to explain.

However, our findings present notable discordance with existing literature, particularly the paper of Kaeck (2018). While our findings are aligned with Goyal and Saretto's paper (2009), it is still important to mention conflicting findings on the topic. For example, in his publication, Kaeck (2018) exposed that *"trading strategies exploiting the difference between the implied and realized variance of the VIX index yield average excess returns of − 24.16% per month, with an alpha of − 16.98% after adjusting for Fama–French and Carhart risk factors."* Although his observation is specifically focused on the VIX and hence does not fully contradicts our findings per-say, it introduces an opposed perspective on the potential of VRP-based strategies in other indexes, related to the S&P 500 too. Indeed, as the VIX is derived from the variations of our sample base, the S&P 500, this contradiction is still pertinent to our cause and it underscores the importance of being cautious with our results. As previously mentioned, the origin of that discordance, apart from the fact that the samples are not the same, might be caused by the simulation process to test the strategies. In addition to the simulation approach, the strategy construction itself plays an important role in the tests, for e.g., whether it's designed to hedge or to obtain the highest return possible, or even the least drawdown possible. Indeed, in financial studies like these, especially involving the empiric validation of strategies, diverse results can indeed emerge for a vast number of reasons. As aforementioned, many parameters need to be considered when evaluating the effectiveness of financial strategies. These parameters can include, among others, the sample period of the study, the market conditions during the period of analysis, the risk tolerance, the investment horizon of the strategy, the funding cost, etc. Lastly, the statistical methods and tools used to empirically test the strategies such as Monte Carlo simulations, historical backtesting, or machine learning algorithms have their own assumptions and limitations, which can influence the conclusions drawn from financial data. All these factors might explain the possible divergences observed with the existing literature. Detailed discussion about the implication of these is specifically described in the next section.

5.6 Discussion

Firstly, after the extensive analysis conducted in section 5.3 to 5.5, we can state that our second null hypothesis is not rejected. Hence, VRP-based trading strategies indeed produce significant profits. The rejection of the later motivates our answer to our research question. To recall, our primary focus is to analyse to what extent strategies leveraging the volatility risk premium can generate positive returns in the US equity options market during the period of 2010 to 2022. As a result, VRP-based trading strategies indeed produce significantly and economically positive and abnormal returns over the observed period.

However, it is crucial to mention the necessity for further refinement in the backtesting methods. Although we attain significant, positive and abnormal profits, as some findings diverge with the existing literature, this opposes a significant limitation to our thesis. Indeed, the source of these divergences mentioned previously might be caused by the failure to include parameters in our simulation. That might have resulted in upward or downward bias of our result in some instances. For example, our results would have been different if the transaction costs were considered, or also and most importantly, the margin requirements. If we consider the inclusion of the transaction costs in our simulation, then our observed returns may be not significantly different from zero anymore. Content on the effect of the exclusion of transaction cost is extensively discussed by Guo (2000), where he finds that the profits are not significantly different from zero in most trading strategies using the VRP after including transaction costs (assumed to be 1% in his paper).

In addition to this, our unique sample to the existing literature might also be the cause of our diverging results. It would have been beneficial for the accuracy of our results to also consider other type of options rather than only ATM call and puts. Including different maturity dates would also have been useful for our analysis in order to construct straddles and strangles portfolios, alike Goyal and Saretto did. In the same way, we could have followed the methodology of Carr and Wu (2016) whom analysed the effect of the VRP over every type of options, and also proved VRP to be a significant predictor.

As a result, upon the individual strategy discussion already formulated in the previous corresponding sections, our findings and their implications diverge with some existing literature but also join some other researchers. That means that the true nature and potential of VRP-based trading strategies is still debated among researchers. However, given our results, it is clear to us that VRP-based strategies have a potential to provide consistent, abnormal and positive returns. Indeed, it was first significantly observed by Goyal and Saretto (2009) over a sample from 1996 to 2006, then by us on our 2010-2022 sample.

To give a real-life example of such strategies profiting from incorrectly priced assets, a similar trading strategy to ours, based on the divergence in observed and theoretical values in the bond market, was used in the 2000s by a hedge fund named LTCM. This hedge fund created an arbitrage strategy, discovered by the founders of LTCM, among whom was Myron Scholes, co-researcher of the Black-Scholes model (Black and Scholes, 1973). Motivated by their findings, they put that strategy in place in 1995 and was very profitable as it yielded $3.5 billion to its investor in the year 1998 alone. However, in 1999, as the Russian bond market collapsed, the strategy collapsed as well. LTCM was fortunately bailed out by the US government in order to avoid systematic contagion. The riskiness of that strategy came from the fact that they piled positions on bonds, by purchasing some on a daily basis, thus accumulating many bonds at once. This relates to our strategies, as both involves buying options when the VRP is negative. As such, while the VRP is negative, the options holdings accumulate each other, leading to massive risks if the economy collapses. Indeed, the piling has a great leveraging effect, thus participating to increased riskiness. The LTCM example gives a clear warning about such arbitrage strategies involving piling positions, and to be careful with our observed results.

Finally, moving forward, further development of such strategies will be crucial in developing investment approaches more thoroughly validated (inclusion of more parameters). Our study successfully opens the way for further research that could enhance our understanding and implementation of volatility risk premium (VRP) strategies. One significant improvement would be the integration of machine learning process in our analysis framework. That enhancement is inspired by Dierckx et al. (2022), whom successfully leveraged ML model to use VRP in the foreign exchange market. Applied to the equity option market, this process could potentially improve the accuracy VRP-based trading strategies.

CHAPTER 6 Conclusion

In conclusion, this thesis explores the potential of using the Volatility Risk Premium (VRP) into option trading strategies to generate positive returns in the U.S. equity options market. With a particular focus on the period from 2010 to 2022, we successfully demonstrate that VRP-based trading strategies can be effectively used to produce not only positive returns, but also abnormal returns by surpassing our benchmark performance over the studied period.

By conducting our analysis on these differences between implied and historical volatilities, we explore both the theoretical and practical aspects of VRP. This deep understanding, coupled with rigorous literature reviews and accurate empirical analysis to understand its mean-reverting properties and its potential as a reliable source of excess returns, convey a solid research paper. We thoroughly test the strategy first proposed by Goyal and Saretto (2009), with a methodological innovation by extending its applicability across a larger time frame, from 2010 to 2022. This period, characterized by both economic booms and recessions, provides a robust testing ground for our VRP-based trading strategies. By effectively leveraging the HV-IV gap into trading strategies, this research managed to emphasize the fact that options can be incorrectly priced as volatility can be over / under-estimated. The findings exposed in this paper are coherent with most of the literature on one main point: VRP can be leverage to establish working and profitable trading strategies.

This academic research not only contributes to enhance VRP's knowledge and implication but also offers practical strategies that can be used for individuals seeking to exploit volatility for superior returns. Lastly, this thesis not only supportes existing theories regarding the potential of the VRP but also extends them by applying methodologies and insights to a recent and diverse dataset.

REFERENCES

Alankar, A., Maymin, P., & Scholes, M. (2023). Middle out: only extreme deciles matter. *Evolving Practices in Public Investment Management*, 251.

Baker, M., & Wurgler, J. (2007). Investor sentiment in the stock market. *Journal of economic perspectives*, *21*(2), 129-151.

Bakshi, G., & Kapadia, N. (2003). Delta-hedged gains and the negative market volatility risk premium. *The Review of Financial Studies*, *16*(2), 527-566.

Black, F., & Scholes, M. (1973). The pricing of options and corporate liabilities. *Journal of political economy*, *81*(3), 637-654.

Bollerslev, T., Tauchen, G., & Zhou, H. (2009). Expected stock returns and variance risk premia. *The Review of Financial Studies*, *22(11),* 4463-4492

Cao, Z., Chelikani, S., Kilic, O., & Wang, X. (2024). Implied volatility spread and stock mispricing. *Journal of Behavioral Finance*, *25*(1), 79-91.

Carr, P., & Wu, L. (2016). Analyzing volatility risk and risk premium in option contracts: A new theory. *Journal of Financial Economics*, 120(1), 1-20.

Chung, S. (2024). Modelling and Forecasting Energy Market Volatility Using GARCH and Machine Learning Approach.

Constancio, V. (2012). Contagion and the European debt crisis. *Financial Stability Review*, 16(2012), 109-121.

Cox, J. C., Ross, S. A., & Rubinstein, M. (1979). Option pricing: A simplified approach. *Journal of financial Economics, 7(3),* 229-263.

De Long, J. B., Shleifer, A., Summers, L. H., & Waldmann, R. J. (1990). Noise trader risk in financial markets. *Journal of political Economy*, *98*(4), 703-738.

Della Corte, P., Ramadorai, T., & Sarno, L. (2016). Volatility risk premia and exchange rate predictability. *Journal of Financial Economics*, *120*(1), 21-40.

Dierckx, T., Davis, J., & Schoutens, W. (2022). Trading the FX volatility risk premium with machine learning and alternative data. *The Journal of Finance and Data Science*, 8, 162-179.

Edirisinghe, C., Naik, V., & Uppal, R. (1993). Optimal replication of options with transactions costs and trading restrictions. *Journal of Financial and Quantitative Analysis*, *28(1),* 117-138.

Endri, E., Aipama, W., & Septiano, R. (2021). Stock price volatility during the COVID-19 pandemic: The GARCH model. *Investment Management & Financial Innovations*, *18*(4), 12.

Fadaei, R. (2012). AN ANALYSIS OF STANDARD AND POOR'S DOWNGRADE OF THE US CREDIT RATING. *Journal of Business and Behavioral Sciences, 24(3),* 184.

Feunou, B., Lopez Aliouchkin, R., Tédongap, R., & Xi, L. (2017). *Variance premium, downside risk and expected stock returns* (No. 2017-58). Bank of Canada.

Fouque, J. P., Papanicolaou, G., & Sircar, K. R. (2000). Mean-reverting stochastic volatility. *International Journal of theoretical and applied finance*, 3(01), 101-142.

Ge, W. (2016). A survey of three derivative-based methods to harvest the volatility premium in equity markets. *The Journal of Investing*, *25(3),* 48-58.

Goyal, A., & Saretto, A. (2009). Cross-section of option returns and volatility. *Journal of Financial Economics*, *94*(2), 310-326.

Guo, D. (2000). Dynamic volatility trading strategies in the currency option market. *Review of derivatives research*, 4(2), 133-154.

Han, B., & Zhou, Y. (2012). Variance risk premium and cross-section of stock returns. *unpublished paper, University of Texas at Austin*, 8, 34.

Kaeck, A. (2018). Variance-of-variance risk premium. *Review of Finance*, 22(4), 1549-1579.

Lee, W. Y., Jiang, C. X., & Indro, D. C. (2002). Stock market volatility, excess returns, and the role of investor sentiment. *Journal of banking & Finance*, 26(12), 2277-2299.

Novy-Marx, R. (2015). Backtesting strategies based on multiple signals. National Bureau of Economic Research.

Vasquez, A. (2017). Equity volatility term structures and the cross section of option returns. *Journal of Financial and Quantitative Analysis*, *52*(6), 2727-2754.

APPENDIX A: Companies' Tickers forming our sample

MSFT	CAT	BLK	SHW	VLO	PRU	FANG	ANSS	TTWO	SYF	ALB	LKQ	GNRC
AAPL	ABT	SYK	TGT	NSC	IQV	KDP	EIX	WDC	RF	VRSN	SJM	PAYC
NVDA	INTU	UPS	MCK	GM	GWW	KVUE	GPN	NTAP	PFG	ENPH	UDR	MTCH
AMZN	GE	BSX	EQIX	TFC	IDXX	GEHC	AVB	HPE	TDY	JBL	KMX	FMC
META	AMAT	DE	CVS	HLT	STZ	IT	CHTR	GPC	COO	EXPE	CRL	FOXA
GOOGL	TXN	ADI	TDG	EW	HUM	DAL	FTV	DECK	CMS	SWKS	NI	HAS
GOOG	VZ	REGN	PH	AZO	LHX	CTSH	EBAY	BALL	AVY	AMCR	CPT	MKTX
BRK.B	DHR	CB	SLB	MCHP	MRNA	DD	WEC	HUBB	STX	SNA	PODD	CZR
LLY	AMGN	BA	BDX	TRV	DOW	XYL	WST	PPL	BAX	NDSN	MGM	RHI
AVGO	PFE	CI	PYPL	NEM	OXY	MPWR	CBRE	ES	DRI	CAG	ALLE	FRT
JPM	CMCSA	ADP	CEG	SRE	CNC	ED	GLW	SBAC	LH	CCL	JNPR	
XOM	PM	MMC	CSX	WMB	MNST	HAL	WTW	PTC	UAL	BBY	EPAM	
TSLA	IBM	LMT	ITW	DXCM	CTVA	ADM	CHD	AXON	NTRS	POOL	TAP	
UNH	NEE	PLD	NXPI	F	GIS	RCL	KEYS	VLTO	DPZ	AKAM	BBWI	
V	NOW	MDLZ	NOC	AEP	PAYX	FICO	FITB	CTRA	ILMN	TRMB	UHS	
PG	UNP	KLAC	GD	SPG	LEN	BIIB	HPQ	CPAY	HOLX	L	INCY	
MA	GS	PANW	MPC	CPRT	LULU	ODFL	GRMN	WAT	CLX	CF	AOS	
JNJ	COP	FI	EMR	OKE	CMI	BKR	AWK	BLDR	J	KEY	HRL	
COST	RTX	BMY	ABNB	URI	OTIS	DVN	DOV	BRO	LVS	SWK	FFIV	
HD	SPGI	BX	USB	ADSK	SMCI	RMD	LYB	FE	ATO	DOC	HII	
MRK	AXP	CMG	PNC	DLR	AME	PPG	MTB	STLD	TXT	JBHT	CTLT	
CVX	MU	SBUX	HCA	KMB	PWR	MTD	TROW	MOH	EXPD	IP	CHRW	
ABBV	UBER	AMT	MCO	TEL	FAST	HSY	TRGP	FSLR	TSN	VTRS	REG	
CRM	HON	SO	PSX	O	RSG	HWM	DLTR	TYL	IEX	PNR	AAL	
NFLX	ISRG	TMUS	FDX	ROST	MSCI	EL	ZBH	HBAN	CFG	LYV	TFX	
BAC	LOW	SNPS	ORLY	AFL	YUM	EXR	PHM	INVH	ESS	GEN	NWSA	
WMT	ETN	GILD	CTAS	JCI	PCG	ON	IFF	LDOS	FDS	AES	MOS	
PEP	BKNG	MO	MSI	FIS	FTNT	DFS	CAH	ULTA	LUV	WBA	QRVO	
KO	INTC	DUK	ECL	ALL	EXC	EA	NVR	ALGN	K	ROL	HSIC	
AMD	ELV	ANET	MAR	MET	SYY	ROK	DTE	AEE	EG	RVTY	DAY	
TMO	PGR	CL	AON	D	MLM	XEL	RJF	CBOE	EQT	HST	TPR	
WFC	MS	CDNS	PCAR	HES	COR	VICI	ETR	VTR	PKG	WRK	WYNN	
ADBE	T	WM	APD	BK	KMI	DG	BR	TER	ZBRA	LNT	AIZ	
LIN	C	ZTS	ROP	GEV	IR	VST	APTV	OMC	WRB	TECH	APA	
QCOM	LRCX	ICE	AIG	DHI	PEG	HIG	NDAQ	MKC	CE	EVRG	CPB	
CSCO	NKE	APH	WELL	PSA	ACGL	EFX	STT	ARE	MAA	KIM	BF.B	
MCD	TJX	CME	MMM	CCI	CSGP	CDW	STE	CNP	DGX	JKHY	BWA	
DIS	SCHW	EOG	COF	AMP	VMC	WAB	IRM	NRG	BG	LW	BXP	
ACN	MDT	FCX	CARR	A	KR	TSCO	EQR	WBD	MAS	IPG	SOLV	
ORCL	VRTX	TT	AJG	NUE	VRSK	KHC	WY	CINF	MRO	EMN	PNW	

Note. List covering nearly the entire SPDR S&P 500 ETF Trust (SPY) composite, replicating the performance of the S&P 500. List retrieved from www.slickchart.com, sorted by their respective weight in the index.