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Optimizing Pick-up Locations and Bus Allocation in Transit-Based Evacuation Planning by Minimizing Maximum Driving Time and Allowing Multi-Point Service under Demand Uncertainty

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Abstract

In this thesis, we address the problem of planning transit-based evacuations under demand uncertainty. Building upon the model proposed by Kulshrestha et al. (2014), which selects designated pick-up points and plans bus trips to transport evacuees to shelters, we propose two significant extensions. First, we introduce an alternative objective function that minimizes the maximum driving time across buses, promoting a more balanced workload distribution and ensuring a quicker evacuation process. Second, we relax the constraint that limits each bus to a single pick-up point, allowing for more flexible routing and efficient use of bus resources. Our computational experiments, conducted on the Sioux Falls network, confirm that these extensions enhance the efficiency and robustness of evacuation plans. The modified objective function leads to quicker and more equitable evacuations, while the relaxed pick-up point constraint optimizes resource allocation and reduces total evacuation time. Future research directions include the development of more efficient algorithms, integration of dynamic updates, and incorporation of socio-economic factors to further enhance the model's applicability and effectiveness.

1 Introduction

Humanity has always had to deal with the effects of disasters, both natural and man-made. The highest priority in the event of a disaster is preventing as many casualties as possible. Evacuation of the area is often regarded as a reliable option in response to a disaster. This has, however, become an increasingly complicated challenge in the past decades as the population, size and therefore complexity of cities have continued to grow. This is evidenced by the fact that currently more than half of the global population lives in cities (Yao, Yao, Cenci, Liao & Zhang, 2023).

Cities have residents who are dependent on public transit for both their normal commute and evacuation. It is therefore crucial to develop an evacuation plan specifically for these transit-dependent populations. This planning involves two key aspects: deciding where evacuees should gather and scheduling the buses to transport them to safety. Furthermore, the increasing complexity of cities has led to evacuation planning receiving a lot of attention from additional angles, such as evacuees' behaviour (Thompson, Garfin & Silver, 2017) and logistical planning (Ozdamar & Ertem, 2015). Most of the research, however, assumes the demand to be deterministic, whilst in reality, it is very difficult to know exactly how many people are present in each area at the time of evacuation. Not taking into account this uncertainty when planning an evacuation can have disastrous effects on the efficiency and effectiveness of the evacuation.

Kulshrestha, Lou and Yin (2014) were the first to combine these three aspects. They define a model that chooses the points at which the evacuees gather to be picked up and allocates the available vehicles to these points. It also incorporates a capacity limit on the evacuation centres, so the model determines the number of trips each bus must take from its allocated point to each evacuation centre. The model does this under the assumption of uncertainty in the evacuation demand. They propose a cutting plane method to solve the model and test it on the Sioux Falls network. The method is shown to be efficient and effective.

In this thesis, we propose an extension to the model developed by Kulshrestha et al. (2014) in the form of a new objective value. Their model minimizes the total driving time, but we propose minimizing the maximum driving time across buses. This adjustment aims to ensure a more balanced distribution of evacuation workload among buses, preventing scenarios where some buses remain idle while others are overburdened. By focusing on the maximum driving time, we enhance the efficiency and equity of the evacuation process, ensuring that all evacuees are transported within the shortest possible time frame for the entire fleet.

This thesis provides a second extension on the model proposed by Kulshrestha et al. (2014) by addressing the limitation that each bus is restricted to serving only a single pick-up point. This constraint can lead to underutilisation of available bus resources and inefficiencies in evacuation time. By allowing buses to serve multiple pick-up points, the extended model aims to optimize resource allocation and reduce total evacuation time. This enhancement could significantly improve the effectiveness of evacuation strategies, ensuring a more flexible and efficient response to emergencies. The extended formulation is too large to be solved by a commercial solver in a reasonable timeframe, so a simplified version is used to evaluate the potential of the extension.

Effective and efficient evacuation planning has significant economic implications. Efficient evacuation planning not only saves lives but also reduces the economic impact of disasters. Minimizing the maximum driving time can lead to significant cost savings in terms of fuel consumption, vehicle wear and tear, and overall operational efficiency. Additionally, optimizing resource allocation by allowing buses to serve multiple pick-up points can reduce the need for additional vehicles and personnel, leading to further cost reductions.

Moreover, improved evacuation strategies can enhance public confidence in disaster management plans, potentially boosting economic stability in affected areas. Businesses and residents are more likely to invest and remain in regions with reliable and effective evacuation plans. Therefore, the extensions proposed in this thesis have the potential to contribute to both the immediate and long-term economic resilience of urban areas facing disaster risks.

The remainder of this thesis is structured as follows. In Section 2, we give a short review of the related scientific literature. In Section 3 we give a description of the problem that this thesis aims to solve. In Section 4 we give the model formulations for the problems and in Section 5 we describe how we obtain our solutions. The results are presented in Section 6 and we conclude in Section 7.

2 Literature Review

An efficient evacuation is crucial in response to disasters. There have been many studies performed around this topic, for example in crowd behaviour (Haghani & Sarvi, 2018), pedestrian guidance in case of emergency (Yang, Yang, Wang, Kang & Pan, 2020), building evacuation (Liu, Xu, Lu & Zhang, 2018) and shipwreck evacuation (Kang, Zhang & Li, 2019). Cities and other urban areas have a dense population and therefore a higher number of people that need to be evacuated in case of emergency. The evacuation of these crowded areas has received additional attention (Cristobal-Salas et al., 2019; Tu, Tamminga, Drolenga, de Wit & van der Berg, 2010; Loehner, Haug, Zinggerling & Onate, 2018; Goto et al., 2012). These urban areas also contain residents who do not own a car and rely on public transportation for their normal commute. From them, transit-based evacuation plans need to be put in place in order to enable them to evacuate the endangered area.

Bish (2011) introduced a model to solve this problem: the bus evacuation problem (BEP). He noticed that a transit-based evacuation problem is very similar to a vehicle routing problem (VRP), with key differences in the objective (evacuation time minimisation vs cost minimisation) and network structure (shelter capacities). He proposes two model formulations for the BEP and provides a framework to incorporate a variety of objective functions in the models. Finally, two heuristics are designed to solve the BEP. The BEP proposed by Bish (2011) formed the basis for transit-based evacuation planning, after which many papers have studied and extended this model.

Goerigk, Gruen and Hessler (2013) propose a branch and bound framework to solve the BEP. They describe multiple methods to find upper and lower bounds on the maximum driving time, which they use in the branch and bound framework. They also discuss some branching rules and pruning techniques. Their computational experiments show that the solution times are greatly improved when using their approach compared to a commercial solver. Dikas and Minis (2016) provide an alternative formulation to the BEP and investigate the casualty evacuation problem (CEP). The objective of the CEP is to minimise the sum of arrival times, instead of the sum of total transportation times in the BEP. This essentially means the CEP minimises the average transport time of a casualty to the medical centre. They implement a hybrid two-phase algorithm to solve the BEP, as they note that BEPs are problems of high complexity and instances of realistic size may not be solved to optimality using exact approaches. Their first phase consists of a combination of a Large Neighbourhood Search (LNS), a mixed integer programming program (MIP) and a travelling salesman problem (TSP) to generate a good initial solution. The second phase uses a linear relaxation and a Variable Neighborhood Search (VNS) to improve on the previous solution found.

The above-mentioned papers assume the locations of the shelters and the points where the evacuees assemble, the pick-up points, are fixed and known in advance. Goerigk, Gruen and Hessler (2014) extend the BEP introduced by Bish (2011) by relaxing the assumption that the pick-up locations are known in advance. Their proposed model integrates the choice of pick-up locations so that it is combined with the creation of bus routing plans to achieve a minimal evacuation time. They implement a branch-and-price approach to solve the model and show that it is still tractable to solve realistic instances, despite the increase in computational complexity. Zhang and Chang (2014) propose an alternative way of solving the extended BEP. They split the problem into two phases. The first phase selects the pick-up points from the network, after which the second phase optimises the bus scheduling. The pick-up point selection is made by forming clusters of demand points and designating pick-up points within each cluster. An overlapping clustering algorithm is used to assign demand points to multiple clusters. The second phase is solved with a MIP that takes into account a time-dependent arrival rate of evacuees. Gao, Nayeem and Hezam (2019) apply the same concept of decomposition on this problem but solve the first step with an integer programming problem (IP) that minimises the total walking time of evacuees. Their second stage is also split up, where they first assign evacuees to shelters and then assign buses to shelters based on these assignments. Finally, they create the scheduling to perform the evacuation of each pick-up point within its given time window.

An additional problem that is faced when planning for evacuation is uncertainty, as it is very difficult to know exactly how many people are present in each area. The research in this area is still somewhat limited, but Kulshrestha et al. (2014) were the first to study the situation where there was uncertainty in the evacuation demand when planning for transit-based evacuation. They observe that it is practically impossible to know the exact distribution of the demand, so they decide on using an uncertainty set to optimise over. They implement a cutting plane algorithm to solve efficiently over all demand scenarios in the uncertainty set. Goerigk, Deghdak and T'Kindt (2015) expanded upon this uncertainty set by also considering uncertainty in the travel times and the number of available buses. They define a few robustness criteria and propose an iterative solution approach to optimise over these criteria. Finally, Goerigk and Gruen (2014) considered the situation where the exact information on the evacuation demand is delayed. Their proposed MIP model has to choose whether to send out buses now with the information on a few demand scenarios or wait some time to get the precise number. They propose a tabu search framework to find heuristic solutions.

Although there have been some papers that have considered demand uncertainty in transit-based evacuation planning, they have used a simplified version of the BEP as their model base. Kulshrestha et al. (2014) for example, limit buses to a single pick-up point and only plan the number of round trips the buses perform to each shelter. The contribution of this thesis is the incorporation of a larger decision model in the demand uncertainty framework. This is done by expanding the model formulated by Kulshrestha et al. (2014) by relieving the constraint that limits the buses to a single pick-up point.

3 Problem Description

We look at the setting in which a residential area has to be evacuated because of a disaster. This area has residents who are dependent on public transport for their evacuation. The evacuees gather at designated pick-up points, where buses will pick them up and bring them to safe shelters. The evacuees all reside at given points (demand points) on the map, which coincide with the possible locations for pick-up points. When the pick-up points are selected, we assume the evacuees gather at the pick-up point that is closest to their current location. The buses have a known capacity, as do the shelters. The capacity of the buses and shelters can differ from one another. We assume the buses have no influence on the congestion on the roads, so we assume the travel times over the road network to be fixed and known. We also assume the buses have no loading and unloading time.

We will look at two different scenarios regarding the behaviour of the buses. The first assumes that buses can only pick up evacuees from a single pick-up point. They can visit different shelters, but they always return to the same pick-up point. The routes of the buses can therefore be seen as round trips to different shelters, starting from their designated pick-up point. The second alleviates this restriction, meaning that buses can pick up evacuees from multiple pick-up points. Both scenarios, however, assume that the evacuees gather at the pick-up points before the start of the evacuation process. This is a realistic assumption, as the evacuees could use the time it takes for the buses to reach the pick-up points to gather at these points.

4 Model Formulations

This section discusses the model formulations for the problem described above. First, we describe the model proposed by Kulshrestha et al. (2014). Then, we provide a formulation for the situation where the bus assignment constraint is lifted. Finally, we provide an alternative objective function and we modify the two models to incorporate the new objective.

4.1 Demand Uncertainty

In the ideal scenario, we would want to know exactly how many residents are located at each point when planning an evacuation. In reality, however, it is very difficult to know the exact number of residents in each area. We, therefore, need to incorporate this uncertainty into our model. As mentioned earlier, it is very difficult to know the exact distribution of the evacuation demand at each point, so we will have to work with some form of approximation.

The method chosen by Kulshrestha et al. (2014), which this thesis also adopts, is to select the demand of each point from a finite list of possible values. Let $d_s^u, s = 1, 2, \ldots, S^i$ be the possible values of the demand at point $i \in I$, where d_1^i corresponds to the normal or most likely value. A way to hedge against the uncertainty would be to optimize the location-allocation plan for the worst-case scenario, in which the maximum demand value is realised at each demand point. This optimisation would, however, result in a plan that is very conservative. It is very unlikely that each demand point will simultaneously receive its largest demand.

This thesis thus assumes a limit on the number of demand points that can deviate from its nominal value. Let this limit be denoted by Γ, also known as the degree of pessimism. This name reflects the relationship between the parameter and the decision-maker's attitude towards risk, where a higher Γ implies a higher risk aversion.

Let z_s^i be a binary variable that indicates if demand scenario s is realised at point $i \in I$. Since only one value can be realised at each point, we have

$$\sum_{s=1}^{S^i} z_s^i = 1.$$

Additionally, at most Γ locations can deviate from their nominal values. Since $z_1^i = 1$ indicates the nominal value is realised, we have

$$\sum_{i \in I} \sum_{s=2}^{S^i} z_s^i \leq \Gamma.$$

Let d^i be the demand realised at point $i \in I$ and d the vector containing the realised values of each demand point. We can then define the uncertainty set for the transit-based evacuation demands as

$$D = \left\{ d \middle| d^i = \sum_{s=1}^{S^i} d_s^i z_s^i,\ \sum_{s=1}^{S^i} z_s^i = 1,\ \sum_{i \in I} \sum_{s=2}^{S^i} z_s^i \leq \Gamma, z_s^i \in \{0, 1\}\ \forall i \in I, s \in S^i \right\}.$$

D is the set containing all possible combinations of demand given the possible demand values at each demand point. The bus allocation planning has to be optimised over all scenarios, so the choice of D can have a great impact on the solving time and the quality of the solution.

4.2 Single Pick-up Allocation

We first look at the formulation proposed by Kulshrestha et al. (2014). This formulation is built upon the assumption that each bus is allocated to a single pick-up point and can only evacuate the residents that have assembled at this point.

Consider a network $G(N, A)$, where N represents the set of nodes and A represents the set of undirected arcs. Let $I \subset N$ denote the set of nodes that have an evacuation demand, and let $J \subset N$ denote the set of nodes designated as shelters. Each shelter $j \in J$ has a capacity K_j. We assume that buses are the only available mode of public transportation. Let B be the set of available buses, with individual capacities β_b. The round-trip travel time from pick-up point i to shelter j is denoted as T_{ij}, and the maximum time a bus is allowed to drive as $T_{\max}$. The distance between nodes i and $p \in N$ is denoted by C_{ip}, and the walking distance for each demand point i to its nearest pick-up point is limited by a maximum ω.

We introduce the following decision variables for our model:

- $y_i = \begin{cases} 1, & \text{if demand point } i \text{ is selected as a pick-up point, } i \in I \\ 0, & \text{otherwise} \end{cases}$
- $\mu_b^i = \begin{cases} 1, & \text{if bus } b \text{ is allocated to pick-up point } i,\ b \in B, i \in I \\ 0, & \text{otherwise} \end{cases}$
- $\delta_p^i = \begin{cases} 1, & \text{if the evacuees of demand point } i \text{ are assigned to pick-up point } p,\ i \in I, p \in I \\ 0, & \text{otherwise} \end{cases}$
- X_{bij} : the number round trips bus b performs between pick-up point i and shelter j, $b \in B,\ i \in I,\ j \in J$
- q_i^d : the demand accumulated at pick-up point i under demand scenario d, $i \in I,\ d \in D$
- w_i : the walking distance of demand point i to its nearest pick-up point, $i \in I$

Using these notations, we formulate the robust transit pick-up location and bus allocation (RTPL) model as follows:

$$
\begin{aligned}
\min \quad & \sum_{b\in B}\sum_{i\in I}\sum_{j\in J} T_{ij}X_{bij} && && (1)\\
\text{s.t.} \quad & \sum_{b\in B}\sum_{j\in J} \beta_b X_{bij} \geq q_i^d && \forall i \in I, d \in D && (2)\\
& \sum_{b\in B}\sum_{i\in I} \beta_b X_{bij} \leq K_j && \forall j \in J && (3)\\
& \sum_{i\in I} \mu_b^i = 1 && \forall b \in B && (4)\\
& \sum_{b\in B} \mu_b^i \leq |B| y_i && \forall i \in I && (5)\\
& \sum_{j\in J} X_{bij} \leq M\mu_b^i && \forall i \in I, b \in B && (6)\\
& W_i = \sum_{p\in I} C_{ip}\delta_p^i && \forall i \in I && (7)\\
& W_i \leq C_{ip}y_p + M(1-y_p) && \forall i \in I, p \in I && (8)\\
& \sum_{p\in I} \delta_p^i = 1 && \forall i \in I && (9)\\
& \delta_p^i \leq y_p && \forall i \in I, p \in I && (10)\\
& q_p^d = \sum_{i\in I} d^i \delta_p^i && \forall p \in I, d \in D && (11)\\
& W_i \leq \omega && \forall i \in I && (12)\\
& \sum_{i\in I}\sum_{j\in J} T_{ij}X_{bij} \leq T_{\max} && \forall b \in B && (13)\\
& y_i, \delta_p^i, \mu_b^i \in \{0,1\} && \forall i \in I, p \in I, b \in B && (14)\\
& q_i^d, W_i \geq 0 && \forall i \in I, d \in D && (15)\\
& X_{bij} \in \mathbb{Z}^+ && \forall i \in I, j \in J, b \in B, && (16)
\end{aligned}
$$

where M is a sufficiently large number. The objective of this model is to minimise the total evacuation time. Constraints (2) ensure that the accumulated demand at each pick-up point for each demand scenario can be transported by the buses. Constraints (3) ensure that the capacity of each shelter is not exceeded. Constraints (4) limit the allocation of each bus to a single pick-up point and constraints (5) make sure the point they are allocated to is selected as a pick-up point. Constraints (6) ensure that buses can only make round trips starting from their allocated pick-up point. Constraints (7) and (8) calculate the walking distance for each demand point to its nearest pick-up point. Constraints (9) ensure all evacuees of the same demand point gather at the same pick-up point and constraints (10) ensure that the point where they gather is a designated pick-up point. Constraints (11) calculate the demand accumulated at each pick-up point. Constraints (12) ensure that the walking distance from each demand point to its closest pick-up point is at most a certain maximum. Constraints (13) limit the total driving time of each bus to a certain maximum. Finally, constraints (14), (15) and (16) define the domains of the decision variables.

4.3 Multiple Pick-up Allocation

We now look at the situation where multiple buses can transport evacuees from multiple pick-up points. Relieving the constraint maintained by Kulshrestha et al. (2014) gives more freedom in the planning of the bus trips, but increases the complexity of the model.

We introduce the concept of a pair $r = \{i, j\}$, which consists of a pick-up point $i \in I$ and a shelter $j \in J$. Let R be the set containing all possible pairs of the network. Instead of being assigned to a pick-up point, a bus will be assigned to pairs. Being assigned to a pair means the bus will perform trips between the pick-up point and the shelter of that pair. After finishing the round trips over that pair, the buses can now also travel between pairs. This means that after dropping the last evacuee at the shelter, it travels to the pick-up point of a new pair instead of going back to the pick-up point it came from. This transfer is called a shuttle, and the distance travelled during the shuttle from pair $r \in R$ to $t \in R$ is denoted by w_{rt}. We define the following decision variables:

- $\psi_{brt} = \begin{cases} 1, & \text{if bus } b \text{ services pair } t \text{ after pair } r,\ b \in B,\ t \in R, r \in R \\ 0, & \text{otherwise} \end{cases}$
- $f_{br} = \begin{cases} 1, & \text{if pair } r \text{ is the first pair that bus } b \text{ services, } b \in B, r \in R \\ 0, & \text{otherwise} \end{cases}$
- $l_{br} = \begin{cases} 1, & \text{if pair } r \text{ is the last pair that bus } b \text{ services, } b \in B,\ r \in R \\ 0, & \text{otherwise} \end{cases}$
- λ_{br} : integer variable that denotes the position of pair $r \in R$ in the route of bus $b \in B$

In order to adapt the model described in Section 4.2, we modify the notation of the following parameters and variables:

- $T_{ij} \rightarrow T_r^{\text{round}}$: round trip travel time for pair $r \in R$
- $X_{bij} \rightarrow X_{br}$: integer decision variable that indicates how often bus $b \in B$ services pair $r \in R$
- $\mu_b^i \rightarrow \mu_b^r$: binary decision variable that takes value 1 if bus $b \in B$ services pair $r \in R$, 0 otherwise
- T_r^{single} : single trip travel time from pick-up point $i \in r$ to shelter $j \in r$ of pair $r \in R$

We formulate the extended model as

$$
\begin{aligned}
\min \quad & \sum_{b\in B}\sum_{r\in R}(T_r^{\text{round}}(X_{br}-\mu_b^r)+T_r^{\text{single}}\mu_b^r+\sum_{t\in R}w_{rt}\psi_{brt}) && && (17)\\
\text{s.t.} \quad & (7)\text{-}(12) && && \\
& \sum_{b\in B}\sum_{r\in R:i\in r}\beta_b X_{br}\geq q_i^d && \forall i\in I, d\in D && (18)\\
& \sum_{b\in B}\sum_{r\in R:j\in r}\beta_b X_{br}\leq K_j && \forall j\in J && (19)\\
& \sum_{b\in B}\sum_{r\in R:i\in r}\mu_b^r\leq My_i && \forall i\in I && (20)\\
& X_{br}\leq M\mu_b^r && \forall b\in B, r\in R && (21)\\
& X_{br}\geq \mu_b^r && \forall b\in B, r\in R && (22)\\
& \sum_{r\in R}(T_r^{\text{round}}(X_{br}-\mu_b^r)+T_r^{\text{single}}\mu_b^r+\sum_{t\in R}w_{rt}\psi_{brt})\leq T_{\max} && \forall b\in B && (23)\\
& \sum_{r\in R}f_{br}\leq 1 && \forall b\in B && (24)\\
& \sum_{r\in R}l_{br}=\sum_{r\in R}f_{br} && \forall b\in B && (25)\\
& f_{br}+\sum_{t\in R}\psi_{btr}=\mu_b^r && \forall b\in B, r\in R && (26)\\
& l_{br}+\sum_{t\in R}\psi_{brt}=\mu_b^r && \forall b\in B, r\in R && (27)\\
& \lambda_{bt}-\lambda_{br}+|R|\psi_{brt}\leq |R|-1 && \forall b\in B, r\in R, t\in R && (28)\\
& \lambda_{br}\geq (1-\mu_b^r)(|R|+1) && \forall b\in B, r\in R && (29)\\
& \lambda_{br}\leq f_{br}+M(1-f_{br}) && \forall b\in B, r\in R && (30)\\
& y_i, \delta_p^i, \mu_b^r, \psi_{brt}, f_{br}, l_{br}\in\{0,1\} && \forall i,p\in I, b\in B, r,t\in R && (31)\\
& q_i^d, W_i\geq 0 && \forall i\in I, d\in D && (32)\\
& X_{bij}\in\mathbb{Z}^+ && \forall i\in I, j\in J, b\in B && (33)\\
& \lambda_{br}\in\{1,2,\dots,|R|+1\} && \forall b\in B, r\in R, && (34)
\end{aligned}
$$

where M is a sufficiently large number. The objective function minimises the total driving time. The first term calculates the time needed for all round trips on a pair, the second adds the time needed for the final trip to the shelter of a pair and the last term calculates the travel time for the shuttles. Constraints (18), (19), (20) and (23) correspond to constraints (2), (3), (5) and (13) of the RTPL model. Constraints (18) ensure that the accumulated demand at each pick-up point for each demand scenario can be transported by the buses. Constraints (19) ensure the capacity of each shelter is not exceeded. Constraints (20) ensure that buses are only assigned to pairs of which the demand point is chosen as a pick-up point. Constraints (21) ensure that buses can only make trips on pairs that it is assigned to. Constraints (22) force buses to perform at least one trip on each pair it is assigned to. Constraints (23) ensure the total driving time of each bus is below the allowed maximum. Constraints (24) ensure that each bus can only start its route at one pair. Constraints (25) ensure the number of ending pairs is equal to the number of starting pairs for each bus. Constraints (26) ensure that if a bus is assigned to a pair, that pair is either the starting pair or it has a predecessor. Equally, constraints (27) ensure each assigned pair has a successor or is the final pair of that bus. Constraints (28) prevent the creation of sub-cycles by ensuring subsequent pairs have subsequent route position numbers. If a bus services pair t after pair r, these constraints ensure that the position of pair t in the route of the bus is exactly one higher than the position of pair r. Constraints (29) fix the route positions of pairs that are not visited. Here, a value outside the normal range is selected. Constraints (30) set the route position of the starting pair to 1. Constraints (31)-(34) set the domains of the decision variables.

This model implicitly assumes that when a bus has to perform multiple trips on a pick-up point and shelter pair, it will perform all of these round trips before going to the next pair. Although this might lower the quality of the solution, we find that acceptable as relieving this assumption results in an increase in the size of the model and the solving time.

4.4 Alternative Objective

The objective function of the model proposed by Kulshrestha et al. (2014) aims to minimize the total evacuation time. However, this objective may lead to a scenario where one bus is required to make numerous trips while others remain idle after completing their assignments. This situation is suboptimal in an evacuation context, as it is more desirable to utilize all available buses to evacuate everyone as quickly as possible. Therefore, we propose modifying the objective function to minimize the maximum travel time across all buses. This adjustment aims to ensure that the evacuation process is completed in the shortest possible time for all evacuees.

Let Ω be the maximum travel time across buses. The objective function that minimizes the maximum travel time is formulated as

$$\min \quad \Omega. \tag{35}$$

4.4.1 Single Pick-up Allocation

Ω needs to be defined in the RTPL model, which this is done by adding the following two constraints to the model:

$$\Omega \geq \sum_{i \in I} \sum_{j \in J} T_{ij} X_{bij} \qquad \forall b \in B \tag{36}$$

$$\Omega \geq 0 \tag{37}$$

Constraints (36) ensure that Ω is at least the travel time for each bus. Constraint (37) ensures that Ω is non-negative.

4.4.2 Multiple Pick-up Allocation

In the MP-RTPL we update Ω by adding

$$\Omega \geq \sum_{r \in R} (T_r^{\text{round}}(X_{br} - \mu_b^r) + T_r^{\text{single}} \mu_b^r + \sum_{t \in R} w_{rt} \psi_{brt}) \qquad \forall b \in B \tag{38}$$

$$\Omega \geq 0 \tag{39}$$

to the model, where constraints (38) ensure that Ω is at least the total travel time for each bus and constraint (39) ensures Ω is non-negative.

5 Solution Method

The RTPL and MP-RTPL are modelled as MIP, but constraints (2), (11) and (18) are written for each demand scenario in the uncertainty set. Due to the combinatorial nature of the uncertainty set, the number of constraints could become exponentially large. This could make the model difficult to solve in a reasonable time frame. For this reason, Kulshrestha et al. (2014) have implemented a cutting plane algorithm to solve the model. The algorithm consists of solving a restricted version of the RTPL and MP-RTPL for a subset of the demand scenarios and using that solution to solve a worst-case demand model (WCD). The WCD checks if there are demand scenarios for which the given solution is not feasible. A solution to an RTPL or MP-RTPL is infeasible if there are evacuees remaining at the pick-up locations because the bus capacity is insufficient. If such a demand scenario is found, it is added to the uncertainty set and the restricted RTPL (R-RTPL) or MP-RTPL (R-MP-RTPL) are resolved.

5.1 Single Pick-up Allocation

Let $\bar{D} \subset D$ be a subset of the uncertainty set that contains a much smaller number of demand scenarios. The R-RTPL is then moddeled as

$$\min \quad \sum_{b \in B} \sum_{i \in I} \sum_{j \in J} T_{ij} X_{bij} \tag{40}$$

$$\text{s.t.} \quad \sum_{b \in B} \sum_{j \in J} \beta_b X_{bij} \geq q_i^d \qquad \forall i \in I, d \in \bar{D} \tag{41}$$

$$q_p^d = \sum_{i \in I} d^i \delta_p^i \qquad \forall p \in I, d \in \bar{D} \tag{42}$$

$$(3) - (10), (12) - (16).$$

This R-RTPL is the same as the RTPL, but it is written for only a small subset of the demand scenarios. The R-RTPL is therefore considerably easier to solve because of the lower number of constraints, and it can be solved to optimality. Let $(\hat{y}, \hat{\delta}, \hat{X})$ be the optimal solution to the R-RTPL. If this solution is feasible for all demand scenarios in the uncertainty set it is also the optimal solution to the entire RPTL. This is checked with the WCD, for which we introduce the following decision variables:

- e_i : the excess demand at pick-up point $i \in I$

- η_i : binary slack variable indicating whether the demand at point $i \in I$ deviates from its nominal value

The WCD is then modelled as

$$
\begin{aligned}
\max \quad & \sum_{i \in I} e_i && && (43)\\
\text{s.t.} \quad & e_i \leq M\eta_i && \forall i \in I && (44)\\
& e_i - (q_i - \sum_{b \in B} \sum_{j \in J} \beta_b \hat{X}_{bij}) \leq M(1 - \eta_i) && \forall i \in I && (45)\\
& q_p = \sum_{i \in I} d^i \hat{\delta}^i_p && \forall p \in I && (46)\\
& d^i = \sum_{s=1}^{S^i} d^i_s z^i_s && \forall i \in I && (47)\\
& \sum_{s=1}^{S^i} z^i_s = 1 && \forall i \in I && (48)\\
& \sum_{s=2}^{S^i} z^i_s \leq \Gamma && \forall i \in I && (49)\\
& e_i \geq 0 && \forall i \in I && (50)\\
& z^i_s, \eta_i \in \{0, 1\} && \forall i \in I, s \in \{1, 2, \ldots, S^i\}, && (51)
\end{aligned}
$$

where M is a sufficiently large number. The objective function maximises the excess demand over all points. Constraints (44) and (45) calculate the excess demand at each pick-up point. Constraints (46) calculate the demand that is accumulated at each pick-up point. Constraints (47) calculate the demand realised at each demand point. Constraints (48) ensure only one demand value is realised at each demand point. Constraints (49) ensure the number of demand points that deviate from their nominal demand values is below the allowed maximum. Finally, constraints (50) and (51) set the domains of the decision variable.

If the objective function corresponding to the optimal solution of the WCD is non-zero, it means that there is a demand scenario in the uncertainty set for which the current optimal solution to the R-RTPL is not able to evacuate everyone. The optimal solution to the WCD is the demand scenario for which the excess demand is the highest, the so-called worst-case scenario. This demand scenario $\hat{d}$ is then added to the restricted uncertainty set, after which the R-RTPL is resolved over the extended uncertainty set $\bar{D} \cup \{\hat{d}\}$. The entire cutting plane algorithm is shown in Algorithm 1. Because the number of demand scenarios in the uncertainty set is finite, the algorithm will terminate.

Algorithm 1 Cutting Plane

```
Choose an initial demand scenario d_0 ∈ D. Set n = 1 and D̄ = {d_0}
while Optimal solution RTPL not found do
    Solve R-RTPL over subset D̄
    (ŷ, δ̂, X̂)^n ← Optimal solution R-RTPL
    Solve WCD using (ŷ, δ̂, X̂)^n as parameters
    V ← Optimal objective function WCD
    if V == 0 then
        Optimal solution RTPL found
    else
        d_n ← optimal solution WCD
        D̄ = D̄ ∪ {d_n}
        n = n + 1
    end if
end while
return Optimal solution RTPL
```

5.2 Multiple Pick-up Allocation

The same cutting plane procedure is applied to solve the MP-RTPL, but because its formulation is different from the RTPL we need to alter a few constraints. The R-MP-RTPL is modelled as

$$\min \quad \sum_{b\in B}\sum_{r\in R}(T_r^{\text{round}}(X_{br}-\mu_b^r)+T_r^{\text{single}}\mu_b^r+\sum_{t\in R} w_{rt}\psi_{brt}) \tag{52}$$

$$\text{s.t.} \quad \sum_{b\in B}\sum_{r\in R: i\in r}\beta_b X_{br} \geq q_i^d \qquad \forall i\in I, d\in \bar{D} \tag{53}$$

$$(7)-(10),(12),(19)-(34),(42).$$

The WCD for the MP-RTPL is modelled as

$$\max \quad \sum_{i\in I} e_i \tag{54}$$

$$\text{s.t.} \quad e_i-(q_i-\sum_{b\in B}\sum_{r\in R: i\in r}\beta_b \hat{X}_{br})\leq M(1-\eta_i) \qquad \forall i\in I \tag{55}$$

$$(44),(46)-(51).$$

5.3 Simplified Multiple Pick-up Allocation

As Goerigk et al. (2014) and Zhang and Chang (2014) have noted, this type of formulation is too large to be solved by simply putting it into a commercial solver. For this reason, we propose a simplified version of the MP-RTPL (S-MP-RTPL) to evaluate the potential of the extension. This model relieves constraints (4) to allow each bus to be assigned to two pick-up points instead of one. To calculate the driving times correctly we introduce the following variables and parameters:

- T_{ip}^{single} : travel time between pick-up points $i \in I$ and $p \in I$
- θ_{bip} : auxiliary binary decision variable that takes value 1 if bus $b \in B$ services both pick-up points $i \in I$ and $p \in I$, 0 otherwise

The simplified R-MP-RTPL (SR-MP-RTPL) is then modeled as

$$
\begin{aligned}
\min \quad & \sum_{b\in B}\sum_{i\in I}\Big(\sum_{j\in J} T_{ij}X_{bij} + \sum_{p\in I:p>i} T^{\text{single}}\theta_{bij}\Big) && && (56)\\
\text{s.t.} \quad & \sum_{i\in I}\mu_b^i \le 2 && \forall b \in B && (57)\\
& \sum_{i\in I}\mu_b^i \ge 1 && \forall b \in B && (58)\\
& \theta_{bip} \le \mu_b^i && \forall i \in I, p \in I, b \in B && (59)\\
& \theta_{bip} \le \mu_b^p && \forall i \in I, p \in I, b \in B && (60)\\
& \theta_{bip} \ge \mu_b^i + \mu_b^p - 1 && \forall i \in I, p \in I, b \in B && (61)\\
& (3), (5) - (10), (12) - (16), (41) - (42).
\end{aligned}
$$

The objective minimises the total driving time and adds a transfer trip for the buses that service 2 pick-up points. Constraints (57) limit the service of each bus to two pick-up points. Constraints (58) ensure that each bus services at least one pick-up point. These constraints have no effect on the objective value but aid in solving efficiency. Constraints (59), (60) and (61) set the values for the auxiliary variables.

This formulation assumes that buses return to their first pick-up point before going to the second pick-up point. Because of the triangle inequality, it can be beneficial to drive directly to the new pick-up point after the last shelter visit. Therefore, after Algorithm 1 is performed using the SR-MP-RTPL, a post-processing algorithm is performed that calculates the maximum driving time after correcting for the triangle inequality. The algorithm uses a second correction algorithm on each bus that finds the best order in which to service the pick-up points and which shelter should be visited last before travelling to the new pick-up point. This is done by checking every combination of shelter and pick-up point to determine the shortest travel time. The post-processing algorithm is described by Algorithm 2 and the second algorithm is described by Algorithm 3.

Algorithm 2 Travel Time Post-processing

Initialise maxTravelTime = 0
for $b \in B$ **do**
  tt ← Travel time based based on solution given by Algorithm 1
  if Bus b services two pick-up points **then**
    cor ← *Bus Correction(b)*
    travelTime ← tt - cor
  else
    travelTime ← tt
  end if
  if travelTime > maxTravelTime **then**
    maxTravelTime = travelTime
  end if
end for
return maxTravelTime

Algorithm 3 Bus Correction

Initialise maxCorrection = 0
for Shelters j to which round trips are performed from the first pick-up point **do**
  timeSave ← Time saved from travelling directly from the shelter to the second pick-up point
  if timeSave > maxCorrection **then**
    maxCorrection = timeSave
  end if
end for
for Shelters j to which round trips are performed from the second pick-up point **do**
  timeSave ← Time saved from travelling directly from the shelter to the first pick-up point
  if timeSave > maxCorrection **then**
    maxCorrection = timeSave
  end if
end for
return maxCorrection

6 Results

This section presents the results of the application of the proposed models on an example network. The tests are conducted on an AMD Ryzen 7 7735HS CPU at 3.20 GHz having 16 GB of RAM using a Windows 11 operating system and all MIPs are solved using Gurobi v11.0. The models are tested on the Sioux Falls network, as is done by Kulshrestha et al. (2014). However, Kulshrestha et al. (2014) have not been entirely clear on the network parameters they use. They mention that the travel times for the buses have been adjusted to account for delays caused by congestion during the evacuation, but they do not specify how they have been adjusted. We can therefore not replicate the exact results that have been obtained by Kulshrestha et al. (2014) and compare our results. For the sake of this experiment, we will multiply the bus travel times in the network by a factor of 3, which we believe is a reasonable delay in times of evacuation (Naghawi & Wolshon, 2012). The same issue arises for the initialisation of $\bar{D} = \{d_0\}$. For all tests, we will initialise d_0 to be the nominal scenario. All other parameters used in this numerical example are taken from Kulshrestha et al. (2014).

In order to further evaluate the solutions found, they are assessed against a set of simulated demand scenarios. In the simulated scenarios, the values for each demand point are chosen with equal probability for all possible values. For each solution, 10,000 scenarios are generated. This results in a simulation score (Sim) that depicts the percentage of simulated scenarios for which the solution was feasible.

We will first present the results obtained after replicating the numerical example presented by Kulshrestha et al. (2014) and compare our results to theirs. Then, we will compare the results of the original model to the results obtained with the alternative objective. Finally, we will evaluate the effectiveness of relieving the single pick-up allocation constraint by looking at the results obtained from the simplified MP-RTPL.

6.1 Single Pick-up Allocation

Table 1 shows the results obtained after solving the RTPL. Our objective values (Obj) differ slightly from the results found by Kulshrestha et al. (2014), which was expected because of the ambiguity in their travel times. After comparing the solutions, we also observe a difference in our optimal solutions compared to those found by Kulshrestha et al. (2014). This suggests that the relative distances in their network are different from ours, meaning that they did not multiply the distance of each arc with the same value.

Despite these differences, we observe similar patterns in our results, namely that the simulation score increases greatly with a small increase in the degree of pessimism. The increase in objective value is also very similar, doubling at $\Gamma = 3$ with respect to the nominal case. However, we notice that our simulation scores are lower than those of Kulshrestha et al. (2014), with the biggest difference at $\Gamma = 2$ with 26.84% and 34.88% respectively. Although there is randomness involved in the simulation, the fact that our scores are consistently lower means that their construction of the network parameters leads to a more robust solution.

Additionally, the trend in running times appears to be similar, despite Kulshrestha et al. (2014) only displaying the running times of three degrees of pessimism. The running time of solving for $\Gamma = 3$ is larger than for the nominal and worst-case scenarios. This can be explained by the fact that it takes more iterations of the cutting plane algorithm to solve for $\Gamma = 3$ than for the nominal and worst-case scenarios (10, 1 and 2 respectively for our solutions). The lower running times for our solution are most likely due to a faster computer and more efficient solver being used compared to those used by Kulshrestha et al. (2014).

Table 1: Single pick-up point allocation results

	Our results			Kulshrestha et al. (2014)		
Γ	Obj(min)	Sim(%)	CPU(s)	Obj(min)	Sim(%)	CPU(s)
0 (nominal)	654.0	1.58	0.24	612.0	2.13	1.56
1	924.0	26.84	2.17	782.0	34.88	-
2	1086.0	71.54	5.02	965.6	76.42	-
3	1224.0	90.16	4.18	1128.8	97.94	8.76
4	1338.0	98.71	8.45	1181.0	99.59	-
5	1404.0	99.86	10.1	1227.4	100.00	-
6	1410.0	100.00	4.70	1227.4	100.00	-
7	1410.0	100.00	3.01	1227.4	100.00	-
8	1410.0	100.00	3.83	1234.2	100.00	-
15 (worst-case)	1410.0	100.00	1.68	1234.2	100.00	2.40

6.2 Alternative Objective

Table 2 shows the result of solving the RTPL with two different objective functions, one that minimises the total driving time and one that minimises the maximum driving time across all buses. In addition to the objective and simulation score, the value for the opposite objective obtained from the solution is presented. The first thing we notice is that when minimising the total driving time, the maximum driving time in the optimal solution (Max) is 180 minutes for each degree of pessimism. This implies that constraint (13), which limits the total driving time of each bus, is binding in each solution.

If we compare the results of the two objectives, we see that the solutions produced with the alternative solutions have lower maximum driving times. The difference is 10% for the worst-case scenario and increases to 50% for the nominal scenario. The total driving times, on the other hand, do not have such a large increase when using the alternative objective. The increase is at most 6%, for $\Gamma = 6$, and is even zero for $\Gamma = 2$. The total driving times fluctuate when using the alternative objective, which most likely means that there are multiple solutions with the same optimal objective value. Finally, we see that the simulation scores are equal or better when using the alternative objective. This indicates that using the alternative objective leads to a quicker and more robust evacuation plan at a small cost of total driving time.

The reason for this improvement is most likely because of a better spread of the workload over the buses. The summary statistics of the bus driving times are presented in Table 3. As expected, we see that when using the original objective there are solutions in which some buses drive the maximum time allowed whilst others do not drive at all. To compare, the difference between the minimum and maximum driving time at $\Gamma = 2$ with the alternative objective is 54 minutes, whereas the use of the original objective leads to a difference of 180 minutes. Table 3 shows that the driving times are much more evenly spread when using the alternative objective.

Table 2: Normal and alternative objective function

	Original objective			Alternative objective		
Γ	Obj(min)	Sim(%)	Max(min)	Obj(min)	Sim(%)	Total(min)
0	654.0	1.58	180	90	2.22	678.0
1	924.0	26.84	180	120	36.60	954.0
2	1086.0	71.54	180	126	71.54	1086.0
3	1224.0	90.16	180	144	90.16	1224.0
4	1338.0	98.71	180	150	99.49	1386.0
5	1404.0	99.86	180	156	99.86	1422.0
6	1410.0	100.00	180	162	100.00	1494.0
7	1410.0	100.00	180	162	100.00	1476.0
8	1410.0	100.00	180	162	100.00	1458.0
15	1410.0	100.00	180	162	100.00	1458.0

Table 3: Bus driving time summary statistics

	Original objective				Alternative objective			
Γ	Mean	Min	Median	Max	Mean	Min	Median	Max
0	65.4	0	36	180	67.8	36	72	90
1	92.4	0	84	180	95.4	42	102	120
2	108.6	0	114	180	108.6	72	108	126
3	112.4	48	123	180	122.4	84	123	144
4	133.8	60	156	180	138.6	108	144	150
5	140.4	60	156	180	142.2	126	144	156
6	141.0	60	156	180	149.4	108	156	162
7	141.0	60	159	180	147.6	96	156	162
8	141.0	60	144	180	145.8	114	147	162
15	141.0	66	159	180	145.8	120	144	162

6.3 Multiple Pick-up Allocation

The S-MP-RTPL with the alternative objective is solved to assess the potential of the MP-RTPL. A three-hour time limit was set on the cutting plane algorithm. The results obtained after performing the post-processing algorithm are presented in Table 4, as well as the results of the single pick-up with the alternative objective for comparison. We see that relaxing the pick-up allocation constraint in most cases leads to better objective values, with an improvement of 4.2% for $\Gamma = 3$ and 7.4% for the worst-case scenario. We also see that the objectives are never worse than those of the alternative objective model, which is to be expected as these solutions are feasible for the multiple pick-up model.

The table does, however, show that the improvement in the objective value comes at a large cost in solving time. For some degrees of freedom, the model could not be solved within the two-hour time limit. The solving times also do not exhibit the same trend as the model with the alternative objective. The reason for this is not entirely clear, as the number of cutting plane iterations is the same or very similar for both models. The time needed to solve each iteration does vary a lot in the multiple pick-up model, ranging from 1-90 minutes. This could mean that each specific combination of constraints could make the model more difficult to solve for the commercial solver.

Table 4: Results of solving S-MP-RTPL with alternative objective

	Alternative Objective			Simplified multiple pick-ups		
Γ	Objective	CPU(s)	Sim(%)	Objective	CPU(min)	Sim(%)
0	90	4.01	1.58	72	1.6	2.22
1	120	37.16	36.60	108	55.9	34.17
2	126	54.01	71.54	126	174.4	71.54
3	144	67.47	90.16	138	77.7	92.35
4	150	73.74	99.49	-	-	-
5	156	111.43	99.86	-	-	-
6	162	110.47	100.0	150	125.6	100.0
7	162	104.92	100.0	150	60.0	100.0
8	162	61.90	100.0	-	-	-
15	162	9.23	100.0	150	2.03	100.0

7 Conclusion

In this paper, we have addressed the problem of planning for a transit-based evacuation under demand uncertainty. We have replicated and extended upon the model proposed by Kulshrestha et al. (2014) that selects designated pick-up points and plans bus trips to transport the evacuees to shelters. The demand uncertainty is modelled in the form of possible demand scenarios. A cutting plane algorithm is used to solve the model for the different degrees of pessimism in the uncertainty. The first extension this paper proposes on the model is an alternative objective function. The original model used by Kulshrestha et al. (2014) minimises the total driving time, but we propose to minimise the maximum driving time across buses. The use of the alternative objective will lead to a solution where the work is more evenly spread between buses, resulting in a quicker evacuation. The second extension provided by this paper is the relaxing of the constraints that limit the service of each bus to a single pick-up point. This allows for more flexibility in the routing of the buses and a more efficient use of the available bus resources.

Firstly, our results of the replication are very similar to those obtained by Kulshrestha et al. (2014). We observe the same trend where a small increase in the degree of pessimism leads to a large increase in the robustness of the solution. The increased robustness does come at the cost of total driving time. The replication also illustrates the importance of the construction of the network. With approximately the same network scale, the solutions obtained by Kulshrestha et al. (2014) perform better in the simulations. This means that exact scaling Kulshrestha et al. (2014) have performed on the network has a significant impact on the solution.

Next, our results show that using the alternative objective that minimises the maximum driving time across buses leads to quicker evacuations. The solutions obtained using the alternative objective have total driving times that are similar to the original objective and have almost identical performance in the simulation. The better spread of the bus driving times has the additional benefit that the solution is more robust against bus and road failures. This leads us to conclude that in the time of evacuation, the use of the alternative objective is preferred over the one proposed by Kulshrestha et al. (2014).

Finally, the results of solving the S-MP-RTPL show that relaxing the constraints that limit the buses to a single pick-up point leads to quicker evacuation plans. This illustrates the potential of a model where buses are allowed to service multiple pick-up points. Our suspicions, however, are confirmed that such an extended model is too large to solve exactly. A simplified version of the extended model already has an exponential increase in solving time, so for the complete model, a different method is needed to obtain good solutions.

There are several promising directions for future research based on the findings of this thesis. One area worth exploring is the development of more efficient algorithms to handle the increased complexity of the extended model. Techniques such as heuristic or metaheuristic approaches, including genetic algorithms or simulated annealing, could be investigated to provide good quality solutions within a reasonable timeframe. Additionally, future work could focus on integrating real-time data and dynamic updates into the evacuation planning model, allowing for more responsive and adaptive evacuation strategies.

Lastly, future research could investigate the socio-economic impacts of evacuation plans and the incorporation of equity considerations. Ensuring that evacuation plans are not only efficient but also equitable, providing fair access to transportation resources for all residents, is crucial. Developing models that account for the varying needs and vulnerabilities of different population groups can lead to more inclusive and effective evacuation strategies.

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A Code Explanation

All code is written in Java and is included in the .zip file provided. The zip file contains the following classes.

Main.java

This is the main class that is run to obtain the results. It contains the parameters needed for the results that are not included in the Sioux Falls network. It calls the functions that import the Sioux Falls network, creates the necessary models and calls the functions that solve the models. If the user runs the class, it will get instructions on how to obtain the results they desire. The options presented are "single/multiple pick-up allocation" and for the single pick-up "original/alternative objective".

CSVReader

This class is used to import the Sioux Falls network from the .csv files obtained from the GitHub page. The class produces a list of nodes and a matrix of distances.

ShortestPath.java

This class contains the method to find the shortest distance between two nodes in the network. Dijkstra's algorithm is used to find these shortest distances.

FixedBusModel.java

This class contains the code for the RTPL model with original and alternative objective function and the code to solve this model with the cutting plane algorithm. Creating an instance of the class sets all parameters and running the *solve()* function will run the cutting plane algorithm.

FlexibleBusTestModel.java

This class contains the code for the S-MP-RTPL model and the cutting plane algorithm to solve the model. It is initiated and solved in the same manner as the FixedBusodel.java.

Pair.java

This class represents the concept of a pick-up - shelter pair.

FixedBusModel.java

This class contains the code for the MP-RTPL, but as the model is too large to be solved the cutting plane algorithm is not implemented. The model can be run for a single iteration of the R-MP-RTPL and is left in the .zip file to be available for future reference to the model.