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Exploring Over-Diversification in ETF Portfolios

Author: J.S. Luijendijk
Student number: 584040
Thesis supervisor: P. Messow
Second reader: G. Cocco
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ABSTRACT

This thesis studies the notion of over-diversification and whether ETFs are over-diversified. This was done by creating portfolios of different sizes made up of companies in the Russell 3000 index. These portfolios were then analyzed in multiple different ways. This paper finds that diversification beyond a certain point does not negatively influence portfolio performance, but diversification benefits diminish after this point.

Keywords: ETF; Portfolio; Diversification

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CHAPTER 1 Introduction

In recent decades, the landscape of investing has been constantly changing. One of the most notable changes is the rise of the popularity of Exchange Trades Funds (EFTs). First created in 1990, ETFs form a way to obtain a diversified portfolio quickly. Regularly made up of more than 500 different stocks, ETFs are very diversified. It builds on the Modern Portfolio Theory, which was constructed by Markowitz (1952) and laid the foundation for investing in the upcoming decades. Since then, diversification has been hailed as a cornerstone strategy for risk-averse investors seeking to construct portfolios that either optimize returns within acceptable risk parameters or minimize risk for a specific level of return. Since Markowitz created this theory, diversification has been seen as a predominantly positive contribution to any portfolio. This thesis seeks to critically examine the notion of over-diversification, questioning whether there exists a point at which the benefits of diversification begin to diminish or even reverse, potentially leading to suboptimal portfolio outcomes paired with suboptimal ETF performances.

While previous literature on over-diversification is relatively limited, Goetzmann and Kumer (2008) have made significant strides in investigating diversification in general. Their study, among other findings, suggests that some individual investors under-diversify due to superior information, hinting at the possibility of over-diversification in the presence of such information. This research, therefore, aims to add a unique perspective to the existing body of knowledge by focusing specifically on the concept of over-diversification within ETF portfolios. Zaimović et al. (2021) performed literature research on diversification. They found that evaluating the number of assets that lead to optimal diversification is complicated as it is impacted by several different factors. The factors stated include but are not limited to the investors' characteristics and the model used to measure diversification. Statman (1987) finds that a portfolio is well-diversified when it contains more than 30 stocks. Benjelloun (2010) expanded on research by Evans and Archer (1968) and found that 40-50 stocks are sufficient to obtain a well-diversified portfolio.

The study of over-diversification within ETF portfolios presents a timely and crucial avenue for research in modern portfolio management. As the popularity of ETFs continues to soar, understanding their role in portfolio diversification is essential for investors and financial professionals alike. By examining the potential existence and implications of over-diversification in ETF-based portfolios, this research addresses the following question:

Does over-diversification exist, and how does it impact the portfolio performance of ETFs?

Answering this question will not only enhance one's understanding of the dynamics of portfolio diversification but also provide valuable insights into the evolving role of ETFs in shaping investment decisions and outcomes. To the best of my knowledge, this topic has not been specifically studied thus far.

This study will employ a quantitative approach to investigate the research question regarding the possible existence of over-diversification within ETF portfolios. The main objective of this study is to find and explain possible over-diversification of ETFs and see if these ETFs can be optimized to create a more efficient portfolio. The primary data sources will include historical ETF holdings and market performance data from financial databases. Specifically, the ETF that will be used for this research is the Vanguard Russell 3000 ETF. This ETF consists of all 3000 assets in the Russell 3000 index, representing 98% of the American market. Composed of 3000 assets, this ETF might be exposed to over-diversification and is thus a viable ETF for this research. Multiple different portfolios will be constructed, all with varying profiles of diversification. The portfolios will vary in amount of assets, the smallest being only five different assets, the largest containing all 3000 assets.

For each portfolio, different performance and risk metrics will be calculated. Then, multiple regressions will be made to examine the effect of portfolio size on the performance and risk metrics. Then, regression analysis will be conducted to investigate the impact of portfolio size on the performance and risk metrics. This research will examine the effect of diversification levels on risk-adjusted returns and volatility. Suppose such a point of optimal diversification is found. In that case, this research will determine whether a portfolio based on the Vanguard ETF but made up of fewer stocks is more efficient than the complete Vanguard ETF portfolio. By combining quantitative analysis findings with previous literature insights, this study aims to provide a comprehensive understanding of the potential implications of over-diversification in ETF investing, offering recommendations for investors.

I anticipate finding nuanced insights into the relationship between ETF diversification and portfolio performance and the potential existence of over-diversification within ETF portfolios. Literature is somewhat divided; some research claims to have found a specific number of stocks that result in a well-diversified portfolio. Others state that a particular number cannot be found since too many factors contribute to diversification. I hypothesize that ETFs are, however, over-diversified. That is, there exists a point of diversification at which more assets do not further lower the risk of a portfolio and only lower returns. I expect this amount to be smaller than 500. Most ETFs are almost tenfold (500+) larger than the optimum number of stocks stated in the literature (±40). I hope to find a range of stocks that optimize the risk-return trade-off by optimizing diversification.

This paper finds that over-diversification doesn't necessarily exist. There are no signs of portfolio performance decreasing after a certain point of diversification. This paper does, however, find statistical evidence that the benefits of diversification are very minimal beyond a certain point, which can be named as over-diversification. This point is most likely between 100 and 300 stocks. Since most ETFs are made up of more than 300 stocks, we can say that they are over-diversified. Similar results can be realized by randomly selecting a certain number of stocks from that ETF. This specific number has not been found in this paper.

The remainder of this thesis is structured as follows. Chapter 2 discusses relevant literature and provides a theoretical background to help you fully understand this thesis. Chapter 3 introduces the dataset used for this research. Chapter 4 discusses how the empirical research in this paper is conducted. Chapter 5 shows the results of this research and discusses these results. In chapter 6, this thesis is concluded, and the research question is answered.

CHAPTER 2 Theoretical Framework

2.1 Diversification: Background

Diversification is essential to most trading strategies and aims to spread risk across various assets to mitigate unsystematic risk. Unsystematic risk is risk specific to a company or industry that can be diversified away. Systematic risk is inherent to the market; this type of risk cannot be overcome via diversification. While diversification will decrease risk, over-diversification can dilute high-performing assets, thus resulting in sub-optimal portfolio performance (Benjelloun, 2010; Statman, 2004). According to the Modern Portfolio Theory (MPT), constructed by Harry Markowitz, an optimal portfolio balances return and risk through sufficient diversification (Markowitz, 1952). MPT suggests but does not imply, that adding assets beyond a certain point no longer reduces risk but only negatively affects portfolio performance.

Correlation plays an important role in diversification. Proper diversification requires selecting assets with low or negative correlation to reduce risk. When assets with a high positive correlation are in a portfolio, the risk will not be reduced to the same extent. Over-diversification often introduces assets with high correlation, which thus diminishes the gains of diversification (Lintner, 1965; Elton & Gruber, 1977). The idea of diminishing returns in portfolio diversification states that the marginal benefit of adding more stocks decreases as more stocks are added. Leading to increased costs and management complexity without adequate risk reduction (Evans & Archer, 1968).

2.2 Performance Measurements

2.2.1 Sharpe Ratio

Performance measurements are vital to any portfolio analysis. Portfolio performance can be measured using several metrics, the Sharpe ratio being the most well-known. Introduced by economist W. F. Sharpe in 1966 as the reward-to-variability ratio, the Sharpe ratio compares an investment's return to its risk (Sharpe, 1966). When comparing different portfolios, those with higher Sharpe ratios are believed to perform better than those with lower Sharpe ratios. The Sharpe Ratio of a portfolio can be calculated as follows:

$$Sharpe\ Ratio = \frac{R_P - R_F}{\sigma_P}$$

Rp is the portfolio's return, Rf is the risk-free return, and σ_p is the portfolio's standard deviation. An annual risk-free rate of 0,87% will be used during this research. The is calculated by averaging the risk-free rates of each year[1] from 2010 until 2024. Although the Sharpe Ratio is one of the most popular methods of measuring portfolio performance, it does have some flaws. The ratio can be manipulated relatively easily by lengthening the return measurement interval, which results in lower estimates for the standard deviation and a higher Sharpe Ratio.

Another problem is that the Sharpe Ratio is calculated assuming the portfolio returns are normally distributed. Therefore, this ratio does not differentiate between the upside and downside risk. In reality, however, rational investors care more about the downside risk than the upside potential (Marhfor, 2016). To combat this problem, the Sortino Ratio can be used.

[1] Risk-free rates according to: https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

2.2.1 Sortino Ratio

The Sortino Ratio, initialized by Sortino and Price (1994), is an adjusted ratio based on the Sharpe Ratio. It measures risk using downside deviation rather than standard deviation. The Sortino Ratio compares each return with a so-called target return. It only includes the returns that fall below the target return when calculating the downside deviation. The Sortino Ratio can be calculated as follows.

$$\textit{Sortino Ratio} = \frac{(R_P - R_f)}{TDD}$$

$$TDD = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(Min(0, X_i - R_T))^2}$$

RP is the portfolio's return, R_f is the risk-free rate, R_T is the target return, and TDD is the target downside deviation. TDD is the standard deviation of all returns that fall below the target return (Rollinger et al., 1980). Once again, a risk-free rate of 0,87% will be used. This paper will use a target return of 0%, so only negative returns are considered when calculating the TDD. The TDD will thus be calculated as follows.

$$TDD = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(Min(0, X_i))^2}$$

2.3 Previous studies on over-diversification

Empirical analysis of diversification starts with the works of Evans and Archer (1968). This paper investigates the effects of diversification on reducing the dispersion of portfolio returns. It shows that as portfolio size grows, the standard deviation decreases significantly. This effect is most apparent for portfolio sizes between one and twenty stocks. Beyond this point, the incremental effect diminishes. Evans and Archer conclude that approximately ten stocks are enough to make a diversified portfolio. The paper provides empirical evidence on the effect of diversification on portfolio performance.

Revisiting the works of Evans and Archer (1968), Bellejoun (2010) studies whether their findings still hold up forty years later. The study confirms that the principle of diminishing returns in diversification still holds. It also finds that forty to fifty stocks are all needed to achieve adequate portfolio diversification. In conclusion, Bellejoun states that Evans and Archer (1968) still deserve recognition as they provided a sound methodology for portfolio diversification.

Statman (2004) finds that, according to mean-variance portfolio theory, the optimal size of a portfolio exceeds 300 stocks. Statman notes that the average investor typically only holds a few stocks and is, thus, severely under-diversified. This notion is agreed upon by Goetzmann and Kumar (2008). To explain this significant difference between theory and practice, Statman employs Behavioural Portfolio Theory (BPT). BPT suggests that investors construct their portfolios as a pyramid. The lower layers comprise low-risk bonds for protection from poverty, mutual funds in the middle layer, and individual stocks and lottery tickets in the top layer for great upside potential. This study shows that investors' behavior is driven by more than just risk aversion; the hope for large profits plays a significant role in portfolio construction.

Newbould and Poon (1996) show that the minimum number of stocks needed to achieve diversification is much higher when market-weighted portfolios are used instead of equally weighted portfolios. In portfolios that use portfolio market-weighting, stocks with higher market capitalization make up a larger fraction of the portfolio. Newbould and Poon do not provide an exact number of stocks that reaches diversification with market weights.

Considering the referenced literature, the following hypotheses are created.

H_1: Portfolio performance will decrease after a certain threshold of portfolio size has been reached.

H_{1a}: Portfolio performance will not decrease after a certain threshold of portfolio size has been reached

H_2: Portfolios with more than a certain number of stocks will show a higher correlation among their stocks, thus reducing the effects of diversification.

H_{2a}: Portfolios with more than a certain number of stocks will not show a higher correlation among their stocks, thus not affecting the effects of diversification.

CHAPTER 3 Data

To execute this research, data was gathered from all companies currently in the Russell 3000, specifically adjusted closing stock prices on the last trading day of each month. Adjusted closing stock prices differ from regular closing stock prices in that the prices are adjusted for stock splits and dividend payout. Stock splits occur when a company increases its number of outstanding shares, but the market capitalization of the company stays the same. This improves the stocks' liquidity, and the stock price will decrease proportionally. When, for example, a company introduces a 2-to-1 split, every shareholder receives two shares for each share they own, and the stock price will halve. Dividend payouts also influence stock prices. Historical stock prices are downward adjusted by the amount of dividends paid. Adjustments for dividend payouts and stock splits are crucial to comparing historical stock prices over time. Adjusted prices give a clearer understanding of the stock's performance.

The data spans the period from January 2010 until April 2024. The dataset has been sourced from an online financial dataset provider, *www.barchart.com*. This dataset provides daily updated stock prices from all companies currently listed in the Russell 3000. The reliability of the data has been ensured by manually cross-checking several data entries. This dataset does, however, come with some limitations. The main limitation is that companies only currently present in the Russell 3000 are included in the data. Since companies are frequently added and removed from the Russell 3000, some companies in our dataset were not in the Russell 3000 for a portion of our sample period. This leads to a survivorship bias, which will be explained later in this thesis.

The variables taken from the dataset to use in the research were the following: *Mtdate* specifies the date in '*YYYYmM*' format (2024m4 denotes April 2024). *Symbol* gives the ticker of the company. *Adjclose is* the adjusted closing price of the stock in question. Next, the total return is calculated for each company using the following equation:

Total return = (final price – initial price) / initial price

The variable *final price* denotes the most recent available stock price, and the *initial price* is the first available stock price.

Furthermore, a variable called *years* will be introduced that stores the number of years a stock is represented in the Russell 3000. Using this variable, the annualized return of each company will be calculated using the following equation:

$$Annualized\ return = ((1 + Total\ return)\ ^\wedge (1 / \ Years)) - 1$$

Table 3.1 displays the summary statistics of the variables used.

Table 3.1 Summary Statistics on the companies and their adjusted closing prices.

Variable	Obs.	Mean	Std. dev.	Min.	Max.
Company	358,003	1330.46	765.60	1.00	2659
Adjclose	358,003	60.38	580.44	0.00	127860

Notes: Numbers are rounded to two decimal places. The variable 'company' assigns an index to each company. Therefore, the mean and standard deviation of the variable 'company' have no meaningful interpretation. Data is collected from January 2010 until April 2024. Obs. = Observations; Std. dev. = Standard deviation; Adjclose = Adjusted Closing price.

The summary statistics provide an overview of the dataset, including the number of observations, mean, standard deviation, minimum and maximum values for each variable. The dataset is extensive, with over 358 thousand observations for the adjusted closing price.

The main shortcoming of this dataset is the apparent survivorship bias. Survivorship bias is the bias of looking only at data entries that passed a particular real-life selection process. This type of selection bias can lead to overly optimistic results on data because multiple data entries are not present in the dataset. This dataset only includes stocks currently listed in the Russell 3000. Since the Russell 3000 regularly changes its constituents, several companies were removed from the dataset from 2010 until 2024. Companies can be removed from the Russell 3000 due to multiple reasons. The most usual reasons for removal are mergers and acquisitions and market capitalization changes. When two companies merge or one company acquires another, a stock will disappear from the Russell 3000. This can have substantial effects on the index's price, depending on the size and market perception of the companies involved. If a company is removed due to a change in market capitalization, it is reasonable to assume the removal of these companies negatively influences the index's return. Declining market capitalization is typically a result of poor performance relative to other companies in the market. Not including these entries in our dataset will give an overly optimistic view of calculated returns.

The Russell 3000 index has been specifically chosen to minimize the effects of survivorship bias. When using the S&P 500 index, this bias significantly affected the results of this research. Due to the larger size of the Russell 3000 index, this effect will still be apparent but smaller in size.

CHAPTER 4 Method

To continue the research, multiple portfolios of different sizes will be constructed by randomly choosing stocks from the dataset. Portfolio sizes range from 10 to 2700, with increments of 10 from 5 to 100, 50 from 100 to 1000, and 100 from 1000 to 2700. The increments for portfolio sizes had been found based on the following rationale. Small increments are used for the smaller portfolio sizes (10 to 100) to increase the level of detail of the results. When portfolio sizes get larger (100 to 500), increments of 50 were selected. As the number of stocks increases, the marginal benefit of adding each additional stock decreases. The small increments at smaller portfolio sizes provide insights into the initial benefits of diversification, while the broader increments at larger portfolio sizes help in understanding the diminishing returns of diversification. Thus, larger increments can capture possible trends while maintaining computing efficiency.

For each portfolio size, 2000 portfolios are created by randomly selecting stocks from the dataset. By creating large amounts of portfolios for each portfolio size, we can ensure that the results are not driven by any outliers in the dataset but truly capture the desired effect. This large number of portfolios provides a more reliable estimate of the average returns and risks associated with each portfolio size. By doing this, 90000 data entries are created, each representing a unique portfolio.

Then, a few risk and performance measurements will be calculated for each portfolio. Specifically, total return, annualized return, standard deviation, Sharpe Ratio, and the Sortino Ratio will be calculated for each portfolio. Annualized returns will be calculated to remove some of the effects of the survivorship bias. The effect of this bias will accumulate over time and will, therefore, have a more significant impact on the total return than on the annualized return.

To analyze this data, we will employ several analytical methods. First, the effect of portfolio size on annualized returns will be graphically displayed. If the earlier stated first hypothesis is correct, there will be signs of annualized returns decreasing as a certain portfolio size threshold is passed. Multiple possible breakpoints will be selected based on the graphical analysis in this research and previous research. These breakpoints are potential points of optimal diversification and allow for detailed statistical analysis.

Secondly, multiple linear regressions will be performed to further analyze the effects of portfolio size on return and risk measurements. Three different Ordinary Least Squared (OLS) regressions will be fitted on the data to quantify the influence of portfolio size on performance and risk. The dependent variables will be total return, annualized return, Sharpe Ratio, and Sortino Ratio, with portfolio size as the independent ratio. Each regression will be performed twice per breakpoint, once for all the data below and once for all the data above the breakpoint. The regressions will be fitted as follows.

Regression 1:

$$Annualized\_Return_i = \beta_0 + \beta_1 * Portfolio\_Size_i + \varepsilon_i$$

Regression 2:

$$Sharpe\_Ratio_i = \beta_0 + \beta_1 * Portfolio\_Size_i + \varepsilon_i$$

Regression 3:

$$Sortino\_Ratio_i = \beta_0 + \beta_1 * Portfolio\_Size_i + \varepsilon_i$$

In the above equations, the subscript *i* denotes each portfolio in our dataset.

A correlation analysis will be conducted to test the second hypothesis that larger portfolios have a higher correlation among constituents, thus reducing the effects of diversification. The mean correlation will be calculated for every portfolio. This average correlation will provide insights into the relationship between the number of assets and the degree of correlation in the portfolio. To identify the point where over-diversification occurs, the average correlation will be plotted against portfolio size.

CHAPTER 5 Results & Discussion

5.1 Portfolio Characteristics

First, the portfolios need to be constructed to analyze the results. The annualized return, standard deviation, and target downside deviation (TDD) will be calculated for each portfolio. These metrics are then used to calculate each portfolio's Sharpe and Sortino ratios. For more information on calculating these ratios, consult the theoretical framework. Table 5.1.1 shows some summarizing statistics for the performance and risk measurements.

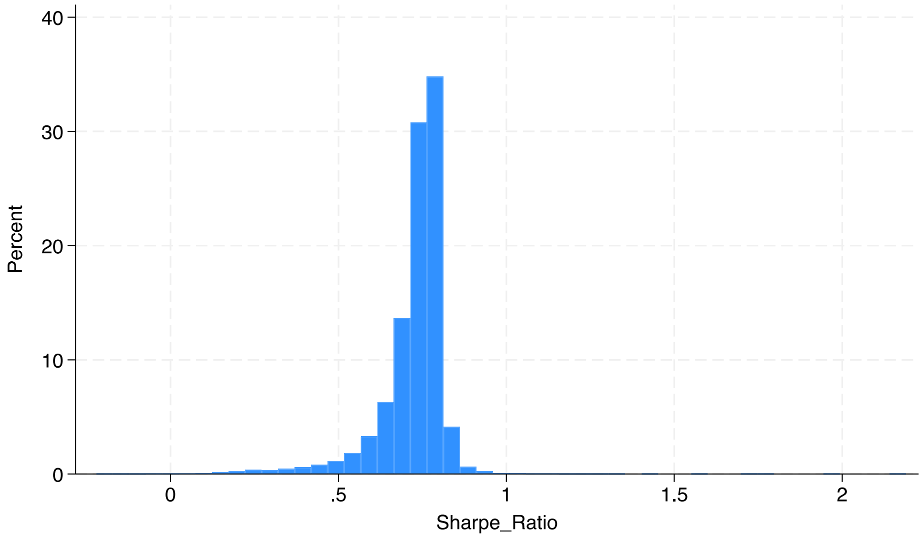
Table 5.1.1 Summary Statistics for the created portfolios.

Variable	Obs.	Mean	Std. dev.	Min.	Max.
Ann. Return	90,000	0.175	0.025	-0.169	2.318
Std. dev.	90,000	0.241	0.217	0.147	20.616
TDD	90,000	0.154	0.009	0.102	0.392
Sharpe ratio	90,000	0.724	0.100	-0.221	2.190
Sortino ratio	90,000	1.079	0.166	-0.396	6.270

Notes: Each observation belongs to an individual portfolio. The first column contains variables with information on each portfolio. Obs. = Observations; Std. dev = Standard deviation; TDD = Target downside deviation.

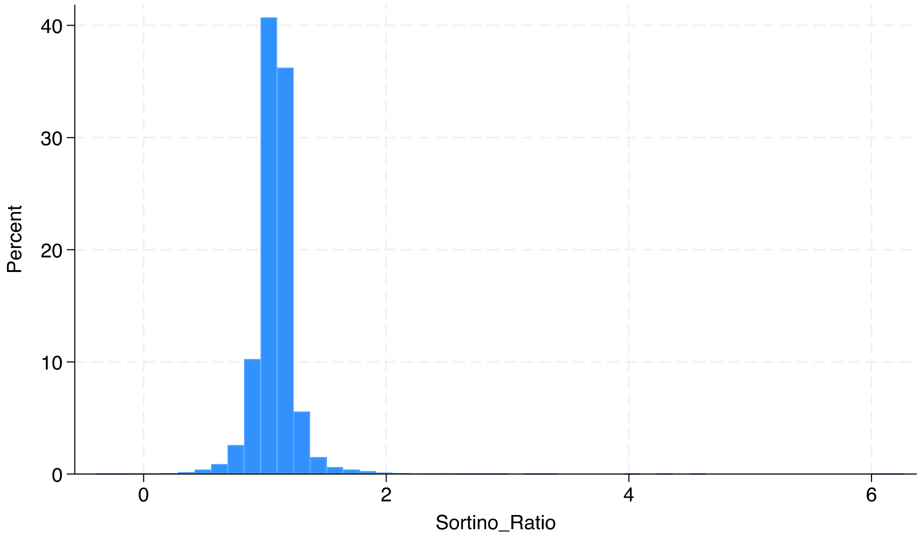
Figure 5.1.1 shows the distribution of the Sharpe ratio among the portfolios. Figure 5.1.2 shows the distribution of the Sortino ratio among the portfolios. Sharpe ratios above one are usually considered adequate, while Sharpe ratios below one are considered poor. In the case of the Sortino ratio, any portfolio with a ratio greater than two is considered a portfolio with adequate risk-adjusted returns.

Figure 5.1.1 Sharpe Ratio Distribution



Notes: This figure depicts the distribution of the Sharpe Ratio across the portfolios. A bar that reaches 40 on the Y-axis means that the corresponding value on the X-axis belongs to 40% of all portfolios.

Figure 5.1.2 Sortino Ratio Distribution

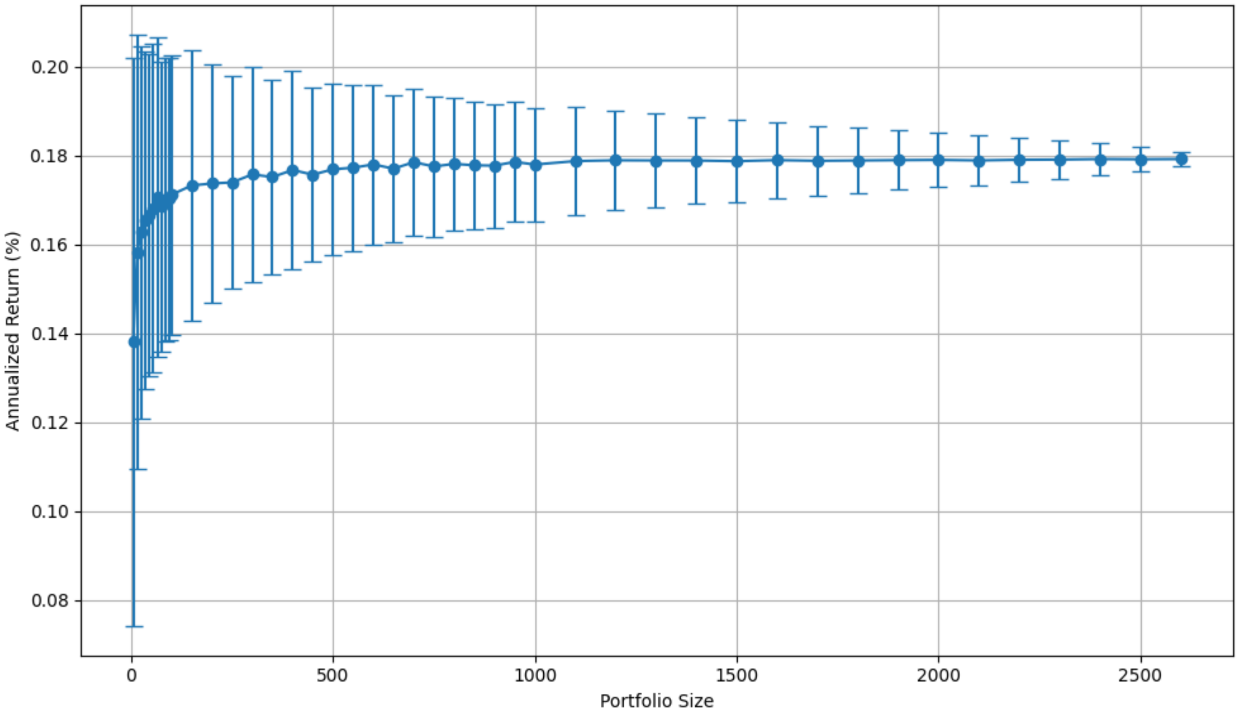


Notes: This figure depicts the distribution of the Sortino Ratio across the portfolios. A bar that reaches 40 on the Y-axis means that the corresponding value on the X-axis belongs to 40% of all portfolios.

5.2 Graphical Analysis

Figure 5.2.1 graphically presents the relationship between portfolio size and the mean and standard deviation of annualized returns per portfolio size. This graph provides several insights. First and foremost, the diminishing effects of diversification are apparent. The function in the graph mimics an asymptotic function, where the y-value approaches a limit as the x-value increases. In this case, as portfolio size increases, the mean annualized returns approach a limit of approximately 18%. Note that the average annual returns of the Vanguard Russell 3000 ETF are approximately 12%. The earlier-mentioned survivorship bias most likely causes this difference.

Figure 5.2.1 Mean and Standard Deviation of Annualized Returns



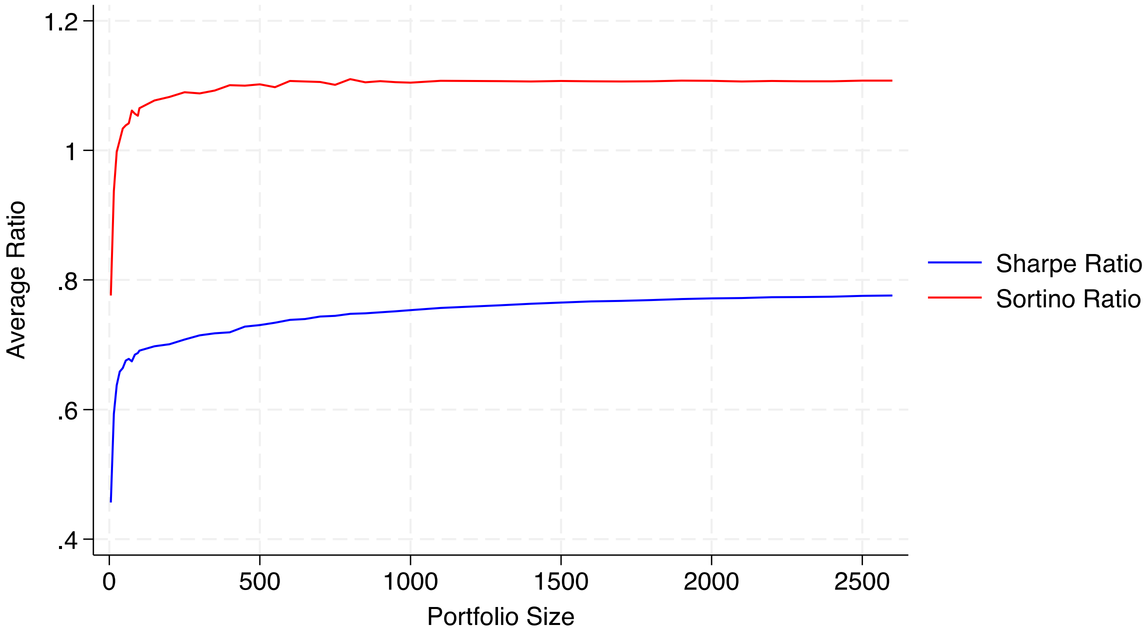
Notes: The data points represent the mean annualized returns for each portfolio size. The error bars give a visual representation of the variability of the annualized returns for each portfolio size.

The stabilization in performance suggests that there is a point beyond which further diversification no longer considerably benefits portfolio performance. The most significant benefits of diversification are present in smaller portfolio sizes. The initial phase of diversification shows a rapid decrease in standard deviation and a considerable increase of mean annualized returns towards the asymptote of 18%. The risk, measured as standard deviation, keeps decreasing as portfolio size increases, but the decrease becomes less pronounced as portfolio size increases.

Even though there is no sign of annualized returns decreasing after a certain point of diversification, one can still argue that over-diversification is present. Over-diversification can be seen in the diminishing decrease in standard deviation and the diminishing increase in annualized returns. It looks like after the point of 500 stocks, the increase in returns is very minimal.

Figure 5.2.2 shows the effect of portfolio size on a portfolio's Sharpe and Sortino ratios. The relationship between portfolio size and these two ratios seems very similar to the relationship between portfolio size and annualized returns in Figure 5.2.1. Once again, as portfolio size increases, the increase in both the Sharpe and Sortino ratios seems to diminish. The Sortino ratio comes close to its limit around a portfolio size of five hundred stocks. In comparison, the Sharpe ratio approaches its limit around a portfolio size between five hundred and one thousand stocks. This again suggests that diversification beyond a certain point loses its benefits.

Figure 5.2.2 The relationship between Portfolio Size and both Sharpe and Sortino Ratio



Notes: Averages Sharpe and Sortino Ratio have been calculated and displayed for each portfolio size.

Two potential breakpoints are chosen based on this graphical analysis and previous research. The first breakpoint is based on Bellejoun's (2010) research. He states that forty to fifty stocks achieve sufficient diversification. Based on Figure 5.2.1, Figure 5.2.2, and Statman's (2004) research, the second breakpoint is at five hundred stocks. This means twelve regressions will be performed: two per breakpoint and two breakpoints per dependent variable. Only two breakpoints were chosen for efficiency reasons. More breakpoints can be used to get more accurate results.

5.3 Statistical Analysis

This section looks to provide an answer to hypothesis 1. Hypothesis 1 was defined as follows:

H_1: Portfolio performance will decrease after a certain threshold of portfolio size has been reached.

H_{1a}: Portfolio performance will not decrease after a certain threshold of portfolio size has been reached

The earlier-mentioned breakpoints of 50 and 500 stocks will be further examined to analyze the optimal point of diversification. As a reminder, the regressions that will be performed are listed below. Every regression will be performed four times, with data selected according to the breakpoints of 50 and 500 stocks. For example, regression 1, which examines the effect of portfolio size on annualized return, will be performed once for portfolio sizes smaller than 50 (1), once for portfolio sizes larger than 50 (2), once for portfolio sizes smaller than 500 (3), and once for portfolio sizes larger than 500 (4). The number in between brackets corresponds to the column number in the regression results. Regression 2 and 3 will be performed similarly.

Regression 1:

$$Annualized\_Return_i = \beta_0 + \beta_1 * Portfolio\_Size_i + \varepsilon_i$$

Regression 2:

$$Sharpe\_Ratio_i = \beta_0 + \beta_1 * Portfolio\_Size_i + \varepsilon_i$$

Regression 3:

$$Sortino\_Ratio_i = \beta_0 + \beta_1 * Portfolio\_Size_i + \varepsilon_i$$

Table 5.3.1 shows the regression results of the regressions with annualized returns as the dependent variable. First, remember that the variable annualized return is used in decimal notation, so an annual return of 2% is displayed as 0.02. Furthermore, the results of an OLS regression can be interpreted as follows: What is the effect of an increase of 1 unit in the independent variable on the dependent variable? If the first regression is taken as an example, it can be concluded that as the portfolio size increases with one stock, the annualized return of that portfolio rises with 6.10e-4 or 0.00061%. Even though that is a minimal number, if extrapolated, that would translate to an increase in annualized returns of 6% if 100 stocks are added to the portfolio.

Table 5.3.1 further shows that the effect of portfolio size on annualized returns decreases as we move from the left side of a breakpoint to the right side. On the left side of the breakpoint of 50 stocks, the effect is 6.10e-4; on the right side of this breakpoint, the effect drops to 3.08e-6. That is a decrease of 99.5%. This significant drop is still apparent when looking at the breakpoint of 500 stocks. On the left side of this breakpoint, the effect is 3.64e-5 and drops to 8.64e-7 on the right side. That is once again a considerable decrease of 97.7%

Table 5.3.1 Regression results of all regression performed with Annualized returns as the dependent variable.

	Dependent Variable: Annualized Returns			
Variable	**(1)**	**(2)**	**(3)**	**(4)**
Portfolio Size	6.10e-4***	3.08e-6***	3.64e-5***	8.64e-7***
	(3.51e-5)	(8.64e-8)	(1.17e-6)	(7.76e-8)
Constant	0.14***	0.17***	0.16***	0.18***
	(1.01e-3)	(1.11e-4)	(2.67e-4)	(1.22e-4)
Observations	10,000	80,000	38,000	52,000

*Notes: Results in columns (1), (2), (3), and (4) correspond to the regressions performed on portfolio sizes smaller than 50, larger than 50, smaller than 500, and larger than 500, respectively. All numbers are rounded to two decimals. The standard errors are displayed in brackets. The asterisks denote significance. * p < 0.1, ** p < 0.05, *** p < 0.01*

Table 5.3.2 shows the effects of portfolio size on the Sharpe ratio. Similar to the effects on annualized return, significant decreases are once again present when moving from the left side of the breakpoint to the right side. At the breakpoint of 50 stocks, the effect drops from 4.08e-3 to 3.65e-5, a drop of 99.11%. When looking at the breakpoint 500 stocks, the effect drops from 2.50e-4 to 1.91e-5, a decrease of 92.36%. It is still a significant drop, but not as large as around the breakpoint of 50 stocks.

Table 5.3.2 Regression results of all regression performed with Sharpe Ratio as the dependent variable.

	Dependent Variable: Sharpe Ratio			
Variable	**(1)**	**(2)**	**(3)**	**(4)**
Portfolio Size	4.08e-3***	3.65e-5***	2.50e-4***	1.91e-5***
	(1.16e-4)	(3.00e-7)	(4.24e-6)	(2.27e-7)
Constant	0.48***	0.70***	0.63***	0.73***
	(3.33e-3)	(3.84e-4)	(9.70e-4)	(3.57e-4)
Observations	10,000	80,000	38,000	52,000

*Notes: Results in columns (1), (2), (3), and (4) correspond to the regressions performed on portfolio sizes smaller than 50, larger than 50, smaller than 500, and larger than 500, respectively. All numbers are rounded to two decimals. The standard errors are displayed in brackets. The asterisks denote significance. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$*

Table 5.3.3 shows the regression results for the regressions, with the dependent variable being the Sortino ratio. Once again, the effect of portfolio size on the Sortino ratio decreases considerably when moving from the left side to the right side of the breakpoint. In the case of the breakpoint of 50 stocks, a drop of 99.72% (from 5.94e-3 to 1.64e-5) occurs when moving from the left to the right side. At the breakpoint of 500 stocks, the effects drop from 2.97e-4 to 1.45e-6, a decrease of 99.51%.

Table 5.3.3 Regression results of all regression performed with Sortino Ratio as the dependent variable.

	Dependent Variable: Sortino Ratio			
Variable	**(1)**	**(2)**	**(3)**	**(4)**
Portfolio Size	5.94e-3***	1.64e-5***	2.97e-4***	1.45e-6***
	(2.18e-4)	(5.69e-7)	(7.68e-6)	(4.96e-7)
Constant	0.80***	1.08***	0.99***	1.10***
	(6.25e-3)	(7.27e-4)	(1.76e-3)	(7.80e-4)
Observations	10,000	80,000	38,000	52,000

*Notes: Results in columns (1), (2), (3), and (4) correspond to the regressions performed on portfolio sizes smaller than 50, larger than 50, smaller than 500, and larger than 500, respectively. All numbers are rounded to two decimals. The standard errors are displayed in brackets. The asterisks denote significance. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$*

These results confirm hypothesis H_1. They show that once a certain threshold of diversification is passed, the positive effects of diversification diminish. The regression results show large drops in the effects of portfolio size on all three risk and performance metrics when a breakpoint has been passed. Comparing the two breakpoints, it is apparent that the drop in effect is larger around the breakpoint of 50 stocks compared to the breakpoint of 500 stocks. This indicates that the optimal point of diversification is more likely to be 50 than 500.

Note that one can never pinpoint the optimal number of stocks that comprise a perfectly diversified portfolio. As can be seen in the graphs of section 5.2, the effects of diversification are continuous. Choosing an absolute border between under and over-diversification is thus impossible. The previously discussed results, however, do give an insight into over-diversification and indicate that a portfolio of 500 stocks does not gain the same benefits from diversification as a portfolio of 50 stocks.

It is also possible that the difference between two other breakpoints is even more significant than the two used in this paper. However, as mentioned earlier, this paper only uses two breakpoints for efficiency reasons. It is also outside the scope of this article to pinpoint the exact point of optimal diversification but only to research the existence of over-diversification.

5.4 Correlation Analysis

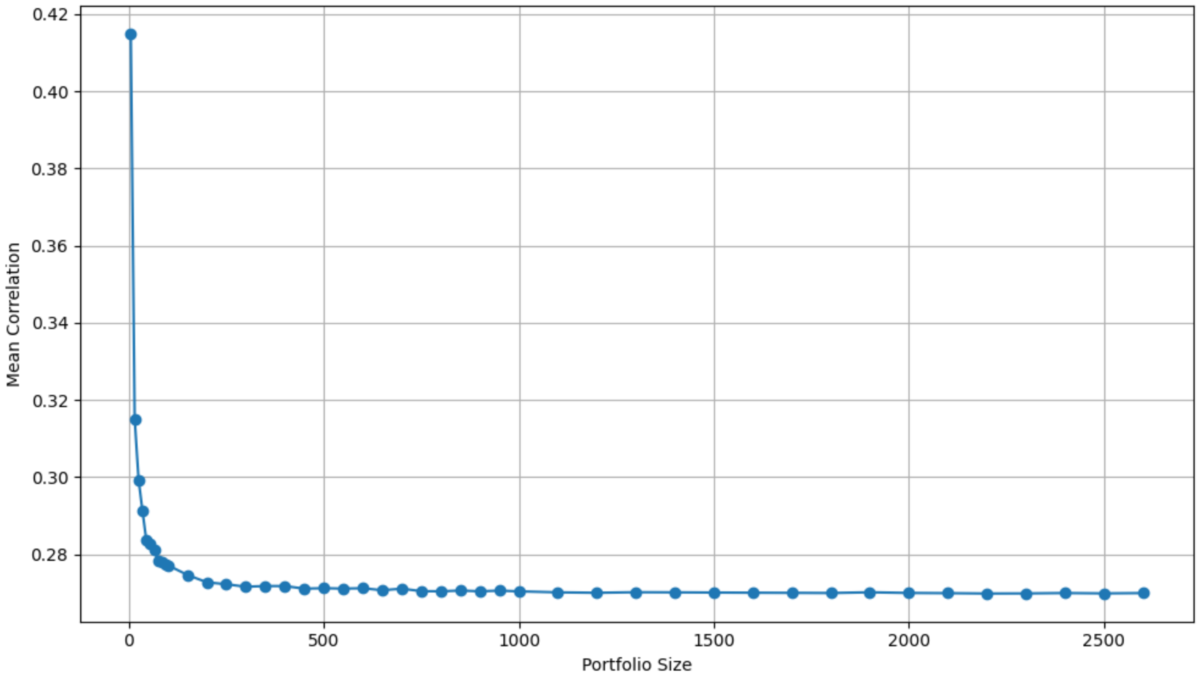
This chapter aims to answer the second hypothesis of this paper. The second hypothesis was previously stated as follows:

H_2: Portfolios with more than a certain number of stocks will show a higher correlation among their stocks, thus reducing the effects of diversification.

H_{2a}: Portfolios with more than a certain number of stocks will not show a higher correlation among their stocks, thus not affecting the effects of diversification.

This analysis will be conducted similarly to the previous analysis. Random companies will be used to create multiple portfolios of different sizes. The mean correlation will be calculated and plotted against each portfolio size. Figure 5.4.1 shows the relationship between correlation and portfolio size.

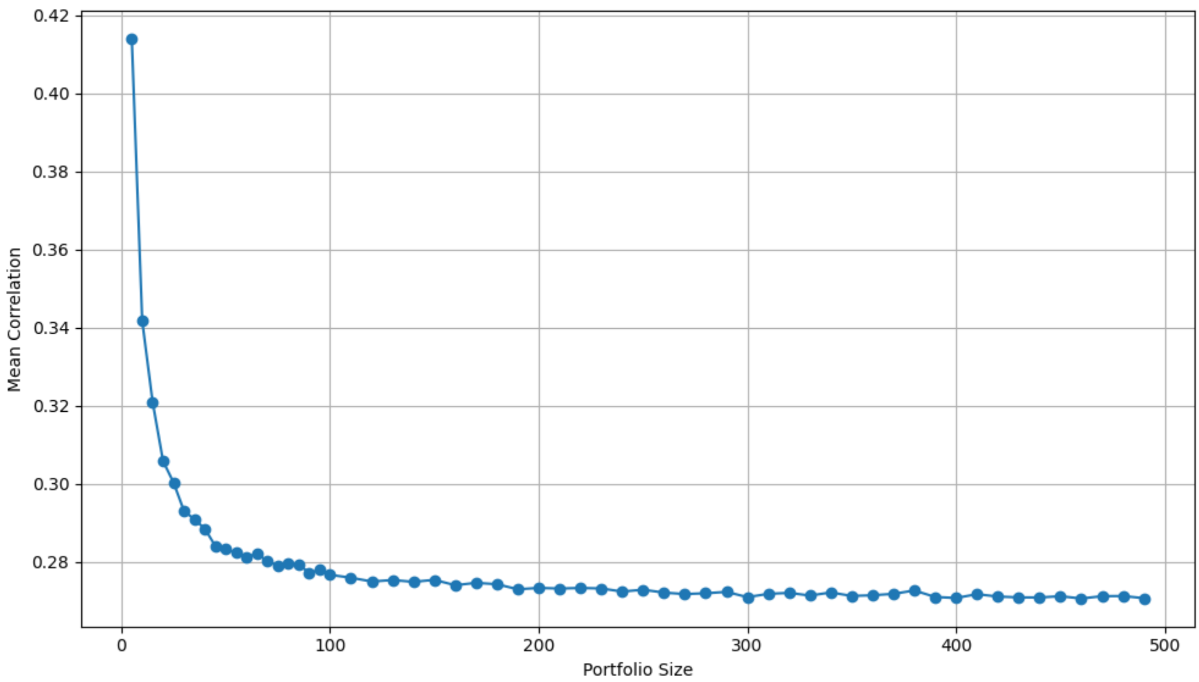
Figure 5.4.1 Mean Correlation per Portfolio Size



These results contradict the second hypothesis of this paper. The correlation of companies in a portfolio seems to decrease as portfolio size increases. One explanation for this negative relationship is the following. As the number of stocks in the portfolio increases, the diversity of industries and sectors represented in the portfolio also increases. Stocks in the same industry are likely to have a high correlation. A portfolio with more industries will, therefore, have a lower correlation among constituents on average. This reduced correlation can be beneficial for an investor. As some stocks move up, others may move down, thus reducing overall volatility.

The effect of portfolio size on correlation follows a similar trajectory to the effect of portfolio size on performance and risk measurements. The benefits of diversification are high in the initial phase, but after a certain point, they start to diminish, and the graph slowly approaches a limit. Figure 5.4.2 shows a concentrated version of Figure 5.4.1, focusing on the initial phase of diversification. Figure 5.4.2 shows that the benefits of further diversification are very minimal after around 100 stocks.

Figure 5.4.2 Mean Correlation per Portfolio Size Concentrated



Notes: This figure shows the same data as figure 5.4.1. This figure is concentrated on portfolio sizes 0 to 500 for clarification purposes.

CHAPTER 6 Conclusion

This thesis explored the notion of over-diversification of ETFs, focussing specifically on Vanguard's Russell 3000 index ETF. This paper aims to answer the following research question:

Does over-diversification exist, and how does it impact the portfolio performance of ETFs?

To answer this question, random portfolios of different sizes were created. These portfolios were all made up of a random selection of companies from the Russell 3000 index. This index spans approximately 96% of the investable United States equity market. Using multiple different analyzing methods, the following conclusions can be made. In all cases, when looking at either risk and performance metrics or correlation, the effects of diversification are at their highest in the initial phase. More specifically, all figures shown in this thesis show that after a breakpoint of around 100 to 300 stocks, most benefits from diversification have been enjoyed. Diversification beyond this point doesn't necessarily negatively influence the performance of a portfolio, but the benefits gained by this extra diversification are minimal. The regression results shown in this thesis also support this fact. Using two different breakpoints, it can be concluded that the benefits of diversification are higher when the portfolio size is smaller than this breakpoint than when the portfolio size is larger than this breakpoint.

If over-diversification is formulated as decreasing performance as portfolio size grows, then this research finds no evidence to support the existence of over-diversification. This thesis does, however, find statistical evidence that when diversifying beyond a certain point, the benefits of said diversification strongly diminish. This research found that the optimal point of diversification lies somewhere between 100 and 300 stocks. Hence, most ETFs are not necessarily over-diversified, but similar portfolio results can be realized when randomly selecting the optimal number of stocks from this ETF. For investors, this research -carefully- suggests that focusing on a well-diversified portfolio of stocks can increase returns and reduce risks without holding overly extensive large portfolios.

6.1 Limitations and further research

One of the main limitations of this research is the selected data, which has inherent survivorship bias. By having only currently active companies in the dataset, the results are most likely positively biased. For further research, one could spend more time and effort in realizing a more accurate and complete dataset of the Russell 3000 index over time.

Another limitation of this research is the number of selected possible breakpoints. Due to efficiency reasons, this paper could only test two possible breakpoints, points of optimal diversification. More breakpoints should be used to get a more accurate indication of the point of optimal diversification.

In further research, one could dive more deeply into the different risk measurements. In this research, only quantities of the second moment are considered. One could look more into the third and fourth moments with skewness and kurtosis. Examining these risk measurements could give insights into the tail risk and asymmetry of the portfolio returns.

Lastly, this paper focuses purely on the quantitative side of diversification. However, a large part of the reason an investor diversifies may be due to more qualitative factors such as personal preferences or market behaviour. In further research, one could expand the quantitative research with some qualitative research, such as interviewing professional investors or investment funds.

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