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The Role of Commodities in Traditional Stock and Bond Portfolios: A Comparative Study

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Abstract

The following study analyses stock-bond portfolios with the addition of commodities (as represented by indices) under 60/40, 1/N, Risk Parity and Minimum Variance allocation strategies between 1993-2024 to assess if the asset class enhances a portfolio's risk adjusted returns. Although allocation to commodities smaller than 15% can diversify a portfolio to lower annualized standard deviation by 9 basis points (bps) and Value-at-Risk by 8 bps, this is not found to increase the overall Sharpe ratio of the portfolios. In fact, it is shown to decrease Sharpe ratios by an average of 0.1. Exposure to industrial and precious metals are seen as favourable within commodity subsectors due to their favourable returns profile. However, the general cyclicality of commodities performance throughout subsections of the sample period do not provide significant proof of them being beneficial for a long-term stock and bond portfolio.

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Introduction

While the 60/40 portfolio, popularized by Markowitz's consideration of risk adjusted returns has proved to reach beneficial portfolio performance, excluding asset classes like commodities may overlook diversification benefits. Commodities, particularly gold after the dollar was unpegged in 1971, are often seen as a hedge against inflation (Spirdijk and Umar, 2013). Following the increased inflationary pressures observed from 2021 onwards, they have regained significance in portfolio allocation discussions. McKinsey & Co highlighted a 1.7x increase in trading commodities from 2019-2023, as higher credit costs impacted profits from equities and bonds, emphasizing their importance in challenging macroeconomic environments (McKinsey & Co, 2023). Commodities on average also experience more volatility as they are prone to geopolitical risks, supply chain bottlenecks and other macro focused factors. The holistic portfolio impact of a greater diversification through exposure to commodities, rivalled by higher volatility serves as a key motivation behind the study of this topic.

Prior research in commodity investing has shown varying results. Bessler & Wolff (2015) explored implementing a commodity portion to the multi-asset portfolio and found a higher Sharpe ratio for the addition of industrial/precious metals and energies. An improvement of the Sharpe ratio with livestock and agriculture was rarely observed. Another area of focus was the varying results in each subperiod, which Conover et al., (2010) contribute to by suggesting commodities are only beneficial in times of Federal Fund Rate increases. They also find commodities to be beneficial to all types of equity investors - but only with a larger allocation than 5% within the portfolio. An agreeable result comes from Belousova & Dorfleitner (2012), who found that the industrial metals benefit in reducing risk, but precious metals and energies reduce risk and increase potential returns. These three papers showcase and boast the strength of commodities in a multi-asset portfolio. Contrastingly, Daskalaki & Skiadopoulos, (2011), found that under mean variance optimal portfolios, classic investment splits perform better than those who have a commodity allocation, suggesting inclusion should depend on investors' allocation preferences.

From the literature above, not only is there a stark difference in some studies' results – but most used only American indexes for the equity and bond portions of the traditional portfolio. As a non-American investor, there is motivation to observe the results of the allocation strategies using world data for all the samples. Additionally, a recent hiking cycle from worldwide central banks to tackle higher inflation provides crucial new data which hasn't been tapped by recent literature to further observe the relationship. With a highlight on subsection analyses to explore the strategy's consistency, and risk measures to provide an alternative measure of success to Sharpe ratio's, the thesis will seek to explore to what extent do commodities' inclusion in classic portfolio allocation strategies yield excess risk adjusted returns over the stock and bond portfolio?

Data from 1993-2024 will be used to ensure a large sample size and minimize any temporary shocks or cycles that may occur. Performance of a stock-bond portfolio will be evaluated using classic portfolio allocation strategies including 1/N, 60/40, Risk Parity and Minimum Variance. The performance of each of these portfolios will then be compared to the portfolios with the addition of all commodities subgroups, and each commodity group individually. To evaluate the role of commodities in a stock-bond portfolio further, the most interesting asset allocation strategy (based on results from the prior part and its industry popularity) will be singled out for a subsection analysis to understand the strategy's consistency since 1993. Finally, a Value-at-Risk analysis will be completed for this strategy to understand the max drawdowns of the approach compared to simply stocks and bonds.

Supply and demand shocks, geopolitical tensions, wars and macroeconomic trends can all have a harsh price effect on commodities, often in the opposing direction to stocks and bonds. For these reasons, it is expected to find a benefit of diversification in the study for most of the asset allocation strategies. This will be assessed by comparing Sharpe ratios over the 31-year period. It is also assumed that diversification benefits will be uneven throughout the period because of so-called commodity 'super-cycles' where changes in global economies – like China's expansion in the late 1990's and 2000's - cause a sustained elevated price of commodities. This will be addressed through the subsection analysis, where it can be tested if the returns are significantly different from each other. Finally, it is hypothesized that the Value-at-Risk will be higher for the portfolio with inclusion of commodities due to the inherently greater standard deviation of commodities.

The rest of the study offers a theoretical framework about commodities investing, a data section, in-depth methodology and empirical results. It will then be rounded off with a conclusion of the main findings and discussion of the importance and limitations of the study.

Theoretical Framework

The following section aims to provide an understanding of past research to do with the paper's topic, specifically regarding (1) the reasons for investing in commodities as an asset class, (2) a review of their performance in economic cycles and crises periods, and (3) the empirical understanding of their dynamic within an investment portfolio.

Reasons for investing in commodities.

The reasons for investing in commodities according to prior research is centralized to three main topics:

Firstly, as with the inclusion of almost any new asset class, investing in commodities provides profound diversification benefits. The correlation of commodities with stock and bond returns is very low, and negative with longer holding periods (Gorton and Rouwenhorst, 2006). This suggests that commodities provide space for diversifying a portfolio when investing in the longer term. Interestingly, Cheung and Miu (2010), find that although diversification is indeed found, the large statistical reasoning comes behind irregular upswings in commodity prices, and the correlation apart from this is in fact higher between the assets. They also find that the diversification is less applicable in resource heavy economies (e.g. Canada), making for an interesting topic of how world stock and bond data may affect the diversification benefits instead of the common American centric view.

Secondly, commodities are often seen as a hedge against rising inflation. A discussion paper by Spirdijk and Umar (2013) conclude that commodities have had an increasingly effective role as a hedging vehicle between 1970 and 2011. They credit this to the economic reasoning that periods with low (expected) inflation are met by a similarly low demand for real assets. This negative demand shock thus results in lower commodity returns, which operate in the contrasting manner when expected inflation is higher – increasing the commodity returns and thus their use as a hedging tactic. When focusing in on which commodity sectors provide the best hedges, current research points to industrial and precious metals as being the first-class choices, with the best results being attained in sustained, longer periods of inflation (Liu et. Al, 2023).

A third focused reason why investors look at commodity investing is their potential at achieving higher yields to other investments. Research by Gorton & Rouwenhorst shows that over a period of 45 years, an equal weighted commodity portfolio exceeded the returns of US T-bills by 5%, which displays a similar risk premium of the stock market in that period – with a lower standard deviation. Certain academic literature however disagrees with the prospects of commodities' high returns, explaining that investor sentiment to commodities has been skewed by periods of rapid price increases such as in 2004-2008 (see Irwin et. al. 2020).

Historical performance of commodities.

Commodities, being a real asset, have a unique trait of having so called 'super-cycles' in which we observe elevated price levels for a period of 10-30 years. This is usually a result of economically stimulated supply-demand disparity (Erten and Ocampo, 2013). This imbalance is largely caused by industrialization led demand, which has been observed during Europe's and Japan's reindustrialization after the World Wars in the 50's and 60's, as well as the sustained economic growth of China in the 1990's-2010's. Nevertheless, it can still occur that supply based shocks cause a super-cycle. For example, the oil embargo set on many western countries in the early 70's had risen oil prices by close to 300%, and the supply bottlenecks from the Iranian revolution had sustained them at the high for a number of years, enough to be referred to as a supply caused super-cycle.

Historical prices of commodities have also been sensitive to monetary conditions. Baffes & Savescu view the effects of decreasing interest rates as both appreciative and depreciative for their own reasons. On the appreciative front, prices of commodities can be expected to rise after rate cuts occur due to the extrapolation commodity demand will increase as the economy is stimulated. Furthermore, as most commodity futures contracts and transactions are denominated in US Dollars (USD), the depreciation of the dollar resulting from lower interest rates makes the commodities relatively cheaper for foreign investors, stimulating upwards pressure on the price. On the contrasting view, lower interest rates also signal muted expected inflation rates, which is generally linked to lower commodity prices. Finally, the lower rates also indicate that a supply catch-up is in reach as costs of capital-intensive mining and oil extracting activities are decreased (Baffes & Savescu, 2014). The overall effect of monetary policy is therefore very varied but will be observed throughout this study to explain portfolio performances from a macro standpoint.

Another important factor of commodities past performances has to do with their reactions to financial market turmoil. Current research shows evidence against diversification to commodities during fear in financial markets. One study investigates the performance of commodities, and their correlation to the equity market during two recent market crashes – the subprime mortgage crisis of 2007-8 and the European sovereign debt crisis in the early 2010's. They directly advise against the coupling of commodities in a portfolio for the aim of diversification as they find commodities have fading segmentation characteristics which renders them less useful in absorbing other assets' poor performance (Tzeng & Shieh, 2016). Creti, Joets and Mignon (2013) explain this increased correlation post crisis in the long run as a result of increasing global demand and reindustrialization. They also stress that all commodities cannot be seen as homogenous, as commodities can be used for many uses (e.g., safe havens, speculatory instruments etc.) which changes their price reactions in crises periods.

Commodities in traditional portfolios.

This sub-section will expand on the papers mentioned in the introduction regarding commodity performance in portfolios and outline a few more academic findings. Bessler & Wolff (2015) found an improvement of portfolio performance with the addition of broad commodity indices and most individual commodities except agriculture and livestock. The results however greatly vary per commodity and sub-period which is analysed. They also find that their out-of-sample approach provides less risk adjusted returns than previously studied in-sample analysis, hinting that in-sample analyses exaggerate the return possibilities of commodities.

Conover et al., (2010) find a commodity inclusion of 10% can significantly decrease the risk of a portfolio, and allocation must be higher than 5% at least to have an overall positive effect. They also expand on our discussion of monetary policy by stating that the portfolios only really saw advantages of commodities when interest rates were hiked – in line with our prior economic reasoning. Furthermore, the most successful portfolio they analysed was one that tactically invested 15% in commodities during times of restrictive monetary policy (rate hiking) and 100% in equities during expansionary policy. This strategy proved effective over the whole 37-year sample period.

Belousova & Dorfleitner (2012) agree with Bessler & Wolff (2015) that individual commodities have very broad usefulness in diversification of a portfolio. They find that agriculture, livestock and industrial metals have the most diversification benefits, making them apt for investors that follow a variance minimization approach such as the Minimum Variance portfolio or Mean Variance portfolio. An added benefit of their study looks at the performance under both bull and bear markets. They find that although industrial metals are highlighted as a source of increased risk adjusted returns in many studies, it provides almost no benefit during bull markets. While many commodities provide diversification gains in bull markets, only silver and heating oil simultaneously decreased risk and increase the return profile of the portfolio.

These past studies on commodity's performance, price relationship to monetary policy and the motivations of commodity investors leads this study to explore the following hypothesis:

1. Commodities significantly improve the risk adjusted returns, measured as Sharpe ratio, of a stock-bond portfolio under most portfolio allocation strategies during the selected period (1993-2024).

2. Performance diversification benefits offered by commodities are not equal throughout subsections and varies over time.
3. The stock-bond portfolio with addition of commodities has a higher Value-at-Risk than the standalone stock-bond portfolio.

Data

The only data required for the study is price data of all assets incorporated into the portfolios that will be analysed, which is then converted to daily and monthly returns. The data for all assets was retrieved from Bloomberg Terminal and spans from 04/01/1993 till 23/05/2024, providing over three decades of data. The reasoning behind using Bloomberg Terminal as a data source comes from its extensive industry use and comprehensive, as well as real-time access to financial data. The assets used to represent the stock and bond allocation are the 'MSCI World Index' and 'Bloomberg GlobalAgg Index' respectively. The stock index, although representing a large allocation to US equities of 70%, contains 1,465 constituents from 23 Developed Market (DM) countries – showcasing a global exposure. The Bloomberg GlobalAgg (bond index) holds corporate and sovereign debt, as well as securitized debt including Mortgage and Asset Backed Securities (MBS & ABS) from 28 currencies, giving a worldwide debt exposure for the forthcoming portfolios.

For the commodity inclusion into the portfolios, indices were selected from the family of Bloomberg Commodity Indexes. Price data was selected for the Agriculture, Energy, Industrial Metals, Livestock and Precious Metals sector subindices, as well as the overarching Bloomberg Commodity Index to represent allocation to all commodities.[1] The methodology behind the indexes aims to track exchange-traded futures and physical commodities, reflecting an accurate representation of the market pricing. The weight of each subindex is made to reflect the liquidity and production of the respective commodity and can be seen in Table 1.

Once daily data for each index since 04/01/1993 was collected, a list of daily and monthly dates was made to correspond to days in which all asset classes have a reading. As some indexes did not report on a particular day, this results in the last day of the month being the nearest day in which all indexes reported. Finally, daily and monthly returns data was calculated from price data using the following formula to be expressed as a percentage:

$$R_t\ (\%) = \frac{P_t}{P_{t-1}} - 1$$

Where P_{t-1} represent the index price for either the day or month prior, depending on the calculation of daily or monthly returns.

[1] Respective tickers; MIWO00000PUS for the MSCI World, LEGATRUU for the Bloomberg GlobalAgg Index, BCOMAGTR for Agriculture, BCOMENTR for Energy, BCOMINTR for Industrial Metals, BCOMLITR for Livestock, BCOMPRTR for Precious Metals and BCOM for the index of all commodities.

Table 1: Subindices weightings

	Sector subindices				
	Energy	**Agriculture**	**Industrial Metals**	**Precious Metals**	**Livestock**
Aluminum	-	-	26.28%	-	-
Brent Crude Oil	26.75%	-	-	-	-
Chicago Wheat	-	9.33%	-	-	-
Coffee	-	11.91%	-	-	-
COMEX Copper	-	-	35.29%	-	-
Corn	-	18.55%	-	-	-
Cotton	-	5.19%	-	-	-
Gold	-	-	-	75.90%	-
USL Diesel	6.80%	-	-	-	-
Kansas City Wheat	-	6.22%	-	-	-
Lead	-	-	11.83%	-	-
Lean Hogs	-	-	-	-	42.34%
Cattle	-	-	-	-	57.66%
Low Sulphur Gas Oil	9.19%	-	-	-	-
Natural Gas	22.57%	-	-	-	-
Nickel	-	-	-	-	-
RBOB Gasoline	8.75%	-	-	-	-
Silver	-	-	-	24.10%	-
Soybean Meal	-	11.39%	-	-	-

Table 1 reflects the weight of each subindex allocated to individual commodities through futures contracts made available by Bloomberg. The weight is reflective of the indexes balances as of May 2024, and is subject to change in the future and throughout the sample period.

Table 2 describes the summary statistics of the data for daily returns. Namely, the annualized mean returns and standard deviation, maximum, minimum, 95% Value-at-Risk (VaR), and observation count. The VaR represents the maximum possible daily drawdown of the assets given a 95% confidence interval, the measure is a simple way to overview the risk of the asset and will also be used to evaluate the risk of individual portfolios.

It can be seen from the summary statistics that commodities usually have a higher standard deviation and VaR than stocks and bonds, without necessarily outperforming stocks on the returns front. This implies a higher risk of holding the asset, especially on a risk adjusted return basis. The daily data includes 7857 observations, which translates to 375 monthly datapoints – a rich dataset for exploring the trends and effects of commodities' incorporation to a stock-bond portfolio. The correlation between all assets can also be found in the Appendix.

Table 2: Summary statistics of daily returns data

	Stocks	**Bonds**	**All commodities**	**Industrial Metals**	**Precious Metals**	**Energies**	**Agriculture**	**Livestock**
Annualized Mean Returns	7,31%	3,79%	1,62%	6,59%	5,45%	1,28%	2,80%	-3,87%
Annualized Standard Deviation	15,46%	6,50%	15,08%	20,92%	18,50%	30,10%	17,41%	15,17%
Max	9,52%	3,92%	5,81%	11,28%	9,41%	10,53%	6,79%	5,75%
Min	-9,92%	-2,28%	-6,20%	-9,24%	-9,86%	-13,66%	-6,52%	-6,08%
VaR (95%)	-1.60%	-0.67%	-1.56%	-2.17%	-1.92%	-3.12%	-1.80%	-1.57%
Number of Observations	7857	7857	7857	7857	7857	7857	7857	7857

Note:Mean returns are annualized by multiplying by 252, and standard deviation is annualized by multiplying by $\sqrt{252}$*. VaR is calculated using the delta neutral method which is explained in depth in the Methodology.*

Methodology

To achieve an impartial result in viewing the effects of commodities to a stock-bond portfolio, multiple asset allocation techniques must be utilized. This is due to varying return, risk and diversification profiles when determining the weight associated to each asset. Four commonly used and studied allocation tactics will be used; an adjusted 60/40 portfolio, the 1/N portfolio, a risk-parity portfolio (RPP), and the minimum-variance portfolio (MVP). Portfolio weights are calculated for the last day of the month where each asset is tradable and accordingly rebalanced, ensuring no heavy deviation from the allocation tactic. The first four months of daily data is excluded from the portfolios but used for the construction and 'training' of the more advanced RPP and MVP, as they need covariance data to appropriately allocate the weights (more on this later). For each portfolio allocation technique, the performance of a two-asset stock and bond portfolio is first measured, followed by a three-asset portfolio through the inclusion of each individual commodities sub-sector index. This will result in seven portfolios to be measured in each allocation tactic. The following section of the methodology goes into detail of how each tactic is approached and calculated:

60/40 portfolio

The 60/40 portfolio is a strategic allocation involving a 60% allocation to stocks and 40% allocation to bonds. The tactic originated from Harry Markowitz's "Portfolio Selection" paper (Markowitz, 1952), where he introduced the groundbreaking concepts of Modern Portfolio Theory (MPT) and the Efficient Portfolio Frontier (EPF). Though the 60/40 allocation was never directly stated, it is seen as a practical application of his theoretical model through the focus on risk adjusted return of the two assets. It has long been popularized in retail investing and has acted as a "base case" portfolio for many academic research papers.

As the weights are not dynamic, or specific to the risk characteristics of the assets, the portfolio weights are rebalanced to 60% allocation in stocks and 40% allocation in bonds for the two-asset portfolio at the end of every month. For the inclusion of commodities, the weights are adjusted to 55% stocks, 35% bonds and 10% in the respective commodity – also rebalanced every month. The allocation structure allows for equal dilution of the other two assets, without changing the 'overweight' sentiment to stocks, and diversification to bonds.

1/N portfolio

Viewed as a naïve portfolio allocation tactic, the 1/N strategy incorporates allocating an equal weight to all N number of assets in the portfolio. Hence, a 1/2 weight in the two-asset portfolio and a 1/3 weight for each commodity portfolio. Although this is not an advanced allocation tactic, it represents one of the simplest allocations, and provides high levels of diversification given that individual assets are not highly correlated.

Risk-Parity portfolio

Popularized by Ray Dalio's hedge fund Bridgewater Associates, the RPP aims to allocate weights of the assets in a way that gives equal risk contribution to all assets. This is in the aim of creating a portfolio that can withstand most economic and macro circumstances, hence Dalio's use of it in the "All Weather" fund at Bridgewater Associates. To calculate this in the portfolio weights for the study, the following 3-step method was used:

Firstly, the covariance matrix needs to be calculated between all assets to show to which extent they statistically vary from one another. This will be done utilizing the following formula:

$$Cov(i,j) = \Sigma_{ij} = \frac{\sum_{k=1}^{n}(R_{k,i} - \bar{R}_i)(R_{k,j} - \bar{R}_j)}{n-1} \tag{1}$$

Where:

Σ_{ij} is the covariance between assets i and j.

n represents the number of datapoints.

$\bar{R}_i$ is the mean returns of asset i.

$R_{k,i}$ is the returns of asset i at time k.

To annualize the covariance, the study also multiplies formula (1) by 252, as we assume 252 trading days in a year. It is then necessary to initialize a starting weight, from which iterations will be made to reach the RPP weights. The starting weight of 1/N is set for each asset, where N represents the number of assets (in short – equal weights are set).

As the RPP aims to equalize the risk contribution amongst assets, we now need to calculate each risk contribution. This is done through the following formula:

$$RC_i = \frac{w_i \cdot (\Sigma w)_i}{\sigma^2} \tag{2}$$

Where:

w_i is the weight of asset i, in the same iteration.

$(\Sigma w)_i$ is the matrix multiplication of the covariance matrix and weight for asset i.

σ^2 is the variance of the portfolio given the weights at the current iteration.

Intuitively, the numerator provides the individual asset's risk contribution. Firstly, the output of the matrix multiplication Σw gives the weighted sum of the assets' covariance with the rest of the portfolio, giving the marginal contribution of the asset to the total risk. Further multiplying this by the weight of asset i gives the actual risk contribution of the asset. Finally, dividing the equation by the variance (σ^2) will normalize the equation and make sure that all the risk contributions sum to one. This is because the variance is calculated as the sum of all assets' individual risk contributions, $w_i \cdot (\Sigma w)_i$. Simply speaking, we divide the single assets risk by the total portfolio risk, to obtain a percentage, comparable risk contribution RC_i.

Finally, new weights are iteratively calculated. This is done by applying the following formula:

$$w_i^{new} = w_i - 0.1(RC_i - \frac{1}{N}) \qquad \textbf{(3)}$$

Which reduces the new weight of the asset if the risk contribution is larger than 1/N (iteratively through a factor of 0.1), and likewise increases it if the risk contribution is smaller than 1/N. After the new weight is calculated, the process of calculating the risk contribution from formula (2) and new weights from formula (3) are repeated until all risk contributions are equal. The final weights will thus represent the portfolio which equalizes risk of every asset – the risk-parity portfolio.

In practice, the whole process is completed through a Visual Basics for Application (VBA) code on Microsoft Excel, through a User Defined Formula (UDF). These weights are calculated at every month end with all daily returns preceding that date. The weights of previous months are then applied to the current month's returns to calculate the portfolio returns.

Minimum-Variance portfolio

Similarly to the 60/40 portfolio, the origins of the MVP come from Markowitz's famous paper "Portfolio Selection" (Markowitz, 1952). The paper introduced the concept of Mean-Variance Optimization (MVO), which aims to minimize the variance of a portfolio given a certain expected return, or alternatively maximize the return given a volatility constraint. The concept of the MVP does not deviate far from this, it simply aims to allocate weight to assets in a way that will minimize the overall variance of the portfolio, given the covariance matrix. To implement this in a dynamic, monthly rebalanced portfolio, the further methods of Robert Merton are used. Merton expanded on Markowitz works, by providing the analytical techniques to calculate all forms of optimal portfolios, often through inverse covariance matrices to minimize the total portfolio's variance (Merton, 1972).

For the MVP, calculations start off similarly to the RPP by calculating the covariance matrix of all assets using the same formula (1). Following Merton's method, it must then be transformed into an inverse covariance matrix. This is done by solving the inverse which multiplies with the covariance matrix to create the identity matrix, or simply using the MINVERSE() formula on Microsoft Excel.

Accordingly to Merton's method, we create an e-vector of ones, and an h-vector to calculate an alpha factor that will be directly linked to the derivation of the portfolio weights:

$$e - vector = \begin{vmatrix} 1 \\ 1 \\ . \\ . \\ 1 \end{vmatrix} \tag{4}$$

$$h - vector = \Sigma^{-1} \cdot e \tag{5}$$

$$\alpha - factor = \sum_{i=1}^{N} h_i \tag{6}$$

Where:

Σ^{-1} represents the inverse covariance matrix.

e represents the e-vector, a vector of N-number of ones.

The h-vector represents the weights that should be allocated to each asset, meaning that it needs to be normalized by dividing by its sum α to obtain a weight for all assets which sums to one.

$$w_i = \frac{h_i}{\alpha} \tag{7}$$

The whole MVP weight formula can also be expanded, for a detailed overview as the following formula:

$$w_i = \frac{h_i}{\alpha} = \frac{(\Sigma^{-1}e)_i}{e^T\Sigma^{-1}e} \tag{8}$$

Where:

e^T is the transposed vector of ones, acting as a summation of the vector output of $\Sigma^{-1}e$.

As in the RPP, the weights of the MVP are calculated and rebalanced at month end but with daily returns data, to which the returns of the next month are then applied to simulate the portfolio performance.

Comparison metrics

To provide an in-depth analysis of each portfolio, the annualized returns, cumulative return, annualized standard deviation and Sharpe ratio will be used to measure the performance. The formula that will be used for the Sharpe ratio is simplified to:

$$SR_i = \frac{r_i}{\sigma_i} \quad \textbf{(9)}$$

Where:

r_i represents the annualized returns of the portfolio.

σ_i represents the annualized standard deviation of the portfolio.

The factor of a risk-free-rate in the Sharpe ratio which is typically utilized has purposely been left out. As the portfolios will only be compared to each other within the same timeframes, the numerator of the Sharpe ratio would always be subtracted by the same risk-free-rate value – thus bringing no benefit to the measure apart from comparison to external portfolio performances, which lays outside the scope of this study.

Cumulative returns provide another measure to annualized returns, displaying the return of a single dollar invested at the time of portfolio creation by multiplying past months dollar value by one plus the current months return. This offers a better visualization of portfolio performances in different periods, important for the analysis of commodity or stock market shocks on overall portfolios.

Sub-section analysis

As commodities prices tend to work on a supercyclic basis, the overall portfolio performance will be supplemented by a sub-section analysis consisting of 6 period, each spanning around 5 years (62 months). The date ranges can be seen in Table 3.

Table 3: Subsection dates

Sub-section	Start	End
1	30/04/1993	29/05/1998
2	30/06/1998	31/07/2003
3	29/08/2003	30/09/2008
4	31/10/2008	29/11/2013
5	31/12/2013	31/01/2019
6	28/02/2019	28/03/2024

The sub-section analysis will go more in depth regarding the individual portfolio performances, this will provide a layer of robustness by ensuring the overall full period results are not causal to a singular section or environment.

Value-at-Risk analysis

Finally, a Value-at-Risk analysis will be completed to provide data on the historical, and simulated risk profile of the stock-bond portfolio in comparison to the portfolios supplemented by the “All commodities” BCOM index. Two measures of VaR will be calculated, firstly, the historical VaR provides a max drawdown percentage under a confidence interval of 75%, 90%, 95% and 99%. Secondly, the Monte Carlo method for simulating portfolio returns will also provide a VaR.

VaR will be calculated using the delta-neutral method, the following formula:

$$VaR_{x\%} = \sigma * Z_{\alpha} \quad \textbf{(10)}$$

Where:

σ represents the standard deviation of the sample (not annualized).

Z_{α} represents the critical value at the α level of confidence.

Intuitively the historical VaR will use past data standard deviations. The Monte Carlo VaR however will use standard deviations obtained from 10,000 simulated monthly returns. The simulated returns are calculated using the following formula:

$$R_s = \bar{R} + (\sigma * Z_r) \quad \textbf{(11)}$$

Where:

R_s represents one of the simulated returns.

$\bar{R}$ represents mean returns over the sample.

σ represents the standard deviation of the sample.

Z_r represents the critical value of an evenly distributed random value between 0 and 1.

The delta neutral method will then be applied to the 10,000 simulated monthly returns to provide an alternative measure. The Monte Carlo simulation has the added benefit of stress testing historical returns to extreme scenarios in both directions that may not yet be present in the data.

Empirical results

Portfolio performances

The performance of monthly rebalanced stock-bond portfolios and all respective multi-asset portfolios under four allocation strategies between April 1993 to May 2024 are presented in Table 4.

Table 4: Results of all portfolios between 1993-2024.

Portfolio	Annualized return	Annualized Standard deviation	Sharpe ratio	Historical VaR (95%)
60/40 (55/35/10)				
Stock-Bond	5,78%	10,04%	0,58	-2,54%
Stock-Bond-All commodities	5,40%	**9,89%***	0,55	-2,80%
Stock-Bond-Industrial Metals	**5,91%***	10,32%	0,57	-2,64%
Stock-Bond-Precious Metals	5,77%	**9,80%***	**0,59***	**-2,35%***
Stock-Bond-Energies	5,34%	10,35%	0,52	-3,23%
Stock-Bond-Agriculture	5,53%	**9,82%***	0,56	**-2,61%***
Stock-Bond-Livestock	4,83%	**9,37%***	0,52	-2,94%
1/N				
Stock-Bond	5,43%	8,96%	0,61	-2,00%
Stock-Bond-All commodities	4,14%	9,28%	0,45	-3,55%
Stock-Bond-Industrial Metals	**5,87%***	11,03%	0,53	-3,28%
Stock-Bond-Precious Metals	5,40%	9,65%	0,56	-2,59%
Stock-Bond-Energies	3,96%	12,86%	0,31	-6,70%
Stock-Bond-Agriculture	4,58%	9,60%	0,48	-3,38%
Stock-Bond-Livestock	2,27%	**7,95%***	0,28	-4,32%
Risk parity				
Stock-Bond	4,69%	7,37%	0,64	-1,42%
Stock-Bond-All commodities	3,83%	7,65%	0,50	-2,52%
Stock-Bond-Industrial Metals	**5,05%***	8,16%	0,62	-1,71%
Stock-Bond-Precious Metals	**4,84%***	8,01%	0,60	-1,80%
Stock-Bond-Energies	4,11%	8,17%	0,50	-2,66%
Stock-Bond-Agriculture	4,27%	7,69%	0,55	-2,11%
Stock-Bond-Livestock	2,48%	**6,60%***	0,38	-2,99%
Min-Variance				
Stock-Bond	4,13%	6,61%	0,62	-1,35%
Stock-Bond-All commodities	3,62%	**6,58%***	0,55	**-1,84%***
Stock-Bond-Industrial Metals	**4,30%***	6,64%	**0,65***	**-1,20%***
Stock-Bond-Precious Metals	3,99%	6,64%	0,60	-1,51%
Stock-Bond-Energies	4,06%	6,65%	0,61	-1,46%
Stock-Bond-Agriculture	3,96%	**6,55%***	0,60	-1,47%
Stock-Bond-Livestock	2,77%	**5,94%***	0,47	-2,15%

Note: Any figures in bold and with a "" next to it aim to represent a better performance to the benchmark portfolio which is shown as "Stock-Bond" in the table.*

The results show that under no allocation strategy, does the addition of the broad commodity index BCOM (nicknamed "All commodities"), improve the risk adjusted returns of a portfolio measured by Sharpe ratio. Additionally, no individual commodity index consistently increased the Sharpe ratio of the stock-bond portfolio, nor decreased the VaR.

Two assets stand out when observing the annualized return and standard deviation. Firstly, industrial metals have increased the returns of a portfolio consistently in each allocation strategy. This is no surprise as industrial metals have the highest annualized return (6.59%) amongst all commodities analysed. Unfortunately, this is not able to convert to a higher Sharpe ratio in three out of four allocation strategies – with the exception being the Minimum Variance Portfolio, which by definition allocates weights to minimize the portfolio impact of industrial metals' high standard deviation. Secondly, the livestock index has decreased the standard deviation of every portfolio in which it performed, resulting in the lowest standard deviation for each allocation strategy. Despite this, the extremely low (negative) return profile of livestock still made it the worst performing risk return portfolio in all strategies. This holistic overview suggests that no commodity inclusion in a global stock-bond portfolio is beneficial for extended holding periods. The inclusion of industrial metals however can consistently aid enhance the return profile if an investor has little consideration for more variance in portfolio returns.

For a better understanding of weights provided to individual commodities in each of the three-asset portfolios, a summary can be found in Table 5.

Table 5: Descriptive statistics of portfolio weightings

		All commodities	Industrial Metals	Precious Metals	Energies	Agriculture	Livestock
60/40							
	Max	10%	10%	10%	10%	10%	10%
	Mean	10%	10%	10%	10%	10%	10%
	Min.	10%	10%	10%	10%	10%	10%
1/N							
	Max	33.33%	33.33%	33.33%	33.33%	33.33%	33.33%
	Mean	33.33%	33.33%	33.33%	33.33%	33.33%	33.33%
	Min.	33.33%	33.33%	33.33%	33.33%	33.33%	33.33%
RPP							
	Max	38.60%	24.20%	28.39%	19.79%	29.75%	27.42%
	Mean	24.56%	17.95%	19.98%	13.49%	20.84%	23.71%
	Min.	21.12%	15.98%	17.85%	12.35%	19.31%	22.23%
MVP							
	Max	43.56%	16.33%	22.80%	8.17%	24.87%	19.75%
	Mean	14.55%	6.23%	4.79%	3.57%	9.61%	14.87%
	Min.	7.97%	2.79%	0.00%	2.15%	6.66%	12.85%

As can be seen above, the average allocation provided to individual commodities in three-asset portfolios hovers around 20% for the risk parity portfolio and doesn't have a single average over 15% in the minimum-variance portfolio. In fact, allocations to the precious metals and energy indexes averages below 5% (4.79% and 3.57% respectively). This is likely due to their higher standard deviation and covariance to stocks and bonds.

Interestingly, the average bond allocation in three-asset portfolios stands at 54.72% in the risk parity portfolio and 77.25% in the minimum variance portfolio. Stock allocations were much lower at 25.2% average for the risk parity portfolio and 13.82% in the minimum variance portfolio. This finding makes sense as bonds have the lowest covariance to other asset, making it a safer position to reduce overall portfolio variance. As for the risk parity portfolio, the lower covariance means a higher weight needs to be allocated to bonds to balance the risk contribution the index has.

These results suggest that allocation strategies which assign higher weights to commodities are generally worse off on a risk adjusted basis. As can be seen by the 1/N and RPP (which assign the highest weights to commodities), commodities are not able to sufficiently boost returns in a portfolio for the increase in annualized standard deviation. For both strategies, the inclusion of 'All commodities' not only increased the annualized standard deviation, but also decreased returns. This is less pronounced in the 60/40 and MVP as the lower allocation to commodities diversifies the portfolio more efficiently (as seen through a lower standard deviation).

To better understand the returns profiles and performance of each commodity, cumulative return graphs have been mapped for each allocation tactic bellow (Figures 1-4). Cumulative returns can be interpreted as the progressive value of $1 invested at the beginning of the period, alternatively, the progressive percentage value of the initial investment. Figures displaying only the benchmark portfolio and the one supplemented by All commodities can be found in Appendix B.

There are two strikingly obvious observations that can be detected in each of the four strategies. Firstly, the long run underperformance of portfolios which include the livestock index, making them the worst performer in each strategy. This is not surprising as the index has lost approximately 80% of its value over the observed period, eating away at any gains made in stock and bonds. This widescale decline in lean hog and cattle futures prices has generally been attributed to the oversupply through increased efficiency in the mass production of livestock and technological developments in genetical engineering (Holechek, 2009). Secondly, it can be observed that the top three performers in each graph are the two-asset portfolio, and the industrial metals and precious metals supplemented portfolios, usually with some margin on cumulative returns to the runner-ups. The key factor behind price appreciation of metals has historically been industrial production (Baffes and Savescu, 2014). It is thus likely that the sustained demand growth of developed countries and industrial expansion in emerging markets (e.g. China's role in commodity super cycle of the 1990's-2010's) has been a vital contributor to the elevated price of metals, and thus their performance in the portfolios.

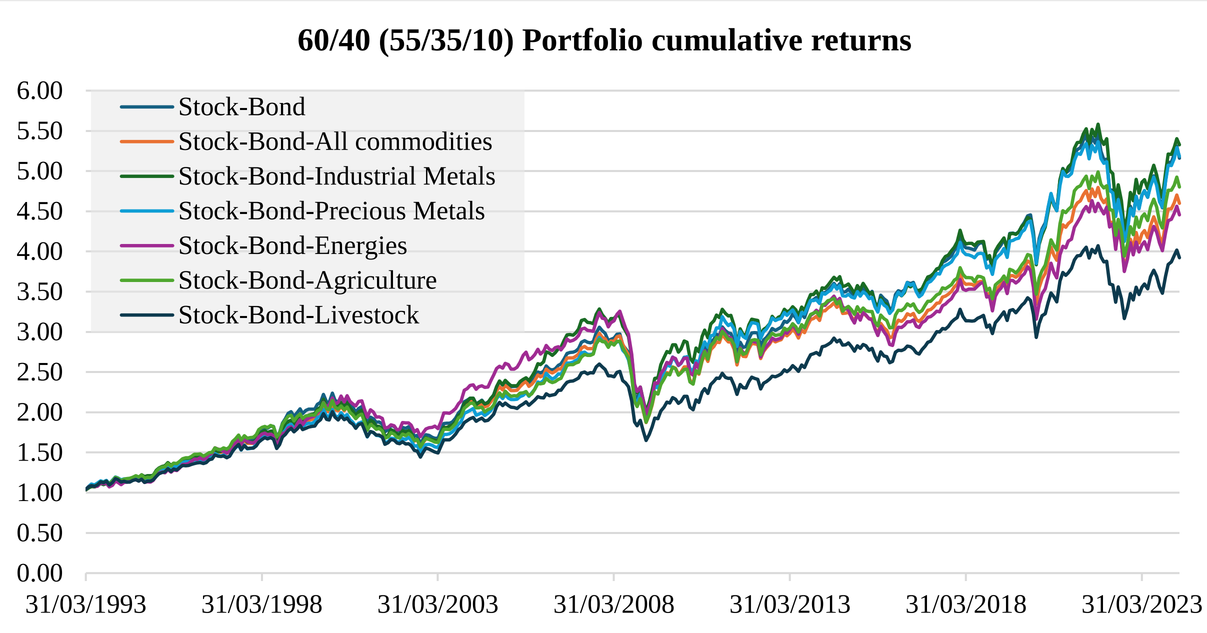

Fig. 1. Cumulative returns of the stock/bond and commodity portfolios under 60/40 allocation.

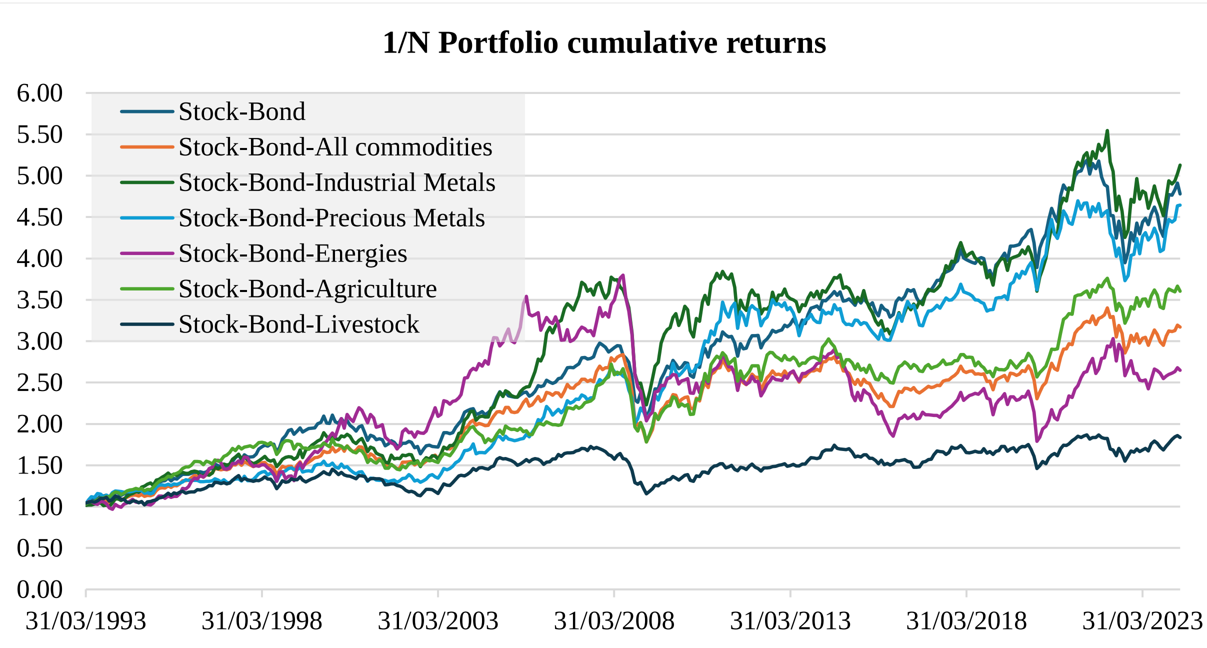

Fig. 2. Cumulative returns of the stock/bond and commodity portfolios under 1/N allocation.

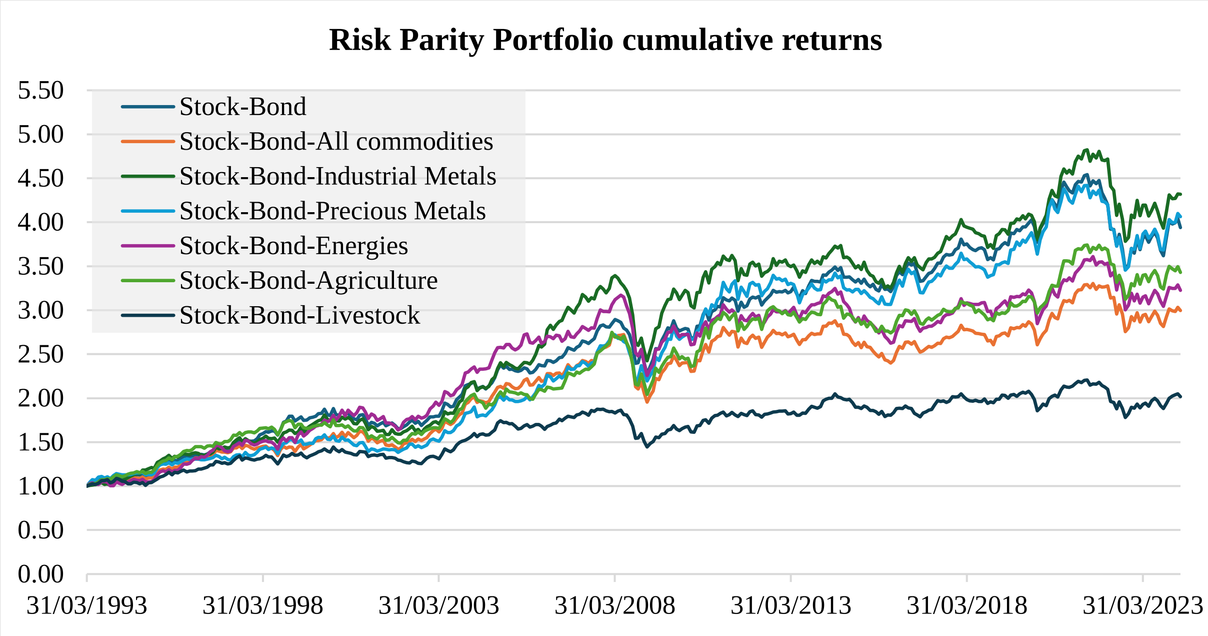

Fig. 3. Cumulative returns of the stock/bond and commodity portfolios under RPP allocation.

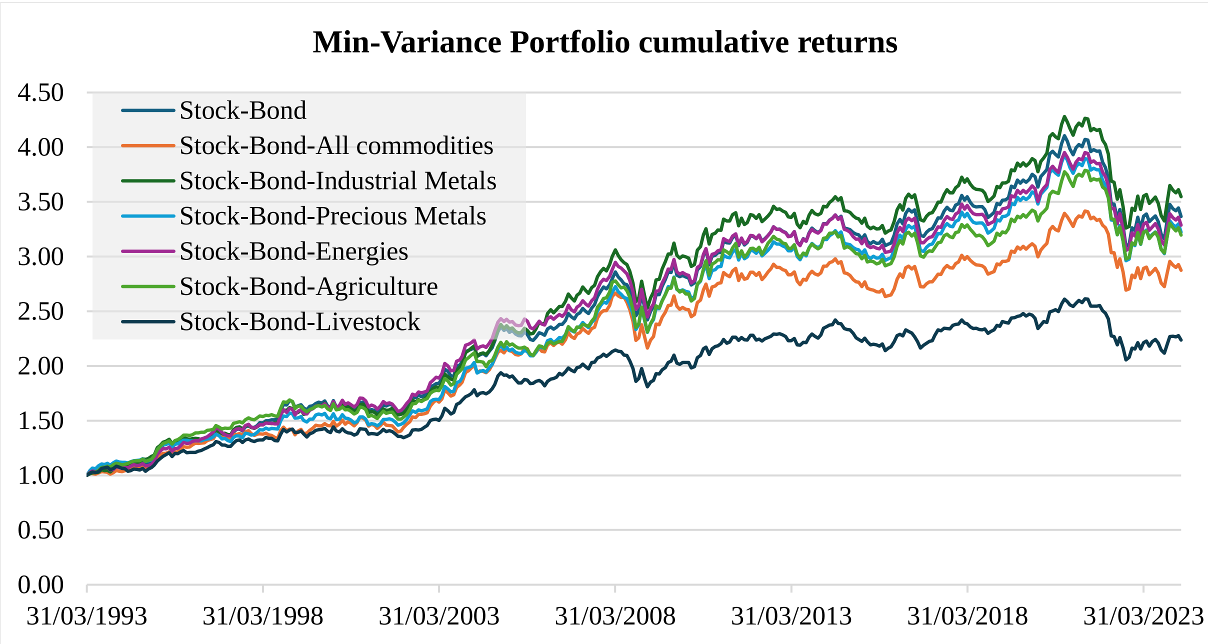

Fig. 4. Cumulative returns of the stock/bond and commodity portfolios under MVP allocation.

Portfolio performances causal to individual commodity index moves can best be observed under the 1/N portfolio (Figure 2) due to the high allocation, but evidently effect all portfolios. Namely, energies saw a rally in from 2003-08 following America's invasion of Iraq and the increasing global demand for oil. The volatility of energies can also be pictured by the much sharper drop in cumulative returns resulting from the GFC in 2008 compared to all other portfolios. Similar volatility shocks occur in agriculture and metals after Russia's invasion of Ukraine in 2022.

These events show that although individual commodities prices do appreciate due to structural supply constraints, idiosyncratic shocks are large dominators of their performance. These compliment negatively to a portfolio that relies on either steady corporate earnings growth in stocks, or coupon and principal obligations in bonds.

Subsection analysis

As suggested in the methodology, two portfolios have been selected to create a subsection analysis. The two portfolios selected based on earlier results were the 60/40 portfolio and MVP. This is due to their better performance over the whole period, and smaller allocation to commodities which would be better representative of an institutional or even retail investor's weighting. The Sharpe ratios of both portfolios in all six subsections is summarized in Table 6, the same tables for the annualized return and standard deviation can be found in Appendix C.

Table 6: Sharpe ratios for all portfolios in 60/40 and MVP by subsection.

		04/1993-05/1998	06/1998-07/2003	08/2003-09/2008	10/2008-11/2013	12/2013-01/2019	02/2019-03/2024
60/40							
	Stock-Bond	1.49	0.11	0.83	0.50	0.46	0.51
Stock-bond +	All commodities	1.47	**0.14***	**0.85***	0.41	0.32	**0.52***
	Industrial Metals	1.49	**0.13***	**0.96***	0.42	0.42	**0.52***
	Precious Metals	1.36	0.10	**0.94***	**0.53***	0.42	**0.54***
	Energies	1.35	**0.31***	0.80	0.31	0.20	0.49
	Agriculture	**1.67***	0.03	0.83	0.47	0.32	**0.58***
	Livestock	1.34	0.04	0.74	0.45	0.42	0.47
Min-variance							
	Stock-Bond	1.56	0.74	1.03	0.57	0.32	0.01
Stock-bond +	All commodities	1.41	0.68	**1.07***	0.43	0.15	**0.03***
	Industrial Metals	**1.59***	**0.76***	**1.19***	0.53	0.30	**0.02***
	Precious Metals	1.35	0.68	**1.09***	**0.58***	0.32	0.01
	Energies	1.45	**0.82***	**1.05***	0.49	0.23	0.01
	Agriculture	**1.99***	0.53	1.01	0.52	0.18	**0.08***
	Livestock	1.18	0.52	0.87	0.43	0.20	-0.07

The results do not deviate much from the overall sample period, but some takeaways are worth noting. Firstly, we can note heavy deviations in which portfolios outperform from one period to another, in fact almost every period saw a different commodity mix 'win' over the other portfolios. Otherwise, not a single portfolio outperformed the stock-bond portfolio between December 2013 and January 2019. This is due to the superior annualized returns. Although some commodity portfolios had a lower standard deviation during the period, not a single beat the benchmark portfolio in terms of returns. Furthermore, most portfolios – including the one supplemented by 'All commodities' – beat the benchmark in the last period, likely induced by supply shocks from geopolitical tensions (e.g. Russia's invasion of Ukraine) and structurally higher interest/inflation rates. Lastly, although livestock's Sharpe ratio is generally unimpressive, it has been able to decrease the standard deviation of every portfolio in all 6 periods – a feat worth noting. The 'All commodities' supplemented index only managed to achieve this in half of the subsections.

Value-at-Risk analysis

The results of the Value-at-Risk analysis is displayed in Table 7. Results which have a lower VaR in the portfolio supplemented by commodities are highlighted in bold and have the addition of a "*". These can be interpreted that they have a lower maximum monthly drawdown given the stated level of confidence.

Table 7: Value-at-Risk analysis

	Historical VaR		**Monte - Carlo VaR**	
	Stock-Bond	+ All commodities	Stock-Bond	+ All commodities
60/40				
VaR				
1%	-6.74%	**-6.64%**	-6.76%	**-6.63%**
5%	-4.77%	**-4.70%**	-4.78%	**-4.69%**
10%	-3.71%	**-3.66%**	-3.72%	**-3.65%**
25%	-1.96%	**-1.93%**	-1.96%	**-1.92%**
1/N				
VaR				
1%	-6.01%	-6.24%	-6.00%	-6.23%
5%	-4.25%	-4.41%	-4.24%	-4.41%
10%	-3.31%	-3.44%	-3.31%	-3.43%
25%	-1.74%	-1.81%	-1.74%	-1.81%
Risk Parity				
VaR				
1%	-4.95%	-5.14%	-4.95%	-5.16%
5%	-3.50%	-3.64%	-3.50%	-3.65%
10%	-2.73%	-2.83%	-2.73%	-2.84%
25%	-1.44%	-1.49%	-1.44%	-1.50%
Min-variance				
VaR				
1%	-4.44%	**-4.42%**	-4.43%	**-4.42%**
5%	-3.14%	**-3.13%**	-3.13%	**-3.12%**
10%	-2.45%	**-2.44%**	-2.44%	**-2.43%**
25%	-1.29%	**-1.28%**	-1.29%	**-1.28%**

The VaR was decreased with the addition of the BCOM index (All commodities) in two out of the four portfolios. This matches the prior observations of increased variance and general fluctuation for the 1/N and Risk Parity portfolios which assign a higher weight to commodities. However, this shows that small allocations to commodities can aid the risk profile of the overall portfolio.

The results from both the historical and Monte Carlo methods are very similar. The accuracy of the latter however can be seen as higher due to the large number of simulations which cover a larger variety of extreme cases. This is of course under the assumption that monthly returns of the portfolios are normally distributed.

Conclusion

Through extensive analyses of 28 portfolios with varying allocation strategies and constituent assets, it can be said with confidence that the allocation of commodities in a stock-bond portfolio in smaller weights (<15%) can indeed diversify to achieve a lower overall risk on both a VaR and standard deviation measure. Standard deviation was down 9 basis points[2] (bpt), and Monte Carlo VaR down 8 bps when this criterion was reached. However, it lacks the returns profile make the allocation worthwhile as a measure of risk adjusted returns. On average between the four allocation strategies, the inclusion of the BCOM index lowered the Sharpe ratio by 0.1, largely due to the average 9 bps decrease in annualized returns. Thus, we reject the first hypothesis of significant improvements in the Sharpe ratio as can be evidently shown through commodities underperformance in Table 4.

Commodities unique standing of being driven mainly by supply-demand disruptions instead of the steady earnings growth or bond obligations seen in the benchmark portfolio, means that their performance is more cyclical. The subsection analysis shows a different winner in almost every one of the six periods for Sharpe ratio, the only benchmark outperformance was between 2014-19 where it averaged 0.1 over all other portfolios. The only consistency is livestock beating the benchmark for standard deviation, decreasing it by an average of 65 bps. We thus accept the second hypothesis of diversification benefits being unequal throughout subsections.

Though the findings of the Value-at-Risk analysis did show a decrease in the max portfolio drawdowns for the two allocation tactics with less weight on commodities, the same cannot be said for the 1/N and Risk Parity portfolios. On average for all portfolios, the inclusion of commodities increases both the historical and Monte Carlo VaR by 0.075%. This very small increase and inconsistency between allocations means we cannot reject nor accept the third hypothesis of higher VaR in commodity portfolios.

A few findings also came from the study which should be mentioned. Commodities are shown to decrease the standard deviation of a stock-bond portfolio when the allocation remains small (see Table 4 or Appendix C). Thus, the competing return profiles of industrial and precious metals (6.6% and 5.4% respectively annualized) make them the leading commodities to include in smaller stakes for achieving higher risk adjusted returns, this is supported by both the cumulative returns of the portfolios and the Sharpe ratios of the metals throughout the study. Additionally, the underperformance of livestock from 1993-2024 (-3.8% annualized return) does not provide any evidence of it being a beneficial addition to a multi-asset portfolio under any allocation strategy or weight.

[2] Basis points (bps) represent a tenth of a percentage point. One bps is 0.01%.

To conclude, commodities variance in performance over periods and individual sub-indexes suggest that inclusion of them in a portfolio is better suited in times of known supply-demand imbalances, inflationary pressures or geopolitical risk. The inclusion of them intrinsically for the sake of diversification has been shown to not benefit investors, across multiple allocation strategies on a risk adjusted returns basis.

Discussion

The study provided a comprehensive overview of commodity performance since 1993. Most importantly, it adds to literature by tapping into data from the most recent inflationary pressures and high interest rates (most previous studies used data ending in the early 2010's). This study further adds to academic literature by using stock and bond data, which is not central to the United States - providing valuable insights to investors who come with a global perspective. Finally, commodities as an asset class do show opportunities for high growth and returns potential, studying their involvement in a portfolio through back-testing allows for better advised and safer investment decisions to be made.

A limitation of this study is the utilization of a single commodity family of indexes. The Bloomberg Commodity Index family does provide a succinct look at commodity prices, but any differences in methodology to other indexes would imply large price inconsistencies over the period of 30 years. Furthermore, indexes cannot be directly invested into, and their analysis ignores the transaction costs and management fees that may be charged through alternatives like Exchange Traded Funds (ETFs). Holding all fees as equal for the different asset classes, the study still reaches it's aim of comparing a regular stock bond portfolio to one supplemented with commodities. Finally, as previously mentioned, the derivation of the RPP and MVP both used all data available before the derivation date – going back to the four months of training data. This means that recent portfolios are trained on 30 years of covariance data, and the earliest ones on four months. It should be explored whether the use of rolling data, or other methods would be better at isolating clusters of volatility that can skew results in the long run.

The result of this study also provides the foundational work for what could be noteworthy research in allocation of multiple sub-groups of commodities (e.g., Precious and Industrial Metals or All Commodities Excl. Livestock) in a portfolio. Additionally, performance of portfolios which allocate only during periods of supply shortage, central bank tightening or inflationary pressures would all expand on the basis of these results quite well and would help explain the performance of some portfolios included.

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Appendix A

Table A1: Correlation of all indices

	Stocks	Bonds	All commodities	Industrial Metals	Precious Metals	Energies	Agriculture	Livestock
Stocks	1.00							
Bonds	0.041	1.00						
All commodities	0.328	0.097	1.00					
Industrial Metals	0.351	0.073	0.578	1.00				
Precious Metals	0.129	0.347	0.463	0.345	1.00			
Energies	0.227	0.005	0.835	0.264	0.187	1.00		
Agriculture	0.209	0.057	0.638	0.296	0.248	0.267	1.00	
Livestock	0.140	-0.034	0.230	0.118	0.053	0.116	0.141	1.00

Appendix B

Appendix B shows the cumulative returns of just the stock-bond portfolio and one supplemented with the BCOM index in all 4different strategies.

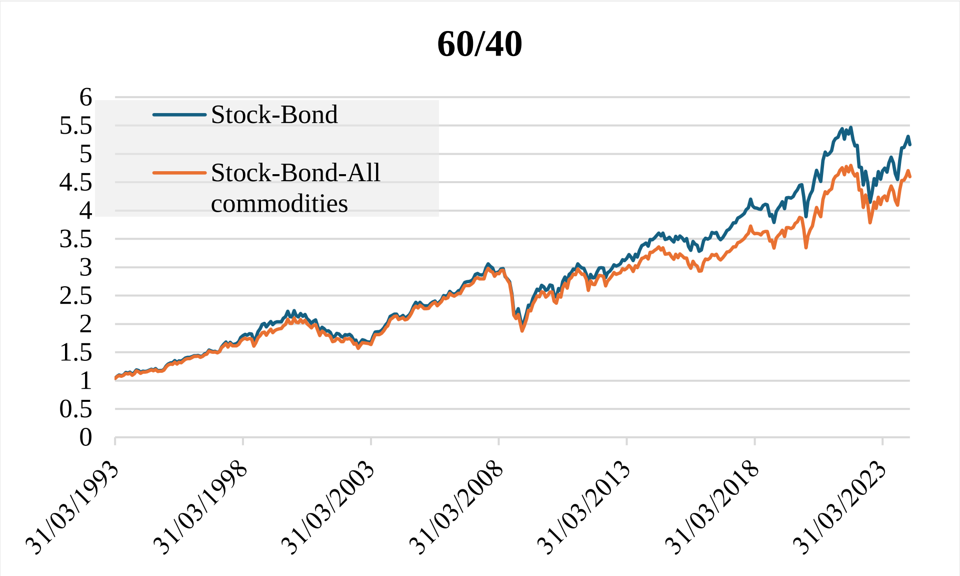

Figure B1. Cumulative returns of just stock/bonds against the supplement of BCOM in 60/40.

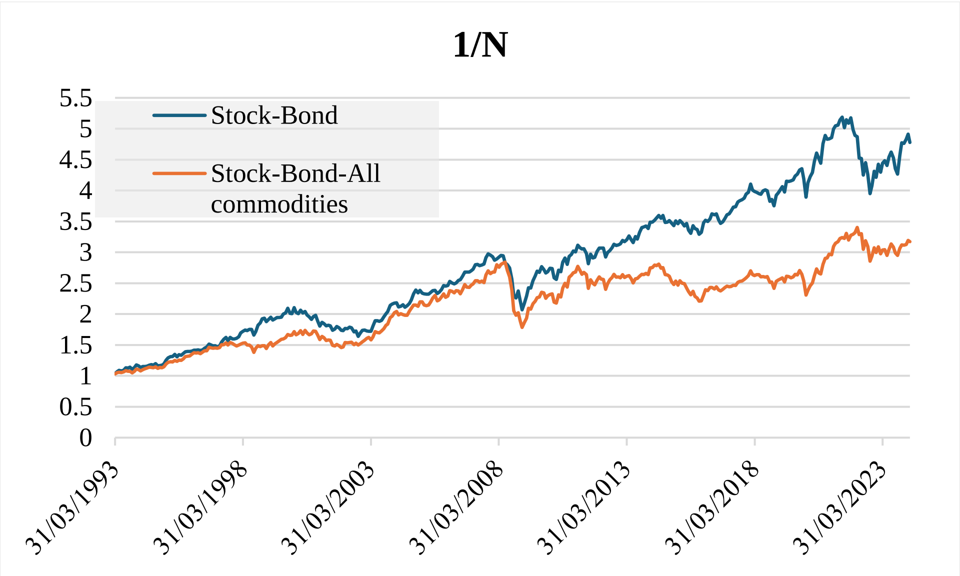

Figure B2. Cumulative returns of just stock/bonds against the supplement of BCOM in 1/N.

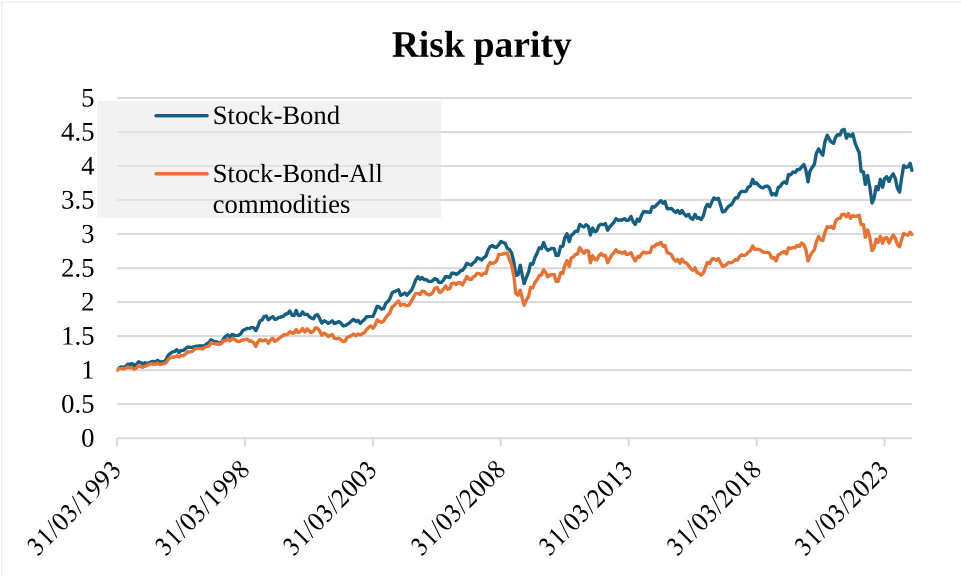

Figure B1. Cumulative returns of just stock/bonds against the supplement of BCOM under RPP.

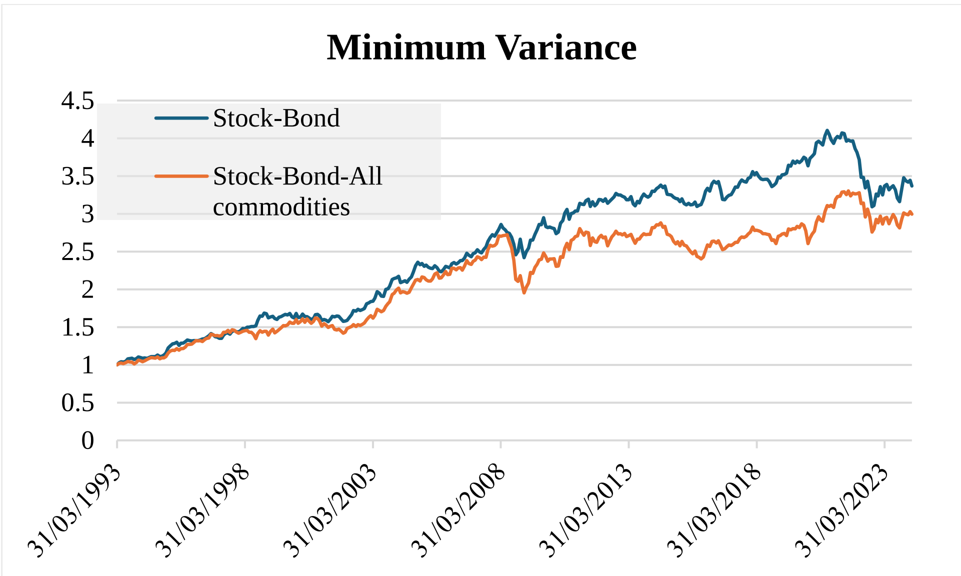

Figure B1. Cumulative returns of just stock/bonds against the supplement of BCOM in MVP.

Appendix C

Appendix C displays the annualized standard deviation and returns for the subsection analysis.

Table C1: Annualized return of sub-section analysis

		04/1993-05/1998	**06/1998-07/2003**	**08/2003-09/2008**	**10/2008-11/2013**	**12/2013-01/2019**	**02/2019-03/2024**
60/40							
	Stock-Bond	10.95%	1.23%	6.23%	6.58%	3.31%	6.37%
Stock-bond +	All commodities	10.11%	**1.45%***	**6.46%***	5.57%	2.26%	**6.40%***
	Industrial Metals	10.47%	**1.39%***	**7.90%***	5.84%	3.08%	**6.60%***
	Precious Metals	9.61%	1.05%	**7.15%***	**7.00%***	2.96%	**6.66%***
	Energies	10.07%	**3.39%***	**6.45%***	4.21%	1.56%	6.27%
	Agriculture	**11.00%***	0.26%	**6.42%***	6.33%	2.24%	**6.84%***
	Livestock	9.37%	0.36%	5.22%	5.42%	2.79%	5.65%
Min-variance							
	Stock-Bond	8.01%	4.88%	6.17%	4.57%	1.55%	0.11%
Stock-bond +	All commodities	6.20%	4.71%	**6.65%***	3.58%	0.73%	**0.25%***
	Industrial Metals	7.66%	**4.89%***	**7.66%***	4.34%	1.48%	**0.19%***
	Precious Metals	6.93%	4.42%	**6.76%***	**4.68%***	1.55%	0.11%
	Energies	7.69%	**5.72%***	**6.27%***	3.94%	1.15%	0.07%
	Agriculture	**8.55%***	3.41%	**6.40%***	4.39%	0.86%	**0.59%***
	Livestock	5.78%	3.16%	4.71%	2.95%	0.91%	-0.51%

Table C2: Annualized standard deviation of sub-section analysis

		04/1993-05/1998	**06/1998-07/2003**	**08/2003-09/2008**	**10/2008-11/2013**	**12/2013-01/2019**	**02/2019-03/2024**
60/40							
	Stock-Bond	7.36%	10.70%	7.52%	13.11%	7.16%	12.61%
Stock-bond +	All commodities	**6.86%***	**10.22%***	7.62%	13.44%	7.16%	**12.21%***
	Industrial Metals	**7.02%***	**10.65%***	8.21%	14.05%	7.30%	12.77%
	Precious Metals	**7.06%***	**10.05%***	7.63%	**13.09%***	**6.98%***	12.24%
	Energies	7.44%	10.78%	8.04%	13.60%	7.88%	12.80%
	Agriculture	**6.58%***	**10.22%***	7.78%	13.58%	**7.00%***	**11.75%***
	Livestock	**6.98%***	**9.81%***	**7.04%***	**12.11%***	**6.69%***	**11.96%***
Min-variance							
	Stock-Bond	5.14%	6.63%	5.98%	7.99%	1.55%	8.22%
Stock-bond +	All commodities	**4.40%***	6.92%	6.23%	8.31%	**0.73%***	**7.77%***
	Industrial Metals	**4.82%***	**6.40%***	6.46%	8.12%	**1.48%***	8.22%
	Precious Metals	5.15%	**6.50%***	6.22%	8.06%	1.55%	8.22%
	Energies	5.29%	6.95%	5.98%	8.02%	**1.15%***	**7.99%***
	Agriculture	**4.30%***	**6.46%***	6.35%	8.51%	**0.86%***	**7.69%***
	Livestock	**4.89%***	**6.11%***	**5.39%***	**6.91%***	**0.91%***	**7.33%***