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More efficient electric water heater models for demand response optimization

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Abstract

Demand response programs are widely used to consume electricity more efficiently and to decrease the electricity bill of households. Henggeler Antunes et al. (2022) proposes a formulation for different appliances, the appliance operation model is computationally expensive, mainly caused by the model for the electric water heater. This paper compares two alternative approaches for modelling the electric water heater. An adapted formulation is described and a simulated annealing heuristic is created. These approaches do not result in any improvement over the original model for the electric water heater. However, when the overall model with other appliances is considered the simulated annealing heuristic manages to obtain a better solution is less computation time. The original model still outperforms both approaches with more computation time.

1 Introduction

The growing usage of renewable energy sources is decreasing the dependence of fossil fuels in order to generate electricity for households. As of 2020 around 29% of global electricity is produced from renewable sources, not including nuclear (International Energy Agency, 2021). For the foreseeable future the generated electricity will be a mix of renewable sources and fossil fuels.

This leads to an increasing complexity of the electricity supply. The supply of electricity generated from fossil fuels is static and does not depend on external factors, while the supply of electricity generated from renewable sources is dynamic and depends on factors such as the time of day or the weather. In order to combat the fluctuations in energy supply many demand response programs have been developed.

These programs can be divided into incentive based, which pays consumers who decrease their electricity usage during peak hours, and price based, which charges different electricity prices to the consumer, depending on the electricity supply cost (Jordehi, 2019). Incentive based programs can be further divided into into direct load control, load curtailment, demand bidding emergency demand reduction, and price based programs can be further divided into time of use, real time pricing, critical peak pricing and inclining block rate. This paper focuses on one of these programs, namely time of use pricing, where the cost per kWh paid by the consumer depend on the time of day. The main question this paper aims to answer is 'How can we develop a more computationally efficient method for appliance operation, specifically focusing on electric water heaters, that maintains accuracy in meeting comfort and safety constraints within demand response programs?' In order to answer this question this paper describes two alternative approaches to the electric water heater model within the appliance operation model from Henggeler Antunes et al. (2022). One of these approaches is an adapted linear programming model, which uses a different formulation for calculating the water temperature, as well as formulating the requirement to heat the water in order to kill harmful bacteria in a different way. The second approach is a simulated annealing heuristic, which does not use a solver to obtain the optimal solution, but instead computes a good solution quickly using simulated annealing. The efficiency of these models is compared on their own to the original electric water heater model as well as in part of the overall appliance operation model.

Section 2 provides a review of relevant literature, Section 3 describes the problem this paper aims to solve in more detail. Section 4 provides an overview of the used data. Section 5 describes the methodology used in this paper to obtain the results. Section 6 summarises the results which have been obtained from the implemented methodology. Section 7 provides a conclusion on the paper.

2 Literature review

Demand response programs and their applications have been extensively studied and many different methods have been developed, this has lead to a general decrease in the load of the power grid in regions that are able to make use of these programs. These demand response programs are typically implemented in developed countries, where the infrastructure allows for such programs, but even in emerging countries there are opportunities to implement demand response programs (Martinez & Rudnick, 2012). Examples of methods for demand response that have been implemented include inverse optimization to design incentive based demand response programs (Murakami et al., 2017).

Home energy management systems are used by households in order to make use of the different response programs. These systems determine when to operate what appliances based on factors such as the current electricity price and user comfort variables. A widely used approach of minimising the electricity price using home energy management systems is by means of a Mixed Integer Linear Program (MILP). A straightforward implementation views all appliances as shiftable loads, which operation time can be shifted to a certain degree and is determined by the model, the model proposed in Javadi et al. (2021) describes one of these models, which can be solved efficiently. These models however lack the complexity to model some of the more complex appliances, such as the air conditioner or electric water heater, which operation cannot be shifted without impacting external factors. The appliance operation model described in Henggeler Antunes et al. (2022) accounts for these appliances by proposing a modular set of constraints for shiftable loads, an air conditioner, an electric water heater and a static and electric vehicle battery, as well as the option of selling back energy to the grid. The model schedules the operation times of the appliances such that the electricity cost is minimized whilst ensuring a certain comfort level is met, e.g. a minimum and maximum water temperature level. The electricity pricing structure consists of a time of use tariff, supplemented with a fixed price paid per power level, which increases as more electricity is used in total throughout the day.

The model proposed by Henggeler Antunes et al. (2022) is effective in decreasing the electricity bill of households, but it is computationally expensive. Implementing this model in household management systems would require expensive hardware, which increases the prices and limits the implementation of these household management systems, especially in less developed regions where it is crucial to keep the costs of these home energy management systems as low as possible.

The complexity of the model is caused by the electric water heater, without the electric water heater the model would be solved to optimality within seconds (Henggeler Antunes et al., 2022). Much research has been done on the modelling of electric water heater and alternative models and methods have been developed, such as a genetic algorithm (Lin et al., 2017), which shows a huge decrease in electricity costs compared to conventional electric water heater systems. Wu et al. (2019) describes a linear approximation of a non-linear model for electric water heaters, which can be solved within a minute. The improved performance of the model from Wu et al. (2019) is partially caused by a decrease in realism. The model from Henggeler Antunes et al. (2022) assures the water inside the tank is heated up above a certain temperature in order to kill harmful bacteria, while this model does not account for this. There is currently no optimization model for the electric water heater which is as realistic as Henggeler Antunes et al. (2022) and can be solved efficiently.

3 Problem statement

This paper aims to find a more efficient appliance operation model over the model proposed in Henggeler Antunes et al. (2022) where the distinction is made between four different appliance types. These types are shiftable loads, electric water heater, air conditioner and static and electric vehicle battery. These appliances are combined into one model, with the objective to satisfy a certain level of service for the household which these appliances reside in, while minimising the total energy costs. The energy cost follows a time of use tariff, where the price paid per kWh of electricity is determined by the time of day and is known beforehand. The model considers a time span of one day, and determines the operation of the appliances per minute. The following paragraphs explain each of the appliances and the complete appliance operation model in more detail.

Shiftable loads

Shiftable loads are determined by appliances which only need to be operational for a set time and their performance is independent on the time they operate. These appliances also have a uninterruptible operation cycle. Appliances of these types include a laundry machine and dishwasher for example. The model determines when the appliances should be used under the constraints that each appliance should only be used within a comfort time slot, as well as accounting for differing electricity draws for each operation stage of each appliance.

Electric water heater

The model for the electric water heater determines when the heater should be enabled or disabled, whilst ensuring the temperature of the water is always between predefined bounds. The water temperature is only allowed to fall below the minimum temperature if the electric water heater is enabled and similarly the water temperature is only allowed to rise above the maximum temperature if the electric water heater is disabled. This distinction is necessary, as the model would otherwise be infeasible if the starting temperature would be too high or if the temperature would be too low due to excessive water consumption. The model does also ensure the water is heated up to a certain temperature for a certain amount of time in order to kill harmful bacteria such as legionella.

Air conditioner

The model determines when the air conditioner should be enabled or disabled, it ensures the indoor temperature stays between defined boundaries, the model does also account for the thermal characteristics of the building. Additionally the air conditioner is only allowed to start operating if the indoor temperature falls below the minimum allowed temperature, and is only allowed to stop operating once the indoor temperature has exceeded the maximum allowed temperature. This restriction limits the amount of operation switches and increases the lifespan of the air conditioner.

Static and electric vehicle battery

The model allows the static and electric vehicle battery to be used to store energy, the electric vehicle battery is only available when the vehicle is present and has to be charged to a certain amount before departure, the amount of energy that the electric vehicle uses on its trip is known. The static battery is available at any time and does not require a certain charge level.

Appliance operation model

The overall model ensures all conditions for the appliances are met, whilst minimizing total costs, additionally the model accounts for the possibilities of local electricity generation, e.g. through solar panels, and to sell electricity back to the grid for a constant price. The amount of electricity that is locally generated differs depending on the time of day, but is known beforehand. The price at which electricity can be sold back to the grid is a fraction of the weighted average buying price.

4 Data

In order to accurately compare the efficiency of the model this paper makes use of the same data as in Henggeler Antunes et al. (2022). This data is obtained from field audits and actual equipment technical specifications and contains information on the pricing structure, consumption and comfort parameters of the household, temperature and parameters of the different types of appliances. This paper assumes temperature and household consumption are deterministic and known beforehand.

5 Methodology

The demand response models for shiftable loads, air conditioner and static and electric vehicle batteries are equivalent to the models described in Henggeler Antunes et al. (2022). As these can already be solved to optimality within seconds, there is currently little need to improve these models. The model for the entire household management system, which schedules the appliance operation, as well as a power component remains unchanged as well, this model obtains the optimal solution in seconds without considering the electric water heater (Henggeler Antunes et al., 2022), but takes over an hour including the electric water heater model. In order to improve the efficiency of the this model it is intuitive to improve the most inefficient aspect, which is clearly the electric water heater. This paper develops an adapted formulation and a simulated annealing heuristic for the electric water heater model. The following sections provide the models of the appliances used in Henggeler Antunes et al. (2022) in more detail, as well as the two alternative approaches for the electric water heater. The parameters T, Δt and C_t^{buy} are used in all models and denote the time horizon, the time step size and the buying price of electricity respectively.

The alternative approaches are compared to the original electric water heater model from Henggeler Antunes et al. (2022) as well as their implementation in the overall model, a more detailed description of how these different models are compared is given in section 5.8.

5.1 Shiftable loads model

Parameters

$$\begin{aligned} J &: \text{number of shiftable loads} \\ T(j) &: \text{allowed operation times for load } j \\ Tstart(j) &: \text{allowed starting times for load } j \\ R(j) &: \text{number of stages for load } j \\ T_{l_j} &: \text{earliest allowed starting time of load } j \\ g_{jr} &: \text{power requirement of load } j \text{ at stage } r \end{aligned}$$

Variables

$$\begin{aligned} s_{jt} &: \begin{cases} 1 \text{ if load } j \text{ starts operating at time } t \\ 0 \text{ otherwise} \end{cases} \\ P_{jt}^{Sh} &: \text{required power for load } j \text{ at time } t \end{aligned}$$

Formulation

$$\min \sum_{t=1}^{T} \sum_{j=1}^{J} C_t^{buy} P_{jt}^{Sh} \Delta t \tag{1}$$

$$\sum_{t \in Tstart(j)} s_{jt} = 1 \qquad j = 1...J \tag{2}$$

$$P_{jt}^{Sh} = \sum_{r \in R(j); r \leq t \wedge r \leq t+1-T_{L_j}} g_{jr} s_{j(t-r+1)} \qquad j = 1...J, t \in T(j) \tag{3}$$

$$P_{jt}^{Sh} = 0 \qquad j = 1...J, t \notin T(j) \tag{4}$$

$$s_{jt} \in \{0, 1\} \qquad j = 1...J, t \in Tstart(j) \tag{5}$$

Equation (1) is the objective function, which aims to minimize the total costs of operating the shiftable loads, equation (2) ensures every shiftable load is started exactly once. equation (3) sets the correct power level for each appliance at every time within the allowed operation times, for all other times the power level is set to zero, this is done by equation (4).

5.2 Electric water heater model

Parameters

$$
\begin{aligned}
P^R &: \text{power draw of the resistive heating element} \\
A &: \text{area of the tank} \\
U &: \text{heat transfer coefficient of the tank} \\
M &: \text{capacity of the tank} \\
\tau_t^{amb} &: \text{ambient temperature at time } t \\
m_t &: \text{water withdrawal at time } t \\
\tau^{net} &: \text{inlet water temperature} \\
c^p &: \text{specific heat of water} \\
\tau^{min} &: \text{minimum allowed water temperature} \\
\tau^{max} &: \text{maximum allowed water temperature} \\
t^{req} &: \text{required time to heat water to kill bacteria} \\
\tau^{req} &: \text{required temperature to kill bacteria}
\end{aligned}
$$

Variables

$$
\begin{aligned}
&v_t : \begin{cases} 1 \text{ if the electric water heater is turned on at time } t \\ 0 \text{ otherwise} \end{cases} \\
&P_t^{losses} : \text{amount of heat lost at time } t \\
&\tau_t : \text{water temperature at time } t \\
&n_t : \begin{cases} 1 \text{ if } \tau_t > \tau^{req} \text{ for the first time at time } t \\ 0 \text{ otherwise} \end{cases}
\end{aligned}
$$

Formulation

$$\min \sum_{t=1}^{T} C_t^{buy} P^R v_t \Delta t \tag{6}$$

$$P_t^{losses} = AU(\tau_t - \tau_t^{amb}) \qquad t = 1...T \tag{7}$$

$$\tau_{t+1} = (\frac{M - m_t}{M}\tau_t + \frac{m_t}{M}\tau^{net}) + \frac{P^R v_t - P_t^{losses}}{Mc^p}\Delta t \qquad t = 0...T-1 \tag{8}$$

$$\tau_t \geq \tau^{min} - Nv_t \qquad t = 1...T \tag{9}$$

$$\tau_t \leq \tau^{max} + N(1 - v_t) \qquad t = 1...T \tag{10}$$

$$\sum_{t=1}^{T-t^{req}+1} n_t = 1 \tag{11}$$

$$\tau_t \geq \sum_{t'=1(t' \leq t)}^{t^{req}} \tau^{req} n_{t-t'+1} \qquad t = 1...T \tag{12}$$

$$v_t, n_t \in \{0, 1\} \qquad t = 1...T \tag{13}$$

$$P_t^{losses} \geq 0 \qquad t = 1...T \tag{14}$$

$$\tau_t \geq 0 \qquad t = 1...T \tag{15}$$

Equation (6) is the objective function, which minimizes the total costs. Equation (7) calculates the correct amount of energy lost based on the water temperature and the ambient temperature, equation (8) sets the correct value of the water temperature at every time. Equation (9) ensures the water temperature can only fall below the minimum temperature if the electric water heater is turned on, equation (10) works similarly, but instead ensures the water temperature is only allowed to exceed the maximum temperature if the electric water heater is turned off, N is a sufficiently large number. Equation (11) ensures that there is exactly one time period where the water temperature is high enough to kill harmful bacteria for the first time. Equation (12) requires the temperature of the water to be above the required temperature for at least t^{req} periods before n_t is allowed to equal one.

5.3 Air conditioner model

Parameters

P^{AC} : power of the air conditioner heating element

β : heat loss coefficient, defined as $(UA/C)\Delta t$,

where UA is the thermal conductance of the building and C the thermal capacity

θ_t^{ext} : outside temperature at time t

γ : efficiency of the air conditioner

θ^{min} : minimum allowed indoor temperature

θ^{max} : maximum allowed indoor temperature

Variables

$$s_t^{AC} : \begin{cases} 1 \text{ if the air conditioner is turned on at time } t \\ 0 \text{ otherwise} \end{cases}$$

$$\theta_t^{in} : \text{indoor temperature at time } t$$

$$z_t : \begin{cases} 1 \text{ if } \theta_t^{in} > \theta^{min} \\ 0 \text{ otherwise} \end{cases}$$

$$y_t : \begin{cases} 1 \text{ if } \theta_t^{in} < \theta^{max} \\ 0 \text{ otherwise} \end{cases}$$

Formulation

$$\min \sum_{t=1}^{T} C_t^{buy} P^{AC} s_t^{AC} \Delta t \tag{16}$$

$$\theta_t^{in} = (1-\beta)\theta_{t-1}^{in} + \beta\theta_{t-1}^{ext} + \gamma P^{AC} s_{t-1}^{AC} \qquad t = 1...T \tag{17}$$

$$\theta_t^{in} \geq \theta^{min} - M s_t^{AC} \qquad t = 1...T \tag{18}$$

$$\theta_t^{in} \leq \theta^{min} + M z_t \qquad t = 1...T \tag{19}$$

$$\theta^{in} \geq \theta^{max} - M y_t \qquad t = 1...T \tag{20}$$

$$z_t + y_t - s_{t-1}^{AC} + s_t^{AC} \leq 2 \qquad t = 1...T \tag{21}$$

$$z_t + y_t + s_{t-1}^{AC} - s_t^{AC} \leq 2 \qquad t = 1...T \tag{22}$$

$$\theta_t^{in} \leq \theta^{max} + M(1 - s_t^{AC}) \qquad t = 1...T \tag{23}$$

$$s_t^{AC}, z_t, y_t \in \{0, 1\} \qquad t = 1...T \tag{24}$$

Equation (16) is the objective function, which aims to minimize the total costs of operating the air conditioner. Equation (17) sets the correct indoor temperature based on the previous temperature, the outside temperature and the enabled status of the air conditioner. Equation (18) ensure the air conditioner is turned on when the indoor temperature is too low. Equations (19) to (22) only allow the air conditioner to switch enabled status if the temperature falls outside the allowed range. Equation (23) ensure the air conditioner is turned off when the indoor temperature is above the maximum temperature.

5.4 Static and electric battery model

Parameters

$$\begin{aligned}
x &: \text{type of battery, either static } (B) \text{ or electric vehicle } (V)\\
\eta_x^{ch} &: \text{charging efficiency of battery } x\\
\eta_x^{dch} &: \text{discharging efficiency of battery } x\\
E_x^{min} &: \text{minimum allowed charge level for battery } x\\
E_x^{max} &: \text{maximum allowed charge level for battery } x\\
P_x^{ch_max} &: \text{maximum charging power for battery } x\\
P_x^{dch_max} &: \text{maximum discharging power for battery } x\\
E_B^{req} &: \text{required charge level at the end of the time horizon for the static battery}\\
E_V^{req} &: \text{required charge level at the time of departure for the electric vehicle battery}
\end{aligned}$$

Variables

$$\begin{aligned}
&P_t^{x2H} : \text{power flow from battery } x \text{ to home grid}\\
&P_t^{H2x} : \text{power flow from home grid to battery } x\\
&E_{x,t} : \text{charge level of battery } x\\
&s_t^{x2H} : \begin{cases} 1 \text{ if battery } x \text{ is discharging}\\ 0 \text{ otherwise}\end{cases}\\
&s_t^{H2x} : \begin{cases} 1 \text{ if battery } x \text{ is charging}\\ 0 \text{ otherwise}\end{cases}
\end{aligned}$$

Formulation

$$\min \sum_{t=1}^{T} C_t^{buy} P^{H2x} \Delta t \tag{25}$$

$$E_{x,t} = E_{x,t-1} + (\eta_x^{ch} P_t^{H2x} \Delta t) - (\frac{P_t^{x2H} \Delta t}{\eta_x^{dch}}) \qquad t \in T_x, x \in \{B, V\} \tag{26}$$

$$E_x^{min} \leq E_{x,t} \leq E_x^{max} \qquad t \in T_x, x \in \{B, V\} \tag{27}$$

$$0 \leq P_t^{H2x} \leq P_x^{ch\_max} s_t^{H2x} \qquad t \in T_x, x \in \{B, V\} \tag{28}$$

$$0 \leq P_t^{x2H} \leq P_x^{dch\_max} s_t^{x2H} \qquad t \in T_x, x \in \{B, V\} \tag{29}$$

$$s_t^{H2x} + s_t^{x2H} \leq 1 \qquad t \in T_x, x \in \{B, V\} \tag{30}$$

$$E_{B,T} \geq E_B^{req} \tag{31}$$

$$E_{V,t_d} \geq E_V^{req} \tag{32}$$

$$s_t^{H2x}, s_t^{x2H} \in \{0, 1\} \qquad t \in T_x, x \in \{B, V\} \tag{33}$$

Equation (25) is the objective function, which aims to minimize the total costs, equation (26) sets the correct charge level for the batteries. Equation (27) ensures that the charge level does not exceed the bounds, equations (28) and (29) sets the power flow to be at most the maximum allowed flow, and only allows power to flow to the batteries if it they are charging and power to flow to the home if the batteries are discharging. Equation (30) ensures that the batteries cannot both charge and discharge at the same time, equations (31) and (32) make sure that the charge level of the battery is at least the required level at the end of the time horizon or at the time of departure.

5.5 Appliance operation model

Parameters

C^{sell} : price of selling electricity back to the grid

L : set of power levels

c_l^{Cont} : price of power level l

P_l^{Cont} : maximum allowed power for power level l

P^{G_max} : maximum power flow

B_t : base load at time t

P_t^{PV} : local power generation at time t

J : set of shiftable loads

P^{AC} : power of the air conditioner heating element

P^{R} : power of the electric water heater heating element

Variables

P_t^{G2H} : power flow from grid to home at time t

P_t^{H2G} : power flow from home to grid at time t

$$u_l^{Cont} : \begin{cases} 1 \text{ if power level } l \text{ is the maximum power level} \\ 0 \text{ otherwise} \end{cases}$$

$$s_t^{G2H} : \begin{cases} 1 \text{ if power is flowing from grid to home} \\ 0 \text{ otherwise} \end{cases}$$

$$s_t^{H2G} : \begin{cases} 1 \text{ if power is flowing from home to grid} \\ 0 \text{ otherwise} \end{cases}$$

P_{jt}^{Sh} : power required by shiftable load j at time t

$$s_t^{AC} : \begin{cases} 1 \text{ if the air conditioner is turned on} \\ 0 \text{ otherwise} \end{cases}$$

$$v_t : \begin{cases} 1 \text{ if the electric water heater is turned on} \\ 0 \text{ otherwise} \end{cases}$$

P_t^{H2B} : power flow from home to static battery at time t

P_t^{B2H} : power flow from static battery to home at time t

P_t^{H2V} : power flow from home to electric vehicle battery at time t

P_t^{V2H} : power flow from electric vehicle battery to home at time t

Formulation

The formulation of the appliance operation model consists of equations (34) to (42) which models the power component and the power flow from the grid to the home, as well as equations (2) to (5) for the shiftable loads, equations (7) to (15) for the electric water heater, equations (17) to (24) for the air conditioner and equations (26) to (33) for the static and electric vehicle battery.

$$\min \sum_{t=1}^{T}((C_t^{buy}P_t^{G2H}\Delta t) - (C^{sell}P_t^{H2G}\Delta t)) + \sum_{l=1}^{L}(c_l^{Cont}u_l^{Cont}) \tag{34}$$

$$P_t^{G2H} \leq \sum_{l=1}^{L} P_l^{Cont}u_l^{Cont} \qquad t = 1...T \tag{35}$$

$$\sum_{t=1}^{L} u_t^{Cont} = 1 \tag{36}$$

$$0 \leq P_t^{G2H} \leq P^{G\_max}s_t^{G2H} \qquad t = 1...T \tag{37}$$

$$0 \leq P_t^{H2G} \leq P^{G\_max}s_t^{H2G} \qquad t = 1...T \tag{38}$$

$$s_t^{G2H} + s_t^{H2G} \leq 1 \qquad t = 1...T \tag{39}$$

$$P_t^{G2H} - P_t^{H2G} + P_t^{PV} =$$

$$B_t + \sum_{j=1}^{J} P_{j,t}^{Sh} + P^{AC}s_t^{AC} + P^{R}v_t + (P_t^{H2B} - p_t^{B2H}) + (P_t^{H2V} - p_t^{V2H}) \qquad \forall t \in T_v \tag{40}$$

$$P_t^{G2H} - P_t^{H2G} + P_t^{PV} =$$

$$B_t + \sum_{j=1}^{J} P_{j,t}^{Sh} + P^{AC}s_t^{AC} + P^{R}v_t + (P_t^{H2B} - p_t^{B2H}) \qquad \forall t \in T \setminus T_v \tag{41}$$

$$u_t^{Cont} \in \{0, 1\} \qquad l = 1...L \tag{42}$$

$$\tag{43}$$

Equation (34) is the objective function, which minimizes the total cost of operating the appliances, the costs consist of three parts, the bought electricity, the sold electricity and the power level. Equations (35) and (36) are responsible for setting the correct power level, equations (37) and (38) ensure the amount of flow does not exceed the maximum flow. Equation (39) makes sure that flow is only flowing in one direction at a time. Equations (40) and (41) set the correct amount of power to be used, these equations only differ depending on the availability of the electric vehicle.

5.6 Adapted Electric water heater model

The alternative model for the electric water heater is described in equations (44) to (52), the calculation of the water temperature is based on the model from Wu et al. (2019), which uses linearization techniques to transform a non-linear electric water heater model to a MILP problem. In order to maintain the same level of realism as the model from Henggeler Antunes et al. (2022) additional constraints to ensure harmful bacteria are killed by high water temperatures are added to the model. The model is specified as follows:

Parameters

$$
\begin{aligned}
c^p &: \text{specific heat of the water} \\
\rho &: \text{density of water} \\
R &: \text{water heater thermal resistance} \\
M &: \text{tank capacity} \\
\eta &: \text{efficiency of the electric water heater} \\
P^R &: \text{power of the resistive heating element} \\
\delta &: \text{the length of one time unit} \\
m_t &: \text{water withdrawal at time } t \\
\tau^{max} &: \text{the maximum allowed temperature of the water} \\
\tau^{net} &: \text{the inlet water temperature} \\
\tau^{req} &: \text{the required temperature to kill bacteria} \\
t^{req} &: \text{the required time the water has to be above the required temperature to kill bacteria}
\end{aligned}
$$

Variables

$$
\begin{aligned}
&\tau_t : \text{the temperature of the water at time } t \\
&v_t : \begin{cases} 1 \text{ if the electric water heater is turned on} \\ 0 \text{ otherwise} \end{cases} \\
&\alpha_t : \begin{cases} 1 \text{ if } \tau_t > \tau^{req} \\ 0 \text{ otherwise} \end{cases} \\
&\beta_t : \begin{cases} 1 \text{ if } \tau_t > \tau^{req} \text{ for period } t \text{ and all the } t^{req} - 1 \text{ following periods} \\ 0 \text{ otherwise} \end{cases}
\end{aligned}
$$

Formulation

$$\min \sum_{t=1}^{T} C_t^{buy} p^R \Delta t v_t \tag{44}$$

$$\tau^{max} \geq \tau_t \geq \tau^{net} \qquad t = 1...T \tag{45}$$

$$\tau_{t+1} =$$

$$(1 - \frac{\delta}{c^p \rho R M})\tau_t + \frac{\Delta t \mu P^R}{c^p \rho M} v_t + \frac{\delta}{c^p \rho M R}\tau_t^{amb} + \frac{\delta}{M}(\tau^{net} - \tau_t)m_t \qquad t = 0...T-1 \tag{46}$$

$$\tau_t \geq \tau^{min} - N v_t \qquad t = 1...T \tag{47}$$

$$\tau_t - \tau^{req} \leq N \alpha_t \qquad t = 1...T \tag{48}$$

$$\tau^{req} - \tau_t < N(1 - \alpha_t) \qquad t = 1...T \tag{49}$$

$$\sum_{i=1}^{t^{req}-1} \alpha_{t+i} \geq t^{req} \beta_t \qquad t = 1...T - t^{req} + 1 \tag{50}$$

$$\sum_{t=1}^{T-t^{req}+1} \beta_t \geq 1 \tag{51}$$

$$v_t, \delta_t, \alpha_t, \beta_t \in \{0, 1\} \qquad t = 1...T \tag{52}$$

The only parameter values that are not defined in the dataset (Henggeler Antunes et al., 2022) are δ and R. δ is the length of one time unit, as the time horizon is divided into 1440 equal units each unit has a length of one minute. So the value of δ equals 1. The water heater thermal resistance R is set as 17.922 Kday/kWh, this value has lead to the best results in previous research (Nel et al., 2018). In the formulation described above equation (44) is the objective function, which aims to minimize the total costs of operating the electric water heater throughout the day. Equation (45) limits the temperature of the water to be between the temperature of the inlet water and maximum allowed temperature of the water inside the tank. Equation (46) calculates the temperature of the water inside the tank for the next time interval. The constraints account for heat loss through the tank and inefficiency of the electric water heater unit. Equation (47) sets the minimum temperature of the water and allows the temperature to be lower if the electric water heater is enabled, it is not possible to simply replace τ^{net} by τ^{min} in Equation (45) as the water temperature falls sharply at times of consumption. Equations (48) and (49) ensure that the value of α_t is set correctly for every time period. Equation (50) only allows β_t to equal 1 if the temperature of the current period and all $t^{req} - 1$ periods afterwards is above the required temperature to kill the bacteria. Equation (51) forces the solution to have at least one sequence of t^{req} periods where the temperature is high enough to kill the bacteria. Equation (52) simply puts the binary constraints on the variables.

5.7 Simulated annealing heuristic

The second approach used to efficiently obtain a solution for the electric water heater is a simulated annealing heuristic. The general procedure of this heuristic is to first obtain an initial feasible solution, which is set as the current best solution, after which new solutions are generated which are slightly different from the current best solution in order to find better solutions, a new solution is always accepted as the new current best objective if the solution is feasible and if the objective value is lower, in order to escape a local minimum the solution also has a chance to be accepted if it is feasible, but does not have a lower objective value. Infeasible solutions are discarded and a new solution is generated to replace the infeasible solution. The probability of a worse feasible solution being accepted is generally high at the start, but decreases over time. This decreasing likelihood of accepting worse solutions is aimed to escape local minima and instead end up in the global minima. This method mimics methods in metallurgy, on which the simulated annealing heuristic was originally based (Tsallis & Stariolo, 1996). Simulated annealing typically outperforms heuristics where the objective value is only allowed to decrease, such as local search.

Algorithm 1 describes the procedure of the heuristic in more detail. The heuristic uses the following parameters:

- T_0: The initial temperature of the heuristic
- α: the cooling rate, i.e. the rate at which the temperature decreases

Algorithm 1 Simulated annealing heuristic for the electric water heater

```
bestSolution = getInitialSolution()
bestObjective = getObjectiveValue(bestSolution)
T = T_0
while Stopping condition not met do
    candidateSolution = getCandidate(bestSolution)
    if isFeasible(candidateSolution then
        candidateObjective = getObjectiveValue(candidateSolution)
        if candidateObjective < bestObjective then
            bestSolution = candidateSolution
            bestObjective = candidateObjective
        else
            Δ = candidateObjective - bestObjective
            if random() < exp(−Δ / T) then
                bestSolution = candidateSolution
                bestObjective = candidateObjective
            end if
        end if
    end if
    T = (1 − α)T
end while
```

The initial solution is generated by calculating the water temperature at time 1 if the electric water heater would be turned off at that time, if the temperature would fall below the required temperature to kill harmful bacteria the electric water heater is turned on at that time, this procedure is repeated for all following time periods. This ensures a feasible solution as long as the problem itself is feasible.

Candidate solutions are generated by enabling or disabling the electric water heater and random time points, each time a candidate solution gets generated there is a probability that the candidate solution attempts to fix any time periods where the water temperature is below the minimum temperature or above the maximum temperature, this increases the likelihood of a candidate solution being feasible and thus being considered as a possible improvement. The procedure of fixing a candidate solution is similar to generating the initial solution, the water temperature is iteratively calculated and the electric water heater is turned enabled or disabled if the water temperature exceeds the predefined bounds. The heuristic will continue to search for better solutions until the stopping condition is met, in order to easily compare the heuristic approach to the MILP models the heuristic continues searching for solutions until a predetermined time has passed.

5.8 Comparison of models

The adapted model and the simulated annealing heuristic are implemented as standalone models for the electric water heater, and they are also implemented in an overall model which includes shiftable loads, an air conditioner and a static and electric vehicle battery as well. The adapted model simply replaces the original electric water heater model in the overall model from Henggeler Antunes et al. (2022). The implementation of the simulated annealing heuristic into the overall model is less straightforward, as the heuristic and the overall model cannot be solved simultaneously. Instead the simulated heuristic runs first for a fraction γ of the allowed solving time. The obtained solution for the electric water heater is passed to the overall model, which sets the found solution for operating the electric water heater as the initial solution and then solves the appliance operation model with the remaining fraction $1 - \gamma$ of the available solving time to compute a solution.

In order to accurately compare the efficiency of the adapted formulation and the simulated annealing heuristic to the original electric water heater model their objective value and MILP gap are compared after solving times of 5, 10, 30 and 60 seconds, as well as 5, 10 and 60 minutes. These solving times provide a clear view of how the objective changes over time, which is essential as the efficiency of the models not only depend on the found objective value, but also on the time it takes to find good objective values.

6 Results

All the models have been implemented in Java using the Gurobi solver, the computations have been performed with an AMD Ryzen 7 5700U, 1.80 GHz, 16GB RAM. A brief description of the code used to obtain these results and the performed runs can be found in the appendix.

Table 1 displays the objective values for the shiftable loads, air conditioner and static and electric vehicle battery models. All of these models were solved to optimality within less than 5 seconds. The found objective values are equal to those of the same models in Henggeler Antunes et al. (2022) which serves as validation that these models are correct and that they can be used in the appliance operation models.

Table 1: Objective value of original models

Shiftable Loads	**Air Conditioner**	**Battery**
0.470	2.442	1.720

6.1 Electric Water Heater

The Electric Water Heater model from Henggeler Antunes et al. (2022) is computationally expensive, thus two alternative approaches for modelling the Electric Water Heater have been developed. The simulated annealing heuristic makes use of the parameters T_0 and α for the initial temperature and the cooling rate. The values 0.01 and $1 \cdot 10^{-5}$ for T_0 and α respectively are used as the initial values for the simulated annealing heuristic, these values ensure that the probability of accepting a worse solution is high enough at the start and decreases slowly. In order to obtain the optimal parameter values the heuristic is run with an allowed solving time of 60 seconds with these parameter values, as well as the values which differ 5, 10, 15 and 20 percent from the initial values. The results can be found in table 2, as can be seen from this table, the parameter values 0.0085 for T_0 and $1.15 \cdot 10^{-5}$ for α provide the best results, these values are thus further used for the simulated annealing heuristic.

Table 3 displays the objective value of the original model after different solving times, as well as the adapted model and the simulated annealing heuristic. The percentages indicate the MILP gap of these models after the given solving time has passed. The simulated annealing heuristic does not consist of an linear programming model and thus no lower bounds are calculated, which are necessary to compute the gaps, instead the gaps for the simulated annealing heuristic are calculated based on the lower bound obtained by the original model with the same solving time. The efficiency of the electric water heater, denoted by μ is set to 100% in order to match the heating power of the original model.

Table 2: Objective value simulated annealing heuristic with different parameters

		$\alpha \cdot \mathbf{10^{-6}}$								
		8	**8.5**	**9**	**9.5**	**10**	**10.5**	**11**	**11.5**	**12**
	0.008	1.309	1.312	1.312	1.323	1.318	1.305	1.310	1.314	1.320
	0.0085	1.307	1.310	1.306	1.305	1.312	1.321	1.305	**1.299**	1.316
	0.009	1.310	1.312	1.312	1.310	1.309	1.316	1.308	1.312	1.309
	0.0095	1.307	1.308	1.306	1.303	1.313	1.312	1.307	1.305	1.314
$\mathbf{T_0}$	**0.01**	1.313	1.307	1.306	1.323	1.311	1.314	1.310	1.321	1.307
	0.0105	1.320	1.311	1.314	1.314	1.313	1.313	1.305	1.327	1.307
	0.011	1.311	1.309	1.314	1.317	1.307	1.312	1.308	1.309	1.317
	0.0115	1.305	1.311	1.310	1.302	1.319	1.307	1.309	1.301	1.303
	0.012	1.303	1.311	1.307	1.311	1.318	1.305	1.307	1.310	1.315

The optimal values of the original model and the adapted model will most likely slightly vary, as both models use separate formulations for calculating the water temperature. Comparing the MILP gaps of these two models shows that the original model outperforms the adapted model for all solving times. The simulated annealing heuristic does not manage to find a better solution than the original model, this is most likely caused by the candidate solutions generated by the heuristic not being diverse enough to escape local minima or by the temperature and cooling value not allowing for different neighbourhoods to be accepted, further tuning of the heuristic and the set parameters could prevent this.

Table 3: Objective value of electric water heater models

Solving time	**Original Model**	**Adapted Model**	**Simulated Annealing**
5 sec	1.349 (6.95%)	1.704 (21.34%)	1.360 (7.72%)
10 sec	1.285 (1.95%)	1.698 (20.94%)	1.311 (3.89%)
30 sec	1.279 (0.21%)	1.401 (4.19%)	1.319 (3.26%)
60 sec	1.279 (0.19%)	1.386 (3.16%)	1.305 (2.14%)
5 min	1.279 (0.16%)	1.377 (1.03%)	1.309 (2.44%)
10 min	1.279 (0.16%)	1.375 (0.55%)	1.302 (1.92%)
60 min	1.279 (0.16%)	1.377 (0.98%)	1.297 (1.54%)

6.2 Appliance operation Model

In order to compare the different models the correct selling price of electricity needs to be determined first, the selling price is a fixed price (Henggeler Antunes et al., 2022). This value is determined as a fraction of the weighted average buying price. Table 4 displays the objective value of the original appliance model for different fractions of the selling price, these values have been obtained with a running time of 5 minutes each. The objective value of 4.098, obtained with a selling price fraction of 0.875 is the closest to the found objective in Henggeler Antunes et al. (2022), so this fraction is further used for the appliance operation models.

Table 4: Objective value of original model with different selling price fractions

Selling price fraction	**Objective value**
≤ 0.65	4.518
0.675	4.513
0.7	4.500
0.725	4.436
0.75	4.382
0.775	4.327
0.8	4.273
0.825	4.213
0.85	4.161
0.875	4.098
0.9	4.041
0.925	3.977
0.95	3.911
0.975	3.820
1	3.722

Figures 1 to 3 displays the found solution for the original appliance operation model with a solving time of 60 minutes. Figure 3 matches exactly with the results from Henggeler Antunes et al. (2022) whereas figures 1 and 2 are slightly different. These different results will most likely be caused by the unspecified selling price in Henggeler Antunes et al. (2022). This paper studies the efficiency of different approaches for the electric water heater model, which is unaffected by the power consumption of the appliance operation model and the battery charge levels, these differences are thus not problematic for the comparisons between electric water heater approaches.

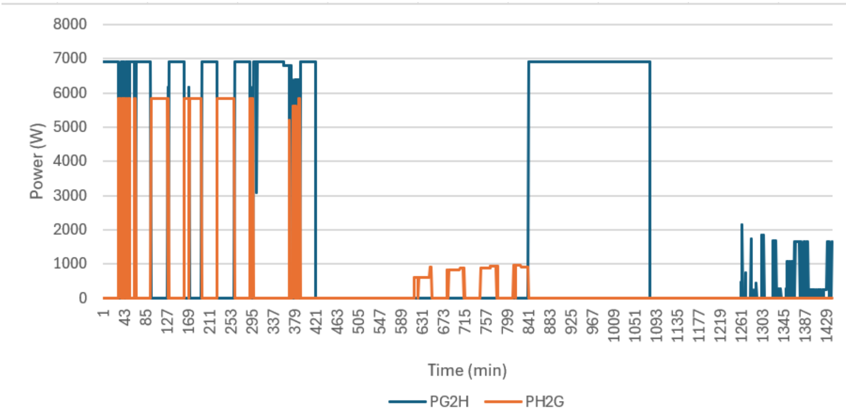

Figure 1: Power flow to and from the grid over time

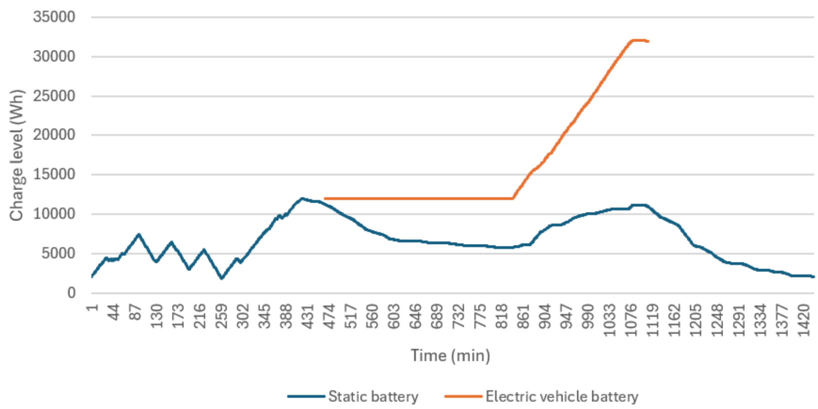

Figure 2: Charge level of static and electric vehicle battery over time

The appliance operation model with the simulated annealing heuristic for the electric water heater makes use of the variable γ in order to determine the fraction of time that should be spend on the heuristic, the remaining time will be spend on the appliance operation model. As can be seen from table 3 the simulated annealing heuristic continues finding improvements with longer solving time, however the improvements are minimal, and will most likely be outweighed by the appliance operation model. The majority of the solving time should thus be dedicated to the appliance operation model, the fraction of the time spend on the heuristic is set as 0.3.

Table 5 displays the objective value of the appliance operation models, as well as the MILP gap after each solving time. Both the adapted model and the simulated annealing model outperform the original model for solving times under five minutes. For longer solving time the original model performs generally better, this complies with the results from Table 3, as the original electric water heater model outperforms the two alternative approaches.

Figure 4 visualises the objective values for the three appliance operation models for the first 30 seconds. This figure further shows that the adapted model and the simulated annealing model outperform the original appliance operation model significantly with short time limits. Especially the simulated annealing model finds a good solution within a small time limit.

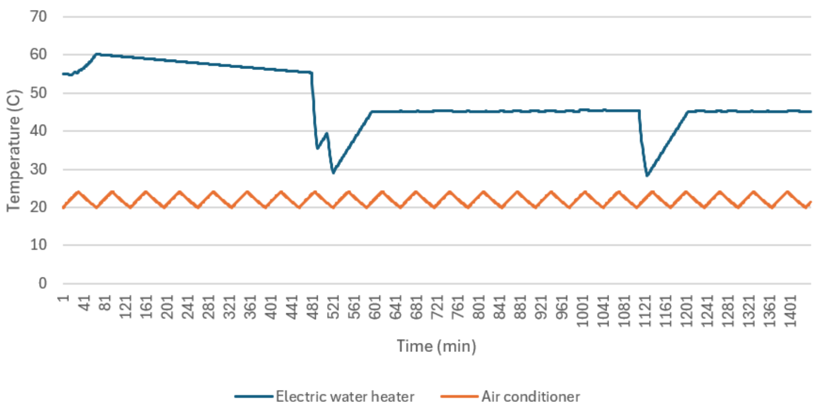

Figure 3: indoor and water temperature over time

Table 5: Objective value of overall models

Solving time	Original Model	Adapted Model	Simulated Annealing
5 sec	27.080 (85.66%)	27.080 (85.40%)	4.520 (-)
10 sec	27.080 (85.16%)	5.267 (22.46%)	4.345 (-)
30 sec	4.139 (1.78%)	4.412 (6.77%)	4.192 (3.33%)
60 sec	4.114 (1.07%)	4.401 (6.54%)	4.129 (1.53%)
5 min	4.104 (0.41%)	4.273 (3.73%)	4.105 (0.85%)
10 min	4.104 (0.40%)	4.273 (3.73%)	4.108 (0.92%)
60 min	4.104 (0.26%)	4.272 (3.72%)	4.107 (0.25%)

Table 6 displays the found objective values for the three appliance models with different comfort parameters, these parameter changes correspond with those from Henggeler Antunes et al. (2022). These values have been obtained with a solving time of 5 minutes, as can be seen from this table the simulated annealing performs slightly better than the original model.

Table 6: Results for different comfort parameters

Solving time	Original	Adapted	Simulated Annealing
Original model	4.104	4.273	4.105
$\theta^{min} = 19$	4.030	4.182	4.030
$\tau^{min} = 43$	4.040	4.125	4.036
$E_V^{req} = 30$	3.832	3.931	3.830
$\theta^{min} = 19$, $\tau^{min} = 43$	3.966	4.064	3.964
$\theta^{min} = 19$, $\tau^{min} = 43$, $E_V^{req} = 30$	3.694	3.845	3.694

7 Conclusion

This paper proposes two different approaches for the electric water heater model from Henggeler Antunes et al. (2022), which aim to improve the efficiency of this model. The first approach is a different linear programming formulation of the problem, the seconds approach is a simulated annealing heuristic to determine when the electric water heater should operate. Neither approaches have managed to obtain a better solution when only considering the electric water heater itself. When other appliances, which includes shiftable loads, an air conditioner, a static

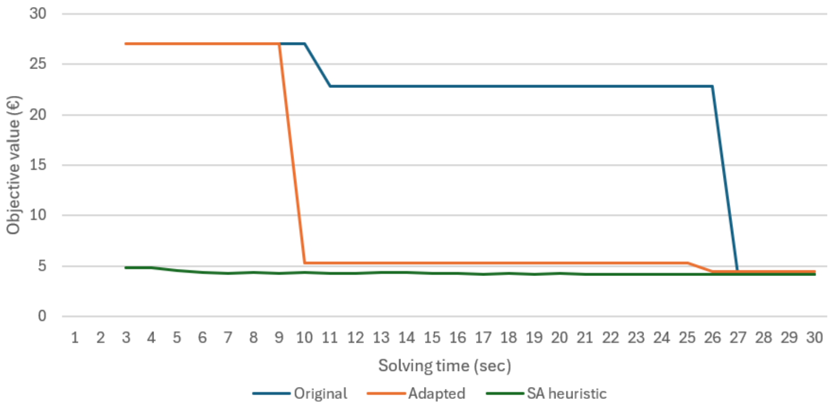

Figure 4: Objective value of overall models over time

and electric vehicle battery and a power component, are taken into account the simulated annealing heuristic outperforms both the original and the adapted model with less computation times, with longer computation times the original model still performs the best. Which model is the most efficient to use thus depends on how limited the available computational power is. With limited computation time the simulated annealing model can be used to more efficiently obtain solutions for the appliance operation. This approach could be further researched, perhaps with different heuristics for the electric water heater, which may in turn lead to even better results for the overall model. The simulated annealing heuristic itself could also be improved, as it currently does not manage to find the global minimum of the electric water heater.

A Explanation of code

The results from this paper have been obtained with use of Java and the commercial Gurobi solver. All data from Henggeler Antunes et al. (2022) has been manually copied to an Excel document, which can be loaded with the DataReader class. Once loaded in all the data is stored within the DataSet class and can be accessed easily. The parameters are stored in subclasses in order to make the code more readable. Each of the implemented models is its own class which inherits of the general Model class. This class consists of a Gurobi model and provides all the functionality that is shared by all MILP models, this has been done to reduce the amount of repetitive code. This class holds the Gurobi model variables and provides a clear interface for the SolutionWriter class to write a solution to a file. Each model can be optimised by using the solve method, and automatically reports the found solution, the objective value and the MILP gap in an excel file if a solution has been found. The simulated annealing model does not consist of a Gurobi model and is thus implemented separately. The public methods are the same as for the other models. The provided code has been set up in such a way that simply running the program will compute all results in Tables 1 to 6 and figures 1 to 4, this will however take over 9 hours.

B Performed runs

In order to obtain the results from tables 1, 3 and 5 all 9 models have been run for 5 seconds, 10 seconds, 30 seconds, 60 seconds, 5 minutes, 10 minutes and 1 hour. The solving times below 60 seconds have been chosen to accurately determine how the efficiency of the different models differ for low time limits, the solving times of 5 and 60 minutes serve to make comparisons to Henggeler Antunes et al. (2022) easier, as the same time limits are used in that paper.

The results from tables 2, 4 and 6 have been obtained with a solving time of 60 seconds, 5 minutes and 5 minutes respectively. The random seed, which is used in the simulated annealing heuristic is set as 0 for all of the runs. The results in Figure 4 has been obtained by running the original appliance operation, adapted appliance operation and the simulated annealing model for every time $t \in 1...30$ and plotting the results.

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