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What is the most efficient way to heat houses with a heat pump in the Netherlands with fluctuating energy prices?

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Abstract

Producing efficient schedules for heating houses is time consuming, but also of great importance for households and society. With increasingly fluctuating energy prices due to the transition to renewable energy resources, an efficient distribution of the energy is necessary. Not only is it financially important, the efficient distribution of energy creates great environmental benefits. In this paper a model is presented that demonstrates how to efficiently heat Dutch terraced houses. The model is a Mixed Integer Linear Program that can create useful heating schedules for Dutch houses within 2 hours. The final model is an extension of the model presented in Antunes et al. (2019), the model is made suitable for heat pumps, different temperature profiles throughout the day, and for a limited number of on/off switches within a day. In this paper results are presented that are more than 40% better than results retrieved from the old model. This research therefore provides great insights for the future and is most relevant in the current time.

1 Introduction

There are around 8 million houses in the Netherlands, and keeping these houses at a liveable temperature demands great energy. It is therefore of great importance that we effectively do this with as much renewable energy as possible. That is why this research will focus on using heat pumps to keep houses at temperature in an optimal way.

Optimally producing heating schedules demands lots of work, which is why Mixed Integer Programming models are designed to solve this problem. The goal is to create a model that can come up with useful schedules in a limited time so that tomorrow's heating schedules can be created once the day-ahead energy prices are known.

In this research, I will extend the research done in Antunes et al. (2019) on thermostatic loads of air conditioning systems in large buildings. I will focus on the Dutch market, specifically standard Dutch terraced houses. The overarching extension of my research will be the switch from Air Conditioning (AC) systems to Heat Pumps (HP), and besides that, there will be extensions to increase the effectiveness and applicability of the model. The main research question will therefore be:

What is the most efficient way to heat houses with a heat pump in the Netherlands with fluctuating energy prices?

I chose this direction for my research because heat pumps are more representative of the future of this field than air conditioning systems. The Coefficient Of Performance (COP) of heat pumps is higher than the COP of AC systems, which means that it is more sustainable and with that also less expensive. The research done in the Antunes et al. (2019) paper is useful but misses some applicability for the future, emphasising this research's relevance.

In Sect. 2, the previous literature on this topic is discussed and reviewed. The models used in this research are presented and explained in Sect. 3. In Sect. 4 information is provided on the Data used to generate the results presented in Sect. 5. In Sect. 6 the conclusions of the research are stated together with suggestions for further research.

2 Literature Review

Much research has already been conducted on how to effectively heat houses, and on how to make schedules for this. This is due to the relevance and the possibilities with it. The problem is often described as a Mixed Integer Linear Programming (MILP) problem due to the nature of the problem. There is always a device or system that is used for heating and this system is either on or off, that is where a binary variable brings integers into the problem. For example, the research done in Halvgaard, Poulsen, Madsen and Jørgensen (2012) which also considered heat pumps for heating, or the research done in Ali, Jokisalo, Siren, Safdarian and Lehtonen (2015) where a model is proposed in which a loss function is minimised that is dependent on the electricity prices, thermal comfort and exposure to riskiness in price. In both works the problem is dealt with as a Mixed Integer Problem.

There is much research in this domain with MILP that is focused on large buildings, these models do mostly not use heat pumps but air conditioning systems. In Yoon, Kang and Moon (2020) for example is a model introduced to minimise the electricity expense for HVAC, this is a model for large office buildings. Also in Antunes et al. (2019) the model is built for large buildings. This research will focus on using the techniques and ideas of this research but then adjusting it to terraced houses to gain insight into how to effectively heat.

Much scientific research is available emphasising the benefits of heat pumps in different situations. For example Qiao et al. (2020), in which the performance of a ground-source heat pump is presented, and the work of Bakirci and Yuksel (2011) that presents the performance of heat pumps for residential heating. In Fei, Li, Shilin and Xu (2010) they compare heat pumps to air conditioning systems and evaluate the applicability for large buildings. In all scientific work that is presented on this subject, it is mentioned how important it is to switch to this way of heating. This has multiple reasons, of which the first is that heat pumps are fully electric. With the decreasing prices for wind and solar power, heating houses with electricity is the best option for the future both financially and morally. Besides that, the COP of a heat pump is higher than the COP of an Air Conditioning system.

In this research I build further on the work done in Antunes et al. (2019) because of the promising results that were presented. More specifically, I will extend the third MILP formulation the authors present. In the paper, multiple models are presented and evaluated. The third model proposes a model in which the heating system can be exploited in five power settings, 20%, 40%, 60%, 80%, and 100%. The model is tested with variable energy tariffs and shows that we can benefit greatly from this. We will extend this model to see if for heating houses we can also benefit from the varying electricity prices.

The research in this paper is relevant because it will extend a model of the Antunes et al. (2019) paper with promising results to a more useful and effective model for the future. It also investigates further how variable prices can be beneficial and used efficiently to heat houses. The problem being dealt with is of great importance for the future of renewable energy and with that the planet.

3 Models

In the upcoming subsections, the main idea of the extensions will be presented. The first subsection explains how the third model of Antunes et al. (2019) can be transformed into a suitable model for heat pumps in Dutch terraced houses. Then the second subsection provides an idea of extending the model to one with possible different temperature profiles throughout the day, and in the last subsection, an extension of the model is presented that limits the on/off switching of the heat pump.

3.1 Air Conditioning to Heat Pumps

To change the model from air conditioning systems to heat pumps, we need to analyse the differences. The conclusion is that the difference is in the model parameters, an air conditioning system does not have the same parameters as a heat pump, but the model stays the same. In this subsection, an explanation of these parameters, and how they change, is provided.

Underneath this section, an outline of the model is presented. The only visible change here opposed to the model presented in Antunes et al. (2019), is that we now consider the P^{HP}, the heat pump's power. However, there are other changes hidden in the parameters α, β, and γ, these adaptations will be discussed in the next paragraph. For the power of an average heat pump present in a standard Dutch terraced house, we consider a value of 6 kW, the power of the Remeha Elga Ace 6kW.

Besides the parameter changes due to the heat pump we change another term in equation 2, we add the wind speed at time t to get the effective outside temperature at time t, θ_t^{extEFF}, as this determines better the heat loss of a house than just the outside temperature. More about this effective temperature is explained in the Data section.

$$min \sum_{t=1}^{T} (c_t P_t^{HP}) \Delta t \tag{1}$$

s.t:

$$\theta_t^{in} = \alpha\theta_{t-1}^{in} + \beta\theta_{t-1}^{extEFF} + \gamma P_{t-1}^{HP} \qquad t = 1, ..., T \tag{2}$$

$$P_t^{HP} = (0,2\delta_1^t + 0,4\delta_2^t + 0,6\delta_3^t + 0,8\delta_4^t + \delta_5^t)P_{nom}^{HP} \qquad t = 1, ..., T \tag{3}$$

$$\delta_1^t + \delta_2^t + \delta_3^t + \delta_4^t + \delta_5^t \leq 1 \qquad t = 1, ..., T \tag{4}$$

$$\theta_t^{in} \geq \theta^{min} \qquad t = 1, ..., T \tag{5}$$

$$\theta_t^{in} \leq \theta^{max} \qquad t = 1, ..., T \tag{6}$$

$$\delta_r^t \in \{0, 1\} \qquad t = 1, ..., T \quad r = 1, ..., 5 \tag{7}$$

$$\theta_0^{in} = 20 \tag{8}$$

To analyse parameters α, β, and γ, I refer to the original version of equation 2 from Antunes et al. (2019). This equation, adapted to this new case, is stated below and displays the parameters we can adjust.

$$\theta_t^{in} = (1 - \frac{U.A}{C}\Delta t)\theta_{t-1}^{in} + (\frac{U.A}{C}\Delta t)\theta_{t-1}^{extEFF} + \frac{\Delta t}{C} COP\ P_{nom}^{HP}$$

This equation only holds for small time intervals, Δt, as this is a discretisation of a differential equation. The term $U.A$ represents the heat transfer from outside based on the surface of the exterior walls in kW/°C. The value C represents the thermal capacity of the building based on the formula, $C = \rho c_p V$. In this formula, the c_p represents the specific heat of indoor air at constant pressure in kJ/(kg °C), the ρ represents the mass density in kg/m^3, and V is the volume of the object being heated in m^3. The last new term is the COP term which represents the coefficient of performance of the heat pump.

All these parameters are adapted to this new case with a Dutch terraced house with a heat pump. The exact values we use and the explanation for it is discussed in Sect. 4.

3.2 Different Temperature Profile

People have different preferences for temperature throughout the 24 hours of the day. Some people prefer to sleep in a lower temperature than the temperature they like to spend their evening in for example. Being able to apply different temperature profiles throughout the day is therefore a valuable extension to the model. Providers of heat pumps often claim that keeping the demanded temperature constant is the most efficient way. For example on the website of *Daikin*, www.daikin.be, they claim that the most efficient way is to change the demanded temperature maximal one degree during the day and to keep heating at night at 18 degrees. The reason is that if you let your house cool down too much, it takes a lot of energy to heat back up. The reason for the extension in this subsection is to check the validity of this claim as this claim could be invalid with fluctuating energy prices.

I propose to extend the model in such a way that users can assign different temperatures for the morning, afternoon, evening, and night. Besides that users can choose themselves from what time till what time these periods last. I start therefore by introducing parameter x_i with $i = 1, ..., 4$, which represents the point in time where each part of the day ends. So $t = 0$ where the user decides the night starts, and x_1 is the point in time where the night ends. We use this parameter in the model to determine at each point in time what part of the day we are in.

Instead of having only the two constraints 5 and 6, we now need two equations for every part of the day separately. That is why I introduce constraints 9 til 16, with a for each part of the day a separate variable, that ensures that the constraints are only active at the right part of the day.

$$\theta_t^{in} \geq \theta_{night}^{min} - My_t \qquad t = 1, ..., T \tag{9}$$

$$\theta_t^{in} \leq \theta_{night}^{max} + My_t \qquad t = 1, ..., T \tag{10}$$

$$\theta_t^{in} \geq \theta_{morning}^{min} - Mz_t \qquad t = 1, ..., T \tag{11}$$

$$\theta_t^{in} \leq \theta_{morning}^{max} + Mz_t \qquad t = 1, ..., T \tag{12}$$

$$\theta_t^{in} \geq \theta_{afternoon}^{min} - Mu_t \qquad t = 1, ..., T \tag{13}$$

$$\theta_t^{in} \leq \theta_{afternoon}^{max} + Mu_t \qquad t = 1, ..., T \tag{14}$$

$$\theta_t^{in} \geq \theta_{evening}^{min} - Mk_t \qquad t = 1, ..., T \tag{15}$$

$$\theta_t^{in} \leq \theta_{evening}^{max} + Mk_t \qquad t = 1, ..., T \tag{16}$$

The variables y_t, z_t, u_t, and k_t that control the constraints are defined below. M represents a value of 1000, this is to ensure that the constraint is always met when a variable is equal to 1.

$$y_t = \begin{cases} 1, & \text{if } t > x_1 \\ 0, & \text{if } t \leq x_1 \end{cases}$$

$$z_t = \begin{cases} 1, & \text{if } t \leq x_1 \text{ or } t > x_2 \\ 0, & \text{if } x_1 < t \leq x_2 \end{cases}$$

$$u_t = \begin{cases} 1, & \text{if } t \leq x_2 \text{ or } t > x_3 \\ 0, & \text{if } x_2 < t \leq x_3 \end{cases}$$

$$k_t = \begin{cases} 1, & \text{if } t \leq x_3 \text{ or } t > x_4 \\ 0, & \text{if } x_3 < t \leq x_4 \end{cases}$$

By implementing these variables and constraints in the previous model with the heat pump, a new model can also handle different temperature profiles during the 24 hours of the day. The reason I model it with these additional variables that deactivate constraints is because of the implementation in the AIMMS software as you can avoid having dynamic sets.

3.3 Limiting On/Off Switching

In the results section of the Antunes et al. (2019) paper the graphs show a lot of on/off switching. With our heat pump adaptation, the model will likely do this even more. This is because when the heat pump is active it has a greater effect on the temperature than the air conditioning system, and therefore will be sooner on a desired temperature level. In this subsection, we will present another model extension that sets a maximum to the on/off switching in a single 24-hour schedule. This is a valuable extension because a lot of on/off switching results in large temperature fluctuations which could lead to cracks in the metal parts of the heat pump.

To implement this restriction we need to find a way to keep track of how many times the system goes on and off. That is why we add lines 17 and 18, variable s_t keeps track of the number of times the system turns on or off. I added table 1 to show the working of line 17. If the system is either on or off at both t and $t-1$ then both summations are the same and s_t equals zero. If the system switches from off to on or the other way around, then the term inside the square becomes 1 or minus 1, so s_t becomes 1. The inclusion of line 17 makes the problem quadratic.

The line 18 is the actual restriction, this line makes sure that the number of times that the system gets switched on or off is smaller than a value S. This value S is therefore two times the limit of turning on the heat pump. If we want the heat pump to turn on 15 times maximum, then we choose $S = 30$.

$$s_t = (\sum_{i=1}^{5} \delta_i^t - \sum_{i=1}^{5} \delta_i^{t-1})^2 \qquad t = 1, ..., T \tag{17}$$

$$\sum_{t=1}^{T} s_t \leq S \tag{18}$$

Powersetting	On at t	Off at t
On at t-1	0	1
Off at t-1	1	0

Table 1: Values of s_t based on the situation

The solver we use does not have a problem with this quadratic variable, but despite that, we still present another formulation which is non-quadratic. Lines 19, 20, and 21 present the extension that does not make the problem quadratic. In this case when the summations result in a value of -1, the variable s_t will be automatically set to zero. So we now only count the on switches and not both the on and off switches. If we want to limit this to 15 on switches we set $S = 15$ and not double it like in the previous formulation.

$$s_t \geq \sum_{i=1}^{5} \delta_i^t - \sum_{i=1}^{5} \delta_i^{t-1} \qquad t = 1, ..., T \tag{19}$$

$$\sum_{t=1}^{T} s_t \leq S \tag{20}$$

$$s_t \geq 0 \qquad t = 1, ..., T \tag{21}$$

4 Data

In this section, I explain the values I have chosen for all the fixed parameters in the model. Thereafter, the weather data and electricity prices that are used in the research will be presented.

4.1 Model Parameters

The values for all the parameters of the models are presented in table 2 together with the values used in Antunes et al. (2019). I chose a value of 4,5 for the COP as this is the COP at an outside temperature of 7 degrees for a Remeha Elga Ace. I retrieved this information from the brochure of the Remeha Elga Ace Remeha (2017), I also use the nominal power of this heat pump, 6 kW. The values for c_p and ρ are based on the physics at room temperature and constant pressure. We assume a volume of 357.5 cubic meters for an average terraced house in the Netherlands this is based on 2.5 floors of 10 × 5.5 × 2.6 meters. We chose this volume because the average for a single-family home lies somewhere between 125 and 151 square meters Bunschoten (2013). The $U.A$ value we use is based on U-values of 0,35 for the facades and 0.30 for the roof in W/m^2°C, these values are retrieved from the Antunes et al. (2019) paper. It is debatable whether these values are trustworthy, more on this is discussed in Sect. 5. Combining the values with the average exterior wall and roof surface gives us a $U.A$ value of 0,039 kW/°C. This is based on 57m^2 facades, $\approx 5.5 \times 2.6 \times 2 \times 2$, and 63m^2 roof, $\approx 5.5 \times 10 \times 1.15$. This is an approximation because we now assume that there is more roof than surface because of a slanted roof. We assume a terraced house so the facades are only the front and the back of the house. We assume that there is no heat transfer from or to the neighbours.

Research	ρ	c_p	V	$U.A$	COP
This Research	1.204	1.005	357.5	0.039	4.5
Antunes Research	1.19	1.007	675	0.129	2.5

Table 2: Parameter values

Using these parameters we get the following table for the possible values of the parameters α, β, and γ, computed for a time interval of one minute.

parameter	$\Delta t = 1$ minute
α	0.9946
β	0.0054
γ	0.624

Table 3: Model parameters

These parameters imply a certain heat loss that is made visible in Fig. 1. The blue line in the graph is the temperature progression, and the orange line is the outside temperature. The graph is based on a starting temperature of 20°C and an outside temperature of 12°C. In three hours, there is a temperature loss of almost 5 degrees. The temperature loss is high but this is not very important for the relative comparison of the models and this research.

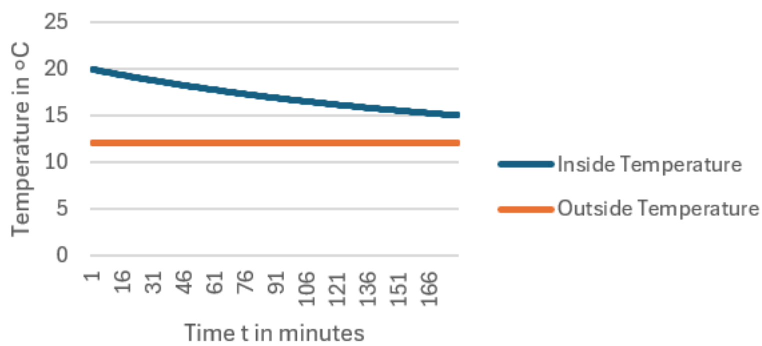

Figure 1: Temperature Loss

4.2 Temperature and Energy Prices

In this research, I choose to use the same time intervals as in the Antunes et al. (2019) paper, 1-minute intervals. In table 4 the electricity prices are presented in €/kWh for each hour on the 19th of March 2023, retrieved from ENTSO-E. For the outside temperature, I use the data on temperature from the KNMI measured in a central Dutch place called De Bilt. I will not just use the outside temperature, but the data on the effective outside temperature. This is the temperature corrected for the influence of the wind at that moment. The formula for the effective temperature in a certain period is stated in 22 Wever (2008).

$$T_{eff} = T_{gem} - \frac{2}{3} u_{gem} \tag{22}$$

In this formula T_{eff} is the effective temperature, T_{gem} is the average temperature in a time interval, and u_{gem} is the average wind speed in m/s in a time interval. In our model, this effective external temperature, θ_{t-1}^{extEFF} , is modelled as $(\theta_{t-1}^{ext} - \frac{2}{3} ws_{t-1})$. For the inside temperature variable, θ_0^{in}, we choose a value of 20, so at $t = 0$ the temperature inside is 20°C.

P_1 [1,60]	P_2 [61,120]	P_3 [121,180]	P_4 [181,240]	P_5 [241,300]	P_6 [301,360]
0.110	0.106	0.100	0.099	0.099	0.103
P_7 [361,420]	P_8 [421,480]	P_9 [481,540]	P_{10} [541,600]	P_{11} [601,660]	P_{12} [661,720]
0.100	0.102	0.101	0.102	0.101	0.099
P_{13} [721,780]	P_{14} [781,840]	P_{15} [841,900]	P_{16} [901,960]	P_{17} [961,1020]	P_{18} [1021,1080]
0.097	0.094	0.095	0.099	0.106	0.135
P_{19} [1081,1140]	P_{20} [1141,1200]	P_{21} [1201,1260]	P_{22} [1261,1320]	P_{23} [1321,1380]	P_{24} [1381,1440]
0.145	0.153	0.145	0.135	0.133	0.130

Table 4: Day ahead electricity prices in euros, taxes excluded

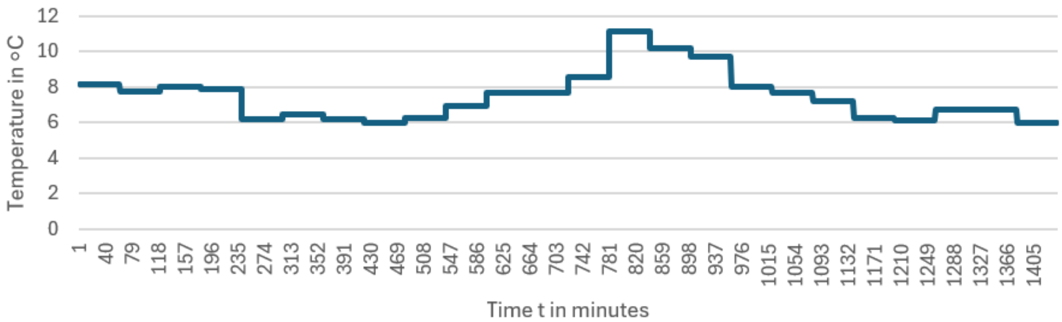

Figure 2: Effective Temperature, 19^{th} of March, De Bilt

5 Results

This section will present the results gained from the research conducted. For each extension, results from different contexts are also presented to show the profit from these extensions. The first subsection discusses the reproduction of the Antunes et al. (2019) paper. The results for the heat pump model are presented in the second subsection. The results of the heat pump model with different temperature profiles are displayed and explained in the third subsection. The last subsection presents the model's results when extended with the on/off switch limitation. The results are gained by making use of the AIMMS software version 24.4.2.3, this software uses CPLEX 22.1 for solving MIP problems. This software is executed on a computer with an 11^{th} Gen Intel(R) Core(TM) i7-1165G7 @ 2.80GHz 2.80 GHz processor. In the appendix more information on the code is provided and information on which results are generated with which program.

5.1 Reproduction

This subsection presents the results that were gained from the reproduction of the third model from the Antunes et al. (2019) paper. Table 5 shows the results of the reproduction next to the original results. The aim was to gain the same results as in the original paper to ensure that the extension is directly extending the original model. Through that way, I would be able to directly compare the models and draw conclusions. I used the same temperature bounds, so a minimum temperature of 20°C, a maximum temperature of 24°C and a starting temperature of 20°C.

Model:	Solution in €	Relative Gap in %	Solving Time in s
Reproduction Model 3	1.9416	0,56	7232.35
Original Results Model 3	1.94445	0,57	7200

Table 5: Results Reproduction

The results resemble the results in the Antunes et al. (2019) paper, and if we look at Fig. 3 we see the same temperature progression the authors found. There are minor differences, but they can be explained by the hardware the experiments are processed on and the different versions of the CPLEX software that are used. The two peaks in the temperature progression can be explained by the low electricity prices in the periods before the peaks. In Fig. 4 a part of the energy price table of the Antunes et al. (2019) paper is shown. In the temperature progression graph you see that it starts heating extra right before the price shifts up.

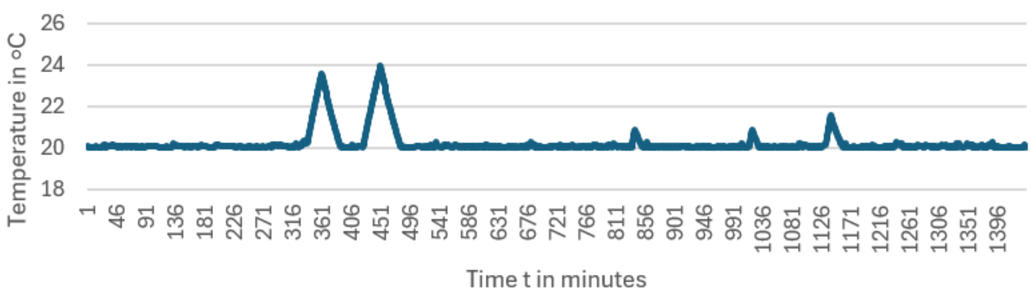

Figure 3: Temperature Progression Reproduction

Price sub-period	P_1 [1,120]	P_2 [121,360]	P_3 [361,450]	P_4 [451,630]
c_t (€/kWh)	0.1	0.075	0.12	0.24

Figure 4: Energy Prices used in Antunes et al. (2019) paper

5.2 Heat Pumps

In this subsection, the results of the heat pump model will be presented. Besides that, some other results will be presented to make a more valid comparison and conclusion. For example, the original model from the previous subsection uses normal temperature instead of effective temperature, and it uses the temperature of a specific day in Portugal and we use data from the Netherlands for different buildings.

I will start by using the original Air Conditioning model with the effective temperature from the 19^{th} of March for a standard Dutch terraced house. I will use the same minimum and maximum temperature as in the original paper, a minimum temperature of 20°C and a maximum temperature of 24°C. The nominal power of the air conditioning system will be 1.5 kW, like in the Antunes et al. (2019) paper. The results for this experiment are visible in table 6 in the row of "Air Conditioning". In Fig. 5 and Fig. 6 the temperature and power progression, respectively, are visualised.

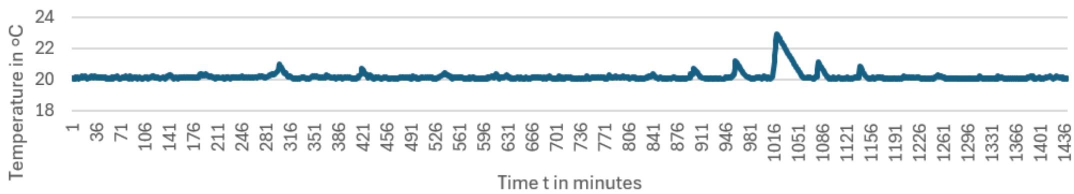

Figure 5: Temperature Progression Air Conditioning

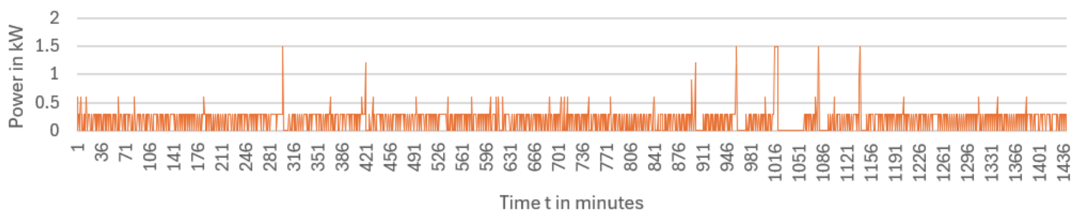

Figure 6: Power Progression Air Conditioning

Now that the benchmark results are generated, the extension can be presented. In table 6, the results of the first extension are presented in the "Heat Pump" row. In Fig. 7 and Fig. 8 the progressions of the temperature and power are presented. From the table becomes clear how great of a difference a heat pump makes in comparison to a regular Air Conditioning system. The cost of heating decreases by 40.5% just because of the transition from air conditioning to using a heat pump. The reason for this is the great difference in the COP. In the temperature graphs, we see that turning on the Heat Pump has a greater positive impact on the temperature than turning on the air conditioning system with approximately the same power use.

What also stands out is that the model only uses the lowest power output except for the moments short before the prices go up. This shows that the model effectively uses the fluctuating prices to minimise the costs. This phenomenon is also visible in the power progression of the air conditioning experiment.

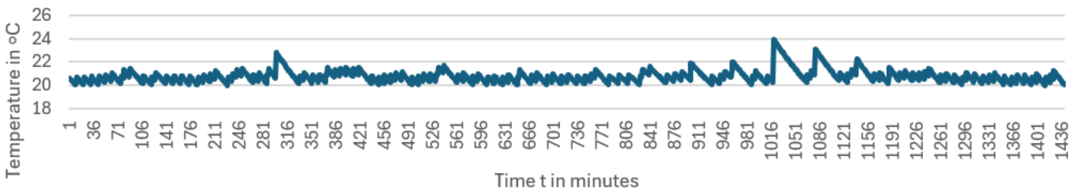

Figure 7: Temperature Progression Heat Pump

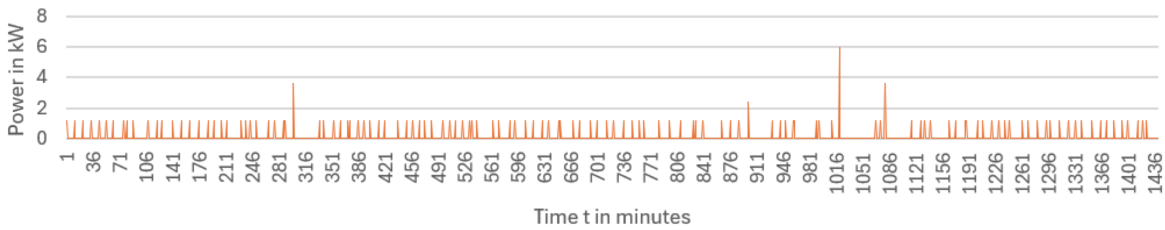

Figure 8: Power Progression Heat Pump

Model:	Solution in €	Relative Gap in %	Solving Time in s
Air Conditioning	0.5158	0.64	7207.09
Heat Pump	0.3071	2.76	7200.30

Table 6: Results First Extension

5.3 Different Temperature Profiles

In this subsection, the results of the second extension are presented. This is the same model as in the previous subsection but then extended with the possibility of four different temperature settings throughout the day. This model is tested for the day-ahead prices on the 19^{th} of March and for a constant price. For this constant price, an average of the day-ahead prices is used. I do this extra test to show the benefit of reacting to fluctuating prices with different temperature profiles. In table 7, the four temperature profiles used in the experiments are stated.

Part of the day:	Min Temperature	Max Temperature	from:	to:
Night	16	20	00:01	08:00
Morning	19	23	08:01	11:30
Afternoon	18	22	11:31	18:00
Evening	20	24	18:01	00:00

Table 7: Caption

In table 8, the results are stated for both a fixed electricity price and day-ahead prices. In Fig. 9 the temperature progression is shown, and in Fig. 10 the power progression is shown. These figures both relate to the model with day-ahead prices. It is visible that the temperature profiles are different throughout the day. In the graph with the power progression, we can see that the model adjusts its pattern to the price progression. A power peak is visible in the interval [961,1020], right before the prices go up. The temperature shift needed for the evening is at $t = 1080$, so 18:00, but it starts heating earlier to save money. Several small peaks can be explained in the same way, for example, the peaks before $t = 300$ and before $t = 900$.

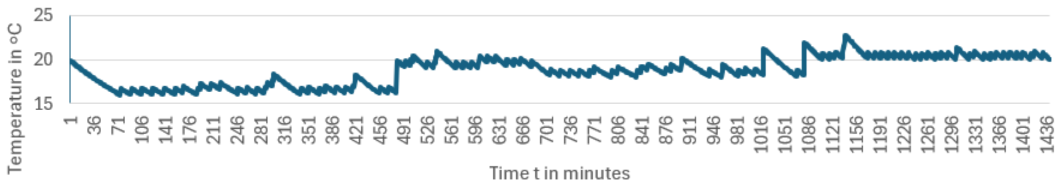

Figure 9: Temperature Progression Different Temperature Profiles, Day-Ahead Prices

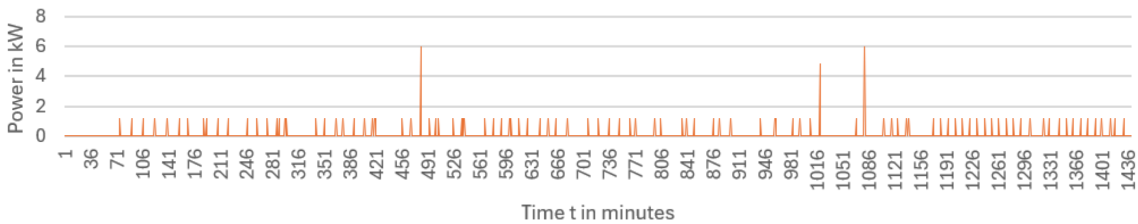

Figure 10: Power Progression Different Temperature Profiles, Day-Ahead Prices

As mentioned, we also did a test with fixed energy prices. As a fixed tariff we chose to use a value of 0.14005. This value is based on the average of the day-ahead prices, 0.11204 €per kWh, and that the electricity provider would stay above this average price to compensate for the fact that energy use will be higher at moments when the supply will be more scarce. The average tariff is multiplied by 1.25 as a buffer for these risks the provider is taking.

As visible in Fig. 11 there is no clear peak in the power usage apart from when the minimum temperature goes up, this is because the model is now not influenced by fluctuating prices. The system turns on only when the energy is needed at that moment and does not consider the future. Table 8 states the solutions for the fixed price and the day-ahead price. It turns out that the model benefits from the day-ahead prices and gained a solution 19.1% cheaper.

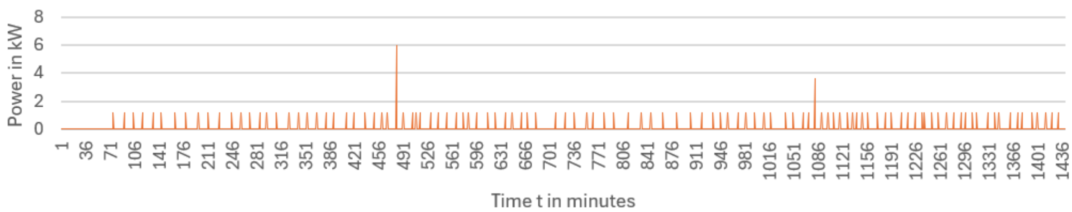

Figure 11: Power Progression Different Temperature Profiles, Fixed Prices

Model:	Solution in €	Relative Gap in %	Solving Time in s
Day-Ahead Prices	0.2651	2.43	7220
Fixed Price	0.3277	3.42	7216.31

Table 8: Results Different Temperature Profiles

Fig. 12 shows the results for more fixed price settings to provide additional insight into the difference between fixed and variable prices. The figure shows the relationship between the optimal solution and the factor which we multiplied by the average variable price. The blue line represents the fixed-price solutions, and the orange line represents the day-ahead price solutions. It is shown that at a fixed price of 1.05 times the average variable price, the fixed-price setting is outperformed by the variable price model.

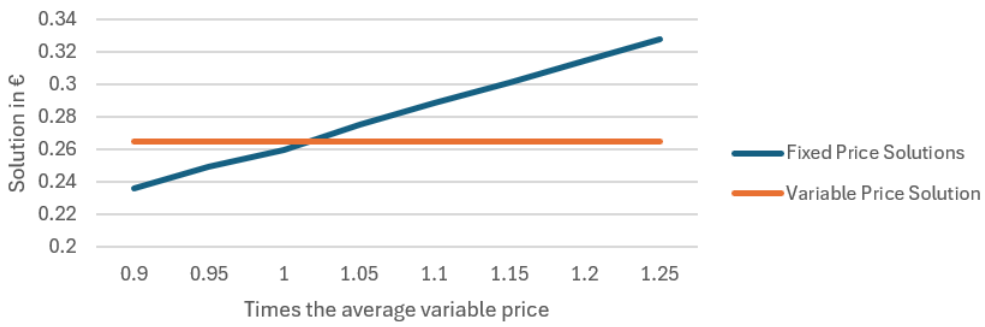

Figure 12: Optimal solutions based on fixed energy price

The model also has maximum temperatures to prevent the model from providing a solution that heats as much as possible when the prices are low. The temperature progression shows that the model stops heating at specific times to prevent passing the maximum temperature. It is therefore also interesting to see how much the model benefits from larger intervals. I raised the maximum temperature of all four intervals by both 1 and 2 degrees to see if this yields better results. The results of this experiment are shown in table 9.

Model:	Solution in €	Relative Gap in %	Solving Time in s
4 °C above minimum	0.2651	2.43	7220
5 °C above minimum	0.2644	2.23	7208,16
6 °C above minimum	0.2640	2.16	7213,48

Table 9: Results with higher maximum temperatures

The results show that as expected, the cost decreases if the interval increases. The reason is that the system can heat extra when the prices are low so that it does not have to heat when prices become high. The only disadvantage is that the temperature can become unpleasantly high if you enlarge this interval too much.

To show the working of this, Fig. 13 is added. This provides a visual insight into why higher maximum temperatures could decrease cost. Assume there is a price increase at $t = 7$. The orange and the blue line start heating till $t = 7$, but the orange line stops at 24 °C and the blue line at 26 °C. We see that the orange line then has to start heating around $t = 16$ and they both end on the same level at $t = 20$. Now the blue line has a lower cost because of the price increase after $t = 7$. It is true that the blue line uses more kWh because it has to heat from 24 °C to 26 °C, but because of the price increase after $t = 7$, it could still be cheaper. This is a brief explanation on why we see a cost reduction in table 9.

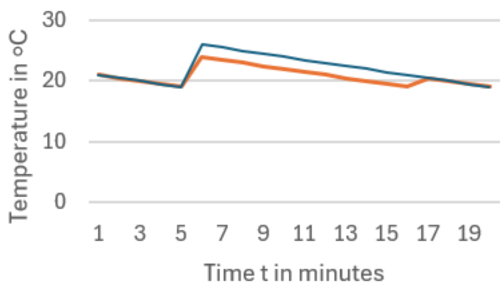

Figure 13: Blue line is 6-degree interval and Orange line is 4-degree interval

5.4 Limiting On/Off Switching

In this subsection, the results are presented of the same model but limited to a fixed number of on/off switching. We choose $S = 100$, this means that the model is limited to switching on the heat pump 50 times at most in the quadratic formulation. In the previous results, the model switched on 100 times, so we half this.

In Fig. 15 and Fig. 14 the progression of the temperature and power are visible. Table 10 shows the results and compares them to the previous model. It is visible that the model still does not keep the heat pump on for long periods but it turns on the heat pump on a higher level. We see therefore more peaks in the temperature progression. The model probably does this because there is so little of a cost difference between two periods at half power or one period at full power. It could also be that the model needs more time to compute solutions of this kind. Later in this section, we see that other outcomes are possible with the non-quadratic form.

This model provides a more expensive solution, but because there is less on/off switching, the lifetime of the heat pump will be longer. All heat pump providers stress the importance of limiting the on/off switching especially to save the compressor. The website technea.nl, states, like other providers, that you should limit on/off switches to 6 per hour max.

Figure 14: Power Progression Limited On/Off

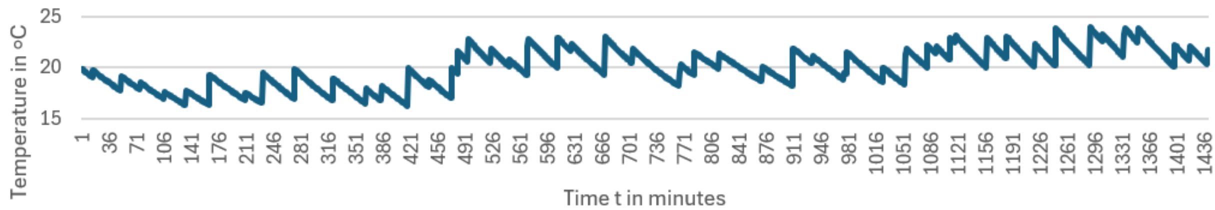

Figure 15: Temperature Progression Limited On/Off

Model:	Solution in €	Relative Gap in %	Solving Time in s	on/off switches
Limited On/Off	0.3039	15.37	7241.1	100
Without Limit	0.2651	2.43	7220	200

Table 10: Results Limited On/Off

The reason the heat pump switches on so often is that there is a large loss of heat. That is also the reason that if we choose a lower value of S we do not get a feasible solution. If we change the U-values we use in this research to lower values it is possible to reduce this value S.

In the Netherlands, a passive building quality mark can be requested at PassiefBouwen-Keur®. This is a quality mark for the most efficiently insulated buildings. The minimum required U-values for walls and roofs are 0.15 W/m^2°C and 0.10 W/m^2°C, respectively. These values result in a $U.A$ value of 0.01455 kW/°C. The parameters that belong to this $U.A$ value are stated in table 11. In Fig. 16 a graph shows the heat loss in this situation with a starting temperature of 20 °C and an outside temperature of 12 °C.

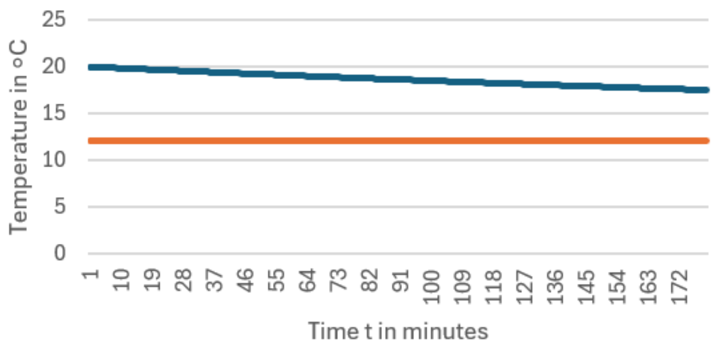

Figure 16: Heat loss Passive Building Quality Mark

parameter	$\Delta t = 1$ minute
α	0.99798
β	0.00202
γ	0.624

Table 11: Model parameters

If we apply these parameters to the model with the limited on/off switching, we can choose lower values for S. In Fig. 17 the power progression is shown of a schedule produced with $S = 40$, so a maximum of 20 uses. Here the suggested constraint of 3 uses per hour is met.

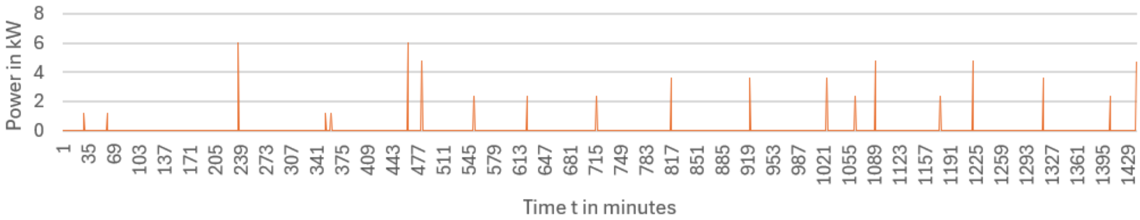

Figure 17: Power Progression with S = 40

If we use the non-quadratic formulation we get the power progression presented in Fig. 18. What is interesting is that this solution is better and keeps the heat pump on for longer periods. This solution has a cost of 0.10232 and the solution in Fig. 17 had a cost of 0.12688. This is probably because computation takes more time in a quadratic model and to get the same solution it should run for a longer period. The relative gap of the non-quadratic solution was 4.9%, and the relative gap of the quadratic solution was 21.3%. We also see that in this solution the model keeps on the system for multiple periods on a low level instead of single periods on a high level. I did an experiment where I relaxed the binary constraint of the deltas, the power progression of that experiment is shown in Fig. 19. In this experiment, it is visible that the heat pump is on for long periods at a low level. So keeping on the pump for multiple periods on a low level turns out to be beneficial. The non-quadratic formulation gets closer to the optimal solution within a short time and also creates solutions with a switched-on heat pump for longer periods. These two facts lead to the conclusion that it is better to use the non-quadratic form.

This, however, does not change major things in the conclusion that we need to limit the on/off switching of the heat pump. It is best to limit the number of on/off switches and ideally keep the heat pump on for longer periods on a low level.

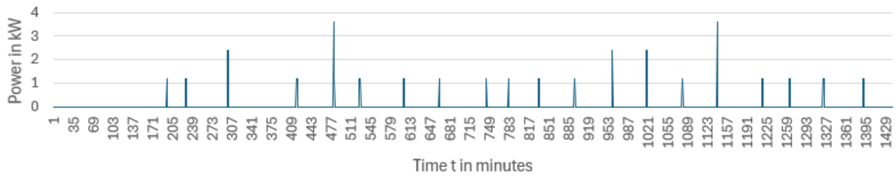

Figure 18: Power Progression with S = 20

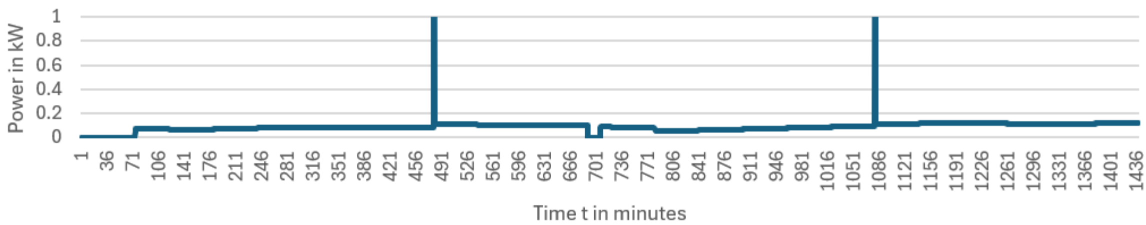

Figure 19: Power Progression of the relaxation

6 Conclusion

This paper presents models to create heating schedules for heat pumps in standard Dutch terraced houses. The models create these schedules based on hourly changing energy prices aiming to minimise the costs. The purpose of these models is to gain insight into what methods are best to keep Dutch terraced houses at temperature and be able to draw conclusions from it.

Firstly a model is presented that extends a model that is made for Air Conditioning Systems, this new model is made such that it is suitable for heat pumps. A cost reduction of 40.5% is realised with that adaption. So the first conclusion was that it is useful to switch to heat pumps if the possibility is there.

After that, the model was extended to a version where different temperature profiles could be assigned to different parts of the day. Another cost reduction of 13.7% was realised by that adaption, but this reduction is obviously dependent on how different the temperature profiles are. If you allow a night temperature which is 5 degrees colder than the morning temperature this yields a greater saving than 2 degrees.

The results of this model are also compared with a fixed energy price level to see how much the model profits from the varying prices. With an estimated fixed price at 1.25 times the average day-ahead price, a 23.6% higher price was obtained. A list of additional tests showed that the model benefits from the varying prices as long as the fixed price is at least 1.05 times the average day-ahead price. The reason the fixed price has to be higher is that heating can be done at every point in time. With variable prices there is often a peak at dinnertime for example, and that is also a time that a warm house is demanded. An energy provider will likely be well above this price of 1.05 times the average day-ahead price for a fixed tariff to cover the risks. We therefore conclude that the model profits from the variable prices in optimising the cost.

We also showed that it helps to enlarge the acceptable temperature intervals. That does not mean that the minimum temperature needs to be lower, but it means that if you allow a higher maximum temperature, the cost could be lower. This is because it creates the opportunity to heat more while prices are low and creates a greater buffer for when prices are high.

Lastly, we looked at limiting the on/off switching of the heat pump to increase the lifetime of the heat pump. We concluded that the possibility of limiting this is dependent on the heat loss of the building. When heat loss is high it is not possible to create large buffers for multiple hours. With good insulation, it is possible to limit the on/off switching to max 6 times an hour which is recommended by the heat pump providers.

The conclusion out of this research is that it is wise to use heat pumps instead of air conditioning systems, preferably with different temperature profiles throughout the day. Besides that, it is helpful to use larger intervals, so higher maximum temperatures, to generate lower expenses, and to limit the on/off switching of the heat pump to extend its lifetime.

For further research, it would be interesting to focus on reducing the computation time for solving these models. This model is designed to give a useful solution within 2 hours that can be used for the next day. Ideally, you would have a model that can change at any time depending on how the weather forecast changes. With this model that is not possible, but it would be interesting to see what the possibilities are by for example relaxing some constraints.

Another thing that is interesting to look at is more realistic heat loss parameters. In this research, we used the same heat loss parameters as in the Antunes et al. (2019) paper. As visible in the Data section, these parameters result in a higher heat loss than you would expect. The model would be more realistic if more research would be done into these parameters. With additional attention to the right parameters, one could experiment with these models in reality.

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A Programming code

This section of the appendix provides a brief explanation of my programming code. I did the programming in AIMMS version 24.4.2.3. In the zip file attached, there are 4 programs, one with the code for the reproduction, one with the code for the heat pump extension, one with the code for the extension with the different temperature profiles, and one for the last extension with the limited on/off switches.

A.1 Reproduction

In this code, I tried to reproduce exactly what was done in the Antunes et al. (2019) paper. I created the same variables and parameters as in the paper and initialised a mathematical program for minimising this problem. All the variables in the code have suggestive names and can be easily linked to the problem as stated in the paper.

A.2 Heat Pump Extension

In this code, there are two things different opposed to the Reproduction. The first thing is the values of the coefficients, these are adapted to the situation of a Heat Pump. The Second difference is the new parameter *Windspeed(t)*, which contains the average wind speed at every time interval. This parameter is then used in the *InsideTemperature(t)* variable to correct the influence of outside temperature with the effect of the wind.

Apart from these two adaptions, the code works the same as the code from the reproduction. The initialised mathematical program minimises the *TotalCost* variable while adhering to the constraints as stated in Sect. 3.

A.3 Different Temperature Profiles Extension

This third model is bigger because of the amount of constraints that were needed. Instead of one constraint for the minimum temperature and one for the maximum temperature, there is now a set of two constraints for each of the four parts of the day.

Another difference is the number of parameters, there are four parameters with binary values that determine whether the constraints are active, these parameters are labelled as *Morning(t), Afternoon(t), Evening(t),* and *Night(t).* There is also a parameter M to ensure that constraints are met, and extra parameters specify the minimum and maximum temperature in each part of the day.

As in the previous models, all constraints, variables, and parameters have suggestive names and can be easily linked to the formulation stated in Sect. 3.

A.4 Limited On/Off Switching

For this last model, we included the non-quadratic form in the zip file. Two variables are introduced, one that becomes 1 if the model is turned on at time t and -1 if it is turned off at time t, and one that is constrained to only include the on switches. We created a constraint that limits the summation of this variable to a value S, and we created a constraint that makes sure to only count the on switches.

To get the quadratic form we only square the first variable and keep the first constraint, besides that we double the value of S because there are on and off switches. All the other things added can then be removed.

I included the non-quadratic form because this is the best model and the model that is most useful for further research.