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Erasmus School of Economics
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Corporate Bonds in a Spectral Factor Realm

Daniël van der Korst (578436)

Supervisor:	Dr. M. Grith
Second assessor:	S. van Meer
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Abstract

We use spectral factor models to allow systematic risk to vary across frequencies, thereby studying the cross-sectional pricing effects of cycles of different lengths, that would otherwise be hidden. The factor models that we study are common in recent asset pricing literature and include the stock, bond CAPM, and multifactor stock and bond models. We bridge the gap between spectral factor models in the equity and bond market and set out to broaden the existing set of test assets and factor models to which a spectral approach is applied. We find that low-frequency components are subject to heavy change when the estimation method differs slightly. Test assets and factor models require constant adjusting of tuning parameters for optimal spectral models. Furthermore, we find that the pricing abilities of the bond CAPM can benefit from a spectral approach and that the spectral multifactor bond model leads to economically interpretable results.

1 Introduction

This research is based on Bandi et al. (2021), who focus on so-called **spectral factor models**. Spectral factor models extend 'regular' factor models using a link between systematic risk and frequencies. Research on the topic of spectral factor models is highly relevant, because current risk assessments lack the interpretation on the role of frequencies. As Bandi et al. (2021) conclude, spectral factor models may lead to better model selection and may also lead to dimensionality reduction of the factor space. People who perform risk assessments, such as risk or portfolio managers, financial analysts, and investment advisors, can benefit from the extra interpretability of spectral factor models, as these models allow them to differentiate between long and short term effects. When frequency is of importance, but one does not allow systematic risk to differ across frequencies, high-frequency effects can hide low-frequency effects (Bansal and Yaron (2004)). The generalized framework of risk, style, and frequency, covered in this paper, is capable of disentangling these effects.

The purpose of this research is to focus on the empirical evaluation of the beta-representation and the cross-sectional pricing abilities of each spectral model. We address this main research question in a (i) replication part and an (ii) extension part. We try to replicate the results of Bandi et al. (2021) using our own code and data (and extending it until December 2022). The spectral approach of Bandi et al. (2021) uses the stock CAPM, with only the stock market excess return as a factor. Then, we extend their research in two ways: (1) We shift the spectral approach to the bond market (bond CAPM) using portfolio and traded factor data from Dickerson et al. (2023), and (2) we research two spectral multifactor models, namely the Fama-French five-factor stock model (Fama and French (2015)), and the BBW bond model (with bond market excess return, downside, credit and liquidity risk) from Bai et al. (2019).

Our main contribution is that we broaden the set of test assets and factor models on which spectral methods are performed. We pave the way for further research on spectral factor models in the bond market and on spectral multifactor models. Also, we contribute to existing literature on the economic interpretation of these spectral models. The main finding of this research is that small changes in the estimation method lead to large changes in the low-frequency components of the spectral models. This follows from the fact that the exact replication of the results from Bandi et al. (2021) is not possible due to: (1) State variables and return data that differs slightly, (2) the restriction of zero Granger-causality that is not implemented in their code, and (3) that they do not state every decision made in their research. Furthermore, we find that the bond CAPM performs well in cross-sectionally pricing different sets of portfolios, giving stable prices of risk, low constant estimates and large $\mathbb{R}^2$ values compared to the spectral stock CAPM. Regarding the multifactor models, the spectral form of the BBW model leaves us with results that are easy to economically interpret.

We will proceed as follows. In Section 2 we will elaborate on the existing literature regarding the topic of spectral factor models, and in Section 4 we will discuss the data used for both 'types' of spectral factor models (stock and bond). Section 3 will cover the procedure to obtain spectral components, and Section 5 lays out the results that follow from the methodology. Lastly, in Section 6 we will summarize the relevant and main conclusions, together with possibilities for further research. In the Appendix, the reader can find extra tables and figures that are not explicitly discussed in the text, as well as a description of the programming code (and differences with the code of Bandi et al. (2021)), and derivations to obtain the spectral components. The exact data and code are to be found in a zip-file enclosed with this research.

2 Literature Review

In this section we will give an overview of the literature preceding this research, and how we extend the existing literature. Specifically, we will expand on the asset pricing context of this work, highlighting stock factor models, bond factor models, and frequency-specific risk.

2.1 Factor Models

There exists extensive research on the topic of factor models used for asset pricing purposes in equity markets, starting with the findings of Sharpe (1964) and Lintner (1965), who proposed the capital asset pricing model (CAPM) for stocks. The CAPM is used to calculate systematic risk, by projecting excess stock portfolio returns on excess market returns. Extensions are the consumption CAPM, which uses a consumption beta (Breeden (1979)), and the intertemporal CAPM, that extends the one-period nature of the normal CAPM (Merton (1973)). Other famous linear stock factor models include the arbitrage pricing theory by Ross (1976) and the Fama-French three- and five-factor models (Fama and French (1993); Fama and French (2015)). It was for corporate bonds only later that the independent search for relevant factor models started, as Dickerson et al. (2023) state. Most existing bond market factor models do not perform well cross-sectionally in predicting prices, because they are not based on bond-specific information, but on stock factor models and on macroeconomic variables. Differences between the stock market and the bond market, such as over-the-counter trades that bring higher liquidity risk, higher exposure to downside risk, and buy-and-hold strategies, call for the independent identification of bond risk factors (Bai et al. (2019)). An example of a bond-specific factor model is the DEFTERM model by Fama and French (1993), which consists of a default factor (DEF) and term spread (TERM). Dickerson et al. (2023) show that outperforming the bond market factor (the bond CAPM), for example by adding the BBW factors from Bai et al. (2019) (i.e. the downside, credit, and liquidity risk factor), is very challenging and does not come with an increase in pricing ability.

However, Bandi and Tamoni (2022) draw the important notion that for all CAPM's (stock & bond) and the other factor models above, meaningful conclusions about investment effects based on the length of time cannot be drawn, because betas are assumed to be constant over time.

2.2 Frequency-Specific Risk

Consequently, most recent literature shifts the attention more from the relation between risk and style, towards the relation between risk and frequencies (Chaudhuri and Lo (2015); Neuhierl and Varneskov (2021)). But, a framework for factor models that links risk, style and frequencies, remains relatively unexplored. The methods of Bandi et al. (2021) yield promising results, that substantiate the analysis of short- and long-term dynamics in asset pricing. They propose a methodology to obtain frequency-specific components and analyze the cross-sectional pricing abilities of these components. Bandi et al. (2021) find that the 'business cycle' component belonging to cycles between 32 and 64 months leads to economically interpretable and stable pricing results in a stock context. Their research builds upon four different streams of literature on asset pricing. Firstly, re-balancing frequency of portfolios can cause betas to change simultaneously (Kothari et al. (1995)). Secondly, cross-sectional pricing contains time effects that can be studied using temporal aggregation (Hawawini (1983)). Thirdly, filters can split returns and factors to get frequency-specific analyses of risk, as in Kang et al. (2017) who also price assets using different time-weighted betas. The filters that Bandi et al. (2021) use to move from the regular Wold decomposition to the extended Wold decomposition are the Haar filters, which are the only 'symmetric compactly supported orthonormal wavelets' (Gençay et al. (2001)). Fourthly, Neuhierl and Varneskov (2021) show that the stochastic discount factor has frequency characteristics. Also Ortu et al. (2020) and Bandi et al. (2019) use the extended Wold composition that is necessary to get the component effects of a stationary time series. The first shows that in two applications (for a realized variance and a yield-Treasury bonds analysis) the spectral approach can eliminate noise and can leave us with better economic interpretation. The second shows that for economic uncertainty as a factor, the restriction of no changing effects across frequencies, is too strong. Several others have, since the introduction of frequency approaches, used the same spectral forms for other applications (Bandi and Tamoni (2023); Cerreia-Vioglio et al. (2023); Piccotti (2022)).

This research can be placed in the streams of the literature above, as it uses the methodology from Bandi et al. (2021) on modern (multi)factor models. Specifically, we propose a spectral approach for the corporate bond models from Dickerson et al. (2023). We increase the spectral research on potentially useful test assets and factor models, using Wold decompositions and Haar wavelet filters, and adding frequency-specific interpretations to existing and thoroughly researched factor models.

3 Methods

This section will discuss the transformation of our time series from a *regular* Wold decomposition into an *extended* Wold decomposition that we will estimate using a Least Squared method (after cutoff), which then looks like:

$$y_t = \alpha + \sum_{j=1}^{\infty} \beta^{(j)} x_t^{(j)} + u_t, \tag{1}$$

where y_t can be some excess asset/portfolio return (for bond or stock) and $x_t^{(j)}$ denotes the jth component of our factor. This is the univariate case, but spectral approaches also apply to multivariate factor models, which we will discuss here. Both y_t and x_t are covariance-stationary time series. The betas now correspond with frequency-specific dependence and u_t exhibits standard properties. The notation and methodology closely follow Ortu et al. (2020) and Bandi et al. (2019). It is the restructured and multivariate case of the methodology and notation from Bandi et al. (2021). Both the replication and the extension part follow the same steps, but differ in the test asset/portfolio return and factors, which will be explained in more detail in Section 4.

3.1 Factor Structure

Firstly, like Bandi et al. (2021), we write our time series in the form of $\boldsymbol{x_t} = (y_t, \boldsymbol{\tilde{x}_t^T})^T \in \mathbb{R}^k$, with $y_t \in \mathbb{R}$ and $\boldsymbol{\tilde{x}_t} \in \mathbb{R}^{k-1}$. Here, y_t is the excess return of some asset or portfolio at time t and $\boldsymbol{\tilde{x}_t}$ is a vector that can contain (multiple) factors and state variables, also known at time t. Both y_t and $\boldsymbol{\tilde{x}_t}$ will change, according to the model that we are trying to evaluate. By assumption, $\boldsymbol{x_t}$ is a Vector Autoregressive (VAR) process of lag p (we use $p = 18$ like Bandi et al. (2021)):

$$\boldsymbol{x_t} = A_1 \boldsymbol{x_{t-1}} + \ldots + A_p \boldsymbol{x_{t-p}} + \boldsymbol{\epsilon_t}, \tag{2}$$

where $A_1, \ldots, A_p$ are $k \times k$ matrices and $\boldsymbol{\epsilon_t}$ follows a white noise process with a covariance matrix Σ_k. We can rewrite Equation 2 into a VAR(1) process and estimate using Least Squares:

$$\boldsymbol{X_t} = A \boldsymbol{X_{t-1}} + \boldsymbol{U_t}, \tag{3}$$

using the fact that $\boldsymbol{X_t} = (\boldsymbol{x_t}^T, \ldots, \boldsymbol{x_{t-p+1}}^T)^T$. We recall that X_t is stable if $det(I_{kp} - Az) \neq 0$ $\forall |z| \leq 1$ (Heij et al. (2004)).

In the companion matrix A above, just like Bandi et al. (2021), we implement the following structure:

$$y_t = a_y + A_{1,y} Y_{t-1} + A_{2,y} \cdot \boldsymbol{\tilde{X}_{t-1}} + \epsilon_t^1, \tag{4}$$

$$\boldsymbol{\tilde{x}_t} = a_x + A_{2,x} \cdot \boldsymbol{\tilde{X}_{t-1}} + \boldsymbol{\epsilon_t^2}, \tag{5}$$

where $Y_{t-1} = (y_{t-1}, \ldots, y_{t-p})^T$ and $\boldsymbol{\tilde{X}_{t-1}} = \{\boldsymbol{\tilde{x}_{t-1}^T}, \ldots, \boldsymbol{\tilde{x}_{t-p}^T}\}^T$. Also, $(\cdot)$ denotes the element-by-element inner product, and $A_{2,y}$ and $A_{2,x}$ are of size $1 \times p$ and $k \times p$, with k being equal to the total number of factors and state variables. Importantly, $A_{1,x} Y_{t-1}$ is missing, because we set $A_{1,x} = \mathbf{0}$. This corresponds with the assumption that y does not Granger-cause $\boldsymbol{\tilde{x}}$ across all lags, and the interpretation that we do not let the test asset return influence the factors or state variables. In Appendix B, we provide the detailed structure of the companion matrix A.

3.2 Wold Decomposition

Then, as Bandi et al. (2021) point out, by investigating time-series that are stable, our processes have a time-invariant mean and variance/covariance matrix, meaning that they are covariance stationary and can be written in the form of a Wold representation. Using the Wold decomposition theorem from Wold (1938), applied to our multivariate case (with a zero-mean), we get for any $t \in \mathbb{Z}$:

$$\boldsymbol{x_t} = \begin{pmatrix} y_t \\ \boldsymbol{\tilde{x}_t} \end{pmatrix} = \sum_{k=0}^{\infty} \begin{pmatrix} \alpha_k^1 & \boldsymbol{\alpha_k^2} \\ \boldsymbol{\alpha_k^3} & \boldsymbol{\alpha_k^4} \end{pmatrix} \begin{pmatrix} \epsilon_{t-k}^1 \\ \boldsymbol{\epsilon_{t-k}^2} \end{pmatrix} = \sum_{k=0}^{\infty} \boldsymbol{\alpha_k \epsilon_{t-k}}, \tag{6}$$

where $\boldsymbol{\epsilon} = \{(\epsilon_t^1, \boldsymbol{\epsilon_t^2})^T\}_{t\in\mathbb{Z}}$ is again our white noise process from Section 3.1, $\sum_{k=0}^{\infty} tr^{1/2}(\boldsymbol{\alpha_k^T \alpha_k}) < \infty$ and $\alpha_0 = I_k$ (Bandi et al. (2021)). What follows from this, is that we can set the matrix A, which consists of the different A_i (see Appendix B) with $i = 1, \ldots, p$ from Equation 2 equal to the $\boldsymbol{\alpha_k}$ from Equation 6 to get

$$\boldsymbol{\alpha_k} = A^k, \tag{7}$$

from which we can draw the conclusion that we can obtain the Wold-coefficients $\boldsymbol{\alpha_k}$ from our VAR estimation (Bandi et al. (2021)).

3.3 Extended Wold Decomposition

Furthermore, the *extended* Wold decomposition, as also in Ortu et al. (2020), allows us to write Equation 6 in a spectral form:

$$\boldsymbol{x_t} = \begin{pmatrix} y_t \\ \boldsymbol{\tilde{x}_t} \end{pmatrix} = \sum_{j=1}^{\infty} \sum_{k=0}^{\infty} \boldsymbol{\psi_k^{(j)} \epsilon_{t-k2^j}^{(j)}} = \sum_{j=1}^{\infty} \boldsymbol{x_t^{(j)}}, \tag{8}$$

with $\boldsymbol{x}^{(j)}$ the j-th component of $\boldsymbol{x}$. We obtain the innovations from the extended Wold decomposition, the same way as Bandi et al. (2021), as follows:

$$\boldsymbol{\psi_k^{(j)}} = \frac{1}{\sqrt{2^j}} \left(\sum_{i=0}^{2^{j-1}-1} \boldsymbol{\alpha_{k2^j+i}} - \sum_{i=0}^{2^{j-1}-1} \boldsymbol{\alpha_{k2^j+2^{j-1}+i}} \right), \tag{9}$$

$$\boldsymbol{\epsilon_t^{(j)}} = \frac{1}{\sqrt{2^j}} \left(\sum_{i=0}^{2^{j-1}-1} \boldsymbol{\epsilon_{t-i}} - \sum_{i=0}^{2^{j-1}-1} \boldsymbol{\epsilon_{t-2^{j-1}-i}} \right), \tag{10}$$

where $\boldsymbol{\psi_k^{(j)}}$ for any $j \in \mathbb{N}$ is a $k \times k$ matrix and $\boldsymbol{\epsilon_t^{(j)}}$ is a $k \times 1$ vector. Both are the unique Haar transforms (Haar (1909)). It can be proven that the extended Wold decomposition from Equation 8 has components that are orthogonal at all moments and lags in time. This corresponds with the expression, as Bandi et al. (2021) state,

$$\mathbb{E}[\boldsymbol{x_{t-m2^j}^{(j)}}, \boldsymbol{x_{t-n2^k}^{(k)}}^T] = 0, \tag{11}$$

$\forall j \neq k$, $\forall m, n \in \mathbb{N}_0$, $\forall t \in \mathbb{Z}$. Every component j is defined on the support $S^{(j)} = \{t - k2^j : k \in \mathbb{Z}\}$, and has white noise shocks, capturing patterns between 2^{j-1} and 2^j periods[1].

3.4 Evaluation

Lastly, we get the spectral factor model in a form that can be estimated (using a Least Squares method), which gives us the frequency-specific interpretation of the original model:

$$y_t = \alpha + \sum_{i=1}^{k-1} \sum_{j=1}^{\infty} \beta_i^{(j)} x_{i,t}^{(j)} + u_t, \tag{12}$$

where again y_t is a test excess asset/portfolio return (that can also be represented in its component form) and $i = 1, \ldots, k-1$ correspond with the different factors and state variables that are in $\tilde{\boldsymbol{x}}_{\boldsymbol{t}}$. Important to note is that the series can be broken down into *any* number of J components, which boils down to

$$y_t = \alpha + \sum_{i=1}^{k-1} \sum_{j=1}^{J} \beta_i^{(j)} x_{i,t}^{(j)} + \sum_{i=1}^{k-1} \beta_i^{(J+1)} \pi_{i,t}^{(J)} + u_t. \tag{13}$$

Here, $\pi_{i,t}^{(J)} = x_{i,t} - \sum_{j=1}^{J} x_{i,t}^{j}$ ($\forall i = 1, \ldots, k-1$), which can be interpreted as a residual component. We refer to Bandi et al. (2021), who state as a theorem that the traditional beta, without splits into components, 'is the same as a weighted average of spectral betas with weights directly related to the relative informational content of the corresponding frequencies.' The same applies to only a range of frequencies.

4 Data

Here, we will discuss where the data for our spectral models (stock/bond CAPM, BBW model and Fama-French five-factor model), regular factor models for performance comparison and portfolio returns are retrieved from. Furthermore, we will elaborate on the exact implementation of the methods from Section 3.

4.1 Stock Spectral Factor Models

Data for spectral evaluation of the stock factor models ranges from January 1967 through December 2022, and includes the stock CAPM (with only the market risk premium MKTS) and the Fama and French (2015) five-factor model (with the market risk premium MKTS, 'small-minus-big' SMB, 'high-minus-low' HML, 'robust-minus-weak RMW, and 'conservative-minus-aggressive' CMA). It is retrieved from the following sources, in the same way as Bandi et al. (2021) describe:

- `https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html` (Kenneth French's data library)

[1] For monthly data, we get components that correspond with 1-2 months, 2-4 months, 4-8 months, 8-16 months etc.

- https://www.aqr.com/Insights/Datasets (AQR's data library)
- https://global-q.org/index.html (Hou, Xue, and Zhang's data library)
- https://faculty.chicagobooth.edu/michael-weber/research/data (Weber's data library)

Factor models used for evaluation (without decomposition into components) also include the Fama-French three-factor model (Fama and French (1993)), which contains the first three factors of the five-factor model, and the four-factor model from Hou et al. (2015), which includes the market risk premium, and difference returns on small/big stocks (ME), investment (IA), and profitability stocks (ROE). In Table 1 (constructed using Dickerson et al. (2023) with our stock data), we report summary statistics for each of the stock factors. All means are statistically significant at the 5% level, even after correcting for stock market risk, and are relatively large compared to MKTS. Only SMB and ME remain statistically insignificant. Then, most of the bias-adjusted squared Sharpe ratios are statistically significant at the 5% level, excluding SMB. We see that most of the factors can keep up with the squared Sharpe ratio of MKTS. We conclude that all of the factor models, the Fama-French three- and five-factor model and the four-factor model by Hou et al. (2015) perform well and outperform each of the factors independently. MKTS, SMB, HML, and ME show high standard deviations.

Table 1: Summary statistics for the stock factors. Based on Dickerson et al. (2023). In panel (a) are the mean, single-factor bond market alphas, squared Sharpe ratios and the standard deviations for the stock factors. Panel (b) reports the squared Sharpe ratios for the stock multifactor models. Data are from January 1967 through December 2022. Values statistically significant at the 5% level are in bold using Newey-West standard errors with three lags, except for the p-values of the alphas, which use a different heteroskedastic test (see Dickerson et al. (2023)).

Panel (a): Statistics and squared Sharpe ratios								
	MKTS	SMB	HML	RMW	CMA	ME	IA	ROE
Mean	**0.561**	0.172	**0.307**	**0.302**	**0.319**	**0.272**	**0.396**	**0.532**
Alpha	-	0.061	**0.387**	**0.352**	**0.414**	0.170	**0.483**	**0.599**
Sh^2	**0.013**	0.002	**0.009**	**0.016**	**0.022**	**0.006**	**0.036**	**0.040**
SD	4.593	3.077	3.035	2.265	2.077	3.052	2.039	2.599
Panel (b): Model squared Sharpe ratios								
		FF3		FF5		HXZ		
Sh^2		**0.029**		**0.098**		**0.149**		

From the sources above, the relevant portfolio returns are retrieved for: The 25 Fama-French size and book-to-market portfolios, the 25 Fama-French size and operating profitability portfolios, the 25 Fama-French size and investment portfolios, 48 anomaly portfolios from French (together with 'betting-against-beta' and 'quality-minus-junk' from AQR's data library), 24 anomaly portfolios from Hou et al. (2020) and 10 duration portfolios from Weber (2018).

4.2 Bond Spectral Factor Models

Monthly data for the spectral evaluation of the bond factor models (i.e. the bond CAPM and the BBW model) is constructed by Dickerson et al. (2023) and is retrieved from https://data.mendeley.com/datasets/n66rp59tr7/1, which covers the period of August 2004 through December 2021. The data of the authors is on traded-factor models, and consists of the bond CAPM and the BBW model. Additionally, we use the following 'regular' factor models: The default and term structure model (DEFTERM), and the intermediary capital model (HKM). DEFTERM, introduced by Fama and French (1993), uses default risk (DEF) and term structure risk (TERM) as factors, where the first is defined as the return difference between the market portfolio of long-term corporate bonds and long-term government bonds, and the second is the return difference between long-term government bonds and the one-month T-Bill rate. The intermediary capital HKM from He et al. (2017), uses the stock market factor (MKTS) and the value-weighted equity excess return for the New York Fed's primary dealer sector (CPTLT). Dickerson et al. (2023) make use of a set of filters on clean bond prices from the Trade Reporting and Compliance Engine (TRACE) and the Mergent Fixed Income Securities Database (FISD), to calculate monthly bond returns at time t as:

$$R_{i,t} = \frac{P_{i,t} + AI_{i,t} + C_{i,t}}{P_{i,t-1} + AI_{i,t-1}} - 1, \tag{14}$$

where $C_{i,t}$ is the coupon, $AI_{i,t}$ is the accrued interest and $P_{i,t}$ is the clean bond price for bond i at time t. These returns are then transformed into excess returns by subtracting the one-month U.S. T-bill rate of return. Table 2 is from Dickerson et al. (2023) and gives summary statistics on the different bond factors. The means of most factors are statistically significant at the the 5% level, except for the mean of DEF. This significance is removed when we adjust for bond market risk, looking at the alpha values, and the mean of DEF becomes statistically significant. It seems that only DEF is not to spanned by the bond market factor. MKTB and LRF report the highest bias-adjusted squared Sharpe ratios, followed by MKTS, DRF and TERM. When looking at the full models, BBW slightly outperforms the bond market factor, while DEFTERM and HKM do not outperform the bond market factor. Also, most factors are very noisy compared to MKTB, and only LRF has a lower standard deviation. In line with the conclusions of Dickerson et al. (2023), we conclude from the summary statistics that the bond market factor entails a lot of information and is difficult to outperform. This underlines the importance of investigating the spectral effects of the MKTB factor.

Table 2: Summary statistics for the bond factors. Table from Dickerson et al. (2023). In panel (a) are the mean, single-factor bond market alphas, squared Sharpe ratios and the standard deviations for the bond factors. Panel (b) reports the squared Sharpe ratios for the bond multifactor models. Data are from August 2004 through December 2021. Values statistically significant at the 5% level are in bold using Newey-West standard errors with three lags, except for the p-values of the alphas, which use a different heteroskedastic test (see Dickerson et al. (2023)).

Panel (a): Statistics and squared Sharpe ratios								
	MKTB	DRF	CRF	LRF	DEF	TERM	MKTS	CPTLT
Mean	**0.446**	**0.625**	0.412	**0.330**	0.059	**0.456**	**0.893**	0.622
Alpha	-	-0.008	0.093	0.119	**-0.266**	0.292	0.340	-0.006
Sh^2	**0.054**	**0.034**	0.012	**0.055**	-0.004	**0.016**	**0.037**	0.003
SD	1.832	3.165	3.122	1.340	2.117	3.171	4.320	6.844
Panel (b): Model squared Sharpe ratios								
		BBW			DEFTERM		HKM	
Sh^2		**0.061**			**0.021**		**0.050**	

Also, the authors gathered data on 32 bond sorted portfolios based on bond characteristics and Fama-French industry classifications, which are 5 bond rating portfolios, 5 maturity portfolios, 10 credit spread portfolios, and 12 Fama-French industry portfolios. These will be used as our set of bond portfolios.

4.3 Implementation

Regarding the implementation, for every factor model that we transform into a spectral factor model, we follow the reasoning of Bandi et al. (2021) to take away market return predictability by adding three state variables: The yield spread between long and short-term bonds (TY), the market's price-dividend ratio (PE), and the small-stock value spread (VS). This means that $\tilde{\boldsymbol{x}}$ from Section 3 is a time series consisting of the relevant factors and the aforementioned state variables. These state variables are constructed, as Bandi et al. (2021) do, guided by Campbell and Vuolteenaho (2004). Firstly, the yield spread is the yield difference between ten-year constant-maturity taxable bonds and short-term taxable notes (in annualized percentage points) by Global Financial Data. Secondly, the price-earnings ratio is the logarithm of the CRSP price index divided by a one-year trailing moving average of dividends. Thirdly, the small-stock value spread is the logarithm of the book-to-market ratio of small value stocks divided by the book-to-market ratio of small growth stocks. The book-to-market ratios are from Kenneth French, and follow from the intersection of two size portfolios and three book equity to market equity portfolios. Data on the state variables is monthly and ranges from January 1967 through December 2022. In Appendix A, we report summary statistics on our state variables and the state variables that Bandi et al. (2021) obtain.

5 Results

This section will discuss the results of spectral decomposition for the stock factor models, the bond factor models, and a comparison between both types.

5.1 Stock Spectral Models

Throughout this section on stock spectral models, it is important to note that we make use of our own collected data and code. In Appendix D, we highlight the specific code and data differences with Bandi et al. (2021). The main differences are: (1) The data on returns and state variables differ, (2) the restriction in 3.1 is not implemented in the code from Bandi et al. (2021), (3) our sample ranges through December 2022. We adhere to our own code and data to ensure the most recent, closely followed replication.

5.1.1 Empirical Evaluation of the Beta-Representation

Firstly, we delve into an example of the covariance and beta decomposition for two different portfolios: A 'value' (high book-to-market) portfolio and a 'growth' (low book-to-market) portfolio, which are respectively the first, and the tenth decile value-weighted portfolios formed on book-to-market by Kenneth French. We estimate simple regressions (on excess returns) and obtain the following results for January 1967 until December 2022:

$$R_{value,t} = \alpha + 1.130 * R_{m,t} + u_t, \quad \mathbb{R}^2 = 0.68,$$
$$(t\text{-stat} = 37.49)$$
$$R_{growth,t} = \alpha + 1.059 * R_{m,t} + u_t, \quad \mathbb{R}^2 = 0.87.$$
$$(t\text{-stat} = 66.53)$$

This confirms that the portfolios have a market beta close to one (statistically significant at the 5% level) and that the estimated market beta is higher for the value portfolio than for the growth portfolio, due to higher exposure to market risk. Here, the first difference with Bandi et al. (2021) becomes apparent, as they report a variance of 15.25% per year for the excess market returns (January 1967 through December 2018), while we report a variance of 20.14% per year for the same sample period. This of course increases our market beta estimates above.

Then, in Table 3, we provide spectral covariances that are defined as: $\hat{\mathbb{C}}_j = \hat{\mathbb{C}}(\hat{R}_m^{(j)}, \hat{R}_p^{(j)})$ (with $p = \{value, growth\}$ and $j = 1, \ldots, 6$). In the last four rows we specify the spectral betas together with weights that are calculated as:

$$\hat{v}^{(j)} = \frac{\hat{\mathbb{V}}(\hat{R}_m^{(j)})}{\hat{\mathbb{V}}(\hat{R}_m)}. \tag{15}$$

What becomes apparent from the table is that the numerical results differ from Bandi et al. (2021) although some patterns are the same. As in their research, the spectral covariances of most high frequency components are higher for the value portfolio than for the growth portfolio, which suggests that our estimation method gives high frequency market components that move more in line with the value portfolio. For lower frequencies, this effect dies out. Again, as our excess market returns report higher variance, our spectral covariances are also higher. Then, regarding the spectral betas, we observe higher dispersion among low frequency components and that the weighted sum comes very close to the estimated betas from the simple regression at the beginning of this section. The spectral betas show similar patterns as in Bandi et al. (2021), where they increase at $j = 2$ for the value portfolio and decreases at $j = 2$ for the growth portfolio. The effects of $j > 6$ is different, however, and the growth portfolio seems to be exposed more to the residual component than the value portfolio is exposed to its residual component. Another important notion is that the weights for the portfolios are the same, due to the restricted Granger causality. The weights of Bandi et al. (2021) differ among the value and growth portfolios, because it seems that they do not implement this restriction in their code.

Table 3: Book-to-market sorted portfolios: covariance and beta decomposition. This table shows estimation results of the spectral covariances and the betas, after decomposition into six components (seven frequencies), for a value and a growth portfolio. Data is from January 1967 through December 2022.

	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$	$j>6$	$\sum_{j=1}^{7}\hat{\mathbb{C}}_j$
Spectral cov.								
Value	10.467	7.225	3.135	1.967	0.928	0.146	0.055	22.486
Growth	10.444	6.413	2.453	1.748	0.912	0.252	0.209	22.431
	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$	$j>6$	$\sum_{j=1}^{7}\hat{v}^{(j)}\hat{\beta}^{(j)}$
Spectral betas								
Value	1.059	1.119	1.354	1.214	1.240	1.259	0.631	1.136
weight	0.467	0.294	0.110	0.081	0.040	0.010	0.007	
Growth	1.071	1.041	1.058	1.034	1.005	0.957	1.337	1.065
weight	0.467	0.294	0.110	0.081	0.040	0.010	0.007	

Table 4, shows the results when we sum high frequency components $j = 1, 2, 3, 4$ and the remaining low frequency components, for both portfolios. We should observe that the estimated betas are close, but only see that this is the case for the high frequency betas and that the estimates differ more for the low frequency component. This suggests that the estimated low-frequency components are not fully orthogonal, meaning that effects of components cannot be singled out completely. We can draw the conclusion that small changes in estimation can lead to large changes in low-frequency components.

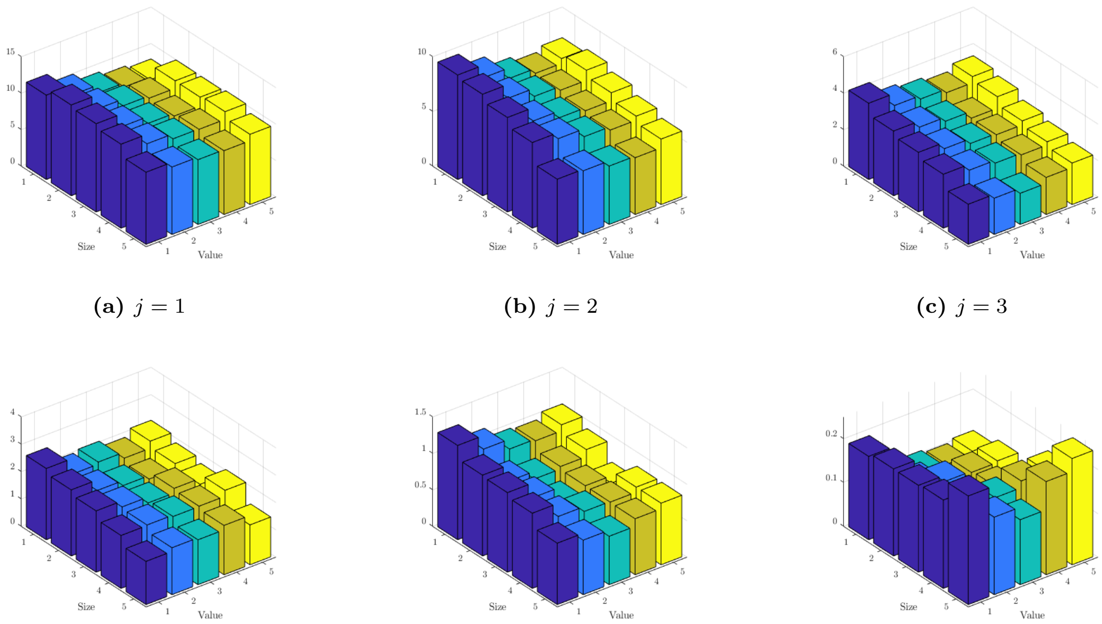
Table 4: Simple and multiple regressions for high- and low-frequency components. Check for the orthogonality property. The estimated betas in columns 1 and 2 should match the estimated betas in columns 3 and 4. Data are from January 1967 through December 2022.

	Simple regression		**Multiple regression**	
	β^{LF}	β^{HF}	β^{LF}	β^{HF}
Value	0.905	1.139	1.163	1.121
	(20.200)	(36.185)	(8.589)	(33.221)
Growth	1.136	1.066	1.033	1.072
	(56.575)	(66.399)	(15.288)	(63.611)

5.1.2 Cross-Sectional Pricing

Then, in this section, we will analyse broader sets of portfolios, starting with the 25 Fama-French book-to-market and size sorted portfolios, for which the average returns and pricing errors from a spectral CAPM model (where $j = 6$) can be seen in Figure 7 in Appendix 7. In Figure 1, we observe the spectral covariances of every component for the 25 book-to-market and size portfolios of Fama-French. The first four components seem to show a similar pattern as in Bandi et al. (2021), but for $j = 5$ and $j = 6$, we observe smaller spectral covariances than their research shows. Their figure is likely to be a mistake for components $j = 5$ and $j = 6$, as the code that they provided also gives components that have decreasing covariances, which can be seen in Figure 9 in the appendix.

Contrary to Bandi et al. (2021), the shift in our spectral covariances is from small size and small value portfolios towards big size and high value portfolios, whereas they report a shift in the covariance towards small size and high value portfolios.

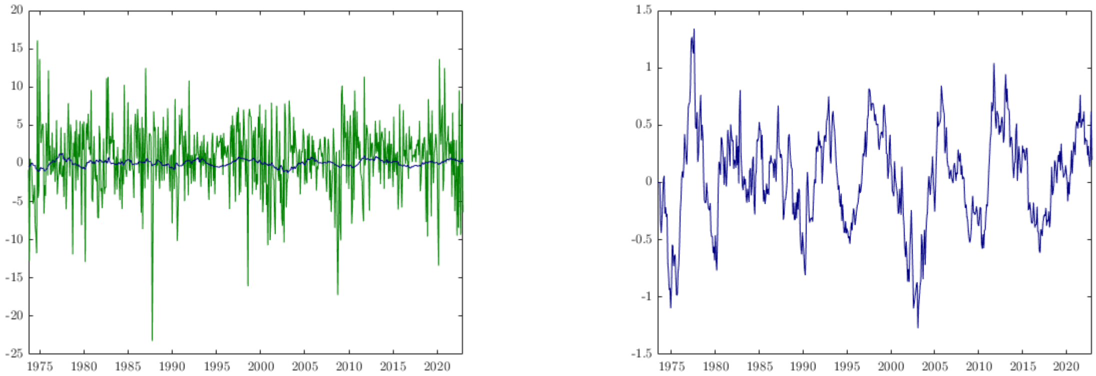


(a) $j = 1$ (b) $j = 2$ (c) $j = 3$

(d) $j = 4$ (e) $j = 5$ (f) $j = 6$

Figure 1: Spectral covariances for every component of the 25 book-to-market and size portfolios. The spectral covariances belonging to each component ($j = 1, \ldots, 6$), for every portfolio from the 25 book-to-market and size portfolios of Fama-French. We observe a shift towards higher value and smaller size portfolios with overall decreasing covariances. Data is from January 1967 until December 2022.

The $j = 6$ market component, which corresponds to frequencies between 32 and 64 months, has the interpretation of the 'business cycle' component. This component can help to determine the effects of the business cycle on excess portfolio returns. For the remainder of the analysis of stock spectral factor models, we will use the business cycle component of our factors. As Bandi et al. (2021) point out, the low weights for this component in Table 3 imply that when we do not use a spectral approach, the effect of the business cycle component is hidden by the effects of other components. In Figure 2, we see the excess market return together with the business cycle component. Both time series show a similar pattern as in Bandi et al. (2021), where the business cycle component is also slow moving and only moves little together with the excess market return. The business cycle component is, however, consistently smaller in magnitude than for Bandi et al. (2021), which is an important difference, as this also influences the pricing abilities of the component.



(a) Excess market return and the $j = 6$ market component.

(b) The 6th 'business cycle' market component.

Figure 2: Time series of the excess market return and the sixth market component. Time series of both the excess market return and the sole $j = 6$ 'business cycle' market component. We observe similar patterns as Bandi et al. (2021), but the magnitude for both time series is smaller in our case. Data are from January 1967 through December 2022.

Table 5: Results of the two-pass Fama and MacBeth regression. In this table are the results of the cross-sectional regression $\overline{R_i^e} = \lambda_0 + \lambda^{(6)}\hat{\mathbb{C}}_i^{(6)} + \xi_i$. Together with Fama and MacBeth (1973) standard errors in parentheses, Kan et al. (2013) model misspecification-robust standard errors in braces, the Root Mean Squared Error (RMSE), the Mean Absolute Percentage Error (MAPE) and the $\mathbb{R}^2$ with its standard error. Bold means that the estimate is statistically significant at the 10% level or lower. Data are from January 1967 through December 2022 (June 2014 for the duration portfolios).

Constant	$\lambda^{(6)}$	RMSE	MAPE	$\mathbb{R}^2$
	Panel (a): 25 size and book-to-market portfolios			
1.559	**-2.975**	1.391	1.119	0.63
(0.245)	(0.747)			(0.20)
{0.351}	{1.060}			
	Panel (b): 25 size and profitability portfolios			
1.511	**-2.755**	1.363	1.089	0.57
(0.293)	(0.965)			(0.33)
{0.413}	{1.392}			
	Panel (c): 25 size and investment portfolios			
1.524	**-2.727**	1.210	0.884	0.68
(0.240)	(0.717)			(0.17)
{0.333}	{1.074}			
	Panel (d): 24 portfolios			
1.007	**-0.565**	2.841	2.202	0.04
(0.113)	(0.699)			(0.11)
{0.146}	{0.865}			
	Panel (e): 48 portfolios			
1.465	**-2.717**	2.481	2.114	0.33
(0.202)	(0.722)			(0.24)
{0.370}	{1.379}			
	Panel (f): 10 duration portfolios			
2.165	**-8.135**	1.262	0.837	0.89
(0.244)	(1.467)			(0.26)
{0.883}	{4.472}			

In Table 5 are different sets of portfolios and the price risk of the business cycle component. The portfolios that are researched, are the 25 size and book-to-market portfolios (panel (a)), the 25 size and profitablity portfolios (panel (b)) and the 25 size and investment (panel (c)) portfolios by Fama-French. Also, 24 anomaly portfolios from Kenneth French (panel (d)), the 48 anomalies from Hou et al. (2020) (panel (e)) and the 10 duration portfolios from Weber (2018) (panel (f)). We can observe that, contrary to the results of Bandi et al. (2021), the price of risk associated with the business cycle component turns out to be negative, meaning that exposure to (or a larger covariance with) the business cycle component has a negative influence on the mean excess return of every portfolio. The second-pass regression has some explanatory power, based on the $\mathbb{R}^2$ of every model. It is much smaller, however, than the $\mathbb{R}^2$ of Bandi et al. (2021) and we report larger values for the RMSE/MAPE. For the 24 anomaly portfolios, the business cycle component has the least risk-based explanatory power, while for the 10 duration portfolios it has the strongest explanatory power. The large estimates for the constants suggest that the business cycle component has a large excess mean return that it cannot explain across all of the different portfolios. This is underlined by the fact that the standard errors of the $\mathbb{R}^2$ are quity large. We observe that Bandi et al. (2021) report stable constants around 0 and stable prices of risk of around 3. From this, we conclude that adding the restriction of Granger causality has a large (negative) impact on the business cycle component of the stock CAPM, the covariance and subsequently on the price of risk. In Table 11 in Appendix A, we provide the differences between $\mathbb{R}^2$ of different models, with the spectral CAPM and the regular CAPM as benchmark models.

5.1.3 Multifactor Spectral Approach

Lastly, as an extension for the stock factor models, we propose the spectral decomposition of the Fama-French five-factor model. Figure 3 shows time series of the business cycle components of each of the five factors. We see that the business cycle component of the excess market return moves in opposite direction with the other factors, that, on their turn, move very much in line. We see that just like their overall factors, the business cycle component of the excess stock market return is lower in economic downturns, and that of the other factors is higher in economic downturns, as small/value/robust profitability and conservative stocks can be viewed as safer in those periods.

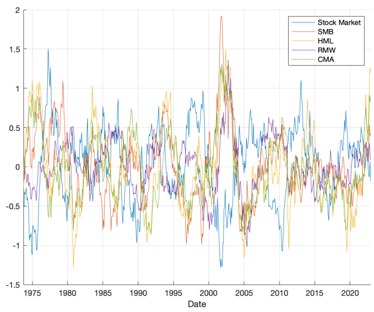


Figure 3: Fama-French five-factor model. The business cycle component for the Fama-French five-factor model: the stock market excess return, SMB, HML, RMW and CMA. Data are from January 1967 through December 2022.

The results from Figure 3, cause the estimates in Table 6 to be statistically insignificant at the 10% level. It seems that overfitting is the issue here, as the large standard errors take away the significance of the market business cycle component. The explanatory power is large, due to a low constant estimate and a large $\mathbb{R}^2$, but it is unsure how the effects differ among the components. The multifactor approach for the stock portfolios seem to be difficult to interpret, whereas the single factor stock CAPM has better economic interpretation.

Table 6: Results of the two-pass Fama and MacBeth regression for the multifactor stock model. The table reports covariance risk for business cycle components of the Fama-French five-factor model (panel (a) with the Fama-French 25 size and book-to-market portfolios) together with standard errors in parentheses, the Root Mean Squared Error and $\mathbb{R}^2$. Bold means statistically significant at the 10% or lower.

25 size and book-to-market portfolios							
Constant	$\lambda^{(6)}_{mkts}$	$\lambda^{(6)}_{smb}$	$\lambda^{(6)}_{hml}$	$\lambda^{(6)}_{rmw}$	$\lambda^{(6)}_{cma}$	RMSE	$\mathbb{R}^2$
0.868	-0.730	-0.430	0.029	1.756	0.962	0.112	0.74
(0.289)	(2.696)	(0.814)	(2.354)	(1.042)	(4.223)		

5.2 Bond Spectral Models

As an extension, we perform the same spectral methodology to bond factor models. The estimation procedure regarding the state variables and Wold innovations is the same. We empirically evaluate two portfolios, report pricing abilities for a larger set of portfolios and decompose the multifactor BBW model.

5.2.1 Empirical Evaluation of the Beta-Representation

What follows firsty is the empirical evaluation of two bond portfolios, one based on long maturity bonds from the highest quintile and one based on short maturity bonds from the lowest quintile (as constructed by Dickerson et al. (2023)). The simple regression of excess returns on excess bond market returns gives:

$$\begin{aligned} R_{short,t} &= \alpha + 0.543 * R_{m,t} + u_t, \quad \mathbb{R}^2 = 0.77. \\ &\quad (t\text{-stat} = 26.30) \\ R_{long,t} &= \alpha + 1.492 * R_{m,t} + u_t, \quad \mathbb{R}^2 = 0.86, \\ &\quad (t\text{-stat} = 36.14) \end{aligned}$$

The long maturity bond portfolio gives higher excess returns when exposed to the bond excess market returns, due to higher risk exposure. Together with the results from Table 7, we see that we can approximate the bond market effect of the equations above by multiplying the weights with the spectral betas, and summing them. Table 7 reports low spectral covariances due to low variance of returns overall. The bond market excess return only has a variance of 3.35% per annum, which also makes that the spectral covariances are small. Just like the simple regression estimate, the covariances of the long maturity bond portfolio are consistently larger than the short maturity bond portfolio.

Regarding the spectral betas, we see estimates that remain somewhat constant over the first four components. After the fourth component, the effects drop. At $j = 6$, we see a spike in exposure for the long maturity bond portfolio, and a declining exposure for the short maturity bond portfolio.

Table 7: Maturity sorted bond portfolios: covariance and beta decomposition. This table shows estimation results of the spectral covariances and the betas, after decomposition into six components (seven frequencies), for a long and short maturity bond portfolio. Data is from August 2004 through December 2021.

	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$	$j>6$	$\sum_{j=1}^{7}\hat{\mathbb{C}}_j$
Spectral cov.								
Short	0.867	0.611	0.156	0.326	0.113	0.151	0.199	2.422
Long	2.327	1.788	0.424	0.688	0.137	0.206	0.156	5.726
	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$	$j>6$	$\sum_{j=1}^{7}\hat{v}^{(j)}\hat{\beta}^{(j)}$
Spectral betas								
Short	0.563	0.545	0.589	0.542	0.486	0.231	-0.198	0.607
weight	0.463	0.334	0.086	0.170	0.041	0.055	0.057	
Long	1.512	1.667	1.479	1.125	0.303	0.568	0.190	1.631
weight	0.463	0.334	0.086	0.170	0.041	0.055	0.057	

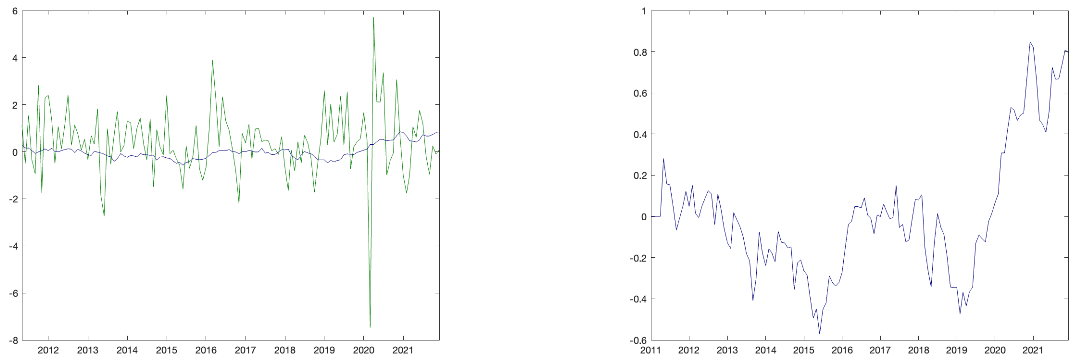
Table 8 reports results that are somewhat problematic. By summing high- and low frequency components into two (with a cutoff at $j = 4$), we should get beta estimates that are the same for simple and multiple regressions. For the short and long maturity bond portfolio, this is only the case for the high frequency component. The low frequency component gives beta estimates that differ tremendously. The estimation procedure from Bandi et al. (2021) should ensure that components are orthogonal and are not correlated with each other, but this does not hold for the maturity bond portfolios. The results from Table 8 make us conclude that due to potential correlation among spectral components, we should be cautious when interpreting their effects.

Table 8: Simple and multiple regressions for high- and low-frequency components of the maturity bond portfolios. Check for the orthogonality property of the high- and low-frequency components for a long and short maturity bond portfolio. The estimated betas in columns 1 and 2 should match the estimated betas in columns 3 and 4. Data are from January 1967 through December 2022.

	Simple regression		**Multiple regression**	
	β^{LF}	β^{HF}	β^{LF}	β^{HF}
Short	0.878	0.394	0.067	0.335
	(53.354)	(15.834)	(1.705)	(14.929)
Long	1.014	1.614	0.336	1.554
	(49.278)	(25.839)	(2.684)	(21.791)

5.2.2 Cross-Sectional Pricing

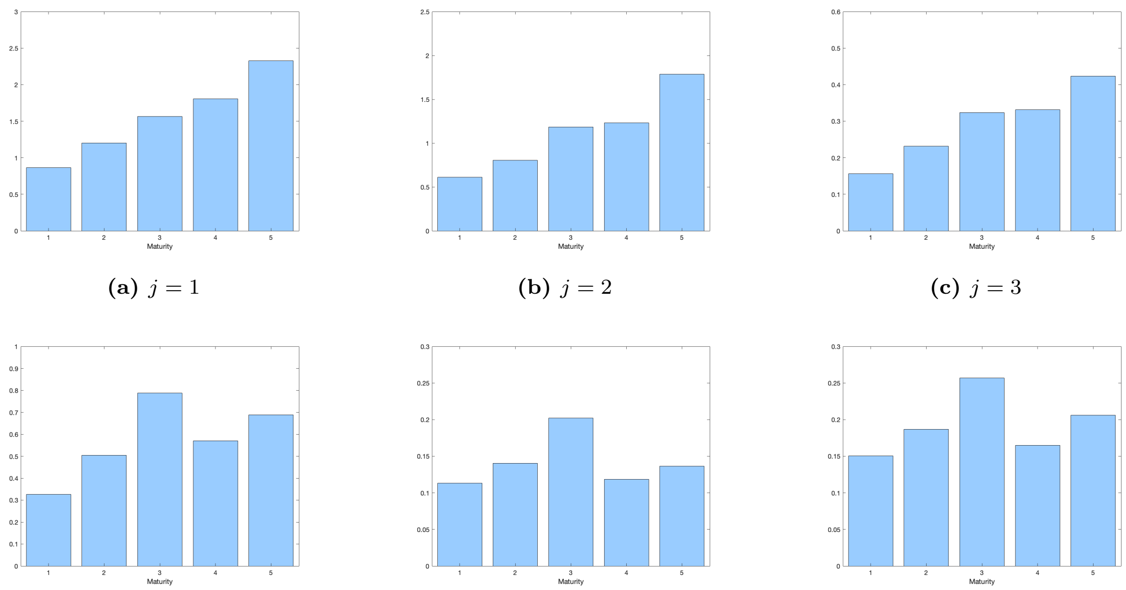
Then, for the $N = 32$ bond portfolios from Dickerson et al. (2023), we also perform a cross-sectional pricing analysis. We start with Figure 4, which shows us the excess bond market return, together with the independent $j = 6$ business cycle bond market component. It becomes clear that the business cycle component is a less volatile variant of the overall excess bond market return and follows the declines around 2015 and the increase from 2019 onwards.



(a) Excess bond market return and the $j = 6$ market component.

(b) The 6th 'business cycle' bond market component, estimated independently.

Figure 4: Time series of the excess bond market return and the sixth bond market component. Time series of both the excess bond market return and the sole $j = 6$ 'business cycle' bond market component. Data are from August 2004 through December 2021, but figure only depicts from 2011, due to the estimation period.



(a) $j = 1$ **(b)** $j = 2$ **(c)** $j = 3$

(d) $j = 4$ **(e)** $j = 5$ **(f)** $j = 6$

Figure 5: Spectral covariances for every component of 5 maturity sorted bond portfolios. The spectral covariances belonging to each component ($j = 1, \ldots, 6$), the five maturity sorted bond portfolios of Dickerson et al. (2023). Data is from August 2004 until December 2021.

The 5 portfolios sorted on maturity are particularly interesting, because of large differences in returns. The long maturity bond portfolios, as shown earlier in this section, have higher average returns. When decomposed into 6 components, Figure 5 shows a shift towards lower maturities for the spectral covariances. Especially for the middle quintile, the spectral covariance turns out to be the largest among the different maturities. This suggests that it moves most in line with the business cycle bond market component, because it does not entail very extreme returns.

Table 9: Results of the two-pass Fama and MacBeth regression for the bond portfolios. In this table are the results of the cross-sectional regression $\overline{R_i^e} = \lambda_0 + \lambda^{(6)} \hat{\mathbb{C}}_i^{(6)} + \xi_i$. Together with Fama and MacBeth (1973) standard errors in parentheses, Kan et al. (2013) model misspecification-robust standard errors in braces, the Root Mean Squared Error (RMSE), the Mean Absolute Percentage Error (MAPE) and the $\mathbb{R}^2$ with its standard error. Bold means that the estimate is statistically significant at the 10% level or lower. Data are from August 2004 through December 2021 from the $N = 32$ bond portfolios of Dickerson et al. (2023).

Constant	$\lambda^{(6)}$	RMSE	MAPE	$\mathbb{R}^2$
	Panel (a): 12 industry portfolios			
0.317	**1.111**	0.564	0.463	0.68
(0.126)	(0.944)			(0.34)
{0.126}	{1.920}			
	Panel (b): 10 credit spread portfolios			
0.195	**2.036**	1.282	1.071	0.86
(0.105)	(0.824)			(0.09)
{0.112}	{0.865}			
	Panel (c): 5 maturity portfolios			
0.038	**3.202**	1.101	1.059	0.43
(0.084)	(1.270)			(0.57)
{0.251}	{1.998}			
	Panel (d): 5 rating portfolios			
0.172	**2.124**	0.353	0.278	0.98
(0.121)	(0.918)			(0.04)
{0.128}	{0.963}			

Table 9 reports the price of risk for the different bond portfolios estimated with the Fama and MacBeth regression for the set of 12 industry portfolios, 10 credit spread portfolios, 5 maturity portfolios, and 5 rating portfolios from Dickerson et al. (2023). The $\lambda^{(6)}$ estimate gives the effect of the business cycle component on the mean excess return of every set of portfolios. The business cycle component performs well in explaining the mean excess return. The constants are close to zero, and the RMSE and MAPE, for especially the rating portfolios, are low. Exposure to the business cycle component comes with an increase in mean excess return. The $\mathbb{R}^2$ of every set of portfolios is high and has low standard errors, except for the 5 maturity portfolios. The standard errors by Fama and MacBeth (1973) and by Kan et al. (2013) are on the high side for the lambda estimates, but effects remain positive. We can conclude that the business cycle component adds to economic interpretation when it comes to the cross-sectional pricing of bonds. This is also explained by Table 12 in Appendix A, which tells us that the business cycle component has relatively high explanatory power regarding beta risk, also compared to other factor models (using the spectral bond CAPM and the regular bond CAPM as benchmark models).

5.2.3 Multifactor Spectral Approach

Lastly, we propose a multifactor spectral approach for the BBW model from Bai et al. (2019). In Figure 6, we report the business cycle components for each of the different factors of the BBW model. It becomes clear that each of the factors show the same pattern as the business cycle component of the excess bond market return with slightly higher and lower volatility. Especially LRF has higher volatility than the other factors.

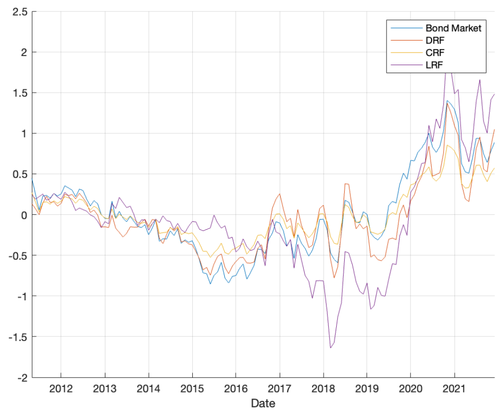


Figure 6: Multifactor decomposition for bonds (the BBW model). The business cycle component for each of the BBW factors from Bai et al. (2019): the bond market excess return, DRF, CRF and LRF. Data are from August 2004 through December 2021.

Looking at the cross-sectionally pricing abilities of the components in Table 10, we conclude that the model performs well in explaining mean excess portfolio returns. Only the excess market bond factor seems to be statistically insignificant, which can be explained with our results from Table 2, which shows that most of the other factors are spanned by the excess bond market factor. This seems to also be the case for the business cycle components. Exposure to LRF seems to have a positive effect on the mean excess portfolio returns. With a low RMSE and a high $\mathbb{R}^2$, a multifactor spectral approach results in good interpretation of business cycle effects.

Table 10: Results of the two-pass Fama and MacBeth regression for the multifactor bond model. The table reports covariance risk for business cycle components of the BBW model with the 32 bond portfolios of Dickerson et al. (2023)) together with standard errors in parentheses, the Root Mean Squared Error and $\mathbb{R}^2$. Bold means statistically significant at the 10% or lower.

32 bond portfolios						
Constant	$\lambda^{(6)}_{mktb}$	$\lambda^{(6)}_{drf}$	$\lambda^{(6)}_{crf}$	$\lambda^{(6)}_{lrf}$	RMSE	$\mathbb{R}^2$
0.236	1.782	**-17.268**	**-4.608**	**35.131**	0.099	0.78
(0.034)	(2.166)	(7.549)	(1.209)	(10.122)		

5.3 Comparison Stock vs. Bond Spectral Models

Concluding this section, we compare the spectral analysis of the stock factor models with the spectral analysis of the bond factor models. Regarding the empirical evaluation of two portfolios, we obtain similar results, as we come close to beta estimates of each of the simple regressions. The smaller spectral covariances that are obtained for the bond portfolios are caused by the smaller variance that is in the excess returns of those portfolios. In both applications, we see that the low frequency components are not fully orthogonal, which is mostly the case for the bond spectral components. This can be due to our restriction of zero Granger-causality or due to a misspecification that follows from undisclosed decisions by Bandi et al. (2021). We can conclude, however, that small changes in the estimation procedure can lead to large differences in the low frequency components. Overall, it seems that our spectral approach and the state variables are a better fit for the spectral bond CAPM, as the Fama and MacBeth regressions from Table 9 perform better than the spectral stock CAPM in Table 5. We report constants that are closer to 0, positive and stable prices of risk, and larger values of the $\mathbb{R}^2$. Then, regarding the multifactor spectral decomposition, we see that the BBW model performs better than the Fama-French five-factor model. Even though in Figure 10 in Appendix A, the Fama-French five-factor model is closer to the mean-variance frontier, the business cycle decomposition of the BBW model, results in pricing effects that are economically easier to interpret.

6 Conclusion

In this research paper, we set out to use the spectral methodology as proposed by Bandi et al. (2021) to obtain frequency-specific effects of factor models. This is done, by moving from a regular Wold representation to an extended Wold representation using Haar wavelet filters. The spectral components that follow from the procedure cover the effects of cycles of different lengths and should be orthogonal. The models are not forced to have equal risk among frequencies, but can have varying effects. This adds to the interpretation of the models, especially, when business cycle components (with lengths between 32 and 64 months) are studied.

We try to replicate the results of Bandi et al. (2021), and extend their research using more recent data, and the spectral decomposition of bond (multi)factor models. Whereas Bandi et al. (2021) find business cycle components of the excess stock market return that are capable of pricing different sets of portfolios effectively, we find business cycle components in the stock context that perform significantly worse and are not fully orthogonal. This is likely attributable to the following: (1) Data on returns and state variables differ from their description, (2) their proposed restriction of no Granger-causality is not in their code, and (3) low-frequency components change rapidly with small changes in the estimation procedure. Model misspecification seems to be the problem here, as the factors are chosen empirically and lack theoretical foundation. The spectral approach works well for the bond case, as the business cycle component of the bond CAPM can price different portfolios from Dickerson et al. (2023) relatively well. Also, the spectral multifactor BBW model gives interpretable results, but orthogonality still remains an issue. Bandi et al. (2021) underline that their research is purely methodological and needs adjustment of tuning parameters. This is not ideal for investors that seek models that are applicable to a variety of cases. But, spectral factor models with effects of low-frequency components that are easy to grasp and without too many factors, can decrease rebalancing costs. From our results and an investor viewpoint, we can conclude that using our methodology, the bond CAPM can benefit from a spectral approach, as pricing abilities are well and business cycle effects can be distinguished. The multifactor BBW model can also benefit, but needs more parameter tuning.

For future research, we also propose that more test assets and factor models should be evaluated. It would be valuable to increase the small sample size of the bond portfolios and do research on what the exact tuning of parameters should be, depending on the assets, factors and state variables. Furthermore, we deem Principal Component Analysis (PCA) to be relevant, as it can rule out orthogonality issues of the spectral components. Lastly, different Least Squares estimation methods, such as GLS, can be considered when using Fama MacBeth regressions in the spectral context.

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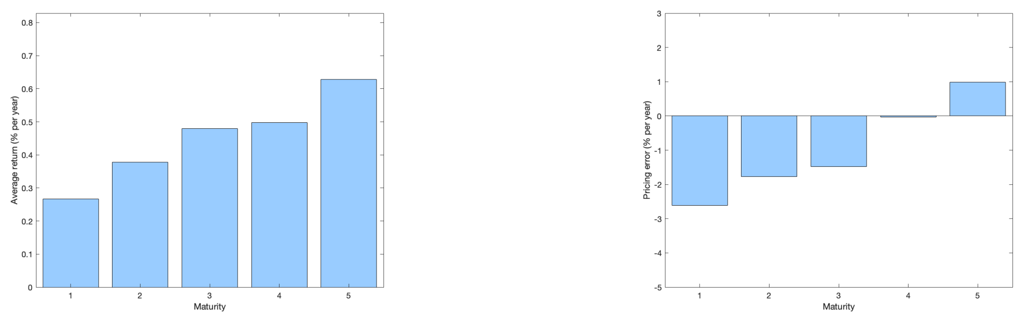
A Extra Tables and Figures

(a) Average returns **(b)** Pricing errors

Figure 7: Average returns and pricing errors for the 25 book-to-market and size portfolios. The figures plot the average returns of the 25 book-to-market and size portfolios of Fama-French and the pricing errors of a spectral CAPM model that uses $j = 6$ as a cutoff. Data is from January 1967 through December 2022.

Table 11: Differences between the $\mathbb{R}^2$ of the stock (spectral) factor models. The difference between the $\mathbb{R}^2$ (of the second-pass regression) for the spectral factor model as a benchmark model and the CAPM as a benchmark model. Data is from January 1967 through December 2022. The portfolios are the same as in Table 5, excluding the 10 duration portfolios of Weber (2018).

	Panel (a): Spectral factor model vs. alternative models						
	Benchmark model		(1)	(2)	(3)	(4)	(5)
	Spectral factor model	versus	7 freq.	CAPM	FF3	FF5	HXZ
25 size-B/M portfolios			-0.599	-0.040	-0.598	-0.692	-0.671
25 size-OP portfolios			-0.833	0.010	-0.593	-0.896	-0.902
25 size-inv portfolios			-0.637	-0.018	-0.650	-0.654	-0.661
24 portfolios			-0.346	-0.021	-0.499	-0.657	-0.670
48 portfolios			-0.405	-0.157	-0.331	-0.545	-0.578
	Panel (b): CAPM vs. alternative multifactor models						
	CAPM	versus			FF3	FF5	HXZ
25 size-B/M portfolios					-0.559	-0.652	-0.632
25 size-OP portfolios					-0.595	-0.897	-0.902
25 size-inv portfolios					-0.633	-0.637	-0.644
24 portfolios					-0.477	-0.635	-0.648
48 portfolios					-0.174	-0.388	-0.421



(a) Average returns **(b)** Pricing errors

Figure 8: Average returns and pricing errors for the 5 maturity sorted bond portfolios. The figures plot the average returns of the 5 maturity sorted bond portfolios (from Dickerson et al. (2023)) and the pricing errors of a spectral bond CAPM model that uses $j = 6$ as a cutoff. Data is from August 2004 through December 2021.

Table 12: Differences between the $\mathbb{R}^2$ of the bond (spectral) factor models. The difference between the $\mathbb{R}^2$ (of the second-pass regression) for the spectral factor model as a benchmark model and the CAPM as a benchmark model. Data is from August 2004 through December 2021. The portfolios are from Dickerson et al. (2023).

	Panel (a): Spectral factor model vs. alternative models						
	Benchmark model		(1)	(2)	(3)	(4)	(5)
	Spectral factor model	versus	7 freq.	CAPMB	BBW	DEFTERM	HKM
32 bond portfolios			-0.225	-0.178	-0.208	-0.145	-0.161
	Panel (b): CAPM vs. alternative multifactor models						
	CAPMB	versus			BBW	DEFTERM	HKM
32 bond portfolios					-0.030	0.033	0.017

Table 13: Results of the two-pass Fama and MacBeth regression with data and code from Bandi et al. (2021). In this table are the results of the cross-sectional regression $\overline{R_i^e} = \lambda_0 + \lambda^{(6)}\hat{\mathbb{C}}_i^{(6)} + \xi_i$ using the code and data from Bandi et al. (2021) from panel a and e. Together with Fama and MacBeth (1973) standard errors in parentheses, Kan et al. (2013) model misspecification-robust standard errors in braces, the Root Mean Squared Error (RMSE), the Mean Absolute Percentage Error (MAPE) and the $\mathbb{R}^2$ with its standard error. Bold means that the estimate is statistically significant at the 10% level or lower. Data are from January 1967 through December 2018 (June 2014 for the duration portfolios).

Constant	$\lambda^{(6)}$	RMSE	MAPE	$\mathbb{R}^2$
	Panel (a): 25 size and book-to-market portfolios			
0.109	**3.039**	1.593	1.087	0.52
(0.245)	(0.825)			(0.23)
{0.426}	{1.478}			
	Panel (b): 25 size and profitability portfolios			
0.621	**2.611**	1.181	0.907	0.64
(0.215)	(0.806)			(0.21)
{0.360}	{1.192}			
	Panel (c): 25 size and investment portfolios			
0.727	**2.221**	1.864	1.276	0.27
(0.210)	(0.930)			(0.26)
{0.305}	{1.442}			
	Panel (d): 24 portfolios			
0.601	**2.222**	2.569	2.036	0.27
(0.185)	(0.0.710)			(0.22)
{0.315}	{0.1.324}			
	Panel (e): 48 portfolios			
0.535	**3.557**	1.415	1.214	0.61
(0.249)	(0.920)			(0.21)
{0.532}	{2.048}			
	Panel (f): 10 duration portfolios			
0.218	**5.746**	1.539	1.204	0.84
(0.420)	(1.161)			(0.15)
{1.183}	{2.885}			

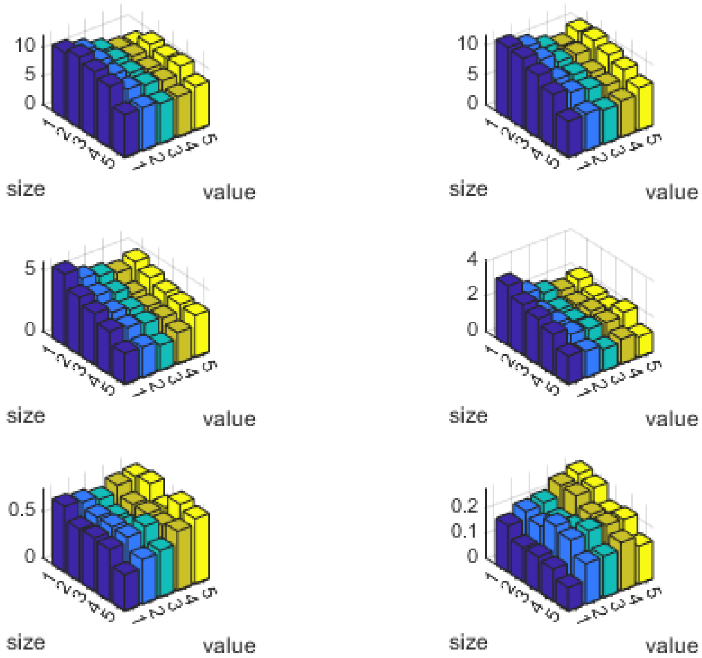
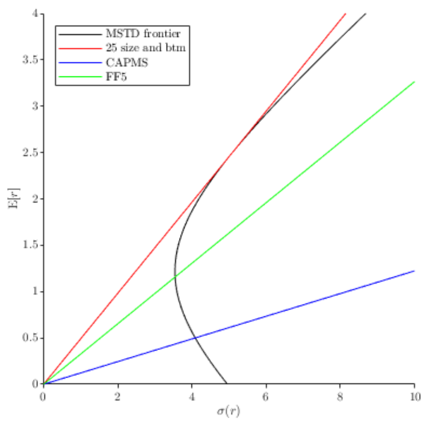
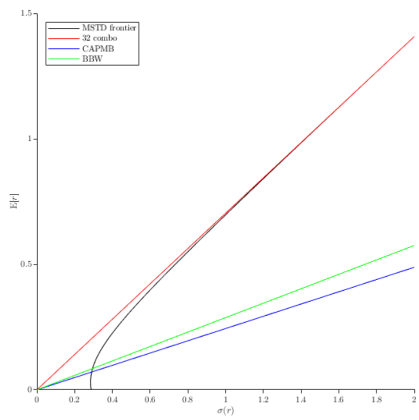


Figure 9: Components of the 25 size and book-to-market portfolios of Fama-French. Figure that follows from the code of Bandi et al. (2021) on the six components of the 25 size and book-to-market portfolios of Fama-French. Data is from January 1967 through December 2018.



(a) MSTD frontier and stock factor models.



(b) MSTD frontier and bond factor models.

Figure 10: Mean-standard deviation frontier and maximum Sharpe ratios for stocks and bonds. Right figure is from Dickerson et al. (2023), left figure is with data of Bandi et al. (2021). The figures plot the mean-standard deviation frontier and maximum Sharpe ratios for stocks and bonds. Figure on the left shows the MSTD frontier for the 25 size and book-to-market portfolios of Fama-French, the maximum Sharpe ratio of the stock CAPM and the maximum Sharpe ratio of optimally combining the Fama-French five-factors (data from January 1967 through December 2022). Figure on the right shows the MSTD frontier for the 32 portfolios of Dickerson et al. (2023), the maximum Sharpe ratio of the bond CAPM and the maximum Sharpe ratio of optimally combining the BBW five-factors (data from August 2004 through December 2021).

B Derivation of the Companion Matrix

We follow the notation from Section 3 and we define the time series vector as $\boldsymbol{x_t} = (y_t, \boldsymbol{\tilde{x}_t^T})^T \in \mathbb{R}^k$, with $y_t \in \mathbb{R}$ and $\boldsymbol{\tilde{x}_t} \in \mathbb{R}^{k-1}$. With the assumption that the whole process follows a VAR(p) process, we can write it as follows, just like Bandi et al. (2021) do:

$$y_t = a_y + A_{1,y} Y_{t-1} + A_{2,y} \cdot \boldsymbol{\tilde{X}_{t-1}} + \epsilon_t^1, \tag{16}$$

$$\boldsymbol{\tilde{x}_t} = a_x + A_{1,x} Y_{t-1} + A_{2,x} \cdot \boldsymbol{\tilde{X}_{t-1}} + \boldsymbol{\epsilon_t^2}, \tag{17}$$

where $Y_{t-1} = (y_{t-1}, \ldots, y_{t-p})^T$ and $\boldsymbol{\tilde{X}_{t-1}} = \{\boldsymbol{\tilde{x}_{t-1}^T}, \ldots, \boldsymbol{\tilde{x}_{t-p}^T}\}^T$. Also, $(\cdot)$ denotes the element-by-element inner product, and $A_{2,y}$ and $A_{2,x}$ are of size $1 \times p$ and $k \times p$. We set $A_{1,x} = 0$, due to restricted Granger causality. Then, by construction:

$$\boldsymbol{X_t} = A\boldsymbol{X_{t-1}} + \boldsymbol{U_t}, \tag{18}$$

where $\boldsymbol{X_t} = (\boldsymbol{x_t}^T, \ldots, \boldsymbol{x_{t-p+1}}^T)^T$ (size $kp \times 1$), $\boldsymbol{X_{t-1}} = (\boldsymbol{x_{t-1}}^T, \ldots, \boldsymbol{x_{t-p}}^T)^T$ (size $kp \times 1$), and $\boldsymbol{U_t} = (\boldsymbol{\epsilon_t}^T, 0, \ldots, 0)^T$ (size $kp \times 1$) with $\boldsymbol{\epsilon_t} = ((\epsilon_t^1)^T, (\boldsymbol{\epsilon_t^2})^{\boldsymbol{T}})^T$. Then, the companion matrix A is of size $kp \times kp$, and we can conclude that:

$$A = \begin{pmatrix} A_1 & A_2 & \ldots & A_{p-1} & A_p \\ I_k & O & \ldots & O & O \\ O & I_k & \ldots & O & O \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ O & O & \ldots & I_k & O \end{pmatrix}, \tag{19}$$

where I_k and O are $k \times k$ identity and zero matrices respectively. Using Equation 16 and 17, we know that every matrix A_i ($k \times k$), due to the restricted Granger-causality, looks as follows:

$$A_i = \begin{pmatrix} a_{i,11} & a_{i,12} & \ldots & a_{i,1k} \\ 0 & a_{i,22} & \ldots & a_{i,2k} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & a_{i,k2} & \ldots & a_{i,kk} \end{pmatrix}. \tag{20}$$

C Programming Code

In this section, I will explain what the zip file with my code and data consists of, which parts I have retrieved and which parts I have written myself. Important to note is that every piece of code only needs to be run once and in Matlab (preferably R2023a). The zip file consists of two folders:

1. The first folder called 'replication_package_bandi2021' consists of the replication package that F. Bandi has sent me and my fellow students. This folder contains data on the portfolio returns, factor models and state variables ('dataBCLT.mat' and 'dataBCLThxz.mat'), that he has used for his paper Bandi et al. (2021). These files are respectively for 'mainT4PanelA.m' and 'mainT4PanelE.m'. This corresponds to the code for Figure 9, Table 13 panel (a) and (e) in my paper and Table 4 panel (a) and (e) in Bandi et al. (2021). I used this code and my own data ('data_bandi.xlsx') for the other panels in Table 13 of which the outcomes follow when running the file 'replicate4.m'.

2. The second folder 'own_research' is the folder that consists of the code and data for the biggest part of my research. The data is in two separate files: (1) 'data_bandi.xlsx', which I have gathered myself as described in the data section. It contains data on the state variables 'DataVAR', the portfolio returns 'Value_growth' through '10PF_Weber' and the stock factor models 'FamaFrench3', 'FamaFrench5' and 'FourFactorModel' for the period of January 1967 through December 2022. Only the 10 duration portfolios reach until August 2014. (2) The second data file is 'data_dickerson.xlsx' which contains the data from Dickerson et al. (2023), which is completely retrieved through `https://data.mendeley.com/datasets/n66rp59tr7/1`. It contains monthly data on the 32 bond portfolio returns, and the traded factors for the period of January 2004 through December 2021. Then, the files in the folder 'LibraryKRS' are from the replication package of Bandi et al. (2021) above, and so are the files 'ComputeMVWoldCOmponentsFromMA.m', 'ComputeWoldComponentsFromVAR.m' (to which I added the code snippet from Appendix D) and 'RedundantHaarlso.m'. These are all used to get the Wold decomposition from a factor model. The files 'csrgl.m', 'csrw.m', 'grs.m', 'nested_r2.m', 'nested.m', 'nonnested_r2.m', 'nonnested.m' and 'nw.m' are also retrieved from Dickerson et al. (2023) through the url above. They are used for the summary statistical analysis of the factor (models). The other files have been written by myself: 'ComputeCovariances.m', 'ComputeCovariancesBonds.m', 'replication.m', 'extension.m', 'frontier.m', and 'summarystatistics.m'. 'frontier.m' and 'summarystatistics.m' are based on files from Dickerson et al. (2023) to get the mean variance frontier and summary statistics of the factor models, and contain parts of their code. 'replication.m' contains the code used for pricing risk from the replication package of Bandi et al. (2021). Running 'replication.m' results in Table 3, Table 4, Figure 1, Figure 2, Table 5, Figure 7, and Table 11. Running 'extension.m' results in Table 1, Table 2, Figure 3, Table 6, Table 7, Table 8, Figure 4, Figure 5, Table 9, Figure 6, Table 10, Figure 8, Table 12, and Figure 10.

D Differences with Bandi et al. (2021)

In this Section, I will highlight the important differences in code and data with Bandi et al. (2021). Important to note is that the replication package of Bandi et al. (2021) only contains the code for my Table 13 panel (a) and (e), and their Table 4 panel (a) and (e). The rest is written by myself.

Firstly, I followed their description on obtaining the data, but it becomes apparent that the data on the state variables differs. This can be seen in Table 14 and Table 15. Also, even when following their description, returns data differ. This can be seen from the files in the replication package enclosed with this research, by comparing the data from the 'data_bandi.xlsx' file with the return data in their replication package.

Table 14: Summary statistics for our state variables. Summary statistics of the state variables that we use in the VAR-estimation: The yield spread between long-term and short-term bonds (TY), the market's price-dividend ratio (PE) and the small-stock value spread (VS). Data are monthly from January 1967 through December 2022.

	TY	PE	VS
Mean	0.795	6.102	1.615
Minimum	-2.010	5.234	1.283
Maximum	2.840	6.978	2.206
SD	0.896	0.394	0.184

Table 15: Summary statistics for the state variables of Bandi et al. (2021). Summary statistics of the state variables that Bandi et al. (2021) use in the VAR-estimation: The yield spread between long-term and short-term bonds (TY), the market's price-dividend ratio (PE) and the small-stock value spread (VS). Data are monthly from January 1967 through December 2018.

	TY	PE	VS
Mean	1.938	3.647	1.478
Minimum	-3.650	2.843	1.162
Maximum	4.550	4.546	1.957
SD	1.469	0.388	0.167

Another difference is that I structure the market components together with the portfolio components, and then use those when estimating the covariances, the weights and the spectral betas. Another option, that Bandi et al. (2021) do not expand on but maybe implement at times, is to decompose the market components independently from the excess portfolio returns. Lastly, a very important difference is that I was unable to find where in their code, Bandi et al. (2021) implement the restriction of no Granger-causality from Section 3.1. In my 'ComputeWoldComponentsFromVAR.m' file, I implement this as follows:

```
Mdl = varm(nvars, nlags);
for i = 1:nlags
    Mdl.AR{i}(2:nvars, 1) = 0;
end
VAR_p_model = estimate(Mdl, z);
const = VAR_p_model.Constant;
betaVAR = [const]';
for i = 1:nlags
    betaVAR = [betaVAR; VAR_p_model.AR{1,i}'];
end
```