Optimal lot sizing with market selection and stochastic demand.

Bachelor Thesis

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Abstract

This thesis presents an extention of the market selection problem. In this problem the demand and revenue are said to be known, but in this thesis we present a model for the market selection problem with stochastic demand and revenue. Because this problem is NP-hard, we have to find heuristics to solve this problem for large amount of time periods, scenarios and markets within a reasonable amount of time. Different profit criteria are introduced, so a supplier can choose the profit criterium he likes. We also provide a computational analysis to test the heuristics compared to the mixed integer programming formulation for the different profit criteria.

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1 Introduction

A supplier has to decide the amount of items to produce in every time period in order to satisfy the demand of the market or customer. This is known as the lot sizing problem. This problem is described in [14] and a lot of research has done to solve this problem with minimum costs. In real life the markets or customers to supply are not fixed. So a supplier first has to choose between certain markets. Given this set of markets and the demand and revenue per market, the total demand per time unit can be calculated and the lot sizing problem can be solved. This is known as the "Market Selection Problem" (MSP). This problem has been researched in [5]. Here they came up with a given demand, but because the future is unknown, no one knows the demand and revenue for each market in the coming periods. Therefore we have to extend the MSP where we take into account demand and revenue in different scenarios.

The concept with a stochastic parameter in a model is known as "Stochastic Programming". This was introduced by Dantzig in [3]. In [1], [2], [6] and [9] the problem is extended and algorithms to solve this problem were found in [8] and [11].

The concept as descriped above is part of "two-stage stochastic integer problem with recourse". This means that the first stage consists of an unknown parameter, but in the second stage this parameter is known. This concept is analysed in [7] with an example of electricity distribution, in [13] with binary variables in the first stage of the problem and in [4], [10] and [12] they did a theoretical analysis.

The MSP problem is a NP-hard problem, so we are not able to solve the problem for large amounts of markets and time periods in a reasonable amount of time. So we have to search for one or more greedy algorithms with lower- and upperbounds, to obtain a value for a specific algorithm. Although the problem is NP-hard, we can find a mathematical programming formulation and run this for a small amount of markets and a small amount of time periods and compare this solution to the solutions of the algorithms.

The goal of this thesis is to find a mathematical programming formulation for the market selection problem with stochastic demand and revenue. Also we have to find different heuristics for this problem and check how good they are for different choices of profit criteria, by running the heuristics for different amounts of markets, time periods and scenarios. Different profit criteria are average profit, worst case profit and N-reliability profit.

This thesis is organized as follows. In Chapter 2 we analyse different models to come up with our own model for the different profit criteria. Three different heuristics are descriped in chapter 3. In chapter 4 we make some assumptions on the data we use to obtain the results. At the end we come up with some conclusions.

2 Model formulation

To find a mathematical programming formulation for the market selection problem with stochastic demand and revenue we have to start at the beginning. In this chapter we first analyse the uncapacitated lot-sizing problem to understand the basics of the lot sizing problem.

Secondly we extend this problem to the market selection problem where the selection of markets has to be determined in order to get a profit at large as possible.

At last we come to our own problem and find a model formulation for the market selection problem with stochastic demand and revenue.

2.1 Uncapacitated Lot-Sizing problem (ULS)

The goal of the lot-sizing problem is to make a production plan for a single product to supply a single market. There will be no upper bound on x_t, so the model is uncapacitated. The basic model as described in [14] is:

min	$\sum^{n}_{t=1} (f_t y_t + p_t x_t + h_t s_t)$		(1)
s.t.	$s_{t-1} + x_t = d_t + s_t$	for t = 1, ..., T	(2)
	$x_t \leq M y_t$	for t = 1, ..., T	(3)
	$s_0 = 0$		(4)
	$s_t , x_t \geq 0$	for t = 1, ..., T	(5)
	$y_t \in \{ 0, 1 \}$	for t = 1, ..., T	(6)

Sets:

{1, ..., T} is the set of time periods indexed by t

Parameters:

f_t is the fixed cost of producing in period t

p_t is the unit production cost in period t

h_t is the unit storage cost in period t

d_t is the demand in period t

s_0 is the initial stock

M is a large number

Variables:

x_t is the amount produced in period t

s_t is the stock at the end of period t

$y_t = 1$ if production occurs in period t and $y_t = 0$ otherwise

Constraints:

(1) is the objective function; Minimizing the total costs

(2) is the "stock constraint"; Stock from previous period + production in the current period must be equal to the demand in the current period + the stock in the current period

(3) is the "production constraint"; x can only be larger than 0 if production occurs in the current period

(4) The initial stock must be equal to zero

(5) s_t and x_t are larger or equal to zero

(6) y_t must be binairy

The uncapacitated lot sizing problem can be solved by dynamic programming. This is explained in [14] with the help of proposition 5.1 and observation 5.4.

Proposition 5.1 *(i) There exists an optimal solution with $s_{t-1}x_t = 0$ for all t. (Production takes place only when the stock is zero.)*
(ii) There exists an optimal solution such that if $x_t > 0$, $x_t = \sum_{i=t}^{t+k} d_i$ for some $k \geq 0$. (If production takes place in t, the amount produced exactly satisfies demand for periods t to t+k.

Observation 5.4 *As $s_t = \sum_{i=1}^{t} x_i - d_{1t}$, the stock variables can be eliminated from the objective function giving $\sum_{t=1}^{n} p_t x_t + \sum_{t=1}^{n} h_t s_t = \sum_{t=1}^{n} p_t x_t + \sum_{t=1}^{n} h_t (\sum_{i=1}^{t} x_i - d_{1t}) = \sum_{t=1}^{n} c_t x_t - \sum_{t=1}^{n} h_t d_{1t}$ where $c_t = p_t + \sum_{i=t}^{n} h_i$. This allows us to work with the modified cost function $\sum_{t=1}^{n} c_t x_t + 0 \sum_{t=1}^{n} s_t + \sum_{t=1}^{n} f_t y_t$, and the constant term $\sum_{t=1}^{n} h_t d_{1t}$ must be substracted at the end of the calculations.*

The term d_{1t} is the demand satisfied until time t. If we let H (k) be the minimum cost of a solution for period k, then the forward recursion is:

$H(k) = \min_{1 \leq t \leq k} \{ H(t-1) + f_t + c_t d_{tk} \}$, where k is the current period, t the last period in which production occurs and $H(0) = 0$.

2.2 Market selection problem (MSP)

The market selection problem consists of 2 problems. First decide which markets to select from M potential markets all with revenue R_m. Secondly, making a production plan to satisfy the selected markets in the first step.

The MSP assumes that market m has a known demand in each period. If a specific market is selected, the market's demand must be satisfied in all periods. If a market is rejected, none of the market's demand must be satisfied and all revenue is lost.

The steps as stated above can be combined to one mixed integer programming problem which is described in [5]:

$$\max \quad \sum^{M}_{m=1} R_m z_m - \sum^{T}_{t=1} \left(f_t y_t + \sum^{t}_{i=1} \sum^{M}_{m=1} c_{i,t,m} x_{i,t,m} \right) \tag{1}$$

$$\text{s.t.} \quad \sum^{t}_{i=1} x_{i,t,m} = z_m \qquad \text{for } t = 1, ..., T;\ m = 1, ..., M \tag{2}$$

$$x_{i,t,m} \leq y_i \qquad \text{for } i,t = 1, ..., T,\ i \leq t;\ m = 1, ..., M \tag{3}$$

$$x_{i,t,m} \geq 0 \qquad \text{for } i,t = 1, ..., T,\ i \leq t;\ m = 1, ..., M \tag{4}$$

$$y_i,\ z_m \in \{0, 1\} \qquad \text{for } i = 1, ..., T;\ m = 1, ..., M \tag{5}$$

Sets:

{1, ..., M} is the set of markets indexed by m

{1, ..., T} is the set of time periods indexed by t and i

Parameters:

R_m is the revenue of market m

f_t is the fixed cost of producing in period t

$c_{i,t,m} = d_{t,m} (p_i + \sum_{j=i}^{t-1} h_j)$ is the cost of satisfying market m in period t using production in production period i

$d_{t,m}$ is the demand of market m in period t

p_i is the unit production cost in production period i

h_j is the unit storage cost in period j

Variables:

$x_{i,t,m}$ is the fraction of demand of market m produced in production period i used for the demand of market m in period t

y_i = 1 if production occurs in production period t and y_t = 0 otherwise

z_m =1 if market m is selected and z_m = 0 otherwise

Constraints:

(1) is the objective function; Maximizing total revenue minus total production costs

(2) is the total amount produced in all production periods for a certain period for a certain market has to be equal to 1 if the market is selected and 0 otherwise

(3) is the "production constraint"; x can only be larger than 0 if production occurs in the current production period

(4) $x_{i,t,m}$ is larger or equal to zero

(5) y_i and z_m must be binairy

This problem is not easy to solve. Therefore this problem is analysed in [5]. If one of the binary variables is known, the problem is easy to solve. If z_m is known the problem becomes a normal lot sizing problem as described before. If y_i is known this means that the production plan is known. Then the problem is easy to solve too, because for every market you can calculate the production and holding costs. If these production and holding costs are more than the revenue you get from that specific market you do not want to choose this market, but if the production and holding costs are less than the revenue of that market you will choose that market.

2.3 MSP with stochastic demand and revenue

The MSP with stochastic demand and revenue consist of S scenarios. All these scenarios will be put in the model as $d_{t,m,s}$ and $R_{m,s}$, where {1, ..., S} is the set of scenarios indexed by s. So the demand not only depends on the time and market but also on the scenario. Similarly the revenue not only depends on the market but also on the scenario.

Not only the demands and revenues are dependent on a scenario, also the decision variables y_i and $x_{i,t,m}$ are related to a specific scenario so they become $y_{i,s}$ and $x_{i,t,m,s}$.

The market selection problem consists of 2 stages. In the first stage the market selection is chosen, while in the second stage the production plan is determined after the demand and revenue is set. In the market selection problem with stochastic demand and revenue the production plan is made for every scenario. Then for every scenario we calculate the profit of the problem. Then we take the average profit as the profit of this combination of markets if we use the "average profit criterium". If we use the "worst case criterium" we take the smallest profit of all scenarios as the profit of the combination of markets. At last, we can sort the profits in decreasing order and take the N^{th} profit. This is called the "N-reliability criterium". With this criterium you can exclude the worst "S – N" profits and so create a "N / S" certainty on this profit. This profit will be maximized over all the possible combination of markets for every criterium and then we have the "maximum average profit", the "maximum worst case profit" and the "maximum N-reliability profit" of the problem.

These criteria are chosen so the supplier can choose the criterium according to their needs. If a supplier want to be 100% sure of archieving a specific profit, this supplier chooses the worst case profit criterium. If a supplier wants to obtain a maximum average profit, it takes the average profit criterium. And last if a supplier wants to obtain in at least 90% of the scenarios a larger profit than the maximum profit, it choses the N-reliability profit.

There are more criteria, but we choose to take these 3 as example and determine the models and heuristics of these criteria.

2.3.1 Average profit criterium

The average profit criterium MSP is just a normal MSP, where the revenues of all scenarios are added up and devided by the amount of scenarios.

The model is as follows:

$$\max \quad (\sum^{M}_{m=1} \sum^{S}_{s=1} R_{m,s}z_m - (\sum^{S}_{s=1} \sum^{T}_{i=1} f_i y_{i,s} + \sum^{t}_{i=1} \sum^{M}_{m=1} c_{i,t,m,s}x_{i,t,m,s})) / S \qquad (1)$$

$$\text{s.t.} \quad \sum^{t}_{i=1} x_{i,t,m,s} = z_m \qquad \text{for } t = 1, ..., T;\ m = 1, ..., M;\ s = 1, ..., S \qquad (2)$$

$x_{i,t,m,s} \leq y_{i,s}$	for i,t = 1, ..., T, $i \leq t$; m = 1, ..., M; s = 1, ..., S	(3)
$x_{i,t,m,s} \geq 0$	for i,t = 1, ..., T, $i \leq t$; m = 1, ..., M; s = 1, ..., S	(4)
$y_{i,s}$, $z_m \in \{ 0, 1 \}$	for i = 1, ..., T; m = 1, ..., M; s = 1, ..., S	(5)

Sets:

{1, ..., M} is the set of markets indexed by m

{1, ..., T} is the set of time periods indexed by t, i and j

{1, ..., S} is the set of scenarios indexed by s

Parameters:

$R_{m,s}$ is the revenue of market m in scenario s

f_i is the fixed cost of producing in production period i

$c_{i,t,m,s} = d_{t,m,s} (p_i + \sum_{j=i}^{t-1} h_j)$ is the cost of satisfying market m in period t using production in production period i in scenario s

$d_{t,m,s}$ is the demand of market m in period t in scenario s

p_i is the unit production cost in production period i

h_j is the unit storage cost in storage period j

Variables:

$x_{i,t,m,s}$ is the fraction of demand of market m produced in production period i used for the demand of market m in period t in scenario s

$y_{i,s} = 1$ if production occurs in production period i in scenario s and 0 otherwise

$z_m = 1$ if market m is selected and 0 otherwise

Constraints:

(1) is the objective function; Maximizing average revenue minus average productioncosts

(2) is the total amount produced in all production periods for a certain period for a certain market has to be equal to 1 if the market is selected and 0 otherwise

(3) is the "production constraint"; x can only be larger than 0 if production occurs in this production period

(4) $x_{i,t,m,s}$ is larger or equal to zero

(5) $y_{i,s}$ and z_m must be binairy

In this model, we only determine which markets to select to obtain maximum average profit. We do not determine a production plan, because in every scenario an other production plan is possible. When we have to make a production plan the demand of the chosen markets is known, so the problem becomes a normal lot sizing problem.

If demand is still unknown when making a production plan we have to find one production plan that is optimal for every scenario. This is the case when the set up periods has to be determined before the demand is known, for example when a reservation has to be made for using a machine. Therefore we have to adapt the model to:

$$\max \quad \Big(\sum_{m=1}^{M} \sum_{s=1}^{S} R_{m,s} z_m - \Big(\big(\sum_{i=1}^{T} f_i y_i \big) + \sum_{s=1}^{S} \sum_{t=1}^{T} \sum_{i=1}^{t} \sum_{m=1}^{M} c_{i,t,m,s} x_{i,t,m,s} \Big) \Big) / S \qquad (1)$$

$$\text{s.t.} \quad \sum_{i=1}^{t} x_{i,t,m,s} = z_m \qquad \text{for } t = 1, ..., T;\ m = 1, ..., M;\ s = 1, ..., S \qquad (2)$$

$$x_{i,t,m,s} \leq y_i \qquad \text{for } i,t = 1, ..., T,\ i \leq t;\ m = 1, ..., M;\ s = 1, ..., S \qquad (3)$$

$$x_{i,t,m,s} \geq 0 \qquad \text{for } i,t = 1, ..., T,\ i \leq t;\ m = 1, ..., M;\ s = 1, ..., S \qquad (4)$$

$$y_i,\ z_m \in \{ 0, 1 \} \qquad \text{for } i = 1, ..., T;\ m = 1, ..., M; \qquad (5)$$

The only difference with the MSP model with stochastic demand and revenue with known demand after stage 1 is that the y-variable has no index s. So it does not change in every scenario. With this model you inmediately have an optimal set of set up periods. In this paper we do not use this model, because we assume that the demand is known after making a market selection and before making a production plan, so the lot sizing problem is solved with known demand.

2.3.2 Worst case profit criterium

The worst case profit criterium consists of one specific scenario, namely the one with the worst profit. The model is as follows:

$$\max \quad \delta \qquad (1)$$

$$\text{s.t.} \quad \sum_{i=1}^{t} x_{i,t,m,s} = z_m \qquad \text{for } t = 1, ..., T;\ m = 1, ..., M;\ s = 1, ..., S \qquad (2)$$

$$x_{i,t,m,s} \leq y_{i,s} \qquad \text{for } i,t = 1, ..., T,\ i \leq t;\ m = 1, ..., M;\ s = 1, ..., S \qquad (3)$$

$$\delta \leq \sum_{m=1}^{M} R_{m,s} z_m - \Big(\big(\sum_{i=1}^{T} f_i y_{i,s} \big) + \sum_{t=1}^{T} \sum_{i=1}^{t} \sum_{m=1}^{M} c_{i,t,m,s} x_{i,t,m,s} \Big) \qquad \text{for } s = 1, ..., S \qquad (4)$$

$x_{i,t,m,s} \geq 0$ for i,t = 1, ..., T, i ≤ t; m = 1, ..., M; s = 1, ..., S (5)

$y_{i,s}, z_m \in \{0, 1\}$ for i = 1, ..., T; m = 1, ..., M; s = 1, ..., S (6)

Compared to the MSP with stochastic demand (average profit) the following changes have been made:

Additional variables:

δ is the worst case profit

Constraints:

(1) is the objective function; The worst case profit

(2) is constraint (2) in the MSP with stochastic demand (average profit)

(3) is constraint (3) in the MSP with stochastic demand (average profit)

(4) is the "worst case constraint"; The worst case profit has to be smaller or equal to the profit of every scenario so if we maximize the worst case profit in the objective function, this profit is equal to the smallest scenario profit.

(5) is constraint (4) in the MSP with stochastic demand (average profit)

(6) is constraint (5) in the MSP with stochastic demand (average profit)

2.3.3 N-reliability profit criterium

The N-reliability profit criterium also consists of one specific scenario. This time it is the scenario according to the N^{th} best profit. The model is as follows:

max δ (1)

s.t. $\sum^{t}_{i=1} x_{i,t,m,s} = z_m$ for t = 1, ..., T; m = 1, ..., M; s = 1, ..., S (2)

$x_{i,t,m,s} \leq y_{i,s}$ for i,t = 1, ..., T, i ≤ t; m = 1, ..., M; s = 1, ..., S (3)

$\delta \leq \sum^{M}_{m=1} R_{m,s} z_m - ((\sum^{T}_{i=1} f_i y_{i,s}) +$

$\sum^{T}_{t=1} \sum^{t}_{i=1} \sum^{M}_{m=1} c_{i,t,m,s} x_{i,t,m,s}) + (1 - w_s) L$ for s = 1, ..., S (4)

$\sum^{S}_{s=1} w_s = N$ (5)

$x_{i,t,m,s} \geq 0$ for i,t = 1, ..., T, i ≤ t; m = 1, ..., M; s = 1, ..., S (6)

$y_{i,s}, z_m, w_s \in \{0, 1\}$ for i = 1, ..., T; m = 1, ..., M; s = 1, ..., S (7)

Compared to the MSP with stochastic demand (worst case profit) the following changes have been made:

Additional Parameters:

L is a large number; this number has to be larger than the difference between the worst case profit and the N-reliability profit, but because the N-reliability profit is unknown, you can choose L for example equal to the average revenue times the amount of markets.
N is the reliability, so the N-th largest profit scenario is chosen to obtain an optimal market selection.

Additional variables:

w_s is 1 if the scenario is in the top N of largest profits

Constraints:

(1) is the objective function; The worst case profit, after awarding the (S - N) smallest profits.
(2) is constraint (2) in the MSP with stochastic demand (worst case profit)
(3) is constraint (3) in the MSP with stochastic demand (worst case profit)
(4) is constraint (4) in the MSP with stochastic demand (worst case profit), but now we award the (S – N) smallest profits with L so the S-N+1th scenario is the new worst case scenario and so the N-reliability scenario.
(5) is the "N-reliability constraint"; The total amount of scenarios not awarded by L has to be equal to N
(6) is constraint (5) in the MSP with stochastic demand (worst case profit)
(7) is constraint (6) in the MSP with stochastic demand (worst case profit) plus w_s is also binary

3 Heuristics

Because the MSP is NP-hard, the MSP with stochastic demand is also NP-hard. For large amount of time periods, markets and scenarios the models can not be solved. Therefore we came up with some heuristics. In the following section we propose three heuristics and we test these heuristics in chapter 4.

3.1 Greedy heuristic

The first heuristic is a greedy heuristic, where we sort the markets according to their average revenue and average demand. The markets with a large average revenue and a small average demand are likely to be in the model because they make a lot of profit. That is the idea of the greedy heuristics.

We came up with two different greedy heuristics, the normal greedy heuristic and the inverse greedy heuristic. We thought it was a good plan to combine these two in 1 total greedy algorithm to not only obtain a local maximum but also a global maximum.

3.1.1 Normal greedy heuristic

In the normal greedy heuristic the average revenues for each market are divided by the average demand for each market. This will result in what we call "the marketprofit". The marketprofits are sorted in decreasing order, so the market with the relatively highest average profit and the smallest average demand is in the first place.

The heuristic is as follows:

1. Start with an initial empty set of markets
2. For every market take the average revenues over all scenarios and the average demand over all time periods and scenarios. Devide this revenue by the demand and store this in a variable "marketprofit". Sort these marketprofits in decreasing order.
3. Add the first market with the highest "marketprofit" to the set of markets.

4. With this set of markets an optimal production plan can be made according to the lot-sizing problem and we can calculate the profit for all scenarios.
5. If this profit is higher than or equal to the last profit go again to step 3 to add another market. If this profit is smaller than the last profit and the last profit is larger than 0, the last set of markets is a local maximum. If this profit is smaller or equal to 0, selecting no market results in a higher profit.

In step 4 we calculate the profit for all scenarios. Here we can apply different criteria as described before. For the average profit criterium we add the profit for all scenarios and devide this by the amount of scenarios. This profit we use to go to step 5.

For the worst case profit criterium we just take the smallest profit over all scenarios and use this profit to go to step 5.

For the N-reliability profit criterium we sort the profits over all scenarios and take the N^{th} profit and use this profit to go to step 5.

At the end we have local optimal market selections for each of the different criteria.

3.1.2 Inverse greedy heuristic

The second greedy heuristic is the inverse greedy heuristic. In this heuristic we again calculate the “marketprofits”, but now we sort the markets in non-decreasing order. The difference with the normal greedy heuristic is that we start by selecting all markets and in step 3 delete the first market with the smallest “marketprofit”. This heuristic gives also for every criterium a local optimum market selection.

3.1.3 Total greedy heuristic

A combination of the 2 greedy heuristics can be archieved by implementing the normal greedy heuristic and continue until all markets are selected.
So only step 5 chances and we add step 6, so the heuristic becomes:

5. Store this profit with according market selection and go again to step 3 to add the next market. If all markets are selected, go to step 6.

6. Select the market selection with the highest profit. If this profit is larger than 0, this set of markets is a global maximum. If this profit is smaller or equal to 0, selecting no market results in a higher profit.

This heuristic gives a global maximum instead of a local maximum, but takes longer to find it than the 2 other greedy heuristics.

3.2 Iterative heuristic

In [5] this heuristic is proposed. They stated the MSP is easy to solve given a selection of markets and the MSP is easy to solve with a known production plan. That is why they suggest an alternating heuristic which for a given set of markets calculates the optimal production plan. With this production plan the optimal set of selected markets is calculated etc. Until the market selection and optimal production plan is not changing anymore. The profit of this solution is again a local optimum. In [5] they started in their heuristic with an initial production plan and then determine the optimal market selection. But because in the MSP with stochastic demand and revenue there is for every scenario a different production plan, we decided to start with an initial market selection and then calculates the optimal production plan. Otherwise we have to specify different production plans for each scenario.

The iterative heuristic consists of the following steps:

1. Start with an initial market selection z.
2. Given this selection of markets, determine the optimal production plan y = y(z)
3. Given this plan, find the optimal set of markets z = z(y)
4. Go back to step 2 until the market selection is the same as last iteration.

In each iteration an optimal production plan has to be determined. For the different criteria the production plans are different. For the worst case profit criterium or the N-reliability profit criterium, we take the production plan according to the worst case profit or the N-reliability profit. For the average case it is more complicated, because the average profit criterium has no corresponding scenario and therefore no corresponding production plan. We came up with the plan to take the rounded average production plan. So if in a certain time period production takes place in more than half of the scenarios, then in the average production plan production takes place in this time period. If in a certain time period production takes place in no more than half of the scenarios, then in the average production plan production does not take place in this time period.

After the production plan is determined, an optimal market selection has to be calculated. Also this is different for the different criteria. For worst case or N-reliability it is again very easy, because we take the demand and revenues according to the worst case scenario or N-reliability scenario in the step before and calculate the optimal market selection. For the average case we decided to take the average demand over all scenarios to calculate the optimal market selection.

The calculation of the optimal market selection will be done according to the following procedure. Given a production plan, a market m is selected if the variable production and holding costs are less than the revenue. The fixed costs are not taken into account because they are already determined by the production plan. They cannot change anymore.

We have to start with an initial market selection. Different market selection are possible, but most of the time a local optimum will come out of this heuristic. If zero markets are selected as initial market selection, the production plan is obvious; do not produce at all. This will not change, so that is the outcome. This is obviously a local optimum. If we take different initial market selections and compare these after the heuristic has been runned, the probability of finding a global optimum will increase.

But how do we have to select the different initial market selections. It is not desirable to try all possible combinations, because then we do not need a heuristic anymore. This takes way too long. In our implementation we use the same amount of initial market selections as markets. The first initial market selection consists of one random market, the second initial market selection consists of two random markets, the n^{th} initial market selection consists of n random markets. So the last initial market selection consists of all markets. With this system of initial market selections we have a large probability to find the global optimum.

3.3 Rounding procedure

The last heuristic is a rounding procedure. This heuristic is also proposed in [5].

1. Solve the problem as LP problem. Sort the markets in decreasing order of z-value.
2. Select the first market and solve the lot sizing problem.
3. Select also the next market until all markets are selected.
4. Choose the market selection with the highest (average, worst case, N-reliability) profit.

The LP problem is solved in AIMMS. The MSP with stochastic demand and revenue model is used and the y and z variables are relaxed. The N-reliability profit criterium has an extra integer variable, so we have to choose if we want to relax this variable too. We have choosen to look at both cases, namely to relax the w variable and not to relax the w variable. If we do not relax the w variable, the problem is still a mixed integer problem, but this problem is much easier to solve, because we only have S integer variables. The other problems are normal LP problems.

AIMMS provides the z-values of the different markets. These z-values will be used in the heuristic. Only the values with the highest z-values are rounded to 1, the others to 0. So there is some treshold that determines if a z-value becomes 1, or becomes 0.

4 Computational results

In this section we present the results of the performance of the heuristics. First we explain how we generated the data. After that, the results are discussed of the heuristics for every criterium.

4.1 Data generation

Some assumptions on the data are made in the first place. In all computational results we assumed the costs parameters as time invariant. This means that the fixed costs of producing f_i, the unit production costs p_i and the unit storage costs h_j are not dependent on time, so they become f, p and h.

Second we have to determine the size of the sets. We let the amount of markets, time periods and scenarios change so we can see how these sets influences the optimal solution and the solution time. The amount of markets, time periods and scenarios can not be too large because we want to compare the outcomes of the heuristics with the outcome of the mixed integer programming solution, so the mixed integer programming problem must give a solution. Also we have to decide how many set up periods we want on average. The amount of set up periods we can use to calculate a value for the fixed set up costs. We call the amount of set up periods alpha. The higher the alpha, the higher the fixed set up costs, the lower the amount of production periods, because it is expensive to produce.

We decided to have one initial data set with 12 markets, 6 time periods, 10 scenarios and alpha equal to 2. Then we in- or decrease the size of one set to see how the size of this set influences the solution and solution time. So we have the initial datasets and 8 other datasets we test as we see in table 4.1.

Now we have to determine which parameters stay the same all these runs and which change. Of all parameters the unit production costs and the unit holding costs do not depend on the amount of markets, scenarios or time periods in the datasets, so we set p = 10 and h = 1 in all runs. Similarly the demand does not depend on the amount of markets, scenarios or time periods, but for every run they are generated from an integer uniform distribution over the interval [0, 100], so the mean of the demand equals 50 in all runs.

For the fixed costs of producing we need the alpha as discussed before. Because the fixed cost of producing determines the amount of times production takes place. The right formula for the fixed costs of producing is: $f = alpha \cdot M \cdot dmean$, where M is the amount of markets and dmean is the mean of the demand (= 50).

It is obvious that if we do not want to have only all markets selected or no market selected (straightforward solutions), the average revenue has to change over the dataruns, because if the amount of time periods is higher, the totalcosts are higher too. So we have found a expression for the average revenue we can use to generate the revenues from an integer uniform distribution over the interval [0, 2 · average revenue]. The totalcosts consist of three subcosts, namely the fixed costs of producing (kmean), the unit production costs (pmean) and the unit storage costs (hmean). We found a good expression so these costs are divided over the markets.

$$kmean = ((T / alpha) \cdot f) / (1/2M)$$

T is the amount of time periods, alpha as discussed above, f is the fixed costs of producing and M is the amount of markets, so the total production costs will be devided by half of the markets.

$$pmean = p \cdot dmean \cdot T$$

p is the unit production costs, dmean is the average demand and T is again the amount of time periods, so the this is the expected total unit costs for each market in each scenario.

$$hmean = \sum_{i=1}^{alpha-1} (i (dmean \cdot h)) (T / alpha)$$

h is the unit storage costs, so this is the expected total unit storage costs for each market in each scenario.

If we sum up these costs we have the total costs for each market in each scenario, so if we set this equal to the average revenue, we expect zero profit if we choose half of the markets.

For the N-reliability profit criterium we need 2 more parameters, namely the large number L and the N. The L has to be larger than the difference between the worst case profit and the N-reliability profit, but because the N-reliability profit is unknown, we choose L equal to the difference between the average case profit and the worst case profit. The N is the amount of reliability. If you want to be n% sure of a certain profit, the N is equal to the amount of scenarios times n%. This number has to be integer.

4.2 Results

So we have 9 datasets to test. In table 4.1 we see the diffent sets and parameters as explained in the last section.

Run	# Markets	# Time periods	# Scenarios	alpha	f	Rmean
1	12	6	10	2	1200	7500
2	15	6	10	2	1500	7500
3	10	6	10	2	1000	7500
4	12	8	10	2	1200	10000
5	12	4	10	2	1200	5000
6	12	6	12	2	1200	7500
7	12	6	8	2	1200	7500
8	12	6	10	1	600	7200
9	12	6	10	3	1800	8100

For every datarun:	
p	10
h	1
dmean	50
N	# Scenarios - 1

Table 4.1: Different datasets

For every profit criterium these datasets are runned. The outcomes are presented in the sections below. Every run consists of a mixed integer programming run with AIMMS 3.8 and a run for each of the heuristics with MATLAB 2006. The LP relaxation in the rounding procedure heuristic is also done in AIMMS 3.8.

We have tried to test the heuristics for larger amounts of markets, scenarios and time periods, but because the computer on which we runned the mixed integer programming problem can not handle these amounts of variables. The solution times become very large, so maybe for future research we have to find a computer that is a lot faster and has more memory. Although the solution time is very high at large problems we have tested one dataset in section 4.2.4.

4.2.1 Results on average profit criterium

In this section the results of the average profit criterium are presented. In table 4.2 you see the most important statistics.

	Straightforward ?				Sol. Time		Gap				Solution optimal ?				
Run	MIP	GH	IH	RP	MIP	LP	MIP	GH	IH	RP	MIP	GH	IH	RP	LP
1	no	no	yes	no	0.66	1.38	0.00%	0.00%	10.90%	0.00%	yes	yes	no	yes	yes
2	no	no	yes	no	0.89	2.31	0.00%	0.00%	100.00%	0.00%	yes	yes	no	yes	yes
3	no	no	yes	no	0.84	1.09	0.00%	0.00%	100.00%	0.00%	yes	yes	no	yes	yes
4	no	no	yes	no	2.56	3.53	0.00%	0.00%	0.07%	0.00%	yes	yes	no	yes	no
5	no	no	yes	no	0.2	0.45	0.00%	0.00%	27.66%	0.00%	yes	yes	no	yes	yes
6	yes	yes	yes	yes	1.08	1.95	0.00%	0.00%	0.00%	0.00%	yes	yes	yes	yes	yes
7	no	no	yes	no	0.41	0.99	0.00%	0.00%	28.13%	0.00%	yes	yes	no	yes	yes
8	yes	yes	yes	yes	4.45	1.3	0.00%	0.00%	0.00%	0.00%	yes	yes	yes	yes	no
9	no	no	yes	no	0.67	1.67	0.00%	0.00%	11.16%	0.00%	yes	yes	no	yes	yes
Total	7/9 no	7/9 no	0/9 no	7/9 no	1.31	1.63	0.00%	0.00%	30.88%	0.00%	9/9 yes	9/9 yes	2/9 yes	9/9 yes	7/9 yes

Table 4.2: Results average profit criterium

Column “Straightforward ?” present if the solution of the run is straightforward. Straightforward is selecting no markets at all or all markets. “MIP” is the mixed integer programming solution calculated with AIMMS. “GH” is the solution of the greedy heuristic, “IH” is the solution of the iterative heuristic and “RP” is the solution of the rounding procedure. In column “Sol. Time” are the solution times in seconds of the MIP and LP relaxation in AIMMS. We have not calculated the solution times of the heuristics, but they all runned within one second. Column “Gap” shows for the MIP the LP gap and for the heuristics it gives the gap with the MIP solution calculated as:

$$GapHeuristic = (Solution\ MIP - Solution\ Heuristic) / Solution\ MIP$$

In the column “Solution optimal ?” is stated if the solution of the MIP or heuristics is optimal. Column “LP” shows if the LP relaxation in the rounding procedure provides an optimal solution. In the bottom row are the totals of the columns. So the first part counts how many times the solution of the MIP or heuristic is straightforward. The second part gives the average solution times of the MIP and LP relaxation. The following part gives the average gap, the fourth part gives the amount of time the MIP or heuristics give an optimal solution and the last part counts how many times the LP relaxation gives the optimal solution.

Table 2 shows that in all cases the iterative heuristic gives a straightforward solution, while in the mixed integer programming solution in only two cases there is a straightforward solution, so the iterative heuristic finds in only two cases the optimal solution while the greedy heuristic and the rounding procedure found in all cases the optimal solution. The average gap of the iterative heuristic is 30.88%. In seven instances the LP relaxation gives the optimal solution, so for the rounding procedure it is easy to calculate the optimal solution.

The best algorithms for the average profit criterium are the greedy heuristic and the rounding procedure because they find in all cases the optimal solution.

4.2.2 Results on worst case profit criterium

The performance of the heuristics on the worst case profit criterium can be found in table 4.3.

	Straightforward ?				Sol. Time		Gap				Solution optimal ?				
Run	MIP	GH	IH	RP	MIP	LP	MIP	GH	IH	RP	MIP	GH	IH	RP	LP
1	no	no	yes	no	39.39	1.91	0.00%	20.16%	100.00%	4.09%	yes	no	no	no	no
2	yes	yes	yes	yes	0.98	2.17	0.00%	0.00%	0.00%	0.00%	yes	yes	yes	yes	yes
3	yes	yes	yes	yes	0.14	0.55	0.00%	0.00%	0.00%	0.00%	yes	yes	yes	yes	yes
4	yes	yes	yes	yes	31.38	6.03	0.00%	0.00%	0.00%	0.00%	yes	yes	yes	yes	no
5	yes	yes	yes	yes	0.11	0.36	0.00%	0.00%	0.00%	0.00%	yes	yes	yes	yes	yes
6	no	yes	yes	yes	15.67	2.33	0.00%	100.00%	100.00%	100.00%	yes	no	no	no	no
7	yes	yes	yes	yes	3.47	0.94	0.00%	0.00%	0.00%	0.00%	yes	yes	yes	yes	no
8	no	yes	yes	no	5.44	1.5	0.00%	100.00%	100.00%	30.94%	yes	no	no	no	no
9	yes	yes	yes	yes	0.38	1.26	0.00%	0.00%	0.00%	0.00%	yes	yes	yes	yes	yes
Total	3/9 no	1/9 no	0/9 no	2/9 no	10.77	1.89	0.00%	24.46%	33.33%	15.00%	9/9 yes	6/9 yes	6/9 yes	6/9 yes	4/9 yes

Table 4.3: Results worst case profit criterium

The columns present the same statistics as table 4.2, which is explained in section 4.2.1.

In the worst case profit criterium a lot of straightforward solutions are optimal, so this criterium can not be reviewed very well. Only three cases are non-straightforward, but in none of these cases one of the heuristics find the optimal solution. In four cases the LP relaxation finds the optimal solution. The rounding procedure gives the smallest gap, but has the same optimal solutions as the other heuristics.

Therefore we can conclude that none of the heuristics works better than the other in the worst case profit criterium. We have searched for good data to obtain some non-straightforward solutions but they are very hard to find and therefore we can not test this criterium in detail.

4.2.3 Results on N-reliability profit criterium

In table 4.4 the results of the N-reliability profit criterium are presented.

	Straightforward ?					Solution time			Gap				
Run	MIP	GH	IH	RP	RP2	MIP	LP	MIP 2	MIP	GH	IH	RP	RP2
1	no	no	yes	no	no	600	0.72	5.45	228.00%	11.84%	78.02%	12.76%	-33.21%
2	no	yes	yes	yes	no	600	1.17	7.38	265.00%	-100.00%	-100.00%	-100.00%	-101.48%
3	yes	yes	yes	yes	yes	4.88	0.63	2.51	0.00%	0.00%	0.00%	0.00%	0.00%
4	no	yes	yes	yes	no	600	3.89	13.8	939.00%	86.26%	86.26%	86.26%	-44.79%
5	no	yes	yes	yes	no	19.17	0.23	1.16	0.00%	100.00%	100.00%	100.00%	0.00%
6	no	no	yes	no	no	11.56	2.63	4.02	0.00%	59.74%	94.19%	82.02%	0.00%
7	no	no	yes	no	no	600	0.7	2.45	334.00%	-66.67%	100.00%	-66.67%	-66.67%
8	no	no	yes	no	no	63.34	1.25	4.13	0.00%	45.01%	68.35%	3.92%	0.00%
9	no	yes	yes	yes	no	600	1.2	3.28	778.00%	-100.00%	-100.00%	-100.00%	-113.31%
Total	8/9 no	4/9 no	0/9 no	4/9 no	8/9 no	344.33	1.38	4.91	282.67%	4.02%	36.31%	2.03%	-39.94%

	Solution optimal ?						
Run	MIP	GH	IH	RP	RP2	LP	MIP2
1	no	no	no	no	yes?	no	no
2	no	no	no	no	yes?	no	no
3	yes	yes	yes	yes	yes	no	yes
4	no	no	no	no	yes?	no	no
5	yes	no	no	no	yes	no	no
6	yes	no	no	no	yes	no	no
7	no	yes	no	yes?	yes?	yes	no
8	yes	no	no	no	yes	no	no
9	no	no	no	no	yes?	no	no
Total	4/9 yes	2/9 yes	1/9 yes	2/9 yes	9/9 yes	1/9 yes	1/9 yes

Table 4.4: Results N-reliability profit criterium

Also table 4.4 has the same contents as table 4.2 and 4.3, only extra columns are added with “RP2” and “MIP2”. “RP” is the normal rounding procedure with all variables relaxed and “RP2” is the rounding procedure with only the market selection variable and the production variable relaxated, so the w_s variable is still binary, so the problem is still a mixed integer programming problem. “MIP2” is this according problem. In the N-reliability model the N is in every run 1 less than the amount of scenarios, so the solution has to be near the solution of the worst case. This is not true as you see in table 4.4. In only 1 case there is a straightforward solution. As presented in the table the mixed integer programming problem not always gives the optimal solution, because we set the maximum runtime to 600 seconds. The average gap of this problem is 282.67%. In only four cases the “MIP” finds an optimal solution. Surprisingly the “RP2” finds in every case at least an equal solution to the “MIP”. In some cases the “RP2” performs better. We do not know if these solutions are optimal, because we do not know the optimal solution. This explains the negative gaps in the table. The solution time of the “MIP2” is about four times as long as the solution time of the LP relaxation, so for larger instances this would not become a problem.

We can conclude that in the N-reliability profit criterium the rounding procedure where we do not relax the w_s variable performs best. It performs even better and a lot faster than the mixed integer programming problem.

4.2.4 Results on a large problem

In this section we test a larger problem. We consider 25 markets, 15 time periods, 20 scenarios and an alpha of 5, because this is the maximum capacity of the memory on the computer.

The fixed costs of producing are $25 \cdot 5 \cdot 50 = 6250$, the unit and holding costs are still 10 and 1. The average revenue is 10500 and the average demand is still 50.

Unfortunately we have a straightforward solution for the worst case profit criterium and the 18-Reliability profit criterium, so we have only results for the average profit criterium.

In the opimal solution of the average profit criterium we select 20 of the 25 markets. We have a solving time of over 4800 seconds with a LP gap of 0.24%. The greedy heuristic and the rounding procedure give the same solution as the “MIP”. Only the iterative heuristic does not give this solution, but gives again a straightforward solution of selecting all markets and this solution has a gap of about 11% compared to the solution obtained with the “MIP”. The LP relaxation solves the problem in 4.47 seconds with the method of “Barrier”, but gives no optimal integer solution. The rounding procedure after the LP relaxation does give the same solution as the “MIP” within one second. Also the greedy heuristic runs within one second.

So because the solving time of this problem is very high, we would not go on with larger instances in this thesis but recommend for future research to look at these problems on a faster computer.

5 Conclusion & future research

With the help of the uncapacitated lot sizing model and the market selection problem model we found a nice model for the market selection problem with stochastic demand and revenue. For three specific profit criteria, namely the average profit, worst case profit and N-reliability profit, we found different models, so a supplier can choose which profit criteria to use.

Furthermore we found three heuristics to solve the model for large instances of scenarios, time periods and markets. To test these heuristics we use smaller amounts of scenarios, time periods and markets, so we can compare the solutions of the heuristics to the solutions of the mixed integer programming solution.

The results show that for different criteria, different heuristics are the best. For the average profit criterium the greedy heuristic and the rounding procedure are the best, while for the N-reliability profit criterium only the rounding procedure where we do not relax the w_s variable is the best. For the worst case profit criterium the heuristics found in the same amount of cases an optimal solution. The iterative heuristic is in none of the criteria the best heuristic while this heuristic was the best to solve the market selection problem with known demand and revenue.

For future research we can look at larger instances of markets, scenarios and time periods. For these larger problems we need a high speed computer with a lot of memory to obtain a solution for the mixed integer programming problem and maybe an other solver in AIMMS. If these larger instances are runned, we have to measure the solving time of the heuristics to compare the speed of the heuristics. Also we have to find an alternative definition for the revenues, fixed production costs and demands, so there will be no straightforward solutions at the worst case profit criterium. Then we can evaluate the worst case profit criterium better. Also there may be other criteria to look at, in stead of the three we evaluated.

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