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Justifying Mass Voting Epistemically

Research Master's Thesis

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Chapter 1

Introduction

1.1 Background and Context

In the European Union, the average size of a democratic electorate composed of all possible eligible voters is roughly fourteen million individuals. The average size of an actual electorate (so counting only those voters who cast a vote, and not all those who have the right) is roughly ten million. The number of individuals involved in democratic voting procedures is thus high. A question now arises: is having so many voters vote *ideally*[1] justified if our primary aim is to maximize the common good? This thesis, through an extensive reflection on an argument against mass participation, intends to show why this *can* be the case.

Modern democracies — as we just saw — oftentimes consist of millions of citizens, who have the right to vote. In fact, in a democratic state, every citizen who satisfies some basic requirements has this right. Universal participation — that given the population size of modern democracies translates to *mass-participation* — is usually considered one of the necessary conditions to call a form of government truly democratic. The standard, and intuitive, reason behind it is that universal participation is the most straightforward way to ensure that a democratic system embodies a set of desirable values.[2] For instance, it guarantees political equality, it recognizes human dignity, and it preserves liberty (Riker 1978). These are

1. The content of this thesis, and all the arguments in it, position themselves in the camp of ideal theory. What I am going to discuss relates to how democracies *could* function in ideal circumstances and under simplifying modeling assumptions. There is no ambition to offer a complete and fully realistic picture of how actual democracies work. The following discussion — I believe — may nonetheless reveal precious insight that could help us draw (partial) conclusions about democratic decision-making in the real world.

2. Universal participation has here a precise meaning: first, every citizen should have an *equal right* to participate the the process of decision-making; and, second, it is desirable that everyone actually participate.

typical non-instrumental justifications for democracy. But it can also be justified as a means, for instance, to pledge social justice (Christiano 2018). Despite being all interesting in their own respects, this thesis will not engage in any of these justifications.

There could in fact be another line of justification which is purely *instrumental*: democracies can be justified *epistemically*; i.e., democratic electorates may be justifiable because they ensure that or increase the probability that the correct choice is made. This line of contemporary argumentation in favor of democracy emerged, thanks to the work of Jules Coleman and John Ferejohn (Coleman and Ferejohn 1986), as a response to an influential publication by W. Riker (Riker 1978), who put democracy in front of the challenges raised by Social Choice Theory. The notion has been then fleshed out and refined by Joshua Cohen (Cohen 1986).

Epistemic democrats basically affirm that democratic social arrangements are justified due to the epistemic virtues of democratic decision-making. Democratic decision-making — they claim — has the tendency to lead to the correct decision. And, arguably, making the correct decision in a democracy can greatly impact society in terms of expected benefits. Hence, we may well want to arrange our society democratically. The CJT offers a formal argument in favor of this claim.

1.2 The CJT and Epistemic Democracy

The CJT is the basic starting point of this thesis. A "Jury Theorem" is a mathematical theorem concerning the ability of a group — traditionally, a *jury* — to make epistemically correct decisions under majority rule. Despite being slightly diverse in their assumptions, all such theorems originate from a formal argument first put forth by Nicolas de Condorcet in the second half of the eighteenth century (De Condorcet 2014). Fundamentally, the CJT is a theorem about the performance of collective decision-making based on the aggregation of individual judgments. Roughly, according to the CJT, if the members of a group of voters are minimally competent, then the group's accuracy in choosing correctly increases as the size of the group increases, and approaches perfection if the size of the group goes to infinity.

Curiously, the CJT has been long lost to history. Its rediscovery occurred only in the second half of the Twentieth century, and it gained popularity in economics and political science just in its very last decades. Since then, the CJT has often been used as an argument for the "Wisdom of the Crowds", and is a central argument for the epistemic justification of democracies (e.g., in (Barry and Rees 1964), (Dagger 1997), (Goodin 2003), (Grofman and Feld 1988), (List and Goodin 2001)).

Being a formal theorem, the CJT provides a relevant basis to structure and

assess the epistemic performance of a collective, be it a jury or a democratic electorate, in the camp of *ideal* theory. The CJT — and this should be clear throughout this thesis — does not want to capture all aspects of how democratic decision-making, let alone politics, works. There may be aspects not fully considered by the CJT, some less important and some more. Remarkably, the CJT still offers sophisticated tools of analysis and helps us draw insightful conclusions for democracies. It shows that, under a precise set of assumptions, large groups are likely to be correct, and — *ceteris paribus* — outperform smaller groups and single individuals.

Despite these promising results, not all epistemic democrats consider the CJT as a good argument to support their claims. For example, David Estlund believes that the problems of the CJT as a good justification for democracies are rooted in the logic of the theorem itself (Estlund 2009, Ch.12), (Goodin and Estlund 2004). He has famously argued that the main problem of the theorem is its *converse*. Namely, if instead of assuming that all individuals are more likely to vote correctly than not, we assume the opposite, then the probability that will increase is that of at least a majority of voters voting for the *incorrect* alternative. And this probability still tends to 1 in the limit. Since, according to him, we are everything but certain that voters are indeed more likely than not to vote correctly, the CJT is a "too shaky" basis to argue for democracies epistemically (Estlund 2009, 223). Essentially, Estlund says that what the CJT really tells us is that we should be suspicious about majorities, as we cannot be sure that the individuals are competent.

Similar concerns regarding the CJT are expressed by Landemore, another prominent epistemic democrat. Landemore and Estlund have in fact together argued for other approaches to epistemic democracy, such as that of epistemic deliberation (Estlund and Landemore 2018). Delving into the epistemic values and merits of epistemic deliberation and other epistemic approaches to democracy is nevertheless out of the scope of this thesis. In fact, the basic fundamental assumption of this thesis will remain that, in some of its forms — perhaps not in all, as we will see — jury theorems tell us something valuable about democracies and their assumptions can find appropriate justifications if correctly spelled out.

Interestingly enough, Jason Brennan has found room to claim that, if we integrate the CJT positive results with a few additional assumptions about voting, one can be on its basis committed to democratic decision-making on small scales, but not to democratic mass voting (Brennan 2011). If Brennan is right, this appears a serious blow for all those epistemic democrats who rely on the CJT to recommend mass democracies.

In the next section of the present Introduction, I am going to briefly summarise what Brennan's argument says and how I plan to deal with it in the thesis.

1.3 The Target and the Argumentative Strategy

Brennan's argument extends the implications of the typical epistemic considerations derived from CJT to some value-related questions regarding the "common good" of a society. Through a simple formal model, it basically aims to show that by mass voting we will not maximize societal expected utility (Brennan 2011). Essentially, Brennan theorizes a simple model to calculate the expected marginal value to the common good of adding an additional voter to a group of voters who have already voted. The model is quite straightforward because it is based on simple calculations of a small set of variables. Importantly, the model presupposes the CJT: one of its variables is nothing more than the difference in group competence[3] between the new, larger, group of voters and the former. As we will see, this difference is always positive but it becomes insignificant quite quickly, according to the standard CJT.[4]

The conclusions Brennan reaches through it are striking, as they would force us to accept that having more than 100,000 voters vote would not be socially efficient; namely, it would not maximize societal expected utility. Part of this thesis consists of carefully reconstructing this argument, because Brennan himself does not always appear so clear in the conclusion he wants to draw — especially, it is *not* clear how such 100,000 voters are selected. As we will see in greater detail in Section 3.5, Brennan sometimes assumes that his conclusions refer to *any* 100,000 voters and fewer times to 100,000 *randomly selected* voters. These are two different claims that will be treated separately in the thesis. In fact, the thesis will mostly counter the former claim as it is the strongest one. Only conjectures will be made about dismissing the latter.

This argument has gotten extremely low resonance in the field: it has been touched by scholars only in passing. No advocate of the CJT as an epistemic justification for mass democracies addresses this problem. For example, in their book *An Epistemic Theory of Democracy*, Goodin and Spiekermann write: "[t]he CJT says that more votes are typically better, *from an epistemic point of view* — subject of course to the CJT's assumptions being satisfied"[emphasis mine] (Goodin and Spiekermann 2018, 226). They add the following footnote:

> Maybe not *much* better, once we already have a large number of voters maybe the epistemic gains to the community are not worth the costs those additional voters incur in turning out to vote. But that is a separate issue. From a purely epistemic perspective, more is better.[5]

3. Calculated accordingly with the CJT.

4. The reader should keep this point in mind, as the argument's negative upshots for mass voting massively hinges on this.

5. As far as we were able to check, this is the only relevant citation about Brennan's paper,

(Goodin and Spiekermann 2018, 22)

They explicitly cite Brennan here. And, in a sense, they are correct. On a purely epistemic ground, the greater the size of the electorate the more accurate it will be. Nevertheless, they also recognize it as a (separate) *issue*. But, I shall argue, it is a rather prominent and not-so-separate issue; especially if, as many epistemic democrats do, they want the CJT to be applied to real-world settings and to justify large-scale democracies. Brennan's argument cannot be dismissed by simply ignoring it. Especially, it seems naive to say "more is better", no matter what. Brennan may be in fact right in saying that, as large as it can be the social value of making the correct choice in an election, there is some point after which adding extra voters *ad infinitum* does not come to the benefit of the common good — and Goodin and Spiekermann seem to recognize this. The question is then if this point comes as early as Brennan claims, or if under more appropriate assumptions it comes in a way so that we can still make sense of mass voting. If epistemic democrats want the CJT to effectively recommend mass voting, they cannot ignore Brennan's challenge.

Moreover, what is peculiar about this argument is that Brennan does not *a priori* deny the CJT as a justification for democratic decision-making, but he assumes it. And — allegedly, at least — he does so with charitable Condorcetian parameters. Brennan here does not directly attack democracy with the explicit scope of endorsing some other forms of government.[6] So, he concedes something to epistemic democrats and shows that even in this case the CJT cannot recommend mass participation. The CJT can in fact recommend democratic decision-making for small groups, but no one can be committed to mass democracies on the basis of the CJT. The upshot is therefore that "Condorcet's Jury Theorem might say that democracy is a good thing, but we do not need a large dose of it" (Brennan 2011, 61).

I will tackle Brennan's argument according to the following argumentative strategy. I will firstly argue that the assumptions built in the classical version of the CJT — on which Brennan's argument partially relies — do not offer a plausible representation of mass voting. I will secondly refine these assumptions in a way in which they are more plausible for mass voting, but also so that they still retain the CJT's epistemic results. I will finally show that under this refined representation (i) Brennan's conclusions — spelled out in terms of 100,000 voters — no longer follow, and (ii) mass voting can be restored; in the sense that I will

besides him citing himself.

6. In this thesis, I am not going to confront directly the typical detractors of epistemic democracy *tout-court*, such as Althaus or Caplan, or Brennan himself in other works, who have long argued for the ignorance and irrationality of the average democratic voter, and thus proposed different forms of government based on this (Althaus 2003) (Caplan 2007)(Brennan 2016).

show that it can maximize societal expected utility. My arguments will then always have two different conclusions to show. For, simply showing that Brennan's conclusions do not hold is different from reaching a justification for mass voting. The argument I am going to propose can also be briefly rendered as follows: if the standard CJT definitely cannot be used to recommend mass voting (as Brennan shows), that is also because its assumptions offer an implausible representation of mass voting itself; with some refined jury theorem's assumptions that also offer a more plausible representation of mass voting, some form of jury theorem *can/may* do so.[7] The two assumptions I will be focusing on are the Independence assumption and the Competence assumption, whose meaning will be clarified in the next Chapter. For now, the reader should bear in mind that my aim in Chapters 4 and 5 is to provide an argument of the given form for each of the two assumptions, respectively.

As a final point, despite, as said earlier, Brennan's argument is a combination of epistemic and value-related considerations, my arguments will only focus on the former component. Reflections on individual and aggregated utility functions, that may well be brought into the game, will be bracketed throughout the thesis and left open for future research.[8]

In the next section, which concludes this Introduction, I will give an overview of the structure of the thesis.

1.4 Structure of the Thesis

After the present Introduction, Chapter 2 will introduce and explain the CJT, its assumptions, and some of its variants. Specifically, Section 2.1 introduces the CJT and its assumptions informally; Section 2.2 proceeds with a formal statement. Section 2.3 reviews some of the main extensions of the theorem. Section 2.4 finally explains how the CJT will be treated in this thesis.

Chapter 3 is devoted to explaining in detail Brennan's argument and its implications. After an introduction to the chapter (Section 3.1), Section 3.2 and Section 3.3 deal with Brennan's argument informally and formally, respectively. Section 3.4 distinguishes it from a similar line of argumentation. Section 3.5 summarises the argument and illustrates how I plan to deal with it. Finally, Section 3.6 contains some important notes for a broader appreciation of Brennan's and

7. I am limiting my conclusions to a possibility result because, under my new models, positive conclusions for mass voting depend on the chosen parameters. It is already important to say, though, that the chosen parameters are not *ad hoc*, but supported by reasons and motivations throughout the thesis. My argument will nonetheless remain an *if...then* statement. If my assumptions with those parameters hold, then mass voting is justified.

8. Section 3.6 elaborates more on this.

mine arguments.

Chapters 4 and 5 contain my counterarguments to Brennan. Chapter 4 is divided as follows. After an introduction to the chapter, in Section 4.1, Section 4.2 focuses in detail on the Independence Assumption. Section 4.3 investigates the plausibility of this assumption in large electorates. Section 4.4 explains how the CJT can deal with dependencies between votes and then Section 4.5 examines a simple case against Brennan, based on a single common influencing factor that creates dependence between votes. Section 4.6 proposes a more plausible model of mass voting with dependent votes and Section 4.7 analyzes the implications of this model for Brennan's argument. Section 4.8 concludes.

Chapter 5 follows a similar structure. Section 5.1 is an introduction to the chapter; Section 5.2 delves into the Competence Assumption and some of its plausible refinements. Section 5.3 clarifies Brennan's use of Competence. Section 5.4 introduces informally and formally the idea of Topic-Specific Competence as a plausible representation of Competence in mass democracies. Section 5.5 shows the implication of this refined Competence model on Brennan's conclusions. Section 5.6 concludes with some final remarks.

Chapter 6 summarises, goes through some limitations and objections, and points out possible avenues of future research.

Chapter 2

The Condorcet Jury Theorem

2.1 CJT: Framework and Intuitive Explanation

In Section 1.2 of the Introduction, I have already laid down some of the basic intuitions that stand behind the Condorcet Jury Theorem. Essentially, the CJT tells us under what conditions "crowds are wise". Let me now delve a little more into the reasons why and in what sense this is the case.

First, the CJT usually deals with groups of voters. The vote is typically the instrument that individuals use to express their judgments. And the CJT tells us when a specific type of majority voting procedure is justified *epistemically*. Importantly, voters are not assumed to express their *preferences* over a set of alternatives. They vote according to their *beliefs* about the world. Voters are not asked which alternatives they prefer, but which alternatives they believe to be the correct (or the best) one. Moreover, the epistemic tasks typically involved in jury theorems are binary[1] decisions; namely, voters have only two alternatives from which to choose. And one of them is by assumption true/correct/best. This is the CJT's metaphysical assumption.[2] For this reason, in its stricter interpretation, the CJT applies to decisions that are matters of fact. It works with propositions that can be either *true* or *false*. A recurrent example is that of a jury that has to decide whether an accused person is guilty or not. The proposition "person x has committed a crime" is either factually true or factually false. All decisions of this kind are suitable to a CJT's setting. The CJT, instead, cannot be applied to

1. "Typically" because, as we will see in Subsection 5.3, Goodin and List have provided a generalization of the theorem for k alternatives (Goodin and Spiekermann 2018).

2. Discussing this assumption is out of the scope of this thesis. Assuming the existence of an independent standard of correctness is vital for the CJT and for Epistemic Democracy in general. In fact, investigating if we can really assume this would require an independent piece; thus, for the sake of the argument, I am going to take such a metaphysical statement to be true. The interested reader may nonetheless find a few arguments in its favor in the Appendix A.

decision problems that involve value judgments (such as preferences), as they are commonly understood by non-cognitivist views.[3] Think about an electorate that has to decide on whether it is beneficial to allow the construction of nuclear power plants within their country's borders. Under a non-cognitivist view, deciding what is beneficial is a matter of value judgment, it is not a question with a factual or objective answer. To problems like these, the applicability of the CJT is far less straightforward, if not impossible.

But what does the CJT exactly say about this process of truth-tracking? Informally, the classical CJT shows that, under the appropriate assumptions, (i) the probability of a majority of people tracking the truth increases as the size of the group increases, and (ii) this probability converges to 1 as the size of the group approaches infinity. From (i) we can derive the less general results that groups of people perform better (i.e. have higher probability) than single individuals in choosing the correct option from a pair.

The appropriate assumptions are the six, jointly sufficient, following conditions:

(a) **Simple Majority Voting**: every juror votes on a dichotomous choice under simple majority rule.

(b) **Correctness**: one choice is correct/true/objectively better.

(c) **Competence**: every juror has a higher-than-random (> 0.5) probability of voting for the correct option.

(d) **Homogeneity**: these probabilities are equal for all jurors.[4]

(e) **Independence**: these probabilities are independent of each other.

(f) **Sincerity**:every juror must vote sincerely.

Of these assumptions, (c)-(d), and (e), are usually considered by the literature the CJT's *core* assumptions.[5] (d) and (e) form what is normally called the Competence assumption; (f) constitutes the Independence assumption. These are both statistical assumptions involving probabilistic reasoning. Competence assumes that each voter is more likely to get it right than to get it wrong, each with the same probability. Independence requires that in casting their votes, voters do not influence each other nor are influenced by some other factors: every vote must count as a single independent piece of information when they are eventually aggregated. The two statistical assumptions (Competence and Independence)

3. These views assume a meta-ethical perspective according to which value judgments are neither true nor false.

4. Note that in the standard CJT (c) and (d) usually go together under the name "Competence Assumption".

5. See (Goodin and Spiekermann 2018, 44–46) in their Introduction.

built into the theorem are essential to obtain encouraging conclusions about the wisdom of crowds. If any of them does not hold, we obtain undesirable results. Competence and Independence will be the core of my analysis with respect to how mass participation should be modeled within the CJT's framework. Accordingly, let me make a final remark on them. Competence and Independence should not be confused. As Goodin and Spiekermann say: "[i]ndependence is a claim about the relation of the votes to one another, while competence is a claim about the probability of individuals each voting correctly in relation to the state of the world" (Goodin and Spiekermann 2018). The confusion can stem from the fact both assumptions can be violated by similar factors. For example, it may be the case that many voters are affected by the same systematic bias. If racism is spread among one portion of a population, it is not implausible to believe that, in that portion of the population, both competence and independence might be negatively affected. Essentially, some set of common causes might exist that can undermine both Competence and Independence. If a group of people systematically read fake news on some social network and revise their beliefs about the world accordingly, they will likely be more often wrong than right when they vote, and, since they are all influenced by the very same source, their votes will not be independent either. These are nonetheless two distinct phenomena, and they do not necessarily happen together.[6]

Intuitively, the CJT works as follows. One can conceive each vote as the toss of a coin. This coin is known to be biased. However, it is not known which is the biased side. We aim to discover that. Conditional on the bias, the outcome of each toss is stochastically independent of any other. The more the coin is tossed the higher the probability that a majority of the lands will be on the biased side. In the limit, this probability approaches 1. As one can notice, in its essential form, the CJT's asymptotic result is nothing more than an application of the law of large numbers.

In the next section, I will illustrate the CJT's assumptions and conclusions in a more formal fashion.

2.2 Statement of the Theorem

Suppose to have an infinite population of individuals $N = \{1, 2, \dots\}$. I will use i as a placeholder for members of N. The standard CJT considers — out of this population — a group of individuals of size n (i.e. a jury, the electorate of a modern democracy with voting rights), with $n \geq 2$ and assumed to be odd for mathematical convenience. The group has to select an alternative from the

6. In the Appendix B the reader can find a graphically illustrated explanation of how the violation of either Competence or Independence can impact group competence.

unordered pair $A = \{0, 1\}$ by majority voting. By assumption, one of the two alternatives in A is correct. We call it the state of the world $\mathbf{x}$, which is a random variable that takes values in A. Hence, we have two possibilities: either $\mathbf{x} = 0$ or $\mathbf{x} = 1$. Moreover, individual i's vote, labeled as $\mathbf{v}_i$, is also a random variable taking values from the same set A. Accordingly, we can define the event that individual i votes correctly, R_i, as when $\mathbf{v}_i$ coincides with $\mathbf{x}$. Individual competence is defined as $Pr(R_i) = p_i$; i.e., the probability that individual i's vote matches x. Finally, M_n denotes the event that a majority of out of the first n voters votes correctly — which is the event we are interested in. And $Pr(M_n) = p_{M_n}$ is the probability that a majority of n votes correctly.

The CJT is a single theorem, an *if... then* statement. It tells us that, *if* the following three statements hold:[7]

- **Independence**: the events $R_1, R_2, \ldots, R_n$ are stochastically independent (conditional on the state of the world $\mathbf{x}$);
- **Competence**: the probability p that, for any i and for any $\mathbf{x}$, $\mathbf{v}_i$ coincides with $\mathbf{x}$ is greater than $\frac{1}{2}$ and the same for every i;
- **Sincerity**: voters do not vote strategically; i.e. they always vote for the alternative they believe to be correct;

then the following results can be proven:

- **Group Competence**: the probability that a majority of the group is correct is strictly greater than the probability of each individual being correct;
- **Monotonicity in Group Size**: the probability that a majority of the group is correct strictly increases in the group size n;
- **Asymptotic Perfection**: the probability that a majority of the group is correct approaches 1 as n goes to infinity.

Here is the formal statement of the Theorem.

Condorcet Jury Theorem 1. *If an individual's vote* $\mathbf{v_i}$ *is an independent Bernoulli trial with a fixed probability* $p > 0.5$*, and* $\vec{v}$ *is a vector of* n *such trials, with* n *being odd, then:*

$$p_{M_n} > p_i \tag{2.1}$$

7. I borrowed the following notation from (Dietrich and Spiekermann 2013). The three following statements offer a precise understanding of the conditions (c)-(f) of Section 2.1.

$$p_{M_{n+2}} > p_{M_n} \tag{2.2}$$

$$\lim_{n\to\infty} p_{M_n} = 1 \tag{2.3}$$

To calculate p_{M_n}, the following formula based on the sum of the probability mass functions of the binomial distribution can be derived:

Condorcet's Majority Decision Probability Formula 1.

$$\mathcal{C}(n,p) = \sum_{k=\frac{n+1}{2}}^{n} \binom{n}{k} \cdot p^k \cdot (1-p)^{n-k} \tag{2.4}$$

Now that the CJT has been fully introduced in its standard version, in the next Section, I will go through some of its main extensions.

2.3 Main Extensions

Some of the assumptions of the classical CJT can be relaxed while preserving all or some of its conclusions. In this section, I will go through some of the most straightforward relaxations. Relaxation of Competence and Independence will be discussed first, as they are already introduced and will both have a dedicated chapter later on. I will then move on to review models of jury theorems that are compatible with choices over more than two alternatives, that drop objective uncertainty in favor of subjective uncertainty, and that allow for strategic voting.

2.3.1 Relaxation of Competence

The standard Competence assumption assumes equally competent voters. Competence homogeneity, however, is not a mathematical requirement for the CJT to work — at least in its asymptotic conclusion. The asymptotic conclusion (that is, group competence approaches 1 as n goes to infinity) is retained as long as the *average* competence $\overline{\mathrm{p}}$ is — as usual — above 0.5.

- **Average Competence**: for any $\mathbf{x}$, the average probability $\overline{\mathrm{p}} := \lim_{n\to\infty}(p_1 + \cdots + p_n)/n$ that, for any i, $\mathbf{v}_i$ coincides with $\mathbf{x}$ exists and it is greater than $\frac{1}{2}$.

Average competence thus allows for competence *heterogeneity* among voters, and preserves the infallibility conclusion (Dietrich 2008). Therefore, it is no longer needed neither (i) that each voter has the same competence level, (ii) nor that each voter is competent.

Moreover, if individual competence levels are symmetrically distributed around the mean competence value, monotonicity in group size is also preserved (that is, group competence strictly increases in the group size n). As a reminder, monotonicity in group size guarantees that the more voters the higher group competence. It is clear why we cannot conclude the same if competence levels are not symmetrically distributed. If we were to add some voters to a population of size n, it might be the case that less competent members are added and that the new, larger, population performs worse than the original smaller group.

This, however, does not happen if the mean value is also the median one, as it occurs in a symmetrical distribution. For, in this latter case, this symmetry ensures that for every voter who, with a certain probability ϵ, is less likely than the mean to vote correctly (competence $\overline{\mathrm{p}}-\epsilon$), there is another voter who, by the same probability ϵ, is more likely to vote correctly to the same extent (competence $\overline{\mathrm{p}}+\epsilon$). The average competence of any two voters is thus $\overline{\mathrm{p}}$ (Goodin and Spiekermann 2018).

Unfortunately, whenever competence levels are asymmetrically distributed around the mean, monotonicity in group size does not necessarily hold. In fact, it could be the case that group competence is not a monotonic function of n, and that group competence is lower than $\overline{\mathrm{p}}$ (Dietrich and Spiekermann 2019).

2.3.2 Relaxation of Independence

We also saw that the CJT can live with some non-extreme degree of dependence between votes (Estlund 1994), (Spiekermann and Goodin 2012), (Goodin and Spiekermann 2018). Extreme dependence occurs when all the individuals in a population unconditionally follow one and the same influencing factor. "Unconditionally" means voters follow it with certainty. Again, if that were the case, we would be counting the judgment of such opinion leader n times, and the CJT would obviously not obtain. Throughout the text, I will assume that extreme dependence occurs if and only if the following two conditions are jointly satisfied: there exists a common cause c such that, (i) for every individual i, c influences i's vote, (ii) and does so with certainty. If one of these two conditions is violated, I will refer to that as a case of non-extreme dependence. In such cases, voters follow a common influencing factor only with probability $\pi < 1$, or the common influencing factors are multiple.

Let me stick here to the scenario where there is still one single influencing factor, but $\pi < 1$. I will introduce (and argue for) the scenario where the common influencing factors are many in Chapter 4. If an individual's probability π of following a common influencing factor (such as an opinion leader) is sufficiently low compared to their private competence level, the asymptotic result of the CJT might still hold. Spiekermann and Goodin (Spiekermann and Goodin 2012) prove

that collective competence still converges to 1, just as long as

$$\pi < \frac{(p_i - \frac{1}{2})}{p_i} \tag{2.5}$$

Where, as said, π is the probability that i follows the opinion leader, and p_i is the probability that i votes correctly when she does not follow the opinion leader.[8] If the above inequality (2.5) holds, then the asymptotic conclusion of the CJT also holds. Collective competence, nevertheless, increases at a *slower rate* in the group size. And the closer π gets to $\frac{(p_i - \frac{1}{2})}{p_i}$, the slower collective competence increases. If, instead, π exceeds the threshold given by $\frac{(p_i - \frac{1}{2})}{p_i}$, group competence increases in n, but it converges to the competence of the opinion leader, not necessarily to 1.

For instance, suppose there is a population of 500 voters, where each voter's $p_i = 0.55$. Suppose also that every voter follows an equally competent opinion leader with a given probability π. If $\pi = 0$ (i.e. conditional Independence is satisfied), collective competence, $p_{M_{500}}$, is roughly 1. Instead, if $\pi = 0.05$ (a number that still satisfies 2.5), collective Competence will be (only) around 0.9, assuming that the opinion leader is as likely as any other voter to be correct.

2.3.3 CJT for More than Two Alternatives

Democratic decisions do not always involve binary choices. Binary choices might exist for some sort of polarized political elections, such as those that typically take place in the US, or some sort of yes/no referendum. For instance, a CJT-like decision problem could be that of choosing whether to build nuclear power plants within a country's borders, as it happened in Italy in 1987, or that of choosing whether to stay or leave some international organization, as it happened in 2016 with the Brexit.

One, however, may still argue that many political decisions involve more than two alternatives. How does the CJT deal with this limitation? One simple reply is that, given one set of alternatives A and given the fact that one of them is correct by assumption, it is in principle possible to force any choice into a binary one of the form ϕ or ϕ^c, where ϕ is the subset of A containing the correct alternatives and ϕ^c its complement.

It is also possible to leave the choice in its original many-alternatives form. This is the case if, instead of the majority rule, we use the *plurality rule* to determine the winner of the voting process. Given a population N and a set of alternatives A, a profile P specifies, for each member of N, a strict ordering over the alternatives

8. More generally, p_i indicates individual's competence levels without the influence of the specific common causal factors under discussion.

in A. For each profile P, according to the Plurality Rule, the winner(s) in that profile is (are) the alternative(s) with most first places. The plurality rule can be thought of as a natural extension of the majority rule, since every majority winner is also a plurality winner.

With two alternatives the plurality rule simply coincides with the majority rule. With more than two alternatives, the CJT still obtains even if Competence is lower. For the k-alternatives case, the Competence assumption requires that the probability that each of the voters is correct is greater than $\frac{1}{k}$ and it is also higher than the probability that she votes for any other alternatives. Essentially, given a set A of k alternatives, with t being the correct one, and e labeling any alternative different from t, p_t must exceed any independent probability p_e that a voter votes for each of the other alternatives. Competence thus takes the following form (Goodin and Spiekermann 2018).

- **k-alternatives Competence**: for any i and for any $\mathbf{x}$, $p_t > p_e$ for any $e \neq t$.

Note that this implies that $p_t > \frac{1}{k}$. The Independence assumption remains unchanged, as well as Sincerity.

Accordingly, we can define p_{PV_n} as the probability that the correct alternative is the plurality winner; i.e., group competence. Hence, *if* **k-alternatives Competence**, Independence and Sincerity hold, *then* the following results can be proven (List and Goodin 2001):

- **Reliability Result**: $p_{PV_n} > p_{\neg PV_n}$; namely, the correct alternative is more likely to be the plurality winner than any other alternative.
- **Asymptotic Perfection for k-alternatives**: $\lim_{n\to\infty} p_{PV_n} = 1$; namely, as the number of voters tends to infinity, the probability that the correct alternative is the plurality winner approaches 1.

2.3.4 Further Extensions

This subsection goes through a few other relevant extensions of the CJT. I will eschew here the technical details to concentrate more on their philosophical implications.

First, in the standard CJT, the probability function Pr relative to some state of the world $\mathbf{x}$ represents *objective* uncertainty. Krishna K. Ladha, has famously proven a Jury Theorem where objective uncertainty about the world is substituted with the subjective uncertainty of an external social planner (Ladha 1993). The social planner has here to decide between two alternatives and, in doing so, has to choose whether to *trust* what a majority says. The planner is, however, uncertain about voters and the state of the world. Therefore, Competence p_i is no

longer the probability that an individual i's vote matches the true state of the world. It is a measure of the planner's subjective belief that i is right; namely, it represents how much she trusts i's vote. Moreover, Ladha also observes that the social planner does not plausibly know anything about the voters. Votes are thus assumed to be exchangeable, in the sense that the joint probability distribution of the sequence of votes (which are random variables) does not change when the positions of the variables in the sequence are altered. Under this setting, Ladha proves that a majority of voters is more likely to be correct than single individuals, but shows that this value does not generally converge to 1 (Ladha 1993). The reason is that the assumption of exchangeability — that the sequence of votes remains probabilistically the same under any permutation — implies a lack of any inherent ordering or distinguishable difference in the reliability among voters from the planner's perspective. Therefore, while a majority can statistically be more likely to choose correctly than any single individual by the virtue of aggregating diverse opinions, the absence of any qualitative distinction in voter reliability (from the planner's subjective perspective) means that increasing the number of voters doesn't systematically push the probability of a correct majority decision towards certainty (1).

Also, there are studies that investigate the results of dropping the sincerity assumptions and allowing for strategic voting. Those theorems assume that voters do not cast their vote for the alternative they believe to be correct, but that they vote *strategically*. They typically assume that before casting their votes, voters reason about what others would vote for. This involves game theoretic analyses. What is important to highlight is that, from a game theoretic perspective, sincere voting is not necessarily a Nash equilibrium (Austen-Smith and Banks 1996), (Feddersen and Pesendorfer 1998). However, due to game theoretic complexities, epistemic strategic voting will be out of the scope of this thesis. Sincerity will be thus always assumed to be satisfied.

Finally, recent work has studied the interaction between the CJT and another important element of epistemic democracy: epistemic deliberation (Dietrich and Spiekermann 2022), (Pivato 2017). Deliberation appears in principle a good way to obtain some form of epistemic enhancement. Nevertheless, it has been traditionally considered a menace to the CJT since the interaction between voters that it necessarily presupposes is seen to undermine Independence. These studies, in contrast to that, aim to show that, in some cases, deliberation can work as a competence enhancement and still be compatible with the CJT. They tackle the problem both from a game-theoretic and a social-choice-theoretic approach. Despite clearly enriching the picture, considerations about the role and the benefits of deliberation will be set aside, as in mass participation contexts, they severely complicate the analysis and the modeling exercise.

2.4 The CJT in This Thesis

As stated in the Introduction (Chapter 1), this thesis takes place within the perimeter offered by the CJT, as Brennan's argument which will be analyzed in the next Chapter is partially based on the CJT as well. In the next chapters, however, non-standard versions of the theorem will be considered. In fact, my main criticisms of Brennan are based on plausible refinements of the CJT's core statistical assumptions: Competence and Independence. The standard assumptions on which Brennan's argument conclusions heavily rely do not offer a sufficiently realistic model of mass voting. Some of the relaxations here introduced will be thus important later on, and others will be introduced in due time. Some other assumptions however will not be touched. Throughout the thesis, I will constantly assume that the decision problem is binary and that a common correct option exists (assumptions a, and b). I will also assume that voters are sincere (assumption f) and that any of the probability functions discussed represents objective uncertainty about the world.

Moreover, this thesis wishes to investigate questions about mass voting. The number of voters n will always be assumed to be substantially high. Again, Jury theorems are here always interpreted not simply as mere interesting mathematical results, but as possible ideal justifications for mass democratic voting. In fact, under the standard CJT's assumptions, even quite small electorates would be almost infallible at this task (Goodin and Spiekermann 2018). At first glance — precisely because group competence increases in group size — this seems an incredibly promising way of providing strong justification for mass-participation democracies — at least in the form of mass voting. Allowing large electorates to vote would be the best way to secure correct decisions. This might appear a solid point for epistemic democrats, who advocate for mass voting. Jason Brennan, however, disagrees. He maintains that, even under charitable numerical assumptions, while the CJT could be used to justify democratic decision-making within small groups, like juries or expert boards, it cannot do so for large electorates, such as those who currently vote in modern democracies worldwide (Brennan 2011) — unless society is willing to accept a massive utility loss.

In the next Chapter, I will introduce and explain Brennan's argument in greater detail. The discussion of this argument constitutes the second step of this thesis: while investigating whether Brennan's claims are plausible and defendable, I will try to analyze to what extent and under what conditions the CJT can indeed recommend mass voting.

Chapter 3

Jason Brennan's CJT-Based Model Against Mass Voting

3.1 Introduction to the Chapter

Jason Brennan's argument against the CJT as an effective justification for mass voting will be the opponent of this thesis. However, before delving into its details, it is worth contextualizing his argument in its broader work about the topics that we are going to discuss.

Brennan is a libertarian political philosopher known for his controversial takes on different political matters. First, as a libertarian, he is a promoter of any process of regulated commodification.[1] He sees in the markets an institution that, under the appropriate conditions, can be extended to encompass a broader range of goods and services than what happens in most political economies. Among those, there are problematic goods and services that are usually deemed *immoral* to trade. But, somewhat paradoxically, Brennan argues that, exactly because of markets' moral virtues, if these goods and services were traded, they would become less problematic. These issues, however, are clearly out of the scope of this thesis.

In this thesis, Brennan appears as one of the most obstinate critics of democracy. To be clear, Brennan does not want to purport to show that democracies are *in toto* undesirable. He acknowledges that modern democracies are often positively correlated with good results and valuable outcomes. He even affirms that sometimes we have evidence to claim that this might not simply be correlation but also causation. In the end — he notices — the current best places in the world where to live (in terms of liberties, rights, and economic prosperity) are democratic states (Brennan 2016, 8).

Nevertheless, Brennan also argues against the so-called "democratic triumphal-

1. See (Brennan and Jaworski 2022) for a famous work in support of that.

ism" (Brennan 2016, 6), according to which democracy and universal suffrage are undisputed inherently good systems. Democracy should be evaluated only in terms of the outcomes it produces. And the truth, he says, is that it often produces outcomes far from the optimum and sometimes even awful. The main reason he adduces to support this claim is that there is a widespread lack of knowledge and irrationality among voters. He suggests that this ignorance can lead to poor decision-making and governance, as voters often make choices that are not in their own best interests or in the best interests of society. He also backs up these claims with empirical data. Surveys supposedly show that the mean, median, and modal voters are ignorant on basic political issues, and completely blind on topics that require a higher social science background (Brennan 2016, 10). Even more, voters are not just ignorant, but sometimes even misinformed and thus systematically wrong. Their beliefs about politics are deeply irrational. And they act on those beliefs.

To overcome that, Brennan has famously argued to substitute democracy with some form of "epistocracy" (Brennan 2016, Chapter 8). Epistocracy is a system of government where political power is proportionate to knowledge. In his book, Brennan argues for various models of epistocracy, such as a system where votes are weighted according to knowledge or one where only individuals who pass a competency exam are allowed to vote. In an epistocracy, political power is formally distributed unequally, to reflect the unequal competence each citizen has. We restrict political power to those who use it well. Epistocracy means the rule of the knowledgeable. As said, he believes that such systems could lead to better outcomes than mass-participation democracy.

In more recent times, Brennan has partially departed from his strong position on epistocracy for a more nuanced and slightly different approach based on enlightened preferences (Brennan 2022). Roughly speaking, they refer to the choices that individuals would make if they had all relevant factual information, were able to evaluate this information in a clear-headed manner, and were fully aware of the consequences of their decisions. Brennan's discussion on this topic aligns with a broader philosophical inquiry into how people's preferences might change under conditions of perfect information and rational reflection. This approach suggests a form of decision-making that tries to implement policies that people would choose under these ideal conditions rather than merely aggregating their actual, often under-informed, or misinformed judgments.

To sum up, while reading this Chapter, the reader should bear in mind Brennan's overall political project. The argument I am going to discuss can be conceived as part of the *pars destruens* of such a project, where Brennan mostly aims to show that democracies are instrumentally suboptimal systems. In this case, however, its scope is quite narrow. With the present argument, his goal is simply

to show that the CJT cannot be used to recommend mass voting, as it is often done. In the next sections of this Chapter, we will analyze the argument in detail. Understanding what Brennan says, what are his assumptions, and what are their implications is fundamental in order to grasp the criticism that I will propose in the rest of the thesis.

3.2 Brennan's Argument

Brennan's reflection on the application of the CJT to mass-participation democracies starts from the following question: is the CJT a *good* (if at all) justification for mass voting? With respect to that, he claims that

> [r]ealistically, when the conditions of Condorcet's Jury Theorem hold, even in very high stakes elections, having more than 100,000 citizens vote does no significant good in securing good political outcomes. On the Condorcet model, unless voters enjoy voting, or unless they produce some other value by voting, then the cost to most voters of voting exceeds the epistemic benefits to the common good of their casting a vote.
>
> (Brennan 2011, 55)

The intuition of Brennan's arguments is then the following. Mass-participation democracies usually consist of millions of voters. We know that, on the CJT's model, every vote cast by a competent voter is a positive contribution to the competence of the group. That is, each additional vote increases to some numerical value the probability that the group as a whole will make the correct choice. This is a voter i's *marginal contribution* to group competence. This contribution is positive for every voter when — faithfully to the original version of the CJT — every vote has (the same) higher than random probability of being correct and is fully independent of any other. But it can also be negative or null should one of the CJT's assumptions not be satisfied.

Suppose now every voter orderly casts her vote and puts it in the same ballot box. Under the CJT's assumptions, the second cast vote will increase[2] the probability that a majority of them will be correct, the third cast vote will further increase this probability, and so on.[3] But how much does each vote increase this probability? Or, in other terms, how much does each vote contribute to group competence? On the one hand, this depends on the competence level of the voters

2. I am assuming here that voters are competent; i.e. $Pr(R_i) > \frac{1}{2}$.
3. Note that to have a strictly increasing probability, we should talk about pair of votes.

— which for Brennan is fixed and the same for all voters. The higher the competence of the voters the greater their marginal contribution.[4] On the other hand, the marginal contribution of each vote to group competence is also a function of how many votes have already been collected. And this function is monotonically decreasing. The "later" a vote enters the ballot box, the smaller will be its relative contribution to group competence. For example, if 500 votes have already been collected, the 501^{th} vote will contribute to group Competence less than the 251^{th} had the already collected votes been only 250.

It is commonly known that, if the standard CJT's assumptions are satisfied, group competence converges to perfection quite quickly as the population increases — even if individuals' competence is relatively low. For instance, the majority of a population of 100 voters, where each voter's competence equals 0.55, already has a 0.8 probability of making the correct choice. This feature of the CJT is quite striking and it is rather important for Brennan's argument. Even with very low competence values, only a few hundred (perhaps a few thousand with very low competence levels) votes are needed to secure a group Competence very close to perfection. Obviously, it is also true that each additional vote will always increase by some — perhaps extremely small — value collective competence. Moreover, making the correct decision in a modern democracy can be of really high value to the common good. For simplicity, we can express this value in monetary terms, and assume that money is a good proxy for societal utility. In these terms, it is not absurd to assume that the correct decision may be sometimes worth trillions of Euros. In some countries, the outcome of a democratic decision can massively impact the aggregate welfare of the population. Choosing right is thus important, often crucial. Therefore, it seems tempting to conclude that, since every additional competent voter increases the likelihood of making correct decisions as a collective, the more voters the better. In the end, it is in society's interest to minimize the probability of choosing wrongly and losing the money surplus given by making the correct choice.

Hence, to resume the point so far, Brennan assumes that correct majority voting goes to the benefit of the common good; and that this benefit is worth a lot — in monetary terms. By the CJT we know that, if the assumptions are satisfied, group competence quickly approaches perfection (that is, 1), even with low parameters for n and individual competence p_i. However, under the very same assumptions, the more voters we have the higher the probability of securing that benefit, as the *growing-reliability* conclusion shows. Therefore, one might be tempted to conclude that the more voters the better.

However, Brennan notes also that having an individual vote comes at a societal

4. Note that here I am using both the terms "voters" and "vote" for writing convenience. Formally, however, it would be more correct to talk only about "votes".

cost. And this may well be true. For each cast vote, this very same voter could have spent her time and effort in doing some other socially beneficial activity. And society could have not implemented the facilities to make her vote. Given this, is having *all* voters vote a good investment for society? Brennan argues that this is not the case. From a societal point of view, it is a good investment only to have a relatively low number of voters.[5] The reason is that, after this given number, the expected marginal contribution to group competence[6] is outscored by the opportunity cost of voting, resulting in a *loss* of total social utility. Therefore, once the optimal number is reached, it is better for society that citizens benefit the common good in some other ways, and not by casting their votes.

Importantly, the opportunity cost of voting is not to be conceived as a mere individual cost — as it happens in some motivational arguments against mass voting that we will discuss later in the thesis. If that was the case, we were subtracting an individual cost from the collective benefit of making the correct choice. This subtraction would surely appear problematic in some sense. Rather, in this context, the opportunity cost of voting should be understood as a social cost — something that society pays by having an individual cast her vote. Society should be happy to pay this cost up to a certain extent, after which there is a waste: the societal investment of paying this cost for all citizens has in fact negative returns, according to Brennan. If society aims at economic efficiency — in the sense of allocating resources so as to maximize the common good — it should steer citizens to address the same effort to something else.

Now that I have provided its main intuitions, in the next section, I will explain and analyze the formal aspects of Brennan's model.

3.3 Brennan's Formal Model against Mass Voting

Here's Brennan's argument a bit more formally. Suppose we have an electorate of size n who has to decide on a dichotomous choice by majority voting. Making the correct choice guarantees a surplus of money to the electorate, which would be wasted otherwise. Suppose also that the CJT's assumptions hold, but that each individual act of voting comes with a societal cost. This cost is expressed in monetary terms.

Imagine $n - 1$ voters have already voted. Now, Brennan asks: what is the

5. As we will see later, what exactly this number is is a function of a set of different variables.

6. In other terms, how much a vote helps the collective to achieve the societal goal of making the correct choice.

expected marginal value of the n^{th}-voter's vote? He calculates that by equation 3.1

$$E_{\mathbf{v}_n} = \Delta P_{M_n}(V_C - V_W) - C_0 \tag{3.1}$$

(Brennan 2011, 58)

where $E_{\mathbf{v}_n}$ is the expected marginal value of voting for n^{th}-voter; ΔP_{M_n} is her marginal contribution to group Competence (that is, the difference between the probability that a group of size n makes the correct choice and the probability that a group of size $n-1$ makes it, given that the probability that each individual votes correctly is $Pr(R_i) > \frac{1}{2}$). V_C and V_C are the expected monetary value of the "correct" option and the "incorrect" one, respectively; C_0, finally, is the n^{th}-voter's opportunity cost of voting. In Brennan's framework, $Pr(R_i), V_C - V_W$, and C_0 are the same for all voters and their numerical value is externally determined. $E_{\mathbf{v}_n}$ is thus a real-valued function f of $(Pr(R_i), V_C - V_W, C_0)$. Brennan's argument says that, even under very charitable assumptions about the actual value of each of the variables in the domain, $E_{\mathbf{v}_n}$ quickly becomes negative as the number of voters increases, making it socially undesirable to have the n^{th} voter voting. As pointed out earlier, the costs society incurs to her vote could have been spent on something else with a positive impact on the common good.

In the above, we saw that Brennan claims to be charitable towards Condorcetian. What does it mean in practice? It means that he attaches charitable values to the parameters of equation 3.1. In the sense that $Pr(R_i)$, $V_C - V_W$, and C_0 are purposefully chosen to inflate the value of $E_{\mathbf{v}_n}$. First, each individual is only slightly better than random at making the correct choice: $p_i = Pr(R_i) = 0.51$. In this case, for $n = 10.001$, $Pr(M_n) = 0.977$. Second, by assumption, the decision to make is a really high-stakes one: $V_C - V_W = 10^{13}$€; this value is arbitrarily high. It is chosen to convey the point that it is rather important to avoid mistakes. For this reason, if $n = 50.003$, equation 3.1 tells us that the expected value of the 50.003rd voter's vote is still pretty high: her vote is 8.099€ worth of competence to the common good. But, third, voting costs (only) 1€ to each voter. If that is the case, equation 3.1 tells us that $E_{\mathbf{v}100,003} = -0.74$€. Namely, the $100,003$rd voter's vote has a negative expected value: it would be socially preferable to have her doing something else — unless we attach some other value to her vote. From this, Brennan concludes that to secure a good political outcome, even in the most unfavorable cases, only approximately 100,000 voters are needed. Having more voters results in an "epistemic overkill" and goes to the detriment of the common good.

In sum, what Brennan's model does is setting a threshold of voters, given a set of parameters. The threshold is passed as soon as the n^{th} voter's expected marginal contribution becomes negative. Clearly, once the threshold is reached, the more voters are added the higher the societal loss. Also, where the threshold

lies depends on the chosen parameters. It is in fact in principle possible to assume a value for p_i so close to 0.5 that even very large electorates would be justified under equation 3.1. In *theory*, there exists some p_i such that also the 100 millionth voter expected marginal contribution would be positive. This is, of course, a logical possibility. But, as Brennan notes,

> this is little consolation to anyone who wishes to use Condorcet's Jury Theorem as a defence of mass democracy. To use Condorcet's Jury Theorem as an epistemic defence of democracy, we need to be confident that $p_i > 0.5$. If we do not know what the p_i of voters is, we cannot be sure whether democracy is good or bad.

As argued before, Brennan chooses these parameters as those which are maximally charitable to the Condorcetian without being excessively artificial. For the sake of the argument, I will follow Brennan's approach in the rest of the thesis.

Therefore, Brennan's results seem to be alarming for epistemic democrats, since current mass-democracies electorates are usually made up of many more than a hundred thousand voters. Under his model of mass voting based on the standard CJT, we should conclude that, every time they vote, modern democracies are plausibly facing an enormous utility loss. The claim that the CJT provides good reason to justify mass-voting seems now less straightforward.

Now that Brennan's argument has been explained, before moving to the critical part, it is important to distinguish it from another somehow similar, but much more common, line of argumentation.

3.4 One Relevant Distinction

It is rather important, as Brennan himself does so (Brennan 2011, 58), to differentiate his argument from a family of motivational arguments against mass voting. This line of argumentation generally refers to the Paradox of Voting, first spelled out by Anthony Downs: "if voting entails costs to individuals whereas the benefit from voting is essentially a collective benefit only very weakly dependent on any individual's vote, individuals may find it in their interest not to vote" (Downs 1957). The situation here described is a collective action problem that results in a tragedy of the commons (Hardin and Cullity 2020). Since every citizen's vote in a large democracy has a very low chance of being pivotal (that is, a vote that swings the final outcome in favor of one side or another), rational and self-interested voters would maximize their expected utility choosing not to vote.[7] But this means that if voters are rational the actual participation in the voting process should drastically decrease.

7. Given the fact the voting as an opportunity cost.

Elizabeth Anderson clearly explains this scenario:

> Democracy would collapse if the people did not go to the polls, so, assuming democracy is good for the people, mass voting can be regarded as a public good. Moreover, from the perspective of the supporters of any particular candidate on the ballot, the election of that candidate is also a public good. That is, supporters suppose that the election of their candidate would be better for society, not just for themselves, and may well prefer their candidate for this public-spirited reason. Yet when deciding whether to vote, each partisan who accepts the principle of expected utility reasons as follows: regardless of how my fellow partisans vote, the chances that my ballot will make a difference to the outcome of the election are negligible. Therefore, if there is the slightest inconvenience to me (or — thinking altruistically now — inconvenience to others!) from my voting, this will certainly outweigh the expected marginal positive impact of my voting. But there is always some inconvenience. So I ought not to vote. The conclusion follows not because each partisan is selfish, but because each partisan correctly reasons that her marginal impact on the outcome each prefers from a public-spirited standpoint is negligible.
>
> (Anderson 2001, 26)

Although Anderson talks about candidates of an election, it is straightforward that the same argument can be generalized to a dichotomous choice as the one involved in the Jury Theorems' setting.

And even if one does not want to frame the problem as a game-theoretic situation, it is sufficient to say that motivational arguments simply take the perspective of the single voter's reasons to cast her vote. Namely, they try to answer the following question: if a voter can safely assume that so many others are already voting, then will she be motivated to vote herself, rationally speaking?

Brennan, and I accordingly, are facing instead another problem. We are concerned with the *societal*, *third-person*, perspective; not with the single voter's motivation. The problem we aim to solve is thus the following. Given the assumptions of the CJT, and the fact that each vote costs something to society as a whole: should *we* have all citizens vote? We are therefore asking if, from the point of view of society, it is beneficial to have mass voting. In other terms, we investigate whether and under what conditions under what conditions mass participation can be both justified on an epistemic ground and yet maximize societal expected utility.

In the next Section, I will summarise Brennan's argument, make some relevant remarks, and explain how I am planning to deal with it.

3.5 Summary and Plan of Attack

Very briefly, Brennan's argument can be summarised in premise-conclusion form as follows.

P1 Society must aim at economic efficiency: allocating resources so as to maximize the common good;

P2 Voting is societally costly and its *only* value is to steer society to a correct decision;

P3 Brennan's model implies that — under maximally charitable parameters — the marginal contribution to the common good of each voter is quickly exceeded by the cost of voting;

C4 Mass voting leads to utility loss for society after *any* 100,000 voters have cast their vote.

First, an important remark should be made. In some passages of his paper, Brennan seems to draw from his argument a different conclusion from [C4]. Namely, that, to not go at the detriment of the common good, it would be best to *draw* 100,000 voters *at random* and have only these voters vote (Brennan 2011, 11). This reference to lottocracy[8] would make it a stronger case than simply having *any* k-number of voters voting. If voters are truly drawn at random, it may well be the case that to secure the best outcome only a certain, relatively small, number is needed. And, therefore, mass voting would lead to a utility loss.

The resulting two statements that I am going to confront can be resumed as follows:

- **General Utility Loss (GUL)** (GUL): *Given any 100,000 voters, adding more voters would lead to a utility loss for society.*
- **Random Utility Loss (RUL)** (RUL): *Given 100,000 randomly selected voters, adding more voters would lead to a utility loss for society.*

Note that RUL is strictly weaker than GUL. The reason is that GUL, which refers to *any* 100,000 voters, is more general and also encompasses RUL itself. GUL basically says that any combination of 100,000 voters (thus also randomly selected) will lead to a societal utility loss. Therefore, to prove that GUL does not follow it is sufficient to show that for one of the possible combinations of voters,

8. With this term I refer to political systems in which decision-making power is allocated randomly or through a lottery.

society does not incur a loss of utility. RUL, instead, is a more specific claim about randomly chosen voters. Due to its lesser generality, it is a weaker conclusion to draw.

Given these statements, I will counter Brennan as follows. I will argue that some of Brennan's assumptions seem untenable in the context of mass participation: they are unrealistically oversimplifying. Once shown this, I will argue that, given somewhat more plausible and realistic assumptions that are still compatible with some form of jury theorems, one can show that the two statements presented above — spelled out in terms of $100,000$ voters — no longer follow,[9] and that mass voting becomes justified.

More specifically, I will argue that one hidden assumption that Brennan makes is particularly problematic and unrealistic. Brennan's point relies on a precise feature of the standard CJT, which is directly implied by the standard versions of Competence and Independence. Namely, that any two votes are interchangeable. Let me stress this point a bit further. Votes' interchangeability can be understood as follows. We know that from a mathematical perspective, votes in jury theorems are random variables. A sequence of random variables is interchangeable if their joint probability distribution is invariant to any finite permutation. Every voter has one vote to cast, so we have as many votes as voters. $\mathbf{v}_i$ is the vote cast by individual i. Hence, every election comprises a sequence of votes of size n, $(\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n)$. This sequence is finite if the population N consists of a finite number of individuals. Interchangeability of votes means that for any permutation $(\mathbf{v}_{i_1}, \mathbf{v}_{i_2}, \ldots, \mathbf{v}_{i_n})$, and for any subgroup $J \subseteq N$, it is equally likely that only the votes in J are correct as it is that only votes in $(i_j : j \in J)$ are correct. In other words, votes are interchangeable if they are perfectly symmetric in their probabilities.

Crucially, in the standard CJT, a special case of interchangeability is assumed. Every random variable $\mathbf{v_i}$ is independent and identically distributed. That is, votes fully satisfy the Independence assumption and voters are equally competent. This type of interchangeability makes it an even stronger assumption, as some forms of interchangeability, as we will see, are compatible with correlated votes.

It is exactly this type of special interchangeability that allows Brennan to say that *realistically* "having more than 100,000 citizens vote does no significant good" (Brennan 2011). In the next chapters, I will argue that this is hardly the case in a mass democratic context. Especially, I will argue that, if votes are not independent or identically distributed, as it happens in more refined versions of Jury Theorems, Brennan's conclusion might cease to hold; or, at least, might cease to be too drastic for mass voting. Accordingly, in Chapter 4, I will tackle the assumption that every vote is statistically independent of any other. And, in Chapter 5, I will tackle the

9. Or, at least, that one of them (GUL) no longer follows.

assumption that all voters are equally competent, and being so in any topic.

I will now conclude this chapter with some additional reflection on Brennan's argument in its entirety, which appears important to delimit the scope of the project.

3.6 Final Notes on Brennan

First, in the Introduction[10] to this thesis, we have already said that Brennan's point is not solely epistemic. It is not exclusively based on the CJT as an epistemic model that captures some voting dynamics. Brennan's model does more. It extends the purely epistemic inquiry concerning mass voting to some value-related questions concerning the common good. This surely adds additional elements to the picture, making it more complex. To simplify, and in line with Brennan, I will use money as a proxy for utility, and as the reference metric to address such value-related issues is money. Monetary values will thus be the metric for societal costs and benefits. Again, all these are costs and benefits incurred by society *as whole*. Thus, when we say that the opportunity cost of voting is 1€, what we mean is that having an individual vote costs 1€ to society, and this is the case for every individual.[11]; and society, instead of paying that cost, could have had that individual doing something different which benefitted it more. This way of conceiving costs and benefits makes it possible to avoid reasoning on the agents' utility functions. For these reasonings would acquire a prominent sense if the costs incurred and the benefit obtained were those of the single agent, as it is, for instance, in the motivational argument presented in Section 3.5.[12] Note also, however, that Brennan's argument — and hence mine — does not "change the metric" of the CJT. In his model, the CJT remains a mathematical result about a probability value; namely, the probability that a majority of voters of a group chose correctly. Then, Brennan's model uses these probabilities (as well as other variables) to calculate the *expected* marginal value to add another voter. In his model — this is true — the metric changes. But this does not appear a misuse or a substantial reformulation of the theorem.

Second, looking at Brennan's argument and, especially, at the parameters he endorses one could ask why not push on his very same direction and make his conclusions even stronger, instead of making an argument *for* mass voting. In the end, he is assuming parameters which are *charitable* to the Condorcetian, but this is not necessarily needed. With different parameters, it would have surely

10. See Section 1.3.

11. This is clearly simplifying as it is not absurd to argue that some voters may cost more than others to society. I will not investigate this in this thesis, though.

12. I am here distancing myself from views that state the societal costs and benefits are the mere aggregation of individuals' costs and benefits.

been possible to conclude that even less than $100,000$ voters would suffice to have a societally efficient electorate. Note, however, that Brennan presents the one analyzed in this chapter as an extreme case; i.e., the most charitable possible to Condorcetian without making absurd claims. He thus concludes that — even in this case — the CJT *cannot* recommend mass voting. The argument I am going to present should be seen as a way to say that, some other — more plausible — jury theorems' assumptions, instead, can do so. In other terms, the charitable starting point that Brennan endorsed with the intention of fully dismissing Condorcetian epistemic democrats will be kept; and I will then see if making some assumptions more plausible and less oversimplified it is possible to reach different conclusions. There is, in fact, a difference between saying that the jury theorem *cannot* recommend mass voting; that it *can* do so; and that it is the *best* justification we have for mass voting. Brennan argues for the first claim. I am arguing for the second and *not* for the third. Yet, the fact that, keeping other things equal, only by making Brennan's assumptions more plausible, the CJT can recommend mass voting, and thus Brennan's argument is not definitely conclusive is something that is relevant to acknowledge, both if one wants to argue for or against mass voting. In this thesis, I am showing that this is actually the case.

In accordance with the argumentative structure provided in Section 3.5, in the next Chapter, I will start to analyze how Brennan's argument deals with the Independence assumption.

Chapter 4

Refining Independence

4.1 Introduction to the Chapter

In this Chapter, I will offer my first critique of Brennan. This critique is based on the fact that Brennan models mass voting according to the standard CJT, and thus his conclusions on the optimal number of voters rely on the assumption of full independence between votes. This is an implausible assumption to make on large electorates. Votes are unlikely to be probabilistically independent. I will in fact propose a refined model of mass voting that drops the assumption of full independence between votes, and assume instead that there are common causes that threaten voters' independence.[1]

Here my main target will be Brennan's General Utility Loss conclusion. In brief, the argument I will present in this Chapter will take the following form.

P1 Brennan's *General Utility Loss* conclusion relies crucially on the implicit assumption of full Independence between voters.

P2 A more plausible model of mass voting, still compatible with some form of jury theorem, is that in which voters come in *clusters* of dependence.

C1 Under this new model, *General Utility Loss*, spelt out in terms of $100,000$ voters, no longer follows, *ceteris paribus*.

C2 Under this new model, the number $100,000$ should be increased for *General Utility Loss* to be valid, to the point that mass voting is now justified, *ceteris paribus*.

1. The model I am proposing is inspired by Spiekermann and Goodin's discussion on opinion leaders (Spiekermann and Goodin 2012).

Accordingly, the Chapter will be structured as follows. First, I will give a close-up look at the Independence assumption. I will then explain why assuming full Independence in the case of mass voting is implausible, and show that if Independence is violated Brennan's conclusions no longer hold, but some form of Jury Theorem may still hold; i.e. group competence grows at a slower rate, but the epistemic properties of increasing the size of the electorate are retained. Concerning this, we are going to first see what happens to Brennan's conclusion in one extremely simplified case. Starting from this, I will propose a more refined CJT-based model of mass voting in which Independence is partially violated, and according to which Brennan's GUL no longer holds.

4.2 The Independence Assumption

The Independence assumption has been the more contested one among the CJT's three main assumptions[2] (see (Dietrich and Spiekermann 2013, 92–93), or (Ladha 1993, 70). For instance, John Rawls, in his influential *Theory of Justice*, writes:

> [t]hus we might be tempted to suppose that if many rational persons were to try to simulate the conditions of the ideal procedure and conducted their reasoning and discussion accordingly, a large majority anyway would be almost certainly right. This would be a mistake. We must not only be sure that there is a greater chance of a correct than of an incorrect judgment on the part of the representative legislator, but it is also clear that the votes of different persons are not independent. Since their views will be influenced by the course of the discussion, the simpler sorts of probabilistic reasoning do not apply.[3]
>
> (Rawls 2017)

Rejecting the CJT by appealing to inevitable failures of Independence among voters may be tempting; after all, the Independence assumption appears crucial for the CJT to obtain. Nevertheless, assuming that because voters necessarily interact with each other, Independence is therefore violated would be mistaken. In fact, Independence does not mean the absence of interaction among voters. Goodin and Spiekermann illustrate that interpreting Independence as such would be too permissive on the one hand, and too restrictive on the other (Goodin and Spiekermann 2018). Too permissive because sources of dependence are to be

2. See Section 2.2.

3. In this passage, Rawls is commenting on the majority rule as the best procedure to ensure just laws and policies. However, he does not consider the CJT as a reliable justification for such a voting rule — as the citation above clearly shows.

found way beyond mere voters' interaction. For instance, the votes of voters who read the same newspaper are likely to be positively correlated: this is a violation of independence due to a common influencing factor. Too restrictive because mere interaction among voters does not necessarily create patterns of dependence. The interaction of voters and stochastic Independence of votes are two different things that may well coexist. Notwithstanding this remark, Independence still appears a quite demanding requirement. Before sentencing the unsuitability of the CJT to any real-world setting due to systematic failures of Independence, a full appreciation of what this assumption actually requires is needed.

As well as the Competence assumption, Independence is a statistical assumption involving probabilistic reasoning. In probability theory, two events A and B are said to be stochastically independent if and only if $P(A|B) = P(A)$ (to be read the probability that A occurs given the fact that B also occurs is the same as the probability that A occurs). In the classical CJT, this applies as follows. Suppose to have a population of voters of size n. Independence assumes that, for any i and any j the probability that individual i votes correctly given that individual j does so is equal to the probability that i votes correctly alone. Formally,

$$Pr(R_i|R_j) = Pr(R_i) \tag{4.1}$$

. However, as Dietrich points out (Dietrich and Spiekermann 2019), votes cannot be *unconditionally* independent. If voters are competent, then they must be casually and positively influenced by the state of the world $\mathbf{x}$. The state of the world is exactly that source of causal dependence between votes that makes it possible to assume that each voter has a higher than random probability of tracking the truth. If — as a spectator — you observe individual A voting for 1, and you are subsequently asked to place a bet on individual B's vote, you are rationally required to bet on B voting for 1 too, given the fact that by, assumption, all voters are more likely to get it right than to get it wrong. Therefore, a less naive formulation of the CJT requires Independence *conditional* on the state of the world.

$$Pr(R_i|R_j, \mathbf{x}) = Pr(R_i|\mathbf{x}) \tag{4.2}$$

From now on, unless explicitly stated otherwise, when I will be referring to Independence I will mean Independence conditional on $\mathbf{x}$, and so in the sense of 4.2.

Now that the meaning of Independence has been clarified, in the next section, I will put the role of Independence in Brennan's argument under closer inspection.

4.3 Independence and Mass Democracies

Brennan's argument fully endorses the Independence assumption. He implicitly assumes that every vote is independent conditional on the state of the world; im-

plicitly, because the Independence assumption is not even mentioned in his paper. Recall Brennan's central point: "*realistically*, when the conditions of Condorcet's Jury Theorem hold, even in very high stakes elections, having more than 100,000 citizens vote does no significant good in securing good political outcomes" [emphasis mine] (Brennan 2011, 55). This conclusion is based on the fact that votes are uncorrelated.

This is a relevant feature of Brennan's argument, whose drastic conclusions for mass voting largely depend on the fact that group competence quickly approaches perfection. Full Independence (i.e., absence of correlation) among votes ensures that this happens even with low individual competence levels. With it, we are assuming that, given a population of n voters, we are pooling n *different* pieces of information. That means that the standard CJT assumes maximal *diversity* in individual judgments; where diversity can be understood as "plurality in sources and backgrounds: individuals reason differently, hold different perspectives, use different information, etc" (Dietrich and Spiekermann 2019). Conversely, if individuals were not at all diverse, then the correctness event R_1, R_2, ..., R_n will coincide and Independence will be fully violated. In summary, Brennan's argument assumes that votes are fully independent and thus that large electorates are maximally diverse.

Large electorates, however, are likely *not* to be maximally diverse. That is, for a large, non-artificial, electorate it is implausible that for any i, and for any j, $Pr(R_i) = Pr(R_j)$. Interestingly, this is the case even if we assume that votes are *causally independent*, where causal dependence occurs when a vote is directly influenced by another vote. Assuming causal independence between votes is, in fact, not implausible as long as votes are secret. Secrecy prevents direct influencing between votes.[4]

Nevertheless, causal independence is not sufficient to secure the Independence assumption because it is extremely unlikely that there is no common influencing factor that violates *probabilistic* independence among votes. These influencing factors are essentially *common causes* that create positive correlations between random variables (i.e. votes), even if these random variables do not directly influence each other. This happens, for instance, when voters are exposed to similar or even the same evidence, when they are exposed to common contextual elements, and when they share perspectives based on common features. To sum up, causal independence is not probabilistic independence. The latter is satisfied only if no common causes that create a correlation between votes exist. If such common causes exist — and, arguably, in large electorates, they are numerous and varied — then Independence is violated.

4. Visualizing it with a directed acyclic graph (DAG), causal independence occurs when there is no direct edge connecting two different votes (See Figure 4.1).

Figure 4.1: How a common influencing factor (common cause) may threaten Independence between votes.

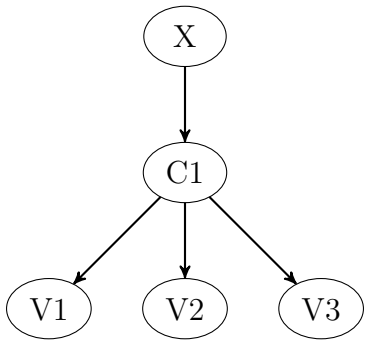


Figure 4.1 visually illustrates why the existence of common influencing factors would threaten Independence. Suppose, for simplicity, to have a population of n voters. There is a newspaper, $c1$, that is very much read by a subset of the population composed of three voters, labeled $v1$, $v2$, $v3$. This newspaper provides opinions on the decision at stake. Since all three voters may vote according to what the newspaper $c1$ signals, their votes are positively correlated. And this happens despite the fact that the three voters do not interact with each other, as no arrow connects them directly.

Note finally that my aim is not that of simply dismissing Brennan and the CJT altogether in light of systematic failures of Independence in large electorates. In the next sections, I will argue, instead, for an appropriate refinement of how to frame the Independence assumption in mass voting, which is still compatible with some form of jury theorem. I will claim that with this refinement (i) we can offer a more realistic, CJT-compatible, representation of mass voting, under which (ii) Brennan's conclusions no longer follow, and (iii) mass voting can be justified.

4.4 CJT with Non-Extreme Dependence

In the previous section, we saw how Independence between votes can be contravened due to the existence of common influencing factors. Reichenbach's *common cause principle* (CCP) clearly illustrates the phenomenon and tells us how to restore Independence (Dietrich and Spiekermann 2013, 95):

- CCP: Any probabilistic dependence between phenomena which do not causally affect each other is due to common causes, and these phenomena become probabilistically independent once we conditionalize on their common causes.

Thus, to have Independence the probability that each voter is correct, $Pr(R_i)$, should be conditional not only on the state of the world $\mathbf{x}$, but also on any common influencing factor c (Dietrich and Spiekermann 2013, 95). Nevertheless, I previously argued that, when an electorate is large, common influencing factors are probably several. But there is also another element: they may be only common among certain subgroups of the whole group of voters. This would render this process of conditionalization implausible, as we should be able to detect any pattern of dependence between voters that could be at work in that specific decision problem and then conditionalize on it. Also, conditionalization on all common causes is problematic for Competence. As Ladha, points out, once we conditionalize on all the information and evidence voters share, "we could not assume that p_i exceeds 0.5" (Ladha 1993). Or — at least — this assumption is harder to justify.

To state this very same point a little bit more formally, when an electorate votes, it is facing the decision problem δ of individuating the true state of the world under certain circumstances. Following (Dietrich 2008), I interpret such circumstances as common influencing factors. One can then think of each δ as a pair composed of the state of the world and the relative common influencing factors $< \mathbf{x}, c >$. When $Pr(R_i)$ is conditionalized on the decision problem δ, we have Independence conditional on both the state of the world and all common causes. Conditionalization on δ is thus a way to solve issues of Independence. But we mentioned that it is in principle hard to assume conditionalization on all c in the case of very large bodies of individuals; and that, even if that were the case, full conditionalization would, in turn, make competence untenable, because, with Independence now conditional on the problem, a problem specific notion of competence is also needed.[5] Another possibility to solve the problem is accepting that some non-extreme influence by common influencing factors is there. In the remainder of the section, I will pursue this point.

In fact, in Chapter 2, we saw that the CJT can live with non-extreme cases of dependence. Extreme dependence occurs when all the individuals in a population unconditionally (i.e. $\pi = 1$) follow one and the same influencing factor.[6] But it seems there should not be too much to worry about extreme dependence, since — Condorcet himself already observed — such cases of slavish following are somewhat unlikely to happen on a large scale.

From Spiekermann and Goodin (Spiekermann and Goodin 2012) we also know

5. The reader should go to the Appendix D to see how Dietrich and Spiekermann in (Dietrich and Spiekermann 2013) attempt to solve this problem.

6. The reader should remember that π is the probability that a voter follows c. When she does not, she votes according to her private competence signal.

that collective competence still converges to 1, just as long as

$$\pi < \frac{(p_i - \frac{1}{2})}{p_i} \tag{4.3}$$

It nevertheless does so at a slower rate compared to the standard CJT. When instead π exceeds the threshold given by $\pi < \frac{(p_i - \frac{1}{2})}{p_i}$, group competence converges to the competence of the opinion leader. We can refer to this as the "tolerance" threshold for π.

In the next section, I will re-evaluate Brennan's conclusions in light of this new basic setting, that allows simple correlation between votes. I will do this by first looking at a very simple case and then pass to a more plausible one.

4.5 A Simple Case against Brennan

Let me now re-examine Brennan's argument under this new setting,[7] in which Independence is partially violated by the existence of one common influencing factor c. Suppose first to have a population of n voters. I will investigate what happens under Brennan's original competence parameter; hence, $p_i = 0.51$. I also assume that $p_c = p_i = 0.51$; so the factor that influences the voters is as "competent" as any voter. We know that, if the probability that individuals follow such a common cause is sufficiently low,[8] then the CJT's conclusions survive. Under Brennan's individual competence parameter, the tolerance threshold for π is roughly 2%; that is, the CJT is retained as long as individuals do not follow c more than roughly 2% of the times. More precisely, I will assume that $\pi = 0.019$. This is a very low probability; but note that one could argue that here we are moving away from Brennan's setting only slightly, as π is so low that individuals mostly vote according to their private signal.

Suppose now $100,001$ voters have already voted. With this new setting, p_{M_n} (i.e. group competence) is equal to 0.79: a surprisingly low number compared to that obtained with the standard CJT formula (higher than 0.99).[9] One should ask now what is the expected value of adding one voter to the electorate, given the fact that making the correct choice is worth 10^{13}€ and voting costs 1€. Using

7. In this Section and in Section 4.7, I will provide results based on analytical calculations and Monte-Carlo simulations. The formulas that ground the analytical results and the basic routine on which simulations are based can be found in Appendix C. They are both derived from the "Supplementary Material" of (Spiekermann and Goodin 2012).

8. The "tolerance" threshold is defined by 2.5.

9. All group competence results in this Section are analytical results obtained calculating formula 1 in *Mathematica*. The expected marginal value of the n^{nt} is calculated by resolving equation 3.1 also on *Mathematica*.

Brennan's equation, we can estimate that the expected marginal value for the $100,003^{rd}$ voter is approximately 1.9×10^6€. Table 4.1 resumes this result.

p_i	p_c	π	$p_{M_{100,001}}$	$E_{v_{100,003}}$
0.51	0.51	0	0.99	-0.74€
0.51	0.51	0.019	0.79	1.9×10^6€

Table 4.1: Change in the expected marginal value of the 100,003rd after introducing partial dependence between votes caused by a single influencing factor c. The first line shows Brennan's result. the second line shows the result with dependence. The other Brennan's parameters are unchanged.

Interestingly — using the same parameters — for $10,000,001$ voters collective competence is higher but still "far" from perfection: $Pr(M_{10,000,001}) = 0.987$. Ten million voters is a number that is not far from the actual electorates of some European democracies. For instance, it is greater than the current size of the electorates of countries like Belgium and Switzerland and not distant from that of the Netherlands.[10] then, what is the expected marginal value to add an extra voter to the electorate? The expected value for the $10,000,003^{rd}$ is still conspicuously high. Her vote will contribute $2,8420.6$€ of competence to the common good.[11] Hence, her vote is beneficial for society.

Therefore, these simple calculations show that introducing a common factor that creates a tolerable dependence between votes invalidates Brennan's conclusions and justifies an electorate of the size of a small democratic country. Under this more refined setting and with the aforementioned parameters, mass voting can be justified.

One may, however, argue that setting π so low is artificial. It is nonetheless sufficient to slightly depart from such a low competence value as that assumed by Brennan to be able to also increase π to a more realistic standard. Suppose now that the population is more competent. Individuals tend to get it right 65% of the time. They follow c 23% of the time. So despite being more competent, the voters nonetheless tend to be more reliant on c. They do not follow their private competence signal quite often. Suppose also that p_c is still 0.51. So, the common influencing factor is less competent than voters. With news parameters for p_i and p_c, $E_{v_{100,003}} = 4,890$€, other things being equal. Hence, it would still be desirable to enlarge the electorate beyond Brennan's conclusion. Nevertheless, we would expect a negative E_v earlier than the previous case. Therefore, with these

10. These are rough data taken from "Eurostat" website. Their scope is purely illustrative.

11. Clearly, these numbers would change if p_c was higher or π lower. In both cases, group competence would increase at a faster rate in n. But if p_c is not really high and/or π really close to 0, then similar results are still obtained. For instance, with $p_c = 0.60$, $E_{v_{100,003}} = 1 \times 10^6$€.

parameters, we obtain the first result that Brennan's conclusion no longer holds, but mass voting would perhaps not yet be justified.

Note, finally, that these results would invalidate not only GUL, but also RUL. The reason is simply that under this setting in which there is only one common influencing factor followed by everyone with the same probability, votes are still fully interchangeable, despite being correlated. Thus the first n voters are equivalent to a random sample of n voters.

The results just provided, however, present different issues. First, large electorates are justified only if an artificially low π is assumed. Realistically, π can expected to be much higher: we can often fairly assume that voters tend to follow cues quite frequently. Second, we assumed that the influencing factor affecting voters is only one. The fact that everyone in a democracy is influenced by the same common cause appears implausible. In fact, we acknowledge that, in large electorates, the common influencing factors shaping dependencies between individuals' votes are likely to be numerous and varied. In the next section, I will propose to revise Independence according to this latter, more plausible, scenario, where common influencing factors are many and the probability that voters follow them is reasonably high.

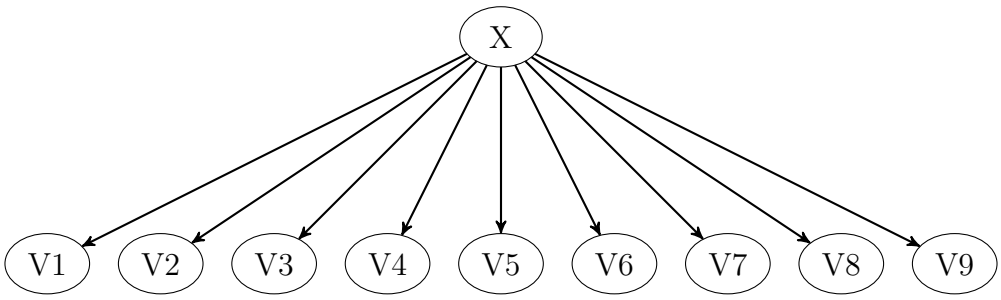
4.6 A More Realistic Model of Mass Voting

Two preliminary considerations should be made. First, I argued before that when the number of votes involved is reasonably high, the number of influencing factors that may lead to a violation of Independence is likely high as well. It is implausible to assume the existence of only one c.

Now, remember again that Brennan assumed votes to be fully independent given the state of the world. Common influencing factors are not taken into account in his model of mass voting. For a small population of 9 voters, Brennan's model is exemplified in Figure 4.2.

Contrary to Brennan, I suggest conceiving votes in a mass-participation democracy as coming in *clusters of dependence*. Different subpopulations of voters, I assume, have access to different, *independent*, influencing factors. This scenario offers a much more appropriate and plausible representation of how mass voting could look like. One can imagine various reasons why different voters may be reached by different pieces of information. For instance, one subpopulation could be that of voters who are influenced by a certain newspaper x. This subpopulation would be different and independent from that of voters influenced by newspaper y, and so on. That is, within each group, voters are influenced by the same common cause which is different from that of other groups of voters. The idea is not fully dissimilar to that of epistemic bubbles (typical of social media), where agents'

Figure 4.2: Small scale Brennan's model.



beliefs within each bubble are based on shared evidence, rely on the same sources, and are thus highly correlated; they are nonetheless typically insulated from the beliefs of agents of different bubbles (Nguyen 2020). That being said, I propose to flesh out this intuition as follows.

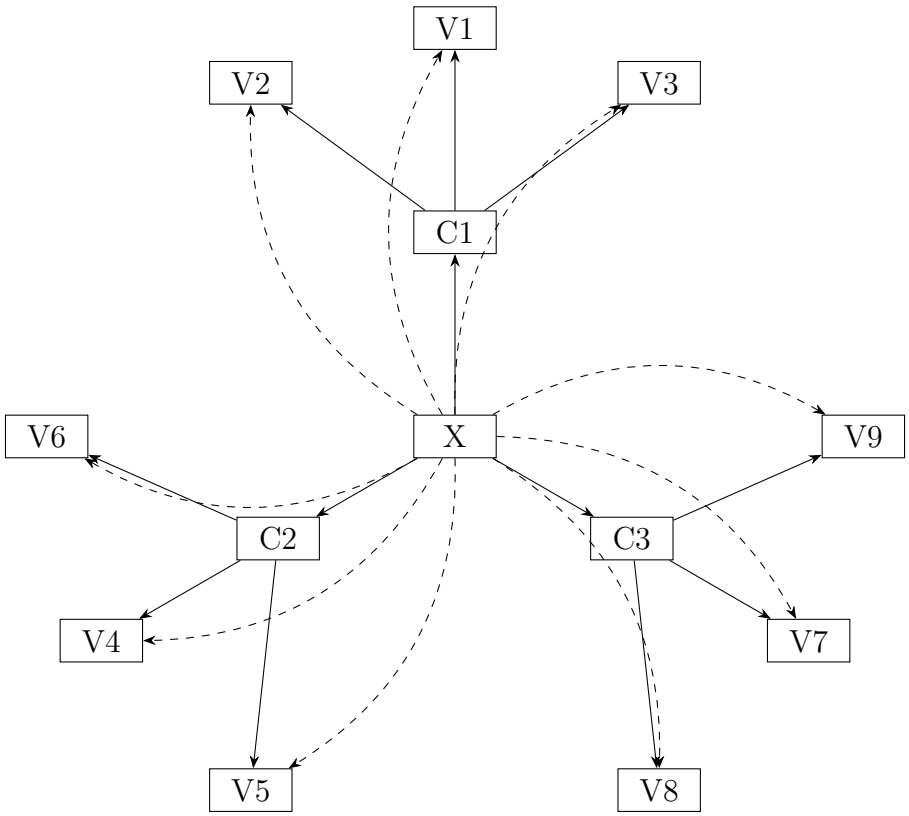
Let n be the total number of voters; let k denote the total number of independent[12] influencing factors c that may influence the electorate. Each voter may follow only one influencing factor c with probability π, which is assumed to be the same for all voters. All the voters that may follow the same influencing factor represent a subpopulation of n; that is, a cluster of voters. So, let $r = k$ be the number of subpopulations of n, each of size $\frac{n}{k}$ (i.e., for convenience, voters are assumed to be evenly split among influencing factors). That means that each influencing factor c gives rise to a cluster of voters and these clusters are mutually exclusive and jointly exhaustive, relative to the population. When individuals do *not* follow the cue of their respective shared influencing factors, they vote according to their assumed competence level p_i (that is, individuals vote according to their private judgments); when they do, the probability that their votes are correct is simply the probability that the influencing factor is correct (labeled p_{c_i}).

On a small scale, this model of mass voting is rendered in Figure 4.3. Essentially, here we still have 9 voters ($n = 9$). We also have three independent influencing factors that create three clusters of three voters each; so $k = r = 3$. Votes are thus dependent within clusters and independent between clusters.

The model I am presenting is still simplified. It is, however, a step towards a more realistic representation of how mass voting could work in real-life settings, whereas Brennan's assumption of full Independence was clearly overly simplifying the picture. Moreover, relaxing Independence in the way I have just proposed is still compatible with some form of jury theorem, as long as certain conditions are

12. This is a further assumption I make. But one can conjecture that fully independent factors are not a strict requirement.

Figure 4.3: Small scale model of mass voting with voters coming in clusters of dependence. c_1, c_2, c_3 are independent factors that influence three voters each. Dotted lines represent the possible direct influence from the state of the world to the voter, that happens when $\pi \neq 1$.



satisfied. That is, the growing reliability and the asymptotic perfection conclusions may be retained.

In the next Section, I will explain when this happens, highlighting the consequences for Brennan's argument.

4.7 Implications for General Utility Loss

In this section, I will investigate the effects on Brennan's conclusions of assuming that votes come in clusters of dependence (as shown in Figure 4.3, instead of assuming standard Independence between votes (Figure 4.2).

First of all, two important remarks. As done before, I am assuming here that each influencing factor is "competent"; namely, it is more likely to be correct than

not with respect to the issue at stake (for every c, $p_c > 0.5$).[13] Moreover, I also suggested that in mass-participation democracies, the average voter may tend to follow common influencing factors with a high probability. Internet, newspapers, TV channels seem all to point in this direction: they exert much influence on voting behavior.[14] π is thus expected to be reasonably high; perhaps as high as 50%, or even more.

I will now introduce some general implications of the model previously explained. The first general result, also obtained by (Spiekermann and Goodin 2012) in their discussion of the CJT and opinion leaders, is that the more independent the influencing factors there are, the better for group competence, *ceteris paribus*. Table 4.2 illustrates this result.[15]

p_i	k	p_c	π	$p_{M_{100001}}$
0.51	0	0.51	0.5	0.999
0.51	10	0.51	0.5	0.642
0.51	1000	0.51	0.5	0.876
0.51	5000	0.51	0.5	0.996

Table 4.2: Variations in group competence for $100,001$, given different values of k. With $k = 0$, we obtain the standard CJT. Also, if $k \neq 0$, the higher k the better for $p_{M_{100001}}$.

The reason behind these results is that different independent influencing factors provide more independent points of judgment, which are beneficial for group competence.[16] In this setting the classical CJT would be the trivial case where $n = k$ (there are as many influencing factors as voters), $p_c > 0.5$ and $\pi = 1$; or, alternatively, whenever $p_i > 0.5$, and either $\pi = 0$ or $k = 0$.

Importantly, however, in non-trivial cases, group competence, after a small initial increase, would stop increasing, should the number of voters increase while the number of influencing factors remains steady. The increasing number of voters is balanced by the increasing size of the clusters of dependent voters. There are more voters, but also more dependencies. Table 4.3 illustrates this.

Starting from these general considerations, it is possible to appreciate that, besides offering a more realistic picture of how the CJT may hold in mass-participation democracies, the above setting presents also some issues to Brennan's argument.

13. But perhaps this is not even needed, depending on the values of other parameters.

14. Empirical evidence seems to support this claim; see, for instance, (Lata 2020).

15. All probability results in this section are derived through Monte Carlo simulations run in Python.

16. These results also depend on the value of π, which here is set equal to 0.5. The higher is π the slower group competence increases.

n	p_i	k	p_c	π	p_{M_n}
$1,001$	0.51	10	0.51	0.5	0.57
$10,001$	0.51	10	0.51	0.5	0.64
$50,001$	0.51	10	0.51	0.5	0.66
$1,000,001$	0.51	10	0.51	0.5	0.65
$5,000,001$	0.51	10	0.51	0.5	0.65

Table 4.3: Change in group competence, if n increases and k remains steady, *ceteris paribus*.

In fact, Brennan's conclusion that having more than *any* 100,000 citizens vote does no significant good in securing good political outcomes hinges, we know, on the rapid convergence of group competence to perfection, which is granted by the CJT's standard assumptions, and thus by the fact that votes are fully independent. The plausible introduction of dependence among voters complicates the scenario: patterns of dependencies, as they are introduced above, violate votes' independence (and also interchangeability) and reduce the growth rate of group competence, thus questioning Brennan's conclusions. Let's get back to the parameters used until now (which are inspired by Brennan's work): $n = 100,001$, $p_i = 0.51$, $V_c - V_w = 10^{13}$€, $C_0 = 1$€. We also add that for every c, $p_c = 0.51$, and $\pi = 0.5$. Now, suppose that it is plausible that in a group of $100,000$ voters there are 10 big influencing factors ($c = 10$) that may threaten Independence. Voters are also evenly split among such factors: each factor exerts influence on 10.000 voters. With these parameters $p_{M_n} = 0.65$. This number is arguably low. And this might be problematic for the common good, as we are all but certain to secure the money surplus given by making the correct choice. Brennan's GUL appears to no longer follow. In fact, one could hence expect the expected marginal contribution of adding a new voter to the electorate to be really high. Calculating the expected marginal value of the next voter, however, might not be the most straightforward move to make in this refined setting.

How then can we ensure that we make the correct choice with reasonably high probability, provided that making the right choice is worth a lot to the common good, and that we aim at societal efficiency? Namely, how can we increase group competence without incurring a waste of money and resources? A more fruitful approach according to this setting seems that of calculating the expected value of adding a new group S of m voters and not single individuals to the electorate, and then asking if adding this new group is worth to the common good. I propose to calculate this by slightly revising Brennan's equation, as follows.

$$E_{\mathbf{v}_S} = \Delta P_{M_S}(V_C - V_W) - (C_0 \times m) \tag{4.4}$$

Where $E_{\mathbf{v}_S}$ is the expected value of adding the new group S of m voters; ΔP_{M_S} is the difference in group competence between n and $n-m$ voters; while $(V_C - V_W)$ and C_0 remain unchanged.

Let me now add two additional elements to the picture. First, I endorse the additional assumption that the number of *independent* influencing factors k must increase together with the number of voters. This is not an unrealistic assumption. In a sense, this might be interpreted as if by enlarging the electorate we increase diversity in influencing sources, in a way similar to that presented by Landemore (Landemore 2011), (Landemore 2012). Numbers are a proxy for more independent common causes and thus for more clusters of dependence. It seems, in fact, plausible to assume that, as long as random new voters are introduced in the electorates, influencing factors also tend to increase. Every time new groups of voters are given voting rights these groups also bring in new patterns of dependence. That is to say, the larger the electorates, the higher the number of influencing factors that serve as independent judgment points.

Second, following Goodin and Spiekermann (Goodin and Spiekermann 2018), I conjecture that, if the number of voters and the number of independent influencing factors grow at the same rate, collective competence grows, and in the limit still reaches 1. In this case, the CJT's conclusions are retained.

Conjecture 4.7.1. *if n and k grow at the same rate, then "Monotonicity in Group Size" and "Asymptotic Perfection" are retained.*

Suppose Conjecture 4.7.1 is true, and that it is actually the case that n and k grow at the same rate. Table 4.4 shows what happens to group competence, starting with Brennan-inspired parameters, and then increasing n and k at the same rate.

n	p_i	k	p_c	π	p_{M_n}
$100,001$	0.51	10	0.51	0.5	0.65
$5,000,001$	0.51	500	0.51	0.5	0.83
$10,000,001$	0.51	$1,000$	0.51	0.5	0.90
$100,000,001$	0.51	$10,000$	0.51	0.5	~ 1

Table 4.4: Increase in group competence with n and k growing at the same rate.

Now, given the collective competence results shown in Table 4.4, a new question arises. Is, under this new setting, mass voting still unjustified? Or, similarly, is mass voting beneficial for society? First, let's assume that that $15,000,001$ voters can be safely considered mass participation. Now, we know that, with the parameters described above, for $n = 10,000,001$ and $k = 1,000$, $p_{M_n} = 0.90$. Suppose now that we want to calculate the expected value of adding to the

electorate a group S of size $m = 5,000,000$ (so that $n = 15,000,001$), assuming that Conjecture 4.7.1 is true; i.e., in the resulting electorate, $k = 1,500$. Following 4.4, we obtain that, with the parameters previously defined, $E_{\mathbf{v}_S} = 3.9 \times 10^{11}$€. The expected value of adding $5,000,000$ voters to an already existing electorate of $10,000,001$ is largely beneficial to society. Table 4.5 resumes this final result.

p_i	k	p_c	π	$p_{M_{n-m}}$	E_{v_S}
0.51	1,000	0.51	0.5	0.90	3.9×10^{11}

Table 4.5: Expected (monetary) value of adding to an electorate of size $n - m = 10,000,0001$ a new group of voters S of size $m = 5,000,000$, given the assumption that n and k grow at the same rate. Other parameters remain unchanged.

To conclude, under this refined model of mass voting, not only GUL spelled out in terms of $100,000$ voters is false, but mass voting can be justified too.

4.8 Concluding Remarks

Informally, one can generalize the results discussed in the previous section by saying that it is better to have a large electorate whose groups of voters are affected by many *independent* influencing factors than a small electorate affected by fewer common causes.

In assessing the counterargument to Brennan presented in this Chapter, some remarks should be made. First, the proposed model drops the implausible assumption that votes are interchangeable. Votes of different clusters of dependence are clearly not interchangeable. Intuitively, three votes from different clusters have a higher chance of tracking the truth, than three votes within the same clusters.

Second, this more realistic setting invalidates GUL. Moreover, I pointed out that the simple model with just one common influencing factor does invalidate RUL. However, if the more refined model presented in Section 4.6 invalidates RUL to the point of justifying mass voting needs further research.[17]

Third, when, in Section 4.6, I introduced the model with multiple influencing factors, I assumed these factors to be independent of each other. I added in the footnote 12 that this is not even necessary, perhaps, to invalidate GUL. Let me elaborate on this conjecture a bit more. To see why this might hold, suppose to have 100,000 voters that have already voted. Suppose also that these voters are influenced by a few bunch of overlapping common causes. Thus, group competence

17. For now, one can probably conjecture that the model somehow impacts on RUL as well. The reason is that, as group competence depends both on k and n, if $n = 100,000$, and k is reasonably smaller than n, group competence would still be relatively low. And this would suggest that perhaps adding more voters is beneficial.

is expected to be widely affected by this. Now, it seems reasonable to assume that if we add a voter to the electorate who is not influenced by any of the factors already present, this new voter would make a great contribution to group competence and hence to the common good.

Let me conclude with a final remark on the model that proposed and supported in this Chapter. As I have already mentioned, that model does not want to offer a fully complete and realistic picture of how mass voting can be represented in a real-world setting. There are still elements that are both idealized and simplified. This, however, should not surprise the reader, as this discussion aims to remain in the field of ideal theory. Even in this, nevertheless, some statements appear to be completely untenable. And Brennan's assumption of full Independence between votes in large electorates seems to fall in this category. What I here proposed is to drop such an assumption in favor of a more plausible one, which offers a better representation of mass voting. Even if the proposed model is still not fully realistic, I wished to persuade the reader that a step toward realism has been made.

My analysis of Independence is thus concluded. In the next Chapter, I will challenge Brennan's argument from the perspective of another CJT's core assumption: Competence.

Chapter 5

Refining Competence

5.1 Introduction to the Chapter

In this Chapter, I will offer my second critique of Brennan. While in the previous Chapter, I criticized Brennan's argument from the standpoint of how it exploits the Independence assumption, in the present one I will offer a critique based on Competence. The argumentative structure will be quite similar to that of the previous Chapter. I will first argue that Brennan's argument employs an implausible representation of the Competence assumption in the context of mass voting. I will then argue for a refined and more plausible interpretation and modelization of Competence for large electorates. I will finally show that under this refined interpretation of Competence, Brennan's GUL conclusions no longer necessarily follows.[1]

The refined interpretation that I am going to offer is based on the idea that Competence is "Topic-Specific". In brief, with this, I mean that different individuals are competent in different topics. Competence levels thus vary across different topic-individual pairs. As said, I will argue that this assumption provides a better representation of what mass voting could look like.

In summary, the argument I will present in this Chapter will take the following form.

P1 Brennan's *General Utility Loss* conclusion relies crucially on the implausible (especially for large electorates) Standard Competence Assumption.

P2 A more plausible model of mass voting, still compatible with some form of jury theorem, is that in which Competence is topic-specific.

C1 Under this new model, *General Utility Loss*, spelt out in terms of $100,000$ voters, no longer follows, *ceteris paribus*.

1. And some impacts on RUL can be also conjectured.

C2 Under this new model, the number $100,000$ should be increased for *General Utility Loss* to be valid, to the point that mass voting is now justified, *ceteris paribus*.

Accordingly, the Chapter will be structured as follows. First, I will re-state in detail what the Competence assumption is about and what are some plausible relaxations of it. I will then analyze how Brennan introduces the Competence assumption in his argument and paper, pointing out that, there, he already seems to make some contradictory statements. After this, I am going to first offer the basic intuitions and motivations that stand behind the introduction of Topic-Specific Competence, and second give a formalization of these institutions. I will finally demonstrate that with competence being topic-specific mass voting can be restored.

5.2 The Competence Assumption

The Competence Assumption is the second statistical assumption of the CJT. Before casting her vote, each voter forms a belief on which is the correct alternative. A voter[2] is competent if her belief has a higher than random probability of matching the true state of the world.

Let me now resume what the standard Competence assumption says more formally. We have a set of alternatives $A = \{0, 1\}$. Then we have a state of the world $\mathbf{x}$ which is a random variable taking values in A. Moreover, individual i's vote $\mathbf{v}_i$ is also a random variable taking values in A. We then define R_i as the event in which $\mathbf{v}_i$ matches the true state of the world. In other terms, R_i is the event in which individual i votes correctly.

The Competence assumption simply requires that, for any $\mathbf{v}$ and for any $\mathbf{x}$, R_i is more probable than $\neg R_i$, and that this probability is equal for every i:

- **Competence** (p_i): the probability Pr that, for any i and for any $\mathbf{x}$, $\mathbf{v}_i$ coincides with $\mathbf{x}$ is greater than $\frac{1}{2}$ and the same for every i;.

Notably, Pr is a conditional probability. Since we assumed in previous chapters that each vote is independent *conditional* on the state $\mathbf{x}$, Competence above is an assumption on $Pr(R_i|\mathbf{x})$.

Furthermore, as we have seen in Chapter 2, the Competence assumption can be relaxed to "competence on average"; namely, it is sufficient that individuals' probability of voting correctly is only *on average* greater than 0.5. We may then

2. But note that, to be precise, the mathematical object to which the Competence Assumption refers are the *votes*, not the *voters*.

have, depending on the specific problem subject to the election, some voters who are competent and some others who are not, but still be above random on average (see Subsection 2.3.1 for a formal statement of Average Competence). Unfortunately, we also know that, only if individual competence levels are symmetrically distributed, Average Competence only preserves the Asymptotic Perfection conclusion, which, from a political perspective, is the more appealing one.

One should also note that competence should not be equated to *knowledge*. Competence is simply the tendency to get it right. Clearly, it is undeniable that, despite being two different concepts, competence should (partly) depend on individuals' pieces of knowledge and on the information they possess. But, if almost everyone would agree with the claim that many voters are *not* knowledgeable in many political and factual respects, harder to support is the claim that voters are systematically mistaken. A voter that does not know anything about the issue at stake is ignorant. Being ignorant seems to mean having no clue. But if that is the case then we would expect such a voter to vote randomly; i.e. the probability that he makes the correct choice is nothing more and nothing less than 0.5. Making systematic mistakes implies that voters systematically form their beliefs based on misleading evidence, which they are not able to classify as such. Those who claim that voters' beliefs about the world are systematically wrong about any kind of decision problem seem thus to have the burden of proof.

In summary, in this section, the structure and the meaning of the Competence assumption have been recovered and clarified, and its first straightforward relaxation has been recalled.

In the next section, we will have a closer inspection to the role that Competence plays in Brennan's argument.

5.3 Brennan's Use of Competence

Competence is crucial for the CJT to obtain. Without it, other things being equal, group competence would not increase, but decrease, and in the limit will approach 0. Conversely, as we already noted in previous chapters, even quite low individual competence levels are sufficient to reach epistemic perfection rather quickly. Again, this is quite fundamental to Brennan's argument. However, the way the argument he supports in (Brennan 2011) deals with Competence is for the rest unclear. This section aims to make some clarifications about it.

When introducing the CJT's setting and assumptions, Brennan mentions the Competence Assumption as follows: "[voters] are *on average* more likely than not to make the right choice" [emphasis mine] (Brennan 2011). He thus acknowledges that the standard Competence Assumption can be relaxed to Average Competence. Note, however, that a few lines later he claims: "[l]et us assume, on behalf of the

Condorcetian, that *every* voter has a greater than 0.5 chance of being correct. If so, it follows that every additional voter increases, by some amount, the probability that the electorate will make the correct choice" [emphasis mine] (Brennan 2011). This very last claim seems to contradict the previous one. And, more generally, without further specifications, the first claim seems inconsistent with the rest of his statements. One should in fact note two things.

First, Brennan cannot claim — as he does — that (i) competence is on average and is equal to 0.51;[3] (ii) *every* voter has a higher than random probability of making the correct choice; (iii) competence levels below 0.51 are artificial and *ad hoc*, and thus should not be assumed.[4] The reason is the following. Suppose that, consistently with Average Competence, 0.51 is a mean value. But Brennan also says that *every* voter has a *greater than 0.5* chance of being correct. This would imply keeping fixed 0.51 as a mean value and letting other competence levels be distributed above and below this value, having 0.5 as a lower bound. This seems extremely artificial, as the variation would be minimal, and one of the advantages of having Average competence would be dismissed: the fact that it allows some voters to be below the critical competence ($Pr(R_i) < 0.5$) level. Moreover, Brennan takes 0.51 as the most charitable (to Condorcetian) value because he himself considers assuming values below this too artificial. However, with this setting, he is forced to assume that some voters have these competence levels, as the variation in competence levels would occur between 0.5 and 0.51; otherwise, without variation, we fall back into standard Competence. Therefore, as a first conclusion, we can claim that Brennan's statements about Competence are contradictory.

Second, and more importantly, Average Competence is *not* sufficient for his argument to work. Arguably, assuming Average Competence seems in principle more plausible in the case of large electorates. Competence homogeneity is a too strict requirement to be credible. However, as the previous statement highlights, he needs the growing reliability conclusion (i.e., monotonicity in group size) to make his argument work. In other terms, we must be able to claim that, from a purely epistemic point of view, the more voters the better. The reader should, in fact, note that swapping standard competence for average competence would already invalidate GUL, if we let competence levels vary below 0.5. One cannot just pick *any* $100,000$ voters and let them vote to secure quasi-perfection in group competence. If, for instance, one were to pick only voters from the first percentile of a distribution that has 0.51 as a mean value and 0.2 as a lower bound, then group competence would arguably never reach perfection, but would quickly converge to

3. The reader should remember that 0.51 is the competence parameter on which Brennan builds his argument.

4. Brennan spends quite some time to show this: "Using Condorcet's Jury Theorem in an argument on behalf of real-life democracy decision-making becomes less and less plausible as $p_c \to 0.5^+$" (Brennan 2011, 60).

0, *ceteris paribus*. Similarly, if voters from the last percentile only are to be picked, then way less than 100,000 would be needed.

To make his argument work while assuming Average Competence (leaving for the moment aside his contradictory statements about competence levels), Brennan needs a further assumption: competence levels that are symmetrically distributed around the mean. A scenario in which, we know, both the CJT's main conclusions are retained. That is, it is still true that adding more voters increases group competence. Nevertheless, such a relaxation of competence is not even mentioned in the paper. For this reason, I will assume that Brennan really means that indeed $Pr(R_i) = 0.51$ for every i (that is, the standard competence assumption with a fixed low value for individual competence). In the end, despite his contradictory statements, that standard Competence is what he assumed becomes quite clear in the calculations he makes with Equation 3.1.

There is nevertheless a final remark that has to be made. The results of his calculations presuppose — as said in previous chapters —that votes are independent and *identically distributed* (which is in fact guaranteed by Independence and Standard Competence).

However, due to another property that Competence might have — especially when applied to large electorates that face decision problems on many different topics — this form of interchangeability is likely to be violated at the expense of Brennan's conclusions. In the next section, I will pursue this point, introducing a further notion of Competence.

5.4 Topic-Specific Competence

In this Section, I am going to argue for individual competence to be "topic-specific".[5]

Dietrich and Spiekermann point out that Competence can vary across problems[6] of the same reference class (Dietrich and Spiekermann 2013). Some decision problems are more difficult than others. It seems fair to assume that there are some cases in which it is less likely that a voter's vote matches the true state of the world. But, if this is the case, then the conditional competence assumption, that states that for *every decision problem* each voter has a higher than random probability of making the correct choice,[7] seems unlikely to hold. When problems

5. Something along these lines seems to be suggested by Goodin and Estlund when they say: "[t]he CJT's basic conclusion — that democracy is a good truth-tracker — holds just so long as the mean voter is 'better than random' on each topic. There is no need for them to be equally competent across all topics" (Goodin 2003, 135). They do not add on that, though.

6. See Section 4.4, Chapter 4) for the definition of a decision problem.

7. Note that the same reasoning applies also to the average competence assumption.

are intrinsically difficult this is less likely to happen.[8]

There is nevertheless another way according to which Competence can vary. It can vary across different topics. This kind of variation is not based on the nature of the problem, but it is, instead, "*ad hominem*" (Goodin and Spiekermann 2018). It depends on the features of the agents more than on those of the problem.

First of all, it seems quite reasonable to assume that the range of possible topics about which a democratic decision has to be taken is quite large. The standard Competence assumption — endorsed by Brennan — implicitly assumes that each individual is equally competent in any possible topic. Looking at the composition of actual democratic electorates, this appears extremely implausible.

It is more realistic to say that, given a specific voter, her competence level varies across different topics based on her specific expertise, experience, interests, and prior knowledge. Obviously, when these traits are more present, individual competence is higher, and vice versa. A railway engineer is expected to be more competent in finding the truth about a newly constructed railway, than the truth about a specific tropical bird species. And it is also reasonable to assume that, even without being a railway engineer, a railway enthusiast might be more competent than the average citizen with respect to the railway issue. Different individuals are competent in some specific topics and issues, and not in others.

What is more, the set of topics of expertise that each individual might have is plausibly small. In modern liberal societies, voters are not just voters. They are usually citizens who primarily act in order to pursue their private interests by, for instance, working and taking care of their dear ones. Presumably, the satisfaction of one's private interests occupies a large part of a democratic citizen's life. The proper exercise of citizenship and political duties is only a marginal part of the average citizen's life - in terms of time and effort dedicated. Therefore, it seems implausible, if not fully unrealistic, that citizens have broad expertise on many issues that could be subject to a CJT-like majority voting procedure. For building an expertise that leads to competent voting seems to some extent to require gathering and selecting information of good quality. Even if being minimally competent in a given topic is not an excessively time-consuming and demanding task *per se*, it can become as such considering the large number of possible topics about which a decision has to be taken. Being competent in many of those respects appears thus a problematic assumption, due to the fact that the epistemic resources of each voter are to some extent limited. Therefore, I will assume that constrained epistemic resources make each voter competent on a small subset of topics out of all the possible topics subject to the decision-making process.

8. Appendix C offers a more detailed explanation of the formulation of this refined notion of competence by Dietrich and Spiekermann.

Furthermore, not only is it reasonable to expect that each voter is competent in a small specific subset out of the set of all possible topics; but also that, for each voter, these subsets present a quite high degree of diversity. If voters are not perfect copies of each other, their areas of expertise are likely to present differences. Individual i may be more knowledgeable on one specific kind of topic, whereas individual j may be more knowledgeable on another one. Moreover, it could also be the case that, while being informed on some topics of her interests, individual i is *ignorant* on j's areas of knowledge.[9] Possibly, it may be the case that there are some topics that "attract" more interests and are thus within the interest area of more voters, whereas other topics could be expected to be more niche. One can nonetheless still think about the former point as the citizenship/electorate being diverse in its areas of expertise. I am going to assume this kind of diversity.

If we (plausibly) assume that the set of all possible topics related to democratic decision-making is quite large, having scarce epistemic resources and being diverse in interest/expertise lead to a case in which each topic falls in the competence area of just a relatively small number of voters, compared to the full size of the population. As an ideal result of this conceptualization, we would have a small subset of the electorate competent for each possible topic subject of the decision-making process.

Let me provide a formal statement of these intuitions. First — as always — suppose to have an infinite reservoir of individuals N. We also have a finite set of possible topics subject to the election $\mathbf{\Pi} = \{\Pi_1, \Pi_2, \ldots, \Pi_n\}$. Resuming here Dietrich and Spiekermann's work, each topic Π can be conceived as a reference class of decision problems on a similar issue (Dietrich 2008, 64), (Dietrich and Spiekermann 2019, 389). Examples could be the classes of infrastructure problems or public health problems.

Under the topic-specific setting, individual competence is indexed by the topic. Let me define $p_i^{\Pi_j} = Pr(R_i|\Pi_j)$ as individual i's *topic-specific competence*; i.e., $p_i^{\Pi_j}$ refers to the probability that i votes correctly given topic Π_j. Similarly, $\overline{p}^{\Pi_j}$ is the average probability that each individual is competent in that topic. A voter is *competent* in a topic if $p_i^{\Pi_j} > 0.5$.

In this context, Brennan's assumption is that everyone is always equally competent in any topic. This would result in the following *implausible* assumption.

- **Competence over All Topics (CAT)** (CAT): for all topics Π_j, for any i and for any $\mathbf{x}$, the probability that $\mathbf{v}_i$ coincides with $\mathbf{x}$ is greater than $\frac{1}{2}$ and the same for every i; i.e. given a topic Π^j, $p^{\Pi_j} > 0.5$.

Instead, above we argue for competence being topic-specific in the following sense: (i) voters have scarce epistemic resources, and (ii) voters are diverse in their

9. Ignorance means voting at random.

areas of expertise. Thus, voters are competent just in a small subset of topics $T_i \subset \Pi$, and the competence distribution T_i varies significantly among voters, reflecting diverse expertise. T_i labels i's competence area. Figure 5.1 illustrates this scenario with a simplified representation.

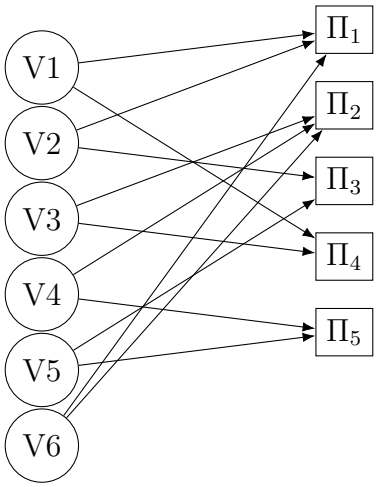


Figure 5.1: Topic-Specific Competence in the presence of scarce epistemic resources and diversity of interest areas.

In Figure 5.1, we have 6 voters, labeled $V1 - V6$, and five topics, $\Pi_1 - \Pi_5$. The arrows from a voter to a topic signal the fact that the voter is competent in that topic. Each voter is competent only in two topics out of six (scarcity in epistemic resources is thus reflected in the cardinality of T_i). Moreover, voters show diversity in their areas of interest/expertise, as arrows are not only directed toward a restricted subset of topics, but every topic is reached by a similar proportion of arrows.

Now, another thing should be noted: claiming that we are able to clearly identify the competence area of each individual appears excessively demanding. Similarly to what (Romeijn and Atkinson 2011) assumes with respect to individual competence levels more broadly, it seems more plausible to say that we cannot exactly know on which topics individuals are competent. As Dietrich and Spiekermann also acknowledge, this appears to be a quite realistic assumption to make with respect to individual-specific competence levels (Dietrich and Spiekermann 2019).[10]

Accordingly, to generalize the ideas of scarcity of epistemic resources and diversity in interest areas — without making claims about the specific individuals — we define $\mu_i^{\Pi_j}$, as the probability that a randomly selected topic Π_j is within

10. Section 5.6 elaborates more on this.

the competence set T_i of voter i. Basically, μ_i is the probability that a voter is competent in any given topic, and, for simplicity, is assumed to be the same for all voters. The higher μ_i is, the higher the number of competent voters we should expect for each topic. However, to reflect our previous assumptions of scarcity and diversity, μ_i is expected to be a *low* probability value. If this is the case, then for any randomly selected topic the proportion of competent voters out of the total size of the population is expected to be quite small, and numerically similar across different topics (but likely containing diverse topics). Yet, as long as $\mu_i > 0$, we would expect some competent voters for each topic Π_j. Therefore — as modeled here — μ_i ensures that, given a random topic, there is always a relatively small subset of individuals who are competent in that topic.

Essentially, the μ_i parameter tells us how many competent voters we should approximately expect for every topic. To get what it means, suppose to have an urn full n balls of k different colors. Each ball has a $\frac{1}{k}$ probability of being of any given color.[11] We are interested in balls of one specific color (say "red"). $n \times \frac{1}{k}$ tells us how many red balls we should approximately expect to draw. So, if we have $100,000$ balls of 100 different colors (one of which is red), we should expect to draw approximately $1,000$ red balls. In our case, one should think of red balls as competent voters. As said, in what follows — to highlight the plausible assumption that individuals have scarce epistemic resources — I am going to assume $\mu_i^{\Pi_j}$ to be low.

What is more, in what follows I will assume that voters who are not competent in a topic are ignorant in the sense that they vote randomly; i.e., for all i and all j, if $\Pi_j \notin T_i$, then $p_i^{\Pi_j} = 0.5$. Having no clue about a possible decision to make seems to have as a straightforward implication that of voting with the same probability for each of the two options. Namely, each vote would *not* be different from the toss of an unbiased coin. As I have already noted, such votes would neither add anything significant to the electorate in terms of competence nor they would detract from it. Ignorant voters can be statistically interpreted as randomly distributed "noise". Hence, they cancel out (Goodin and Spiekermann 2018, 88). That means that only the subgroup of topic-specific competent voters is a determinant of the decision on that topic. Clearly, another possibility is that some voters are systematically wrong.[12] However, in section 5.2, we have already mentioned the fact that this scenario is supposedly less plausible to assume. Claiming that voters are as bad as they systematically vote wrongly is a rather severe statement. As we said, those who claim this appear to have the burden of proof. For these reasons — and also for modeling simplicity — I will assume that whenever a topic Π_j is not in voter

11. This is essentially the μ parameter rephrased.

12. It appears possible to conjecture that the model I am presenting here could be adjusted to account for incompetent voters. For now, I leave it open as a possibility.

i's competence area T_i, i is ignorant on Π_j.

Fundamentally, we are saying that each individual is competent on a small subset of topics and ignorant otherwise. This scenario is consistent with the following assumption:

- **Average Competence over All Topics (ACAT)** (ACAT): for all topics Π_j, for any $\mathbf{x}$, the average probability $\overline{p}^{\Pi_j} := \lim_{n\to\infty}(p_1^{\Pi_j} + \cdots + p_n^{\Pi_j})/n$ that, for any i, $\mathbf{v}_i$ coincides with $\mathbf{x}$ exists and it is greater than $\frac{1}{2}$. And its value (partially) depends on μ_i

Namely, as long as for all i and all j, if $\Pi_j \notin T_i$, then $p_i^{\Pi_j} = 0.5$, and $\mu_i > 0$, then a group is expected to be on average competent on each topic.

Note that ACAT is essentially a reformulation of Average Competence.[13] Note also, though, that my representation of how competence is topic-specifically distributed in a democratic electorate is consistent with a precise instantiation of ACAT. Without making absurd statements, we are assuming ACAT to be satisfied (i.e., $\overline{p}^{\Pi_j} > 0.5$), but its actual value to be close to 0.5 — a possibility that is acknowledged also by Estlund and Goodin in (Goodin 2003, 140).[14]

To sum up, voters are competent in just a small subset of topics out of all the possible topics subject of an election; these subsets are likely quite diverse in their elements; we cannot make precise claims on voter's expertise, and thus we assume that, given a random topic, there is only a very small probability that, for any voter, this topic is in her competence subset. This more plausible picture is consistent with TSAC.

In the next Section, I will investigate the implications of this refined and more plausible way of modeling Competence in large electorates on Brennan's argument and conclusions. As done before, the aim of this investigation is twofold. The first aim is to check whether under this refined model GUL and RUL still hold. The second aim is to check whether under this more refined model mass voting is justified.

5.5 Implications for General Utility Loss

Let's get back to Brennan's argument and see what changes with Competence being topic-specific, starting by a simplified example. Suppose — as always — that 100.001 voters have already voted, but that, instead of having $p_i = 0.51$ for every i for any topic Π_j, Competence is indexed by topic, and whenever an individual is

13. Consistent, thus, with the CJT's asymptotic conclusion.

14. The reason is that, for each topic, only a small subset of voters (the actual size depends on μ) is competent, while the rest is ignorant.

not competent on a topic, she is ignorant. $p_g^{\Pi_j}$ labels the probability that a topic-specific ignorant voter makes the correct choice — which is by definition always 0.5; $p_w^{\Pi_j}$ labels the probability that a topic-specific competent voter makes the correct choice. Suppose now, as an extremely simplified case, that 99,998 of those voters who have already voted are *not* competent with respect to the topic subject to the decision at stake. Namely, for each of them, $p_g^{\Pi_j} = 0.5$. The remaining three voters are instead competent: for them $p_w^{\Pi_j} = 0.51$. If this is the case, as ignorance voters cancel out, we need to calculate a majority out of the competent voters is correct: $p_{M_3} = 0.515$. Imagine that the next voter that is going to be added to the electorate is a competent one. At this point, in accordance with 3.1, if the n^{th} voter is known to be a competent one, the expected marginal value of adding that voter to the electorate is 1.87×10^{10}€. This simple example already invalidates GUL, as, according to it, it would not be the case that after any 100.000 voters, adding an extra voter to the electorate would go to the detriment of the common good.[15] The rationale behind this result is straightforward. Being indexed by topic, Topic-Specific Competence is no longer consistent with the interchangeability of votes. To increase group competence we need to have those voters who are competent in that topic. We cannot simply take any voter and let her vote to obtain the desired result.

In this simplified example, however, we are assuming that we know the topic-specific competence of the voters. As argued in the previous Section, this is unlikely the case. In fact, the model exposed in Section 5.4 avoids claims about the specific individual. In what follows, I will conduct an analysis of Brennan's argument fully consistent with that new model.

Suppose that the possible topics that can be subject to a CJT-like democratic procedure are roughly 1,000. That is, $|\mathbf{\Pi}| = 1,000$. Suppose also that is reasonable to assume that each voter is competent on approximately 5 different topics.[16] This means that these topics are the voter's area of interest/expertise. Formally, for every i, $|T| = 5$. Each voter is thus competent only in the 0.5% of all possible topics. Thus, the probability μ_i that a voter i is competent in any given topic is equal to 0.005. Let's as usual suppose that 100,001 voters have already voted. Given a randomly selected topic, we would expect approximately[17] 500 voters to be competent on that topic.[18] Let's continue using Brennan's original parameters as

15. All group competence results in this Section are analytical results obtained calculating formula 1 in *Mathematica*. The expected marginal value of the n^{nt} is calculated by resolving equation 3.1 also in *Mathematica*.

16. Following the original spirit of Brennan's argument, these are charitable parameters.

17. "Approximately" because, the number of competent voters for any given topic can be modeled using a binomial distribution (n, p), which implies some variance in the result to be considered.

18. I am, here and later as well, going to assume that these are odd numbers. For example, in

a benchmark. If, for competent voters, $\overline{p}_w^{\Pi_j} = 0.51$ (the competent voters' average topic-specific competence), or — more simply[19] — $p_w^{\Pi_j} = 0.51$, then collective competence[20] $p_{M_n}^{\Pi_j} = 0.67$. This is a low competence value which suggests that adding more voters to the electorate would be beneficial for society. To verify this claim, it is again more fruitful — as we did in Section 4.7 of Chapter 4 — to calculate the expected value of adding a new entire group of voters S of size m, instead of calculating the expected marginal value of adding a single individual. The main reason is that given our parameters for n and μ_i we would expect 500 competent voters only on *average*; this value is in fact subject to some variance that can be calculated using the binomial distribution. Accordingly, it is permissible to assume that adding only one voter would potentially not make any difference. To grasp the potential benefit of enlarging the electorate it is more interesting to ask what happens if we add another group of voters all with a very low probability of being competent. I am going to do that according to Equation 4.4.

Suppose then to add a new group of $1,000$ voters to the electorate, such that $n + m = 101,000$. What is the expected value, $E_{\mathbf{v}_S}$, of adding S to the electorate, given that, as before, $V_c - V_w = 10^{13}$€, and that $C_0 = 1$€. Equation 4.4 tells us that $E_{\mathbf{v}_S} = 6.4 \times 10^9$€. Table 5.1 resumes these results.

$p_w^{\Pi_j}$	$\lvert\mathbf{\Pi}\rvert$	μ_i	$p_{M_n}^{\Pi_j}$	E_{v_S}
0.51	$1,000$	0.005	0.67	6.4×10^9€

Table 5.1: Expected value of adding to an electorate of size $n = 100,001$ a new group of voters S of size $m = 1,000$, given the assumption of Topic-Specific Competence and the aforementioned parameters.

Therefore, these results allow us to conclude that, under this more plausible representation of competence, GUL no longer follows.

As we did in Section 4.7 of Chapter 4, we are not simply interested in checking the validity of GUL under a new set-up, but we also want to see if mass voting then becomes justified. As we did there, I will assume that having roughly $15,000,000$ voters vote can be said to be mass voting.

Suppose then that — instead of $100,001$ — $10,000,001$ voters have already voted. Keeping fixed the previous parameters concerning the total number of topics and the probability that a voter is competent on any given topic, we can calculate that for any randomly selected topic, we should expect to have approximately

this case, I will take the number of competent voters to be 501.

19. The reader should remember that not-competent voters are ignorant and thus cancel out. There is no voter below the critical level and then assuming Average Competence (with symmetrical distribution) or Standard Competence does not make a difference.

20. That is, the probability that a majority of roughly 500 competent voters votes correctly.

$50,000$ competent voters (whose $p_w^{\Pi_j} = 0.51$ by assumption). What is the expected value of adding a new group of $5,000,000$ voters to the electorate, such that the newly formed electorate would consist of $15,000,001$ voters? Table 5.2 illustrates the result of the calculations, based on the usual parameters inspired by Brennan's work (Brennan 2011, 58).

$p_w^{\Pi_j}$	$\lvert\mathbf{\Pi}\rvert$	μ_i	$p_{M_n}^{\Pi_j}$	E_{v_S}
0.51	$1,000$	0.005	0.99	3.8×10^7€

Table 5.2: Expected value of adding to an electorate of size $n = 10,000,001$ a new group of voters S of size $m = 5,000,000$, given the assumption of Topic-Specific Competence and the aforementioned parameters.

Enlarging the electorate from 10 million to 15 million voters is thus still highly beneficial for society. This increase is worth a lot of competence to the common good. Therefore, we can thus conclude that, if competence is topic-specific, with the parameters previously illustrated, mass voting is both epistemically justified and maximizes societal expected utility.

5.6 Concluding Remarks

In section 4.7, I have shown that, if competence is topic-specific, then having more than any 100,000 voters vote does not necessarily imply GUL, *ceteris paribus*. Notably, one could conjecture that something similar could happen to RUL as well.[21] A full reflection on the impact of Topic-Specific Competence on RUL needs however further investigation.

A further point can be made. The results obtained in the analysis that I proposed (see Tables 5.1, 5.2) partially hinge on the assumed very low societal cost of voting. This, however, is already part of the original argument made by Brennan, whose parameter, whenever possible, I kept fixed to not depart from his original — charitable — picture. In any case, increasing the societal cost of voting up to a

21. As long as we know the total number of possible topics and what is the probability that a voter is competent in any given topic (where competent voters have a 0.51 probability of making the correct choice), the reasoning delineated with respect to the first/any n voters holds also for a randomly selected sample of n voters. In fact, with our usual parameters, we would need a sample of roughly $20,000,000$ voters to be able to claim that adding other voters would come at a societal cost. The reason is that only with such a number of voters we would be reasonably sure to have roughly $100,000$ competent voters for each topic — which is the number identified by Brennan. Looking at this, we could re-state Brennan's point by saying that what goes to the detriment of the common good is having more than approximately $100,000$ competent voters for any given topic. But this is well consistent with mass voting that thus results justified.

reasonable amount, would not excessively impact these results. For the results also hinge on another parameter assumed by Brennan: the very high positive value of choosing right. This value remains a few orders of magnitude higher than the cost society incur in having voters vote, even if this cost is increased. The reader should, in fact, remember that — for the sake of the argument — our discussion, along the same lines as Brennan, started from the following two suppositions: "suppose that (i) making the right choice is *very much worth* to the common good, and that (ii) individual voting comes at a quite *low* societal cost". Furthermore, as it happened for the Independence model presented in Chapter 4, my refinement of Competence does not claim to represent mass voting in all its realist aspects. It claims to be more plausible and (perhaps) realistic than what Brennan originally assumed. In the end, moving from a case in which all individuals are equally competent in any given topic, to a case in which individuals are competent just in a restricted subset of topics while assumed to be ignorant on others, and in which individuals differ from each other in their areas of expertise seems exactly to go in that direction.

Last but not least, one may ask, based on the fact that Competence is topic-specific, why would not be more socially efficient to individuate for each topic only those voters that are competent and let them vote. That is, given a topic Π we would have only competent voters to vote on that topic. Note, however, that in order to be fully effective, this procedure should be extended to any possible topic; namely, for all elements of $\mathbf{\Pi}$. If this procedure of topic-specific competence identification were really cheap for society, then it may well be the case that this would be more cost-efficient. And the result would be a sort of topic-specific epistocracy. In general, if society were able to easily and with certainty detect and select a reasonably large group of competent individuals (or even experts, perhaps), it may well be the case that — on a pure *instrumental* ground[22] — having only those people vote would be more efficient. Note, however, that whether this selection procedure is indeed less costly for society than having everyone vote — for similar levels of group competence, to be clear — is an empirical question. And, from a purely theoretical perspective, though, the answer is anything but obvious.

With these considerations, we can conclude this Chapter, and also the analysis of Brennan's argument in terms of Independence and Competence refined to be more plausible to be applied to large electorates. The last Chapter of this thesis will be devoted to reviewing the main arguments advanced, offering a summary of the conclusions reached, and sketching possible avenues of future research.

22. Because, politically, there may well be other reasons why we do not want this to happen.

Chapter 6

Conclusion

6.1 Summary and Upshot of the Thesis

In this thesis, I reconstructed and critically assessed an argument made by Jason Brennan against mass voting. This argument essentially concludes that epistemic democrats cannot use the CJT to recommend mass voting as society would incur a utility loss. More precisely, Brennan shows that approximately 100,000 voters would suffice to maximize societal expected utility.

In the thesis, I nevertheless argued that the aforementioned argument hinges on some peculiar assumptions of the Classical CJT; assumptions that appear however implausible to model mass voting. Accordingly, I argued that under more refined and plausible jury theorem's assumptions mass voting can maximize societal expected utility.[1]

More precisely, in Chapter 2, I introduced the CJT in its classical versions, examined its most fundamental assumptions, and went through some relaxations of some of these assumptions. In Chapter 3 I offered a detailed reconstruction of Brennan's argument, showed its implications for mass voting, and distinguished it from the motivational argument against mass voting. In Chapter 4, I gave my first counterargument to Brennan. There, I argued that assuming full Independence among votes, as Brennan does, is an implausible way of modeling mass voting. I thus showed that, if we allow for some patterns of correlations between votes, we may still retain the CJT's conclusions, but be also able to justify mass voting (see Table 4.5). Similarly, in Chapter 5 I presented a counterargument based on Competence. Brennan assumes that all voters are standardly competent. I argued for a more refined topic-specific notion of the Competence Assumption. With this new notion — I concluded — the CJT can plausibly recommend mass voting (see Table 5.2).

1. The reader should remember that most of my claims refer to GUL and not to RUL, though.

In essence, this thesis has tried to restore mass voting through more plausible Condorcetian assumptions. The reader could also look at the arguments I proposed as a way to find Condorcetian sufficient conditions for mass voting to maximize societal expected utility, showing that these conditions are also more plausible than the original ones from which Brennan starts.

This is important. For this thesis is not a move from an unrealistic picture to a realistic one. It is supposed to be a step from a *less* plausible modelization of mass voting to a *more* plausible one. One could think about it as an initial step towards realism; a step that already shows that the CJT *can* recommend mass voting.

However, the arguments made in the thesis present obvious limitations and some questions still remain unanswered. In the next concluding section, I will go through some of these.

6.2 Limitations and Further Research

First of all, one cannot buy into my arguments, unless they accept some form of jury theorems to be a relevant tool to answer questions about democracies. If one rejects the CJT altogether, then it is not possible to make sense of the arguments. For all of them presuppose the CJT. The jury theorem's assumptions are obviously idealized and simplified. If one believes that no truth or correct answer exists, one cannot work with jury theorems. In this thesis, however, I assumed that jury theorems are indeed relevant tools to make claims about democracies. And in Chapter 2 I gave reasons why the literature — and I with it — believes that is not impermissible to do so.

Second, another potential problem that lies at the essence of the discussion. One might say that we should answer questions only in the purely epistemic lane with jury theorems' models, which are already quite simplified in capturing the epistemic properties of, say, a voting process. Any traversing to value-related questions must fail, because the epistemic assumptions are then becoming even more heroic. This would be a remark on Brennan's project in its entirety. The point that we are asking the CJT to do more than it is usually asked is however not fully true. The jury theorems and their assumptions remain intact in their explanatory power. What I and Brennan are doing is combining them with some other additional value-related assumptions, which — for the sake of the argument — are extremely simplified. This does not mean that these assumptions cannot be somehow enriched. Also, it does not mean that we are changing the original epistemic metric of the CJT. And, in any case, I believe that there is some clear sense in which his argument poses some unanswered issues to epistemic democrats who really rely on the CJT as a prime justification. Even if by contemplating some

simplifications, these issues should be confronted.

Another debatable point is the perspective according to which Brennan tackles the problem. Any benefit and any cost are seen from the perspective of society, not from that of the individual agent. There are quite some assumptions behind this, such as that some form of society as a collective agent exists, and that "societal costs" are *not* merely the aggregate of individual costs. The opposites of these claims are not obviously true, though. It seems fair to make these assumptions for the sake of the argument, without entering such debates.

These were general remarks on the very basic setting of the thesis. Let me now turn to some concerns related to my arguments more specifically. Firstly, throughout the paper, I made several assumptions about how mass voting can and should be modeled to offer a more plausible picture of it. Whenever I did that, I also tried to provide convincing motivations and justifications. I nonetheless acknowledge that these are all theoretical claims that lack empirical support. However, this thesis is theoretical in nature: checking the empirical validity of some of the assumptions made is a different kind of project.

Secondly, the results presented in Tables 4.5 and 5.2 partially hinge on the choice of some specific parameters. This is true. But these parameters are not arbitrarily chosen. First, whenever that was possible, I stuck to Brennan's original parameters, which were purposefully chosen by him to be "maximally charitable" to Condorcetian. Second, whenever this was *not* possible or whenever new variables were included, I tried to give justifications for the chosen values, and explicitly stated if, for some preliminary calculations, some parameters were artificial. Despite this, it is true that under different parameters, different results would have been obtained, and also that, perhaps, the process of finding the most accurate and realistic parameters was not the prime concern of this thesis. Seeing how, with different parameters, our conclusion for mass voting change could be an interesting avenue of future work.

Thirdly, my discussion — as already said — was mostly focused on GUL. Although I made some conjectures about how the arguments I supported could affect RUL as well, this weaker conclusion has not been properly addressed within the thesis. Investigating whether appealing to lottocracy is indeed instrumentally superior to mass voting is surely one of the biggest left-open questions of this work, and thus a clear motivation for future research.

Finally, the two arguments I gave in Chapters 4 and 5 have been presented as two *independent* arguments against Brennan. This, however, leaves open the question of whether and how they can relate to each other and be interconnected. Inquiring how clusters of dependencies among votes and Topic-Specific Competence can be made compatible and interact requires further investigation, as only then the picture of mass voting offered in this thesis will fully cohere.

Glossary

ACAT Average Competence over All Topics. 55

CAT Competence over All Topics. 52

CJT Condorcet Jury Theorem. iii, 2–18, 20–22, 24, 25, 27–36, 41–43, 47, 48, 50, 51, 55, 56, 60, 61, 68, 73, 77

GUL General Utility Loss. 26, 27, 31, 38, 42, 44, 46, 49, 55–58, 60, 62

RUL Random Utility Loss. 26, 27, 38, 44, 46, 55, 58, 60, 62

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Appendix A

The Metaphysical Assumption

The traditional CJT is built on a debated metaphysical assumption: one alternative is assumed to be correct or objectively better than the other(s). Most of the literature on jury theorems buys into that leaving a thorough and fully satisfactory justification of it for other branches of philosophy. The actual existence of an independent truth or, in Cohen's terms, an independent standard of correctness is, however, quite fundamental for the CJT and, more generally, for the whole literature on epistemic democracy (Cohen 1986). If there is no truth to track then the CJT loses its appeal. This might be even more problematic in the case of political issues: according to the predominant non-cognitivist view, they are a matter of value judgments which are far from any definition of factual correctness. In this section, I will then provide some replies that the Condorcetian might adduce to ease the conceptual appraisal of the assumption that some sort of truth exists.

As said earlier, the typical case used to explicate the CJT is that of a jury that has to decide on the destiny of a person on trial. This is an "guilty or not-guilty" choice and only one of the two can be factually correct. To some, this seems far from what is usually at stake in political elections — which, incidentally, are the main interest of this thesis. But all is not lost.

First, the CJT's metaphysical assumption is compatible with some ethical schools of thought. *Moral realism* is one of them. According to moral realists, ethical and political value judgments are propositional knowledge about the world. They are facts and can be thus either true or false. Under this view, voters would have a truth to track. For various reasons, however, moral realism, while being defended and developed by many philosophers, is definitely not commonly endorsed and without problems (Talisse 2021).

Interestingly enough, the truth we are looking for should not necessarily be a matter of facts. It could be a socially constructed truth. As Estlund and Landemore point out, the "truths" we aim to track are not necessarily only objectives facts about the world, but they can also be "more intersubjective, culturally-

dependent, and temporarily construct (about more socially constructed facts or moral questions)" (Estlund and Landemore 2018). This approach is usually as *moral conventionalism*, and assumes that voters are not searching for facts about the world but for facts about themselves as a social entity.

Lastly, List and Spiekermann have conjectured a jury theorem for voter-specific truth (List and Spiekermann 2016). This result is of a slightly different nature because it does not technically deny that there is a truth. It simply says that such truths are many: every voter has her own. What is fully dropped is the underlying assumption that voters share a common goal. As Ladha says: "[w]hen members pursue a common goal, what is better for one is better for all" (Ladha 1993). In the voter-specific truth setting, List and Spiekermann's jury theorem tracks the truth that is "preferred" by most of the voters. And under an accordingly refined version of Competence and independence, it conjectures that the probability that the mostly chosen option is the truth preferred by most of the voters increases in n and approaches 1 in the limit. In this thesis, I will nonetheless stick to the original metaphysical assumption.

Appendix B

A Visual Close-Up on Competence and Independence

Figure B.1 and Figure B.2 illustrate the possible behaviors of Competence and Independence and their impact on group competence, for a population of 200 voters. In figure B.1, there are two lines parallel to the x-axis. The upper line represents a case in which individual competence may be satisfied but independence is fully violated: every individual follows with 100% probability one opinion leader who is also competent (but does not vote). Each voter votes according to what the opinion leader would do, and thus group competence is reduced to the competence of the opinion leader. We are not pooling the information of n independent voters, but we are simply pooling the information possessed by one individual n times. The bottom parallel line represents the corresponding scenario in which independence is still violated but the opinion leader is *not* competent. The probability that a majority is correct still "converges" to that of the opinion leader being correct.

In Figure B.2, the s-shaped curve shows how group competence varies with changes in individual competence levels, keeping the population number fixed and assuming that votes are independent. Instead of letting the population size vary, here individuals' competence level varies. It can be noted that group competence quickly converges to 1 as individuals get more competent, and, symmetrically, it quickly converges to 0 as individuals get more incompetent — provided, as we know, that individuals are competent if $Pr(R_i) > 0.5$, and incompetent if $Pr(R_i) < 0.5$.

Figure B.1 thus reminds us that just having an individual competence level above 0.5 is not sufficient for the CJT to hold, as independence must be satisfied as well. Also, above we mentioned those cases in which Independence is just partially violated, but which nonetheless preserve the CJT's nice conclusions. This can be thought of as an extension of the theorem which involves some relaxation of the

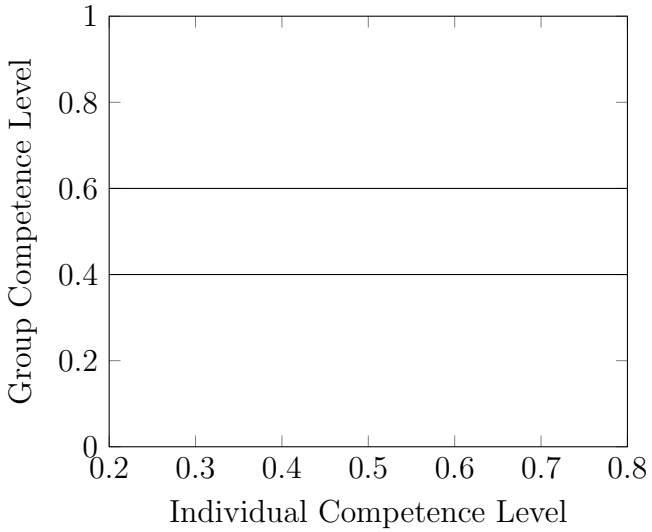

Figure B.1: How group competence changes for 200 voters in the presence of one common influencing factor which is followed by everyone with certainty.

Independence assumption. In the next section, this and some other straightforward extensions of the theorem will be examined.

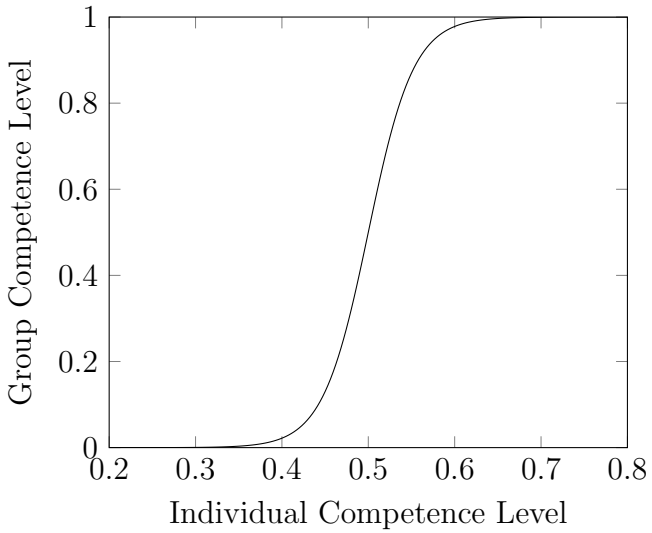

Figure B.2: How group competence changes for 200 voters depending on individual competence levels, with Independence being satisfied.

Appendix C

Calculations and Formulas

C.1 Group Competence with One Common Cause

I here explain the logic behind the calculations made in Section 4.5, where Standard Independence is threatened by the presence of *one* common influencing factor c. What I am going to illustrate is heavily reliant on the "Supplementary Material" of (Spiekermann and Goodin 2012) (see Section 2), who in turn built on (Boland, Proschan, and Tong 1989)' discussion.

Let me start by laying down again the basics of the CJT. Suppose to have an infinite population of individuals $N = \{1, 2, \dots\}$. I will use i as a placeholder for members of N. The standard CJT considers — out of this population — a group of individuals of size n (i.e. a jury, the electorate of a modern democracy with voting rights), with $n \geq 2$ and assumed to be odd for mathematical convenience, that has to select an alternative from the unordered pair $A = \{0, 1\}$ by majority voting. By assumption, one of the two alternatives in A is correct. We call it the state of the world $\mathbf{x}$, which is a random variable that takes value in A. Hence, we have two possibilities: either $\mathbf{x} = 0$ or $\mathbf{x} = 1$. Moreover, individual i's vote, labeled as $\mathbf{v}_i$, is also a random variable taking value from the same set A. Accordingly, we can define the event that individual i votes correctly, R_i, as when $\mathbf{v}_i$ coincides with $\mathbf{x}$. Individual competence is defined as $Pr(R_i) = p_i$; i.e. the probability that individual i's vote matches x. Finally, M_n denotes the event that a majority of out of the first n voters votes correctly - which is the event we are interested in. And $p_{M_n} = Pr(M_n)$ is the probability that a majority of n votes correctly. We assume that $Pr(R_i) = p_i > 0.5$, and $q_i = 1 - p_i$ is its converse. All probabilities are also independent conditional on the state of the world $\mathbf{x}$. We can thus arrive at the following CJT formula to calculate the probability that a majority of voters

is correct (see Section 2.2, Chapter 2.):

$$\mathcal{C}(n,p) = \sum_{k=\frac{n+1}{2}}^{n} \binom{n}{k} \cdot p^k \cdot (1-p)^{n-k} \tag{C.1}$$

Let there be a common influencing factor c. Each individual i has probability π of following c. She follows her private signal with probability $1-\pi$. Similarly to before, R_c labels the event that c is correct, and $Pr(R_c) = p_c$ is the probability that c is correct. $q_c = 1 - p_c$ is its converse. The probability of a correct vote depends on the following probability distribution.

$$Pr(R_i|R_c) = \pi + (1-\pi)p_i \tag{C.2}$$

$$Pr(R_i|\neg R_c) = (1-\pi)p_i \tag{C.3}$$

$$Pr(\neg R_i|R_c) = (1-\pi)q_i \tag{C.4}$$

$$Pr(\neg R_i|\neg R_c) = \pi + (1-\pi)q_i \tag{C.5}$$

Given this, the probability that a majority of voters is correct in the presence of one common influencing factors that may influence each voter is calculated by summing up the case in which the common cause is correct and that in which the common cause is incorrect. The resulting formula is as follows:

$$Pr(n, p_i, p_c, \pi) = p_c \cdot \mathcal{C}[n, \pi + (1-\pi)p_i] + q_c \cdot \mathcal{C}[n, (1-\pi)p_i]. \tag{C.6}$$

C.2 Group Competence with Multiple Uncorrelated Common Causes

In Sections 4.6 and 4.7, I presented a refined model of mass voting with multiple correlated common causes. I am here going to show how group competence is calculated in this setting, where we have n voters and k independent common causes that are each followed with probability π by $\frac{n}{k}$ voters. Each voter is influenced by one common cause only. Let me start with the simple cause of two common causes. Group competence is here a function of the competence of the two causes, p_{c_1} and p_{c_2}, of the size of the two groups of followers, n_1 and n_2, the competencies of the voters, p_{i_1} and p_{i_2}, and the probabilities of following π_1 and π_2. I now introduce the notion of a winning tuple $< w_1, w_2 >$, which is given by the number of correct

votes $w-1$ in group 1, and the number of correct votes w_2 in group 2, such that $w_1 + w_2 > \frac{n}{2}$. $\mathcal{W}$ is the set of all winning tuples. The probability that a specific winning tuple occurs is defined as follows:

$$\begin{aligned}&\Pr(p_{c_1}, p_{c_2}, n_1, n_2, p_{i_1}, p_{i_2}, \pi_1, \pi_2, < w_1, w_2 >) = \\ &\quad \big[p_{c_1}\mathcal{B}(n_1, w_1, \pi_1 + (1-\pi_1)p_{i_1})(1-p_{c_1})\mathcal{B}(n_1, w_1, \pi_1 + (1-\pi_1)p_{i_1}))\big]\cdot \\ &\quad \big[p_{c_2}\mathcal{B}(n_2, w_2, \pi_2 + (1-\pi_2)p_{i_2})(1-p_{c_2})\mathcal{B}(n_2, w_2, \pi_2 + (1-\pi_2)p_{i_2}))\big]. \qquad \text{(C.7)}\end{aligned}$$

where $\mathcal{B}$ stands for the binomial probability mass function:

$$\mathcal{B}(n, k, p) = \binom{n}{k} \cdot p^k \cdot (1-p)^{n-k} \qquad \text{(C.8)}$$

Group competence (i.e., the probability that a majority is correct) is given by the sum of the probabilities of all winning tuples:

$$\begin{aligned}&\Pr_{\text{maj}}(p_{c_1}, p_{c_2}, n_1, n_2, p_{i_1}, p_{i_2}, \pi_1, \pi_2) = \\ &\qquad \sum_{\forall <w_1, w_2> \in \mathcal{W}} \Pr_{\text{maj}}(p_{c_1}, p_{c_2}, n_1, n_2, p_{i_1}, p_{i_2}, w_1, w_2). \qquad \text{(C.9)}\end{aligned}$$

Due to poor handling in calculating analytical results for a large number of k, I relied on Monte Carlo simulations to calculate group competence (following, as always, (Spiekermann and Goodin 2012)). Here is the Monte Carlo routine on which my Python code is based:

Algorithm 1 Monte Carlo Simulation

```
1: success ← 0
2: for r times do
3:     Set votes of all opinion leaders by random draw according to their competence
4:     for each voter i do
5:         Determine by random draw according to probability of following if follows
6:         if voter i follows then
7:             Vote(i) = vote opinion leader(i)
8:         else
9:             Draw vote(i) according to competence of i
10:        end if
11:        Count correct votes
12:        if correct votes ≥ majority threshold then
13:            Increase success
14:        end if
15:    end for
16: end for
17: Return success/r
```

Appendix D

Tendency to Competence

Remind first that, when voters vote, they face a decision problem δ. One can think of a decision problem δ as a pair composed of the state of the world and the relative common causes $< \mathbf{x}, c >$ (Dietrich 2008).[1] As it has been introduced in this thesis, the CJT'a assumptions are defined over a variable decision problem, as the state of the world $\mathbf{x}$ is also a random variable. That is, Independence and Competence are defined over a reference class of relevant problems that the electorate might face. They are not defined given a specific problem.

Dietrich and Spiekarmenn point out that Competence can vary across problems of the same reference class (Dietrich and Spiekermann 2013). Some decision problems are more difficult than others

Take again the case of a jury that has to decide whether a potential criminal is guilty or not. It appears legitimate to assume that in some cases this decision is harder to make. So, decision problems of the class "guilty-not guilty" can be ordered according to their complexity. Is it still possible to call the jury as a group competent when decision problems vary in their level of complexity? Detrich and Spiekerman showed that we can call such a jury "competent" as long as every juror is more likely to face easier problems than harder ones - where easier problems are exactly those where a voter's specific competence level is higher than random (Dietrich and Spiekermann 2013). Jurors must *tend* to be competent.

More precisely, suppose to have a non-artificial reference class Π containing all problems π of the same kind. The competence assumption is now problem-specific. That is, it pertains to the individual's competence level with respect to a randomly drawn π from Π. Formally, a voter's i problem-specific competence p_i^π is defined as $Pr(R_i|\pi)$. For a voter to be competent, this probability must have a tendency to exceed $\frac{1}{2}$. The competence assumption thus revisited is the following:

- **Tendency to Competence**: problem specific competence p_i^π (i) tends to

1. The reader could go to Section 4.4 of Chapter 4 to look for a better explanation of this.

exceed $\frac{1}{2}$, (ii) is the same for every i.

As I said earlier, "having a tendency to exceed $\frac{1}{2}$" means, bluntly, that each voter is more likely to face an easy problem than a tough one. Or, in other terms, easy problems are more likely to be drawn from Π. Formally, in a discrete distribution, $Pr(p_i^\pi = \frac{1}{2} - \epsilon) \leq Pr(p_i^\pi = \frac{1}{2} + \epsilon)$, for any $\epsilon > 0$.

For completeness, it is worth mentioning that Independence conditional on the problem π and Tendency to Competence have led Dietrich and Spiekermann to prove a new Jury Theorem (Dietrich and Spiekermann 2013).

- **D&S's Jury Theorem**: Assume problem-conditional independence and tendency to competence. As the group size increases, the probability that a majority votes correctly (i) increases, and (ii) converges to a value which is less than one, unless $Pr(p^\pi > \frac{1}{2}) = 1$.

According to this jury theorem's asymptotic conclusion, group competence approaches the value corresponding to the probability that each voter faces an easy problem time half of the probability that each voter faces a neither-easy-nor-difficult problem (i.e. those problems where $Pr(R_i|\pi) = 0.5$).

This section is important as it shows one way in which competence can vary depending on the intrinsic nature of the problem. Within a specific reference class — we said — some problems are intrinsically harder than others. Problems of a similar kind thus vary in terms of their intrinsic difficulty to be dealt with successfully.