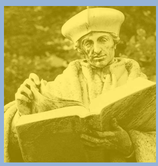


On the edge of distress

An alternative approach to distress valuation

Roman Maďar

Erasmus School of Economics

Master Thesis Economics & Business

Financial Economics
Student ID: 333163
Supervisor: Sebastian Gryglewicz
2nd Supervisor: Tiantian Huang

ERASMUS UNIVERSITEIT ROTTERDAM

August 2010

Acknowledgements

At this place I would like to thank my thesis supervisor Mr. Sebastian Gryglewicz as well as my co-supervisor Ms. Tiantian Huang for all their time and effort invested into this thesis. Furthermore, I would like to thank Mr. van der Boom, who gave me helpful insights on accounting principles and underlying relations within the company financial statements. I am also grateful to all my professors, teachers and other important people who paved my way during the completion of my studies.

Of course, my deepest gratitude belongs to my family, parents and siblings who have supported me through my entire life and made my studies abroad possible. Finally, I would like to thank my roommates as my motivating study sparing-partners and to all my friends which make my life enjoyable.

Executive Summary

This thesis investigates the problem of corporate valuation of problematic companies on the edge of the rating cliff. Initially, the thesis analyzes the distress valuation from the perspective of conventional valuation methodologies and summarizes the most frequent shortcomings mentioned in the literature. The brief literature review is followed by a development of the alternative valuation model based on the Monte Carlo simulation approach. The model is principally a risk-adjusted adaptation of the original Schwartz & Moon (2001) valuation model. Its stochastic core is based on the geometric Brownian motion and the arithmetic Ornstein-Uhlenbeck mean reverting process. In contrast to its ancestor, it forecasts the company performance based on a set of company financial statements, which are interlinked. Therefore, an overall company value is determined based on the information which fully reflects the effects of the forecasted performance. The subsequent case study illustrates the application of the model including the overall process of parameter estimation and output interpretation.

Keywords: distress, Monte Carlo simulation, risk-adjusted valuation, valuation methodologies

(JEL: C63, G17, G31, G33, G34)

Contents

List of Tables

List of Figures

1 Introduction

The detection of financial difficulties has been a popular subject of theoretical and empirical studies over fifty years due to its prime importance in different fields of economics and finance. An early bankruptcy prediction helps economists, supervisors, creditors and investors to preserve the economic value embedded in their assets. Even though the research on application of bankruptcy prediction in corporate valuation is still young, but it is emerging. Academics and practitioners realized that conventional valuation techniques designed for valuation of healthy firms tend to provide (even fundamentally) inaccurate value estimates, since they simply ignore the possibility of distress. Moreover, the recent fluctuating market conditions boost the development in this field even more.

On the other hand, distress itself, is no more perceived only as a synonym of the value destruction but more and more investors start to recognize it as a potentially highly profitable opportunity rather than difficulty. The number of wealthy individuals investing into distressed assets dramatically rose over the last period. While downturn still lasts and recovery is still highly questionable, without any doubt a list of crisis corporate victims (potential opportunities) will most likely grow further. Thus, the valuation of companies on the edge of the rating cliff remains the hot topic for skilled investors formed in vulture and equity groups.

1.1 Problem Statement

The main topic of the thesis is the analysis of issues surrounding the valuation of the problematic companies and the identification of alternatives for accurate valuation of a company on the edge of distress situation. The overall problem statement is addressed through the following set of research questions.

Research questions:

- What are the key issues of valuation under distressed conditions?
- In which way, may a substantial likelihood of default (distress) affect the value of a company?
- The Normal distribution is commonly accepted (assumed) as a reasonable predictor in many fields of finance, including the corporate valuation. Are there any consequences to normality assumptions linked with higher default probabilities?
- Academics as well as practitioners claim that under certain circumstances (e.g. higher default probabilities) conventional valuation techniques may over / understate the inherent value of the company. Are there any alternative theoretically feasible solutions to this problem?

1.2 Delimitations

The thesis is naturally limited in scale and scope only to areas and aspects which are directly related to our topic of research. However, we excluded several subparts of text during the process of writing since they significantly departed from our research topic; the reader may look for our suggestions in references and footnotes. Additionally, we would like to address one general assumption which should be taken into account, during the reading of this thesis. Alternative default probability predictions based on conventional methodologies as the Z-score, O-score and Credit Ratings, are used as the available benchmarks for validation of the model distress probabilities. Any questions about their accuracy are beyond the scope of the thesis.

1.3 Methodology summary

Our model could be tagged as an adjusted DCF valuation based on the Monte Carlo simulation. The underlying stochastic processes used within a simulation are adaptations of geometric Brownian motion and arithmetic Ornstein-Uhlenbeck mean reverting process. The stochastic outputs are subsequently transformed through the company financial statements into the forecasts and the company enterprise value is determined. The model calibration is performed with the aid of alternative default prediction techniques such as the Altman's Z-Score model, Ohlson's O-Score model and Credit ratings.

1.4 Thesis Structure

The rest of the thesis proceeds in the following order. Chapter 2 introduces a theoretical background underlying the corporate valuation and particularly the valuation on the edge of a distress. Chapter 3 describes the simulation methodology fundamentally based on the model of Schwartz & Moon, while Chapter 4 presents a step-by-step description of the model parameters estimation on a corporate case study of Norske Skog. The case study section concludes with a sensitivity analysis and calibration of the model with alternative methodologies. The final section summarizes the main findings and their interpretations, provides a clear comparison of the model advantages and disadvantages and identifies possible directions for further research.

2 Theoretical Background

This chapter presents the important theoretical background necessary for complete understanding of the valuation process and a construction of the subsequent valuation model. The chapter begins with a broader introduction of the traditional company valuation, followed by the problem of distress valuation in particular. After a brief elaboration on the main issues which commonly arise during the distress valuation, some of the alternative valuation techniques are presented. The theoretical chapter concludes with a sample of techniques used particularly for the estimation of distress probabilities. The chapter is inspired by a broad academic literature, for explicit sources please check the footnotes and references within the text.

2.1 Introduction to corporate valuation

The proper value determination of a company presents a complex practical problem of enormous importance in corporate business. However, the term valuation is commonly associated as a process of value determination for purpose of trading; there is a multitude of other important business activities which may be matched as objective of valuation. Since, the valuation may be besides obvious trading purposes used as a strategic managerial tool for identification of main company value drivers, assisting to decision making processes (R&D, acquisition) or even as supporting evidence during fundraising (IPO, company listing etc.). Thus, the valuation would be perceived in a broader perspective as a strategic guideline for the financial analysts, investors, consultants and managers.

Recalling back the initial trading purpose of valuation, it is desirable to emphasize that even though the valuation analysis can provide a solid starting point for business deals; the final price is formed over the negotiations and critically depends on actual market conditions, timing, subjective motivations and alternative options of both bargaining parties. Following, Arzac's notion of value with regard to company valuation: *"…the term value determines the price investors would be willing to pay in a competitive capital market (Arzac, 2005, pp.9)."* One may point out that any valuation is subjective and it is not possible to point out a single absolutely correct value. Fortunately, there are several different valuation methodologies available, which are tagged as 'fundamentally correct' and accepted by the majority of practitioners operating on a field of corporate finance. However, no one can state that any of them are absolutely fundamentally and practically superior to all others. Therefore, particular method selection essentially differs from case-to-case and depends on the specific circumstances, such as data availability and reliability[1], time constrains, valuation purpose and of course on personal preferences (experiences) of the person performing the valuation. Following *Table 2-1* presents the overview of conventional valuation techniques, commonly used by practitioners for valuation of a stand-alone company or business unit.

Table 2-1: Conventional valuation methodologies[2]

Step	DCF	APV	Multiples
I	Study the Target and Determine Key Performance Drivers	Study the Target and Determine Key Performance Drivers	Select the Universe of Comparable Companies
II	Project Free Cash Flow Path	Project Free Cash Flow Path	Locate the Necessary Financial Information
III	Calculate Weighted Average Cost of Capital	Calculate Unlevered Cost of Capital	Spread Key Statistics, Ratios and Trading Multiple
IV	Determine Terminal Value (based on perpetuity)	Determine the Unlevered Value and the Value of Tax benefits	Benchmark the Comparable Companies
V	Calculate Present Value and Determine Valuation	Calculate Present Value and Determine Valuation	Determine Valuation

Source: *Arzac (2009), Pearl & Rosenbaum (2009) and own compilation*

Discounted cash flow methodology (DCF) is the most common methodology of company valuation broadly used by practitioners (like investment bankers, corporate officers, private investors etc.) and academics. The cornerstone of DCF methodology lies in the approach that intrinsic value of a company can be obtained through the present value of its projected expected free cash flows (FCF). The FCFs are derived based on a set of assumptions and judgments about individual FCF components (such as revenues, COGS, CAPEX, etc.) with regard to the past company performance and business plan (Pearl & Rosenbaum, 2009). After projection period, the year-to-year projection is replaced by one (perpetuity-based) estimation due to inherent forecasting limitations. Eventually, the FCFs and perpetuity value are discounted accordingly to company specified weighted average cost of capital (WACC) and company enterprise value is determined.

[1] With the word expression 'data reliability' author refers to a situation when a company undergoes significant structural changes therefore the past company track and performance cannot provide adequate information for any analysis.

[2] For more comprehensive definition of presented methodologies please check the *Appendix – Theory*.

Adjusted present value approach (APV) became popular among practitioners due to its 'relative superiority' over DCF. While within the basic DCF valuation framework it is assumed that a leverage ratio stays constant over the whole projection period, reality shown that this assumption is frequently violated.[3] Thus, APV was designed in order to accurately capture the company value even under excessive fluctuations in capital structure. According to Koller et al. (2005) the APV model separates the calculation of enterprise value into two components: (1) into the value of operations as if the company was all-equity financed and (2) the value of tax benefits that arise from debt financing. Thanks to this unique insight, APV equips the analysts with necessary modeling flexibility in order to facilitate an adequate valuation of complex financial maneuvers.

Comparable multiples analysis (MA) closing the portfolio of traditional methodologies used for valuing companies. Fundamental idea behind this simple approach is that the assets with similar characteristics should be traded on the market at similar prices. Thus, as it was shown within a brief summary above, enterprise value under MA is extrapolated according to industry benchmark metrics - multiples.[4] However, this methodology is the most simplifying valuation technique presented; it can provide very useful insights into the industry trends and support other valuation methodologies. By the words of Arzac (2005, pp.63): *"Although valuation based upon multiples is only indicative and not a substitute for a careful projection and valuation of free cash flows, it has a role as a complement and check of DCF valuation."*

Despite the fact that various methodologies were formulated in order to help the analysts tackle with company valuation, as it was previously presented, the real crux of valuation lies in the uncertainties embedded in the future company prospects. This problem stems from the fact that valuation techniques are critically dependent on the underlying assumptions and judgments, so their accurate value determination may become questionable with increasing uncertainties.

"While valuation has always involved a great deal of 'art' in addition to time-tested 'science', the artistry is perpetually evolving in accordance with market developments and conditions."

Pearl & Rosenbaum (2009)

[3] In reality, companies often tend to adapt their capital structures and depart from their target debt levels in order to provide a fundraising for the upcoming large investment outlay (like R&D projects, acquisitions, etc.) or simply because of deterioration in cash flow generation (Arzac, 2005, pp.89).

[4] For concrete examples of multiple metrics, please check the *Appendix – Theory*.

2.2 Problematic of distress valuation

Following section introduces to the reader the controversy of distress valuation and presents the most obvious underpinnings of the conventional valuation methodologies employed in a case of distress (or close to distress). Moreover, the current section also presents the standpoint that distress may be perceived not only as a synonym of difficulties but also as an opportunity in the phenomenon of so-called vulture investment. The section is mainly inspired by Altman & Hotchkiss (2006), Damodaran (2002) and Moyer (2005).

2.2.1 Shortcomings of traditional valuation approaches

Although, a company valuation as it was briefly described above seems quite simple, or at least not difficult, the close sentence in the previous section pointed out the obvious limitations of the traditional valuation frameworks. The valuation techniques work well in the ordinary situations, but if considerable uncertainties take place, assumptions about the future prospects start to depart from reality, which may significantly affect the quality of valuation. Of course, the uncertainties are present in all valuations, but their magnitudes are important. There are two common problematic examples frequently mentioned in literature, the valuations of young start-up companies and the valuations of mature declining companies. Both groups share some similar characteristics like questionable growth potential, declining or even negative margins and extensive debt burden as a result of substantial financial leverage. Under these circumstances the valuation turns out to be extremely challenging, because the probability of distress used to be considerable.

Following ordinary DCF framework, an analyst has to project the representative cash flow path (or paths) which should reflect the potential prospects of the company. But the challenging task is: how to incorporate the probability that a company may fall into distress? It is desirable to emphasize that under conventional valuations, a company is valuated as a *going concern*; in other words it is assumed that all projected FCFs will materialize. Moreover, a perpetuity approach used in order to determine the company termination value implicitly assumes that a company will last *infinitely*. Without any doubt, there is a substantial probability that both assumptions may be violated in a case of a company which is on the edge of distress. More generally it can be said that conventional valuation techniques commonly fail to quasi-accurately assess the enterprise value of a company when there is a considerable likelihood that a company will default in the close future[5]. Damodaran (2009, pp.6) identified the following key shortcomings of the conventional valuation framework:

- The pre-projected cash flow paths do not take into account a possibility that some cash flows may not materialized at all.
- The substantial part of enterprise value comes from terminal value, which is based on questionable infinite company growth and existence.
- Discount factors usually tend to be biased in the case of distress, either upwards or downwards, according to the estimation technique used.
- Within the valuation framework the access to capital is constantly unconstrained, i.e. market liquidity cannot dry up, whereas in reality this cannot hold.

To sum up all the remarks, distress within the traditional DCF framework does not affect the market value of company assets in place likewise the value of expected cash flows. However, the general experience has shown that, the value of a distressed company is just a proportion of its healthy counterpart of the same size and within the same industry.[6] In a real case the decline in value of distressed company flows through several 'channels' simultaneously.

The first evident value loss usually reflects the fact that the distressed company cannot sell any future potential. Thus, the substantial part of enterprise value representing the future cash flow generation instantly disappears in a moment when a company becomes distressed. Another and the most tangible value loss occurs in the resale value of the assets in place as a result of time constraints on urgent asset liquidation in order to acquire funds for the claim-holders. The urgency puts the distressed company into unfavorable subordinate position which causes a loss of bargaining power and consequently the resale value of the assets is usually just a proportion of their real (market) value. Furthermore, the resale counterparties often negotiate additional value discounts as a compensation for the future costs associated with the legal issues and agreement process with present claim holders. Eventually the enterprise value of distressed company really counts for only a fraction of the healthy counterpart. Heretofore, the topic of distress valuation was discussed without any underlying motivation so the following question should arise. Why would companies on the edge of distress be worth the effort of valuation?

[5] Analyst should take into account a case that a company may default over projection period as well as over perpetuity period. Since under both mentioned scenarios, the enterprise value calculated in traditional way is likely to be biased.

[6] For example Andrade & Kaplan (1998) estimated that distress creates instant 10-20% adverse shock on firm value. While, Altman (1984) found that cumulative shortfall in three years before bankruptcy may reach even 25% of the initial stock value.

2.2.2 Introduction distress valuation and investment

At first glance, investing capital into a company which may potentially defaults in a close future looks contra-intuitive or even irrational; nevertheless some investors seek for such opportunities and even generate excess profits on it. What is their secret formula? Essentially, they approach to analysis of a distressed company from a clearly economical perspective. They look for the reasons underlying a distress and judge a company in a sense of value creation.

It is obvious that the companies fall into distress for different reasons. Some of them fall into difficulties due to the fact that they are value consuming. These companies are, of course, worthless and default is desirable. On the other hand, there are also many distressed companies which are capable to create value but unfortunately, due to e.g. poor managerial decisions[7] made in the past they are unable to sustain current operational expenses or cover obligations arising from debt repayments. According to the Crystal & Mokal (2006, pp.2) there are basically two types of distressed companies, the companies which are (1) economically distressed and the companies which are only (2) financially distressed. The simple way how to recognize the type of distressed company is (according them) an answer on the question: Is the business as a going concern worth more, or less than re-sale (liquidation) market value of assets in place? If a company turns out to be economically distressed, business is not viable anymore and operational expenses are higher than sales, then liquidation is the most rational move. On the other hand, a company might be only financially distressed and that may indicate profitable opportunity for a skillful investor.

However, this particular type of investment is often tagged with hostile title 'vulture investments' and investors operating in this field are usually described as corporate raiders ('vultures' or hyenas), empirical studies shown (e.g. Hotchkiss & Mooradian, 1996) that presence of vulture agents in a company have predominantly positive influence on a distressed company. Vulture investors usually enter a distressed company by purchasing a significant proportion of company debt claims. In this way vultures acquire a strong bargaining position within a company, because as a major creditor they may force equity holders to swap distressed debt into equity stake. Consequently, as an equity holder they may actively participate on management turnover and restructuring and it is not rare that a vulture investor becomes member of the company board or even CEO. Eventually, when the company successfully rehabilitates from the severe distress conditions vulture investors find a suitable strategic investor for the healthy company and exit it with a profit.[8] However, this particular type of strategy requires a substantial active participation, there

[7] As examples could be mentioned the failure of important investment project or a substantial leverage acquired in the wealthier phase of business but recently overwhelming, etc.

[8] While equity stake in healthy company has significantly higher value than initially acquired distressed debt.

are also other vulture strategies focused on less active approaches. Altman & Hotchkiss (2005) divided vulture strategies into three main classes with own characteristics as following *Table 2-2* shows:

Table 2-2: Investment strategies in Distress investment

Characteristic	Active Control	Active / non-Control	Passive
Initial investment	Requires substantial ownership of debt (30-50%)	Senior secured and senior unsecured debt claims	Investment in undervalued distressed bonds
Entering strategy	Take control of company through debt/equity swap	Active participation during restructuring	Buy low (50-60% discount), Sell high (healthy company)
Control	Active participation, seat in board and management	Active participation, seat in board	No active participation in board and management
Strategy	Additional equity infusion, Rehabilitate company, Exit	Company turnaround, Exit	Trading / speculative oriented (sometimes restricted)
Holding period	2 - 3years	1 - 2years	6 -12months
Targets	Large or mid-cap companies	Large or mid-cap companies	Not specified
Target return	20 - 25%	15 - 20%	12 - 20%

Source: *Altman & Hotchkiss (2005, pp.189)*

Following this description the distress investment strategy may look more like a subpart of private equity business. Indeed private equity and distress investment businesses share some common traits, like active control strategy, substantial debt involvement, short holding period and high expected returns. On the other hand, there is at least one distinguishing trait; the vulture investors usually enter a company as the debt-holders while private equity groups primary focus on buying company equities. Anyway, investment in distressed companies either as vulture or private equity investor requires superior proactive investment skills in order to turn out risky distress involvement into a rewarding profit. In this perspective the distress can be perceived more like a potentially highly profitable opportunity rather than difficulty, however very challenging. Besides the identification of such opportunity and proper control capabilities during turnaround process, investor has to be able accurately estimate the 'true' company value, which may be extraordinary difficult as it was shown above. Thus, following section elaborates more on the valuation under the distress conditions.

"While the valuation of businesses is a difficult task at the best of times, particular complexities arise when the business to be valued is in distress."

Crystal & Mokal (2006)

2.3 Alternative approaches to distress valuation

The previous section points out the fact that a distress may present an extraordinary investment opportunity, it likewise suggests enough arguments that the conventional DCF framework tends to fail in accurate estimation of the enterprise value in such a case. On the other hand, the number of successful deals signed under distress circumstances is evidence[9] that investors found the solution also for this problem. Let's take a closer look on several alternative methodologies used by practitioners, which are frequently only modifications of their conventional ancestors. The current section is based mainly on Arzac (2005), Damodaran (2009) and Smit & Trigeorgis (2004).

2.3.1 Modified DCF approach

The cornerstone of the DCF valuation framework was briefly described in *Subsection 2.1*. Let's make a short recap. The valuation process under DCF approach proceeds within five steps. Starting with projection of FCFs, through determination of the terminal value and concluding by discounting of all expected value determinants at company WACC. Even though that the previous section has clearly shown the obvious shortcomings of this framework, over the course of time academics and practitioners suggested some adjustments which may significantly improve the ability of the DCF to quasi-accurately determine the market value of a company on the edge of distress. Under Modified DCF approach, as it is commonly called, the adjustments to the distress situation are primarily concentrated within the FCF estimation and determination of discount factors. Damodaran (2009) pointed out that the appropriate way of incorporating distress into the FCF projections would be a scenario analysis ranging from the most optimistic to the most pessimistic scenario in combination with period and scenario specified probabilities of distress. Thus, according to this adjusted framework the scenario projection of FCFs can be expressed as follows:

$$FCFs = \sum_{j=1}^{j=n} \pi_{j,t}(Cash\ flow_{j,t}) \qquad (2.1)$$

Where $\pi_{j,t}$ is the cumulative probability under scenario *j* that company will 'survive' a period t, while $Cash\ flow_{j,t}$ is the cash flow generated over a particular period and scenario. The application of distress probabilities is not a brand new idea. Nevertheless, the key innovative improvement of this approach lays in the fact, that instead of one unifying probability figure which represents the

[9] According to Altman & Hotchkiss (2005, pp. 183) the size of market with distressed securities grew five times over the last decade.

possibility of distress, a range of probabilities is used.[10] Moreover, the cumulative probability approach is employed in order to make the valuation process more authentic to the real situation. The following equation presents the way how the cumulative 'survival' probabilities are determined:

$$\pi_{j,t} = \prod_{n=1}^{n=t}(1 - \pi_{distress,j,n}) \quad (2.2)$$

While $\pi_{distress,j,t}$ is the probability that a company falls into the distressed under scenario *j* during a period *t*. Furthermore, Damodaran (2002, pp.10) suggested that the common approaches of *β-eta* estimation for companies with considerable likelihood of distress tend to be biased. Thus, he recommends using the bottom-up unlevered betas for the cost of equity and default spread premium over the risk free rate as a more accurate proxy for the cost of debt, instead of regression betas and synthetic rates.[11]

The most evident drawbacks of this approach are difficulties which arise during an estimation of the numerous probabilities for every period and scenario. In addition, the problems may appear also during the cost of capital determination due to the blurred target debt ratios and the possible recapitalization efforts. Either way, the Modified DCF approach can provide a significantly more reliable valuation of distressed company than its conventional version.

2.3.2 'Going concern' valuation with distress adjustments

The probability estimation difficulties of the Modified DCF approach usually deters analysts from valuation in this manner. Despite this fact, many of them realized that the Modified DCF concept may be under some simplifying assumption transformed into a less challenging model. The basic idea roots essentially from the fact that a difficulty level is proportional to the number of scenarios incorporated within a valuation. Thus, the common practice is to separate a valuation into two representative scenarios and valuate a company as a 'going concern' with a distress adjustment. As a result the number of probabilities which has to be estimated instantly slumps. Under this valuation concept the enterprise value (EV) can be determined in a following manner:

$$EV = Going\ concern\ value * (1 - \pi_{distress}) + Distress\ sale\ value \bullet \pi_{distress} \quad (2.3)$$

[10] Under modified DCF approach, analysts have to estimate specified default probabilities for every scenario and also every period.
[11] For detailed derivation of Cost of Capital please check Damodaran (2009, pp. 37).

Where $\pi_{distress}$ is the cumulative probability over the valuation period, $Going\ concern\ value$ represents the scenario when company 'survives' the estimation period, since the negative effect of potential distress is embedded within $Distress\ sale\ value$.

The major drawbacks of this approach are obvious simplifications as a trade off to the reduction of estimation complexity. One may wonder, why would be desirable to reduce the key advantage[12] of Modified DCF over traditional DCF. Nevertheless, the proposed adjustment can reduce the valuation time significantly, while the accuracy remains superior to traditional DCF. Although, the variable reduction eliminates some difficulties, the question of proper estimation of the distress probabilities as well as the distress sale value still remains. Both problems are further discussed in *Sections 2.4* and *3.5* for distress probabilities and distress sale value, respectively.

2.3.3 APV with special focus on the leverage effects

The linkage between capital structure and company value is a popular subject of corporate research over half of a century. The numerous research papers and studies which elaborated on this particular topic are evidence that capital structure plays an important role in company value determination. A cross-sectional view over the past research shows that paper of Modigliani & Miller (1958) and subsequent correction (1963) laid the foundations of capital structure theory. Thenceforward, research in capital structure has emerged in various ways, ranging from empirical studies to further theoretical adaptations.[13]

Among the traditional valuation approaches mentioned in *Subsection 2.1* only the APV methodology takes into account the effects of capital structure explicitly through the calculation of tax shields. However, the conventional APV approach embraces the leverage effects especially in a positive way, the past experience has shown that leverage may also introduce adverse effects. The Arzac (2005, pp.89) recognized two important consequences of increasing financial leverage: (1) a value enhancing effect in a form of tax shields partially offset by (2) the increase in the cost of equity because leverage magnifies a systematic risk. Particularly, latter effect becomes an issue for the companies which future prospects are blurred with uncertainty. Although, the conventional APV approach works well in ordinary situations, for a company on the edge of distress a calculus should reflect the overall effect of leverage and also take into account the adverse effects of financial

[12] The key difference between traditional DCF and Modified DCF approach is the number of scenarios incorporated into valuation process. Under Modified DCF approach the enterprise value is determined over numerous scenarios, while conventional DCF usually take into account one (or just a few) scenario. Thus, the number of scenarios can be perceived as the key advantage, since it improves the overall valuation accuracy.

[13] For instance studies of Weston (1963), Altman (1984), and Chen et al. (1995) may be mentioned. For further references concerning topics related to capital structure studies please check Altman & Hotchkiss (2006).

leverage in particular. Thus, under mentioned proposition the original APV could be rearranged in order to improve its accuracy in distress as follows:

$$EV = V_{unlevered} + V_{Leverage\ effects}$$
$$V_{Leverage\ effects} = PV(Tax\ Benefits) - Financial\ Distress\ Costs \quad (2.4)$$

Where, the first element of the leverage effect equation represents the value enhancing effects, namely the aggregate present value of tax deductions. While, the second element covers all adverse effects especially the costs linked with corporate distress. However, the idea of tax shields as deductions of interest costs from profits seems clear; the universal quantification formula for tax shields remains ambiguous. Nevertheless, further discussion on the topic of tax shields would be beyond the scope of this thesis, a reader may check several alternative references attached in the footnote.[14]

On the other hand, the classification within the costs of financial distress onto direct costs and indirect costs is generally accepted. Direct costs are defined as all immediate out-of-pocket expenses as fees for lawyers, accountants, restructuring advisers, turnaround specialists, expert witnesses and other professionals. Indirect costs include mainly unobservable opportunity costs as loss of potential sales and profits, loss in market share, increased overheads as a result of higher debt costs, more rigorous terms with suppliers and loss of key employees (Altman & Hotchkiss, 2006, pp.93). Generally, the financial distress costs could be determined as follows:

$$Financial\ Distress\ Costs = Distress\ Probability \bullet Distress\ Costs$$
$$Distress\ Costs = Direct\ costs + Indirect\ costs \quad (2.5)$$

Likewise in previous approaches the problem associated with estimation of distress probabilities arises also in the recent one. Moreover, at least one anomaly which may occur during a valuation of distressed company under APV approach should be mentioned and emphasized. In general, a stream of negative earnings is not unusual trait of distressed companies; ergo this fact may not be neglected while tax benefits are calculated.[15] Essentially, practitioners agree that the APV

[14] The studies of Modigliani & Miller (1958, 1963) provide starting point in topic of tax shields. Alternatively, may be mentioned the studies of Miles & Ezzell (1985), DeAngelo & Masulis (1980) and the study of Kemsly & Nissim (2002), where authors elaborated the complete overview of empirical studies dedicated to the topics concerning a capital structure and tax shields in particular.

[15] Negative earnings fundamentally indicate also no taxes expenses over operations and therefore no tax deduction. In a result a company cannot count with any leverage benefits at all.

methodology is suitable approach for the valuations of companies which are financially distressed as a result of over-leverage, and not for operationally distressed companies with unsustainable operational processes.

2.3.4 Real Option approach

Heretofore, the presented alternative methodologies were only various modifications of the conventional frameworks with special attention to distress. On the other hand, the valuation approach proposed by Fisher Black and Myron Scholes in 1973 and subsequently expanded by Robert C. Merton in 1974 formed the unique stream in corporate valuation.

The fundamental idea underlying the option-pricing approach, as it is commonly called, stems from the notion that the corporate liabilities can be viewed as a combination of corporate options. While following Black and Scholes (1973) option is defined as follows: *"An option is a security giving the right to buy or sell asset, subject to certain conditions within a specified period of time".*[16] Moreover, Black and Scholes (BS) introduced a model[17] specified for valuing the European options on non-dividend paying stock defined over a following set of 'ideal conditions':

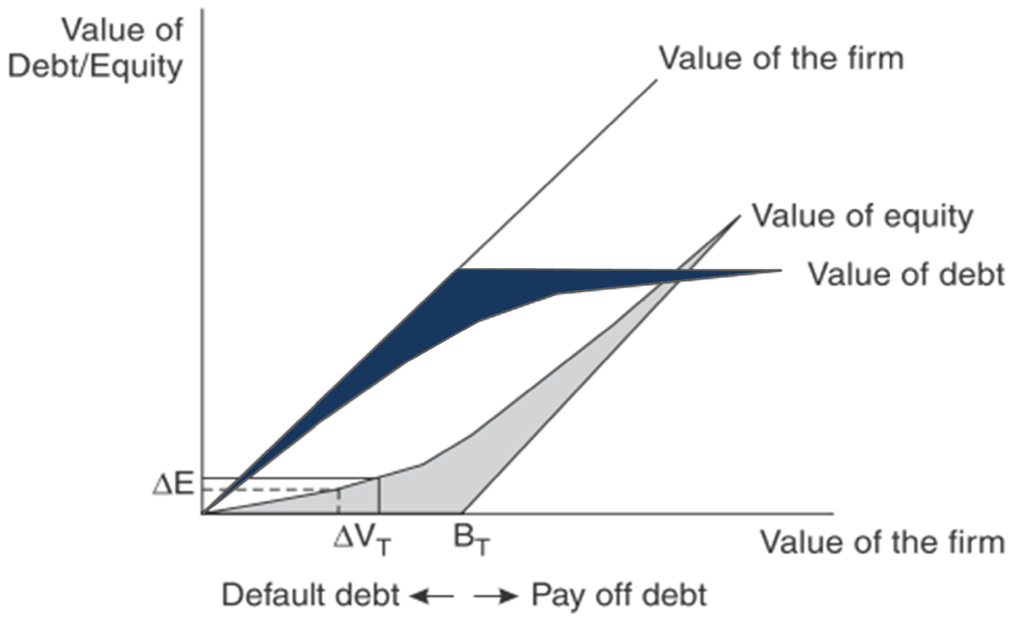
a) The short-term interest rate is known and is constant over the time.
b) The stock price follows a random walk in continuous time with a variance rate proportional to the square of the stock price. The distribution of possible stock prices is lognormal and the variance rate of the return on the stock is constant.
c) The stock price pays no dividends.
d) The option is of a "European" style – it can be exercised only at the maturity.
e) There are no transaction costs in buying or selling the stock or the option.
f) It is possible to borrow any fraction of the price of a security to buy it or to hold it, at a short-term interest rate.
g) There are no penalties to short selling.

Afterwards, Robert C. Merton improved the original BS model by incorporating also the effect of dividend payments. The initial breakthrough was suddenly followed by further research and formed independent valuation stream known as real options.

[16] Furthermore, according to the exercise conditions options may be assigned as European and American type of option. Since, the former one can be exercised only on a pre-specified date, and the latter one can be exercised at any time up to the maturity date.
[17] For a more detailed description of Black & Scholes model please check *Appendix – Theory*.

The underlying insight of the real options is an idea that holding a firm's equity is equivalent to holding a call option on the firm's assets with an exercise price equals to the value of the company's debt; alternatively, holding a risky-company debt is equivalent to holding a risk-free debt and writing a put option on the company's assets, also known as a 'equity-call' model (Diba et. al, 1995). Consequently, the fact that any option cannot be worth less than zero equity-call model authentically imitate the limited liability of equity-holders in reality. One of the main advantages under the option valuation methodology is the risk neutrality of valuation. In a result, valuation is absolutely independent of the investors' risk preferences. This specific feature of the real options based on possibility of risk-free replication[18], eliminates all difficulties with estimation of discount factors. The following figure illustrates the corporate liabilities as options and their payoff structure for both equity-holders as well as debt-holders.

Figure 2.1: Corporate securities as options on the firm's value



Source: *Smit & Trigeorgis (2004, pp. 101)*

The payoff scenarios can be from the model interpreted as follows. On the debt maturity if the value of the firm (V) exceeds the face value of the debt (B), V ≥ B, the shareholders will repay the debt in full (D = B). However if the value of the firm turns out to be lower than the face value of the debt (V < B), the shareholders are only liable for the funds invested in the firm and would exercise

[18] Replication idea enabling option pricing is that one can construct a portfolio consisting of particular number of shares and borrowing against them an appropriate amount of risk free rate. This portfolio would exactly replicate option payoff in any state of nature, thus value of any option may be determined in a similar replication manner (Smit & Trigeorgis, 2004 pp.156).

their option to default (Smit & Trigeorgis, 2004, pp. 100). Thus the payoff structure at maturity can be mathematically expressed as below:

$$Equity\ value = Max[debt\ repayed(\boldsymbol{V} - \boldsymbol{B}), company\ default(\boldsymbol{0})]$$
$$Debt\ value = Min[company\ default\ value(\boldsymbol{V}), debt\ principal(\boldsymbol{B})] \quad (2.6)$$

Although, the real option approach possesses many advantages, mainly the possibility to incorporate the managerial flexibility in a form of various options[19] into the process of valuation, the major limitations arise in valuation of private companies. In addition, the strict maturity determination (European option) can be perceived as another obvious disadvantage. Therefore, several alternative approaches were developed in order to solve this limitation, which can be classified as: analytical, binomial, numerical and simulation models. The further discussion about these models is beyond the scope of this thesis but the reader may check the footnote for specific literature references.[20]

2.3.5 Simulations

All previous models may be tagged as deterministic, because all of them use several predetermined 'ordinary' variables which reflect the analyst's expectations about the company prospects. Generally, it could be said that these variables form a prospect path (or paths) which determines the outcome of valuation – the enterprise value. However, this approach is sufficiently accurate in ordinary valuation cases; the obvious path dependency may introduce the implausible limitations[21] into the valuation when deterministic variables are embraced by high uncertainties (like in a distress situation). An alternative solution to these deterministic point-estimate approaches can be the statistical simulation, under which probability distributions are introduced in order to reflect the uncertainty embedded in variables.

In practice, an analyst converts the deterministic model into a stochastic model and by application of the Monte Carlo (MC) approach determines the enterprise value of the company. The

[19] Smit & Trigeorgis (2004) refers to following types of real options: Option to defer, Option to expand or contract, Option to abandon, Switching option and Compound option.

[20] Alternative model references: binomial approach was originally proposed by Cox, Ross & Rubinstein (1979), in a case of numerical approach reader is referred to 'Greeks' approximation models (e.g. Gamma, Vega, Theta and Rho), Arzac's approach (2008, pp. 109-111) may be mentioned as analytical modifications of the original B&S, while simulation models are represented mainly by Monte Carlo approach.

[21] Under word *'limitation'* author in this case means the numerous scenarios which have to be considered under traditional *DCF* framework in order to adequately capture the uncertainties in company prospects.

MC simulations are traditionally performed with the aid of AI, so it is reasonable that the valuation under this approach is often defined in the quasi-algorithmic way as follows:

The valuation under MC approach may be structured into three main phases: *Phase 1 – Model Creation, Phase 2 – Simulation and Phase 3 – Outcome Interpretation*. Moreover, every phase is further segmented to one or more steps, which proceed in exact sequence.

Phase 1 – Model Creation

Step 1: The valuation under MC starts with a proper analysis of the company and industry as it is common for most of the valuation techniques. An analyst should review the past performance as well as future prospect of the company and industry, the key aim of this analysis is the identification of the company specified dynamics and drivers. In this way, an analyst reveals the uncertainties embedded within the company dynamics and selects which of them will be transformed into a stochastic form. At this stage, it is important to keep in mind that the number of variables in the stochastic form also implies a level of difficulty and computation time per iteration. Thus, only the most critical variables should be transformed into the stochastic ones (e.g. Revenues, COGS, SG&A, etc).

Step 2: In the second step, the analyst determines the probability distributions or stochastic processes which may represent the dynamics within pre-selected variables. However, numerous probability distributions exist not all are suitable for this purpose. In general, the distribution selection should aim on 'statistical' fit of distribution with a particular variable dynamics. Of course, more factors play a role during this distribution-determination stage[22] , but most of distributions can be tailored with the aid of mathematics. So, the most important characteristic of distribution is its tractability. Over the course of time, some distributions and processes have become popular among practitioners using MC approach; the following *Table* 2-3 presents examples of both.

Table 2-3: Sample of probability distributions and stochastic processes

Probability distributions	Stochastic processes
• *Normal distribution*	• *Geometric Brownian Motion (with drift)*
• *Exponential distribution*	• *Mean-reversion process (with jumps)*
• *Geometric distribution*	• *Two / Three factor model*
• *Lognormal distribution*	• *Markov chain process*

Source: *Various mathematic and finance literature*

[22] For instance, the range of feasible outcomes can be mentioned (Damodaran 2002, pp. 7).

Step 3: Up to this stage, the analyst identified key sources of uncertainty in the company business dynamics, determined the most critical variables and their stochastic forms. Now, it is desirable to estimate the input parameters for all distributions and processes incorporated within the valuation model. Parameters are usually determined from historical data with regard to the expected prospects. This estimation must be performed with the rigour and prudence because as Damodaran (2002, pp.9) stated, when simulation inputs are made carelessly or randomly, the output from the simulation may look impressive but actually conveys no valuable information. This is also the stage when the company default and survival conditions are specified in a model environment. Moreover, due to the fact that all calculations and simulations are performed over the AI, the strict valuation procedures have to be defined for a survival case as well as for a default case.

Phase 2 – Simulation

Step 4: Over the first three steps, the valuation model is constructed so meanwhile everything concerning the model construction should be ready for the MC simulation. Within this stage of simulation, the values of stochastic variables are randomly drawn from the corresponding probability distributions for all pre-determined variables from the initial analysis. In some cases it may be desirable to introduce also correlations into the variable drawing in order to mimic the interconnections within the variable dynamics.[23]

Step 5: Next, previously drawn variables are filled into the model and transformed into the corresponding accounting measures and financial ratios, which form a particular company scenario. Thereafter, scenario is classified as a default case or a survival case and the enterprise value is calculated according to the scenario-specified valuation procedure.

Step 6: Then, scenario outcome – the enterprise value is recorded as well as the scenario characteristics[24] under which the valuation was performed and simulation continues back to ***Step 4***. In this way a computer repeats the previous two steps a sufficient number of times, while the number of iterations unusually dip under ten thousand, and gathers a vast sample of potential realizations.

[23] For example, in reality the evident relationships between Revenues and COGS exist, although, the magnitude of such a link is likely to be company specific. Thus, these cross-links should be revealed by appropriate analysis and incorporated into the model.

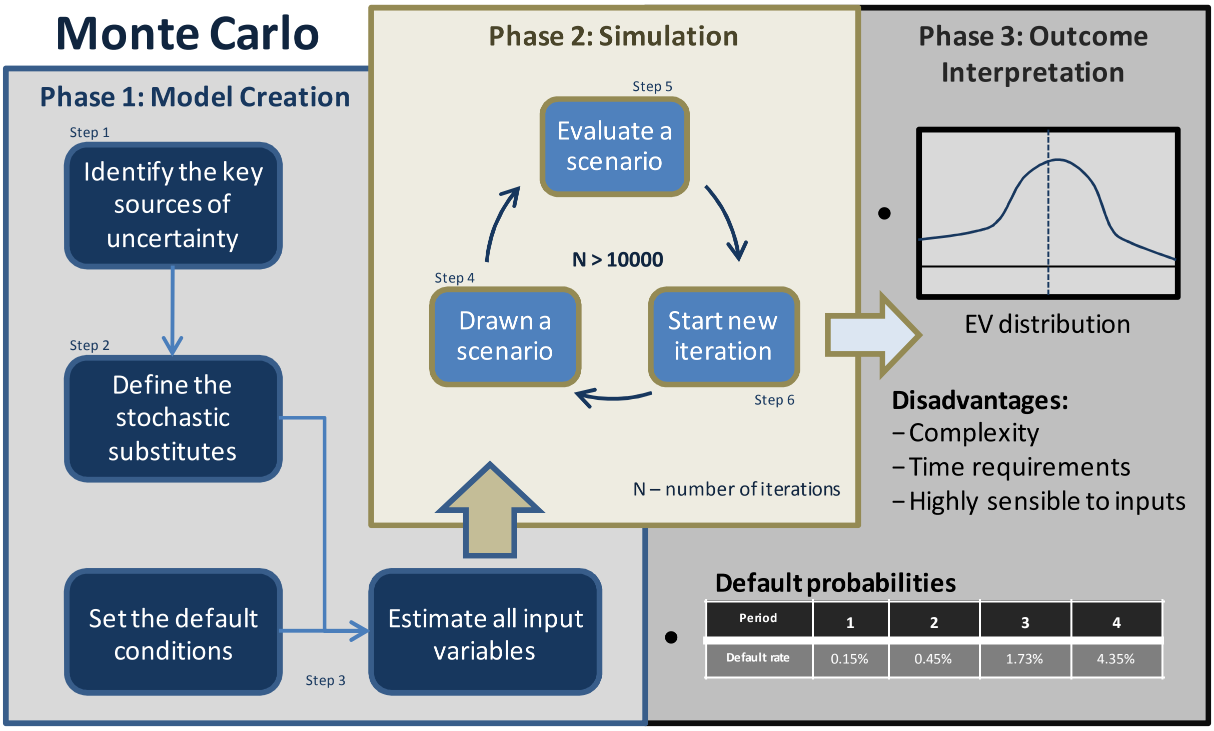
[24] By scenario characteristics author means a scenario classification (survival or default) and in case of default also the type of default.

Phase 3 – Outcome Interpretation

Step 7: Eventually, when the number of iterations reaches some desirable level, the recorded results may be interpreted. Thanks to the vast number of realizations it is possible to create an entire distribution of potential company enterprise value. In addition, the recorded scenario characteristics may also provide a useful company statistics like period-specified default rates and based on prevailing type of default also reveal the company weakness.[25]

The overall process of valuation under the simulation (Monte Carlo) methodology is illustrated by the following scheme:

Figure 2.2: Valuation process under Monte Carlo approach



***Source**: Own compilation*

Despite the fact that the MC simulation method may provide superior outcomes in comparison with one representative figure typical for the conventional methodologies; it is less popular among practitioners. Generally, the simulation methodologies are more complex and require far more background knowledge than other methodologies. Moreover, as it was also stated before, the merit of simulation critically depends on the quality of the model and estimations made over the course of

[25] At least analyst may identify to which type of weakness (operational or financial) is a company more vulnerable.

model construction. The last frequently mentioned drawback of simulation method is time consumption, while a simulation of more complex models may require even several hours.

The recent section has shown that even in a distress situation the quasi-accurate estimation is possible and practitioners over the course of time developed numerous methodologies for this purpose. Unfortunately, the methodology improvements also introduced new challenges for analysts. For example the proper estimation of distress probabilities may be mentioned. Thus, it would be also desirable to introduce the reader more into this problem.

"Since the intrinsic or true value of the firm is unobservable, we must rely on various methodologies that have been accepted as useful approaches to estimating value."

Altman & Hotchkiss (2006)

2.4 Overview of methodologies for estimating distress probability

Among the academics and practitioners active in the field of corporate finance the assessing of default – distress probabilities (DP) [26] became an important issue. A motivation to estimate the exact probability that a company is going to default in upcoming period was clearly visible in previous section. Therefore, the following section presents an overview of the methodologies developed over the time in order to measure the default (distress) probability of a particular company based on its present and past performance. However, the section itself should only introduce a reader into the topic of default prediction, not explain the methodologies in full details. The particular methodologies used as calibration benchmarks in the valuation model are explicitly specified within a case study part. Literature resources of this section cover numerous articles published on a topic of default prediction, especially Altman (1968, 2000), Ohlson (1980), Crosbie & Bohn (2003) and Falkenstein et al. (2000).

2.4.1 The origin of default prediction

William H. Beaver with his study (1966) is recognized as a first pioneer with significant results in a field of default prediction.[27] His effort in identification of the best single predictor of default initiates subsequent wave of research in this topic. Many different approaches were formulated over the course of time and a literature suggests following classification for DP measures:

- Accounting-based DP measures
- Market – based DP measures
- Hybrid DP measures

Since early 30's, researchers have found an apparent linkage between accounting (also financial) measures and likelihood of bankruptcy. Consequently, the first type of DP measures was established based on the notion that selected accounting measures may be used as the proxies for the probability of default. Among the most recognizable DP measures in this group belong Altman's Z-Score model (1968) and Ohlson's O-Score model (1980). With the breakthrough of Black & Scholes option-pricing model in 1973 a new alternative way of DP prediction emerged. People realized that the stock market provides an alternative and potentially superior source of information regarding DP, because it aggregates information from the financial statements as well as from other sources; and

[26] While it is necessary to fundamentally distinguish between the terms distress and default, the default probabilities assessed by default prediction methodologies are commonly used as reliable proxies also for distress probabilities. Thus, terms distress - default probabilities may be used interchangeably.
[27] Literature (e.g. Sobehart & Stein, 2000, pp.14) shows that perhaps, Fitzpatrick (1931) may be the first researchers conducting the research in this field, but prediction power of his model was not significant.

option-pricing model provided a natural way how to extract this information. (Hillegeist et al., 2003, pp.2). Black-Scholes-Merton model for DP prediction (BSM) was formulated based on this approach and subsequently (gradually[28]) extended into Kealhofer-Merton-Vasicek model (KMV). The last hybrid class of DP measures intuitively comprises the combination of both market information as well as accounting information. Underlying idea to this approach is that the combination of both information sources may improve the predictive power and robustness of DP measures. For example the Moody's RiskCalc could be mentioned as a representative DP measure founded on a hybrid model idea. The section continues with a brief description of several previously mentioned measures, enhanced with extensive comparative discussion.

2.4.2 Z-Score & ZETA Score models

Initial Beaver's idea of single best default predictor defined over the accounting and financial measures laid the foundations for further research in a topic of default prediction. Beaver in his study analyzed the sample of 30 financial ratios and assessed their default prediction power. The results of his study indicated that not all ratios have the same prediction power, but the most powerful predictor according to this study is Cash flow / Total debt ratio. Even though that Beaver's approach was innovative it was criticized mainly for sometimes contradicting implications of different ratios.[29]

Altman recognized the evident deficiencies of univariate analysis and proposed a multivariate discriminant analysis (MDA) as a more suitable approach for ratio examination. He applied MDA to a mixed sample of defaulted and non-defaulted manufacturing companies and designed the Z-score model (1968). His main contribution to previous research was the fact that the model comprised of several different ratios into one composite score which indicated the likelihood of distress. The original Z-score model had the following form:

$$Z = 0.012X_1 + 0.014X_2 + 0.033X_3 + 0.006X_4 + 0.999X_5 \tag{2.7}$$

Where: *X_1, Working Capital / Total Assets*; *X_2, Retained Earnings / Total Assets*; *X_3, EBIT / Total Assets*; *X_4, Market value of equity / Book value of total liabilities*; *X_5, Sales/ Total Assets*

[28] It is desirable to use word 'gradually', because initially based on the idea of BSM was designed the Vasicek-Kealhofer model as mid-step which was afterward improved into the Kealhofer-Merton-Vasicek model.

[29] For instance, Altman (2000) explained the problem of contradicting signals by the following example: "A firm with a poor profitability and/or solvency record may be regarded as a potential bankrupt. However, because of its above average liquidity, the situation may not be considered serious."

According to his study Altman set the cutoff score on a level of 2.675, while a company with score below this level is going to default in close future.[30] The Z-score model meant a breakthrough in default prediction techniques but got also a lot of criticism. For instance, Johnson (1970) expressed doubts about several ratios used in model; from his point of view some of them had no predictive power at all. In a response to the criticism, Altman (1970) conducted more robust test of his methodology on a sample of 2000 companies and brought more supporting evidence for his model. In later periods, Altman also defined similar discriminant formulas for privately held companies (Z'-score) and for non-manufacturing companies (Z''-score).[31]

Altman, Haldeman and Narayanan (1977) stepped even further and expanded the original Z-score model into a new generation ZETA model. Unfortunately, the ZETA model is treated as a proprietary knowledge; therefore the exact form of the equation could not be disclosed. Nevertheless, authors stated that the ZETA model was adjusted in order to also capture the following company features and evolution in financial and research practices into a creditworthiness score (Altman, 2000, pp.).

- Company size
- Time decay of data relevancy
- Broader applicability over all company types
- Changes in accounting practices
- Progressive aspects of MDA

Moreover, the authors of the ZETA model increased the number of ratios incorporated in a model from 5 to 7 and also replaced some of the original ratios with more suitable substitutes. In a result the prediction accuracy of the ZETA model outperformed the original Z-score model.[32] Although, the accuracy of these models is highly reliable, models cannot provide direct probability measures of default. The models' outputs can be interpreted more like dummy variables (default or non-default) and this fact obstructs its direct application in the valuation process.

2.4.3 O-Score model

After the initial Z-type model boom over the late 60's and 70's, further research developed into a new O-score model presented by Ohlson in 1980. Ohlson is considered to be the first to design a default predictive model based on multiple-logistic regression – also known as Logit. He pointed out

[30] This level was subsequently adjusted, by Altman's own words: "I advocate using the lower bond of the zone-of-ignorance 1.81 as a more realistic cutoff Z-score that the score 2.675". (Altman, 2000)
[31] For explicit formulas of both Z'-score and Z''-score model please check *Appendix – Theory.*
[32] For a more comprehensive comparison of these models *check Appendix – Theory*, *A-T. 2.*

several obvious limitations of Z-type models, namely timing issues, technical constraints, outcome format and arbitrary character of MDA matching criteria.[33] He identified four fundamental factors with a significant predictive effect which influenced him during a ratio selection: (1) the size of the company, (2) the financial structure measures, (3) the performance measures and (4) the current liquidity measures. Subsequently, employing the Logit methodology Ohlson avoided all mentioned Z-type problems and provided a similar creditworthiness score as the preceding Z-type models. The O-score model was defined over the nine different accounting and financial rations as follows:

$$O_{score} = -1.32 - 0.407Q_1 + 6.03Q_2 - 1.43Q_3 + 0.0757Q_4 - \\ -2.37Q_5 - 1.83Q_6 + 0.285Q_7 - 1.72Q_8 - 0.521Q_9 \quad (2.8)$$

Where:

Q_1, size factor	*Q_5, Net Income / Total Assets*
Q_2, Total Liabilities / Total Assets	*Q_6, EBITDA / Total Liabilities*
Q_3, Working Capital / Total Assets	*Q_7, Sign indicator of net income in the last year*
Q_4, Current Liabilities / Current Assets	*Q_8, Sign indicator of book value of equity in the last year*

Q_9, [Net Income(t) – Net Income(t-1)] / [|Net Income(t)|– |Net Income(t-1) |]

In addition, the O-score model has another important advantage over Z-type models the fact that obtained O-score can be instantly transformed into a default probability measure using the logistic transformation[34] and therefore the final model outcome between 0 and 1 may be directly applied in a valuation process.

[33] By the expressions 'timing issues' Ohlson pointed out concerns about matching the exact date of default with company financial statements; 'technical constraints' represent the models limitations like facts that the variance-covariance matrices of the predictors should be the same for both groups and also requirement of normally distributed predictors; 'outcome format' reflect the Z-type bankruptcy dichotomy (defaulted or non-defaulted) and pre-selective 'matching process' within MDA where creator has to arbitrary select the predictor pairs of companies (Ohlson, 1980).

[34] The logistic transformation formula has a following form: $P_{default} = \frac{e^{O_{score}}}{1+e^{O_{score}}}$

2.4.4 BSM model

Development of the option-pricing model (1973) brought up an important insight that a stock market bears also alternative (to financial statements) and potentially useful information for corporate default prediction.[35] Subsequent Merton's expansion (1974) of the original BS model and introduction of equity-call model equipped researchers with necessary apparatus. The underlying idea of the equity-call model was explained before; therefore reader is referred to *Subsection 2.3.4*.

Following this methodology, company defaults when the market value of assets, is less that the face value of the liabilities at date of liabilities maturity. Based on the Black – Scholes –Merton 'ideal world conditions' the DP can be defined over the BS model as it was shown in McDonald (2002, pp.604), the probability that $\mathrm{V_A(T)} < X$:

$$\pi_{BSM-default} = N\left(-\frac{\ln\left(\frac{V_A}{X}\right) + \left(\mu - \delta - \frac{\sigma_A^2}{2}\right)T}{\sigma_A\sqrt{T}}\right) \quad (2.9)$$

Where alike to BSM model, V_A is the market value of assets, X is the face value of the debt maturing at time T, σ_A is the standard deviation of asset returns, δ is the continuous dividend yield expressed in terms of V_A, μ is the continuously-compounded expected return on assets and $N(\cdot)$ refers to the standard cumulative normal distribution. The further derivation of outcomes as well as overall BSM model is behind the scope of this thesis, for more details and references please check the footnote.[36]

The frequently mentioned advantage of this approach is the fact that the BSM model estimates default probability with regard to market information whereas the accounting-based models tend to use backward looking financial statements. On the other hand, the restrictive maturity limitation casts the considerable doubts about model's reality-mimicking capabilities. Further research of this methodology showed that BSM model would provide an important conceptual framework at a field of default prediction.

---

[35] This notion roots from the Efficient Market Hypothesis (EMH), while according to EMH all publicly available information is instantly incorporated into the market price.
[36] For more detailed description of BSM model and outcome derivation, please check *Appendix – Theory*.

2.4.5 KMV model

As it was mentioned in the section introduction, the original BSM model was further developed. Initially, Vasicek and Kealhofer extended BSM model in 1984, but its later modification known as Kealhofer – McQuown – Vasicek model (KMV) attracted far more popularity and publicity.

The KMV model roots from the similar option nature and market oriented intuition as the BSM model. Crosbie & Bohn (2003, pp.10) emphasized that KMV does not rely on the idea of perfect market efficiency, but according to them is very difficult to consistently beat the market accuracy. Thus, it is reasonable to use market prices as predictors, since the information embedded within the prices has superior default-predictive capabilities in comparison with other alternative sources. Distinctive feature of KMV model is the unique definition of default. Following the KMV model framework a company defaults on its obligations when the market value of asset falls below the default point. However, the default point may significantly vary among various companies[37], authors - based on empirical studies - identified that the most accurate proxy for default point should lie somewhere between total liabilities and current liabilities.[38]

Consequently, the likelihood of default is determined under KMV methodology through the company specified 'Distance-to-Default' (DD). While DD represents the number of standard deviations the market value of assets is away from the default point. Crosbie & Bohn (2003) defined DD through BSM framework in a following way:[39]

$$Distance\ to\ default = \frac{\ln\left(\frac{V_A}{X}\right) + \left(\mu - \delta - \frac{\sigma_A^2}{2}\right)T}{\sigma_A\sqrt{T}} \qquad (2.10)$$

Eventually, the KMV employs a historical database in order to transform the particular DD measure to a concrete default probability rate. However, the KMV approach became a target of criticism and even skepticism, further empirical research (e.g. Kealhofer, 2003) indicated that KMV provides comparable accuracy to original Merton's model whereas the implementation differences

[37] Since the practice shown that some companies may remain active while technically insolvent.

[38] Falkenstein (2000) stated that KMV use the default point at level of current liabilities plus a half of long term liabilities, what should be in line with a distribution of recovery rates on default bonds.

[39] Alternatively may be the Distance-to-Default defined in financial terminology as a ratio: *DD = [(Market value of assets – Default point) / (Market value of assets • Asset volatility)]*, (Crosbie & Bohn, 2003).

are significant (in favor of KMV). The obvious dispute[40] between KMV and Moody's about the superiority of their competing approaches was eventually resolved by a Moody's take-over of KMV and implementation of the KMV model into Moody's MKMV model in April 2002.

2.4.6 Credit Rating approach - Moody's RiskCalc

The convention in benchmarking individuals, companies or even countries, against the 10 – grade rating scheme is the oldest and the most intuitive way of indicating the one's creditworthiness. The credit rating, as this approach is commonly called, emerged in the beginning of 21st century.[41] Over the course of time, three agencies became globally respected, namely Fitch, Standards & Poor's and Moody's as reliable providers of credit ratings. Due to the fact that models underlying the credit ratings are treated as proprietary knowledge the exact comparison with other valuation techniques is difficult. But for the purpose of demonstration, the original Moody's RiskCalc v1.0 is further presented.[42]

RiskCalc v1.0 for public firms was developed by Sobehart and Stein in 2000 as a combination of various credit risk models. Although, its exact form is publicly unknown, the authors stated that model is not based on Merton's model but uses the different data sources including accounting and financial statements, credit ratings as well as information from equity markets. As Falkenstein et al. (2000) affirmed, RiskCalc estimates the credit rating according to 17 input variables which are transformed into 9 financial ratios and company size factors, while these ratios may be further segmented into 6 indicator subcategories. The following *Table 2-4* presents a summary of model inputs and their relative influence on outcome.

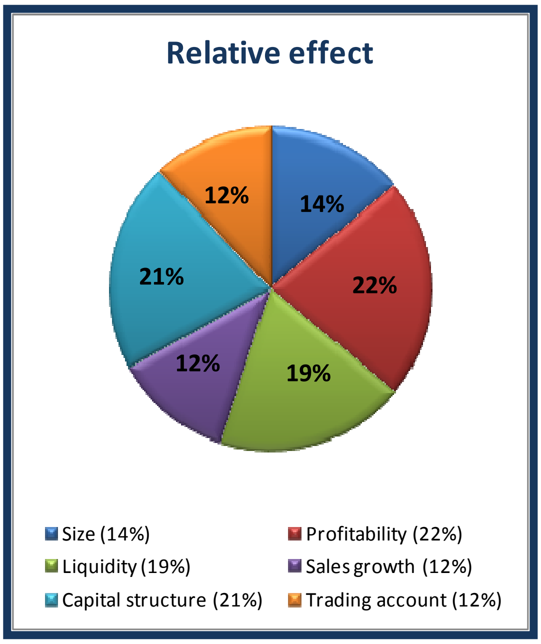
[40] For instance, Sobehart & Stein (1999) claimed that all variants of Merton model have the fundamental limitations whereas Kealhofer & Kurbat (2001) opposed to these critics and provided empirical evidence that credit rating models and accounting measures do not add predictive power to the option-based approaches. For further discussions please check also Stein (2000), Sobehart & Stein (2000), Sobehart and Keenan & Stein (2000).

[41] J.K. Fitch founded the Fitch Publishing company in 1913 and introduced his 10-grade rating system in 1924. J. Moody published his first manual in 1900 and in 1909 began publishing the analytical information concerning value of securities. H. V. Poor is believed as a forerunner in securities analysis publishing the first study in 1860 and formulating Standard Statistics in 1906 (various internet sources).

[42] However, the more recent version 3.1 of RiskCalc is available, the original v1.0 version is presented as more representative example of credit rating approach, mainly due to the fact that its latter versions are partially based on KMV model (since 2002).

Table 2-4: Summary of model variables and inputs with relative effect measure (RiskCalc)

Inputs (17)		Ratios (10) (categorized)
Assets (2y)	↘	
Cost of Goods Sold	→	Assets / CPI
Current Assets	→	Inventories / COGS
Current Liabilities	→	Quick Ratio
Inventory	→	Cash / Assets
Liabilities	→	Liabilities / Assets
Net Income (2y)	→	Retained Earnings / Assets
Retained Earnings	→	Debt Coverage Ratio
Sales (2y)	→	Net income growth
Cash & Equivalents	→	Net income / Assets
EBIT	→	Sales Growth
Interest Expense	↗	
Extraordinary Items	↗	



***Source**: Falkenstein et al. (2000) and own compilation*

Following the RiskCalc methodology, the financial ratios and company size factors are transformed through probit-type[43] model (undisclosed) into intermediate outcome. Model outcome is further mapped through the Moody's database and the final credit rating verdict is determined according to Expected Default Frequency (EDF). The overall creditworthiness outcome is then transformed into the credit grade ranging from 'Aaa' rate (highest quality) to 'C' rate (default).[44] Moody's declared that the key advantage of RiskCalc model is the fact that model was developed especially for the midcap private companies and therefore is capable to deliver superior performance for various types of companies in comparison with alternative models. Subsequently, the implementation of KMV model after KMV takeover in April 2002, improved the Moody's accuracy even more, a latter comparative research showed (e.g. Dwyer et al. 2004) that a new version 3.1 of RiskCalc outperforms all previously presented models.[45]

> *"Prior to default, there is no way to discriminate unambiguously between firms that will default and those that will not. At best we can only make probabilistic assessments of the likelihood of default."*
>
> *Crosbie & Bohn (2003)*

[43] Probit model is a type of econometric regression analysis where the dependent variable using a cumulative normal curve as opposed to a commonly used logistic one, with outcome ranging from 0 to 1.
[44] The mentioned Moody's grade scale from 'Aaa' to 'C' is used for long-term ratings. For short-term scale as well as more comprehensive description of all grades, please check the *Appendix – Theory, A-T. 4* .
[45] For detailed comparison of DP models please check *Appendix – Theory, A-T. 3.*

In this chapter, the reader was introduced to the theoretical background underlying the corporate valuation process. Starting with the fundamental motivations and conventional methodologies, chapter dragged the reader almost instantly into the problem of distress valuation. Recalling, the underlying assumption of a conventional valuation framework the chapter clearly pointed out its obvious underpinnings and explained difficulties related to the valuation under distressed conditions. The subsequent brief introduction of the phenomenon called 'vulture investment' should convince the reader that even distressed companies may be worth the effort of valuation. Therefore, alternative valuation methodologies were presented, starting with various modification of the conventional DCF framework, through the innovative real-option approach to the complex Monte Carlo simulation. Accurate determination of default likelihood became the issue of the prime importance over the course of the chapter. Thus, chapter logically concluded with a short overview of methodologies particularly developed for the default prediction.

The key point of this chapter (and also the main goal of this thesis) was to convince the reader that accurate valuation is possible even in the distress situation and also is worth the effort. In the following chapter, we are going to present a valuation technique which partially combines the key insights of previously introduced methodologies into one comprehensive model in order to provide a fundamentally accurate valuation of a problematic company.

3 Methodology

The current chapter presents to reader the construction of an alternative valuation approach fundamentally based on Monte Carlo simulation in full detail. Starting with the underlying motivation reflecting our intentions to build an alternative valuation model, we then move to the initial brief description. Afterwards, we introduce to the reader a stochastic aspect of our model as the key building block which forms the projections of Revenues, Cost of Goods Sold and Operational Expenses. After an extensive elaboration about the model dynamics we move through the projection of stable (deterministic) variables to the formulation of specific model relations and rules. The chapter concludes with a specific valuation calculus under which a model output is recalculated. The text in the current chapter is extensively accompanied by figures, formulas and tables in order to present our ideas in a comprehensive way.

3.1 Motivation and purpose

Although, the theoretical chapter should provide enough support for motivation to create an alternative approach to the conventional valuation frameworks; we add few more words in order to bolster our effort. While we are aware, that all previously presented methodologies have advantages as well as disadvantages we decided to combine their key insights in order to create a model which may be potentially superior to them. The main purpose of our model is to provide a reasonable and reliable valuation of underperforming and potentially undervalued companies as targets for experienced investors. Since, investors' (human) expectations are commonly driven by 'auto-optimism' approach we surrounded them with a stochastic uncertainty which allows (also) for potential failure and keeps our valuation model more objective.

3.2 Model description (sources of uncertainty)

Essentially our model is based on an approach suggested by Schwartz & Moon (2001). The original model was built in order to provide a reasonable valuation in a rapidly emerging virtual industry. The main idea of Schwartz-Moon (SM) model lies in the fact that it takes into account the extreme uncertainties determining a cash flow generation which stems from specific nature of e-business. In comparison, high uncertainty in our case roots primarily from uncertain prospect of a restructuring process given by a significant likelihood of default. Following the SM approach the future performance uncertainties were formulated as the specific stochastic processes driven by a set of predetermined input variables. Eventually, SM used a Monte Carlo simulation for generating various scenarios and after adjusting for taxes, depreciation and performing a risk-neutral valuation, they have arrived to the enterprise value. Since we are going to adapt the original risk-neutral approach to DCF methodology, our value driver selection follows the conventional free cash flow scheme:

Table 3-1: Conventional Free Cash Flow calculation scheme

	Revenues
-	Cost of Goods Sold (COGS)
=	**Gross Profit**
-	Selling General and Administrative expenditures (SG&A)
=	**Earnings before Interest, Taxes, Depreciation and Amortization (EBITDA)**
-	Depreciation & Amortization (D&A)
=	**Earnings before Interest and Taxes (EBIT)**
-	Taxes
=	**Net Operating Profit after Tax (NOPAT)**
-	Capital Expenditures (CAPEX)
-/+	Δ Net Working Capital
+	Depreciation & Amortization (D&A)
=	**Free Cash Flow**

Source*: Arzac (2005), Pearl & Rosenbaum (2009) and own compilation*

By combining the original SM approach and the conventional free cash flow scheme we decided to replace following three company value drivers by stochastic processes: Revenues, Rate of growth in Revenues and COGS. Furthermore, Operational costs stay proportionally dependent (dynamic) on the level of company Revenues. On the other hand, we assume that Depreciation & Amortization, CAPEX levels and Tax & Interest payments are essentially pre-determined with low inherent uncertainty. Subsequent sections provide an extensive description of all mentioned variables.

3.2.1 Revenues and revenues growth rate

On top of the company models usually stands the expected revenue generation, while it clearly describes the company expected performance. Thus, determination of revenues is the suitable starting point of our model construction. Since we are aware that future company revenues are highly uncertain, we decided to define them in a stochastic form. Following the original SM methodology, we defined the Revenues as a geometric Brownian motion (with drift) which may be in a continuous form mathematically represented by a differential equation:

$$\frac{dR(t)}{R(t)} = \mu(t)dt + \sigma(t)dz_1 \tag{3. 1}$$

Where, the drift, $\mu(t)$ is the expected rate of growth in revenues, $\sigma(t)$ is the unanticipated change (volatility) in revenues and dz_1 stands for a random normal variate drawn from standard normal distribution[46]. Moreover, it is assumed that expected rate of growth in revenues $\mu(t)$ follows a mean reverting (log-normal diffusion) process under which converges to a long-term average growth rate $\bar{\mu}$. This dynamic replicates the fact that the initial rate of growth – which may be even negative during distress – gradually returns to more reasonable levels as a result of a restructuring process.

$$d\mu(t) = k_2(\bar{\mu} - \mu(t))dt + \eta(t)dz_2 \tag{3. 2}$$

While, k_2 is the mean-reversion coefficient interpreting the rate of convergence to pre-determined long-term equilibrium, $\eta(t)$ is the unanticipated change (volatility) in expected growth rate and dz_2 stands again for random normal variate drawn from standard normal distribution. Furthermore, following the SM methodology it is assumed that volatilities presented within revenue dynamics also (deterministically) stabilize over time at more normal levels or even zero, while they used to be considerable during distress. Following formula table shows alternative convergence settings for both revenues volatility $\sigma(t)$ as well as revenue growth volatility $\eta(t)$.

Table 3-2: Alternative convergence settings (volatilities)

Converge to:	Predetermined level		Zero	
Revenues volatility	$d\sigma(t) = k_1(\bar{\sigma} - \sigma(t))dt$ •	or	$d\sigma(t) = -k_1\sigma(t)dt$	(3. 3)
R - growth volatility	$d\eta(t) = k_3(\bar{\eta} - \eta(t))dt$	or	$d\eta(t) = -k_3\eta(t)dt$ •	

Source: *Schwartz & Moon (2001) and own compilation*

Where, k_i-coefficients stands again for volatility-specific convergence rates and variables with line accents ($\bar{\sigma}$, $\bar{\eta}$) denote the long-term levels of revenue volatility and revenue growth volatility, respectively. However, the original SM model applied the setting indicated in the previous table by blue dots, the model gives us flexibility to adapt convergence settings to specific company or industry conditions. Finally, we also assume that stochastic variables may be correlated since both are related to the same dynamic process.

$$dz_1 dz_2 = \rho_{12}dt \tag{3. 4}$$

[46] Standard normal distribution is normal distribution with zero mean and unit variance.

3.2.2 Cost of Goods Sold (COGS)

The second stochastic variable in our model represents the Cost of Goods Sold (COGS), which stands for all direct costs related to the production of goods that a company sells. In other words, this variable includes all raw material costs as well as costs of labor attributable to a production process. Since, COGS are commonly highly correlated with Revenues they used to be expressed by percentage relationship term with regard to Revenues as follows:

$$COGS_t = y(t)\, R(t) \tag{3. 5}$$

While $y(t)$ stands for proportional term, which implicitly reflects a company gross margin. Because the fact that company margin significantly depends on company conditions (i.e. production effectivity, bargaining power, hedging activities etc.) as well as market conditions (i.e. prices of raw materials, competition, market fragmentation etc.) it is reasonable to assume that this proportion can change over time. Therefore we assume that $y(t)$ should follow a mean-reverting process and converge towards long-term industry equilibrium as a result of external competition pressures.

$$dy(t) = k_4(\bar{y} - y(t))dt + \varphi(t)dz_3 \tag{3. 6}$$

Providing

$$d\varphi(t) = k_5\big(\bar{\varphi} - \varphi(t)\big)dt \tag{3. 7}$$

Where alike to revenue growth process, k_i-coefficients stand for particular convergence rates, $\varphi(t)$ is the unanticipated change (volatility) in COGS which is assumed to converge towards a pre-determined level[47], variables with line accents ($\bar{y}, \bar{\varphi}$) are the long-term levels of COGS and volatility of COGS, respectively. The COGS dynamic is again driven by the stochastic variable dz_3 drawn from standard normal distribution. Since, COGS dynamics are closely related with Revenues dynamics it is reasonable to allow the correlation between volatilities:

$$dz_1 dz_3 = \rho_{13}dt \qquad \text{and} \qquad dz_2 dz_3 = \rho_{23}dt \tag{3. 8}$$

[47] Here it is reasonable to assume that volatility is always present (and will be) while market generates unanticipated shocks, even hedging is temporary and moreover it is not possible to hedge all the risks.

3.2.3 Operational expenses

The last quasi-stochastic variable of our model represents the rest of company operational overheads such as Selling General and Administrative expenses (SG&A) and other mainly fixed overheads (e.g. rents, insurance, maintenance etc.). Therefore we assume that these costs can be classified into a fixed and a variable part. Where the variable part is proportional to Revenues but in contrast to COGS, we assume that this proportion relationship stays constant over the time, so the term quasi-stochastic assigned to this variable is more appropriate. Under this proposition the Operational expenses (OE), as we denominated them, may be expressed with following formula:

$$OE(t) = \alpha(t) + \beta(t) \tag{3. 9}$$

Providing

$$\alpha(t) = \alpha = const. \qquad \text{and} \qquad \beta(t) = b\,R(t) \tag{3. 10}$$

Heretofore, the valuation model and variables were defined in continuous form in order formulate a model in a purely general way. However, the continuous form represents the absolute flexibility because a model output may be delivered at any point in time, it also incorporates inherent complexities.[48] Moreover, like with the most financial data, the historical figures are usually published in a discrete manner, thus proposed continuous model should be without any doubt transformed into a discrete version. The transformation of continuous model into discrete has to be performed with absolute mathematical rigour in order to retain a quality of model outputs. Although, Schwartz and Moon (2001) showed that presented stochastic variables may be transformed into a discrete form, they aimed transformation to risk-neutral conditions.[49] On the other hand, we are interested in real (risk-adjusted) valuation so we have to adapt our discrete model to this requirement.

In the case of geometric Brownian motion we apply a logarithm transformation and Itô's Lemma[50], while for the mean-reverting processes we follow the Kloeden & Platen (1992, pp.118) explicit solution of particular differential equations and subsequently apply Dixit & Pindyck (1994, pp.74) approach to acquire its discrete form. Eventually, all stochastic variables presented in our model are defined in discrete forms as it is stated in the following *Table 3-3*.

[48] Like timing issues, e.g. matching the tax deductions with accounting practices, in reality the deductions may be claimed at certain date (moment) over a year, etc.

[49] Since their approach was based on option pricing manner as risk-neutral valuation.

[50] Japanese mathematician Kiyoshi Itō introduced in 1951 methodology known as Itō's calculus which was subsequently applied to a broad spectrum of mathematical problems mainly related to randomness. For more information please check e.g. Dixit & Pindyck (1994, chapter 3).

Table 3-3: Discrete forms of all stochastic variables

	Stochastic process		Discrete form
Revenue dynamics	R(t)	Revenues	$R(t+\Delta t) = R(t)e^{\left\{[(\mu(t)-\frac{1}{2}\sigma(t)^2)\Delta t+\sigma(t)\varepsilon_1\sqrt{\Delta t}\right\}}$
	$\sigma(t)$	Unexpected element of revenues	$\sigma(t) = \sigma_0 e^{-k_1 t} + \bar{\sigma}(1 - e^{-k_1 t})$
	$\mu(t)$	Rate of R-growth	$\mu(t+\Delta t) = e^{-k_2\Delta t}\mu(t) + (1 - e^{-k_2\Delta t})\bar{\mu} + \sqrt{\frac{1-e^{-2k_2\Delta t}}{2k_2}}\eta(t)\varepsilon_2$
	$\eta(t)$	Unexpected element of revenues growth	$\eta(t) = \eta_0 e^{-k_3 t}$
COGS dynamics	γ(t)	Cost of Goods Sold (COGS)	$\gamma(t+\Delta t) = e^{-k_4\Delta t}\gamma(t) + (1 - e^{-k_4\Delta t})\bar{\gamma} + \sqrt{\frac{1-e^{-2k_4\Delta t}}{2k_4}}\varphi(t)\varepsilon_3$
	$\varphi(t)$	Unexpected element of COGS	$\varphi(t) = \varphi_0 e^{-k_5 t} + \bar{\varphi}(1 - e^{-k_5 t})$

Source: *Schwartz & Moon (2001) and own compilation*

The OE costs variable is omitted from previous overview since it is not a stochastic variate therefore formula (*3. 9*) may be used in the simulation model without any further transformation. The comprehensive definition of all processes in their continuous as well as discrete forms together with variables' legend could be found in *Appendix – Methodology*. Up to this point we defined stochastic variates which create a model dynamics; next sections specify the rest of model variables.

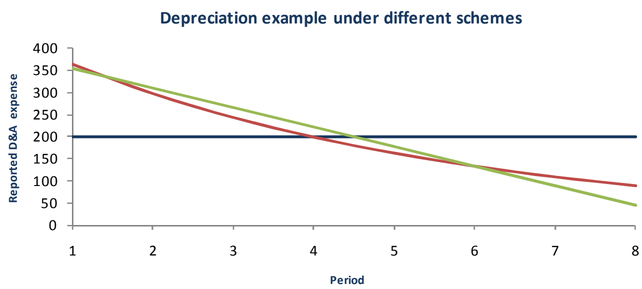
3.2.4 Depreciation, Amortization and CAPEX

Proceeding downwards in the FCF calculus scheme (*Table 3-1*) we arrived to the phase where Depreciation and Amortization is required. At this stage, we decide to define also Capital expenditures as these three accounting items together reflect the movements in company's tangible and intangible assets.

Depreciation & Amortization (D&A) expenses represent the company non-cash expenses which reflect the reduction of the book value of a company's tangible and intangible assets. Although D&A are fundamentally non-cash expenses they reduce the operational earnings, so they have to be projected and incorporated within a valuation model. Fundamentally, there are two alternative ways for projection of D&A expenses. Firstly, providing we have an access to company insider information the D&A may be explicitly determined according to depreciation scheme, asset life and salvage value of each company asset. Unfortunately, this type of information is usually restricted so we have to use approximation approach based on D&A historical levels. Practitioners used to estimate D&A expenses as a percentage of tangible (PP&E) and intangible assets for depreciation and amortization, respectively. Sometimes it is also assumed that D&A can be determined as a percentage of revenues but it may be easily shown that this approach can provide incorrect estimates (e.g. Koller et al., 2005, pp. 242) in certain situations. Following the estimation approach based on (intangible & tangible) asset-drivers D&A expenses are typically estimated by OLS regression over the historical ratios. However, this approach looks simple it is also important to take into account the facts that companies are allowed to use different depreciation (amortization) schemes as well as evident period dissonance between assets within a company. As a result the OLS regression should be adjusted in accordance with company depreciation characteristics. The Following *Table 3-4* presents three 'mainstream' groups of depreciation schemes together with a visual example.[51]

Table 3-4: Traditional D&A scheme types

Scheme type		
Linear (*straight-line*)		●
Accelerated	*Geometric*	●
	Sum of digits	●



Source: *Own compilation*

Under linear (straight-line) depreciation method it is assumed that assets' value deteriorates at uniform pace over its useful lifetime so D&A expenses are projected at a constant (nominal) value per period. On the other hand, most of depreciation schemes belong to the category of accelerated depreciation schemes where it is assumed that an asset's value diminishes more rapidly in early years of its lifecycle (i.e. D&A expenses are higher in early years). Moreover, we can further sort the accelerated depreciation schemes to a geometric type and so called 'sum-of-digits' type. While the geometric scheme assumes that assets value depreciate by a fixed percentage of its remaining value each period, the 'sum-of-digits' method determines D&A expense according to remaining lifetime and predetermined fixed multiple (calculated as initial asset value divided by sum of years of use). Consequently, as it was shown in *Table 3-4*, asset loses most of its value in the early periods and depreciation reduces as the remaining asset value approaches its salvage value. However the correct scheme identification is a critical prerequisite of an accurate D&A projection, analysts have to also

[51] For complete depreciation calculations of presented schemes please check *Appendix – Methodology, A-M. 1.*

take into account a company's capital expenditure and ensure that the implied D&A assumptions are consistent across historical records (Pearl & Rosenbaum, 2009, pp.120).

Capital Expenditures (CAPEX) are according to Pearl and Rosenbaum (2009) defined as the funds (expenditures) that a company uses to purchase, improve, expand or replace physical assets such as buildings, equipment, facilities, machinery and other assets. In other words, we can say that CAPEX plays an important role as an offset to asset deterioration represented by D&A. In general practitioners recognize two types of CAPEX spending, maintenance and growth. Maintenance spending represents the capital required to sustain existing assets at their current output levels. On the other hand, growth CAPEX is primarily used to not only retain the assets' current levels but also to purchase new assets or expand the existing asset base in order to improve potential of future company growth (Pearl & Rosenbaum, 2009, pp.170). The CAPEX forecasts are usually reported within a company's annual reports or 10-K / 10-Q fillings as a part of a company long-term strategy, alternatively historical records may provide reliable proxy levels. Since we assume that our model should be applied to companies close to distress it is reasonable to presume that a company with such difficulties will retain CAPEX spending at lower – only maintenance – levels.

3.2.5 Debt, Interest and Taxes

Another group of financial items which have to be determined (forecasted) during valuation are company's Interest and Tax expenses. While, Taxes as well as Interest payments are in their nature static items which can be forecasted without any significant biases, we decided to define them in ordinary (non-stochastic) way. Although, the definition of debt dynamics itself will not be included within this subsection, we decide to mention it in order to emphasize a close relation between amount of debt and interest payments.

Interest expense (income) of a company are directly related to the liabilities (assets) bearing the interest. Thus net financing costs[52] (interest expense – interest income) are typically projected according to the debt load of a company. In reality companies tend to finance their operations by a broad spectrum of debt instruments which bears a different level of risk. As a result, we can organize debt instruments into debt tranches according to the expected level of return required by lending entities. The company interest expense can be then projected as a sum of interests required over all debt tranches. However the previously described approach may provide highly accurate estimate of future financing costs, the real crux lies in forecasting of future required returns. To avoid these complexities which arise from application of the complete debt structure, we introduce a practical

[52] Since, we are assuming that the model will be applied on a non-financial company, it is reasonable to assume that interest expense will be higher than interest income.

simplification under which we recognize only two debt tranches, short-term and long-term. The specific discrimination rule as well as determination of required returns is extensively discussed in *Subsection 3.3.1*. Furthermore, we also allow for changes in capital structure and these dynamics are further discussed within *Section 3.3*.

Taxes paid by a company usually depend on three critical factors of corporate tax law. Taxes are determined according to corporate tax rate (1) which is usually defined as a percentage proportion of tax base (2). While a company tax base is determined according to the set of strict rules embedded within corporate law which allows also reductions to tax base known as tax deductions or tax shields (3). However it is important to respect and incorporate all three aspects of tax law into the model due to the matter of simplification we decided to follow the common valuation practice and apply the following simplifying approach of tax determination in our simulation model.

$$Taxes_t \begin{cases} = Max[(EBIT_t - NFC_t) \bullet \pi, 0] & \text{if } EBIT_t > 0 \\ = 0 & \text{if } EBIT_t \leq 0 \end{cases} \quad (3.11)$$

Where π is the corporate tax rate in a particular country and NFC stands for company's net financing costs (i.e. interest expense paid decreased by interest income received). Because the fact that corporate tax law varies over the countries and even industries we cannot determine an absolutely universal rule, so the explicit way of determination of company taxes has to be adjusted case-to-case.[53]

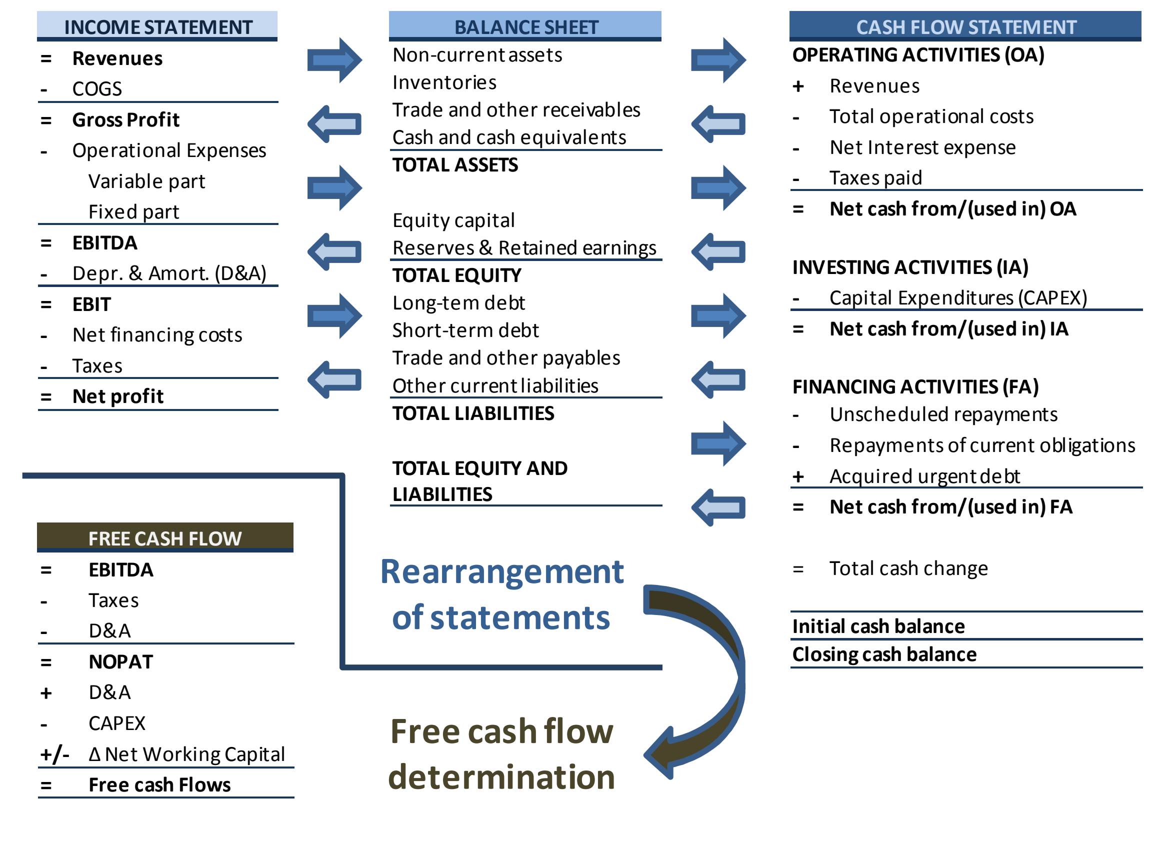
3.2.6 Main financial statements & intrinsic relations

There are many research papers written and valuation models built on the Monte Carlo methodology, but most of them were formulated in the simplest straight-forward approach. Where, the value of a company is based only on a simulation of FCFs with a 'blurred' assumption that all individual financial items presented in a model are in line with each other. These assumptions are usually made on purely theoretical grounds, whilst we are convinced that even an observation of intrinsic relations may provide the interesting insights. In order to breach this 'virtual' boundary we decided to build our model on three key company statements, Income statement, Cash Flow statement and Balance sheet even though we are aware that this decision introduces a lot of difficulties.

[53] As an example the Dutch tonnage tax may be mentioned.

Although, we claimed our incentive to incorporate three financial statements into a model in order to decrease the level of assumptions and facilitate a thorough analysis, we cannot avoid simplifications at all. After a proper analysis we decided to re-state the financial statements into the form demonstrated in *Figure 3.1*.

Figure 3.1: Re-stated financial statements and relations



Source: *Own compilation*

Because the comprehensive description of all processes and relations in our model may be excessively complex we decided to use a principal approach for explanation of the model dynamics. All decisions within our model are based on three key principles.

- Operational profit is primarily used for interest and principal payments. After repayment of all required obligations capital expenditures may be realized and subsequently even unscheduled (early) debt repayments are allowed.
- In the case, that operational profit and reserves are not sufficient to cover company's current obligations (interest and/or principal payments) company may acquire an urgent short-term debt in order to temporary bridge the gap in profits.
- The option to acquire the urgent loan or announce a default is conditional to a specific bankruptcy rule which is extensively discussed in *Section 3.4*.

Since we are aware that these principle rules may appear vague, we extensively discuss the decision process applied within our model in the *Subsection 4.3.4.* The rest of relations presented in the model (company) but not mentioned so far, may be easily defined with common sense and basic accountancy knowledge. Moreover most of these relations should be adjusted case-to-case in order to concisely reflect the specific company reality, thus further discussion and definition of any general framework would be beyond the scope of this thesis. However, the reader may check the *Section 4.3* or alternatively Palepu et al. (2007) for examples and suggestions how to cope with particular relations.

Heretofore, we (re)defined stochastic and non-stochastic variables which create important model dynamics but as our model starts to significantly depart from original SM one, extensive modifications have to be introduced within the model in order to authentically reproduce the reality. The remaining sections in this chapter cover these adjustments in an intuitive sequence.

3.3 Cost of capital

The first important difference between our model and that presented by Schwartz and Moon (2001) is the valuation of risk-adjusted returns in contrast to the original SM risk-neutral valuation. Our decision to move away from the risk-neutral framework towards risk-adjusted can be motivated by the following arguments. First of all, from the theoretical point of view, application of the risk-neutral valuation framework requires many critical assumptions. Foremost, it requires the correction of simulated FCFs (i.e. all sub-components which form the FCF) to risk-neutral settings. In order to solve this problem, Schwartz and Moon (2001) introduced a new parameter the 'market price of risk' *(λ)*. Unfortunately, since the λ is practically unobservable they were forced to make other questionable assumptions to construct an 'applicable' proxy. [54] Secondly, from a practical perspective, risk-neutral valuation is fundamentally based on the notion that valuation is independent of investors' risk preferences – as we have presented in the *Subsection 2.3.4*. Despite the fact that risk-neutral assumptions are theoretically feasible, its real application and especially interpretation to a broader (less qualified) audience is extremely difficult. To summarize our findings, the application of a theoretically simpler approach (risk-neutral) would be in our case significantly more challenging and questionable than the application of a risk-adjusted one.

[54] For instance, they assumed that only revenue has an associated risk premium and the rest of elements (like revenue growth, COGS etc.) are not associated with any risk premium. A comprehensive description of the whole problem would be beyond the scope of the thesis but an interested reader may check Schwartz & Moon (2001, pp.6).

Consequently, the current section presents how to estimate the rates of return required by debt-holders (Cost of Debt) as well as equity-holders (Cost of Equity) and determine the overall valuation discount factor (Weighted Average Cost of Capital).

3.3.1 Cost of debt

We start from debt-side perspective since the Cost of Debt is also one of the necessary prerequisites for forecasting of future debt interest payments. As it was previously mentioned in *Subsection 3.2.5*, we apply a simple discriminant rule for distribution of different debt instruments into short-term and long-term tranche. Recalling, this approach is applied in order to avoid inherent complexities of forecasting the required returns for all specific kinds of debt instruments. Generally speaking, debt instrument is considered as short-term if economic obligation arising from instrument is required to be met within the next 12 months and it is considered as long-term if this obligation is required to be met within a period exceeding 12 months. Following this decomposition, we assume also two different levels of required returns (cost of debt), one for each tranche.

Cost of Debt (short-term tranche) should reflect yield of various short-term debt instruments. While yield difference among maturities of up to 12months is usually negligible, we follow the common practice and use the 6-months (median) interbank interest rate plus some spread as a relevant proxy for the cost of company's short term debt.

$$CoD_S = Interbank\ rate^{6M} + Spread \tag{3. 12}$$

Cost of Debt (long-term tranche), is commonly estimated as the yield-to-maturity on company long-term bonds or alternatively on corporate bonds of a comparable company. Although, this approach can be applied without any doubt in ordinary valuations, academics and practitioners warn to its evident shortcoming in a case of a company with the 'below-investment-grade' debt. For example, Koller et al. (2005, pp. 324) claimed: "*Technically speaking, yield to maturity is only a proxy for expected return, because the yield is actually a promised rate of return on a company's debt (it assumes all coupon payments are made on time and the debt is paid in full).*" Therefore, it is obvious that yield-to-maturity approach does not count for potential non-materialization of payments which might critically underestimate the real cost of debt in a case of company on the edge of distress.

Intuitively, considering the debt as any other tradable asset, application of the Capital Asset Pricing Model (CAPM) would be a suitable solution to this problem. Unfortunately, as also Damodaran (2009, pp. 36) mentioned, the application of OLS regression (estimation of debt beta) on skewed debt returns may lead again to terribly biased outcomes. Instead of these approaches, Damodaran recommended to use the default spread over the risk-free rates as a less biased and fundamentally accurate estimate for Cost of Debt. While, default spread should be determined as the (long-term) average return-difference between the risk-free proxy[55] and index consisting of so-called high-yield bonds. This proposition for estimation of the Cost of Debt for long term tranche (CoD_L) can be expressed as follows:

$$CoD_L = r_f + Default\ spread_{Bond-based} \tag{3. 13}$$

It is a general agreement, in order to estimate the risk-free rate (r_f) to use as a proxy the average yield-to-maturity on government default-free bonds with longer maturities from 10 – 30 years. Because the application of different maturities may significantly affect the overall outcome, since for example the difference between yield-to-maturity at 10-year and 30-year U.S. Treasury bonds is not negligible, the maturity of proxy used should reflect the investment horizon and nature of asset for which a risk-free rate is estimated.

3.3.2 Cost of equity

As opposed to Cost of Debt, in case of estimating the Cost of Equity (CoE) it is difficult (however, not necessarily impossible[56]) to create any synthetic risk premium, bolstered by reasonable arguments, which can essentially beat the traditional CAPM approach. Therefore, in CoE estimation we rely on conventional approach defined as follows:

$$CoE = r_f + \beta_E(r_m - r_f) \tag{3. 14}$$

Where r_f is the risk free rate determined as it was stated previously in *Subsection 3.3.1*, r_m is the market return usually based on average return of regional or global indices and finally β_E stands for beta of equity representing a stock's sensitivity to the market. The beta of equity is calculated according to the variance of market returns and covariance of stock-to-market returns as follows:

$$\beta_E = \frac{Covar(r_{market}, r_{stock})}{Var(r_{market})} \tag{3. 15}$$

[55] Of course, the risk-free proxy should be consistent over the whole valuation.
[56] The Damodaran's bottom-up betas (Damodaran, 2009) may be mentioned as an example, but all alternative methods usually require extremely specific inputs and assumptions, to be applied.

3.3.3 Weighted Average Cost of Capital

Eventually, following the traditional DCF methodology, we can combine previously defined cost components into the Weighted Average Cost of Capital (WACC) as overall discount factor of our valuation model. According to Koller et al. the utilization of WACC is a simple, accurate and robust method of corporate valuation. If, however, the company's target capital structure is expected to change significantly, for instance in a leveraged buyout, a constant WACC can over/under state the company value (Koller et al., 2007, pp. 297). As we allow for (early) debt repayments as well as further debt loading in our model, it is reasonable to reflect these capital dynamics also within the discount factor. Therefore, our model dynamically recalculates the 'actual' WACC for each period according to simulated scenario circumstances:

$$WACC_t = \frac{E_t}{TA_t} \cdot CoE + \frac{SL_t}{TA_t} \cdot CoD_S + \frac{LL_t}{TA_t} \cdot CoD_L \quad (3.16)$$

Providing

$$TA_t = E_t + SL_t + LL_t \quad (3.17)$$

Where TA_t is the book value of Total Assets, E_t is the book value of Equity, SL_t is the book value of Short term liabilities and LL_t is the book value of Long term liabilities all in period *t.* Whilst, CoE, CoD_S and CoD_L stand for cost of particular type of capital. In the end, it is desirable to add that we do not assume any further equity boosts (additional external increase of equity capital) in our model.

3.4 Bankruptcy thresholds

Although, Schwartz and Moon (2001) have shown that their model can indicate a default, they concentrate on the valuation purpose and presented default predictions only as a minor model contribution. In addition, their bankruptcy threshold was a quite simple rule where a company defaults when the amount of cash available reaches some predetermined negative amount. While cash available was defined as follows (Schwartz & Moon, 2001, pp.6):

$$dX(t) = \{r_f \cdot X(t) + Y(t) - Dep(t) - Capx(t)\}dt \quad (3.18)$$

Where $Y(t)$ is the company net profit, $Dep(t)$is the depreciation, $Capx(t)$ stands for planned capital expenditures and $r_f \cdot X(t)$ reflects the assumption that initial unassigned cash earns the r_f risk free interest over a period. As authors also admitted, this assumption was certainly not in line with reality. Even brief analysis reveals at least three important drawbacks of this approach. Firstly, the depreciation and amortization expenses have fundamentally a non-cash character. Secondly, assumption that a company will realize planned investments like capital expenditures in any state of nature (even in periods of poor performance) could be misleading. Lastly, the predetermined negative limit of company cash representing the possibility of future refinancing is defined in a vague maximalization way without apparent link to the real situation. But it is desirable to add, that original SM model was published as a conceptual framework and presented on companies with negligible likelihood of default[57] so the default threshold was not a critical issue.

On the contrary, in our valuation case a default plays the prime role. Therefore, due to obvious limitations of the SM rule and differences of models, we have to define a new default rule. In this effort, we decided to inspire ourselves by definitions of rating agencies in order to stick to the reality as much as possible. Moody's definition (2007) of a default is intended to capture events that change the relationship between debt holders and the debt issuer from the relationship which was originally contracted, and which subjects the bondholder to an economic loss. Moody's recognizes three types of such events: (1) missed or delayed disbursement of interest and/or principal, (2) bankruptcy, administration, legal receivership or other legal blocks to the timely payment of interest and/or principal and (3) issuer's offering of new securities with lower seniority or longer maturity with obvious purpose to avoid default (Moody's, 2007, pp. 54). In line with this definition and inspired also by terminology and definitions presented within Gryglewicz (2010), we formulated the following bankruptcy thresholds for our model which may properly represent mentioned events.

Bankruptcy threshold (1): Financial illiquidity, a company becomes financially illiquid when it misses or delays its current debt obligations (payment of interest or principal). Since, financial illiquidity does not always lead to instant default in reality; we improve this threshold with the possibility of future refinancing (acquirement of a short term debt). Presented illiquidity threshold is in line with the distress dichotomy presented within the *Subsection 2.2.2* and stands for the situation of financial distress. Nevertheless, it would be irrational to assume that a company can finance its obligations by cumulating new debts infinitely; ergo we introduce an absolute limit on urgent loans. Consequently, in our model a company declares liquidity default when it is illiquid to its current obligations and simultaneously reaches its urgency loan limit. This threshold is represented by two independent nodes in our model, [1] Interest payment node presented in Income statement and [2] Principal payment node presented in Balance sheet. However, interest and principal payment in reality present two different financial operations; from the valuation perspective both occur at the same time, at the end of the valuation period. In our MC model we check both conditions separately but at the same time (at the end of each simulation period). Therefore, our model is capable of indicating

[57] Schwartz and Moon (2001, pp. 17) stated 3.4% default rate for the illustrative example presented within their research paper.

whether liquidity default occurs or not, but cannot distinguish between the type of liquidity default. In addition to the threshold defined above, we also incorporate another one in our model.

Bankruptcy threshold (2): Economic insolvency occurs when the book value of equity reaches zero as a result of an abnormally negative level of Reserves & Retained earnings. Following the Gryglewicz (2010, pp.6): In such a case, the equity holders may voluntarily deem a default since the company operations are not profitable enough to run the company. This situation may occur in case of an extremely leveraged company with highly volatile revenues when an instant one period loss totally erases the equity wealth. Thus, insolvency threshold controls the long-term trend of economical sustainability of the company operations and reveals the potential value consumption. However, we can say that it is obviously improbable, since a company with such a poor performance foremost becomes financially illiquid. On the other hand, we are aware that low probability does not automatically mean impossibility. Thus, we introduced in Balance sheet another, [3] Cumulative loss node to cover for this possibility as well. Although, we believe that our description of both thresholds should be sufficient for the reader to realize the differences between the financial illiquidity and the economic insolvency, we refer to the research paper of Gryglewicz (2010) which extensively covers this topic in particular.

In conclusion of this section, it is desirable to remind our assumption that if a default occurs, company fails its restructuring process and goes instantly into liquidation. Proceeding further within the model, once the scenario is classified we can move towards the key objective of our model, the company valuation.

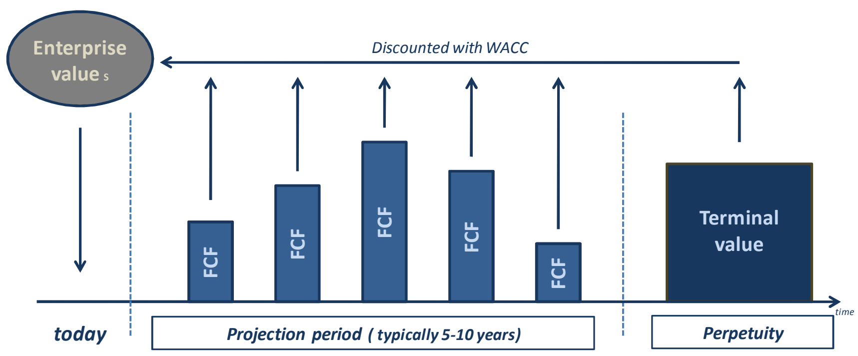
3.5 Enterprise value calculus

Previous sections provide us with the necessary foundations (inputs) and we finally arrived at the terminal stage of our valuation model, the value determination itself. Since our model fundamentally distinguishes between a default and a non-default case we define two valuation methodologies in accordance with appropriate scenario conditions. In a case of non-default scenarios we evaluate a company as a stable going concern, whereas default scenarios are evaluated in clearly liquidation manner as we declared in the conclusion of *Section 3.4*.

3.5.1 Going concern value

As it was previously claimed a value of a company under non-default (survival) scenarios is determined as for any other stable company, in other words the valuation of a survival case follows the traditional DCF framework as it was presented in *Table 2-1.* Starting with the calculation of free cash flows through the estimation of terminal value to the final net present value determination according to the company weighted average cost of capital. It may appears contra-rational to use the traditional DCF approach since we have previously showed in *Subsection 2.2.1* its obvious shortcomings, but it is important to realize that thanks to our specific projection approach we (partially) avoided some critical assumptions. For example, application of bankruptcy thresholds clearly introduces into our valuation model a possibility that some of FCFs may not materialized at all. Moreover, discount factors are estimated with special attention to considerable likelihood of default and finally in the case of determination of perpetuity value we decided to apply a conservative convergence formula. The *Figure 3.2* concisely illustrates the overall 'going concern' valuation concept followed by further specification of its elements.

Figure 3.2: Valuation of non-default case



Source: *Drs. Hans Haanappel, lecture slides pp. 10*

Where:

Free cash flows (FCFs) for non-default scenarios are recalculated in a way presented within the *Figure 3.1* according to the simulated projections of their sub-components (like Revenues, COGS, OE, etc.).

Terminal Value (TV) is estimated according to the convergence formula as it was mentioned previously. We decided to apply convergence formula as it was shown in Arzac (2005, pp.20). Since we are assuming that a company which arises from difficulties, fundamentally does not have any competitive advantage over its competitors after a projection period, therefore Return on Capital (ROC) should converge to WACC. As a result, the company value after the projection period is calculated as follows:

$$Terminal\ value_j = \frac{NOPAT_{j,T+1}}{WACC_{j,T}} \qquad (3.19)$$

Subsequently, all financial outflows are discounted with scenario and period specific $WACC_{j,t}$ which are calculated as it was defined in *Section 3.3* and non-default scenario outcome is calculated in the following way:

$$Enterprise\ Value_{S,j} = NPV(FCF_{j,t}) + NPV(Terminal\ value_j) \qquad (3.20)$$

3.5.2 Value of defaulting company

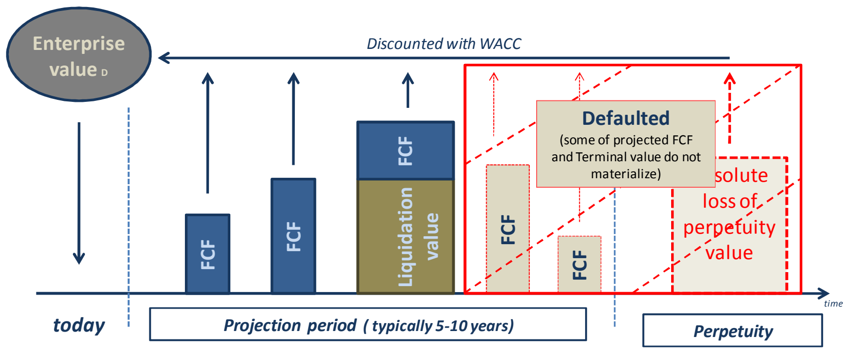
While, the valuation of non-default scenarios could be defined in a quite simple DCF manner, the proper valuation framework for default scenarios presents a real crux. The key issues concerning the process of liquidation, especially almost instant value loss, were discussed in *Subsection 2.2.1*. In order to clarify our approach, we summarize all relevant assumptions already declared, written or even just thought related to our process of company default.

- Model was built for clearly investment (vulture) purposes in order to provide a valuation of underperforming and potentially undervalued targets.
- We assume that if a default occurs, company fails its restructuring process and goes instantly into liquidation. Consequently, any further resale to other vulture agent or investor is not assumed.
- It is assumed that default and subsequent liquidation process significantly deteriorates the 'original' enterprise value as it was described in *Subsection 2.2.1*. To shed more light at this assumption, we assume that equity became instantly worthless as well as all intangible assets. From our point of view, only tangible assets may present a company value, however, only proportion of their real (fair market) value. [58]
- Since, any further resale is not assumed; expected free cash flows which not materialized up to the moment of default becomes also worthless. However, all cash flows materialized prior to default are in their net present value added to the overall enterprise value of defaulting company.

Considering all these assumptions we define the framework for estimation of company enterprise value in default scenarios which is essentially based on the DCF model with several adjustments as it is presented (on the next page) in *Figure 3.3*.

[58] Capitalization of intangible assets during liquidation process is at least difficult if possible at all; only specific patents and licenses may present valuable intangible assets which could be sold. Furthermore, urgent asset liquidation puts the distressed company into unfavorable bargaining position. In a result, the funds acquired by liquidation usually fail to cover even company liabilities therefore equity holders experience an absolute loss.

Figure 3.3: Valuation of default case



Source: *Own compilation*

Where:

Free cash flows (materialized) for default scenario are (similarly to non-default scenario) recalculated in a way presented within the *Table 3-1* according to the simulated projections of their sub-components (like Revenues, COGS, OE, etc.).

Liquidation value (LV) more frequently labeled as Recovery Rate can be in its simplest essence defined as the market price of the security just after the default. The proper estimation of LV is among researchers (and practitioners) considered of equal importance as default prediction, since both present the critical inputs for estimating the company default value. We reviewed various approaches based on (1) structural models as well as (2) reduced-form models but as most of them turn out to be implausibly complex[59] we decided to use empirical (historical) recovery rates as appropriate proxies for LV.

As reliable sources of historical recovery rates may be mentioned for example special reports published by rating agencies (like Moody's "Corporate Default and Recovery Rates") or alternatively by researchers (like Altman & Karlin, 2008). Because, we are aware that seniority and security are important determinants of recovery rates as it was shown in Acharya et al. (2003), these characteristics are also considered. Moreover, what past experience also shown is the significant systematic relationship between economic cycles and recovery rates. While, periods of economic boom are usually periods of higher recovery rates (as well as lower default rates), economic downturn means lower recovery rates (and higher default rates). This effect was closely examined (also) by Altman & Hotchkiss (2005) who claimed that bond recoveries might in economic recession decline even to low 25% in corporate bond market depending also on the specific industry. By combining all these aspects of recovery rates we decided to apply the historical overview published

[59] Implausibly complex to incorporate them into our dynamic model

by Altman and Karlin (2008) where the recovery rates are segmented by seniority and major industry sectors. In a result, the company LV is determined in a similar way as it is shown in the next example.

Example: Company XYZ active in general manufacturing with following Balance sheet structure defaulted. What is the (expected) liquidation value of Company XYZ?

Balance Sheet (Company XYZ)

Non-current Assets	**900**	**Liabilities**	**700**
Tangible Assets	750	Senior Secured	300
Intangible Assets	150	Senior Unsecured	150
Current Assets	**100**	Senior Subordinated	100
		Subordinated	100
		Discount	50
		Equity Capital	**300**
Total Assets	**1000**	**Equity & Liabilities**	**1000**

Historical Recovery Rates

(in General Manufacturing industries)

Seniority	Mean	STD
Senior Secured	40.7	25.24
Senior Unsecured	39.09	22.03
Senior Subordinated	32.71	23.81
Subordinated	34.95	21.15
Discount	19.04	24.04

$$Liquidation\ value = Sen.Sec \cdot RR_1 + Sen.Unsec \cdot RR_2 + Sen.Subor \cdot RR_3 + Subor \cdot RR_4 + Discount \cdot RR_5 \quad (3.\ 21)$$

Where RR_i are percentual recovery rates of particular debt classes determined as stochastic variates drawn from Normal distribution with historical mean and standard deviation. The reference table with historical recovery rates for all types of industries and debt classes is attached in *Appendix – Methodology*, *A-M. 2*. Finally, all financial outflows (FCFs and LV) are discounted with scenario and period specific $WACC_{j,t}$ which are calculated as it was defined in *Section 3.3* and default scenario outcome is calculated in the following way:

$$Enterprise\ Value_{D,j} = NPV(materialized\ FCFs) + NPV(Liquidation\ value) \quad (3.\ 22)$$

Eventually, when the model decides whether the actual simulation scenario is defaulted or non-defaulted and calculates the company enterprise value according to the particular valuation approach, the overall outcome as well as a scenario characteristic is stored. Subsequently, all intermediate are erased and a new iteration starts with a drawing of new random variates. In this way our Monte Carlo approach generates any desirable number of realizations – as it was presented in the *Subsection 2.3.5*. In the end, the number of realizations facilitates to present the outcome in a distributional nature which provides us with the important insights of the company dynamics.

3.6 General discussion on parameter estimation

It is evident that the presented valuation model requires numerous input parameters for its practical application. However, many of them may be directly observable from publicly available accounting and financial company reports, others have to be estimated with thorough industry analysis or even substituted with applicable proxies. In this subsection, we briefly elaborate on the parameter estimation in general, suggest some alternative options and indicate the (subjective) difficulty rank for every parameter. We decided to organize the parameters according to their relative estimation difficulty among the three groups, starting from the simplest to the most complicated as follows.

1st Difficulty Rank – Directly observable

The first group of parameters is formed by the simplest ones which are commonly directly observable from the accounting and financial reports, available as public market information or even specified by the Corporate Tax Code of a particular country. Moreover, this group also covers two parameters which represent the basic model settings which can be without any doubt arbitrary selected by the analyst.

Table 3-5: Estimation guide (1)

	Parameter	*Notation*	Proposed Estimation Procedure
Accounting	Initial level of revenues	R_0	Observable from company annual report (income statement)
	Initial rate of growth in revenues	μ_0	Determined according to the company track (past and actual income statements)
	Initial COGS/revenues proportion	γ_0	Observable from company annual report (income statement)
	Capital expenditures	***CAPEX***	Commonly estimated in line with annual reports and analysts' future projections
	Depreciation & Amortization	***D&A***	Commonly estimated in line with annual reports and analysts' future projections
Market / Law	Risk free rate	r_f	Long-term average YTM of 10 year U.S Treasury Bond or other alternative proxy
	Market risk premium	***MRP***	Observable as a spread between r_f and market return (represented by applicable index e.g. MSCI-W, S&P 500 etc.)
	Default spread (bond)	***DS***	Observable as a spread between r_f and return on high yield bond market (represented by applicable index e.g. Bloomberg High Yield Corporate Bond Index)
	Urgent debt spread	***UDS***	Arbitrary determined by analyst according to the market conditions
	Corporate tax rate	τ	Determined according to the applicable Corporate Tax Code
Setting	Time increment	Δt	Chosen according to data availability (quarterly, yearly)
	Projection period	T	Determined in line with analyst's preferences and expectations

Source: *Schwartz & Moon (2001) and own compilation*

2^{nd} Difficulty Rank –Indirectly observable

The second group consists of parameters which cannot be directly observed, however they can be relatively easily estimated by any semi-experienced analyst. The required toolbox necessary for the estimation of these parameters includes a basic statistical calculus (like mean, variance etc.) and OLS regressions.

Table 3-6: Estimation guide (2)

	Parameter	*Notation*	Proposed Estimation Procedure
Accounting	Initial volatility of revenues	σ_0	Estimated as standard deviation of percentage change in revenues over the recent past
Accounting	Initial volatility of COGS/revenues proportion	φ_0	Estimated as standard deviation of percentage change in COGS / revenues proportion over the recent past
Analysis / Peer / Forecast	Long-term volatility of the rate of growth in revenues	σ_{LT}	Estimated based on industry specific volatility of percentual change in revenues measured over peer group analysis or alternatively forecasted with regard to analyst's expectations
Analysis / Peer / Forecast	Long-term rate of growth in revenues	μ_{LT}	Estimated based on industry specific rate of growth in revenues measured over peer group analysis or alternatively forecasted with regard to analyst's expectations
Analysis / Peer / Forecast	Long-term COGS/revenues proportion	γ_{LT}	Estimated based on industry specific COGS / revenues proportion measured over peer group analysis or alternatively forecasted with regard to analyst's expectations
Analysis / Peer / Forecast	Fixed component of OE	α	Determined based on extensive company information or alternatively estimated as a slope coefficient in OLS regression over applicable period
Analysis / Peer / Forecast	Variable component of OE	b	Determined based on extensive company information or alternatively estimated as a variable coefficient in OLS regression over applicable period
Setting	Long-term volatility of COGS/revenues proportion	φ_{LT}	Estimated as standard deviation of percentage change in COGS / revenues proportion over the longer period or alternatively assumed with regard to analyst's expectations
Setting	Mean-reverting coefficients	k_i	Estimated based on assumptions (expectations) about the company restructuring process
Market	Beta factor	β	Estimated as beta of equity according to the variance of market returns (e.g. MSCI index) and covariance of stock-to-market returns. Alternatively may be adjusted with smoothing (mean-reverting) formula

Source: *Schwartz & Moon (2001) and own compilation*

3^{th} Difficulty Rank – Unobservable

Even though the last group consists of only two parameters, it requires the analyst's prime attention during the process of estimation. Since, parameters gathered in this group may be jointly labeled as practically unobservable. It means that we cannot simply observe or infer their value from any data available. To be more specific, we 'talk' about (1) the initial volatility of expected rates of growth in revenues - η_0 and (2) the limit of urgent debt. These parameters require a thorough analysis which extracts the required information from various sources.

Initial volatility of expected rates of growth in revenues (η_0), the parameter originally suggested by Schwartz and Moon (2001), presents a real crux of this model because it is practically unobservable. Fortunately, authors proposed a simple solution to this. According to Schwartz and Moon (2001, pp. 13) η_0 may be substituted by the proxy inferred from the volatility of the stock.

Urgent debt limit (UDL) indicates the maximum level of short-term debt which may be acquired by the company in the urgent situation – as it was explained in the *Section 3.4*. It is obvious that debt capacity is absolutely company-specified characteristic which has to be estimated case-to-case. However, we cannot suggest any universal approach for its estimation, we believe that the *UDL* could be estimated with the aid of alternative default prediction methodologies presented within the *Section 2.4*. Since providing that, all other parameters are accurately estimated, the model likelihood of default should reach the similar likelihood levels as the probabilities estimated thorough the alternative methodologies. The illustrative application of this approach is more extensively presented within the *Subsection 4.3.5*.

Finally please bear in mind that, as inputs among companies differ, the approach which is applied during the process of estimation should reflect the situation and specifications of the company which is being evaluated. Therefore it is desirable to make any assumptions prudently and ideally bolstered with the understanding of industry attributes and dynamics.

In this chapter, we presented a step-by-step construction of our alternative Monte Carlo model focused on valuation of close-to-distress companies. Following the Schwartz-Moon approach (2001) we started with the identification of future uncertainties in the company value drivers. By adaptation to the free cash flow calculus we have arrived to the initial model selection of stochastic and non-stochastic (deterministic) variables. Subsequent formal definitions of all variables together with the required modifications to risk-adjusted settings of our model were followed by the introduction of our alternative 3-statement concept. Following our concept, we introduced additional model rules which specify the company's bankruptcy conditions and the cost of capital. As a result, the valuation calculus is logically specified to default and non-default scenarios. Eventually, the chapter concludes with a brief general discussion on parameter estimation. Since the model is completely defined, now it is time to show and prove its applicability on the real case.

4 Case Study

In this chapter we present an application of the valuation model which was described in the previous *Chapter 3* in order to assess the model on the real case. The brief company selection is immediately followed by initial parameter estimation and set of applicable assumptions. Subsequent sensitivity analysis reveals all critical inputs while the benchmark overview formed with the aid of alternative methodologies, previously presented within the *Section 2.4*, provides essential information for the final model calibration. The chapter concludes with results obtained from the Monte Carlo simulation after all calibrations and brief discussion focused on comparison of the model results with the alternative estimates calculated based on the conventional DCF methodology.

For the purpose of our case study we use the Bloomberg's bond search engine, through which we obtain an initial sample of corporate bonds. Our selection criteria are: (1) Country of origin – Western European Countries, (2) Rating grade[60] – between BB+ and CC+, and (3) Denomination in EUR. The initial sample provided by Bloomberg database consists of 369 different corporate bonds.

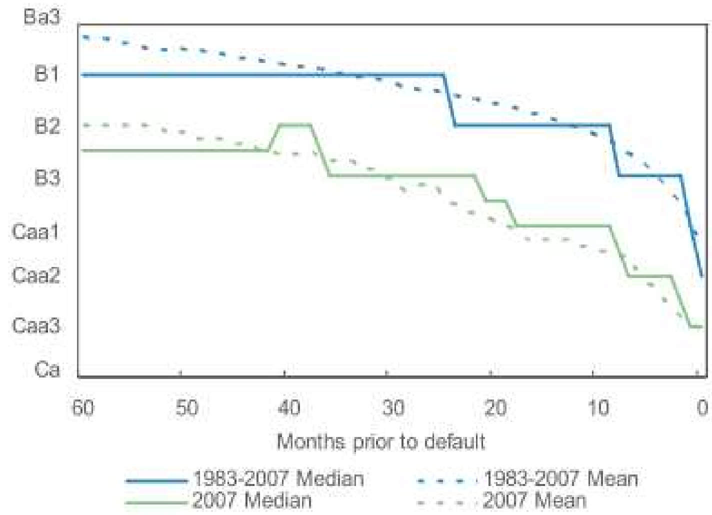
In the second step, we split this sample into two separate groups. The group of 185 corporate bonds issued by companies, generally labeled as financial (e.g. Banks, Special Purpose Entities, Investment and Insurance Companies, etc.) and the group of 184 corporate bonds issued by companies engaged in other (non – financial) business activities.[61] Since we are aware of the fact that companies operating in finance frequently use the so-called 'off-balance items' which may critically bias our simulation results, we decided to exclude them from our sample.

Thirdly, we realize that most of the companies tend to issue more than one batch of bonds, thus we identify 65 companies from the residual sample of 184 corporate bonds. Sample of 65 companies is still too big for individual processing therefore we decided to introduce another selection criterion. Following the Moody's historical credit rating statistics as an effective predictor of default we determine a narrower grade range from B to CCC+. The rationale behind this selection can be interpreted from the following *Figure 4.1* (on the next page), which clearly shows that mentioned range represents the mean grade for the companies which defaulted within following 12 to 24 months. While, we are looking for the companies 'on the edge of distress' this criterion helps us to isolate the sample of 26 companies.

[60] We took into account actual rating grade on June 2010. Decision to select the grade range from BB+ to CC+ was inspired by grading scale itself, *Appendix – Case Study, A-C. 1*. While, BB+ is first grade labeled as „speculative“ and CC+ represents the lower-bound of „prior-to-default“ zone.

[61] For more comprehensive bonds statistics please check *Appendix – Case Study A-C. 2*.

Figure 4.1: Ratings Prior to Default (1983-2007)



Source: *Emery et al. – Moody's (2008)*

Eventually, applying all above mentioned selection steps and also with regard to certain minimal requirements for the company data series we pick the Norwegian paper manufacturer Norske Skog as a reasonable candidate for our case study.

4.1 Company & industry introduction

Norske Skog, Norwegian company with headquarters in Lysaker - Norway founded in 1962, has over the years developed into one of the world's leading producers of paper. The group develops activities principally in two segments of paper business, (1) the production of newsprint and (2) magazine paper. Company formed by a group of 14 paper mills and with 5670 employees all over the world covers around 10 percent of the global market in mentioned industry segments, which amounts to 60 million tonnes of paper per year.

Over the 90's the relatively small company has grown into a big market player through numerous acquisitions. Unfortunately, emerging global crisis has revealed the accumulated oversupply in the paper industry which turned out to be a real problem. Besides the overall decline, the significant debt burden mounted in the years of relative prosperity; has become another critical issue of Norske Skog. During the years 2007 and 2008, the group underwent important cuts, downsizing and divestments. The factories in Norway (Skien), Czech Republic (Steti) and South Korea were closed. Presently, analytics did not rule out the possibility of distress or even bankruptcy. Even though that the company revenues still suffer by a global meltdown (mounting to NOK 20,362 mil. in 2009), the company financial situation stabilized mainly due to the capital obtained by divestments. The following actions performed by the management will resolve the company future.

4.2 Data sources

This section briefly lists all relevant data sources used in our case study of Norske Skog. All data used within a study may be divided into two groups: (1) company data, include all financial statements and various company reports, and (2) market data, ranging from stock prices, bond yields to indices. For the company data and financial statements we use exclusively the official annual reports issued by the company itself. On the other hand, market data are gathered from various sources. The following list covers all relevant sources for market data:

- Bloomberg Finance
- DataStream
- Board of Governors of the Federal Reserve System
- European Central Bank

Please, check the references and footnotes within the text for explicit match of data and source used, as well as for specific data period and subsequent treatment of data.

4.3 Parameter estimation and model calibration

The current section elaborates more on the estimation of concrete input parameters required by our model, which was formulated within the previous *Chapter 3*. We merge the estimation of parameters into three blocks based on the parameters' inherent relations. Each of the next three subsections presents one of the blocks and extensively discusses the way in which certain parameters could be estimated. The chapter then proceeds to additional assumptions which are applied within the model. Eventually, we conclude the chapter with a proper sensitivity analysis and calibration with alternative benchmarks in order to identify the most critical inputs and bolster the model accuracy and relevancy.

4.3.1 Revenue dynamics

In the first block, we focus on the parameters which through the stochastic processes form the simulation forecasts of company revenues. Since, we already extensively elaborated on the assumptions and motivations underlying the formulation of revenue dynamics in the *Subsection 3.2.1*, the current subsection does not explain all these issues again and concentrates only on the estimation of related input parameters. In many cases, the estimation procedures follow the original techniques suggested by Schwartz and Moon (2001).

Revenues

Briefly reminding the *Equations (3.* **1**) and *(3.* 3), which describe the stochastic processes forming company Revenues.

$$\frac{dR(t)}{R(t)} = \mu(t)dt + \sigma(t)dz_1$$
$$d\sigma(t) = k_1(\bar{\sigma} - \sigma(t))dt$$

Starting with the simplest parameter, the initial level of the revenues (R_0) can be easily observable from the actual annual report of the company, which amounts to NOK 20,362 million. The rest of the parameters, like volatilities and rates of growth, are derived from the historical data series. Initial volatility of revenues (σ_0) is derived as a standard deviation of company revenues from 2002 to 2009. For the long-term volatility ($\bar{\sigma}$) we decided to use data excluding the last year results, since they were artificially low due to divestments. The revenues volatilities estimated in this way are 11.09% and 6.19% for initial and long-term, respectively. The estimation of all mean-reverting coefficients (k_1) is jointly discussed within the *Subsection 4.3.3.*

Table 4-1: Company historical revenues and growth rates

Period	**2002**	**2003**	**2004**	**2005**	**2006**	**2007**	**2008**	**2009**
Revenue	23,471	24,068	25,302	25,726	28,812	27,118	26,468	20,362
% growth		2.54%	5.13%	1.68%	12.00%	-5.88%	-2.40%	-23.07%
Standard deviation(2002-2008): **6.19%**					Standard deviation(2002-2009): **11.09%**			

Source*: Company annual reports and Own compilation*

Revenues growth

The growth dynamics in revenues (μ) were in methodology part described by *Equations (3.* **2**) and *(3.* 3), which characterize two separate stochastic processes.

$$d\mu(t) = k_2(\bar{\mu} - \mu(t))dt + \eta(t)dz_2$$
$$d\eta(t) = -k_3\eta(t)dt$$

Because of the fact that initial revenue growth rate should reflect the company capabilities to grow in following period, we realize that application of historical growth rate from 2009 (-23.07%) can be inaccurate estimate of company potential. Since, significant proportion of revenue decline in 2009 was caused by reduction of production capacity as the consequence of company divestment activities (and not only by industry downturn).[62] Although we are still aware of the fact that the company as well as the whole paper industry undergoes tough years; we cannot simply overlook the negative reality. As a solution, we determine the initial revenue growth (μ_0) as an average company growth over the last five years, at level -3.53%. On the other hand, we believe that the company may after restructuring process return to its previous growth potential and sustain a 3% annual growth rate ($\bar{\mu}$) in a long run (after 2014). Unfortunately, initial annual volatility of the growth rate (η_0) is directly unobservable, but as Schwartz and Moon (2001, pp.13) stated that in this case, the volatility derived from a stock price may be used as a proxy. Following this approach we calculate the daily volatility of Norske Skog (NSG.OL) stock price over the long period, from 1st January 2003 to 30th June 2010 and convert it into the annual one.[63] In this way, we arrive to the initial annual volatility of the growth rate equal to 16.17%.

4.3.2 Costs & Expenses dynamics

The second block covers all cash as well as non-cash costs and expenses which the company requires for its operations; excluding the interest expenses (will be discussed in *Subsection 4.3.3*). Alike in the previous block, we avoid the repetition of underlying ideas which were mentioned within the methodology chapter and concentrate this subsection only on procedures directly related to the parameter estimations.

Cost of Goods Sold and Operational Expenses

Cost of Goods Sold (COGS), covering all direct production costs like raw material and labor costs were defined in the methodology chapter by the *Equation (3.* **5**) as a proportion to company revenues following stochastic processes expressed by the *Equations (3.* **6**) and *(3.* **7**).

$$COGS_t = y(t)\,R(t)$$
$$dy(t) = k_4(\bar{y} - y(t))dt + \varphi(t)dz_3$$
$$d\varphi(t) = k_5\big(\bar{\varphi} - \varphi(t)\big)dt$$

The examination of historical trends within a cost and expense structure of Norske Skog over the last eight years provides the necessary information for the estimation of all inputs required for the simulation of COGS. The initial COGS proportion (γ_0) can be easily deduced based on the company figures from 2009 and corresponds to 65.45% of revenues. Similarly, its long-term level (γ_{LT}),

[62] Therefore, this unprecedented decline will not repeat again in next period.
[63] For conversion of daily volatility to annual, we used following formula: $\sigma_{annual} = \sigma_{daily} \cdot \sqrt{252}$

calculated as an average over the eight-year period, equals to 59.76%. These estimations are also in line with the company strategic intentions in reduction of the COGS (increase of gross margin). Company Board of Directors stated that reduction should be achieved by the application of the cost saving plan and stabilization of the raw material prices. The initial volatility of the COGS proportion (φ_0) is determined in a similar way as in the case of revenues volatilities. However, we realize that in 2004 the company evidently underwent changes in accounting principles and a part of operational expenses was moved to COGS (Please, check the *Table 4-2*). In order to avoid a potential bias which may this accounting shift introduce, we calculate the initial volatility of the COGS proportion from the period after 2004. Following this approach we obtain a volatility of 4.51%.

Table 4-2: Company historical Cost of Goods Sold and Operational Expenses

Period	**2002**	**2003**	**2004**	**2005**	**2006**	**2007**	**2008**	**2009**
Revenue	23,471	24,068	25,302	25,726	28,812	27,118	26,468	20,362
COGS	10,765	11,450	15,325	16,006	18,121	17,470	18,287	13,326
% of Revenues	45.87%	47.57%	60.57%	62.22%	62.89%	64.42%	69.09%	65.45%
COGS statistics	Arithm. average(2002-2009): **59.76%**				Standard Deviation(2005-2009): **4.51%**			
Oper. Expenses	8520	7849	5687	6033	6471	5716	5679	5247
% of Revenues	36.30%	32.61%	22.48%	23.45%	22.46%	21.08%	21.46%	25.77%

***Source**: Company annual reports and Own compilation*

Operational Expenses (OE), corresponding to the rest of the company operational overheads, mainly administrative and insurance costs, divided into a fixed and variable part were defined by a linear relationship in the *Equation (3. 9)*.

$$OE(t) = \alpha + b\,R(t)$$

We use a standard OLS regression to determine the fixed part (α) and the variable coefficient (b). Due to obvious reasons (accounting shift) mentioned in the previous paragraph, we use a sample from 2005 to 2009. The following *Table 4-3* reports derived outputs together with the related regression statistic.[64]

Table 4-3: Regression results for Operational Expenses

Period	**2005**	**2006**	**2007**	**2008**	**2009**		
Revenue	25,726	28,812	27,118	26,468	20,362	α	2782.78
OE	6,033	6,471	5,716	5,679	5,247	***b***	11.86%
						R^2	69.24%

***Source**: Company annual reports and Own compilation*

[64] All regressions were performed in econometric software Eviews.

Depreciation & Amortization and CAPEX

Depreciation & Amortization (D&A), is the single non-cash group of company expenses, however it has to be estimated because it affects the operational earnings as well as the value of assets. Initially, we tried to estimate the D&A expenses according to the linear and accelerated schemes, previously discussed in the methodology part. Unfortunately, the results were quite inaccurate, the R^2 for geometric scheme reaches only 46.85% and the simple linear estimate was even worse. Consequently, we decided to apply the conventional approach of regressing the historical D&A expenses to the absolute sum of all assets which are affected by either depreciation or amortization. The regression formula is defined as follows.

$$D\&A_t = \delta_0 + \delta_1(Deteriorating\ Assets'_{t-1})$$

As inputs we use the historical data obtained from company annual reports after 2005.[65] Based on the reports we isolate two groups of deteriorating assets, tangible fixed assets and intangible assets. Of course, the application of one period lag is necessary in order to correctly match the expenses with appropriate level of assets. The following *Table 4-4* presents the outcomes:

Table 4-4: Regression results for Depreciation & Amortization

Period	2005	2006	2007	2008	2009		
D&A Assets	41,151	36,819	28,544	25,499	17,897	$\boldsymbol{\delta_0}$	1293.22
D&A expense	3,072	3,226	2,878	2,623	2,465	$\boldsymbol{\delta_1}$	4.67%
						$\mathbf{R^2}$	89.24%

***Source**: Company annual reports and Own compilation*

Maintenance CAPEX: We derive the maintenance level of capital expenditures from the company annual report. Since, as the company board claimed:*"Capitalized investments in 2009 amounted to NOK 580 million were pure maintenance investments."* (Annual Report 2009, pp. 24). In line with this statement we set the maintenance level of CAPEX at NOK 600 million.

Retaining CAPEX: Likewise, it is reasonable to assume that the company would postpone any spending in times of financial troubles, it should be clear that the company will use the excessive capital in favorable years in order to slacken the reduction of asset value. Following this notion and with regard to our estimate of D&A, we introduce also 'retaining CAPEX' at level of NOK 1,600 million. The specific rule under which models decide to lower or higher level of CAPEX is further discussed within the *Subsection 4.3.4*.

[65] We decided to use 5 year period since it reflects a trend which will most likely prevail in next few years over projection period.

4.3.3 Miscellaneous inputs

In the third block, we merge the rest of inputs which enter our model. The block begins with the mean-reverting coefficients determined according to the assumed convergence period. After the model convergences and correlations we reach the part describing the estimation principles applied to cost of capital.

Convergences and Correlations

Due to a matter of simplification we assume that all mean-reverting processes converge toward their long-term levels at a same pace, $k_1 = k_i\,;\,_{i=1,2,3,4,5}$. This assumption is identical to one used by Schwartz and Moon (2001, pp.14) which stated that the overall speed of adjustment should be determined in line with assumptions about the convergence rate of revenue growth. Since, this convergence factor has the highest impact on the overall result. Thus, following our previous assumption about revenues growth (*Subsection 4.3.1*) we set the convergence period equal to projection period. That reflects our beliefs that company drivers will converge to their long-term levels in the end of the projection period (2014). Below we summarize the long-term levels towards which all mean-reverting processes used in this model converge. (where, T = 5 simulation periods)

$$\sigma(T) = \bar{\sigma} \qquad \varphi(T) = \bar{\varphi}$$
$$\mu(T) = \bar{\mu} \qquad \eta(T) = 0$$

In the initial settings of our model we assume that all three random variates are uncorrelated, $\rho_{12} = \rho_{13} = \rho_{23} = 0$. However, the effect of correlations will be further discussed and measured within the sensitivity analysis of the model in *Subsection 4.3.5*.

Cost of Capital and Taxes

Cost of Debt - short-term tranche (CoD_S), was defined in methodology by the *Equation (3.* 12).

$$CoD_S = Interbank\ rate^{6M} + Spread$$

Where, the 6-month interbank rate is determined as the average 6M EURIBOR rate over the period starting from December 1998 to July 2010, obtained through DataStream. Since, urgent debt to the company is obviously associated with a higher risk than standard interbank deals; we decided to add a spread of 100 bps over the average interbank rate. In this way, we obtain the average interbank rate of 3.13% and CoD_S of 4.13%.

Cost of Debt - long-term tranche (CoD_L), was defined in methodology by the *Equation (3.* **13**).

$$CoD_L = r_f + Default\ spread_{Bond-based}$$

Where, r_f is derived as the average yield-to-maturity (YTM) of 10Y U.S. Treasury Bonds over the last 20 years, downloaded from the Board of Governors of the Federal Reserve System. The bond default spread is determined as a spread between mentioned YTM of 10Y U.S. Treasury Bonds and YTM on High Yield U.S. Corporate Bond Index provided by Bloomberg. In this way we obtain the r_f equal to 5.29% and spread of 3.89%, so the CoD_L reaches the rate of 9.17%.[66]

Cost of Equity (CoE), was formulated in methodology by the *Equation (3.* 14).

$$CoE = r_f + \beta_{E-adj}(r_m - r_f)$$

In the case of CoE, we use the same r_f rate, which was derived above. The market return (r_m) is estimated as the average return of MSCI World Index over the last 20 years, equal to 12.20%. The raw beta of the stock (β_E) is calculated against MSCI World Index in a way described by the *Equation (3.* **15**). We calculated the raw beta from the daily returns (MSCI WRD vs. NSG.OL) over the period starting on 1st January 2003 to 30th June 2010, resulting in a beta of 1.17. Afterwards, we transform this raw beta of stock into adjusted beta (β_{E-adj}) applying the Bloomberg's smoothing formula[67], resulting in adjusted beta of 1.12. Thus, in a result we obtain a CoE of 13.03%.

Eventually following the *Equation (3.* **16**), we calculate actual discount factor (WACC) according to previously derived costs of different classes of capital. The WACC implied in this way (10.94%) fits to the rough company estimate (8.5 – 11.7%) which was mentioned in the company annual report (Annual Report 2009, pp. 55).

[66] It is good to add, that we calculate both risk-free rate as well as default spread from monthly data. Additionally, the High Yield U.S. Corporate Bond Index was established at August 2002, so in this case we were limited to data period from September 2002 to June 2010.

[67] The idea underlying Bloomberg's smoothing formula assumes that company true (future) beta will move towards the market equilibrium – one. Therefore, we adjsuted the raw beta obtained from historical company in the following way: $\beta_{E-adj} = 1/3 + 2/3\beta_E$

Corporate Tax: Since, Norske Skog is a Norwegian company we follow the Norwegian Commercial Law and set the level of corporate tax rate at 28%. This assumption is also in line with a rate applied by the company for results presented within the annual reports.[68] Up to this point we defined all input variables, for the inputs overview please check the *Appendix – Case Study, A-C. 3.*

4.3.4 Closing assumptions

The current subsection covers several additional assumptions mainly with regard to the sequence of cash spending which has to be determined within the model. The *Table 4-5* below presents the sequence of cash spending in which the model flows and makes decisions. The whole process is divided into three stages based on absolute 'survival' priority of the company.

Table 4-5: Sequence of cash spending

Priority	Company obligations
Step 1	Total Operational Costs
Step 2	Net Financing Costs
Step 3	Taxes
Step 4	Current Debt Obligations
Step 5	Acquire Urgent debt?
Step 6	CAPEX growth or CAPEX maintenance
Step 7	Unscheduled Debt Repayment

Source: *Own Compilation*

Red zone (Critical)

The red zone is formed by absolutely minimal level of cash which is required to cover company operational activities. Namely, it presents the cash required to cover Cost of Goods Sold, Operational Expenses, Interest Expense, Taxes and Current Debt Obligations. The company has to withstand this minimal level of expenses at any circumstances, otherwise it directly busts. In case the company misses the cash to cover its operational obligations an urgent loan may be acquired to overlap actual gap in earnings. Total amount of cash acquired from the loan is moreover increased by the optimal level of cash. So, after the repayment of all obligations, company has the minimal level of cash for ongoing operational expenses in the following period. We determine the optimal level of cash based on historical (pre-crisis) levels of Cash & cash equivalents at amount of NOK 600 million[69].

[68] *"Results are presented net of tax, using the Norwegian statutory rate of 28%."* (Annual Report 2009, pp. 66)

[69] The optimal amount of cash was determined according to the pre-crisis levels of company. We used pre-crisis data since the level of cash was over 2008-2009 artificially high due to significant divestment activities.

Yellow zone (Recommended)

The yellow zone stands for investment outlays (CAPEX spending) which are in fact optional, but also a necessary prerequisite for a future revenue growth. Following our proposition from the *Subsection 4.3.2* the model distinguishes between the two types of CAPEX levels. The maintenance level (1), representing the minimal level of investment outlay in order to sustain the current assets in operational status and the growth level (2) which covers besides basic maintenance also additional replacement of deteriorating assets. The model selects the level of CAPEX according to the company financial situation. Basically, if the level of company remaining cash (after withstanding of all operational expenses) exceeds the CAPEX plus optimal level of cash, the certain level of CAPEX takes place.[70] The requirement on optimal level of cash can be motivated with the same arguments as in the previous paragraph.

Green zone (Optional)

Eventually, in case the level of cash is still excessive after the withstanding of all operational expenses and CAPEX, the unscheduled repayment of debt may take place. However, it is reasonable to assume that company prefers to hold certain amounts of cash reserves above the optimal level of cash for the unexpected adverse situations. Therefore, after thorough analysis of upcoming company obligations we decided to set this level to NOK 2000 million. At this stage it is desirable to add that optimal level of cash and reserves has to be determined case-to-case. Lastly, we assume that debt may be repaid in any amount without sanctions.[71]

4.3.5 Sensitivity analysis and Benchmark calibration

Sensitivity analysis

After the estimation of all the model variables described in the previous sections we focus on the measurement of their individual influence on overall output in order to identify the most critical ones and reveal the potential 'red flags' in our model. We evaluate the relative parameter influence based on sensitivity analysis which proceeds in the following order:

1. Perform simulation with originally estimated parameters and record output.
2. Select one of the model parameters.
3. Change the value of selected parameter and ceteris paribus.

[70] Of course, the model tests this condition for both levels of CAPEX, then decides for one type.
[71] We believe that this assumption cannot significantly violate the overall model outcomes. However, the model may be easily adjusted to any repayment setting but this requires additional information.

4. Perform a new simulation with 'adjusted' parameter value and record a new output.
5. Evaluate the parameter influence by comparison against the original output.
6. Return the parameter into original value and select another parameter or setting.

In this way we measure the relative influence of 11 most critical and controversial parameters, while our sensitivity simulations are based on 10 000 iterations. In order to make observations comparable we measure the influence of each parameter under identical shift settings (-50%,-25%, +25%, +50%).[72] The influence is then expressed in the percentual change for both outputs, (1) Enterprise value - EV as well as for (2) Cumulative default probability - CDP. Based on these observations we are able to identify the three most influential (critical) parameters, likewise we can assure ourselves that some of assumptions which had to be made with the lack of relevant information have almost no influence on the results.

It is no surprise that long-term COGS/Revenues proportion (γ_{LT}) turns out to be the most critical parameter of our simulation. The reasoning is quite simple, since this proportion indirectly implies the level of company gross margin. Therefore, even a small change may set a company into existential problems. For illustration, the relative influence on the company EV measured by our sensitivity analysis is +117, +58%, -37% and -43% for our sensitivity settings respectively. The CDP reacts even stronger and reaches almost 100% for the +50% setting. As expected, the second most influential parameter is the long-term estimate of growth in revenues (μ_{LT}) which has relatively lower influence than γ_{LT} but is still highly significant. The sensitivity changes lead to a shift of –6, -2%, +3% and +6% in the EV, while shifts in the CDP are opposite.[73] And the third most influential parameter is the debt limit (*DL*) which affects mainly the CDP. To be more specific, sensitivity changes turns out to +57, +25%, -29% and -55% of CDP, respectively. Furthermore, we also measure the influence of other parameters like all volatilities, correlations, optimal levels of cash and reserves; however most of them prove have low or negligible influence. The comprehensive presentation of all the results could be excessive, thus the interested reader may check the *Appendix – Case Study, A-C. 4* for the complete outcome of the sensitivity analysis. Since, the sensitivity analysis proves the unsurprising fact that a limit for urgent debt is one of the most critical parameters in our model we elaborate more on this topic in following paragraph.

[72] The correlations are exemption; since they were originally set to 0 we test them at levels of 0.2, 0.5 and 0.8.

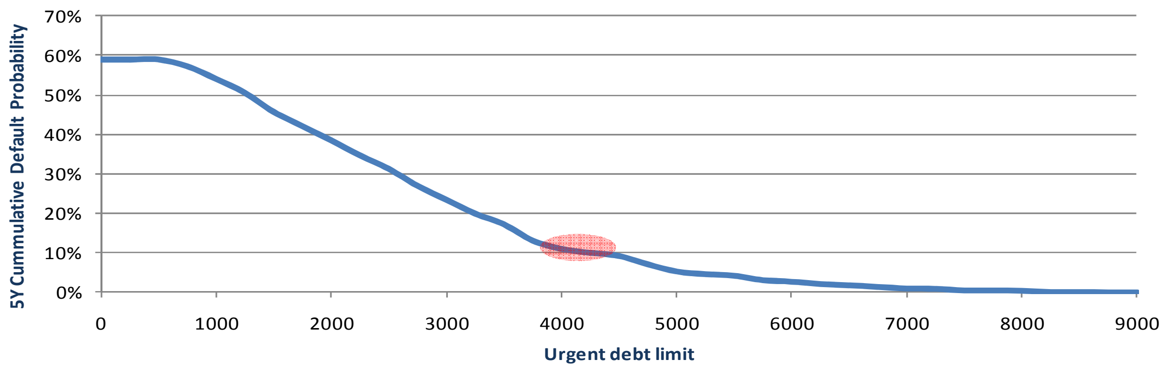
[73] It is no surprise that shifts in the parameters values result in the changes on the EV and CDP which are negatively correlated. This may be easily explained, because these results just confirm the obvious underlying relationship between the company enterprise value and its default rate. The value of the liquidated company is certainly lower than the value of going concern, so higher default rate means also lower expected enterprise value and vice versa.

Alternative benchmark calibration

The previous analysis has clearly shown that the limit for urgent debt (*DL*) is one of the key parameters determining the simulation outcome. In this paragraph, we try to convince the reader that our initial estimate of *DL* parameter is not determined by the rule of thumb but its value is reasonable and in line with reality.

We decided to examine the problem of company debt capacity with mathematical rigour. In order to determine the optimal level of *DL* we firstly perform a new set of sensitivity simulations, focused exclusively on *DL*. The sample of observation obtained in this way reveals the inherent relationship between the level of *DL* and company cumulative default probability. Following *Figure 4.2* presents the results of our investigation.

Figure 4.2: Relationship between CDP and DL



Source: *Own Compilation*

The results are not surprising, the obvious breakeven zone (marked with red zone) formed around the level of critical scheduled debt repayment, which based on the company annual report amounts to NOK 4037 million in 2012. Another important source to our analysis is the overview of default predictions determined by alternative methodologies, which were presented within the theoretical *Section 2.4*. The benchmark overview consists of predictions based on following three methodologies (1) Altman's Z''-Score model, (2) Ohlson's O-Score model and (3) Credit Ratings.

Altman's Z''-Score model was previously presented within the *Subsection 2.4.2*. However, the original model provides only 'dummy' output without any exact measure, Altman & Hartzell (1995) performed the empirical research and introduced the transformation table through which it is possible to approximately convert the company Z''-Score into the credit rating.[74] Thus, we calculated the company Z''-Score based on actual company data which equals to 4.69. That would be according to mentioned table approximately converted into the credit rating of BB / B+.

[74] Mentioned transformation table is attached in *Appendix – Case Study, A-C. 1*.

Ohlson's O-Score model was another default prediction model presented within the theoretical *Subsection 2.4.3*. On the contrary to Altman's model, the O - score model is capable to directly determine the default probability through the logistic transformation formula which was mentioned previously. Unfortunately, the probability measure provided by this model is more general so we cannot exactly determine the 5-year cumulative default probability. The calculation of the company O – Score required actual company data and for determination of the size factor we use the GNP Implicit Price Deflator (1968=100), obtained from the U.S. Department of Commerce: Bureau of Economic Analysis. The O – Score calculated in this way equals to -1.98 which refers to expected default probability of 15.96%.

Credit Ratings Methodology is the last approach incorporated in our calibration (*Subsection 2.4.6*). In this case, we follow two research papers Altman & Karlin (2008) and Emery et al. (2008) which basically provide comparable outcomes. Both reports provide the tables which indicate the historical cumulative as well as the marginal default rates, against a company credit rating. The Norske Skog was on the 31th of December 2009 rated by Moody's – B3 and by S&P – B, both with negative outlook. These grades may be transformed into the 5-year expected cumulative default probability of 35.501%. The following *Table 4-6* provides an overview of default probabilities determined by our analysis.

Table 4-6: Alternative benchmarks for default probabilities

Altman's Z''-Score[75]		Ohlson's O-Score		Credit Ratings[76]	
Z''- Score	4.69	O – Score	-1.98	Moody's Rating	B3 (neg.)
Rating Equivalent	BB / B			S&P Rating	B (neg.)
Cum. default prob.	11.90% / 27.54%	Default prob.	15.96%	Cum. default prob.	35.50%

Source: *Own Compilation*

To summarize our effort, the presented overview and *Figure 4.2* provide some reasonable insights which help us to determine the reasonable limit for the urgent debt. While, Ohlson's O – Score model provides only an 'informative hint', the Altman's Z''-Score and (Moody's) credit rating provide an indicative default range. Therefore, our initial estimate of NOK 2500 million for urgent debt limit turns out to be realistic, since this model setting results in CDP around 31% (average of range, 27.54% – 35.50%).

[75] The cumulative default probability was determined according to Altman & Karlin (2008), corresponding table is attached in the *Appendix – Case Study, A-C. 5.*
[76] The cumulative default probability was determined according to Emery et al. (2008–Moody's), corresponding table is attached in the *Appendix – Case Study, A-C. 6.*

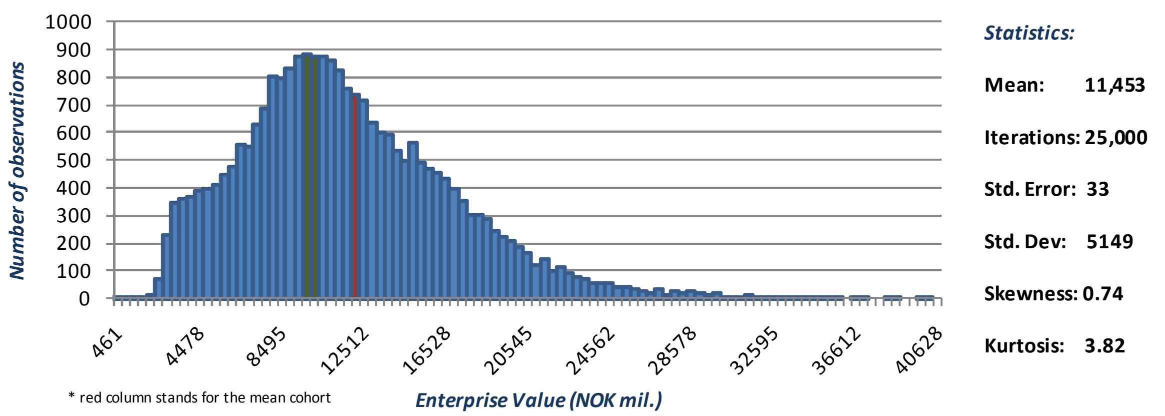
5 Results

The following chapter discusses the result of (risk-adjusted) valuation of Norske Skog's based on our simulation approach.

5.1 Monte Carlo simulation

We designed our valuation model in Microsoft Excel as it was shown in the methodology *Chapter 3*, while the enterprise value is determined according to valuation calculus presented in the *Subsection 3.5*. Estimation of the company-specified input variables follows particular procedures described in *Chapter 4*. Subsequently, when all model prerequisites are fulfilled we perform the Monte Carlo simulation with the aid of Risk AMP v 3.06 (Excel's add-on designed for Monte Carlo simulations). Following *Figure 5.1* presents the distribution of Norske Skog enterprise value obtained by simulation based on 25 000 iterations.

Figure 5.1: Distribution of Norske Skog Enterprise value (simulated)



Source: *Own Compilation*

As can be seen from the figure, the enterprise value of Norske Skog absolutely does not follow the standard Normal distribution. The statistical characteristics even more highlight this fact, since distribution skewness reaches 0.74 (peak) and kurtosis is equal to 3.82 (obvious asymmetry). Furthermore, it can be said that the (logical) downside limitation of the company value cuts the negative tail of distribution. Thanks to the distributional nature of the simulation outcome, we have a relative flexibility in the outcome interpretation. While, conventional DCF methodologies provide one point estimate; through the distribution our model provides us more relevant information underlying this outcome. From a purely statistical point of view, the mean of simulation (NOK 11,453 million) should be our best guess estimate of the company enterprise value.

On the other hand, we can move further and use additional information provided by the distribution. There are at least two other important findings which have to be taken into account. The percentile histogram can show that in more than 55% of observations the company enterprise value does not exceed (or even reach) the mean value as a result of a considerable default probability. The second finding is also linked to the distribution asymmetry and mainly to the contrast between the significant fat upside-tail and the limited downside-tail. It is obvious that this feature affects the position of the mean (upside bias). Therefore, from purely investment point of view, the results may be interpreted in a less frequent way, based on the maximum likelihood approach (and not mean based). According to this approach, the columns (green) which form a histogram plateau indicate the most probable range of the company enterprise value. Following this notion we would interpret the results into a more conservative estimate of NOK 10,808 million.

In order to assess our estimate we perform also two comparable estimations based on the (1) Conventional DCF methodology and (2) Going Concern methodology. Both comparables are estimated with exactly the same underlying assumptions (used in MC simulation) and in the case of Going Concern approach we apply the historical cumulative default probabilities published by Altman & Karlin (2008).[77] Eventually, we add also some publicly available estimates. The following *Table 5-1* presents the overview of results obtained from various sources.

Table 5-1: Comparison of results (NOK mil.)

Methodology / Source	Enterprise Value
Conventional DCF	11,001
Going Concern DCF’	10,787
MC Simulation (statistical)	11,453
MC Simulation (investment)	10,808
DataStream estimate’	11,232
Thomson One Banker estimate’	11,845

Another outcome of our simulation is the overview of the company marginal as well as cumulative default probabilities. Since, the company has specific debt repayment schedule with one extremely big repayment mounting to NOK 4037 million, most of the simulation default occurred due to this obligation. Additionally, the company possesses high initial cash balance gathered from previous divestment activities which provides a strong liquidity cushion for initial periods of simulation. These two facts certainly affected the overall default distribution. The cumulative default probability over the projection period reached 30.81% (for 25 000 iterations). These outcomes draw

[77] The corresponding table is attached in the *Appendix – Case Study, A-C. 5.*

a logical conclusion that rating agencies rate the company with such a low grade since analysts are afraid of mentioned debt repayment. Even though that we do not need any complex model to realize this issue, it shows that our model is in line with reality.

5.2 Further discussion

Although we believe that our model can provide an estimate potentially superior to the conventional valuation methodologies, we are also aware its disadvantages. People used to blame Monte Carlo approach for time requirements on the model construction. From our perspective, this is not entirely justified; construction of the model could be quite swift if the person designing the model has a clear model concept. On the other hand, it is true that principal formulation of the model is truly time consuming, but most of the analyses performed by us during the designing process belong to necessary (everyday) analysts' toolbox.

Anyway, we are aware the fact that our model is not absolutely perfect and we believe that there is a space for modifications to new directions as well as also for potential improvements mainly with regard to some of assumptions (estimations) made during the designing process.

The first potential issue which has not been addressed yet is the seasonality, while performance in certain industry segments is significantly affected by the business cycle. Therefore, the estimation of input variables as well as application of our model in a way we described previously would be for troublesome. This direction would be a suitable topic for subsequent adaptation of the model.

Furthermore, during the course of input estimation we made several controversial assumptions due to a lack of relevant information or essentially based on alternative proxies. There are at least two potential 'red flags' which may require additional improvements, estimation of volatility of revenue growth and the determination of company debt capacity. In the case of revenue growth volatility, we can suggest an interesting starting point - the research paper of Litterman & Winkelmann (1998) dealing with an alternative approach to estimating the covariance matrices.

Lastly, some additional effort can be made towards a more precise approach of estimation of the debt capacity. Though, our approach provides a reasonable estimate, it is entirely dependent on the assumption that all input variables are estimated accurately and moreover that a credit rating is the effective and accurate predictor of a company default rate. Thus, an alternative (independent) approach would bolster the overall model quality.

6 Conclusions

The valuation of problematic companies presents in business practice a complex problem of prime importance. In this thesis we attempted to construct a dynamic valuation model which is capable to estimate a company enterprise value more accurately in comparison to conventional methodologies. Initially, we examined the most controversial aspects of distress valuation with special attention to traditional valuation techniques. Equipped with a fundamental understanding of key issues of distress valuation we rebuilt and extended the original Schwartz & Moon valuation model. On the contrary to original model, we designed our model in order to perform the risk-adjusted valuation for a company on the edge of distress.[78]

Afterwards, we prove the model applicability by performing the illustrative case valuation of the Norwegian paper producer Norske Skog. Based on publicly available data and following our methodology we built a complete Monte Carlo model and estimated the company specified input parameters. Subsequently, performed simulation which consisted of 25,000 iterations proved our initial expectations that value of the company with considerable likelihood of default does not follow the Normal distribution. Although, the statistical outcome did not differ a lot from simple DCF estimate, the additional information provided by the simulation shown that a prudent investor should prefer considerably lower estimate of the company value.

During the process of construction we have had to make many critical assumptions which form the overall model outcome. Although, this feature (critical assumptions) is quite common among all valuation methodologies, we have done our best to determine all required inputs absolutely conservatively and objectively.

Eventually, since we believe that our model should be perceived more as a conceptual framework than a strict valuation procedure, we mentioned several suggestions for the further improvement and research.

[78] The Schwartz & Moon model provides risk-neutral valuation and is designated to the valuation of high growth companies.

References

Literature references:

Acharya, V.V., Bharath, S.T. and Srinivasan A., 2003, *"Understanding the Recovery Rates on Defaulted Securities"*, Working Paper, London Business School.

Andrade, G., and Kaplan, S.N., 1998, *"How Costly is Financial (Not Economic) Distress?- Evidence from Highly Levered Transactions that Became Distressed"*, The Journal of Finance, Vol. 53, No.5, pp. 1443-1493.

Altman, E.I., 1968, "Financial Ratios, Discriminant Analysis and the Prediction of Corporate Bankruptcy*"*, Journal of Finance, pp 189-209.

Altman, E.I., 1970, *"Ratio Analysis and the Prediction of Firm Failure: A Reply"*, The Journal of Finance, Vol. 25, No. 5, pp. 1169-1172

Altman, E.I., 1984, "*A Further Empirical Investigation of the Bankruptcy Cost Question",* Journal of Finance (September).

Altman, E.I, Haldeman, R. and Narayanan, P., 1977, *"ZETA Analysis: A New Model to Identify Bankruptcy Risk of Corporations"*, Journal of Banking and Finance.

Altman, E.I. and Hotchkiss, E., 2006, *"Corporate Financial Distress and Bankruptcy: Predict and Avoid Bankruptcy, Analyze and Invest in Distress Debt"*, John Wiley & Sons Inc.

Altman, E.I. and Karlin, B.J, 2008, *"Defaults and Returns in the High-Yield Bond Market: The Year 2007 in Review and Outlook"*, Stern School of Business.

Arzac, E.R., 2005, *"Valuation for Mergers, Buyouts, and Restructurings",* John Wiley & Sons Inc.

Beaver, W., 1966, *"Financial Ratios as Predictors of Failures, Empirical Research in Accounting",*

Journal of Accounting Research, January 1967.

Black, F., Scholes, M., 1973, *"The Pricing of Options and Corporate Liabilities",* Journal of Political Economy, Vol. 81 (3), pp. 637-654.

Chen, Y., Weston, J.F., Altman, E.I., 1995, "*Financial Distress and Restructuring Models"*, Financial Management 2 (Summer), pp. 57-75.

Cox, J.C., Ross, S.A., and Rubinstein, M., 1979, *"Option Pricing: A Simplified Approach"*, Journal of Financial Economics, Vol. 7, pp. 229-263.

Crosbie, P., Bohn, J., 2003, *"Modeling Default Risk: Modeling Methodology"*, Moody's KMV Company.

Crystal, M., and Mokal, R.J., 2006, "*The Valuation of Distressed Companies – A Conceptual Framework",* International Corporate Rescue, Issues 2 and 3.

Damodaran, A., 2002, *"Dealing with Distress in Valuation"*, Stern School of Business.

Damodaran, A., 2006, *"The Cost of Distress: Survival, Truncation Risk and Valuation"*, Stern School of Business.

Damodaran, A., 2004, *" Equity Risk Premiums"*, Stern School of Business.

Damodaran, A., 2009, *"Valuing Distressed and Declining Companies"*, Stern School of Business.

DeAngelo, H. and Masulis, R.W., 1980, *"Optimal capital structure under corporate and personal taxation",* Journal of Financial Economic, Vol. 8, pp. 3-29

Diba, B., Guo, C.H. and Schwartz, M., 1995, *"Equity as a Call Option on Assets: Some Tests for Failed Banks"*, Economic Letters, Vol. 48, pp. 389-397.

Dixit, A.K., and Pindyck, R.S, 1994, *"Investment under Uncertainty"*, Princeton University Press.

Dwyer, D., Kocagil, A. and Stein, R., 2004, *"The Moody's KMV EDF™ RISKCALC™ v3.1 Model Next Generation Technology or predictive private firm credit default risk"*, Moody's KMV Company.

Emery, K., Ou, S. and Tennant, J., 2008, *"Special Comment: Corporate Default and Recovery Rates 1920 – 2007"*, Moody's Investors Service.

Falkenstein, E., Boral, A. and Carty, L.V., 2000, *"RiskCalc for private companies: Moody's default model",* Moody's Investors Service Global Credit Research.

Gibson, R. and Schwartz. E.H., 1990, "*Stochastic Convenience Yield and the Pricing of Oil Contingent Claims*", Journal of Finance, vol.45, no 3, pp.959-976.

Gryglewicz, S., 2010, *"A theory of corporate financial decisions with liquidity and solvency concerns"*, Journal of Financial Economics, forthcoming

Hillegeist, A.S., Keating, E.K., Cram, D.P. and Lundstedt, K.G., 2003, *"Assessing the Probability of Bankruptcy"*, Review of Accounting Studies, No.9, pp 5-34.

Litterman, R. and Winkelmann, K., 1998, *"Estimating Covariance Matrices"*, Goldman Sachs & Co.

Kealhofer, S. and Kurbat, M., 2001,*"Predictive Merton Models"*, Risk (February), pp. 66-71.

Kealhofer, S. and Kurbat, M., 2001, *"The Default Prediction Power of the Merton Approach, Relative to Debt Ratings and Accounting Variables"*, KMV LLC.

Kemsly, D. and Nissim, D., 2002, *"Valuation of the debt tax shield"*, The Journal of Finance, Vol. 57, no 5, pp. 2045–2073

Kloeden, P.E. and Platen E., 1992, *"Numerical Solution of Stochastic Differential Equations"*, Springer.

Koller, T., Goedhart, M., and Wessels, D., 2005, *"Valuation, Measuring and Managing the Value of Companies"*, John Willey & Sons Inc.

McDonald, R., 2002, *"Derivatives Markets"*, Northwestern University, 1st edition.

Miles, J.A. and Ezzell, J.R., 1985, *"Reformulating tax shield valuation: A note"*, The Journal of Finance, Vol. 40, no. 5, pp. 1485–1492.

Modigliani, F. and Miller, M., 1958, *"The Cost of Capital, Corporate Finance and the Theory of Investment"*, American Economic Review (June).

Modigliani, F. and Miller, M., 1963, *"Corporate Income Taxes and the Cost of Capital: A Correction"*, American Economic Review (June).

Moyer, S.G., 2005, *"Distressed Debt Analysis: Strategies for Speculative Investors"*, J.ROSS Publishing .

Ohlson, J.A., 1980, *"Financial Ratios and the Probabilistic Prediction of Bankruptcy"*, Journal of Accounting Research, Vol. 18, pp. 109-130.

Palepu, K.G., Healy, P.M., Bernard, V.L. and Peek E., 2007, *"Business Analysis and Valuation: IFRS Edition"*, Cengage Learning.

Rosenbaum, J. and Pearl J., 2009, *"Investment Banking: Valuation Leveraged Buyouts and Mergers & Acquisitions"*, pp. 181.

Schwartz, E.S. and Moon, M., 2000, *"Rational Pricing of Internet Companies",* The Financial Analyst Journal May/June 2000, pp. 62-75.

Schwartz, E.S. and Moon, M., 2001, *"Rational Pricing of Internet Companies Revisited"*, The Financial Review, Vol. 36, pp. 7-26.

Smit, H.T.J and Trigeorgis, L., 2004, *"Strategic Investment, Real Options and Games",* Princeton University Press.

Sobehart, J. and Keenan, S., 2001, *"Measuring default accurately"*, Credit Risk Special Report, Risk 14,

pp. 31-33.

Sobehart, J., Keenan, S. and Stein, R., 2000, *"Benchmarking Quantitative Default Risk Models: A Validation Methodology"*, Moody's Investors Service.

Sobehart, J., Stein, R., Mikitkyanskaya V. and Li, L., 2000, *"Moody's Public Firm Risk Model: A Hybrid Approach to Modeling Default Risk"*, Moody's Investors Service Special Comment (March).

Weston, J.F., 1963, *"A Test of Cost of Capital Propositions"*, Southern Economic Journal (October)

Data resources:

- Bloomberg Finance via www.eur.nl/ub
- DataStream via www.eur.nl/ub
- Board of Governors of the Federal Reserve System via http://research.stlouisfed.org/
- U.S. Department of Commerce: Bureau of Economic Analysis via http://research.stlouisfed.org/
- European Central Bank via www.ecb.int
- Norske Skog Annual reports 2002 – 2009 via www.norskeskog.com

Appendices

Appendix – Theory

Conventional valuation methodologies (step-by-step formula sheet)

Discounted cash flow methodology (DCF)

Enterprise value formula under DCF

$$Enterprise\ Value = NPV(FCFs) + NPV(Terminal\ value) \quad \text{(A.1)}$$

Discounting process (net present value)

$$NPV(FCFs) = \frac{FCF_1}{(1+WACC)^1} + \frac{FCF_2}{(1+WACC)^2} + \ldots + \frac{FCF_n}{(1+WACC)^n} \quad \text{(A.2)}$$

Weighted Average Cost of Capital formula

$$WACC = r_d(1-t) \bullet \frac{D}{V} + r_e \bullet \frac{E}{V} \quad \text{(A.3)}$$

Alternative perpetuity formulas

A-T. 1: Perpetuity formulas with underlying assumptions

Model Type	Formula	Assumptions
Aggressive growth formula	$TV = \frac{NOPAT}{WACC - g}$	• The company generates a return on new investment which is approaching infinity • Unrealistic and rarely used
Value driver formula	$TV = \frac{NOPAT(1 - \frac{g}{RONIC})}{WACC - g}$	• The company generates a return on new investment higher than the WACC • Only be used if the company is expected to maintain strong competitive advantage
Convergence model	$TV = \frac{NOPAT}{WACC}$	• The company generates a return on new investment equal to the WACC. • To be used in competitive industries which move towards a equilibrium

Source: *Drs. Hans Haanappel, lecture slide 20.*

Adjusted present value approach (APV)

Enterprise value formula under APV

$$Enterprise\ value = \sum \frac{FCF}{1 + WACC_{Unlevered}} + \sum \frac{Tax\ shields}{1 + r_{Tax\ shields}} \quad (A.4)$$

Weighted Average Cost of Capital - unlevered

$$WACC_{Unlevered} = r_f + \beta_{Unlevered} \bullet (r_m - r_f) \quad (A.5)$$

Unlevered Beta determination

$$\beta_{Unlevered} = \frac{(\beta_E \cdot E) + (\beta_D \cdot D)}{E + D} \quad (A.6)$$

Comparable multiples analysis (MA)

Simple multiple ratio categorization

- *Earnings Multiples* – relate value of company to profitability (PE, EV/EBITDA, EV/EBIT)
- *Sales Multiples* – relate value to sales (P/Sales, EV/Sales)
- *Market to Book Multiples* – relate value to book value (Price to Book, EV/Operating Capital)
- *Industry Specific Multiples* – relate value to some industry metric (e.g. Value per Subscriber).

Source*: Drs. Hans Haanappel, lecture slides*

Black – Scholes – Merton model

The Black – Scholes - Merton model for valuing equity as a European call option is defined by the following formula and consequently the company default probability can be derived in following way (Hillegeist et al., 2003):

$$V_E = V_A e^{-\delta T} N(d_1) - X e^{-rT} N(d_2) + \left(1 - e^{-\delta T}\right) V_A \quad (A.7)$$

While:

$$d_1 = \frac{\ln\left(\frac{V_A}{X}\right) + \left(r - \delta + \frac{\sigma_A^2}{2}\right) T}{\sigma_A \sqrt{T}} \quad (A.8)$$

$$d_2 = d_1 - \sigma_A \sqrt{T} = \frac{\ln\left(\frac{V_A}{X}\right) + \left(r - \delta - \frac{\sigma_A^2}{2}\right) T}{\sigma_A \sqrt{T}} \quad (A.9)$$

Where:

$V_A = current\ market\ value\ of\ assets$
$V_E = current\ market\ value\ of\ equity$
$\sigma_A = standard\ deviation\ of\ asset\ returns$
$X = face\ value\ of\ debt\ maturing\ at\ time\ T$
$r = continously\ compounded\ risk - free\ rate$
$\delta = continous\ dividend\ rate\ expressed\ in\ terms\ of\ V_A$
$N(\cdot) = \ standard\ cumulative\ normal\ distribution$

Assuming that the natural log of future asset values is distributed normally as follows, while μ is the continuously-compounded expected return on assets:

$$lnV_A(t) \sim N\left[lnV_A + \left(\mu - \delta - \frac{\sigma_A^2}{2}\right)t, \sigma_A^2 t\right] \tag{A.10}$$

Then default probability can be determined as follows:

$$\pi_{BSM-default} = N\left(-\frac{\ln\left(\frac{V_A}{X}\right) + \left(\mu - \delta - \frac{\sigma_A^2}{2}\right)T}{\sigma_A\sqrt{T}}\right) \tag{A.11}$$

Providing:

$$\sigma_E V_E = e^{-\delta T} N(d_1)\sigma_A V_A \tag{A.12}$$

And

$$\mu(t) = max\left[\frac{V_A(t) + Dividents - V_A(t-1)}{V_A(t-1)}, r\right] \tag{A.13}$$

Default prediction methodologies

Z-type alternatives

Alternative modifications of Altman's Z-Score model are defined as follows (Altman, 2000):

Z' Score (for privately held companies)

$$Z' = 0.717X_1 + 0.847X_2 + 3.107X_3 + 0.420X_4 + 0.998X_5 \tag{A.14}$$

Where:

$$X_1 = \frac{Working\ capital}{Total\ assets} \qquad X_4 = \frac{Market\ value\ of\ equity}{Book\ value\ of\ total\ liabilities}$$

$$X_2 = \frac{Retained\ earnings}{Total\ asset} \qquad X_5 = \frac{Sales}{Total\ assets}$$

$$X_3 = \frac{EBIT}{Total\ assets}$$

Z'' Score (for non-manufacturing companies)

$$Z'' = 6.56X_1 + 3.26X_2 + 6.72X_3 + 1.05X_4 \qquad \text{(A.15)}$$

$$X_1 = \frac{Working\ capital}{Total\ assets} \qquad X_4 = \frac{Market\ value\ of\ equity}{Book\ value\ of\ total\ liabilities}$$

$$X_2 = \frac{Retained\ earnings}{Total\ asset} \qquad X_3 = \frac{EBIT}{Total\ assets}$$

The X_5 - Sales/Total assets ratio was in Z''- Score model omitted in order to minimize the potential industry specified turnover effect.

Comparative models statistics

A-T. 2:Comparable accuracy summary between the ZETA and Z-score models (% accuracy)

Years prior to bankruptcy	ZETA model		Z-score model		Z-score over ZETA sample	
	Bankrupt	Non-bankrupt	Bankrupt	Non-bankrupt	Bankrupt	Non-bankrupt
(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**
1	96,2	89.7	93.9	97.0	86.8	82.4
2	84.9	93.1	71.9	93.9	83.0	89.3
3	74.5	91.4	48.3	N/A	70.6	91.4
4	68.1	89.5	28.6	N/A	61.7	86.0
5	69.8	82.1	36.0	N/A	55.8	86.2

Source: Altman (2000)

A-T. 3:Model prediction accuracy (Moody's RiskCalc)

	Percentage of defaults		RiskCalc v3.1		RiskCalc v1.0		Z-Score	
Horizon	1Y	5Y	1Y	5Y	1Y	5Y	1Y	5Y
1993	1.20%	3.20%	68.20%	45.10%	64.50%	43.50%	59.40%	33.60%
1994	3.30%	6.70%	57.40%	44.70%	55.40%	43.20%	52.50%	34.90%
1995	5.60%	11.60%	56.60%	38.30%	53.00%	35.70%	44.60%	26.20%
1996	7.10%	16.20%	60.50%	36.10%	60.10%	31.50%	53.00%	22.00%
1997	11.30%	19.80%	47.70%	33.60%	44.20%	28.20%	36.30%	22.20%
1998	20.80%	19.00%	38.90%	37.40%	35.00%	31.10%	28.30%	26.20%
1999	23.60%	14.60%	44.60%	42.30%	39.60%	35.90%	33.50%	31.70%
2000	19.80%	8.80%	49.00%	48.50%	43.60%	40.90%	36.00%	34.80%

***Source:** Dwyer (2004)*

Moody's grading scale

A-T. 4:Moody's grading scale

Category	Long-term	Short-term
Investment Grade	Aaa	Prime 1
	Aa1	Prime 1
	Aa2	Prime 1
	Aa3	Prime 1
	A1	Prime 1
	A2	Prime 1 / Prime 2
	A3	Prime 1 / Prime 2
	Baa1	Prime 2
	Baa2	Prime 2 / Prime 3
	Baa3	Prime 3
Speculative Grade	Ba1	Not Prime
	Ba2	Not Prime
	Ba3	Not Prime
	B1	Not Prime
	B2	Not Prime
	B3	Not Prime
	Caa1	Not Prime
	Caa2	Not Prime
	Caa3	Not Prime
	Ca	Not Prime
	C	Not Prime

Aaa: Obligations rated Aaa are judged to be of the highest quality, with minimal credit risk.

Aa1, Aa2, Aa3: Obligations rated Aa are judged to be of high quality and are subject to very low credit risk.

A1, A2, A3: Obligations rated A are considered upper-medium grade and are subject to low credit risk.

Baa1, Baa2, Baa3: Obligations rated Baa are subject to moderate credit risk. They are considered medium grade and as such may possess certain speculative characteristics.

Ba1, Ba2, Ba3: Obligations rated Ba are judged to have speculative elements and are subject to substantial credit risk.

B1, B2, B3: Obligations rated B are considered speculative and are subject to high credit risk.

Caa1, Caa2, Caa3: Obligations rated Caa are judged to be of poor standing and are subject to very high credit risk.

Ca: Obligations rated Ca are highly speculative and are likely in, or very near, default, with some prospect of recovery of principal and interest.

C: Obligations rated C are the lowest rated class of bonds and are typically in default, with little prospect for recovery of principal or interest.

Source: *Moody's - Rating Symbols & Definitions (2009)*

Appendix – Methodology

MODEL DYNAMICS (continuous form)

	PROCESSES		DEFINED AS EQUATION	PROVIDING	VARIABLES
REVENUES	**Revenues**	$R(t)$	$\frac{dR(t)}{R(t)} = \mu(t)dt + \sigma(t)dz_1$	$d\sigma(t) = k_1(\bar{\sigma} - \sigma(t))dt$	$\mu(t)$ – expected rate of growth in revenues $\sigma(t)dz_1$ – unexpected change in revenues dz_1 – stochastic variable drawn from N(0,1) k_1 – mean-reversion coefficient
	Rate of growth in revenues	$\mu(t)$	$d\mu(t) = k_2(\bar{\mu} - \mu(t))dt + \eta(t)dz_2$	$d\eta(t) = -k_3\eta(t)dt$	k_2 – mean-reversion coefficient $\bar{\mu}$ – long-term rate of growth in revenues $\eta(t)dz_2$ – unexpected change in rate of growth in revenues dz_2 – stochastic variable drawn from N(0,1) k_3 – mean-reversion coefficient
COGS	**Proportion**	$y(t)$	$dy(t) = k_4(\bar{\gamma} - y(t))dt + \varphi(t)dz_3$	$d\varphi(t) = k_5\big(\bar{\varphi} - \varphi(t)\big)dt$	k_4 – mean-reversion coefficient $\bar{\gamma}$ – long-term rate of variable part of COGS $\varphi(t)dz_3$ – unexpected change in variable part of COGS k_5 – mean-reversion coefficient $\bar{\varphi}$ – long-term rate of unexpected part of var. costs
OE	**Fixed part**	$\alpha(t)$	$\alpha(t) = \alpha$	Constant over whole period	$\alpha(t)$ – fixed component of Operational Expenses
	Variable part	$\beta(t)$	$\beta(t) = bR(t)$	Proportionally constant	$\beta(t)$ – variable component of Operational Expenses b – proportion of variable component of OE

MODEL DYNAMICS (discrete form)

	PROCESSES		DEFINED AS EQUATION	VARIABLES
REVENUES	**Revenues**	$R_t(R_{t-1},\mu_{t-1},\sigma_{t-1},\varepsilon_1,\Delta t)$	$R(t+\Delta t)=R(t)e^{\{[(\mu(t)-\frac{1}{2}\sigma(t)^2)\Delta t+\sigma(t)\varepsilon_1\sqrt{\Delta t}\}}$	R_t – level of revenues μ_{t-1} – expected rate of growth in revenues $\sigma_{t-1}\varepsilon_1$ – unexp. part of revenues ε_1 – stochastic variable drawn from N(0,1)
	Volatility of unexpected element of revenues	$\sigma_t(\sigma_0,\bar{\sigma},k_1,t)$	$\sigma(t)=\sigma_0 e^{-k_1 t}+\bar{\sigma}(1-e^{-k_1 t})$	σ_0 – initial volatility of revenues $\bar{\sigma}$ – long-term volatility of revenues k_1 – mean-reverting coefficient
	Rate of growth in revenues	$\mu_t(\mu_{t-1},\bar{\mu},\eta_{t-1},k_2,\varepsilon_2,\Delta t)$	$\mu(t+\Delta t)=e^{-k_2\Delta t}\mu(t)+(1-e^{-k_2\Delta t})\bar{\mu}+\sqrt{\frac{1-e^{-2k_2\Delta t}}{2k_2}}\eta(t)\varepsilon_2$	μ_{t-1} – expected rate of growth in revenues $\bar{\mu}$ – expected long-term rate of growth in revenues $\eta_{t-1}\varepsilon_2$ – unexp. change in revenues ε_2 – stochastic variable drawn from N(0,1) k_2 – mean-reverting coefficient
	Volatility of unexpected element of R-growth	$\eta_t(\eta_0,k_3,t)$	$\eta(t)=\eta_0 e^{-k_3 t}$	η_0 – initial volatility of R-growth k_3 – mean-reverting coefficient
COGS	**Proportion**	$\gamma_t(\gamma_{t-1},\bar{\gamma},\varphi_{t-1},k_4,\varepsilon_3,\Delta t)$	$\gamma(t+\Delta t)=e^{-k_4\Delta t}\gamma(t)+(1-e^{-k_4\Delta t})\bar{\gamma}+\sqrt{\frac{1-e^{-2k_4\Delta t}}{2k_4}}\varphi(t)\varepsilon_3$	γ_{t-1} – expected rate of variable part of COGS $\bar{\gamma}$ – long-term rate of variable part of COGS $\varphi_{t-1}\varepsilon_3$ – unexp. change of variable part of COGS k_4 – mean-reverting coefficient

COGS'	Volatility of unexpected element of var. part of COGS	$\varphi_t(\varphi_0, \bar{\varphi}, k_5, t)$	$\varphi(t) = \varphi_0 e^{-k_5 t} + \bar{\varphi}(1 - e^{-k_5 t})$	φ_0 – initial volatility in variable part of COGS $\bar{\varphi}$ – long-term volatility in variable part of COGS k_5 – mean-reverting coefficient
OE	**Fixed part**	α_t	$\alpha(t) = \alpha = const.$	α – fixed component of Operational expenses
	Variable part	$\beta_t(R)$	$\beta(t) = bR(t)$	b – coefficient of variable component of OE

Further features of model dynamics

Convergences:

Volatilities in 'long-term' converge to:

$$\sigma(T) = \bar{\sigma}$$

$$\eta(T) = 0$$

$$\varphi(T) = \bar{\varphi}$$

Growth rate in 'long-term' converges to:

$$\mu(T) = \bar{\mu}$$

That implies that the revenue process in 'long-term' converges to:

$$\frac{dR(T)}{R(T)} = \bar{\mu}dt + \bar{\sigma}dz_1$$

Correlations:

Unanticipated changes in R-growth rate and unanticipated changes in revenues drift.

$$dz_1 dz_2 = \rho_{12} dt$$

Unanticipated changes in variable costs of COGS and revenues.

$$dz_1 dz_3 = \rho_{13} dt$$

Unanticipated changes in variable costs of COGS and growth in revenues.

$$dz_2 dz_3 = \rho_{23} dt$$

A-M. 1: Depreciation tables (example)

Linear				**Geometric**			
Initial	2000	Annually	200	Initial	2000	Annually	18%
Salvage	400			Salvage	400		
Period	Deprec.	%	Remai. V	Period	Deprec.	%	Remai. V
1	200	10.00%	1800	1	364.47	18.22%	1635.53
2	200	11.11%	1600	2	298.05	18.22%	1337.48
3	200	12.50%	1400	3	243.74	18.22%	1093.75
4	200	14.29%	1200	4	199.32	18.22%	894.43
5	200	16.67%	1000	5	163.00	18.22%	731.43
6	200	20.00%	800	6	133.29	18.22%	598.14
7	200	25.00%	600	7	109.00	18.22%	489.14
8	200	33.33%	400	8	89.14	18.22%	400.00

Sum of Digits				
Initial	2000	Year multiple:		44.44
Salvage	400			
Period	Useful life	Deprec.	%	Remai. V
1	8	355.56	17.78%	1644.44
2	7	311.11	18.92%	1333.33
3	6	266.67	20.00%	1066.67
4	5	222.22	20.83%	844.44
5	4	177.78	21.05%	666.67
6	3	133.33	20.00%	533.33
7	2	88.89	16.67%	444.44
8	1	44.44	10.00%	400.00

Source: *Own Compilation*

A-M. 2: Recovery Rates by Industry and By Seniority (1971 - 2007) 1/2

Industry	Seniority	# of	Mean	Weight.	Median	STD	Min	Max
Auto/Motor Carrier								
	Senior Secured	14.00	30.70	25.56	24.00	23.75	7.00	92.00
	Senior Unsecured	36.00	35.32	44.36	20.00	26.47	6.50	92.50
	Senior Subordinated	17.00	27.68	20.48	27.00	18.57	3.00	71.00
	Subordinated	4.00	34.28	26.71	27.00	21.26	18.00	65.13
	Senior Sub +Sub	21.00	28.93	21.27	27.00	18.73	3.00	71.00
	All	71.00	35.62	35.04	27.00	23.74	3.00	92.50
Conglomerates								
	Senior Unsecured	3.00	44.92	47.94	53.38	14.65	28.00	53.38
	Senior Subordinated	1.00	71.00	71.00			71.00	71.00
	Subordinated	2.00	11.50	15.19	11.50	9.19	5.00	18.00
	Senior Sub +Sub	3.00	31.33	22.52	18.00	34.96	5.00	71.00
	All	167.00	38.13	40.93	40.69	25.10	5.00	71.00
Energy								
	Senior Secured	36.00	63.83	65.62	70.13	32.70	2.00	104.50
	Senior Unsecured	70.00	43.41	45.52	43.50	21.40	10.00	86.38
	Senior Subordinated	28.00	37.16	46.16	34.69	23.70	1.00	107.75
	Subordinated	25.00	25.56	25.51	21.25	12.40	9.50	55.00
	Discount	1.00	45.26	45.26			45.26	45.26
	Senior Sub +Sub	53.00	31.69	41.98	28.00	19.92	1.00	107.75
	All	160.00	44.13	52.60	37.38	26.53	1.00	107.75
Financial Services								
	Senior Secured	15.00	34.48	24.32	14.00	25.17	14.00	94.00
	Senior Unsecured	75.00	45.28	51.16	35.00	31.41	1.00	100.00
	Senior Subordinated	19.00	33.13	30.40	28.00	24.15	1.00	92.00
	Subordinated	19.00	27.51	22.91	30.00	27.60	1.00	103.00
	Senior Sub +Sub	38.00	30.32	27.29	29.00	25.74	1.00	103.00
	All	128.00	39.57	40.56	33.00	29.76	1.00	103.00
Leisure & Entertainment								
	Senior Secured	28.00	55.95	60.69	53.25	26.51	7.00	106.00
	Senior Unsecured	18.00	51.44	54.89	45.25	28.92	3.75	100.00
	Senior Subordinated	30.00	31.55	30.52	22.50	26.95	4.00	99.00
	Subordinated	21.00	46.17	55.78	36.50	29.79	7.00	112.00
	Discount	2.00	18.88	20.74	18.88	9.73	12.00	25.76
	Senior Sub +Sub	51.00	37.57	38.44	29.00	28.79	4.00	112.00
	All	99.00	44.91	49.32	39.00	29.11	3.75	112.00
General Mfg								
	Senior Secured	62.00	40.70	39.57	39.25	25.24	1.75	106.75
	Senior Unsecured	150.00	39.09	36.71	36.63	22.03	3.00	99.50
	Senior Subordinated	125.00	32.71	29.52	28.50	23.81	0.75	106.00
	Subordinated	65.00	34.95	27.87	32.00	21.15	2.00	90.88
	Discount	11.00	19.04	31.23	12.00	24.04	0.75	66.50
	Senior Sub +Sub	190.00	33.48	29.00	29.00	22.90	0.75	106.00
	All	413.00	36.22	33.52	32.00	23.28	0.75	106.75
Healthcare								
	Senior Secured	2.00	91.25	97.38	91.25	10.25	84.00	98.50
	Senior Unsecured	12.00	46.90	49.61	55.50	15.38	8.75	56.00
	Senior Subordinated	31.00	25.58	21.77	18.00	23.37	2.00	86.00
	Subordinated	10.00	23.77	17.57	23.50	13.41	4.75	39.00
	Discount	2.00	21.05	27.07	21.05	18.32	8.09	34.00
	Senior Sub +Sub	41.00	25.14	21.07	18.00	21.23	2.00	86.00
	All	57.00	31.89	32.39	29.00	24.25	2.00	98.50

Source: *Altman & Karlin (2008), originally from NYU Default Database*

A-M. 3: Recovery Rates by Industry and By Seniority (1971 - 2007) 2/2

Industry	Seniority	# of	Mean	Weight.	Median	STD	Min	Max
Misc Industries								
	Senior Secured	30.00	43.80	45.87	40.75	27.61	5.00	93.50
	Senior Unsecured	50.00	50.40	53.97	48.50	26.41	8.00	99.00
	Senior Subordinated	31.00	37.02	37.81	28.00	26.00	1.00	96.00
	Subordinated	8.00	38.67	17.59	34.07	23.53	4.00	88.00
	Discount	5.00	4.37	4.27	4.03	2.79	0.75	8.00
	Senior Sub +Sub	39.00	37.36	36.36	30.00	25.22	1.00	96.00
	All	124.00	42.85	44.90	39.50	27.34	0.75	99.00
Real Estate & Construction								
	Senior Secured	5.00	44.60	41.26	40.00	34.91	3.00	82.50
	Senior Unsecured	18.00	40.06	31.65	30.00	27.45	6.00	100.50
	Senior Subordinated	18.00	30.67	22.82	17.50	29.37	2.00	95.50
	Subordinated	11.00	35.98	25.21	21.88	33.65	2.00	98.25
	Discount	3.00	13.61	20.97	13.63	12.40	1.21	26.00
	Senior Sub +Sub	29.00	32.69	23.55	19.50	30.58	2.00	98.25
	All	55.00	35.14	28.83	25.50	29.38	1.21	100.50
Retailing								
	Senior Secured	26.00	44.95	38.24	43.50	25.57	2.50	90.00
	Senior Unsecured	146.00	46.47	46.89	43.50	19.93	3.00	98.50
	Senior Subordinated	73.00	31.04	28.15	24.00	21.61	0.50	87.90
	Subordinated	36.00	27.27	25.44	20.00	18.96	3.38	70.00
	Discount	3.00	35.85	35.51	20.00	33.26	13.48	74.06
	Senior Sub +Sub	109.00	29.79	27.57	23.00	20.76	0.50	87.90
	All	284.00	39.82	36.29	41.00	22.31	0.50	98.50
Communications & Media								
	Senior Secured	47.00	38.49	35.23	33.00	29.90	1.00	99.00
	Senior Unsecured	244.00	29.28	25.32	23.75	20.97	1.38	95.75
	Senior Subordinated	42.00	39.45	32.65	37.35	25.61	3.00	97.00
	Subordinated	15.00	34.53	38.73	25.63	26.53	6.50	89.00
	Discount	95.00	28.50	26.94	20.00	23.40	0.42	102.50
	Senior Sub +Sub	57.00	38.16	33.27	33.00	25.70	3.00	97.00
	All	443.00	31.23	27.14	24.00	23.46	0.42	102.50
Transport (non-auto)								
	Senior Secured	121.00	50.23	60.59	44.00	26.49	2.00	106.13
	Senior Unsecured	69.00	28.22	26.96	23.75	18.69	6.00	99.00
	Senior Subordinated	5.00	44.45	24.75	45.50	28.07	13.00	83.75
	Subordinated	8.00	29.89	20.50	26.06	21.63	10.00	65.50
	Senior Sub +Sub	13.00	35.49	21.81	39.25	24.29	10.00	83.75
	All	203.00	41.80	44.73	33.50	26.03	2.00	106.13
Utilities								
	Senior Secured	27.00	56.57	39.27	56.25	24.69	2.00	99.88
	Senior Unsecured	27.00	78.34	65.03	84.00	18.34	28.88	98.63
	Senior Subordinated	2.00	43.88	52.92	43.88	36.95	17.75	70.00
	Subordinated	2.00	44.00	44.00	44.00	0.00	44.00	44.00
	Discount	1.00	68.00	68.00			68.00	68.00
	Senior Sub +Sub	4.00	43.94	51.51	44.00	21.33	17.75	70.00
	All	59.00	65.87	51.83	77.00	24.36	2.00	99.88

Source: *Altman & Karlin (2008), originally from NYU Default Database*

Appendix – Case Study

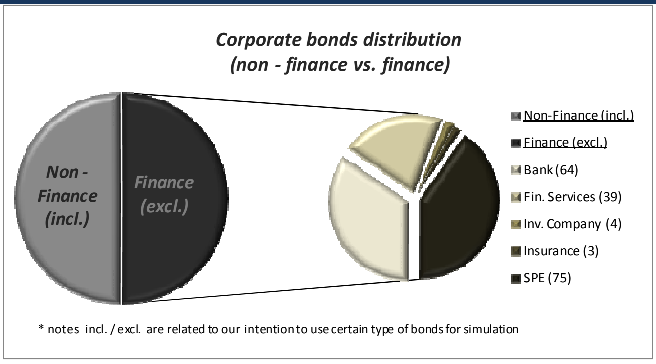
A-C. 1: Rating equivalent to Z'' – Score

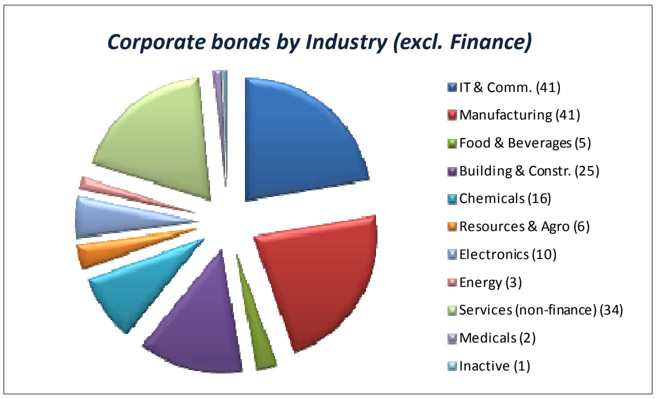
	Rating agency			Definition	Z''-Score
	Moody's	S&P	Fitch		
Investment Grade	Aaa	AAA	AAA	Highest quality	8.15
	Aa1	AA+	AA+	Very High Quality	7.62
	Aa2	AA	AA		7.36
	Aa3	AA-	AA-		7.00
	A1	A+	A+	High Quality	6.85
	A2	A	A		6.65
	A3	A-	A-		6.40
	Baa1	BBB+	BBB+	Medium Grade	6.25
	Baa2	BBB	BBB		5.85
	Baa3	BBB-	BBB-		5.65
Speculative Grade	Ba1	BB+	BB+	Speculative	5.25
	Ba2	BB	BB		4.95
	Ba3	BB-	BB-		4.75
	B1	B+	B+	Highly Speculative	4.50
	B2	B	B		4.15
	B3	B-	B-		3.75
	Caa1	CCC+	CCC+	Substantial Risk	3.20
	Caa2	CCC	CCC		2.50
	Caa3	CCC-	CCC-		1.75
	Ca	CC	CC	Extremely Speculative / Default	n/a
	C	C	C		n/a
	-	D	D		0.00

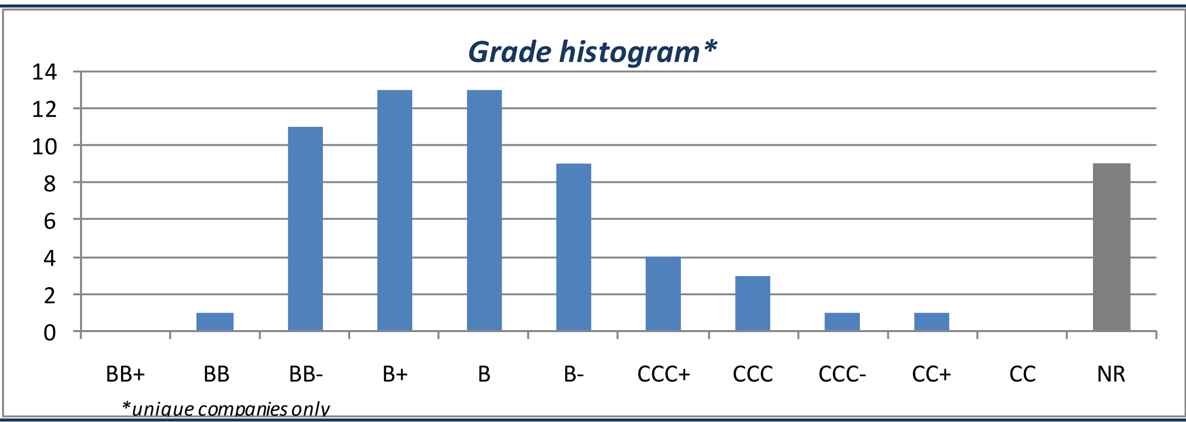
Source: *Altman & Karlin (2008) and Pearl & Rosenbaum (2009)*

A-C. 2: Corporate bonds statistics – industry & grade

Industry / Sample group	# of Bonds
Bank	64
Fin. Services	39
Inv. Company	4
Insurance	3
SPE	75
Finance	**185**
IT & Comm.	41
Manufacturing	41
Food & Beverages	5
Building & Constr.	25
Chemicals	16
Resources & Agro	6
Electronics	10
Energy	3
Services (non-finance)	34
Medicals	2
Inactive	1
Non-Finance	**184**
Total	**369**







Source: *Own Compilation*

A-C. 3: Inputs overview

	Input	Variable	Estimate
Revenues	Initial revenues	R_0	20,362
	Initial rate of growth in revenues	μ_0	-3.53%
	Long-term rate of growth in revenues	μ_{LT}	3%
	Initial volatility of revenues	σ_0	11.09%
	Long-term volatility of the rate of growth in revenues	σ_{LT}	6.19%
	Initial volatility of expected rate of growth in revenues	η_0	16.17%
Costs & Expenses	Initial COGS/revenues proportion	γ_0	65.45%
	Long-term COGS/revenues proportion	γ_{LT}	59.76%
	Initial volatility of COGS/revenues proportion	φ_0	4.51%
	Long-term volatility of COGS/revenues proportion	φ_{LT}	2.26%
	Fixed component of Operational expenses	α	2782.78
	Variable component of Operational expenses	b	11.86%
	Fixed component of D&A	δ_0	1293.22
	Variable component of D&A	δ_1	4.67%
Cost of Capital	Risk free rate	r_f	5.29%
	Market risk premium	MRP	6.92%
	Default spread (bond)	DS	3.89%
	Urgent debt spread	UDS	100bps
	Beta factor	β	1.12
Miscellaneous	Mean-reverting coefficient	k_i	1.11
	Corporate tax rate	τ	28%
	Time increment	Δt	1
	Projection period	T	5
Company specified	Optimal level of cash	$OL\$$	600
	Optimal level of reserves	OLR	2000
	CAPEX maintenance	$CAPEX_m$	600
	CAPEX growth	$CAPEX_g$	1600
	Urgent debt limit	UDL	2500

Source: *Schwartz & Moon (2001) and Own Compilation*

A-C. 4: Results of sensitivity analysis

Input Parameter or Assumption	Notation	Original Estimate	Effect to	Relative effect of change -50.00%	-25.00%	25.00%	50.00%
Long-term rate of growth in revenues	μ_{LT}	**3.00%**	**Enterprise Value**	**-6.16%**	**-1.75%**	**3.11%**	**5.75%**
			Cum. default %	**10.97%**	**2.56%**	**-6.11%**	**-12.91%**
Initial volatility of revenues	σ_0	11.09%	Enterprise Value	-1.37%	-0.95%	1.68%	3.19%
			Cum. default %	-15.52%	-4.86%	6.92%	13.09%
Long-term volatility of the rate of growth in revenues	σ_{LT}	6.19%	Enterprise Value	0.18%	0.18%	0.79%	1.01%
			Cum. default %	-9.29%	-2.12%	1.37%	4.80%
Initial volatility of expected rates of growth in revenues	η_0	16.17%	Enterprise Value	-3.27%	-1.70%	3.05%	6.22%
			Cum. default %	-16.46%	-5.80%	6.17%	10.22%
Long-term COGS/revenues proportion	γ_{LT}	**59.76%**	**Enterprise Value**	**117.00%**	**57.98%**	**-37.03%**	**-42.59%**
			Cum. default %	**-98.13%**	**-80.49%**	**147.32%**	**210.16%**
Initial volatility of COGS/revenues proportion	φ_0	4.51%	Enterprise Value	0.38%	0.22%	0.61%	0.82%
			Cum. default %	-2.74%	-0.87%	2.24%	3.43%
Urgent debt spread	*UDS*	100	Enterprise Value	0.19%	0.49%	0.49%	0.49%
			Cum. default %	-3.37%	0.00%	0.00%	0.00%
Optimal cash level	*$*	600	Enterprise Value	-0.91%	0.05%	1.07%	1.18%
			Cum. default %	-6.17%	-1.43%	-1.68%	3.12%
Optimal reserve level	*ORL*	2000	Enterprise Value	2.90%	1.67%	-0.83%	-0.62%
			Cum. default %	-0.87%	1.56%	-2.43%	-6.98%
Debt limit	***DL***	**2500**	**Enterprise Value**	**-2.78%**	**9.96%**	**-0.01%**	**3.50%**
			Cum. default %	**57.42%**	**24.56%**	**-28.87%**	**-54.49%**
Correlations (*results for 0.2, 0.5, 0.8)	ρ_1, ρ_2, ρ_3	0	Enterprise Value		1.60%	1.57%	2.70%
			Cum. default %		-3.55%	1.93%	5.92%

Source: *Own Compilation*

A-C. 5: Default Rates by Bond Rating (All Rated Corporate Bonds 1971-2007)

	Period	1	2	3	4	5	6	7	8	9	10
AAA	Marginal	0.00%	0.00%	0.00%	0.00%	0.04%	0.02%	0.01%	0.00%	0.00%	0.00%
	Cumulative	0.00%	0.00%	0.00%	0.00%	0.04%	0.06%	0.07%	0.07%	0.07%	0.07%
AA	Marginal	0.00%	0.00%	0.29%	0.13%	0.02%	0.02%	0.00%	0.00%	0.04%	0.01%
	Cumulative	0.00%	0.00%	0.29%	0.42%	0.44%	0.46%	0.46%	0.46%	0.51%	0.51%
A	Marginal	0.01%	0.07%	0.02%	0.05%	0.05%	0.08%	0.05%	0.21%	0.08%	0.04%
	Cumulative	0.01%	0.08%	0.10%	0.15%	0.20%	0.28%	0.33%	0.54%	0.62%	0.66%
BBB	Marginal	0.31%	3.08%	1.29%	1.21%	0.70%	0.29%	0.23%	0.17%	0.11%	0.38%
	Cumulative	0.31%	3.38%	4.63%	5.78%	6.44%	6.71%	6.93%	7.08%	7.19%	7.54%
BB	Marginal	1.13%	2.39%	4.28%	2.22%	2.48%	1.24%	1.63%	1.09%	1.69%	3.42%
	Cumulative	1.13%	3.49%	7.62%	9.69%	11.90%	13.01%	14.42%	15.36%	16.79%	19.63%
B	Marginal	2.78%	6.72%	7.28%	8.44%	5.98%	4.30%	3.91%	2.36%	1.94%	0.95%
	Cumulative	2.78%	9.22%	15.83%	22.93%	27.54%	30.65%	33.36%	34.93%	36.20%	36.80%
CCC	Marginal	7.88%	15.31%	18.68%	11.67%	4.10%	9.32%	5.75%	5.65%	0.82%	4.66%
	Cumulative	7.88%	21.98%	36.56%	43.96%	46.26%	51.37%	54.07%	56.68%	57.02%	59.02%

Source: *Altman & Karlin (2008)*

A-C. 6: Cumulative Default Rates by Rating (1983-2007)

Rating	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10
Aaa	0.00%	0.00%	0.00%	0.04%	0.08%	0.13%	0.19%	0.19%	0.19%	0.19%
Aa1	0.00%	0.00%	0.00%	0.10%	0.15%	0.17%	0.17%	0.17%	0.17%	0.17%
Aa2	0.00%	0.01%	0.05%	0.11%	0.22%	0.26%	0.32%	0.38%	0.45%	0.52%
Aa3	0.02%	0.04%	0.07%	0.12%	0.18%	0.23%	0.27%	0.28%	0.29%	0.34%
A1	0.00%	0.08%	0.21%	0.30%	0.37%	0.44%	0.50%	0.54%	0.61%	0.70%
A2	0.02%	0.08%	0.21%	0.39%	0.56%	0.72%	0.90%	1.08%	1.24%	1.34%
A3	0.03%	0.15%	0.31%	0.42%	0.56%	0.74%	0.89%	1.06%	1.18%	1.23%
Baal	0.15%	0.39%	0.66%	0.89%	1.13%	1.35%	1.63%	1.82%	1.96%	2.09%
Baa2	0.14%	0.42%	0.79%	1.39%	1.91%	2.42%	2.89%	3.34%	3.88%	4.59%
Baa3	0.31%	0.84%	1.47%	2.16%	2.97%	3.77%	4.44%	5.08%	5.56%	5.90%
Ba1	0.69%	1.81%	3.21%	4.62%	6.02%	7.43%	8.38%	9.08%	9.62%	10.27%
Ba2	0.79%	2.21%	4.03%	5.94%	7.58%	8.76%	9.97%	11.19%	12.40%	13.49%
Ba3	1.78%	5.01%	9.06%	13.14%	16.49%	19.66%	22.60%	25.28%	27.76%	30.26%
B1	2.56%	7.10%	11.95%	16.36%	20.86%	25.10%	29.39%	33.16%	36.19%	38.53%
B2	4.33%	9.86%	14.98%	19.34%	22.83%	25.86%	28.96%	31.26%	33.81%	35.95%
B3	8.50%	16.10%	23.25%	29.67%	35.50%	41.33%	45.45%	48.93%	51.27%	53.04%
Caa1	10.50%	20.88%	30.42%	38.37%	44.78%	48.97%	50.76%	51.12%	51.12%	51.12%
Caa2	18.43%	27.67%	34.84%	40.96%	44.79%	48.80%	52.20%	56.39%	62.91%	70.04%
Caa3	25.58%	37.59%	44.53%	49.58%	54.65%	54.99%	54.99%	54.99%	54.99%	54.99%
Ca-C	32.91%	43.08%	51.51%	56.86%	63.29%	66.17%	70.86%	75.52%	75.52%	75.52%
Investment-Grade	0.07%	0.21%	0.40%	0.62%	0.83%	1.05%	1.24%	1.41%	1.56%	1.71%
Speculative-Grade	4.48%	9.01%	13.41%	17.29%	20.62%	23.57%	26.15%	28.32%	30.18%	31.83%
AH Rated	1.59%	3.18%	4.69%	5.98%	7.04%	7.94%	8.69%	9.30%	9.80%	10.23%

Source: *Emery et al. (2008)*