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THE DUTCH TREASURY YIELD CURVE

NELSON & SIEGEL BASED ESTIMATIONS FROM 1952 UNTIL 2001

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PREFACE AND ACKNOWLEDGEMENTS

This master thesis presents my final work for the Master Financial Economics of the study Economics & Business at the Erasmus University Rotterdam. This thesis enables me to finalize my studies and my student life (for one part). It is time to move forward, but not before I have thanked my parents for allowing me to follow my studies and pursue the activities next to it, that made my student life so enjoyable. Next, I want to thank my girlfriend Linda for supporting me during the process of writing this thesis.

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ABSTRACT

The risk free rate is an important variable in financial analyses and it is commonly assumed that this risk free rate can be derived from the yield of Government bonds. However, at any point of time only a finite number of Government bonds are observed in the market. This means that not all maturities have a known yield.

This study has aimed to estimate the Government bond yield curve for the Netherlands for the period 1952 – 2001. Before the start of this thesis, this has not yet been done. For the estimations two models are used; the Nelson & Siegel model and the Extended Nelson & Siegel model. The research is done on forty-nine annual datasets, which consist of bullet and callable bonds. Next, the datasets are checked for illiquid bonds and are cleared of these bonds. Again the yield curves are estimated. The findings show that the Nelson & Siegel model give very similar results. When corrections are made for callable bonds; revalue callable bonds to bullet bonds, it can be seen that this improves the fit of the estimations.

The cleared dataset significantly increases the results of the estimations. The Nelson & Siegel model gives in most cases a satisfactory fit. Only in some cases the fit of the estimations are significantly higher with the Extended Nelson & Siegel model.

JEL Keywords:
Financial statistics, term structure of interest rates, yield and interest

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1. Introduction

In financial theory, the risk free rate is a recurring variable that is used in various analyses, for instance in valuation exercises. It serves as an input when estimating the cost of capital that is used in the discounting of cashflows. It is also commonly used as a benchmark rate for financial securities like corporate bonds and swaps.

In fact, the risk free rate is a theoretical concept, which does not actually exist. As an approximation of the risk free rate, the interest on Government bonds is used, but even for these securities there is a certain degree of default risk (e.g. the recent worries on the Greek financial stability). For very short-term maturities, the Euribor or Libor rate can be used. Financial institutions use these rates when they lend money to each other. These institutions have very low default risk.

Government bonds are considered to be risk free, because it is commonly assumed that Governments do not default. The reasoning is that if a Government can no longer fulfil its bond obligations, it will ‘print’ the additional funds needed. In present times, money creation by Governments is no longer common practice. Central Banks in Europe have become independent institutions with a focus on low inflation. Only enormous political pressure and (financial) crisis will move central banks to expand money supply. In cases where monetary expansion does take place, it is usually used as stimulus as opposed to debt pay down. This argumentation for considering Government bonds risk-free therefore does not hold. Still, Government bonds are considered as very low risk investments. One of the arguments is that Governments have the ability to increase taxes in order to fulfil future obligations, while corporations are far less flexible (due to the competitive nature of their business).

The focus of this study is on the interest Governments pay on the bonds that they issue. Government bonds exist for different maturities and vary in interest rates. The relation between bond maturities and interest rates, is called the term structure of interest rates. This research focuses on how the term structure of the yield on issued Government bonds is calculated (for the Netherlands) and how this information is used in practice. The analysis of the term structure of Government bonds provides valuable information on the way markets see Governments and the general economic environment. It gives an estimation of the risk free interest rate for different maturities. The term structure also gives an overview of the present macroeconomic situation and the market’s expectation about the future economic situation through the forward interest rates (Min ea., 2005).

Plotting a term structure on actual bonds only provides a rough estimate of the relation between maturity and interest rate. At any point in time only a finite number of Government bonds are observed in the market, providing data points for only a number of combinations, while a continuum of data would be desired to obtain the most accurate relation. Thus, issued bonds do not cover all possible maturities.

The yield curve is often used as representation of the term structure, but it gives an inexact image. A yield rate is a complex average of spot rates. Hence, the yield is not equal to the current spot rate with the same maturity. Chapter 3.4 discusses this problem more extensively. To calculate forward rates, spot rates are needed, which are derived from the yield structure.

To solve the issue of limited actual data points, researchers have found multiple solutions. In this thesis, the published research by Nelson & Siegel (1987) and Svensson (1994) is used as a guideline. Their research proposes a way to estimate interest yields for non-existent (market) maturities using data of available bonds. With their models a smooth term structure can be constructed. This has already been done for example for a United States based dataset (Gurkaynak ea., 2006), a UK dataset (Anderson ea., 1999) and a German dataset (Schich, 1997), but up to now, not for the Netherlands.

The goal of this thesis is to present the term structure of interest rates in the Netherlands through time. The obtained yield curve estimates are then compared to estimates by official institutions.

In order to come to the results, this thesis has the following structure. In chapter 2 bond pricing theory is discussed. This is followed by the basics of the term structure of interest rates concept in chapter 3. The data and methodology of the research are discussed in chapter 4. The results are discussed in chapter 5 and 6. Chapter 5 presents the results of all annual datasets, where chapter 6 elaborates on one dataset with special properties. Chapter 7 discusses two annual yield curve estimations, in comparison with estimations by financial institutions. The research concludes in chapter 8.

2. Bond pricing

This chapter offers the basic fundamentals of bond pricing. The purpose of this exercise will be to provide the reader with the necessary knowledge to understand the variables used in bond pricing. These elements are later used in further research.

The bond price is obtained through a calculation with the essential elements of a bond: maturity, coupon rate and face value. Put simply, the bond price is a calculation of the Present Value (PV) of (future) cash flows. Some special features of a bond influence the price, such as early redemption and semi-annual coupons. These features are described in chapter 4.

A discount bond or a zero coupon bond is a bond that pays the face value at maturity and is issued at a discounted price. It does not pay any periodical coupons. The investor's interest is equal to the discount percentage distributed over the years to maturity. The zero coupon bond price can then be calculated as follows:

$$PV_t = \frac{F}{(1+r_m)^m} \qquad (2.1)$$

$P_{m,t}$:	Present Value of bond at t	$r_{m,t}$:	Interest rate for maturity m at time t
F :	Face Value	m:	Maturity of the bond

At this point a new assumption is added: there exists complete certainty about future interest rates (Kroon, 1990). The market aggregates all available information, which is reflected in the interest rates. Bond prices with different maturities and coupons are such, that no arbitrage profits can be made. This is also known as the Efficient Market Hypothesis, by Fama (1970). The theory predicts that all outstanding bonds, regardless of their maturity, must produce identical returns over any given interval of time (Modigliani & Sutch, 1966). With these assumptions, the m term interest rate is calculated as follows:

$$\left(1 + \mathrm{r}_{m,t}\right)^{\mathrm{m}} = \left(1 + r_{1,t}\right) \cdot \left(1 + f_{1,t+1}\right) \cdot (\ldots) \cdot \left(1 + f_{1,t+m-1}\right) \qquad (2.2)$$

Equation 2.2 states that an investment with maturity m pays an interest rate of r_m at t. Received interest payments are reinvested in the bond. The total interest received per year is $(1 + r_{m,t})$. Because of the assumption of no arbitration, this interest return equals the product of every year's one-year spot/forward rate until the year of maturity.

A coupon bond pays each period a coupon and pays face value at maturity. To calculate the price of this bond, every coupon payment is treated as a zero coupon bond. Each payment to the investor is discounted by its own spot rate (with a maturity of now until the payment moment). The equation to price the bond is given in formula 2.3.

$$PV_t = \sum_{\mathrm{i}=1}^{\mathrm{m}} \frac{C_i}{(1+r_i)^{\mathrm{i}}} + \frac{F_i}{(1+r_i)^{\mathrm{i}}} \qquad (2.3)$$

C_i :	Coupon (cash flow) for period i

To calculate coupon bond prices, interest rates with a maturity matching the cash flow moments are necessary. Because interest rates are not directly observable for every maturity, equation 2.3 cannot be used without any adjustment. The equation then uses the yield-to-maturity. The yield to maturity is the internal rate of return of a bond, assuming that the bond is held until maturity. All coupon payments and the face value at maturity are discounted against the same rate: the yield to maturity. The sum of all cash flows discounted by the yield-to-maturity equals the bond price.

The features of a bond such as the coupon rate, maturity and face value are known elements. The bond price is also known, since it is quoted on the financial markets. With this information the yield-to-maturity can be calculated. The bond price calculation with a yield-to-maturity (y_m) is shown in equation 2.4.

$$PV_t = \sum_{\mathrm{i}=1}^{\mathrm{m}} \frac{C_i}{(1+y_m)^{\mathrm{i}}} + \frac{F}{(1+y_m)^{\mathrm{i}}} \qquad (2.4)$$

y_m :	Yield-to-maturity at time t

Equations 2.1, 2.3 and 2.4 discount the future cash flows of the bond against the appropriate interest rate or yield, which results in a bond price (net present value) at time t. In these equations, annual compounding is used.

In this thesis bonds are priced with periodical compounding and not with continuous compounding. Continuous compounding is used by older research, e.g. Nelson & Siegel (1987). Since periodical compounding is financial reality, this is the preferred manner.

3. Term structure of interest rates

This chapter addresses the term structure of interest rates on basis of the Nelson & Siegel model and the Extended Nelson & Siegel model. The aim is to present a theoretical framework for the empirical part of this thesis in chapter 5. The first section will deal with an introduction on the term structure of interest rates in general. Section two focuses on the history of term structure estimations. The two following sections will discuss the two term structure estimation models used in this research.

3.1 Introduction on term structure of interest rates

The term structure of interest rates is a concept which was brought forward by Durant (1942). Since then, many authors have contributed to the objective of fitting and creating a smooth term structure. The term structure gives the relation between interest rates (or yield) and time to maturity. The graphical representation of the term structure, the curve, exists in various forms. These various types of term structure reflect future expectations in financial markets. For example, a steep yield curve implies that interest on long term bonds are relatively high (compared to a 'normal' situation) meaning that financial markets expect an improvement of the wider economy that will lead to an increase in interest rates. Other forms are the 'humped' curve and the 'inverted' curve. Please refer to Appendix A for detailed background.

As said, this study focuses on Government bonds of the Netherlands. The reason for this is that by determining the yield curve on these bonds we can approximate the risk free rate[1]. If corporate bonds would have been used, the resulting yield would include a premium for the assumed risk (of default) related to investing in such securities and would distort the approximation of a risk free yield curve.

In practise, the term structure of Government bonds is used to extract useful information:

- Estimation of the effects of monetary policy decisions by central banks. The ECB aims to maintain a stable price level with a maximum inflation level. Changes made in the official interest rate by the ECB influence LIBOR, EURIBOR and the yield of Government bonds. Central Banks can observe changes in the yield curve as the financial market's response to their policy changes.
- Bond yields are also important for debt policy decisions by the Government. The amount, maturity and coupon interest rate on (to be) issued Government bonds affect the term structure of interest rates. Investors reveal their expectations of the capital market through prices of Government bonds. Forward rates of interest can be derived from the term structure. These forward rates give valuable information on expectations of the financial markets towards future interest rates
- Companies also derive useful information from the term structure. Current and expected interest rates directly influence return calculations and therefore investment decisions. Interest rates are also used in calculating prices of derivatives and hedging strategies.

Spot rates are directly observable through T-bills and zero coupon bonds. These securities do not yield intermediate payments (cashflows) but only pay face value at maturity. The current bond price is found by discounting the face value by the spot rate. By observing the current bond price, one can easily obtain the implied spot rate.

[1] Only Government bonds of developed countries can be assumed to be risk free. If there exists doubt on the credibility of a country, the required will be higher than the risk free rate. The Netherlands can be assumed to pay the risk free rate on its Government bonds.

As T-bills generally do not have maturities beyond one year and zero coupon bonds for longer maturities are rare, other approaches have to be used to estimate spot rates for longer maturities. Coupon bonds which do offer various longer maturities are commonly available on financial markets. However, spot rates of coupon bonds cannot be directly observed. The reason is that coupon bonds have multiple cashflows at multiple dates. The spot rates for the different maturities are unknown and cannot be calculated separately. This issue is overcome by calculating the yield-to-maturity, which is the constant discount rate that can be used to discount all of the cashflows related to the coupon bond that yields the bond price. When all available bond data on yield and maturity are graphed in a figure, the yield curve is observed. This is an overview of all (available) yield-to-maturities at a certain point in time. For coupon bonds with maturity m, the yield is a complicated average of different interest rates over the full interval [t, t + m].

For example, if a bond has twenty-five years left to maturity, it will be likely that less than half of its value will be due to the principal. The remaining value of the bond is based on the intermediate coupon payments. When the yield is calculated and especially for longer maturities, the yield becomes an average without a meaningful shape[2] (McCulloch, 1971). This so-called coupon-effect is a problem of the yield curve. The problem is that spot rates vary with maturity. When the yield is used to value coupon payments and the bond face value, it is constant to maturity. The term structure of yield rates is thus not a perfect representation of the term structure of spot rates.

The estimation of spot rates is done by way of a term structure model. As input for the term structure model the yield of (zero and coupon) bonds is used. The principal task of a term structure model is to decompose yields of coupon bonds into spot rates (Vasicek & Fong, 1982). With the spot rates a term structure of spot rates can created and forward rates be estimated.

Tao Wu (Wu, 2003) shows in a short paper that three factors are responsible for 99% of the shape changes in the yield curve: level, slope and curvature.

- Level change: increases the rate for all maturities by almost identical amounts. The entire yield curve then shifts up with the fixed amount of the level increase.
- Slope change: if an external shock has a larger impact on short term interest rates then on long term interest rates, the increase in short term rates will be higher than the long term interest rates. The slope would in this case decrease and the steepness in yield curve would decline.
- Curvature change: the medium term interest rates are most affected by an external variable. This would affect the ‘hump’ of the yield curve.

3.2 Previous studies

Over time multiple theories have been developed to estimate the yield term structure. One of the first studies on yield curves was by David Durant (Durant, 1942). In this paper the author wanted to augment the knowledge on the structure of interest rates, which was then largely limited to long term bond yields. Durant presents basic yield estimates of corporate bonds for the period 1900 - 1942. The yield curve is a “free hand trend line, so fitted that it passes below most of the yields on the chart but usually above a few isolated low yields”. This trend line represents the yield curve for the highest quality bonds.

[2] In the long term the yield stabilizes to a constant. McCulloch means that this constant does not represent the spot rate at this point in the far future, but that this point is an average of spot rates. This long term yield is according to McCulloch not meaningful.

After 1970, modelling of term structures got more attention of researchers and more theories were developed. In 1971 McCulloch (McCulloch, 1971) publishes an influential paper on estimating the term structure of interest rates. The author fits the discount function with quadratic and cubic splines, also known as polynomial splines. Estimates of the discount function are a continuous function of time.

Vasicek & Fong (Vasicek & Fong, 1982) argue that polynomial splines, as used by McCulloch, have negative properties, which result in unstable forward rates and undesirable asymptotic properties. Vasicek & Fong propose an alternative method called exponential spline splitting to smooth the interest rate term structure data. Since the authors do not accompany empirical results to fundament their theory, Gary Shea tested whether the use of exponential spline method improves the term structure estimates (Shea, 1985). Shea finds that modelling with exponential splines also gives forward rates that are unstable and fluctuate much like forward rates obtained with polynomial splines.

In 1987 Nelson & Siegel created a term structure model, which in turn is adjusted to the Extended Nelson & Siegel model in 1995 by Svensson. These two models are commonly used by central banks and monetary policy makers. In 2005 the Bank for International Settlements found that eight out of thirteen researched central banks used a Nelson & Siegel or an Extended Nelson & Siegel model to estimate the term structure (Bank for International Settlements, 2005). Because of the widely spread use of these models by respected institutions, this thesis uses these models to estimate the term structure of the Netherlands. The objective is to create a term structure that fits the Dutch data and results in a smooth term structure. The following two paragraphs discuss the Nelson & Siegel model and the Extended Nelson & Siegel model.

3.3 Parsimonious modelling of yield curves (Nelson & Siegel, 1987)

Nelson & Siegel intend to create a simple and parsimonious model, which is capable of representing the general range of shapes that come with yield curves. These shapes are monotonic, humped and S-shapes.

A coupon bond can be seen as a portfolio of zero coupon bonds of different maturities, the price of a coupon bond can be calculated. Each coupon payment is seen as a zero coupon bond. The yield-to-maturity is the internal rate of return for a bond. This is the constant interest rate at which the present value of the face value and coupon payments of the bond is equal to the price of the bond.

Implied forward rates are easy to get from the yield-to-maturity on zero coupon bonds. But if a bond is a coupon bearing bond, then the forward rate calculation gets more complicated. Almost all bonds with a maturity greater than twelve months are coupon bonds. Nelson & Siegel state that “if spot rates are generated by a differential equation, then forward rates, being forecasts, will be the solution to the equations”. Estimating forward rates from coupon bonds is done in two steps:

1. Implied spot rates are estimated from yields to maturity on coupon bonds
2. Implied forward rates are computed from implied spot rates

The author describes the relationship between spot rates and forward rates further. The instantaneous forward rate is the forward rate with an infinite small investment period after the settlement date. This instantaneous forward rate (e.g. forward overnight rate) is the marginal increase in total return after a marginal increase in the length of the investment. As a result, the spot rate can be represented as an average of all instantaneous forward rates with settlement between the trade date t and the maturity date T.

The instantaneous forward rate is noted as f(x) in equation 3.1. The yield to maturity is noted as y(m), which is the average of the forward rates until maturity.

$$y(m) = \frac{\int_{x=0}^{m} f(x)dx}{m} \qquad (3.1)$$

y(m)	=	yield to maturity	f(x)	=	instantaneous forward rate

The forward rate is obtained by the following equation, as suggested by Nelson & Siegel. It has the capability to fit most of the specific features of a yield curve. The τ characteristic in the formula is a time constant (Nelson & Siegel, 1987). It determines the rate at which the regressor variables decay to zero. The τ values have a range between 0.03 and 20 years, with equal intervals. In total, there are 45 τ values. The τ variable is important for the rate of decay of the regressors. A small value of variable τ will make the regressors decay fast, making the estimated curvature better fit at low maturities. A large τ variable on the other hand slows down the decay of the regressors, giving the curvature a better fit for longer maturities.

$$f(m) = \beta_0 + \beta_1 \cdot exp\left(-\frac{m}{\tau}\right) + \beta_2 \cdot \left(\frac{m}{\tau}\right) \cdot exp\left(-\frac{m}{\tau}\right) \qquad (3.2)$$

When the two equations for f(m) and y(m) are combined, the yield-to-maturity is calculated as a function from the forward rates from zero to maturity m and divided by maturity m.

B_0, β_1, β_2	=	Coefficients	f(m)	=	instantaneous forward rate
τ	=	time constant			

$$y(m) = \beta_0 + (\beta_1 + \beta_2) \cdot \frac{1-\exp\left(-\frac{m}{\tau}\right)}{\left(\frac{m}{\tau}\right)} - \beta_2 \cdot exp\left(-\frac{m}{\tau}\right) \qquad (3.3)$$

When equation 4.2 and 4.3 are combined, equation 4.4 is obtained.

$$y(m) = \beta_0 + \beta_1 \cdot \left[\frac{1-\exp\left(-\frac{m}{\tau}\right)}{\left(\frac{m}{\tau}\right)}\right] + \beta_2 \cdot \left[\frac{1-\exp\left(-\frac{m}{\tau}\right)}{\left(\frac{m}{\tau}\right)} - exp\left(-\frac{m}{\tau}\right)\right] \qquad (3.4)$$

This formula gives the relationship between the maturity and yield. The formula uses three betas. Each of these elements between the brackets has other features to the maturity. This enables the yield curve to represent a humped or s-shaped yield structure as can be seen on the financial markets. The β_0 variable is the long term yield component. The β_1 variable indicates the medium term component and the β_2 component gives the short term component. This can also be seen in the following two equations:

$$\lim_{m\to\infty} y(m) = \beta_0 \qquad (3.5)$$

Equation 3.5 shows that an 'endless' maturity results in a yield of β_0. Equation 3.6 shows that when the maturity decreases to an infinite small period, the yield is equal to subtraction of the components $\beta_0 - \beta_2$. The medium term component β_1 only exists for maturities between infinite small and endless long.

$$\lim_{m\to 0} y(m) = \beta_0 - \beta_2 \qquad (3.6)$$

Nelson and Siegel test this relationship on thirty-seven monthly samples from 1981 through 1983 on U.S. treasury bills. Per dataset, a grid of τ[3] values is made and for each τ value, β values are estimated through an ordinary least squares regression. The regression with the τ value which has the best overall fitting of the curve against the data, is used. Nelson and Siegel recall that τ is a time constant that determines the rate at which the regressor variables decay to zero.

Nelson and Siegel find that their model is able to characterize the basic shape of the treasury bill term structure. Their model is also able to predict yield and prices at maturities beyond the range of the sample. The writers acknowledge that with their model and set of parameters, no perfect fit of the data is obtained, but that is not their goal either. A reason for the model not to have a perfect fit is that no continuous trading of bills exists. Secondly, some bills exist that have specific features, which make them sell at a discount or premium. The model does not correct for these features. Gimeno & Nave (2006) argue that the Nelson & Siegel results can only be used for monetary policy, because the results are not accurate enough. Diebold & Li (2006) on the other hand find that the Nelson & Siegel model does give accurate term structure forecasts.

In comparison with a cubic polynomial model, Nelson and Siegel notice that their model has the same number of parameters. They also observe that a cubic polynomial model fits the sample data better. Outside the sample interval, the cubic polynomial model has no predictive power. The authors summarize the use of the cubic polynomial model as “a function may have the flexibility of to fit data over a specific interval, but may have very poor properties when extrapolated outside that interval”.

3.4 Extended Nelson & Siegel model (Svensson, 1995)

Svensson observed an increased use of forward interest rates for monetary policy decisions. In his study the author shows a convenient method to estimate implied forward rates from existing financial instruments, especially bonds. The starting point is that the estimation method must be simple and robust. The care for precision is less important, because the purpose is to contribute to monetary policy analysis.

Svensson (1995) argues that using the yield-to-maturity as a representation of the term structure of interest rates is wrong. He states that the yield of a coupon bond is an average of spot rates up to the time of maturity. It does not show the independent spot rates for the different maturities of the cash flows. Secondly, for a given term structure of spot rates, the yield to maturity for a bond will depend on its coupon rate. An example: for a given term structure of spot rates, two identical bonds exist, except for that the coupon rate of bond A is higher than bond B. In this case, the yield of bond A is greater than bond B. The reason is that a higher coupon rate implies that the share of early payments increases, which gives more weight to short maturities in the determination of the yield to maturity. In this situation, the yield curve thus misrepresents the term structure. Svensson argues that yield-to-maturity should not be used as a representative variable for the term structure. Instead, the spot rates of bonds should be used. Because spot rates are not directly available from coupon bonds, the rates should be derived from yields of coupon bonds.

As starting point, Svensson uses the Nelson and Siegel model. He argues that in many cases the Nelson & Siegel model gives a satisfactory fit. However, in some more complex cases an extended model should be used. To accomplish this, Svensson adds an extra term to the Nelson and Siegel model:

[3]Nelson and Siegel use a grid of τ values from 10 to 200 in increments of 10 and also the τ values 250, 300 and 365.

$\beta_3 \cdot \frac{m}{\tau_2} \cdot exp\left(-\frac{m}{\tau_2}\right)$

The extra term improves the fit and adds extra flexibility. It is a U-shaped term with two additional parameters β_3 and τ_2. This adjusted Nelson & Siegel model is labelled as the "Extended Nelson & Siegel model" by Svensson. The forward rate function becomes:

$$f(m) = \beta_0 + \beta_1 \cdot exp\left(-\frac{m}{\tau_1}\right) + \beta_2 \cdot \frac{m}{\tau_1} \cdot exp\left(-\frac{m}{\tau_1}\right) + \beta_3 \cdot \frac{m}{\tau_2} \cdot exp\left(-\frac{m}{\tau_2}\right) \qquad (3.7)$$

The author integrates the forward function in the spot rate function. Because the spot rate is not directly available, this is also the equation with which the yield curve is fitted. The spot rate can be calculated as follows:

$$y(m) = \beta_0 + \beta_1 \left[\frac{1-exp\left(-\frac{m}{\tau_1}\right)}{\frac{m}{\tau_1}}\right] + \beta_2 \left[\frac{1-exp\left(-\frac{m}{\tau_1}\right)}{\frac{m}{\tau_1}} - exp\left(-\frac{m}{\tau_1}\right)\right] + \beta_3 \left[\frac{1-exp\left(-\frac{m}{\tau_2}\right)}{\frac{m}{\tau_2}} - exp\left(-\frac{m}{\tau_2}\right)\right] \qquad (3.8)$$

Equation 3.9 shows that an 'endless' (average) maturity results in a yield of β_0. The long term component is β_0.

$$\lim_{am \to \infty} y(am) = \beta_0 \qquad (3.9)$$

Equation 3.10 shows that when the maturity decreases to an infinite small period, the yield is equal to subtraction of the components $\beta_0 - \beta_2 - \beta_3$. The medium term component β_1 only exists for maturities between infinite small and endless long. In comparison with the Nelson & Siegel model, the Extended Nelson & Siegel model adds an extra term β_3 to the short term interest.

$$\lim_{am \to 0} y(am) = \beta_0 - \beta_2 - \beta_3 \qquad (3.10)$$

Svensson finds that in many cases the Nelson & Siegel model gives a satisfactory fit. However, in some more complex cases of the term structure, the Nelson & Siegel model gives an unsatisfactory fit. The extended model improves the fit considerably.

4. Data and methodology

This chapter focuses on the empirical part of this thesis. The first section will illustrate how the data set is constructed. The second section describes the methodology.

4.1 Data

The dataset is for a large part obtained through the Dutch ministry of finance and the 'Prijscourant'[4]. Other data incorporated in this research are the redemption scheme of different bonds over the research period and the prices of the bonds. Data such as maturity and yield are calculated in this research. There is no data obtained via Datastream, because other information sources already contributed to a complete dataset. Also, the use of Datastream data has a disadvantage. The Datastream yield calculation does not bear in mind different day count conventions as used through time.

4.1.1 Number of bonds

The dataset contains 218 Dutch state issued bonds for the period 1950 to 2001. The records include key variables of every year's last trade of Government bonds of the Netherlands. The smallest outstanding number of bonds in the dataset is in 1950, with eight bonds. The largest number of bonds outstanding is in 1988, with 140 bonds. After eliminating bonds, which do not fulfil the criteria as described in the next sections, the minimum and maximum number of outstanding bonds decrease to respectively six bonds in 1952 and 120 bonds in 1984.

4.1.2 Bonds

All bonds in the dataset are issued by the Government of the Netherlands. The dataset contains three sorts of issued bonds: normal bonds ('*NL obligaties*'), ledger bonds ('*grootboek obligaties*') and certificates ('*beleggingscerticaten*'). The certificates are treated as normal bonds. Of the total of four ledger bonds, three bonds are perpetual (See appendix F for an overview of the dataset).

Each bond in the dataset qualifies as a bullet loan, sinking fund or a no-redemption bond. If the bond is a bullet loan, the full loan (complete issued amount) is paid back at maturity. Bullet loans are issued since January 1986. If the bond is a sinking fund, the bond is gradually redeemed according to a predefined schedule. Normally, the redemptions start ten years after the issue of a bond. The to be redeemed bonds are selected through a lottery. The following sinking fund types are present in the dataset: equal sinking fund and unequal sinking fund. The first type redeems a bond in equal yearly amounts, thus every year a fixed percentage or fixed amount is paid back to the investors. The other option is that every year an unequal amount is paid to bond investors. Since 1989, the Dutch treasury no longer issues sinking fund bonds. Bonds can also be perpetual bonds. This type of bonds has no redemptions and no final maturity, which means that the loan is unredeemable. These loans (the ledger bonds) are rare and can only be terminated by the state buying this type of bonds back from the financial markets. In general, all issued bonds since 1999 have a maturity between three and ten years.

Because of the special qualities of the perpetual bonds, these are excluded from the calculations. The remaining ledger bond has missing redemption data and is therefore also excluded from the calculations.

[4] Prijscourant is the official journal of the Amsterdam Exchange. All price information is publiced in this journal.

Another type of bonds are STRIPS[5], these are stripped bonds where principal and coupon are separated. The payments are separated into two cash flows: separate series of coupon payments and face value of the bond. The principal has become a zero-coupon bond. Since 1999 all issued bonds are allowed to be converted to STRIPS (Ministerie van Financien). One specific difference with coupon bonds is that, because the principal of the STRIPS is a zero coupon a bond, its yield is equal to its spot rate. Because the found value is a spot rate, it cannot be used in the calculations with coupon bond yields. STRIPS are excluded in this research. However, STRIPS in general are very useful to find spot rates.

Bonds in the dataset may also qualify as a callable bond or a convertible bond. These are extra properties and are mentioned in the prospectus. A callable bond is a bond with a call option on it, to be exercised by the issuer. The state can call (redeem) the entire loan, after which the state retires the loan and pays the investor the face value of the bond. Normally, the state can exercise its call option ten years after the original bond issue. For some older bonds, exceptions to this rule may apply. Almost all bonds issued by the Dutch state in the period 1911 until 1969, qualify as callable bonds. The last callable bond was issued in 1987. The Dutch state exercised its call option on bonds for the first time in 1985. After 1997 (in the dataset) the Dutch treasury stopped exercising call options on bonds. The call option can be exercised on specified dates before maturity, normally at the coupon dates of bonds. This type is known as Bermudan call options.

Some non-callable bonds qualify as a convertible bond. On maturity a bullet bond is retired. If a bond is convertible, the bond may be converted by the state and continued into a new bond with the same properties as the retired bond, but with an increased maturity. The bonds, which qualify as callable or convertible, are treated as if they were normal bonds. All callable bonds are assumed to remain outstanding until maturity. These assumptions are made to keep it simple. If a bond is called or converted, the bond is removed from the dataset.

An overview of the bond types in the dataset is given in table 1. The columns show the different redemption properties of the bonds. The bottom row shows the total of bonds for each redemption property. Each bond may also qualify as a callable bond or a convertible bond. These bonds are specified in the rows. In total there are 218 bonds in the dataset.

Table 1 - This table shows the different kind of bonds present in the dataset. The horizontal row gives the main categories of the bonds. The vertical column gives the extra properties of a bond.

TABLE 1 Number of bond types in dataset

	Bond types				
Extra properties	Bullet	Sinking fund - Equal	Sinking fund - Unequal	Perpetual	Totals
None	63	1	64	0	128
Callable	2	35	42	3	82
Convertible	0	0	8	0	8
Totals	65	36	114	3	218

Sinking fund

Some remarks on the sinking fund bonds are made. As said, the redemption scheme is the expected course of redemption for sinking fund bonds. A lottery selects series of bonds which will be redeemed, a series of bonds is thus redeemed in full and not a part of each bond. Redemptions occur

[5] Seperate Trading of Registered Interest and Principal Securities

randomly over all outstanding bonds. Therefore this thesis assumes that an investor's portfolio of bonds follows the redemption scheme. Investors are assumed to have certainty about future redemptions on their bonds. These assumptions can be made because of the size of investments in bonds by institutional investors.

External data is used to create an overview of a redemption scheme. This external data is necessary because redemptions cannot simply be calculated as the negative mutation between issued amount and outstanding amount of a bond. If this method is used to create redemption data, not only redemptions are incorporated, but also bonds that are bought on financial markets by the state. This problem also arises when a bond issue is reopened or enlarged, which changes its amount outstanding. Data for this redemption scheme is directly obtained from the Government bond register and with this information a redemption scheme for the period 1952 until 2001 is created.

A bond can be a sinking fund bond and also qualify as a callable bond. If a bond is called by the state before maturity, the bond is retired. The original redemption scheme for that bond to its maturity changes subsequently. The redemption scheme can be adjusted with hindsight to the historical correct redemption information. Pricing of bonds is based on available information on a certain moment of time. Because the calling of a bond is information which originally was not available to investors, the original published bond redemption scheme is used. From the moment on which a bond is fully (and early) called and redeemed, the bond is no longer used in the calculations.

4.1.3 Price, accrued interest and yield of bonds

Price data of bonds are obtained via different sources. A large part of the data originates from ABN AMRO and the 'Prijscourant'[6]. The bond prices in the dataset are clean prices, which do not incorporate accrued interest. In this thesis, the accrued interest for each bond is computed.

Yields of bonds are calculated from the dataset and are not externally obtained. To calculate the yield to maturity, an overview of cash flows is made for each bond. Thereupon the accrued interest plus the clean bond price is set equal to the present value of cash flows of a bond. The yield is then calculated by trial-and-error. In estimating the term structure models, only bonds with yield between zero and fifteen percent are included in the calculations.

4.1.4 Coupons and day count convention

The bonds in the dataset pay an annual or semi-annual coupon interest. The coupons are paid out on coupon date(s), which are specified in the prospectus. Normally, the coupon dates are on the 1st or 15th day of the month. Some older bonds have as coupon date the 2nd, 16th or 30th day of the month. Semi-annual coupons pay the second coupon six months after the first coupon halve. In May 1956 the last semi-annual coupon bond was issued.

Calculations in this research on the dataset are done in line with the day count convention as used on the researched date. Until December 31, 1998 the 30/360 day count convention is used. This implies for the calculations of present values, cash flows and days of interest, that a month and year counts 30 days, respectively 360 days. In 1999 a new day count convention was introduced by Euronext: actual/actual. This convention became effective for each bond on the first coupon date following January 1st in 1999. In the actual/actual day count convention computations are made with the actual number of days in a month and year.

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[6] This information was supplied by Dr. Smant of the Erasmus Universiteit Rotterdam.

4.1.5 Tax effects

Government and commercial bonds have the same features, the bonds only differ in their issuer. Governments do not just issue bonds, but also have the power to differentiate in tax treatment between Government bonds en corporate bonds. Different sets of rules apply for these two types of bonds. Government bonds are treated on more favourable terms and tax treatment of Government bonds will be more sympathetic. Governments justify this with the thought that participating in a state issued bond, the common good is served. In the United States income earned on Treasury bills and bonds are exempt from state and local income taxes[7]. Corporate bond revenues are subject to local and state income taxes, which lower its effective interest. Because of the tax exemption, US Government bonds quote a lower interest rate before tax than corporate bonds, but earn a higher effective interest rate for investors. In other countries no differentiation between bonds is made and all revenues are subject to taxes.

If various tax treatments exist for different types of bonds, then Government bonds should be adjusted for the tax effect in order to be comparable. Imagine a Government bond and a corporate bond with both the same risk of default. In case of a lower tax on Government bond revenues, the in-all-other-ways-the-same corporate bond pays a higher interest to compensate for the higher tax paid over the corporate bond revenues. An investor will not invest in a corporate bond, if a higher net revenue is made by investing in a Government bond. A term structure for risk free interest rates obtained through Government bonds should then be adjusted upwards for tax effects.

This study researches Government bonds issued by the Netherlands. Over time Dutch tax laws have changed. Since 1964 until 2001 the capital tax was in force. The capital tax regime[8] did tax at the start of every year the actual (capital) property at a seven percent tax. This was added to your income, which at the maximum taxes tariff level could be taxed at 60%. In effect the capital property would be taxed by a maximum of 4,2%. The Netherlands used to tax bonds with a capital tax and an interest tax, no differentiation was made between types of bonds. In 2001 a new tax system came in force , the “box” regime, which also doesn’t make differentiations between bond types.

In the Netherlands no such adjustment is necessary because there exists no tax distinction between Government and corporate bonds. In the Netherlands all bonds and their coupon revenues are treated in the same way.

4.2 Methodology

The calculations are based on the framework as described in section 3.3 (Nelson & Siegel model) and section 3.4 (Extended Nelson & Siegel model). As a basis for this research, the following key variables are calculated: maturity, yield and cash flow. The general overview and the calculations of all key variables are done in Excel. The complete dataset consists of data over different years. A dataset is differentiated by its year.

4.2.1 Maturity

At each dataset date two maturities are calculated. In these computations the appropriate day count convention is taken into account. The first calculated value is the maturity of each bond, which is the end date of the bond minus the researched date. This maturity is used in the Nelson & Siegel and Extended Nelson & Siegel estimations. The second calculated values are the maturities of the individual cash flows (coupons and principal payment) for each bond. This cash flow maturity is

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[7] See http://www.treasurydirect.gov/indiv/products/prod_tbonds_glance.htm
[8] The property tax is called ‘vermogensbelasting’ in the Netherlands

obtained by subtracting the cash flow date from the researched date. Each cash flow uses its ´own´ maturity to convert the cash flow to a present value. These are supporting calculations in order to compute the yield of the bonds at the research date.

The nominal maturity is just one factor of importance. In this study the assumption is made that a bond is bought and then held until maturity. As described, multiple bonds in the dataset qualify as a sinking fund. If the Dutch state redeems a part of the outstanding amount of a bond, before its final maturity, than this redemption decreases the overall maturity of the principal of the bond. A sinking fund thus lowers the maturity of a bond, because the investor gets a portion of his investment repaid before final maturity. The average maturity is calculated as follows:

$$am_t = \sum_{t=1}^{m} \left(\frac{\delta_{t+1} - \delta_t}{\delta_{t_1}}\right) \cdot m_t \qquad (4.1)$$

am	=	average maturity	δ_t	=	outstanding part bond at t
δ_{t+1}	=	outstanding part bond at t+1	$m_{(t-t_0)}$	=	maturity of share

On a specified date equation 4.1 calculates the average maturity of a bond. Redemptions on bonds follow the redemption scheme. On the research date the outstanding percentage of a bond, δ_t is 100%. Redemption values are kept as a percentage of the original issuance. If a bond has no early redemptions, the maturity is the maturity date minus research date. A (scheduled) early redemption makes the redeemed part to have a maturity until its redemption date. Also, if an investor buys the bond at a moment where the bond has already been partially redeemed, then from the eyes of an investor, he buys a full bond. In the calculations the outstanding percentage is thus 100% at the start of each research date (e.g. If at research date 31/12/1998, 50% of the original issue still exists, this is used as the 100% mark). The average maturity is used in the term structure model estimations.

4.2.2 Yield calculation

For the calculation of the yield of bonds, the following information is necessary: price, accrued interest, coupons, face value and maturity of the bonds. These elements are known or are calculated in an earlier stage from known information. Some assumptions are made. Transaction costs, tax restrictions and market imperfections do not exist. Investors act rational. These assumptions are common and necessary to create a pure framework for bond pricing and yield calculation.

The yield is computed by setting the present value of cash flows of the bond equal to the price. The discount rate used is the yield. The equation to calculate the present value of cash flows (or 'dirty' bond price) is:

$$P_{dirty} = PV_t(cash\ flows) = \sum_{i=1}^{m} \frac{C_i}{(1+y)^i} + \frac{F}{(1+y)^i} \qquad (4.2)$$

At the moment a bond is sold, the buyer has to pay the seller accrued interest. That is interest, which is earned on the bond since the last coupon payment, but has not yet been paid out. Financial markets report clean bond prices, which are bond prices without accrued interest. To calculate the clean price, the accrued interest has to be subtracted from the dirty bond price. The clean price of the bond summed up with accrued interest enables us to calculate the yield. First, the accrued interest is computed with equation 4.3.

$$AI = \frac{C}{n_c} \cdot \frac{D_p}{D_c} \qquad (4.3)$$

AI	=	Accrued interest	D_p	=	days in current coupon period
C	=	Coupon rate	D_c	=	coupon period in days
n_c	=	Number of coupons per year			

The yield discounted value of cash flows of the bond is equal to the value of the clean price and the accrued interest. See the following formula.

$$P_{clean} = P_{dirty} - AI \qquad (4.4)$$

The yield of a bond can now be calculated.

4.2.3 Estimation of yield curve (Nelson & Siegel model)

After the calculation of the yield data of the existing bonds, all necessary data to estimate a yield curve with the Nelson & Siegel model is present. A smooth yield curve can now be estimated for each date in the dataset. The next step in the estimation process is to estimate the beta's in the following equation. Equation 4.5 is equal to the equation Nelson & Siegel (1987) used in their research. This equation is a third order model.

$$y(am) = \beta_0 + \beta_1 \cdot \left[\frac{1-\exp\left(-\frac{am}{\tau}\right)}{\left(\frac{am}{\tau}\right)}\right] + \beta_2 \cdot \left[\frac{1-\exp\left(-\frac{am}{\tau}\right)}{\left(\frac{am}{\tau}\right)} - exp\left(-\frac{am}{\tau}\right)\right] \qquad (4.5)$$

The values of β_0, β_1 and β_2 are estimated with an ordinary least squares regression with a 95% confidence level. Each dataset is regressed for 45 possible τ values. The τ values are in years, and start at a value of 10 days (converted in 0.03 years). The maximum τ value is 20 years[9]. This procedure generates different beta outcomes. As a measure of the quality of the found results, the R^2 of the regression is used. The results of the regression and the accompanying τ value with the best R^2 are utilized. The resulting estimates of β_0, β_1 and β_2 with their best τ-value are shown in the next chapter.

In their research Nelson & Siegel state that no set of variables gives a perfect fit of the (estimated) yield curve with the real yield data. Their objective is to make general statements about the interest, not to make perfect estimations.

There exists an error term, which is shown in the results by the standard error or standard deviation of the error term. These errors can be caused by "transaction costs, tax exemption, the capital gains treatment of deep discount bonds, callability, convertibility, ineglibility for commercial bank purchase and imperfect arbitrage" (McCulloch, 1971).

4.2.4 Estimation of yield curve (Extended Nelson & Siegel model)

After the process described in the section before, the beta variables of the Extended Nelson & Siegel model are estimated. Equation 4.6 is a fourth order model. The following equation is used:

$$y(am) = \beta_0 + \beta_1 \cdot \left[\frac{1-\exp\left(-\frac{am}{\tau_1}\right)}{\left(\frac{am}{\tau_1}\right)}\right] + \beta_2 \cdot \left[\frac{1-\exp\left(-\frac{am}{\tau_1}\right)}{\left(\frac{am}{\tau_1}\right)} - exp\left(-\frac{am}{\tau_1}\right)\right] + \beta_3 \cdot \left[\frac{1-\exp\left(-\frac{am}{\tau_2}\right)}{\left(\frac{am}{\tau_2}\right)} - exp\left(-\frac{am}{\tau_2}\right)\right] \qquad (4.6)$$

[9] All following values are converted to years. The range of τ values start at a value of 10 days and increase with an interval of 10 to 200 days. Then this increases in 16 interval steps (with 50 days) to 1000 days. Also nine τ values between 3 and 20 years are taken into account.

The values for τ_1 and τ_2 are the same as the τ values with the Nelson & Siegel model. They both have values between 0.03 and 20 years. If τ_1 and τ_2 have the same value, then β_2 and β_3 are perfectly collinear because the β_2 component is the same as the β_3. In the estimation process, τ_1 and τ_2 are not allowed to have the same value. Each dataset is regressed on all possible τ_1 and τ_2 combinations (2025) to find the beta estimates. The result with the highest R^2 and its appropriate τ_1 and τ_2 value are presented.

5. First results

The results of this thesis are discussed in two chapters; chapter 5 discusses results for all forty-nine annual datasets. Chapter 6 proposes some adjustments to the datasets, based on the results of chapter 5. The results in the latter chapter are based on only one year.

Firstly, this chapter discusses the analyses of the Nelson & Siegel model. Secondly, the analyses of the extended Nelson & Siegel model are presented. In the third section, an adjusted dataset is presented to improve results. A combined estimation results of Nelson & Siegel and the Extended Nelson & Siegel model on this dataset are also discussed in this section.

5.1 Results Nelson & Siegel model

The results of the Nelson & Siegel model are separated into three subsections: normal estimations, median τ estimations and second order model estimations.

Normal estimations

The results of the Nelson & Siegel estimation model are shown in table 2. Forty-nine annual datasets have been researched. The researched annual dataset is presented in column 1, column 2 represents the best-fitting value of τ. The standard deviation of the residuals (standard deviation) is shown in column 4 in basis points (hundredths of a percent). The last columns presents the R^2 of the regressions. The last row gives the median value of the results over the 49 datasets.

The best fitting τ values of the Nelson & Siegel estimation vary between 0.03 and 20. The results show that six τ-values are found on the boundaries of the allowed τ-values range (marked in the table with *). It can be observed that the median of the best τ -values of the original Nelson & Siegel model is 1.11. The best R^2 is obtained with the dataset of December 28, 2001 where R^2 is 97.46% and τ is 1.01. The worst fit of the model is obtained with the dataset of December 29, 1989 with a R^2 of 3.64% and a τ-value of 1.67. The worst and best dataset and their best fitted yield curve estimations are plotted in figure 1 and 2.

The 'worst fitted' dataset of December 29, 1989 is plotted in figure 1. The graph also shows the estimated yield curve with the Nelson & Siegel model. The reason that the estimated yield curve is shown and not the estimated forward rate curve, is that the graph gives a better indication of the fit of the yield estimation against the current yield data. The figure indicates that on December 29, 1989, bond yields are divided in three groups: a group which follows the estimated curve, a group which is situated below the estimated curve and a group which is situated above the estimated term structure. The low R^2 of 3.64% is mainly caused by the group of outliers situated below the yield curve (see chapter 6). Most of these outliers are callable bonds.

A visual indication of the dataset of December 28, 2001 and the estimated yield curve are presented in figure 2. The fit of the model on this dataset is 97.46%. The residuals have a standard deviation of 10.55 basis points. It can be seen that the estimated term structure fits the data very well. The yield curve of the dataset of December 28, 2001 shows a much better fit than estimation for the dataset of December 29, 1989. An explanation for these results, is that in the 1989 dataset multiple types of bonds co-existed (such as callable bonds, convertible bonds, sinking fund bonds and bullet bonds), where in the 2001 dataset only one type of bonds exists (bullet bonds). Different types of bonds may have premiums of discounts included in their bond prices. This may be the case for the dataset of 1989. In chapter 5.3 and chapter 6, this data issue is discussed.

Median τ estimations

In the term structure estimations, each annual dataset is fitted with an individual best τ value. This individual time constant is added to get a better fit of the estimations with the datasets. To check for the added value of fitting each annual dataset with an individual τ value, the median τ value (1.11) is used on all datasets and the yield curves are estimated again with the Nelson & Siegel model. The results show that little precision is lost. Column 5 of table 2 shows that the median of the standard deviation of residuals almost stays the same, with an increase to 31.99 from 31.97 basis points. This is an increase of just 0.02 basis points in standard deviation in the case where not every dataset is allowed to have its own best fitting τ -value. In cases where the best τ value fit was 20, which now uses the τ -value of 1.11, the standard deviation increases with a maximum of 1.18 basis points. This is 0.000118 in nominal yield. The R^2 median decreases from 49.35% to 43.35%. Only six of the forty-nine researched datasets show a decrease of 10% points in R^2. The datasets of December 29, 1967 and December 30, 1968, show a decrease in their fit by respectively 8.05 percentage points and 5.83 percentage points. Most of the datasets have a best τ value close to 1.11. Effects of the use of the median τ are small in those cases. A larger effect is found when the best fitting τ value 20 is replaced by a value of 1.11. The τ – value, which determines the rate of decay of the regressors, is then too small for the longer maturities, which decreases the fit. An exception for this results in the dataset of December 30, 1953. This dataset contains six bonds, which makes fitting the curve more simple.

The overall results show that the median of the standard deviation changes by a minimal amount. The median R^2 decreases by 5 percentage points. This research finds that there may be little profit of fitting an individual best τ to each dataset, with the knowledge of the median value. However, on forehand, the 'standard' τ - value to be used in all estimations on the datasets, is not known. This value has to be found first, before it can be used in all estimations. In addition, some estimations show a worse fit with the median τ – value used. Therefore, the conclusion is that the each dataset needs an individual τ – value.

Second order model estimations

The advantage of equation 4.5 (third-order model or original Nelson & Siegel model) is that it is able to generate hump shaped curves. It is supposed to give extra flexibility to the alternative of a second-order function. The question is whether this extra flexibility really contributes to an improved fit above a second-order equation model. This is tested by setting β_2 to zero in equation 4.5. The results are presented in the last three columns of table 2. Column 7 presents the best τ value for each dataset, column 8 presents the standard deviation of the residuals and column 9 gives the R^2.

The results show a median τ-value of 1.11. The median of the standard deviation increases slightly to 32.69 basis points, where it was 31.97 with the original Nelson & Siegel model estimations. The median of R^2 decreases from 49.35% to 37.12%. Eighteen datasets show a decrease in R^2 of more than 10 percentage points, of which nine datasets show a decrease of over 20 percentage points. The dataset of December 29, 2000 for example has an original R^2 of 89.30%, but when the second-order model is used, this decreases to 55.22%. The second-order model misses an extra hump component, which makes that not all possible (short term) types of term structures can be presented.

The conclusion is that the second-order Nelson & Siegel model produces weaker results than the original Nelson & Siegel estimation. The β_2 component contributes to a better fit of the term structure estimation. The second-order model certainly gives a rough estimation of the term structure, but more precision is obtained with the third-order model.

Table 2 - Results of the Nelson & Siegel regressions. For each dataset, presented results are the best outcome of regressions with all possible τ values.

TABLE 2 Measures of Model Fit (Nelson & Siegel)

	Original model					second-order model		
Date Dataset	Best τ - value	St. Dev. at best τ	R^2	St. Dev. at τ = 1,11	R^2 at τ = 1,11	Best τ - value	St. Dev. at best τ	R^2
30/12/1952	20.00 *	11.83	49.35	12.11	46.91	20.00 *	10.38	48.00
30/12/1953	1.53	11.78	72.45	11.80	72.34	0.08	14.95	46.74
30/12/1954	2.08	14.76	68.20	14.88	67.69	0.03 *	17.10	50.23
29/12/1955	0.69	9.23	69.75	9.27	69.48	0.14	12.32	39.42
28/12/1956	3.00	14.29	63.58	15.20	58.80	20.00 *	14.57	57.93
30/12/1957	10.00	22.92	64.36	27.52	48.60	20.00 *	29.04	36.41
30/12/1958	10.00	8.97	65.03	11.39	43.55	20.00 *	13.22	16.31
29/12/1960	7.50	8.08	91.77	13.63	76.60	0.08	18.67	53.18
28/12/1961	7.50	7.23	91.48	9.50	85.29	0.03 *	10.19	82.06
28/12/1962	12.50	5.38	89.77	6.13	86.68	1.01	5.97	86.68
30/12/1963	5.00	5.10	97.09	6.64	95.06	0.03 *	7.08	94.12
30/12/1964	20.00 *	7.01	32.62	8.19	7.92	0.83	8.04	7.67
30/12/1965	20.00 *	12.07	46.00	14.51	21.92	20.00 *	14.43	19.76
29/12/1966	1.11	10.29	26.82	10.29	26.82	0.03 *	10.82	16.22
29/12/1967	20.00 *	12.04	16.02	12.61	7.98	0.08	12.67	4.15
30/12/1968	20.00 *	11.34	44.04	11.92	38.20	0.03 *	11.85	37.13
30/12/1969	7.50	19.97	31.11	21.22	22.24	0.03 *	22.33	11.64
30/12/1970	4.00	96.69	6.35	98.13	3.53	0.17	97.65	2.35
30/12/1971	0.11	22.80	35.06	23.65	30.12	4.00	23.88	27.31
29/12/1972	1.39	31.97	57.29	31.99	57.24	1.39	31.67	57.29
28/12/1973	7.50	36.38	25.74	38.95	14.87	0.83	38.66	14.56
30/12/1974	0.53	36.71	70.54	36.98	70.10	0.53	36.37	70.53
30/12/1975	0.83	38.37	62.32	39.02	61.04	5.00	42.84	52.21
30/12/1976	1.01	29.78	58.76	29.80	58.71	4.00	30.47	56.11
29/12/1977	0.83	49.63	31.37	49.67	31.25	5.00	50.07	29.06
29/12/1978	0.17	62.66	27.28	63.77	24.68	1.11	63.33	24.68
28/12/1979	0.11	58.99	16.69	60.26	13.05	1.25	59.89	13.03
30/12/1980	2.78	78.95	26.46	79.05	26.27	1.01	78.60	26.26
30/12/1981	0.83	112.13	19.33	112.13	19.32	0.83	111.54	19.33
30/12/1982	0.42	69.67	37.73	69.99	37.17	1.67	69.69	37.12
29/12/1983	0.31	64.71	21.74	65.93	18.74	1.67	65.72	18.55
28/12/1984	0.14	72.94	29.03	73.35	28.22	0.47	72.74	28.81
30/12/1985	1.39	86.30	8.26	86.32	8.22	20.00 *	86.22	7.61
30/12/1986	0.03 *	85.29	7.95	85.86	6.71	10.00	85.80	6.00
30/12/1987	0.19	49.89	55.22	50.16	54.74	0.53	49.97	54.69
29/12/1988	5.00	58.93	13.73	59.01	13.48	5.00	58.68	13.73
29/12/1989	1.67	81.44	3.64	81.46	3.58	0.56	81.16	3.44
31/12/1990	1.01	54.33	14.07	54.34	14.04	0.19	54.84	11.62
30/12/1991	0.22	65.44	19.51	66.59	16.64	1.25	66.27	16.61
30/12/1992	2.08	41.16	35.07	41.86	32.82	0.47	42.71	29.28
30/12/1993	0.69	34.11	67.89	35.72	64.79	0.08	44.47	44.69
30/12/1994	0.42	51.40	61.70	57.49	52.09	4.00	61.33	44.60
28/12/1995	1.11	37.84	78.82	37.84	78.82	7.50	41.91	73.53
30/12/1996	1.11	29.45	90.15	29.45	90.15	4.00	32.69	87.59
30/12/1997	0.69	34.35	64.60	35.57	62.05	7.50	40.86	48.72
30/12/1998	1.94	12.99	89.14	15.92	83.69	20.00 *	20.80	71.42
30/12/1999	0.69	17.22	93.66	19.72	91.69	4.00	24.52	86.74
29/12/2000	1.11	11.67	89.30	11.67	89.30	0.03 *	23.39	55.22
28/12/2001	1.01	10.55	97.46	10.98	97.24	4.00	18.07	92.19
Median	1.11	31.97	49.35	31.99	43.55	1.11	32.69	37.12

Note - Standard deviations are in Basis Points

* Best fit realized at boundary of range of τ

Figure 1 - Worst fit of Nelson & Siegel model (R^2 = 3.64%), dataset December 29, 1989 with τ = 1.67

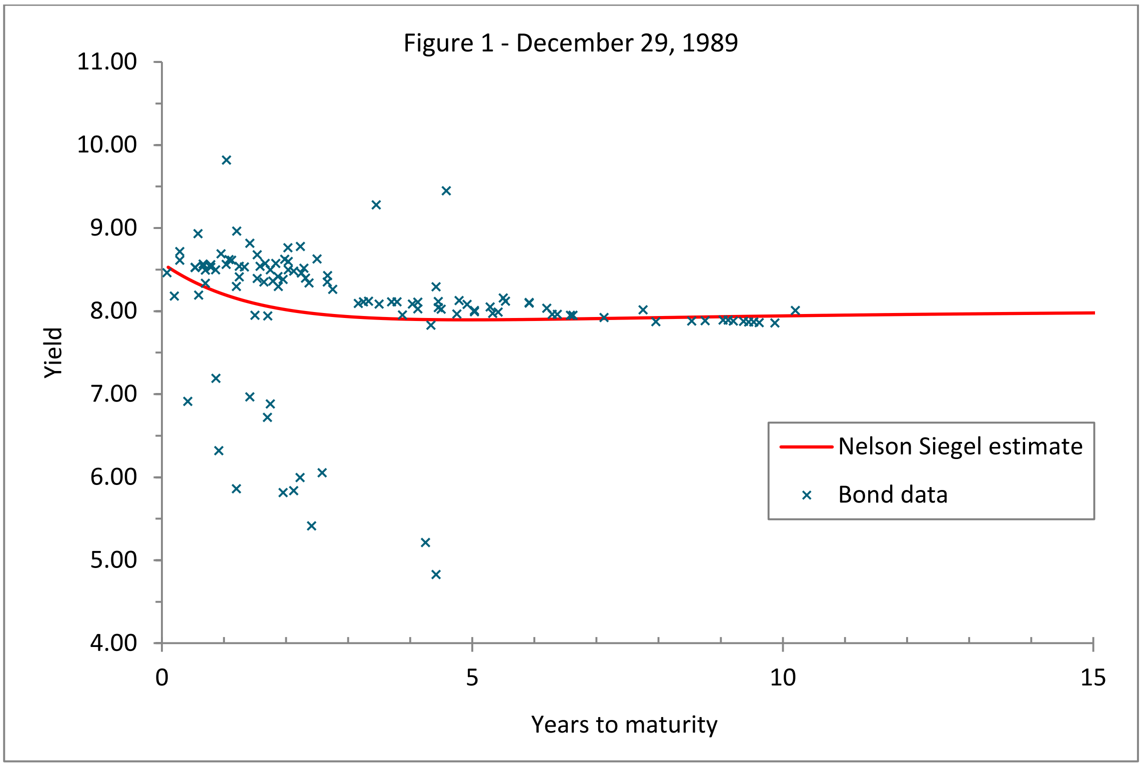
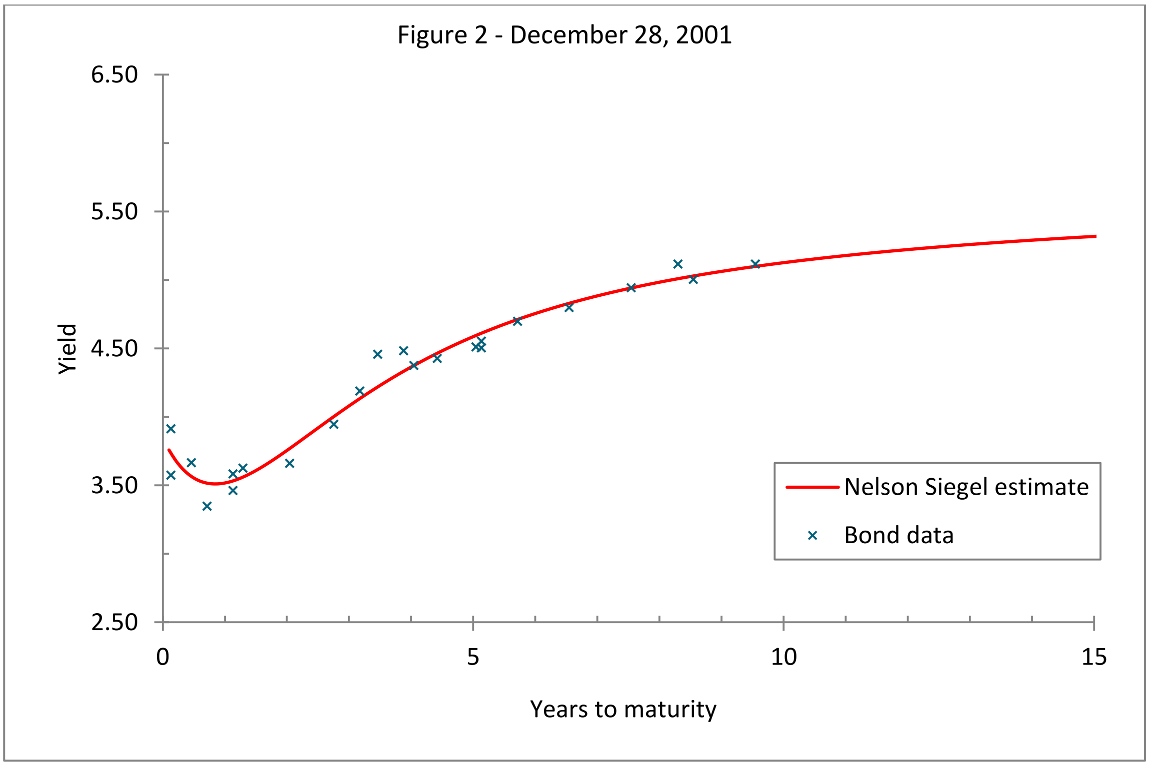


Figure 2 - Best fit of Nelson & Siegel model (R^2 = 97.46%), dataset December 28, 2001 with τ = 1.01



5.1.1 Adjusted maturity

The previous paragraph provides the results of the yield curve estimations of all outstanding bonds of the dataset, also including the bonds with a short maturity (zero to two years). When a bond enters its last years of maturity, problems may arise. Since bonds with short maturities are often illiquid bonds (Jeffrey, Linton, & Nguyen, 2006), this may influence bond price, price variation and bond yield. These bonds may show extreme results and should therefore be excluded from the dataset. To test whether this effect is present, the short-term maturities from zero to two years are excluded from each annual dataset. This is followed by yield curve estimations with the Nelson & Siegel model. The results of this analysis are shown in appendix C.

The results indicate that the median of the standard error (of the limited maturity dataset) improves because it decreases 8.94 basis points, as compared to the original regressions with the complete dataset. On the other hand, the R^2 median of the adjusted dataset decreases to 34.18%, which was 49.35% when bonds with all maturities were included.

This leads to the conclusion that omitting bonds with maturities between zero and two years does not improve the results. An explanation is that the Nelson-Siegel model gets explanatory power due to its ability to include reasonable outlier bond data. Bonds in the dataset that qualify as outlier, were already omitted from the dataset, see section 4.1.3. Crucial explanatory data for the regression is lost when bonds with maturities between zero and two years are omitted. The explanatory power of the regression becomes less.

5.1.2 Differences with the original research by Nelson & Siegel

The results of the analyses performed in this research are different than the results of the Nelson & Siegel study. The results yield a lower fit of the model estimations to the real data as compared to the Nelson & Siegel study. The best τ-values also have different outcomes. Several explanations could explain the differences between this research and Nelson & Siegel's research: firstly, this study researches Dutch bonds and not U.S. bonds. At any point of time there are more U.S. Government bonds outstanding than Dutch Government bonds. More bonds implicate more information for the model estimations (assumed that all data is good data). Secondly, the researched bonds in this thesis are Government bonds and not T-bills, as in the study of Nelson & Siegel. Government bonds have maturities up to thirty years, where T-bills have a maximum maturity of 360 days. Because of the more widely spread maturities and the limited number of available bonds in this thesis' dataset, the model estimations may be influenced in a negative way.

5.2 Results Extended Nelson & Siegel model

After the Nelson & Siegel results, this section discusses the results of the Extended Nelson & Siegel model estimations. The outcomes of the estimations are separated into two subsections: normal estimations and median τ estimations.

Normal estimations

Table 3 shows the outcome of the regressions of the Extended Nelson & Siegel model. Again, forty-nine dates have been researched. Column 1 represents the researched dataset date, column 2 shows the best-fitting value of τ_1, column 3 represents the best-fitting value of τ_2, column 4 presents the standard deviation of residuals (standard error) in basis points (in hundredths of a percent) and column 5 gives the R^2 of the regression results. The last row gives the median value of the results over the 49 datasets.

Table 3 - Results of the Extended Nelson & Siegel regressions. The presented results for each dataset are the best outcomes of regressions with all possible τ_1 and τ_2 values.

TABLE 3 Measures of Model Fit (Extended Nelson & Siegel)

	Original model					
Date Dataset	Best τ_1 - value	Best τ_2 - value	St. Dev. at best τ	R^2	St. Dev. At τ_1 = 1,39 & τ_2 = 0,83	R^2 at τ_1 = 1,39 & τ_2 = 0,83
30/12/1952	10.00	2.78	10.13	75.26	12.12	46.87
30/12/1953	20.00 *	2.78	5.95	94.37	11.78	72.43
30/12/1954	20.00 *	2.64	11.29	84.51	16.15	68.28
29/12/1955	17.50	20.00 *	8.17	79.29	9.84	69.95
28/12/1956	20.00 *	2.50	9.83	84.69	13.91	69.32
30/12/1957	2.64	1.25	23.49	66.71	25.22	61.64
30/12/1958	4.00	1.39	9.37	65.66	10.80	54.29
29/12/1960	2.36	2.64	7.82	92.81	9.08	90.31
28/12/1961	1.67	0.69	6.88	92.72	7.05	92.37
28/12/1962	5.00	20.00 *	5.45	90.08	5.78	88.85
30/12/1963	2.78	15.00	4.74	97.61	5.92	96.27
30/12/1964	0.36	7.50	6.24	48.75	6.83	38.66
30/12/1965	5.00	20.00 *	12.03	48.35	14.02	29.92
29/12/1966	4.00	20.00 *	9.52	39.66	10.49	26.72
29/12/1967	4.00	17.50	12.11	17.88	12.41	13.73
30/12/1968	2.78	15.00	10.94	49.48	12.08	38.41
30/12/1969	1.39	0.56	19.47	36.20	21.23	24.16
30/12/1970	1.81	12.50	95.68	10.34	97.00	7.84
30/12/1971	20.00 *	1.01	22.93	35.67	23.22	34.00
29/12/1972	0.42	0.19	30.72	61.36	31.11	60.36
28/12/1973	1.39	0.22	33.07	39.82	37.01	24.62
30/12/1974	3.00	0.69	36.33	71.68	36.66	71.18
30/12/1975	0.83	0.17	37.34	64.95	38.70	62.35
30/12/1976	1.01	0.39	29.47	60.32	30.07	58.68
29/12/1977	0.50	0.22	49.49	32.81	50.02	31.37
29/12/1978	5.00	0.83	61.75	30.37	62.25	29.25
28/12/1979	0.25	0.17	58.28	19.72	59.18	17.20
30/12/1980	0.31	0.17	78.99	27.24	79.43	26.41
30/12/1981	0.33	0.17	112.11	20.20	112.62	19.49
30/12/1982	0.14	0.08	69.58	38.50	70.21	37.38
29/12/1983	0.08	0.06	64.09	23.93	65.47	20.61
28/12/1984	0.17	2.64	72.24	30.97	72.65	30.19
30/12/1985	1.11	0.22	86.53	8.60	86.69	8.26
30/12/1986	0.56	0.11	83.68	12.20	86.25	6.74
30/12/1987	2.78	12.50	49.97	55.49	50.37	54.77
29/12/1988	0.03 *	20.00 *	59.12	13.93	59.22	13.65
29/12/1989	0.56	5.00	81.64	4.03	81.76	3.74
31/12/1990	0.42	0.08	54.15	15.41	54.60	14.02
30/12/1991	0.42	0.03 *	65.64	19.81	66.52	17.66
30/12/1992	0.14	1.94	40.74	37.07	41.09	35.98
30/12/1993	1.81	0.03 *	29.73	75.93	34.54	67.51
30/12/1994	0.03 *	0.25	47.22	68.18	53.42	59.28
28/12/1995	0.25	1.67	32.85	84.33	37.81	79.24
30/12/1996	2.08	0.17	29.48	90.36	29.71	90.20
30/12/1997	0.03 *	0.28	17.94	90.58	35.02	64.09
30/12/1998	1.81	0.14	8.64	95.32	14.05	87.63
30/12/1999	0.08	12.50	9.54	98.12	18.30	93.07
29/12/2000	2.36	0.03 *	5.19	97.97	11.36	90.28
28/12/2001	1.11	0.11	10.49	97.60	10.67	97.51
Median	1.39	0.83	29.48	55.49	31.11	46.87

Note - Standard deviations are in Basis Points

* Best fit realized at boundary of range of τ search

The best fitting τ_1 and τ_2 values vary between 0.03 and 20. The best estimation is obtained with the dataset of December 30, 1999, which has a R^2 of 98.12%. The worst estimation is found on the dataset of December 29, 1989, which has a R^2 of 4.03%. This dataset also resulted in the lowest R^2 in the Nelson & Siegel estimations performed earlier. On December 29, 1988 the best fitting τ_1 and τ_2 -values are both found on the outer boundaries of the allowed τ_1 and τ_2 -range. In total there are fourteen datasets with a τ_1 and/or a τ_2 – value, which is on the outer limits of the allowed τ_1 and τ_2 ranges.

The R^2 median of the Extended Nelson & Siegel results increases from 31.97% to 55.49% compared to the results obtained with the original Nelson & Siegel model. The median of τ_1 and τ_2 values are 1.39 and 0.83. Because the Extended Nelson & Siegel model uses an extra τ-variable, the median of τ_1 and τ_2 are not directly comparable with the Nelson & Siegel model results. But it is interesting to see that the median values are close to the median τ value (1.11) result of the Nelson & Siegel model estimations. The τ-values of the Extended Nelson & Siegel model thus averagely remain the same.

The worst estimation of the Extended Nelson & Siegel model is plotted in figure 3, which is for dataset of December 29, 1989. The estimated yield curve has a fit of 4.03%. The figure clearly shows outliers, situated above and below the estimated curve, which are responsible for the low fit of the curve. The outliers are further discussed in section 5.3 and chapter 6.

The best estimation of the model is on dataset December 20, 1999, which is graphed in figure 4. This figure shows the estimated term structure curve and the four separate components from which the yield curve is built of. It shows that the long-term interest (component one) is influenced in the short and medium term by a declining positive component two and a negative to zero increasing component three. The long-term is determined by component one and four with an long term interest rate of six percent. The result is characterized as a normal yield curve.

Median τ estimations

As performed for the Nelson & Siegel model, an additional analysis of the Nelson & Siegel model is performed to test for the contributed value of an individual best τ_1 and τ_2 value for each annual dataset. The τ_1 and τ_2 values are set equal to the found median τ_1 and τ_2 values in column 1 and 2 of table 3. Column 6 and 7 show the results when the median values of τ_1 and τ_2 are used on all dataset estimations.

The median of the standard error increases with 1.63 basis points to 31.11 basis points, compared to the situation where each estimation is fitted with an individual best τ_1 and τ_2 value. The R^2 median decreases with 8.62 percent points from 55.49 to 46.87 percent. This result is slightly less than the Nelson & Siegel estimations, where the use of the median τ –value led to a R^2 reduction with 5.80 percent points. The dataset of December 30, 1997 shows the largest increase in standard error with 17.08 basis points and a decrease of R^2 of 26.49 percent points. The largest R^2 decrease is found on December 30, 1952 with a decrease of 28.39 percentage points to 46.87 percent and a standard error increase with 1.99 basis points. It can thus be concluded that the use of a single τ_1 and τ_2 value for all estimations decreases the fit of the estimations. The use of median τ values does improve the calculation time: not all possible τ_1 and τ_2 combinations for each regression on a dataset need to be calculated. However, to find these standard values, there are still all the calculations needed. Therefore an individual value of τ_1 and τ_2 for each dataset is preferred.

To conclude, the Extended Nelson & Siegel model results in a better fit of the model estimations than the Nelson & Siegel estimations. This is true for the estimations where τ_1 and τ_2 are allowed to take a unique, best value for each dataset. It is also true when the median τ_1 and τ_2 values are used on all dataset estimations.

Figure 3 - Worst fit of Extended Nelson & Siegel model (R^2 = 4.03%), dataset December 29, 1989 with τ_1=0.56 and τ_2=5.00.

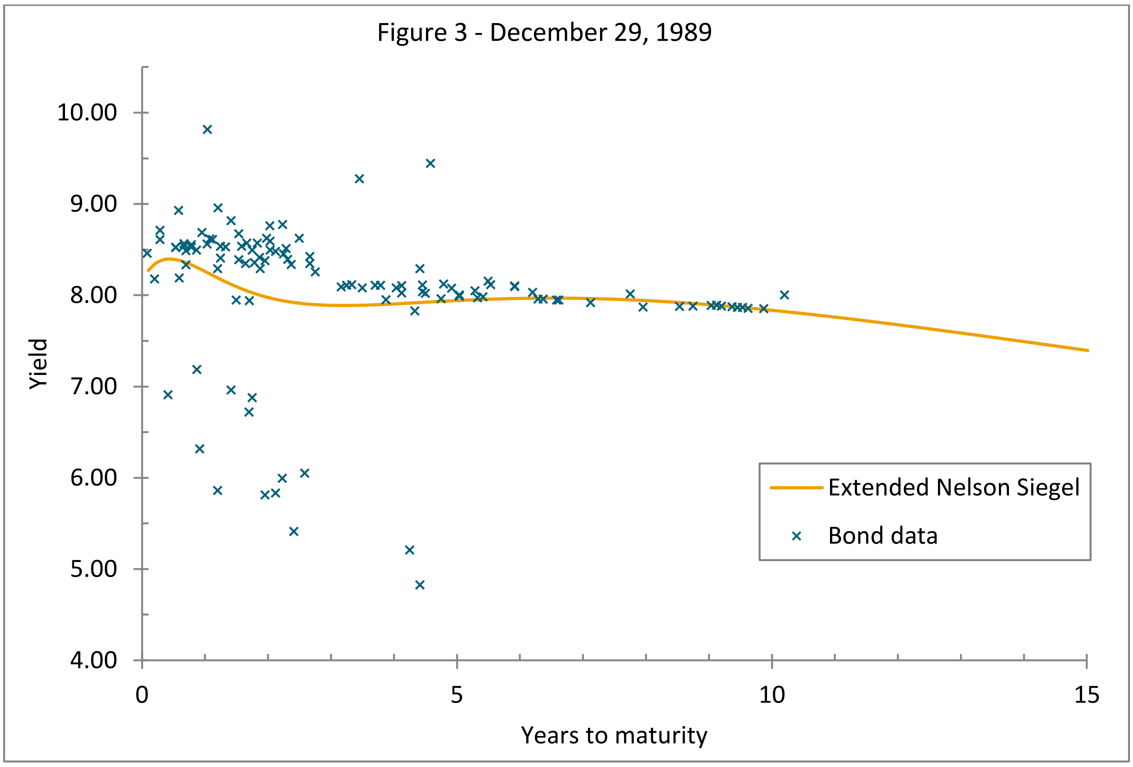
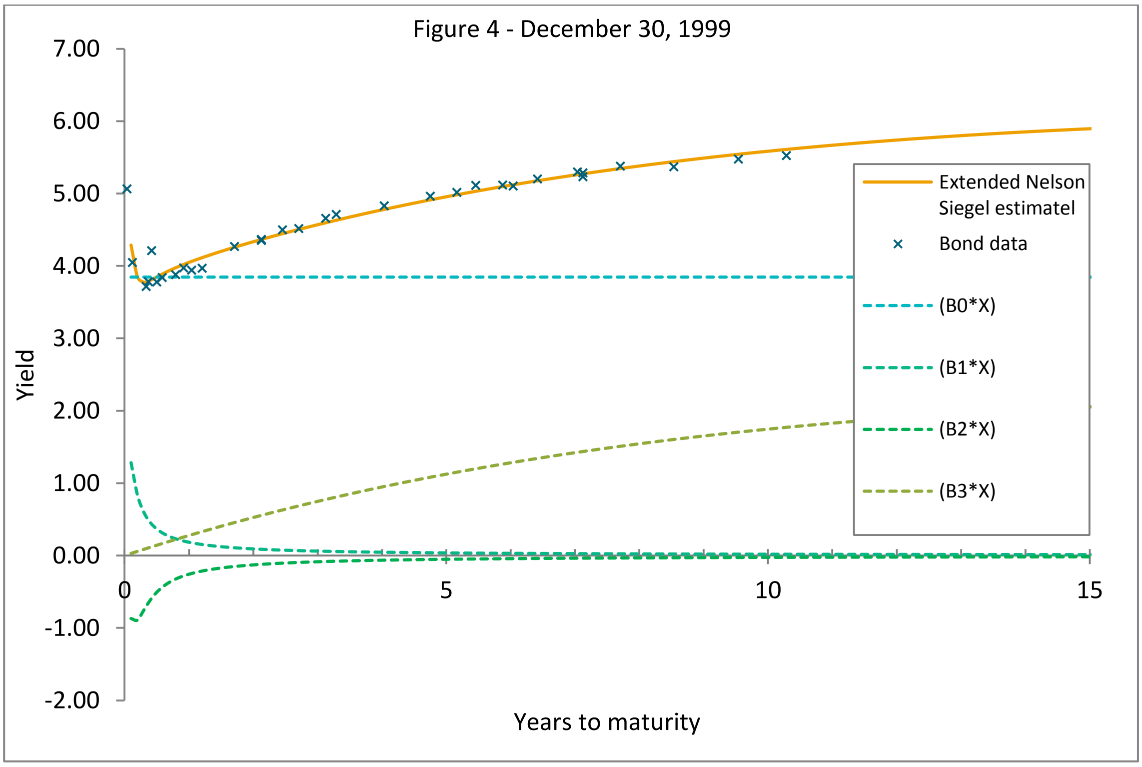


Figure 4 - Best fit of Extended Nelson & Siegel model (R^2 = 98.12%), dataset December 30, 1999 with τ_1=0.08 and τ_2=12.50. Also each individual component of the Extended Nelson & Siegel yield curve estimate is shown.



5.3 Liquidity

The results of the Nelson & Siegel estimations (section 5.1) and the Extended Nelson & Siegel estimations (section 5.2) show that in the period 1967 – 1991 lower R^2 results are obtained. This period contains twenty-five annual datasets of which sixteen Nelson & Siegel estimations produce a R^2 lower than 30%. With the Extended Nelson & Siegel estimations, twelve estimations show a R^2 lower than 30%. This may be an indicator that some bonds in the dataset have special properties which are earlier overlooked and causes estimates with a lower fit. In this section, firstly, the liquidity of bonds is discussed. Secondly, the results of the estimations on the adjusted dataset are presented.

Dataset & liquidity

The dataset of December 29, 1989 showed the worst estimations, for both the Nelson & Siegel model and the Extended Nelson & Siegel model. The dataset and the estimations are shown in figure 1 and figure 3. The scatter plot clearly shows three bond yields which are situated above the estimated yield curves, and fourteen bond yields which are situated below the estimated yield curves. These findings require a more thorough inspection of the data. When the outliers are specified, the results show that the major share of the outliers are yields of callable bond, but also bullet bond yields. This would not be expected with bullet bonds, because these bonds have no special properties which results in positive or negative premiums of the yield. To find a reason for the outliers, properties of the outlier bonds are re-examined.

In the chapter on data and methodology, the redemption scheme of each sinking fund bond is incorporated in the yield computations. The redemption scheme is a measure of the bond amount outstanding as a percentage of the original amount outstanding. It does not show the (nominal) amount outstanding in Euro's. In this re-examination of bond properties, the bond amount outstanding in Euro's is researched. The reason for this is explained with the dataset of December 29, 1989.

On December 29, 1989 there exist fourteen bonds, which are issued before 1964. Almost all of these bonds are sinking funds and during time their outstanding nominal amount has decreased. In general, bonds issued before 1964 have an initial issued amount of less than € 400 mln. While in 1989, twelve Government bonds have been issued, of which the minimum issued amount was € 2 bln. The dataset of December 29, 1989 contains 118 Government bonds with a total amount outstanding of € 179.8 bln. On average, each bond has € 1.5 bln outstanding, but in reality some bonds only have € 5 mln outstanding. These bonds with a very low nominal amount outstanding are assumed to be illiquid. The price and yield of illiquid bonds are based on static information, and therefore in accurate. The dataset is cleared of the illiquid bonds.

If a bond falls under a critical amount outstanding, the assumption is made that it has become illiquid. Therefore, the bond its price data is inadequate to use in the calculations. The minimum value of an outstanding amount to be found illiquid is (arbitrarily) set on ten percent of the average outstanding amount of the bonds. The critical amount is calculated for each annual dataset. All datasets are adjusted with this criterion and illiquid bonds are removed from the dataset. The 'new' dataset is referred to as adjusted dataset (or dataset A).

Results

In the dataset of December 29, 1989, the critical outstanding amount for a bond to be illiquid is set on € 150 mln or less. After the dataset is adjusted with the critical amount outstanding, only 75 bonds remain in the dataset. Six bonds in the remaining dataset are callable bonds. Figure 5 graphs the remaining bonds and the excluded bonds. The graphed Nelson & Siegel yield curve and the

Extended Nelson & Siegel yield curve are estimated with the adjusted dataset. The R^2 rises to 57.08% for the Nelson & Siegel estimation, respectively to 57.20% for the Extended Nelson & Siegel estimation.

Figure 5 - The figure shows the results for the adjusted dataset December 29, 1989. The Nelson & Siegel estimate (R^2 = 57.08%, τ= 1.014) and the Extended Nelson & Siegel estimate (R^2 = 57.20%, τ_1 = 0.08 and τ_2 = 0.83) are both graphed. Illiquid bonds are excluded and illustrated with a red triangle

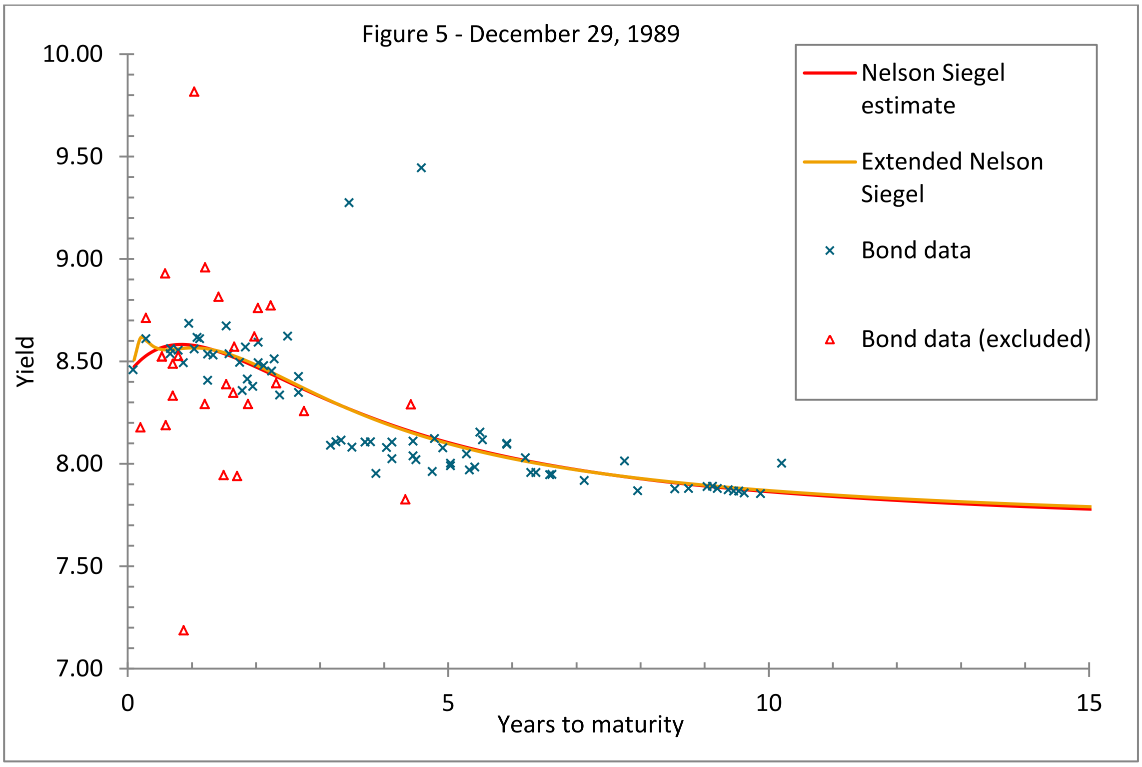


Table 4, columns 2 to 4, show the results for the annual adjusted datasets for the Nelson & Siegel estimations. The estimations show a standard error median of 16.07 basis points and a R^2 median of 62.10%. Compared to the original Nelson & Siegel yield curve estimation, the median R^2 has increased by 12.75 percent points. Of the forty-nine datasets, nine annual datasets in the period 1967 – 1986 are fitted with yield curve estimations with a R^2 of less than 20%.

Table 4, columns 3 to 7, show the results for the annual adjusted datasets for the Extended Nelson & Siegel estimations. The estimations show a standard error median of 13.84 basis points and a R^2 median of 70.05%. Compared to the original Extended Nelson & Siegel yield curve estimation, the median R^2 has increased by 14.56 percent points. Of the forty-nine datasets, eight annual datasets in the period 1967 – 1986 are fitted with a yield curve estimation with a R^2 of less than 20%.

It is remarkable that the results of the Nelson & Siegel estimations and Extended Nelson & Siegel estimations are very similar. The Extended Nelson & Siegel estimations show in most cases a one to three percent points improvement in R^2, compared to the Nelson & Siegel estimations. Only on fifteen datasets, the R^2 of the yield curve estimations increases by 5% or more with the use of the Extended Nelson & Siegel model. And only for seven datasets, this improvement is 10% or more. These results show that the Nelson & Siegel model can estimate the yield curve in most cases on a similar level as the Extended Nelson & Siegel mode. Only in more complex cases, the Extended Nelson & Siegel model will yield a better result.

When all annual yield curve estimations for the Extended Nelson & Siegel model and yield data are graphed in figures, some special yield curves can be seen. Six remarkable yield curves are presented in Appendix E. These figures show that the Extended Nelson & Siegel model gives reliable estimates for the yield curve in the maturity range of the available yield data, but not always for out of dataset range maturities. It can be seen that the model may have a R^2 of 99.99% for December 30, 1952, but that the yield estimates give strange results for short maturities outside the available maturity range. This result is not unexpected, because the model can only fit to available yield data and has no other information of yields outside the available yield and maturity range. The results do show that for longer maturities, reasonable estimates are created. These findings do also apply to the Nelson & Siegel estimations.

It can be concluded that the low fit of yield curve estimates, as described in the previous two sections, is largely solved by removing illiquid bonds of the dataset. This results in a better dataset and better fits of the yield curve estimates. In most cases, the Nelson & Siegel model will return estimations comparable with the Extended Nelson & Siegel model. In more complex cases, the use of the Extended Nelson & Siegel model is preferred.

Table 4 - results of the Nelson & Siegel regressions and Extended Nelson & Siegel regressions on the adjusted dataset. The presented results for each dataset are the best outcomes of regressions with all possible τ_1 and τ_2 values.

TABLE 4 — Measures of Model Fit (adjusted dataset)

	Nelson & Siegel model			Extended Nelson & Siegel model			
Date Dataset	Best τ - value	St. Dev. at best τ	R^2	Best τ_1 - value	Best τ_2 - value	St. Dev. at best τ	R^2
30/12/1952	20.00 *	4.54	94.10	17.50	3.00	0.19	99.99
30/12/1953	1.11	12.14	76.55	20.00 *	2.78	6.42	95.08
30/12/1954	1.81	15.95	69.00	20.00 *	2.64	12.62	84.48
29/12/1955	0.69	9.40	72.33	17.50	20.00 *	8.61	80.13
28/12/1956	3.00	12.24	59.89	7.50	2.08	7.32	87.43
30/12/1957	15.00	7.89	87.30	1.01	15.00	7.15	90.87
30/12/1958	15.00	4.52	80.76	4.00	20.00 *	4.62	82.13
29/12/1960	7.50	7.94	92.57	1.81	0.83	7.51	93.83
28/12/1961	7.50	7.01	92.40	2.78	20.00 *	6.59	93.72
28/12/1962	15.00	3.21	96.26	20.00 *	1.01	3.29	96.30
30/12/1963	5.00	5.21	97.11	2.78	15.00	4.85	97.62
30/12/1964	20.00 *	6.79	38.83	0.36	7.50	5.84	56.76
30/12/1965	20.00 *	9.54	67.15	5.00	20.00 *	9.29	70.05
29/12/1966	1.39	10.11	28.72	4.00	20.00 *	9.43	40.32
29/12/1967	20.00 *	12.14	14.30	4.00	17.50	12.23	16.00
30/12/1968	20.00 *	11.14	45.57	2.78	15.00	10.75	50.81
30/12/1969	7.50	20.06	32.04	1.39	0.56	19.53	37.29
30/12/1970	4.00	97.73	6.44	1.81	12.50	96.72	10.44
30/12/1971	0.25	21.02	41.68	0.39	7.50	21.06	42.65
29/12/1972	1.53	31.43	59.25	0.42	0.19	29.98	63.67
28/12/1973	5.00	36.90	20.12	1.01	0.33	33.53	35.39
30/12/1974	0.56	36.47	71.42	0.83	0.25	35.96	72.74
30/12/1975	20.00 *	36.61	66.68	4.00	0.44	35.46	69.31
30/12/1976	1.01	29.72	59.64	0.83	0.36	29.44	61.07
29/12/1977	1.01	48.16	34.93	0.19	0.14	48.09	36.16
29/12/1978	0.17	58.53	33.01	5.00	0.83	58.10	34.94
28/12/1979	0.11	56.77	19.89	0.19	0.14	56.15	22.66
30/12/1980	2.64	75.83	29.80	0.31	0.17	75.85	30.61
30/12/1981	20.00 *	107.79	13.32	0.17	7.50	108.16	13.69
30/12/1982	4.00	65.53	22.87	2.78	12.50	65.67	23.33
29/12/1983	0.11	59.18	14.80	0.11	0.31	59.20	15.61
28/12/1984	20.00 *	71.00	11.14	2.08	10.00	69.85	14.89
30/12/1985	1.25	91.45	8.43	4.00	0.53	91.74	8.90
30/12/1986	0.83	92.32	7.71	2.78	12.50	92.51	8.53
30/12/1987	0.22	38.10	69.91	0.50	0.11	37.46	71.32
29/12/1988	0.19	42.12	13.73	0.56	5.00	42.14	14.88
29/12/1989	1.01	21.65	57.08	0.08	0.83	21.77	57.20
31/12/1990	0.44	12.44	50.17	0.19	0.06	12.40	51.11
30/12/1991	1.11	30.26	62.10	0.14	0.11	26.46	71.41
30/12/1992	1.67	16.07	69.65	0.03 *	2.78	15.05	73.79
30/12/1993	0.47	28.67	69.17	0.03 *	20.00 *	20.73	84.14
30/12/1994	0.39	14.86	95.25	0.14	0.56	13.84	95.96
28/12/1995	0.83	15.78	96.50	1.53	0.03 *	6.17	99.48
30/12/1996	1.39	6.08	99.56	1.53	0.39	5.49	99.65
30/12/1997	3.00	3.37	99.56	17.50	20.00 *	3.24	99.60
30/12/1998	1.81	12.17	90.96	1.94	0.19	6.15	97.75
30/12/1999	0.56	17.05	93.94	0.11	12.50	5.46	99.40
29/12/2000	1.11	12.42	89.33	2.36	0.03 *	5.49	98.02
28/12/2001	0.83	10.74	97.71	1.01	0.11	10.53	97.91
Median	1.53	16.07	62.10	1.53	2.08	13.84	70.05

Note - Standard deviations are in Basis Points

* Best fit realized at boundary of range of τ

5.4 Conclusions

In this chapter, the Nelson & Siegel and the Extended Nelson & Siegel model are used to research the Dutch term structure of interest rates. The intent of this study is to present a term structure, which gives useful information on future interest expectations. The results show that the Nelson & Siegel model and the Extended Nelson & Siegel model give in most cases good estimates of the Dutch yield curve.

Nelson & Siegel Model

The Nelson & Siegel model estimations on forty-nine annual datasets result in a R^2 median of 49.35%. Each dataset is fitted with an individual τ value (time constant). The best estimation is obtained on dataset December28, 2001 with a R^2 of 97.46%. The worst fit is obtained on dataset December 29, 1989 with a R^2 of 3.64%.

To test for the added value of an individual best τ-value for each dataset, the median τ-value of the first results is used for all estimations on the annual datasets. The median R^2 then decreases to 43.55%. An explanation for this result is that a fixed time constant variable (τ) removes some of the flexibility of the Nelson & Siegel model to fit each dataset. It is better to find an individual τ -value for each separate dataset.

To check for the explanatory power of the Nelson & Siegel model components, the third order component is removed. In this form the Nelson & Siegel model performs worse than the earlier obtained results. The median R^2 falls to 37.12%. The third order component has additional explanatory power and should thus not be removed.

Extended Nelson & Siegel model

The research is continued with the Extended Nelson & Siegel model. In comparison with the Nelson & Siegel model, an extra model component is added. The estimation results show that the median R^2 increases to 55.49%.

Again, a check is made for the added value of individual best τ_1 and τ_2 -values for each dataset. The median τ_1 and τ_2 values of the first results are used for all estimations on the annual datasets. The median R^2 then decreases to 46.87%. An explanation for this result is that fixed time constant variables (τ_1 and τ_2) remove some of the flexibility of the Nelson & Siegel model to fit each dataset. A dataset with long maturity bonds needs a higher τ_1 and τ_2 value, than a dataset with short maturity bonds. The results show that it is better to find individual time constants for each separate dataset.

Liquidity

The results of the earlier sections showed that some datasets contained outliers. These outliers are for a large part caused by illiquid bonds, which results in inaccurate data and estimations. The illiquid bonds are removed and yield curve estimations are computed for all annual datasets.

The results of the adjusted dataset show that removing illiquid bonds of the dataset, contributes to better fits of the yield curve estimates, for both the Nelson & Siegel model and the Extended Nelson & Siegel model. The median R^2 increases to 62.10%, respectively 70.05%

For most datasets, the results of the Nelson & Siegel and the Extended Nelson & Siegel yield curve estimation are very similar. Only in some cases, the use of the Extended Nelson & Siegel model shows larger improvements in fitting the estimation to the dataset.

Overall results

Overall, it can be seen that the estimations for the period 1967 – 1991 show lower R^2 results. This period contains twenty-five datasets of which sixteen Nelson & Siegel estimations produce a R^2 lower than 30%. With the Extended Nelson & Siegel estimations, twelve estimations show a R^2 lower than 30%.

If the performance of the Nelson & Siegel model is compared to the Extended Nelson & Siegel model, it can be seen that the Extended Nelson & Siegel model has more explanatory power. The results with the adjusted dataset show that the use of the Nelson & Siegel model on most datasets gives a satisfactory fit. Also, very similar results are obtained with the Extended Nelson & Siegel model. Only in more complex cases, the Extended Nelson & Siegel model has true added value. Because complex yield curves are not known on forehand, the use of the Extended Nelson & Siegel model is preferred. Yield curve estimations for available maturity data in the annual datasets give the anticipated results. Estimations for longer-term maturities, outside the available dataset range, gives in most cases the expected results. Estimations in the case of short-term maturities, outside the available maturity range of the data, may give extreme yield estimations and are not reliable. To conclude in general, the yield curve estimations are reliable to be interpreted for maturities longer than the shortest maturity available in the dataset

6. Results: callable bonds

This research uses Dutch state issued bonds with various qualities: normal bonds, callable bonds and convertible bonds. In chapter 4, the assumption is made that callable and convertible bonds will remain outstanding until maturity. This implies that callable bonds are valued in the same way as normal bonds. The results of chapter 5 show that this may not always be appropriate. Estimations of term structures for datasets in the period 1967 – 1991 show lower fits of the yield curve estimations. A closer inspection shows that outliers in the dataset often are callable bond yields that may be responsible for the low fit of the estimations (see figure 6). This chapter investigates possible solutions to adjust the dataset to obtain a higher fit of the term structure estimates. These solutions are all tested on the adjusted dataset of December 29, 1989. This dataset shows the lowest fits in the earlier performed estimations (on the unadjusted dataset). Due to time constraints, solutions are only shown for this dataset. From here on, the word 'dataset' refers to the adjusted dataset of section 5.3. The words 'original dataset' refer to the dataset without the liquidity criteria as used in section 5.1 and 5.2.

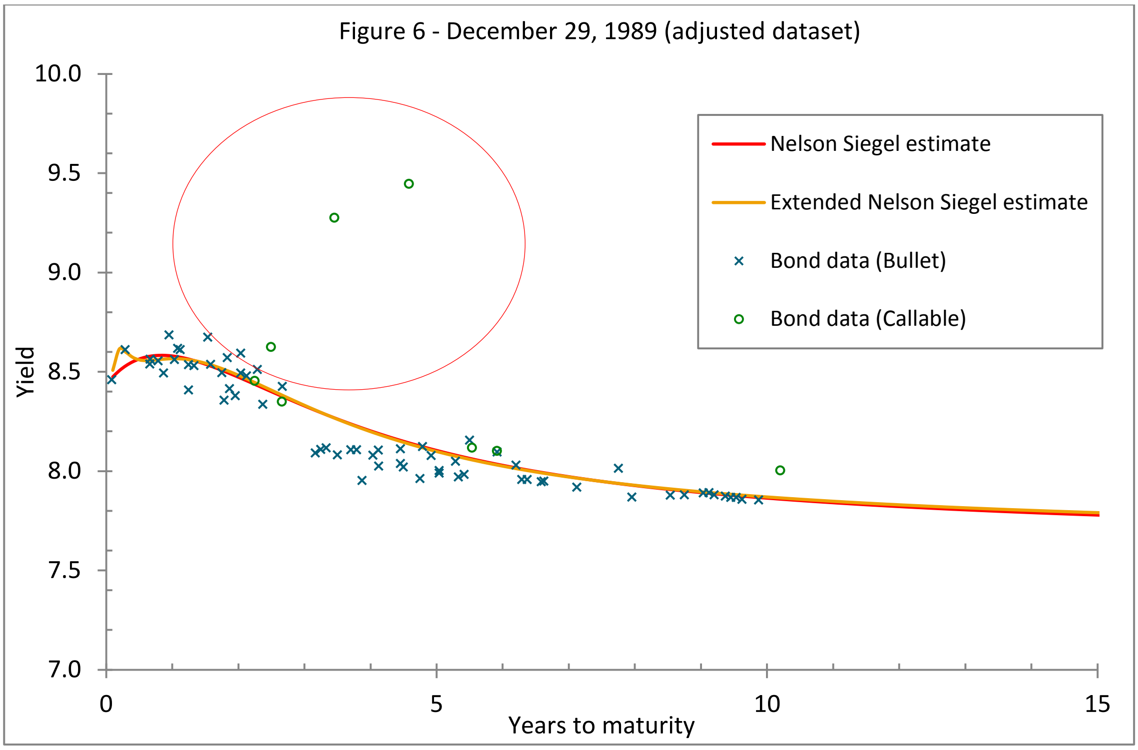
The assumption that callable bonds are valued in the same way as normal bonds is understandable. Before the 1970s investors actions conformed to this assumption. Several reasons can be mentioned. Until 1970 all bonds issued by the Government were callable bonds. There is no need to compare callable and non-callable bonds, if all bonds are callable. In addition, until 1984 the Dutch state (in the dataset) never exercised a call option on a bond. Other institutions, such as the United States Treasury department didn't call any bonds since 1962. Investors knew the properties of callable bonds, but did not expect it to be called. In 1985, the Dutch state called some of its callable bonds for the first time. Another reason to assume that callable bonds and bullet bonds are equal, is that the Dutch term structure was stable, meaning that interest rates were less volatile, which decreases the value of embedded options. Fernandez (2005) concludes that investors before the 1980s traditionally ignored the callable features of bonds, mainly because of the difficulty to value it.

In the beginning of the 1970s, interest rates became more volatile. Furthermore in the late 1970s and early 1980s interest rates started rising. For example, in 1970 the Dutch state paid on new bond issues an average coupon interest of 8%. In 1981, this peaked to coupon rates of 12%. As long as interest rates rose, issuers did not want to refinance their issued bonds and few investors paid attention to the call features. However, when interest rates started to drop, the Dutch state in 1985 called some bonds and refinanced these bonds at lower interest rates (Fernandez, 2005).

When valuing a bond, all future cash flows are discounted against its yield or spot rates. This process is also used when valuing a callable bond. The issuer, the Dutch state, has the right to redeem prior to maturity under certain conditions. The state will call a callable bond if it is paying a higher coupon then the current interest rates. After a bond is called, the state will reissue the redeemed bond at a lower coupon and save on interest payments. After a bond is called, the investor receives the exercise price and accrued interest or a final interest payment (depending on the clauses). The exercise of the call option by the issuer, will truncate the future cash flows for the investor. As a result investors of callable bonds face the risk of reinvesting money at a lower rate. Investors therefore require a higher yield on callable bonds. Whether an embedded option will be exercised, depends on the future course of interest rates. Since the future course of interest rates is a priori uncertain, callable bonds should have higher returns to compensate investors for the call risk.

Figure 6 shows the dataset of December 29, 1989 and the corresponding Nelson & Siegel – and Extended Nelson & Siegel estimations. The dataset is divided into two categories: (i) bullet bonds, that is bullet and convertible bonds grouped together, and (ii) callable bonds. Assumed that the estimated yield curves represent the term structure, it can be shown that there are three outliers. These are encircled in red.

Figure 6 - Fit of Nelson & Siegel model (R^2 = 57.08%) and Extended Nelson & Siegel model (R^2 =57.20%), adjusted dataset December 29, 1989. The dataset is split up in bullet bonds and callable bonds. Outlier bond data is encircled in red.



A closer glance shows that the outliers are callable bonds. The callable bonds may have some special properties which are overlooked earlier in this research.

Because the results in the previous chapter and figure 7 indicate that callable bonds may be the source of the low fits of the yield curve estimates, solutions are mainly focused on correcting aspects of callable bonds. The following solutions are considered: (i) removal of callable bonds from dataset, (ii) use of yield-to-call and (iii) valuing embedded options.

6.1 Solution: removal of callable bonds from dataset

As stated in the previous section, the assumption that bullet bonds and callable bonds are priced with the same technique, may give problems. A simple solution to the problem of mixed bond types is to eliminate all callable bonds. Only normal coupon bonds will remain in the dataset. A problem with this solution is that this method cannot be used for each dataset. Datasets from before 1969 will have few bonds left to make estimations, since almost all bonds issued before 1969 were callable bonds. As a result, a regression will not give very useful results.

The dataset of December 29, 1989 consists of 75 bonds. After elimination of eight callable bonds in this dataset, 67 normal coupon bonds remain. This is followed by yield curve estimations on this dataset with the Nelson & Siegel model and the Extended Nelson & Siegel model.

The results are shown in table 5, under the heading “remove callable bonds” (page 39), together with earlier estimation results for this annual dataset. The Nelson & Siegel estimation shows a large improvement with a R^2 of 92.39%, where this was 57.08% with the previous estimation of the adjusted dataset. The τ –value becomes 0.69. When the Extended Nelson & Siegel model is used, the

R^2 improves to 93.15%, where this was 57.20% when the adjusted dataset was used. The τ_1 and τ_2 values in this estimation are respectively 2.22 and 10.00. The Extended Nelson & Siegel model gives a slight better fit than the Nelson & Siegel model.

The results speak for themselves: the removal of the callable bonds from the dataset leads to a huge improvement of estimations and their fit. In datasets with a sufficient number of bonds, not being callable bonds, this is good way to improve estimations. In the datasets used in this research, especially before 1970, this method is not an option. First, almost all bonds issued before 1970 are callable bonds. No bonds would remain to use in computations. Secondly, all bonds issued before 1970 suffer from the same 'error': they are all callable bonds. When all bonds are callable bonds, this makes them equal to bullet bonds. Certainly, if the callable bonds have never been called before. This justifies the assumption that in this period no special adjustments should be made for being callable (remember that only in 1985 the first bond was called by the Dutch state). The results of the Extended Nelson & Siegel estimations for the period 1952 – 1965 also suggest this; all estimations in this period have a R^2 higher than 66%. Dashkin finds that investors in US treasury bonds used to assume that callable bonds would remain outstanding until maturity, until the US treasury department announced to call a bond in 1992 (Daskin & Kulkarni, 1993).

6.2 Solution: yield-to-call

When a callable bond is quoted above par, prior to maturity, it is likely to be called by the state. The bond is selling at a premium, which means that the coupon rate is higher than the required interest rate by investors. At the same time the call is made, the state will issue a new bond at a lower coupon rate to refinance the called bond. An exercised call option reduces cash flows of the callable bond to a final coupon and face value payment. The original expected cash flows are reduced to cash flows until the earliest call moment. When the yield-to-maturity is used for this bond, it will return a yield that is too high; it does not incorporate the fact that the bond will be called prior to its maturity. The used yield should in this case be the yield-to-call. This section discusses this solution.

The yield-to-call is the yield of a bond calculated until the bond's first possible call date. The yield-to-call is only used when a callable bond is quoted above par and its exercise price. A bond has to be callable at the first upcoming coupon date (that is within a one year period). Uncertainty about future bond prices restrains the use of the yield-to-call for bonds with later exercise dates.

The adjusted dataset of December 29, 1989 contains 75 bonds. Eight bonds qualify as a callable bond. Of these callable bonds, four bonds are excluded of the yield-to-call because they are not callable within a one year period. Of the remaining four bonds, three bonds are quoted below par and exercise price and do not qualify to use the yield-to-call. Only one callable bond is quoted above par and its exercise price. This bond's yield-to-maturity is adjusted to a yield-to-call and is used in the Nelson & Siegel and Extended Nelson & Siegel term structure estimations.

The results of the estimations with the yield to call are stated in table 5 under the heading "yield-to-call" (page 39). In comparison with the earlier Nelson & Siegel estimation, the fit of the dataset improves with 7.81 percent point to a R^2 of 64.89%. In comparison with the original estimation of the Extended Nelson & Siegel model, the fit improves from a R^2 of 57.20% to 64.95%. In both estimations the standard error almost remains the same.

The usage of yield-to-call, instead of yield-to-maturity, when a bond is quoted above par does have a positive impact on the fit of the estimations in this dataset. However, the results do not show huge improvements. This has to do with the fact that in this dataset only one bond out of forty callable bonds qualifies for the yield-to-call option. Also, a possible problem with the yield-to-call method is that it ignores the potential value of waiting, because the present value of not exercising now may have a higher value than the current exercise value of a call option.

6.3 Solution: valuation of embedded call options

In this section solution three is discussed: valuation of embedded call options. Before this solution is explained, a short elaboration on callable bonds is presented. Then the solution is discussed, which is followed by a short review of the option pricing model used to come to the results. After that the results for the term structure estimations on the dataset of December 29, 1989 are presented.

Callable bonds

In the previous sections the assumption is made that callable bonds will remain outstanding until maturity. This assumption is not (always) appropriate. A callable bond is a bond with an embedded call option for the state. The embedded option gives the issuer the possibility to call a bond before maturity and redeem it early. The state will only exercise its call option on a bond, if this bond pays a higher coupon rate than the current spot rate. Whether a callable bond is exercised, depends on the uncertain future course of interest rates. The state will then refinance the called bond by a new bond issue at a lower rate than the called bond.

When a call option on a bond is exercised, future cash flows of that bond will be truncated. Investors loose the future coupon payments of that bond. Future cash flows of a callable bond are thus uncertain. The received face value of the early redeemed bond can only be reinvested at a lower rate than that of the redeemed bond. This is called ‘call risk’. Investors only want to invest in callable bonds, if they are compensated for the call risk. A callable bond rents a higher coupon than bullet bonds. This results in higher yields until maturity (lower prices) for callable bonds compared to bullet bonds. The callable bonds have to be adjusted for this higher yield.

Solution

As mentioned, callable bonds have higher yields than bullet bonds (see Figure 6 for an illustration). Callable bonds thus have to be revalued to take away the difference between bullet bond and callable bond. This is done by valuing the embedded call option of the callable bond. The found value is used to revalue the callable bond to a bullet bond.

Kalotay et al state that any bond should be thought of as a package of cash flows (Kalotay, Williams, & Fabozzi, 1993). The authors see each cash flow as a zero coupon bond, maturing on the date it will be received. Option bearing bonds are viewed in the same way, plus a package of options on those cash flows. A position in a callable bond can then be seen as:

$$\text{Value of callable bond} = \text{value of option free bond} - \text{value of call option on that bond}$$

This can be rewritten as

$$\text{Value of option free bond} = \text{value of callable bond} + \text{value of call option on that bond}$$

The price of a callable bond is thus equal to the price of a non-callable bond, minus the value of the call option on it. The value of the callable bond is reduced because the investor implicitly sold an option to the issuer of the bond. A bullet bond has a higher value, because it does not face the risk of early redemption when interest rates are decreasing. To revalue a callable bond to a bullet bond, the value of the embedded option needs to be calculated. With this new ‘callable’ bond value, a new yield can be calculated and used in the yield term structure estimations.

Option pricing model

Pricing of (embedded) call options is complicated. Multiple models exist, of which the most famous models are by Cox, Ross & Rubinstein and Black-Scholes.

The Black-Scholes model has two assumptions that are in conflict with the aims of this research. The model assumes constant interest rates and also assumes that options can only be exercised upon expiration. The goal of this thesis is to estimate the term structure of interest rates, which makes interest rates by definition a dependent variable. For that same reason risk free interest rates from other sources cannot be used. Furthermore, the assumption of constant interest rates is a simplification that is inadequate. Also, the Black-Scholes model is used for pricing of European options. The callable bonds in the dataset are assumed to be American-European type call options: call options which can be exercised before expiration and only on certain dates. These type of options, with an exercise date before expiration, are not in line with the assumptions of Black-Scholes' model.

The option valuation model by Cox, Ross & Rubinstein uses a binomial tree method. Essential assumptions in this model are that expected returns have to equal the risk free rate and that future cash flows are discounted against the risk free rate. These assumptions are the basis of the risk neutral world of Cox. Cox thus uses the 'already known' risk free interest rate (term structure) in the core of his calculations. With this data, the probabilities for an up- and downstate in binomial model nodes are determined. Since the goal of this chapter is to adjust callable bond prices to normal bond prices, in order to find adjusted yields and then to create a term structure, there exists no fixed risk free rate which can be used in the calculations for this research. The risk free rate in this thesis is an unknown variable.

Option pricing model by Fernandez (2005)

Despite the popularity of the mentioned models, this thesis uses an (embedded) option price model based on the method used by Fernandez (2005). This model is adjusted to fit this chapter's purposes and presented in multiple steps. Fernandez uses a method that doesn't model the direct evolution of bond prices (as Cox et al), but models the evolution of interest rates. A temporary interest rate term structure is constructed by means of a binomial tree that shows the future course of interest spot rates. With this fictive interest rate term structure, cash flows of a bond are discounted against the term structure. Each node of the binomial tree represents a point in time and has its own course specific term structure. This method must be used for each bond in the dataset. The methodology is explained in multiple steps.

Step 1

The basis data of a bond is collected: coupon rate, coupon dates, cash flows and the one year risk free rate. The one year risk free rate is a known and independent variable. This value is used for the all bonds in the annual dataset. Also, each bond has an individual interest rate volatility. The volatility is needed to determine the future path of interest rates. Since this volatility cannot not directly be observed, the implied volatility is calculated in step 5. Its initial value is set to zero.

Step 2

A binomial interest rate term structure tree is created. The binomial tree projects possible future paths of one-year-forward interest rates. The interest rate in the first node, period t to t + 1, is the current one year spot rate. For each next node, interest rates have a fifty percent probability on an up or a down movement. The size of the up or down movement is determined by the volatility of the interest rate. The volatility of interest rates tends to decrease for longer maturities (Fernadez, 2005)[10]. Therefore the assumption is made that each year, the effect of the volatility decreases with ten percent points, until a minimum of ten percent of the interest rate volatility is reached. E.g., in year six the upside interest rate is [σ * (100 % - 50%)].

Step 3

In this step, the value of a bullet bond is calculated. This bond has the same properties as the researched callable bond, but without the call option. Future cash flows of the bond are discounted against appropriate spot and forward rates. Since the values of the discounted cash flows vary by different interest rate paths of the binomial interest tree, the value of the discounted cash flows vary by these paths. The discounted cash flows are presented in a tree. The valuation process of the bullet bond starts at the final node of the bond tree and is calculated backwards. The present value of a bond in each node can be described with equation 6.1.

$$PV_{node\ i} = CF_i + \frac{1}{2}\left(\frac{PV_{node(+)\ i+1} + PV_{node(-)\ i+1}}{1+f_{1,node\ i}}\right) \qquad (6.1)$$

The present value of the bond consists of full cash flows received in node i, coupons, and the average present value of the up and down state values in the next node. The values of node i + 1 are discounted with the forward rate in node i. Since the probability for the up and down state is 50%, the discounted next node values are summed up, and divided by two.

Step 4

At this point the cash flows of the bullet bond are discounted to a present value. Upon next, the value of the call option on this bullet bond is calculated. Again, a tree is calculated which presents the value of the call option in the different nodes. The option valuation is described in equation 6.2.

$$PV_i = Max\left[PV_{bond,node\ i} - P_{exercise};\ \frac{1}{2}\left(\frac{PV_{node(+),i+1} + PV_{node(-),i+1}}{1+f_{1,node\ i}}\right)\right] \qquad (6.2)$$

The valuation of the call option begins at the final node and is calculated backwards. The value of the call option is the higher value of the exercise value or the maintenance value. The exercise value is the positive excess value of the bond price, minus the exercise value of the call option. The maintenance value is the discounted average of the present values of the call option in node i + 1. Negative option values do not exist and are priced at zero.

Step 5

The present value of the bullet bond and the value of the call option on this bond is known. With this information, the present value of the callable bond is calculated with equation 6.3.

$$PV_{callable\ bond} = PV_{bullet\ bond} - PV_{embedded\ option} \qquad (6.3)$$

The present value of the callable bond is not yet equal to the current callable bond price. By adjusting the volatility of the interest rate through trial-and-error, the present value of the callable

[10] Quote is from Fernandez, but the rate of influence decay of the volatility is an assumption made in this thesis.

bond is set equal to the current callable bond price. The resulting present value of the bullet bond is assumed to be the bullet bond price of the callable bond. This price is used to calculate a new yield to maturity for the bond.

Step 1 to 5 is repeated for all callable bonds present in the annual dataset. The new yield values for the 'bullet' bonds substitute the yield values of the callable bonds. With the updated dataset, the yield curve is estimated with the Nelson & Siegel model and the Extended Nelson & Siegel model.

An example of a transforming a callable bond into a bullet bond is presented in Appendix D.

Data

As mentioned earlier, this model is applied to the dataset of December 29, 1989. For each callable bond on this date, a bullet bond is created with the same properties as the callable bond, but without the call option. As starting point of the binomial interest rate tree, the one-year spot rate is used, which is 8% on December 29, 1989. Node dates are set on coupon dates of the researched bond. If a callable bond qualifies as a sinking fund, this property is taken into account in the calculations.

Callable bonds are often not callable for the first few years of their life. Only after this lockout period, the embedded call option can be exercised. Callable bonds issued before the 1970s have exercise prices fixed at par. Later issued callable bonds have exercise prices set at a premium. Some callable bonds have an exercise price declining over time. After a bond has become callable, the assumption is made that the embedded call option can only be exercised on coupon dates, with no announcement requirements. When a bond is called, the investor is redeemed the exercise price and any accrued interest.

Results

The results for the dataset are presented in table 5 under the heading "callable bonds repriced". Earlier obtained results are included to compare between the results. The outcome shows that the transformation of callable bonds to bullet bonds leads to a R^2 of 88.10% for the Nelson & Siegel estimation, where this was 57.08% with adjusted dataset. In case of the Extended Nelson & Siegel estimation the R^2 rises to 89.10%, where this was 57.20% with the adjusted dataset. It can be concluded that repricing has a positive effect on the estimations. Figure 7 graphs the Nelson & Siegel and Extended Nelson & Siegel estimations with the adjusted and repriced dataset of December 29, 1989.

Table 5 - This table shows all Nelson & Siegel and Extended Nelson & Siegel estimations done on dataset December 29, 1989. The estimations Remove callable bonds, yield-to-call and callable bonds repriced are discussed in this chapter. Other results are discussed in chapter 5.

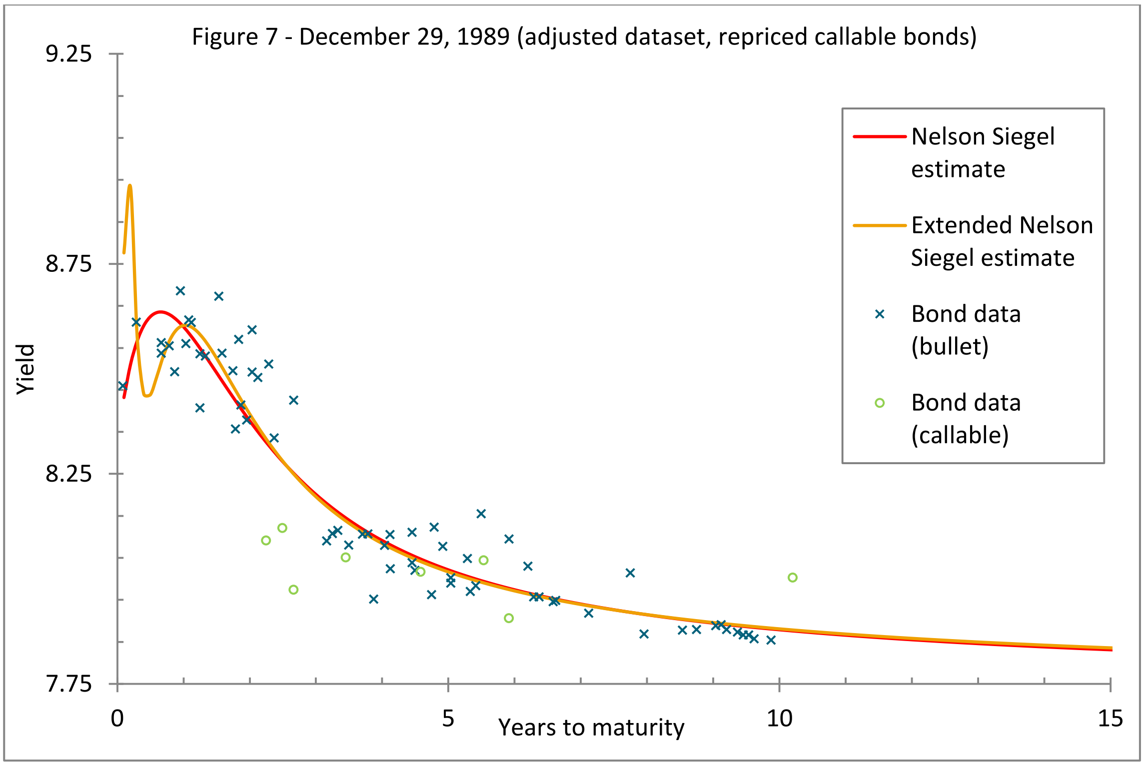
TABLE 5	Adjusted dataset December 29, 1989, comparison of different estimation techniques					
Model	Estimation	Best τ_1 - value	Best τ_2 - value	St. Dev. at best τ values	R^2	# observations
Nelson & Siegel	Third order model (o)	1.667	-	81.44	3.64	115
	Second order model (o)	0.556	-	81.16	3.44	115
	Maturity > 2 yrs (o)	0.167	-	82.70	1.29	67
	Adjusted dataset (a)	1.014	-	21.65	57.08	75
	Remove Callable bonds (a)	0.694	-	7.65	92.39	67
	Yield-to-call (a)	0.694	-	19.54	64.89	75
	Callable Bonds repriced (a)(r)	0.556	-	9.24	88.10	75
Extended Nelson & Siegel	Original (original dataset)	0.556	5.000	81.64	4.03	115
	Adjusted dataset (a)	0.083	0.833	21.77	57.20	75
	Remove Callable bonds (a)	2.222	10.000	7.31	93.15	67
	Yield-to-call (a)	0.556	0.056	19.66	64.95	75
	Callable Bonds repriced (a)(r)	0.361	0.056	8.90	89.10	75

Note - Standard deviations are in Basis Points

(o) - original dataset, (a) - adjusted dataset, (r) - callable bonds repriced

For both the Nelson & Siegel estimate and the Extended Nelson & Siegel estimate, the R^2 fit is somewhat less than in the case where all callable bonds are excluded. This solution concerning the repricing of callable bonds is preferred because it gives a good approximation of the repriced bond yields.

Figure 7 - Fits of Nelson & Siegel model (R^2 = 88.10%) and Extended Nelson & Siegel model (R^2 = 89.10%), Adjusted dataset (illiquid bonds removed, callable bonds repriced to normal bonds) December 29, 1989. The plotted dataset separately shows bonds and repriced



It can be concluded that the solution of repricing callable bonds, leads to a huge improvement in fitting the curve to the current yield data, compared to the chapter 5 Nelson & Siegel and Extended Nelson & Siegel estimations.

6.4 Conclusions

The results in chapter 5 show that certain datasets have very low estimation results. This chapter presented three solutions to improve these results. Because this chapter is intended as addition to the thesis, the solutions are only tested on the adjusted dataset of December 29, 1989 (referred to as dataset). This dataset was chosen because of its low R^2 results in section 5.1 and 5.2. Because the results of the solutions are very close with the results of the Extended Nelson & Siegel estimations, this conclusion only states results of the Nelson & Siegel estimations.

In total, three solutions have been suggested to correct for possible errors.

- The first solution, removes all callable bonds from the dataset. The result improves from the original R^2 57.08% to 92.39%. This method has as a disadvantage that it removes information from the dataset.
- The second solution uses the yield-to-call instead of the yield-to-maturity on applicable callable bonds. The fit of the yield curve estimate improves to 64.89%. Because of the criteria, it is only applicable to certain bond yields, leaving other callable bond yields the same.
- After that, all callable bonds in the dataset are transformed to a bullet bond value. Their yield is recalculated and used in yield curve estimations. The R^2 result of the Nelson & Siegel estimation increases to 88.10%. The results show a very good improvement compared to the adjusted dataset results

It can be concluded that of the three solutions, repricing callable bonds gives good results and is the preferred solution.

7. Comparison with estimations by institutions

The main goal of this thesis is to estimate the Dutch yield and forward curve through time. The results in chapter 6 show that after adjustments of the dataset, good estimates of the yield curve are obtained. The question, which arises next, is how other institutions estimate the Dutch yield curve and how these estimations differ from the estimations obtained through this research.

To compare the results of other institutions, two dates are discussed: December 29, 1989 and December 31, 2001. The first date is chosen because it has been reviewed in chapter 7. The other dataset used is the most recent dataset in this research. The Nelson & Siegel and the Extended Nelson & Siegel estimations both use an adjusted dataset with repriced bonds[11].

Two institutions in the Netherlands publish publicly available information on their estimates of the yield curve. These institutions are the Dutch Central bank or DNB (‘*De Nederlandse Bank*’) and the Central Statistical Office or CBS (‘*Centraal Bureau voor de Statistiek´*). Both institutions use other techniques to estimate the yield curve. The estimate of the DNB is only shown for the year 2001, because data for 1989 is not available.

Techniques

The CBS presents yield estimations for eleven maturities between two and ten years. The quoted yield of all bonds on the specified date is selected, after which the CBS computes the average yield for eight maturity groups (e.g. the 2-3 years yield is the average yield of all bond yields with a maturity between two and three years). The maturities of the groups vary between two-three years and nine-ten years. In total, eleven yield-to-maturity estimations are available for both datasets.

DNB has been assigned a legal task to compute an interest rate term structure. This term structure is used for financial regulation and supervision on pension funds. For this reason, DNB uses a more ‘sophisticated’ model than CBS. The model, introduced in 2004, is based on European Swap rate data (De Nederlandse Bank, 2005). The estimates of the term structure are computed for maturities up to fifty years. The estimated result is a zero coupon spot curve, and not a yield curve. For further information on the technique used by DNB, see the reference. The data for this method only goes back to the year 2001.

Dataset of December 29, 1989

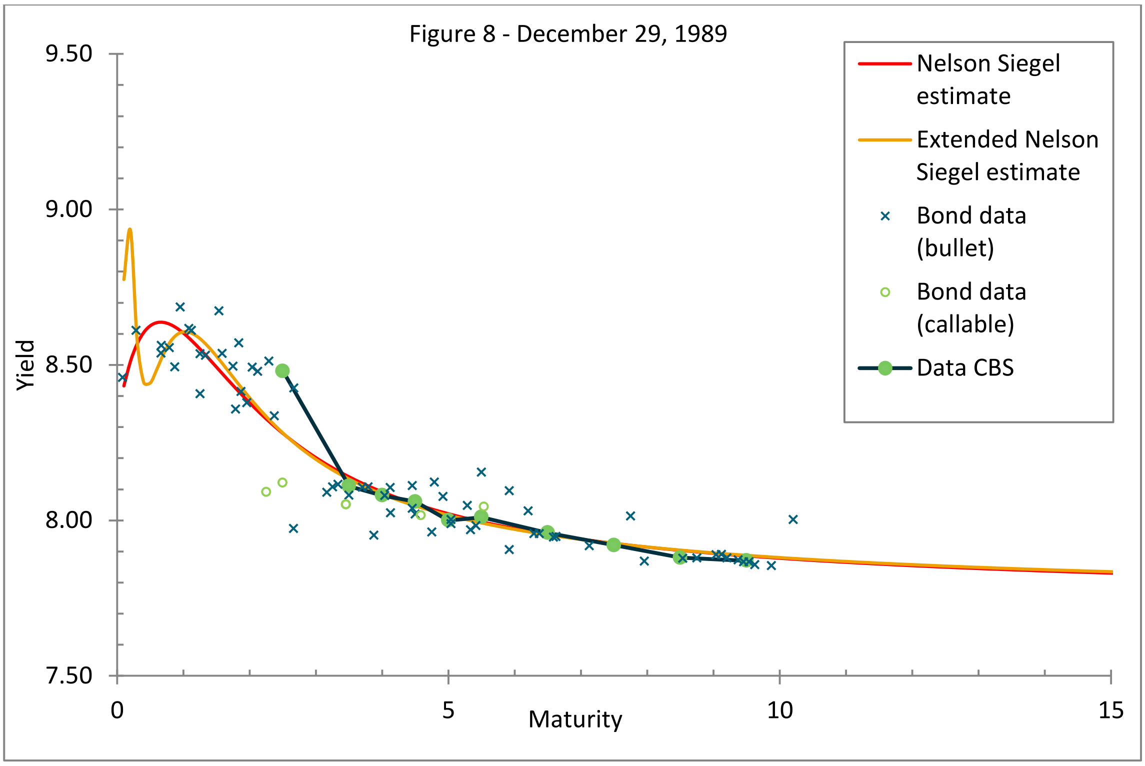
Figure 8 shows the term structure estimates of the Nelson & Siegel model and Extended Nelson & Siegel model. It also shows the scatterplot of bond yields on December 29, 1989 and the CBS estimated yield curve. It can be seen that the CBS data is very similar this thesis’ estimations. Only in the short term, one data point gives a higher result than the Nelson & Siegel or Extended Nelson & Siegel estimations.

The CBS method has advantages and disadvantages. In cases where sufficient data is present, good ‘estimates’ are obtained. All data is based on real data. The results are averages of yield data and do not predict a yield curve. However, in cases where less information is available and spread over longer maturities, for example for the annual dataset of 1952, this method will not give useful results. The ability of the CBS model to estimate a yield curve for longer maturities or with very few

[11] The dataset of December 31, 2001 does not have any corrections for callable bonds, because no callable bonds were outstanding at that date (the Dutch state no longer issues these types of bonds).

information is low. Therefore, the Nelson & Siegel based models are to be used for term structure estimations.

Figure 8 - Nelson & Siegel model (R^2 = 88.10%) and Extended Nelson & Siegel model (R^2 = 89.10%), Adjusted dataset with repriced callable bonds (illiquid bonds removed, callable bonds repriced to bullet bonds) December 29, 1989. Comparison with estimated yield curve by CBS.



Dataset of December 31, 2001

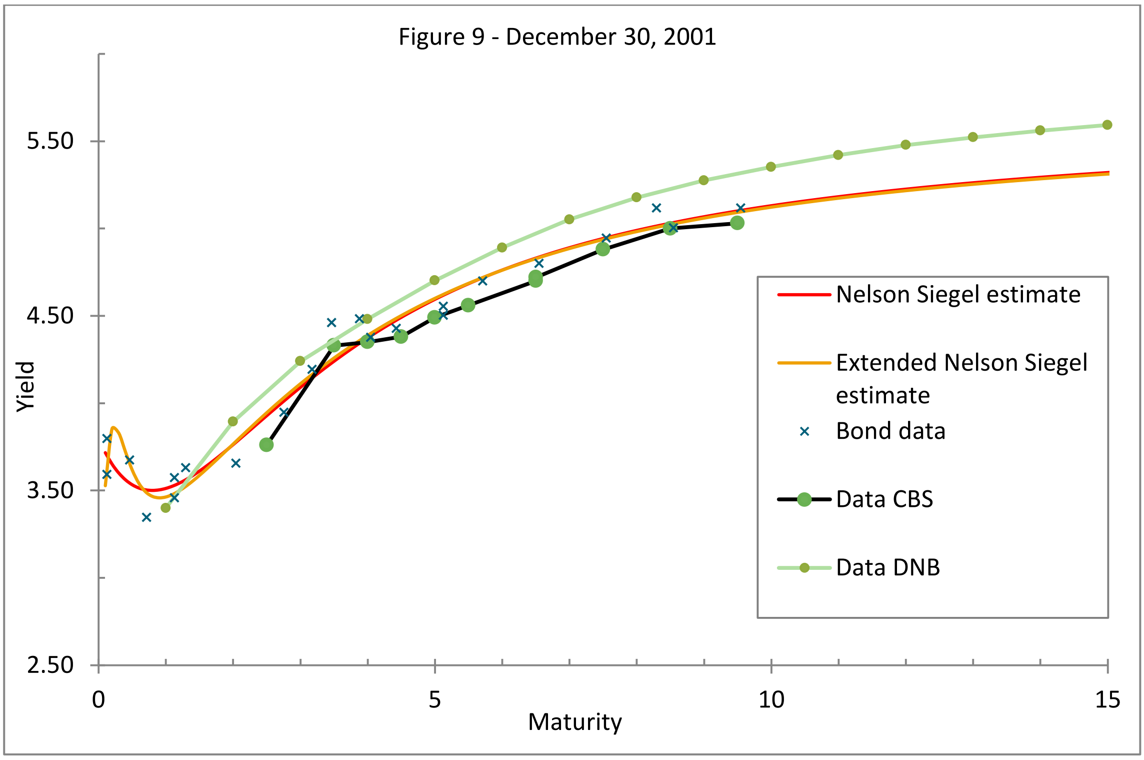
Figure 9 shows the results for dataset December 28, 2001. Again, all bond yield data is plotted and the estimated Nelson & Siegel and Extended Nelson & Siegel term structures are graphed. On the research date, no callable bonds were present in the dataset.

It is shown that the CBS estimate is in the same range as the estimates of this research. The CBS disadvantages, as mentioned earlier, do also apply here. The CBS estimate is only able to give a yield curve with current data. The results are useful, but the CBS method is not able to estimate a complete term structure.

The DNB estimated curve is situated above this thesis' estimates, especially for the longer maturities. This is to be expected, because the DNB curve is a zero-coupon curve. It estimates the spot rate term structure, where the Nelson & Siegel and the Extended Nelson & Siegel estimates present an estimated yield term structure. The results cannot be compared without transforming the yield curve estimations to forward rates, which in turn would have to be transformed to spot rates.

The Nelson & Siegel based models use the coupon bond yields as input and produce yield curve estimations. When the yield of zero coupon bonds would be used, the same as spot rates, the estimated curves would present a spot rate curve. Since 1999, a large part of the issued bonds are STRIPS. The principal is similar to a zero coupon bond. More recent estimates would use zero coupon yields, and thus create estimates that are better comparable with the DNB estimates.

Figure 9 - Nelson & Siegel model (R^2 = 97.71%) and Extended Nelson & Siegel model (R^2 = 97.91%), Adjusted dataset (illiquid bonds removed, callable bonds repriced to normal bonds) December 28, 2001. Comparison with estimated yield curves by CBS and DNB.



Conclusions

To conclude, the results show that the CBS method gives good estimates of the yield curve. The disadvantage is that these estimates can only be done for datasets with sufficient bonds in the researched maturity range. The Nelson & Siegel model and the Extended Nelson & Siegel model are better capable of yield curve estimates in cases of few maturities. The DNB model returns estimates of the term structure of spot rates. Therefore, the results of this research and the DNB results are not fully comparable. Further research is needed to check for performance of the models over time.

8. Conclusions

This thesis' original intention was to create interest term structures through time for the Netherlands, because this was not yet structurally estimated. To accomplish this, two term structure model are used, which are also commonly used by central banks. The models are based on the Nelson & Siegel model and the Extended Nelson & Siegel model. These two models are discussed and applied to estimate the Dutch term structure of Government bonds.

Firstly, the term structure is estimated with the third-order Nelson & Siegel model. For each yield curve estimation, a dataset is regressed with 45 different τ values in order to find the best fit of the yield curve. The term structure estimates, of which the goodness of fit is measured by the R^2, show varying results. The median R^2 of the Nelson & Siegel estimations is 49.35%, which is good. However, some low fits are obtained of which the annual dataset of 1989 performed least effectively. The results also show that an individual τ value for each dataset significantly contributes to the fit of the estimation to the data. In addition, the value of a third-order model is compared to a second-order model, by leaving the third-order element out. The fit of the estimations falls and show that the third-order element has a contributed value in the estimations.

Then, the Extended Nelson & Siegel model is used to estimate the Dutch term structure. For all datasets, better R^2 values are found than the results with the Nelson & Siegel estimations. However, some datasets again give low R^2 results.

To find an explanation for the low R^2 results in some of the datasets, a closer inspection is performed on the data. The outcome shows that the earlier researched datasets contain illiquid bonds. Yields and prices of illiquid bonds are not representative, because of their low trading volumes. The dataset is cleared of the illiquid bond data and the term structures are again estimated for the adjusted annual datasets. For both the Nelson & Siegel model and the Extended Nelson & Siegel model, the estimations show a very large improvement of the goodness of fit to the data. However, still some datasets show lower fits than expected. It is also seen that the Nelson & Siegel model gives in most cases a satisfactory fit to the data. Only in more complex datasets, the Extended Nelson & Siegel model can increase the fit with ten percentage points or more, compared to the Nelson & Siegel fit. In general, the yield curve estimations are reliable to be interpreted for maturities longer than the shortest maturity available in the annual dataset.

With the adjusted dataset, good results are obtained. However, some datasets in the period 1967 – 1991 keep resulting in low fits of the yield curve estimations. When these datasets are investigated in detail, it is shown that callable bonds are responsible for the low fits. Because of premiums due to call risk, these bonds report a higher yield than bullet bonds. Three solutions are tested to correct for callable bond effects in the dataset: (i) removal of callable bonds from dataset, (ii) use of yield-to-call and (iii) valuing embedded options. Because of time constraints, the solutions are only tested on the adjusted dataset of December 29, 1989.

The removal of callable bonds from the dataset, results in a very a good fit of the estimations to the dataset. However, it also removes data, which is not always an option, because there exists too few bullet bonds in a dataset to perform regressions on. Secondly, the yield-to-maturity of apllicable callable bonds is replaced by the yield-to-call. The estimation results show an increase in fit to the data, but not all callable bonds are corrected. Then the third solution is applied: a binomial option pricing model is used to price the embedded call option value of callable bonds. Callable bonds prices are revalued to bullet bonds and their yields are recalculated. The subsequent results are very promising. Where we obtained a R^2 of 57.08% for the Nelson & Siegel estimations on the adjusted dataset, this has now improved 88.10%.

To conclude the estimation research, the Dutch yield curve can be estimated with very high fits to the data using a dataset which removes illiquid bonds and reprices callable bonds. The results of the Nelson & Siegel model show that in most cases the fit is equivalent to results of the Extended Nelson & Siegel model. Only in datasets with a more complex term structure, the use of the Extended Nelson & Siegel model is preferred. The results are compared for two datasets with estimations of the CBS and the DNB. It shows that the estimation results are close. The DNB results are zero coupon results and cannot directly be compared with the yield structure estimations.

A few recommendations for future research are made. This research only tests commonly used models. However, it would be interesting to see a more extensive review of available models and their performance against each other. The models based on Nelson & Siegel perform well, but how do newer models perform, and should banks use other estimation models. Another recommendation is to change the data selection criteria. In future research STRIPS should be included, for one because STRIPS are actively supported by the Dutch treasury department. In addition, information from STRIPS may contain more information than coupon bond yields. Resulting estimates will give term structure estimates of spot rates. As last recommendation, a note is made on the use of τ_1 and τ_2 values in the Extended Nelson & Siegel model. These time constants need stricter rules. In this thesis τ_1 and τ_2 are not allowed to take the same value. In further research, the τ_1 and τ_2 should also have a minimum time distance between τ_1 and τ_2. Multicolinearity problems with the Extended Nelson & Siegel estimations will not arise. This will limit β outlier estimations.

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Appendix A - types of yield curves

Normal yield curve

Under normal conditions, short-term bonds pay less interest than long-term bonds. The difference in interest levels is explained by the smaller risk of not being paid back for a short-term bond. Because the longer the money is invested, the more risk is involved. The investor is compensated for this risk. Risks are numerous, e.g. rising interest rates, inflation, Government changes and central bank's policy actions.

Steep Yield Curve

Normally, there is a regular bandwidth between the interest rates of the short- and long-term bonds. In the US, the interest rate of the 30-year Treasury bill is in general three percentage points above the 3-month Treasury bond. When this variation becomes larger than the typical difference, the slope of the yield curve increases sharply. Long term bond investors expect the economy to improve in the near future. When this expectation is right, money demand will increase and interest rates go up. Long-term bond holders act by requiring a higher return. They do not want their money locked in for a long period at a too low rate. This type of term structure typically occurs after an recession, and at the start of an economic expansion (Fidelity).

Humped or flat yield curve

The term structure can also become such, that the short-term and long-term interest rates are the same. The medium term shows a little hump or is also equal. A humped term structure can be observed when investors expect the interest rates to rise over the next period, but expect a decline after that period (M. Choudry, 2003). This curve could become an inverted term structure.

Inverted yield curve

The inverted yield curve shows a higher interest rate for short-term bonds than for long-term bonds. This is the other way around, but the explanation is logical. In this situation investors expect that the economy in the future will become worse, and thus that the interest rates will become lower than they already are. The investors accept the lower long-term yields, because they think that yields in coming years will be even lower. This type of curve can be observed in times of a (starting) recession.

Appendix B - Theories on the term structure of interest rates

Expectations theory

The expectations theory is probably the best-known (and argued) theory on the term structure of interest rates. Many authors have contributed to this theory. The core of the theory is that long-term interest rates are an average of future short-term interest rates. The forward rates are assumed to be unbiased estimates of the future spot rates. This assumption is made under the condition that all securities are the same in all aspects, except for the maturity. As a result, the expected return for holding short term securities or one long term security for the same period of time, is equal (Kessel, 1971).

For example, a premium will arise if demand for a loan with maturity m exceeds the supply of these loans. Lenders will have to be compensated to come out of their natural habitat. So a premium has to be offered.

“Such premiums or discounts would tend to bring about shifts in funds between different maturity markets, both through the ‘speculation’ of investors tempted out of their natural habitat by the lure of higher expected returns and through ’arbitrage’ by intermediaries induced ‘to take a position’ by borrowing in the maturity range where the return is low, and lending where the expected return is high”.

Lots of authors contributed to the expectations theory. Authors to be named are Hicks (1939), Dodds & Ford (1974), Lutz (1940), Fisher (1930) and Keynes (1930).

Liquidity preference theory (Keynes, 1936)

In his book Keynes disagrees with the (at that time) common view that the rate of interest is the balancing factor between the supply of savings and demand of savings. According to Keynes, the mistake made by other theories is that choices made by individuals after that the money is saved are neglected.

The core of his theory is simple. Each individual has to make two decisions. First he determines how much of his income he will consume and how much money he will save. Keynes calls savings ‘retaining command over future consumption’. Then the individual decides in what form he will control the saved money from the current income and from previous savings. This can be in an immediate and liquid command, as in cash. However, the individual could also sacrifice this immediate control and liquidity. He would then be rewarded with interest.

Interest is thus the reward for parting liquidity. It is the price at which an equilibrium exists between the desire to hold wealth in the form of cash and the available quantity of cash. As the interest would be high, there would be a surplus of money that nobody wants to hold. And if interest would be zero, everybody would hold their money and it would exceed the available supply. Keynes argues from this that besides liquidity preference, the quantity of money is a determinant of the interest rate. The quantity of money is determined by the liquidity preference function with variable interest rate. $M = L(r)$.

Still, the people tend not to hold all their wealth in a form that yields interest, though interest rates are always positive and a certain profit could be made. Keynes introduces here the missing condition for without the liquidity preference cannot exist: future interest rates are uncertain. As a result, liquidity preference can exist as a transaction motive, a precautionary motive and the speculative motive.

The term structure in a Keynes world is rising. Investors (people saving their money) are risk averse. For securities with a longer maturity, a higher premium is required. The premium exists in greater forward rates. The longer the maturity of the security, the premium increases with a decreasing rate. This is due to decreasing volatility of interest rates as the maturity rises.

Risk premium hypothesis (Hicks, 1946)

In the book 'Value and Capital' Hicks criticizes and adjusts the liquidity premium theory, as presented by Keynes (1936). Hicks tries to find answers for two questions. First, why do people give more for those securities that are more like money than the securities that are not. Second, why is interest paid on securities.

As does Keynes, Hicks recognizes two risk elements why interest is paid. The first reason is the default risk on a security. In addition, the uncertainty of future interest rates. Hicks states that Keynes with his liquidity premium theory takes only in account these pure risk elements. "But to say that the rate of interest on perfectly safe securities is determined by nothing else but uncertainty of future interest rates seems to leave interest hanging by its own bootstraps".

Hicks continues his search for reasons to pay interest with the very short bill. This type of security is the nearest to be money, without being quite money. The author furthermore makes the assumption that short bills are safe. So the reason found to pay interest on this type of security, is the reason for the existence of pure interest.

To become a bill, money has to be converted into a security and that comes with costs. Therefore, it must be the trouble of making transactions, which explains the existence of the short rates of interest. Then Hicks suggests that if you remove the conversion step, then would the interest rates disappear? E.g., pay people in the form of short bills which are perfectly safe. No, a discount rate will still exist. Hicks thus states that the lack of general acceptability of bills creates imperfect 'moneyness' of the short bills, which still is no money. This causes a discount on the bill price.

Bills with a maturity longer than the minimum maturity incorporate the risk of rediscounting. Anyone who purchases a bill faces the possibility that he will have to convert the bill in to money, before the maturity date of the bill. Besides the cost of transaction, he then also faces the risk of rediscounting the value of the bill against a risen or lowered interest rate since the original investment date. So a risk premium in the long interest rates is to compensate for the risk of an unfavourable movement of the interest rates.

For even longer maturities Hicks considers a third risk that if rediscounting becomes necessary, it will only be possible on unfavourable terms. Hicks thus states that the long interest rate will normally exceed the short rate.

Segmentation theory

There is no relationship between short-, medium- and long-term interest rates. This theory has two versions, the partial and the complete segmented hypothesis. Both theories, the preferred habitat hypothesis and the complete segmented market hypothesis, are described here.

Preferred habitat hypothesis (Modigliani & Sutch, 1966)

The authors write their paper in perspective of operation Twist, as launched by the Kennedy administration in 1961. Operation Twist was an attempt to twist the maturity structure of the interest rates. Short-term interest rates on securities were raised and long-term interest rates were lowered or held the same. The aim was to increase in short term rates would stop and at least slow down the outflow of capital from the United States and improve the balance-of-payments. The low long-term interest rates would stimulate the economy by increasing the level of private investments. Modigliani and Sutch reviewed whether this approach for twisting the term structure worked. To do this, they created a theoretical framework.

First, the authors describe the traditional assumption of a world without transaction costs, no taxes, full certainty and rationality. This implies that all outstanding instruments must produce identical returns over any given interval of time. These identical returns make that lenders and borrowers do not have a “special incentive to match the maturity structure of his assets or liabilities to the length of time for which intends to remain a creditor or debtor”. Modigliani and Sutch state that in real life this is different.

In the view of Modigliani and Sutch the term structure of interest rates is determined by the Preferred Habitat Theory, which is a blend of three models: the Pure Expectation Hypothesis, the Risk Premium model and the Market Segmentation Hypothesis. The term structure is in essence controlled by the principle of equality of expected returns and adjusted by risk premiums. Also different market participants have different maturity habitats, like the segmentation theory. Risk aversion from investors does lead to a preferred staying in their own maturity habitat. That contradicts with the assumed preferred short investment horizon. Unless other maturities outside the preferred habitat offer an expected premium, sufficient to compensate for the risk and cost of moving out of one’s habitat, an investor stays in its own habitat.

As a result for a given maturity m, the interest rate could differ from the implied interest rate (by the pure expectation hypothesis). The distortion can be a positive or a negative risk premium. This premium depends on the difference between the supply of funds with habitat m and the “aggregate” demand for the m period loans forthcoming at that rate.

Complete segmented market hypothesis

An investor will never consider a bond with a term to maturity that moves away from his preferred habitat. Bonds with different terms to maturity are seen by all market participants as completely distinctive goods. In this the time value of money is no longer present. (Culbertson, 1957)

Money substitute theory

Kessel (1971) observes that a pure expectations model for the cyclical behaviour of interest rates is inconsistent with the following findings:

1. “Short maturities yield less over the cycle than long maturities; yield curves are more often than not positively sloped.”
2. “Short term rates fail to exceed long term rates at troughs.”
3. “The variance over the cycle in yields of the three month treasury bills is less than the variance of nine to twelve month Government bonds.”
4. “When short term rates are above long term rates, it Is not the shortest term to maturity that bears the highest yield, i.e., yield curves at first rise with term to maturity and then fall.”
5. “Long term rates fail to lead turning points in short term rates.“

The author explains the findings by adding liquidity preference to the model. Consequently, the total return from holding a security is no longer measured by just interest rate return, but also by a “nonpecuniary” return. This nonpecuniary return represents a liquidity income. The shorter the term to maturity, the higher the nonpecuniary return from a security. This nonpecuniary return can also be described as the possibility to transform the security into real money.

Kessel states that the *total* return from holding securities is the same, no matter the maturity. The sum of the interest rate and the nonpecuniary return is thus always the same. The nonpecuniary return is inversely related to the maturity of the bond. The interest rate (pecuniary return) is an increasing function of the maturity. This results in a normal yield curve with an upward sloping (pecuniary) yield curve.

Appendix C - Nelson & Siegel estimation, dataset mat. > 2 years

Annex Table 1 – Results of the Nelson & Siegel estimations. The presented results for each dataset are the best outcome of the possible τ values. Each dataset used in the calculations only contains bonds with a maturity of two years or more.

TABLE - Appendix C	Measures of Model Fit (Nelson & Siegel), Maturity > 2 yrs			
	Third order model			
Date Dataset	Best τ - value	St. Dev. at best τ	R^2	# observations
30/12/1952	20.00 *	11.83	49.35	6
30/12/1953	1.53	11.78	72.45	8
30/12/1954	2.08	14.76	68.20	9
29/12/1955	0.69	9.23	69.75	11
28/12/1956	3.00	14.29	63.58	12
30/12/1957	10.00	22.92	64.36	12
30/12/1958	10.00	8.97	65.03	13
29/12/1960	7.50	8.08	91.77	18
28/12/1961	7.50	7.23	91.48	20
28/12/1962	20.00 *	5.50	40.04	20
30/12/1963	20.00 *	4.86	52.11	23
30/12/1964	20.00 *	7.01	32.62	27
30/12/1965	20.00 *	12.07	46.00	29
29/12/1966	1.11	10.29	26.82	31
29/12/1967	20.00 *	12.04	16.02	34
30/12/1968	20.00 *	11.34	44.04	37
30/12/1969	7.50	19.97	31.11	42
30/12/1970	2.64	97.00	6.42	47
30/12/1971	0.17	23.03	34.18	51
29/12/1972	0.56	31.02	33.65	52
28/12/1973	1.67	33.54	33.42	53
30/12/1974	0.25	36.88	27.06	53
30/12/1975	0.31	37.52	54.31	57
30/12/1976	0.31	29.56	57.21	61
29/12/1977	0.19	49.54	31.22	67
29/12/1978	1.53	62.10	28.68	73
28/12/1979	0.17	58.26	19.70	80
30/12/1980	0.28	79.82	16.77	86
30/12/1981	0.17	113.71	12.01	93
30/12/1982	2.08	70.90	16.23	99
29/12/1983	0.31	66.43	6.53	104
28/12/1984	3.00	75.33	8.11	107
30/12/1985	0.69	92.78	6.92	97
30/12/1986	0.28	92.97	7.37	87
30/12/1987	0.25	49.43	11.80	80
29/12/1988	0.14	69.49	7.11	75
29/12/1989	0.17	82.70	1.29	67
31/12/1990	0.17	52.81	4.84	62
30/12/1991	0.56	51.37	16.55	56
30/12/1992	0.42	39.11	13.74	51
30/12/1993	3.00	24.35	59.33	47
30/12/1994	2.08	9.04	87.07	40
28/12/1995	0.47	25.28	86.69	42
30/12/1996	3.00	21.21	93.77	37
30/12/1997	3.00	2.49	99.62	29
30/12/1998	2.78	6.30	97.80	23
30/12/1999	3.00	3.02	99.61	22
29/12/2000	2.36	4.01	97.53	20
28/12/2001	1.67	8.97	96.87	18
Median	1.67	23.03	34.18	42.00

Note - Standard deviations are in Basis Points

* Best fit realized at boundary of range of τ search

Appendix D - Example of (embedded) option pricing

In some annual datasets, callable bonds are present. Chapter 6 discusses that callable bonds require higher yields than bullet bonds, because of the call risk of callable bonds that investors face. In the period 1967 – 1990, this results in low R^2 values of the term structure estimations. The low outcomes are caused by a mixed dataset with bullet bonds and callable bonds. As a solution, the callable bond is transformed to a bullet bond. This appendix gives an example of how this is done. The transformation process is done in multiple steps, which are described below. The example discusses date December 29, 1989 with callable bond 10.75NL1980I/II – 1986/1995.

Step 1

The first step is to collect key data of the bond and calculations. The callable bond price of € 104.05 and coupon rate of 10.75% are noted. Also the one year spot rate, which is known, is collected. In this case, the callable bond is also a sinking fund. Each following coupon date until maturity, 1/6 of the bond is redeemed (through lottery, see chapter 4). If today a share of the bond is redeemed (t), then the next coupon date (t+1), there is no interest earned on the redeemed part on t+1. The bond properties and cash flows are presented in the step 1a and step 1b table. The cash flows table shows the cash flows at the coupon dates, and are not discounted to a present value

Step 1a - Key data			
Bond	10.75NL1980I/II -1986/1995	1 year spot rate	8.00%
Date	29/12/1989	P callable bond	104.05
Coupon date	15/12	Prob. (up)	50%
Volatility	1.233	Prob. (down)	50%

Step 1b - Cash flow overview							
Cash Flows	29/12/1989	15/12/1990	15/12/1991	15/12/1992	15/12/1993	15/12/1994	15/12/1995
CF - Coupon		10.75	8.96	7.17	5.38	3.58	1.79
CF - Sinking Fund		16.67	16.67	16.67	16.67	16.67	16.67
Maturity (years)		0.96	1.96	2.96	3.96	4.96	5.96
Original issued amount	100%	83%	67%	50%	33%	17%	0%
Accrued Interest	0.448						

Step 2

In this step, the future paths of the term structure are estimated. The nodes are set on coupon dates of the callable bond. The interest rate at node 0 is the current one-year spot rate. Forward rates in other nodes are estimated one-year forward rates. The volatility determines the size of the up or down movement of the forward rate. The weight of the volatility (w) decreases after node 1, each next node with 10 percent points until a minimum of 10%. See equations D.1 for the upside forward rates and D.2 for down side forward rates.

$$f_{i,up} = f_{i-1} * (\sigma * w) \qquad \text{(D.1)}$$

$$f_{i,down} = f_{i-1} / (\sigma * w) \qquad \text{(D.2)}$$

Table Step 2 gives an overview of the estimated rates, calculations start in node 1.

Step 2 - Term structure							
Volatility Weight		100%	90%	80%	70%	60%	50%
Nodes	0	1	2	3	4	5	6
Forward Rates	0.080	0.099	0.119	0.142	0.165	0.188	0.210
		0.065	0.078	0.093	0.108	0.123	0.138
			0.054	0.064	0.074	0.084	0.094
				0.045	0.053	0.060	0.067
					0.039	0.044	0.049
						0.034	0.038
							0.031

Step 3

With the gathered information, the value of the bullet bond is calculated. Each cash flow is discounted against its appropriate forward rate value. The calculations start in latest node and are done backwards to node 0. Each node shows a present value, for the date of that node. See equation D.3.

$$PV_{node\ i} = CF_i + \frac{1}{2}\left(\frac{PV_{node(+)\ i+1} + PV_{node(-)\ i+1}}{1 + r_{node\ i}}\right) \qquad \text{(D.3)}$$

E.g., the present value in node 5, in case of a five time increase in interest rates, is calculated as follows:

$$PV_{node\ 5,5+} = (3.58 + 16.67) + \frac{1}{2}\left(\frac{18.46 + 18.46}{1 + 0.188}\right) = 35.79$$

Step 3 - Bullet bond							
Nodes	0	1	2	3	4	5	6
Discounted cash flows	108.08	112.83	91.11	71.38	53.15	35.79	18.46
		119.88	96.58	75.21	55.40	36.68	18.46
			100.34	77.84	56.93	37.27	18.46
				79.60	57.95	37.66	18.46
					58.63	37.93	18.46
						38.10	18.46
							18.46

Step 4

The bond price is calculated, and now the call option value on that bond can be calculated. The bond in this example has an exercise price of € 103.00. Due to the assumptions, the bond can only be called on the coupon dates and on the research date. At maturity, the bond cannot be called. Because the bond is partially redeemed at node 1, only the not redeemed part can be called by the state. In case of exercise value at node 0, this is corrected for accrued interest.

E.g., in node 1, 16.67% is redeemed. Of the original amount at node 0, only 83.33% remains in node 1. If the call option on this bond is exercised, the exercise price also has to be adjusted to this new outstanding percentage. In this case, € 85.83 = 103 * 83.33%.

The results are presented in table step 4. The calculations start in node 6 and are calculated backwards to node 0. The value is calculated with the following equation:

$$PV_i = Max\left[PV_{bond,node\ i} - P_{exercise};\ \frac{1}{2}\left(\frac{PV_{node(+),i+1}+PV_{node(-),i+1}}{1+r_{node\ i}}\right)\right] \qquad (D.4)$$

Step 4 - Present value call option							
Exercise Price		85.83	68.67	51.50	34.33	17.17	0.00
Nodes	0	1	2	3	4	5	6
Call Option Value	3.59	1.09	0.11	0.00	0.00	0.00	0.00
		6.63	2.29	0.24	0.00	0.00	0.00
			6.05	2.50	0.55	0.00	0.00
				4.27	1.57	0.25	0.00
					2.26	0.51	0.00
						0.68	0.00
							0.00

E.g., the call option value in node 2, with a two time decrease in interest rates, is calculated as follows. If the call option is not exercised in node 2, the call option has a value of

$$Maintaining\ value = \frac{1}{2}\frac{(2.50 + 4.27)}{(1.054)} = 3.21$$

If the call option is exercised in node 2, the option value is the present value of the bond minus the exercise value. At node 2, the value of the bond 100.34, for the call option valuation two values have to be subtracted. First, the sinking fund cash flow of 16.97 and second the coupon cash flow. These cash flows are needed in the calculation of the bond value, but are paid out on that coupon date. The intrinsic value of the bullet bond on node two is 100.34 – 16.97 -6.96 = 76.41. The exercise value of the bond is:

$$Exercise\ value = 100.34 - 68.67 - 16.97 - 6.96 = 6.05$$

The call option value is the higher of the two found values, which in this case is 6.05.

Step 5

The transformed bullet bond value of the callable bond is now calculated:

$$PV_{callable\ bond} = PV_{bullet\ bond} - PV_{embedded\ option}$$

As mentioned in section 6.3, this interest rate volatility is first set on 0%, which would be 1.00 in table step 1. By adjusting the interest rate volatility of the bond, the calculated present value of the callable is set equal to the current callable bond price. In this case the interest rate volatility is set on 1.233.

Callable bond 10.75NL1980I/II – 1986/1995 on December 29, 1989 of 104.05 is now transformed into a bullet bond price. The clean ‘price’ of the transformed bullet bond is.

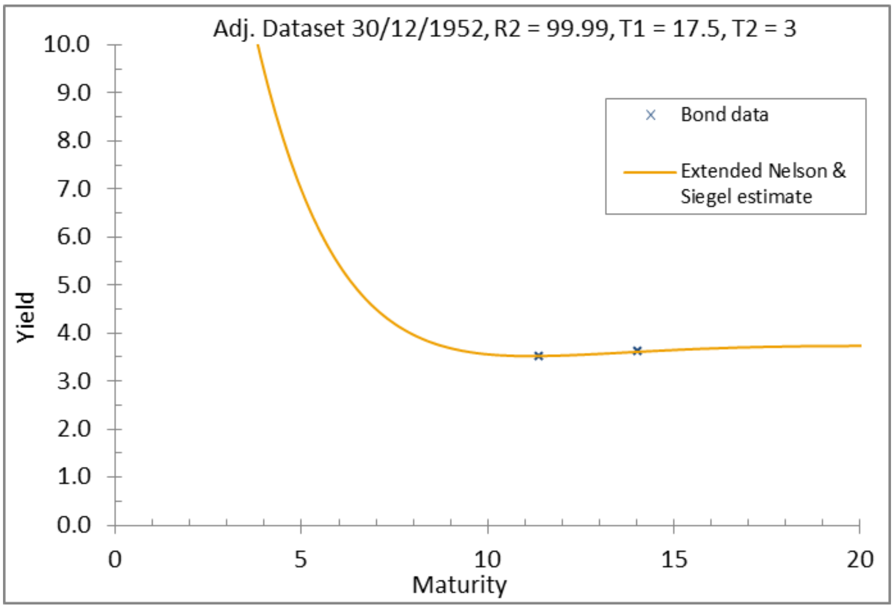
$$PV_{bullet\ bond} = 104.05 + 3.59 = 107.64$$

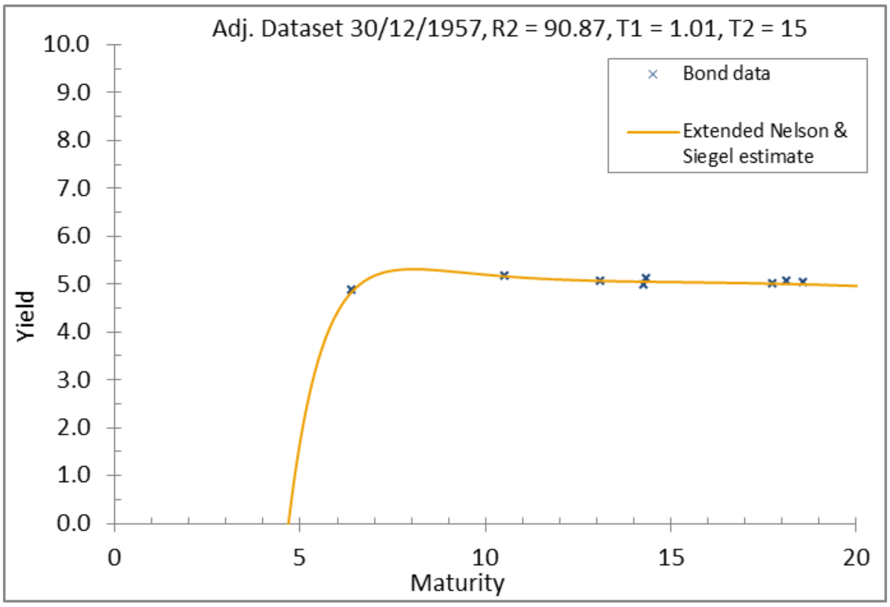
With this information, the yield is recalculated and used in the adjusted dataset.

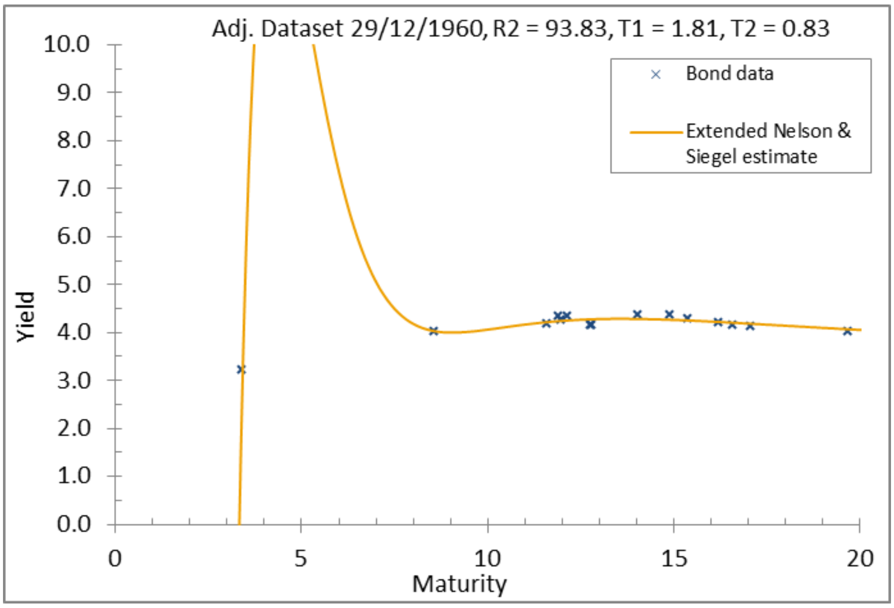
Appendix E - Extended Nelson & Siegel Yield curves

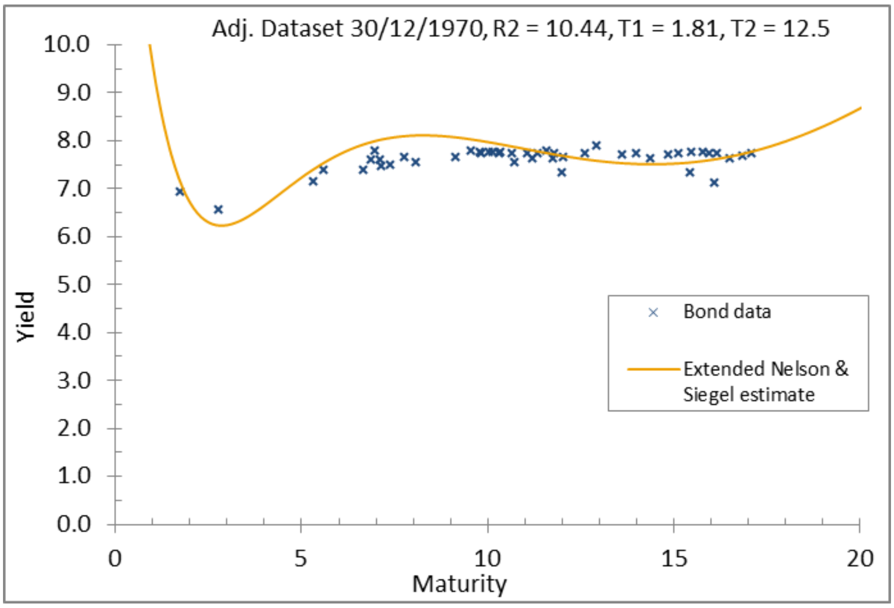
This appendix presents six annual adjusted datasets and their Extended Nelson & Siegel estimations. The results of all estimations are shown in table 4 (see section 5.3). The figures show the R^2, τ_1 and τ_2 values. The datasets presented are chosen to show that the estimations give reliable results for available maturities and their yields. Yield estimations outside the range of available data may give strange results. It is seen that especially missing data for short maturities results in strange yield curve estimations. The estimations for longer maturities, outside the available maturity range, give in most instances normal results.

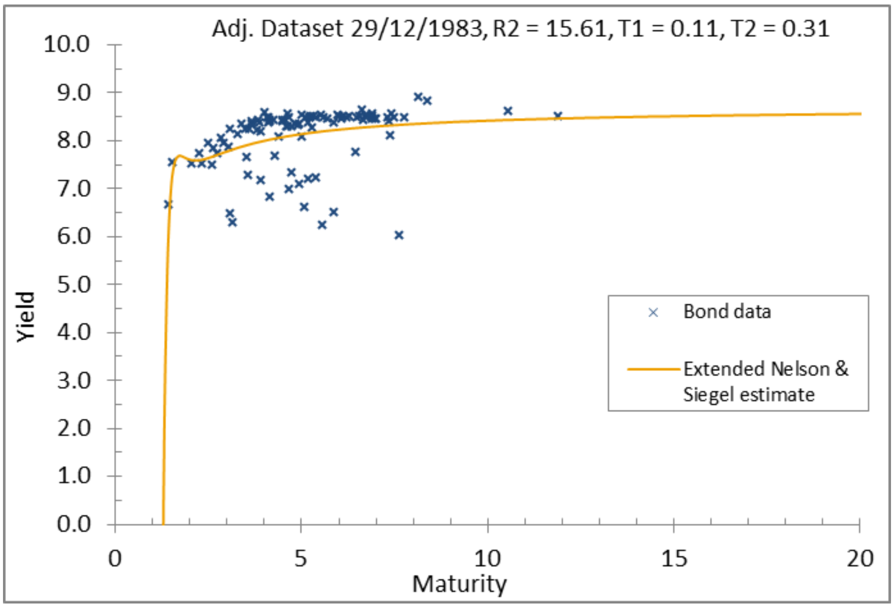
Due to time constraints, this appendix does not individually discuss the below presented yield curve estimations or the specific properties of the datasets involved.

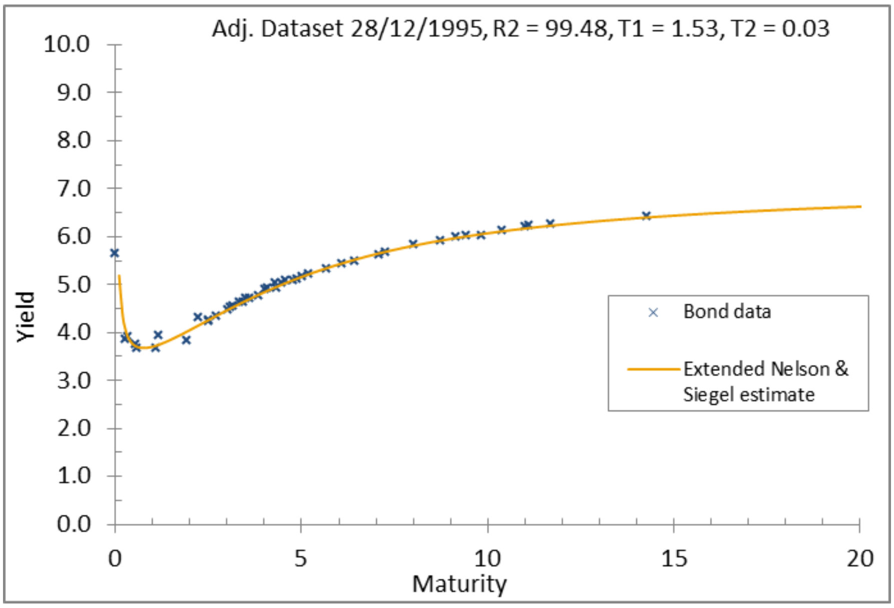





Adj. Dataset 29/12/1960, R2 = 93.83, T1 = 1.81, T2 = 0.83
Bond data
Extended Nelson & Siegel estimate
Yield
Maturity
10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
0
5
10
15
20


Adj. Dataset 30/12/1970, R2 = 10.44, T1 = 1.81, T2 = 12.5
Bond data
Extended Nelson & Siegel estimate
Yield
Maturity
10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
0
5
10
15
20


Adj. Dataset 29/12/1983, R2 = 15.61, T1 = 0.11, T2 = 0.31
Yield
10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
0
5
10
15
20
Maturity
Bond data
Extended Nelson & Siegel estimate


Adj. Dataset 28/12/1995, R2 = 99.48, T1 = 1.53, T2 = 0.03
Yield
10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
0
5
10
15
20
Maturity
Bond data
Extended Nelson & Siegel estimate

Appendix F - Dataset overview

This appendix presents all bonds used in this research. The first column shows the identification number for the bond. The second column shows the name of the bonds. The third column shows the coupon rate of bonds and column four and five show the start date and end date of bonds. The coupon dates are presented in columns six en seven and the number of coupons is column eight. The column redemption scheme shows whether the bond is a sinking fund, and if so, whether the bond is 'sinking' with equal amounts or unequal amounts. Other redemption properties are bullet bonds and perpetual bonds. The final column shows the extra qualities of a bond: normal bond, convertible bond or callable bond.

Easy ID #	Name	coupon rate	Bond start	Final Redemption	coupon date 1	coupon date 2	number coupons	redemption scheme **	bond type *
1	10.00NL1980 -1986/1990	10.00%	15-Jul-80	15-Jul-90	15-Jul		1	SF-e	NO
2	10.00NL1982 -1988/1992	10.00%	1-Oct-82	1-Oct-92	01-Oct		1	SF-e	NO
3	10.00NL1982I -1986/1989	10.00%	1-Jul-82	1-Jul-89	01-Jun		1	SF-e	NO
4	10.00NL1982II -1986/1989	10.00%	1-Nov-82	1-Nov-89	01-Nov		1	SF-e	CO
5	10.00NL1985 -1989/1992	10.00%	1-Nov-85	1-Nov-92	01-Nov		1	SF-e	NO
6	10.25NL1980 -1984/1987	10.25%	15-Nov-80	15-Nov-87	15-Nov		1	SF-e	NO
7	10.25NL1980 -1986/1990	10.25%	15-Oct-80	15-Oct-90	15-Oct		1	SF-e	CO
8	10.25NL1982 -1988/1992	10.25%	1-Jul-82	1-Jul-92	01-Jul		1	SF-e	CO
9	10.25NL1986 -1992/1996	10.25%	15-Oct-86	15-Oct-96	15-Oct		1	SF-e	NO
10	10.25NL1987 -1993/1997	10.25%	1-Jul-87	1-Jul-97	01-Jul		1	SF-e	NO
11	10.50NL1974 -1975/1986	10.50%	1-Sep-74	1-Sep-86	01-Sep		1	SF-e	CA
12	10.50NL1980 -1991/2000	10.50%	1-Jun-80	1-Jun-00	01-Jun		1	SF-e	NO
13	10.50NL1982 -1986/1989	10.50%	1-Sep-82	1-Sep-89	01-Sep		1	SF-e	NO
14	10.50NL1982 -1988/1992	10.50%	1-May-82	1-May-92	01-May		1	SF-e	NO
15	10.75NL1980I/II -1986/1995	10.75%	15-Dec-80	15-Dec-95	15-Dec		1	SF-e	CA
16	10.75NL1981 -1987/1991	10.75%	1-Mar-81	1-Mar-91	01-Mar		1	SF-e	NO
17	11.00NL1981I/II -1985/1988	11.00%	1-Aug-81	1-Aug-88	01-Aug		1	SF-e	NO
18	11.00NL1982 -1988/1992	11.00%	1-Aug-82	1-Aug-92	01-Aug		1	SF-e	NO
19	11.25NL1981I/II -1992/1996	11.25%	1-Aug-81	1-Aug-96	01-Aug		1	SF-e	CA
20	11.25NL1982 -1988/1992	11.25%	1-Apr-82	1-Apr-92	01-Apr		1	SF-e	NO
21	11.50NL1980 -1986/1990	11.50%	15-Apr-80	15-Apr-90	15-Apr		1	SF-e	NO
22	11.50NL1981 -1987/1991	11.50%	15-May-81	15-May-91	15-May		1	SF-e	NO
23	11.50NL1981 -1988/1992	11.50%	15-Jan-82	15-Jan-92	15-Jan		1	SF-e	NO
24	11.50NL1982 -1988/1992	11.50%	15-Feb-82	15-Feb-92	15-Feb		1	SF-e	NO
25	11.75NL1981 -1982/1991	11.75%	1-Sep-81	1-Sep-91	01-Sep		1	SF-e	NO
26	12.00NL1981 -1985/1988	12.00%	15-Jun-81	15-Jun-88	15-Jun		1	SF-e	NO
27	12.00NL1981 -1987/1991	12.00%	15-Apr-81	15-Apr-91	15-Apr		1	SF-e	NO
28	12.25NL1981 -1985/1988	12.25%	1-Nov-81	1-Nov-88	01-Nov		1	SF-e	NO
29	12.50NL1981 -1987/1991	12.50%	1-Oct-81	1-Oct-91	01-Oct		1	SF-e	NO
30	12.75NL1981 -1987/1991	12.75%	1-Dec-81	1-Dec-91	01-Dec		1	SF-e	CO
31	12.75NL1986 -1992/1996	12.75%	1-Dec-86	1-Dec-96	01-Dec		1	SF-e	NO
32	3.00GB1946 -1964/1982	3.00%	1-Nov-46	1-Nov-82	01-Nov	01-May	2	SF-u	CA
33	3.00NL1937 -1952/1981	3.00%	1-Jul-37	1-Jul-81	01-Jul	01-Jan	2	SF-u	CA
34	3.00NL1948 -1962/1964	3.00%	13-May-48	1-Jun-64	01-Jun	01-Dec	2	BU	CA
35	3.00NL1999 -2002/2002	3.00%	15-Feb-99	15-Feb-02	15-Feb		1	BU	NO
36	3.25BC1948 - 1959/1998	3.25%	1-Apr-48	1-Apr-98	01-Apr	01-Oct	2	SF-u	CA
37	3.25NL1948 - 1959/1998	3.25%	1-Jun-48	1-Jun-98	01-Jun	01-Dec	2	SF-u	CA
38	3.25NL1950I/II -1951/1990	3.25%	15-Mar-50	15-Mar-90	15-Mar		1	SF-u	CA
39	3.25NL1954I/II -1955/1994	3.25%	15-Feb-54	15-Feb-94	15-Feb	15-Aug	2	SF-e	CA
40	3.25NL1955I -1956/1995	3.25%	1-Feb-55	1-Feb-95	01-Feb	01-Aug	2	SF-e	CA
41	3.25NL1955II -1956/1985	3.25%	15-Oct-55	15-Oct-85	15-Oct	15-Apr	2	SF-u	CA
42	3.50NL1951 -1952/1976	3.50%	1-Apr-51	1-Apr-76	01-Apr	01-Oct	2	SF-u	CA
43	3.50NL1953I/II -1954/1983	3.50%	1-Sep-53	1-Sep-83	01-Sep	01-Mar	2	SF-u	CA
44	3.50NL1956 -1957/1986	3.50%	1-May-56	1-May-86	01-May	01-Nov	2	SF-u	CA
45	3.75NL1953 -1954/1993	3.75%	1-Apr-53	1-Apr-93	01-Apr	01-Oct	2	SF-e	CA
46	3.75NL1999 -2009/2009	3.75%	15-Jul-99	15-Jul-09	15-Jul		1	BU	NO
47	3-3.50NL1947 -1987/1987	3.50%	15-Feb-47	15-Feb-87	15-Feb	15-Aug	2	BU	CA
48	4.00NL1961 -1962/1986	4.00%	15-Aug-61	15-Aug-86	15-Aug		1	SF-e	CA
49	4.00NL1962 -1963/1992	4.00%	15-Mar-62	15-Mar-92	15-Mar		1	SF-u	CA
50	4.25NL1959 -1960/1984	4.25%	1-Jul-59	1-Jul-84	01-Jul		1	SF-e	CA
51	4.25NL1960 -1961/1990	4.25%	1-Dec-60	1-Dec-90	01-Dec		1	SF-u	CA
52	4.25NL1961 -1962/1991	4.25%	1-Mar-61	1-Mar-91	01-Mar		1	SF-u	CA
53	4.25NL1963I -1964/1993	4.25%	1-Mar-63	1-Mar-93	01-Mar		1	SF-u	CA
54	4.25NL1963II -1964/1993	4.25%	1-Jun-63	1-Jun-93	01-Jun		1	SF-u	CA
55	4.50NL1958 -1959/1983	4.50%	1-Dec-58	1-Dec-83	01-Dec		1	SF-e	CA
56	4.50NL1959 -1960/1989	4.50%	16-Feb-59	16-Feb-89	16-Feb		1	SF-u	CA
57	4.50NL1960I -1961/1985	4.50%	1-Mar-60	1-Mar-85	01-Mar		1	SF-e	CA
58	4.50NL1960II -1961/1990	4.50%	1-Jun-60	1-Jun-90	01-Jun		1	SF-u	CA
59	4.50NL1963 -1964/1993	4.50%	15-Nov-63	15-Nov-93	15-Nov		1	SF-u	CA

Easy ID #	Name	coupon rate	Bond start	Final Redemption	coupon date 1	coupon date 2	number coupons	redemption scheme **	bond type *
60	4.50NL1964 -1965/1974	4.50%	15-Apr-64	15-Apr-74	15-Apr		1	SF-e	NO
61	4.75NL2000 -2003/2003	4.75%	15-Feb-00	15-Feb-03	15-Feb		1	BU	NO
62	5.00NL1964 -1965/1994	5.00%	15-Apr-64	15-Apr-94	15-Apr		1	SF-u	CA
63	5.00NL2001 -2011/2011	5.00%	15-Jul-01	15-Jul-11	15-Jul		1	BU	NO
64	5.25NL1964I -1970/1989	5.25%	1-Aug-64	1-Aug-89	01-Aug		1	SF-e	CA
65	5.25NL1964II -1970/1989	5.25%	1-Nov-64	1-Nov-89	01-Nov		1	SF-e	CA
66	5.25NL1998 -2008/2008	5.25%	15-Jul-98	15-Jul-08	15-Jul		1	BU	NO
67	5.50NL1998 -2028/2028	5.50%	15-Jan-98	15-Jan-28	15-Jan		1	BU	NO
68	5.50NL2000 -2010/2010	5.50%	15-Jul-00	15-Jul-10	15-Jul		1	BU	NO
69	5.75NL1965I -1971/1990	5.75%	1-Aug-65	1-Aug-90	01-Aug		1	SF-e	CA
70	5.75NL1965II -1971/1990	5.75%	15-Nov-65	15-Nov-90	15-Nov		1	SF-e	CA
71	5.75NL1994 -2004/2004	5.75%	15-Jan-94	15-Jan-04	15-Jan		1	BU	NO
72	5.75NL1996 -2002/2002	5.75%	15-Sep-96	15-Sep-02	15-Sep		1	BU	NO
73	5.75NL1997 -2007/2007	5.75%	15-Feb-97	15-Feb-07	15-Feb		1	BU	NO
74	6.00NL1967 -1978/1992	6.00%	1-Jun-66	1-Jun-92	01-Jun		1	SF-u	CA
75	6.00NL1987 -1994/1994	6.00%	1-Jul-87	1-Jul-94	01-Jul		1	BU	NO
76	6.00NL1988 -1994/1994	6.00%	15-Jun-88	15-Jun-94	15-Jun		1	BU	NO
77	6.00NL1988 -1995/1995	6.00%	15-Apr-88	15-Apr-95	15-Apr		1	BU	NO
78	6.00NL1988 -1996/1996	6.00%	15-May-88	15-May-96	15-May		1	BU	NO
79	6.00NL1996 -2006/2006	6.00%	15-Jan-96	15-Jan-06	15-Jan		1	BU	NO
80	6.25NL1966 -1972/1991	6.25%	15-Mar-65	15-Mar-91	15-Mar		1	SF-e	CA
81	6.25NL1967 -1978/1992	6.25%	16-Nov-67	16-Nov-92	16-Nov		1	SF-u	CA
82	6.25NL1986 -1992/1996	6.25%	15-Jun-86	15-Jun-96	15-Jun		1	SF-e	NO
83	6.25NL1986 -1995/1995	6.25%	15-Jan-87	15-Jan-95	15-Jan		1	BU	NO
84	6.25NL1986 -1996/1996	6.25%	1-Aug-86	1-Aug-96	01-Aug		1	BU	NO
85	6.25NL1987 -1997/1997	6.25%	15-Feb-87	15-Feb-97	15-Feb		1	BU	NO
86	6.25NL1987 -1998/2002	6.25%	15-Mar-87	15-Mar-02	15-Mar		1	SF-e	CA
87	6.25NL1987I -1995/1995	6.25%	1-May-87	1-May-95	01-May		1	BU	NO
88	6.25NL1987II -1995/1995	6.25%	1-Jun-87	1-Jun-95	01-Jun		1	BU	NO
89	6.25NL1987III -1995/1995	6.25%	15-Jan-87	15-Jan-95	15-Jan		1	BU	NO
90	6.25NL1988 -1994/1994	6.25%	15-Feb-88	15-Feb-94	15-Feb		1	BU	NO
91	6.25NL1988 -1994/1998	6.25%	15-Mar-88	15-Mar-98	15-Mar		1	SF-e	NO
92	6.25NL1993 -1998/1998	6.25%	15-Jul-93	15-Jul-98	15-Jul		1	BU	NO
93	6.375NL1987 -1997/1997	6.375%	15-Dec-87	15-Dec-97	15-Dec		1	BU	NO
94	6.50NL1968I -1979/1993	6.50%	1-Mar-68	1-Mar-93	01-Mar		1	SF-u	CA
95	6.50NL1968II -1979/1993	6.50%	1-Jul-68	1-Jul-93	01-Jul		1	SF-u	CA
96	6.50NL1968III -1979/1993	6.50%	1-Nov-68	1-Nov-93	01-Nov		1	SF-u	CA
97	6.50NL1968IV -1980/1994	6.50%	2-Jan-69	2-Jan-94	02-Jan		1	SF-u	CA
98	6.50NL1986 -1996/1996	6.50%	15-Apr-86	15-Apr-96	15-Apr		1	BU	NO
99	6.50NL1987 -1994/1994	6.50%	1-Oct-87	1-Oct-94	01-Oct		1	BU	NO
100	6.50NL1988 -1996/1996	6.50%	15-Aug-88	15-Aug-96	15-Aug		1	BU	NO
101	6.50NL1988 -1998/1998	6.50%	15-Jul-88	15-Jul-98	15-Jul		1	BU	NO
102	6.50NL1989 -1999/1999	6.50%	15-Jan-89	15-Jan-99	15-Jan		1	BU	NO
103	6.50NL1993 -2003/2003	6.50%	15-Apr-93	15-Apr-03	15-Apr		1	BU	NO
104	6.75NL1978 -1979/1998	6.75%	1-Jun-78	1-Jun-98	01-Jun		1	SF-e	CA
105	6.75NL1985I/II -1991/1995	6.75%	15-Oct-85	15-Oct-95	15-Oct		1	SF-e	NO
106	6.75NL1986I/II -1992/1996	6.75%	15-Feb-86	15-Feb-96	15-Feb		1	SF-e	NO
107	6.75NL1988 -1998/1998	6.75%	1-Oct-88	1-Oct-98	01-Oct		1	BU	NO
108	6.75NL1989 -1999/1999	6.75%	15-Feb-89	15-Feb-99	15-Feb		1	BU	NO
109	6.75NL1995 -2005/2005	6.75%	15-Nov-95	15-Nov-05	15-Nov		1	BU	NO
110	7.00NL1966I -1977/1991	7.00%	1-Sep-66	1-Sep-91	01-Sep		1	SF-u	CA
111	7.00NL1966II -1978/1992	7.00%	16-Jan-67	16-Jan-92	16-Jan		1	SF-u	CA
112	7.00NL1969 -1980/1994	7.00%	15-Mar-69	15-Mar-94	15-Mar		1	SF-u	CA
113	7.00NL1985 -1992/1996	7.00%	15-Jan-86	15-Jan-96	15-Jan		1	SF-e	NO
114	7.00NL1987 -1993/1993	7.00%	15-Nov-87	15-Nov-93	15-Nov		1	BU	NO
115	7.00NL1989I/II -1999/1999	7.00%	15-Mar-89	15-Mar-99	15-Mar		1	BU	NO
116	7.00NL1989III -1999/1999	7.00%	15-May-89	15-May-99	15-May		1	BU	NO
117	7.00NL1989IV -1999/1999	7.00%	15-Aug-89	15-Aug-99	15-Aug		1	BU	NO
118	7.00NL1993 -2003/2003	7.00%	15-Feb-93	15-Feb-03	15-Feb		1	BU	NO
119	7.00NL1995 -2005/2005	7.00%	15-Jun-95	15-Jun-05	15-Jun		1	BU	NO
120	7.20NL1972 -1983/1997	7.20%	1-Mar-72	1-Mar-97	01-Mar		1	SF-u	CA
121	7.25NL1989 -1999/1999	7.25%	15-Jul-89	15-Jul-99	15-Jul		1	BU	NO
122	7.25NL1994 -2004/2004	7.25%	1-Oct-94	1-Oct-04	01-Oct		1	BU	NO
123	7.50NL1969 -1980/1994	7.50%	15-Jul-69	15-Jul-94	15-Jul		1	SF-u	CA
124	7.50NL1971 -1977/1981	7.50%	15-Dec-71	15-Dec-81	15-Dec		1	SF-e	NO
125	7.50NL1971 -1982/1996	7.50%	1-Mar-71	1-Mar-96	01-Mar		1	SF-u	CA
126	7.50NL1972 -1983/1997	7.50%	1-Jul-72	1-Jul-97	01-Jul		1	SF-u	CA
127	7.50NL1978 -1979/1993	7.50%	1-Mar-78	1-Mar-93	01-Mar		1	SF-e	CA
128	7.50NL1978I -1984/1988	7.50%	15-Apr-78	15-Apr-88	15-Apr		1	SF-e	NO
129	7.50NL1978II -1984/1988	7.50%	15-Oct-78	15-Oct-88	15-Oct		1	SF-e	NO
130	7.50NL1983I -1987/1990	7.50%	1-Feb-83	1-Feb-90	01-Feb		1	SF-e	NO
131	7.50NL1983II -1987/1990	7.50%	15-Apr-83	15-Apr-90	15-Apr		1	SF-e	CO
132	7.50NL1984 -1991/2000	7.50%	15-Jan-85	15-Jan-00	15-Jan		1	SF-e	CA
133	7.50NL1985I -1991/1995	7.50%	1-Mar-85	1-Mar-95	01-Mar		1	SF-e	NO
134	7.50NL1985II -1991/1995	7.50%	1-Jul-85	1-Jul-95	01-Jul		1	SF-e	NO
135	7.50NL1986 -1990/1993	7.50%	15-Apr-86	15-Apr-93	15-Apr		1	SF-e	NO
136	7.50NL1989I -1999/1999	7.50%	15-Jun-89	15-Jun-99	15-Jun		1	BU	NO

Easy ID #	Name	coupon rate	Bond start	Final Redemption	coupon date 1	coupon date 2	number coupons	redemption scheme **	bond type *
137	7.50NL1989II -1999/1999	7.50%	15-Nov-89	15-Nov-99	15-Nov		1	BU	NO
138	7.50NL1993 -2023/2023	7.50%	15-Jan-93	15-Jan-23	15-Jan		1	BU	NO
139	7.50NL1995 -2010/2010	7.50%	15-Apr-95	15-Apr-10	15-Apr		1	BU	NO
140	7.75NL1970 -1977/1978	7.75%	1-Sep-70	1-Sep-78	01-Sep		1	SF-e	NO
141	7.75NL1971 -1982/1996	7.75%	15-Dec-71	15-Dec-96	15-Dec		1	SF-u	CA
142	7.75NL1973 -1984/1998	7.75%	15-Jul-73	15-Jul-98	15-Jul		1	SF-e	CA
143	7.75NL1977 -1978/1992	7.75%	15-Sep-77	15-Sep-92	15-Sep		1	SF-e	CA
144	7.75NL1977 -1978/1997	7.75%	1-Nov-77	1-Nov-97	01-Nov		1	SF-e	CA
145	7.75NL1982 -1989/1993	7.75%	15-Jan-83	15-Jan-93	15-Jan		1	SF-e	NO
146	7.75NL1985 -1991/2000	7.75%	1-Jun-85	1-Jun-00	01-Jun		1	SF-e	CA
147	7.75NL1990 -2000/2000	7.75%	15-Jan-90	15-Jan-00	15-Jan		1	BU	NO
148	7.75NL1995 -2005/2005	7.75%	1-Mar-95	1-Mar-05	01-Mar		1	BU	NO
149	8.00NL1969 -1975/1976	8.00%	15-Nov-69	15-Nov-76	15-Nov		1	SF-e	NO
150	8.00NL1969 -1980/1994	8.00%	15-Nov-69	15-Nov-94	15-Nov		1	SF-u	CA
151	8.00NL1970 -1976/1977	8.00%	15-Feb-70	15-Feb-77	15-Feb		1	SF-e	NO
152	8.00NL1970 -1981/1995	8.00%	15-Feb-70	15-Feb-95	15-Feb		1	SF-u	CA
153	8.00NL1970I -1971/1985	8.00%	1-Jun-70	1-Jun-85	01-Jun		1	SF-u	NO
154	8.00NL1970II -1981/1985	8.00%	1-Sep-70	1-Sep-85	01-Sep		1	SF-e	CA
155	8.00NL1970III -1981/1985	8.00%	15-Dec-70	15-Dec-85	15-Dec		1	SF-e	CA
156	8.00NL1971 -1982/1996	8.00%	15-Oct-71	15-Oct-96	15-Oct		1	SF-u	CA
157	8.00NL1976I/II -1977/1991	8.00%	15-Mar-76	15-Mar-91	15-Mar		1	SF-e	CA
158	8.00NL1977 -1978/1987	8.00%	1-Aug-77	1-Aug-87	01-Aug		1	SF-e	NO
159	8.00NL1977 -1978/1997	8.00%	15-Jun-77	15-Jun-97	15-Jun		1	SF-e	CA
160	8.00NL1978 -1984/1988	8.00%	1-Sep-78	1-Sep-88	01-Sep		1	SF-e	NO
161	8.00NL1983 -1989/1993	8.00%	15-May-83	15-May-93	15-May		1	SF-e	NO
162	8.00NL1985 -1991/1995	8.00%	1-Apr-85	1-Apr-95	01-Apr		1	SF-e	NO
163	8.25NL1976 -1978/1997	8.25%	15-Jan-77	15-Jan-97	15-Jan		1	SF-e	CA
164	8.25NL1977 -1979/1993	8.25%	15-Jan-78	15-Jan-93	15-Jan		1	SF-e	CA
165	8.25NL1977I/II -1978/1992	8.25%	15-Mar-77	15-Mar-92	15-Mar		1	SF-e	CA
166	8.25NL1979 -1985/1989	8.25%	15-Feb-79	15-Feb-89	15-Feb		1	SF-e	NO
167	8.25NL1983 -1984/1993	8.25%	15-Jun-83	15-Jun-93	15-Jun		1	SF-e	NO
168	8.25NL1984 -1990/1994	8.25%	15-May-84	15-May-94	15-May		1	SF-e	NO
169	8.25NL1985 -1991/1995	8.25%	1-May-85	1-May-95	01-May		1	SF-e	NO
170	8.25NL1990 -2000/2000	8.25%	15-Feb-90	15-Feb-00	15-Feb		1	BU	NO
171	8.25NL1992I -2002/2002	8.25%	15-Feb-92	15-Feb-02	15-Feb		1	BU	NO
172	8.25NL1992I -2007/2007	8.25%	15-Feb-92	15-Feb-07	15-Feb		1	BU	NO
173	8.25NL1992II -2002/2002	8.25%	15-Jun-92	15-Jun-02	15-Jun		1	BU	NO
174	8.25NL1992II -2007/2007	8.25%	15-Sep-92	15-Sep-07	15-Sep		1	BU	NO
175	8.50NL1975I -1976/1990	8.50%	15-Aug-75	15-Aug-90	15-Aug		1	SF-e	CA
176	8.50NL1975II -1977/1991	8.50%	15-Jan-76	15-Jan-91	15-Jan		1	SF-e	CA
177	8.50NL1978 -1980/1989	8.50%	15-Jan-79	15-Jan-89	15-Jan		1	SF-e	NO
178	8.50NL1978 -1984/1993	8.50%	1-Dec-78	1-Dec-93	01-Dec		1	SF-e	CA
179	8.50NL1979 -1980/1989	8.50%	15-May-79	15-May-89	15-May		1	SF-e	NO
180	8.50NL1983 -1990/1994	8.50%	15-Jan-83	15-Jan-94	15-Jan		1	SF-e	NO
181	8.50NL1984I -1988/1991	8.50%	15-Mar-84	15-Mar-91	15-Mar		1	SF-e	CO
182	8.50NL1984I -1990/1994	8.50%	15-Feb-84	15-Feb-94	15-Feb		1	SF-e	NO
183	8.50NL1984II -1988/1991	8.50%	15-Jun-84	15-Jun-91	15-Jun		1	SF-e	NO
184	8.50NL1984II -1990/1994	8.50%	1-Oct-84	1-Oct-94	01-Oct		1	SF-e	CO
185	8.50NL1984III -1988/1991	8.50%	1-Aug-84	1-Aug-91	01-Aug		1	SF-e	NO
186	8.50NL1987 -1992/1995	8.50%	15-Mar-87	15-Mar-95	15-Mar		1	SF-e	NO
187	8.50NL1989 -1995/1999	8.50%	1-Oct-89	1-Oct-99	01-Oct		1	SF-e	NO
188	8.50NL1991 -2001/2001	8.50%	15-Mar-91	15-Mar-01	15-Mar		1	BU	NO
189	8.50NL1991 -2006/2006	8.50%	1-Jun-91	1-Jun-06	01-Jun		1	BU	NO
190	8.75NL1975I -1981/1990	8.75%	1-Jun-75	1-Jun-90	01-Jun		1	SF-e	CA
191	8.75NL1975II -1976/1990	8.75%	15-Nov-75	15-Nov-90	15-Nov		1	SF-e	CA
192	8.75NL1976 -1977/1996	8.75%	15-Dec-76	15-Dec-96	15-Dec		1	SF-e	CA
193	8.75NL1979 -1980/1994	8.75%	1-Apr-79	1-Apr-94	01-Apr		1	SF-e	CA
194	8.75NL1979 -1985/1989	8.75%	1-Nov-79	1-Nov-89	01-Nov		1	SF-e	NO
195	8.75NL1984 -1990/1994	8.75%	1-Sep-84	1-Sep-94	01-Sep		1	SF-e	NO
196	8.75NL1990I -2000/2000	8.75%	1-May-90	1-May-00	01-May		1	BU	NO
197	8.75NL1990II -2000/2000	8.75%	1-Aug-90	1-Aug-00	01-Aug		1	BU	NO
198	8.75NL1991 -2001/2001	8.75%	15-Sep-91	15-Sep-01	15-Sep		1	BU	NO
199	8.75NL1992 -2007/2007	8.75%	15-Jan-92	15-Jan-07	15-Jan		1	BU	NO
200	9.00NL1975 -1976/2000	9.00%	1-Mar-75	1-Mar-00	01-Mar		1	SF-e	CA
201	9.00NL1979I/II -1985/1994	9.00%	1-Jul-79	1-Jul-94	01-Jul		1	SF-e	CA
202	9.00NL1983 -1989/1993	9.00%	15-Oct-83	15-Oct-93	15-Oct		1	SF-e	NO
203	9.00NL1990I/II -2000/2000	9.00%	15-May-90	15-May-00	15-May		1	BU	NO
204	9.00NL1990III -2000/2000	9.00%	1-Jul-90	1-Jul-00	01-Jul		1	BU	NO
205	9.00NL1990IV -2000/2000	9.00%	16-Oct-90	16-Oct-00	16-Oct		1	BU	NO
206	9.00NL1991I/II -2001/2001	9.00%	15-Jan-91	15-Jan-01	15-Jan		1	BU	NO
207	9.25NL1979I/II -1985/1989	9.25%	15-Dec-79	15-Dec-89	15-Dec		1	SF-e	NO
208	9.25NL1990 -2000/2000	9.25%	30-Nov-90	30-Nov-00	30-Nov		1	BU	NO
209	9.50NL1976 -1977/1991	9.50%	15-Aug-76	15-Aug-91	15-Aug		1	SF-e	CA
210	9.50NL1976I/II -1977/1986	9.50%	1-Aug-76	1-Aug-86	01-Aug		1	SF-e	NO
211	9.50NL1980 -1986/1990	9.50%	1-Sep-80	1-Sep-90	01-Sep		1	SF-e	NO
212	9.50NL1980 -1986/1995	9.50%	1-Mar-80	1-Mar-95	01-Mar		1	SF-e	CA
213	9.50NL1983 -1987/1990	9.50%	15-Jul-83	15-Jul-90	15-Jul		1	SF-e	CO

Easy ID #	Name	coupon rate	Bond start	Final Redemption	coupon date 1	coupon date 2	number coupons	redemption scheme **	bond type *
214	9.50NL1986 -1990/1993	9.50%	15-Jul-86	15-Jul-93	15-Jul		1	SF-e	NO
215	9.75NL1974 -1975/1999	9.75%	1-Dec-74	1-Dec-99	01-Dec		1	SF-e	CA
216	GB2.50 1814 (PERPETUAL)	2.50%	01-01-1814	perpetual	01-Jan	01-Jul	2	PE	CA
217	GB3.00 1884 (PERPETUAL)	3.00%	03-01-1884	perpetual	01-Mar	01-Sep	2	PE	CA
218	GB3.50 1911 (PERPETUAL)	3.50%	1-Jun-11	perpetual	01-Jun	01-Dec	2	PE	CA

** bond type:*	
NO	*normal bond*
CA	*callable bond*
CO	*convertible bond*

*** redemption scheme:*	
BU	*bullet*
SF-e	*sinking fund with equal amounts*
SF-u	*sinking fund with unequal amounts*
PE	*perpetual bonds*