Density Forecasts of Inflation with Principal Component Regression, a Time-Varying Level and Stochastic Volatility

Master Thesis

Econometrics & Management Science

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Abstract

This thesis provides an answer to the following research question: *Does the use of a regression model that includes macroeconomic factors, a time-varying level and stochastic volatility lead to more accurate density forecasts for inflation compared to benchmark models?* In order to answer this research question, an empirical analysis has been performed. The benchmark models that are used are the Random Walk model and the Autoregressive model. The macroeconomic factors are estimated with principal components and used in a principal component regression. The models are estimated using a rolling window with a Metropolis-within-Gibbs MCMC algorithm. The density forecasts are assessed by the probability integral transform, the Berkowitz LR test, serial autocorrelation plots and the Kullback-Leibler information criterion (KLIC). The three main findings can be summarized as follows. First, none of the Principal Component Regression, time-varying constant and stochastic volatility models outperforms both benchmark models. Second, models that capture the volatility dynamics appear to provide accurate density forecasts. And third, the density forecasts show that the real density appears to have positive skewness in both the 12- and 24-month forecast horizon in the period 1994-2008 which indicates that the inflation series suffers from upside risk.

Keywords: *Inflation, Principal component regression, Time-varying constant, Stochastic volatility, Density forecasts*

Contents

Chapter 1

Introduction

Inflation - the general increase in the prices of goods and services - is an important measure of economic development. The effects of inflation on the economy can be positive and negative at the same time. Negative effects of inflation include a decrease in the real value of money over time, while uncertainty about future inflation may discourage investments and savings. Positive effects of inflation include a mitigation of economic recessions (Hummel (2007)) and debt relief by a reduction of the real level of debt. In addition to the positive and negative effects of inflation, inflation targeting should be mentioned. Inflation targeting is applied by central banks. They set a target inflation rate, which also is made public, and subsequently attempt to steer the actual inflation rate towards the target inflation rate through the use of interest rate changes and other monetary policy tools. All together, forecasting inflation is an important part of economic analysis.

During the past decades, the focus on forecasting inflation has intensified, generating a multitude of studies on applying new models and different forecasting types. Amongst others, models used in forecasting inflation are: the Random Walk (RW) model (Fisher *et al.* (2002), Atkeson and Ohanian (2001)), the Autoregressive (AR) model (Stock and Watson (2008), Gillitzer and Kearns (2007)), the Philips curve model (Stock and Watson (2008), Fisher *et al.* (2002), Atkeson and Ohanian (2001)), the Vector Autoregressive (VAR) model (Clark (2009), Orphanides and Wei (2010), Cogley *et al.* (2003)) and the Principal Component Regression (PCR) model (Gavin and Kliesen (2008), Stock and Watson (2002), Gillitzer and Kearns (2007)). In general roughly three possible forecast types are distinguished: point-forecasts, interval forecasts and density forecasts. Not all possible combinations of models and types of forecasting have been evaluated yet. As the level of forecast accuracy varies, there is always room for improvement. This thesis provides the assessment of a new combination of one of these models with a forecasting type, namely PCR with density forecasts.

The PCR model uses a large number of predictor variables in order to make forecasts of inflation. Gavin and Kliesen (2008) show that using the PCR model in forecasting inflation[1] is useful at both the 12- and 24-month forecast horizon. Both Stock and Watson (2002) and Gillitzer and Kearns (2007) show that adding lagged inflation in the PCR model dramatically improves the forecasts. The studies of Gavin and Kliesen (2008), Stock and Watson (2002) and Gillitzer and Kearns (2007) on principal component regression focus on *point forecasts*.

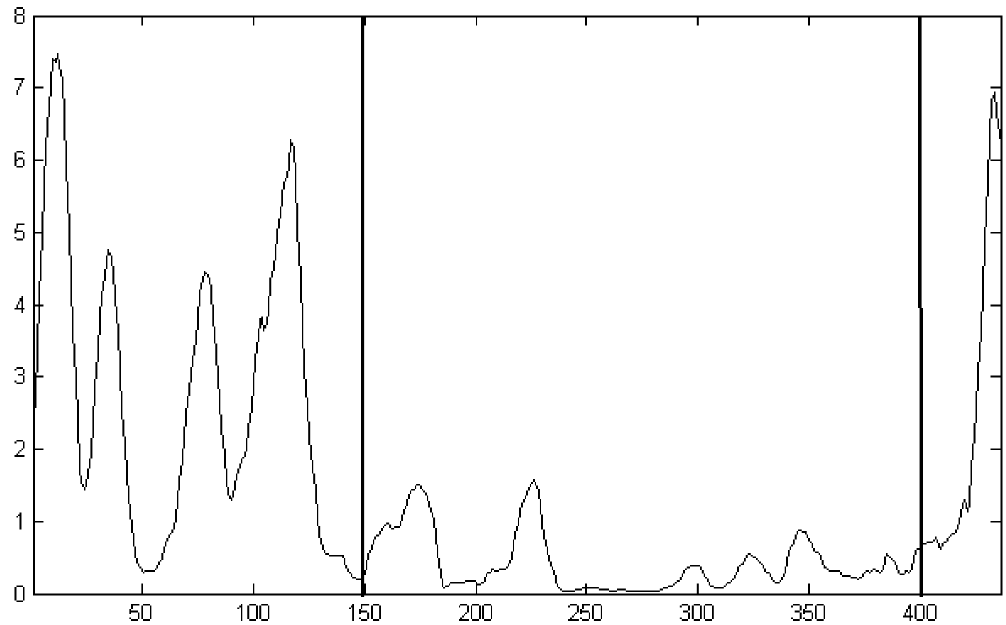

Figure 1.1: Two years sample variance of inflation.

[1]defined here with the widely used measure of inflation: Consumer Price Index (CPI), see Gavin and Kliesen (2008)

The density type of forecasting provides information on the future value *as well as information on the future volatility*. Density forecasts of inflation is used or analyzed by The Bank of England, the National Institute of Economic and Social Research (NIESR), Clark (2009) and Cogley *et al.* (2003), amongst others. The density forecasts of the Bank of England are based on the deliberations of the Monetary Policy Committee (MPC). The density forecasts of NIESR are produced by NiGEM, a large-scale macroeconometric model.

Inflation has had different magnitudes of volatility over time. Figure 1.1 shows this; inflation in the the mid-1980s and 1990s (mid part in Figure 1.1) is much less volatile than it was in the 1970s or the early 1980s (left part in Figure 1.1). On the other hand, due to the recent credit crisis, volatility of inflation increased sharply (right part in Figure 1.1). Inflation also has had different levels over time. Figure 1.2 shows these different levels of inflation.

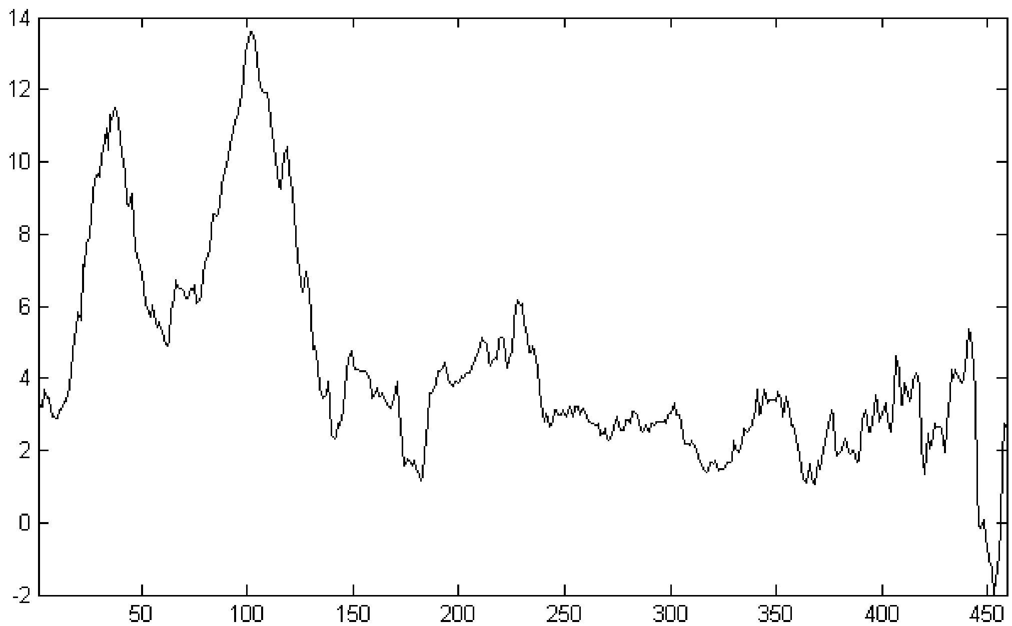

Figure 1.2: 12-month CPI-all measure of Inflation over the period 1970-2008

Shifts in volatility have the potential to result in forecast densities that are either far to wide or too narrow. While shifts in the level of inflation have the potential to result in forecast densities that are centered too high or too low. Several studies (Groen *et al.* (2009), Kohn (2007), Stock and Watson (2006), Clark (2009), Cogley and Sargent (2005) and Cogley *et al.* (2003)) have shown the importance of both time-varying coefficients and stochastic volatility in the (density) forecasting of inflation.

The combination of PCR with density forecasts has not been evaluated yet. The objective of this thesis is to combine a principal component regression, time-varying level and stochastic volatility model with density forecasts in order to forecast inflation. This leads to the following research question:

Does the use of a regression model that includes macroeconomic factors, a time-varying level and stochastic volatility lead to more accurate density forecasts for inflation compared to benchmark models?

When the proposed models indeed provide better forecasting accuracy than the benchmark models, they could be applied in practical applications of forecasting inflation. In order to answer the research question, this thesis performs an empirical analysis.

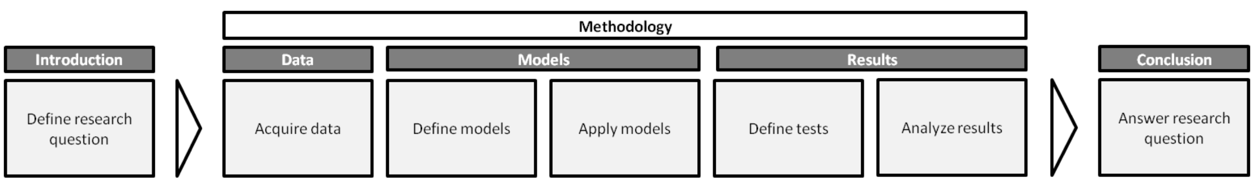

Figure 1.3: Approach of the research

The remainder of this thesis is organized following the structure as depicted in Figure 1.3. Chapter 2 describes the dataset that is used in this thesis. Chapter 3 defines the models that are used in this thesis. Chapter 4 describes the application of the models to the data. Chapter 5 defines the tests that are used to evaluate the forecasts. Chapter 6 analyzes the empirical results. Chapter 7 provides a conclusion and discussion of the research. The appendices provide technical details.

Chapter 2

Data

The dataset that is used in this thesis is an updated version of the dataset that was used by Stock and Watson (2005). This chapter describes the dataset of Stock and Watson (2005) and the processing of this dataset, following the process as depicted in Figure 2.1.

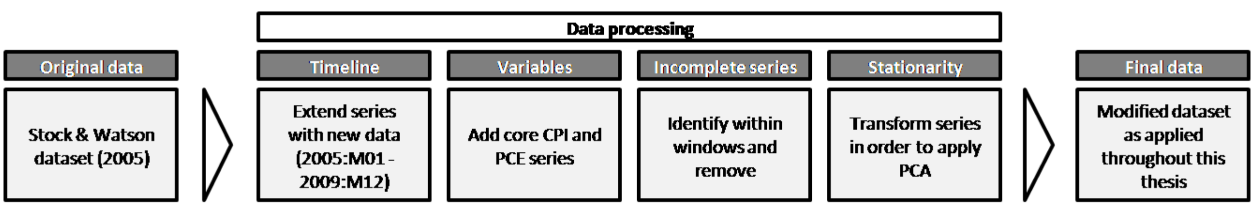

Figure 2.1: Process concerning the changes to the data

2.1 Original dataset

The original dataset has been used in the Stock and Watson (2005) article concerning the application of principal component regression in order to (among others) forecast inflation. In the article, the dataset has shown its value in this kind of analysis[1]. Hence, the Stock and Watson (2005) dataset is a fair choice for use in this thesis.

The dataset of Stock and Watson (2005) consists of monthly observations on U.S. macroeconomic variables over the period 1959:M01 through 2003:M12. The dataset is a balance of series that represent different aspects of the entire economy. The dataset includes 128 (+2)[2] different predictor variables that fall into 14 different categories, a summary of which is presented in Table 2.1. A full list of variables per category is listed in Appendix A.

Table 2.1: Categories of predictor variables

Category name	**# of series**
Real output and income	15
Employment and hours	29
Real retail	1
Consumption	1
Housing starts and sales	10
Real inventories	3
Orders	7
Stock prices	4
Exchange rates	5
Interest rates and spreads	17
Money and credit quantity aggregates	11
Prices indexes	21 (+2)
Average hourly earnings	3
Consumer expectations	1

[1]Stock and Watson (2005) screened automatically for outliers and observations exceeding 10 times the interquartile range from the median and replace them by missing values. Regarding outliers and observations exceeding 10 times the interquartile range from the median, this thesis leaves the dataset intact.

[2]The addition of these two variables in price indexes is explained in section 2.3.

2.2 Timeline

The Stock and Watson (2005) dataset covers the timeline up to 2003:M12. As new data is available, the dataset can be extended. This should be done for three reasons. First, extending the dataset provides a more up to date view of the appropriateness of the methodology. Second, the crisis that recently occurred provides additional information in the behavior of the models. Third, more data over time provides more forecasts to evaluate and therefore enhanced results. The dataset of Stock and Watson (2005) is updated up to December 2009[3]. The sources of the data are listed in Appendix B.

2.3 Variables

This thesis focuses on the evaluation of forecasts of the CPI-All measure of inflation. However, there are other measures of inflation: Core-CPI, PCE-All and Core-PCE. The latter two, core-CPI and core-PCE are not available in the dataset of Stock and Watson (2005). To use the information of these measures as well, Core CPI and Core PCE have been added to the dataset. The sources of core CPI and core PCE are listed in Appendix B. Although not applied in this thesis, it is possible to evaluate the Core-CPI, PCE-All and Core-PCE measures of inflation as well.

2.4 Incomplete series

Some series contain insufficient data in order to fill the complete timeline (e.g. in one series, the last four months of data lacks, and in another series, the last ten months of data is absent). This incompleteness is caused by two reasons. The first cause concerns data that is simply no longer measured[4]. The second cause concerns data that is not available yet (e.g. because it is not measured yet).

[3] Special thanks to Peter Exterkate for providing the updated Stock and Watson (2005) dataset.

[4] An example of a series that is no longer measured is the Dollar-Guilder exchange rate, it is no longer measured due to the introduction of the Euro.

Procedures of dealing with such incomplete series include the imputation and deletion. Imputation can be implemented through the expectation maximization algorithm. However, this thesis uses a deletion procedure. This procedure is implemented through excluding an incomplete series for further analysis as soon as it enters a window (the rolling window is explained in section 4.1). Analysis performed on previous windows is not affected by this exclusion. For clarity, the procedure is illustrated in Figure 2.2. Table 2.2 lists the excluded series with the first date of occurrence.

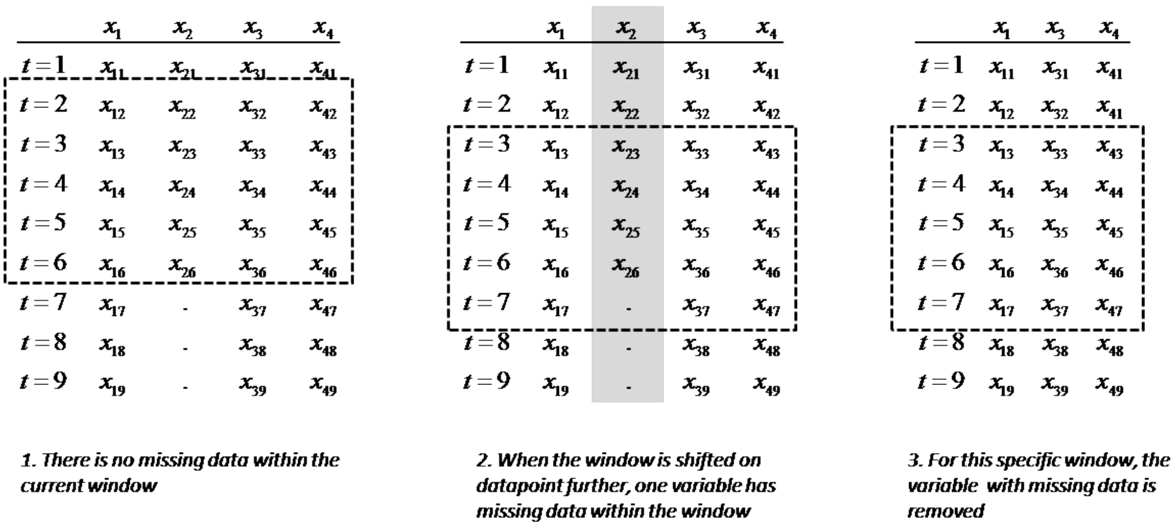

Figure 2.2: Procedure of handling incomplete series

Table 2.2: Exclusion of series

Series name	**Occurrence of first exclusion**
S&P's Common Stock Price Index: Industrials $(1941-43=10)$	2004:M01
Index Of Sensitive Materials Prices $(1990=100)$(Bci-99a)	2004:M05
Money Stock: M3(M2+Lg Time Dep,Term Rp's&Inst Only Mmmfs)(Bil$,Sa)	2006:M01

2.5 Stationarity

Stationarity is a requisite for applying principal component analysis. In order to make the series stationary, the series are transformed by taking logarithms and/or first and second differences. The kind of transformation for each variable is given in Appendix A. The transformations are the same kind of transformations as Stock and Watson (2005) use.

2.6 Final dataset

To summarize, data processing is applied to the original dataset; extending the timeline, adding new variables, dealing with incomplete series and transformation for stationarity. These four modifications lead to he final dataset. The next chapter defines the models that are used in this thesis.

Chapter 3

Models

This chapter describes the models that are used in this thesis. Section 3.1 describes the benchmark models. Section 3.2 describes the proposed models. Section 3.3 provides a description of the density forecasts.

3.1 Benchmark models

Two well known models are used as benchmark models. These models are the Random Walk model and the Autoregressive model. The Autoregressive model is also used by Gavin and Kliesen (2008), Stock and Watson (2002) and Gillitzer and Kearns (2007) as a benchmark model. The Random Walk model is only used by Gavin and Kliesen (2008) as a benchmark model. This section describes both benchmark models.

3.1.1 Random Walk model

The random walk is a mathematical formalization of a series that consists of taking consecutive random steps. The system of equations that belongs to the random walk is:

$$\pi_{t+h}^h = \pi_t^h + \sigma_{t+h}\varepsilon_{t+h} \tag{3.1}$$

$$\ln\left(\sigma_{t+1}^2\right) = \ln\left(\sigma_t^2\right) + \eta_{t+1} \tag{3.2}$$

Where π_t^h is the h month growth rate of inflation, $\varepsilon_t \sim N(0,1)$, $\eta_t \sim N\left(0, \sigma_\eta^2\right)$, and $Cov(\eta_t, \varepsilon_t) = 0$. Equation (3.2) shows a random walk process for the volatilities, known as stochastic volatility, and is explained in section 3.2.3.

3.1.2 Autoregressive model

The autoregressive model is typically applied to autocorrelated time series data. The system of equations that belongs to the autoregressive model is:

$$\pi_{t+h}^h = \alpha_0 + \beta(L)\pi_t + \sigma_{t+h}\varepsilon_{t+1} \tag{3.3}$$

$$\ln\left(\sigma_{t+1}^2\right) = \ln\left(\sigma_t^2\right) + \eta_{t+1} \tag{3.4}$$

Where π_t^h is the h month growth rate of inflation, $\beta(L)$ is a lag polynomial in nonnegative powers of L with $L < \infty$, π_t is the 1-month growth rate of inflation, $\varepsilon_t \sim N(0,1)$, $\eta_t \sim N\left(0, \sigma_\eta^2\right)$ and $Cov(\eta_t, \varepsilon_t) = 0$. Equation (3.4) shows a random walk process for the volatilities, known as stochastic volatility, and is explained in section 3.2.3.

3.2 Proposed models

The system of equations that belongs to the proposed models is:

$$\pi^h_{t+h} = \alpha_{\pi,t+h} + \beta(L)\pi_t + \gamma(L)F_t + \sigma_{t+h}\varepsilon_{t+h} \tag{3.5}$$

$$X_t = \Lambda F_t + e_t \tag{3.6}$$

$$\ln(\sigma^2_{t+1}) = \ln(\sigma^2_t) + \eta_{t+1} \tag{3.7}$$

Where π^h_t is the h month growth rate of inflation, $\beta\,(L)$ and $\gamma(L)$ are lag polynomials in nonnegative powers of L with $L < \infty$, π_t is the 1-month growth rate of inflation, $\varepsilon_t \sim N\,(0, 1)$, F_t is a $k \times 1$ vector of principal components, X_t is an $N \times 1$ vector of predictor variables, e_t is the $N \times 1$ vector of idiosyncratic disturbance, $\eta_t \sim N\left(0, \sigma^2_\eta\right)$ and $Cov\,(\eta_t, \varepsilon_t) = 0$. The following sections describe the separate parts of this system.

3.2.1 Principal component regression

This section describes equation (3.5) and (3.6) in the system of equations. By using equation (3.5) and (3.6), one assumes that π_t and X_t admit a dynamic factor representation (Stock and Watson (2002)). Principal Component Analysis (PCA) is used for estimating equation (3.6). Subsequently, Principal Component Regression (PCR) is used for estimating equation (3.5).

Forecasting a variable of interest using a large number of macroeconomic variables is not feasible in using a linear regression model due to overfitting. PCA can be applied when some of the large number of macroeconomic variables are correlated. PCA involves a mathematical procedure that transforms the possibly correlated variables into uncorrelated variables. These uncorrelated variables are called principal components. The main feature of PCA is that only a few principal components account for a large proportion of the variance of the macroeconomic variables.

The first principal component accounts for as much of the variance in the data as possible. Each following component accounts for as much of the remaining variance as possible. When applying this method, it is possible to account for a high percentage of the variance in the data, using only a fraction of the initial number of variables. With the resulting smaller set of variables, an ordinary least squares regression can be performed in order to forecast the variable of interest. The combination of PCA and ordinary least squares is called principal component regression (PCR).

The remainder of this section describes the mathematical specification of PCA, and explains how the obtained principal components can be used in a regression.

Principal component analysis

The objective of PCA is to obtain a linear combination of the original variables with maximum variance. Let $\mathbf{X}$ be the datamatrix including the k variables of (full) rank k, this requires no perfect multicollinearity among the observed variables. This matrix $\mathbf{X}$ has to be standardized. Let $\mathbf{u}$ be the linear combination that has to be found. The next step is to choose $\mathbf{u}$ in order to maximize the variance of $\mathbf{z} = \mathbf{Xu}$, which can be written as

$$Var\left(\mathbf{z}\right) = \mathbf{u}'\mathbf{R}\mathbf{u}.$$

Because $\mathbf{X}$ is standardized, $\frac{1}{(n-1)}\mathbf{X}'\mathbf{X} = \mathbf{R}$ is the sample correlation matrix and the covariance matrix.

The solution of the problem is not unique, it has in fact infinitely many solutions. To solve this problem, a restriction of unit length is imposed to $\mathbf{u}$, that is $\mathbf{u}'\mathbf{u} = 1$. The objective can now be stated as:

$$\text{Choose } \mathbf{u} \text{ to maximize } \mathbf{u}'\mathbf{R}\mathbf{u},$$

$$\text{such that } \mathbf{u}'\mathbf{u} = 1.$$

This optimization problem can be solved by the Lagrangian, it is given by

$$\mathrm{L} = \mathbf{u}'\mathbf{R}\mathbf{u} - \lambda\left(\mathbf{u}'\mathbf{u} - 1\right).$$

Taking the derivative of L with respect to the elements of $\mathbf{u}$ results in

$$\frac{\partial \mathrm{L}}{\partial \mathbf{u}} = 2\mathbf{R}\mathbf{u} - 2\lambda\mathbf{u}.$$

Setting this equation equal to zero and solving this obtains the following condition

$$\mathbf{R}\mathbf{u} = \lambda\mathbf{u}.$$

This is a simple *eigenvalue - eigenvector problem*, where the vector $\mathbf{u}$ is an eigenvector and the scalar λ is called an eigenvalue. Solving this problem leads to k eigenvectors $\mathbf{u_1}, \mathbf{u_2}, \ldots, \mathbf{u_k}$ with k corresponding eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_k$.

It is interesting to see that the variance that is accounted for by the principal components is

$$Var\left(\mathbf{z}\right) = \mathbf{u}'\mathbf{R}\mathbf{u} = \mathbf{u}'\lambda\mathbf{u} = \lambda.$$

This can be interpreted as the variance of a principal component. The selection of a number of principal components can be made on the basis of these eigenvalues. The component corresponding to the largest eigenvalue is often referred to as the *first* principal component, the component corresponding to the second largest eigenvalue is referred to as the *second* principal component, and so on. Suppose the first r principal components are selected for further usage. These first r principal components explain $\frac{\sum_{i=1}^{r}\lambda_i}{\sum_{i=1}^{k}\lambda_i}$.

Let $\mathbf{D}$ be the diagonal matrix with the eigenvalues, sorted, on the diagonal and let $\mathbf{U}$ be the matrix of corresponding, sorted, eigenvectors. The principal components consist of component scores; the expression of the influence of an eigenvector on a specific sample. The component scores can be computed as follows:

$$\mathbf{F_t} = \left(\mathbf{D}^{-\frac{1}{2}}\mathbf{U}'\right)\mathbf{X}.$$

The principal components with component scores are required for PCR.

Principal component regression

The next step is to use these principal components in a regression. This leads to the following expression which is also depicted in equation (3.5):

$$\beta(L)\pi_t + \gamma(L)\mathbf{F_t}.$$

A principal component regression model with k principal components, p lags of the principal components and q lags of the dependent variable is written as PCR(k, p, q). Generally, next to PCR(k, p, q) two restricted versions are analylized (Gavin and Kliesen (2008), Stock and Watson (2002)). The first restriction concerns $p = q = 0$; no lags of the dependent variable and no lags of the principal components. The second restriction concerns $p = 0$; no lags of the principal components.

3.2.2 Time varying level

Chapter 1 stated that different levels are present in the inflation series. In order to take this feature into account, this thesis follows Orphanides and Wei (2010) and uses exponential smoothing as a time varying level. Exponential smoothing allocates exponentially decreasing weights, as observations get older. Hence, recent observations are assigned relatively more weight in forecasting than older observations. The mathematical representation of the exponential smoothing in this thesis is given by

$$\alpha_{\pi,t+h} = \left(\sum_{i=0}^{M-1} v^i\right)^{-1}\left(\sum_{i=0}^{M-1} v^i \pi_{t-i}\right), \tag{3.8}$$

where M is the number of observations used to determine the time varying level. The parameter v controls how aggressively the weights of older observations decrease.

3.2.3 Stochastic volatility

Chapter 1 stated that shifts in volatility are present in the inflation series. Stochastic volatility allows for continuous changes in the conditional variance of the shocks. It has shown it importance in the density forecasts of inflation, Clark (2009): "Compared to models with constant variances, models with stochastic volatility have lower RMSEs, significantly more accurate interval forecasts (coverage rates), probability integral transforms (PITs) that are closer to uniformity, normalized forecast errors (computed from the PITs) that are much closer to a standard normal distribution, and average log predictive density scores that are much lower."

Following Stock and Watson (2006), stochastic volatility is modeled in equation (3.7)[1] with a random walk process. The next step in the research is specifying the type of forecasting, which is provided in the next section.

3.3 Density forecasts

Following Cogley and Sargent (2005), Cogley *et al.* (2003) and NIESR, the density to be forecasted is assumed to be normally distributed. A density forecast of this type requires a mean and a variance. To model the mean of the density, equation (3.5) is used. Please refer to sections 3.2.2 and 3.2.1 for further description of this equation. To model the volatility of the density, equations (3.2), (3.4) and (3.7) are used. Which is written again for clarity:

$$\ln(\sigma^2_{t+1}) = \ln(\sigma^2_t) + \eta_{t+1}$$

This equation allows volatility to change over time. The next chapter describes the empirical application of the models that are defined in this chapter.

[1] and (3.2), (3.4) in the Benchmark models

Chapter 4

Applying the models

This chapter describes the empirical application of the models as defined in chapter 3. Section 4.1 describes the estimation procedure. Section 4.2 describes the specification of the models. Section 4.3 describes the actual application of the models.

4.1 Estimating the models

The models are estimated using a rolling window with a Metropolis-within-Gibbs MCMC algorithm as described by Jacquier *et al.* (1994). This method was successfully used by, among others, Cogley and Sargent (2005), Cogley *et al.* (2003), Brandt and Jones (2005) and Jacquier *et al.* (2004). This section describes the estimation procedure: The rolling window, and the specification of the Metropolis-within-Gibbs MCMC algorithm.

4.1.1 Rolling window

The forecasts in this thesis are out-of-sample. For the out-of-sample forecasting of inflation and in order to use the same amount of information for each forecast, a rolling window forecasting methodology is employed. This section explains the rolling window methodology.

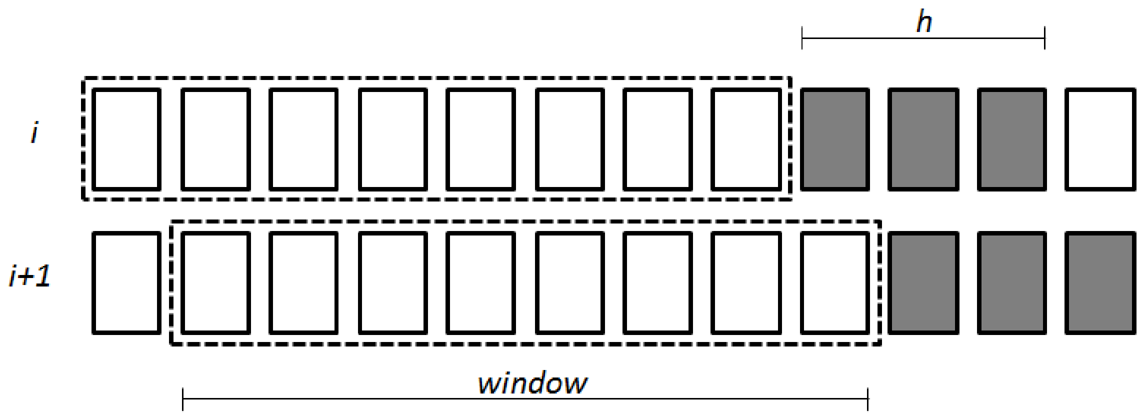

Figure 4.1: Illustration of the i-th and (i+1)-th eight period rolling window with a forecast horizon of $h = 3$.

The first window contains the first T observations. Estimate the model for the first window and obtain the h-period ahead forecast, shift the window *one* period forward and follow the procedure again. The rolling window procedure is illustrated in Figure 4.1 for the i-th and (i+1)-th windows with an eight period rolling window and a forecast horizon of $h = 3$.

4.1.2 Metropolis-within-Gibbs MCMC sampler

The stochastic volatility model contains unobserved variables, σ_t^2. Bayesian estimation methods can easily deal with unobserved variables. Jacquier *et al.* (1994) describe a Bayesian estimation method for models with stochastic volatility. The specific estimation method that they describe and that is (slightly adapted) used in this thesis is called the Metropolis-within-Gibbs MCMC sampler. This estimation method concerns M-H samplers for the σ_t^2 variables and Gibbs samplers for the remaining variables.

Prior specification

Following Cogley and Sargent (2005), Cogley *et al.* (2003), natural conjugate priors are chosen for β and σ_η^2, making the priors proper. The priors are assumed to be independent across the different blocks, resulting in the following prior specification: $p\left(\beta, \sigma_\eta^2, \sigma_0^2\right) = p\left(\beta\right) p\left(\sigma_\eta^2\right) p\left(\sigma_0^2\right)$,

$p\left(\beta\right) \sim N\left(b, B\right),$

$p\left(\sigma_\eta^2\right) \sim IG\left(\delta_0/2, \nu_0/2\right),$

The algorithm

The algorithm for obtaining posterior results is illustrated by a flowchart in Figure 4.2. Details on the different steps are given in Appendix C.

The posterior results are used to compute the posterior mean of π_{t+h}^h and σ_{t+h}^2, which together result in the density forecast $N\left(\hat{\pi}_{t+h}^h, \hat{\sigma}_{t+h}^2\right)$. In order to use the model that is described in this section in an actual application, there should be some choices made. The choices that are made for this research are listed and motivated in the next section.

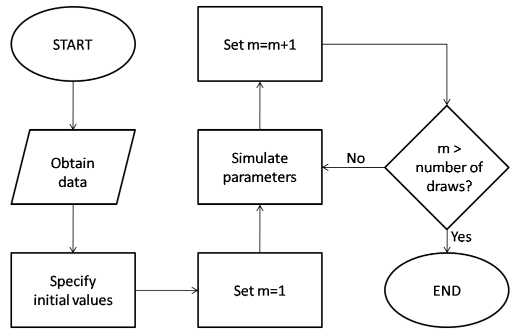

Figure 4.2: Flowchart of the Metropolis-within-Gibbs MCMC algorithm

4.2 Specifying the models

This section lists and motivates the choices that are made for the actual application of the model. These choices concern consecutively the priors, the initial values, the number of lags and number of components, the forecast horizon and growth rates, the rolling window, the time-varying level, the variance of the Random Walk sampler and the number of draws. Refer to Appendix C.1 for an introduction to the notation that is used in this section.

4.2.1 Priors

At this point, the values of the priors as defined in section 4.1.2 need to be filled in. The priors of this thesis follow Cogley and Sargent (2005) and Cogley *et al.* (2003).

The prior for β is chosen $p(\beta) \sim N\left(\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}'\pi^*, 1000 \times \mathbf{I}\right)$, the variance of this prior is very large, making it a diffuse prior.

The prior for σ_η^2 is chosen $p\left(\sigma_\eta^2\right) \sim IG\left(\frac{(0.01)^2}{2}, \frac{1}{2}\right)$, the scale parameter δ_0 of this prior is very small, making it a diffuse prior.

4.2.2 Initial values

The initial values of σ_t^2 in the first window are $\log\left(\pi_t^* - X_t\beta_0\right)^2, t \in \{1, \ldots, T\}$, where $\beta_0 = \left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}'\pi^*$. The initial value for σ_0^2 is the initial value of σ_1^2. That is, $\sigma_0^2 = \log\left(\pi_1^* - \mathbf{X}_1 * \beta_0\right)^2$. In the second and subsequent windows, the posterior means of the previous window are used as initial values. That is, the posterior mean of σ_1^2 of the previous window is used as initial value for σ_0^2 in the current window, the posterior mean of σ_2^2 of the previous window is used as initial value for σ_1^2 in the current window, etc. Since there is no posterior mean for σ_{T+1}^2 in the previous window, the value of σ_T^2 in the previous window is used as initial value for σ_T^2 (as well as initial value for σ_{T-1}^2).

4.2.3 Number of lags and number of components

AR

Figure D.1 in Appendix D shows that for both the 12- and the 24-month growth rate of inflation, the partial autocorrelations are significantly different from zero for up to one and two lags. The partial autocorrelations are not significantly different from zero for up to three lags or more. With this in mind, an AR(2) model is estimated as benchmark model.

PCR

Stock and Watson (2002) show the value of using the complete dataset in principal component regression. Following Stock and Watson (2002), the complete dataset of the macroeconomic variables is used to compute the principal components. The number of components is chosen by both Gillitzer and Kearns (2007) and Stock and Watson (2002) through BIC. To save computation time, the number of components in this thesis is fixed. The numbers of components analyzed are one, five and ten. The variance explained with one principal components is on average 19%. The average variance explained for the model with five principal components is 46%. The average variance explained for the model with 10 principal components is 61%. Consequently, by adding more principal components, approximately 25% more variance of the macroeconomic variables is taken into account.

Stock and Watson (2002) analyze zero, one and two lags of the principal components. Following them, this thesis analyzes these numbers of lags of the principal components in the PCR models.

With the result of Figure D.1 in Appendix D in mind, as discussed in the previous section, zero and two lags of the dependent variable are included as predictor variable.

4.2.4 Forecast horizon and growth rates

As Stock and Watson (2008) state in their paper, the forecasting of inflation tends to focus on *one*- and *two*-year forecast horizons. Following them, this thesis obtains forecasts with a 12- and 24-month forecast horizon. The $h-$month forecasts of the inflation variables concern growth rates over h months. The h-month annualized growth rate is computed by $\pi_t = \frac{1200}{h}\Delta_h \ln CPI_t$. These growth rates are computed in order to forecast over the whole $(T, T+h)$ period instead of solely the endmost $(T+h-1, T+h)$ period.

4.2.5 Rolling window

Following Stock and Watson (2008), each window contains ten years of data, i.e. 120 observations. The forecasts are made for the period of 1970:M06 through 2008:M08 respectively 1971:M06 through 2007:M08 resulting in a sequence of 459 respectively 435 density forecasts for a $12-$, respectively $24-$month forecast horizon.

4.2.6 Time-varying level

The time varying-constant is computed, in line with the length of the window, over 120 observations. That is, $M = 120$ in Equation (3.8). For the first 120 observations, the available observations up to that point are used to compute the time-varying level. For example; if the first 15 observations are available, $M = 15$.

Orphanides and Wei (2010) try several values for v and find that different values for v obtain similar results. With this in mind, this thesis uses the same value for v as Orphanides and Wei (2010), namely $v = 0.98$.

4.2.7 Variance of the Random Walk sampler

The value is chosen such that the algorithm mixes well while ensuring convergence. Several values for ω have been tried: $\omega = 0.75$ has shown that it works best.

4.2.8 Number of draws

For the first window, 20.000 simulations of the sampler are drawn. The first 15.000 simulations are discarded to allow for convergence in the chain, resulting in a sample of 5.000 simulations from the posterior density. For the second and subsequent windows, 15.000 simulations of the sampler are draw. The first 10.000 simulations are discarded to allow for convergence in the chain, again resulting in a sample of 5.000 simulations from the posterior density. The 15.000 simulations are sufficient due to the initial values of the second and subsequent windows[1]. Panel (a)-(d) of Figure D.2 in Appendix D show that the chains converge. The next section describes the application of the models.

4.3 Applying the models

All the models that are described in chapter 3 have been estimated with matlab. The actual script can be obtained by the writer of this thesis. The estimation in matlab has been carried out by using thirty computers. These computers have been running simultaneously for five days of 9 hours. The result of this estimation has led to an outcome of 28 series containing density forecasts of inflation. The next chapter defines the tests that can be used to analyze the density forecasts.

[1]See section 4.2.2

Chapter 5

Defining the tests

For the assessment of a density forecast, two types of comparisons are considered. The first type is the comparison of the forecasted density, $p_t(\hat{\pi}_t)$, with the true density $f_t(\pi_t)$ of the data generating process. The second type is the comparison of two competing density forecasts $p_{1t}(\hat{\pi}_t)$ and $p_{2t}(\hat{\pi}_t)$.

This chapter is structured as follows. Section 5.1 discusses the probability integral transform, the Berkowitz LR test and the assessment of the serial autocorrelation plots. These tests deal with the first type of comparison. Section 5.2 discusses the Kullback-Leibler information criterion, which deals with the second type of comparison.

5.1 Comparing a density forecast with the true density

Density forecasts can be assessed by comparing the forecasted density with the true density. These two densities are related through the probability integral transform, which is defined as z_t. This section shows that, if the density forecast is correct, the sequence of probability integral transforms is i.i.d. $U(0,1)$ for a one-period forecast horizon. Knowing this, one can compare the forecasted density and the true density by assessing the sequence of probability integral transforms $\{z_t\}$.

5.1.1 Probability Integral Transform

The probability integral transform (Diebold *et al.* (1998), Clements (2004) and Berkowitz (2001)) is the cumulative density function corresponding to the density $p_t(\hat{\pi}_t)$ evaluated at π_t,

$$z_t = \int_{-\infty}^{\pi_t} p_t(u)\,du = P_t(\pi_t).$$

Assume $\frac{\partial P_t^{-1}(z_t)}{\partial z_t}$ continuous and nonzero. The density q_t of z_t[1] is given by

$$q_t(z_t) = \left|\frac{\partial P_t^{-1}(z_t)}{\partial z_t}\right| f_t\left(P_t^{-1}(z_t)\right) = \frac{f_t\left(P_t^{-1}(z_t)\right)}{p_t\left(P_t^{-1}(z_t)\right)}.$$

When the forecasted density is equal to the true density, $q_t(z_t) \sim U(0,1)$ holds. Thus, the forecast densities can be tested by assessing whether $\{z_t\} \sim$ i.i.d. $U(0,1)$. This involves a joint hypothesis of independence and uniformity. Several tests are available to evaluate this joint hypothesis. The uniformity can be checked informally by assessing the PIT histograms. Uniformity can be checked formally with the Berkowitz LR test.

5.1.2 Berkowitz LR

Berkowitz (2001) proposed to take the inverse normal cumulative distribution function transformation of the probability integral transformations z_t. This results in a series $\Phi^{-1}(z_t) = z_t^*$. This obtains the following null hypothesis:

$$H_0 : \{z_t^*\} \sim \text{i.i.d.} N(0,1).$$

Testing for normality is convenient; tests for normality are widely seen as more powerful than tests for uniformity (Mitchell and Hall (2005)). Berkowitz (2001) proposes a three-degree of freedom test of zero-mean, unit variance and independence. The assumption is normality, therefore, a standard likelihood ratio test statistic can be constructed:

$$LR = -2\left[L(0,1,0) - L\left(\hat{\mu}, \hat{\sigma}^2, \hat{\rho}\right)\right],$$

[1]Using the change of variable formula, $p_t(\pi_t) = \frac{\partial P_t(\pi_t)}{\partial \pi_t}$ and $\pi_t = P_t^{-1}(z_t)$.

where $L\left(\hat{\mu},\hat{\sigma}^2,\hat{\rho}\right)$ is the value of the log-likelihood of a Gaussian AR(1) model:

$$\begin{aligned} L\left(\hat{\mu},\hat{\sigma}^2,\hat{\rho}\right) &= -\frac{1}{2}\log(2\pi)-\frac{1}{2}\log\left[\sigma^2/\left(1-\rho^2\right)\right] \\ &\quad -\frac{\left(z_1-\mu/\left(1-\rho\right)\right)^2}{2\sigma^2/\left(1-\rho^2\right)}-\frac{T-1}{2}\log\left(2\pi\right) \\ &\quad -\frac{T-1}{2}\log\left(\sigma^2\right)-\sum_{t=2}^{T}\left(\frac{\left(z_t-\mu-\rho z_{t-1}\right)^2}{2\sigma^2}\right). \end{aligned}$$

Under the null hypothesis, $LR \sim \chi^2(3)$. The null hypothesis is rejected if the test statistic calculated from the data is greater than the critical value of the $\chi^2(3)$ distribution for some desired probability.

Berkowitz (2001) proposed this test only for a 1-period-ahead forecast horizon; when the forecast horizon is larger, there is serial dependence expected in the sequence of probability integral transforms $\{z_t\}$, and thus in $\{z_t^*\}$. The test can be be generalized to a two-degree of freedom test of zero-mean and unit variance. This generalized berkowitz LR test allows for serial dependence in the sequence of probability integral transforms. This obtains the following null hypothesis:

$$H_0 : \{z_t^*\} \sim N\left(0,1\right).$$

The assumption is normality and therefore, de likelihood ratio test statistic can be constructed as

$$LR = -2\left[L\left(0,1\right)-L\left(\hat{\mu},\hat{\sigma}^2\right)\right],$$

where $L\left(\hat{\mu},\hat{\sigma}^2\right)$ is the value of the log-likelihood:

$$L\left(\hat{\mu},\hat{\sigma}^2\right) = -\frac{T}{2}\log\left(2\pi\right)-\frac{T}{2}\log\left(\sigma^2\right)-\sum_{t=1}^{T}\left(\frac{\left(z_t-\mu\right)^2}{2\sigma^2}\right),$$

where $\hat{\sigma}^2 = \zeta_0 + 2\sum_{j=1}^{h-1}\zeta_j$ for $\zeta_j = E\left[z_t^* z_{t-j}^*\right]$; the Newey-West estimator for $\hat{\sigma}^2$, following Mitchell and Hall (2005) and Giacomini and White (2006). Under the null hypothesis, $LR \sim \chi^2(2)$. The null hypothesis is rejected if the test statistic calculated from the data is greater than the critical value of the $\chi^2(2)$ distribution for some desired probability. Independence can be checked informally by assessing the serial autocorrelation plots.

5.1.3 Assessment of the serial autocorrelation plots

Independence can be checked through assessing the serial autocorrelation plots of the series $\{z_t - \overline{z}\}$, $\{z_t - \overline{z}\}^2$, $\{z_t - \overline{z}\}^3$ and $\{z_t - \overline{z}\}^4$ (Clements (2004) and Diebold *et al.* (1998)). The serial autocorrelations of accurate models fit in between the confidence intervals. The serial autocorrelations of inaccurate models do not fit in between the confidence intervals. Therefore, the serial autocorrelation plots provide information about the deficiencies of density forecasts by contributing in the detection of dependence patterns. There is by construction autocorrelation in the first h lags. There are two ways to take this feature into account. First, divide the forecasted series in h separate series and assess the series. Second, do not take the first h lags into account when assessing the autocorrelation plots. As a result of the relatively large values for h that are used in this thesis, the second way of dealing with the feature is used in this thesis. The following list describes how the assessment of the different serial autocorrelation plots have been used in this research.

- The mean is as measure of accuracy of a probability distribution. Significant serial autocorrelations in $\{z_t - \overline{z}\}$ indicates that the model does not accurately forecasts the mean of inflation.
- The variance is a measure of volatility of a probability distribution. Significant serial autocorrelations in $\{z_t - \overline{z}\}^2$ indicates that the model does not adequately forecasts the volatility of inflation.
- The skewness is a measure of asymmetry of a probability distribution. Because the forecasted distribution is symmetric, significant autocorrelations in $\{z_t - \overline{z}\}^3$ indicates that the real density is possibly not symmetric. Serial correlation in $\{z_t - \overline{z}\}^3$ can be caused by either up- or downside-risk or alternately both up- and downside-risk. This indicates that other densities that capture the difference between upside and downside risk should be considered.
- The kurtosis is a measure of peakedness of a probability distribution. High kurtosis signifies more of the variance is the result of infrequent extreme deviations. Significant serial autocorrelations in $\{z_t - \overline{z}\}^4$ indicates that the density does not capture infrequent extreme deviations. This indicates that other densities that capture the fatter tails should be considered.

Since the conditional mean and conditional skewness are related, significant autocorrelations in the conditional mean cause significant autocorrelations in the conditional skewness. Therefore, two cases should be noted. The first case concerns the event that the pattern in the serial autocorrelation plots of $\{z_t - \overline{z}\}^3$ coincides with the pattern in the serial autocorrelation plots of $\{z_t - \overline{z}\}$. The second case concerns the event that the pattern in the serial autocorrelation plots of $\{z_t - \overline{z}\}^3$ does not coincide with the pattern in the serial autocorrelation plots of $\{z_t - \overline{z}\}$. In the first case, the conditional skewness dynamics are captured by the density forecasts. In the second case, the conditional skewness dynamics are not captured by the density forecasts. The same notes are applicable for the patterns in the serial autocorrelation plots of $\{z_t - \overline{z}\}^2$ and $\{z_t - \overline{z}\}^4$.

The PITs, Berkowitz LR test and serial autocorrelation plots allow comparing the forecasted density to the true density. However, one might also be interested in comparing two competing densities. The Kullback-Leibler information criterion has been used in order to compare competing densities. The Kullback-Leibler information criterion is explained in the next section.

5.2 Comparing two competing density forecasts

The Kullback-Leibler information criterion (KLIC) is a well respected measure of 'distance' between two densities. Mitchell and Hall (2005) proposed to use the KLIC to test for equal predictive performance for two density forecasts. This section describes the method as Mitchell and Hall (2005) proposed, adapted to this thesis.

5.2.1 The Kullback-Leibler information criterion

Suppose there are two competing density forecasts $p_{1t}(\pi_t)$ and $p_{2t}(\pi_t)$. The test constructed is based on the sequence $\{d_t\}$, where d_t is defined as:

$$\begin{aligned} d_t &= [\ln f_t(\pi_t) - \ln p_{1t}(\pi_t)] - [\ln f_t(\pi_t) - \ln p_{2t}(\pi_t)], \\ &= \ln p_{2t}(\pi_t) - \ln p_{1t}(\pi_t). \end{aligned}$$

The null hypothesis of equal accuracy is defined as:

$$H_0 : E\left[d_t\right] = 0.$$

This hypothesis can be evaluated using the sample mean $\overline{d} = \frac{1}{T}\sum_{t=1} T\left[\ln p_{2t}\left(\pi_t\right) - \ln p_{1t}\left(\pi_t\right)\right].$ The test can be constructed based on the fact that $\overline{d}$ has, by the central limit theorem, the following limiting distribution:

$$\sqrt{T}\left(\overline{d} - E\left[d_t\right]\right) \overset{d}{\sim} N\left(0, \Omega\right),$$

where Ω is the covariance matrix given in Mitchell and Hall (2005), allowing for parameter uncertainty. Because parameter uncertainty is asymptotically irrelevant, this reduces to a DM-type test in the absence of parameter uncertainty (Mitchell and Hall (2005)). Under the null hypothesis, the test statistic is standard normally distributed. That is:

$$\overline{d}/\sqrt{S_d/T} \overset{d}{\sim} N\left(0, 1\right),$$

where $S_d = \zeta_0 + 2\sum_{j=1}^{h-1}\zeta_j$ for $\zeta_j = E\left[d_t d_{t-j}\right]$; the Newey-West estimator for S_d (Mitchell and Hall (2005), Giacomini and White (2006)). The null hypothesis is rejected if the test statistic calculated from the data is greater than the critical value of the $N\left(0, 1\right)$ distribution for some desired probability. The next chapter describes the results of the testing.

Chapter 6

Analyzing the results

As stated in chapter 1, inflation has had different sizes of volatility over time. It is therefore interesting to examine the density forecasts of inflation within different periods of time. Following Gavin and Kliesen (2008), this thesis uses 1983:M01 as the date of the structural break in many macroeconomic variables including inflation. The length of the rolling window is ten years, and in order to take solely information after the structural break into account, the forecasting has been started at 1994:M01 respectively 1995:M01 for the 12- respectively 24-month forecast horizon. Consequently, the periods 1994:M01-2007:M08 respectively 1995:M01-2007:M08 for the 12- respectively 24-month forecast horizon have been assessed in addition to the assessment of density forecasts over the whole forecast period.

This chapter is structured as follows. The principal components are analyzed in section 6.1. The estimated mean and variance are analyzed in section 6.2. Results for the 12-month forecast horizon over the period 1970:M06 through 2008:M08 are discussed in section 6.3. Results for the 24-month forecast horizon over the period 1971:M06 through 2007:M08 are discussed in section 6.4. Results for the 12-month forecast horizon over the period 1994:M01-2007:M08 are discussed in section 6.5. Results for the 24-month forecast horizon over the period 1995:M01-2007:M08 are discussed in section 6.6. The chapter concludes with relating the results to findings in the literature in section 6.7.

6.1 Analyzing the principal components

Figure E.1 respectively E.2 show the cross correlation plots of the principal components with the 12- respectively 24-month growth rate of inflation over the whole sample period.

Two results are important based on the cross correlation plots. The first result is that the cross correlations are generally quite small. The second result concerns the unexpected behavior of the cross correlation plots. The choices in section 4.2.3 were made based on two assumptions. The first assumption concerns a higher number of the component results in decreasing cross-correlation with inflation. The second assumption concerns an increasing number of lags of a component results in decreasing cross-correlation with inflation. However, the cross correlation plots show that the cross correlations of the components and lags of components with inflation behave unlike expected and on top of that, the cross correlations are generally quite small. Consequently, in some models variables are included that do not have significant correlations with inflation, which is harmful. It possibly causes even worse forecasts of inflation than not including these variables.

As a result, neither the components nor the lags of components should be chosen by increasing number. It should for example be possible to select the first third and tenth component together with only the third lag of the first component. This is a totally different approach than Gavin and Kliesen (2008), Stock and Watson (2002), Gillitzer and Kearns (2007) carry out. However, it is something that certainly should be investigated.

6.2 Analyzing the estimated mean and variance

This section analyzes the out-of-sample estimated mean and variance for both the 12- and 24-month forecast horizon. The estimated mean and variance for the models are depicted in Figure E.3 - E.6 in Appendix E.

The Random Walk model captures shifts in inflation for both the 12- and 24-month forecast horizon. The AR(2) model does not manages to capture the shifts in inflation. There are no shifts in inflation in the period 1994-2008[1], it is likely that the AR(2) model outperforms the RW model in this period.

[1] 1994 is approximately point 270 in the graphs

The models with the same amount of principal components show the same patterns in the graphs. While the models with different amounts of principal components show different patterns in the graphs. Therefore, the dynamics of the figures are likely determined by the number of principal components and not by the added lags of the dependent variable or number of lags of the principal components. Since in the period 1994-2008 the models show less variation in the mean compared to the real mean than in the period 1971-1994, the models all seem to provide better forecasts of the mean of inflation in the 1994-2008 period.

A striking feature of the estimated variance plots for the 12-month forecast horizon is the fact that the models all seem to have the same level of variance after the mid 1980s[2]. On the other hand, the models with 5 and 10 principal components show in general a lower level of variance in the period up to the mid 1980s while the level of the other models is in general a lot higher in that period. Note that adding more components (and thus more regressors) results per definition in lower variances. Another striking feature is the overestimation of the volatility-peak in the late 1980s[3] in the 24-month forecast horizon by almost every model.

6.3 Analyzing the 12-month forecast horizon

This section analyzes the 12-months-ahead density forecasts over the period 1970:M06 through 2008:M08. The probability integral transform (PIT) provides a general indicator of the accuracy of a density forecast, it is therefore a good point to start in the assessment of the models.

6.3.1 PIT and Berkowitz LR

Uniformity of the PITs is checked by assessing the PIT histograms. Figure E.7 in Appendix E shows the histograms of the analyzed models for the 12-month forecast horizon.

The histograms show clustering of mass on both sides of the distributions of the PITs. This clustering of mass

[2]The mid 1980s are approximately point 150 in the graphs
[3]The late 1980s are approximately point 200 in the graphs

either reflects estimated forecast distributions that are too narrow or the estimated mean is occasionally too high and occasionally too low. Both reflections indicate that there are relatively too much forecasts with a probability near zero or one. The stochastic volatility allows for much freedom in the models. A possible explanation is that this freedom leads to under-estimating the volatility. However, conclusions on this subject can only be made if the models are compared to the same models but without stochastic volatility. This is however beyond the scope of this thesis.

Based on the histograms, it is clear that neither of the models provide an accurate density forecast. This is also indicated by the Berkowitz LR tests that are depicted in Table E.1 in Appendix E; the null hypothesis of uniformly distributed $\{z_t\}$ can be rejected at a 5% significance interval for every model. The independence of the PITs is assessed by examining the autocorrelation plots of $\{z_t - \overline{z}\}, \{z_t - \overline{z}\}^2, \{z_t - \overline{z}\}^3$ and $\{z_t - \overline{z}\}^4$ in the next section.

6.3.2 Assessing the serial autocorrelation plots

The autocorrelation plots of the AR(2) model show high positive autocorrelations in the conditional mean (and skewness) dynamics. This indicates that there is too much persistence in the estimate of the mean of the density forecasts. The autocorrelation plots show negative autocorrelations in the conditional variance (and kurtosis) dynamics. This indicates that the AR(2) model responds in the opposite direction to shifts in the volatility.

The autocorrelation plots of the models with five and ten principal components show high correlations in both the conditional mean (and skewness) and conditional variance (and kurtosis) dynamics. This indicates that there is too much persistence in the estimates of the mean and variance of the density forecasts for these models.

The PCR models with one principal component show somewhat smaller autocorrelations than the models with five and ten components, there is however still too much persistence in the estimates of the mean and variance of the density forecasts for these models.

The Random Walk is the only model that captures the conditional variance (and kurtosis) dynamics. The

Random Walk also performs relatively good on the conditional mean (and skewness) dynamics compared to the other models.

6.3.3 KLIC

The KLIC test statistics are listed in Table E.2 in Appendix E. The associated probabilities are listed in Table E.4. The ranking based on the KLIC is depicted in Table 6.1.

Surprisingly, the models with one principal component perform relatively poorly based on the KLIC while the serial autocorrelation plots show that the one principal component models capture the conditional mean (and skewness) dynamics better than the other PCR models. However, the conditional variance (and kurtosis) dynamics are not captured at all by the one principal component models. This provides a possible explanation for the one principal component models performing relatively poorly based on the KLIC.

Table E.4 shows that adding lags of the principal components or lags of the dependent variable to a PCR(x,0,0), $x \in \{1, 5, 10\}$ model does not result in significantly different density forecasts. Table E.4 also shows that adding more components to a PCR(1,x,y), $x \in \{0, 1, 2\}, y \in \{0, 2\}$ model does not result in significantly different density forecasts. The only significant difference at the 10% level in the density forecasts of the PCR models is between PCR(1,0,0) and PCR(5,0,2). The Random Walk model density forecasts are however significantly different from some of the PCR models at the 10% level.

Table 6.1: KLIC ranking of the 12-months-ahead density forecasts.

1.	RW	8.	PCR(10,1,2)
2.	PCR(10,0,0)	9.	PCR(5,2,2)
3.	PCR(5,0,2)	10.	PCR(10,2,2)
4.	PCR(10,0,2)	11.	PCR(1,2,2)
5.	PCR(5,1,2)	12.	PCR(1,0,2)
6.	PCR(1,1,2)	13.	AR(2)
7.	PCR(5,0,0)	14.	PCR(1,0,0)

6.4 Analyzing the 24-month forecast horizon

This section analyzes the 24-months-ahead density forecasts over the period 1971:M06 through 2007:M08.

6.4.1 PIT and Berkowitz LR

Uniformity of the PITs is checked by assessing the PIT histograms. Figure E.22 in Appendix E shows the histograms of all the analyzed models for the 24-month forecast horizon.

As in the 12-month forecast horizon, there is clustering of mass on the sides of the distributions. The same explanation as in the 12-month forecast horizon is applicable and will therefore not be repeated here.

Based on the histograms, it is clear that neither of the models provide an accurate density forecast. This is again also indicated by the Berkowitz LR tests that are depicted in Table E.5 in Appendix E; the null hypothesis of uniformly distributed $\{z_t\}$ has been rejected at a 5% significance interval for every model. The independence of the PITs is assessed by examining the autocorrelation plots of $\{z_t - \overline{z}\}, \{z_t - \overline{z}\}^2, \{z_t - \overline{z}\}^3$ and $\{z_t - \overline{z}\}^4$ in the next section.

6.4.2 Assessing the serial autocorrelation plots

The Random Walk model shows negative autocorrelations in the conditional mean (and skewness) dynamics of the density forecasts. This negative autocorrelation is probably caused by the delay in the density forecasts that occurs by construction.

The AR(2) models shows positive autocorrelations in the conditional mean (and skewness) dynamics, again indicating that there is too much persistence in the forecasts of the mean. On the other hand, the autocorrelation plots of the AR(2) model shows that the AR(2) model captures the conditional variance (and kurtosis) dynamics quite well.

The autocorrelation plots of the models with five and ten principal components show high correlations in the conditional mean (and skewness) dynamics. The models with five components capture the conditional variance (and kurtosis) dynamics while the models with ten principal components do not capture the conditional variance (and kurtosis) dynamics.

The models with one principal component show in the autocorrelation plots that they capture almost all the dynamics. There is however slightly positive autocorrelations in the early lags.

6.4.3 KLIC

The KLIC test statistics are listed in Table E.6 in Appendix E. The associated probabilities are listed in Table E.8. The ranking based on the KLIC is depicted in Table 6.2.

The models with one principal component and lags of inflation perform relatively good. However, the Random Walk model is still not outperformed.

Table E.8 shows that adding lags of the principal components or lags of the dependent variable to a PCR(x,0,0), $x \in \{5, 10\}$ model does not result in significantly different density forecast. However, adding lags of the principal component and lags of the dependent variable for the PCR(1,0,0) model, does result in significantly different density forecasts.

The only significant difference at the 10% level in the density forecasts of the PCR models is between PCR(1,x,y) and other PCR models. The PCR(5,x,y) and PCR(10,x,y) models do not provide significant different density forecast when compared to each other. In addition, the Random Walk model density forecasts are again significantly different from some of the PCR models at the 10% level.

Other than in the 12-month forecast horizon, the serial autocorrelation plots show that the one principal component models capture both the conditional mean (and skewness) and the conditional variance (and kurtosis) dynamics. It appears that capturing the variance (and kurtosis) dynamics leads to a high ranking in the KLIC.

Surprisingly, the AR(2) model outperforms models that include lagged inflation. However, the AR(2) model does not contain a time-varying level. The time-varying level perhaps does not contribute to more accurate density forecasts.

Table 6.2: KLIC ranking of the 24-months-ahead density forecasts.

1.	RW	8.	AR(2)
2.	PCR(1,2,2)	9.	PCR(5,0,2)
3.	PCR(1,0,2)	10.	PCR(1,0,0)
4.	PCR(1,1,2)	11.	PCR(10,1,2)
5.	PCR(10,2,2)	12.	PCR(10,0,0)
6.	PCR(5,0,0)	13.	PCR(5,2,2)
7.	PCR(10,0,2)	14.	PCR(5,1,2)

6.5 Analyzing the 12-month forecast horizon over the period 1994:M01-2008:M08

This section analyzes the 12-months-ahead density forecasts over the period 1994:M01 through 2008:M08.

6.5.1 PIT and Berkowitz LR

Uniformity of the PITs is checked by assessing the PIT histograms. Figure E.37 in Appendix E shows the histograms of all the analyzed models for the 12-month forecast horizon over the period 1994:M01-2007:M08.

Other than the PITS of the whole sample period, not all models show clustering of mass on both sides of the distributions of the PITs. Some models show only clustering of mass on the left side of the PITs. This indicates that the density forecast still reflect estimated forecast distributions that are too narrow or the estimated mean is sometimes too high. The latter reflection is strengthened by the fact that inflation shows a downward trend in the late 1980s and early 1990s. Concluding, there is definite improvement compared to the PITs of the whole forecast period.

Based on the histograms, it is clear that still neither of the models provide an accurate density forecast. This is also indicated by the Berkowitz LR tests that are depicted in Table E.9 in Appendix E; the null hypothesis of uniformly distributed $\{z_t\}$ has been rejected at a 5% significance interval for every model. The independence of the PITs is assessed by examining the autocorrelation plots of $\{z_t - \overline{z}\}, \{z_t - \overline{z}\}^2, \{z_t - \overline{z}\}^3$ and $\{z_t - \overline{z}\}^4$ in the next section.

6.5.2 Assessing the serial autocorrelation plots

The Random Walk models shows small negative autocorrelations in the conditional mean (and skewness) dynamics. These autocorrelations are again probably caused by the delay in the density forecasts that occurs by construction. The autocorrelations for the conditional variance (and kurtosis) dynamics show small significant positive and negative autocorrelations.

The AR(2) model captures the conditional mean (and skewness) dynamics. It shows slightly negative and positive autocorrelations in the conditional variance while there is no significant autocorrelation in the kurtosis dynamics.

The PCR model with one principal component capture and no lags captures all the conditional dynamics at once. The models with lags of the factors and/or lags of the dependent variable show slightly negative and positive autocorrelations in the conditional variance (and kurtosis).

The conditional mean dynamics are captured by all the models with five and ten principal components. The conditional skewness dynamics however are sometimes positively autocorrelated. This indicates that the real density of inflation might suffer from upside risk. Higher values of inflation are in general less desirable than lower values of inflation. Therefore, if there is indeed skewness in the real density of inflation, upside risk is expected. This reinforces that the direction of the conditional skewness that is found makes sense.

6.5.3 KLIC

The KLIC test statistics are listed in Table E.10 in Appendix E. The associated probabilities are listed in Table E.12. The ranking based on the KLIC is depicted in Table 6.3.

The PCR models with one principal component perform relatively well. This appears also in the 24-month forecast horizon of the whole forecast period, but not in the 12-month forecast horizon of the whole forecast period.

The PCR models with five and ten components perform relatively poorly. This appears also in the 24-month forecast horizon of the whole forecast period, but not in the 12-month forecast horizon of the whole forecast period.

It is striking that neither of the density forecasts are significantly different based on the KLIC.

Table 6.3: KLIC ranking of the 12-months-ahead density forecasts over the period 1994:M01 through 2008:M08.

1.	AR(2)	8.	PCR(5,0,2)
2.	RW	9.	PCR(1,0,0)
3.	PCR(1,2,2)	10.	PCR(10,0,2)
4.	PCR(1,0,2)	11.	PCR(10,1,2)
5.	PCR(1,1,2)	12.	PCR(5,1,2)
6.	PCR(10,0,0)	13.	PCR(10,2,2)
7.	PCR(5,0,0)	14.	PCR(5,2,2)

6.6 Analyzing the 24-month forecast horizon over the period 1995:M01-2007:M08

This section analyzes the 24-months-ahead density forecasts over the period 1995:M01 through 2007:M08.

6.6.1 PIT and Berkowitz LR

Uniformity of the PITs is checked by assessing the PIT histograms. Figure E.52 in Appendix E shows the histograms of the analyzed models for the 12-month forecast horizon over the period 1995:M01-2007:M08.

As also indicated in the 12-month forecast horizon of the period 1994:M01 through 2008:M08, not all models show clustering of mass on both sides of the distributions of the PITs. Some models show only clustering of mass on the left side of the PITs. This indicates that the density forecast still reflect estimated forecast distributions that are too narrow. There is however definite improvement compared to the PITs of the whole forecast period.

Based on the histograms, it is clear that neither of the models provide an accurate density forecast. This is also indicated by the Berkowitz LR tests that are depicted in Table E.13 in Appendix E; the null hypothesis of uniformly distributed $\{z_t\}$ has been rejected at a 5% significance interval for every model. The independence of the PITs is assessed by examining the autocorrelation plots of $\{z_t - \overline{z}\}, \{z_t - \overline{z}\}^2, \{z_t - \overline{z}\}^3$ and $\{z_t - \overline{z}\}^4$ in the next section.

6.6.2 Assessing of the serial autocorrelation plots

The Random Walk models shows negative autocorrelations in the conditional mean (and skewness) dynamics. These autocorrelations are again probably caused by the delay in the density forecasts that occurs by construction. The autocorrelations for the conditional kurtosis dynamics show small significant positive and negative autocorrelations while the autocorrelations for the conditional variance show no autocorrelations.

The AR(2) model captures the conditional variance (and kurtosis) dynamics. It shows slightly negative autocorrelations in the conditional mean while there is no significant autocorrelation in the skewness dynamics.

All the PCR models capture conditional variance (and kurtosis) dynamics. The PCR models with one principal component show significant negative autocorrelation in the conditional mean (and skewness) dynamics, while the PCR models with five components show significant autocorrelations in the conditional mean dynamics but not in the conditional skewness dynamics. Furthermore, the PCR models with ten principal components capture all the conditional dynamics at once. However, all the models show different patterns in the conditional mean dynamics compared to the conditional skewness dynamics. All the PCR models appear to have smaller autocorrelations in the conditional skewness dynamics than in the conditional mean dynamics. This indicates again that the inflation series might suffer from upside risk.

6.6.3 KLIC

The KLIC test statistics are listed in Table E.14 in Appendix E. The associated probabilities are listed in Table E.16. The ranking based on the KLIC is depicted in Table 6.4.

Except for the benchmark models, the ranking appears somewhat the same as the 12-month forecast horizon over the period 1994:M01-2007:M08. Striking is the number one ranking of the AR(2) model. This ranking is especially remarkable since the AR(2) model was ranked last in the 12-month forecast horizon of the whole forecast period.

Just as in the 24-month forecast horizon over the whole forecast period and the 12-month forecast horizon over the period 1994:M01-2007:M08, the models with one principal component perform relatively well. The PCR(1,1,2) model is even favored over the Random Walk model.

Only two pairs of density forecasts are significantly different based on the KLIC. The first pair is PCR(5,0,0) and PCR(10,1,2) with the first models as favored. The second pair is PCR(10,0,0) and PCR(1,1,2) with the second model as favored model.

If it is possible to relate the findings of this research to findings in the literature, it strengthens the results. The next section relates the findings of this research to the findings in the literature.

Table 6.4: KLIC ranking of the 24-months-ahead density forecasts over the period 1995:M01 through 2007:M08.

1.	AR(2)	8.	PCR(5,2,2)
2.	PCR(1,1,2)	9.	PCR(10,0,0)
3.	RW	10.	PCR(1,0,0)
4.	PCR(1,2,2)	11.	PCR(5,1,2)
5.	PCR(1,0,2)	12.	PCR(10,1,2)
6.	PCR(5,0,2)	13.	PCR(10,2,2)
7.	PCR(5,0,0)	14.	PCR(10,0,2)

6.7 Relating findings to the literature

Gillitzer and Kearns (2007) find that adding lags of inflation is required to account for structural change. They consider forecasts in the period 1960:Q3-2005:Q4 and consequently, their results should be compared to the forecast densities that are considered in section 6.3 and section 6.4. As Gillitzer and Kearns (2007), this research finds that adding lags of inflation is required for PCR with one principal component. This research does not find that adding lags of inflation is required for PCR with five or ten components. The latter is not contradictory to the findings of Gillitzer and Kearns (2007) as they consider only models with two principal components and lags of inflation.

Gillitzer and Kearns (2007) find significant improvement of the PCR models with lags of inflation over their AR model. For the same forecasting period, this improvement is also found in this research. Gillitzer and Kearns (2007) find that PCR models with two principal components and no lags of the dependent variable do not outperform AR. The PCR(1,0,0) model in this research does not outperform the AR(2) in both the 12- and 24-month horizon. The PCR(5,0,0) and PCR(10,0,0) models outperform the AR(2) models. The latter is again not contradictory to the findings of Gillitzer and Kearns (2007) as they consider only models with two principal components and lags of inflation. In conclusion, it can be stated that the results of this research are related to the results of Gillitzer and Kearns (2007).

Gillitzer and Kearns (2007), Stock and Watson (2002) also find that adding lagged inflation dramatically improves the forecasts. As described above, this result can be related to this thesis.

Stock and Watson (2002) show that without adding lagged inflation their DI forecasts are actually worse than their autoregressive forecasts. At the 12 month horizon, only the PCR(1,0,0) model is performing worse than the AR(2) model. At the 24 month horizon, the PCR(1,0,0) and PCR(10,0,0) models perform worse than the AR(2) model. The finding of Stock and Watson (2002) can therefore not completely be related to the findings in this research. Nevertheless, the difference can be explained: the forecast period of Stock and Watson (2002) is until 1998:M12, this thesis however analyzes forecasts for ten additional years. When comparing the results of sections 6.3 and 6.4 to the results of 6.5 and 6.6, the conclusion that there is much difference in the performance of the AR(2) model can be drawn. Consequently, the difference of this research concerning the Autoregressive model compared to the research of Stock and Watson (2002) can be explained.

Stock and Watson (2008) find it curious that Gavin and Kliesen (2008) found their AR(12) model outperforming their Random Walk model at the 12-month forecast horizon. The Autoregressive model in this thesis outperforms the Random Walk model at both the 12- and 24-month forecast horizon in the forecasting periods considered in section 6.5 and 6.6. Stock and Watson (2008) presume that this surprising result is either a consequence of including earlier and later data than Atkeson and Ohanian (2001) or indicates some subtle differences between using quarterly data and monthly data. This thesis follows the forecasting period of Gavin and Kliesen (2008) and also uses monthly data. Other than taking over the statement of Stock and Watson (2008) nothing can be concluded here.

Chapter 7

Conclusion & Discussion

This thesis provides an answer to the following research question: *Does the use of a regression model that includes macroeconomic factors, a time-varying level and stochastic volatility lead to more accurate density forecasts for inflation compared to benchmark models?* In order to answer this research question, an empirical analysis has been performed.

The macroeconomic factors are estimated with principal components and used in a principal component regression. The density forecasts of inflation have been obtained through the Random Walk model, the Autoregressive model (both benchmark models) and twelve different Principal Component Regression models. The forecasting of inflation in this thesis focused on 12- and 24-month forecast horizons. The assessment of the density forecasts has been carried out for the periods 1970-2008 and 1994-2008, resulting in four different assessments of the fourteen models. The main results can be summarized as follows.

- Based on the empirical research conducted in this thesis, the proposed model does not increase density forecasting accuracy of inflation compared to the benchmark models. In neither of the four assessments, there is a Principal Component Regression model that outperforms both benchmark models.
- Regarding the benchmark models; the Random Walk model has proven to be the most consistent performing model, it is only outperformed by the Autoregressive Model (twice) and the PCR model

with one principal component, one lag of the principal component and lagged inflation (once). Whereas the Autoregressive model is the least (12-month forecast horizon) and a moderate (24-month forecast horizon) performing model in the 1970-2008 period.

- The density forecasts over the period 1994-2008 are generally more accurate than the density forecasts over the period 1970-2008. This difference is explained by the structural break in many macroeconomic variables (including inflation) around 1983. This structural break has led to less volatile inflation, allowing for more accurate density forecasts.
- The Principal Component Regression models with one principal component generally provide more accurate density forecasts than the Principal Component Regression models with five or ten principal components. Adding lagged inflation in the Principal Component Regression models also improves forecasting accuracy. Both results are likewise found by Gillitzer and Kearns (2007) and Stock and Watson (2002).
- Models that capture the volatility dynamics appear to provide accurate density forecasts. On the other hand, models that capture the mean dynamics in general do not provide accurate density forecasts. Capturing the volatility dynamics appears to be more important for accurate *density forecasts* than capturing the mean dynamics.
- There density forecasts show that the real density appears to have skewness in both the 12- and 24-month forecast horizon over the period 1994-2008. This indicates that inflation suffers from upside risk. This is strengthened by the statement that higher values of inflation are in general less desirable than lower values of inflation.

The findings of this thesis suggest that more empirical and theoretical research is necessary to come to a complete answer of the research question. Five recommendations for further research are given.

1. The time-varying level should be assessed in order to determine whether it provides significant improvement.
2. Other methods of selection components and lags of components should be analyzed in order to determine whether they outperform the benchmark models.

3. Other measures of inflation should be considered; CPI-core, PCE and PCE-core as the core measures of inflation are typically easier to forecast (Fisher *et al.* (2002)).

4. Different forecasting periods should be considered in order to determine the robustness of the models. This is important because forecast accuracy of models tends to differ across forecasting periods.

5. Other probability densities should be considered. The two-piece normal density (Wallis (2004)) is a possible choice as it takes upside (or downside) risk into account.

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Appendix A

Original dataset

Table A.1: Description of the variables in the Stock & Watson dataset

Short Name	Transformation	Description	Category
PI	Δln	Personal Income (AR, Bil. Chain 2000 $) (TCB)	Real Output and Income
PI less transfers	Δln	Personal Income Less Transfer Payments (AR, Bil. Chain 2000 $) (TCB)	Real Output and Income
Consumption	Δln	Real Consumption (AC) a0m224/gmdc (a0m224 is from TCB)	Consumption
M&T sales	Δln	Manufacturing And Trade Sales (Mil. Chain 1996 $) (TCB)	Manufacturing and Trade Sales
Retail sales	Δln	Sales Of Retail Stores (Mil. Chain 2000 $) (TCB)	Real Retail
IP: total	Δln	Industrial Production Index - Total Index	Real Output and Income
IP: products	Δln	Industrial Production Index - Products, Total	Real Output and Income
IP: final prod	Δln	Industrial Production Index - Final Products	Real Output and Income
IP: cons gds	Δln	Industrial Production Index - Consumer Goods	Real Output and Income
IP: cons dble	Δln	Industrial Production Index - Durable Consumer Goods	Real Output and Income
IP: cons nondble	Δln	Industrial Production Index - Nondurable Consumer Goods	Real Output and Income
IP: bus eqpt	Δln	Industrial Production Index - Business Equipment	Real Output and Income
IP: matls	Δln	Industrial Production Index - Materials	Real Output and Income
IP: dble matls	Δln	Industrial Production Index - Durable Goods Materials	Real Output and Income
IP: nondble matls	Δln	Industrial Production Index - Nondurable Goods Materials	Real Output and Income
IP: mfg	Δln	Industrial Production Index - Manufacturing (Sic)	Real Output and Income
IP: res util	Δln	Industrial Production Index - Residential Utilities	Real Output and Income
IP: fuels	Δln	Industrial Production Index - Fuels	Real Output and Income
NAPM prodn	lv	Napm Production Index (Percent)	Real Output and Income
Cap util	Δlv	Capacity Utilization (Mfg) (TCB)	Real Output and Income
Help wanted indx	Δlv	Index Of Help-Wanted Advertising In Newspapers (1967=100;Sa)	Employment and Hours
Help wanted/emp	Δlv	Employment: Ratio; Help-Wanted Ads:No. Unemployed Clf	Employment and Hours
Emp CPS total	Δln	Civilian Labor Force: Employed, Total (Thous.,Sa)	Employment and Hours
Emp CPS nonag	Δln	Civilian Labor Force: Employed, Nonagric.Industries (Thous.,Sa)	Employment and Hours
U: all	Δlv	Unemployment Rate: All Workers, 16 Years & Over (%,Sa)	Employment and Hours
U: mean duration	Δlv	Unemploy.By Duration: Average(Mean)Duration In Weeks (Sa)	Employment and Hours
U $<$ 5 wks	Δln	Unemploy.By Duration: Persons Unempl.Less Than 5 Wks (Thous.,Sa)	Employment and Hours
U 41760 wks	Δln	Unemploy.By Duration: Persons Unempl.5 To 14 Wks (Thous.,Sa)	Employment and Hours

U 15+ wks	Δln	Unemploy.By Duration: Persons Unempl.15 Wks + (Thous.,Sa)	Employment and Hours
U 15-26 wks	Δln	Unemploy.By Duration: Persons Unempl.15 To 26 Wks (Thous.,Sa)	Employment and Hours
U 27+ wks	Δln	Unemploy.By Duration: Persons Unempl.27 Wks + (Thous,Sa)	Employment and Hours
UI claims	Δln	Average Weekly Initial Claims, Unemploy. Insurance (Thous.) (TCB)	Employment and Hours
Emp: total	Δln	Employees On Nonfarm Payrolls: Total Private	Employment and Hours
Emp: gds prod	Δln	Employees On Nonfarm Payrolls - Goods-Producing	Employment and Hours
Emp: mining	Δln	Employees On Nonfarm Payrolls - Mining	Employment and Hours
Emp: const	Δln	Employees On Nonfarm Payrolls - Construction	Employment and Hours
Emp: mfg	Δln	Employees On Nonfarm Payrolls - Manufacturing	Employment and Hours
Emp: dble gds	Δln	Employees On Nonfarm Payrolls - Durable Goods	Employment and Hours
Emp: nondbles	Δln	Employees On Nonfarm Payrolls - Nondurable Goods	Employment and Hours
Emp: services	Δln	Employees On Nonfarm Payrolls - Service-Providing	Employment and Hours
Emp: TTU	Δln	Employees On Nonfarm Payrolls - Trade, Transportation, And Utilities	Employment and Hours
Emp: wholesale	Δln	Employees On Nonfarm Payrolls - Wholesale Trade	Employment and Hours
Emp: retail	Δln	Employees On Nonfarm Payrolls - Retail Trade	Employment and Hours
Emp: FIRE	Δln	Employees On Nonfarm Payrolls - Financial Activities	Employment and Hours
Emp: Govt	Δln	Employees On Nonfarm Payrolls - Government	Employment and Hours
Emp-hrs nonag	Δln	Employee Hours In Nonag. Establishments (AR, Bil. Hours) (TCB)	Employment and Hours
Avg hrs	lv	Avg Weekly Hrs of Prod or Nonsup Workers On Private Nonfarm Payrolls - Goods-Producing	Employment and Hours
Overtime: mfg	Δlv	Avg Weekly Hrs of Prod or Nonsup Workers On Private Nonfarm Payrolls - Mfg Overtime Hours	Employment and Hours
Avg hrs: mfg	lv	Average Weekly Hours, Mfg. (Hours) (TCB)	Employment and Hours
NAPM empl	lv	Napm Employment Index (Percent)	Employment and Hours
Starts: nonfarm	ln	Housing Starts:Nonfarm(1947-58);Total Farm&Nonfarm(1959-)(Thous.,Saar)	Housing Starts and Sales
Starts: NE	ln	Housing Starts:Northeast (Thous.U.)S.A.	Housing Starts and Sales
Starts: MW	ln	Housing Starts:Midwest(Thous.U.)S.A.	Housing Starts and Sales
Starts: South	ln	Housing Starts:South (Thous.U.)S.A.	Housing Starts and Sales
Starts: West	ln	Housing Starts:West (Thous.U.)S.A.	Housing Starts and Sales
BP: total	ln	Housing Authorized: Total New Priv Housing Units (Thous.,Saar)	Housing Starts and Sales
BP: NE	ln	Houses Authorized By Build. Permits:Northeast(Thou.U.)S.A	Housing Starts and Sales
BP: MW	ln	Houses Authorized By Build. Permits:Midwest(Thou.U.)S.A.	Housing Starts and Sales
BP: South	ln	Houses Authorized By Build. Permits:South(Thou.U.)S.A.	Housing Starts and Sales

BP: West	ln	Houses Authorized By Build. Permits:West(Thou.U.)S.A.	Housing Starts and Sales
PMI	lv	Purchasing Managers' Index (Sa)	Orders
NAPM new ordrs	lv	Napm New Orders Index (Percent)	Orders
NAPM vendor del	lv	Napm Vendor Deliveries Index (Percent)	Orders
NAPM Invent	lv	Napm Inventories Index (Percent)	Real Inventories
Orders: cons gds	Δln	Mfrs' New Orders, Consumer Goods And Materials (Bil. Chain 1982 $) (TCB)	Orders
Orders: dble gds	Δln	Mfrs' New Orders, Durable Goods Industries (Bil. Chain 2000 $) (TCB)	Orders
Orders: cap gds	Δln	Mfrs' New Orders, Nondefense Capital Goods (Mil. Chain 1982 $) (TCB)	Orders
Unf orders: dble	Δln	Mfrs' Unfilled Orders, Durable Goods Indus. (Bil. Chain 2000 $) (TCB)	Orders
M&T invent	Δln	Manufacturing And Trade Inventories (Bil. Chain 2000 $) (TCB)	Real Inventories
M&T invent/sales	Δlv	Ratio, Mfg. And Trade Inventories To Sales (Based On Chain 2000 $) (TCB)	Real Inventories
M1	Δ^2ln	Money Stock: M1(Curr,Trav.Cks,Dem Dep,Other Ck'able Dep)(Bil$,Sa)	Money and Credit Quantity Aggregates
M2	Δ^2ln	Money Stock:M2(M1+O'nite Rps,Euro$,G/P&B/D Mmmfs&Sav&Sm Time Dep(Bil$,Sa)	Money and Credit Quantity Aggregates
M3	Δ^2ln	Money Stock: M3(M2+Lg Time Dep,Term Rp's&Inst Only Mmmfs)(Bil$,Sa)	Money and Credit Quantity Aggregates
M2 (real)	Δln	Money Supply - M2 In 1996 Dollars (Bci)	Money and Credit Quantity Aggregates
MB	Δ^2ln	Monetary Base, Adj For Reserve Requirement Changes(Mil$,Sa)	Money and Credit Quantity Aggregates
Reserves tot	Δ^2ln	Depository Inst Reserves:Total, Adj For Reserve Req Chgs(Mil$,Sa)	Money and Credit Quantity Aggregates
Reserves nonbor	Δ^2ln	Depository Inst Reserves:Nonborrowed,Adj Res Req Chgs(Mil$,Sa)	Money and Credit Quantity Aggregates
C&I loans	Δ^2ln	Commercial & Industrial Loans Oustanding In 1996 Dollars (Bci)	Money and Credit Quantity Aggregates
Δ C&I loans	lv	Wkly Rp Lg Com'l Banks:Net Change Com'l & Indus Loans(Bil$,Saar)	Money and Credit Quantity Aggregates
Cons credit	Δ^2ln	Consumer Credit Outstanding - Nonrevolving(G19)	Money and Credit Quantity Aggregates
Inst cred/PI	Δlv	Ratio, Consumer Installment Credit To Personal Income (Pct.) (TCB)	Money and Credit Quantity Aggregates
S&P 500	Δln	S&P's Common Stock Price Index: Composite (1941-43=10)	Stock Prices
S&P: indust	Δln	S&P's Common Stock Price Index: Industrials (1941-43=10)	Stock Prices
S&P div yield	Δlv	S&P's Composite Common Stock: Dividend Yield (% Per Annum)	Stock Prices

S&P PE ratio	Δln	S&P's Composite Common Stock: Price-Earnings Ratio (%,Nsa)	Stock Prices
Fed Funds	Δlv	Interest Rate: Federal Funds (Effective) (% Per Annum,Nsa)	Interest Rates and Spreads
Comm paper	Δlv	Cmmercial Paper Rate (AC)	Interest Rates and Spreads
3 mo T-bill	Δlv	Interest Rate: U.S.Treasury Bills,Sec Mkt,3-Mo.(% Per Ann,Nsa)	Interest Rates and Spreads
6 mo T-bill	Δlv	Interest Rate: U.S.Treasury Bills,Sec Mkt,6-Mo.(% Per Ann,Nsa)	Interest Rates and Spreads
1 yr T-bond	Δlv	Interest Rate: U.S.Treasury Const Maturities,1-Yr.(% Per Ann,Nsa)	Interest Rates and Spreads
5 yr T-bond	Δlv	Interest Rate: U.S.Treasury Const Maturities,5-Yr.(% Per Ann,Nsa)	Interest Rates and Spreads
10 yr T-bond	Δlv	Interest Rate: U.S.Treasury Const Maturities,10-Yr.(% Per Ann,Nsa)	Interest Rates and Spreads
Aaa bond	Δlv	Bond Yield: Moody's Aaa Corporate (% Per Annum)	Interest Rates and Spreads
Baa bond	Δlv	Bond Yield: Moody's Baa Corporate (% Per Annum)	Interest Rates and Spreads
CP-FF spread	lv	cp90-fyff (AC)	Interest Rates and Spreads
3 mo-FF spread	lv	fygm3-fyff (AC)	Interest Rates and Spreads
6 mo-FF spread	lv	fygm6-fyff (AC)	Interest Rates and Spreads
1 yr-FF spread	lv	fygt1-fyff (AC)	Interest Rates and Spreads
5 yr-FF spread	lv	fygt5-fyff (AC)	Interest Rates and Spreads
10 yr-FF spread	lv	fygt10-fyff (AC)	Interest Rates and Spreads
Aaa-FF spread	lv	fyaaac-fyff (AC)	Interest Rates and Spreads
Baa-FF spread	lv	fybaac-fyff (AC)	Interest Rates and Spreads
Ex rate: avg	Δln	United States;Effective Exchange Rate(Merm)(Index No.)	Exchange Rates
Ex rate: Switz	Δln	Foreign Exchange Rate: Switzerland (Swiss Franc Per U.S.$)	Exchange Rates
Ex rate: Japan	Δln	Foreign Exchange Rate: Japan (Yen Per U.S.$)	Exchange Rates
Ex rate: UK	Δln	Foreign Exchange Rate: United Kingdom (Cents Per Pound)	Exchange Rates
EX rate: Canada	Δln	Foreign Exchange Rate: Canada (Canadian $ Per U.S.$)	Exchange Rates
PPI: fin gds	Δ^2ln	Producer Price Index: Finished Goods (82=100,Sa)	Price Indexes
PPI: cons gds	Δ^2ln	Producer Price Index: Finished Consumer Goods (82=100,Sa)	Price Indexes
PPI: int matls	Δ^2ln	Producer Price Index:I ntermed Mat.Supplies & Components(82=100,Sa)	Price Indexes
PPI: crude matls	Δ^2ln	Producer Price Index: Crude Materials (82=100,Sa)	Price Indexes
Spot market price	Δ^2ln	Spot market price index: bls & crb: all commodities(1967=100)	Price Indexes
Sens matls price	Δ^2ln	Index Of Sensitive Materials Prices (1990=100)(Bci-99a)	Price Indexes
NAPM com price	lv	Napm Commodity Prices Index (Percent)	Price Indexes

CPI-U: all	Δ^2ln	Cpi-U: All Items (82-84=100,Sa)	Price Indexes
CPI-U: Core	Δ^2ln	Cpi-U: Core	Price Indexes
CPI-U: apparel	Δ^2ln	Cpi-U: Apparel & Upkeep (82-84=100,Sa)	Price Indexes
CPI-U: transp	Δ^2ln	Cpi-U: Transportation (82-84=100,Sa)	Price Indexes
CPI-U: medical	Δ^2ln	Cpi-U: Medical Care (82-84=100,Sa)	Price Indexes
CPI-U: comm.	Δ^2ln	Cpi-U: Commodities (82-84=100,Sa)	Price Indexes
CPI-U: dbles	Δ^2ln	Cpi-U: Durables (82-84=100,Sa)	Price Indexes
CPI-U: services	Δ^2ln	Cpi-U: Services (82-84=100,Sa)	Price Indexes
CPI-U: ex food	Δ^2ln	Cpi-U: All Items Less Food (82-84=100,Sa)	Price Indexes
CPI-U: ex shelter	Δ^2ln	Cpi-U: All Items Less Shelter (82-84=100,Sa)	Price Indexes
CPI-U: ex med	Δ^2ln	Cpi-U: All Items Less Medical Care (82-84=100,Sa)	Price Indexes
PCE defl	Δ^2ln	Pce, Impl Pr Defl:Pce (1987=100)	Price Indexes
PCE defl: Core	Δ^2ln	Pce, Imple Pr Delf:Core	Prince Indexes
PCE defl: dlbes	Δ^2ln	Pce, Impl Pr Defl:Pce; Durables (1987=100)	Price Indexes
PCE defl: nondble	Δ^2ln	Pce, Impl Pr Defl:Pce; Nondurables (1996=100)	Price Indexes
PCE defl: service	Δ^2ln	Pce, Impl Pr Defl:Pce; Services (1987=100)	Price Indexes
AHE: goods	Δ^2ln	Avg Hourly Earnings of Prod or Nonsup Workers On Private Nonfarm Payrolls - Goods-Producing	Average Hourly Earnings
AHE: const	Δ^2ln	Avg Hourly Earnings of Prod or Nonsup Workers On Private Nonfarm Payrolls - Construction	Average Hourly Earnings
AHE: mfg	Δ^2ln	Avg Hourly Earnings of Prod or Nonsup Workers On Private Nonfarm Payrolls - Manufacturing	Average Hourly Earnings
Consumer expect	Δlv	U. Of Mich. Index Of Consumer Expectations(Bcd-83)	Consumer Expectations

Appendix B

Timeline

Table B.1: Sources of the variables in the dataset

Variable	Source
Personal income (AR, bil. chain 2000 $)	DS
Personal income less transfer payments (AR, bil. chain 2000 $)	DS
Real Consumption (AC) A0m224/gmdc	FRED
Manufacturing and trade sales (mil. Chain 1996 $)	
Sales of retail stores (mil. Chain 2000 $)	DS
INDUSTRIAL PRODUCTION INDEX - TOTAL INDEX	FRB
INDUSTRIAL PRODUCTION INDEX - PRODUCTS, TOTAL	FRB
INDUSTRIAL PRODUCTION INDEX - FINAL PRODUCTS	FRB
INDUSTRIAL PRODUCTION INDEX - CONSUMER GOODS	FRB
INDUSTRIAL PRODUCTION INDEX - DURABLE CONSUMER GOODS	FRB
INDUSTRIAL PRODUCTION INDEX - NONDURABLE CONSUMER GOODS	FRB
INDUSTRIAL PRODUCTION INDEX - BUSINESS EQUIPMENT	FRB
INDUSTRIAL PRODUCTION INDEX - MATERIALS	FRB
INDUSTRIAL PRODUCTION INDEX - DURABLE GOODS MATERIALS	FRB
INDUSTRIAL PRODUCTION INDEX - NONDURABLE GOODS MATERIALS	FRB
INDUSTRIAL PRODUCTION INDEX - MANUFACTURING (SIC)	FRB
INDUSTRIAL PRODUCTION INDEX - RESIDENTIAL UTILITIES	FRB
INDUSTRIAL PRODUCTION INDEX - FUELS	FRB
NAPM PRODUCTION INDEX (PERCENT)	ism.ws
Capacity Utilization (Mfg)	FRB
INDEX OF HELP-WANTED ADVERTISING IN NEWSPAPERS (1967=100;SA)	DS
EMPLOYMENT: RATIO; HELP-WANTED ADS:NO. UNEMPLOYED CLF	DS
CIVILIAN LABOR FORCE: EMPLOYED, TOTAL (THOUS.,SA)	BLS

CIVILIAN LABOR FORCE: EMPLOYED, NONAGRIC.INDUSTRIES (THOUS.,SA)	BLS
UNEMPLOYMENT RATE: ALL WORKERS, 16 YEARS & OVER (%,SA)	FRED
UNEMPLOY.BY DURATION: AVERAGE(MEAN)DURATION IN WEEKS (SA)	FRED
UNEMPLOY.BY DURATION: PERSONS UNEMPL.LESS THAN 5 WKS (THOUS.,SA)	FRED
UNEMPLOY.BY DURATION: PERSONS UNEMPL.5 TO 14 WKS (THOUS.,SA)	FRED
UNEMPLOY.BY DURATION: PERSONS UNEMPL.15 WKS + (THOUS.,SA)	FRED
UNEMPLOY.BY DURATION: PERSONS UNEMPL.15 TO 26 WKS (THOUS.,SA)	FRED
UNEMPLOY.BY DURATION: PERSONS UNEMPL.27 WKS + (THOUS,SA)	FRED
Average weekly initial claims, unemploy. insurance (thous.)	
EMPLOYEES ON NONFARM PAYROLLS - TOTAL PRIVATE	BLS
EMPLOYEES ON NONFARM PAYROLLS - GOODS-PRODUCING	BLS
EMPLOYEES ON NONFARM PAYROLLS - MINING	BLS
EMPLOYEES ON NONFARM PAYROLLS - CONSTRUCTION	BLS
EMPLOYEES ON NONFARM PAYROLLS - MANUFACTURING	BLS
EMPLOYEES ON NONFARM PAYROLLS - DURABLE GOODS	BLS
EMPLOYEES ON NONFARM PAYROLLS - NONDURABLE GOODS	BLS
EMPLOYEES ON NONFARM PAYROLLS - SERVICE-PROVIDING	BLS
EMPLOYEES ON NONFARM PAYROLLS - TRADE, TRANSPORTATION, AND UTILITIES	BLS
EMPLOYEES ON NONFARM PAYROLLS - WHOLESALE TRADE	BLS
EMPLOYEES ON NONFARM PAYROLLS - RETAIL TRADE	BLS
EMPLOYEES ON NONFARM PAYROLLS - FINANCIAL ACTIVITIES	BLS
EMPLOYEES ON NONFARM PAYROLLS - GOVERNMENT	BLS
Employee hours in nonag. establishments (AR, bil. hours)	FRED
AVERAGE WEEKLY HOURS OF PRODUCTION OR NONSUPERVISORY WORKERS ON PRIVATE NONFAR	BLS
AVERAGE WEEKLY HOURS OF PRODUCTION OR NONSUPERVISORY WORKERS ON PRIVATE NONFAR	BLS
Average weekly hours, mfg. (hours)	

NAPM EMPLOYMENT INDEX (PERCENT)	ism.ws
HOUSING STARTS:NONFARM(1947-58);TOTAL FARM&NONFARM(1959-)(THOUS.,SA	FRED
HOUSING STARTS:NORTHEAST (THOUS.U.)S.A.	FRED
HOUSING STARTS:MIDWEST(THOUS.U.)S.A.	FRED
HOUSING STARTS:SOUTH (THOUS.U.)S.A.	FRED
HOUSING STARTS:WEST (THOUS.U.)S.A.	FRED
HOUSING AUTHORIZED: TOTAL NEW PRIV HOUSING UNITS (THOUS.,SAAR)	FRED
HOUSES AUTHORIZED BY BUILD. PERMITS:NORTHEAST(THOU.U.)S.A	FRED
HOUSES AUTHORIZED BY BUILD. PERMITS:MIDWEST(THOU.U.)S.A.	FRED
HOUSES AUTHORIZED BY BUILD. PERMITS:SOUTH(THOU.U.)S.A.	FRED
HOUSES AUTHORIZED BY BUILD. PERMITS:WEST(THOU.U.)S.A.	FRED
PURCHASING MANAGERS' INDEX (SA)	ism.ws
NAPM NEW ORDERS INDEX (PERCENT)	ism.ws
NAPM VENDOR DELIVERIES INDEX (PERCENT)	ism.ws
NAPM INVENTORIES INDEX (PERCENT)	ism.ws
Mfrs' new orders, consumer goods and materials (bil. chain 1982 $)	
Mfrs' new orders, durable goods industries (bil. chain 2000 $)	DS
Mfrs' new orders, nondefense capital goods (mil. chain 1982 $)	
Mfrs' unfilled orders, durable goods indus. (bil. chain 2000 $)	DS
Manufacturing and trade inventories (bil. chain 2000 $)	DS
Ratio, mfg. and trade inventories to sales (based on chain 2000 $)	DS
MONEY STOCK: M1(CURR,TRAV.CKS,DEM DEP,OTHER CK'ABLE DEP)(BIL$,SA)	FRED
MONEY STOCK:M2(M1+O'NITE RPS,EURO$,G/P&B/D MMMFS&SAV&SM TIME DEP(BIL$,	FRED
MONEY STOCK: M3(M2+LG TIME DEP,TERM RP'S&INST ONLY MMMFS)(BIL$,SA)	FRED
MONEY SUPPLY - M2 IN 1996 DOLLARS (BCI)	own calc
MONETARY BASE, ADJ FOR RESERVE REQUIREMENT CHANGES(MIL$,SA)	FRB

DEPOSITORY INST RESERVES:TOTAL,ADJ FOR RESERVE REQ CHGS(MIL$,SA)	FRB
DEPOSITORY INST RESERVES:NONBORROWED,ADJ RES REQ CHGS(MIL$,SA)	FRB
COMMERCIAL & INDUSTRIAL LOANS OUSTANDING IN 1996 DOLLARS (BCI)	DS
WKLY RP LG COM'L BANKS:NET CHANGE COM'L & INDUS LOANS(BIL$,SAAR)	
CONSUMER CREDIT OUTSTANDING - NONREVOLVING(G19)	FRB
Ratio, consumer installment credit to personal income (pct.)	DS
S&P'S COMMON STOCK PRICE INDEX: COMPOSITE (1941-43=10)	DS
S&P'S COMMON STOCK PRICE INDEX: INDUSTRIALS (1941-43=10)	DS
S&P'S COMPOSITE COMMON STOCK: DIVIDEND YIELD (% PER ANNUM)	DS
S&P'S COMPOSITE COMMON STOCK: PRICE-EARNINGS RATIO (%,NSA)	DS
INTEREST RATE: FEDERAL FUNDS (EFFECTIVE) (% PER ANNUM,NSA)	FRB
Cmmercial Paper Rate (AC)	FRB
INTEREST RATE: U.S.TREASURY BILLS,SEC MKT,3-MO.(% PER ANN,NSA)	FRB
INTEREST RATE: U.S.TREASURY BILLS,SEC MKT,6-MO.(% PER ANN,NSA)	FRB
INTEREST RATE: U.S.TREASURY CONST MATURITIES,1-YR.(% PER ANN,NSA)	FRB
INTEREST RATE: U.S.TREASURY CONST MATURITIES,5-YR.(% PER ANN,NSA)	FRB
INTEREST RATE: U.S.TREASURY CONST MATURITIES,10-YR.(% PER ANN,NSA)	FRB
BOND YIELD: MOODY'S AAA CORPORATE (% PER ANNUM)	FRB
BOND YIELD: MOODY'S BAA CORPORATE (% PER ANNUM)	FRB
cp90-fyff	own calc
fygm3-fyff	own calc
fygm6-fyff	own calc
fygt1-fyff	own calc
fygt5-fyff	own calc
fygt10-fyff	own calc
fyaaac-fyff	own calc

fybaac-fyff	own calc
UNITED STATES;EFFECTIVE EXCHANGE RATE(MERM)(INDEX NO.)	DS
FOREIGN EXCHANGE RATE: SWITZERLAND (SWISS FRANC PER U.S.$)	FRED
FOREIGN EXCHANGE RATE: JAPAN (YEN PER U.S.$)	FRED
FOREIGN EXCHANGE RATE: UNITED KINGDOM (CENTS PER POUND)	FRED
FOREIGN EXCHANGE RATE: CANADA (CANADIAN $ PER U.S.$)	FRED
PRODUCER PRICE INDEX: FINISHED GOODS (82=100,SA)	FRED
PRODUCER PRICE INDEX:FINISHED CONSUMER GOODS (82=100,SA)	FRED
PRODUCER PRICE INDEX:INTERMED MAT.SUPPLIES & COMPONENTS(82=100,SA)	FRED
PRODUCER PRICE INDEX:CRUDE MATERIALS (82=100,SA)	FRED
SPOT MARKET PRICE INDEX:BLS & CRB: ALL COMMODITIES(1967=100)	?
INDEX OF SENSITIVE MATERIALS PRICES (1990=100)(BCI-99A)	DS
NAPM COMMODITY PRICES INDEX (PERCENT)	ism.ws
CPI-U: ALL ITEMS (82-84=100,SA)	FRED
CPI-U: APPAREL & UPKEEP (82-84=100,SA)	FRED
CPI-U: TRANSPORTATION (82-84=100,SA)	FRED
CPI-U: MEDICAL CARE (82-84=100,SA)	FRED
CPI-U: COMMODITIES (82-84=100,SA)	BLS
CPI-U: DURABLES (82-84=100,SA)	BLS
CPI-U: SERVICES (82-84=100,SA)	BLS
CPI-U: ALL ITEMS LESS FOOD (82-84=100,SA)	FRED
CPI-U: ALL ITEMS LESS SHELTER (82-84=100,SA)	BLS
CPI-U: ALL ITEMS LESS MEDICAL CARE (82-84=100,SA)	BLS
Pce, Impl Pr De:Pce (1987=100)	FRED
PCE,IMPL PR DEFL:PCE; DURABLES (1987=100)	FRED
PCE,IMPL PR DEFL:PCE; NONDURABLES (1996=100)	FRED

PCE,IMPL PR DEFL:PCE; SERVICES (1987=100)	FRED
AVERAGE HOURLY EARNINGS OF PRODUCTION OR NONSUPERVISORY WORKERS ON PRIVATE NO	BLS
AVERAGE HOURLY EARNINGS OF PRODUCTION OR NONSUPERVISORY WORKERS ON PRIVATE NO	BLS
AVERAGE HOURLY EARNINGS OF PRODUCTION OR NONSUPERVISORY WORKERS ON PRIVATE NO	BLS
U. OF MICH. INDEX OF CONSUMER EXPECTATIONS(BCD-83)	umich.edu
CPI-U: Core	St Louis Fed.
Pce, Imple Pr Delf:Core	St Louis Fed.

Appendix C

Estimating the models

C.1 Rewriting the model for further usage

For convenience, some new notation is introduced. Let $\Pi^T = \left[\pi_1^h, \ldots, \pi_T^h\right]'$ and $E^T = [\sigma_1\varepsilon_1, \ldots, \sigma_T\varepsilon_T]'$. Let $\beta = [\beta(L), \gamma(L)]'.\beta = [\alpha_0, \beta(L)]'$ or $\beta = 1$ depending on the model that is used. Let X_t the vector of pooled predictor variables, including a zero if a constant is used. Let $\pi_t^* = \pi_t^h - \alpha_{\pi,t}$ respectively $\pi_t^* = \pi_t^h$ for models where the time-varying level is included respectively where the time-varying level is not included.

With this new notation, the model can be rewritten as:

$$\pi_{t+h}^* = X_t'\beta + \varepsilon_{t+h} \tag{C.1}$$

$$\ln(\sigma_{t+1}^2) = \ln(\sigma_t^2) + \eta_{t+1} \tag{C.2}$$

C.2 The algorithm

Step 1

Specify the initial values:

$\log \sigma_t^{2(0)}$ and $\sigma_\eta^{2(0)}$,

and set $m = 1$.

Step 2

Simulate $\beta^{(m+1)}$ from $p\left(\beta|\sigma_\eta^{2(m)}, E^T, \Pi^T\right)$ (Normal distribution),

$\beta^{(m+1)} \sim N\left(\left(\mathbf{X}'\mathbf{X} + B^{-1}\right)^{-1}\left(\mathbf{X}'\pi^* + B^{-1}b\right), \overline{\sigma}^{(m)}\left(\mathbf{X}'\mathbf{X} + B^{-1}\right)^{-1}\right)$, where $\overline{\sigma}^{(m)} = \frac{1}{T}\sum_t^T \sigma_t^{(m)}$.

Step 3

The conditional distributions have the following form[1]:

$$
\begin{aligned}
p\left(\log \sigma_t^2 | \log \sigma_{-t}^2, \beta, \sigma_\eta^2, \Pi^T\right) &= \frac{p\left(\log \sigma_t^2, \sigma_{-t}^2, \beta, \sigma_\eta^2, \Pi^T\right)}{p\left(\log \sigma_{-t}^2, \beta, \sigma_\eta^2, \Pi^T\right)} \\
&= \frac{p\left(\Pi^T, \beta | \log \sigma_t^2, \log \sigma_{-t}^2, \sigma_\eta^2\right) p\left(\log \sigma_t^2, \log \sigma_{-t}^2 | \sigma_\eta^2\right)}{p\left(\Pi^T, \beta | \log \sigma_{-t}^2, \sigma\eta^2\right) p\left(\log \sigma_{-t}^2 | \sigma_\eta^2\right)} \\
&\propto p\left(\Pi^T, \beta | \log \sigma_t^2, \log \sigma_{-t}^2\right) p\left(\log \sigma_t^2, \log \sigma_{-t}^2 | \sigma_\eta^2\right) \\
&\propto p\left(\pi_t^*, \beta | \log \sigma_t^2\right) p\left(\log \sigma_t^2 | \log \sigma_{t-1}^2, \sigma_\eta^2\right) p\left(\log \sigma_{t+1}^2 | \log \sigma_t^2, \sigma_\eta^2\right)
\end{aligned}
$$

Direct draws from these distributions is not feasible. Therefore, a Metropolis-hastings accept/reject step is

[1] $\sigma_{-t}^2 = \sigma_1^2, \ldots, \sigma_{t-1}^2, \sigma_{t+1}^2, \ldots, \sigma_T^2$

used. To generate a sample from $p\left(\log\sigma_t^2|\sigma_{-t}^2,\beta,\sigma_\eta,\Pi^T\right)$, a proposal density has to be identified.

This thesis uses a random walk sampler with the proposal density equal to a normal distribution, that is:

$$g\left(\log\sigma_t^{2(m+1)}|\log\sigma_t^{2(m)}\right)\sim N\left(\log\sigma_t^{2(m)},\omega\times\sigma_\eta^2\right),\omega\in(0,1)^{2}. \tag{C.3}$$

Yielding to the following simulation procedure:

Simulate $\log\sigma_t^{2*}$, equation by equation, from $p\left(\log\sigma_t^2|\log\sigma_{-t}^2,\beta^{(m+1)},\sigma_\eta^{2(m)},\Pi^T\right)$ (Normal distribution),

$\log\sigma_t^{2*}\sim N\left(\log\sigma_t^{2(m)},\omega\times\sigma_\eta^2\right)$,

set $\log\sigma_t^{2(m+1)}=\log\sigma_t^{2*}$ with probability α,

set $\log\sigma_t^{2(m+1)}=\log\sigma_t^{2(m)}$ with probability $1-\alpha$,

where

$$\begin{aligned}\alpha &= \min\{\frac{p\left(\log\sigma_t^{2*}|\log\sigma_{-t}^2,\beta^{(m+1)},\sigma_\eta^{2(m)}\right)}{p\left(\log\sigma_t^{2(m)}|\log\sigma_{-t}^2,\beta^{(m+1)},\sigma_\eta^{2(m)}\right)},1\}\\ &= \min\{\frac{p\left(\pi_t^*,\beta^{(m+1)}|\log\sigma_t^{2*}\right)p\left(\log\sigma_t^{2*}|\log\sigma_{t-1}^{2*},\sigma_\eta^2\right)p\left(\log\sigma_{t+1}^{2(m)}|\log\sigma_t^{2*},\sigma_\eta^{2(m)}\right)}{p\left(\pi_t^*,\beta^{(m+1)}|\log\sigma_t^{2(m)}\right)p\left(\log\sigma_t^{2(m)}|\log\sigma_{t-1}^{2(m)},\sigma_\eta^{2(m)}\right)p\left(\log\sigma_{t+1}^{2(m)}|\log\sigma_t^{2(m)},\sigma_\eta^{2(m)}\right)},1\}.\end{aligned}$$

Step 4

Simulate $\sigma_\eta^{2(m+1)}$ from $p\left(\sigma_\eta^2|\beta^{(m+1)},E^T,\Pi^T\right)$ (Inverted Gamma distribution),

$\frac{\frac{1}{T}\sum_{t=1}^T\left(\Delta\ln\sigma_t^{2(m+1)}\right)^2}{\sigma_\eta^{2(m+1)}}\sim\chi^2(T)$.

[2] The variance of the random walk sampler should be smaller than the the innovation variance σ_η. If this is not the case, the sampler will walk around the probability space as a random walk.

Step 5

Simulate σ_{t+h} from

Step 1

Set $j = -h$.

Step 2

Simulate $\log \sigma_{t+j}$ from $p\left(\log \sigma_{t+j} | \log \sigma_{\eta}^{2(m+1)}\right)$ (Normal distribution),

$\log \sigma_{t+j} \sim N\left(\log \sigma_{t+j-1}, \sigma_{\eta}^{2(m+1)}\right)$,

Step 3

Set $j = j + 1$ and go to step 2.

$\log \sigma_{t+h}^{(m+1)} = \log \sigma_{t+h}$ is obtained from $j = h$.

Step 6

Simulate $\pi_{t+h}^{(m+1)}$ from $p\left(\pi_{t+h} | \beta^{(m+1)}, \sigma_{t+h}^{(m+1)}, \Pi^t\right)$ (Normal distribution),

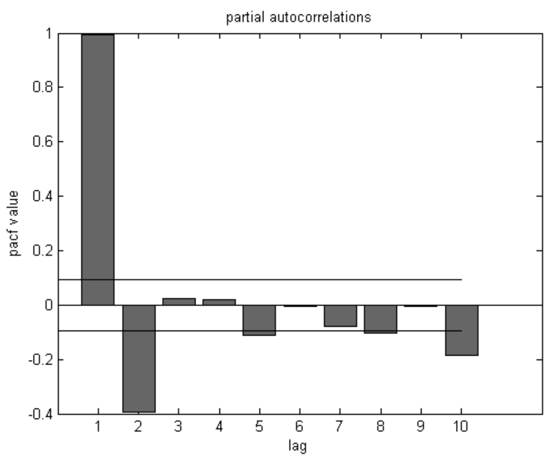
$\pi_{t+h} \sim N\left(\alpha_{\pi,t+h} + X_t' \beta^{(m+1)}, \exp\left(\log \sigma_{t+h}^{(m+1)} + \frac{1}{2}\sigma_{\eta}^2\right)\right)$.

Step 7

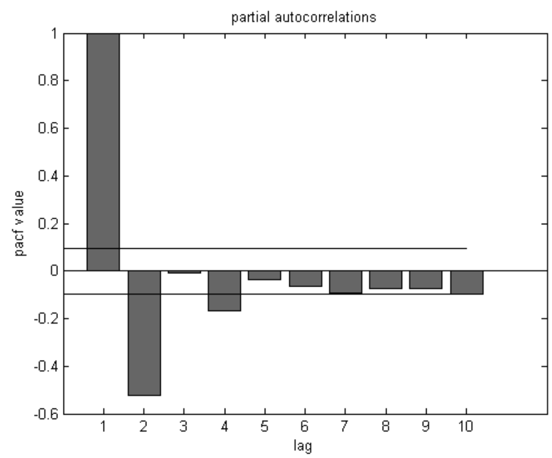
Set $m = m + 1$ and go to step 2.

Appendix D

Applying the models

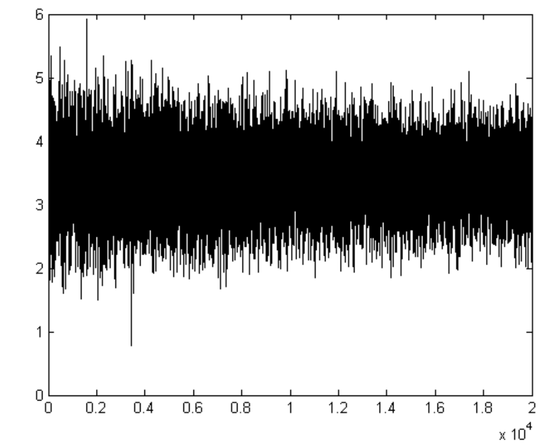


(a) 12-month growth rate

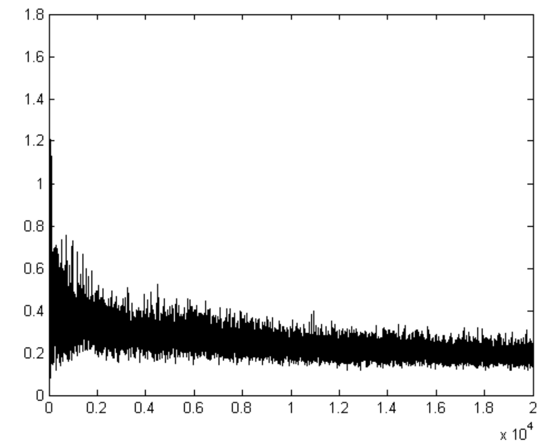


(b) 24-month growth rate

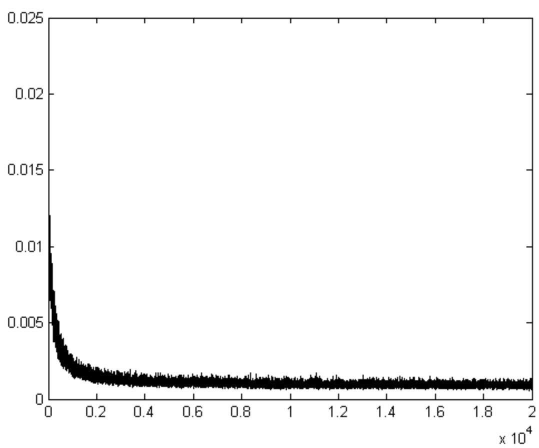
Figure D.1: Partial autocorrelation plots of inflation.



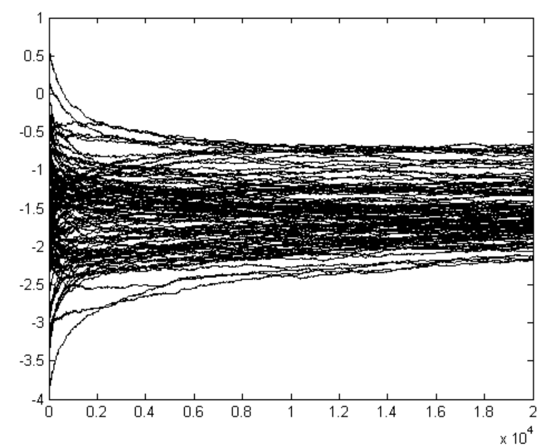
(a) $\hat{\pi}_{t+h}$



(b) $\hat{\sigma}^2_{t+h}$



(c) σ^2_η

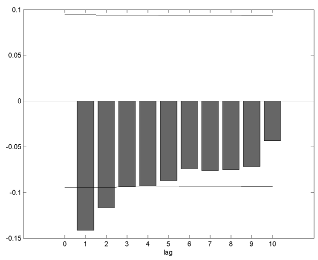


(d) $\log \sigma^2_0, \ldots, \log \sigma^2_T$

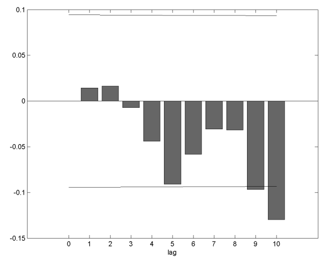
Figure D.2: Draws of different parameters in the sampler.

Appendix E

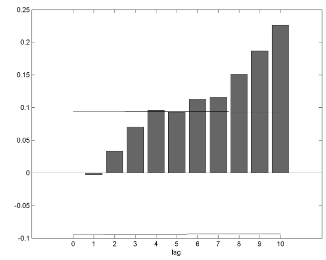
Analyzing the results



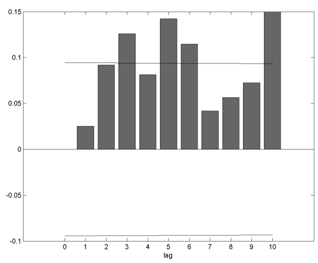
(a) pc1



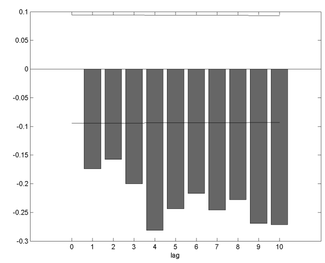
(b) pc2



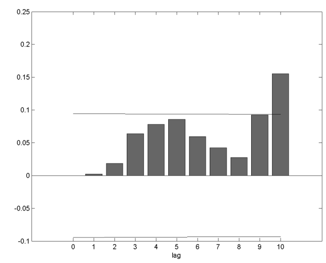
(c) pc3



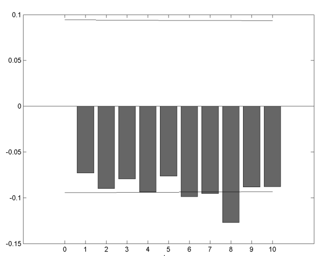
(d) pc4



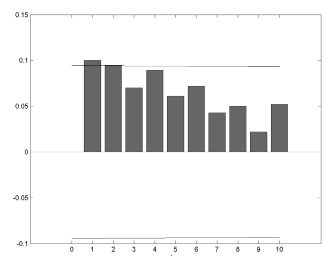
(e) pc5



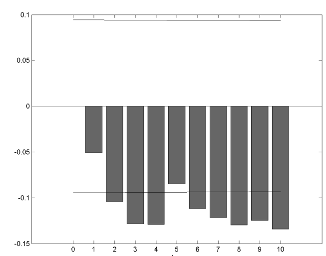
(f) pc6



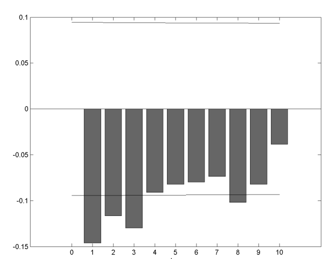
(g) pc7



(h) pc8

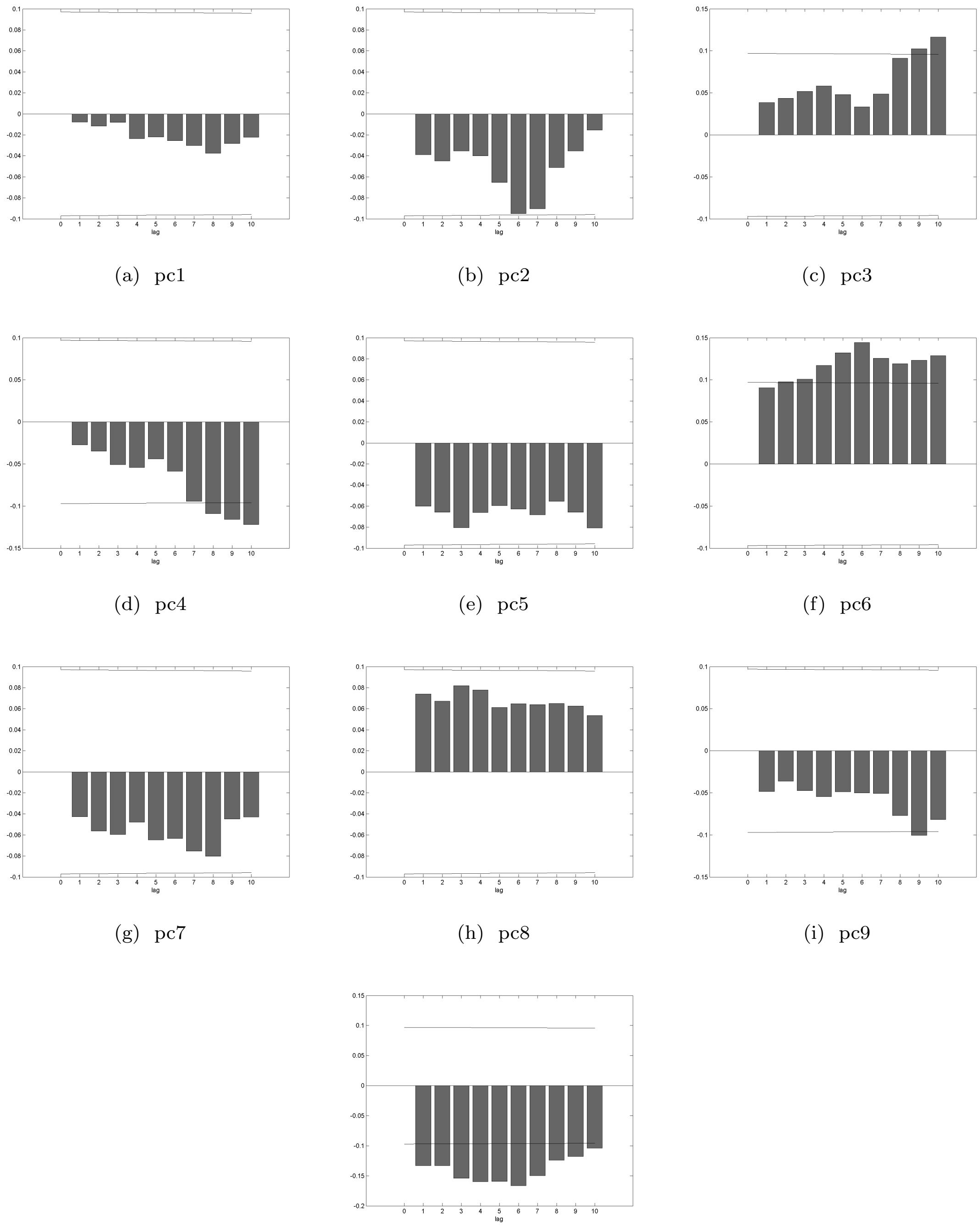


(i) pc9



(j) pc10

Figure E.1: Cross correlations plots of the principal components with the 12-month growth rate of inflation.



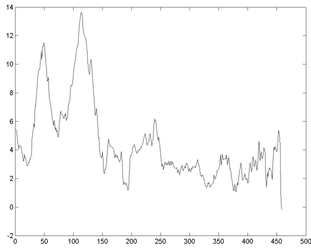
(a) pc1 (b) pc2 (c) pc3

(d) pc4 (e) pc5 (f) pc6

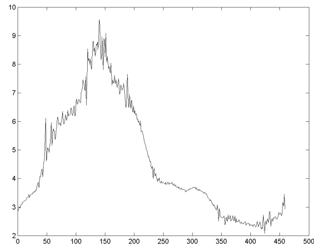
(g) pc7 (h) pc8 (i) pc9

(j) pc10

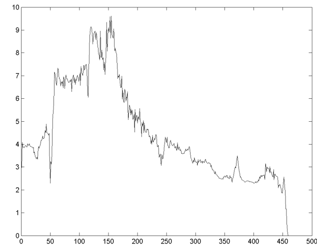
Figure E.2: Cross correlations plots of the principal components with the 24-month growth rate of inflation.



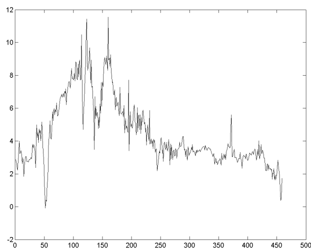
(a) RW



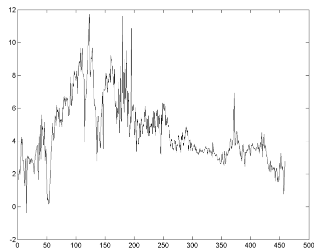
(b) AR(2)



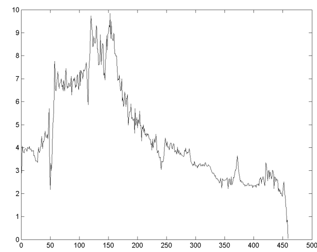
(c) PCR(1,0,0)



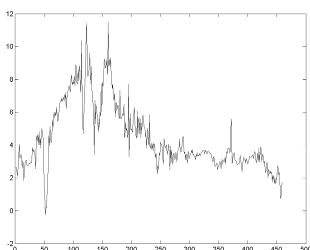
(d) PCR(5,0,0)



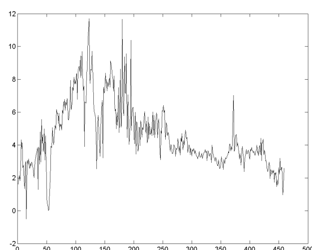
(e) PCR(10,0,0)



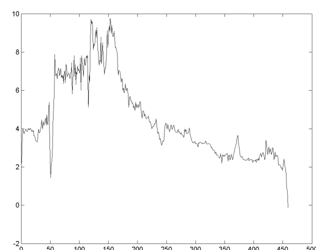
(f) PCR(1,0,2)



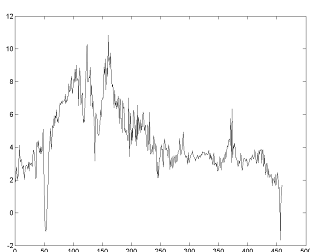
(g) PCR(5,0,2)



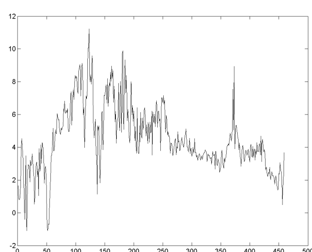
(h) PCR(10,0,2)



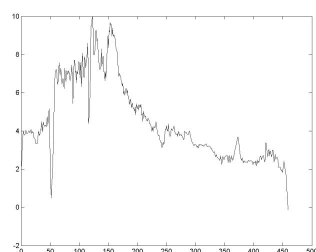
(i) PCR(1,1,2)



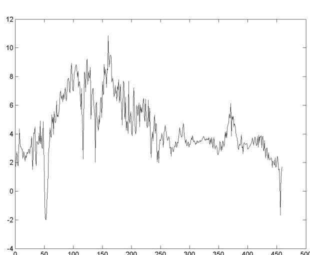
(j) PCR(5,1,2)



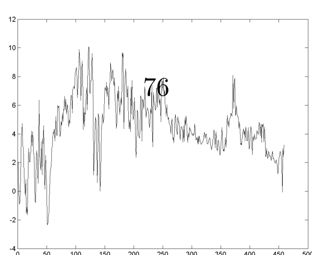
(k) PCR(10,1,2)



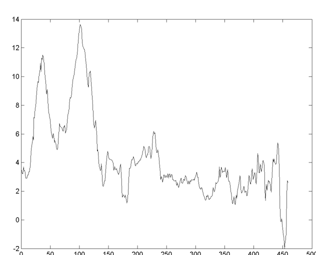
(l) PCR(1,2,2)



(m) PCR(5,2,2)

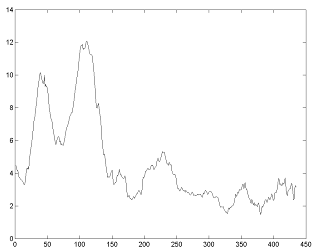


(n) PCR(10,2,2)

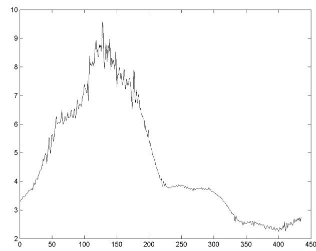


(o) 12-month growth rate of inflation

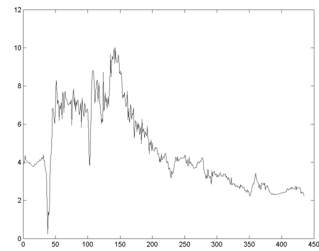
Figure E.3: Estimated mean for $h = 12$.



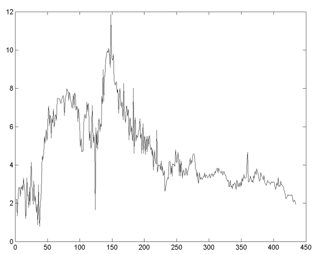
(a) RW



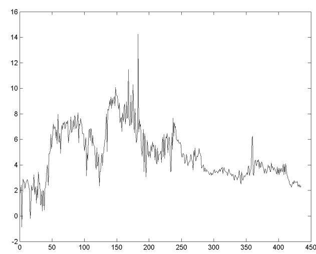
(b) AR(2)



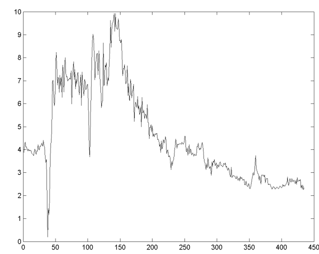
(c) PCR(1,0,0)



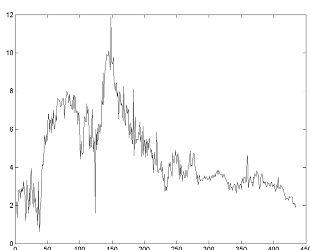
(d) PCR(5,0,0)



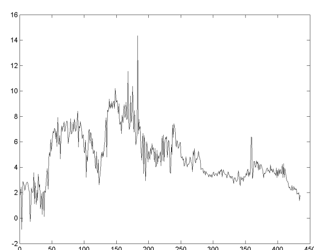
(e) PCR(10,0,0)



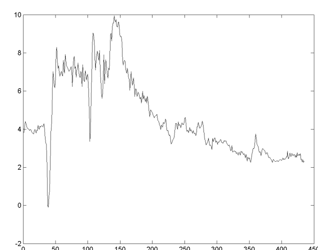
(f) PCR(1,0,2)



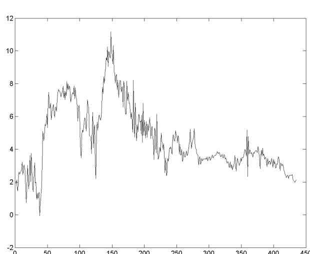
(g) PCR(5,0,2)



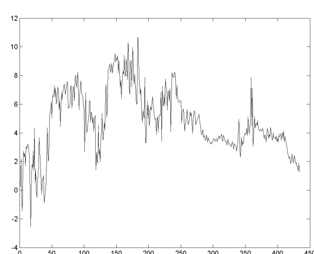
(h) PCR(10,0,2)



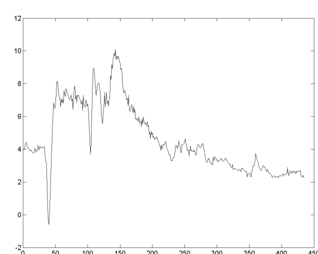
(i) PCR(1,1,2)



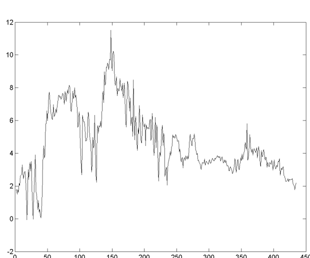
(j) PCR(5,1,2)



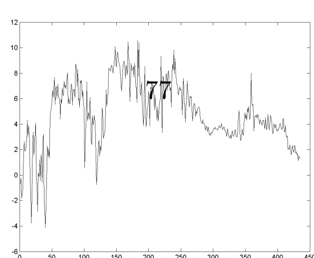
(k) PCR(10,1,2)



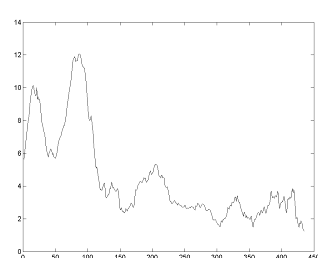
(l) PCR(1,2,2)



(m) PCR(5,2,2)

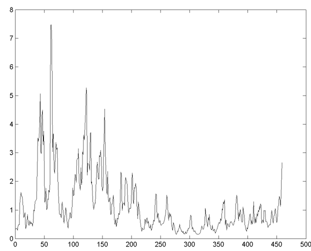


(n) PCR(10,2,2)

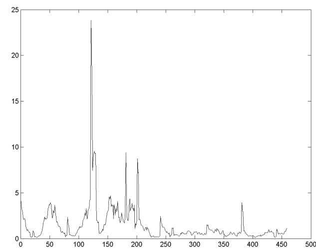


(o) 24-month growth rate of inflation

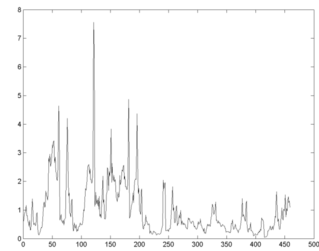
Figure E.4: Estimated mean for $h = 24$.



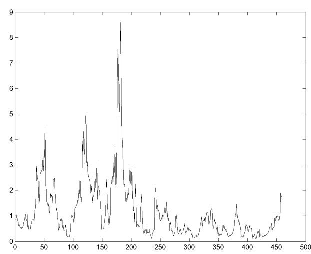
(a) RW



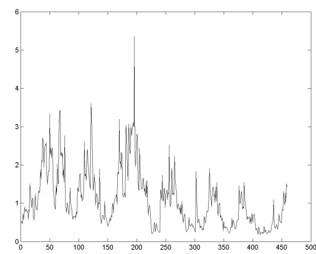
(b) AR(2)



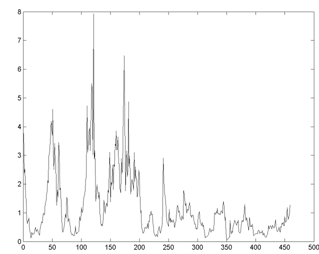
(c) PCR(1,0,0)



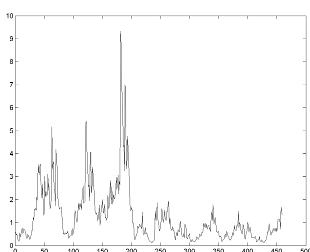
(d) PCR(5,0,0)



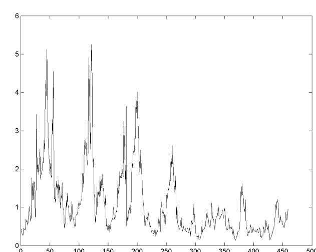
(e) PCR(10,0,0)



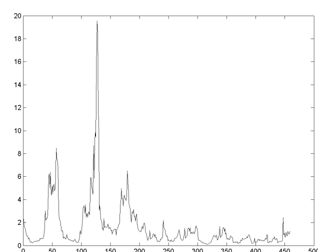
(f) PCR(1,0,2)



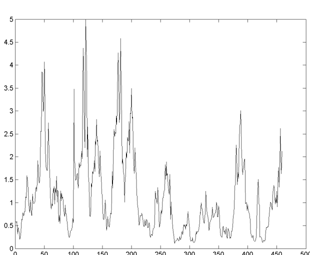
(g) PCR(5,0,2)



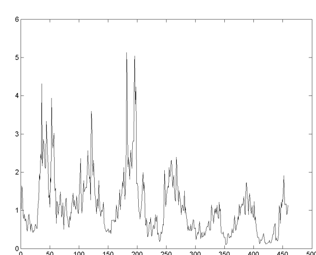
(h) PCR(10,0,2)



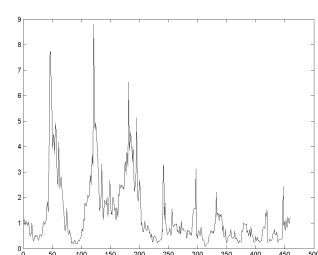
(i) PCR(1,1,2)



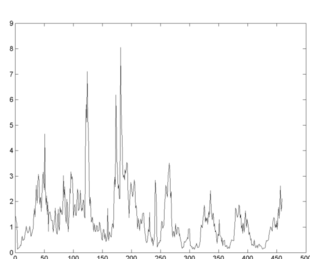
(j) PCR(5,1,2)



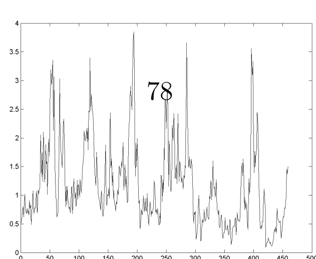
(k) PCR(10,1,2)



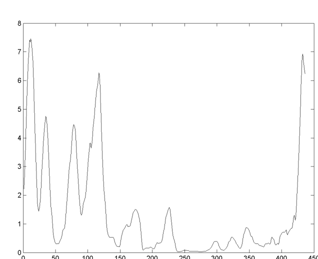
(l) PCR(1,2,2)



(m) PCR(5,2,2)

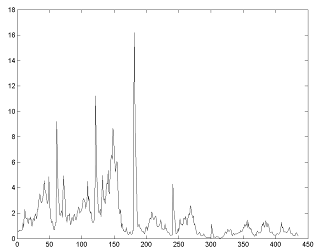


(n) PCR(10,2,2)

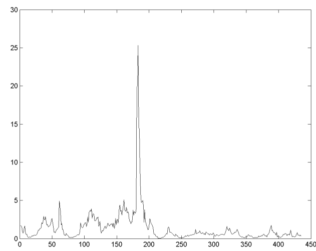


(o) Two years sample variance

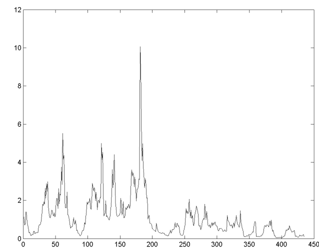
Figure E.5: Estimated variance for $h = 12$.



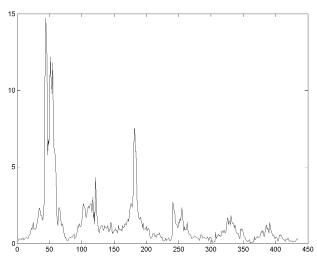
(a) RW



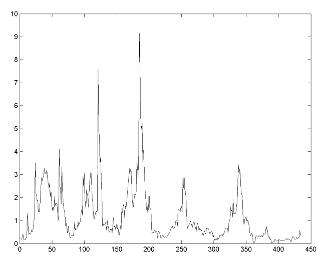
(b) AR(2)



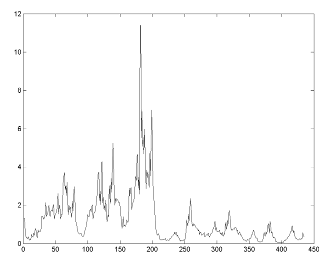
(c) PCR(1,0,0)



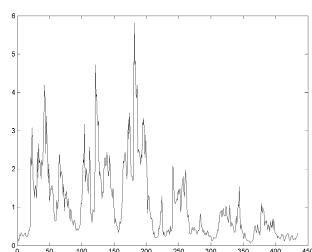
(d) PCR(5,0,0)



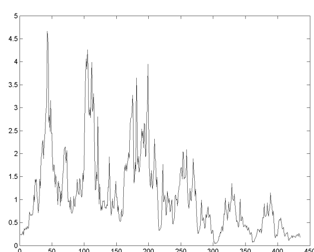
(e) PCR(10,0,0)



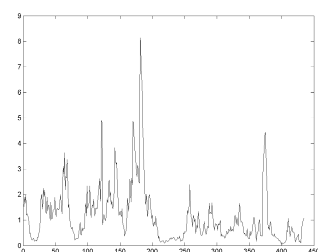
(f) PCR(1,0,2)



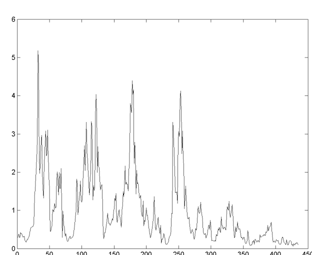
(g) PCR(5,0,2)



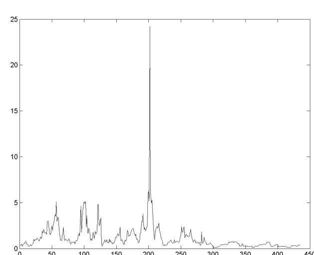
(h) PCR(10,0,2)



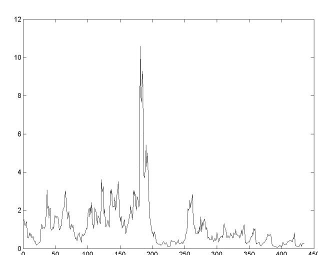
(i) PCR(1,1,2)



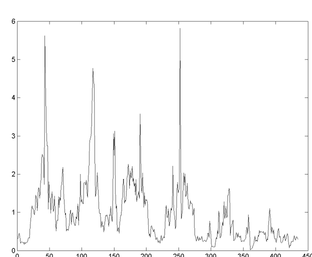
(j) PCR(5,1,2)



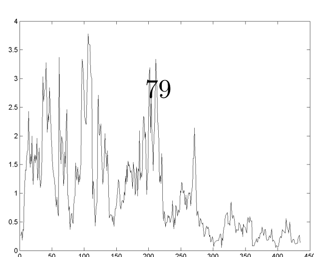
(k) PCR(10,1,2)



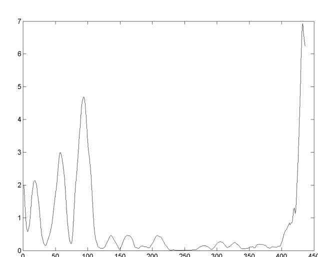
(l) PCR(1,2,2)



(m) PCR(5,2,2)

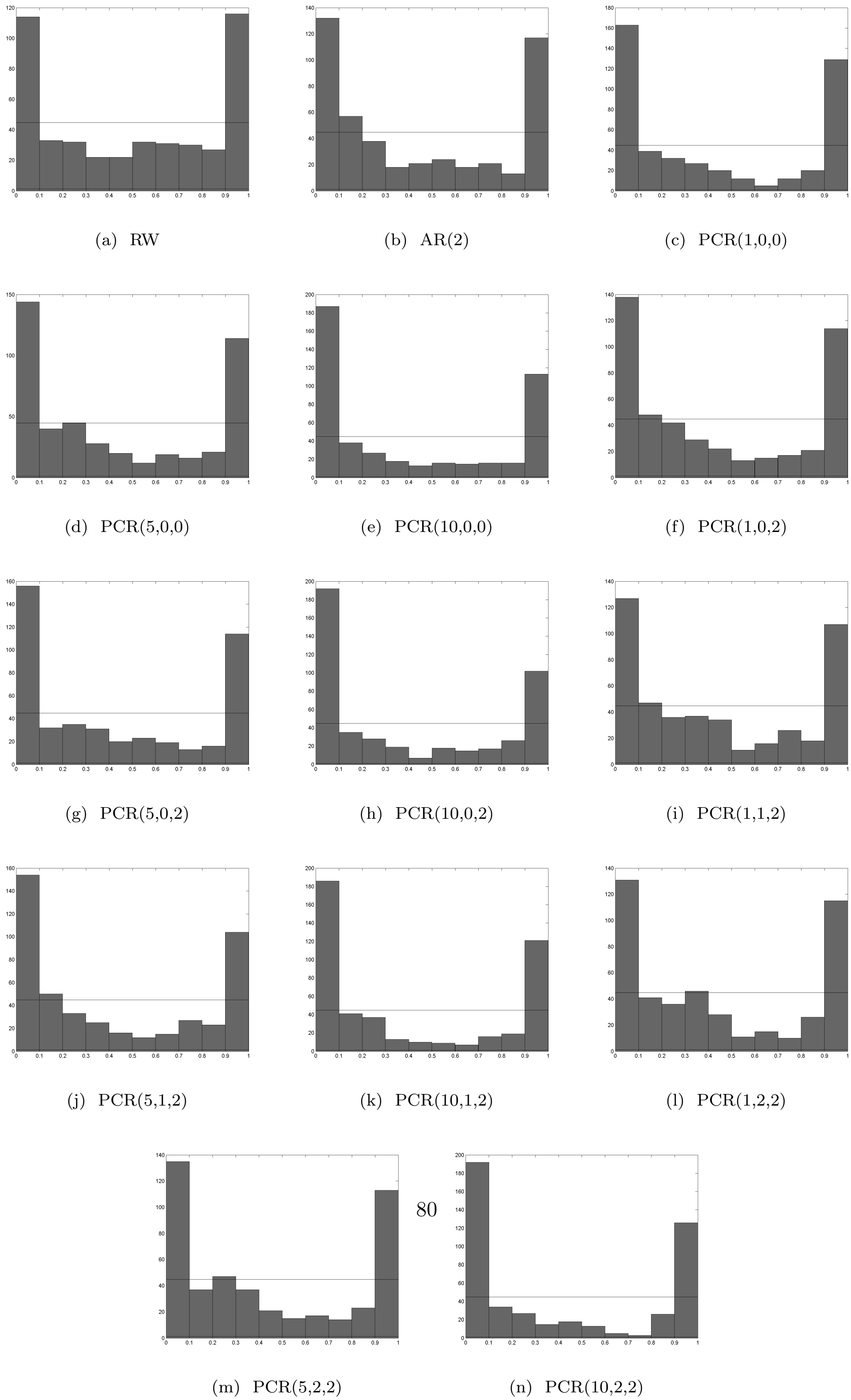


(n) PCR(10,2,2)



(o) Two years sample variance

Figure E.6: Estimated variance for $h = 24$.



(a) RW (b) AR(2) (c) PCR(1,0,0)

(d) PCR(5,0,0) (e) PCR(10,0,0) (f) PCR(1,0,2)

(g) PCR(5,0,2) (h) PCR(10,0,2) (i) PCR(1,1,2)

(j) PCR(5,1,2) (k) PCR(10,1,2) (l) PCR(1,2,2)

(m) PCR(5,2,2) (n) PCR(10,2,2)

Figure E.7: Histogram of the probability integral transforms with $h = 12$.

Table E.1: Berkowitz Likelihood Ratios and p-values for the models evaluated with a $h = 12$.

Model	LR	p-value
RW	1,12E+03	0
AR(2)	1,92E+03	0
PCR(1,0,0)	2,20E+03	0
PCR(5,0,0)	2,79E+03	0
PCR(10,0,0)	2,46E+03	0
PCR(1,0,2)	2,21E+03	0
PCR(5,0,2)	1,98E+03	0
PCR(10,0,2)	3,78E+03	0
PCR(1,1,2)	2,46E+03	0
PCR(5,1,2)	3,16E+03	0
PCR(10,1,2)	4,15E+03	0
PCR(1,2,2)	2,17E+03	0
PCR(5,2,2)	4,33E+03	0
PCR(10,2,2)	5,11E+03	0

Table E.2: KLIC statistics for $h = 12$.

Model	RW	AR(2)	PCR(1,0,0)	PCR(5,0,0)	PCR(10,0,0)	PCR(1,0,2)	PCR(5,0,2)
RW	-	-1,39	-1,84	-1,74	-1,25	-1,91	-1,71
AR(2)	1,39	-	-0,48	0,8	1,1	0,82	1,18
PCR(1,0,0)	1,84	0,48	-	1,28	1,57	1,35	1,74
PCR(5,0,0)	1,74	-0,8	-1,28	-	1,24	-0,66	0,74
PCR(10,0,0)	1,25	-1,1	-1,57	-1,24	-	-1,26	-0,73
PCR(1,0,2)	1,91	-0,82	-1,35	0,66	1,26	-	1,49
PCR(5,0,2)	1,71	-1,18	-1,74	-0,74	0,73	-1,49	-
PCR(10,0,2)	1,21	-0,8	-1,18	-0,49	0,85	-0,67	0,05
PCR(1,1,2)	1,75	-0,93	-1,39	-0,15	0,91	-0,99	0,58
PCR(5,1,2)	1,43	-0,85	-1,31	-0,7	1,07	-0,77	0,16
PCR(10,1,2)	2,04	-0,68	-1,23	0,19	1,8	-0,35	0,87
PCR(1,2,2)	1,49	-0,74	-1,24	0,51	1	-0,38	0,85
PCR(5,2,2)	1,28	-0,63	-0,97	0,03	1	-0,34	0,43
PCR(10,2,2)	1,75	-0,57	-1,06	0,36	1,52	-0,2	0,9

Model	PCR(10,0,2)	PCR(1,1,2)	PCR(5,1,2)	PCR(10,1,2)	PCR(1,2,2)	PCR(5,2,2)	PCR(10,2,2)
RW	-1,21	-1,75	-1,43	-2,04	-1,49	-1,28	-1,75
AR(2)	0,8	0,93	0,85	0,68	0,74	0,63	0,57
PCR(1,0,0)	1,18	1,39	1,31	1,23	1,24	0,97	1,06
PCR(5,0,0)	0,49	0,15	0,7	-0,19	-0,51	-0,03	-0,36
PCR(10,0,0)	-0,85	-0,91	-1,07	-1,8	-1	-1	-1,52
PCR(1,0,2)	0,67	0,99	0,77	0,35	0,38	0,34	0,2
PCR(5,0,2)	-0,05	-0,58	-0,16	-0,87	-0,85	-0,43	-0,9
PCR(10,0,2)	-	-0,27	-0,14	-0,61	-0,51	-0,86	-1,1
PCR(1,1,2)	0,27	-	0,27	-0,26	-0,6	-0,1	-0,35
PCR(5,1,2)	0,14	-0,27	-	-0,6	-0,61	-0,49	-0,85
PCR(10,1,2)	0,61	0,26	0,6	-	-0,11	0,13	-0,21
PCR(1,2,2)	0,51	0,6	0,61	0,11	-	0,17	0
PCR(5,2,2)	0,86	0,1	0,49	-0,13	-0,17	-	-0,44
PCR(10,2,2)	1,1	0,35	0,85	0,21	0	0,44	-

Table E.3: KLIC model favors for $h = 12$. One signifies the model in the row is preferred. Two signifies the model in the column is preferred.

Model	RW	AR(2)	PCR(1,0,0)	PCR(5,0,0)	PCR(10,0,0)	PCR(1,0,2)	PCR(5,0,2)
RW	-	1	1	1	1	1	1
AR(2)	2	-	1	2	2	2	2
PCR(1,0,0)	2	2	-	2	2	2	2
PCR(5,0,0)	2	1	1	-	2	1	2
PCR(10,0,0)	2	1	1	1	-	1	1
PCR(1,0,2)	2	1	1	2	2	-	2
PCR(5,0,2)	2	1	1	1	2	1	-
PCR(10,0,2)	2	1	1	1	2	1	2
PCR(1,1,2)	2	1	1	1	2	1	2
PCR(5,1,2)	2	1	1	1	2	1	2
PCR(10,1,2)	2	1	1	2	2	1	2
PCR(1,2,2)	2	1	1	2	2	1	2
PCR(5,2,2)	2	1	1	2	2	1	2
PCR(10,2,2)	2	1	1	2	2	1	2

Model	PCR(10,0,2)	PCR(1,1,2)	PCR(5,1,2)	PCR(10,1,2)	PCR(1,2,2)	PCR(5,2,2)	PCR(10,2,2)
RW	1	1	1	1	1	1	1
AR(2)	2	2	2	2	2	2	2
PCR(1,0,0)	2	2	2	2	2	2	2
PCR(5,0,0)	2	2	2	1	1	1	1
PCR(10,0,0)	1	1	1	1	1	1	1
PCR(1,0,2)	2	2	2	2	2	2	2
PCR(5,0,2)	1	1	1	1	1	1	1
PCR(10,0,2)	-	1	1	1	1	1	1
PCR(1,1,2)	2	-	2	1	1	1	1
PCR(5,1,2)	2	1	-	1	1	1	1
PCR(10,1,2)	2	2	2	-	1	2	1
PCR(1,2,2)	2	2	2	2	-	2	2
PCR(5,2,2)	2	2	2	1	1	-	1
PCR(10,2,2)	2	2	2	2	1	2	-

Table E.4: KLIC probabilities for $h = 12$.

Model	RW	AR(2)	PCR(1,0,0)	PCR(5,0,0)	PCR(10,0,0)	PCR(1,0,2)	PCR(5,0,2)
RW	-	0,15	0,07	0,09	0,18	0,06	0,09
AR(2)	0,15	-	0,36	0,29	0,22	0,29	0,2
PCR(1,0,0)	0,07	0,36	-	0,17	0,12	0,16	0,09
PCR(5,0,0)	0,09	0,29	0,17	-	0,19	0,32	0,3
PCR(10,0,0)	0,18	0,22	0,12	0,19	-	0,18	0,31
PCR(1,0,2)	0,06	0,29	0,16	0,32	0,18	-	0,13
PCR(5,0,2)	0,09	0,2	0,09	0,3	0,31	0,13	-
PCR(10,0,2)	0,19	0,29	0,2	0,35	0,28	0,32	0,4
PCR(1,1,2)	0,09	0,26	0,15	0,39	0,26	0,24	0,34
PCR(5,1,2)	0,14	0,28	0,17	0,31	0,22	0,3	0,39
PCR(10,1,2)	0,05	0,32	0,19	0,39	0,08	0,38	0,27
PCR(1,2,2)	0,13	0,3	0,19	0,35	0,24	0,37	0,28
PCR(5,2,2)	0,18	0,33	0,25	0,4	0,24	0,38	0,36
PCR(10,2,2)	0,09	0,34	0,23	0,37	0,13	0,39	0,27

Model	PCR(10,0,2)	PCR(1,1,2)	PCR(5,1,2)	PCR(10,1,2)	PCR(1,2,2)	PCR(5,2,2)	PCR(10,2,2)
RW	0,19	0,09	0,14	0,05	0,13	0,18	0,09
AR(2)	0,29	0,26	0,28	0,32	0,3	0,33	0,34
PCR(1,0,0)	0,2	0,15	0,17	0,19	0,19	0,25	0,23
PCR(5,0,0)	0,35	0,39	0,31	0,39	0,35	0,4	0,37
PCR(10,0,0)	0,28	0,26	0,22	0,08	0,24	0,24	0,13
PCR(1,0,2)	0,32	0,24	0,3	0,38	0,37	0,38	0,39
PCR(5,0,2)	0,4	0,34	0,39	0,27	0,28	0,36	0,27
PCR(10,0,2)	-	0,38	0,4	0,33	0,35	0,27	0,22
PCR(1,1,2)	0,38	-	0,38	0,39	0,33	0,4	0,37
PCR(5,1,2)	0,4	0,38	-	0,33	0,33	0,35	0,28
PCR(10,1,2)	0,33	0,39	0,33	-	0,4	0,4	0,39
PCR(1,2,2)	0,35	0,33	0,33	0,4	-	0,39	0,4
PCR(5,2,2)	0,27	0,4	0,35	0,4	0,39	-	0,36
PCR(10,2,2)	0,22	0,37	0,28	0,39	0,4	0,36	-

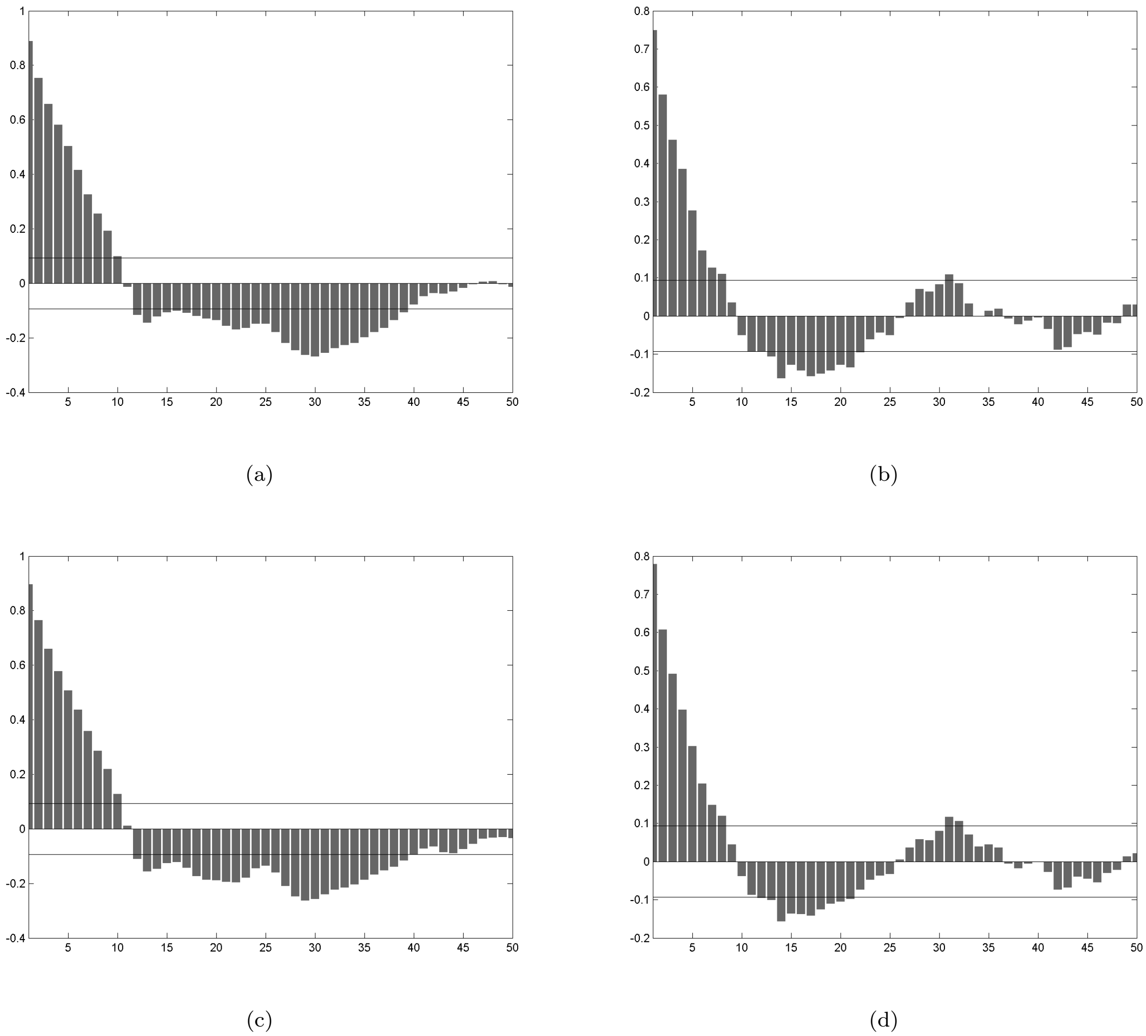
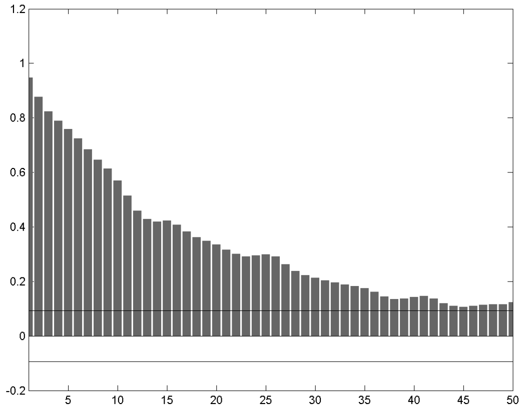
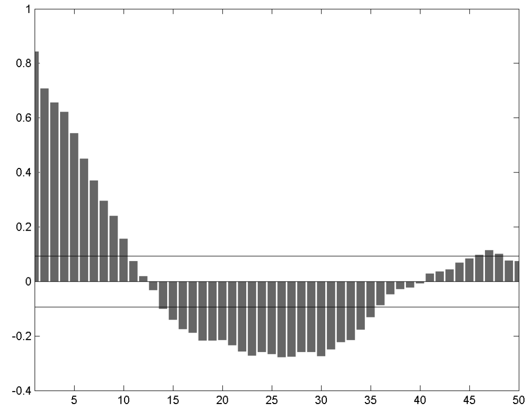


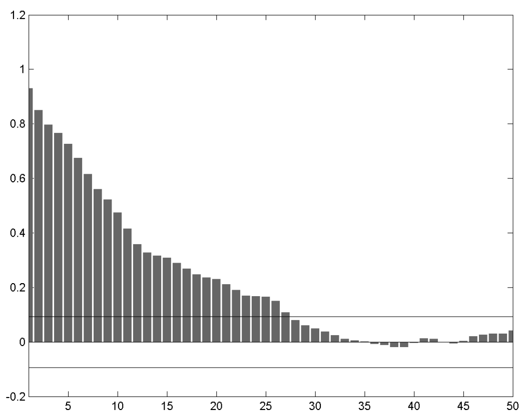
Figure E.8: Panels (a)-(d) show sample autocorrelations of $(z - \overline{z})\,,(z - \overline{z})^2\,,(z - \overline{z})^3$ and $(z - \overline{z})^4$ of RW with $h = 12$.



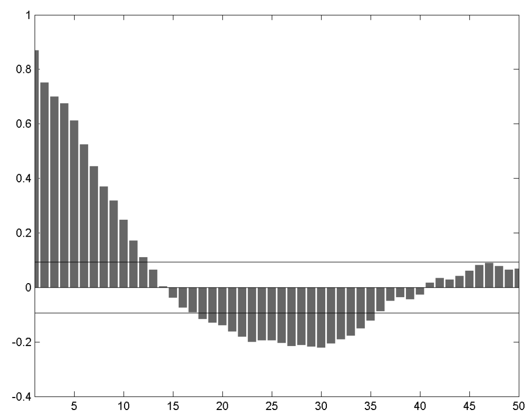
(a)



(b)



(c)



(d)

Figure E.9: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z})\,,(z-\overline{z})^2\,,(z-\overline{z})^3$ and $(z-\overline{z})^4$ of AR(2) with $h=12$.

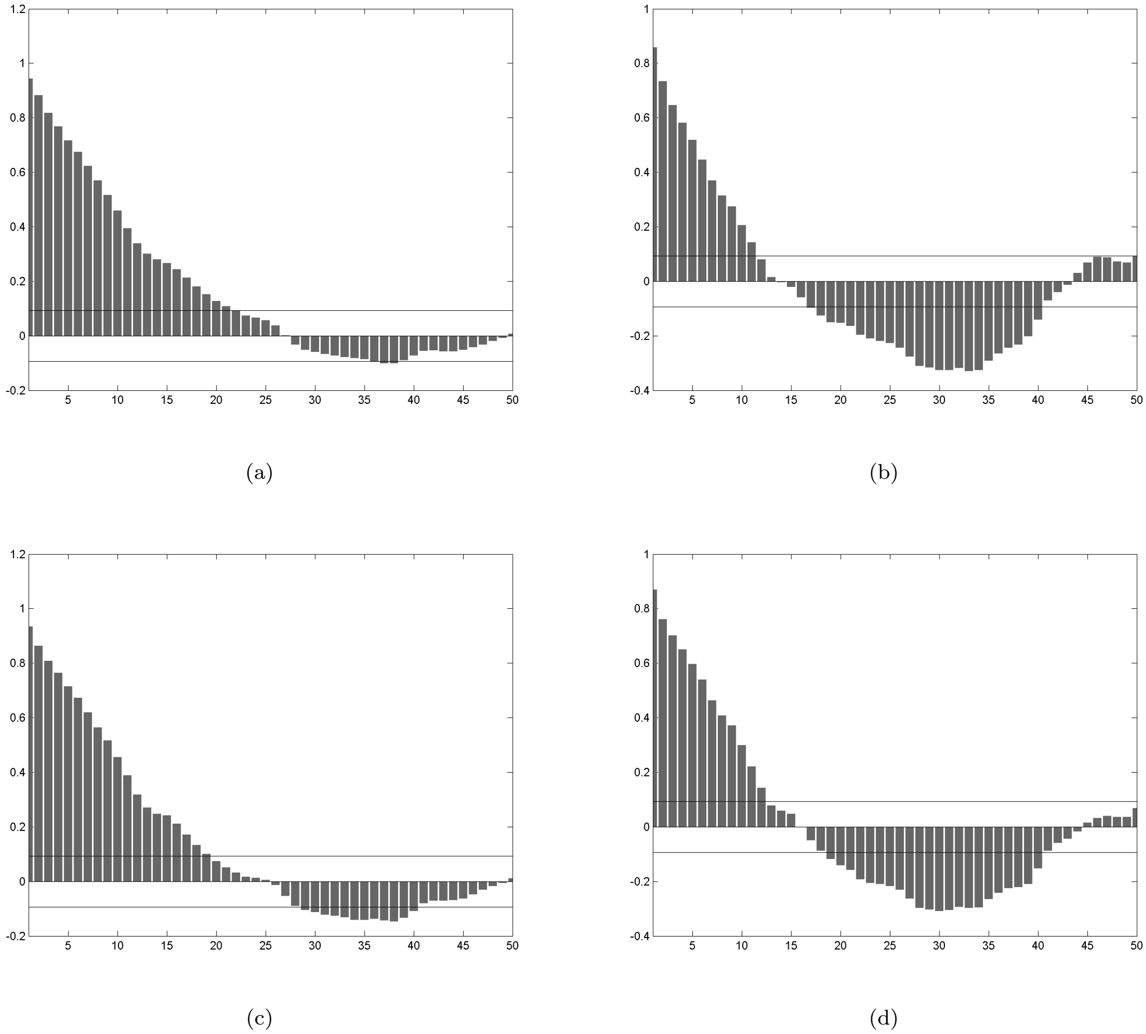


Figure E.10: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z}), (z-\overline{z})^2, (z-\overline{z})^3$ and $(z-\overline{z})^4$ of PCR(1,0,0) with $h = 12$.

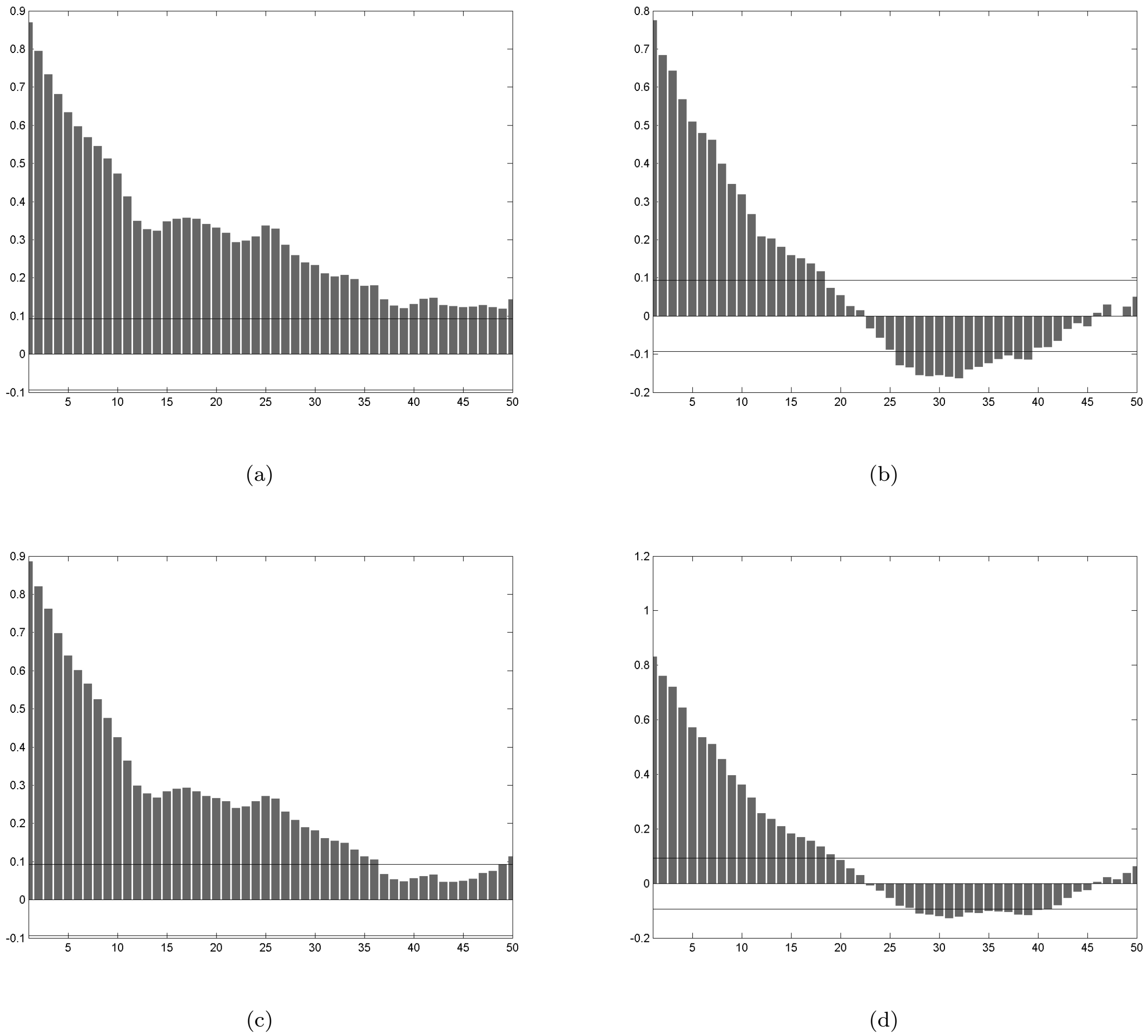


Figure E.11: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z}), (z-\overline{z})^2, (z-\overline{z})^3$ and $(z-\overline{z})^4$ of PCR(5,0,0) with $h = 12$.

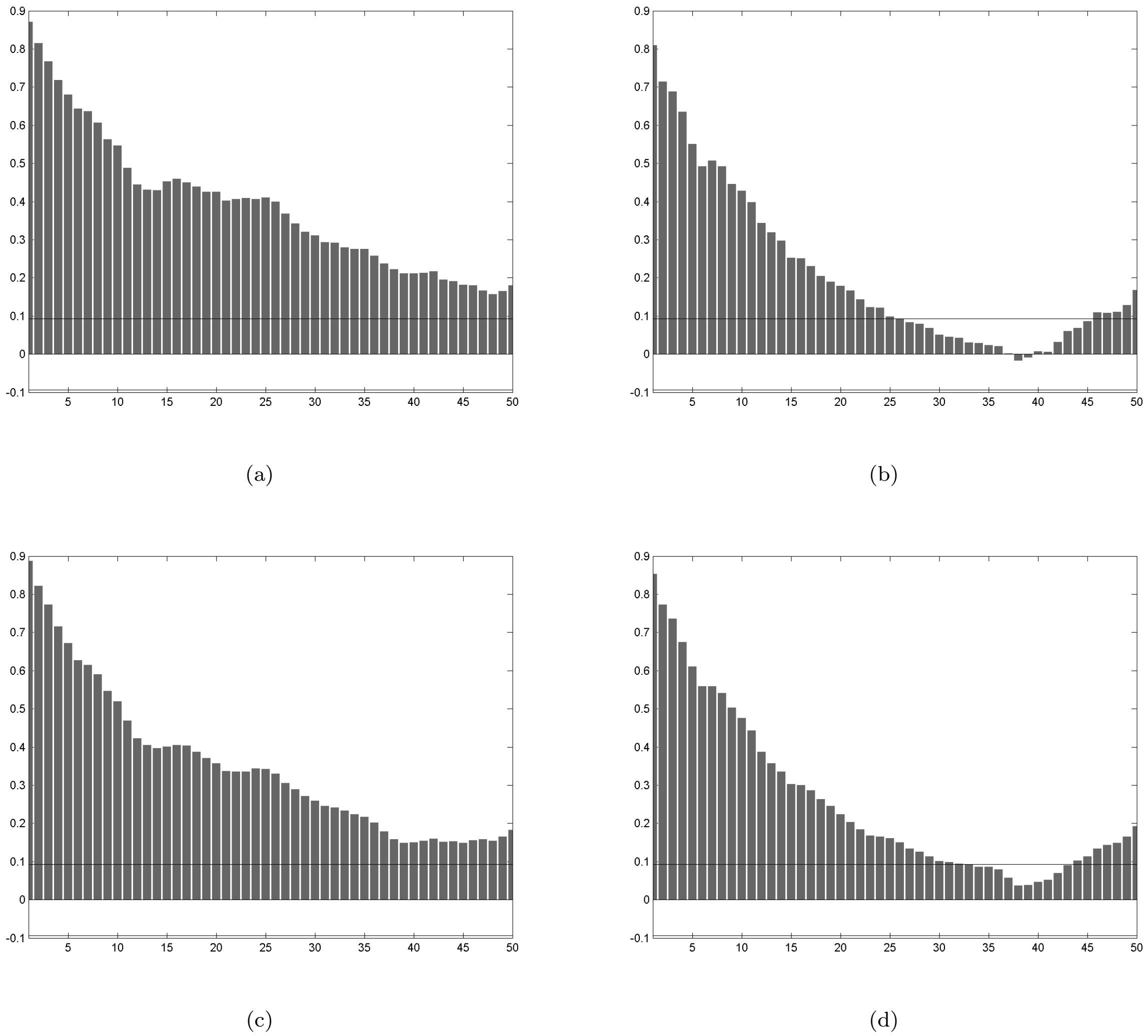


Figure E.12: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z}), (z-\overline{z})^2, (z-\overline{z})^3$ and $(z-\overline{z})^4$ of PCR(10,0,0) with $h = 12$.

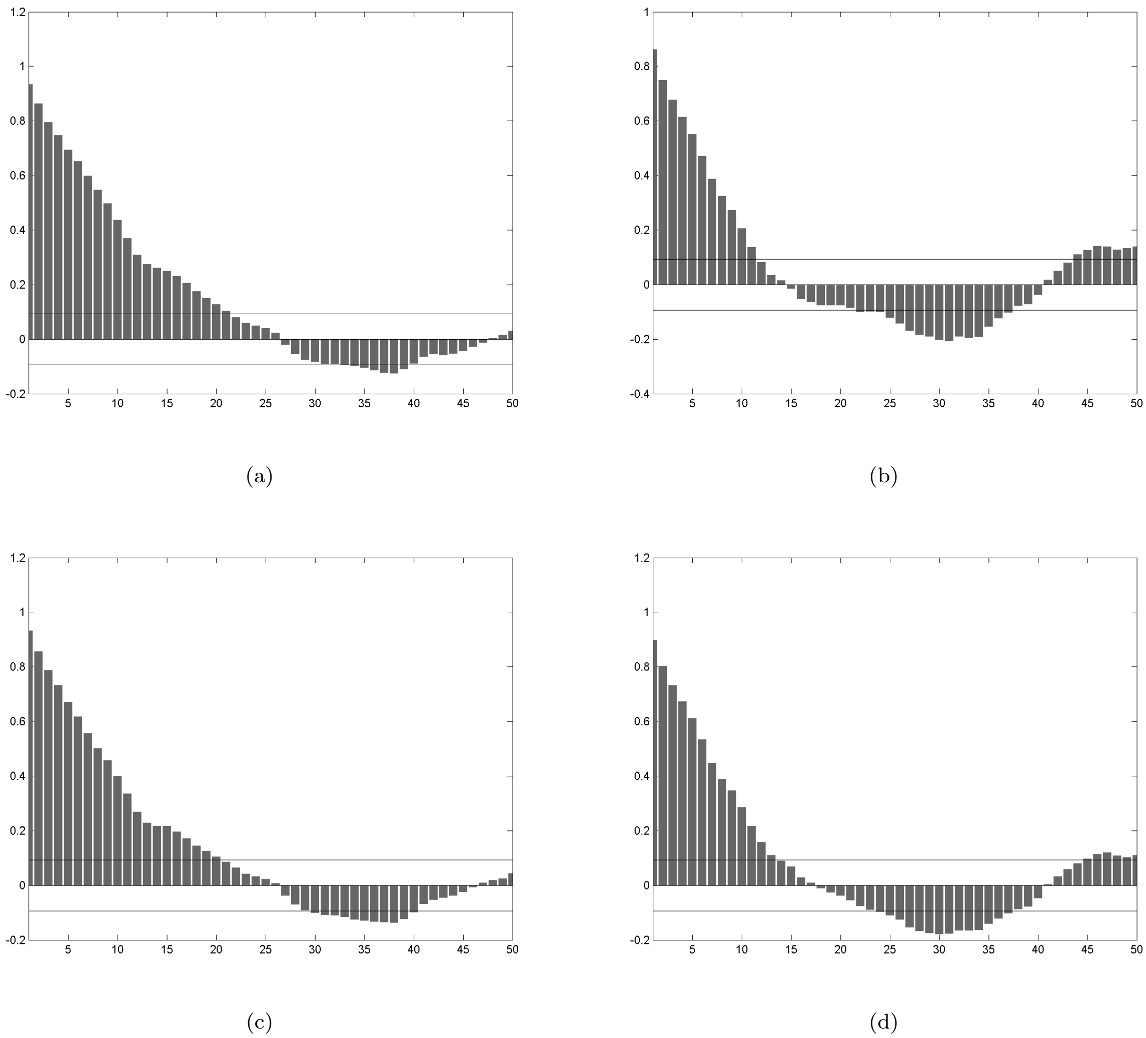


Figure E.13: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z}), (z-\overline{z})^2, (z-\overline{z})^3$ and $(z-\overline{z})^4$ of PCR(1,0,2) with $h=12$.

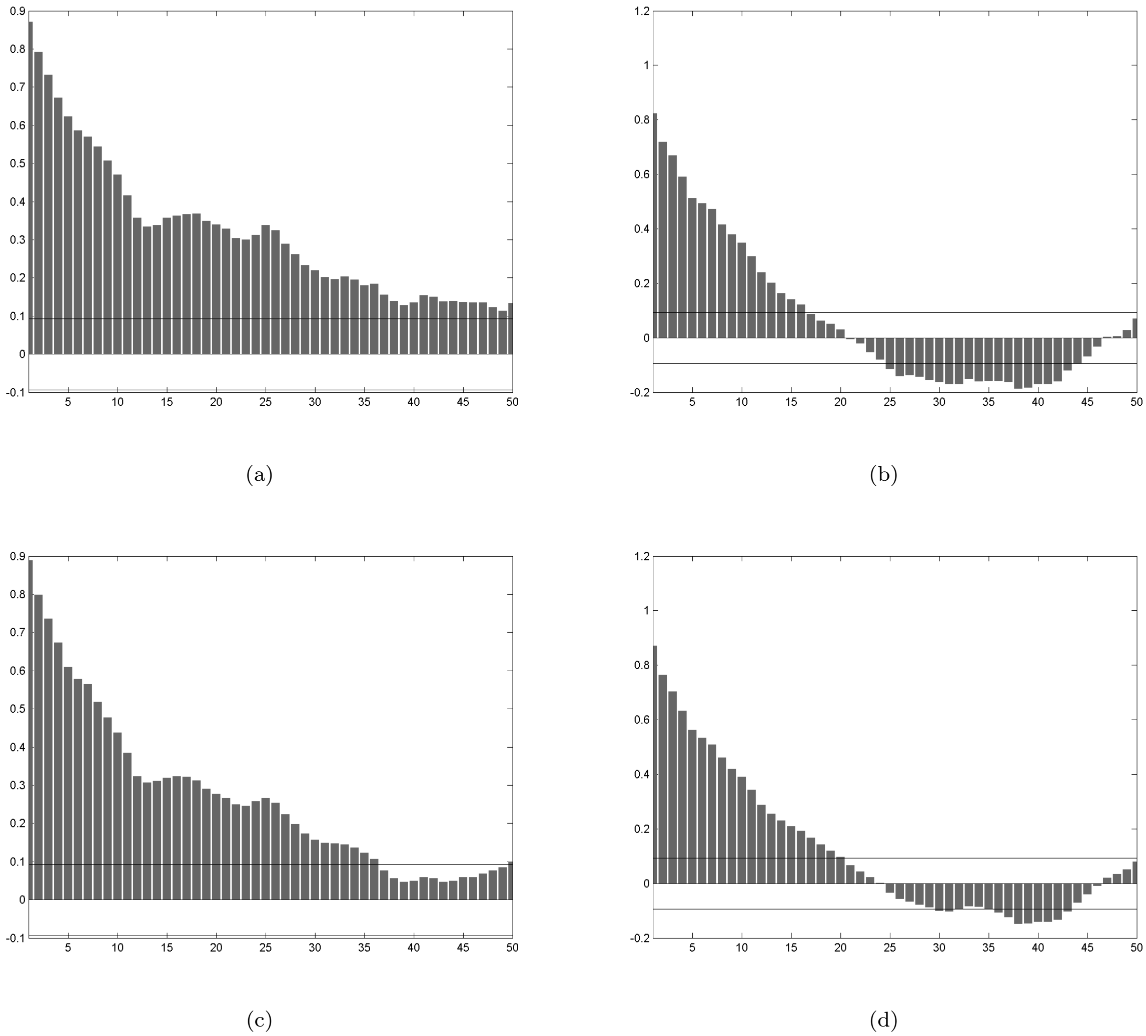


Figure E.14: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z})\,,(z-\overline{z})^2\,,(z-\overline{z})^3$ and $(z-\overline{z})^4$ of PCR(5,0,2) with $h=12$.

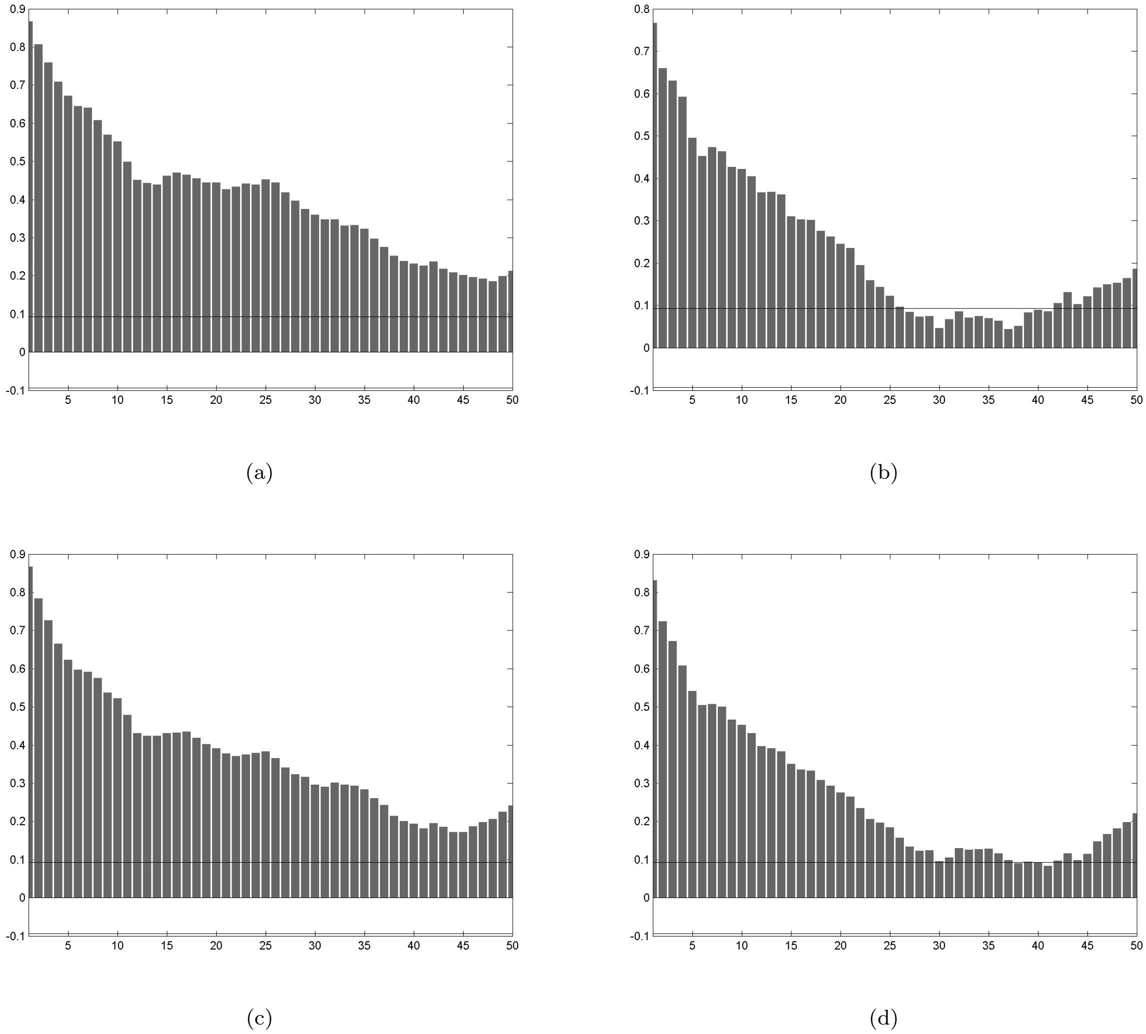


Figure E.15: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z}), (z-\overline{z})^2, (z-\overline{z})^3$ and $(z-\overline{z})^4$ of PCR(10,0,2) with $h = 12$.

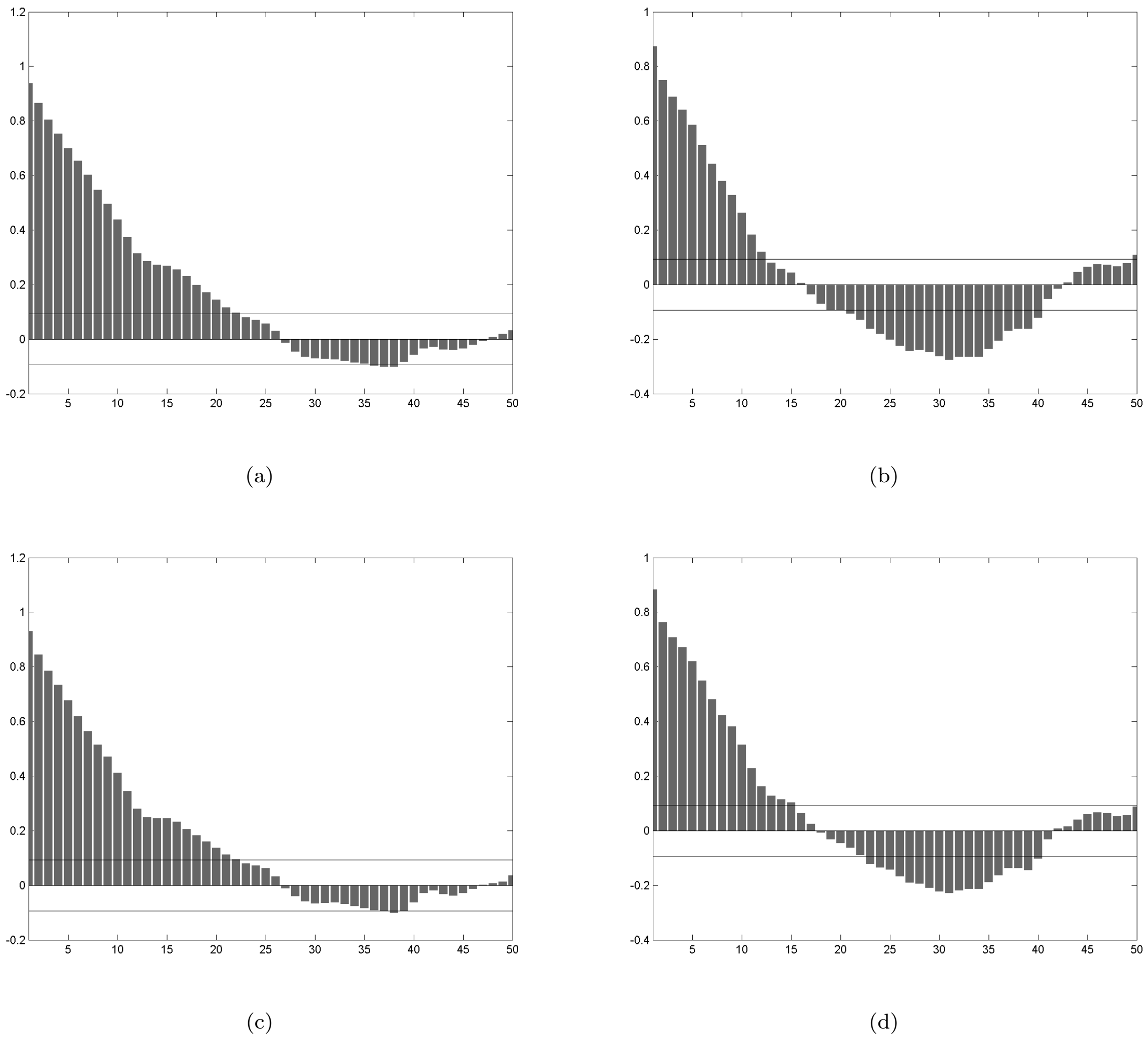


Figure E.16: Panels (a)-(d) show sample autocorrelations of $(z - \overline{z}), (z - \overline{z})^2, (z - \overline{z})^3$ and $(z - \overline{z})^4$ of PCR(1,1,2) with $h = 12$.

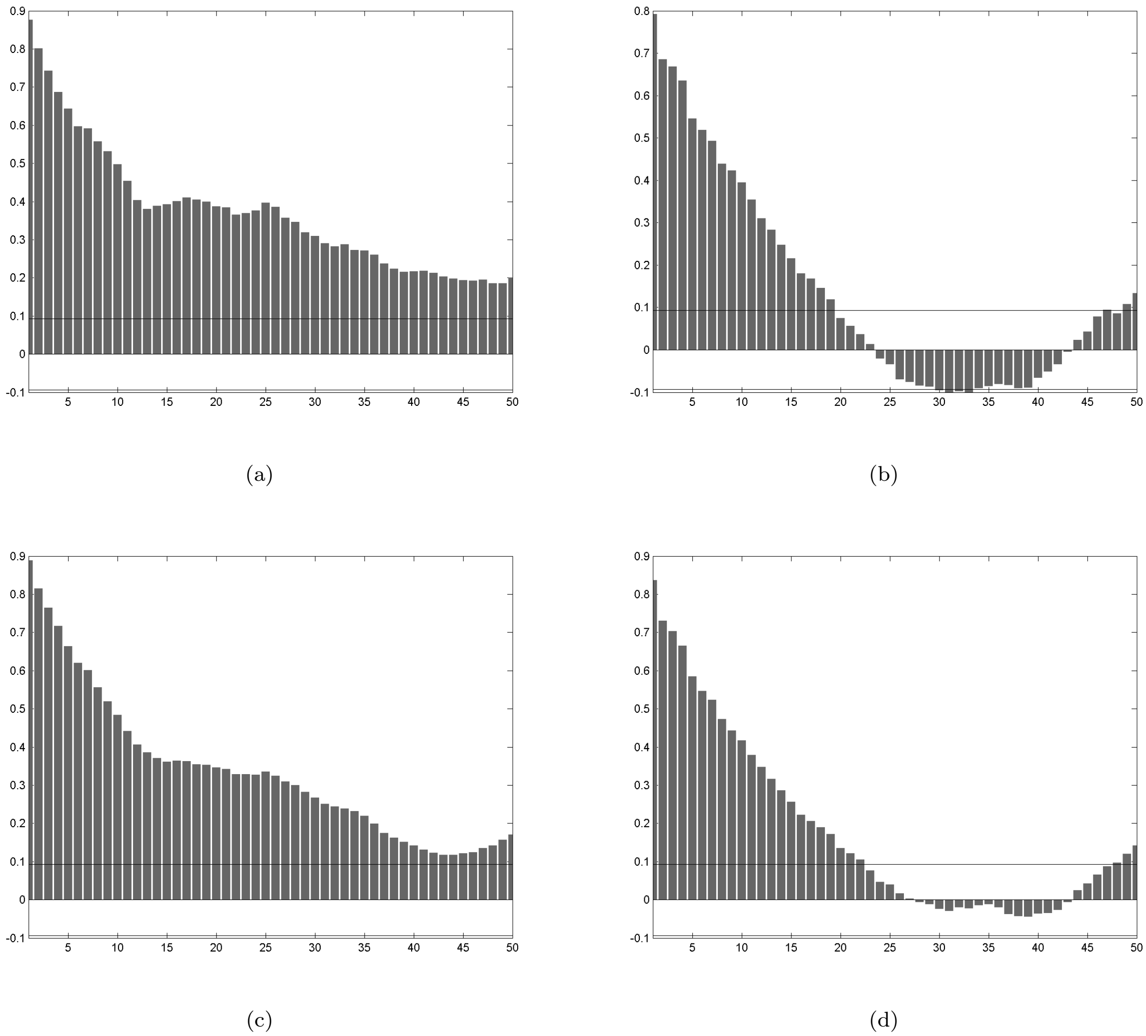


Figure E.17: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z}), (z-\overline{z})^2, (z-\overline{z})^3$ and $(z-\overline{z})^4$ of PCR(5,1,2) with $h=12$.

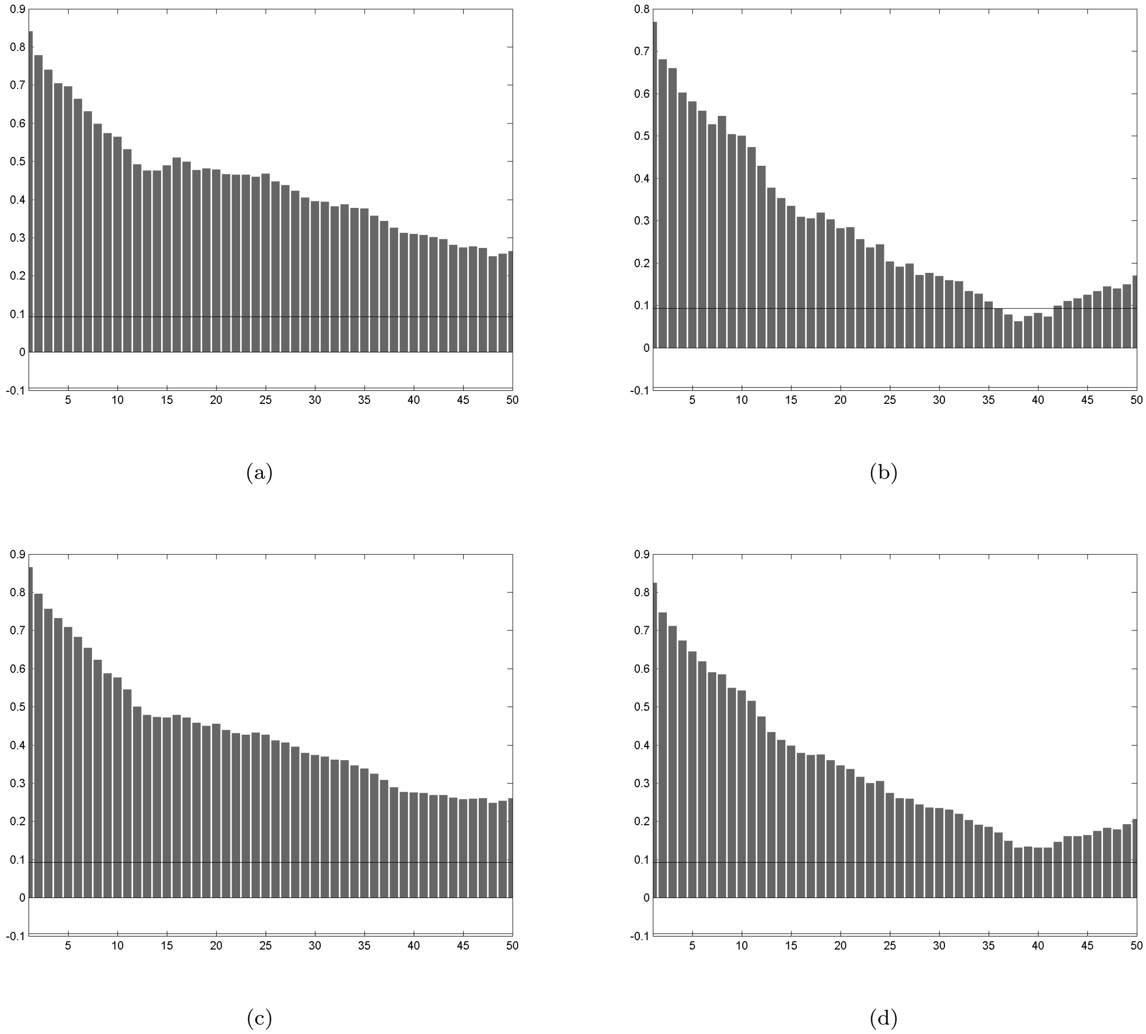
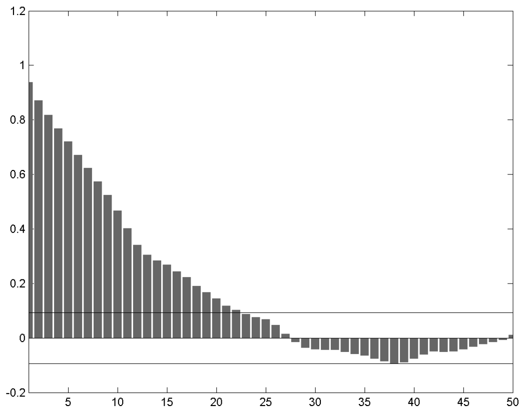
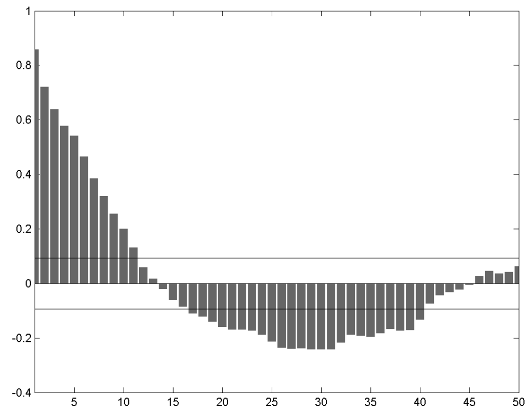


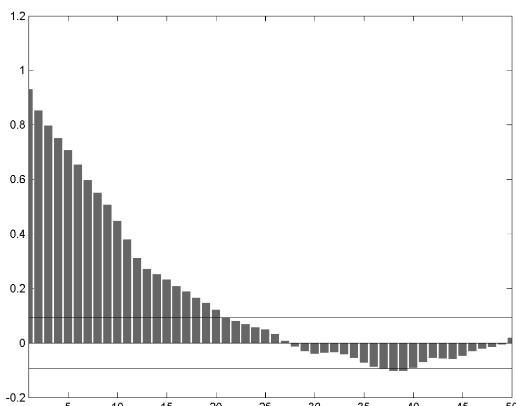
Figure E.18: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z}), (z-\overline{z})^2, (z-\overline{z})^3$ and $(z-\overline{z})^4$ of PCR(10,1,2) with $h = 12$.



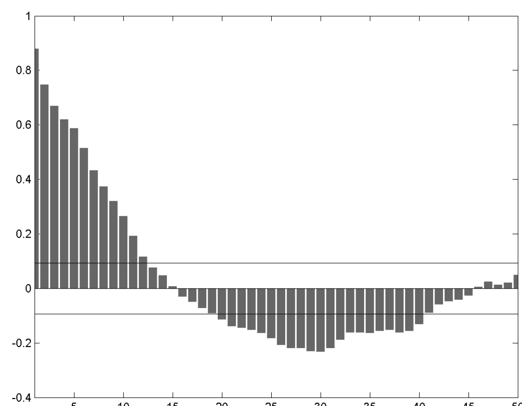
(a)



(b)



(c)



(d)

Figure E.19: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z}),(z-\overline{z})^2,(z-\overline{z})^3$ and $(z-\overline{z})^4$ of PCR(1,2,2) with $h=12$.

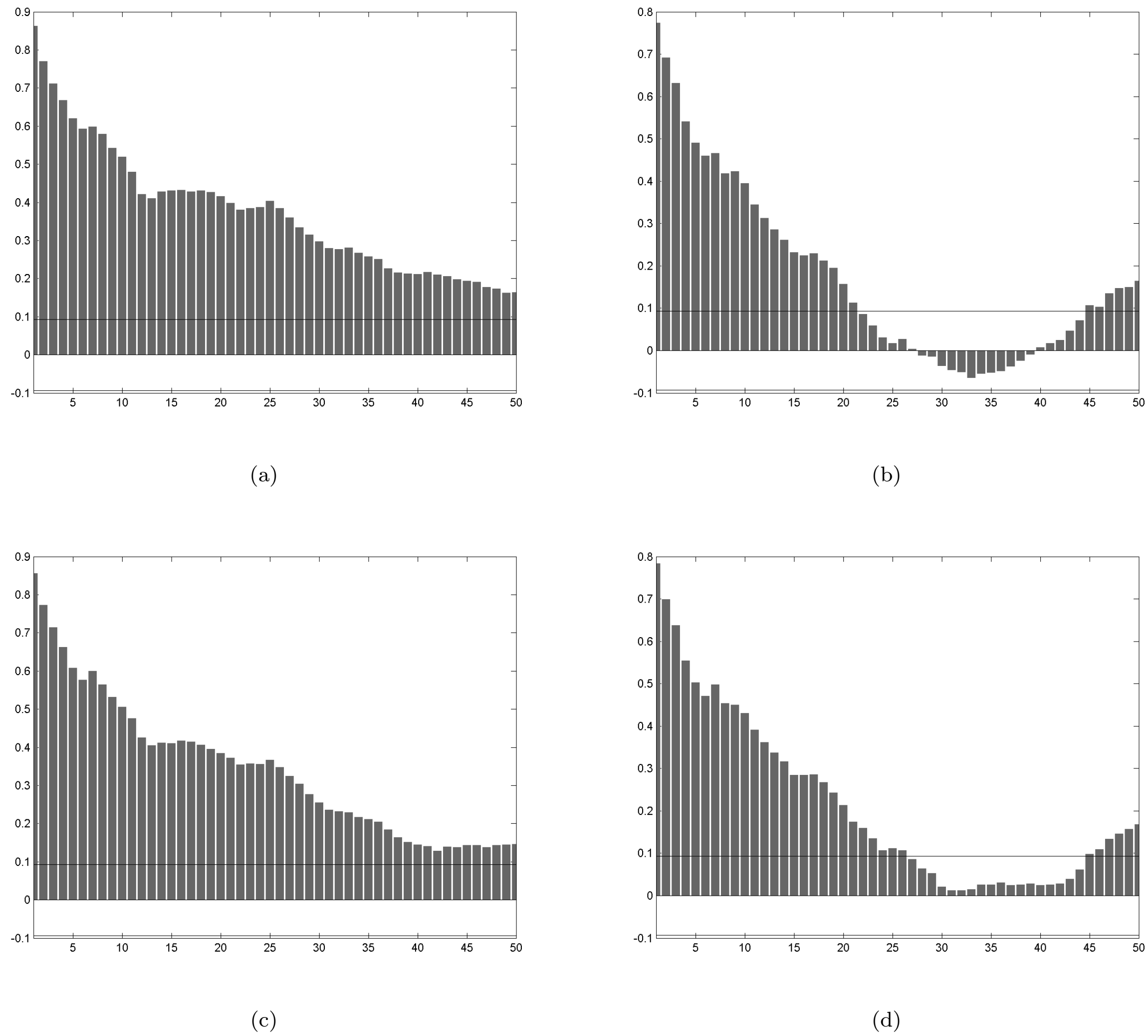


Figure E.20: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z}), (z-\overline{z})^2, (z-\overline{z})^3$ and $(z-\overline{z})^4$ of PCR(5,2,2) with $h = 12$.

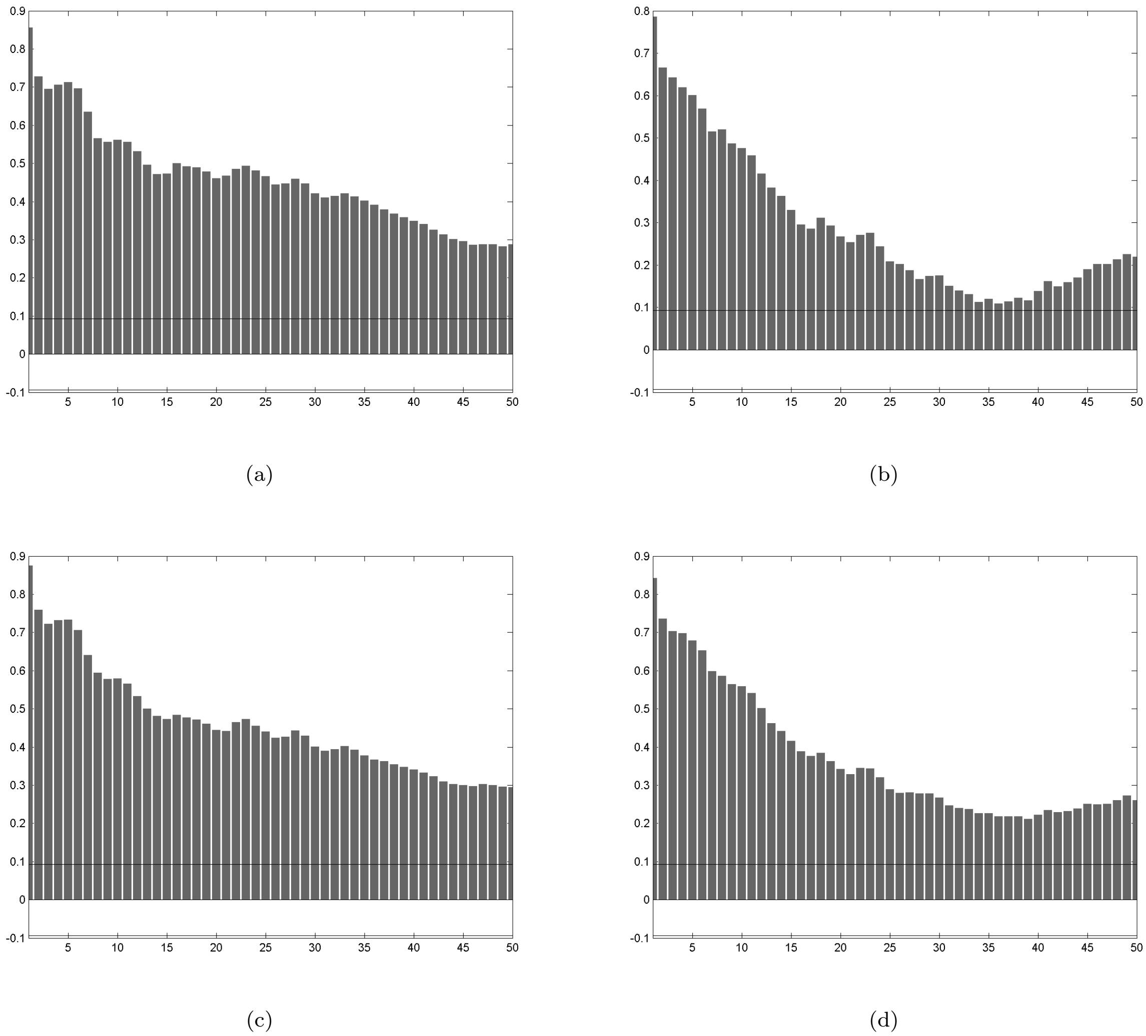
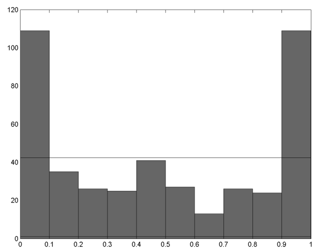


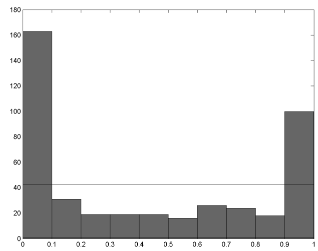
Figure E.21: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z}), (z-\overline{z})^2, (z-\overline{z})^3$ and $(z-\overline{z})^4$ of PCR(10,2,2) with $h = 12$.

Table E.5: Berkowitz Likelihood Ratios and p-values for the models evaluated with $h = 24$.

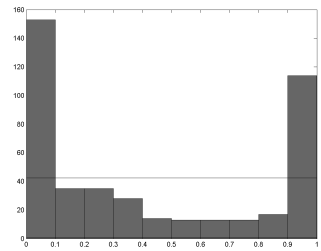
Model	LR	p-value
RW	1,48E+03	0
AR(2)	1,72E+03	0
PCR(1,0,0)	2,37E+03	0
PCR(5,0,0)	5,78E+03	0
PCR(10,0,0)	7,22E+03	0
PCR(1,0,2)	1,95E+03	0
PCR(5,0,2)	5,29E+03	0
PCR(10,0,2)	7,78E+03	0
PCR(1,1,2)	1,91E+03	0
PCR(5,1,2)	7,01E+03	0
PCR(10,1,2)	9,11E+03	0
PCR(1,2,2)	1,78E+03	0
PCR(5,2,2)	6,37E+03	0
PCR(10,2,2)	1,13E+04	0



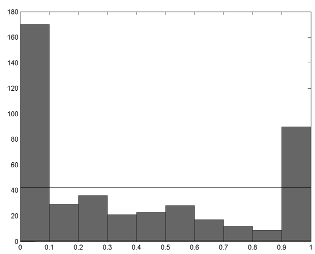
(a) RW



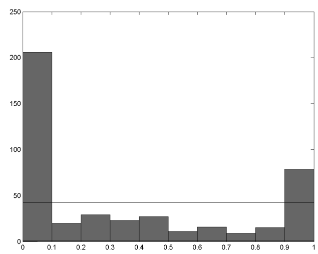
(b) AR(2)



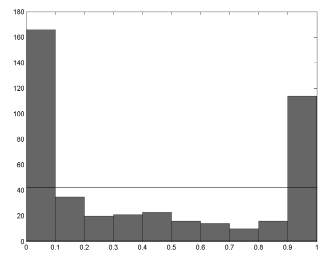
(c) PCR(1,0,0)



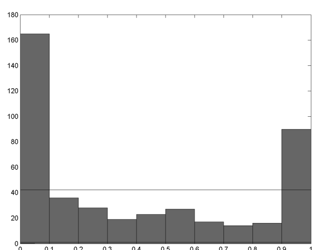
(d) PCR(5,0,0)



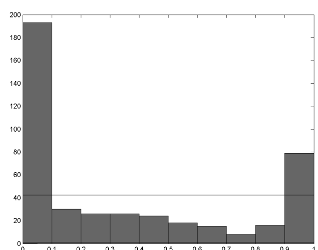
(e) PCR(10,0,0)



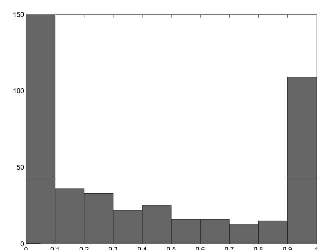
(f) PCR(1,0,2)



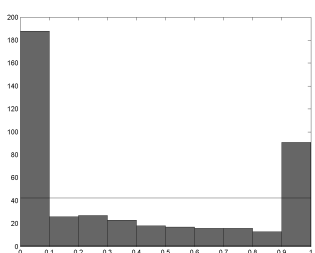
(g) PCR(5,0,2)



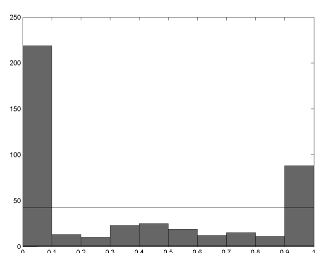
(h) PCR(10,0,2)



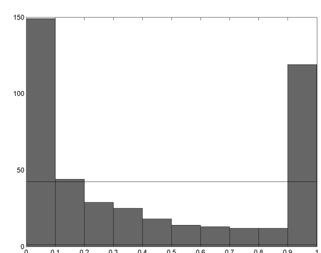
(i) PCR(1,1,2)



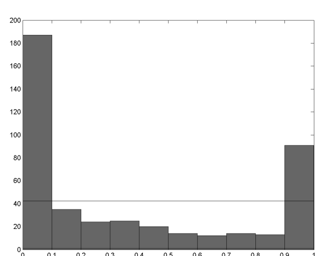
(j) PCR(5,1,2)



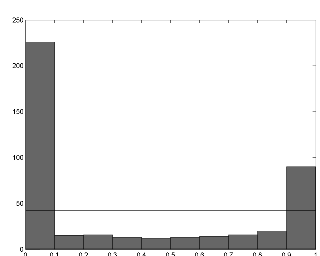
(k) PCR(10,1,2)



(l) PCR(1,2,2)



(m) PCR(5,2,2)



(n) PCR(10,2,2)

Figure E.22: Histogram of the probability integral transforms with $h = 24$.

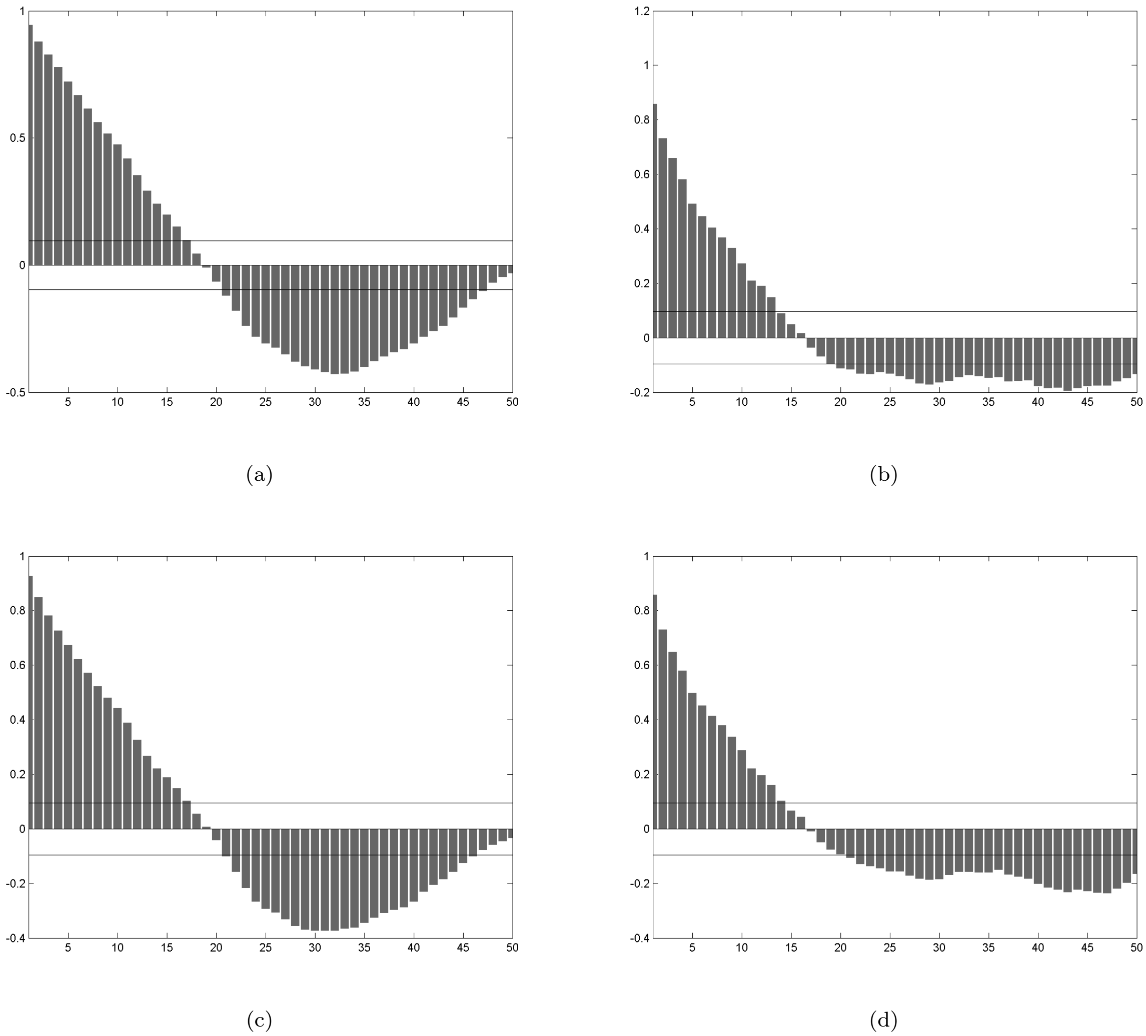


Figure E.23: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z})\,,(z-\overline{z})^2\,,(z-\overline{z})^3$ and $(z-\overline{z})^4$ of RW with $h=24$.

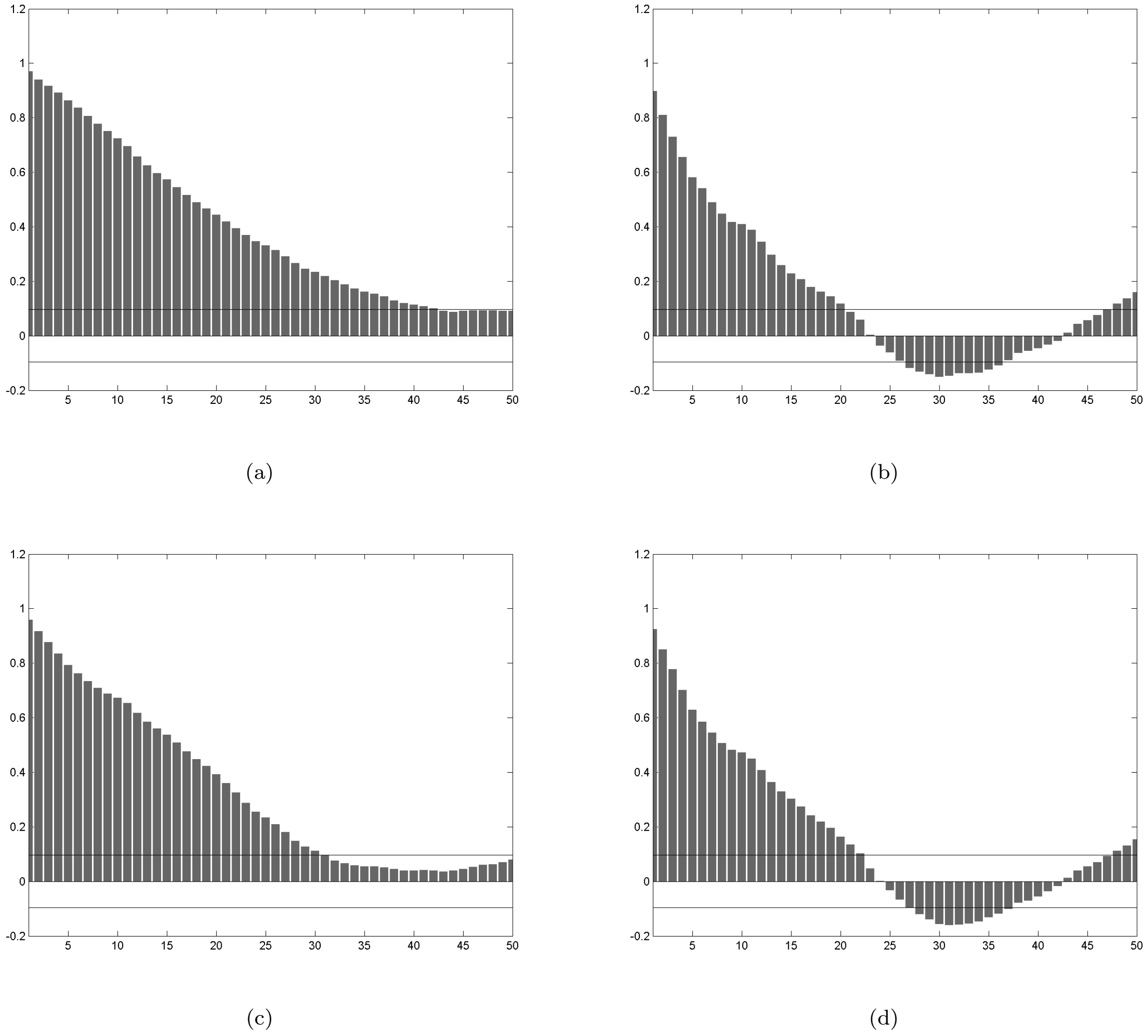


Figure E.24: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z})\,, (z-\overline{z})^2\,, (z-\overline{z})^3$ and $(z-\overline{z})^4$ of AR(2) with $h = 24$.

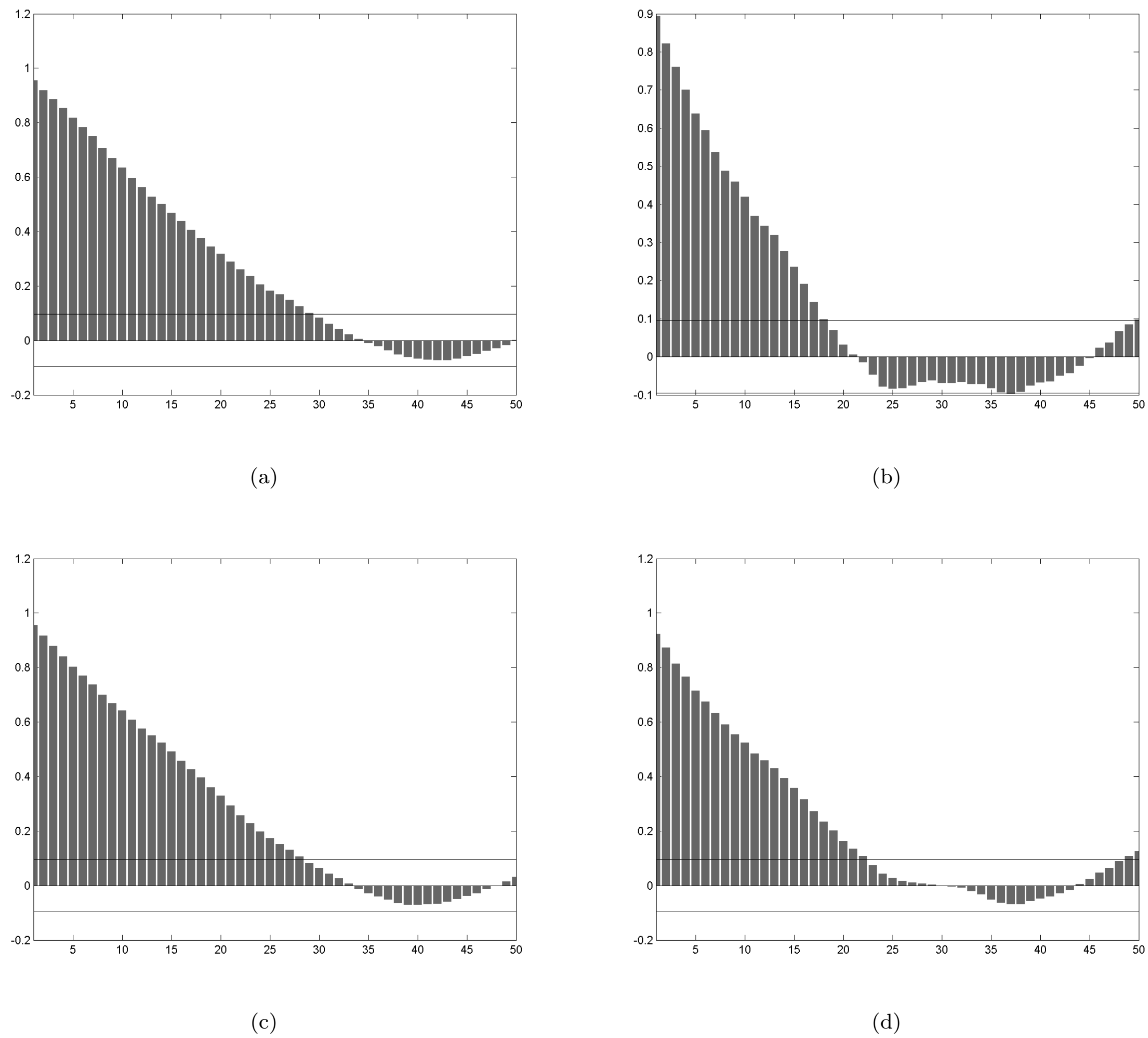


Figure E.25: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z}), (z-\overline{z})^2, (z-\overline{z})^3$ and $(z-\overline{z})^4$ of PCR(1,0,0) with $h = 24$.

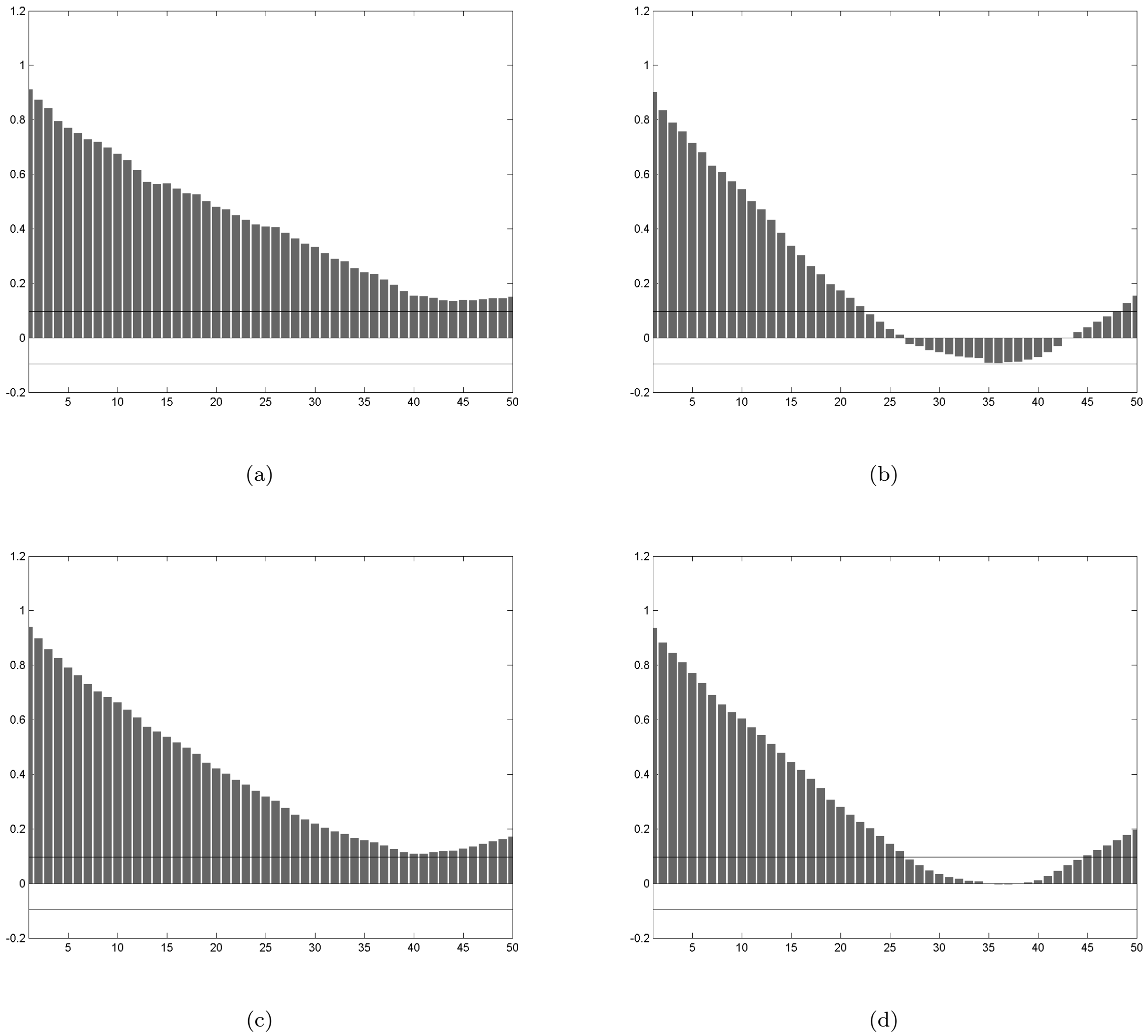


Figure E.26: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z}),(z-\overline{z})^2,(z-\overline{z})^3$ and $(z-\overline{z})^4$ of PCR(5,0,0) with $h=24$.

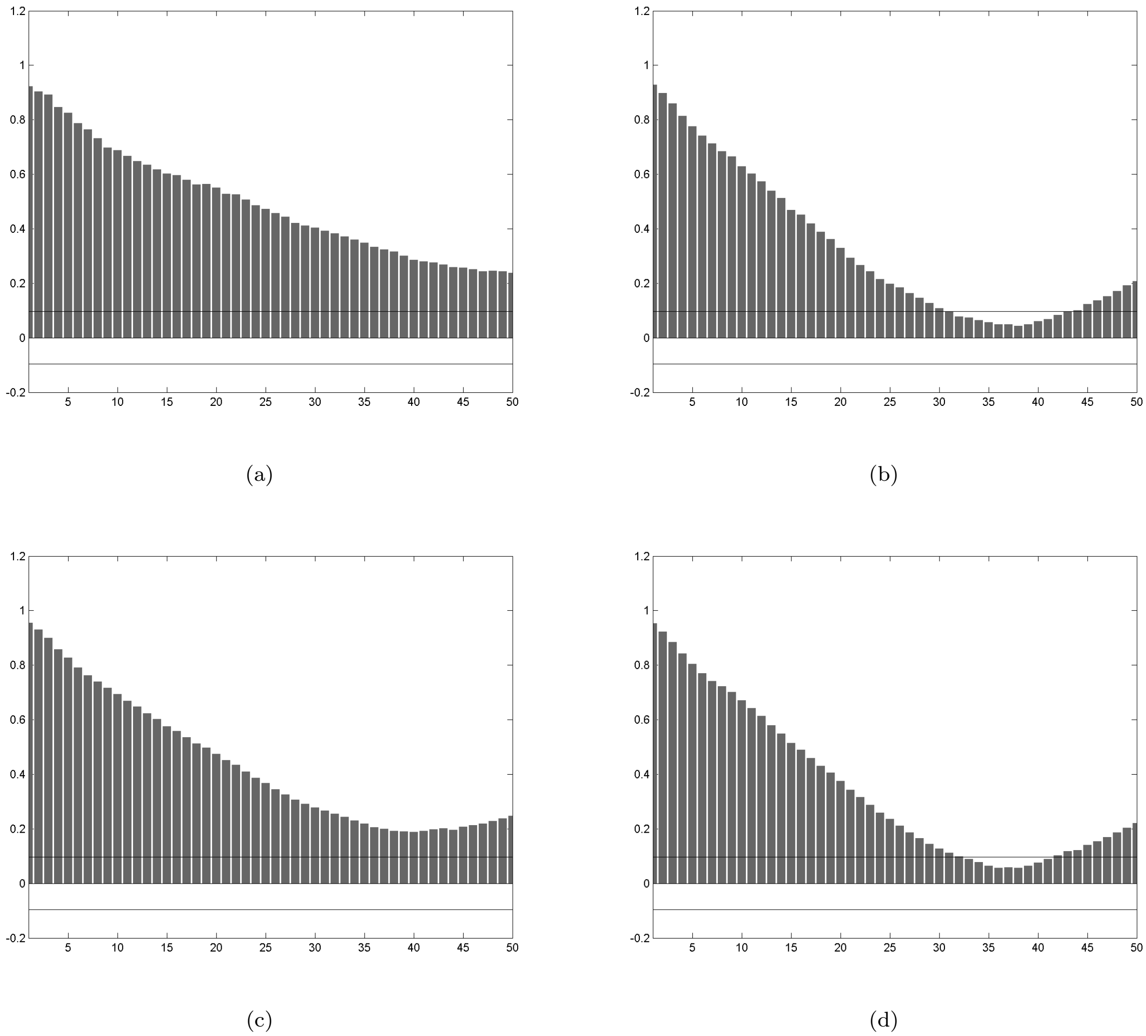


Figure E.27: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z}), (z-\overline{z})^2, (z-\overline{z})^3$ and $(z-\overline{z})^4$ of PCR(10,0,0) with $h = 24$.

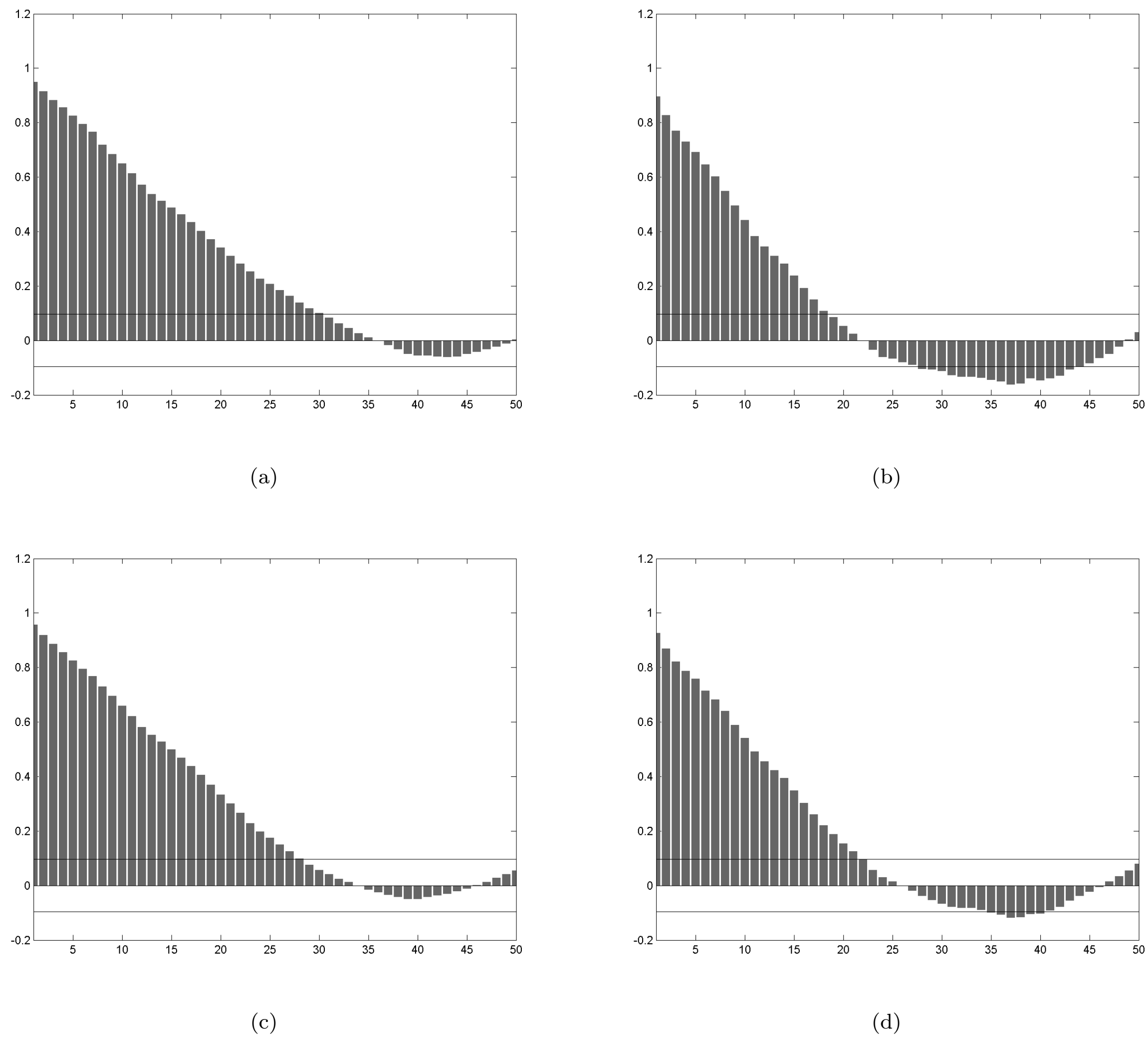
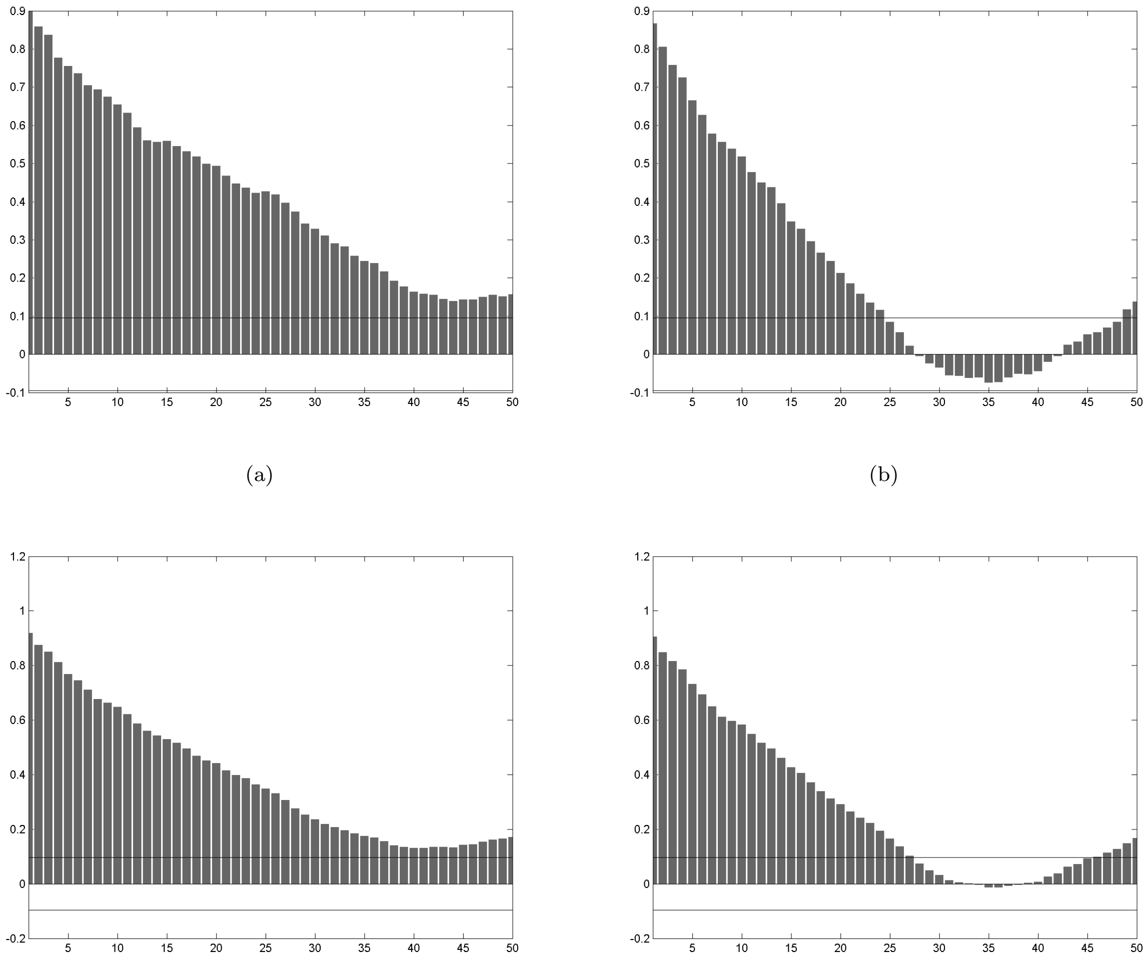


Figure E.28: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z}),(z-\overline{z})^2,(z-\overline{z})^3$ and $(z-\overline{z})^4$ of PCR(1,0,2) with $h=24$.



(a)

(b)

(c)

(d)

Figure E.29: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z})\,,(z-\overline{z})^2\,,(z-\overline{z})^3$ and $(z-\overline{z})^4$ of PCR(5,0,2) with $h=24$.

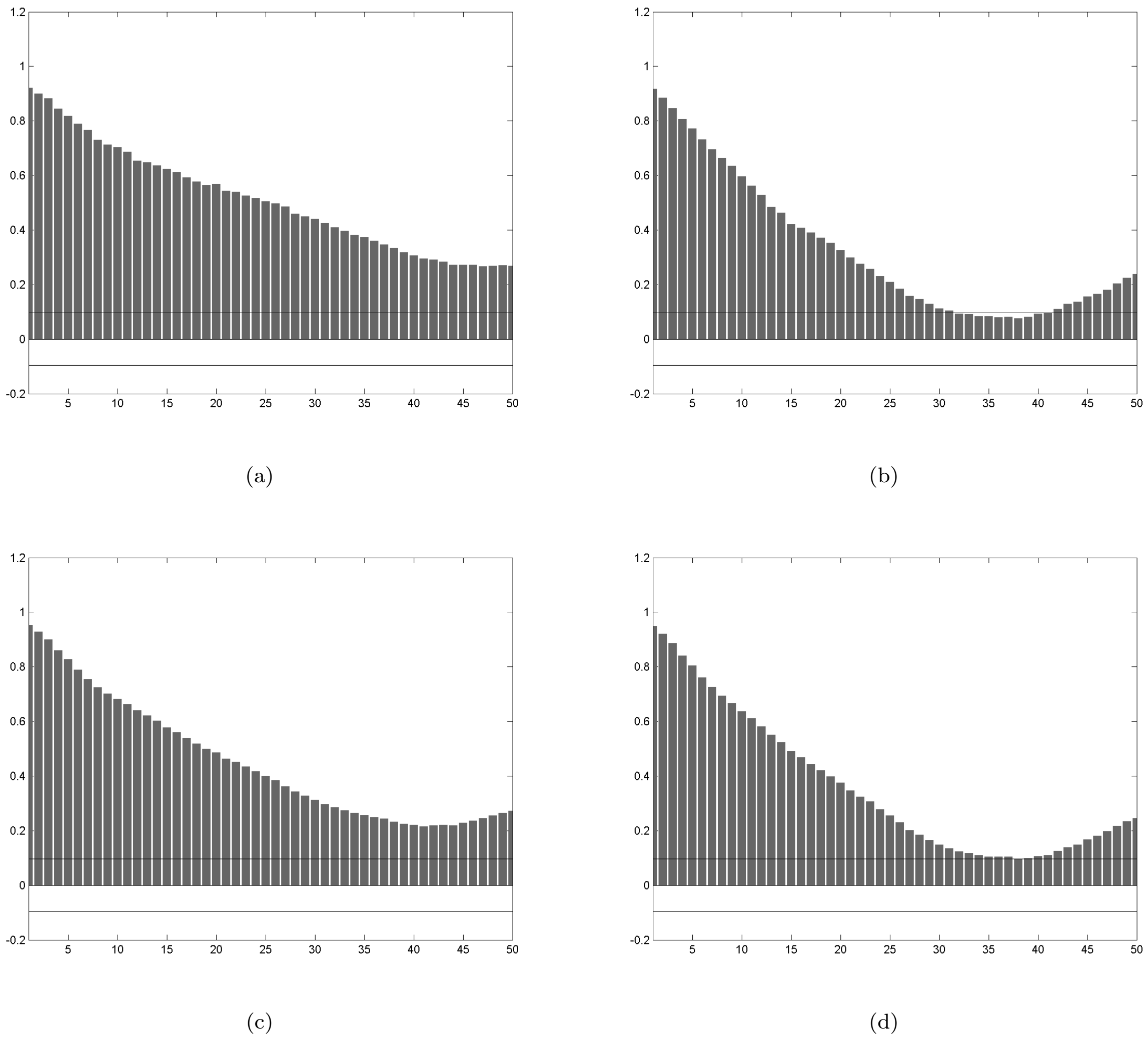


Figure E.30: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z}), (z-\overline{z})^2, (z-\overline{z})^3$ and $(z-\overline{z})^4$ of PCR(10,0,2) with $h=24$.

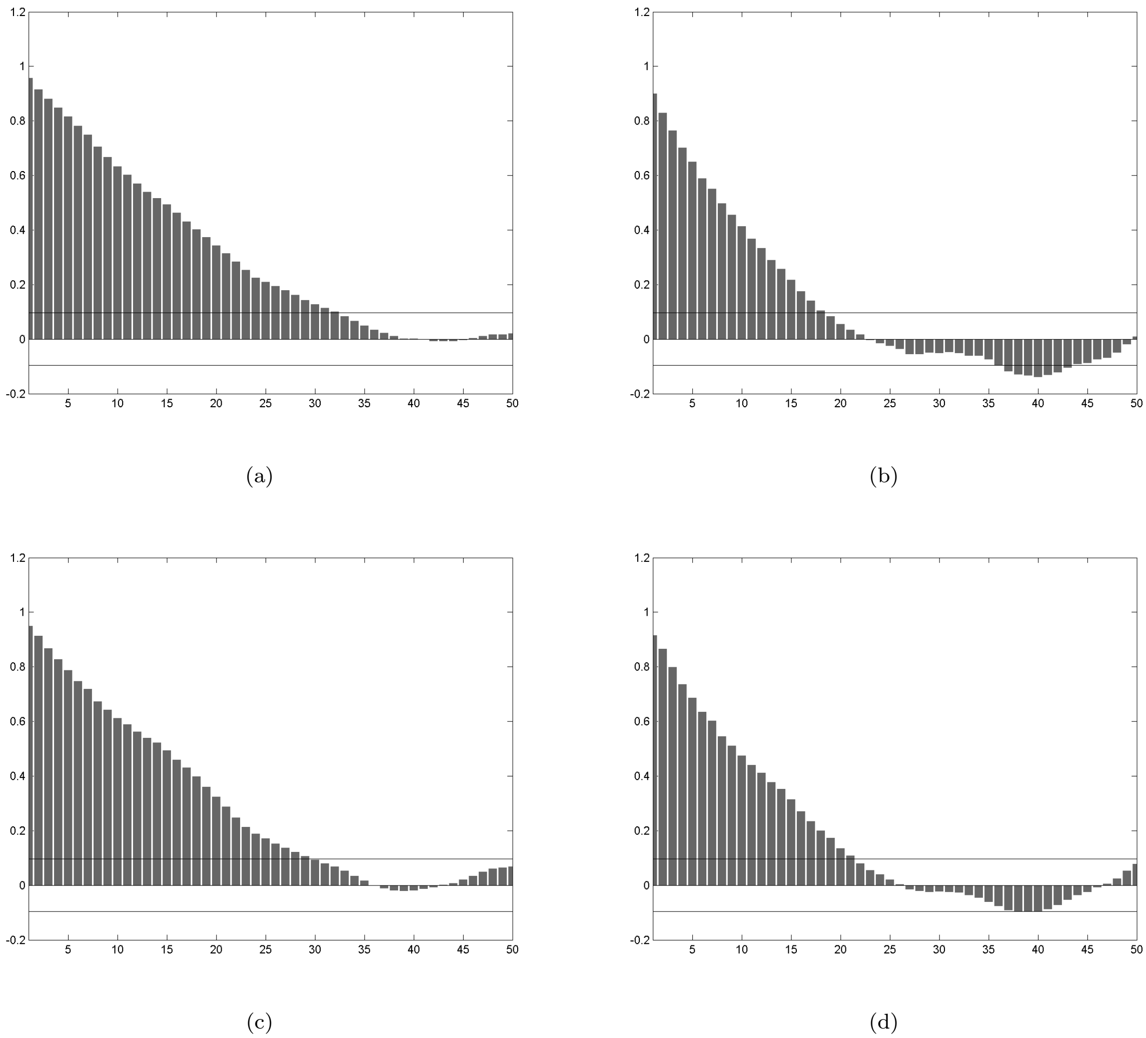


Figure E.31: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z}), (z-\overline{z})^2, (z-\overline{z})^3$ and $(z-\overline{z})^4$ of PCR(1,1,2) with $h = 24$.

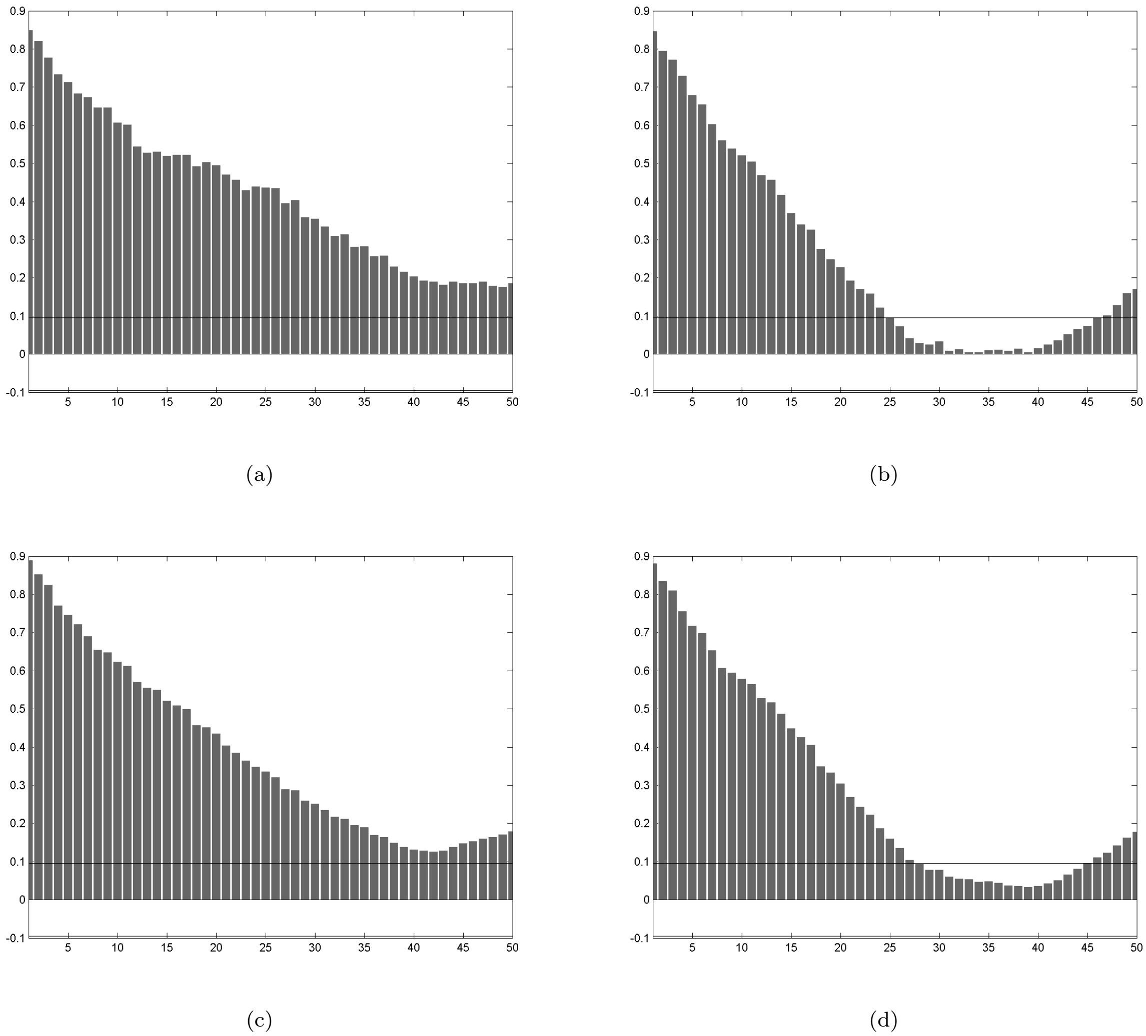
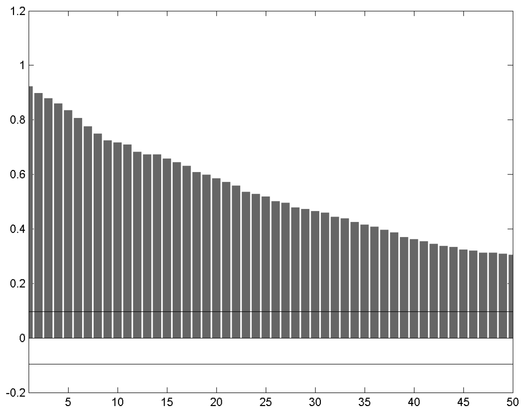
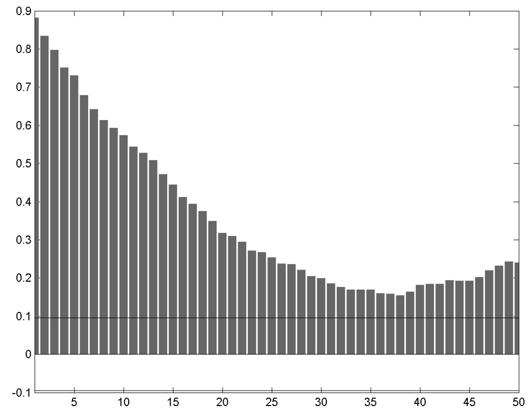


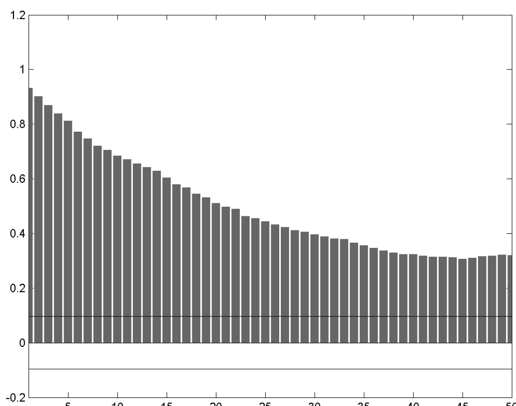
Figure E.32: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z}),(z-\overline{z})^2,(z-\overline{z})^3$ and $(z-\overline{z})^4$ of PCR(5,1,2) with $h=24$.



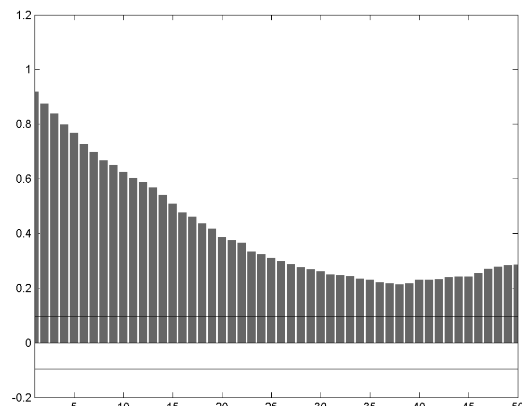
(a)



(b)

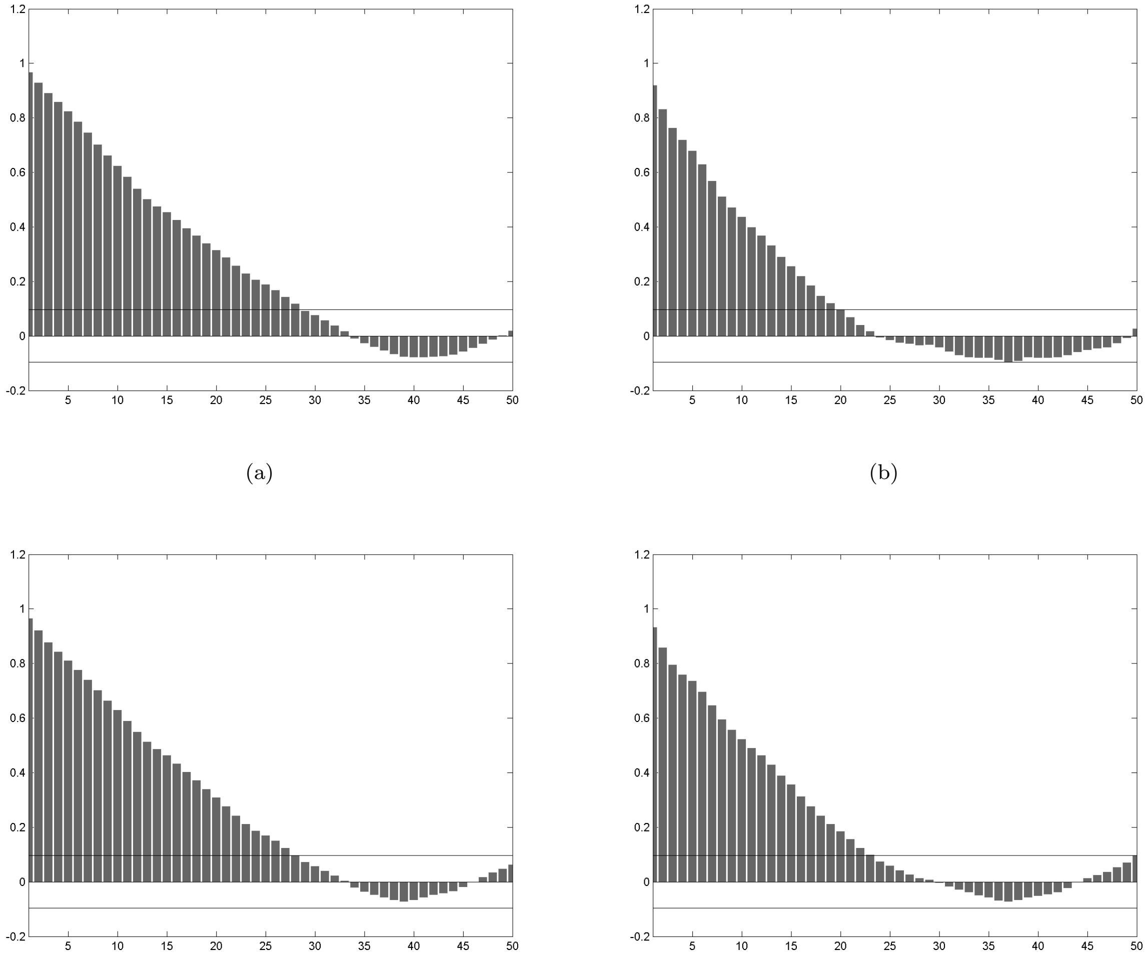


(c)



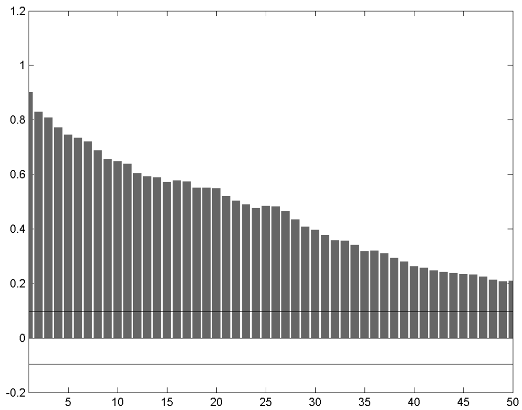
(d)

Figure E.33: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z}), (z-\overline{z})^2, (z-\overline{z})^3$ and $(z-\overline{z})^4$ of PCR(10,1,2) with $h = 24$.

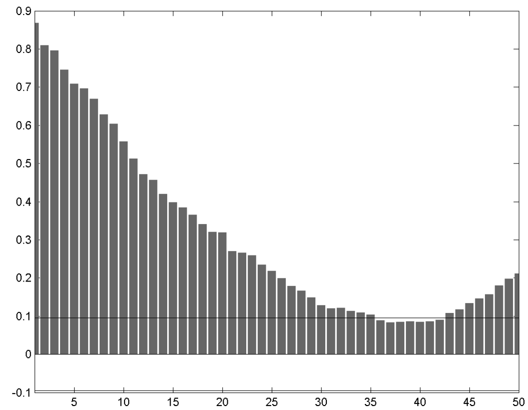


(a) (b) (c) (d)

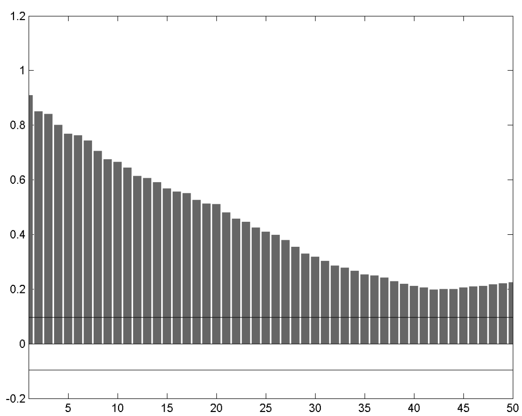
Figure E.34: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z}), (z-\overline{z})^2, (z-\overline{z})^3$ and $(z-\overline{z})^4$ of PCR(1,2,2) with $h = 24$.



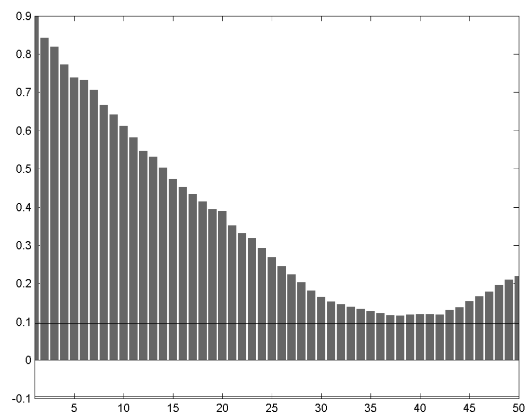
(a)



(b)



(c)



(d)

Figure E.35: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z}),(z-\overline{z})^2,(z-\overline{z})^3$ and $(z-\overline{z})^4$ of PCR(5,2,2) with $h=24$.

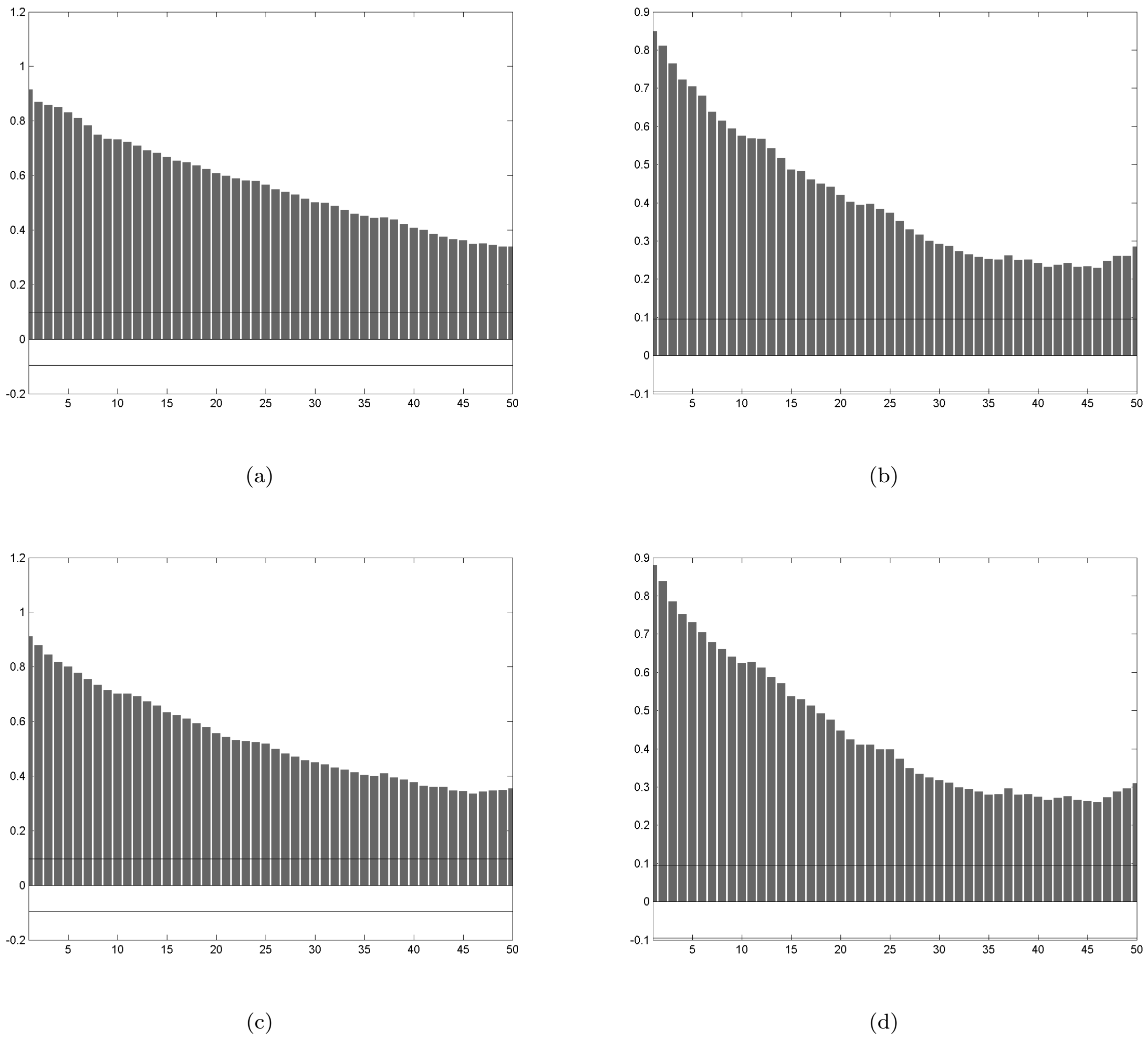


Figure E.36: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z}), (z-\overline{z})^2, (z-\overline{z})^3$ and $(z-\overline{z})^4$ of PCR(10,2,2) with $h = 24$.

Table E.6: KLIC statistics for $h = 24$.

Model	RW	AR(2)	PCR(1,0,0)	PCR(5,0,0)	PCR(10,0,0)	PCR(1,0,2)	PCR(5,0,2)
RW	-	0,18	-0,94	-1,14	-0,54	-1,42	-0,6
AR(2)	-0,18	-	-1,06	-1	-0,66	-1,08	-0,57
PCR(1,0,0)	0,94	1,06	-	0,57	0,87	0,76	0,89
PCR(5,0,0)	1,14	1	-0,57	-	1,14	0,31	1,5
PCR(10,0,0)	0,54	0,66	-0,87	-1,14	-	-0,42	-0,08
PCR(1,0,2)	1,42	1,08	-0,76	-0,31	0,42	-	0,43
PCR(5,0,2)	0,6	0,57	-0,89	-1,5	0,08	-0,43	-
PCR(10,0,2)	0,93	0,9	-0,32	0,54	0,96	0,45	0,96
PCR(1,1,2)	0,96	0,82	-0,5	0,03	0,54	0,36	0,6
PCR(5,1,2)	0,89	0,83	-0,57	-0,15	0,89	0,22	1,04
PCR(10,1,2)	1,21	1,31	-0,53	0,55	1,25	0,74	1,29
PCR(1,2,2)	0,9	0,68	-0,82	-0,77	0,09	-0,57	0,07
PCR(5,2,2)	1,07	1,03	0,01	1,02	1,11	0,76	1,16
PCR(10,2,2)	1,28	1,29	0,13	1,33	1,44	1,04	1,54

Model	PCR(10,0,2)	PCR(1,1,2)	PCR(5,1,2)	PCR(10,1,2)	PCR(1,2,2)	PCR(5,2,2)	PCR(10,2,2)
RW	-0,93	-0,96	-0,89	-1,21	-0,9	-1,07	-1,28
AR(2)	-0,9	-0,82	-0,83	-1,31	-0,68	-1,03	-1,29
PCR(1,0,0)	0,32	0,5	0,57	0,53	0,82	-0,01	-0,13
PCR(5,0,0)	-0,54	-0,03	0,15	-0,55	0,77	-1,02	-1,33
PCR(10,0,0)	-0,96	-0,54	-0,89	-1,25	-0,09	-1,11	-1,44
PCR(1,0,2)	-0,45	-0,36	-0,22	-0,74	0,57	-0,76	-1,04
PCR(5,0,2)	-0,96	-0,6	-1,04	-1,29	-0,07	-1,16	-1,54
PCR(10,0,2)	-	0,24	0,77	-0,07	0,69	-1,24	-1,09
PCR(1,1,2)	-0,24	-	0,06	-0,32	0,97	-0,64	-0,73
PCR(5,1,2)	-0,77	-0,06	-	-0,52	0,54	-1,19	-1,38
PCR(10,1,2)	0,07	0,32	0,52	-	0,88	-0,42	-0,9
PCR(1,2,2)	-0,69	-0,97	-0,54	-0,88	-	-0,94	-1,1
PCR(5,2,2)	1,24	0,64	1,19	0,42	0,94	-	-0,21
PCR(10,2,2)	1,09	0,73	1,38	0,9	1,1	0,21	-

Table E.7: KLIC model favors for $h = 24$. One signifies the model in the row is preferred. Two signifies the model in the column is preferred.

Model	RW	AR(2)	PCR(1,0,0)	PCR(5,0,0)	PCR(10,0,0)	PCR(1,0,2)	PCR(5,0,2)
RW	-	1	1	1	1	1	1
AR(2)	2	-	1	2	1	2	1
PCR(1,0,0)	2	2	-	2	1	2	2
PCR(5,0,0)	2	1	1	-	1	2	1
PCR(10,0,0)	2	2	2	2	-	2	2
PCR(1,0,2)	2	1	1	1	1	-	1
PCR(5,0,2)	2	2	1	2	1	2	-
PCR(10,0,2)	2	1	1	1	1	2	1
PCR(1,1,2)	2	1	1	1	1	2	1
PCR(5,1,2)	2	2	2	2	2	2	2
PCR(10,1,2)	2	2	2	2	1	2	2
PCR(1,2,2)	2	1	1	1	1	1	1
PCR(5,2,2)	2	2	2	2	2	2	2
PCR(10,2,2)	2	1	1	1	1	2	1

Model	PCR(10,0,2)	PCR(1,1,2)	PCR(5,1,2)	PCR(10,1,2)	PCR(1,2,2)	PCR(5,2,2)	PCR(10,2,2)
RW	1	1	1	1	1	1	1
AR(2)	2	2	1	1	2	1	2
PCR(1,0,0)	2	2	1	1	2	1	2
PCR(5,0,0)	2	2	1	1	2	1	2
PCR(10,0,0)	2	2	1	2	2	1	2
PCR(1,0,2)	1	1	1	1	2	1	1
PCR(5,0,2)	2	2	1	1	2	1	2
PCR(10,0,2)	-	2	1	1	2	1	2
PCR(1,1,2)	1	-	1	1	2	1	1
PCR(5,1,2)	2	2	-	2	2	2	2
PCR(10,1,2)	2	2	1	-	2	1	2
PCR(1,2,2)	1	1	1	1	-	1	1
PCR(5,2,2)	2	2	1	2	2	-	2
PCR(10,2,2)	1	2	1	1	2	1	-

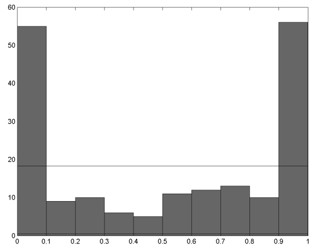
Table E.8: KLIC probabilities for $h = 24$.

Model	RW	AR(2)	PCR(1,0,0)	PCR(5,0,0)	PCR(10,0,0)	PCR(1,0,2)	PCR(5,0,2)
RW	-	0,17	0,12	0,1	0,1	0,17	0,17
AR(2)	0,17	-	0,38	0,4	0,37	0,28	0,4
PCR(1,0,0)	0,12	0,38	-	0,36	0,38	0,1	0,39
PCR(5,0,0)	0,1	0,4	0,36	-	0,24	0,19	0,39
PCR(10,0,0)	0,1	0,37	0,38	0,24	-	0,1	0,18
PCR(1,0,2)	0,17	0,28	0,1	0,19	0,1	-	0,24
PCR(5,0,2)	0,17	0,4	0,39	0,39	0,18	0,24	-
PCR(10,0,2)	0,07	0,4	0,38	0,4	0,33	0,25	0,39
PCR(1,1,2)	0,19	0,33	0,17	0,26	0,03	0,3	0,26
PCR(5,1,2)	0,11	0,33	0,34	0,14	0,3	0,12	0,13
PCR(10,1,2)	0,08	0,39	0,4	0,29	0,38	0,13	0,34
PCR(1,2,2)	0,21	0,25	0,08	0,11	0,05	0,23	0,18
PCR(5,2,2)	0,17	0,38	0,39	0,33	0,4	0,22	0,24
PCR(10,2,2)	0,03	0,4	0,38	0,4	0,34	0,22	0,39

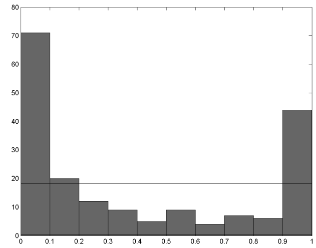
Model	PCR(10,0,2)	PCR(1,1,2)	PCR(5,1,2)	PCR(10,1,2)	PCR(1,2,2)	PCR(5,2,2)	PCR(10,2,2)
RW	0,07	0,19	0,11	0,08	0,21	0,17	0,03
AR(2)	0,4	0,33	0,33	0,39	0,25	0,38	0,4
PCR(1,0,0)	0,38	0,17	0,34	0,4	0,08	0,39	0,38
PCR(5,0,0)	0,4	0,26	0,14	0,29	0,11	0,33	0,4
PCR(10,0,0)	0,33	0,03	0,3	0,38	0,05	0,4	0,34
PCR(1,0,2)	0,25	0,3	0,12	0,13	0,23	0,22	0,22
PCR(5,0,2)	0,39	0,26	0,13	0,34	0,18	0,24	0,39
PCR(10,0,2)	-	0,36	0,3	0,29	0,19	0,35	0,4
PCR(1,1,2)	0,36	-	0,06	0,2	0,2	0,22	0,38
PCR(5,1,2)	0,3	0,06	-	0,33	0,08	0,38	0,31
PCR(10,1,2)	0,29	0,2	0,33	-	0,08	0,39	0,34
PCR(1,2,2)	0,19	0,2	0,08	0,08	-	0,17	0,2
PCR(5,2,2)	0,35	0,22	0,38	0,39	0,17	-	0,37
PCR(10,2,2)	0,4	0,38	0,31	0,34	0,2	0,37	-

Table E.9: Berkowitz Likelihood Ratios and p-values for the models evaluated with a $h = 12$ over the period 1994:M01-2008:M08.

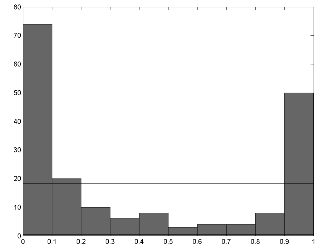
Model	LR	p-value
RW	8,75E+02	0
AR(2)	6,47E+02	0
PCR(1,0,0)	1,01E+03	0
PCR(5,0,0)	1,88E+03	0
PCR(10,0,0)	1,10E+03	0
PCR(1,0,2)	1,07E+03	0
PCR(5,0,2)	1,25E+03	0
PCR(10,0,2)	2,44E+03	0
PCR(1,1,2)	1,94E+03	0
PCR(5,1,2)	1,85E+03	0
PCR(10,1,2)	2,14E+03	0
PCR(1,2,2)	1,20E+03	0
PCR(5,2,2)	3,45E+03	0
PCR(10,2,2)	3,70E+03	0



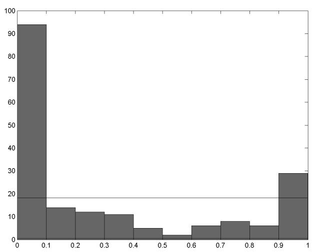
(a) RW



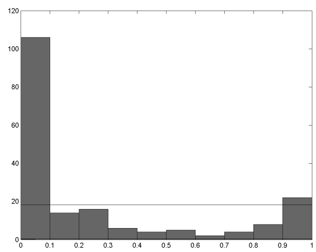
(b) AR(2)



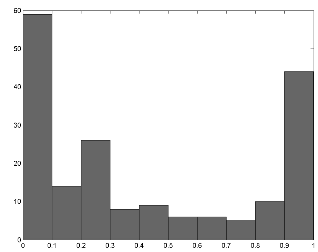
(c) PCR(1,0,0)



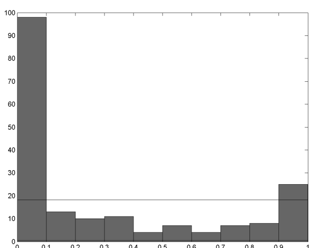
(d) PCR(5,0,0)



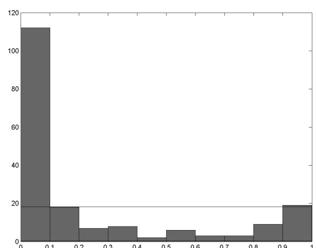
(e) PCR(10,0,0)



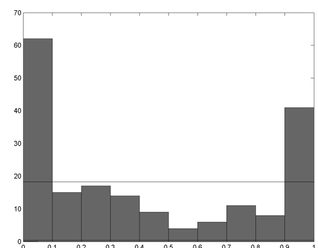
(f) PCR(1,0,2)



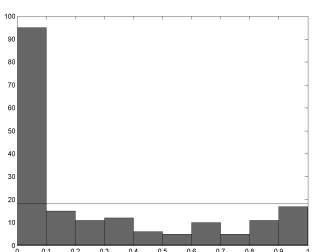
(g) PCR(5,0,2)



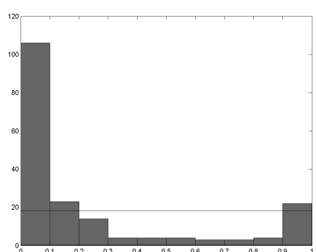
(h) PCR(10,0,2)



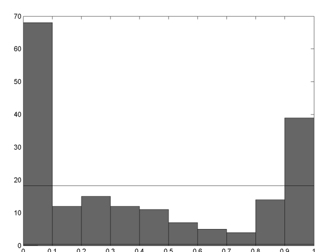
(i) PCR(1,1,2)



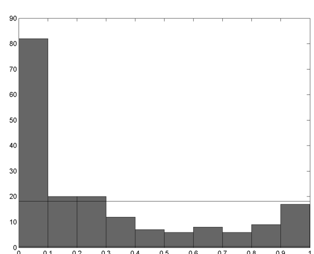
(j) PCR(5,1,2)



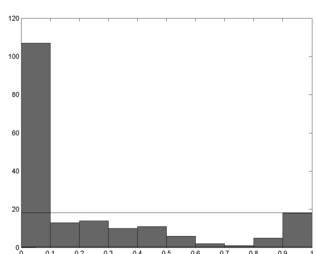
(k) PCR(10,1,2)



(l) PCR(1,2,2)

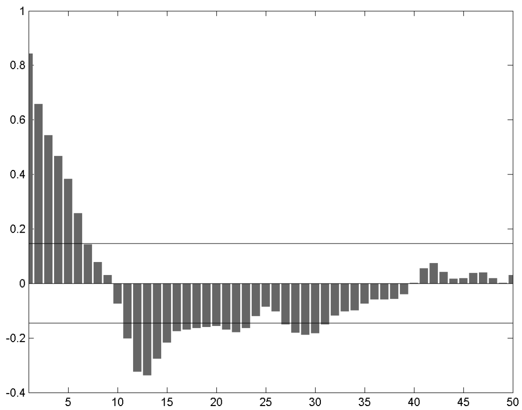


(m) PCR(5,2,2)

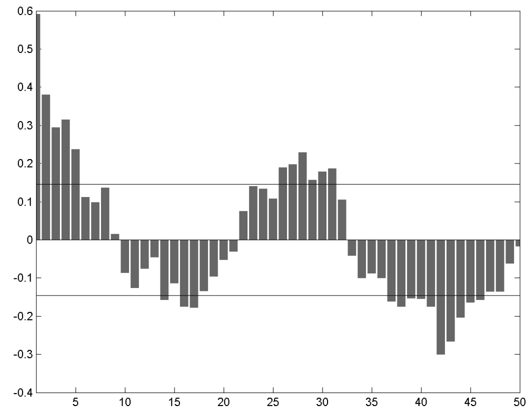


(n) PCR(10,2,2)

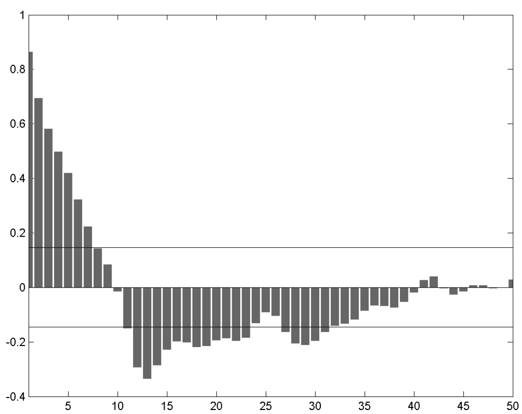
Figure E.37: Histogram of the probability integral transforms with $h = 12$ over the period 1994:M01-2008:M08.



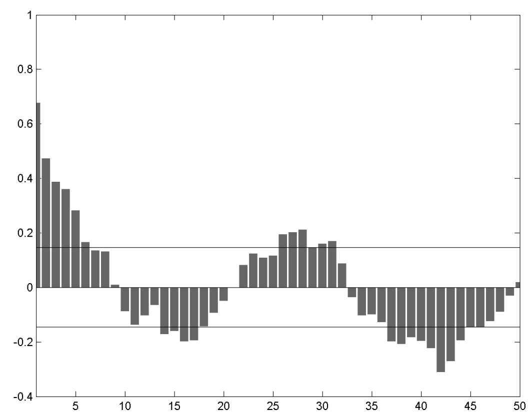
(a)



(b)



(c)



(d)

Figure E.38: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z})$, $(z-\overline{z})^2$, $(z-\overline{z})^3$ and $(z-\overline{z})^4$ of RW with $h = 12$ over the period 1994:M01-2008:M08.

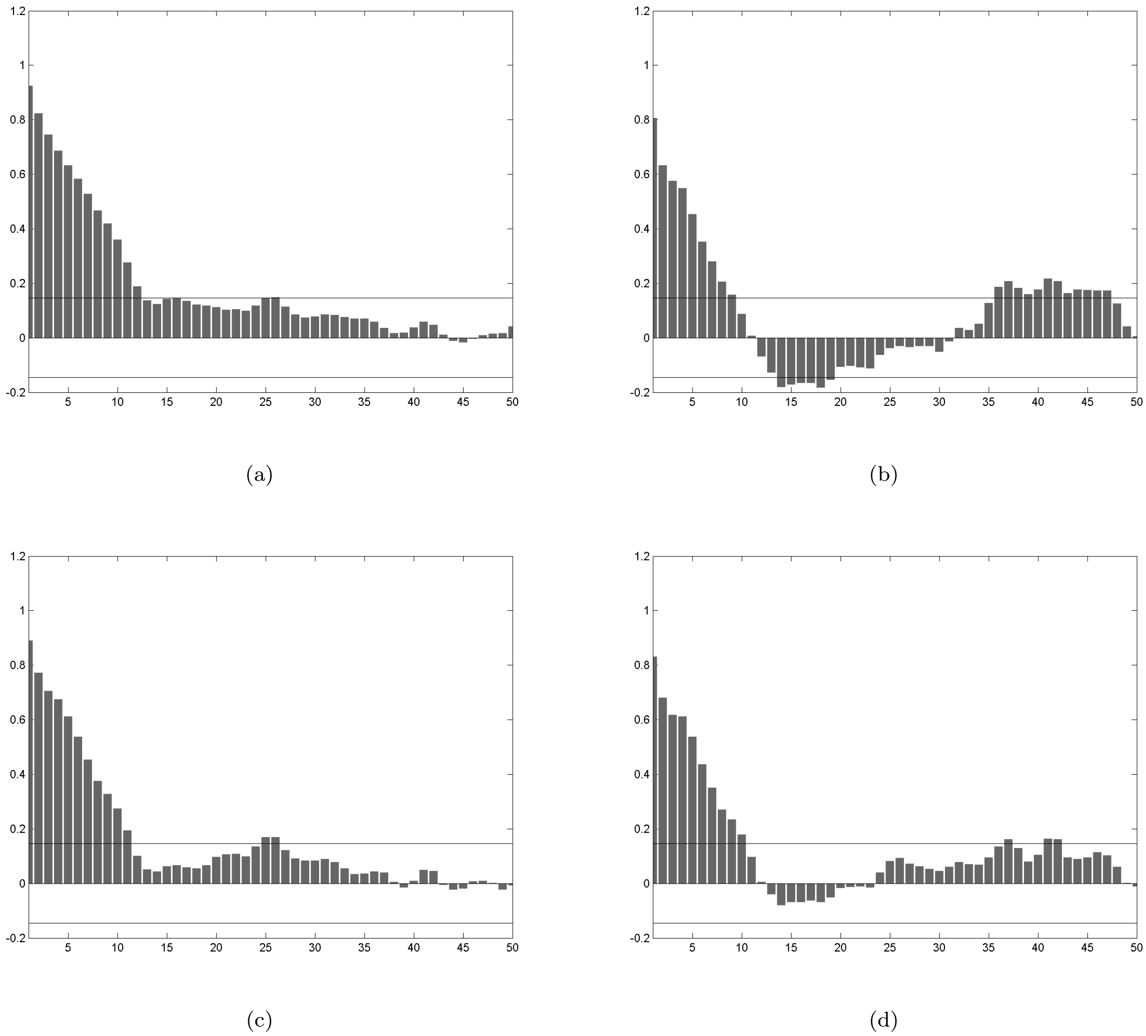


Figure E.39: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z})\,,(z-\overline{z})^2\,,(z-\overline{z})^3$ and $(z-\overline{z})^4$ of AR(2) with $h=12$ over the period 1994:M01-2008:M08.

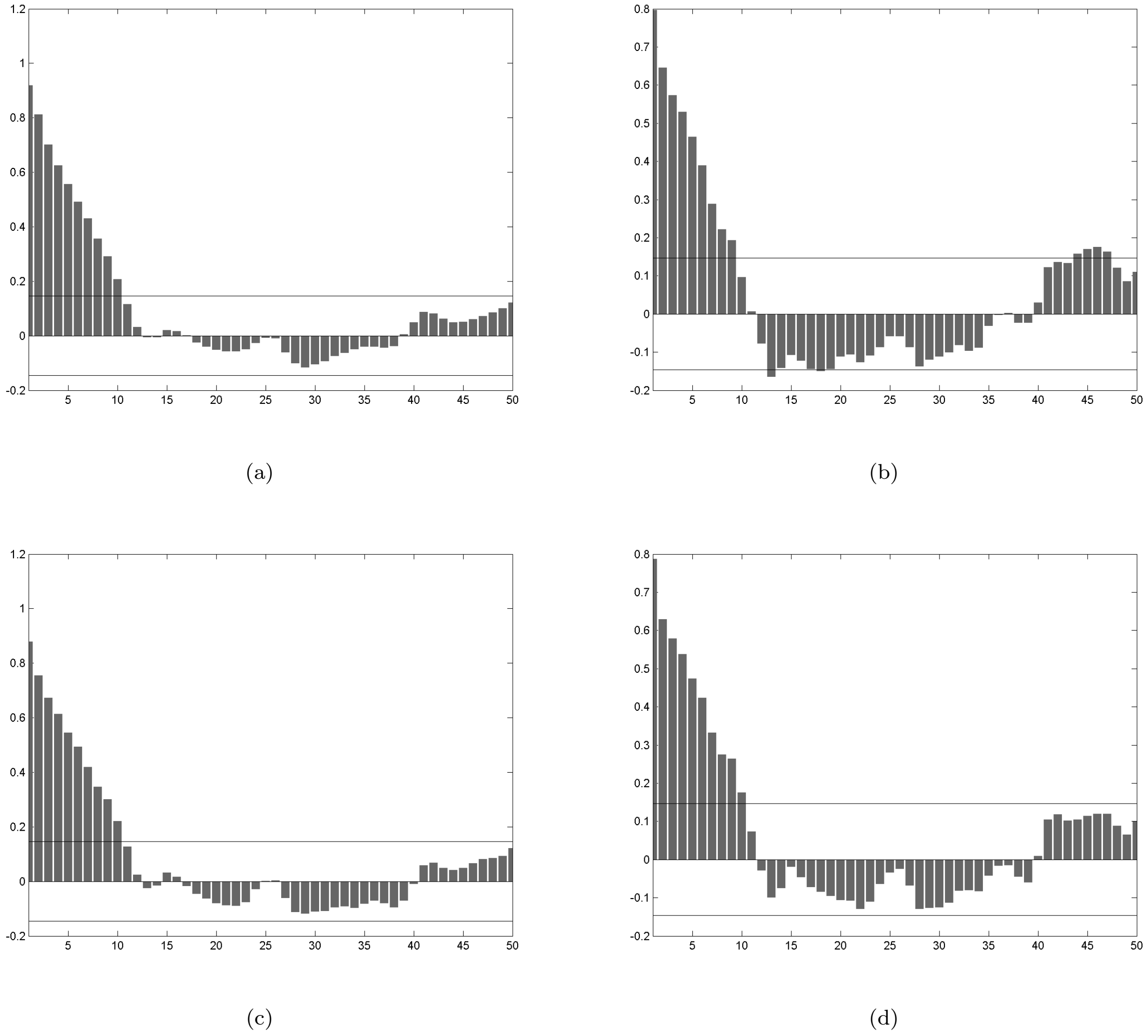
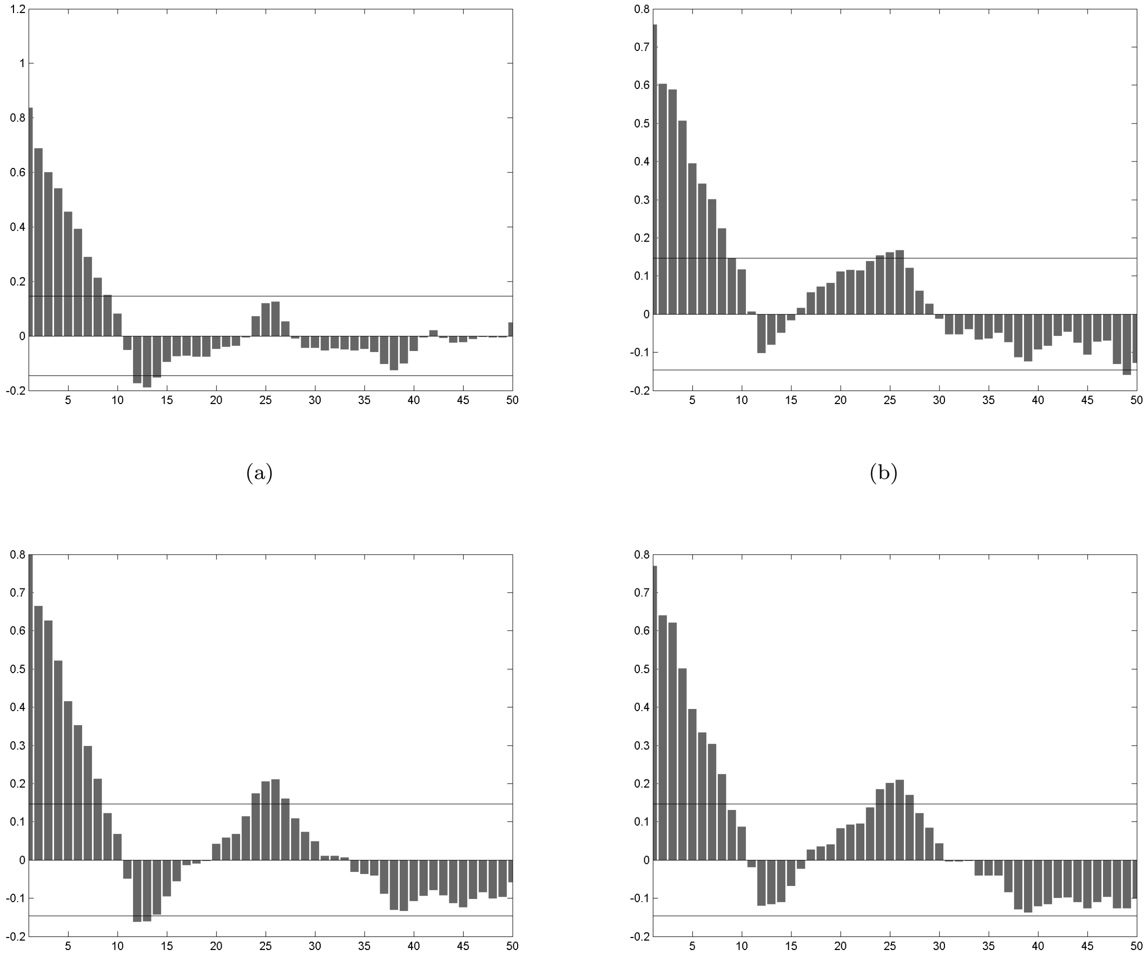


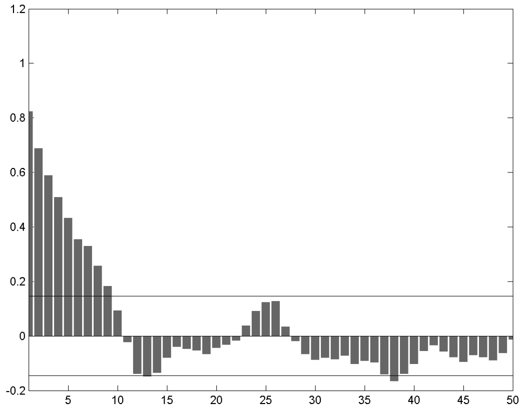
Figure E.40: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z}),(z-\overline{z})^2,(z-\overline{z})^3$ and $(z-\overline{z})^4$ of PCR(1,0,0) with $h=12$ over the period 1994:M01-2008:M08.



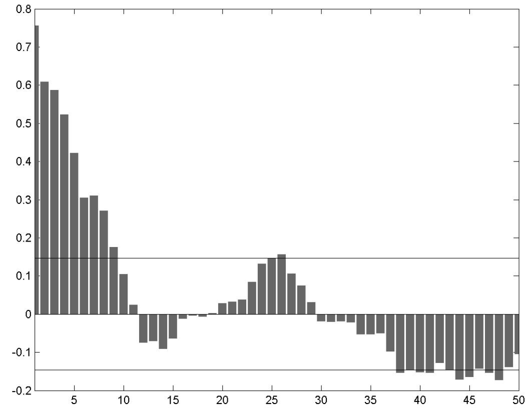
(a) (b)

(c) (d)

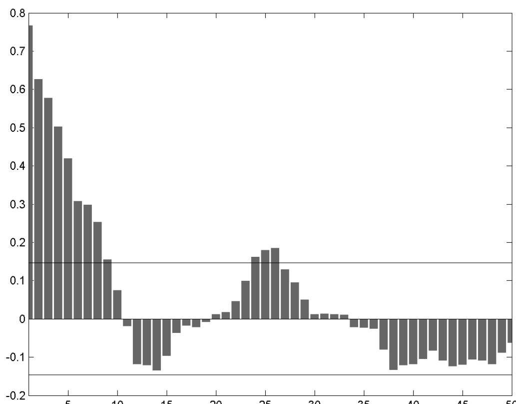
Figure E.41: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z}),(z-\overline{z})^2,(z-\overline{z})^3$ and $(z-\overline{z})^4$ of PCR(5,0,0) with $h=12$ over the period 1994:M01-2008:M08.



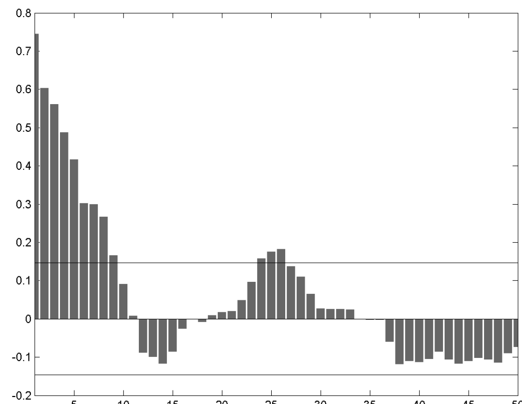
(a)



(b)



(c)



(d)

Figure E.42: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z}), (z-\overline{z})^2, (z-\overline{z})^3$ and $(z-\overline{z})^4$ of PCR(10,0,0) with $h = 12$ over the period 1994:M01-2008:M08.

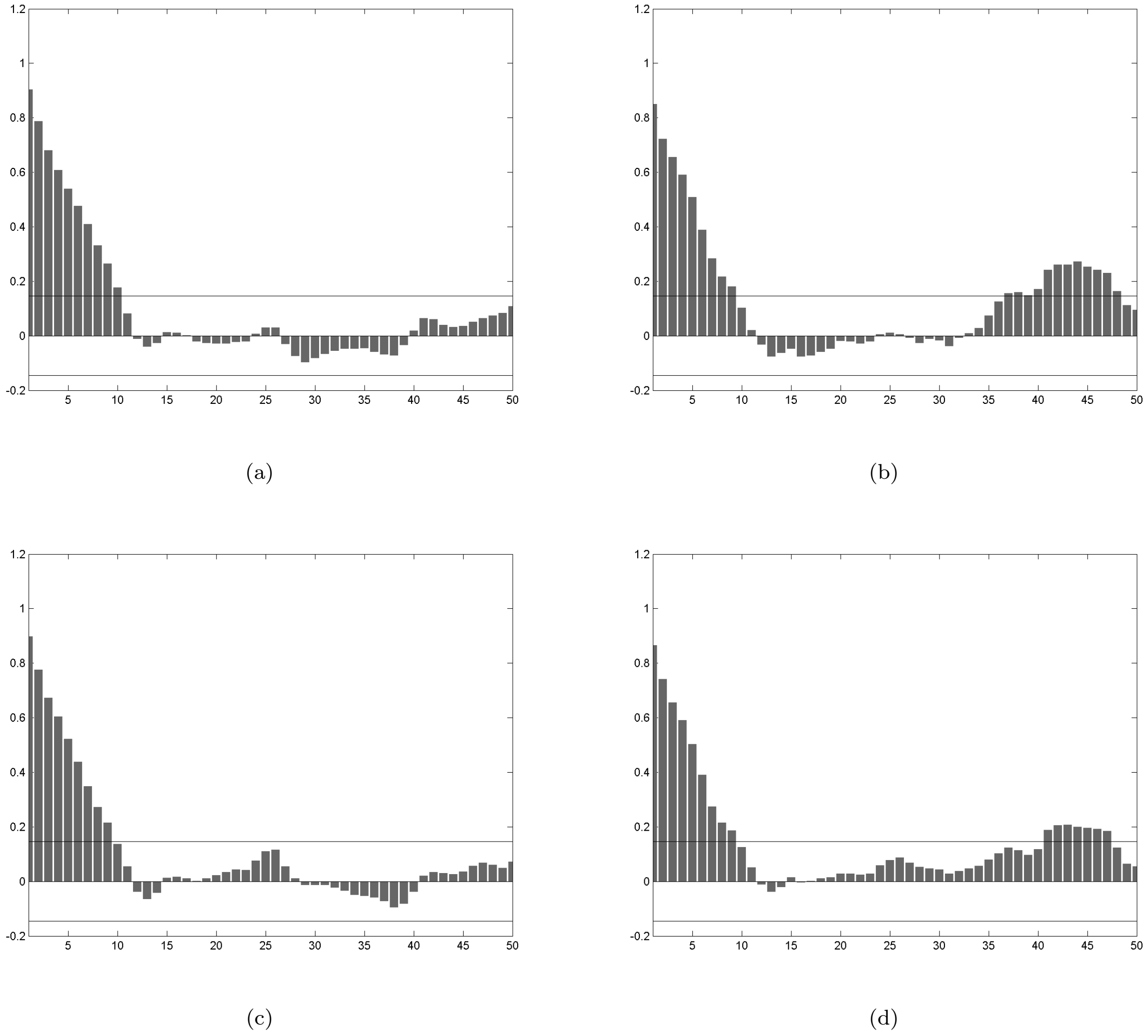


Figure E.43: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z}), (z-\overline{z})^2, (z-\overline{z})^3$ and $(z-\overline{z})^4$ of PCR(1,0,2) with $h=12$ over the period 1994:M01-2008:M08.

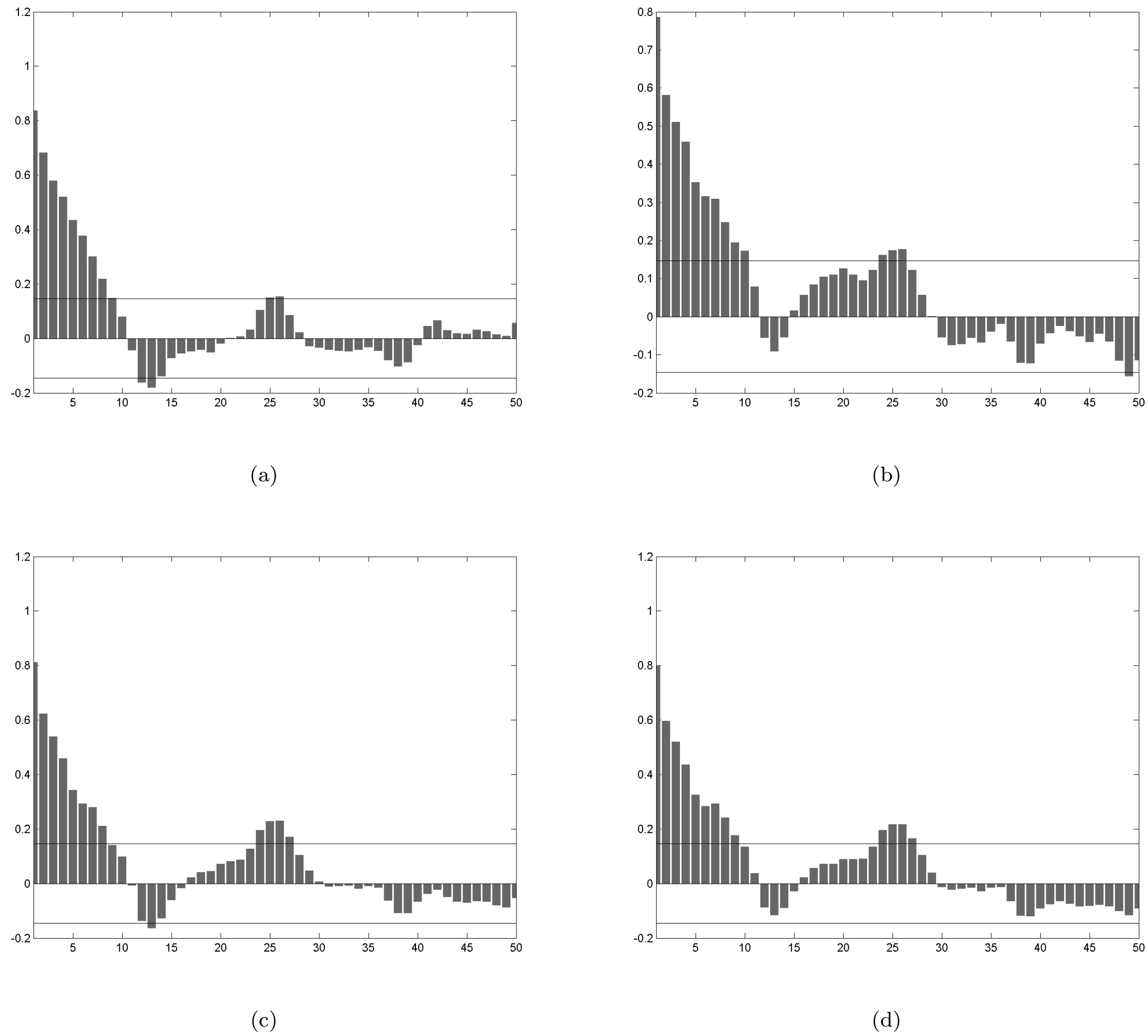
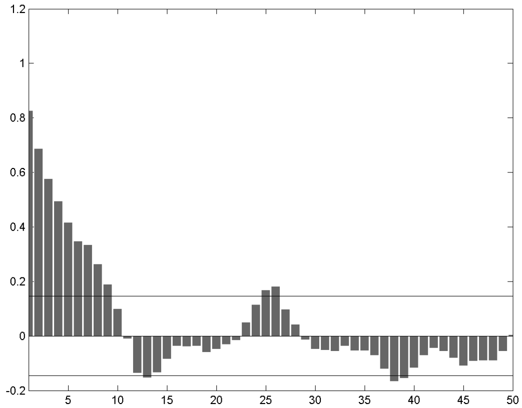
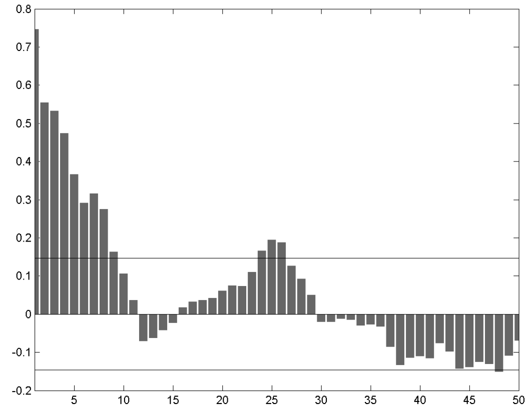


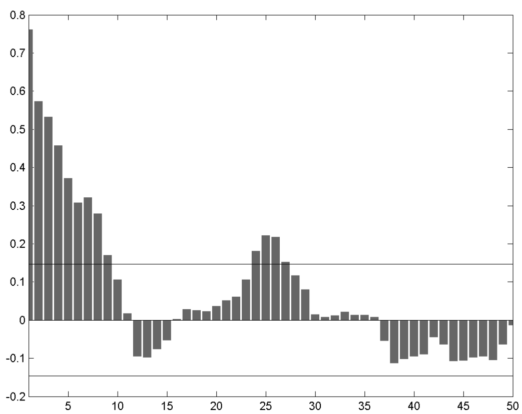
Figure E.44: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z}), (z-\overline{z})^2, (z-\overline{z})^3$ and $(z-\overline{z})^4$ of PCR(5,0,2) with $h = 12$ over the period 1994:M01-2008:M08.



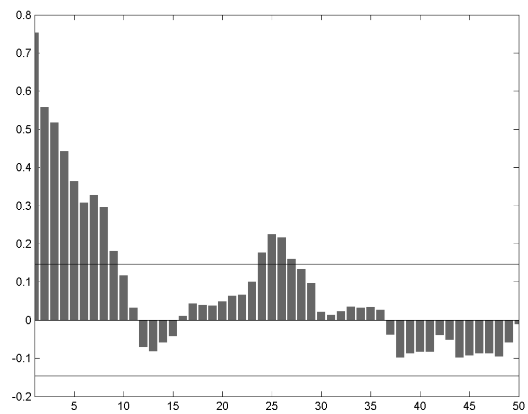
(a)



(b)

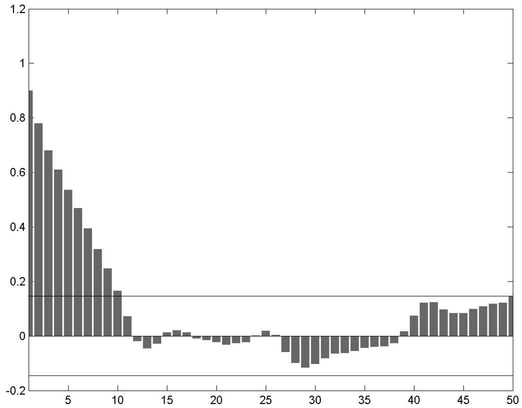


(c)

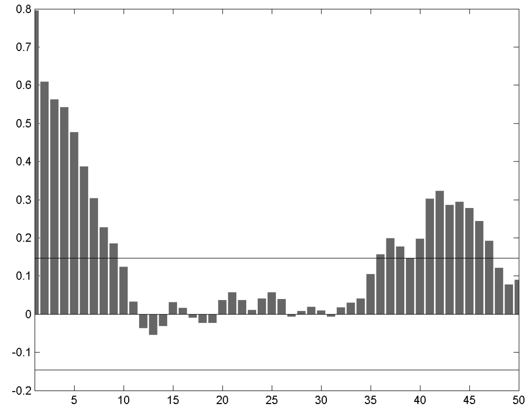


(d)

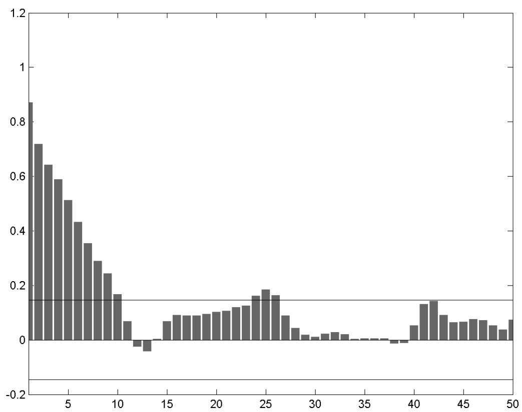
Figure E.45: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z}), (z-\overline{z})^2, (z-\overline{z})^3$ and $(z-\overline{z})^4$ of PCR(10,0,2) with $h = 12$ over the period 1994:M01-2008:M08.



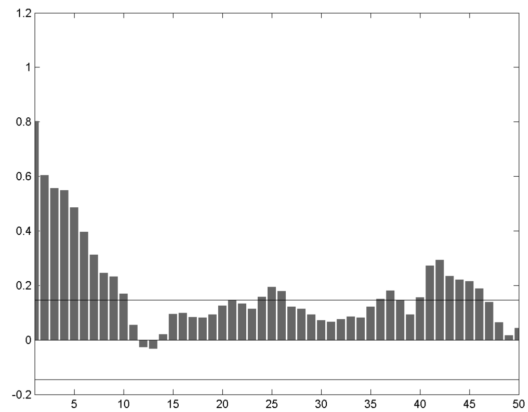
(a)



(b)



(c)



(d)

Figure E.46: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z}), (z-\overline{z})^2, (z-\overline{z})^3$ and $(z-\overline{z})^4$ of PCR(1,1,2) with $h = 12$ over the period 1994:M01-2008:M08.

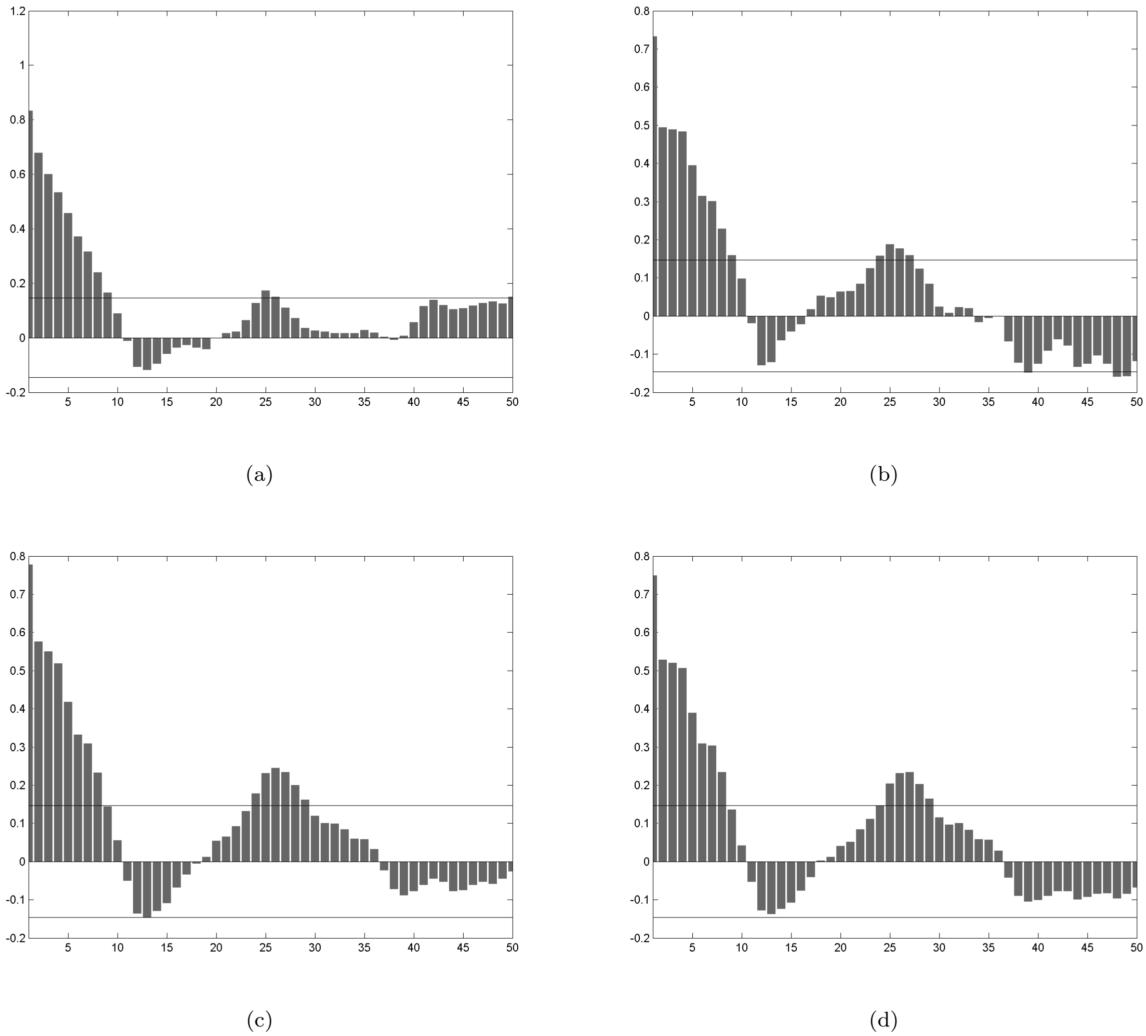
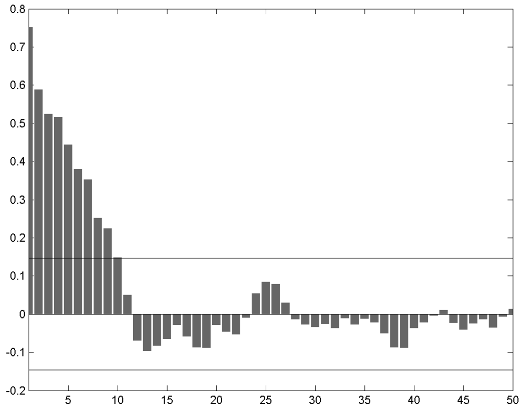
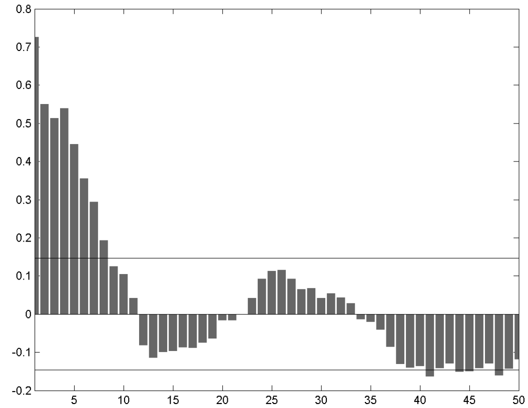


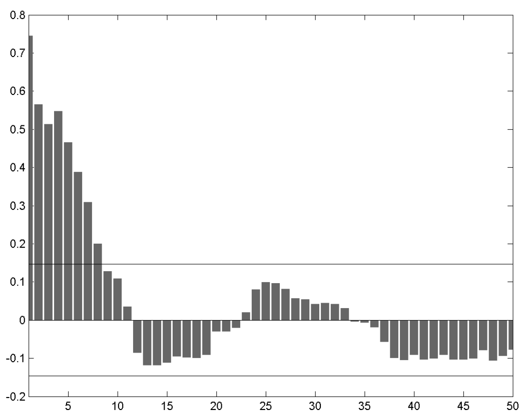
Figure E.47: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z})\,,(z-\overline{z})^2\,,(z-\overline{z})^3$ and $(z-\overline{z})^4$ of PCR(5,1,2) with $h=12$ over the period 1994:M01-2008:M08.



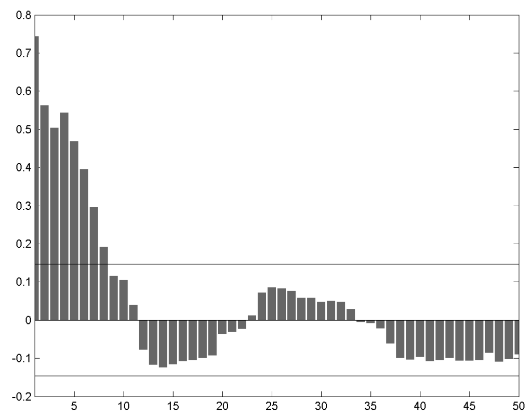
(a)



(b)



(c)



(d)

Figure E.48: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z}), (z-\overline{z})^2, (z-\overline{z})^3$ and $(z-\overline{z})^4$ of PCR(10,1,2) with $h = 12$ over the period 1994:M01-2008:M08.

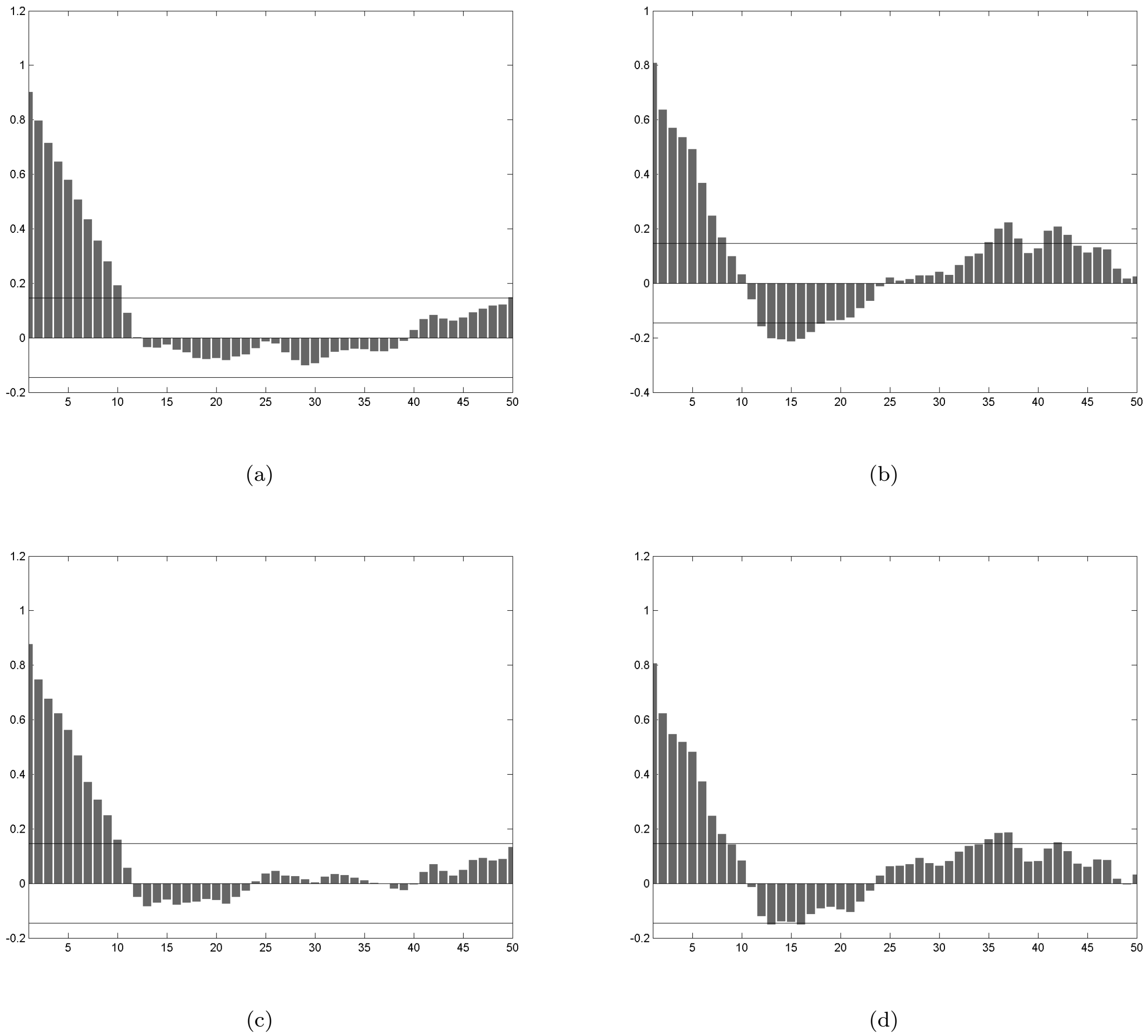
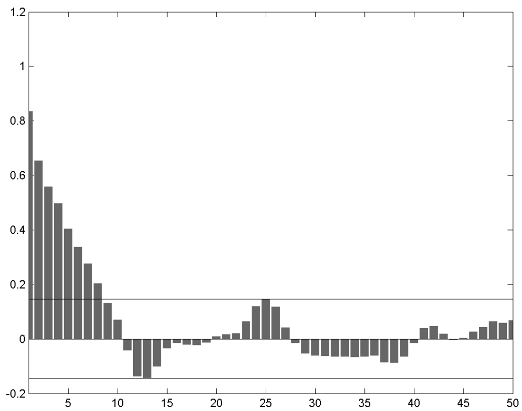
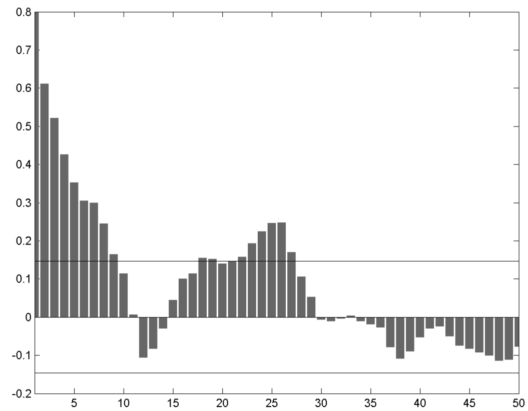


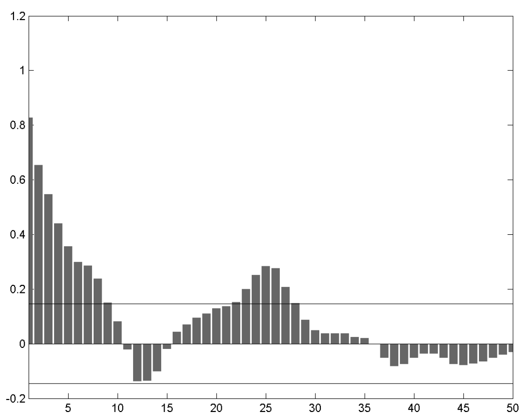
Figure E.49: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z}),(z-\overline{z})^2,(z-\overline{z})^3$ and $(z-\overline{z})^4$ of PCR(1,2,2) with $h=12$ over the period 1994:M01-2008:M08.



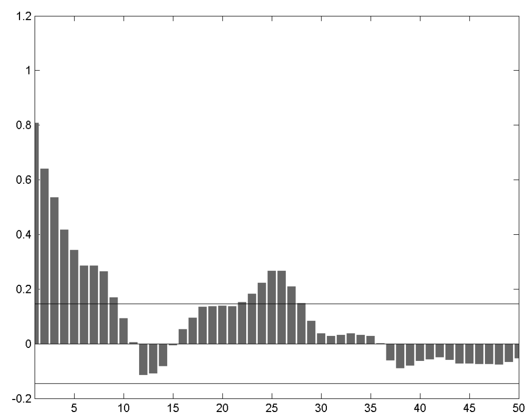
(a)



(b)



(c)



(d)

Figure E.50: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z}),(z-\overline{z})^2,(z-\overline{z})^3$ and $(z-\overline{z})^4$ of PCR(5,2,2) with $h=12$ over the period 1994:M01-2008:M08.

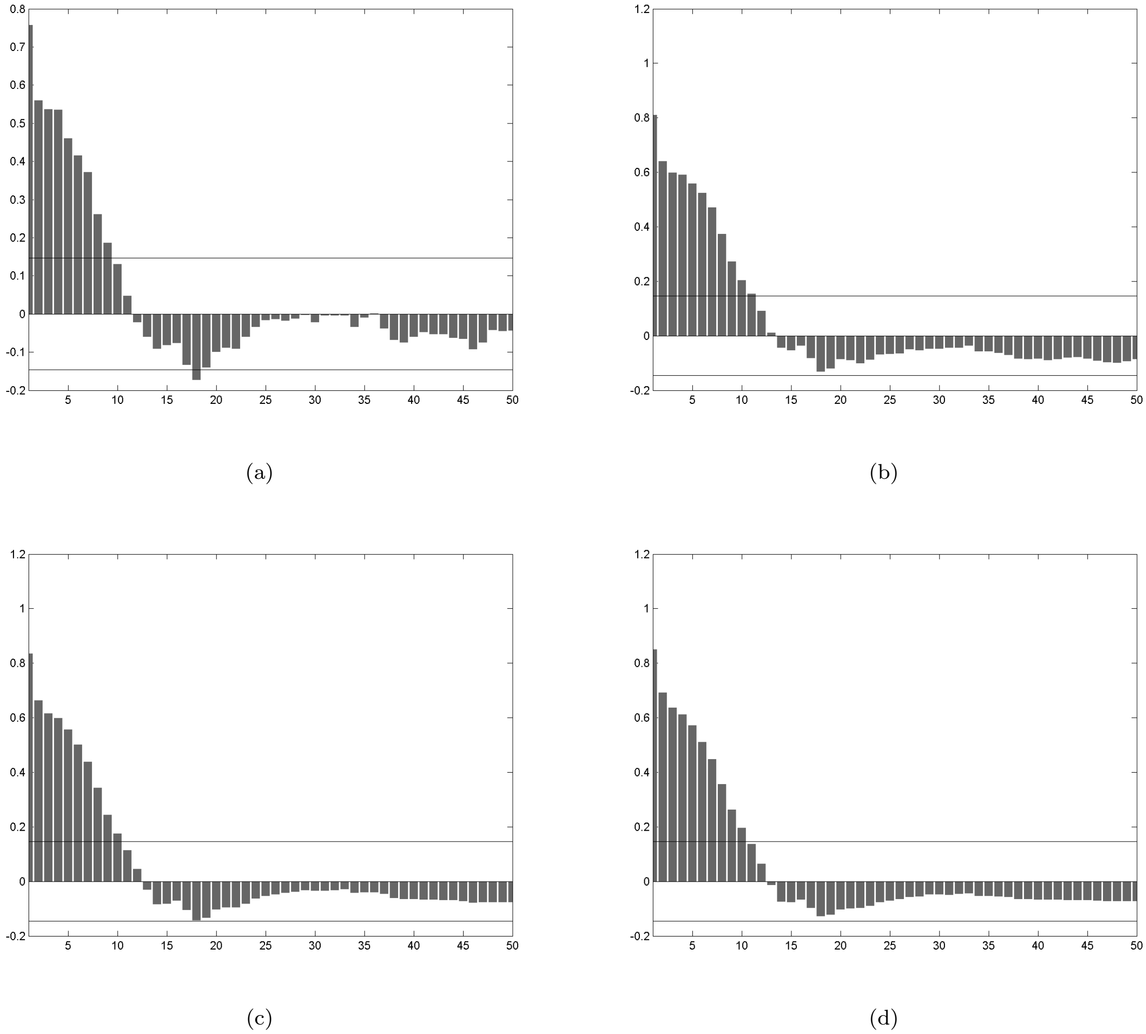


Figure E.51: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z}),(z-\overline{z})^2,(z-\overline{z})^3$ and $(z-\overline{z})^4$ of PCR(10,2,2) with $h=12$ over the period 1994:M01-2008:M08.

Table E.10: KLIC statistics for $h = 12$ over the period over the period 1994:M01-2008:M08.

Model	RW	AR(2)	PCR(1,0,0)	PCR(5,0,0)	PCR(10,0,0)	PCR(1,0,2)	PCR(5,0,2)
RW	-	0,18	-0,94	-1,14	-0,54	-1,42	-0,6
AR(2)	-0,18	-	-1,06	-1	-0,66	-1,08	-0,57
PCR(1,0,0)	0,94	1,06	-	0,57	0,87	0,76	0,89
PCR(5,0,0)	1,14	1	-0,57	-	1,14	0,31	1,5
PCR(10,0,0)	0,54	0,66	-0,87	-1,14	-	-0,42	-0,08
PCR(1,0,2)	1,42	1,08	-0,76	-0,31	0,42	-	0,43
PCR(5,0,2)	0,6	0,57	-0,89	-1,5	0,08	-0,43	-
PCR(10,0,2)	0,93	0,9	-0,32	0,54	0,96	0,45	0,96
PCR(1,1,2)	0,96	0,82	-0,5	0,03	0,54	0,36	0,6
PCR(5,1,2)	0,89	0,83	-0,57	-0,15	0,89	0,22	1,04
PCR(10,1,2)	1,21	1,31	-0,53	0,55	1,25	0,74	1,29
PCR(1,2,2)	0,9	0,68	-0,82	-0,77	0,09	-0,57	0,07
PCR(5,2,2)	1,07	1,03	0,01	1,02	1,11	0,76	1,16
PCR(10,2,2)	1,28	1,29	0,13	1,33	1,44	1,04	1,54

Model	PCR(10,0,2)	PCR(1,1,2)	PCR(5,1,2)	PCR(10,1,2)	PCR(1,2,2)	PCR(5,2,2)	PCR(10,2,2)
RW	-0,93	-0,96	-0,89	-1,21	-0,9	-1,07	-1,28
AR(2)	-0,9	-0,82	-0,83	-1,31	-0,68	-1,03	-1,29
PCR(1,0,0)	0,32	0,5	0,57	0,53	0,82	-0,01	-0,13
PCR(5,0,0)	-0,54	-0,03	0,15	-0,55	0,77	-1,02	-1,33
PCR(10,0,0)	-0,96	-0,54	-0,89	-1,25	-0,09	-1,11	-1,44
PCR(1,0,2)	-0,45	-0,36	-0,22	-0,74	0,57	-0,76	-1,04
PCR(5,0,2)	-0,96	-0,6	-1,04	-1,29	-0,07	-1,16	-1,54
PCR(10,0,2)	-	0,24	0,77	-0,07	0,69	-1,24	-1,09
PCR(1,1,2)	-0,24	-	0,06	-0,32	0,97	-0,64	-0,73
PCR(5,1,2)	-0,77	-0,06	-	-0,52	0,54	-1,19	-1,38
PCR(10,1,2)	0,07	0,32	0,52	-	0,88	-0,42	-0,9
PCR(1,2,2)	-0,69	-0,97	-0,54	-0,88	-	-0,94	-1,1
PCR(5,2,2)	1,24	0,64	1,19	0,42	0,94	-	-0,21
PCR(10,2,2)	1,09	0,73	1,38	0,9	1,1	0,21	-

Table E.11: KLIC model favors for $h = 12$ over the period over the period 1994:M01-2008:M08. One signifies the model in the row is preferred. Two signifies the model in the column is preferred.

Model	RW	AR(2)	PCR(1,0,0)	PCR(5,0,0)	PCR(10,0,0)	PCR(1,0,2)	PCR(5,0,2)
RW	-	2	1	1	1	1	1
AR(2)	1	-	1	1	1	1	1
PCR(1,0,0)	2	2	-	2	2	2	2
PCR(5,0,0)	2	2	1	-	2	2	2
PCR(10,0,0)	2	2	1	1	-	1	1
PCR(1,0,2)	2	2	1	1	2	-	2
PCR(5,0,2)	2	2	1	1	2	1	-
PCR(10,0,2)	2	2	1	2	2	2	2
PCR(1,1,2)	2	2	1	2	2	2	2
PCR(5,1,2)	2	2	1	1	2	2	2
PCR(10,1,2)	2	2	1	2	2	2	2
PCR(1,2,2)	2	2	1	1	2	1	2
PCR(5,2,2)	2	2	2	2	2	2	2
PCR(10,2,2)	2	2	2	2	2	2	2

Model	PCR(10,0,2)	PCR(1,1,2)	PCR(5,1,2)	PCR(10,1,2)	PCR(1,2,2)	PCR(5,2,2)	PCR(10,2,2)
RW	1	1	1	1	1	1	1
AR(2)	1	1	1	1	1	1	1
PCR(1,0,0)	2	2	2	2	2	1	1
PCR(5,0,0)	1	1	2	1	2	1	1
PCR(10,0,0)	1	1	1	1	1	1	1
PCR(1,0,2)	1	1	1	1	2	1	1
PCR(5,0,2)	1	1	1	1	1	1	1
PCR(10,0,2)	-	2	2	1	2	1	1
PCR(1,1,2)	1	-	2	1	2	1	1
PCR(5,1,2)	1	1	-	1	2	1	1
PCR(10,1,2)	2	2	2	-	2	1	1
PCR(1,2,2)	1	1	1	1	-	1	1
PCR(5,2,2)	2	2	2	2	2	-	1
PCR(10,2,2)	2	2	2	2	2	2	-

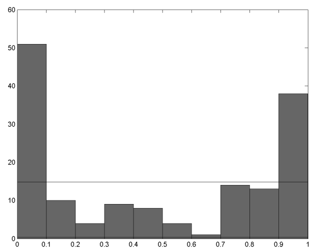
Table E.12: KLIC probabilities for $h = 24$ over the period over the period 1994:M01-2008:M08.

Model	RW	AR(2)	PCR(1,0,0)	PCR(5,0,0)	PCR(10,0,0)	PCR(1,0,2)	PCR(5,0,2)
RW	-	0,39	0,26	0,21	0,34	0,15	0,33
AR(2)	0,39	-	0,23	0,24	0,32	0,22	0,34
PCR(1,0,0)	0,26	0,23	-	0,34	0,27	0,3	0,27
PCR(5,0,0)	0,21	0,24	0,34	-	0,21	0,38	0,13
PCR(10,0,0)	0,34	0,32	0,27	0,21	-	0,36	0,4
PCR(1,0,2)	0,15	0,22	0,3	0,38	0,36	-	0,36
PCR(5,0,2)	0,33	0,34	0,27	0,13	0,4	0,36	-
PCR(10,0,2)	0,26	0,27	0,38	0,34	0,25	0,36	0,25
PCR(1,1,2)	0,25	0,29	0,35	0,4	0,34	0,37	0,33
PCR(5,1,2)	0,27	0,28	0,34	0,39	0,27	0,39	0,23
PCR(10,1,2)	0,19	0,17	0,35	0,34	0,18	0,3	0,17
PCR(1,2,2)	0,27	0,32	0,28	0,3	0,4	0,34	0,4
PCR(5,2,2)	0,22	0,23	0,4	0,24	0,22	0,3	0,2
PCR(10,2,2)	0,18	0,17	0,4	0,16	0,14	0,23	0,12

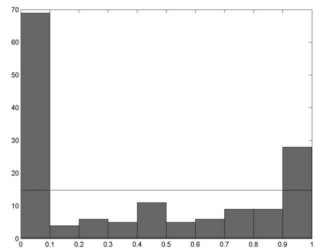
Model	PCR(10,0,2)	PCR(1,1,2)	PCR(5,1,2)	PCR(10,1,2)	PCR(1,2,2)	PCR(5,2,2)	PCR(10,2,2)
RW	0,26	0,25	0,27	0,19	0,27	0,22	0,18
AR(2)	0,27	0,29	0,28	0,17	0,32	0,23	0,17
PCR(1,0,0)	0,38	0,35	0,34	0,35	0,28	0,4	0,4
PCR(5,0,0)	0,34	0,4	0,39	0,34	0,3	0,24	0,16
PCR(10,0,0)	0,25	0,34	0,27	0,18	0,4	0,22	0,14
PCR(1,0,2)	0,36	0,37	0,39	0,3	0,34	0,3	0,23
PCR(5,0,2)	0,25	0,33	0,23	0,17	0,4	0,2	0,12
PCR(10,0,2)	-	0,39	0,3	0,4	0,31	0,18	0,22
PCR(1,1,2)	0,39	-	0,4	0,38	0,25	0,33	0,3
PCR(5,1,2)	0,3	0,4	-	0,35	0,34	0,2	0,15
PCR(10,1,2)	0,4	0,38	0,35	-	0,27	0,37	0,27
PCR(1,2,2)	0,31	0,25	0,34	0,27	-	0,26	0,22
PCR(5,2,2)	0,18	0,33	0,2	0,37	0,26	-	0,39
PCR(10,2,2)	0,22	0,3	0,15	0,27	0,22	0,39	-

Table E.13: Berkowitz Likelihood Ratios and p-values for the models evaluated with a $h = 24$ over the period 1995:M01-2007:M08.

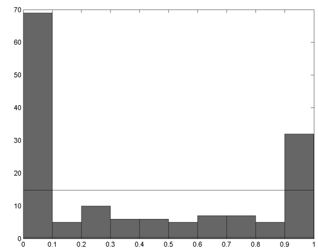
Model	LR	p-value
RW	1,49E+03	0
AR(2)	6,80E+01	0
PCR(1,0,0)	1,28E+03	0
PCR(5,0,0)	3,86E+03	0
PCR(10,0,0)	3,92E+03	0
PCR(1,0,2)	4,81E+02	0
PCR(5,0,2)	3,18E+03	0
PCR(10,0,2)	5,78E+03	0
PCR(1,1,2)	2,53E+02	0
PCR(5,1,2)	4,52E+03	0
PCR(10,1,2)	5,54E+03	0
PCR(1,2,2)	7,14E+02	0
PCR(5,2,2)	4,16E+03	0
PCR(10,2,2)	7,44E+03	0



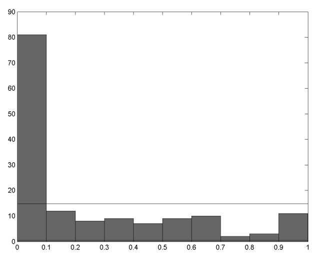
(a) RW



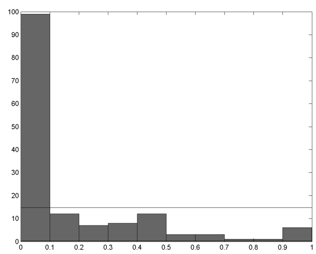
(b) AR(2)



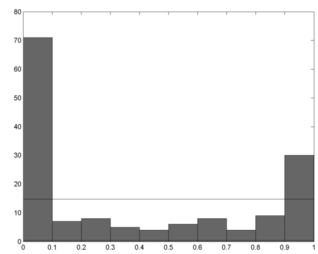
(c) PCR(1,0,0)



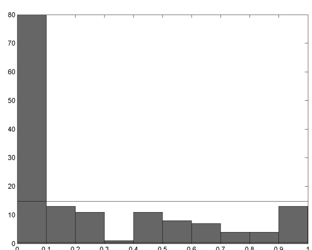
(d) PCR(5,0,0)



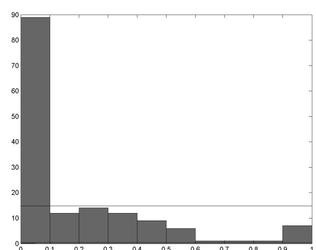
(e) PCR(10,0,0)



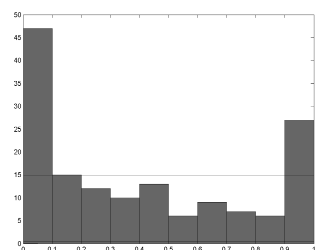
(f) PCR(1,0,2)



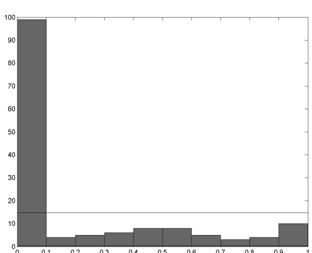
(g) PCR(5,0,2)



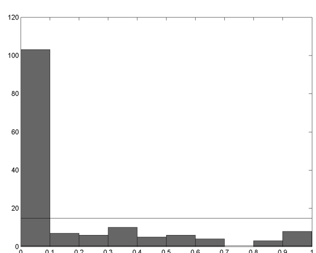
(h) PCR(10,0,2)



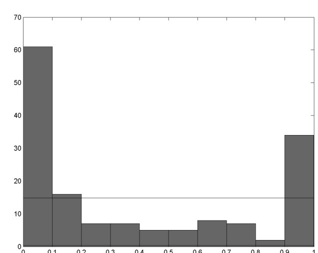
(i) PCR(1,1,2)



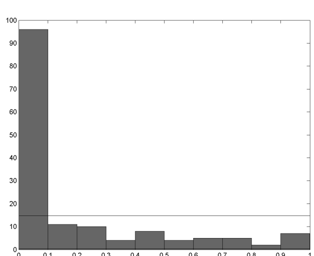
(j) PCR(5,1,2)



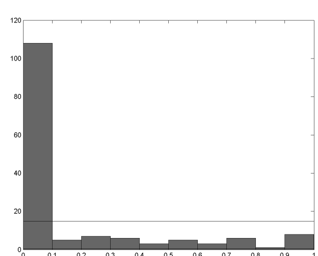
(k) PCR(10,1,2)



(l) PCR(1,2,2)

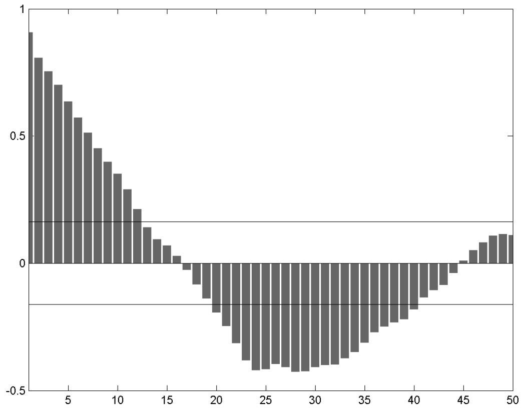


(m) PCR(5,2,2)

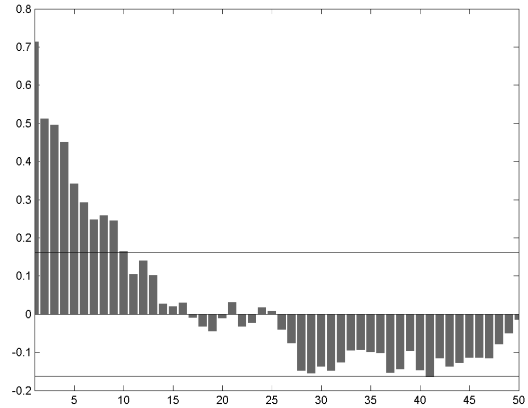


(n) PCR(10,2,2)

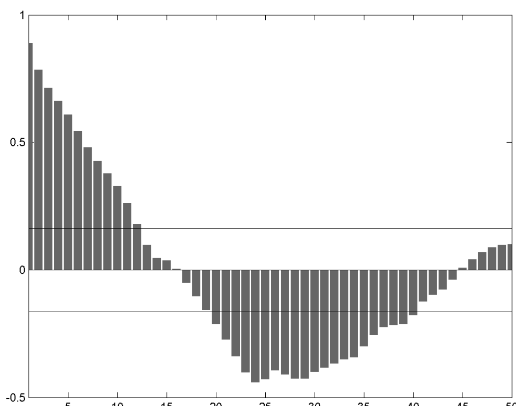
Figure E.52: Histogram of the probability integral transforms with $h = 24$ over the period 1995:M01-2007:M08.



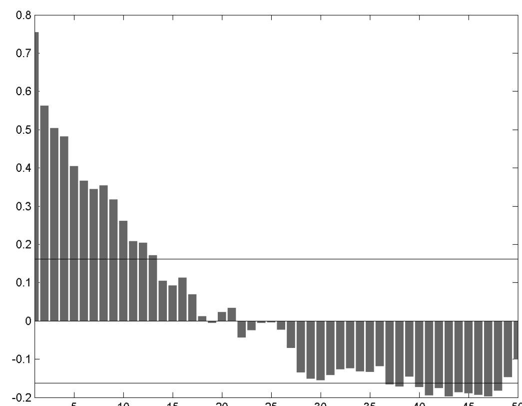
(a)



(b)



(c)



(d)

Figure E.53: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z})\,,(z-\overline{z})^2\,,(z-\overline{z})^3$ and $(z-\overline{z})^4$ of RW with $h=24$ over the period 1995:M01-2007:M08.

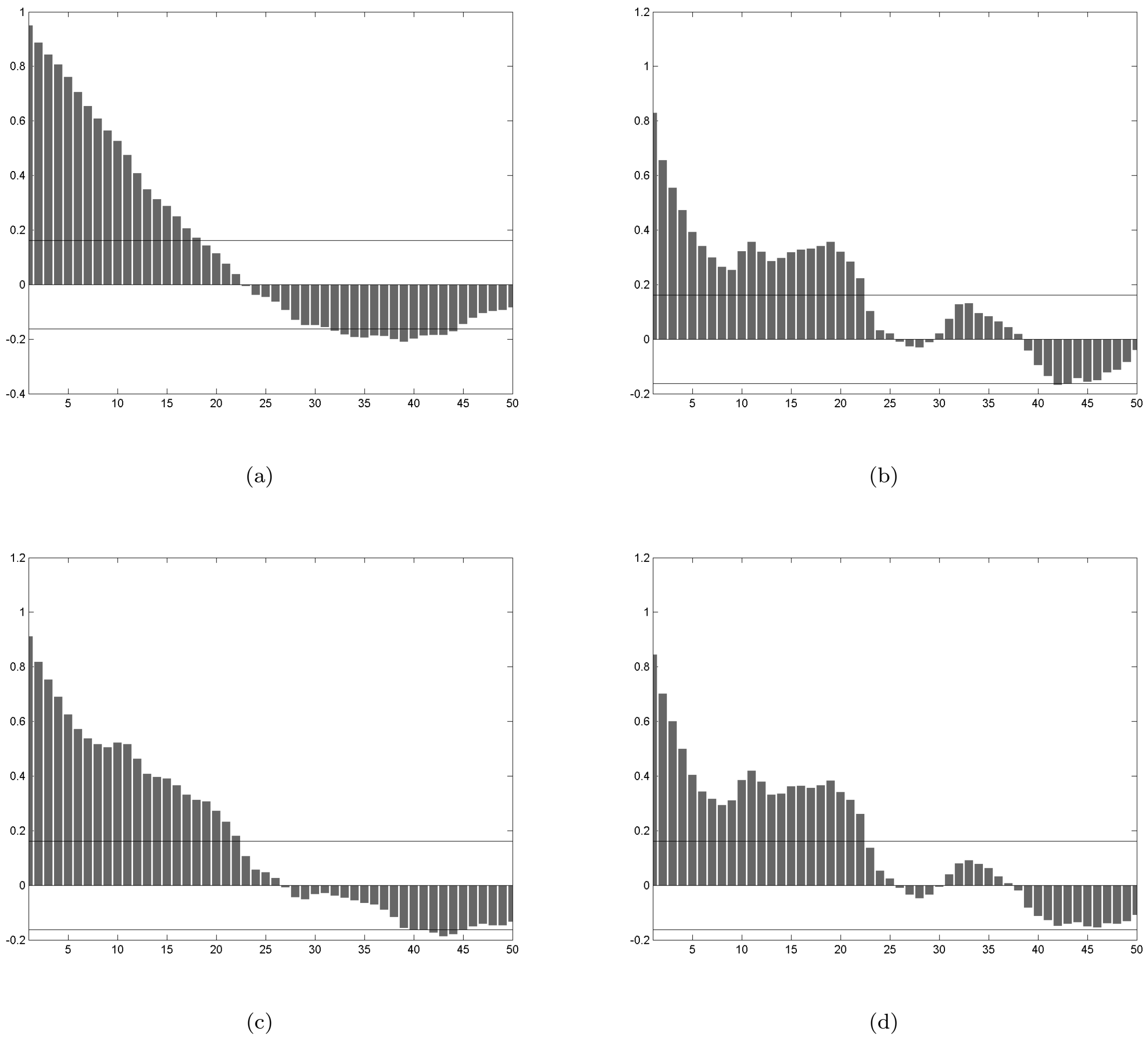


Figure E.54: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z})\,,(z-\overline{z})^2\,,(z-\overline{z})^3$ and $(z-\overline{z})^4$ of AR(2) with $h=24$ over the period 1995:M01-2007:M08.

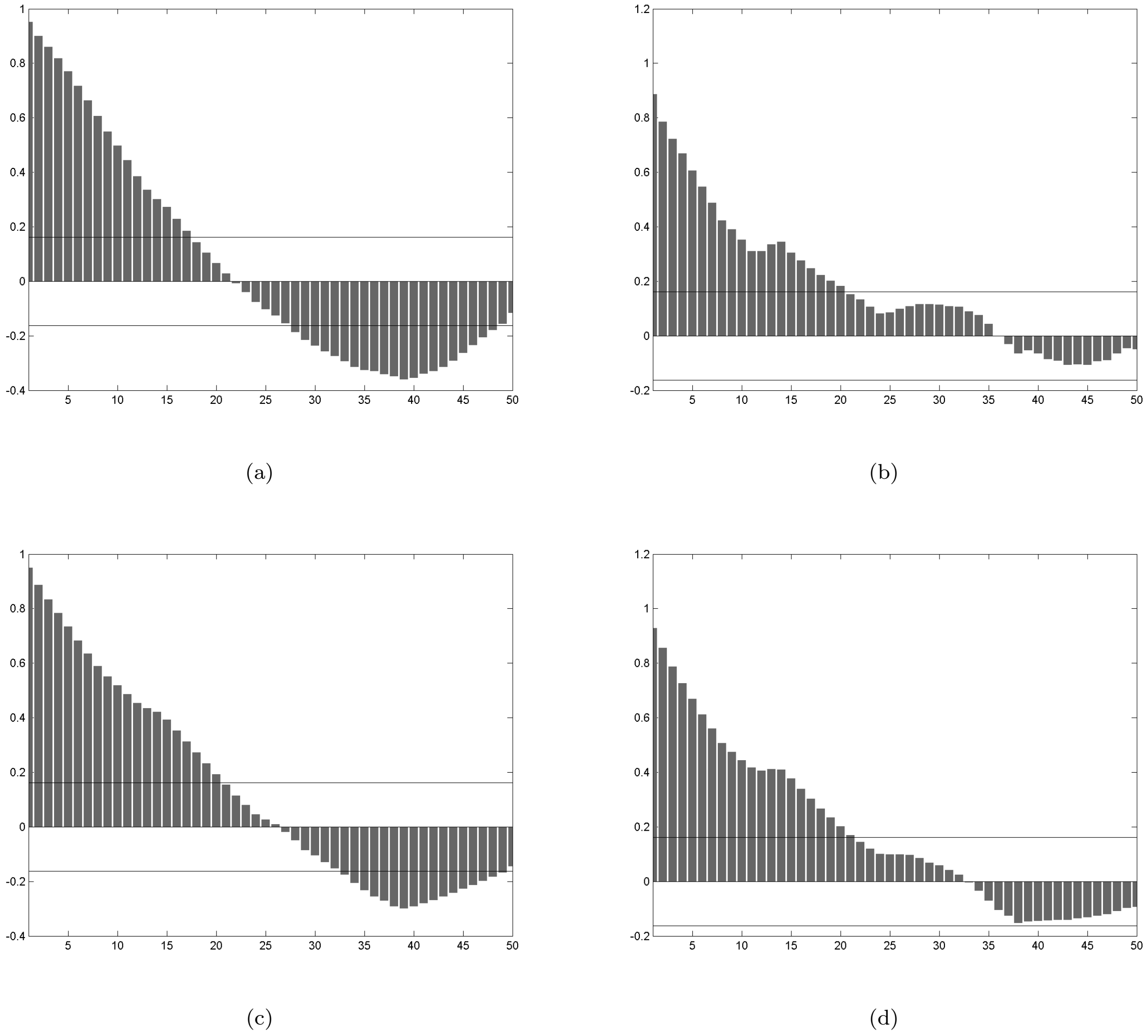
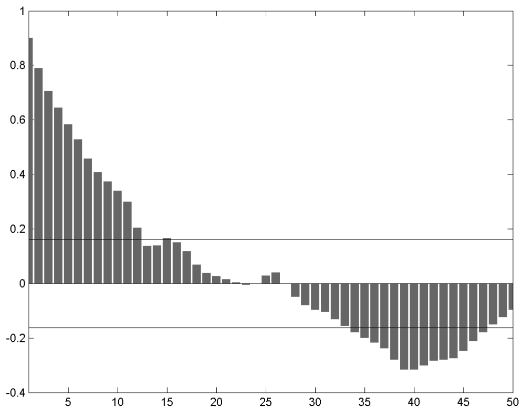
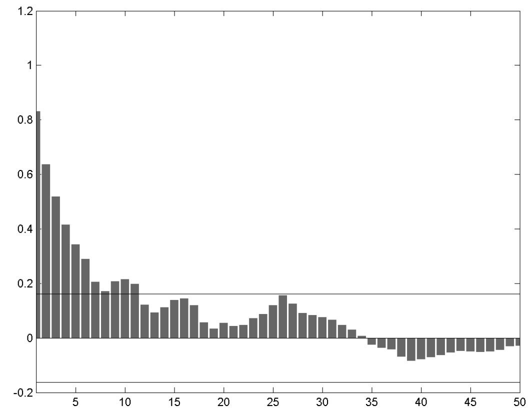


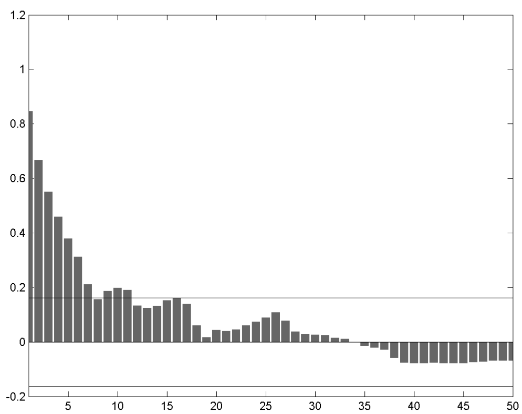
Figure E.55: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z}), (z-\overline{z})^2, (z-\overline{z})^3$ and $(z-\overline{z})^4$ of PCR(1,0,0) with $h = 24$ over the period 1995:M01-2007:M08.



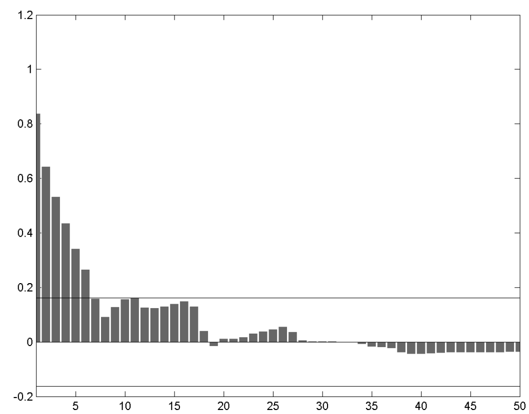
(a)



(b)

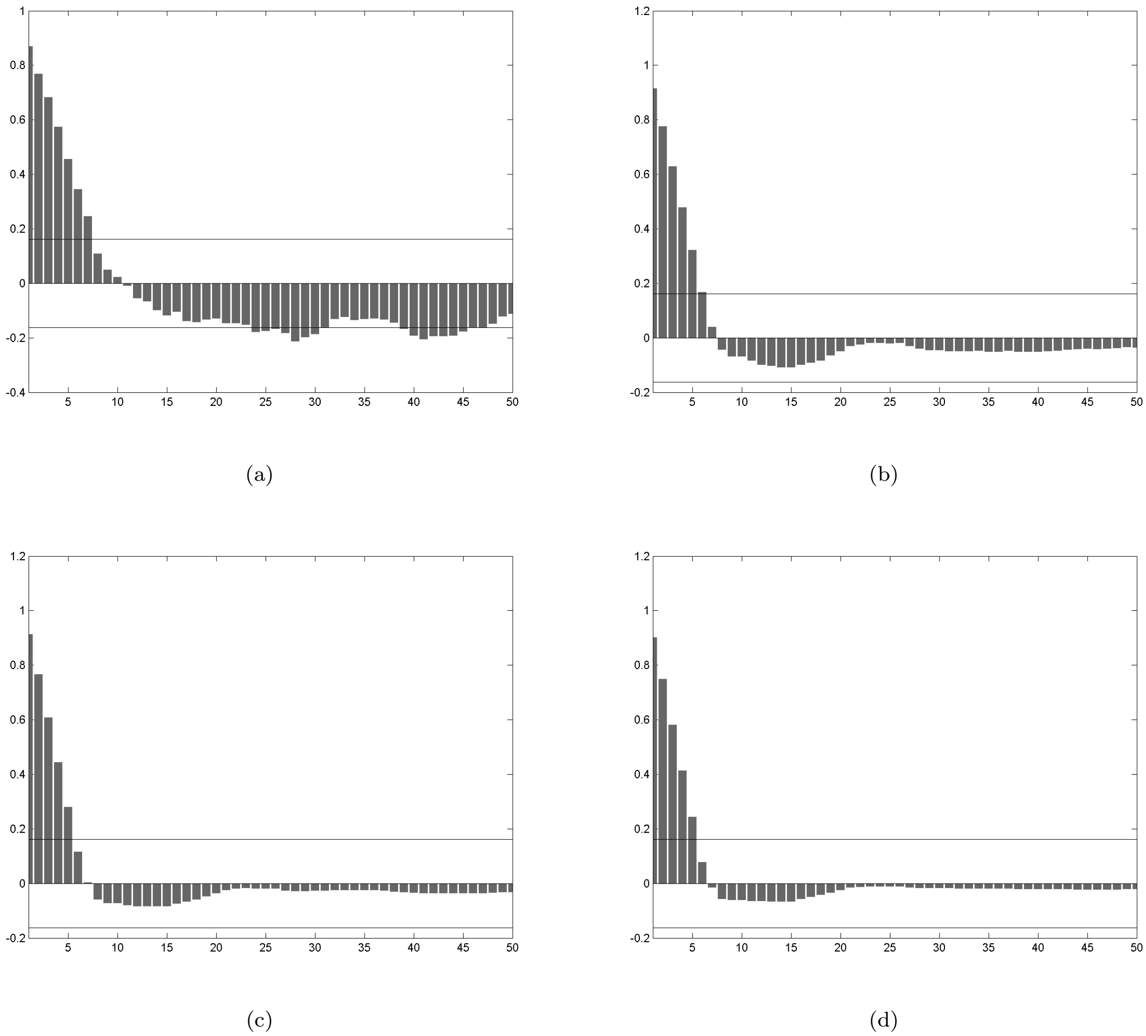


(c)



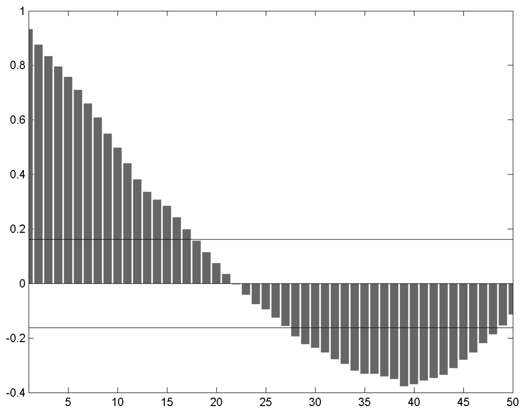
(d)

Figure E.56: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z}), (z-\overline{z})^2, (z-\overline{z})^3$ and $(z-\overline{z})^4$ of PCR(5,0,0) with $h = 24$ over the period 1995:M01-2007:M08.

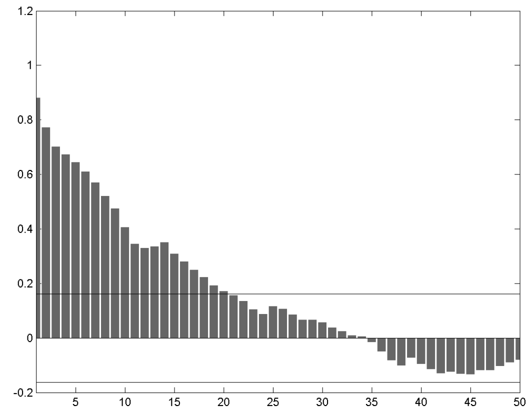


(a) (b) (c) (d)

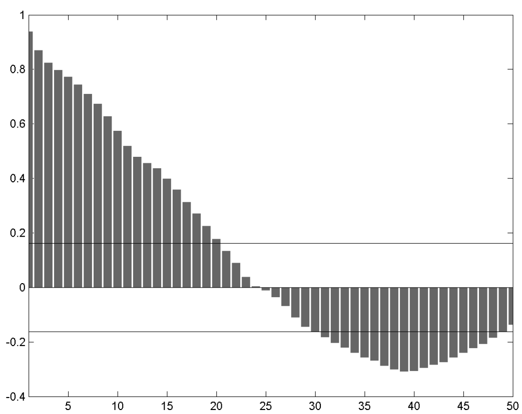
Figure E.57: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z}), (z-\overline{z})^2, (z-\overline{z})^3$ and $(z-\overline{z})^4$ of PCR(10,0,0) with $h = 24$ over the period 1995:M01-2007:M08.



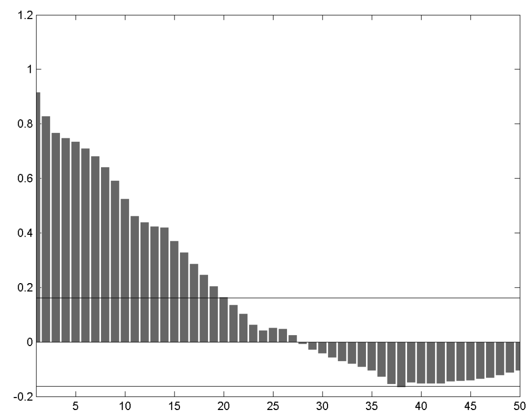
(a)



(b)

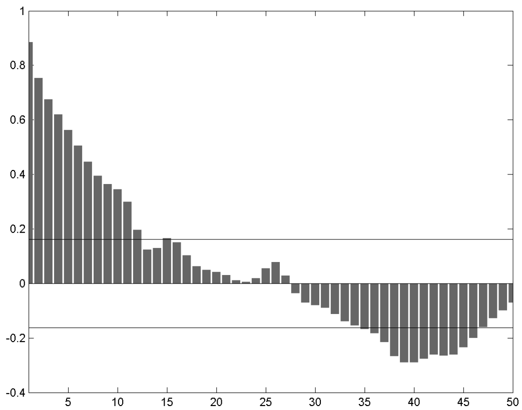


(c)

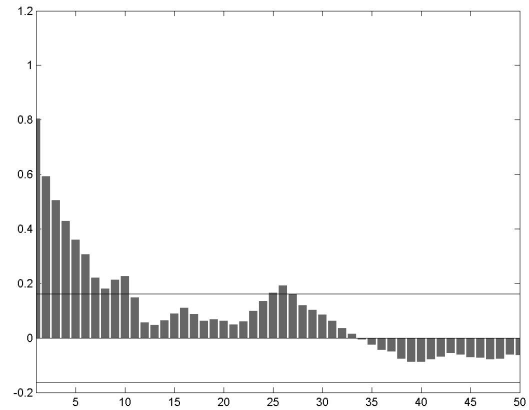


(d)

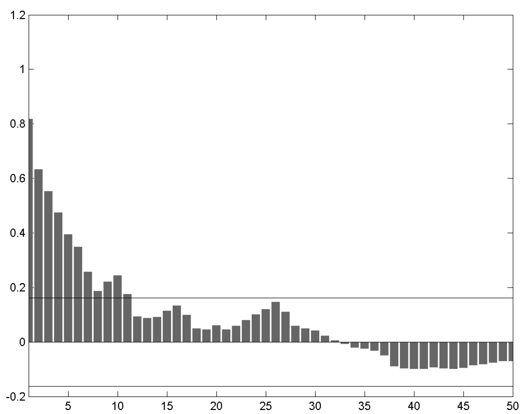
Figure E.58: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z}),(z-\overline{z})^2,(z-\overline{z})^3$ and $(z-\overline{z})^4$ of PCR(1,0,2) with $h=24$ over the period 1995:M01-2007:M08.



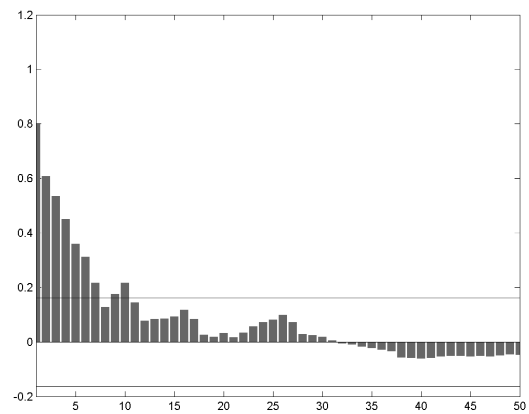
(a)



(b)



(c)



(d)

Figure E.59: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z}), (z-\overline{z})^2, (z-\overline{z})^3$ and $(z-\overline{z})^4$ of PCR(5,0,2) with $h = 24$ over the period 1995:M01-2007:M08.

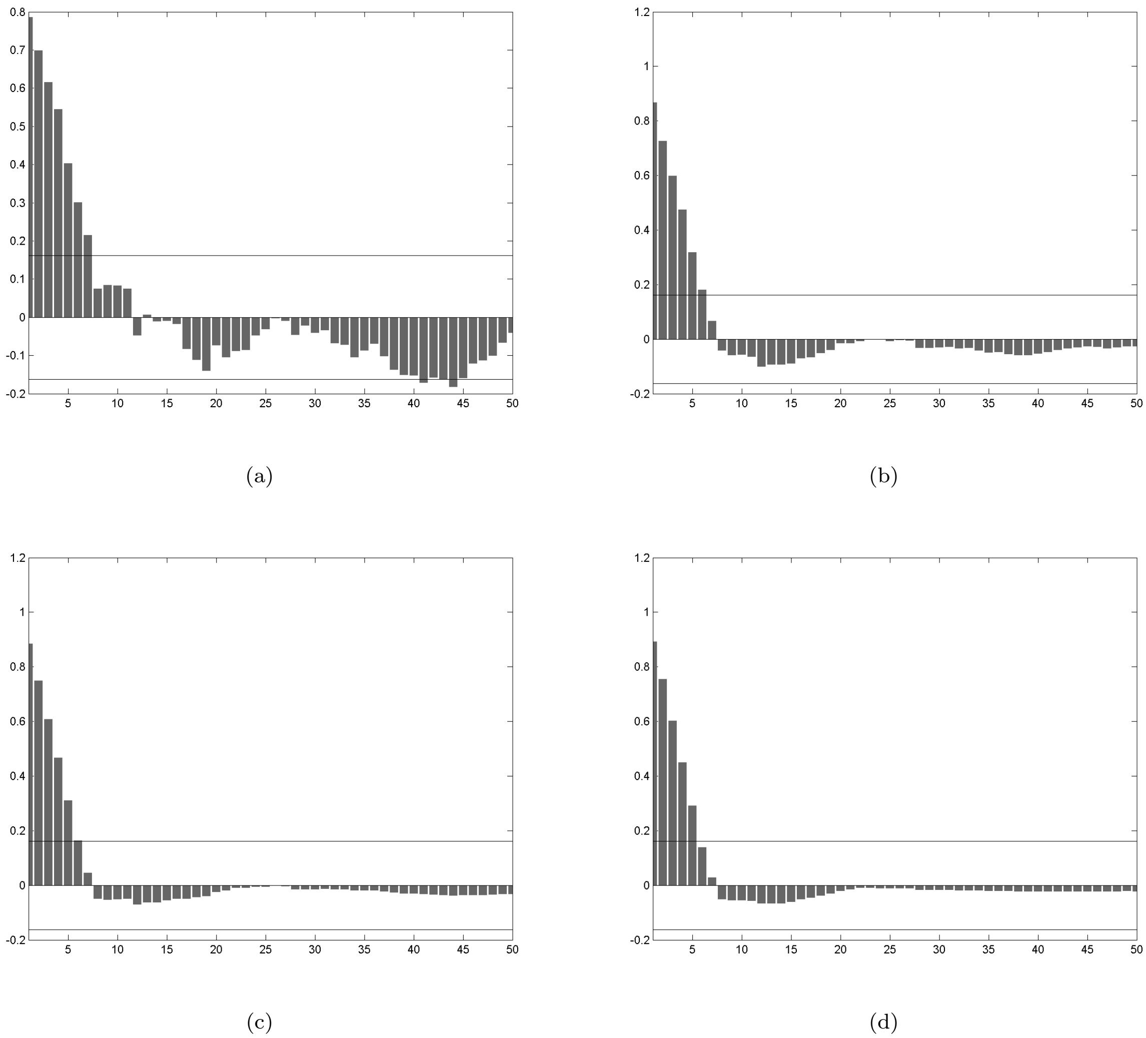
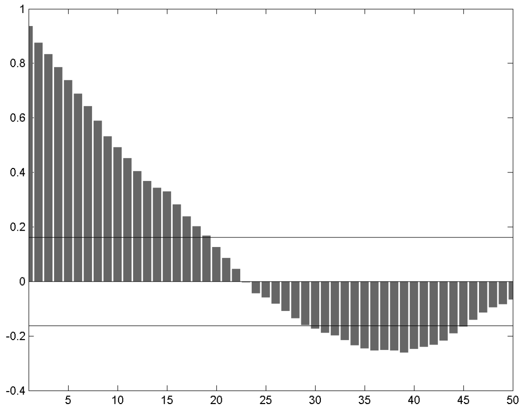
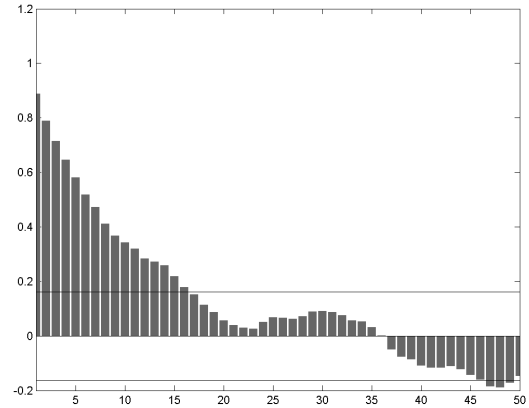


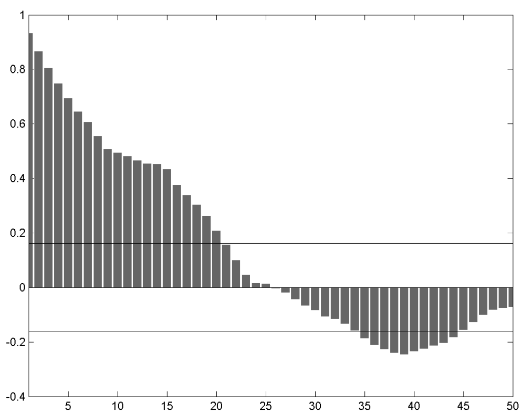
Figure E.60: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z}), (z-\overline{z})^2, (z-\overline{z})^3$ and $(z-\overline{z})^4$ of PCR(10,0,2) with $h=24$ over the period 1995:M01-2007:M08.



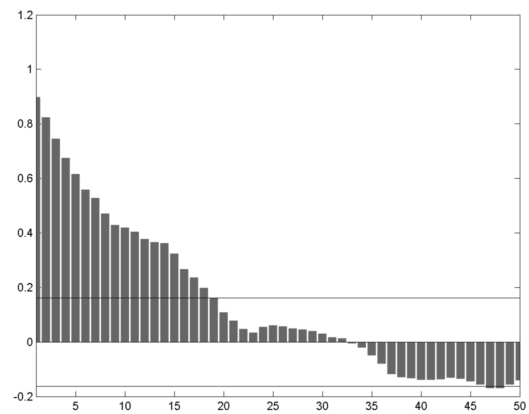
(a)



(b)

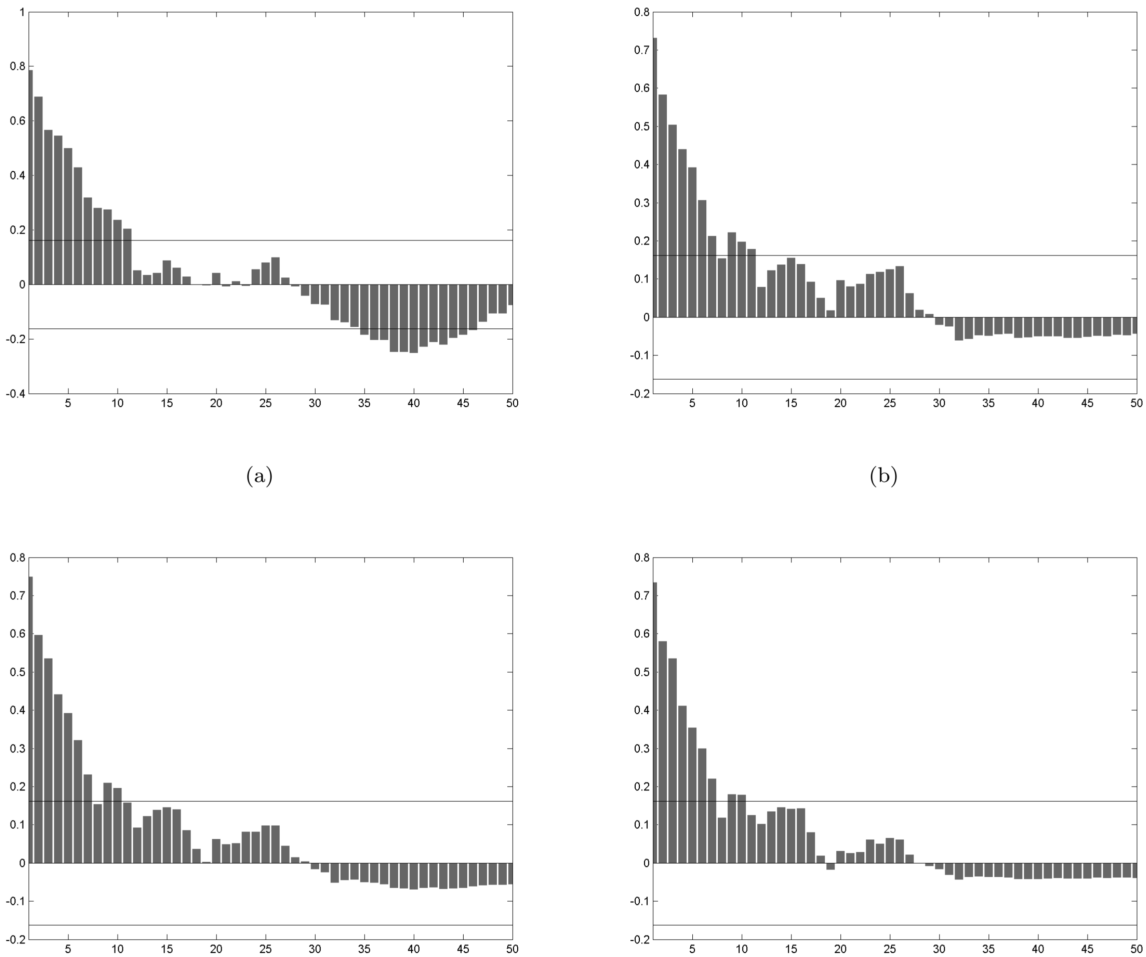


(c)



(d)

Figure E.61: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z}), (z-\overline{z})^2, (z-\overline{z})^3$ and $(z-\overline{z})^4$ of PCR(1,1,2) with $h=24$ over the period 1995:M01-2007:M08.



(a) (b) (c) (d)

Figure E.62: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z})\,,(z-\overline{z})^2\,,(z-\overline{z})^3$ and $(z-\overline{z})^4$ of PCR(5,1,2) with $h=24$ over the period 1995:M01-2007:M08.

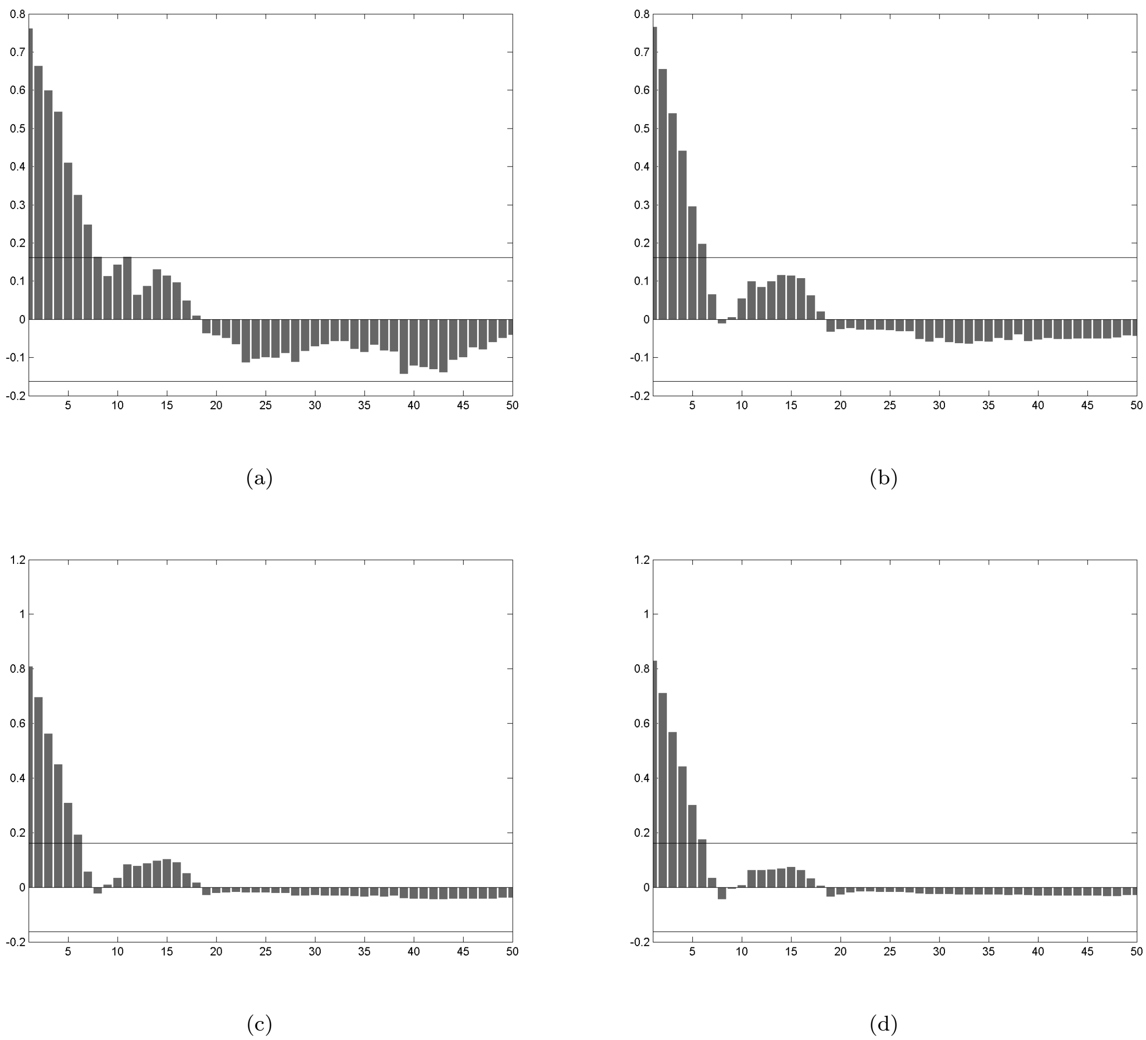
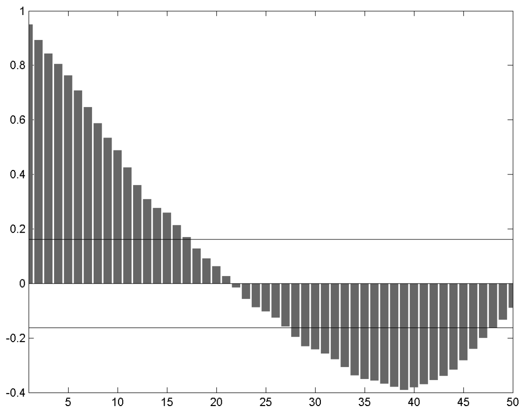
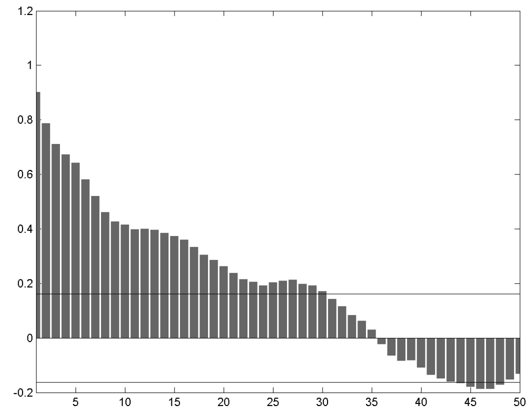


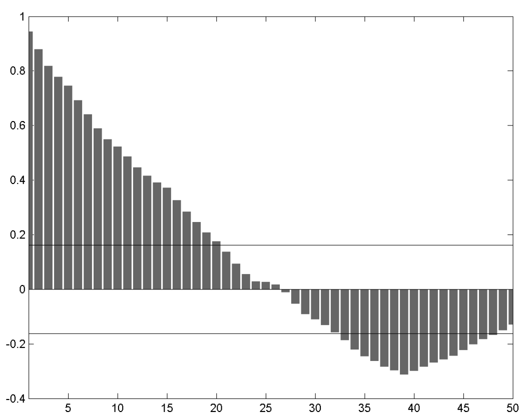
Figure E.63: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z}),(z-\overline{z})^2,(z-\overline{z})^3$ and $(z-\overline{z})^4$ of PCR(10,1,2) with $h=24$ over the period 1995:M01-2007:M08.



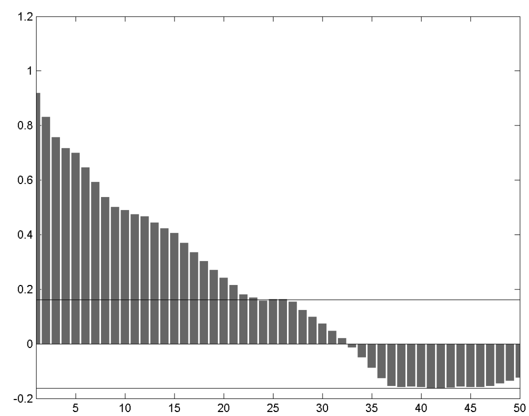
(a)



(b)

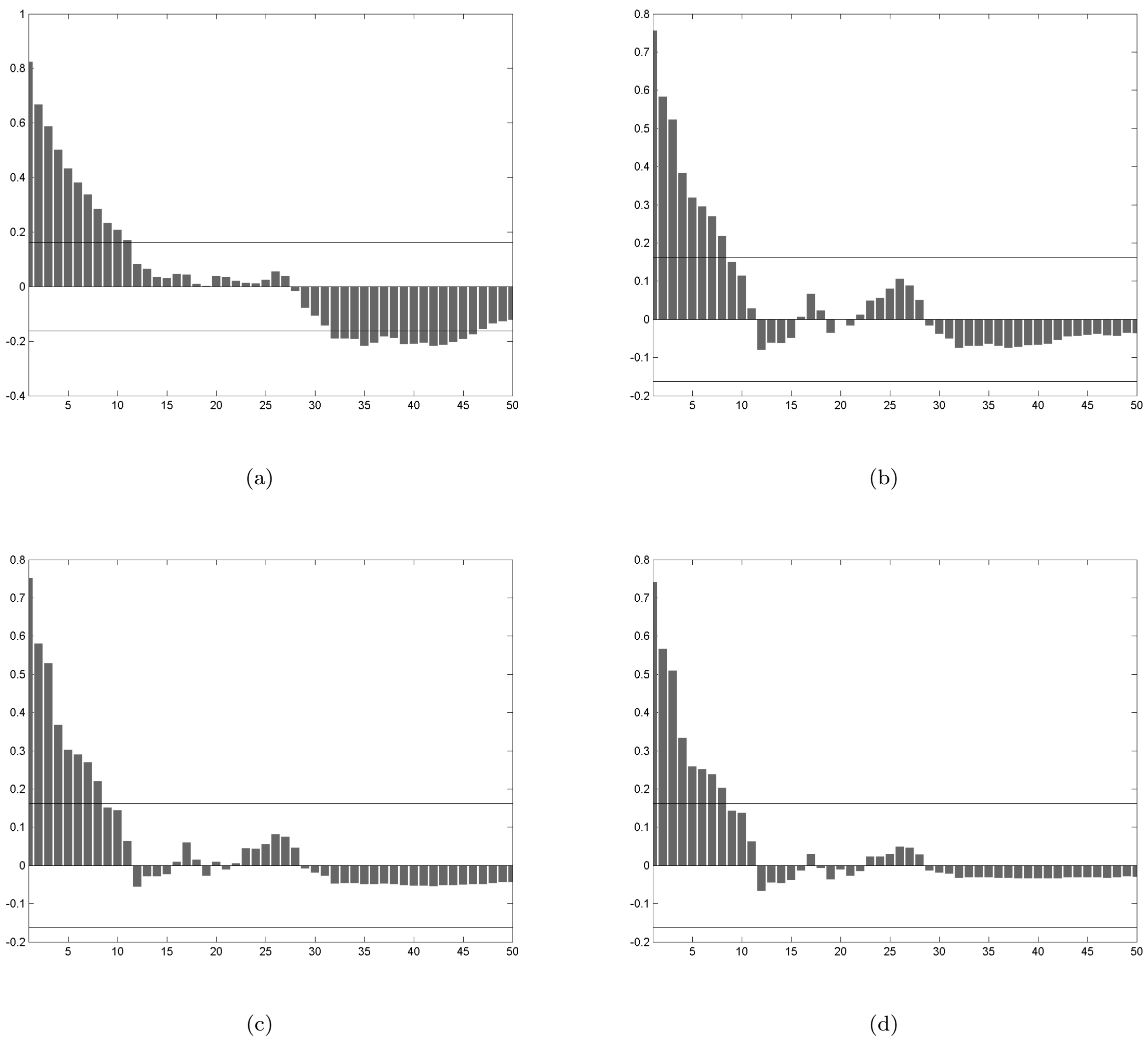


(c)



(d)

Figure E.64: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z})\,,(z-\overline{z})^2\,,(z-\overline{z})^3$ and $(z-\overline{z})^4$ of PCR(1,2,2) with $h=24$ over the period 1995:M01-2007:M08.



(a) (b) (c) (d)

Figure E.65: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z}),(z-\overline{z})^2,(z-\overline{z})^3$ and $(z-\overline{z})^4$ of PCR(5,2,2) with $h=24$ over the period 1995:M01-2007:M08.

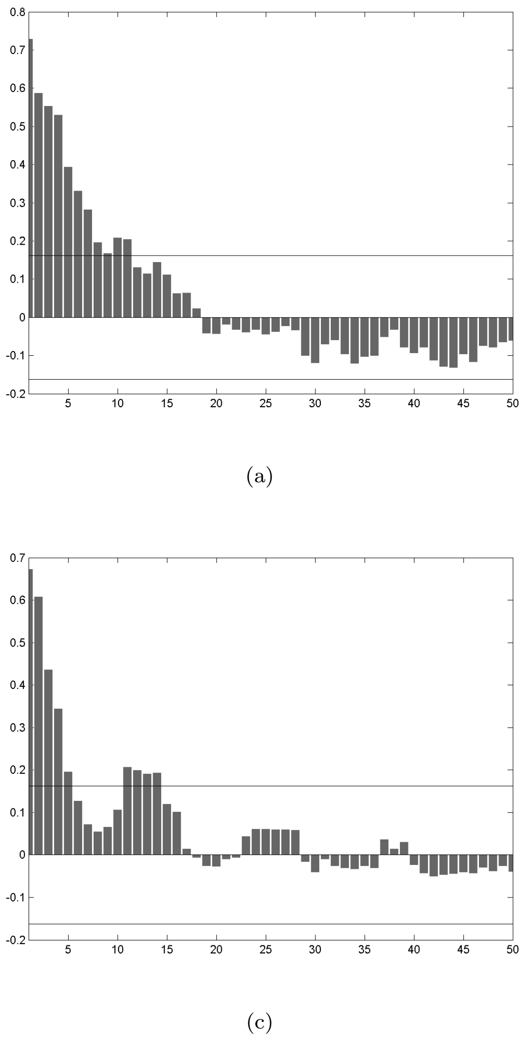


Figure E.66: Panels (a)-(d) show sample autocorrelations of $(z-\overline{z}),(z-\overline{z})^2,(z-\overline{z})^3$ and $(z-\overline{z})^4$ of PCR(10,2,2) with $h=24$ over the period 1995:M01-2007:M08.

Table E.14: KLIC statistics for $h = 24$ over the period 1995:M01-2007:M08.

Model	RW	AR(2)	PCR(1,0,0)	PCR(5,0,0)	PCR(10,0,0)	PCR(1,0,2)	PCR(5,0,2)
RW	-	0,97	-0,7	-1,04	-1,07	-0,28	-0,69
AR(2)	-0,97	-	-1,14	-1,38	-1,63	-0,99	-1,26
PCR(1,0,0)	0,7	1,14	-	0,14	0,03	1,41	0,3
PCR(5,0,0)	1,04	1,38	-0,14	-	-0,31	0,42	1,08
PCR(10,0,0)	1,07	1,63	-0,03	0,31	-	0,96	0,76
PCR(1,0,2)	0,28	0,99	-1,41	-0,42	-0,96	-	-0,24
PCR(5,0,2)	0,69	1,26	-0,3	-1,08	-0,76	0,24	-
PCR(10,0,2)	1,44	1,38	0,44	1,16	0,7	0,87	1,14
PCR(1,1,2)	-0,57	0,93	-1,19	-1,17	-1,68	-1	-1,04
PCR(5,1,2)	0,87	1,3	0,07	0,49	0,23	0,64	1,01
PCR(10,1,2)	1,4	1,49	0,28	1,67	0,66	0,84	1,5
PCR(1,2,2)	0,06	1,13	-1,15	-0,7	-1,3	-0,83	-0,53
PCR(5,2,2)	1,22	1,34	-0,08	0,42	-0,12	0,47	0,71
PCR(10,2,2)	1,32	1,5	0,55	1,47	1,27	1,06	1,62

Model	PCR(10,0,2)	PCR(1,1,2)	PCR(5,1,2)	PCR(10,1,2)	PCR(1,2,2)	PCR(5,2,2)	PCR(10,2,2)
RW	-1,44	0,57	-0,87	-1,4	-0,06	-1,22	-1,32
AR(2)	-1,38	-0,93	-1,3	-1,49	-1,13	-1,34	-1,5
PCR(1,0,0)	-0,44	1,19	-0,07	-0,28	1,15	0,08	-0,55
PCR(5,0,0)	-1,16	1,17	-0,49	-1,67	0,7	-0,42	-1,47
PCR(10,0,0)	-0,7	1,68	-0,23	-0,66	1,3	0,12	-1,27
PCR(1,0,2)	-0,87	1	-0,64	-0,84	0,83	-0,47	-1,06
PCR(5,0,2)	-1,14	1,04	-1,01	-1,5	0,53	-0,71	-1,62
PCR(10,0,2)	-	1,27	0,58	0,7	1,04	1,37	0,01
PCR(1,1,2)	-1,27	-	-1,21	-1,37	-1,53	-1,14	-1,42
PCR(5,1,2)	-0,58	1,21	-	-0,5	0,89	0,25	-1,12
PCR(10,1,2)	-0,7	1,37	0,5	-	1,06	1,56	-1,11
PCR(1,2,2)	-1,04	1,53	-0,89	-1,06	-	-0,75	-1,24
PCR(5,2,2)	-1,37	1,14	-0,25	-1,56	0,75	-	-1,26
PCR(10,2,2)	-0,01	1,42	1,12	1,11	1,24	1,26	-

Table E.15: KLIC model favors for $h = 24$ over the period over the period 1995:M01-2007:M08. One signifies the model in the row is preferred. Two signifies the model in the column is preferred.

Model	RW	AR(2)	PCR(1,0,0)	PCR(5,0,0)	PCR(10,0,0)	PCR(1,0,2)	PCR(5,0,2)
RW	-	2	1	1	1	1	1
AR(2)	1	-	1	1	1	1	1
PCR(1,0,0)	2	2	-	2	2	2	2
PCR(5,0,0)	2	2	1	-	1	2	2
PCR(10,0,0)	2	2	1	2	-	2	2
PCR(1,0,2)	2	2	1	1	1	-	1
PCR(5,0,2)	2	2	1	1	1	2	-
PCR(10,0,2)	2	2	2	2	2	2	2
PCR(1,1,2)	1	2	1	1	1	1	1
PCR(5,1,2)	2	2	2	2	2	2	2
PCR(10,1,2)	2	2	2	2	2	2	2
PCR(1,2,2)	2	2	1	1	1	1	1
PCR(5,2,2)	2	2	1	2	1	2	2
PCR(10,2,2)	2	2	2	2	2	2	2

Model	PCR(10,0,2)	PCR(1,1,2)	PCR(5,1,2)	PCR(10,1,2)	PCR(1,2,2)	PCR(5,2,2)	PCR(10,2,2)
RW	1	2	1	1	1	1	1
AR(2)	1	1	1	1	1	1	1
PCR(1,0,0)	1	2	1	1	2	2	1
PCR(5,0,0)	1	2	1	1	2	1	1
PCR(10,0,0)	1	2	1	1	2	2	1
PCR(1,0,2)	1	2	1	1	2	1	1
PCR(5,0,2)	1	2	1	1	2	1	1
PCR(10,0,2)	-	2	2	2	2	2	2
PCR(1,1,2)	1	-	1	1	1	1	1
PCR(5,1,2)	1	2	-	1	2	2	1
PCR(10,1,2)	1	2	2	-	2	2	1
PCR(1,2,2)	1	2	1	1	-	1	1
PCR(5,2,2)	1	2	1	1	2	-	1
PCR(10,2,2)	1	2	2	2	2	2	-

Table E.16: KLIC probabilities for $h = 24$ over the period over the period 1995:M01-2007:M08.

Model	RW	AR(2)	PCR(1,0,0)	PCR(5,0,0)	PCR(10,0,0)	PCR(1,0,2)	PCR(5,0,2)
RW	-	0,25	0,31	0,23	0,22	0,38	0,31
AR(2)	0,25	-	0,21	0,15	0,11	0,25	0,18
PCR(1,0,0)	0,31	0,21	-	0,4	0,4	0,15	0,38
PCR(5,0,0)	0,23	0,15	0,4	-	0,38	0,37	0,22
PCR(10,0,0)	0,22	0,11	0,4	0,38	-	0,25	0,3
PCR(1,0,2)	0,38	0,25	0,15	0,37	0,25	-	0,39
PCR(5,0,2)	0,31	0,18	0,38	0,22	0,3	0,39	-
PCR(10,0,2)	0,14	0,15	0,36	0,2	0,31	0,27	0,21
PCR(1,1,2)	0,34	0,26	0,2	0,2	0,1	0,24	0,23
PCR(5,1,2)	0,27	0,17	0,4	0,35	0,39	0,32	0,24
PCR(10,1,2)	0,15	0,13	0,38	0,1	0,32	0,28	0,13
PCR(1,2,2)	0,4	0,21	0,21	0,31	0,17	0,28	0,35
PCR(5,2,2)	0,19	0,16	0,4	0,37	0,4	0,36	0,31
PCR(10,2,2)	0,17	0,13	0,34	0,14	0,18	0,23	0,11

Model	PCR(10,0,2)	PCR(1,1,2)	PCR(5,1,2)	PCR(10,1,2)	PCR(1,2,2)	PCR(5,2,2)	PCR(10,2,2)
RW	0,14	0,34	0,27	0,15	0,4	0,19	0,17
AR(2)	0,15	0,26	0,17	0,13	0,21	0,16	0,13
PCR(1,0,0)	0,36	0,2	0,4	0,38	0,21	0,4	0,34
PCR(5,0,0)	0,2	0,2	0,35	0,1	0,31	0,37	0,14
PCR(10,0,0)	0,31	0,1	0,39	0,32	0,17	0,4	0,18
PCR(1,0,2)	0,27	0,24	0,32	0,28	0,28	0,36	0,23
PCR(5,0,2)	0,21	0,23	0,24	0,13	0,35	0,31	0,11
PCR(10,0,2)	-	0,18	0,34	0,31	0,23	0,16	0,4
PCR(1,1,2)	0,18	-	0,19	0,16	0,12	0,21	0,15
PCR(5,1,2)	0,34	0,19	-	0,35	0,27	0,39	0,21
PCR(10,1,2)	0,31	0,16	0,35	-	0,23	0,12	0,21
PCR(1,2,2)	0,23	0,12	0,27	0,23	-	0,3	0,19
PCR(5,2,2)	0,16	0,21	0,39	0,12	0,3	-	0,18
PCR(10,2,2)	0,4	0,15	0,21	0,21	0,19	0,18	-