VOLATILITY AND CONVENIENCE YIELD IN COMMODITY MARKET

An analysis of the predictive power of commodity convenience yield in different volatility environments

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Abstract

With this work we aim to analyze the changing in the accuracy of the fundamental relation between convenience yield and commodity prices in high and low volatility commodity prices environments.
Convenience yield embodies a kind of predictive power of future prices and this research explores its connection with volatility of commodity prices and how and if its predictive power could be influenced by commodity prices volatility.
To develop our research we apply the present value model implemented by Pindyck (1992) to five different commodities: crude oil, natural gas, corn, wheat and gold.
Our results show that the energy prices are particularly reactive to convenience yields movement especially in the high volatility frameworks.

Keywords:
Convenience yield, asset pricing, volatility, commodity markets, futures markets.

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1. Introduction

The commodity market experienced an upturn in volume in 2000s, when commodities started to be traded as a separate asset class. The commodity market consists of both direct physical trading and derivatives trading; in the time interval 2002-2005 the total value of global physical exports of commodities grew up by 17% and the derivatives contracts on commodities exponentially followed that rising. As a matter of fact, considering only the commodity derivatives traded on exchanges, the outstanding notional value of commodity increased more than 200% while the Over-The-Counter derivatives on commodities more than 500%.

The 2008 boom in commodity prices was especially fueled by two elements: the heated demand from China and India and the wild speculation in forward markets. Many investors started to diversify their portfolio by allocating consistent and growing part of their wealth in commodity futures. Usually money managers of pension plans and hedge funds use indexes as their primary investment tool and this contributed to the process of financialization of commodities.

As supported by Tang and Xiong (2009) the channel of index investment as a result of the spillover effects in equity and currency markets is responsible of a large part of commodity price volatility in 2008.

Traditionally the commodity spot prices are linked to the future ones through a relationship involving interest rate, cost of storage and convenience yield. The latter is not an observable quantity but we know that it exists and can be associated to the benefits of holding the physical asset itself instead of the future contract on it. Investors are also interested in holding actual commodities if they think there is a large enough possibility that convenience yield will rise substantially in the future. Convenience yields are traditionally associated to commodities among the other asset classes, due to their particular features linked to shortage, cost of carry and cost of storage which are all factors positively or negatively affecting the convenience yield.

On the light of the recent upturn of commodity markets the aim of this paper is to investigate how the convenience yield is able to reflect the future availability of a commodity and if its performance is linked to the magnitude of the volatility price on the spot market of commodities.

To connect the convenience yield and the commodity price we will apply the present value model, following the approach of Pindyck (1993) whose work is based on Campbell (1987) and Shiller (1987). The underlying idea of present value model is that commodity price's changes are only caused by the present and future convenience yield movements. When we observe abnormal high commodity prices it means that there are bubbles in the market. The causes of this can be recognized in the economic phenomena summarized before. The result is that the large money injection in the commodity market from investors diluted the fundamental relationship between price and convenience yield causing oil and other commodity being beyond their fundamental value.

With this work we aim to analyze the changing in the accuracy of the fundamental relation between convenience yield and commodity prices in high and low volatility commodity prices environments. Indeed, there is a relationship between the volatility of commodity prices and convenience yields.

Volatility directly influences the marginal value of storage, i.e. the flow of benefits from an extra unit of inventory, which is nothing more than the marginal convenience yield (Pindyck, 2004). Although this relationship seems to be economically significant lot of research on this field still needs to be done.

A shared view in the existing literature (Brennan (1986), Fama and French (1987, 1988) is that convenience yield embodies a kind of predictive power of future prices; therefore it is very interesting and useful to study its connection with volatility of commodity prices and how and if its predictive power could be influenced by commodity prices volatility.

To develop our research we apply the present value model implemented by Pindyck (1992) to five different commodities: crude oil, natural gas, corn, wheat and gold. Then the trend of historical volatility of each commodity will be studied in order to identify a period (or two periods as the case of crude oil) of relatively high volatility and a period of relatively low volatility in the correspondent markets. Since commodity prices exhibit wide ranges of variability, we select different periods of analysis for each commodity, according to the trend of its price changes along the time.

In this way we should be able to derive some conclusion on the nature of the connection between convenience yields and volatility in commodity markets by studying the mismatch between convenience yield and commodity prices and detecting if the bubbles originated by abnormal high spot prices are more likely to be observed in high or low volatility frameworks.

Developing an asset pricing model and testing for cointegration we obtained results which confirm a kind of dependence between convenience yield and volatility of the underlying commodity price for crude oil and natural gas; a close conformance to the model is found for crude oil when spot prices are more volatile.

Accordingly to the empirical results oil prices and its convenience yields proved to be cointegrated in periods of high volatility, meaning that prices can be predicted through the relationship with its convenience yield when shocks in prices occur.

The reminder of this paper is organized as follows. In Chapter 2 relevant literature will be presented as a framework for our empirical results. Chapter 3 illustrates the setup of Present Value model and in Chapter 4 we detail the data we used, the assumptions we tested and the empirical results obtained. Finally conclusions are presented in Chapter 5.

2. Literature review

Since our work develops following two main stream, the one concerning the relationship between the convenience yield and the volatility of the commodity prices and the one related to the predictive power of future prices, in this section we present an overview on the existing literature reference for both the search fields.

The concept of convenience yield is introduced for the first time when Brennan (1958) attempts to explain inverse carrying charges in future markets, i.e. when futures prices are below spot prices or prices deferred futures below that of near futures. He observes that the holder of stocks of all goods is provided a kind of compensation for holding the stock; such a compensation has to be deducted from storage costs when calculating properly net storage costs. In equilibrium the gap between futures and spot prices is equal to the marginal cost for storage costs minus the marginal convenience yield of stocks.

Brennan introduces another concept, the risk premium, to link the holding of stocks to price spread and empirically deals with such a phenomenon by applying the theory to agricultural commodities.

The results of Brennan (1958) were thereafter confirmed by the works of Deaton and Laroque (1992), Routledge at al. (2002): convenience yield arise in the interaction between supply, demand and storage.

Another masterpiece in the literature concerning convenience yield is the paper by Fama and French (1987) where it is clarified that there are two main existing approaches to commodity futures prices, The first one sees the interest foregone in storing a commodity, warehousing costs and the convenience yield on inventory as explanatory factors of the spread between contemporaneous spot and futures prices.

This view is known as theory of storage and is supported by Kaldor (1939), Working (1948), Brennan (1958) and Telser (1958). Fama and French (1987) are able to incorporate the theory of storage with the one that splits the futures price into an expected risk premium and a forecast of a future spot price. The latter alternative view is developed by Cootner (1960), Dusak (1973), Breeden (1980) and Hazuka (1984).

The study is conducted among a large number of commodities and the results provide very good evidence of the impact of storage cost variables to futures prices. On the other side only a little evidence that futures prices contain premiums or power to forecast spot prices is provided with their work since on the 21 commodities analysed only 10 exhibit a kind of forecast power and time-varying expected premiums were found for five commodities.

The main contribution of Fama and French's studies remains the fact that the importance of the futures market curve is being emphasized and explained: when the market is in backwardation, longer term maturity futures are being sold against higher prices than their short term counterparts, convenience yield prevail and outweigh the cost of carry. On the contrary, when the futures commodity market is in contango, i.e. when shorter term maturity futures are being sold against a higher market price than their long term counterparts, the cost of carry are higher than the convenience yield, resulting in a very low value of the convenience yield itself.

Starting from the assumption that convenience yield proved to drive the relationship between futures and spot prices of many commodities, Gibson and Schwartz (1990) adopt a different approach by developing a two-factor model for pricing financial and real assets dependent on the price of crude oil.

This work differentiates from the majority of the previous ones based on the fact of a single source of uncertainty related to the price of the commodity; on the contrary Gibson and Schwartz elaborate a more general approach which can be easily applied to the real and financial oil contingent claims. They reject the assumption of constant convenience yield, proving that the mean reverting trend as well as the variability of its changes best fit a stochastic representation in order to provide an accurate pricing process of oil-linked securities. Both short and long term futures contracts are analysed and statistics showed that the former highlighted a better performance.

A strong point of their work is the fact that the model found application also to any more complex payoff structure characterizing the option feature of real and financial oil claims.

Indeed, even if Gibson-Schwartz model can not be ignored from who is willing to study and conduct some research on the convenience yield field, it is not without drawbacks. Carmona and Ludkovski (1991) propose two new extensions of the classical Gibson-Schwartz model namely the time-inhomogeneous extension and the stochastic market price of risk extension. This variants aim to solve the discrepancy with the forward curve of the original model and the introduction of a third parameter allows to achieve a good fit of the cross section of futures prices.

Regarding the metal commodities an interesting research is conducted by Heaney (2000) who elaborates a two periods model in which, at the beginning of each period, the trader can choose between buying the commodity and storing it or taking out long futures contract with delivery of the commodity at the end of the second period. If the spot price is high enough at the end of the first period the trader has the opportunity to sell the inventory. Such a trading strategy requires a change to the standard futures contract arbitrage, according to which traders are supposed to apply a buy and hold strategy to the spot asset and to short a futures contract if the price of the latter is smaller than the cost of carry. On the opposite situation, when the future price is less than the cost of carry, the arbitrager takes a long futures position and shorts the spot asset.

According to the expectations on prices the traders behave opportunistically and to maximize their payouts arbitrageurs need perfect foresight but it is not always the case. This article applies the aforementioned model for copper, lead and zinc and it focuses on the approximation of the value of the convenience yield using a model first advanced by Longstaff (1995) with three input variables: spot price volatility, the futures price volatility and the time to future price volatility.

We have seen that regarding the models to estimate the convenience yield we can choose among a wide collection of literature from Brennan (1986) and the work of Fama and French (1987) to energy commodities-focused spot model of Gibson and Swartz (1993). Following the approach of Ke Tang (2009) our work is mainly based on the findings of Campbell and Shiller (1987) and Pindyck (1993).

The so called present value model explains the pricing of storable commodities: the underlying idea is that from spot and futures prices the stream of payoffs deriving form holding inventories, i.e. the convenience yield, can be directly measured. This means that the present value model is nothing more than a kind of (highly reduced) dynamic supply and demand model.

In his work Pindyck tests the the present value model for four commodities, heating oil, copper, lumber and gold and imposes restrictions on the joint dynamics of spot and futures prices. In the end only the performance of heating oil shows close conformance to the model while for the other three commodities data analysis and result indicate that prices deviate at least temporary deviate form fundamentals.

Previous studies are consistent with this result since they provide several evidence that commodity prices are not always based on fundamentals.

Pindyck himself and Rotemberg (1990a) study different commodities and found high levels of unexplained price correlation across commodities, and this is inconsistent with prices following the fundamentals.

Mentioning the work of Roll (1994), the price movements fro frozen orange juice can be explained by “fundamentals” only partially, taking as fundamental values variables such as the weather. Indeed, both these results may be affected by the possibility that the key variables operating on supply or demand for a broad range of commodities have been underestimated or omitted, but they are still an evidence of non-rational behaviour of prices.

According to this view the findings of rejections of some of the implications of the present value model from Pindyck (1993) provides additional evidence that the pricing of some commodities is indeed driven by fads. This is not true for heating oil, whose prices show a performance close to the one predicted by the model and no evidence of serial dependence in excess returns.

There could be several explanation for this happening, one could say that the high average of heating oil convenience yield makes speculation too costly so that odds are more likely to be found in other commodities markets.

The present value model is a generic asset pricing model which can be applied to basically all the asset classes; the underlying idea is indeed very simple, meaning that the price of an asset is nothing more than the sum of current and expected payoffs that accrue from the ownership of the asset.

The model is indeed very simple but not a few problem arise when testing it, as pointed out by Campbell and Shiller (1987).

In the literature concerning the present value model there are several test procedures including single-equation regression tests, tests of cross-equation restrictions on a vector autoregression (VAR), and variance bounds tests, but is not clear how these alternative approaches are related to one another.

Moreover, it is quite possible that the model explains most of the variation in Pt even if it is rejected at the 5% level of confidence; the authors suggest that a statistical rejection of the model might not have much economic significance and that may be better to focus on the informal evaluation of the fit of the model rather than statistical testing.

Another issue is that the variable Pt usually requires some transformation before the theory of stationary stochastic processes can be applied; to overcome this problem one option is to to remove a deterministic linear trend, but this can result in biased test procedures against the model if in fact Pt is stationary in first differences. When the latter is the case a valid way of testing is the one implemented by Campbell and Schiller recalling for the cointegrated processes. The positive aspects of such a test is that is fully efficient since it tests all the implications of the model and can be interpreted as a single-equation regression or as a test of restrictions on a VAR. Further applications may be the testing of variance bounds and the assessing the economic significance of deviations.

3. Model

This section provides in its first part a description of the Present value model, its predictive power and implications; in the second part we explain how we are going to apply this model to different volatility frameworks of the commodity spot prices.

3.1 Present Value Model

The first element which we need to develop our analysis is a rational asset pricing model. Among the several studies conducted and the wide and diverse literature extended on the approximation of the convenience yield the approach developed by Pindyck (1993) known as the Present Value model convinced us with its simplicity and linearity of the underlying idea.

Originally this model was mainly used for stocks and presents in the following form:

$$P_t = \delta \sum \delta^i E_t \psi_{t+i}; \qquad (1)$$

it says that the price of an asset (P_t) equals the sum of current and discounted expected future payoffs ($\delta^i E_t \psi_{t+i}$), which can be dividends in case of stocks, or benefits from the ownership of the specific asset.

$\delta = 1/(1+\mu)$ where μ is the commodity specific 1-period discount rate, i.e. the expected rate of return an investor would require to hold a unit of the commodity. It is also equals to the sum of the risk-free rate r and a risk premium ρ and for now it is assumed to be constant.

This model not only explains the pricing process of a stock but , since it is a valid and efficient pricing model, can be applied as well to any asset class which yields a payoff stream. Following Pindyck (1993) we will use this model to price commodities: therefore in our work the Present Value model prices the commodity by terms of its flow of benefits, i.e. the convenience yield that accrues from holding inventories.

As the model can be viewed as a reduced version of a dynamic and supply model, it provides a fundamental-based explanation of why rational investors would hold stocks of commodities: they will hold inventories if they think there is a large enough probability that convenience yield would rise in the future.

To exemplify this concept Pindyck chooses the case of gold. Convenience yield of gold is usually small, sometimes it is even not significantly different from zero; this is because inventories are usually held for investment purposes and are very large relative to production. If the metal were someday monetized, the inventories would drop dramatically causing a significant rise in the convenience yield.

At this point we need to fix a set of assumptions for the validity of the Present Value Model:

1) *commodity is well defined*;
2) *commodity is easily traded*;
3) *the aggregate storage is always positive*;
4) *the market is efficient: no arbitrage opportunities*.

We will recall the first three assumptions in the data part; now we want to dwell upon assumption 4. A good point of the Present Value model is that when it is applied to commodities traded on efficient futures markets, it becomes particularly parsimonious in terms of data: only spot and futures prices are required by the model. Indeed, when no arbitrage opportunities exist there are no errors from the relation between spot and futures prices and so convenience yield can be directly measured only from spot and futures prices. As opposite to other popular models estimating convenience yield such as Brennan (1986) and Fama and French (1987) who developed the theory of storage, there is no need of data on inventories, production costs, or other variables that affect supply, demand, or convenience yield.

The framework in which we are going to apply the model is characterized by an economy with three types of assets: the physical commodity with spot price P_t, its associated futures $f_{T,t}$ observed at time t and with maturity T, and the risk-free interest rate r_T:

$$\psi_{t,T} = \left(1 + r_T\right) P_t - f_{T,t}\,, \qquad (2)$$

where Ψ_t is the 1-period per unit marginal convenience yield from the beginning to the end of period t; it is net of storage and insurance costs. In other words it is the flow of benefits deriving from the physical ownership which can be assimilated to the dividends pay out by a stock.

Stochastic return from holding a unit of the commodity from t to $t+T$ is $\Psi_{t,T} + (P_{t+}T\text{-}Pt)$: if another investor shorts a forward (futures) contract at time t, he receives a total return of $\Psi^*_{t,T} + f_{T,t} - P_{t.}$ Since no outflow is required for the forward (future) contract and this total return is not-stochastic it must equal r_tP_t.

In the original set up of the model forward prices are used instead of futures; Since for most commodities future contracts are much more traded than forward contracts, future prices are more readily available than forward ones so that in our research we are going to use future prices instead of the forward prices as theory describes. The result of our work does not suffer because of this substitution because the two contracts differ each others only in the fact that a future contract is “marked to market”, consisting in a settlement and transfer of fund at the end of each trading day and it usually results in only a little difference in the two prices.[1]

3.2 Implications and predictive power

Campbell and Shiller (1987) show that the Present Value model implies that the price of an asset and its payoff stream are cointegrated; Pindyck imposes similar restrictions for the joint dynamics of the spot and futures prices of a commodity.

Following this approach P_t and Ψ_t are both cointegrated of order (*I*).

The cointegrating vector is $1/(1+\mu)'$ and a stationary spread is built up:

$$S'_t = P_t - (1/\mu)\psi_t, \tag{3}$$

so that expected return on a commodity μ could be estimated by running a cointegration regression of P_t and Ψ_t.

Joining (1) and (3) together we obtain:

$$S'_t = (1/\mu)E_t \Delta P_{t+1}, \tag{4}$$

which shows that P_t and Ψ_t are enough to forecast P_{t+1} because they contain all the necessary information.

Substituting (2) and (3) into (4):

$$E_t P_{t+1} = F_{T,t} + (\mu - r) P_t, \tag{5}$$

meaning that future price is a biased predictor of the future spot price and the bias corresponds to the commodity's expected excess return.

Consequently either (4) and (5) can be used to forecast P_{t+1} if μ is known.

Campbell and Shiller also show that (1) and (3) together lead to:

$$\mu S'_t = E_t \sum \delta^i \Delta\psi_{t+i}, \tag{6}$$

where $\mu S'_t$ is the present value of expected future changes in the convenience yield.

Then (4) and (6) explains how futures and spot prices describe the market's expectations on how Ψ_t and P_t will evolve assuming for simplicity that $\mu = r$, which is always approximately true for agricultural commodities and

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[1] This approach is also adopted by Pindyck 1993.

gold.

Three general cases are possible: $S'_t=P_t$, $S'_t>0$, $S'_t<0$.

The case which has as $S'_t = P_t$ a result, shows a convenience yield equals to zero and $E_t(P_{t+1}) = F_{T,t} = (1+r)P_t$; in this situation people hold stock of commodity even if

$\Psi_t = 0$ because they rationally expect price to rise up at the rate of interest because expect convenience yield to rise in the future.

Instead, when $P_t < F_{T,t} < (1+r)P_t$ the spread $S't$ is larger than zero and both price and convenience yield are expected to rise; on the contrary $S'_t<0$ when Ψ_t is large enough so that $F_{T,t} < P_t$. Consequently the present value of expected future changes in is negative and price and convenience yield are expected to fall.

4. Data and Results

4.1 Data

Our data set is obtained from *Bloomberg* and *Datastream* databases and consists of spot prices and futures prices of crude oil, natural gas, corn, wheat and gold and the euribor interest rate.

For all commodity prices, futures and interest rates we use daily data from January 1st 2000 to January 1st 2010, which are approximately 2500 observations for each commodity.

The length of the time series varies because of the volatility constraints: it goes from the longest series of 2039 observations of the low volatility period of corn to the shortest sample of the oil's second high volatility period which counts 190 observations.

Sometimes a contract price is constrained by exchange-imposed limits on daily price movements; in those cases we are using data for the next available date.

To obtain a spot price P_t we use the price on the daily last price of the commodity on the exchange where it is traded. For the futures contract we are using the generic futures prices with maturities 1 month, 3 months and 12 months. All quotations are in US dollars.

The risk free rate is approximated by the 1 month, 3 months and 12 months euribor rates.

Convenience yield is calculated using series of spot prices, futures and euribor rates which apply to the same day for which the futures prices are measured.

While selecting the data set we put particular attention in being faithful to the assumptions mentioned in the methodology part in order to create the best conditions possible for our research.

First constraints is that " *commodity is well defined* "; we then provide a brief but precise description of the underlying commodities of our futures contracts:

- Crude Oil: Brent type traded on the *New York Mercantile Exchange Division light, sweet crude oil futures contract* which is the world's most liquid forum for crude oil trading, as well as the world's largest-volume futures contract trading on a physical commodity. The contract trades in units of 1,000 barrels and the delivering point is Cushing, Oklahoma.

- Natural Gas: contract traded on *New York Mercantile Exchange Division natural gas futures contract*; its liquidity is granted by the fact that natural gas is widely used as a national benchmark price. The future contract trades in units of 10,000 million British thermal units and the price is based on delivery at the Henry Hub in Lousiana.

- Wheat: contract traded on Chicago Board of Trade in units of 5,000 bushels. Wheat is one of the most traded agricultural products and it is also quite diversified; the futures contracts considered are based on the Hard Red Winter Wheat, the largest of the US wheat crops.
  It was not possible to find data for 12 months maturities futures contracts.

- Corn: contract traded on Chicago Board of Trade in units of 5,000 bushels. Corn has a wide range of uses, among which there are animal feed, food, alcohol, ethanol, industrial usage and seed but the grades of corn that are traded on the CBOT are mainly used for animal feed and it of the Yellow type. As for wheat, data insufficiency made not possible to construct the series of 12 months maturity futures contracts.

- Gold: contract traded on *Chicago Mercantile Exchange Globex Division*; this kind of futures is very liquid, since gold is a vital industrial commodity for its nearly indestructible nature, conductor property and because it offers ongoing trading opportunities, since gold prices respond very quickly to political and economic events. Contracts are traded in units of 100 troy ounces and the quality standard is fixed to a minimum of 995 fineness.
  Series of 12 months futures contracts are not available.

To adhere to the second assumption, “ *the commodity is easily traded* ”, and to the efficiency and no-arbitrage hypothesis (assumption 4) we tried to choose liquid futures contracts with a high open interest and with frequently traded maturities: the first month, the first quarter and the first year.

Another constraint of this model is that inventories are assumed to be always positive (assumption 3); this is consistent with our data since the value of the inventories of all our commodities never fall below zero because the products are very homogeneous and well defined, the markets are liquid and with low transaction costs. It is therefore simple to buy and sell rapidly gold, crude oil and natural gas on the market so that firms do not usually experience shortage.

4.2 Cointegration test

As a general rule, non stationary time-series variables should not be used in regression models to avoid the problem of spurious regression. However, there is an exception to this rule. If y_t and x_t are not stationary I(1) variables, then we expect their difference, or any linear combination of them, to be I(1) as well. Congregation implies that y_t and x_t share similar stochastic trends, and, since the difference e_t is stationary, they never diverge too far from each others.

On the basis of Campbell and Shiller (1987) if there is no “rational bubble” , rationale present value models predict a cointegration relationship between stock prices and their dividend if the both of them are cointegrated following a *I(1)* process. This relationship in the stock market has been extended by Pindyck (1993) to commodities, so that the present value model results in the following form:

$$P_t = E_t\left[\int e^{-r(-t)} \psi_t \, dt\right]. \qquad (7)$$

For the present value to hold there should be a cointegration relationship between spot price and the convenience yield, I.e. spot prices and convenience yield should not wander off in opposite directions for very long without coming back to a mean distance eventually. This concept is synthesized by the equation:

$$P_t = \lambda_0 + \lambda_1 \psi_t + \varepsilon_t , \qquad (8)$$

where λ_0 and λ_1 are constants and ε_t is a mean-reverting process.

The convenience yield Ψ_t is calculated following Pindyck (2001):

$$\psi_t = (1+r) P_t - F_{T,t} , \qquad (9)$$

Combining (8) and (9) together we get

$$F_{T,t} = \frac{\lambda_0}{\lambda_1} + \left(1 + r - \frac{1}{\lambda_1}\right) P_t + \frac{1}{\lambda_1} \varepsilon_t , \qquad (10)$$

that leads to the fact that the present value model holds if the above relationship holds.

We run a cointegration test for each maturity futures for the four commodities using the augmented Dickey-Fuller test. The results are discussed in the next session.

Once we have obtained the convenience yield from (9) for each of the commodities considered and for each maturities, we need to know how much of the change of the spot price is due to the change in its fundamental convenience yield movements in different time periods, selected accordingly to the volatility of each commodity prices.

We difference (8) and run

$$\Delta P_t = \alpha_0 + \lambda_1 \Delta \psi_t + \varepsilon_t , \tag{11}$$

and we obtain λ_1; then λ_0 comes from

$$\lambda_0 = E\left[P -_1 \delta\lambda[\,] \right] \tag{12}$$

Since usually λ_1 has a quite small variance we can safely use (12) to estimate λ_0.

If we obtain large value of λ_0 it means that high increase in price cannot been explained by the fundamental convenience yields changes. Therefore, a deviation of the considered variable from its fundamental value it is a signal that we are in presence of a “commodity bubble”.

4.3 Volatility frameworks

Volatility of the underlying commodity is an important element in determining future spot prices; intuitively the volatility of the commodity spot prices should effect somehow the convenience yield.

Heaney (2002) includes the underlying asset price volatility among the factors determining the approximation of the convenience yield and the oil volatility is also the basis for the stochastic convenience yield of Gibson-Schwartz (1990).

We can also treat the case of scarcity of a specific commodity to exemplify this concept. Thinking at the possibility of shortages, it is safe to say that when the volatility of the commodity price is higher, there is more chance of encountering a situation of scarcity of inventories on the market and it should reasonably lead to an increase of spot prices against futures prices. In this case the owner of the physical commodity at issue is entitled of the benefits deriving from the premium price he can ask because of the situation of scarcity. This is why a market in backwardation is associated to high convenience yield; vice versa when spot prices are lower than futures, i.e. the market is in contango, the levels of convenience yield are lower and sometimes even negative.

Therefore the volatility of underlying commodity seems to be a key factor in connecting the level of convenience yield and the trend of future market.

Borrowing the above described present value model we investigate if the disconnection between the commodity prices and their fundamental value is dependent and to which extent on the volatility of the underlying commodity.

We first need to identify periods of low and high volatility of the spot prices for each commodity. Consequently we run the daily log returns of each of the commodity during the time interval considered and then we compute the 22 days volatility. Daily data of the commodity spot prices have been used to compute variance.

$$lnR_t = P_t / P_{t-1};$$

$$(13)$$

$$\sigma^2 = \frac{\sum[(\ln R_t - \ln \bar{R})^2]}{n}. \qquad (14)$$

As noticed by Campbell et al. (2001) this approach does not require a parametric model of the evolution of volatility, a further advantage in addition to its simplicity.

Figure 1 (in the Appendix) shows the results. Identifying periods of low and high volatility of the returns of a commodity is a kind of arbitrary decision, especially when we have included in the sample period periods of market turmoil as 2008-2009 crisis; even though we tried to define some parameter to use in order to make such a selection the most accurate possible, we are conscious that other reasonable solutions could be adopted. Most of markets have been affected by the 2008 financial crisis and the related market turmoil but some differences in volatility trends can still be recognized.

A brief study of the trend of volatilities follows; the data are summarized in Table 1.

Crude oil

Looking at the graph in Appendix 1 the maximum value of oil volatility (0,31%) has been reached on 21st of January 2009 and the minimum level (0,00002%) is on 20th of July 2007.

Basically three periods can be easily recognized: two with relatively high volatility and one of low volatility. Volatility is high between spring 2000 and spring 2003, with 13.84% of the values above the mean and 12.99% in the third quarter of observations, i.e. between 0.053% and 0.057% and then again form the second half of 2008 until April 2009 with above 8% of observations higher than the mean and 6.86% in the third quarter. We consider a period of low volatility the time interval between January 2005 and the first half of 2008 with 1,49% of observations above the mean and 1,07% in the third quarter.

Since the 22days volatility has been computed with a moving mean, it takes some time to reflect the real volatility in the market; as a consequence the time periods identified may not be very faithful to the actual volatility conditions. To solve this problem we extend the periods starting one month in advance for each time interval.

Natural gas

The volatility of natural gas for the considered time interval has a regular trend with periods of values remaining below the mean (approximately 0.2%) alternating with periods of the same length of values above the mean. The highest peak of 4.84% has been reached on 14th March 2003 and the minimum volatility of our sample is place able on 2nd June 2005 with a variance of returns of 0.014%. Between May 2007 and October 2009 however the volatility has been particularly low with a volatility larger than the mean only in the 0.73% of days (and 0.19% above the third quarter). In the other period we can observe a wavy trend with high percentage of observations above the mean (20.35% and 18.05% respectively).

Wheat

Until September 2005 the market of wheat has been characterized by a relatively low volatility with values above the mean in only 4.75% of cases and nearly none observation above the third quarter of 0.07% volatility. Then, the trend became decisively up-wording reaching a peak of 0.6% on 25th March 2008 and with 22.81% of values above the mean and even 21.04% above the third quarter, which is a particularly high ratio.

Corn

The mean of variances of returns of corn in the observed periods is 0.037% with a maximum level of 0.27% reached in 29th October 2008, in the middle of financial crisis. The turning point between a first period of relatively low volatility and a markedly higher one is September 2005, the same as the market of wheat. In the first period only 15.33% and 12.27% of values exceed the mean and the third quarter respectively while more recently the same levels were come through by 18.13% and 12.76% of cases.

Gold

Comparing to the other markets analysed the gold one is the commodity which shows the lowest volatility on average with a mean of 0.013% and a third quarter very close to the mean (0.014%) Again, we distinguish two periods; low volatility until the end of May 2006 with 6.44% of observations above the mean and 5.52% above the third quarter and then a high volatility period of 19.74% of observations above the mean and 18.63% higher than the third quarter. In the last period two peaks can be noticed: one during 2006 and the biggest one in the crisis of 2008.

Since the 22 days volatility is computed on a moving mean base, it takes some time to the variance function to incorporate the data, so that the results we get through (14) may not be very accurate; in order to overcome this problem we consider the sub-periods starting one month before the change in trends which can be recognized by the variances.

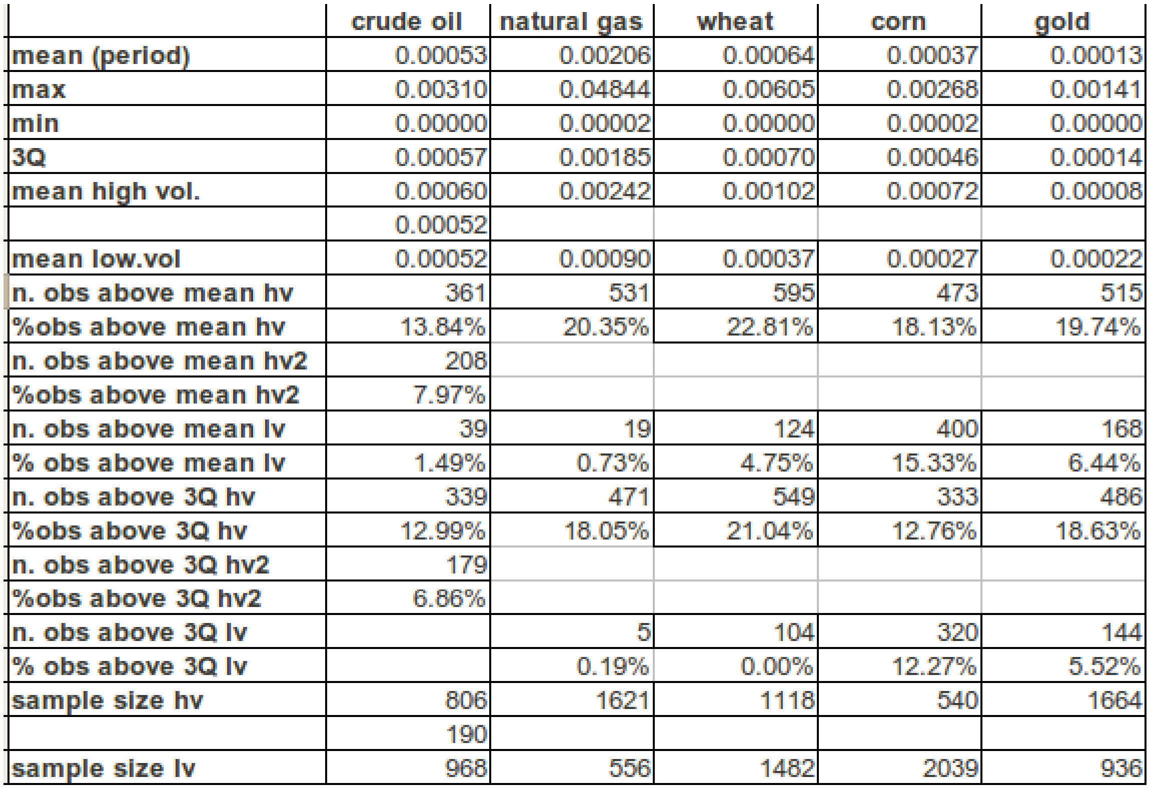
Considering each graph some correspondences with macro-economic events and levels of volatility can be recognized.

The oil market is a rather volatile market, with peaks of volatility corresponding to 11th September 2001 world crisis, the beginning of war of Iraq in 2003; from 2005 prices stabilized on a high level because of several factors, like a weak dollar, the continued rapid growth in Asian economies and their petroleum consumption. Volatility explodes again in autumn 2008 in the context of financial crisis.

Natural gas market has the characteristic to react with some delay to macroeconomic shocks. It is also the most affected commodity by September 2001 with a very high peak only in the winter; in response to higher prices producers increased investments and activities and the supply and demand responses led to a new equilibrium which resulted in a relatively long period of low volatility which remained until 2009.

Regarding wheat and corn markets in summer 2005 the production was hard bit by a severe drought and a heat wave that impacted grain production on a global scale so that as we described before, two periods of volatilities are easily recognized before and after 2005.

Table 1. Volatility analysis



	crude oil	natural gas	wheat	corn	gold
mean (period)	0.00053	0.00206	0.00064	0.00037	0.00013
max	0.00310	0.04844	0.00605	0.00268	0.00141
min	0.00000	0.00002	0.00000	0.00002	0.00000
3Q	0.00057	0.00185	0.00070	0.00046	0.00014
mean high vol.	0.00060	0.00242	0.00102	0.00072	0.00008
	0.00052				
mean low.vol	0.00052	0.00090	0.00037	0.00027	0.00022
n. obs above mean hv	361	531	595	473	515
%obs above mean hv	13.84%	20.35%	22.81%	18.13%	19.74%
n. obs above mean hv2	208				
%obs above mean hv2	7.97%				
n. obs above mean lv	39	19	124	400	168
% obs above mean lv	1.49%	0.73%	4.75%	15.33%	6.44%
n. obs above 3Q hv	339	471	549	333	486
%obs above 3Q hv	12.99%	18.05%	21.04%	12.76%	18.63%
n. obs above 3Q hv2	179				
%obs above 3Q hv2	6.86%				
n. obs above 3Q lv		5	104	320	144
% obs above 3Q lv		0.19%	0.00%	12.27%	5.52%
sample size hv	806	1621	1118	540	1664
	190				
sample size lv	968	556	1482	2039	936

On the light of the analysis conducted and on the latter macro-economic considerations, the sub-periods of volatility which we use for our research are:

- Crude Oil: a first period how high volatility from 20th February 2000 to 22nd March 2003, then a period of low volatility between 2nd December 2004 and 26th August 2008 and a higher one starting on 27th August 2008 and ending on 13th May 2009;
- Natural Gas: a relatively high period of volatility from 1st January 2001 to 21st February 2007 and then a low period between 22nd February 2007 and 5th September 2009;
- Wheat: low volatility from 13th January 2000 to 19th August 2005 and high period from 20th August 2005 to 13th December 2009
- Corn: a first low period between 13th January 2000 and 11th June 2007 and a high period from 12th June 2007 to 13th December 2009.
- Gold: from 13th January 2000 to 3oth April 2006 volatility is relatively low and then it increases for the period starting on 1st May 2006 to 13th December 2009.

4.4 Results

Table 2 shows the results of the regression (8). The estimated values of the coefficients λ_1 are all significant at least at 1% confidence level and are all positive, between 0.41, as the case of three months maturity futures of gold in its low volatility period, and 1.24 reached by oil with maturity 1 month in the low volatility period.

As a first consideration we can say that generally the convenience yield results to be a good estimator of the price, or that the price can be explained by the fundamental convenience yield changes.

Observing closer the outputs, a trend is confirmed by almost all the commodities analyzed: the estimated coefficients λ_1 (except for the one estimated for the one month, low volatility period of crude oil) are higher for the high volatility period compared to the lower one, meaning that the price changes are more explained by the convenience yields movements when the volatility of the price itself is higher. However this is not confirmed by wheat results because they only tell us that there is a positive and significant relationship between prices and convenience yields, for each maturities and volatility frameworks.

Looking at the maturities instead, the coefficients are lower for the nearby maturity futures of 1 month and tent to increase with the maturity; the only exception is again wheat in the high volatility period, when λ_1 estimated for 3 months is lower than the 1 month's coefficient and the one month low volatility of crude oil, which is the highest achieved result.

The R^2 varies consistently among commodities, maturities and periods, with a minimum of 10% in the case of the crude oil, 1 month, low volatility balancing the abnormal large value of its $\lambda_{1;}$ the best performances are given by natural gas, with R^2 always higher than 30% and with a very good outcome for 12 months high volatility regression, where 92% of data are explained by the model. The goodness of fit is particularly low for corn, the 1 month fist period of high volatility of crude oil and the 3 months and 12 months second period of high volatility of crude oil.

Gold and natural gas R^2 are particularly interesting since they really increase in the high volatility period comparing to the low ones.

Continuing our analysis additional information are provided by the estimated coefficient λ_0 . It is derived by equation (9) and its value varies among the different commodities. Its value says something regarding the level of commodity prices excluding the influence of the convenience yield; the fact that the values of λ_0 are higher for the nearby maturities of 1 month than for the further maturities means that the influence of the convenience yield on prices is higher for the long futures maturity.

To derive more definitive conclusions regarding the validity of the present value model in different volatility frameworks we now need to analyze the outputs of the cointegration test.

Table 2. Regression test on the relationship between Spot Price and the Convenience Yield

		Low volatility			High volatility			High volatility 2		
		1m	3m	12m	1m	3m	12m	1m	3m	12m
Oil	λ_0	71,46	73,43	73,35	26,89	26,56	26,54	49,11	53,40	53,31
	λ_1	1.24*	0.73*	0.71*	0.84*	0.74*	0.74*	0.9*	1.22*	1.23*
	$se(\lambda_1)$	0,13	0,04	0,04	0,08	0,03	0,03	0,28	0,21	0,21
	R^2	0,1	0,3	0,3	0,12	0,5	0,5	0,07	0,2	0,21
Natural gas	λ_0	6,45	6,81	7,42	5,75	5,91	5,95			
	λ_1	0.54*	0.58*	0.63*	0.85*	0.9*	0.9*			
	$se(\lambda_1)$	0,03	0,03	0,02	0,02	0,01	0,01			
	R^2	0,42	0,35	0,66	0,67	0,74	0,92			
Wheat	λ_0	308,15	5,46		557,41	-21,03				
	λ_1	0.79*	0.82*		0.83*	0.72*				
	$se(\lambda_1)$	0,03	0,03		0,05	0,04				
	R^2	0,29	0,38		0,23	0,28				
Corn	λ_0	233,30	10,46		433,82	22,96				
	λ_1	0.63*	0.8*		0.93*	1.02*				
	$se(\lambda_1)$	0,04	0,04		0,11	0,11				
	R^2	0,11	0,18		0,12	0,13				
Gold	λ_0	358,00	7,15		795,90	17,54				
	λ_1	0.45*	0.41*		0.51*	0.93*				
	$se(\lambda_1)$	0,02	0,03		0,02	0,03				
	R^2	0,19	0,15		0,47	0,48				

*,** denotes significance with 99% confidence interval

Two series are said to be cointegrated if, once run the regression between them, the residuals are stationary; if this is the case the two series, in our case the commodity price and the convenience yield series, can not wander off in opposite direction for very long without coming back to a mean distance eventually.

Thus, a cointergation test is a test on the stationarity of the residuals. We run an augmented Dickey-Fuller test on the unit root on the residual series with ten lags.

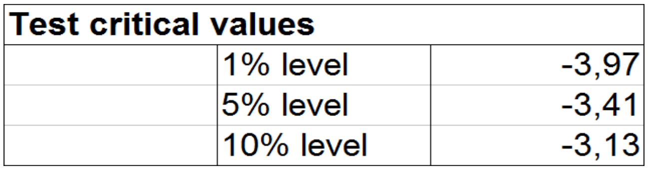
The null and alternative hypothesis are:

$$H_0\text{: the series are not cointegrated} \leftrightarrow \text{residuals are non-stationary}$$

$$H_1\text{: the series are cointegrated} \leftrightarrow \text{residuals are stationary}$$

We then reject the null hypothesis of no cointegration if the t statistics estimated by the test (τ) is lower than or equal to the critical value (τ_c) and we do not reject the null hypothesis that the series are not cointegrated if $\tau > \tau_c$. We refer to critical values using MacKinnon (1996) one-sided p-values; thresholds are shown in Table 3.

Table 3. MacKinnon (1996) critical values

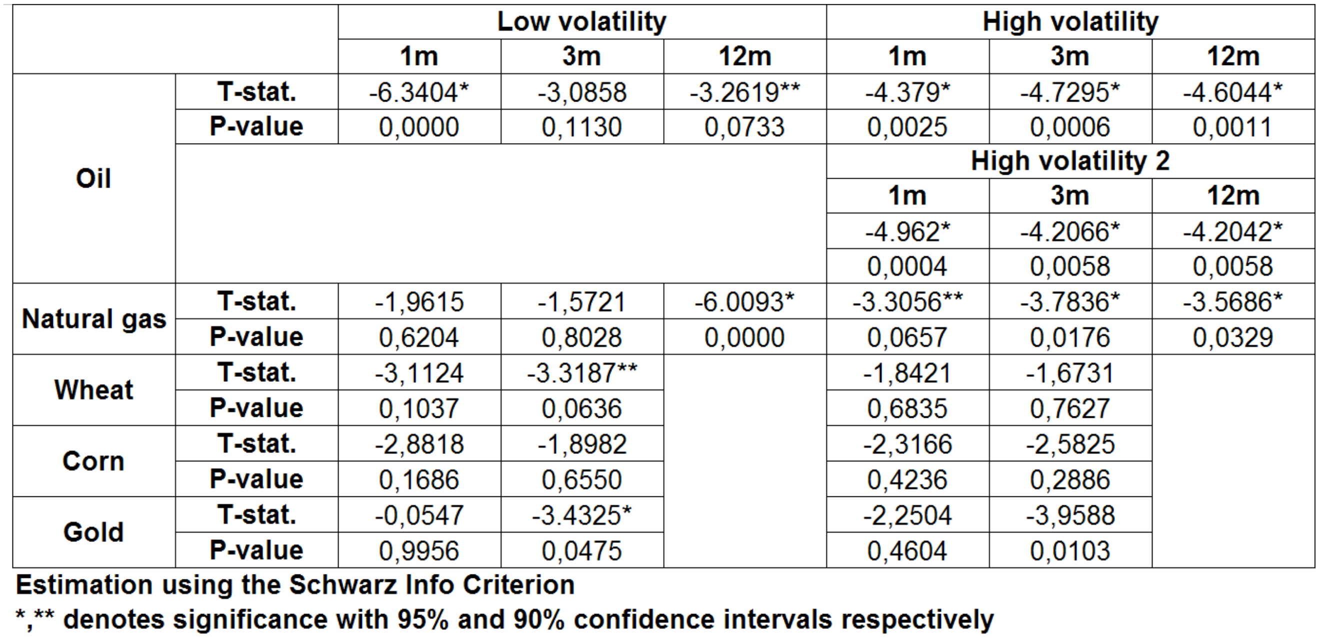


Test critical values		
	1% level	-3,97
	5% level	-3,41
	10% level	-3,13

Since the residuals which we applied the unit root test come from a regression between series with a trend and as the estimation for the intercept don't prove to be significantly equal to zero, Dickey-Fuller test's values refer to an equation with trend and constant term.

Results are summarized in Table 4.

Table 4. Augmented Dickey Fuller Test



		Low volatility			High volatility		
		1m	3m	12m	1m	3m	12m
Oil	**T-stat.**	-6.3404*	-3,0858	-3.2619**	-4.379*	-4.7295*	-4.6044*
	P-value	0,0000	0,1130	0,0733	0,0025	0,0006	0,0011
					High volatility 2		
					1m	**3m**	**12m**
					-4.962*	-4.2066*	-4.2042*
					0,0004	0,0058	0,0058
Natural gas	**T-stat.**	-1,9615	-1,5721	-6.0093*	-3.3056**	-3.7836*	-3.5686*
	P-value	0,6204	0,8028	0,0000	0,0657	0,0176	0,0329
Wheat	**T-stat.**	-3,1124	-3.3187**		-1,8421	-1,6731	
	P-value	0,1037	0,0636		0,6835	0,7627	
Corn	**T-stat.**	-2,8818	-1,8982		-2,3166	-2,5825	
	P-value	0,1686	0,6550		0,4236	0,2886	
Gold	**T-stat.**	-0,0547	-3.4325*		-2,2504	-3,9588	
	P-value	0,9956	0,0475		0,4604	0,0103	

Estimation using the Schwarz Info Criterion
***,** denotes significance with 95% and 90% confidence intervals respectively**

For crude oil the present value model holds for all the maturities futures in both the high volatility periods: indeed the p-values are very low and the t-statistics are small enough to reject the null hypothesis of no cointegration: prices and convenience yields are indeed cointegrated. In the low probability framework we can not reject the null hypothesis of no cointegration for the three months maturity futures but we are allowed to do that for the other maturities, even if with only 10% of level of significance for the 12 months.

We can reject the null hypothesis for the natural gas for all the maturities in the high volatility contest, but we can not do the same in the low one, where only the t-statistics of the 12 months is significant. Poorest results are obtained by the agricultural commodities: we can not reject the null hypothesis of no cointegration for all the maturities of corn, irrespective of the volatility. Results for wheat are not much better, since the present value model holds for only the 12 months maturity in the low volatility, where the null hypothesis can be rejected within an interval of only 90% confidence.

Finally gold series do not show to be cointegrated except for the longest maturity in the low volatility period with a rejection region of 5%.

Energy commodities thus provide the best results both in terms of goodness of fit, parameter estimation and results of the cointegration test and they agree on the evidence that the present value model performs better when the volatility of the underlying commodities of the futures contracts is high. Another phenomenon which emerges from the analysis is that the model is more significant for the longer maturities than for the nearby maturities of 1 month.

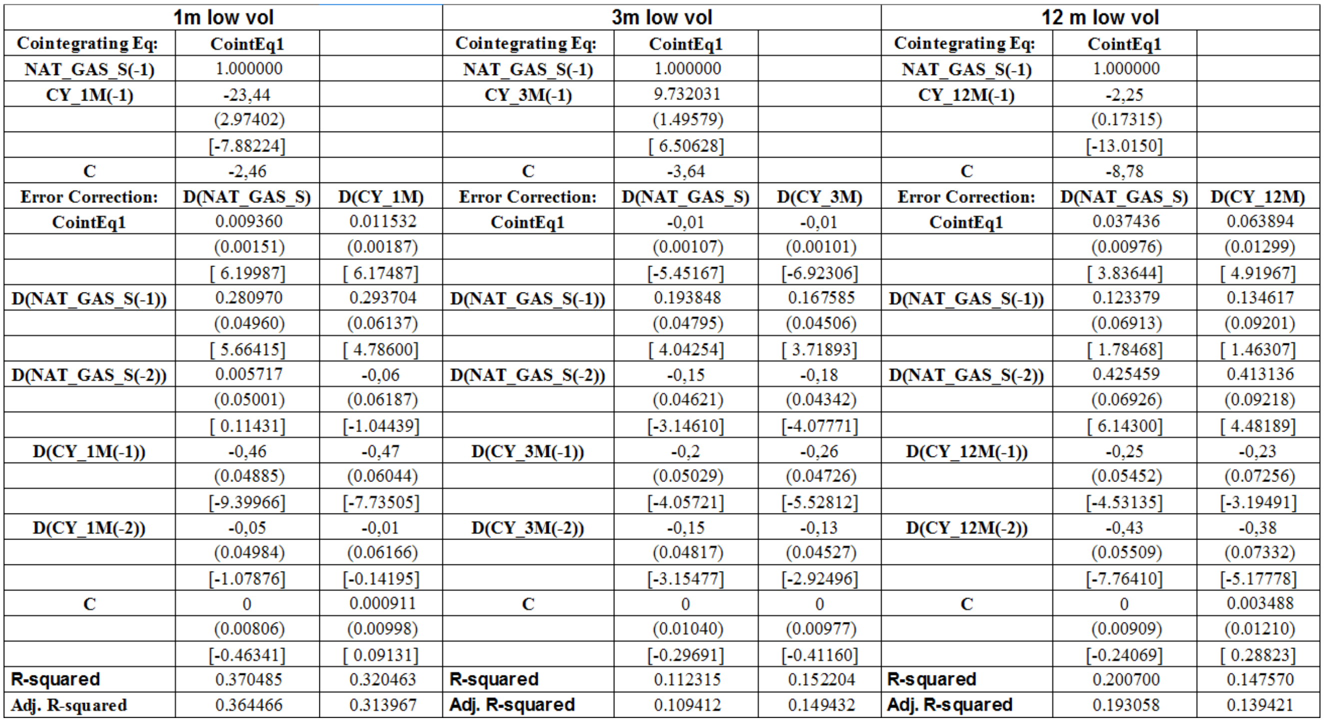
A natural extension of our analysis is to apply a Vector Error Correction model to the series which proved to be cointegrated, which are the natural gas and the crude oil.

A VEC model is useful to ascertain how much a series responds within a time lag or more to the other series, giving a kind of rate of adjustment indicating the series reacting to changes in the other one. More specifically a VEC model relates the change in an I(1) variable (the I(0) variables of changes in the original series) to other I(0) variables, namely the cointegration residuals.

In order to apply a VEC model the error term of the two series being analyzed should be stationary; hence we recall the results obtained with the Augmented Dickey-Fuller test which assessed the cointegration for all the maturities and periods of crude oil and natural gas, with some exceptions in the low volatility framework.

Tables from 5 to 9 illustrate the results.

Table 5: VECM Analyses for spot prices of Natural Gas and convenience yield in the low volatility period.



1m low vol			3m low vol			12 m low vol		
Cointegrating Eq:	**CointEq1**		**Cointegrating Eq:**	**CointEq1**		**Cointegrating Eq:**	**CointEq1**	
NAT_GAS_S(-1)	1.000000		**NAT_GAS_S(-1)**	1.000000		**NAT_GAS_S(-1)**	1.000000	
CY_1M(-1)	-23,44		**CY_3M(-1)**	9.732031		**CY_12M(-1)**	-2,25	
	(2.97402)			(1.49579)			(0.17315)	
	[-7.88224]			[6.50628]			[-13.0150]	
C	-2,46		**C**	-3,64		**C**	-8,78	
Error Correction:	**D(NAT_GAS_S)**	**D(CY_1M)**	**Error Correction:**	**D(NAT_GAS_S)**	**D(CY_3M)**	**Error Correction:**	**D(NAT_GAS_S)**	**D(CY_12M)**
CointEq1	0.009360	0.011532	**CointEq1**	-0,01	-0,01	**CointEq1**	0.037436	0.063894
	(0.00151)	(0.00187)		(0.00107)	(0.00101)		(0.00976)	(0.01299)
	[6.19987]	[6.17487]		[-5.45167]	[-6.92306]		[3.83644]	[4.91967]
D(NAT_GAS_S(-1))	0.280970	0.293704	**D(NAT_GAS_S(-1))**	0.193848	0.167585	**D(NAT_GAS_S(-1))**	0.123379	0.134617
	(0.04960)	(0.06137)		(0.04795)	(0.04506)		(0.06913)	(0.09201)
	[5.66415]	[4.78600]		[4.04254]	[3.71893]		[1.78468]	[1.46307]
D(NAT_GAS_S(-2))	0.005717	-0,06	**D(NAT_GAS_S(-2))**	-0,15	-0,18	**D(NAT_GAS_S(-2))**	0.425459	0.413136
	(0.05001)	(0.06187)		(0.04621)	(0.04342)		(0.06926)	(0.09218)
	[0.11431]	[-1.04439]		[-3.14610]	[-4.07771]		[6.14300]	[4.48189]
D(CY_1M(-1))	-0,46	-0,47	**D(CY_3M(-1))**	-0,2	-0,26	**D(CY_12M(-1))**	-0,25	-0,23
	(0.04885)	(0.06044)		(0.05029)	(0.04726)		(0.05452)	(0.07256)
	[-9.39966]	[-7.73505]		[-4.05721]	[-5.52812]		[-4.53135]	[-3.19491]
D(CY_1M(-2))	-0,05	-0,01	**D(CY_3M(-2))**	-0,15	-0,13	**D(CY_12M(-2))**	-0,43	-0,38
	(0.04984)	(0.06166)		(0.04817)	(0.04527)		(0.05509)	(0.07332)
	[-1.07876]	[-0.14195]		[-3.15477]	[-2.92496]		[-7.76410]	[-5.17778]
C	0	0.000911	**C**	0	0	**C**	0	0.003488
	(0.00806)	(0.00998)		(0.01040)	(0.00977)		(0.00909)	(0.01210)
	[-0.46341]	[0.09131]		[-0.29691]	[-0.41160]		[-0.24069]	[0.28823]
R-squared	0.370485	0.320463	**R-squared**	0.112315	0.152204	**R-squared**	0.200700	0.147570
Adj. R-squared	0.364466	0.313967	**Adj. R-squared**	0.109412	0.149432	**Adj. R-squared**	0.193058	0.139421

Table 6: VECM Analyses of Natural Gas of the spot prices and convenience yield in the high volatility period.

1m high vol			3m high vol			12m high vol		
Cointegrating Eq:	**CointEq1**		**Cointegrating Eq:**	**CointEq1**		**Cointegrating Eq:**	**CointEq1**	
NAT_GAS_S(-1)	1.000000		**NAT_GAS_S(-1)**	1.000000		**NAT_GAS_S(-1)**	1.000000	
CY_1M(-1)	-191,41		**CY_3M(-1)**	9.732031		**CY_12M(-1)**	5.852339	
	(20.1074)			(1.49579)			(1.12704)	
	[-9.51924]			[6.50628]			[5.19265]	
C	3.911239		**C**	-3,64		**C**	-4,25	
Error Correction:	**D(NAT_GAS_S)**	**D(CY_1M)**	**Error Correction:**	**D(NAT_GAS_S)**	**D(CY_3M)**	**Error Correction:**	**D(NAT_GAS_S)**	**D(CY_12M)**
CointEq1	0.000875	0.000918	**CointEq1**	-0,01	-0,01	**CointEq1**	-0,01	-0,01
	(0.00010)	(0.00010)		(0.00107)	(0.00101)		(0.00108)	(0.00116)
	[8.39447]	[9.15870]		[-5.45167]	[-6.92306]		[-5.51760]	[-5.92590]
D(NAT_GAS_S(-1))	0.518565	0.505980	**D(NAT_GAS_S(-1))**	0.193848	0.167585	**D(NAT_GAS_S(-1))**	0.463185	0.532513
	(0.03749)	(0.03602)		(0.04795)	(0.04506)		(0.08036)	(0.08603)
	[13.8314]	[14.0460]		[4.04254]	[3.71893]		[5.76389]	[6.18985]
D(NAT_GAS_S(-2))	-0,08	-0,03	**D(NAT_GAS_S(-2))**	-0,15	-0,18	**D(NAT_GAS_S(-2))**	0.563627	0.623559
	(0.03938)	(0.03783)		(0.04621)	(0.04342)		(0.08130)	(0.08704)
	[-1.95319]	[-0.84687]		[-3.14610]	[-4.07771]		[6.93264]	[7.16430]
D(CY_1M(-1))	-0,65	-0,68	**D(CY_3M(-1))**	-0,2	-0,26	**D(CY_12M(-1))**	-0,47	-0,52
	(0.04234)	(0.04068)		(0.05029)	(0.04726)		(0.07531)	(0.08063)
	[-15.3736]	[-16.5961]		[-4.05721]	[-5.52812]		[-6.26663]	[-6.46423]
D(CY_1M(-2))	-0,23	-0,2	**D(CY_3M(-2))**	-0,15	-0,13	**D(CY_12M(-2))**	-0,79	-0,85
	(0.04308)	(0.04139)		(0.04817)	(0.04527)		(0.07612)	(0.08149)
	[-5.27082]	[-4.84043]		[-3.15477]	[-2.92496]		[-10.4366]	[-10.4700]
C	0	0	**C**	0	0	**C**	-0,01	-0,01
	(0.00928)	(0.00891)		(0.01040)	(0.00977)		(0.01004)	(0.01075)
	[-0.20128]	[-0.02642]		[-0.29691]	[-0.41160]		[-0.60191]	[-0.83087]
R-squared	0.293408	0.299645	**R-squared**	0.112315	0.152204	**R-squared**	0.173104	0.170409
Adj. R-squared	0.291098	0.297355	**Adj. R-squared**	0.109412	0.149432	**Adj. R-squared**	0.170400	0.167697

Table 7: VECM Analyses of Crude Oil of the spot prices and convenience yield in the first high volatility period.

1m high vol 1			3m high vol 1			12m high vol 1		
Cointegrating Eq:	**CointEq1**		**Cointegrating Eq:**	**CointEq1**		**Cointegrating Eq:**	**CointEq1**	
CRUDE_OIL_S(-1)	1.000000		**CRUDE_OIL_S(-1)**	1.000000		**CRUDE_OIL_S(-1)**	1.000000	
CY_1(-1)	-69,92		**CY_3(-1)**	-5,53		**CY_12(-1)**	-5,5	
	(15.3673)			(0.79652)			(0.83365)	
	[-4.54974]			[-6.94244]			[-6.60074]	
C	-46,81		**C**	-24,42		**C**	-24,32	
Error Correction:	**D(CRUDE_OIL_S)**	**D(CY_1)**	**Error Correction:**	**D(CRUDE_OIL_S)**	**D(CY_3)**	**Error Correction:**	**D(CRUDE_OIL_S)**	**D(CY_12)**
CointEq1	-6,79E-006	0.000983	**CointEq1**	0	0.014892	**CointEq1**	0	0.013593
	(0.00030)	(0.00023)		(0.00503)	(0.00433)		(0.00484)	(0.00417)
	[-0.02276]	[4.31702]		[-0.40664]	[3.44086]		[-0.46452]	[3.25764]
D(CRUDE_OIL_S(-1))	0.046824	-0,03	**D(CRUDE_OIL_S(-1))**	-0,02	-0,15	**D(CRUDE_OIL_S(-1))**	-0,02	-0,15
	(0.03787)	(0.02891)		(0.04901)	(0.04220)		(0.04904)	(0.04229)
	[1.23653]	[-1.20569]		[-0.32267]	[-3.55879]		[-0.33243]	[-3.56521]
D(CRUDE_OIL_S(-2))	-0,01	0.001287	**D(CRUDE_OIL_S(-2))**	-0,02	-0,05	**D(CRUDE_OIL_S(-2))**	-0,02	-0,05
	(0.03788)	(0.02892)		(0.04755)	(0.04094)		(0.04757)	(0.04103)
	[-0.27050]	[0.04449]		[-0.37544]	[-1.22080]		[-0.37637]	[-1.21919]
D(CY_1(-1))	-0,14	-0,26	**D(CY_3(-1))**	0.032754	-0,24	**D(CY_12(-1))**	0.032640	-0,25
	(0.05029)	(0.03839)		(0.05830)	(0.05020)		(0.05811)	(0.05012)
	[-2.73287]	[-6.80024]		[0.56177]	[-4.83512]		[0.56165]	[-4.89687]
D(CY_1(-2))	0.055032	-0,08	**D(CY_3(-2))**	0.078849	-0,03	**D(CY_12(-2))**	0.078464	-0,03
	(0.04925)	(0.03760)		(0.05068)	(0.04363)		(0.05064)	(0.04367)
	[1.11749]	[-2.11123]		[1.55597]	[-0.72282]		[1.54932]	[-0.77663]
C	-0,01	0.000653	**C**	-0,01	0	**C**	-0,01	0
	(0.02409)	(0.01839)		(0.02422)	(0.02085)		(0.02422)	(0.02089)
	[-0.21387]	[0.03550]		[-0.21924]	[-0.22366]		[-0.21840]	[-0.23788]
R-squared	0.015096	0.115410	**R-squared**	0.004190	0.172747	**R-squared**	0.004249	0.170444
Adj. R-squared	0.008617	0.109591	**Adj. R-squared**	0	0.167304	**Adj. R-squared**	0	0.164986

Table 8: VECM Analyses of Crude Oil of the spot prices and convenience yield in the second high volatility period.

1m high vol 2			3m high vol 2			12m high vol 2		
Cointegrating Eq:	CointEq1		**Cointegrating Eq:**	CointEq1		**Cointegrating Eq:**	CointEq1	
CRUDE_OIL_S(-1)	1.000000		**CRUDE_OIL_S(-1)**	1.000000		**CRUDE_OIL_S(-1)**	1.000000	
CY_1(-1)	2.272467		**CY_3(-1)**	89.29353		**CY_12(-1)**	108.4006	
	(0.62526)			(10.9165)			(13.2489)	
	[3.63446]			[8.17971]			[8.18183]	
C	-54,9		**C**	162.1287		**C**	197.8076	
Error Correction:	**D(CRUDE_OIL_S)**	**D(CY_1)**	**Error Correction:**	**D(CRUDE_OIL_S)**	**D(CY_3)**	**Error Correction:**	**D(CRUDE_OIL_S)**	**D(CY_12)**
CointEq1	-0,06	0.000918	**CointEq1**	0	0	**CointEq1**	0	0
	(0.01026)	(0.02206)		(0.00057)	(0.00132)		(0.00047)	(0.00108)
	[-5.41221]	[0.04163]		[-8.10364]	[-1.89529]		[-8.06607]	[-1.90809]
D(CRUDE_OIL_S(-1))	-0,26	0.211066	**D(CRUDE_OIL_S(-1))**	-0,53	-0,07	**D(CRUDE_OIL_S(-1))**	-0,52	-0,07
	(0.09935)	(0.21365)		(0.10051)	(0.23065)		(0.10051)	(0.23005)
	[-2.62502]	[0.98793]		[-5.22660]	[-0.30793]		[-5.17721]	[-0.29890]
D(CRUDE_OIL_S(-2))	0.228303	0.442681	**D(CRUDE_OIL_S(-2))**	0.043194	0.209646	**D(CRUDE_OIL_S(-2))**	0.049201	0.213310
	(0.09533)	(0.20499)		(0.09398)	(0.21567)		(0.09394)	(0.21501)
	[2.39482]	[2.15947]		[0.45963]	[0.97207]		[0.52373]	[0.99208]
D(CY_1(-1))	0.223233	-0,12	**D(CY_3(-1))**	0.452078	0.089815	**D(CY_12(-1))**	0.448643	0.088666
	(0.06045)	(0.13000)		(0.07113)	(0.16324)		(0.07110)	(0.16273)
	[3.69262]	[-0.92139]		[6.35554]	[0.55019]		[6.31010]	[0.54486]
D(CY_1(-2))	-0,26	-0,57	**D(CY_3(-2))**	-0,11	-0,41	**D(CY_12(-2))**	-0,11	-0,41
	(0.06060)	(0.13030)		(0.06885)	(0.15800)		(0.06878)	(0.15742)
	[-4.33206]	[-4.37064]		[-1.57445]	[-2.57283]		[-1.63140]	[-2.59585]
C	-0,4	0.258706	**C**	-0,57	-0,01	**C**	-0,56	0
	(0.15739)	(0.33845)		(0.13669)	(0.31370)		(0.13680)	(0.31311)
	[-2.54506]	[0.76439]		[-4.13760]	[-0.01636]		[-4.10145]	[-0.01510]
R-squared	0.493238	0.198678	**R-squared**	0.638736	0.271355	**R-squared**	0.637259	0.272653
Adj. R-squared	0.473128	0.166880	**Adj. R-squared**	0.624401	0.242440	**Adj. R-squared**	0.622865	0.243790

Table 9: VECM Analyses of Crude Oil of the spot prices and convenience yield in the low volatility period.

1m low vol			3m low vol			12 m low vol		
Cointegrating Eq:	**CointEq1**		**Cointegrating Eq:**	**CointEq1**		**Cointegrating Eq:**	**CointEq1**	
CRUDE_OIL_S(-1)	1.000000		**CRUDE_OIL_S(-1)**	1.000000		**CRUDE_OIL_S(-1)**	1.000000	
CY_1(-1)	-22.1883		**CY_3(-1)**	-11.89414		**CY_12(-1)**	-11.49712	
	(1.78599)			(0.84862)			(0.79145)	
	[-12.4236]			[-14.0159]			[-14.5266]	
C	-40.59444		**C**	-65.32407		**C**	-63.77101	
Error Correction:	**D(CRUDE_OIL_S)**	**D(CY_1)**	**Error Correction:**	**D(CRUDE_OIL_S)**	**D(CY_3)**	**Error Correction:**	**D(CRUDE_OIL_S)**	**D(CY_12)**
CointEq1	0.008244	0.009231	**CointEq1**	0.015409	0.021005	**CointEq1**	0.016519	0.021506
	(0.00075)	(0.00135)		(0.00160)	(0.00268)		(0.00167)	(0.00281)
	[11.0574]	[6.83279]		[9.63139]	[7.83876]		[9.87942]	[7.64402]
D(CRUDE_OIL_S(-1))	-0.080022	0.056578	**D(CRUDE_OIL_S(-1))**	0.012230	0.059486	**D(CRUDE_OIL_S(-1))**	0.012138	0.087374
	(0.03739)	(0.06774)		(0.03870)	(0.06481)		(0.03875)	(0.06520)
	[-2.14047]	[0.83519]		[0.31603]	[0.91779]		[0.31326]	[1.34018]
D(CRUDE_OIL_S(-2))	0.313277	0.431149	**D(CRUDE_OIL_S(-2))**	0.394106	0.393534	**D(CRUDE_OIL_S(-2))**	0.383393	0.388379
	(0.03653)	(0.06619)		(0.03828)	(0.06412)		(0.03822)	(0.06432)
	[8.57699]	[6.51426]		[10.2941]	[6.13736]		[10.0299]	[6.03841]
D(CY_1(-1))	0.137683	0.073563	**D(CY_3(-1))**	-0.000358	0.003316	**D(CY_12(-1))**	0.002312	-0.026014
	(0.02632)	(0.04770)		(0.02954)	(0.04948)		(0.02969)	(0.04995)
	[5.23015]	[1.54215]		[-0.01212]	[0.06702]		[0.07787]	[-0.52078]
D(CY_1(-2))	-0.335798	-0.403921	**D(CY_3(-2))**	-0.399928	-0.38463	**D(CY_12(-2))**	-0.391015	-0.381531
	(0.02569)	(0.04656)		(0.02820)	(0.04724)		(0.02813)	(0.04733)
	[-13.0697]	[-8.67591]		[-14.1803]	[-8.14271]		[-13.9021]	[-8.06174]
C	0.053969	-0.026562	**C**	0.044345	-0.022271	**C**	0.045147	-0.024144
	(0.03848)	(0.06973)		(0.03907)	(0.06543)		(0.03904)	(0.06570)
	[1.40240]	[-0.38091]		[1.13507]	[-0.34036]		[1.15633]	[-0.36751]
R-squared	0.419065	0.234917	**R-squared**	0.400657	0.236322	**R-squared**	0.401459	0.235409
Adj. R-squared	0.415925	0.230781	**Adj. R-squared**	0.397417	0.232194	**Adj. R-squared**	0.398223	0.231276

The first parts of the tables represent the equilibrium part: the cointegration in equilibrium condition is confirmed for all the series here.

The error correction is presented in the second parts, where out of equilibrium dynamics are tested. Coint.Eq1 shows the reaction of the two series to a deviation from equilibrium. If the present value holds, the results should provide evidence that the prices adjust to the convenience yields movements, i.e. the coefficient for the changes in the convenience yield should be positive at least under the price columns. The lags show the reminder to out of equilibrium dynamics.

For natural gas, the positive coefficient for delta in the prices thus show that they are moving against the equilibrium value while the positive value for delta in the convenience yield shows that it moves to eliminate the gap between the cointegrated series. For natural gas both price and convenience yield series react positively to change in the spot price and negatively to a change in the convenience yield, either in the low and in the high volatility periods. There are not significant differences among maturities,

Thus the Present Value model can not be confirmed by the VEC model, since the tables show that the convenience yield react to the prices, while it should be in the other way around.

The crude oil instead gives a different picture: in the high volatility frameworks prices generally react positively to changes in the convenience yield within one time lag. More specifically in the first period of high volatility the coefficient for prices relating to changes within one lag in the convenience yield are positive, with the only exception of the 1 month maturity. The results stay positive also going back up to two lags, meaning that the influence of convenience yields on prices remain for two periods at least.

The analyses in the second period of high volatility gives always positive coefficients of prices in relation to changes in the convenience yield within one time lag, and negative for two lags; the R squared is the higher obtained and the levels of significance are satisfying.

Further, the output for the low volatility period confirm the influence of the one lag convenience yield movements for the 1 month and 12 months maturity, even if the coefficients are relatively low if compared to the ones of the periods of high volatility.

Thus, the VEC analyses for crude oil confirms our previous findings: the prices are more connected to the correspondent convenience yield when the volatility of the prices themselves are high.

The Present Value model was originally conceived to explain price movements with “fundamentals”, here represented by convenience yield; our empirical results obtained by performing the model and adjusting to account for cointegration tell that this happens for crude oil, and that such a forecast is accurate when volatility of crude oil prices is high.

This is somehow surprising: the high volatility in oil prices from is in large part due to the channel of index investments and its spillover effects of equity and currency markets so that the fundamental relationship between spot prices and convenience yield should be diluted due to the introduction of disturbing elements, as Tang and Xiong (2009) pointed out.

Anyway, we need to consider that the Present Value model is primarily an asset pricing tool thought to price stocks recalling for the flow of dividends which they originate; extending the model further it resulted to be suitable to price all kind of marketable securities which yield a payoff stream. Among commodities, crude oil is now considered as a special asset allocation tool, almost considered as en extra asset class; these are indeed causes of bubbles in oil prices but also make crude oil commodity closer to marketable securities. This explains why when there is high volatility the present value model witnesses a stronger relationship between the price of the security, the oil prices, and its payoff steam, the convenience yield.

According to this prospective volatility is perceived as a kind of homogenizing element which is able to bring near a commodity to a marketable security, so that the higher the volatility, the stronger the relationship with fundamentals.

5. Conclusions

Commodity market has witnessed some structural changes in the last ten years; in particular the large injection of commodity linked products in hedge funds, exchange-traded funds and structured products contributed both to an upturn in volume in commodity trading and to the process of financialization of commodities.

The 2008 boom in commodity prices is the direct consequence of these elements and of the heated demand of emerging countries and unbridled speculation of forward markets.

With this research we investigated the relationship between commodity prices and their associated convenience yields, as a kind of fundamental value enable to predict the future availability of a commodity through the futures contract prices. The model which we decided to apply to the commodity markets analyzed, the present value model by Pindyck (2003) tells us that when the gap between commodity prices and convenience yield is abnormally high, then there is a bubble in the market.

The results of our analysis confirm that the large money injection in the commodity prices diluted the fundamental relationship between commodity price and convenience yield for corn, wheat and gold.

Agricultural markets were deeply involved by the financialization and the new entrance of varied components of demand and supply introduced elements of disconnection from fundamentals.

Gold plays the role of a safe haven in periods of markets turmoil: in the 2008 credit crunch banks faced difficulties in raising cash, so that they sold their stocks of gold causing a drop in the price reflected in a higher convenience yield.

Energy markets instead confirm the present value model and they also provide significant results regarding the connection between volatility frameworks and convenience yield.

Both crude oil and natural gas results tell that the convenience yield is a better predictor of the commodity price when the volatility of the commodity price itself is higher. In particular the analyses conducted on the out of equilibrium dynamics indicated the crude oil as the only commodity, among the ones considered in this work, which prices directly react to changes in the convenience yield. This result is somehow surprising because the energy market is one of the most affected by the financialization, and the introduction of new elements unrelated to the physical markets, introduce some disturbing elements which could deviate the relation with fundamentals. Apparently this effect is mitigated in periods of financial turmoils, when the volatility of the prices underlying futures contracts is very high.

Another finding is that the present value model provides best results when considering longer maturities, meaning that the relationship between prices and convenience yield is generally stronger with maturites of one year than for shorter maturities like 1 month and 1 quarter.

The field of study connected to the optimal forecast of the futures prices is highly valued by investors, since every findings is an added contribute to the building of optimal investments portfolio and strategy. This work tries to provide investors with a kind of instrument to relate the forecasting of futures prices and the volatility of the market thorough the study of convenience yield. A useful and exploitable finding is that when the volatility of an energy market is particularly high, the convenience yield is a better predictor of futures prices than in a contest of low volatility.

We aimed to find similar conclusions for gold and agricultural commodities, but maybe other pricing models suit this market better and other researches could exploit those markets and different models and assumptions may asses if their prices react to their convenience yields, or not.

Further studies can be made on convenience yield and volatility, but maybe other data, methodology or statistical measure should be use, in order to confirm with added evidences and better explanations our results, or to oppose other findings to support the contrary.

This work was conducted at the best of our possibility and effort and we hope to have added a contribute, however small, to the intricate and fascinating world of prediction of futures prices.

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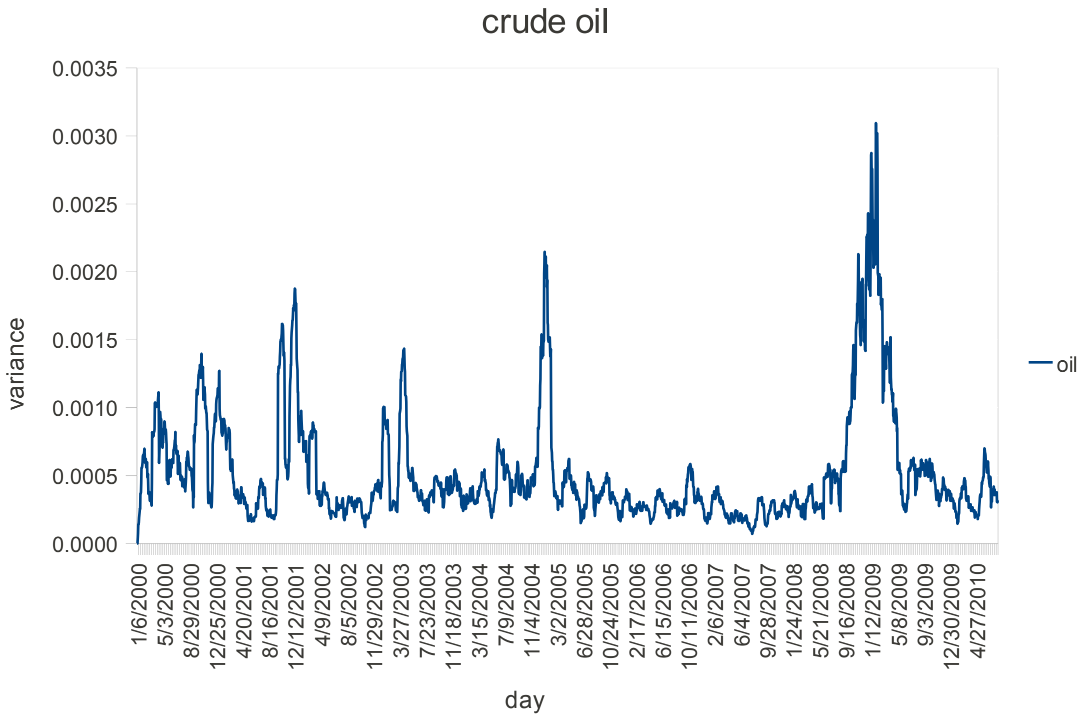
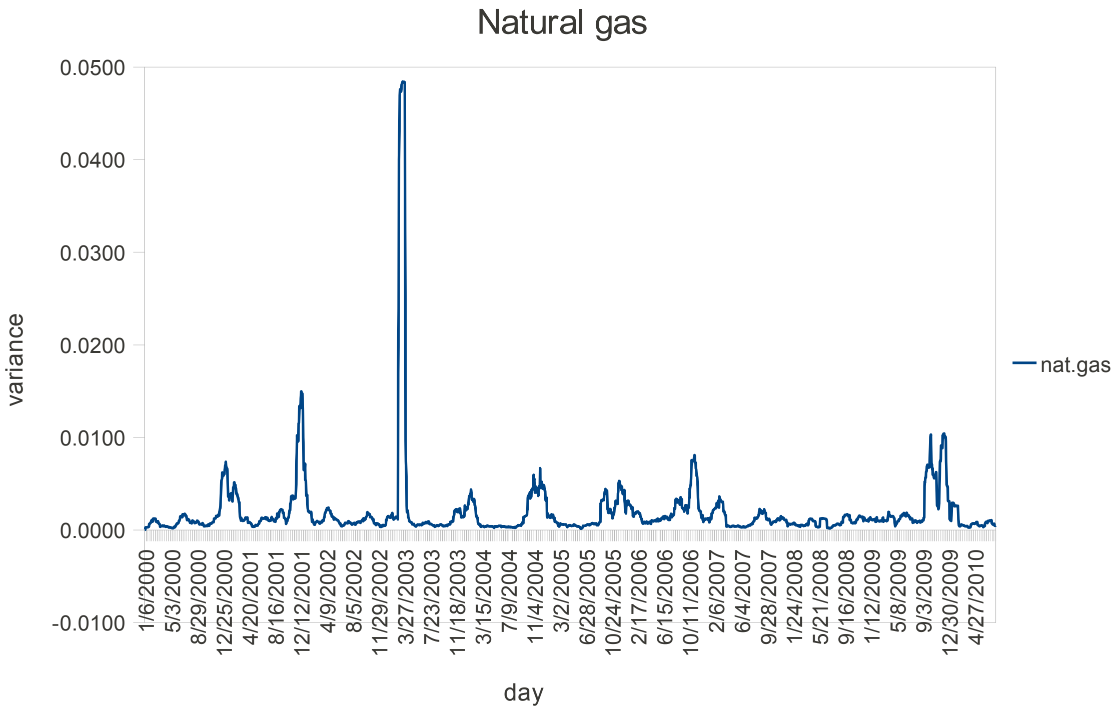
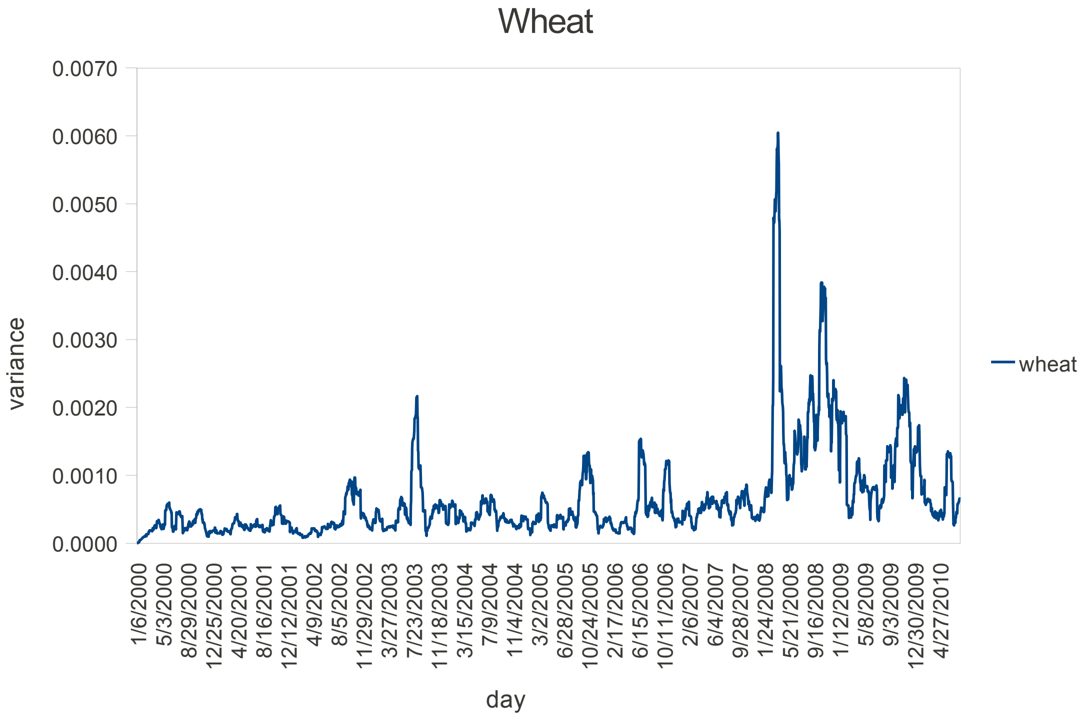
Appendix

Figure 1

22 days Volatility for each commodity from 01/01/2000 to 01/01/2010 (daily data)






Wheat
variance
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0.0060
0.0050
0.0040
0.0030
0.0020
0.0010
0.0000
wheat
1/6/2000
5/3/2000
8/29/2000
12/25/2000
4/20/2001
8/16/2001
12/12/2001
4/9/2002
8/5/2002
11/29/2002
3/27/2003
7/23/2003
11/18/2003
3/15/2004
7/9/2004
11/4/2004
3/2/2005
6/28/2005
10/24/2005
2/17/2006
6/15/2006
10/11/2006
2/6/2007
6/4/2007
9/28/2007
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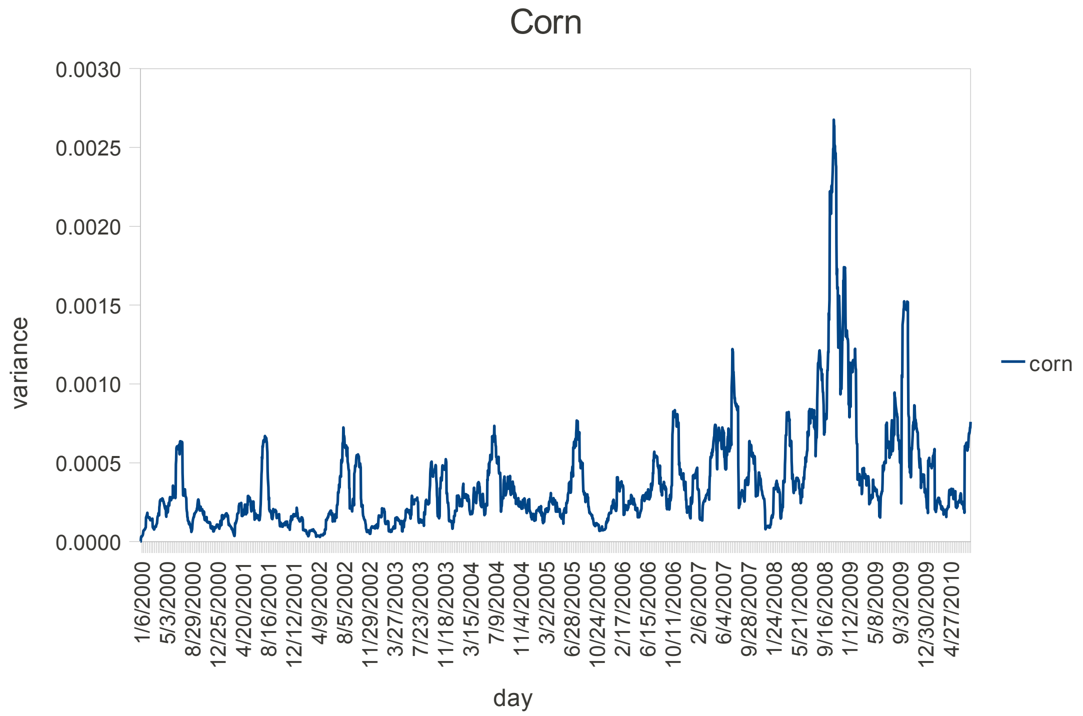
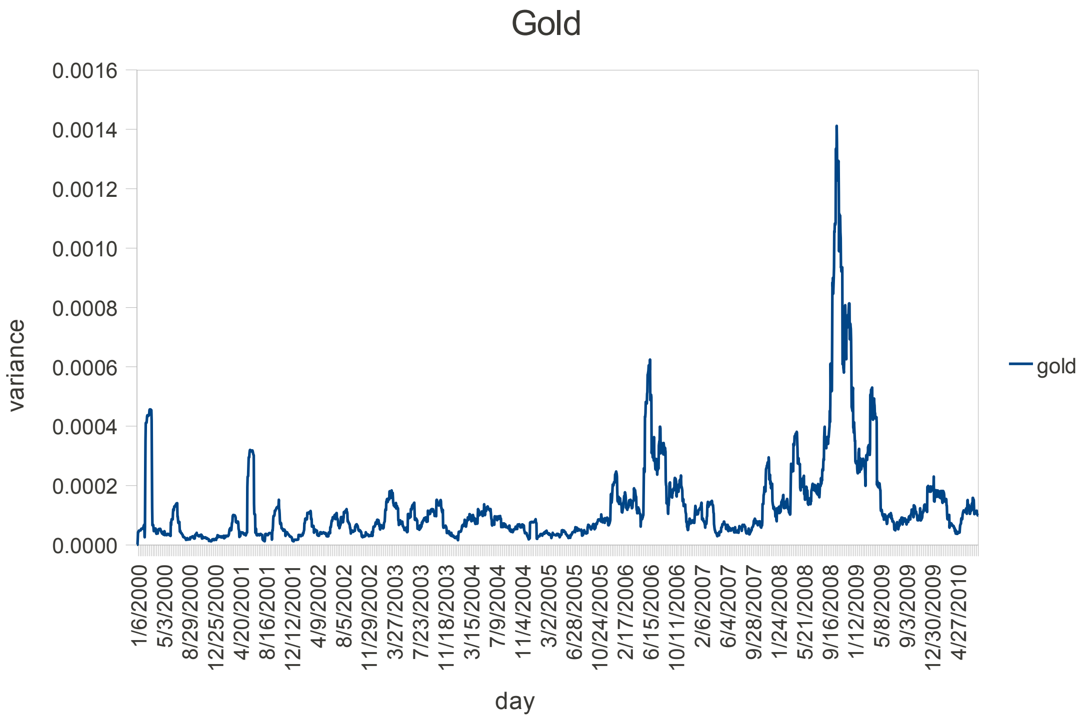
12/30/2009
4/27/2010
day


Corn
variance
0.0030
0.0025
0.0020
0.0015
0.0010
0.0005
0.0000
corn
1/6/2000
5/3/2000
8/29/2000
12/25/2000
4/20/2001
8/16/2001
12/12/2001
4/9/2002
8/5/2002
11/29/2002
3/27/2003
7/23/2003
11/18/2003
3/15/2004
7/9/2004
11/4/2004
3/2/2005
6/28/2005
10/24/2005
2/17/2006
6/15/2006
10/11/2006
2/6/2007
6/4/2007
9/28/2007
1/24/2008
5/21/2008
9/16/2008
1/12/2009
5/8/2009
9/3/2009
12/30/2009
4/27/2010
day


Gold
variance
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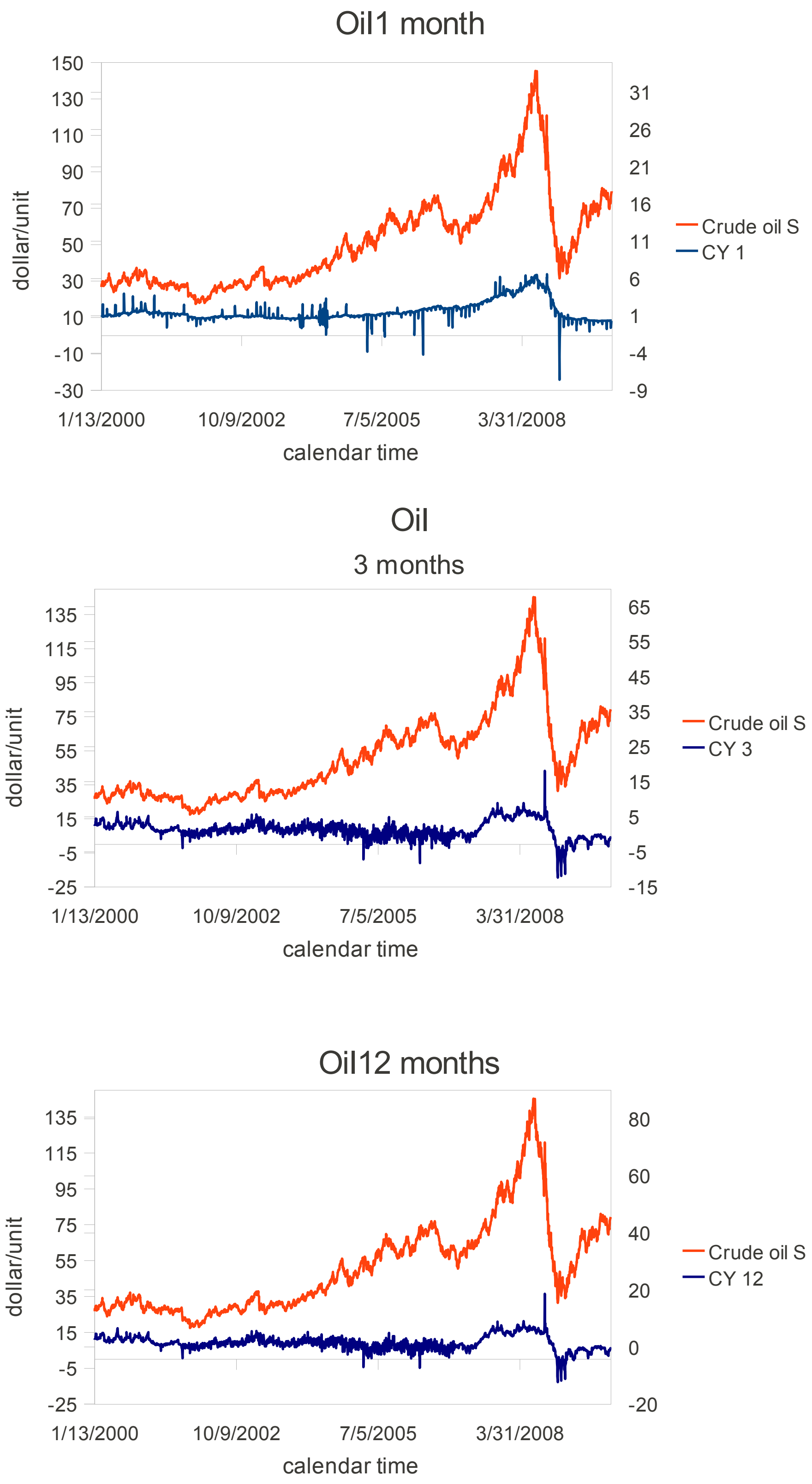
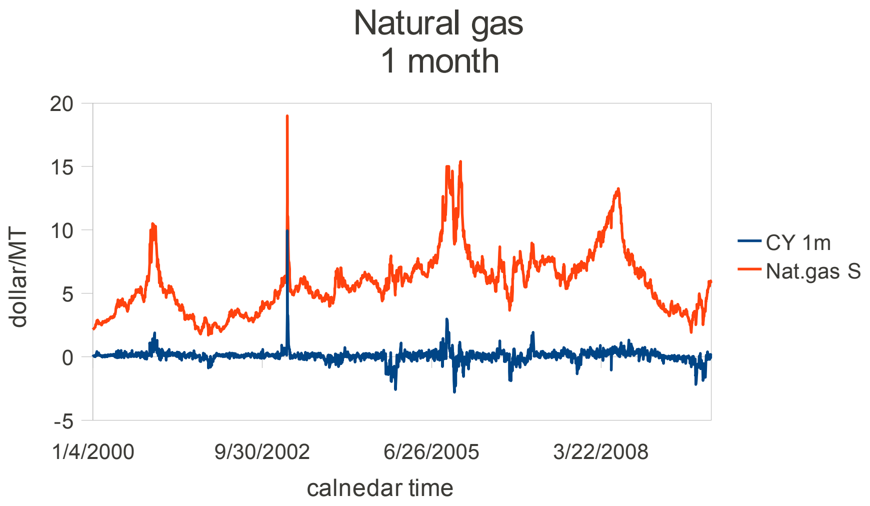
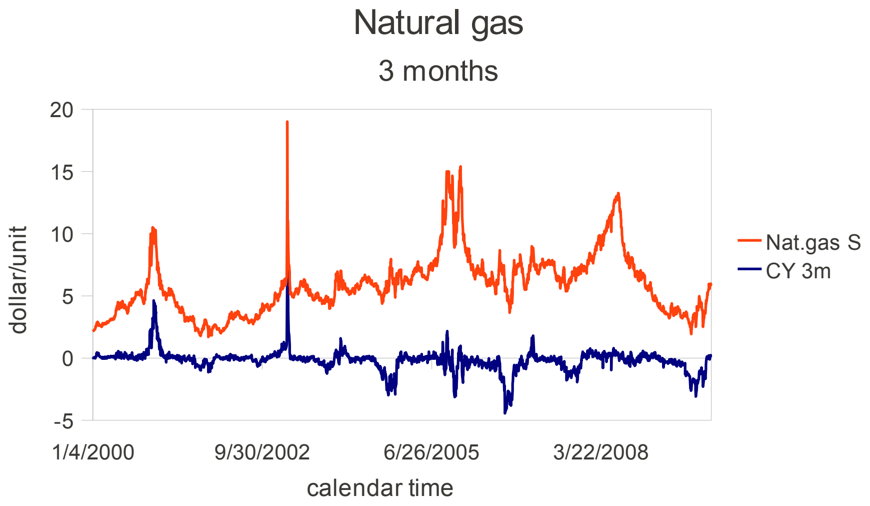
0.0014
0.0012
0.0010
0.0008
0.0006
0.0004
0.0002
0.0000
1/6/2000
5/3/2000
8/29/2000
12/25/2000
4/20/2001
8/16/2001
12/12/2001
4/9/2002
8/5/2002
11/29/2002
3/27/2003
7/23/2003
11/18/2003
3/15/2004
7/9/2004
11/4/2004
3/2/2005
6/28/2005
10/24/2005
2/17/2006
6/15/2006
10/11/2006
2/6/2007
6/4/2007
9/28/2007
1/24/2008
5/21/2008
9/16/2008
1/12/2009
5/8/2009
9/3/2009
12/30/2009
4/27/2010
day
gold

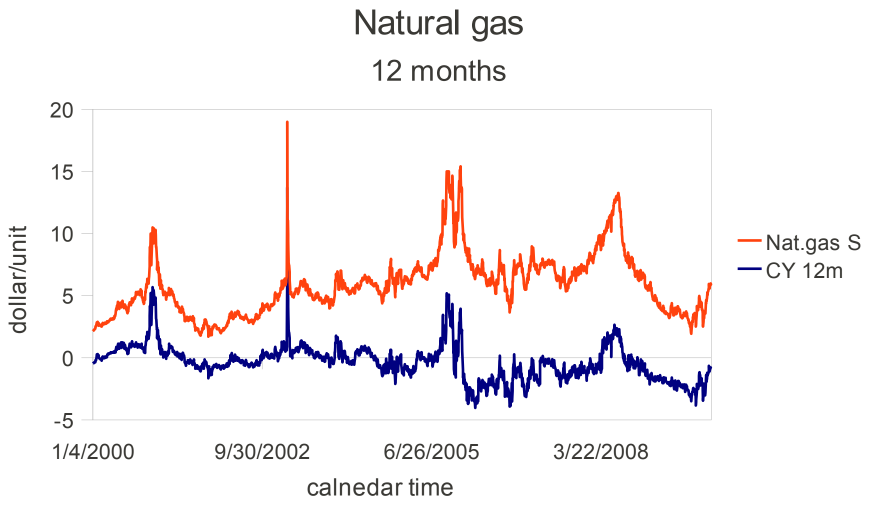
Figure 2

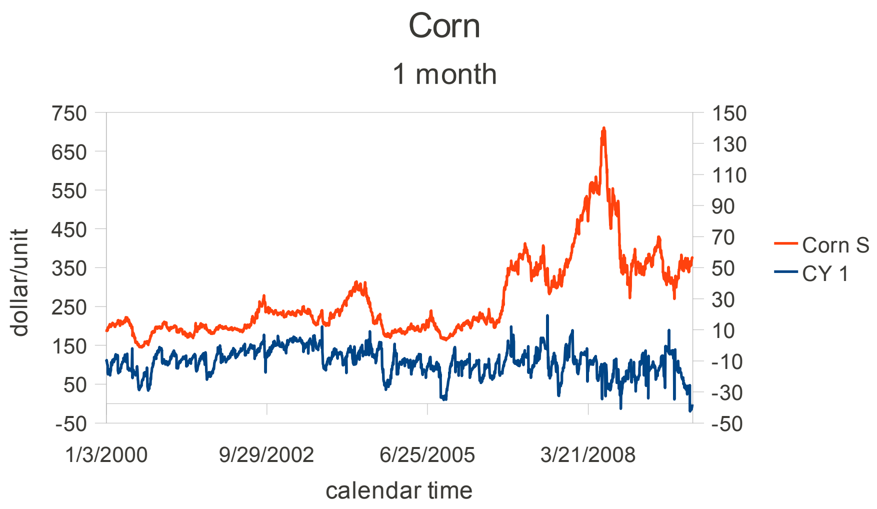
Relationship between Commodity Spot Price and Convenience Yield

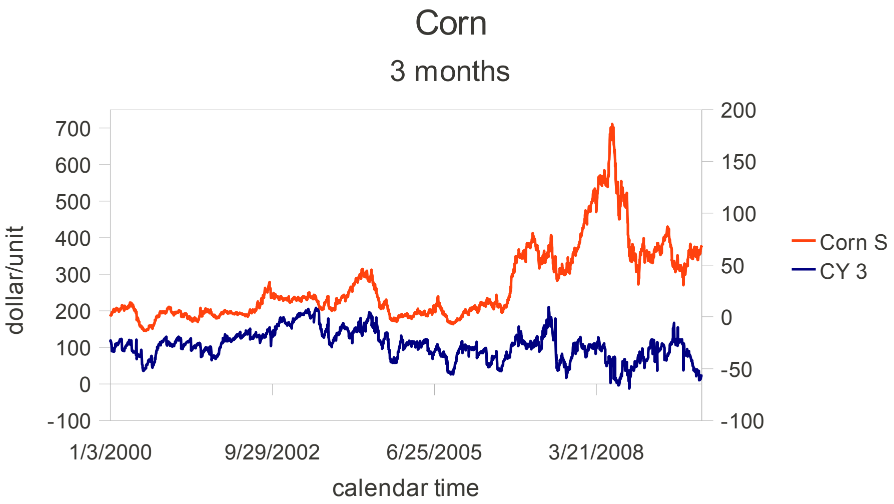


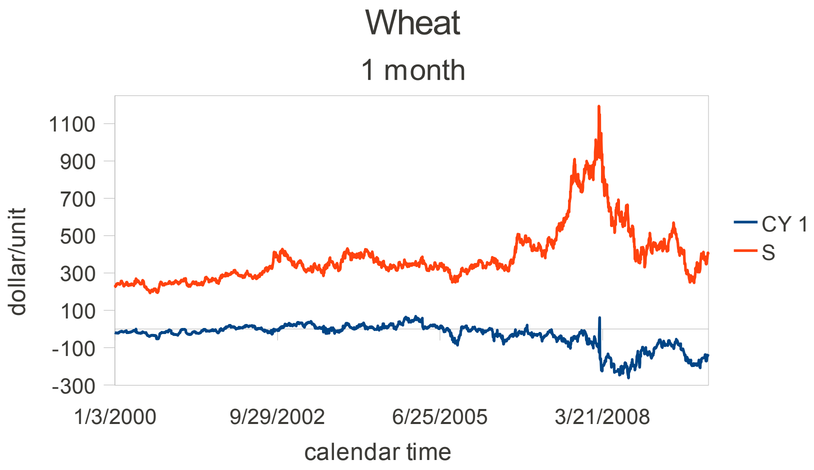

Natural gas
1 month
20
15
10
5
0
-5
dollar/MT
1/4/2000
9/30/2002
6/26/2005
3/22/2008
calnedar time
CY 1m
Nat.gas S

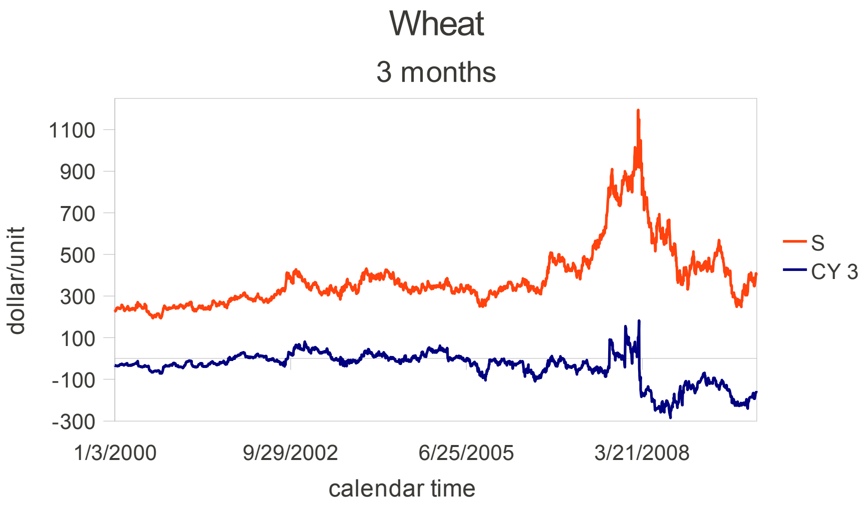

Natural gas
3 months
20
15
10
5
0
-5
dollar/unit
1/4/2000
9/30/2002
6/26/2005
3/22/2008
calendar time
Nat.gas S
CY 3m

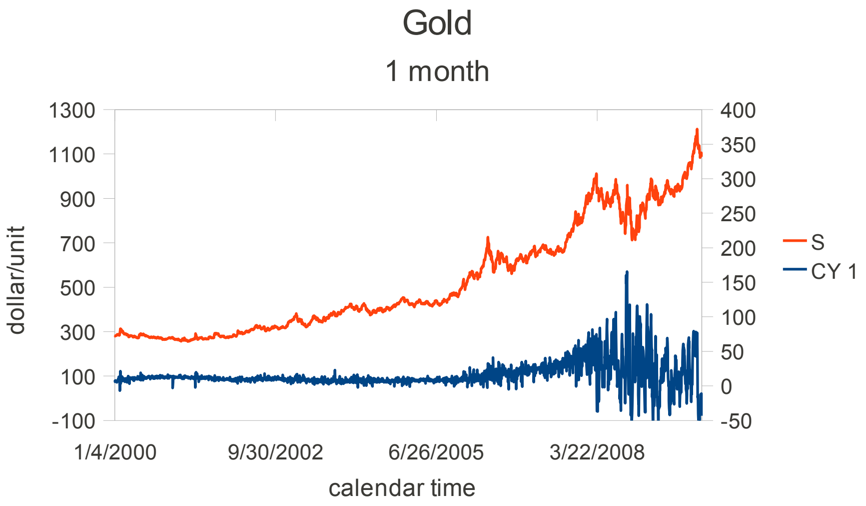

Natural gas
12 months
20
15
10
5
0
-5
dollar/unit
1/4/2000
9/30/2002
6/26/2005
3/22/2008
calnedar time
Nat.gas S
CY 12m

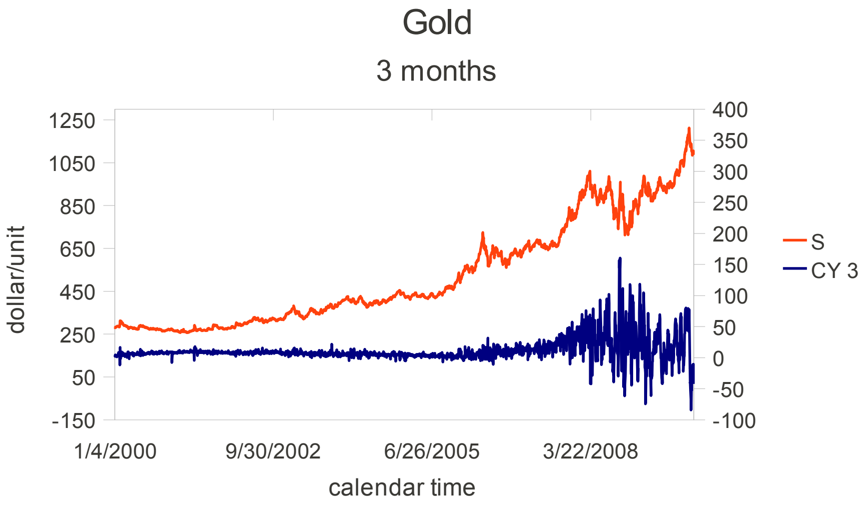

Corn
1 month
750
650
550
450
350
250
150
50
-50
150
130
110
90
70
50
30
10
-10
-30
-50
dollar/unit
1/3/2000
9/29/2002
6/25/2005
3/21/2008
calendar time
Corn S
CY 1


Corn
3 months
700
600
500
400
300
200
100
0
-100
200
150
100
50
0
-50
-100
dollar/unit
1/3/2000
9/29/2002
6/25/2005
3/21/2008
calendar time
Corn S
CY 3


Wheat
1 month
1100
900
700
500
300
100
-100
-300
dollar/unit
1/3/2000
9/29/2002
6/25/2005
3/21/2008
calendar time
CY 1
S


Wheat
3 months
1100
900
700
500
300
100
-100
-300
dollar/unit
1/3/2000
9/29/2002
6/25/2005
3/21/2008
calendar time
S
CY 3


Gold
1 month
1300
1100
900
700
500
300
100
-100
400
350
300
250
200
150
100
50
0
-50
dollar/unit
1/4/2000
9/30/2002
6/26/2005
3/22/2008
calendar time
S
CY 1


Gold
3 months
1250
1050
850
650
450
250
50
-150
400
350
300
250
200
150
100
50
0
-50
-100
dollar/unit
1/4/2000
9/30/2002
6/26/2005
3/22/2008
calendar time
S
CY 3