Social Cohesion and Economic Growth

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1. Introduction

This paper examines whether increased diversity leads to a decrease in social cohesion. By using a two sector endogenous growth model we find that diversity indeed threatens social cohesion but does not unambiguously lead to a demise in social cohesion. Social cohesion can be important for future growth. Population groups need to interact and the government needs to change its behaviour.

In order to keep growing economies have always needed new labourers. When their own population was not enough or did not grow fast enough foreign labourers were attracted. With the inflow of foreign labourers came an increase in the diversity of the population.

In the recent nineties globalization favoured the opening of markets and inflow of foreign workers, increasing the diversity in countries. A good example of today is the movement of Eastern European labourers to Western countries.

The main goal of this thesis is to understand how the inflow of foreign labourers affects the social cohesion of an economy and what its impact is on growth. Peter Taylor-Gooby\(^1\) states that “social cohesion and diversity are simply incompatible”. This thesis will argue that statement. What lacks in current research on this topic is a model that acknowledges the productive side of social cohesion and can combine it with endogenous growth and of course diversity to provide an insight into the future of social cohesion. This thesis will provide those insights.

On a country level, economic literature has focused on what caused some countries to grow much more than others and what determines the fluctuations. One major contributor to growth has been classified as human capital or knowledge. Within that context social capital, the fundament of social cohesion, is important as well. One cannot ignore that labourers are more than a factor of production. Their social side plays an important role in economics. That social side might influence the economy via a political regime chosen or is embedded in the actions of the economic agents. Because it can influence the economy, it can also influence the growth potentials of the economy. Diversity in population can spur growth but it also affects the social setting.

\(^1\)Taylor-Gooby, Peter, “Is the future American? Or can left politics preserve European welfare states from erosion through growing “racial” diversity?”, short research note, University of Kent.
**Social Cohesion**

Social cohesion has been given different definitions as authors use the term differently. One popular definition is by Judith Maxwell\(^2\) and translates social cohesion as follows:

“Building shared values and communities of interpretation, reducing disparities in wealth and income, and generally enabling people to have a sense that they are engaged in a common enterprise, facing shared challenges, and that they are members of the same community.”

But the definition uses social capital characteristics together with social cohesion characteristics. There is a certain causational link between different steps of the social process which could be vicious. Maxwell’s “building shared values” happens before “reducing disparities”. This distinction is important if social cohesion needs to be modelled.

My definition of social cohesion therefore is:

“When groups of people share the same social capital on a national level which allows them to work together and decide on problems in a way that benefits them all and society in the long run.”

**Social Capital**

Social Cohesion at the nation level is created by sharing the same social capital at that national level. But social capital is not a clear concept. It is the sum of different aspects which can all be interpret differently. As a result researchers have given similar, but somehow different interpretations.

The first feature is the network. Networks are the ties between individuals and the way the networks are configurated. Effort needed to use such a tie can also be seen as a part of the network. M. Bolino et al\(^3\) call this the structural part of social capital which indicates that individuals are tied to each other in a social space.

The ties can be on many levels. Starting at the family level networks can range from neighbourhood to state or from company to sector. Ties are created automatically as you

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enter a family, school or job. But they can also be formed out of interest when you join a political movement or other organizations that interests you.

The second feature is norms and values. M. Bolino et al call this the relational social capital which according to P. Paxton⁴ is “subjective and must be of a particular type, reciprocal, trusting, and involving positive emotion.” These types are established via shared norms and values. Sharing norms and values with other individuals is important for coordination and trust between individuals and determines the way you act.

Norms and values are created mostly via education and the family. School helps learning the same history and stories that later on help coordination. The family gives an individual cultural values of the ethnic group it belongs to, which are taught and passed on from generation to generation. Values such as for example religion or opinions about care for the elderly, which often are very different from another ethnic group.

The third aspect is mentioned by Kenneth Newton⁵ as the consequences that the norms and networks have. What is created or produced with the social capital. They can be both material and nonmaterial. When thinking of material goods one might think of voluntary organizations that produce charity work as a result of the shared norms of their members. Of course the social infrastructure that norms and networks create sometimes can exhibit material forms.

But mainly we are talking about nonmaterial goods such as trust and obligations and expectations, although Newton does not mention trust as one. Trust in general on a national level is good for the economy.

Besides these three pillars social capital exhibits three important aspects that influence its outcome on the economy.

The first is the chicken-egg problem. Mentioned by Newton and others is that the norms and the network interact. Norms influence the networks but networks can influence the norms. The question is which was first.

For example, at a national level government policies were once installed to protect our norms and values and keep them safe. Of course this influences our social capital today, especially that of the foreigners who only recently moved in.

Recognizing that norms and networks interact is important for the analysis.

Second, we can have interaction with other groups and levels. Groups and therefore networks can form on many different levels such as the family, neighbourhood, city, and etcetera. That has an effect on the trust within the network as the network can become more abstract. Newton stated that development might have a negative effect on social capital as the networks in which we participate become more complex and abstract. In a lower stage of development of the society you would know what was going on in your city simply because the city is not that big. Later on when the city has grown you have to hear it via the media. You are still in the network but the links are far less clear.

Also, the fact that social capital can be used at different levels gives rise to problems. Because each smaller group operates within a bigger network it is possible that the smaller group can undermine the workings of a bigger network and cause the social capital of the bigger group to decline. A relevant example is the United States. The country as a whole is less socially cohesive than for instance the Netherlands. But the individual associations in the US have stronger links than those in the Netherlands.

Besides this each cultural group has a different kind of social capital due to different norms. This does not matter at the family level but for instance at work the different norms might collide if two groups need to work together. Because “people will always favour their own families and communities” the domestic population might be suspicious of the foreign workers. Transactions costs between individuals in the country will rise as the networks and trust between individuals decrease.

**Economic effects of social capital**

There have been several studies on the effects of social capital and whether they contribute to the growth of the economy.

P.F. Whiteley did an empirical study on the effectiveness of social capital on economic growth. After examining the relationship in a sample of 34 countries from 1970 to 1992


8 Taylor-Gooby, Peter, “Is the future American? Or can left politics preserve European welfare states from erosion through growing “racial” diversity?”, *short research note, University of Kent.*

within a neo-classical framework of economic growth, his conclusion was “that social capital has an impact on growth which is at least as strong as that of human capital or education. And that the effect is the same as the catch-up or ability of poorer nations to adopt technological innovations pioneered by their richer counterparts”.

An interesting note is that the effects are not limited to democratic countries. So social capital is always worth investing in as even a non-democratic country can benefit from it.

The theoretical argumentation is that repeated interactions with others help solve the game theoretical problems of opportunism and free-rider problems\(^\text{10}\). Having a strong network within a community facilitates interaction, thereby increasing the trust between individuals and decreasing transactions costs. Second is the notion that social capital increases economic growth because it allows people to solve problems collectively. Networks facilitate the diffusion of information and coordination of efforts can be easier\(^\text{11}\). Another benefit of social cohesion is that the increased trust in a society allows for more investment because higher trust means lower risk. More investment logically generates more capital.

Given these arguments we must state that social capital is highly important for any country as it has a positive effect on economic growth.

A second study by S. Knack and P. Keeper\(^\text{12}\) also does an empirical analysis and shares the conclusion by Whiteley. They note that “trust and norms are higher in countries that are less polarized and have better formal institutions.” A clear indication that without between group interaction problems can become bigger.

**Modelling social capital**

In the previous section the different aspects of social capital and the economic effects of social capital have been discussed. Now we go into which aspects we want to model of the three basic features, norms, networks and consequences.

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For the analysis the assumption is made that each ethnic group has a specific kind of social capital. Although not waterproof it is a reasonable assumption. The result is that each group can be modelled as a different input into the social sector.

Again for purpose of the analysis we make the additional assumption that a country consists of only one cultural group that is socially cohesive at the national level. Id est, they share social capital.

In the light of the economic background the focus is on the different norms of the domestic population and a group of foreign workers moving into the country and their interaction.

Given that this is a macroeconomic analysis at the national level, the network where the types of social capital come together is a bit more abstract. It is the social platform where the two different groups and their different types of social capital come together and share their social capital to build common social ideas. The platform can range from schooling to public projects and interactions at any level.

Key is that different groups interact with each other at this social platform which is at a higher level than the group is. So the platform does not contain interest groups working solely for themselves as they are too rigid.

It will be modelled as a different sector next to the production sector as a sector that produces shares social capital.

The result of social cohesion is that social capital becomes a production factor as it can increase the economic output with more efficiency as states in earlier chapters. So we model a factor of production. That factor can be material and nonmaterial. The material outcome usually is only limited and does not have a big impact on the economy. The nonmaterial outcome is more likely to have spill over effect which has an effect on economic growth on a macroeconomic level.

With that comes the notion that trust can be trust or general. The latter is more important for growth which will be make a difference for the analysis.

In the next part the model is presented. Followed by the presentation of the results on social cohesion, diversity and economic growth in the third section. Thereafter some extensions are given to highlight the potential threats and benefits of social cohesion. Finally a conclusion is given.
2. The Model

From the theory we know that social cohesion is potentially good for economic growth as it increases the efficiency of labourers via better coordination and trust. Now a model is presented which proves and confirms that social cohesion does the economy good in the long run. The model is a 2 sector endogenous growth model. Most important is the addition of a social sector next to a production sector.

Basic model

We know social capital is a part of human capital and has the same empirical effects on economic growth. We therefore start with an endogenous growth model with human capital and change this into a model worker for social capital.

The model is mostly derived from Romer (1990). To start the economy produces an aggregate good in the production sector according to the function:

\[ Y(t) = [K(t)]^\alpha [L(t)]^{1-\alpha} \]  

(1)

Where,

\[ Y(t) = \text{The production of the aggregate good} \]
\[ K(t) = \text{Capital} \]
\[ L(t) = \text{Labour} \]
\[ \alpha, 1 - \alpha = \text{Elasticity's, with } 0 < \alpha < 1 \]

Besides this the growth of capital is a function of the savings as a percentage of income: \( K(t) = sY(t) \). Because we focus on labour and want to keep the model simple capital exhibits no depreciation. Population grows at a constant rate given by: \( L(t) = nL(t), n \geq 0 \).

Adding foreign workers

In this model we will not act as if all labour is the same. According to Alesina and Ferrara each cultural group has a different specific skill. Foreign labour has a different

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return than domestic labour. So the model needs to be adjusted for a third factor of production. The model still exhibits constant returns to scale as the foreign workers will generally be lower skilled workers. The function becomes:

\[ Y(t) = [K(t)]^\alpha [L_h(t)]^\beta [L_f(t)]^{1-\alpha-\beta} \]  

Where,

\[ L_h(t) = \text{Domestic labour force with } L_h(t) = n_h L_h(t), n_h \geq 0 \]
\[ L_f(t) = \text{Foreign workforce with } L_f(t) = n_f L_f(t), n_f \geq 0 \]
\[ \beta = \text{Elasticity, with } \alpha + \beta < 1 \]

The model so far represents an endogenous growth model without social cohesion taken into account.

**Adding social capital to the production function**

To transform the basic function into one for a society with social cohesion we need to add the effects or consequences of social cohesion as another factor of production to the production function. Two problems in choosing the model need to be worked out, whether social cohesion generates increasing returns to scale for production and how social cohesion affects production and thus where it comes into play in the function.

As noted earlier, social capital consists of norms, networks and trust. All effect the economy in some way, but the most influential on a national level is trust. Moreover, trust is the result of norms and networks. Trust in its turn can be divided in personal and general trust with the notion that personal trust creates general trust.

Social capital can thus be modelled in two ways:

\[ Y(t) = [K(t)]^\alpha [A(t)(1 - a_{hL})L_h(t)]^\beta \left[ (1 - a_{fL})A(t)L_f(t) \right]^{1-\alpha-\beta} \]  

With \( \alpha + \beta < 1 \). Alternatively the model could be formulated as:

\[ Y(t) = [A(t)[K(t)]^\alpha \left[ (1 - a_{hL})L_h(t) \right]^\beta \left[ (1 - a_{fL})L_f(t) \right]^{1-\alpha-\beta} \]  

With \( \alpha + \beta < 1 \). An in both cases

\[ A(t) = \text{Social Capital.} \]
\[ a_{hL}, a_{fL} = \text{The part of domestic and foreign labourers used in the production sector.} \]

Option (3) presents social cohesion in interaction with labour. The workings are the same as human capital, which is in line with the argument that social capital is an aspect of
human capital. The function implies that social capital is owned by the labourers and cannot be used without them. In this way the effect represents the personal trust.

In the case of function (4) social capital is something which is not specifically attributed to labourers. This option represents general trust which increases overall production. Although it is something less in line with the argument of human capital it is a better option as general trust in the society is the most important factor for growth in production. And because the analysis is on a macroeconomic level we look at overall trust, not the trust of one particular person. So the second option seems to fit the best.

Turning to the problem of the returns to scale, the difference between personal and general trust described above has an effect on how we need to model it. Because the choice is for general trust, as it has the most effect on economic growth, constant returns to scale is a logical choice. The reason is that general trust is growth facilitating instead of growth creating. The latter case would mean increasing returns to scale would be possible.

As trust reduces transaction costs we need to think what happens if transaction costs are fully eliminated. No transaction costs do not mean increasing returns to scale according to economics. Social cohesion has now removed a market disturbance. So trust mostly reduces a market imperfection instead of creating market. As P. Whiteley puts it: “it holds down the diseconomies”\(^\text{15}\).

Of course there are arguments for increasing returns to scale as well. In this case social cohesion would be growth creating. Returning to the argument of removing diseconomies one can argue that diseconomies are a fact of life and constant returns to scale can never be achieved by removing them. Instead the economy has adjusted to diseconomies and the removal of transaction costs can bring forth increasing returns to scale.

Another discussion is spill over effects. Social Cohesion has spill over effects which can create increasing returns to scale as the benefits from social cohesion improves more than only one production factor. Following up on the human capital argument, social capital is non-rival. If one person uses a network tie or trust given by another it does not prevent a third person to do the same simultaneously. So social cohesion is non-rival. This makes it likely to have spill over effects.

Both constant and increasing returns to scale are probably a viable option given the discussion. What is most likely is that the returns are time bound. At some point there are constant returns and at a different stage of development there are increasing returns. So for completeness of the analysis the results of both options will be discussed. The functions used will be, when there are constant returns to scale:

\[
Y(t) = [A(t)]^\gamma [K(t)]^\alpha [(1 - a_hL_h(t)]^\beta [(1 - a_fL_f(t)]^{1-\alpha-\beta-\gamma}
\]

With \(\alpha + \beta + \gamma < 1\). Or when there are increasing returns to scale:

\[
Y(t) = [A(t)]^\gamma [(1 - a_hL_h(t)]^\alpha [L_h(t)]^\beta [(1 - a_fL_f(t)]^{1-\alpha-\beta}
\]

With \(\alpha + \beta < 1\) and \(0 < \gamma < 1\).

Adding Social Cohesion

We now change the attention to the social sector. We model the creation of social cohesion which is the sum of several effects according to the literature. The first is the interaction of the two labour groups, second their norms and third the existing social cohesion.

The creation of social cohesion happens in a specific social sector which represents the network. The network links the two labour groups together and causes them to interact and build shared social capital. This is highly important as they cannot produce shared social capital if they do not interact.

Even more, the effect of the existing social cohesion on making new social cohesion is reduced. This will be taken into account via a participation parameter.

The reason for existing social cohesion to have an effect on new social cohesion is the chicken-egg problem. Because the earlier networks influence new networks and norms we cannot overlook that they might be rigid and work against new settings.

Of course the social capital within both groups plays a role as well. There can be a difference between for instance the norms of both groups. When they are very different it will have an effect on the growth of social cohesion because to groups will have more trouble interacting and finding common ground. A distance parameter is therefore taken into account. We will see that it only has a level effect and will die off in time. This makes sense as their interaction brings both groups together as time lingers on.
The second sector produces social capital according to the following function:

\[ A(t) = (1 - \delta)A_t^{\theta - \varphi} \left[ a_{hl}L_h(t) \right]^\theta \left[ a_{fl}L_f(t) \right]^{(1-\rho)} \]  
(7)

Where,

- \( A(t) = \) Social Capital
- \( \dot{A}(t) = \) The production of social capital
- \( a_{hl}, a_{fl} = \) The part of domestic and foreign labour used in the social sector
- \( (1 - \delta) = \) Distance parameter of social capital, where \( \delta = 1 \) means fully different social capital and \( \delta = 0 \) complete overlap. \( \delta > 0 \).
- \( \theta = \) The effect of the current stock of social capital on the production of new social capital. \( \theta \geq 0 \).
- \( \varphi = \) Participation parameter. If either \( a_{hl} \) or \( a_{fl} \) equals 0, \( \varphi = 0.5 \). Otherwise \( \varphi = 0 \).
- \( \rho, (1 - \rho) = \) Elasticity’s. \( 1 > \rho > 0 \)

Note that capital is not used in the social sector. Capital plays a logical role in creating social infrastructure via for example investments in communication. But it does not create interaction or a platform. It can merely facilitate it. More capital invested in the social sector will not lead to people willing to participate more. Therefore we ignore capital in the production of social cohesion.

Furthermore there is no fraction for \( A(t) \), as it can be used everywhere since it is a non-rivalrous factor.

Another noteworthy point concerns the elasticity’s of the two different labour inputs. They together exhibit constant returns to scale as it is logical that we cannot have unlimited growth in social capital. However, combined with social cohesion already present, there can be increasing returns to scale. This is a fair assumption as already established social cohesion can speed up the process of creating common ground.

Regarding \( \theta \), it is crucial that it is smaller than 1 as we cannot have unlimited growth in social cohesion. This condition makes sure that when population is not growing, social cohesion will inevitably slowdown and stop growing.

If it were true that \( \theta = 1 \), social cohesion would be self sustaining. This obviously cannot be as one needs to put effort into social cohesion in order to keep for example the social ties alive.
The participation parameter $\varphi$ in contrast to the distance parameter is a static parameter. How much people participate will not affect the long term growth rate of social capital. This is because ideas are not created, in the long run, by an amount of people but simply by people. There is only a difference between participation or not. The effect of social cohesion increases dramatically with 2 groups working on it.

0.5 is chosen because we use 2 groups and can assume that the lack of one group affects the growth potential by half on a scale from 0 to 1.

Finally, $a_{ht}$ and $a_{jt}$ can represent two things. On a macroeconomic level the fractions represent the number of people that work in either sector as they cannot work in both at the same time. In this case the sectors represent for example the car industry and high school.

On a microeconomic level they represent time per worker. A person only has a limited amount of hours to spend per day and he needs to choose what to do with it. Some hours are needed to work and acquire an income. After this amount he can freely choose to work more or participate in the social sector. The hours needed for home production are assumed to be the same for everybody and hence do not play a role.
3. Results

In this section the results for the long run growth rates of the economy are presented. All the calculations are presented in the Appendix.

Social Cohesion and growth

**Proposition 1a:** With constant returns to social cohesion and all factors of production, the growth rate of the economy without social cohesion is $g_y^{BGP} = g_K^{BGP} = n$. That of the economy with social cohesion is $g_y^{BGP} = g_K^{BGP} = \frac{\gamma g_A(t) + (1-\alpha - \gamma)n}{(1-\alpha)}$. Thus, compared to the economy without social cohesion, the economy with social cohesion is characterized by higher growth because $g_A^{BGP} = \frac{\rho m}{(1-\theta)}$ is always higher than $n$ and positive.

What becomes clear is that social cohesion diversifies the growth of the economy over more inputs. Because $g_A$ uses $n$ more effectively population growth can spur the economy more than without social cohesion. But the growth of the economy is however dependent on $g_A$ to be positive in order to have the same growth rate as the economy without social cohesion. If $g_A$ where to collapse for some reason, growth can be lower. This implies that if people are aware of the benefits of social cohesion they will take better care of the social sector.

**Proposition 1b:** With increasing returns from social cohesion, the growth rate of the economy without social cohesion is $g_y^{BGP} = g_K^{BGP} = n$. That of the economy with social cohesion is $g_y^{BGP} = g_K^{BGP} = \frac{\gamma g_A(t)}{(1-\alpha)} + n$. Thus, compared to the economy without social cohesion, the economy with social cohesion is characterized by higher growth again as $g_A^{BGP} = \frac{\rho m}{(1-\theta)}$ and positive.

As with the case of constant returns to scale the growth of the society without social cohesion is fully dependent on the growth population. The social society on the other hand can also grow via social cohesion.

In this case the growth benefits of social cohesion are even greater. But more important is that the economy is not dependent on $g_A$ to sustain the same growth as the economy
without social cohesion. This makes the effort in social cohesion less necessary which is a threat to the sustainability of social cohesion.

The above makes clear that in the long run the social society will have higher growth rates as $\theta < 1$. The fact that the individual society uses more labour in production does not affect the long run growth rates. In the present time it will have an effect but the effect dies off over time. So social cohesion is good for growth in the long run.

**Diversity and growth**

**Proposition 2**: The growth rate of the economy without foreign workers is $g^B_{y} = g^B_{K} = n$. That of the economy with foreign workers is $g^B_{y} = g^B_{K} = \frac{\beta n_h + (1 - \alpha - \beta) n_f}{(1 - \alpha)}$. Thus, compared to the economy without foreign workers, the economy with foreign workers is characterized by more diversified growth.

In this case the foreign workers reduce the contribution of domestic workers to production. If $n_h = 0$, the case clearly illustrates that the domestic economy cannot grow without the help of foreign workers. This makes the foreign workers so attractive and important.

**Social Cohesion and diversity**

**Proposition 3a**: With constant returns to social cohesion and all factors of production, the growth rate of the social economy without foreign workers is $g^B_{y} = g^B_{K} = \frac{\gamma g_A(t) + (1 - \alpha - \gamma) n}{(1 - \alpha)}$ with $g^B_A = \frac{\rho n}{(1 - \theta)}$ and positive. That of the social economy with foreign workers is $g^B_{y} = g^B_{K} = \frac{\gamma g_A(t) + \beta n_h + (1 - \alpha - \beta - \gamma) n_f}{(1 - \alpha)}$ with $g^B_A = \frac{\rho n_h + (1 - \rho) n_f}{(1 + \varphi - \theta)}$ and positive. Thus, compared to the economy without foreign workers, the economy with foreign workers is characterized by more diversified growth from the foreign workers.

Again, the social society needs $g_A$ to sustain growth. This makes $g_A$ easier to sustain but at the same time poses a threat for the economy.

**Proposition 3b**: With increasing returns from social cohesion, the growth rate of the social economy without foreign workers is $g^B_{y} = g^B_{K} = \frac{\gamma g_A(t)}{(1 - \alpha)} + n$ with $g^B_A = \frac{\rho n}{(1 - \theta)}$ and positive. That of the social economy with foreign workers is $g^B_{y} = g^B_{K} = \frac{\gamma g_A(t) + \beta n_h + (1 - \alpha - \beta) n_f}{(1 - \alpha)}$ with $g^B_A = \frac{\rho n_h + (1 - \rho) n_f}{(1 + \varphi - \theta)}$ and positive. Thus, compared to the
economy without foreign workers, the economy with foreign workers is characterized by more diversified growth from the foreign workers.

But again because the economy can also sustain growth without \( g_A \), \( g_A \) becomes more vulnerable in the case of increasing returns to scale.

Note that \( \varphi \) can be assumed to be 0 if the foreign workers simply participate in the social sector. This allows \( g_A \) to grow at the same rate as without foreign workers assuming that the growth of both groups is equal.

Turning to the population growth rates, a case relevant for Western economies is the one where the domestic population stops growing. There are two options. The first is that growth stagnates for a period and picks up later on, the second is that the stagnation does not stop and growth will eventually come to a complete halt. The United Nations population division estimates that this might be coming true.

In these cases growth is determined by the foreign workers. The participation of foreign workers in the production sector as well as the social sector is necessary to sustain growth in both sectors. This explains why diversity via foreign workers is important for growth. The societies are dependent on the labourers for long term growth. At the same time diversity poses a threat to the social sector.

**Short term growth**

In the short run the society that lacks a social sector can produce more because no factors of production are given up for the production of social cohesion. But at the same time their workers are less efficient due to the loss in social cohesion, especially in the case of constant returns to scale.

Which force is stronger is arguable because an exact figure cannot be given to social cohesion. In the long run it does not matter as the level effects of more production will die off in time. But people are often focused on the short term and governments find it hard to deal with this pressure. So the social sector is under pressure.

Another short term problem arises when the social sector has not already been established. Let us assume that people only care about income growth. In the long run their income will be the highest if the economy sets up a social sector. But in the short run if no social sector is present all factors of production are deployed in the production sector. If the economy wants to set up a social sector it will first have to lower production and income in
order for growth to be higher in the future. So it is difficult to start a social sector. The question is how to convince people to give up some income now for more in the future.

This first analysis clearly indicated that an economy with or without foreign workers benefits from social cohesion in the long run as social cohesion influences the long run growth positively via a better use of \( n \).

Also, foreign workers are important for the economies. But this hinges on a crucial assumption. The assumption that foreign workers will participate in the social sector. What would be the difference if they did not want to spend some of their time in the social sector.
4. Extensions

No foreign workers in the social sector

In the macro-function \( a_{hL} \) and \( a_{fL} \) indicate how many labourers work in the production sector and social sector. The choice whether or not to work in the production sector or social sector when the latter already has been established, depends on your relative marginal contribution, or wage.

On a micro level, image that foreign workers are lower skilled than domestic workers. This means that they get paid less than domestic workers, thus wanting to work more hours to get a certain income. This could translate into foreign workers spending all their time in the production sector not contributing to social cohesion.

On a macro level the number of foreign workers working in the social sector is likely to be very low as that sector is probably not the reason for them to move into this country.

**Proposition 4**: When foreign labourers do not participate in the social sector, the growth rate of the social sector is \( g_A(t) = (1 - \delta)[a_{hL}L_h(t)]^p A_t^{\theta - \phi - 1} \) which is always negative. In the long run there could be a unique value of \( g_A^{BGP} = \frac{\rho \eta_h}{(1 + \phi - \theta)} \), reducing the stock of social cohesion.

This implies that social cohesion will be destroyed and potentially becomes negative. We will not discuss the possibility of negative social cohesion.

As social cohesion becomes negative the growth of the economy which is characterized by \( \gamma g_{t} + \beta n_h + (1 - \alpha - \beta - \gamma) n_f \), is lower than that of the economy without social cohesion and constant returns to scale \( \frac{\beta n_h + (1 - \alpha - \beta) n_f}{(1 - \alpha)} \). If social cohesion would have created increasing returns to scale there the growth rates of both economies would be the same in the long run. In the short term the social society would lose some growth as it has given up production inputs for the sake of a sector that counteracts in growth.

It is immediately clear that the social sector is under pressure.

Firstly the effect of the domestic workers on generating new social capital has already been lowered due to the existence of foreign workers in the economy. They alone cannot sustain social cohesion as the elasticity is smaller than 1.
Second, the contribution of the existing social capital to new social capital deteriorates as \( \varphi \) becomes 0.5 if the participation of a group is not there. Because foreign labourers are attracted, \( n_h \) is likely to be low. Over time \( A_t^\theta \) becomes lower as it accrues to a smaller part of society as foreign labourers do not participate. This also makes it likely that \( \delta \) will be high as no participation is an indicator that the groups are very different. But we have already seen that in the long run this is irrelevant. Looking at the growth rate of social capital in this case, as \( (\theta - \varphi - 1) \) is far below 0, existing social cohesion starts working against new social cohesion. The growth of \( A_t \) is dependent on whether the negative impact can be compensated by effort of the domestic workers. This is highly unlikely. Of course the reasoning behind this can be the other way around. Domestic workers might not let foreigners participate or do not want to participate with them. The result is the same.

So we know that social cohesion, although providing more growth in the long run still is highly dependable on the effort of the population. Even more, with the economy ultimately needing foreign workers problems increase as foreign workers are less likely to put effort into social cohesion.

**Different social capital of domestic and foreign labourers**

In the analysis the effect of different social capital of foreigners was incorporated into \( 1 - \delta \). The analysis was looking for combined social cohesion. But the analysis has also pointed out that it is quite possible that the between social cohesion will not happen. The question is what happens when the two different groups completely polarize. The result would be two different social sectors which have a combined effect on the economy. Two different kinds of social capital are not all that bad. It can be seen as a variety in skills which especially for a big economy might very well be positive. But the fact that the society is divided into two groups outweighs that benefit due to largely increased transaction costs. So we need to model the effects of two different social groups which do not have any between group trust.

One option is to think of the effects social cohesion on production to be a result of the two \( [A_h(t) - A_f(t)]^\theta \). This would imply that the stronger is the foreign social cohesion the lesser the positive effects of the domestic social cohesion become. The groups would
work against each other. It makes sense in reality. The more socially cohesive and closed the foreign groups is, the lesser the economy can benefit from them.

The model would become:

\[
Y(t) = [A_h(t) - A_f(t)]^\gamma [K(t)]^\alpha [(1 - a_{hL})L_h(t)]^\beta \left[(1 - a_{fL})L_f(t)\right]^{1-\alpha-\beta-\gamma} \tag{8}
\]

with, \(\alpha + \beta + \gamma < 1\)

\[
A_h(t) = A_h(t)^\theta [a_{hL}L_h(t)]^\rho \tag{9}
\]

\[
A_f(t) = A_f(t)^\theta [a_{fL}L_f(t)]^\rho \tag{10}
\]

with, \(\theta \geq 0, 1 > \rho > 0\)

One can imagine that \(A_h\) has an effect on \(A_f\) in the social sector. But this needs to be viewed carefully. The more socially cohesive the foreign group is, the more domestic workers will interact with each other. This leads to an increase in \(a_{hL}\) which is only a short term effect. This makes sense in reality as higher trust of one group does not make you trust your group more, instead you interact more with you own which can lead to more trust.

**Proposition 5:** When both labour groups polarize the positive effects of their social cohesion depends on \(\gamma \left( g_{hA}(t) - g_{fA}(t) \right) \) where \(g_{hA} = \frac{\rho a_{hL}}{1-\theta}\) and \(g_{fA} = \frac{\rho a_{fL}}{1-\theta}\). Thus if the foreign population grows more than the domestic or the bigger the foreign group is, the more their polarization leads to no possible gains from social cohesion.

In this case the economy looses as inputs are lost into a social process which will not benefit the economy, in the long run and the short run.

The two labour groups can be seen as closed groups as they do not interact. Although both groups work in the economy they are not used optimally.

This is unfortunate as interaction will lead to more growth and a higher income for both. As this clearly is a problem of aligning social and private returns it is time to look at the government.

**Government**

First off, given that social and private returns need to be aligned a government is a welcome and classical possibility to solve the problems.

Given that social cohesion increases efficiency and thereby economic growth, there might be a possibility where the government reaps some of the benefits of social cohesion via a tax levied on labourers. With those incomes they can try to promote social cohesion and keep
social cohesion in place. Or focus on letting only small amounts of foreigner in or groups that are similar to the domestic population. But this is a very short answer. The government is involved more deeply.

As stated earlier in the thesis there is a causational link between different steps of the social process which could be vicious. Social capital in the beginning determines the course of the government. At a certain point in time the government in its turn is strong enough to influence social capital again. Mostly via its policies which affect the possibilities of the population. In context of the model the government relates to $A_t^θ$ as the government is an unmistakable part of social cohesion. The government chosen in an earlier time period has an effect on the current formation of social cohesion. This brings forth an important insight.

If the government is a part of, or completely represents $A_t^θ$, the effect of its actions are influenced by the participation of its population.

So if the government were to ignore the breakdown of social cohesion and allow a group not to participate in the social sector, the government itself will work against the creation of new social cohesion.

But simultaneously the model can also be seen differently in relation to the government. Social cohesion or that sector can be viewed as the foundation or justification for government policies as the social status of a country determines the ease with which a government is formed or can operate.

As the society is likely to be complex, a breakdown of the social sector inevitably makes politics more difficult as a government will find it more difficult to establish a good long run policy which fits both groups. And with the danger of groups working against each other the course of the government will shift more often. Nowadays we see governments having difficulty imposing their policies as the diversity of the population causes the policy to hurt some groups at all times. Together this will make the government more costly.

So if the government can recognize its own role it can improve the social cohesion. This would result in an increased efficiency of the government, thereby lowering costs of government.
Diseconomies of scale

The analysis thus far has treated social cohesion as a factor of production which kept constant returns to scale intact but could also generate increasing returns to scale. The difference in the outcomes between constant and increasing returns to scale are important as they affect the necessity of social cohesion differently. With increasing returns to scale, social cohesion was not necessary to keep the growth rate of the economy unchanged compared to a society without social cohesion.

A definitive answer is important for further research on the benefits and sustainability of social cohesion. The answer depends on whether social cohesion is viewed as the correction of a market failure or whether it truly is another factor of production that has not been used to its full potential.

But there is another way to look at social cohesion to make the discussion even stronger in favour of increasing returns without starting the discussion on what social cohesion is.

If one states that an economy with social cohesion still has constant returns to scale in all factors of production, social cohesion thus is not an extra factor of production. In this case social cohesion can be viewed as the correction of diseconomies. This implies that the comparable economy without social cohesion has decreasing returns to scale since it misses a crucial part needed for growth.

This means that the production function becomes

\[ Y(t) = [K(t)]^\alpha [(1 - a_L)L(t)]^{1-\alpha-\gamma} \]  \hspace{1cm} (11)

With, \( \alpha + \gamma < 1 \). And for the social economy the functions are

\[ Y(t) = [A(t)]^\beta [K(t)]^\alpha [(1 - a_L)L(t)]^{1-\alpha-\gamma} \]  \hspace{1cm} (12)

With \( \alpha + \beta + \gamma < 1 \) and

\[ A(t) = [a_L L(t)]^\rho A(t)^\theta \]  \hspace{1cm} (13)

With \( \theta \geq 0, 1 > \rho > 0 \). This leads to the following proposition.

**Proposition 6**: When the economy without social cohesion exhibits decreasing returns to scale, the growth rates become

\[ g_Y^{BGP} = g_K^{BGP} = \frac{(1-\alpha-\gamma)n}{(1-\alpha)} \]

That of the economy with social cohesion and constant returns to scale is

\[ g_Y^{BGP} = g_K^{BGP} = \frac{\gamma B A(t) + (1-\alpha-\gamma)n}{(1-\alpha)} \]

with \( g_A^{BGP} = \frac{\rho n}{(1-\theta)} \). Now the differences in growth rates are similar to the differences with increasing returns to scale.
This is an indication that increasing returns to scale in the long run is possibly a better estimate, with more potential gains from social cohesion. But this also means that the sustainability of social cohesion is more difficult.

*Diversity and human capital*

In this model labour could also represent human capital one for one. It is quite clear that the potential benefits of knowledge can barely be reaped if the social infrastructure is not there and knowledge cannot work together. Although social cohesion can prove to be costly to sustain it is worthwhile if it increases the benefits from human capital. This is most important in the sectors for higher educated people as there can be increasing returns to scale to human capital. But it might also be important for the entire economy in the long run, as the optimal functioning of human capital can be important on many levels.
5. Conclusion

This thesis has proven that social cohesion and diversity are both good for long run economic growth. However, the analysis also confirms that social cohesion and diversity combined proves to be a difficult match. It is likely that diversity will work against social cohesion and undue the positive contribution of social cohesion to economic growth if social cohesion is left to the will of the people. With increasing returns to scale from social cohesion the problem is bigger because the necessity of social cohesion is lower.

But as diversity might prove crucial due to the stagnation in population growth in several countries, governments will have to find a way to solve this difficulty. To do so, the government will need to realize that it itself is a lagged variable to society and should focus on getting the population to work together, especially when the dependence on foreign labourers increases. In doing so the government will provide itself a service by lowering its own costs. In the short run they can concentrate on attracting groups that are similar to the own population making participation easier.

By modelling social cohesion in an endogenous model, this thesis has proven that social cohesion and diversity are not unambiguously incompatible. Further research should concentrate on when exactly the potential benefits are big enough for the government to take action. Also a fiscal scheme to grasp the benefits and promote social cohesion needs to be worked out.

Regardless of the economic potentials social cohesion has, it seems social cohesion will inevitably be under pressure due to constant changes in society. Especially an open and dynamic economy will find it hard to deal with different labour groups. But if a society derives utility from the social status, the benefits are easier to grasp and bigger. So in the end the choice to put effort into social cohesion is a choice made by the population. Each culture is different and may want a different solution. Some get positive utility from social cohesion, other countries might not.

Perhaps the answer lies in the famous Korea example. Where during the financial crisis of 1997 people gave up loved family items to try and make things right. Such a self-sacrifice for the benefit of society might be the perfect foundation for long run sustainability.
6. References


17) Taylor-Gooby, Peter, “Is the future American? Or can left politics preserve European welfare states from erosion through growing “racial” diversity?”, short research note, University of Kent.


7. Appendix

Proof of Proposition 1a

To prove the proposition the growth rates on the balanced growth path of capital and the economy are calculated for the economy without social cohesion and with social cohesion. For the latter the growth rate of social capital will be made as well.

Basic economy

\[ Y(t) = [K(t)]^\alpha [L(t)]^{1-\alpha} \]  \hspace{1cm} (A1)

With \( 0 < \alpha < 1 \) and \( K(t) = sY(t) \) and \( L(t) = nL(t) \) with \( n \geq 0 \).

Growth rate of Capital:

1) Substitute the production function into the expression for capital accumulation:
   \[ K'(t) = s[[K(t)]^\alpha [L(t)]^{1-\alpha}] \]

2) Dividing both sides by \( K(t) \) gives the growth rate of \( K(t) \):
   \[ g_k(t) = \frac{K'(t)}{K(t)} = s \left( \frac{L_h(t)}{K(t)} \right)^{1-\alpha} \]

3) Taking logs of both sides and differentiating with respect to time yields:
   \[ \frac{g_k(t)}{g_k(t)} = (1 - \alpha)[n - g_k(t)] \]

4) Solving for the growth rate of capital on a balanced growth path:
   \[ g_k^{BGP} = n \]

Growth rate of the Economy:

1) Take logs on both sides of the production function:
   \[ \ln Y(t) = \alpha \ln K(t) + (1 - \alpha) \ln L(t) \]

2) Differentiate with respect to time:
   \[ g_y(t) = \alpha g_k(t) + (1 - \alpha)g_L(t) \]

3) Simplify with relevant growth rates:
   \[ g_y(t) = \alpha g_k(t) + (1 - \alpha)n \]

4) If on a balanced growth path then \( g_y = g_k \):
   \[ g_y^{BGP} = n \]

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16 As we see the growth rates of the economy and capital are similar if there is a balanced growth path.
Social economy with constant returns to social cohesion

\[ Y(t) = [A(t)]^\gamma [K(t)]^\alpha \left[(1 - a_L)L(t)\right]^{1-a-\gamma} \]  
(A2)

\[ A(t) = [\alpha_L L(t)]^{\theta} A(t)^{\theta} \]  
(A3)

With \( \alpha + \gamma < 1 \). And \( \theta \geq 0, 1 > \rho > 0 \). \( K(t) = sY(t) \) and \( L(t) = nL(t), n \geq 0 \). Which is a simpler form of the extensive social sector formula defined in the text.

Growth rate of Capital and the Economy:

1) Substitute the production function into the expression for capital accumulation:

\[ K(t) = s([A(t)]^\gamma [K(t)]^\alpha \left[(1 - a_L)L(t)\right]^{1-a-\gamma}) \]

2) Dividing both sides by \( K_t \) gives the growth rate of \( K_t \):

\[ g_k(t) = \frac{K(t)}{K(t)} = s(1 - a_L)^{1-a} \left([A(t)]^\gamma [K(t)]^\alpha \left[(1 - a_L)L(t)\right]^{1-a-\gamma}\right) \]

3) Taking logs of both sides and differentiating with respect to time yields:

\[ \frac{g_k(t)}{K(t)} = \left[\gamma g_A(t) + (1 - \alpha - \gamma)n + (\alpha - 1)g_k(t)\right]^{17} \]

4) Solving for the growth rate of capital on a balanced growth path:

\[ g_y^{BGp} = g_k^{BGp} = \frac{\gamma g_A(t) + (1 - \alpha - \gamma)n}{(1 - \alpha)} \]

Growth rate of Social Capital

1) Dividing both sides of \( A(t) \) by \( A(t) \) gives the growth rate of \( A(t) \):

\[ g_A(t) = \frac{A(t)}{A(t)} = \alpha_L L(t)^\rho A(t)^{\theta-1} \]

2) Taking logs of both sides and differentiating with respect to time yields:

\[ \frac{g_A(t)}{g_A(t)} = \rho n + (\theta - 1)g_A \]

3) Solving for the growth rate of social capital on a balanced growth path:

\[ g_A^{BGp} = \frac{\rho n}{(1 - \theta)} \]

As \( \theta \) is smaller than 1, \( \frac{\rho n}{(1 - \theta)} \) is bigger than \( n \), so the economy with social cohesion has a higher growth rate in the long run. If \( g_A(t) \) is stable.

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\(^{17}\) At this point one can check whether there is a unique value to which \( g_k \) converges. This can be done by multiplying \([\gamma g_A(t) + (1 - \alpha - \gamma)n + (\alpha - 1)g_k(t)] \) by \( g_k \). This allows us to see that for high values of \( g_k \) the growth is negative and for small values positive. Implying that there is a unique value to which \( g_k \) converges. The entire analysis has unique values.
Proof of Proposition 1b

With an analogous procedure the growth rates of the economy with increasing returns to scale can easily be defined.

For the social economy the functions are:

\[ Y(t) = [A(t)]^{\gamma} [K(t)]^{\alpha} [L(t)]^{1-\alpha} \quad (A4) \]

\[ A(t) = [\alpha L(t)]^{\rho} A(t)^{\theta} \quad (A5) \]

With \( \alpha < 1, 0 < \gamma < 1 \). And \( \theta \geq 0, 1 > \rho > 0 \). This gives the growth rates

\[ g_{y}^{BGP} = g_{K}^{BGP} = \frac{\gamma \theta A(t)}{1-\alpha} + n \quad \text{and} \quad g_{A}^{BGP} = \frac{\rho n}{1-\theta} \]

So the growth of the economy is even higher.

Proof of Proposition 2

From (A1) we know that \( g_{y}^{BGP} = g_{K}^{BGP} = n \) if there are no foreign labourers. To prove the proposition the comparison must be made with an economy with foreign labourers.

Basic economy with foreign labourers

\[ Y(t) = [K(t)]^{\alpha} [L_{h}(t)]^{\beta} [L_{f}(t)]^{1-\alpha-\beta} \quad (A6) \]

With \( \alpha + \beta < 1 \). \( K(t) = sY(t), L_{h}(t) = n_{h}L_{h}(t), n_{h} \geq 0 \) and \( L_{f}(t) = n_{f}L_{f}(t), n_{f} \geq 0 \).

Growth rate of Capital and the Economy:

1) Substitute the production function into the expression for capital accumulation:

\[ K(t) = s \left( [K(t)]^{\alpha} [L_{h}(t)]^{\beta} [L_{f}(t)]^{1-\alpha-\beta} \right) \]

2) Dividing both sides by \( K(t) \) gives the growth rate of \( K(t) \):

\[ g_{k}(t) = \frac{K(t)}{K(t)} = s \left( \frac{1}{K(t)} \right)^{1-\alpha} \cdot L_{h}(t)^{\beta} L_{f}(t)^{1-\alpha-\beta} \]

3) Taking logs of both sides and differentiating with respect to time yields:

\[ \frac{g_{k}(t)}{g_{k}(t)} = (1-\alpha) [-g_{k}(t)] + \beta n_{h} + (1-\alpha - \beta) n_{f} \]

4) Solving for the growth rate of capital on a balanced growth path:

\[ g_{K}^{BGP} = g_{Y}^{BGP} = \frac{\beta n_{h} + (1-\alpha - \beta) n_{f}}{1-\alpha} \]

In this case \( g_{Y}^{BGP} = g_{K}^{BGP} \) is not necessarily higher than \( n \). But it is more diversified. If \( n_{h} = 0 \) the economy without foreign labourers cannot grow. This clearly demonstrates the benefits from diversity.
Proof of Proposition 3a

To prove the proposition the growth rates of capital and the economy are calculated for the economies with and without social cohesion. Here they will be made for the case of constant returns to scale.

Constant returns to scale and no foreign workers

Again from (A2) and (A3) we already know that the growth rates for a social economy without foreign labourers are \( g_{K}^{BGP} = g_{Y}^{BGP} = \frac{\gamma g_A(t) + (1 - \alpha - \gamma)n}{(1 - \alpha)} \) and \( g_{A}^{BGP} = \frac{m}{(1 - \theta)} \).

Constant returns to scale with foreign workers

\[
Y(t) = [A(t)]^\gamma [K(t)]^\alpha [(1 - a_{hl})L_{h}(t)]^\beta [(1 - a_{fl})L_{f}(t)]^{1 - \alpha - \beta - \gamma} \quad (A7)
\]
\[
A(t) = (1 - \delta)A_{t}^{\theta - \rho} [a_{hl}L_{h}(t)]^{p} [a_{fl}L_{f}(t)]^{(1 - \rho)} \quad (A8)
\]

With \( \alpha + \beta + \gamma < 1, \delta > 0, \theta \geq 0 \) and \( 1 > \rho > 0 \). \( K(t) = sY(t), L_{h}(t) = n_{h}L_{h}(t), n_{h} \geq 0 \) and \( L_{f}(t) = n_{f}L_{f}(t), n_{f} \geq 0 \).

Growth rate of Capital and the Economy:

1) Substitute the production function into the expression for capital accumulation:
\[
K(t) = s \left( [A(t)]^\gamma [K(t)]^\alpha [(1 - a_{hl})L_{h}(t)]^\beta [(1 - a_{fl})L_{f}(t)]^{1 - \alpha - \beta - \gamma} \right)
\]

2) Dividing both sides by \( K_t \) gives the growth rate of \( K_t \)
\[
g_{k}(t) = \frac{K(t)}{K(t)} = c_{k} \left[ \frac{1}{K(t)} \right]^{1 - \alpha} \cdot A(t)^{\gamma} \cdot L_{h}(t)^{\beta} \cdot L_{f}(t)^{1 - \alpha - \beta - \gamma}
\]

3) Taking logs of both sides and differentiating with respect to time yields:
\[
\frac{g_{k}(t)}{g_{k}(t)} = (1 - \alpha)[-g_{k}(t)] + \gamma g_{A}(t) + \beta n_{h} + (1 - \alpha - \beta - \gamma)n_{f}
\]

4) Solving for the growth rate of capital on a balanced growth path:
\[
g_{k}^{BGP} = g_{Y}^{BGP} = \frac{\gamma g_A(t) + \beta n_{h} + (1 - \alpha - \beta - \gamma)n_{f}}{(1 - \alpha)}
\]
Growth rate of Social Capital:

1) Dividing both sides of $\dot{A}(t)$ by $A(t)$ gives the growth rate of $A(t)$ with $c_A = (1 - \delta)a_h L^{\rho} a_{fl}^{\gamma(1-\rho)}$:

$$g_A(t) = \frac{\dot{A}(t)}{A(t)} = c_A A_t^{\theta-\varphi-1} L_h(t)^{\rho} L_f(t)^{(1-\rho)}$$

2) Taking logs of both sides and differentiating with respect to time yields:

$$\frac{g_A'(t)}{g_A(t)} = (\theta - \varphi - 1)g_A + \rho n_h + (1 - \rho)n_f$$

3) Solving for the growth rate of social capital on a balanced growth path:

$$g_A^{BGP} = \frac{\rho n_h + (1 - \rho)n_f}{(1 + \varphi - \theta)}$$

\[\frac{\gamma g_A(t) + (1-\alpha-\gamma)n}{(1-\alpha)}\] compared to \[\frac{\gamma(g_A(t)+\beta n_h + (1-\alpha-\beta-\gamma)n_f}{(1-\alpha)}\] holds no surprises. The economy with foreign workers is more diversified which makes growth more stable.

But $\frac{\rho n}{(1-\theta)}$ compared to $\frac{\rho n_h + (1-\rho)n_f}{(1+\varphi-\theta)}$ is less positive. Although growth in social cohesion can be achieved by more than one input now, both are needed for $\varphi$ to stay 0. The sector now has become dependent on foreign workers their participation.

**Proof of Proposition 3b**

With an analogous procedure the growth rates of both economies with increasing returns to scale can be easily defined.

*Increasing returns to scale and no foreign workers*

Again from (A4) and (A5) we already know that the growth rates for a social economy without foreign labourers are $g_f^{BGP} = g_k^{BGP} = \frac{\gamma g_A(t)}{(1-\alpha)} + n$ and $g_A^{BGP} = \frac{\rho n}{(1-\theta)}$.

*Increasing returns to scale with foreign workers*

Once again by analogous procedure we find that the growth rates of both economies with increasing returns to scale can be easily defined.

The functions are:

$$Y(t) = [A(t)]^{\gamma}[K(t)]^\alpha [(1 - a_h L_h(t))]^\beta [(1 - a_{fl} L_f(t))]^{1-\alpha-\beta}$$ (A9)

$$A(t) = (1 - \delta)A_t^{\theta-\varphi}[a_h L_h(t)]^\rho [a_{fl} L_f(t)]^{(1-\rho)}$$ (A10)
With $\alpha + \beta < 1, 0 < \gamma < 1$. And $\delta > 0$, $\theta \geq 0$, $1 > \rho > 0$. $K(t) = sY(t)$, $L_h(t) = n_hL_h(t)$, $n_h \geq 0$ and $L_f(t) = n_fL_f(t)$, $n_f \geq 0$.

Which gives $g_y^{BGP} = g_k^{BGP} = \frac{\gamma g_A(t) + \theta n_h(1-\alpha-\beta)n_f}{(1-\alpha)}$ and $g_A^{BGP} = \frac{\rho n_h(1-\rho)n_f}{(1+\varphi-\theta)}$.

Comparing the growth rates of the economies with and without foreign labourers reveals the same story as in proposition 3a. Only now the effect of $g_A$ serves as an extra boost. So $g_A$ is once again more vulnerable to neglect.

**Proof of Proposition 4**

To prove the proposition the growth rate of social capital is calculated for the economy with social cohesion when foreign labourers so not participate in this sector. The function is:

$$A'(t) = (1-\delta)[a_{hl}L_h(t)]^\rho A_t^{\theta-\varphi}$$  \hspace{1cm} (A11)

With $\delta > 0$, $\theta \geq 0$, $1 > \rho > 0$. Where $[a_{fl}L_f(t)]^{(1-\rho)}$ has dropped out completely.

Growth rate of Social Capital:

1) Dividing both sides of $A'(t)$ by $A(t)$ gives the growth rate of $A(t)$ with $c_A = (1-\delta)a_{hl}^\rho$:

$$g_A(t) = \frac{A'(t)}{A(t)} = c_A L_h(t)^\rho A_t^{\theta-\varphi-1}$$

2) Taking logs of both sides and differentiating with respect to time yields:

$$\frac{g_A(t)}{g_A(t)} = \rho n_h + (\theta - \varphi - 1)g_A$$

This line is always negative if $n_h$ is 0. If $n_h > 0$ the growth is negative for large values of $g_A$. For small values of $g_A$ there is a small possibility that the growth is positive. But for that to be true $\rho$ needs to be larger than $(\theta - \varphi - 1)$, which implies both $\rho$ and $\theta$ need to be higher than 0,5. This case is not that likely so we can assume that $g_A$ is never positive. For completeness of the analysis as convergence mathematically can be achieved the unique value is presented:

3) Solving for the growth rate of social capital:

$$g_A^{BGP} = \frac{\rho n_h}{(1+\varphi-\theta)}$$
By similar steps the growth rate of the social capital without domestic workers contributing to social capital can be defined as:

\[ g^{BG}_A = \frac{(1 - \rho) n_f}{(1 + \varphi - \theta)} \]

Although the growth of the growth is positive, as the growth is negative we can have a steady negative growth.

**Proof of Proposition 5**

In this case we have two social sectors working next to each other.

\[ Y(t) = [A_h(t) - A_f(t)]^\gamma [K(t)]^\alpha [(1 - a_{hl}) L_h(t)]^\beta [(1 - a_{lf}) L_f(t)]^{1 - \alpha - \beta - \gamma} \] (A13)

\[ A_h(t) = A_h(t)\theta [a_{hl} L_h(t)]^\rho \] (A14)

\[ A_f(t) = A_f(t)\theta [a_{lf} L_f(t)]^\rho \] (A15)

With \( \alpha + \beta + \gamma < 1, \theta \geq 0, 1 > \rho > 0 \). \( K(t) = sY(t), L_h(t) = n_h L_h(t), n_h \geq 0 \) and \( L_f(t) = n_f L_f(t), n_f \geq 0 \).

Growth rate of Capital and the Economy:

1) Substitute the production function into the expression for capital accumulation:

\[ K(t) = s \left([A_h(t) - A_f(t)]^\gamma [K(t)]^\alpha [(1 - a_{hl}) L_h(t)]^\beta [(1 - a_{lf}) L_f(t)]^{1 - \alpha - \beta - \gamma}\right) \]

\[ K(t) = s \cdot (1 - a_{hl})^\beta (1 - a_{lf})^{1 - \alpha - \beta - \gamma} [A_h(t) - A_f(t)]^\gamma K(t)^\alpha L_h(t)^\beta L_f(t)^{1 - \alpha - \beta - \gamma} \]

2) Dividing both sides by \( K_t \) gives the growth rate of \( K_t \)

with \( c_k = s \cdot (1 - a_{hl})^\beta (1 - a_{lf})^{1 - \alpha - \beta - \gamma} \):

\[ g_k(t) = \frac{K(t)}{K(t)} = c_k \left[ \frac{1}{K(t)} \right]^{1 - \alpha} \cdot [A_h(t) - A_f(t)]^\gamma \cdot L_h(t)^\beta \cdot L_f(t)^{1 - \alpha - \beta - \gamma} \]

3) Taking logs of both sides and differentiating with respect to time yields:

\[ \frac{g_k'(t)}{g_k(t)} = (1 - \alpha) [-g_k(t)] + \gamma \left(g_{hA}(t) - g_{fA}(t)\right) + \beta n_h + (1 - \alpha - \beta - \gamma) n_f \]

4) Solving for the growth rate of capital on a balanced growth path:

\[ g^{BG}_f = g^{BG}_K = \frac{\gamma \left(g_{hA}(t) - g_{fA}(t)\right) + \beta n_h + (1 - \alpha - \beta - \gamma) n_f}{(1 - \alpha)} \]
Growth rate of Domestic Social Capital

1) Dividing both sides of $A(t)$ by $A(t)$ gives the growth rate of $A(t)$

$$g_{hA}(t) = \frac{A(t)}{A(t)} = A_h(t)^{\theta-1} [a_{hl}L_h(t)]^\rho$$

2) Taking logs of both sides and differentiating with respect to time yields:

$$\frac{g_{hA}(t)}{g_{hA}(t)} = (\theta - 1) g_{hA} + \rho n_h$$

3) Solving for the growth rate of social capital on a balanced growth path:

$$g_{hA}^{BGP} = \frac{\rho n_h}{(1 - \theta)}$$

Growth rate of Foreign Social Capital

By analogy:

$$g_{fA}^{BGP} = \frac{\rho n_f}{(1 - \theta)}$$

As the growth rates of both social capitals are the same and positive the question whether they cancel out depends on the growth of both labour groups and their size. If the domestic group grows faster its effect will be stronger on the economy and vice versa.

**Proof of Proposition 6: Diseconomies to scale.**

The production function now becomes:

$$Y(t) = [K(t)]^\alpha [L(t)]^{1-\alpha-\gamma} \quad (A12)$$

With $\alpha - \gamma < 1$.

Growth rate of Capital and the Economy

1) Substitute the production function into the expression for capital accumulation:

$$K(t) = s([K(t)]^\alpha [L(t)]^{1-\alpha-\gamma})$$

2) Dividing both sides by $K(t)$ gives the growth rate of $K(t)$:

$$g_k(t) = \frac{K(t)}{K(t)} = sK(t) \gamma K(t) = \gamma L(t)^{1-\alpha-\gamma}$$

3) Taking logs of both sides and differentiating with respect to time yields:

$$\frac{g_k(t)}{g_k(t)} = (\alpha - 1) g_k(t) + (1 - \alpha - \gamma) n$$

4) Solving for the growth rate of capital on a balanced growth path:

$$g_k^{BGP} = g_k^{BGP} = \frac{(1 - \alpha - \gamma) n}{(1 - \alpha)}$$