Real Time Parking

Victor van der Pols
Erasmus University Rotterdam
17-03-2011
Real Time Parking

Master Thesis on ‘Fair’ Parking Tariffs

Author
Victor Pieter van der Pols
Student number: 279571
279571vp@student.eur.nl
06-10524023

Thesis Supervisor
Dr. V. A. Karamychev (Erasmus School of Economics, Erasmus University Rotterdam)

Second Reader
Dr. M. Bøg (Erasmus School of Economics, Erasmus University Rotterdam)

Master Specialisation
International Economics

March 2011
This thesis is the concluding part of my International Economics Master Specialisation at the Erasmus University Rotterdam. In this thesis ‘Real Time Parking’ is studied, especially its effect on consumers. Data from a parking garage in which a test with ‘Real Time Parking’ was held have I obtained via Spark, a consulting company in the field of Parking Policy and Parking Exploitation. In Holland, Real Time Parking is a hot topic for both public and private parking operators, but research on this kind of payment scheme is not yet available. The effects of the introduction of this ‘fair’ parking tariff on the composition of the consumer population are studied in this thesis.

At this coming to an end of my thesis, I would like to thank my supervisor, mister Karamychev. Together we have struggled to get the data workable and get the results into perspective. Special thanks go to Rob Ebbing and his partners at Spark for giving me access to the data and maintaining interest after my work for them was finished. I would like to thank the following reviewers for their useful advice: Leo van der Pols, Rieneke van der Pols, Robin van der Pols and Rob van der Wal. Finally I would like to thank mister Bøg for his role in this thesis process.

Victor van der Pols
March 2011
Preface ................................................................................................................................................................ - 3 -
Executive Summary ................................................................................................................................................. - 5 -
Chapter 1 Introduction ............................................................................................................................................ - 7 -
Chapter 2 Parking .................................................................................................................................................. - 11 -
  Parking and parking behavior ........................................................................................................................ - 11 -
  2.1 Parking Market ........................................................................................................................................ - 11 -
  2.1.1 parking purposes ............................................................................................................................. - 13 -
  2.1.2 private and public parking operators ........................................................................................... - 14 -
  2.2 decision making factors ....................................................................................................................... - 14 -
  2.3 Parking Policy ...................................................................................................................................... - 16 -
Chapter 3 Data ....................................................................................................................................................... - 18 -
  Business case on fairness in parking tariffs ............................................................................................ - 18 -
  3.1 Data ....................................................................................................................................................... - 18 -
  3.2 Descriptive Statistics ........................................................................................................................... - 19 -
Chapter 4 Analysis ................................................................................................................................................. - 23 -
  Business case data tested .......................................................................................................................... - 23 -
  4.1 Standard Distributions ......................................................................................................................... - 23 -
  4.2 Maximum Likelihood Estimation ......................................................................................................... - 25 -
  4.3 Results ................................................................................................................................................... - 29 -
  4.3.1 Time series: total ............................................................................................................................... - 29 -
  4.3.2 Time series: Wednesday .................................................................................................................. - 30 -
  4.3.3 Time Series: Saturday ..................................................................................................................... - 31 -
Conclusion ......................................................................................................................................................... - 32 -
References ......................................................................................................................................................... - 34 -
This thesis analyzes consumer response to a change in a parking pricing scheme. In a garage in Amsterdam a Real-Time Parking test was run in which consumers paid per 3 minutes instead of per hour. From the used standard distributions it becomes clear that the consumer population existed of at least 2 groups. The focus in the analysis is on the population composition and the group specific variables. For two out of three time series significance can be found for changes in one of the variables under investigation. Concluding the following can be stated: The introduction of the new pricing scheme made consumers change their behavior.
CHAPTER 1
INTRODUCTION

Holland has over 16 million inhabitants who altogether own 7.6 million cars. An average car is used for one hour a day and spends the other twenty three hours in a parking place. Sometimes this parking is free, e.g. on one’s own property. But often cars spend time in paid parking. This is a necessity because if parking was free everywhere there would be insufficient parking space on some locations. Based on the willingness to pay allocation takes place.

There are several different characteristics of parking. There is on-street and off-street parking resulting in a possible trade-off between walk-time and surveillance. For both these possibilities different payment schemes are observed. There is paying per time measurement and per factual time. For the first scheme payment can happen in advance or afterwards. Furthermore, there are free parking tickets, seasonal tickets and parking permits.

This thesis analyzes consumer response to a change in the parking payment scheme of a parking garage. The ‘old’ payment scheme was to pay afterwards per time measurement of one hour. This is the normal way of paying in off-street parking in Holland. The introduced scheme is called RTP, Real Time Parking. In this new situation consumers still pay afterwards, but now per 3 minutes instead of per hour.

The research question is: What are the main effects of the change on consumer behavior?

The data on which the analysis is based comes from a city centre garage in Amsterdam. The Stopera garage is located underneath the city hall and the opera house. Based on the location characteristics some filters are used on the raw data. Length of theatre plays e.g. must not influence the outcomes of the analysis so parking entries after three o’clock in the afternoon are filtered out. The fact that the garage is located in the centre of Amsterdam
implies that on car-free days here, there will be almost no activity at all in the garage. These dates are therefore omitted from the data. Also omitted are all season tickets and incomplete or erroneous transactions. In all of these cases, the consumer does not control or does not have an incentive to control his parking behavior. The testing period for RTP was April to September 2008. The total dataset contains 30 months of parking data: 27 months from before April 2008 plus the first 3 months of the testing period.

Apart from the location characteristics other factors influence the research as well. Alongside the introduction of RTP an increase in price was introduced. This increase was partly to cover expected losses but mostly for consumer convenience reasons. The price per hour used to be € 3,80 and was increased to € 4,- so that consumers had to pay € 0,20 per 3 minute time interval. The results on e.g. total numbers of consumers will suffer from mixed effects. Some of the consumers may benefit from the new pricing scheme more than they ‘lose’ on the higher price whereas for other consumers the higher price outruns the RTP benefit. Furthermore there is the effect that trends in consumer behavior cannot be accounted for completely resulting in an even larger deviation from a ceteris paribus situation.

Therefore the focus of this thesis is not on total consumer numbers or average parking times but on the composition of the total population of consumers. The setup of the testing situation is comparable to situations in Renewal Theory: All objects start, but the moment of exit differs. Based on this theory the Erlang distribution is used to determine probability distributions. The graphical comparison between the data and different Erlang distributions points out that the parking population consists of at least two groups of consumers. The Maximum Likelihood Estimation is used for parameter estimation. With the optimal parameters, significance of possible changes can be determined. This is done for the totaled time series as well as for a specific weekday and weekend day.

The results of the analysis lead to different conclusions. For the totaled time series no significance is observed for both population composition and the group specific variables. For the Wednesdays, very large confidence intervals are found for the population composition and the variable of group 1. For group 2 however, significance is found for the
increase in the group specific variable. This increase could stem from the fact that consumers have more control on the tariff they have to pay in the new situation. For parking in the garage in the old situation consumers had a decision moment once an hour, while in the new situation they can decide every three minutes to stay or leave, yielding direct effect on their tariff. For Saturdays significance is found for the population composition variable which denotes the share of consumers that come for one parking purpose. Apparently on Saturdays relatively more consumers come to the garage for one purpose compared to the old situation. The reason for this could be consumers that switch from a two purpose parking stay to a one purpose stay. But also it can be that the garage has become more interesting for ‘new’ consumers.

The rest of the thesis is organized as follows: in Chapter 2 there is a literature study on parking, Chapter 3 gives background and descriptive statistics on the dataset, in Chapter 4 the actual analysis takes place, both analytically and empirically. After this hard content, an interpretation on the results is given in the Conclusion.
CHAPTER 2
PARKING

PARKING AND PARKING BEHAVIOR

The specific market in which this thesis investigates the effects of pricing on performance is the parking branch. Parking is becoming increasingly important in decisions on business, residence and city centre development. The constantly growing amount of cars leads to a growth in the need of parking space of the same proportions, which will not stop in the coming years according to Dargay et al. (2007). For public and private building decisions parking plays a increasingly important role. Parking is a perishable product. When time goes by, an empty parking space can never be sold afterwards. This perishability is an important characteristic of the market.

Recently a Dutch research came up with the top ten nuisances involving parking\(^1\). Ranked second was the price of parking, with over 55% of the surveyed people naming this nuisance. This is not very remarkable for a Dutch research, taking into account the worldwide parking tariffs\(^2\). A global City Business District parking survey found that the four largest cities in Holland are in the top twenty of daily parking rates. Amsterdam was topping this list with $70 per day while the second most expensive district (London – City) had a rate of $56 per day. Apparently Dutch cities are built so densely that prices are as high as in the world’s largest cities.

2.1 PARKING MARKET

The specific market this thesis deals with is the parking market. A parking lot is also a perishable good. If it is used by a car, no other car can use it. But also, if it is not used by a car for an entire day, that day can never be sold afterwards. Hensher (2006), and Bianco et

---

\(^1\) The Great Parking-research, December 2008; Blaauw Research ordered by Park-line.

\(^2\) Collier International; Parking Rates, Global CBD Parking Rate Survey 2009
al. (1997) conclude that the price part of parking is the most influential variable on whether or not consumers, drivers, make use of public transport or not. In the case of Hensher a test was undertaken with different pricing zones and parking curfews. The curfews did not make drivers shift their behavior as the tariff change did. Bianco found that the parking tariff is of greater influence on public transport usage than the improvements in public transport itself.

The parking ‘market’ can be divided in several ways. A lot of different characteristics can be used to group parking locations. There is on-street and off-street parking, paid and free parking, parking in underground garages and parking in garage buildings. Payment can be in advance, afterwards or by season ticket. Parking operators can be private companies but can also be city departments. All of these characteristics can differ between parking locations having their effect on the choice of consumers.

Scientific studies have investigated the role of different factors on the decision to park in a given parking space or not. Thompson (1998) uses the economic assumption that consumers seek to maximize their utility. After implementing several models based on the simulated situation of a city business district, this maximizing behavior is found for drivers too. A driver relies on the utility he expects to get from using a given parking place. The driver aims to have minimal costs in several ways (Figure 1). Between drivers there are differences in the value attached to the different dimensions. A person parking at his residence will for example value costs in the ‘native’ direction higher because the car will be parked for a larger amount of time. A driver who is travelling to a meeting will value the ‘access’ dimension higher because he or she has to be on time.

![Figure 1: Three generalized cost dimensions](image-url)
2.1.1 PARKING PURPOSES

The ‘cost dimensions’ idea of Thompson however is quite general. Other studies provide a more focused view on the parking market. Kelly et al. (2006) for example distinguishes two groups of drivers looking for parking space: business and non-business drivers. Kelly et al. find empirical evidence that drivers with a business purpose behave more price inelastic than drivers with a non-business purpose. Kelly et al. conclude that this is purely based on the absolute price level, they suggest further research on price response when pricing methods are altered.

Marsden (2006) also comes up with the conclusion that commuters (business parkers) are less likely to shift by parking policy. The behavior of leisure and shopping parkers is more easy to target by policy. He therefore pleads for an increased role of parking planning in development decisions concerning shopping malls and city centers. Lastly he states that drivers value out-of-vehicle costs more than in-vehicle costs. So search-time and travel-time are less influential on the decision to park somewhere than ticket fare and walking time.

Hensher and King (2001) come to the same conclusion. Subject of study is an empirical test in the central business district of Sydney. Hensher and King investigate the possible shift of parking behavior due to a further differentiation of the available parking spaces. This focuses on the group Marsden calls ‘commuters’, for which Marsden finds relatively inelastic behavior. Hensher and King however conclude their research stating: “In general there is high sensitivity to parking prices, far higher than one finds for in-vehicle cost and even travel time in mode choice”. Empirical base for this statement is found in the fact that only 3% of the shifted parking resides from parking space availability, 97% of the shift can be deducted to parking prices.

Bain (2002) comes to the same conclusion as Marsden. Car owners have low elasticity’s concerning parking prices, implying they will remain travelling to their destination by car, (almost) regardless of the costs. They have a clear trade-off between convenience and price. Golias et al. (2002) come in the same year with their paper on parking choice sensitivity. In the choice for on- or off-street parking, price again is the key factor. Driver or trip
characteristics however do not have a significant influence according to Golias. This is in contradiction with the articles mentioned earlier, which claim that the purpose of parking is an important factor.

2.1.2 PRIVATE AND PUBLIC PARKING OPERATORS

Not only the purposes of drivers influence the choice of parking location. Differences in the field of operators are to be taken into account as well. Research in this field is about, whether public or private operators yield differences in service and availability of parking space, but also, in which way revenues are influenced.

Matsoukis (1995) studied the effects of a partial privatization of the parking market in Patra (Greece). He finds that this privatization yields more than just an increase in revenue. Because of the increasing focus at payments within the semi-public company, illegal parking is detected more often, leading to an increase in fines. When the number of fines increases the number of people that will try to avoid them will increase to, yielding less illegal parking. Parking duration differs between parking zones in the city, following the different tariffs that are used. All in all, Matsoukis (1995) states that this way of operating parking can be positive in almost any given situation. This is because public goals can be achieved as a side effect of the privately operated parking places.

Tsai and Chu (2006) find the same positive side effects for Taiwan. Privatizing parking operators leads to an increase in total welfare. This does not need implying governmental regulations, but stems from the general maximizing profit idea. If consumers are satisfied, their welfare can be increased, even though the prices may be higher.

2.2 DECISION MAKING FACTORS

Empirical studies on parking often come from parking operators, public departments or scientific researchers. A large study was performed by the Transit Cooperative Research Program, TCRP (1998). This study focuses on finding triggers to get people out of their cars, and into Public Transport. The study targets at working people owning a drivers license. The conclusion is, that parking prices are by far the largest effector of travel behavior.
Public transport is also the topic in the paper by Acierno et al. (2006). Acierno et al. conclude their article in the same direction, pricing of parking to be the key factor in people’s choice between public transport and travelling by car. Because people have a preference for the private car system, policy makers will have to imply a tariff to prevent urban traffic to get congested. If traffic congestion is a concern for a government Glazer (1992) agrees with Acierno that priced parking can play a role. He goes even further, implying priced parking will be beneficial to residents, drivers and governments in case of congestion.

CONSUMER WEIGHS TIME AND MONEY

Okada et al. (2004) conclude that time and money are both perishable goods to consumers. Both are often not used optimal because people do not always have their opportunity costs at hand. Time however is treated differently in different situations. If one has to increase his or her time expenditure, the value of time is ‘decreased’ to justify the use of it. Whereas, if time can be ‘saved’ the value attached to this time is increased. For money this phenomenon is not observed to the same extend, because the value of money cannot be as easily adjusted.

Taken into account search times and parking costs, a classroom game from Guthrie et al. (2005) gives insight in drivers preferences. With enough price differentiation and awareness of factors that are valued, most drivers act rationally. In the discussed cases, the outcome of this experiment is that search time is minimized, revenues optimized and consumer surplus is maximized if prices are differentiated in the correct way.

SUPPLY OF PARKING

Benenson et al. (2008) claim that the increase of the amount of parking places in case of a shortage has only a small effect on drivers search time and walking distance. They form up an agent-based model of parking. With this model they can describe driver behavior dependent on the factors: search time, walking distance from destination and parking costs. In a real-life test of the system in Tel-Aviv they find that in case of parking place shortage, an increase in the amount of places that not fully covers the shortage, the effect is little.
2.3 PARKING POLICY

Gagliano (2009) writes that governments have to determine which target they have with their parking policy. Tariffs can be implemented to prevent urban traffic from congestion, getting the governmental balance straight with some extra income or from environmental point of view. Button (2006) finds that policies very often are not in line with classic economical ideals. This is because of the institutionalizing of the decision making process of policy makers. Formal laws are consulted but also the expected actual outcomes of policies-to-be-implemented. This split of direction leads to a more adjusted economic approach, just to capture all the possible outcomes that can emerge.

In Holland there is currently a debate on the way prices of parking are set. In the parliament questions are asked at the responsible Minister\(^3\). Local governments of cities are responsible for the parking prices, the central government has no direct role in this. However, it is clear that the members of parliament are aware of the nuisance consumer experience concerning parking.

The direct cause of the recent interest of politicians is a research by Kassa that was broadcasted on the 9th of October 2010. Kassa is a television program about consumer rights. The research deals with the issue that prices in off-street garages are, in most cases, rounded off upwards. Part of the research stems from a parking research by the national parking platform of Holland, Vexpan\(^4\). In this research, consumers were asked several questions about parking payments as well. A large majority of the respondents, 80 %, preferred a method of paying per minute over paying per hour. These answers were including a 20% raise of the price if paying per minute was to be implemented. Furthermore, people favored, as before, walking longer over paying more for a parking place.

---

\(^3\) Dijksma C.S.; motion nr 79 on 2011 budget of the Ministry of Economic Affairs; 24 November 2010;
\(^4\) Vexpan; Vexpan Parkeeronderzoek 2010
3 DATA

BUSINESS CASE ON FAIRNESS IN PARKING TARIFFS
CHAPTER 3
DATA

BUSINESS CASE ON FAIRNESS IN PARKING TARIFFS

The data used in the empirical part of this thesis comes from a parking garage which is in maintenance by the city of Amsterdam. The parking garage is located in the centre of Amsterdam, beneath the city hall and the adjacent theatre (Stopera). The supervisor of the garage performed a test on Real-Time Parking (RTP). Here follows the outline of how the data of this test is made up.

3.1 DATA

The data is from an off-street parking garage. In this garage, the normal way of pricing was a fixed price per hour or part of an hour. Paying the tariff takes place at the end of the consumed parking time. Real-Time Parking was applied for the period of April to September 2008. During this period, consumers still paid afterwards, but now they paid for 3 minutes or part of that. The difference between this test period and the normal way was therefore that consumers went from a maximum of 59 minutes ‘overpayment’ to a maximum of 2 minutes. The most clear example to show this is a parker which consumes 61 minutes of parking. In the old situation he pays for 120 minutes (paying per hour or part of an hour), in the new situation he pays for 63 minutes (paying per 3 minutes or part of 3 minutes).

In Figure 2 the ‘overpayment’-difference is the sum of the triangles above and below the blue line. The reason for the blue line to be above the red one sometimes is that the price per hour is lower than the price per 3 minutes times 20. The price in the old situation was € 3,80 per hour, in the new situation this is € 0,20 per 3 minutes which is € 4,00 per hour. This price increase was implemented because of easiness of checkout payment. The overpayment in this example is 5,3 % of total revenues. This is with an increased price, different pricing scheme but constant numbers of customers.
Figure 2: Overpayment, old and new situation

3.2 DESCRIPTIVE STATISTICS

The data from the first 3 months of the testing period is compared to a baseline out of old data. In total data on 27 months from the old situation and 3 months from the test is used. This data is filtered for special occasions and incomplete transactions which could have influenced the results. For example, the garage is situated in the centre of a large city, which occasionally has car-free days. Also, specific time intervals are taken from the data based on the user profile of the consumers of the garage. Near the garage is a theatre which causes a significant peak in the number of entering parkers between 17h and 19h as can be seen in Figure 3.
This group of consumers are filtered out of the data because of the influence fixed durations of theatre plays would have on the parking duration of this group. Also the days that there was an afternoon show at the theatre are removed. Enterers between 8h and 15h are therefore taken into account, the other data is filtered out. Totals of the old and the new situation are compared as well as a specific weekday (Wednesday) and a day in the weekend (Saturday). In the analysis only complete transactions are used. This means, transactions where the ticket received when entering the garage is used to pay and leave the garage. Doing this, parkers with season tickets, free-leave tickets and erroneous transactions are omitted. This group would be a lot less sensitive to the new way of pricing and would therefore influence the results.

In total, the data used contains 6 time series of parked time in a parking garage in the centre of Amsterdam. The data comes from 30 months: 27 months before April 2008 and 3 months in 2008 after implementing Real-Time Parking. For these two periods there are series for totaled numbers unfiltered and Wednesdays and Saturdays numbers including filter. Used filters: entrance time between 8:00-15:00h, only completed transactions (entering, paying and leaving on same ticket), ‘abnormal’ days excluded (car-free days, special events near garage), maximal parked time 10 hours (600 minutes). Time series are in absolute numbers of departures per 10 minutes of parked time.

To get an idea if any changes at all occur due to the change in pricing scheme a basic analysis is performed. Average parking times are compared, as well as, average numbers of consumers and payment timing. The data shows that the average parking time of users stays constant in the old and the new situation. In the old situation (paying per hour) this is 190 minutes, in the new one (RTP) 188 minutes parking time is the average. A difference of 1%, for which however, no significance can be determined. The average number of consumers has increased (for the chosen time intervals and days) from 370 to 420 on Wednesdays and from 330 to 450 on Saturdays. This increase has the opposite direction of the decrease in average parking time. As a measure of payment timing, the minimum and maximum amounts of overpayment are measured. In the old situation if a consumer pays immediately after 60 minutes of parking, overpayment is at a maximum, while paying right before this
boundary of 60 minutes yields a minimum of overpayment. In the old situation there were 8-9\% more payments in the last 10 minutes before a full parking hour than in the first 10 minutes after a full parking hour. In the new situation the difference still exists, but is now 1-2\%.

The impossibility to check significance for some of the numbers stems from the fact that the data is obtained via a data miner. The raw data from the parking machines was made accessible, and some preliminary findings came with the accessible data.

Because significance cannot be determined the above findings are inconclusive. Moreover, the opposite effects of the ‘fair’ pricing scheme and the higher price charged makes that no conclusions on numbers of consumers can be drawn. The composition of the population of consumers will therefore be the focus of the further analysis.
4

ANALYSIS

BUSINESS CASE DATA TESTED
4.1 STANDARD DISTRIBUTIONS

The set-up of the test has the characteristics of a failure-rate distribution. Cox (1962) states that failure-rate distributions are used to determine failure probability for a given moment, based on the condition that the ‘product’ has not failed at that time. The probability of a consumer to leave the parking, can be compared to the probability of a product to fail.

Standard failure rate distributions are the exponential, Erlang, Gamma and Weibull distribution. The exponential distribution has a constant failure rate, which results in a downwards sloping graph approaching zero. For the exact shape, check Figure 4 in which the red line displays the exponential distribution.

The formula however does not yield an outcome that matches the data. This can be seen from the distribution of the data, the blue line, and different failure rate distributions that are plotted in the same graph. The three used distributions are all based on the Erlang probability density function
\[
f(x; k; \lambda) = \frac{\lambda^k x^{k-1} e^{-\lambda x}}{(k-1)!}
\]
The difference between the three is the value of the parameter ‘k’. This parameter is a so called shape
parameter. It changes the shape of a graph rather than shifting or re-sizing it as do location and scale parameters.

Best results are obtained using the Erlang PDF (k=2). The Erlang distribution was introduced for the first time in telephony. The telephone engineer Erlang came up with his formula to increase predictability of phone traffic. Through this ‘traffic engineering’ companies were better capable to adjust supply and demand, preventing calls from queuing up. The idea behind this distribution is found in the assumption of statistic equilibrium. Jensen (1948) states in his commentary on the works of Erlang that: "The processes Erlang uses satisfy the assumption: The probability that an arrival will occur during a given time interval...is asymptotically proportional to the length of the interval with a factor of proportionality...that is independent of time".

The Erlang distribution gives the distribution of the waiting times until the k'th Poisson event. The probability density function for Erlang is: 

$$f(x; k, \lambda) = \frac{\lambda^k x^{k-1} e^{-\lambda x}}{(k-1)!}$$

in which \( \lambda \) is the function specific variable, k is the number of events waited for and x is the time variable. The parameter k is always an integer in the Erlang distribution, but can range from zero up to infinity.

The function is continuous, whereas the data is discrete. Because the Maximum Likelihood Estimation method is the most suitable method to determine significance, the Erlang formula used is transformed into a discrete function. This will lead to a slightly different formula, but this will not influence the outcome of the analysis.

Another deviation is made based on Figure 4. The graphs of k=2 and the not completely comparable to the one from the actual data, so there is probably some deviation from k=2. To see whether this is the case the distribution is recomputed both analytically and empirically consisting of Erlang with k=1 with probability \( \alpha \), and Erlang with k=2 with probability \((1- \alpha)\). This yields a constructed PDF with two user groups: one group in which people behave like they have one purpose and another group in which people behave like they have two purposes.
The Erlang distributions used are pointed out on the individual level. Assume an individual driver. With probability $\alpha \in (0, 1)$, his departure time follows the Erlang distribution with parameter $k = 1$ (exponential distribution):

$$f_1(n|\lambda_1) = \Pr(\text{He departs in time interval } n) = \lambda_1 e^{-\lambda_1 n} \left(1 - e^{-\lambda_1}ight)$$

The variable ‘$n$’ replaces the variable ‘$x$’ from the original Erlang PDF. The second part of the formula, the fraction, is a correction term to make sure total probability is 1.

With probability $(1 - \alpha)$ his departure time follows the Erlang distribution with parameter $k = 2$:

$$f_2(n|\lambda_2) = \Pr(\text{He departs in time interval } n) = \lambda_2^2 e^{-\lambda_2 n} \left(1 - e^{-\lambda_2}ight)^2 = n e^{-\lambda_2 n} \left(1 - e^{-\lambda_2}ight)^2$$

Again, the fraction term is a correction term.

### 4.2 MAXIMUM LIKELIHOOD ESTIMATION

Myung (2003) states that the Maximum Likelihood Estimation method (MLE) is a suitable method of parameter estimation for probability density functions. Through the likelihood function, values for parameters of the PDF’s can be found that are optimal when comparing to the distribution of the sample data.

The likelihood function for an individual is constructed from the two Erlang PDF’s created before:

$$f(n|\alpha, \lambda_1, \lambda_2) = \alpha f_1(n|\lambda_1) + (1 - \alpha) f_2(n|\lambda_2) = \alpha e^{-\lambda_1 n} \left(1 - e^{-\lambda_1}ight) + (1 - \alpha) n e^{-\lambda_2 n} \left(1 - e^{-\lambda_2}ight)^2.$$  

The likelihood function for the total population then is:

$$L = \prod_{n=0}^{\infty} \left(\alpha e^{-\lambda_1 n} \left(1 - e^{-\lambda_1}\right) + (1 - \alpha) n e^{-\lambda_2 n} \left(1 - e^{-\lambda_2}\right)^2 \right)^{k_n},$$
where \( x_n \) is the total number of departures in time slot \( n \). As there are only a finite number of the slots, the following is approximated:

\[
L = \prod_{n=0}^{N} \left( \alpha e^{-\lambda_n} (1 - e^{-\lambda_n}) + (1 - \alpha) n e^{-\lambda_2(n+1)} \left( e^{\lambda_2} - 1 \right)^2 \right)^{x_n}.
\]

Maximum Likelihood Estimation makes use of the log-likelihood function:

\[
F(\alpha, \lambda_1, \lambda_2) = \ln L = \sum_{n=0}^{N} x_n \ln \left( \alpha e^{-\lambda_n} (1 - e^{-\lambda_n}) + (1 - \alpha) n e^{-\lambda_2(n+1)} \left( e^{\lambda_2} - 1 \right)^2 \right) = \sum_{n=0}^{N} x_n \ln f_n.
\]

The formula \( f_n = \left( \alpha e^{-\lambda_n} (1 - e^{-\lambda_n}) + (1 - \alpha) n e^{-\lambda_2(n+1)} \left( e^{\lambda_2} - 1 \right)^2 \right) \) is taken outside the log-likelihood function. This is done for analytical convenience, and reduces the changes on miscalculations. Derivatives are taken from this function \( f_n \):

\[
f_{n,\alpha} = \frac{\partial}{\partial \alpha} f_n = e^{-\lambda_n} (1 - e^{-\lambda_n}) - n e^{-\lambda_2(n+1)} \left( e^{\lambda_2} - 1 \right)^2 \\
f_{n,\lambda_1} = \frac{\partial}{\partial \lambda_1} f_n = \alpha \left( (n+1) e^{-\lambda_1} - n \right) e^{-\lambda_n} \\
f_{n,\lambda_2} = \frac{\partial}{\partial \lambda_2} f_n = (1 - \alpha) n \left( (n+1) - (n-1) e^{\lambda_2} \right) e^{-\lambda_2(n+1)} \left( e^{\lambda_2} - 1 \right)
\]

Computing the first-order derivatives of the log-likelihood function yields:

\[
\frac{\partial F}{\partial \alpha} = \frac{\partial}{\partial \alpha} \sum_{n=0}^{N} x_n \ln f_n = \sum_{n=0}^{N} x_n \frac{\partial F}{\partial f_n} f_{n,\alpha} \\
\frac{\partial F}{\partial \lambda_1} = \sum_{n=0}^{N} x_n f_n \frac{\partial f_n}{\partial \lambda_1} \\
\frac{\partial F}{\partial \lambda_2} = \sum_{n=0}^{N} x_n f_n \frac{\partial f_n}{\partial \lambda_2}
\]

All of these derivatives should yield an outcome of zero when computed with the actual data. Based on these values, the second-order derivatives can be taken. In the second-order derivatives of the log-likelihood function the function \( f_n \) is put back in:
\[ f_{n,\alpha^2} \equiv \frac{\partial^2}{\partial \alpha^2} f_n = 0 \]

\[ f_{n,\alpha \lambda_1} = \frac{\partial^2}{\partial \alpha \partial \lambda_1} f_n = (n+1) \left( e^{-\lambda_1} - n \right) e^{-\lambda_n} \]

\[ f_{n,\alpha \lambda_2} = \frac{\partial^2}{\partial \alpha \partial \lambda_2} f_n = -n \left( (n+1) - (n-1) e^{\lambda_2} \right) e^{-\lambda_n} \left( e^{\lambda_2} - 1 \right) \]

\[ f_{n,\lambda_1^2} = \frac{\partial^2}{\partial \lambda_1^2} f_n = -\alpha \left( (n+1)^2 e^{-\lambda_1} - n^2 \right) e^{-\lambda_n} \]

\[ f_{n,\lambda_2^2} = \frac{\partial^2}{\partial \lambda_2^2} f_n = 0 \]

\[ f_{n,\lambda_1 \lambda_2} = -\frac{\partial^2}{\partial \lambda_1 \partial \lambda_2} f_n = -(n+1) f_{n,\lambda_2} - 2(1-\alpha) n e^{-\lambda_n} \left( (n-1) e^{\lambda_2} - n \right) \]

Computing the second-order derivatives yields \( f_{n,\alpha^2} = f_{n,\alpha \lambda_2} = 0 \) and \( \frac{\partial^2 F}{\partial \alpha \partial B} = \frac{\partial^2 F}{\partial B \partial \alpha} \) are used. 

\[
\frac{\partial^2 F}{\partial \alpha^2} = \frac{\partial}{\partial \alpha} \sum_{n=0}^{N} x_n f_{n,\alpha} = \sum_{n=0}^{N} \left( f_{n,\alpha} \frac{\partial}{\partial \alpha} f_n + x_n \frac{\partial}{\partial \alpha} f_{n,\alpha} \right) = -\sum_{n=0}^{N} \frac{x_n}{f_n} (f_{n,\alpha})^2
\]

\[
\frac{\partial^2 F}{\partial \alpha \partial \lambda_1} = \frac{\partial}{\partial \alpha} \sum_{n=0}^{N} x_n f_{n,\lambda_1} = \sum_{n=0}^{N} \left( f_{n,\lambda_1} \frac{\partial}{\partial \alpha} f_n + x_n \frac{\partial}{\partial \alpha} f_{n,\lambda_1} \right) = -\sum_{n=0}^{N} \frac{x_n}{f_n} (f_{n,\lambda_1} f_{n,\alpha} - f_{n} f_{n,\alpha \lambda_1})
\]

\[
\frac{\partial^2 F}{\partial \alpha \partial \lambda_2} = \frac{\partial}{\partial \alpha} \sum_{n=0}^{N} x_n f_{n,\lambda_2} = \sum_{n=0}^{N} \left( f_{n,\lambda_2} \frac{\partial}{\partial \alpha} f_n + x_n \frac{\partial}{\partial \alpha} f_{n,\lambda_2} \right) = -\sum_{n=0}^{N} \frac{x_n}{f_n} (f_{n,\lambda_2} f_{n,\alpha} - f_{n} f_{n,\alpha \lambda_2})
\]

\[
\frac{\partial^2 F}{\partial \lambda_1^2} = \frac{\partial}{\partial \lambda_1} \sum_{n=0}^{N} x_n f_{n,\lambda_1} = \sum_{n=0}^{N} \left( f_{n,\lambda_1} \frac{\partial}{\partial \lambda_1} f_n + x_n \frac{\partial}{\partial \lambda_1} f_{n,\lambda_1} \right) = -\sum_{n=0}^{N} \frac{x_n}{f_n} \left( \left( f_{n,\lambda_1} \right)^2 - f_{n} f_{n,\lambda_1} \right)
\]

\[
\frac{\partial^2 F}{\partial \lambda_1 \partial \lambda_2} = \frac{\partial}{\partial \lambda_1} \sum_{n=0}^{N} x_n f_{n,\lambda_2} = \sum_{n=0}^{N} \left( f_{n,\lambda_2} \frac{\partial}{\partial \lambda_1} f_n + x_n \frac{\partial}{\partial \lambda_1} f_{n,\lambda_2} \right) = -\sum_{n=0}^{N} \frac{x_n}{f_n} f_{n,\lambda_2} f_{n,\lambda_1}
\]

\[
\frac{\partial^2 F}{\partial \lambda_2^2} = \frac{\partial}{\partial \lambda_2} \sum_{n=0}^{N} x_n f_{n,\lambda_2} = \sum_{n=0}^{N} \left( f_{n,\lambda_2} \frac{\partial}{\partial \lambda_2} f_n + x_n \frac{\partial}{\partial \lambda_2} f_{n,\lambda_2} \right) = -\sum_{n=0}^{N} \frac{x_n}{f_n} \left( \left( f_{n,\lambda_2} \right)^2 - f_{n} f_{n,\lambda_2} \right)
\]
To use the second order derivatives found to determine confidence intervals, they are now calculated statistically. The resulting values are imported into a Hessian Matrix:

\[
H = \begin{pmatrix}
\frac{\partial^2 F}{\partial \alpha^2} & \frac{\partial^2 F}{\partial \alpha \partial \lambda_1} & \frac{\partial^2 F}{\partial \alpha \partial \lambda_2} \\
\frac{\partial^2 F}{\partial \lambda_1 \partial \alpha} & \frac{\partial^2 F}{\partial \lambda_1^2} & \frac{\partial^2 F}{\partial \lambda_1 \partial \lambda_2} \\
\frac{\partial^2 F}{\partial \lambda_2 \partial \alpha} & \frac{\partial^2 F}{\partial \lambda_2 \partial \lambda_1} & \frac{\partial^2 F}{\partial \lambda_2^2}
\end{pmatrix}
\]

By taking the negative inverse of this matrix, the variances matrix can be determined.

\[-H^{-1} = \text{Var} \begin{pmatrix} \alpha \\ \lambda_1 \\ \lambda_2 \end{pmatrix}\]

From the variances, square roots are taken to get standard deviations. Confidence intervals now are created by taking the optimal values of the three variables plus or minus 1.96 times the standard deviation. This gives the 95 % confidence interval.
4.3 RESULTS

What can be seen from the results below is that most of the resulting confidence intervals are quite precise. However, this is not the case for the intervals of $\alpha$ and $\lambda_1$ on Wednesdays both before as within the testing period. The confidence intervals of these parameters are too large, which can be explained by the fact that there might be an absence of people from the Erlang k=1 group for this specific day. Because of this noise in the calculations, $\lambda_2$ can be influenced as well. Therefore, the MLE is run again for this parameter, but now with $\alpha$=0.

The optimal value for $\lambda$ gives the chance at the corresponding amount of events (1 or 2) to happen for a given time slot. On average a $\lambda$ of 0.10 results in a parking time of 100 minutes (10 times bins of 10 minutes): \[ \text{avg\_parking\_time} = \frac{1}{\lambda} \text{min\_in\_timebin} \]

With these optimal values for the variables and the corresponding confidence intervals at hand, the results can be interpreted.

4.3.1 TIME SERIES: TOTAL

<table>
<thead>
<tr>
<th>Total</th>
<th>Before Test</th>
<th>After Test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>value</td>
<td>Confidence Interval</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.07216</td>
<td>0.06809 - 0.07623</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>0.25454</td>
<td>0.23655 - 0.27254</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>0.10195</td>
<td>0.10154 - 0.10235</td>
</tr>
</tbody>
</table>

The optimal values for the time series ‘Total’ show, that at least RTP made the values shift. The problem however is that none of the results for this time series is significant. For all three variables the confidence intervals overlap. This implies that differences between the values of the variables before and after the introduction of the new pricing scheme are not significant at the 5 % level.
This can be the effect of weekdays and weekends that are all part of the ‘Total’ time series. To see if individual workdays or weekend days show significant changes, time series for Wednesdays and Saturdays are analyzed.

### 4.3.2 TIME SERIES: WEDNESDAY

<table>
<thead>
<tr>
<th></th>
<th>Before Test</th>
<th></th>
<th>After Test</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>value</td>
<td>Confidence Interval</td>
<td>value</td>
<td>Confidence Interval</td>
</tr>
<tr>
<td>α</td>
<td>0.21303</td>
<td>-3.92054</td>
<td>4.34660</td>
<td>0.20476</td>
</tr>
<tr>
<td>λ₁</td>
<td>0.10343</td>
<td>-2.01092</td>
<td>2.21778</td>
<td>0.10783</td>
</tr>
<tr>
<td>λ₂</td>
<td>0.10314</td>
<td>0.10126</td>
<td>0.10503</td>
<td>0.10783</td>
</tr>
</tbody>
</table>

As mentioned above, the confidence intervals for α and λ₁ are too large to discuss clearly. The reason for this noise could be that the amount of people acting like Erlang (k=1) is negligibly small. Results for λ₂ can be affected due to this noise. For better results this noise should be removed. Removing α and λ₁ can be done by setting α to zero. The following results are then found for λ₂.

<table>
<thead>
<tr>
<th></th>
<th>Before Test</th>
<th></th>
<th>After Test</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>value</td>
<td>Confidence Interval</td>
<td>value</td>
<td>Confidence Interval</td>
</tr>
<tr>
<td>λ₂</td>
<td>0.11449</td>
<td>0.11357</td>
<td>0.11542</td>
<td>0.11915</td>
</tr>
</tbody>
</table>

Variable λ₂, both with and without the noise, shows an increase in the new situation. Moreover, the change of the variable λ₂ is significant. The confidence intervals before and during the test period do not coincide and therefore it is safe to state that the optimal value of λ₂ has increased significantly. This increase could be explained by the idea that consumers behaving like Erlang (k=2) used to be very price conscious. If they, e.g., were to park for about 3 hours, they might have had the incentive to pay and leave the garage just before their third hour expires. While in the new situation this incentive, avoiding overpayment, becomes a lot smaller. Fully using the amount of time paid for, is less ‘profitable’ so consumers might leave earlier. Another explanation, mentioned above, could be that relatively long parking consumers do not use the garage for two purposes anymore. They
switch to a single-event parking purpose, because of the increased attractiveness and increased influence they have on the tariff they pay. Within the group of Erlang (k=2) the average amount of parked time therefore could become smaller.

4.3.3 TIME SERIES: SATURDAY

<table>
<thead>
<tr>
<th>Saturday</th>
<th>Before Test</th>
<th>After Test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>value</td>
<td>Confidence Interval</td>
</tr>
<tr>
<td>α</td>
<td>0.01743</td>
<td>0.01525</td>
</tr>
<tr>
<td>λ1</td>
<td>1.37661</td>
<td>1.08692</td>
</tr>
<tr>
<td>λ2</td>
<td>0.09267</td>
<td>0.09186</td>
</tr>
</tbody>
</table>

For Saturdays the confidence intervals are sufficiently small, so in this case none of the variables is excluded directly from the analysis. For the variable α, significance is found. The increase in this variable is about 50% and significant. So, the relative amount of consumers behaving like Erlang (k=1) increases on average. Specifically on Saturday this effect occurs, resulting in the idea that on Saturday the group of Erlang (k=1) consumers increases due to RTP. People in this group behave like they prefer a short and one purpose parking time over a longer stay. Another explanation might be found in the idea that consumers who used to behave like an Erlang (k=2) distribution, shift their behavior to Erlang (k=1). The new pricing scheme seems to attract a different type of consumers.

The results from this analysis show that the introduction of Real Time Parking has led to significant changes in the composition of the consumer population. Weekdays and weekend days seem to attract different groups of consumers. At least for Wednesdays and Saturdays this holds. Based on this finding, the insignificance of the findings for the ‘Total’ time series can be explained as well. With at least one insignificant outcome for all variables already for these two days, insignificance in the totaled numbers is almost unavoidable.
CONCLUSION

In this thesis consumer parking behavior is analyzed. Subject of analysis is a new pricing scheme called Real Time Parking in which consumers pay per 3 minutes instead of per hour. The effect RTP had on consumer behavior is contaminated by the fact that there was an increase in price per hour. Also parking behavior cannot be observed ceteris paribus because of trends in total consumer numbers and other exogenous factors. Therefore no analysis on the absolute numbers of e.g. numbers of consumers can take place.

The focus of the thesis therefore is on consumer population composition. At least two groups of consumers are observed in the time series from the business case. The groups are distinguished based on the amount of parking purposes they seem to come for. The ratio of these two groups in the total population and the group specific variables are the measurement tools to determine whether this composition changes. A further differentiation of the population in more groups was not significantly possible with the given data.

The ambiguous effects of the price change and RTP did make a general conclusion for the total time series impossible. What can be stated is that the change in group composition is significant at least for Saturdays. Within the groups, a change in one of the group specific variables is significant as well. Overall it is safe to state that the introduction of Real Time Parking has significant effects on group composition as well as on within group behavior.

The explanatory power of this thesis could have been larger if the data would have been available per minute or even per second. Assumptions and filters are used to create a workable dataset. For example, if the filter of the entrance time would have been excluded, the effect on the total results could have been large. Moreover the size of the dataset is always subject of possible improvement. In this case would a larger dataset probably yield more clear findings because after the testing period the effect of consumer returning to their ‘old’ behavior might have been visible.
Consumers will have a two sided view on RTP. They will be more in control on the tariff they have to pay. However, the system will have some startup costs and operators will aim at keeping overall revenues at least at the same level as in the old situation. Prices will therefore on average increase which is negative for consumers.

For as far as this parking concerns, a consumer survey among the users of the Stopera garage gives the clear outcome that 75 % percent of the consumers think of it as a more fair pricing scheme. Being as it may, the responsible alderman at the city of Amsterdam announced last November that the system is not yet to be installed permanently\textsuperscript{5}. Real-Time Parking has caused their total revenue to decline by 17 % over the entire testing period of six months. Annually this would ‘cost’ the city approximately three million euro’s which is unaffordable in a time that cities have to cut their budgets.

\textit{When Solomon said there was a time and a place for everything he had not encountered the problem of parking his automobile.} Bob Edwards

\begin{footnotesize}
Wiebes, Eric; Bericht van Wethouder Wiebes aan gemeenteraad Amsterdam; 11-11-2010
\end{footnotesize}
REFERENCES


Aczel, A.D., Sounderpandian, J.; Business Statistics; McGraw-Hill/Irwin; 2008

Bain, R.; Improving the quality of city centre parking: will the consumer pay?; Traffic Engineering and Control 43 (5), 175–179; 2002


Brockmeyer, E., Halstrøm H.L., Jensen, A.; The life and works of A.K. Erlang; Danish Academy of Technical Sciences; 1948


Collier International; Parking Rates, Global CBD Parking Rate Survey 2009

Cox, D.; Renewal Theory; Methuen en Co. Science Paperbacks; 1962


Dijksma C.S.; motion on budget of the Ministry of Economic Affairs; (32 500 XIII)Vaststelling van de begrotingsstaten van het Ministerie van Economische Zaken, Landbouw en Innovatie (XIII) voor het jaar 2011, Nr. 79 Motie van het lid Dijksma C.S.); 24 November 2010

Gagliano; Win-win parking strategies; 2009

Glazer and Amihaei; Parking fees and congestion; Regional science and urban economics North-Holland no. 22 1992 page 123-132; 1992


Henscher; Parking demand and responsiveness to supply, pricing and location in the Sydney central business district, 2006


Kelly, J. A., Clinch, J. P.; Influence of varied parking tariffs on parking occupancy levels by trip purpose. Transport Policy, 13(6), 487–49; 2002


Okada, E. M., Hoch, S.J.; Spending time versus spending money; Journal of Consumer Research, 31 (September), 313-323; 2006


Vexpan; Vexpan Parkeeronderzoek 2010