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Estimating Value-at-Risk Conditional on Bull and Bear Predictions: A New Approach to Combining Risk Models

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Abstract

This paper proposes a new risk management strategy that uses combinations of two different models to estimate VaR by conditioning the model choice on the prediction of bull and bear markets. The main goal of this research is to trigger financial risk managers to take a critical look at their internal risk model(s). They should ask themselves whether there is room for further minimization of the capital charges, by conditioning their risk model choice on the prediction of the market condition. We describe various risk models and a pragmatic strategy to predict bull and bear markets, using a binomial logit model. Using a parametric linear Student *t* model with EWMA volatility with an optimized λ leads to the best results for VaR estimation, looking at the number of violations and the average minimum required capital. For the prediction of bull and bear markets, using the Schwarz Information Criterium for variable selection leads to an out-of-sample hitrate of more than 85%. No combinations of two different models are found, that lead to a significant decrease in the average minimum required capital.

Keywords and phrases: Value-at-risk (VaR), parametric linear models, historical simulation models, Monte Carlo simulation models, bull and bear markets, combination of risk models, optimizing strategy.

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1 Introduction

Financial risk management is driven internally by the need for optimal returns on risk-based capital and, ultimately, by the survival of the firm. External drivers include clients and industry regulators, whose objectives are to protect investors and to promote competition, despite their ultimate concern for financial stability in the global economy. In recent years market volatility has been rising as trading activity increasingly utilizes complex instruments whose risks are relatively difficult to assess.

The Basel II Accord (Basel Committee on Banking Supervision, 2006) requires that banks (and other Authorized Deposit-taking Institutions) communicate their risk forecasts to the appropriate monetary authorities at the beginning of each trading day, using one or more risk models to measure Value-at-Risk (VaR). Risk estimates of these models are used to determine capital requirements and associated capital costs of banks, depending in part on the number of previous violations, whereby realized losses exceed the estimated VaR.

Banks are permitted (and encouraged) to use internal models to forecast VaR. In case internal models lead to more violations than could be expected (3 violations per financial year when using a 99% confidence level), a bank is required to hold a higher level of capital. If a bank's VaR forecasts are violated more than 10 times in any financial year, they may be required to use a 'Standardized' approach, instead of their own internal model(s). Such a penalty will not only cause higher capital charges, but it will also damage a bank's reputation. That is why financial risk managers tend to prefer following strategies that are passive and conservative. However, excessive conservatism can have a negative impact on the profitability of a bank as higher capital charges are subsequently required. A bank should seek a strategy, which minimizes the daily capital charges and has less than 10 violations per year.

In this paper we seek such a strategy by choosing sensibly from a variety of risk models, i.e. parametric linear, historical and Monte Carlo risk models, with a separate section devoted to Monte Carlo risk models with copula dependence. A new approach to model selection for predicting VaR is proposed, namely conditioning the model choice on the prediction of bull and bear markets. When dividing the stock market into bull and bear markets, a bear market is likely to capture more extreme events and its return distribution will have thicker tails. Therefore, it is unlikely for one single risk model to perform best in all conditions. During bear markets, it can be optimal to use a different risk model than during bull markets.

The main goal of this research is to trigger financial risk managers to take a critical look at their internal risk model(s). They should ask themselves whether there is room for further minimization of the capital charges, by conditioning their risk model choice on the predicted market condition.

We find that a parametric linear Student t model with EWMA volatility with an optimized λ leads to the best results for VaR estimation, looking at the number of violations and the average minimum required capital. This model and a filtered historical simulation model with GARCH volatility are the only models that have less than 10 violations per year over the whole testing period. With the Schwarz Information Criterium used for variable selection we score highest on statistical accuracy for the prediction of bull and bear markets, namely an out-of-sample hitrate of more than 85%. Unfortunately, no combinations of two different models are found, that lead to a significant decrease in the average minimum required capital. The main reasons for this result are the low number of bear months (only 19% of the whole testing period) and the fact that the best performing models incurred their largest number of violations just before the beginning of these bear months.

The paper proceeds as follows. Section 2 presents the basic methodology applied for the estimation of VaR. In sections 3 to 6 we discuss the methodology of the parametric linear, historical and Monte Carlo risk models respectively, with section 6 devoted to Monte Carlo simulation with copulas.

Section 7 presents a series of backtests to determine the accuracy of every risk model. Section 8 gives an outline of the methods used to predict bull and bear markets. Empirical results are provided in section 9. Finally, concluding remarks are offered in Section 10.

2 Methodology for Estimating Value-at-Risk

The most used measure of financial risk is the Value-at-Risk (VaR). The widespread popularity of VaR (see Jorion, 2000) is due to the adoption of the "1st pillar" in the Basel II agreement. It is defined as "the loss we are fairly sure will not be exceeded if the current portfolio is held over some period of time". If the 1% 1-day VaR = $2 million, this means that we are 99% confident that we would lose no more than $2 million from holding the portfolio for 1 day.

Despite its widespread use and simplicity, Value-at-Risk is highly criticized among academics. It has the disadvantage that it is not a coherent risk measure, i.e. it may not be sub-additive (see Artzner et al., 1999, for details). Moreover, VaR provides no information about the expected size of the loss if a tail event occurs. On the other hand, Expected Shortfall, which is defined as the expected loss given that VaR is exceeded, is coherent and provides information about the size of tail events.

VaR is defined on two parameters, i.e. a holding period, which is the period of time over which we measure our portfolio profit or loss, denoted h, which is traditionally measured in trading days rather than calendar days; and a significance level α (or confidence level $1-\alpha$), which indicates the likelihood that we will get an outcome worse than VaR.

Regulators that review the regulatory capital of banks usually allow this capital to be assessed using an internal VaR model, provided they have approved the model and that certain qualitative requirements have also been met. In this case a 99% confidence level must be applied in the VaR model to assess potential losses over a 2-week risk horizon, i.e. a 99% 10-day VaR. This figure is then multiplied by a factor of between 3 and 4 to obtain the minimum required capital[1].

We estimate VaR for portfolio returns from a banking perspective, although it is not a bank's core business to seek profits through enhanced returns on investments: this is the role of portfolio management. Where banks are required by regulators to measure their risks as accurately as possible, every day, and to hold capital in proportion to these risks, there are no such regulations for the fund management industry. The fund manager does have a responsibility to report risks accurately, but only to his clients. Their confidence level and risk horizon are not set by regulators and thus are likely to be different among different fund management companies. However, all models discussed in this report can be adjusted to every confidence level and risk horizon we like, since they are defined as input parameters.

Investing in a portfolio leaves the choice of the risk/return profile of the investment. The return distribution of a high risk portfolio is likely to capture more extreme events than that of a low risk portfolio. Finding a good risk measure can therefore also depend on the risk-return profile of a portfolio. We create three testing portfolios with different risk/return profiles from *x%* equity index and *y%* bond index:

1. 80% equity index and 20% bond index (high-risk portfolio).

2. 50% equity index and 50% bond index (moderate risk portfolio).

3. 20% equity index and 80% bond index (low-risk portfolio).

[1] Section 7.1 provides more information on the calculation of the minimum required capital.

Note that we want to keep the portfolio weights constant over the whole sample period. This means that we assume that all the holdings are rebalanced whenever the price of one asset changes. This assumption implies that the same risks are faced every trading day during our 10-day risk horizon. This makes it relatively easy to scale a 1-day VaR to a 10-day VaR.

2.1 Data

This paper focuses on the US market and investigates its stock returns and bond returns using the daily returns on the S&P500 Price Index and the Barclays Capital US Aggregate Bond Index from January 1, 1978 to December 31, 2010. The Barclays Capital US Aggregate Bond Index (formerly the Lehman Aggregate Bond Index) was created in 1986, with backdated history going back to 1976. The S&P500 Price Index is obtained from Yahoo! Finance, Barclays Capital US Aggregate Bond Index is obtained from Thompson Datastream.

Every trading day, starting at January 1, 1981, we estimate a 10-day VaR, using a moving window of three years, which contains the 750 most recent daily returns. We use a log approximation to the daily returns. To be more specific, we let

$$r_t = \frac{P_t - P_{t-1}}{P_t} \approx \ln\left(\frac{P_t}{P_{t-1}}\right), \tag{2.1}$$

where P_t denotes the portfolio price at time t. We use log returns, because we want to calculate a 10-day VaR. The 10-day log return is the sum of 10 consecutive daily returns, and therefore these are more convenient than portfolio returns. Although the log returns are not exactly equal to the portfolio returns calculated as $\frac{P_t - P_{t-1}}{P_t}$, for daily returns they will be fairly close.

3 Parametric linear models

A parametric linear model calculates VaR using analytic formulae that are based on an assumed parametric distribution for the asset returns, when the portfolio value is a linear function of its underlying asset returns. The most basic assumption is that the returns on the portfolio are independent and identically distributed with a normal distribution. Unfortunately this assumption is very unrealistic, even for linear portfolios. Therefore, we extend this assumption so that we can incorporate volatility clustering in our model. We calculate VaR estimates of a linear portfolio under this assumption and also when portfolio returns are assumed to have a Student t distribution or a mixture of two normal distributions.

3.1 Volatility estimation

3.1.1 Unconditional volatility

The easiest way to measure volatility is to base the historical volatility estimates on the equally weighted unconditional variance estimate. For instance, denoting the portfolio return at time t by r_t and assuming these returns are i.i.d., the unbiased sample variance estimate based on the most recent T returns is

$$\hat{\sigma}_t^2 = \frac{1}{T-1} \sum_{k=1}^{T} (r_{t-k} - \bar{r})^2 \tag{3.1}$$

where $\bar{r}$ denotes the average daily return over the previous T days.

3.1.2 Volatility clustering

Unconditional volatility is useful for estimating VaR over a long term risk horizon, but it has limited use for estimating VaR over a short term horizon. For instance, if we use three years of data to estimate volatility, the unconditional volatility estimate represents the average sample volatility over the last three years. This may be fine for long term VaR estimation, but short-term VaR estimates are supposed to reflect the current market conditions, and not the average conditions of the past three years. For this we need a forecast of the conditional volatility, or a time-varying estimate of the unconditional volatility.

Exponentially Weighted Moving Average (EWMA) volatilities are more risk sensitive than equally weighted average estimates of the same parameters in a way that they should respond more rapidly to changing market circumstances.

We define two different ways to estimate the EWMA volatility:

- $\hat{\sigma}_t$ equal to the EWMA volatility with $\lambda = 0.94$ (*RiskMetrics*™).

  In this case, no estimation is necessary, which is a huge advantage in large portfolios. The EWMA variance estimate at time t is then defined as

$$\hat{\sigma}_t^2 = (1-\lambda)r_{t-1}^2 + \lambda\hat{\sigma}_{t-1}^2, \qquad t = 2,\dots,T \qquad \lambda = 0.94, \tag{3.2}$$

  which is a function of the previous squared return and the previous variance.

On the other hand, a predefined value for λ does not have to be the optimal value for every return series. Therefore, we also optimize λ by using Maximum Likelihood Estimation:

- $\hat{\sigma}_t$ equal to the EWMA volatility with an optimized λ.

  Here, the EWMA variance estimate at time t is defined as

$$\hat{\sigma}_t^2 = (1-\lambda)r_{t-1}^2 + \lambda\hat{\sigma}_{t-1}^2, \qquad t = 2,\dots,T \qquad 0 < \lambda < 1. \tag{3.3}$$

  The term $(1-\lambda)r_{t-1}^2$ determines the intensity of reaction of volatility to market events: the smaller is λ the more the volatility reacts to the market information in yesterday's return. The term $\lambda\hat{\sigma}_{t-1}^2$ determines the persistence in volatility: irrespective of what happens in the market, if volatility was high yesterday it will be still high today. The closer λ is to 1, the more persistent is volatility following a market shock.

3.2 Calculation of linear VaR

In this section we define the formulae to calculate the parametric linear VaR, when these estimates are based on the assumption that returns are normally distributed, Student t distributed or follow a normal mixture distribution. We also discuss the rules for scaling normal linear VaR under both i.i.d. and autocorrelated returns.

3.2.1 Normal linear VaR

When normal linear VaR estimates are based on daily returns to the portfolio, we obtain a 1-day VaR estimate using the daily mean estimate at time t, $\hat{\mu}_{1,t}$, and the daily volatility estimate at time t, $\hat{\sigma}_{1,t}$. This 1-day VaR estimate should then be scaled to a 10-day VaR estimate. Under the assumption that our daily returns are i.i.d., it follows by the square-root-of-time rule that the $100(1-\alpha)\%$ 10-day VaR at time t is defined as:

$$\mathrm{VaR}_{10,t,\alpha} = \Phi^{-1}(\alpha) \cdot \sqrt{h}\hat{\sigma}_{1,t} + h\hat{\mu}_{1,t} \tag{3.4}$$

with

$\Phi^{-1}(\alpha) =$ The z-value such that, with probability α, a standard normal random variable takes on a value that is less than or equal to z

$\hat{\sigma}_{1,t} =$ The estimated daily return volatility at time t

$\hat{\mu}_{1,t} =$ The average daily return at time t

$h =$ The risk horizon in days $= 10$.

This 10-day VaR estimate is based on the assumption that returns are not only normally distributed but also generated by an i.i.d. process, which is simply not justified for most financial returns. Even when returns are not autocorrelated, the volatility clustering effect we see in most markets can cause the squared returns to be autocorrelated. An EWMA volatility estimate is a constant, in the sense that it is equal for all time horizons. The EWMA model gives time-varying estimates of the unconditional volatility and therefore it will estimate the same average volatility for all time horizons, whether the forecast is over the next day or over the next 10 days. Hence, we can also use (3.4) to calculate a 10-day VaR, with $\hat{\sigma}_{1,t}$ equal to the daily EWMA volatility estimate.

In the standard parametric linear VaR model we cannot calculate a 10-day volatility estimate using a GARCH model, which has time-varying conditional volatility estimates. The problem is that when a return follows a GARCH process we do not know the exact price distribution 10 days from now. We know this distribution when the returns are i.i.d., because it is the same as the distribution we have estimated over a historical sample. However, the 10-day return in a GARCH model is the sum of 10 consecutive daily returns and, due to the volatility clustering it is the sum of non-i.i.d. variables. Instead, we could apply a GARCH model to simulate daily returns over the risk horizon by imposing path dependence, as explained in Sections 4.3 and 5.2.2.

Alexander (2008) proves that when returns are autocorrelated with first order autocorrelation coefficient ρ, then the scaling factor for standard deviation is not $\sqrt{h}$ but $\sqrt{\tilde{h}}$, where

$$\tilde{h} = h + 2\frac{\rho}{(1-\rho)^2}\left[(h-1)(1-\rho) - \rho\left(1-\rho^{h-1}\right)\right]. \tag{3.5}$$

Hence, we should scale our normal linear VaR as

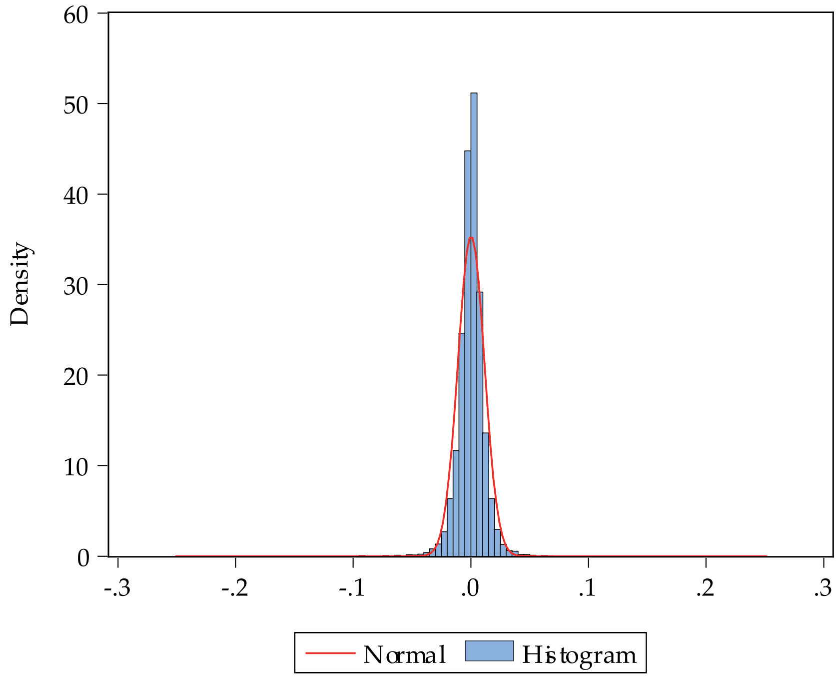
$$\mathrm{VaR}_{h,t,\alpha} = \Phi^{-1}(\alpha) \cdot \sqrt{\tilde{h}}\hat{\sigma}_{1,t} - h\hat{\mu}_{1,t}. \tag{3.6}$$

3.2.2 Student *t* distributed linear VaR

One of the basic stylized facts of (daily) asset returns is that their distribution is not normal (see Taylor, 2005, Chapter 4). Often excess kurtosis and negative skewness are apparent in these data series. With excess kurtosis and negative skewness in return distributions the normal linear VaR formula, as described in section 3.2.1, is likely to underestimate the VaR at high confidence levels.

The daily log returns of the S&P500 Index in our sample (January 1, 1978 - December 31, 2010) have mean equal to 7.75 percent (expressed in annualized percent) and a volatility (=annualized standard deviation) of 17.83 percent. Skewness is negative at -1.202, suggesting that large negative returns occur more often than large positive returns. Kurtosis is equal to 31.371, which is substantially higher than the normal value of 3, suggesting the presence of a high peak and fat tails in the empirical distribution of returns. Both are confirmed by the histogram that is shown Figure 1, together with a normal density with the same mean and variance. The Jarque-Bera test statistic takes a value of 281,269.7, with a p-value of 0.000 such that normality is convincingly rejected. If we omit the daily return on October 19, 1987, the skewness and kurtosis change to -0.214 and 11.986, respectively. In this case normality is still convincingly rejected, with a Jarque-Bera test statistic of 28,074.2, and a p-value of 0.000.

Figure 1: Histogram and theoretical normal density of daily S&P500 returns, Jan 1st, 1978 - Dec 31st, 2010.



Student t distributions are the most well known leptokurtic distributions. When significant positive excess kurtosis is found in empirical financial daily return distributions, the Student t distribution is likely to produce VaR estimates that are more representative of historical behaviour than normal linear VaR. When h is small, a approximate formula[2] for the $100(1-\alpha)\%$ h-day VaR is

$$\text{Student } t \text{ VaR}_{h,t,\alpha,\nu} = \sqrt{\nu^{-1}(\nu-2)\tilde{h}} t_{\nu}^{-1}(\alpha)\sigma_{1,t} + h\mu_{1,t}. \tag{3.7}$$

with $t_{\nu}^{-1}(\alpha)$ the α quantile and ν the number of degrees of freedom of the standard Student t distribution.

The Student t distribution is not a stable distribution and by the central limit theorem the sum of i.i.d. Student t variables converges to a normal variable as the number of terms in the sum increases. With a risk horizon of 10 days, we cannot say beforehand whether the normal linear VaR estimate

[2]The square-root-of-time rule for scaling the standard deviation used in (3.7) does not apply to Student t distributed returns, so this formula is only an approximation.

will be sufficiently accurate or that the Student t VaR estimate is a significant improvement over the normal linear VaR estimate.

The Student t linear VaR model provides a more accurate representation of most financial asset returns, but a potentially significant source of model risk arises from assuming the return distribution is symmetric. By far the easiest way to extend the parametric linear VaR model to accommodate the skewness that is so often evident in financial asset returns is to use the mixture linear VaR model, which is explained in the next section.

3.2.3 Normal mixture linear VaR

A mixture model is designed to capture different market regimes. In a mixture of two normal distributions, there are two regimes for our daily returns: one regime where the daily returns have mean μ_1 and variance σ_1^2 and another regime where the daily returns have mean μ_2 and variance σ_2^2. The parameter π defines the probability of occurrence of regime 1, so regime 2 occurs with probability $1-\pi$.

The estimation of the mixture parameters from historical data is best performed using the EM algorithm, although with only a few parameters they could also be obtained by using a Method of Moments estimation. Then, the distribution function of a mixture of two distributions is defined as:

$$G(x) = \pi F_1(x;\mu_1,\sigma_1) + (1-\pi)F_2(x;\mu_2,\sigma_2), \qquad 0 < \pi < 1, \tag{3.8}$$

which is a probability weighted sum of each of the two distribution functions, where $F_i(x;\mu_i,\sigma_i)$ denotes the normal distribution function of the returns in regime i.

We have

$$P(X < x_\alpha) = G(x_\alpha) = \pi F_1(x_\alpha;\mu_1,\sigma_1) + (1-\pi)F_2(x_\alpha;\mu_2,\sigma_2), \qquad 0 < \pi < 1, \tag{3.9}$$

and when $P(X < x_\alpha) = \alpha$, then x_α is the α quantile of the mixture distribution. Let X_i be the random variable with distribution function $F_i(x;\mu_i,\sigma_i)$. Then

$$F_i(x_\alpha;\mu_i,\sigma_i) = P(X_i < x_\alpha) = P\left(\frac{X_i-\mu_i}{\sigma_i} < \frac{x_\alpha-\mu_i}{\sigma_i}\right),$$

with $\frac{X_i-\mu_i}{\sigma_i} = Y_i = Z_i$, where Z_i is a standard normal variable. Hence,

$$\pi P\left(Y_1 < \frac{x_\alpha-\mu_1}{\sigma_1}\right) + (1-\pi)P\left(Y_2 < \frac{x_\alpha-\mu_2}{\sigma_2}\right) = \alpha. \tag{3.10}$$

The 1-day VaR estimate is equal to x_α, which can be backed out from (3.10). To calculate the 10-day VaR estimate, we simply back out x_α from the following equation:

$$\pi P\left(Y_1 < \frac{x_\alpha-10\mu_1}{\sqrt{10}\sigma_1}\right) + (1-\pi)P\left(Y_2 < \frac{x_\alpha-10\mu_2}{\sqrt{10}\sigma_2}\right) = \alpha. \tag{3.11}$$

4 Historical Simulation models

Historical simulation as a method for estimating VaR was introduced by Hendricks (1996) and is by far the most popular VaR method amongst banks. Perignon and Smith (2010) show that 73 percent of banks that disclosed their VaR method reported using historical simulation. The main advantage of using historical simulation to calculate VaR is that it does not assume any distribution on the portfolio returns and it is relatively easy to implement. However, historical simulation assumes that all possible future variation has been experienced in the past. To ensure enough points in the lower tail of the distribution, the sample size needs to be sufficiently large.

For historical simulation, we rank our daily portfolio returns and pick the worst 1% return, according to their weight.[3] This value is the 99% 1-day VaR estimate. We scale the 1-day VaR estimate to a 10-day horizon. In Section 3.2.1 we stated that for linear VaR the 10-day VaR is equal to the square root of 10 times the 1-day VaR. This square-root-of-time rule applies to linear VaR because it obeys the same rules as standard deviation. However, in the historical simulation model the VaR estimate corresponds to a quantile of some unspecified empirical distribution and quantiles do not obey a square-root-of-time rule, except when the returns are i.i.d. and normally distributed.

Scaling VaR using a historical simulation model can therefore only be performed by making certain assumptions about the distribution. In this case we only have to assume that our portfolio returns have a stable distribution. When a distribution is stable with scale parameter ξ then the whole distribution, including the quantiles, scales as $h^{1/\xi}$. For instance, in a normal distribution $\xi = 2$ and the scale exponent is $\xi^{-1} = \frac{1}{2}$.

Let $x_{h,\alpha}$ denote the α quantile of the h-day log returns. We seek ξ such that

$$x_{h,\alpha} = h^{1/\xi} x_{1,\alpha}. \tag{4.1}$$

In other words, taking logs of the above,

$$\xi = \frac{\ln(h)}{\ln(x_{h,\alpha}) - \ln(x_{1,\alpha})}. \tag{4.2}$$

Then, to estimate the 10-day VaR we take $10^{1/\xi}$ times the 1-day VaR.

4.1 Equal weights to returns

Equal weighting of historical data was the first statistical method for forecasting volatility of financial asset returns to be widely accepted. For many years, it was the market standard to forecast average volatility over the next h days by taking an equally weighted average of squared returns over the previous h days. The weight of the t-th return is defined as

$$w_t = 1/T, \tag{4.3}$$

where

$$T = \text{The number of daily returns in our sample = 750.}$$

In the parametric linear and Monte Carlo VaR models the volatility over the risk horizon can be estimated using an exponentially weighted moving average model. In these cases, previous returns have an exponentially declining effect on the volatility forecast and therefore also on the VaR estimate. In the next section, we describe two different ways of weighting returns for historical simulation models: exponentially declining weights to returns and filtered historical simulation.

[3] In sections 4.1 to 4.3 we discuss several weighting methods.

4.2 Exponentially declining weights to returns

A major problem with all equally weighted risk measure estimates is that extreme market events can influence the risk measure estimate for a considerable period of time. In historical simulation with equally weighted returns, this happens even if the events occurred long ago, since the ordering of observations is irrelevant.

Weighting the returns equally is inconsistent with the nature where there is diminishing predictability of data that are further away from the present. To overcome this, we also use models which apply exponentially declining weights to returns that are further away from the present. That is, as extreme returns move further into the past when the data window moves, they become less important in the average. For this reason exponentially weighting forecasts do not suffer from the 'ghost features' that we find in equally weighted moving averages.

If we assign an exponentially declining weight to the probability of each return in its distribution, the weight of the t-th return is defined as

$$w_t = \frac{\lambda^{T-t}(1-\lambda)}{(1-\lambda^T)}, \tag{4.4}$$

where

$T =$ The number of daily returns in our sample = 750

$\lambda =$ Decay factor = 0.94

Then, we use these probability weights to find the cumulative probability associated with the returns when they are put in increasing order of magnitude. That is, we order the returns, starting at the smallest return, and record its associated probability weight. To this we add the weight associated with the next smallest return, and so on until we reach a cumulative probability of $100\alpha\%$, the significance level for the VaR calculation. To obtain the $100(1-\alpha)\%$ historical VaR, we interpolate between the last return that was taken into the sum and the next smallest return after this last return.

4.3 Filtered Historical Simulation

One problem with using data that span a very long historical period is that market circumstances change over time. Since historical simulation requires a very large sample, the question is how best to employ data, possibly from a long time ago when the market was in a different regime. As a simple example, consider an equity market that has been stable and trending for one or two years, but previously experienced a long period of high volatility. We have little option but to use a long historical sample period for the historical VaR estimate, but we would like to adjust the returns from the volatile regime so that their volatility is lower. Otherwise the current historical VaR estimate will be too high. Conversely, if markets are particularly volatile at the moment but were previously stable for many years, an equally weighted historical estimate will tend to underestimate the current VaR, unless we scale up the volatility of the returns from the previous, tranquil period.

Filtered historical simulation, as introduced by Barone-Adesi et al. (1998), is an extension the idea of volatility adjustment to multi-step historical simulation. A distinct advantage of the filtered historical simulation approach over standard historical simulation is that it combines Monte Carlo simulation based on volatility clustering with the empirical return distribution that has occurred in the past.

The idea is to use a parametric dynamic model of returns volatility, such as a GARCH(1,1) model[4], to simulate log returns on each day over the risk horizon. For instance, suppose we have estimated a symmetric GARCH(1,1) model on the historical portfolio returns r_t, obtaining the estimated model

$$\hat{\sigma}_t^2 = \hat{\omega} + \hat{\alpha} r_{t-1}^2 + \hat{\beta}\hat{\sigma}_{t-1}^2. \tag{4.5}$$

The filtered historical simulation model assumes that the GARCH innovations are drawn from the standardized empirical return distribution. That is, we assume the standardized innovations are

$$\varepsilon_t = \frac{r_t}{\hat{\sigma}_t}, \tag{4.6}$$

where r_t is the historical portfolio return at time t and $\hat{\sigma}_t$ is the estimated GARCH daily standard deviation at time t. To start the one step simulation we set $\hat{\sigma}_0$ to be equal to the estimated daily GARCH standard deviation on the last day of the historical sample, when the VaR is estimated, and also set r_0 to be the last daily return of the historical sample. Then we compute the GARCH daily variance on day 1 of the risk horizon as

$$\hat{\sigma}_1^2 = \hat{\omega} + \hat{\alpha} r_0^2 + \hat{\beta}\hat{\sigma}_0^2,$$

Now the simulated return on the first day of the risk horizon is $\hat{r}_1 = \varepsilon_1 \hat{\sigma}_1$ where a value for ε_1 is simulated from our historical sample of standardized innovations (4.6). This is achieved using a statistical bootstrap. Thereupon we iterate in the same way, on each day of the risk horizon setting

$$\hat{\sigma}_{t+1}^2 = \hat{\omega} + \hat{\alpha} r_t^2 + \hat{\beta}\hat{\sigma}_t^2, \qquad \text{with } r_t = \varepsilon_t \hat{\sigma}_t \qquad \text{for } t = 1, \dots, h,$$

where ε_t is drawn independently of ε_{t-1} in the bootstrap. Then the simulated log return over a risk horizon of 10 days is the sum $\hat{r}_1 + \hat{r}_2 + \ldots + \hat{r}_{10}$. Repeating this for 1000 simulations produces a simulated return distribution, and the $100(1-\alpha)\%$ 10-day VaR is obtained as the α quantile of this distribution.

5 Monte Carlo Simulation models

The process for computing VaR with Monte Carlo simulation models is complete analogous to the estimation of VaR using historical simulation, only now we use Monte Carlo simulations instead of historical simulations. That is, we simulate a distribution for the portfolio's 10-day returns, and the $100(1-\alpha)\%$ 10-day VaR is estimated empirically as α quantile of this distribution.

The main advantage of Monte Carlo simulation over historical simulation is the absence of restrictions on historical sample size. The calibration of the parametric distributions for asset returns can be based on very little historical data, indeed we could just use scenario values for the parameters of the distributions. And if the parameters are calibrated on only very recent history, the Monte Carlo VaR estimates will naturally reflect these market circumstances.

The advantage of Monte Carlo VaR compared to parametric VaR estimates for linear portfolios is the large number of alternative asset return distributions that can be assumed. However, it is important to apply simulations to a dynamic model of returns that captures path-dependent behavior, such as volatility clustering, as well as the essential non-normal features of their conditional distribution. Without such a model, filtered historical simulation (see section 4.3) may be the better alternative.

[4] Using a GARCH model to estimate $\hat{\sigma}_t$ is described in more detail in section 5.2.2

5.1 Unconditional volatility

To simulate 10-day portfolio returns, we need to capture the characteristics of daily returns in the simulation model. For this we need to use a multi-step Monte Carlo framework. For our linear portfolio, this consists of simulating a 10-day log return by summing 10 consecutive daily log returns and then just evaluating the portfolio once, 10 days ahead.

We perform N=1000 simulations[5] based on the assumption of i.i.d. lognormally distributed returns. We use log returns to simulate the price of our portfolio on each day over the risk horizon, starting from the current price, S_t, and ending in 10 days' time with 1000 simulated prices. Hence, we simulate 1000 paths for the daily log returns over the next 10 days. The simulated price in 10 days' time based on one-step Monte Carlo is

$$S_{t+h} = S_t \exp\left(h\hat{\mu}_{1,t} + \hat{\sigma}_{1,t}\sqrt{h}\tilde{Z}_t\right), \tag{5.1}$$

where

$$\begin{aligned} S_t &= \text{Current price} \\ \hat{\mu}_t &= \text{Average daily return at time } t \\ h &= \text{Risk horizon in days} = 10 \\ \hat{\sigma}_t &= \text{Daily return volatility at time } t \\ \tilde{Z}_t &= \text{Generated standard normal random number.} \end{aligned}$$

As described in section 3, the assumption of normally distributed portfolio returns is very unrealistic. Therefore we can also base our Monte Carlo simulation on generated standardized Student t distributed random numbers. The simulated price in 10 days' time is then

$$S_{t+h} = S_t \exp\left(h\hat{\mu}_{1,t} + \hat{\sigma}_{1,t}\sqrt{h}\tilde{T}_t\right), \tag{5.2}$$

where $\tilde{T}_t$ is a generated standardized Student t random number. This number can be obtained by multiplying a standard Student t random number by $\sqrt{\nu^{-1}(\nu-2)}$, with ν the number of degrees of freedom.

After obtaining 1000 random prices for time t+10 we calculate 1000 daily returns from the current and the simulated prices. By sorting these returns, we can calculate the VaR by taking the worst $100\alpha\%$ return.

The historical volatility estimate, $\hat{\sigma}_t$ in (5.1) and (5.2), is based on the equally weighted unconditional variance estimate. For instance, denoting the portfolio return at time t by r_t and assuming these returns are i.i.d., the equally weighted sample variance based on the most recent T returns is

$$\hat{\sigma}_t^2 = \frac{1}{T-1}\sum_{k=1}^{T}\left(r_{t-k} - \bar{r}\right)^2, \tag{5.3}$$

where $\bar{r}$ denotes the average daily return over the previous T days.

5.2 Volatility clustering

One of the most important features of daily returns on equity portfolios is that volatility tends to come in clusters. Certainly at the daily frequency, large returns tend to follow large returns of either sign. Whilst returns themselves may show little or no autocorrelation, there is a strong positive

[5] Performing more than 1000 simulations becomes computationally too intensive.

autocorrelation in squared returns. We refer to this feature as volatility clustering, as markets pass through periods with low and high volatility. A tremendous amount of research has been conducted on volatility clustering, started by Mandelbrot (1963).

In this section we discuss various models to capture this volatility clustering, by using multi-step Monte Carlo simulation. We simulate the price of our portfolio on each day over the risk horizon, starting from the current price, S_t, and ending in 10 days' time with 1000 simulated prices. Hence, we simulate 1000 paths for the daily log returns over the next 10 days. This means that, when we are estimating the risk of a portfolio, the simulated daily log returns can be used to calculate the price tomorrow, the price in 2 days' time, and so on up to the risk horizon of 10 days.

5.2.1 EWMA volatility

The method used to estimate $\hat{\sigma}_t$ in (5.1) and (5.2) equal to the EWMA volatility is already described in detail in section 3.1.2. When based on multi-step Monte Carlo simulations, the EWMA variance estimate $\hat{\sigma}_t^2$ at time t is computed using the recurrence

$$\hat{\sigma}_t^2 = (1-\lambda)r_{t-1}^2 + \lambda\hat{\sigma}_{t-1}^2, \tag{5.4}$$

where λ is a constant called the smoothing constant, and r_{t-1} is the simulated log return in the previous simulation.

In the EWMA model for simulating log returns we set $\hat{r}_t = \hat{\sigma}_t\tilde{Z}_t$ or $\hat{r}_t = \hat{\sigma}_t\tilde{T}_t$ where $\tilde{Z}_t$ and $\tilde{T}_t$ are respectively simulations from a standard normal variable and a standardized Student t variable and $\hat{\sigma}_t$ is computed using (5.4). Hence, the simulated log return over a risk horizon of 10 days is the sum $\hat{r}_1 + \hat{r}_2 + \ldots + \hat{r}_{10}$. Repeating this for 1000 simulations produces a simulated return distribution, and the $100(1-\alpha)\%$ 10-day VaR is obtained as the α quantile of this distribution.

An EWMA volatility estimate is a constant, in the sense that it is equal for all time horizons. The EWMA model will estimate the same average volatility for all time horizons, whether the forecast is over the next day or over the next 10 days. For a risk horizon of 10 days, this doesn't seem as a very good risk model. For this reason we also base our forecasts on a GARCH model, which is described in the next section.

5.2.2 GARCH volatility

In this section we show how to estimate $\hat{\sigma}_t$ by using a GARCH(1,1) model (Bollerslev, 1986). The dynamic behaviour of the conditional variance is given by the following equation:

$$\hat{\sigma}_t^2 = \omega + \alpha r_{t-1}^2 + \beta\hat{\sigma}_{t-1}^2, \qquad t = 2, \ldots, T, \tag{5.5}$$

which is a function of the weighted long run variance, the previous squared return and the previous variance. Compared to the EWMA model it has an additional term for mean reversion (ω). Therefore the forecasts that are made from this model are not equal the current estimate. Instead volatility can be lower or higher than average on short term, but it will converge to the long term volatility as the risk horizon increases. The GARCH error parameter α measures the reaction of conditional volatility to market shocks. When α is relatively large (e.g. above 0.1) then volatility is very sensitive to market events. The GARCH lag parameter β measures the persistence in conditional volatility irrespective of anything happening in the market. When β is relatively large (e.g. above 0.9) then volatility takes a long time to die out following a crisis in the market. The sum $\alpha + \beta$ determines the rate of convergence of the conditional volatility to the long term average level. When $\alpha + \beta$ is relatively large (e.g. above

0.99) then the terms structure of volatility forecasts from the GARCH model is relatively flat. The GARCH constant parameter ω, together with the sum $\alpha+\beta$, determines the level of the long term average volatility, i.e. the unconditional volatility in the GARCH model. When $\omega/(1-\alpha-\beta)$ is relatively large (compared to the squared returns) then long term volatility in the market is relatively high.

We use Monte Carlo simulation to simulate a time series of returns that follow a GARCH process. We first fix the parameters of the GARCH model, by assuming that the conditional distribution is normal or Student t. Next, we simulate a path for the daily log returns over the next 10 days by using the recurrence in (5.5). We set $\hat{r}_t = \hat{\sigma}_t \tilde{Z}_t$ or $\hat{r}_t = \hat{\sigma}_t \tilde{T}_t$ where $\tilde{Z}_t$ and $\tilde{T}_t$ are respectively simulations from a standard normal variable and a standardized Student t variable and $\hat{\sigma}_t$ is computed using (5.5). Hence, the simulated log return over a risk horizon of 10 days is the sum $\hat{r}_1+\hat{r}_2+\ldots+\hat{r}_{10}$. Repeating this for 1000 simulations produces a simulated return distribution, and the $100(1-\alpha)\%$ 10-day VaR is obtained as the α quantile of this distribution.

5.2.3 EGARCH volatility

In the symmetric GARCH model, the effects of upward movements in daily returns on the conditional variance are assumed to be the same as the downward movements in daily returns. However, Black (1976) and many others have pointed out that there appears to be an asymmetry in stock market data: negative innovations to stock returns tend to increase volatility more than positive innovations of the same magnitude. A model that captures asymmetric behavior in the conditional variance is the Exponential GARCH model, or EGARCH model. The EGARCH(1,1) model, as introduced by Nelson (1991), is defined as:

$$\ln\hat{\sigma}_t^2 = \omega + \alpha\left|\frac{r_{t-1}}{\hat{\sigma}_{t-1}}\right| + \gamma\frac{r_{t-1}}{\hat{\sigma}_{t-1}} + \beta\ln\hat{\sigma}_{t-1}^2, \qquad t = 2,\ldots,T, \tag{5.6}$$

where the coefficients α and γ show the asymmetry in response to positive and negative r_{t-1}. This model is appealing because it uses logged conditional variance to relax the positiveness constraint of model coefficients. A 10-day VaR can be estimated by using the algorithm described in section 5.2.2, where the GARCH model is replaced by the EGARCH model.

6 Monte Carlo Simulation with Copulas

Copulas are multivariate distributions with uniform marginals that may be used to construct a huge variety of risk factor return distributions. The copula only models dependence; the marginal distribution of each of the asset returns may be anything we like.

In this section we show how to estimate VaR based on simulated returns for the equity and bond returns, with a dependency structure modeled by copulas. The normal and Student t copulas have a dependency structure that is captured by a correlation matrix, and this makes simulation based on these copulas very easy. We impose the dependency structure by using the Cholesky matrix of the correlation matrix. For Clayton copulas we simulate draws from this copula by using a conditional approach, i.e. conditional sampling. For all four different copulas we can choose the marginal distributions, i.e. normal marginals or Student t marginals. In the following sections we provide a step-by-step plan how to estimate VaR based on the different copulas.

6.1 Elliptical copulas

In this section we will discuss the step-by-step approach of the elliptical copulas (i.e. normal and Student *t* copulas), inspired by Alexander (2008), that are based on the Cholesky matrix to impose the dependency structure.

6.1.1 Normal copulas

The steps we perform for estimating the VaR based on simulation from a normal copula are the following:

1. Simulate two columns of 1000 independent standard uniform random numbers.
2. Transform these numbers into independent standard normal returns, by using the inverse standard normal distribution function.
3. Transform these independent standard normal returns into correlated bivariate standard normal returns, using the Cholesky matrix of the correlation matrix.
4. Apply the standard normal distribution function to the bivariate returns to obtain uniform marginals that have dependence defined by a normal copula.

Then we can impose any marginals we like upon these simulations to obtain simulated returns on financial assets that have these marginals and dependence defined by the normal copula.

6.1.2 Student *t* copulas

The steps we perform for estimating the VaR based on simulation from a Student *t* copula are the following:

1. Simulate two columns of 1000 independent standard uniform random numbers.
2. Transform these numbers into independent Student *t* returns, by using the inverse *t* distribution function with ν_c degrees of freedom, with ν_c the number of degrees of freedom of the copula distribution.
3. Transform these independent Student *t* returns into correlated bivariate Student *t* returns, using the Cholesky matrix of the correlation matrix.
4. Apply the Student *t* distribution function with ν_c degrees of freedom to the bivariate returns to obtain uniform marginals that have dependence defined by a Student *t* copula with ν_c degrees of freedom.

Then we can impose any marginals we like upon these simulations to obtain simulated returns on financial assets that have these marginals and dependence defined by the Student *t* copula.

6.2 Archimedean copulas

To apply Archimedean copulas in simulation we generally require a combination of the conditional copula distribution and the marginal distributions of the random variables. Quantile curves are a means of depicting these. In this section we define expressions for the conditional distributions and q quantile curves for the bivariate Clayton copula.

6.2.1 Clayton copulas

In the bivariate Clayton copula (Clayton, 1978) the conditional distribution of u_2 given u_1 is

$$C_{2|1}(u_2|u_1;\delta) = \frac{\partial}{\partial u_1}(u_1^{-\delta} + u_2^{-\delta} - 1)^{-1/\delta} = u_1^{-(1+\delta)}(u_1^{-\delta} + u_2^{-\delta} - 1)^{-(1+\delta)/\delta}, \tag{6.1}$$

where δ is the dependence parameter.

The q quantile curve of the Clayton copula may thus be written in explicit form, setting (6.1) equal to the fixed probability q and solve for u_2, giving the q quantile curve of the Clayton copula as

$$u_2 = C_{2|1}^{-1}(v|u_1) = (1 + u_1^{-\delta}(q^{-\delta/(1+\delta)} - 1))^{-1/\delta}. \tag{6.2}$$

The steps we perform for estimating the VaR based on simulation from a bivariate Clayton copula are then the following[6]:

1. Simulate two columns of 1000 independent standard uniform random numbers, v_1 and v_2
2. Set $u_1 = v_1$
3. Set $u_2 = (1 + v_1^{-\delta}(v_2^{-\delta/(1+\delta)} - 1))^{-1/\delta}$.

The resulting series, u_1 and u_2, are standard uniform random numbers with Clayton copula dependence. Then we can impose any marginals we like upon these simulations to obtain simulated returns on financial assets that have these marginals and dependence defined by the Clayton copula. A Clayton copula has positive lower tail dependence and zero upper tail dependence.

6.3 Calculation of VaR

In this section we describe the strategy to transform the simulated asset return series from the previous section into 10-day portfolio returns. We impose the characteristics of the daily return series by using unconditional volatility estimates and GARCH or EGARCH volatility estimates.

6.3.1 Unconditional volatility estimates

After simulating the two financial asset returns series, we use the 10-day mean and standard deviation to transform the observations into simulations with 10-day asset returns. Next, we apply the portfolio weights to these simulations and estimate the empirical α quantile of the simulated portfolio return distribution. This gives the $100(1-\alpha)\%$ 10-day VaR as a percentage of the portfolio value.

6.3.2 GARCH/EGARCH volatility estimates

The estimation of VaR and ES with unconditional volatility estimates is based on the underlying assumption of i.i.d. returns. As already described in section 3, this is a very unrealistic assumption, as markets pass through periods with low and high volatility. To capture volatility clustering, we use Monte Carlo simulation to simulate a time series of returns with copula dependence that follow a GARCH process. We first fix the parameters of the GARCH or EGARCH model, by assuming that the conditional distribution is normal or Student t. Next, we simulate a path for the daily log returns of the two asset returns separately over the next 10 days by using the recurrence in (5.5) or (5.6)

[6] See Cherubini et al. (2004), Chapter 6, for more details.

when using GARCH or EGARCH respectively. We set $\hat{r}_t = \hat{\sigma}_t \tilde{Z}_{t,c}$ or $\hat{r}_t = \hat{\sigma}_t \tilde{T}_{t,c}$ where $\tilde{Z}_{t,c}$ and $\tilde{T}_{t,c}$ are respectively simulations from standard normal marginals with copula dependence and a standardized Student t marginals with copula dependence and $\hat{\sigma}_t$ is computed using (5.5) or (5.6).

Hence, the simulated log return for each asset over a risk horizon of 10 days is the sum $\hat{r}_1 + \hat{r}_2 + \ldots + \hat{r}_{10}$. Repeating this for 1000 simulations produces a simulated return distribution for each asset, and we apply the portfolio weights to these simulated 10-day asset returns and construct the simulated portfolio return distribution. Finally, the $100(1-\alpha)\%$ 10-day VaR is obtained as the α quantile of this distribution.

7 Backtesting Methodology

Banking supervisors will only allow internal models to be used for regulatory capital calculation if they provide satisfactory results in backtests. The 1996 Amendment to the 1988 Basel Accord (see Basel Committee on Banking Supervision, 1996) contains a detailed description of the backtests that supervisors will review. The backtests to be applied compare whether the observed percentage of outcomes covered by the risk measure is consistent with a 99% level of confidence. That is, they attempt to determine if a bank's 99th percentile risk measures truly cover 99% of the firm's trading outcomes. The Basel Committee recommends a very simple type of backtest, which is based on a 99% VaR estimate and where the number of violations over the previous twelve months (250 trading days) of data is calculated. Hence, the expected number of violations over these twelve months is 2.5. Regulators wish to guard against VaR models whose estimates are too low.

Most backtests on 1-day VaR estimates are based on the assumption that the returns are generated by an i.i.d. Bernoulli process. A Bernoulli variable may take only two values, which could be labelled 1 and 0, or 'success' and 'failure'. In our context, we would call 'success' a violation of the VaR by the return, and further assign this the value 1. Thus we may define an indicator function $I_{\alpha,t+1}$ on the time series of daily returns relative to the $100(1-\alpha)\%$ 1-day VaR by

$$I_{\alpha,t+1} = \begin{cases} 1 & \text{if } r_{t+1} < \mathrm{VaR}_{1,\alpha,t} \\ 0 & \text{otherwise ,} \end{cases} \tag{7.1}$$

where r_{t+1} is the realized daily portfolio return at time $t+1$ and $\mathrm{VaR}_{1,\alpha,t}$ is the $100(1-\alpha)\%$ 1-day VaR estimate at time t.

It is more difficult to perform backtests on 10-day VaR estimates instead of 1-day VaR estimates. The reason for this is that 10-day VaR estimates are based on overlapping samples, since we estimate the 10-day VaR every day. Hence, in that case we can not use our standard assumption that violations follow an i.i.d. Bernoulli process. Violations would be positively autocorrelated (for instance, one extremely large daily loss would have impact on ten consecutive 10-day returns). For this reason, the backtesting framework described by the Basel Committee involves the use of risk measures calibrated to a 1-day holding period. Banks are thus required to perform the backtests described in the next sections on their 1-day VaR estimates instead of their 10-day VaR estimates.

7.1 Minimum Required Capital

For banks that use an internal VaR model to estimate the Minimum Required Capital (MRC) the general risk charge is calculated as k times the average of the 99% 10-day VaR over last 60 days, or yesterday's VaR on the current portfolio, if this is greater:

$$\text{MRC}_t = -\max\left(\frac{k}{60}\sum_{i=1}^{60}\text{VaR}_{10,0.01,t-i}, \text{VaR}_{10,0.01,t-1}\right). \tag{7.2}$$

To determine the minimum required capital, most regulators allow banks to base this value on 1-day VaR estimates and then scale these estimate up by using a square-root-of-time rule. But this rule is only valid for linear portfolios with i.i.d. normally distributed returns, and since most portfolios have non-normally distributed returns that are not i.i.d., we do not expect this rule to be an accurate scaling method. Therefore, we base our calculations of the minimum required using different risk models on the 99% 10-day VaR estimates, for which we already described more reliable scaling methods.

The multiplier k takes a value between 3 and 4 depending on the model's backtesting results. If backtests reveal statistical inaccuracies in the VaR estimates, k takes a higher value or the VaR model may be disallowed. Table 1 show the value of k for several possible numbers of violations over the previous 250 trading days, set by the Basel Committee. Since these values are based on violations for 1-day VaR estimates, we should not base this test on our 10-day VaR estimates, but on 1-day VaR estimates. Unfortunately this may cause inaccurate risk models to pass the regulatory backtest.

Regulators wish to guard against VaR models whose estimates are too low. Since they are very conservative they will only consider that models having 4 exceptions or less as sufficiently accurate. These so-called *green zone* models have a multiplier of 3, which corresponds to backtesting results that do not themselves suggest a problem with the quality or accuracy of a bank's model. If there are between 5 and 9 exceptions, the model is *yellow zone*, which means it is admissible for regulatory capital calculations but the multiplier is increased and will lie between 3 and 4. A *red zone* model means there are 10 or more exceptions. Then the multiplier takes its maximum of value 4, or the VaR model is disallowed. This backtesting framework should be performed on a quarterly basis, so every quarter the multiplier is updated, based on the number of violations over the previous 250 trading days.

Table 1: Basel Accord Penalty Zones

Zone	Number of Violations	k
Green	0 to 4	3.00
Yellow	5	3.40
	6	3.50
	7	3.65
	8	3.75
	9	3.85
Red	10 or more	4.00

Note: The number of violations is given for 250 trading days. The penalty structure under the Basel II Accord is specified for the number of violations and not their magnitude, either individually or cumulatively.

7.2 Coverage Tests

Unconditional coverage tests, introduced by Kupiec (1995), test if the fraction of violations obtained for a particular risk model, call it π, is significantly different from the promised fraction, α. They may be regarded as a more sophisticated and flexible version of the banking regulators' backtesting rules described above. The idea was both formalized and generalized by Christoffersen (1998) to include tests on the independence of violations (i.e. whether violations come in clusters) and conditional coverage tests (which combine unconditional coverage and independence into one test).

7.2.1 Unconditional Coverage Test

An unconditional coverage test is a test of the null hypothesis that the indicator function in (7.1), which is assumed to follow an i.i.d. Bernoulli process, has a constant 'success' probability equal to the significance level of the VaR, α. We call this the unconditional coverage hypothesis. In other words, the number of violations should be sufficiently close to the number of expected violations. In order to test this we define the likelihood function

$$L(\pi) = (1-\pi)^{n_0}\pi^{n_1}, \tag{7.3}$$

where n_1 and n_0 is the number of violations and non-violations respectively, $(n_0+n_1=n)$.

We can easily estimate π using the observed proportion of violations, $\pi_{obs} = \frac{n_1}{n}$. Plugging the ML estimates back into the likelihood function gives the optimized likelihood as

$$L(\pi_{obs}) = (1-\pi_{obs})^{n_0}\pi_{obs}^{n_1}. \tag{7.4}$$

Under the unconditional coverage null hypothesis that $\pi_{exp} = \pi_{obs}$, where π_{exp} is equal to α, the expected proportion of violations, we have the likelihood

$$L(\pi_{exp}) = (1-\pi_{exp})^{n_0}\pi_{exp}^{n_1}. \tag{7.5}$$

The test statistic of the unconditional coverage hypothesis is a likelihood ratio statistic given by

$$LR_{uc} = -2\ln\left[\frac{L(\pi_{exp})}{L(\pi_{obs})}\right] = \frac{(1-\pi_{exp})^{n_0}\pi_{exp}^{n_1}}{(1-\pi_{obs})^{n_0}\pi_{obs}^{n_1}} \sim \chi_1^2. \tag{7.6}$$

7.2.2 Independence Test

Even if the number of observed VaR violations is fairly close to the expected number of violations, we do not want those violations to occur around the same time. For example, if the 99% VaR gave exactly 1% violations but all of these violations came during a one-week period, then the risk of bankruptcy would be much higher than if the violations came scattered randomly through time. We therefore would very much like to reject VaR models which imply violations that are clustered in time. Clustering of violations indicates that the VaR model is not sufficiently responsive to changing market circumstances. Some of our models do not account for the volatility clustering that we know is prevalent in many markets. Even if these models pass the unconditional coverage test, we could still reject the VaR model if the violations are not independent.

If the VaR violations are clustered then we can essentially predict that if today is a violation, then tomorrow a violation will occur with a probability larger than $100\alpha\%$. This is clearly not satisfactory. In such a situation we should increase the VaR in order to lower the conditional probability of a violation to the promised α.

A test for independence of violations is based on the formalization of the notion that when violations are not independent the probability of a violation tomorrow, given there has been a violation today, is no longer equal to α. As in the unconditional coverage test, we define n_1 and n_0 as the number of violations and non-violations respectively, $(n_0+n_1=n)$. Further, we define n_{ij} to be the number of returns with indicator value i followed by indicator value j. For example, n_{01} is the number of non-violations followed by a violation. It follows that $n_1 = n_{11} + n_{01}$ and $n_0 = n_{10} + n_{00}$.

We assume that the sequence of violations is dependent over time and that it can be described as a so-called first-order Markov sequence with transition probability matrix

$$\Pi_1 = \begin{bmatrix} 1-\pi_{01} & \pi_{01} \\ 1-\pi_{11} & \pi_{11} \end{bmatrix} \tag{7.7}$$

where π_{01} is the proportion of violations, given that the last return was a non-violation, and π_{11} is the proportion of violations, given that the last return was a violation. We can write the likelihood function of the first-order Markov process as

$$L(\Pi_1) = (1-\pi_{01})^{n_{00}}\pi_{01}^{n_{01}}(1-\pi_{11})^{n_{10}}\pi_{11}^{n_{11}}. \tag{7.8}$$

Taking first derivatives with respect to π_{01} and π_{11} and setting these derivatives to zero, one can solve for the Maximum Likelihood estimates

$$\hat{\pi}_{01} = \frac{n_{01}}{n_{00}+n_{01}} \qquad \text{and} \qquad \hat{\pi}_{11} = \frac{n_{11}}{n_{10}+n_{11}}, \tag{7.9}$$

which leads to the matrix of estimated transition probabilities

$$\hat{\Pi}_1 \equiv \begin{bmatrix} \hat{\pi}_{00} & \hat{\pi}_{01} \\ \hat{\pi}_{10} & \hat{\pi}_{11} \end{bmatrix} = \begin{bmatrix} 1-\hat{\pi}_{01} & \hat{\pi}_{01} \\ 1-\hat{\pi}_{11} & \hat{\pi}_{11} \end{bmatrix} = \begin{bmatrix} \frac{n_{00}}{n_{00}+n_{01}} & \frac{n_{01}}{n_{00}+n_{01}} \\ \frac{n_{10}}{n_{10}+n_{11}} & \frac{n_{11}}{n_{10}+n_{11}} \end{bmatrix}. \tag{7.10}$$

If the violations are independent over time, then the probability of a violation tomorrow does not depend on today being a violation or not and we write $\pi_{01} = \pi_{11} = \pi$. Under independence the transition matrix is thus

$$\hat{\Pi} = \begin{bmatrix} 1-\hat{\pi} & \hat{\pi} \\ 1-\hat{\pi} & \hat{\pi} \end{bmatrix}. \tag{7.11}$$

Now we can state the independence test statistic, derived by Christoffersen (1998), as

$$LR_{ind} = -2\ln\left[\frac{L(\hat{\pi})}{L(\Pi_1)}\right] = \frac{(1-\pi_{obs})^{n_0}\pi_{obs}^{n_1}}{(1-\pi_{01})^{n_{00}}\pi_{01}^{n_{01}}(1-\pi_{11})^{n_{10}}\pi_{11}^{n_{11}}} \sim \chi_1^2. \tag{7.12}$$

7.2.3 Conditional Coverage Test

Ultimately, we care about simultaneously testing if the VaR violations are independent and the average number of violations is correct. We can test jointly for independence and correct coverage using the conditional coverage test

$$LR_{cc} = -2\ln\left[\frac{L(\pi_{exp})}{L(\Pi_1)}\right] = \frac{(1-\pi_{exp})^{n_0}\pi_{exp}^{n_1}}{(1-\pi_{01})^{n_{00}}\pi_{01}^{n_{01}}(1-\pi_{11})^{n_{10}}\pi_{11}^{n_{11}}} \sim \chi_2^2. \tag{7.13}$$

8 Prediction of bull and bear markets

In this section we propose a method for identifying and predicting bull and bear periods. We compare three different criteria for selecting the optimal set of predicting variables. First, we examine the identification of bull and bear periods, based on the S&P500 Index. Then, by using a logit model, we investigate which variable selection criterium leads to the highest out-of-sample hitrate. Since we want to base the choice of a risk model on the prediction of the market condition, we have to make sure we can predict this market condition reasonably well. We show that a pragmatic and relatively simple model can be very effective in predicting bull and bear markets.

8.1 Predicting variables

We consider macro-economic and financial variables to predict whether the next month will be a bull month or a bear month. All variables were measured at monthly frequencies over the period 1961M1 to 2010M12, and stock prices were measured by the S&P500 Index at close on the last trading day of each month, obtained from Yahoo! Finance.

Motivated by prior studies (e.g., Pesaran and Timmermann, 1995; Rapach et al., 2005; Chen, 2009; Kole and Van Dijk, 2010) we establish a set of predicting variables over which the search for a "satisfactory" prediction model could be conducted. The set consists of a constant, which is always included in the model, as well as 10 predicting variables, namely inflation rates (consumer prices), narrow money stock (M1), broad money stock (M2), aggregate output (industrial production), unemployment rates, trade weighted exchange rates, 3-month T-Bill rate, yield spreads (difference between the 10-year government bond yield and the 3-month treasury bill rate), credit spread (difference between Moody's BAA and AAA corporate bond yields) from the FRED database of the Federal Reserve Bank of St. Louis and the dividend yield, obtained from Thompson Datastream.

For all the variables mentioned above, unit root tests were conducted to investigate whether these series were stationary. The results are provided in the third column of Table 2. Some of the variables exhibit a unit root, but this does not have to be problematic. Park and Phillips (2000) show that logit models with non-stationary explanatory variables will provide consistent maximum likelihood (ML) estimators but a new phenomenon arises in its limit distribution theory. The estimator consists of a mixture of two components, one of which is parallel to and the other orthogonal to the direction of the true parameter vector, with the latter being the principal component. The ML estimator is shown to converge at a rate of $n^{3/4}$ along its principal component but has the slower rate of $n^{1/4}$ convergence in all other directions.

Nevertheless, we transform some of the predictive variables to ensure stationarity, which is shown in the last column of Table 2. For the T-Bill rate, the trade weighted exchange rate and the dividend yield we construct a stationary series by subtracting 12-month backward-looking average from each observation, used more often in forecasting (see e.g., Campbell, 1991; Rapach et al., 2005). For the unemployment rate we construct yearly differences. We transform the industrial production series to yearly growth rates. For the narrow and broad money growth we take the first difference in the loglevels of the defined money stocks (see Rapach et al., 2005). We do not transform the inflation rate, the yield spread or the credit spread series.

The early studies of stock returns are not always clear on what they consider to be the appropriate time lags between the changes in the business cycle variables and stock returns. Here, following Pesaran and Timmermann (1995), we decide to include the most recently available values of the variables in the base set of regressors. The lag associated with the publication of macroeconomic indicators means that these variables must be included in the base set with a 2-month time lag. The financial variables are included with a 1-month time leg.

The recursive model selection and estimation strategy is based on monthly observations over the period 1961M1 to 2010M12. Every month we update the model parameters, using an moving window of 20 years (240 monthly observations).

Table 2: Characteristics of predicting variables

	AR(1)	ADF	Transformation
Inflation rate	0.825	-3.15	
Narrow money stock	1.001	-0.86	First difference in log-levels
Broad money stock	1.001	-2.05	First difference in log-levels
Industrial production	1.002	-0.56	Yearly growth rate
Unemployment rate	1.001	-2.79	Yearly change
Exchange rate	1.001	-1.33	Difference with a 12-month moving average
3-month T-Bill rate	0.997	-2.64	Difference with a 12-month moving average
Yield spread	0.983	-3.36	
Credit spread	0.995	-2.89	
Dividend yield	0.999	-1.12	Difference with a 12-month moving average

Notes: This table shows the set of predicting variables for which we conduct an adjusted Dickey-Fuller test. AR(1) is the first order autocorrelation coefficient, ADF is the Augmented Dickey-Fuller test statistic. In each test, the null hypothesis is that the series has a unit root. Test critical values for ADF are -3.44 (1%), -2.87 (5%) and -2.57 (10%). Lags in ADF test are chosen by the Modified Akaike Information Criterion. If this hypothesis is not rejected, the last column shows the transformation that is applied to the variable. The series run from January 1981 until December 2010.

8.2 Identification of bull and bear markets

An important aspect of bull and bear markets is that they describe the long-term trend, not short term changes. In other words, within a bear (bull) market there has to be room for small positive (negative) returns. Lunde and Timmermann (2004) define a stock market switch from a bull state to a bear state if stock prices have declined by a certain percentage since their previous (local) peak within that bull state. Likewise, a switch from a bear state to a bull state occurs if stock prices experience a similar percentage increase since their local minimum within that state. This definition does not rule out sequences of negative (positive) price movements in stock prices during a bull (bear) market as long as their cumulative value does not exceed a certain threshold. By abstracting from the small unsystematic price movements that dominate time series as noisy as daily price changes, this definition is designed to capture long-run dependencies in the underlying drift in stock prices.

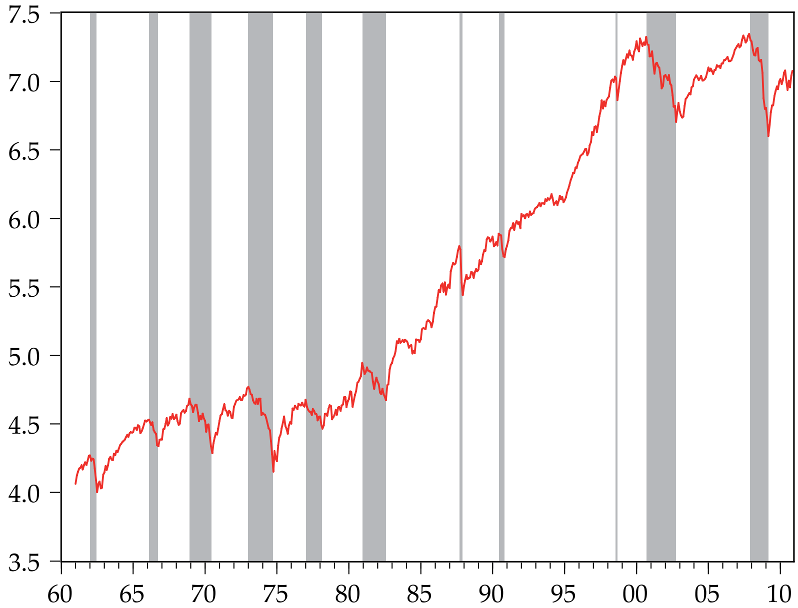
We define λ_1 as the threshold of movements in stock price that trigger a switch from a bear market to a bull market, λ_2 as the threshold for a switch from a bull market to a bear market. The choice of the values for λ_1 and λ_2 should be chosen wisely. The smaller the values at which these parameters are set, the more bull and bear market spells we expect to see. However, there are also limits to how low λ_1 and λ_2 can be set because too-small values will lead our analysis to capture short-term dynamics in stock price movements. Setting $\lambda_1 > \lambda_2$ provides a way to account for the upward drift in stock prices. In our research, finding optimal values for λ_1 and λ_2 is of less importance, so we follow Lunde and Timmermann (2004) by setting λ_1 to 20% and λ_2 to 15%.

After identifying the bull and bear markets, we obtain a binary variable:

$$y_t = \begin{cases} 1 & \text{if bull market at time } t \\ 0 & \text{if bear market at time } t \end{cases}$$

Insight into how the definition described above partitions stock prices into bull and bear periods is provided by Figure 2, which uses the stock price index to show the sequence of consecutive bull and bear markets over the full sample period 1961-2010. Table 3 presents descriptive statistics for bull and bear market durations, which are reported in months. The 20/15 filter splits the sample into 23 bull and bear markets. Many of the bull markets are very long compared to the length of the bear markets, with the longest lasting from 1990 to 1998 (92 months). The longest bear market, which started in 2000 by the burst of the dot-com bubble, lasted for more than two years.

Figure 2: Identification of bull and bear markets.



Notes: This figure shows the identification of bull and bear periods for the US for the period January 1, 1961 to December 31, 2010, based on the S&P500 Stock Price Index. The red line shows the S&P500 Stock Price Index (on log-scale) and the grey shadings track the bear markets derived from this index.

Table 3: Summary Statistics for Bull and Bear Market Durations

Bull					Bear				
Durations	Mean	Median	Min.	Max.	Durations	Mean	Median	Min.	Max.
12	38.42	30	12	92	11	12.64	14	2	25

Notes: This table presents descriptive statistics for the distribution of bull and bear market durations for the US for the period January 1, 1961 to December 31, 2010, based on the S&P500 Stock Price Index.

8.3 Methodology

The next step is to relate the resulting series of bull and bear states to a set of explanatory variables. At each point in time, t, Pesaran and Timmermann (1995) search over a base set of κ regressors to make one period ahead forecasts of y_t using only information that is publicly available at the time. We simulate the search for a forecasting model by applying three different criteria for model selection, to the set of regression models spanned by all possible permutations of the κ regressors $\{x_1, x_2, \ldots, x_\kappa\}$ in the base set. This gives a total of 2^κ different models, M, each of which is uniquely identified by a number, i, between 1 and 2^κ.

Since the dependent variable y_t is binary, it can be forecasted by a logit or probit model. Kole and Van Dijk (2010) opt for a logit model (Berkson, 1944), as this model can be easily extended to a multinomial logit model when more states are present. We consider the following logit model:

$$M_i : P(y_t = 1) = F(\mathbf{X}_{i,t}^{'}\beta) = \frac{\exp(\mathbf{X}_{i,t}^{'}\beta)}{1+\exp(\mathbf{X}_{i,t}^{'}\beta)} \tag{8.1}$$

where $\mathbf{X}_{i,t}$ is a $(\kappa_i + 1) \times 1$ vector of regressors under model M_i, obtained as a subset of the base set of regressors, $\mathbf{X}_t$, plus a vector of ones for the intercept term.

The particular choice of $\mathbf{X}_{i,t}$ to be used in forecasting of y_{t+1} can be based on a number of statistical model selection criteria, such as Akaike's Information Criterion (AIC) (Akaike, 1973), Schwarz's Information Criterion (SIC) (Schwarz, 1978), or the in-sample hitrate. The first two criteria are likelihood-based and assign different weights to the parsimony and fit of the models. The fit is measured by the maximized value of the log-likelihood function ($\widehat{\text{LL}}$), and the parsimony by the number of freely estimated coefficients.

The Akaike and Schwarz model selection criteria are defined as

$$\text{AIC}_{i,t} = \widehat{\text{LL}}_{i,t} - (\kappa_i + 1) \tag{8.2}$$

$$\text{SIC}_{i,t} = \widehat{\text{LL}}_{i,t} - 1/2(\kappa_i + 1)\ln(t) \tag{8.3}$$

Based on these beliefs, we establish a base set of potential forecasting variables and, at each point in time, search for a reasonable model specification, capable of predicting y_{T+1}, across this set. Notably, this procedure assumes that, at each point in time, we use only historically available information to select a model according to a predefined model selection criterion and then use the chosen model to make one-period ahead predictions of the market condition.

In each case the model selection criteria described above were applied to logit models using the S&P500 price index as the dependent variable and subsets of the base set of regressors as the independent variables. For our set of ten regressors, this means comparing $2^{10} = 1024$ models at each point in time, and over the period 1980M12 to 2010M11 this gives a total of 368,640 regressions to be computed.

The model that minimizes the AIC, SIC or maximizes the in-sample hitrate is chosen, and the parameter values estimated with observations over the past twenty years are used to forecast y_{T+1}. The outcome of the logistic regression will be $P(y_{T+1} = 1)$, which is the predicted probability belonging to a bull market at time $T + 1$. The predicted probability belonging to a bear market is $1 - P(y_{T+1} = 1)$. The next step is to use a cut-off value, c, on these probabilities in order to classify them as a bull prediction or a bear prediction. Cramer (1999) shows that in a binary logit analysis with unequal sample frequencies of the two outcomes the less frequent outcome always has lower estimated prediction probabilities than the other outcome. As shown in Figure 2, each window of twenty years contains significantly more bull months than bear months. Therefore we set c to the proportion of bull months in our window. This means c can vary over time.

Kole and Van Dijk (2010) state that to form the one-period ahead prediction for y_{T+1}, the prevailing state at time T is needed. For the rules-based approaches, this information may not be available. In the approach of Lunde and Timmermann (2004), only if the stock price index at time t equals the last observed maximum (minimum), and is a fraction λ_1 above (λ_2 below) the prior minimum is the market surely in a bull (bear) state. So, the market may already have switched, but this will only become obvious later. In that case, the state of the market is known until the period of the last extreme value, which we denote with $T^* \leq T$.

9 Empirical Results

9.1 Prediction of bull and bear markets

As discussed in section 8, we estimated a total of 368,640 models over the 1980M12 to 2010M11 period. Clearly, we cannot supply the reader with all the details of the estimation results. In this section we provide the main results.

Table 4 shows, for every variable, the percentage of months where it was included in the model, according to the three different selection criteria. As to be expected, the average number of variables included in the model is higher for the in-sample hitrate criterion and the AIC than for the SIC. As can be seen in (8.2) and (8.3), the SIC puts a heavier penalty for inclusion of an additional variable than the AIC. It is clear that the in-sample hitrate criterion does not put any restrictions on the inclusion of additional variables. In Appendix A we provide some graphic displays of the inclusion frequency of the variables in the base set under the three different selection criteria.

Table 4: Percentage of periods where a regressor is included in forecasting equations

	In-Sample Hitrate			AIC			SIC		
	Bear	Bull	Total	Bear	Bull	Total	Bear	Bull	Total
Inflation rate	48.8	52.9	51.9	72.3	68.2	69.2	35.3	30.5	31.7
Relative Narrow money growth	51.2	27.9	33.3	18.1	7.2	9.7	00.0	00.0	00.0
Relative Broad money growth	45.2	35.1	37.5	18.7	10.1	11.9	00.0	00.0	00.0
Industrial production yearly growth	77.4	62.0	65.6	68.7	95.3	89.2	63.5	85.5	80.3
Yearly change unemployment rate	66.7	64.1	64.7	60.2	88.8	82.2	37.6	60.4	55.0
Relative trade weighted exchange rate	42.9	29.3	32.5	34.9	26.4	28.3	18.8	12.4	13.9
Relative 3-month T-Bill rate	57.1	44.9	47.8	36.1	18.8	22.8	30.6	17.8	20.8
Yield spread	86.9	84.1	84.7	91.6	88.1	88.9	84.7	78.2	79.7
Credit spread	39.3	54.0	50.6	45.8	49.8	48.9	24.7	35.6	33.1
Relative dividend yield	85.7	79.7	81.1	81.9	92.4	90.0	83.5	84.7	84.4
Average number of variables	5.5	4.8	5.0	4.6	4.8	4.7	3.4	3.7	3.7

Notes: This table shows the percentage of periods where a regressor is included in forecasting equations. The results are based on the selection of variables recursively over the period 1980M12 to 2010M11. Each month the set of regressors that maximizes a given model selection criterium was determined and used to forecast the market condition one month ahead. See Section 8.3 for a definition of the statistical model selection criteria.

Besides the statistical performance, most literature on forecasting of the market condition pays attention to the investment performance of every method analyzed. In our research, we want to condition the risk model choice on the prediction of bull and bear markets, which means our main priority here is to predict as many bull and bear months correctly. Therefore, we only look at the statistical quality of our predictions. We measure the statistical quality by the out-of-sample hitrate and the results are shown in Table 5. Looking at these results, the selection of predicting variables based on the Schwarz Selection Criteria performs best, with a hitrate of more than 85%. However, the hitrates of the other two criteria are only slightly lower. Clearly, there are no significant differences in the performance of the model selection criteria in predicting bull and bear markets.

Table 5: Predictive accuracy

	In-sample hitrate Forecast			**AIC** Forecast			**SIC** Forecast		
	Bear	Bull	Total	Bear	Bull	Total	Bear	Bull	Total
Bear	48	22	70	49	21	70	51	19	70
Bull	36	254	290	34	256	290	34	256	290
Total	84	276	360	83	277	360	85	275	360
Correct	48	254	302	49	256	305	51	256	307
% Correct	57.1	92.0	83.9	59.0	92.4	84.7	60.0	93.1	85.3
% Incorrect	42.9	8.0	6.1	41.0	7.6	15.3	40.0	6.9	14.7

Notes: This table shows the predictive accuracy of the logit model, according to three different selection criteria, namely the in-sample hitrate, the Akaike Selection Criterion (AIC) and the Schwarz Information Criterion (SIC). Every month t a one-step-ahead forecast for month $t+1$ is made for the probability of a bull and of a bear month. The first prediction is made for 1981M1 and the last for 2010M12, giving a total of 360 predictions.

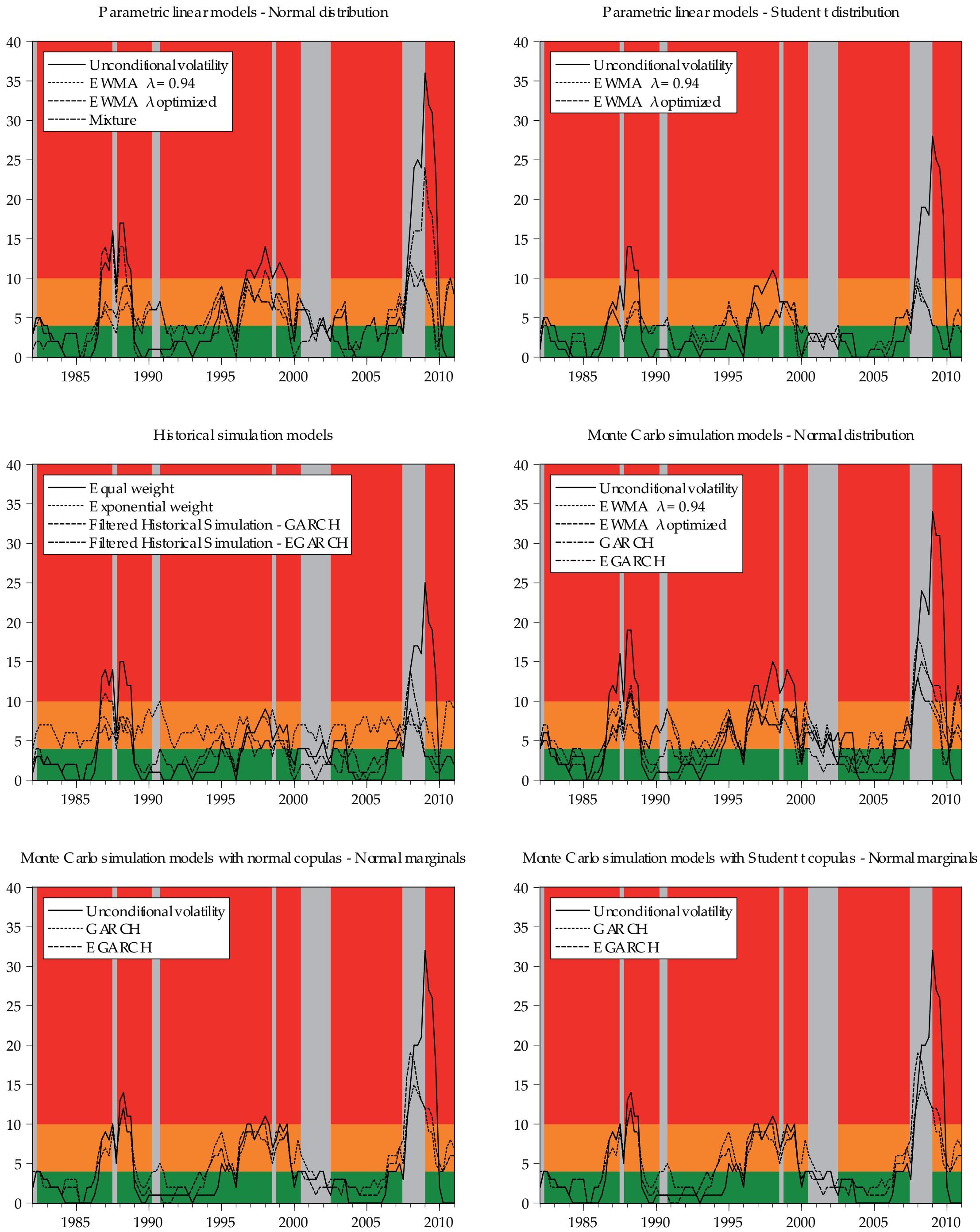
9.2 Backtests VaR - Single risk models

We first check for each model whether the number of VaR violations corresponds to the expected number of violations, which is 2.5 per year for a 99% VaR. In Figure 3, each graph displays the accumulated number of violations of all models for the estimation of the 1-day VaR[7] over the previous 250 trading days, evaluated each quarter. These violations are based on the VaR estimates for the 50/50 portfolio, but the results for the other portfolios, which are shown in Appendix B, are very similar. The colors of the horizontal shaded areas indicate the Basel zones for VaR models (see Table 1) and the vertical shaded areas track the bear markets derived from the S&P500 Index. We see that almost all models have a relatively large number of violations in 1987, which corresponds to the stock market crash on Monday 19, 1987. Furthermore, other peaks in the number of violations occur in 1998 due to the crisis in Russia, in 2000 due to the collapse of the internet bubble and in 2008 due to the recent banking crisis. It is clear that most models do not manage to stay out of the red zone over the whole testing period. When these models reach the number of 10 violations during one financial year, the model enters the red zone and might be disallowed by the banking regulator.

Tables 6 to 16 set out the results for backtesting the 99% 1-day VaR and 99% 10-day VaR for the 50/50 portfolio. The results for the 20/80 portfolio and the 80/20 portfolio are very similar and are provided in Appendix C. The results include the coverage tests, the average minimum required capital and the maximum number of violations for each model over the previous 250 trading days, which corresponds to the highest peak in Figure 3. The results in these tables show that only two models manage to stay out of the red zone for the 1-day VaR estimates during the whole testing period, i.e. the filtered historical simulation model with GARCH volatility and the parametric linear Student t model with EWMA volatility with an optimized λ, each having a maximum of nine violations over the previous 250 trading days, which occurred in 2007 for both models.

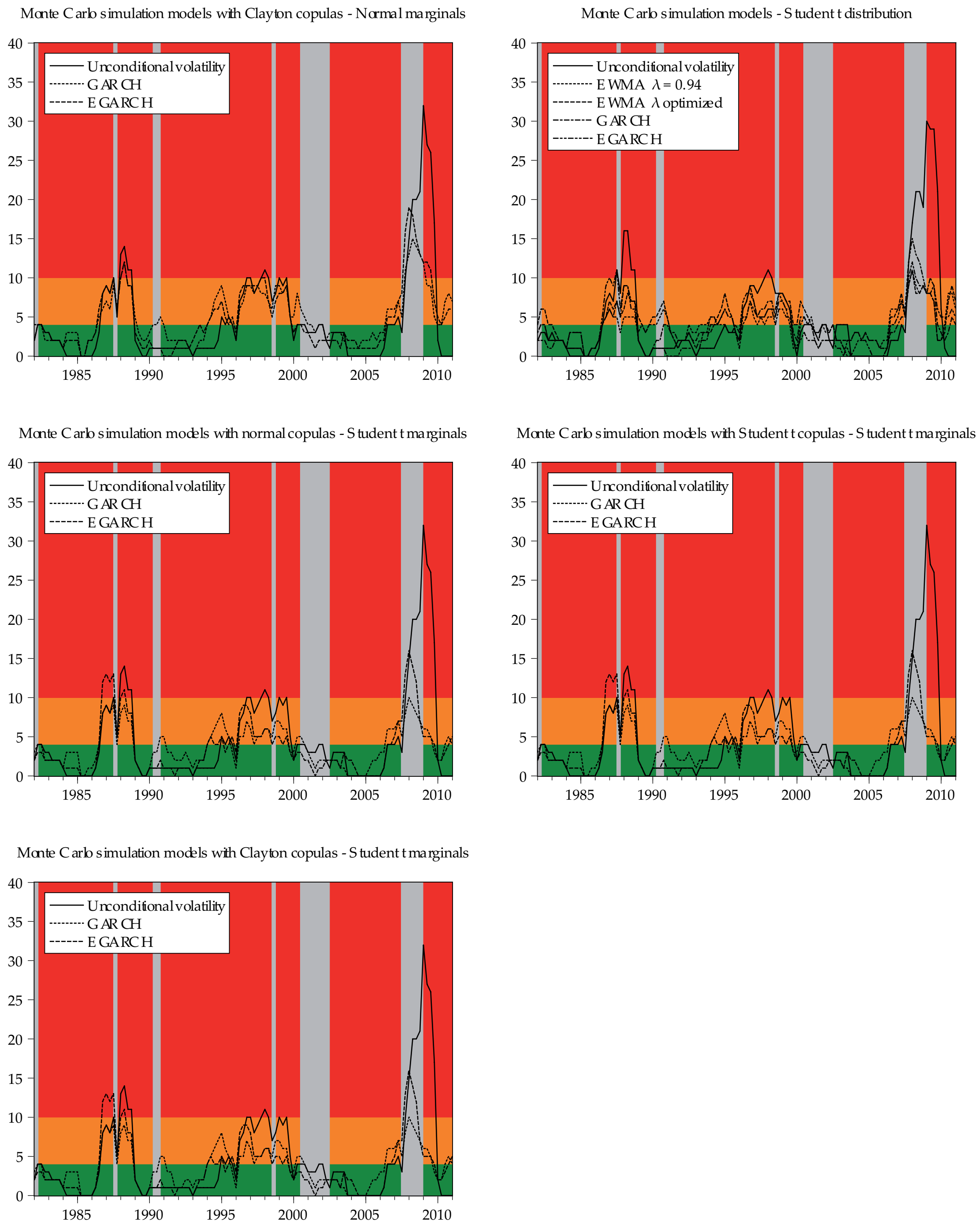
[7] In section 7 we explained why we use the 1-day VaR estimates instead of the 10-day VaR estimates for these backtests.

Figure 3: Number of violations for the 50/50 portfolio accumulated over the previous 250 trading days



Notes: This figure shows graphic displays of the number of violations for the 50/50 portfolio of all models for the estimation of VaR on a 1-day risk horizon accumulated over the previous 250 trading days, evaluated each quarter. The colors of the horizontal shaded areas indicate the Basel zones for VaR models and the vertical shaded areas track the bear markets derived from the S&P500 Index.

Figure 3: Number of violations for the 50/50 portfolio accumulated over the previous 250 trading days - *Continued*



Notes: For figure notes, see the first part of the figure.

For the parametric linear models (Table 6 and 7), the assumption of normally distributed returns leads to an underestimation of VaR. The observed number of violations are almost twice as high as the number of expected violations. The model with a mixture of two normal distributions, which should capture excess kurtosis and negative skewness in the portfolio returns, does indeed lead to a lower number of violations, but this number is still significantly higher than the number of violations we would expect for a 99% confidence level. The assumption of Student t distributed returns logically leads to a more conservative estimation of VaR and therefore also to less violations, compared to the normal assumption. Imposing EWMA volatility leads to more reliable volatility estimates for 1-day VaR estimates, resulting in a lower observed number of violations. However, on a 10-day horizon the EWMA models do not significantly outperform the model with unconditional volatility estimates.

As expected, the number of observed violations for historical simulation models with equal weight or exponential weight do not come close to the expected number of violations. Nevertheless, imposing a volatility clustering structure by using filtered historical simulation results in a significant lower number of violations. Filtered historical simulation with GARCH volatility leads to 92 violations for a 10-day risk horizon, which is not significantly different from the number of expected violations. The EGARCH volatility model overestimates VaR on a 10-day horizon and has an observed number of violations significantly lower than the expected number of violations.

For the estimation of the 1-day VaR with Monte Carlo simulation models, Tables 9 and 13 show that imposing volatility clustering does not lead to significantly better results than assuming unconditional volatility. However, on a 10-day risk horizon, imposing path dependence behavior by using GARCH or EGARCH volatility leads to an observed number of violations relatively close to the expected number of violations. Especially the model with EGARCH volatility estimates, which captures asymmetric behavior in the conditional variance, approaches the expected number of violations reasonably well.

As can be seen in Tables 10 to 12 and 14 to 16, imposing copula dependence does not lead to significantly better results compared to the Monte Carlo simulation models with uncorrelated asset returns. However, to diversificate risk, portfolio managers will hold portfolios containing much more assets than the two-asset portfolios our results are based on. With portfolios containing more assets with stronger dependence, implementing copula dependence is likely to lead to relatively better results.

For the 1-day VaR estimates the number of consecutive violations is relatively low, especially for models which capture volatility clustering. For most of these models the independence test is not rejected. However, estimating VaR on a 10-day horizon leads to a much higher number of consecutive violations. For all models, the hypothesis of independent violations is convincingly rejected for the 10-day VaR estimates. This is a clear indication of positive autocorrelation of the VaR violations.

The average minimum required capital for all models shows that a trade off takes place between the multiplier and the VaR estimate in equation (7.2). High VaR estimates have an upward effect on the MRC, but they also lead to a lower number of violations, and thus to a low multiplier, which has a downward effect on the MRC. However, we should also take into account that a bank is not allowed to use a red zone model. Therefore, in the evaluation of the average MRC, we should only take into account the filtered historical simulation model with GARCH volatility and the parametric linear Student t model with EWMA volatility with an optimized λ. The latter models has the lowest average MRC (24.28% of the portfolio value) and can therefore, based on this criterium, be qualified as best performing model.

Table 6: Backtests on VaR for the 50/50 portfolio - Parametric linear models - Normal distribution

		Unc. vol.	EWMA $\lambda = 0.94$	EWMA $\hat{\lambda}$	Normal mixture
1-day VaR	Expected # violations	76	76	76	76
	Observed # violations	176	146	147	125
	# consecutive violations	17	8	8	13
	Unconditional coverage	<0.001	<0.001	<0.001	<0.001
	Independence	<0.001	0.010	0.011	<0.001
	Conditional coverage	<0.001	<0.001	<0.001	<0.001
	Average MRC	0.0761	0.0729	0.0726	0.0864
	Maximum # violations	36	12	11	24
10-day VaR	Expected # violations	76	76	76	76
	Observed # violations	153	156	157	64
	# consecutive violations	101	103	101	46
	Unconditional coverage	<0.001	<0.001	<0.001	0.165
	Independence	<0.001	<0.001	<0.001	<0.001
	Conditional coverage	<0.001	<0.001	<0.001	<0.001
	Average MRC	0.2304	0.2276	0.2251	0.3258
	Maximum # violations	35	15	16	13

Notes: This table shows the backtest results for 1-day VaR estimates and 10-day VaR estimates for the 50/50 portfolio of different parametric normal linear models, i.e. a model with unconditional volatility (Unc. vol.), a model with EWMA volatility with λ equal to 0.94, a model with EWMA volatility with λ optimized and a model based on a mixture of two normal distributions. For both risk horizons, the first row indicates the expected number of violations, which corresponds to the 99% confidence level on which the VaR estimates are based. The second and third row show the observed numbers of violations and the observed number of consecutive violations, respectively. The *p*-values of the coverage tests are presented in rows 4 to 6. The last two rows give the average minimum required capital over the whole testing period and the maximum number of observed violations over the previous 250 trading days. The backtests are based on VaR estimates made on a daily basis from January 1st, 1981 to December 31st, 2010, giving a total of 7571 estimates to be evaluated for the 10-day VaR estimates and 7580 for the 1-day VaR estimates.

Table 7: Backtests on VaR for the 50/50 portfolio - Parametric linear models - Student *t* distribution

		Unc. vol.	EWMA $\lambda = 0.94$	EWMA $\hat{\lambda}$
1-day VaR	Expected # violations	76	76	76
	Observed # violations	123	95	100
	# consecutive violations	10	4	3
	Unconditional coverage	<0.001	0.033	0.008
	Independence	<0.001	0.040	0.205
	Conditional coverage	<0.001	0.012	0.013
	Average MRC	0.0820	0.0760	0.0755
	Maximum # violations	28	10	9
10-day VaR	Expected # violations	76	76	76
	Observed # violations	114	115	114
	# consecutive violations	71	74	71
	Unconditional coverage	<0.001	<0.001	<0.001
	Independence	<0.001	<0.001	<0.001
	Conditional coverage	<0.001	<0.001	<0.001
	Average MRC	0.2527	0.2450	0.2428
	Maximum # violations	26	14	14

Notes: This table shows the backtest results for 1-day VaR estimates and 10-day VaR estimates for the 50/50 portfolio of different parametric Student *t* linear models, i.e. a model with unconditional volatility (Unc. vol.), a model with EWMA volatility with λ equal to 0.94 and a model with EWMA volatility with λ optimized. For both risk horizons, the first row indicates the expected number of violations, which corresponds to the 99% confidence level on which the VaR estimates are based. The second and third row show the observed numbers of violations and the observed number of consecutive violations, respectively. The *p*-values of the coverage tests are presented in rows 4 to 6. The last two rows give the average minimum required capital over the whole testing period and the maximum number of observed violations over the previous 250 trading days. The backtests are based on VaR estimates made on a daily basis from January 1st, 1981 to December 31st, 2010, giving a total of 7571 estimates to be evaluated for the 10-day VaR estimates and 7580 for the 1-day VaR estimates.

Table 8: Backtests on VaR for the 50/50 portfolio - Historical simulation models

		Equal weight	Exp. weight	FHS - GARCH	FHS - EGARCH
1-day VaR	Expected # violations	76	76	76	76
	Observed # violations	128	194	97	98
	# consecutive violations	12	10	3	4
	Unconditional coverage	<0.001	<0.001	0.019	0.014
	Independence	<0.001	0.044	0.177	0.049
	Conditional coverage	<0.001	<0.001	0.026	0.007
	Average MRC	0.0821	0.0820	0.0773	0.0769
	Maximum # violations	25	10	9	14
10-day VaR	Expected # violations	76	76	76	76
	Observed # violations	149	224	92	55
	# consecutive violations	96	145	58	37
	Unconditional coverage	<0.001	<0.001	0.069	0.012
	Independence	<0.001	<0.001	<0.001	<0.001
	Conditional coverage	<0.001	<0.001	<0.001	<0.001
	Average MRC	0.2569	0.2575	0.2503	0.2935
	Maximum # violations	22	23	16	14

Notes: This table shows the backtest results for 1-day VaR estimates and 10-day VaR estimates for the 50/50 portfolio of different historical simulation models, i.e. a model with equally weighted returns, a model with exponentially weighted returns and filtered historical simulation models with GARCH and EGARCH volatility. For both risk horizons, the first row indicates the expected number of violations, which corresponds to the 99% confidence level on which the VaR estimates are based. The second and third row show the observed numbers of violations and the observed number of consecutive violations, respectively. The p-values of the coverage tests are presented in rows 4 to 6. The last two rows give the average minimum required capital over the whole testing period and the maximum number of observed violations over the previous 250 trading days. The backtests are based on VaR estimates made on a daily basis from January 1st, 1981 to December 31st, 2010, giving a total of 7571 estimates to be evaluated for the 10-day VaR estimates and 7580 for the 1-day VaR estimates.

Table 9: Backtests on 99% VaR for the 50/50 portfolio - Monte Carlo simulation - Normal random numbers

		Unc. vol.	EWMA $\lambda = 0.94$	EWMA $\hat{\lambda}$	GARCH	EGARCH
1-day VaR	Expected # violations	76	76	76	76	76
	Observed # violations	183	166	173	150	145
	# consecutive violations	17	11	6	8	7
	Unconditional coverage	<0.001	<0.001	<0.001	<0.001	<0.001
	Independence	<0.001	0.001	0.349	0.014	0.030
	Conditional coverage	<0.001	<0.001	<0.001	<0.001	<0.001
	Average MRC	0.0746	0.0732	0.0726	0.0724	0.0716
	Maximum # violations	34	13	13	15	18
10-day VaR	Expected # violations	76	76	76	76	76
	Observed # violations	164	166	169	130	66
	# consecutive violations	108	106	103	88	45
	Unconditional coverage	<0.001	<0.001	<0.001	<0.001	0.252
	Independence	<0.001	<0.001	<0.001	<0.001	<0.001
	Conditional coverage	<0.001	<0.001	<0.001	<0.001	<0.001
	Average MRC	0.2263	0.2282	0.2257	0.2291	0.2929
	Maximum # violations	29	16	16	15	15

Notes: This table shows the backtest results for 1-day VaR estimates and 10-day VaR estimates for the 50/50 portfolio of different Monte Carlo simulation models with normal random numbers, i.e. a model with unconditional volatility (Unc. vol.), a model with EWMA volatility with λ equal to 0.94, a model with EWMA volatility with λ optimized and models with GARCH and EGARCH volatility. For both risk horizons, the first row indicates the expected number of violations, which corresponds to the 99% confidence level on which the VaR estimates are based. The second and third row show the observed numbers of violations and the observed number of consecutive violations, respectively. The *p*-values of the coverage tests are presented in rows 4 to 6. The last two rows give the average minimum required capital over the whole testing period and the maximum number of observed violations over the previous 250 trading days. The backtests are based on VaR estimates made on a daily basis from January 1st, 1981 to December 31st, 2010, giving a total of 7571 estimates to be evaluated for the 10-day VaR estimates and 7580 for the 1-day VaR estimates.

Table 10: Backtests on 99% VaR for the 50/50 portfolio - Monte Carlo simulation with Normal Copulas - Normal marginals

		Unc. vol.	GARCH	EGARCH
1-day VaR	Expected # violations	76	76	76
	Observed # violations	140	152	145
	# consecutive violations	13	8	6
	Unconditional coverage	<0.001	<0.001	<0.001
	Independence	<0.001	0.016	0.089
	Conditional coverage	<0.001	<0.001	<0.001
	Average MRC	0.0799	0.0724	0.0713
	Maximum # violations	32	15	19
10-day VaR	Expected # violations	76	76	76
	Observed # violations	163	121	65
	# consecutive violations	107	82	46
	Unconditional coverage	<0.001	<0.001	0.205
	Independence	<0.001	<0.001	<0.001
	Conditional coverage	<0.001	<0.001	<0.001
	Average MRC	0.2277	0.2277	0.2924
	Maximum # violations	29	15	16

Notes: This table shows the backtest results for 1-day VaR estimates and 10-day VaR estimates for the 50/50 portfolio of different Monte Carlo simulation models with normal copulas, based on normal marginals with unconditional volatility estimates, GARCH volatility estimates and EGARCH volatility estimates. For both risk horizons, the first row indicates the expected number of violations, which corresponds to the 99% confidence level on which the VaR estimates are based. The second and third row show the observed numbers of violations and the observed number of consecutive violations, respectively. The p-values of the coverage tests are presented in rows 4 to 6. The last two rows give the average minimum required capital over the whole testing period and the maximum number of observed violations over the previous 250 trading days. The backtests are based on VaR estimates made on a daily basis from January 1st, 1981 to December 31st, 2010, giving a total of 7571 estimates to be evaluated for the 10-day VaR estimates and 7580 for the 1-day VaR estimates.

Table 11: Backtests on 99% VaR for the 50/50 portfolio - Monte Carlo simulation with Student *t* Copulas - Normal marginals

		Unc. vol.	GARCH	EGARCH
1-day VaR	Expected # violations	76	76	76
	Observed # violations	140	152	146
	# consecutive violations	13	8	6
	Unconditional coverage	<0.001	<0.001	<0.001
	Independence	<0.001	0.016	0.094
	Conditional coverage	<0.001	<0.001	<0.001
	Average MRC	0.0799	0.0724	0.0713
	Maximum # violations	32	15	19
10-day VaR	Expected # violations	76	76	76
	Observed # violations	163	122	65
	# consecutive violations	107	83	46
	Unconditional coverage	<0.001	<0.001	0.205
	Independence	<0.001	<0.001	<0.001
	Conditional coverage	<0.001	<0.001	<0.001
	Average MRC	0.2277	0.2278	0.2924
	Maximum # violations	29	15	16

Notes: This table shows the backtest results for 1-day VaR estimates and 10-day VaR estimates for the 50/50 portfolio of different Monte Carlo simulation models with Student *t* copulas, based on normal marginals with unconditional volatility estimates, GARCH volatility estimates and EGARCH volatility estimates. For both risk horizons, the first row indicates the expected number of violations, which corresponds to the 99% confidence level on which the VaR estimates are based. The second and third row show the observed numbers of violations and the observed number of consecutive violations, respectively. The *p*-values of the coverage tests are presented in rows 4 to 6. The last two rows give the average minimum required capital over the whole testing period and the maximum number of observed violations over the previous 250 trading days. The backtests are based on VaR estimates made on a daily basis from January 1st, 1981 to December 31st, 2010, giving a total of 7571 estimates to be evaluated for the 10-day VaR estimates and 7580 for the 1-day VaR estimates.

Table 12: Backtests on 99% VaR for the 50/50 portfolio - Monte Carlo simulation with Clayton Copulas - Normal marginals

		Unc. vol.	GARCH	EGARCH
1-day VaR	Expected # violations	76	76	76
	Observed # violations	140	151	145
	# consecutive violations	13	8	6
	Unconditional coverage	<0.001	<0.001	<0.001
	Independence	<0.001	0.015	0.089
	Conditional coverage	<0.001	<0.001	<0.001
	Average MRC	0.0800	0.0724	0.0714
	Maximum # violations	32	15	19
10-day VaR	Expected # violations	76	76	76
	Observed # violations	163	121	65
	# consecutive violations	107	82	46
	Unconditional coverage	<0.001	<0.001	0.205
	Independence	<0.001	<0.001	<0.001
	Conditional coverage	<0.001	<0.001	<0.001
	Average MRC	0.2279	0.2278	0.2924
	Maximum # violations	29	15	16

Notes: This table shows the backtest results for 1-day VaR estimates and 10-day VaR estimates for the 50/50 portfolio of different Monte Carlo simulation models with Clayton copulas, based on normal marginals with unconditional volatility estimates, GARCH volatility estimates and EGARCH volatility estimates. For both risk horizons, the first row indicates the expected number of violations, which corresponds to the 99% confidence level on which the VaR estimates are based. The second and third row show the observed numbers of violations and the observed number of consecutive violations, respectively. The p-values of the coverage tests are presented in rows 4 to 6. The last two rows give the average minimum required capital over the whole testing period and the maximum number of observed violations over the previous 250 trading days. The backtests are based on VaR estimates made on a daily basis from January 1st, 1981 to December 31st, 2010, giving a total of 7571 estimates to be evaluated for the 10-day VaR estimates and 7580 for the 1-day VaR estimates.

Table 13: Backtests on 99% VaR for the 50/50 portfolio - Monte Carlo simulation - Student t random numbers

		Unc. vol.	EWMA $\lambda = 0.94$	EWMA $\hat{\lambda}$	GARCH	EGARCH
1-day VaR	Expected # violations	76	76	76	76	76
	Observed # violations	140	119	135	107	111
	# consecutive violations	12	4	5	4	5
	Unconditional coverage	<0.001	<0.001	<0.001	<0.001	<0.001
	Independence	<0.001	0.171	0.139	0.089	0.031
	Conditional coverage	<0.001	<0.001	<0.001	<0.001	<0.001
	Average MRC	0.0793	0.0757	0.0764	0.0.0764	0.0759
	Maximum # violations	30	11	11	12	15
10-day VaR	Expected # violations	76	76	76	76	76
	Observed # violations	139	156	161	130	75
	# consecutive violations	91	100	99	84	49
	Unconditional coverage	<0.001	<0.001	<0.001	<0.001	0.935
	Independence	<0.001	<0.001	<0.001	<0.001	<0.001
	Conditional coverage	<0.001	<0.001	<0.001	<0.001	<0.001
	Average MRC	0.2482	0.2330	0.2315	0.2282	0.2672
	Maximum # violations	23	14	16	15	16

Notes: This table shows the backtest results for 1-day VaR estimates and 10-day VaR estimates for the 50/50 portfolio of different Monte Carlo simulation models with Student t random numbers, i.e. a model with unconditional volatility (Unc. vol.), a model with EWMA volatility with λ equal to 0.94, a model with EWMA volatility with λ optimized and models with GARCH and EGARCH volatility. For both risk horizons, the first row indicates the expected number of violations, which corresponds to the 99% confidence level on which the VaR estimates are based. The second and third row show the observed numbers of violations and the observed number of consecutive violations, respectively. The p-values of the coverage tests are presented in rows 4 to 6. The last two rows give the average minimum required capital over the whole testing period and the maximum number of observed violations over the previous 250 trading days. The backtests are based on VaR estimates made on a daily basis from January 1st, 1981 to December 31st, 2010, giving a total of 7571 estimates to be evaluated for the 10-day VaR estimates and 7580 for the 1-day VaR estimates.

Table 14: Backtests on 99% VaR for the 50/50 portfolio - Monte Carlo simulation with Normal Copulas - Student *t* marginals

		Unc. vol.	GARCH	EGARCH
1-day VaR	Expected # violations	76	76	76
	Observed # violations	140	113	110
	# consecutive violations	13	5	6
	Unconditional coverage	<0.001	<0.001	<0.001
	Independence	<0.001	0.036	0.007
	Conditional coverage	<0.001	<0.001	<0.001
	Average MRC	0.0799	0.0761	0.0771
	Maximum # violations	32	10	16
10-day VaR	Expected # violations	76	76	76
	Observed # violations	129	121	67
	# consecutive violations	78	76	47
	Unconditional coverage	<0.001	<0.001	0.305
	Independence	<0.001	<0.001	<0.001
	Conditional coverage	<0.001	<0.001	<0.001
	Average MRC	0.2475	0.2281	0.2659
	Maximum # violations	26	15	15

Notes: This table shows the backtest results for 1-day VaR estimates and 10-day VaR estimates for the 50/50 portfolio of different Monte Carlo simulation models with normal copulas, based on Student *t* marginals with unconditional volatility estimates, GARCH volatility estimates and EGARCH volatility estimates. For both risk horizons, the first row indicates the expected number of violations, which corresponds to the 99% confidence level on which the VaR estimates are based. The second and third row show the observed numbers of violations and the observed number of consecutive violations, respectively. The p-values of the coverage tests are presented in rows 4 to 6. The last two rows give the average minimum required capital over the whole testing period and the maximum number of observed violations over the previous 250 trading days. The backtests are based on VaR estimates made on a daily basis from January 1st, 1981 to December 31st, 2010, giving a total of 7571 estimates to be evaluated for the 10-day VaR estimates and 7580 for the 1-day VaR estimates.

Table 15: Backtests on 99% VaR for the 50/50 portfolio - Monte Carlo simulation with Student *t* Copulas - Student *t* marginals

		Unc. vol.	GARCH	EGARCH
1-day VaR	Expected # violations	76	76	76
	Observed # violations	140	113	110
	# consecutive violations	13	5	6
	Unconditional coverage	<0.001	<0.001	<0.001
	Independence	<0.001	0.036	0.007
	Conditional coverage	<0.001	<0.001	<0.001
	Average MRC	0.0799	0.0761	0.0772
	Maximum # violations	32	10	16
10-day VaR	Expected # violations	76	76	76
	Observed # violations	129	121	68
	# consecutive violations	78	76	47
	Unconditional coverage	<0.001	<0.001	0.365
	Independence	<0.001	<0.001	<0.001
	Conditional coverage	<0.001	<0.001	<0.001
	Average MRC	0.2475	0.2281	0.2661
	Maximum # violations	26	15	15

Notes: This table shows the backtest results for 1-day VaR estimates and 10-day VaR estimates for the 50/50 portfolio of different Monte Carlo simulation models with Student *t* copulas, based on Student *t* marginals with unconditional volatility estimates, GARCH volatility estimates and EGARCH volatility estimates. For both risk horizons, the first row indicates the expected number of violations, which corresponds to the 99% confidence level on which the VaR estimates are based. The second and third row show the observed numbers of violations and the observed number of consecutive violations, respectively. The *p*-values of the coverage tests are presented in rows 4 to 6. The last two rows give the average minimum required capital over the whole testing period and the maximum number of observed violations over the previous 250 trading days. The backtests are based on VaR estimates made on a daily basis from January 1st, 1981 to December 31st, 2010, giving a total of 7571 estimates to be evaluated for the 10-day VaR estimates and 7580 for the 1-day VaR estimates.

Table 16: Backtests on 99% VaR for the 50/50 portfolio - Monte Carlo simulation with Clayton Copulas - Student *t* marginals

		Unc. vol.	GARCH	EGARCH
1-day VaR	Expected # violations	76	76	76
	Observed # violations	140	111	110
	# consecutive violations	13	5	6
	Unconditional coverage	<0.001	<0.001	<0.001
	Independence	<0.001	0.031	0.007
	Conditional coverage	<0.001	<0.001	<0.001
	Average MRC	0.0800	0.0762	0.0773
	Maximum # violations	32	10	16
10-day VaR	Expected # violations	76	76	76
	Observed # violations	128	121	68
	# consecutive violations	76	76	47
	Unconditional coverage	<0.001	<0.001	0.365
	Independence	<0.001	<0.001	<0.001
	Conditional coverage	<0.001	<0.001	<0.001
	Average MRC	0.2475	0.2282	0.2658
	Maximum # violations	26	15	15

Notes: This table shows the backtest results for 1-day VaR estimates and 10-day VaR estimates for the 50/50 portfolio of different Monte Carlo simulation models with Clayton copulas, based on Student *t* marginals with unconditional volatility estimates, GARCH volatility estimates and EGARCH volatility estimates. For both risk horizons, the first row indicates the expected number of violations, which corresponds to the 99% confidence level on which the VaR estimates are based. The second and third row show the observed numbers of violations and the observed number of consecutive violations, respectively. The *p*-values of the coverage tests are presented in rows 4 to 6. The last two rows give the average minimum required capital over the whole testing period and the maximum number of observed violations over the previous 250 trading days. The backtests are based on VaR estimates made on a daily basis from January 1st, 1981 to December 31st, 2010, giving a total of 7571 estimates to be evaluated for the 10-day VaR estimates and 7580 for the 1-day VaR estimates.

9.3 Backtests VaR - Combination of risk models conditional on the market condition

We propose a new approach to model selection for predicting VaR, namely conditioning the risk model choice on bull and bear predictions. When dividing the stock market into bull and bear markets, a bear market is likely to capture more extreme events and its return distribution will have thicker tails. Therefore, it is unlikely for one single risk model to perform best in all conditions. During bear markets, it can be optimal to use a different risk model than used during bull markets.

In the previous sections we discussed the methodology and results of forecasting VaR and we proposed a pragmatic strategy to predict bull and bear markets. In this section we combine these two methodologies and condition the model selection to forecast VaR on the bull and bear predictions. That is, we use one model to estimate VaR during bear markets and another model to estimate VaR during bear markets.

On the first trading day of each month, we predict whether that month will be a bull or a bear month. Then, the risk model belonging to this prediction is used to estimate VaR for the remaining days of the month. A combination of two different models, one performing relatively well during bull months and the other performing relatively well during bear months, is likely to have better VaR estimates than both models used separately.

Unfortunately we did not find a combination of two different models which leads to a maximum number of violations less than 10 and a significant reduction in the average minimum required capital, compared to the parametric linear Student *t* model with EWMA volatility with an optimized λ. The main reasons for this result are the low number of bear months (only 19% of the whole testing period) and the fact that the best performing models incurred their largest number of violations just before the beginning of these bear months.

Figure 3 shows that none of the investigated models incurred a large number of violations during the longest bear period, which started in 2000. Since all models managed to stay out of the red zone, there is no reason to combine two different models to obtain a better result for this period.

During the last bear period, which started in November 2007, many models incurred their largest number of violations. However, the models which performed relatively well during this period incurred their largest number of violations just before the beginning of this bear period, so at the end of a bull period. A reason for this could be the fact that the VaR estimates at the end of this bull period where based on the previous three years, which consisted of only bull months, with an upwarding trend of the S&P500 Index. Just before the start of the bear period in November 2007 there were already some relatively large negative returns and this resulted in an underestimation of VaR and a relatively high number of violations.

10 Conclusion

The Basel II Accord (Basel Committee on Banking Supervision, 2006) requires that banks communicate their risk forecasts to the appropriate monetary authorities at the beginning of each trading day, using one or more risk models to measure Value-at-Risk (VaR). The risk estimates of these models are used to determine capital requirements and associated capital costs of banks, depending in part on the number of previous violations, whereby realized losses exceed the estimated VaR. The minimum capital charges must be set at the higher of the previous day's VaR or the average VaR over the last 60 trading days, multiplied by a factor k, which lies between 3 and 4. Since a bank's main objective is to maximize profits, they wish to minimize their capital charges, while restricting the number of violations in a given year below the maximum of 10 allowed by the Basel II Accord.

In this paper we compared the estimation of Value-at-Risk by a variety of risk models. These models can be divided in three main categories, i.e. parametric linear, historical and Monte Carlo risk models. We considered several methods to make volatility estimates over a 10-day risk horizon. We proposed the idea of constructing a risk management strategy that used combinations of two different risk models for forecasting VaR. This combination was based on the prediction of bull and bear markets, so we used two separate models for bull and bear months.

From the prediction of bull and bear markets we conclude that with a relatively simple and pragmatic strategy a high number of correct predictions can be achieved. We used a binomial logit model with ten predicting variables to predict every month whether next month will be a bull or a bear month. We showed that, by using the Schwarz Information Criterium to determine the optimal subset of predicting variables each month, this results in an out-of-sample hitrate of 85.3%. Using the Akaike Information Criterium of the in-sample hitrate to determine the optimal of variables lead to slightly lower out-of-sample hitrates.

To determine which models perform best for the estimation of VaR, we constructed three different linear portfolios of the S&P500 Index and the Barclays Capital US Aggregate Bond Index. Several backtests were applied and this leads to the conclusion that only two models managed to stay under the maximum of 10 violation per financial year, i.e. the filtered historical simulation model with GARCH volatility and the parametric linear Student *t* model with EWMA volatility with an optimized λ, although several other models had a maximum of 10 or 11 violations per year.

From the combinations of different risk models we conclude that there are no combinations of two different models that lead to a maximum number of violations less than 10 and a significant reduction in the average minimum required capital, compared to the parametric linear Student t model with EWMA volatility with an optimized λ. The main reasons for this result are the low number of bear months (only 19% of the whole testing period) and the fact that the best performing models incurred their largest number of violations just before the beginning of these bear months.

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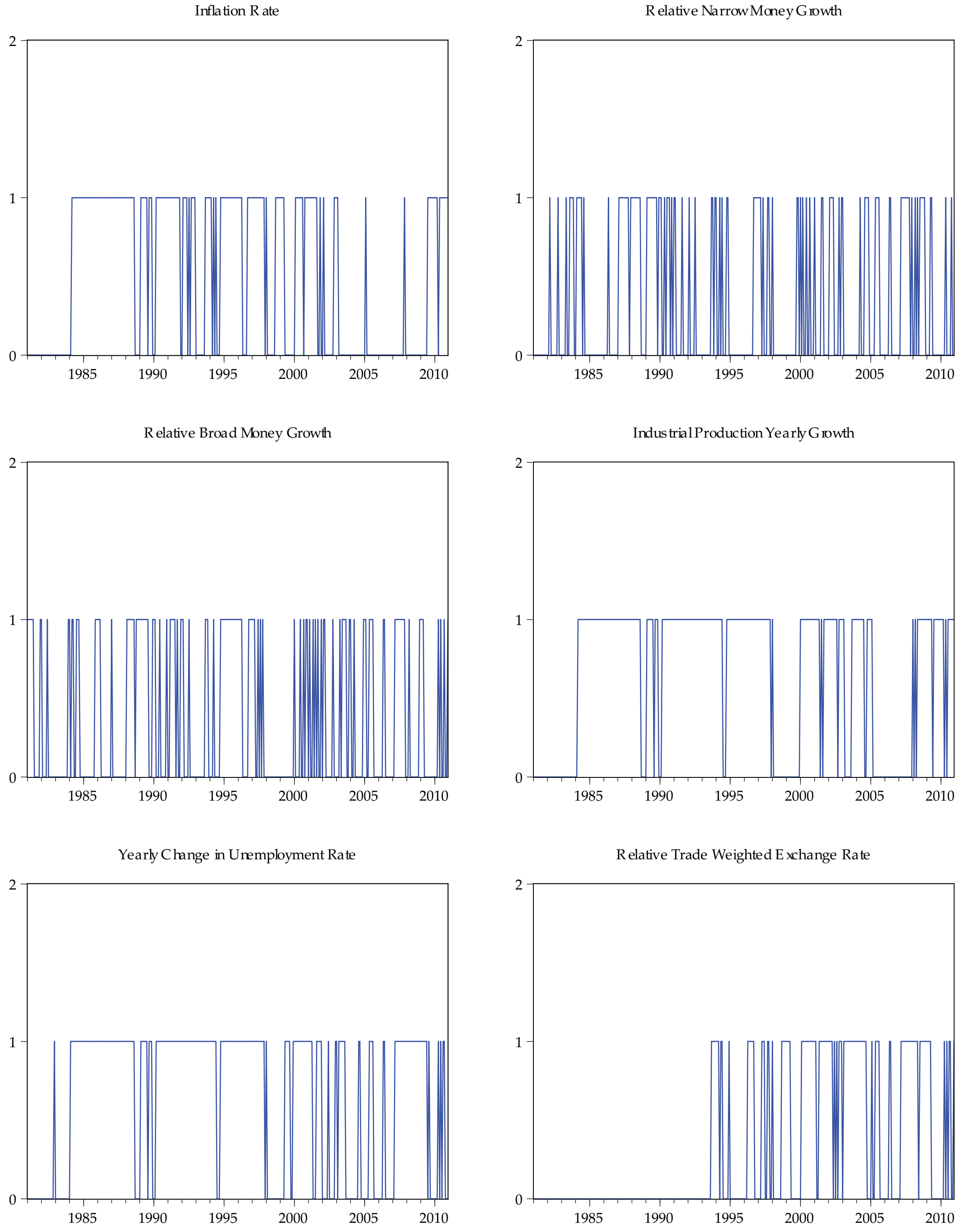
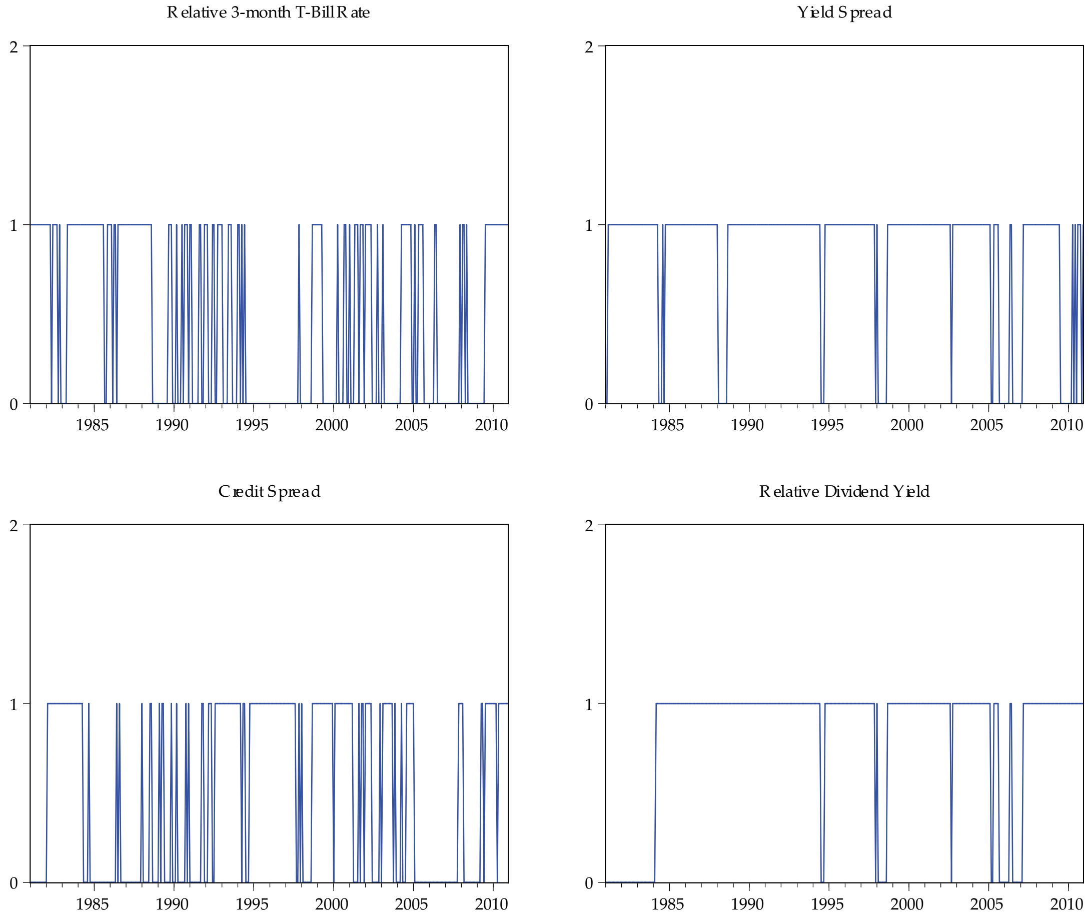
A Graphic displays of the inclusion frequency of the predicting variables

Figure 4: Inclusion frequency of the variables in the base set under the in-sample hitrate selection criterion, 1981M1 to 2010M12



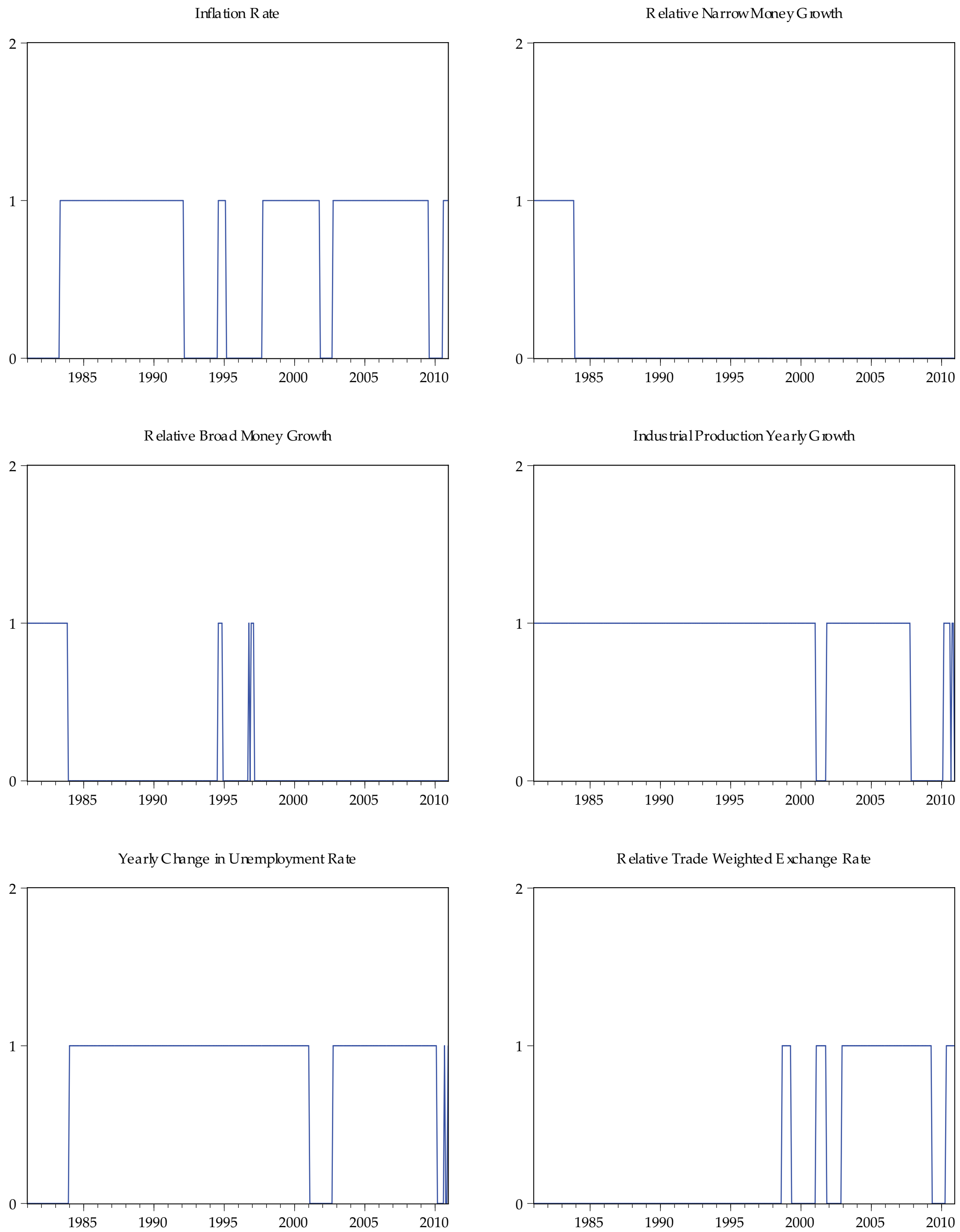
Note: The inclusion of the variable in the regression model is depicted by unity, and zero otherwise.

Figure 4: Inclusion frequency of the variables in the base set under the in-sample hitrate selection criterion, 1981M1 to 2010M12 - *Continued*



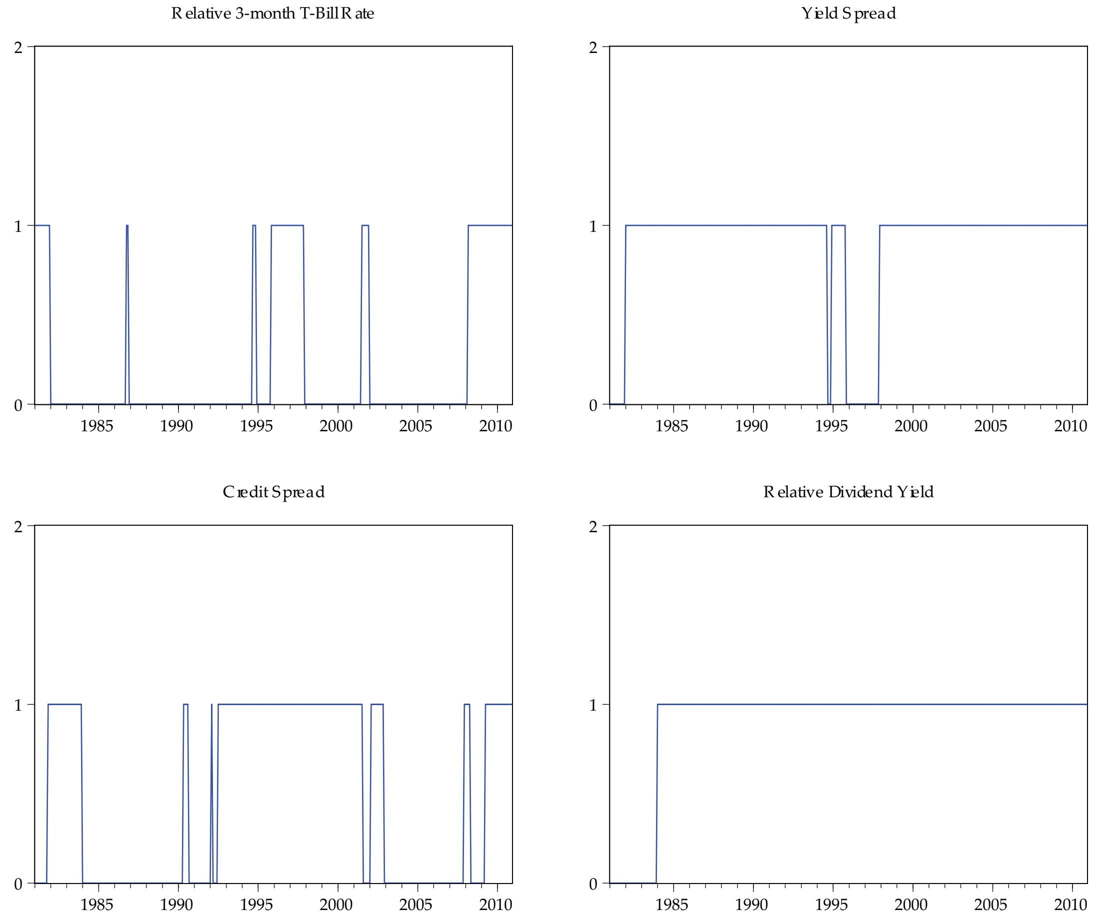
Note: The inclusion of the variable in the regression model is depicted by unity, and zero otherwise.

Figure 5: Inclusion frequency of the variables in the base set under the Akaike selection criterion (AIC), 1981M1 to 2010M12



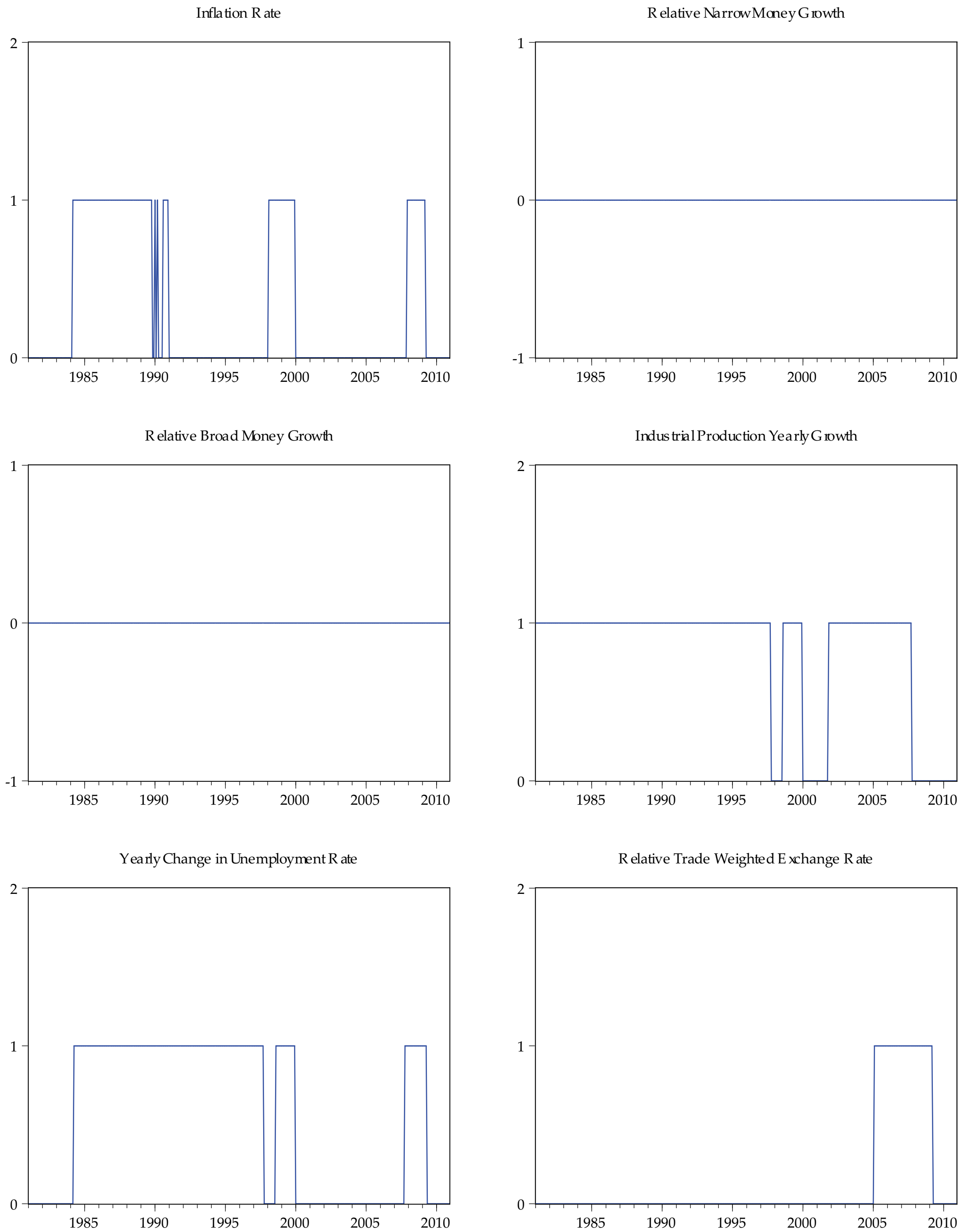
Note: The inclusion of the variable in the regression model is depicted by unity, and zero otherwise.

Figure 5: Inclusion frequency of the variables in the base set under the Akaike selection criterion (AIC), 1981M1 to 2010M12 - *Continued*



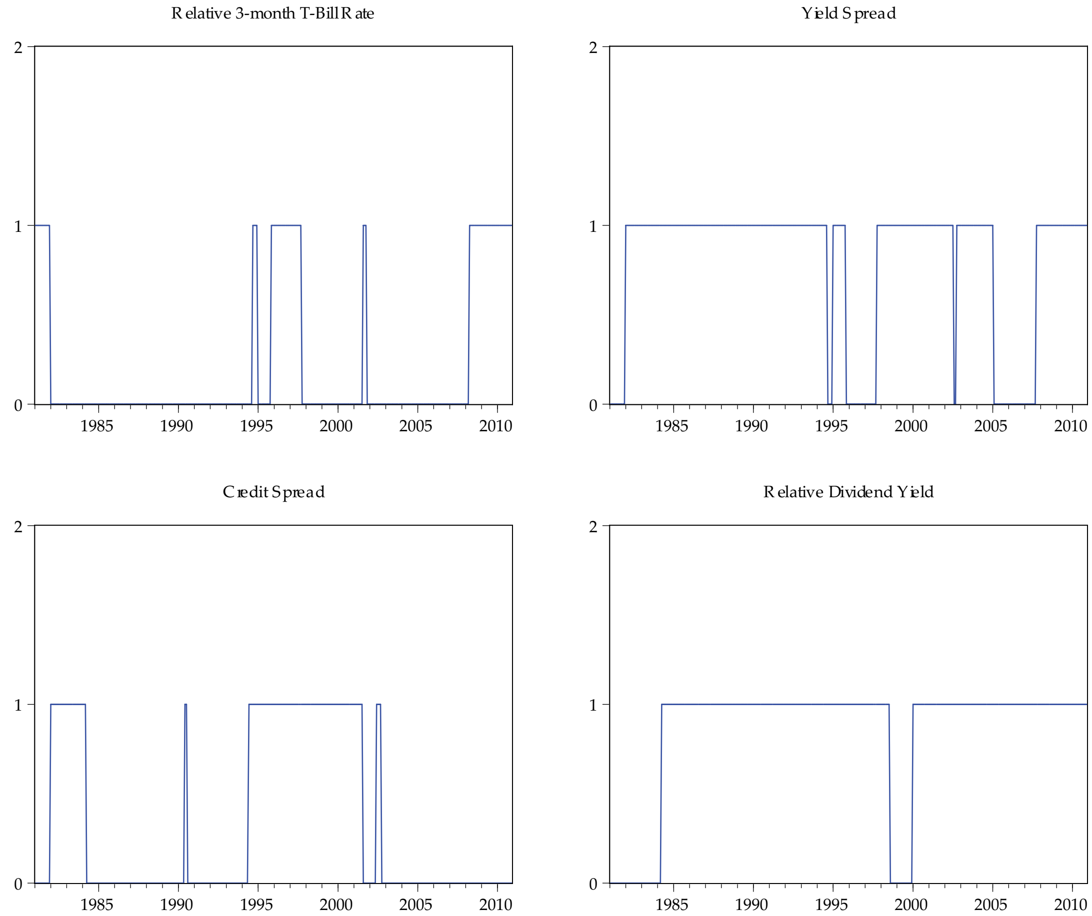
Note: The inclusion of the variable in the regression model is depicted by unity, and zero otherwise.

Figure 6: Inclusion frequency of the variables in the base set under the Schwarz selection criterion (SIC), 1981M1 to 2010M12



Note: The inclusion of the variable in the regression model is depicted by unity, and zero otherwise.

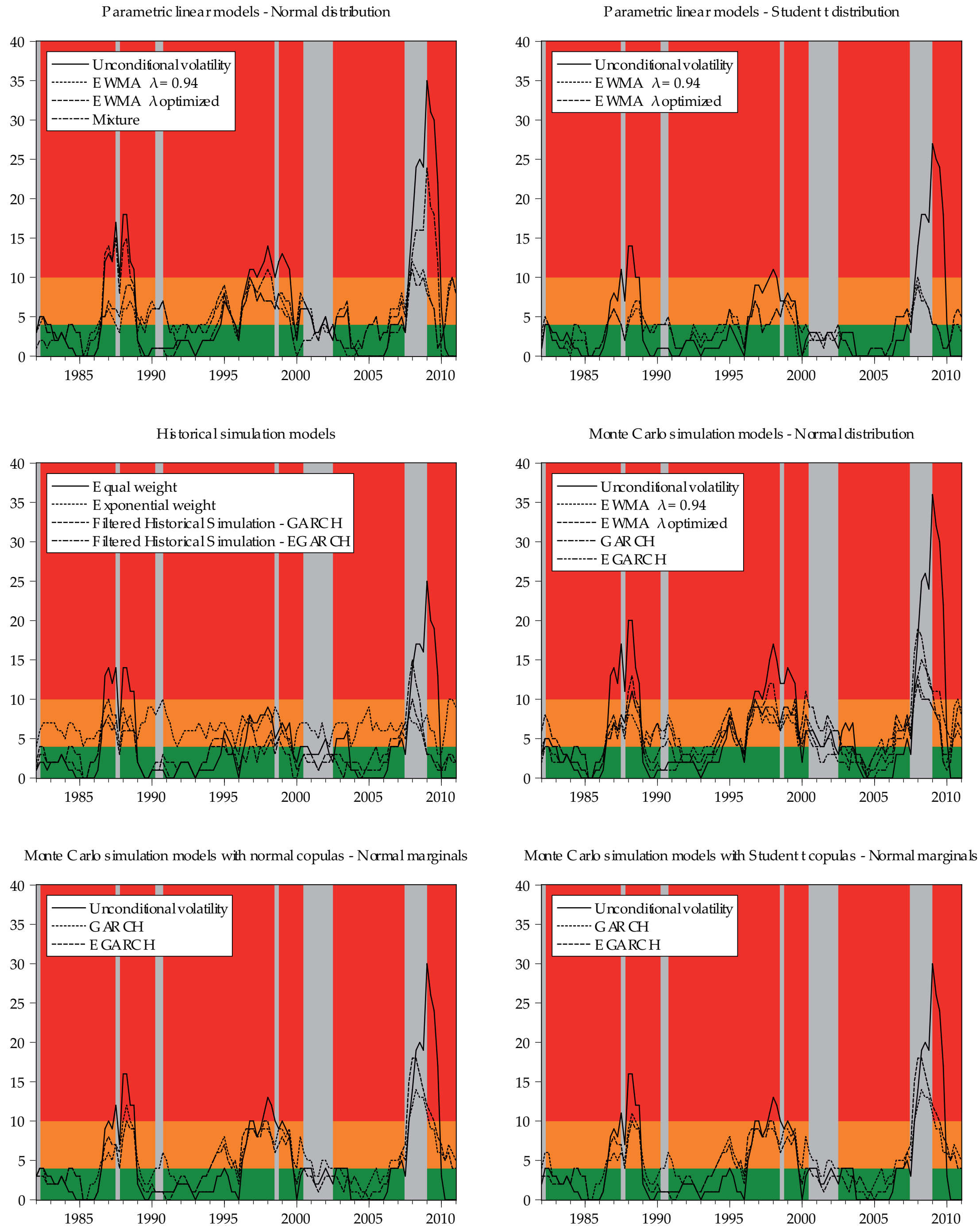
Figure 6: Inclusion frequency of the variables in the base set under the Schwarz selection criterion (SIC), 1981M1 to 2010M12 - *Continued*



Note: The inclusion of the variable in the regression model is depicted by unity, and zero otherwise.

B Graphic displays of the number of violations for the 20/80 and 80/20 portfolio

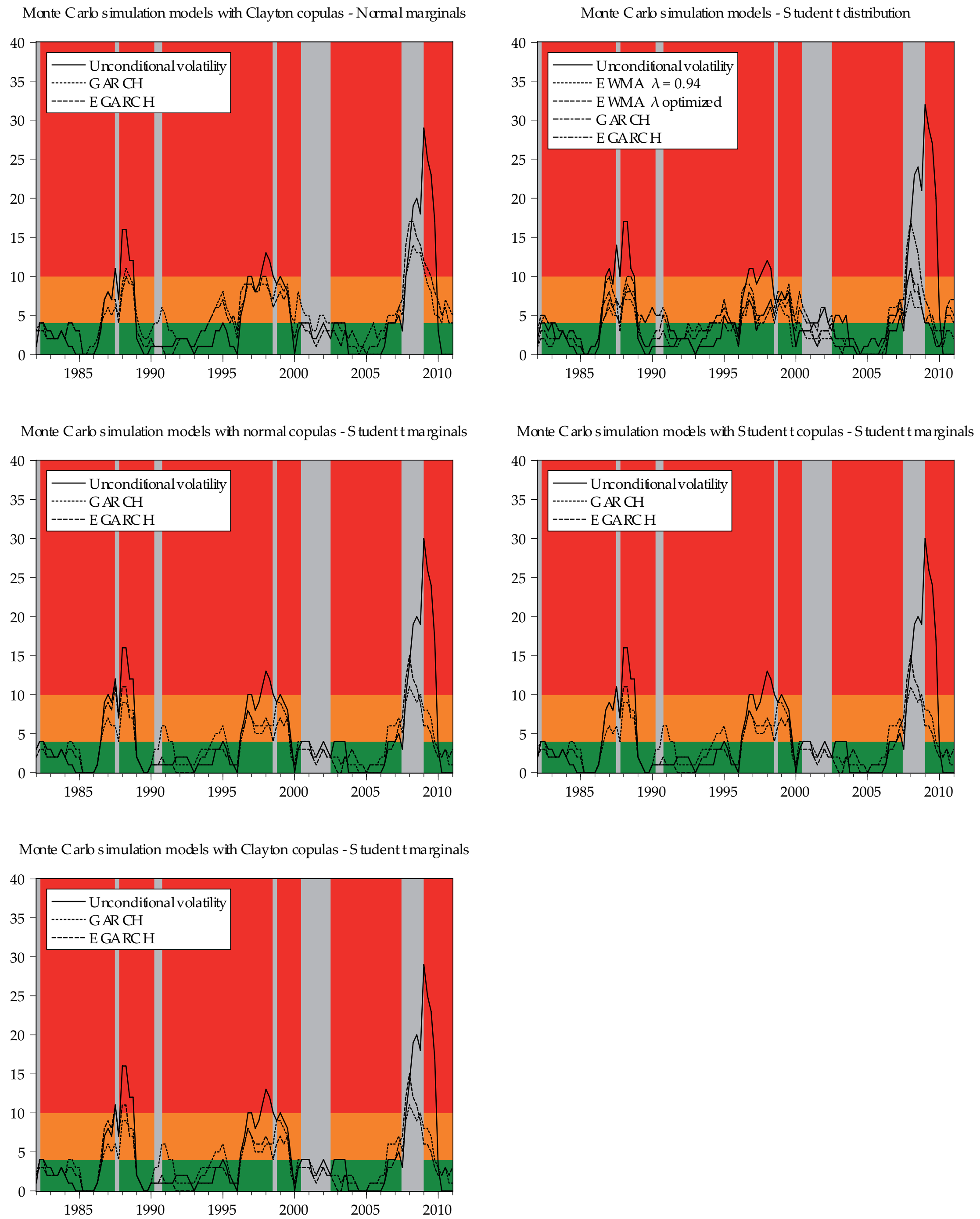
Figure 7: Number of violations for the 20/80 portfolio accumulated over the previous 250 trading days



Notes: This figure shows graphic displays of the number of violations for the 20/80 portfolio of all models for the estimation of VaR on a 1-day risk horizon accumulated over the previous 250 trading days, evaluated each quarter. The colors of the horizontal shaded areas indicate the Basel zones for VaR models and the vertical shaded areas track the bear markets derived from the S&P500 Index.

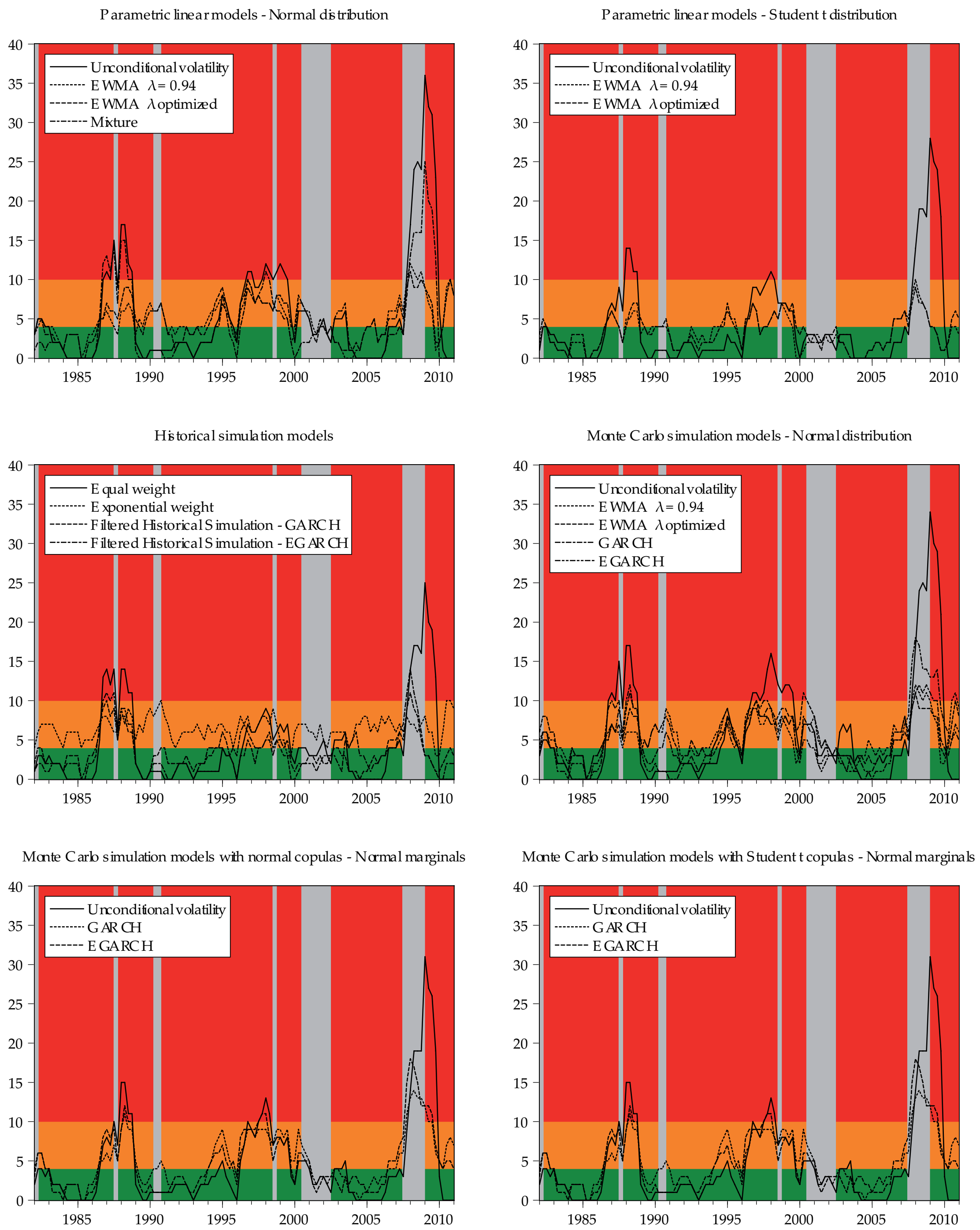
B Graphic displays of the number of violations for the 20/80 and 80/20 portfolio

Figure 7: Number of violations for the 20/80 portfolio accumulated over the previous 250 trading days - *Continued*



Notes: For figure notes, see the first part of the figure.

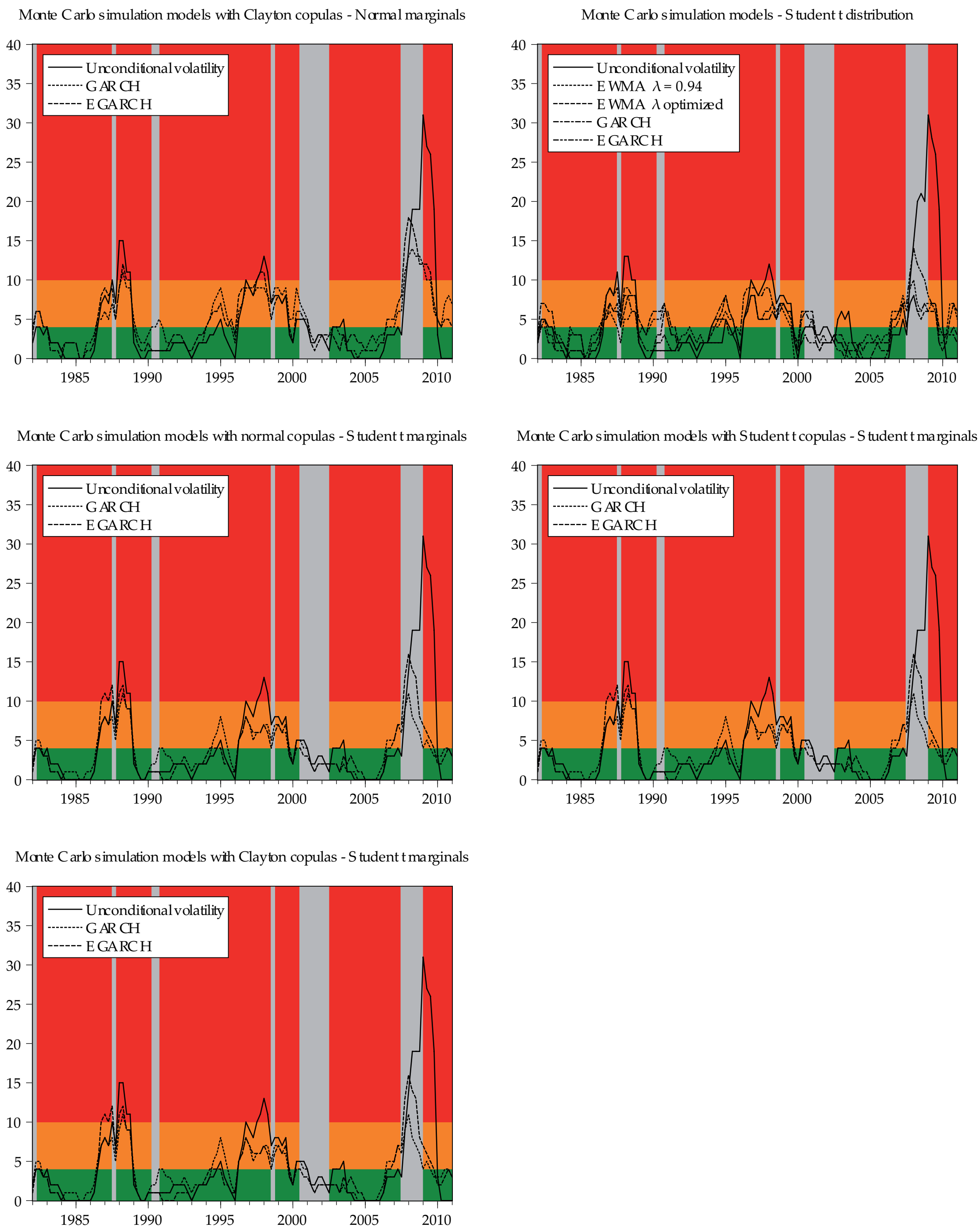
Figure 8: Number of violations for the 80/20 portfolio accumulated over the previous 250 trading days



Notes: This figure shows graphic displays of the number of violations for the 80/20 portfolio of all models for the estimation of VaR on a 1-day risk horizon accumulated over the previous 250 trading days, evaluated each quarter. The colors of the horizontal shaded areas indicate the Basel zones for VaR models and the vertical shaded areas track the bear markets derived from the S&P500 Index.

B Graphic displays of the number of violations for the 20/80 and 80/20 portfolio

Figure 8: Number of violations for the 80/20 portfolio accumulated over the previous 250 trading days - *Continued*



Notes: For figure notes, see the first part of the figure.

C Backtest results on VaR for the 20/80 and 80/20 portfolio

Table 17: Backtests on VaR for the 20/80 portfolio - Parametric linear models - Normal distribution

		Unc. vol.	EWMA $\lambda = 0.94$	EWMA $\hat{\lambda}$	Normal mixture
1-day VaR	Expected # violations	76	76	76	76
	Observed # violations	178	146	147	132
	# consecutive violations	16	7	8	13
	Unconditional coverage	<0.001	<0.001	<0.001	<0.001
	Independence	<0.001	0.032	0.011	<0.001
	Conditional coverage	<0.001	<0.001	<0.001	<0.001
	Average MRC	0.0717	0.0693	0.0689	0.0830
	Maximum # violations	35	12	11	24
10-day VaR	Expected # violations	76	76	76	76
	Observed # violations	158	156	159	67
	# consecutive violations	105	101	102	47
	Unconditional coverage	<0.001	<0.001	<0.001	0.305
	Independence	<0.001	<0.001	<0.001	<0.001
	Conditional coverage	<0.001	<0.001	<0.001	<0.001
	Average MRC	0.2172	0.2156	0.2142	0.3048
	Maximum # violations	34	15	16	13

Notes: This table shows the backtest results for 1-day VaR estimates and 10-day VaR estimates for the 20/80 portfolio of different parametric normal linear models, i.e. a model with unconditional volatility (Unc. vol.), a model with EWMA volatility with λ equal to 0.94, a model with EWMA volatility with λ optimized and a model based on a mixture of two normal distributions. For both risk horizons, the first row indicates the expected number of violations, which corresponds to the 99% confidence level on which the VaR estimates are based. The second and third row show the observed numbers of violations and the observed number of consecutive violations, respectively. The *p*-values of the coverage tests are presented in rows 4 to 6. The last two rows give the average minimum required capital over the whole testing period and the maximum number of observed violations over the previous 250 trading days. The backtests are based on VaR estimates made on a daily basis from January 1st, 1981 to December 31st, 2010, giving a total of 7571 estimates to be evaluated for the 10-day VaR estimates and 7580 for the 1-day VaR estimates.

Table 18: Backtests on VaR for the 20/80 portfolio - Parametric linear models - Student *t* distribution

		Unc. vol.	EWMA $\lambda = 0.94$	EWMA $\hat{\lambda}$
1-day VaR	Expected # violations	76	76	76
	Observed # violations	125	93	101
	# consecutive violations	11	3	3
	Unconditional coverage	<0.001	0.055	0.006
	Independence	<0.001	0.143	0.215
	Conditional coverage	<0.001	0.055	0.010
	Average MRC	0.0774	0.0722	0.0718
	Maximum # violations	27	10	9
10-day VaR	Expected # violations	76	76	76
	Observed # violations	115	114	117
	# consecutive violations	71	73	74
	Unconditional coverage	<0.001	<0.001	<0.001
	Independence	<0.001	<0.001	<0.001
	Conditional coverage	<0.001	<0.001	<0.001
	Average MRC	0.2380	0.2318	0.2318
	Maximum # violations	25	14	14

Notes: This table shows the backtest results for 1-day VaR estimates and 10-day VaR estimates for the 20/80 portfolio of different parametric Student *t* linear models, i.e. a model with unconditional volatility (Unc. vol.), a model with EWMA volatility with λ equal to 0.94 and a model with EWMA volatility with λ optimized. For both risk horizons, the first row indicates the expected number of violations, which corresponds to the 99% confidence level on which the VaR estimates are based. The second and third row show the observed numbers of violations and the observed number of consecutive violations, respectively. The *p*-values of the coverage tests are presented in rows 4 to 6. The last two rows give the average minimum required capital over the whole testing period and the maximum number of observed violations over the previous 250 trading days. The backtests are based on VaR estimates made on a daily basis from January 1st, 1981 to December 31st, 2010, giving a total of 7571 estimates to be evaluated for the 10-day VaR estimates and 7580 for the 1-day VaR estimates.

Table 19: Backtests on VaR for the 20/80 portfolio - Historical simulation models

		Equal weight	Exp. weight	FHS - GARCH	FHS - EGARCH
1-day VaR	Expected # violations	76	76	76	76
	Observed # violations	129	195	91	99
	# consecutive violations	10	10	3	3
	Unconditional coverage	<0.001	<0.001	0.089	0.011
	Independence	<0.001	0.047	0.128	0.195
	Conditional coverage	<0.001	<0.001	0.074	0.016
	Average MRC	0.0778	0.0779	0.0730	0.0737
	Maximum # violations	25	10	10	15
10-day VaR	Expected # violations	76	76	76	76
	Observed # violations	150	229	95	67
	# consecutive violations	97	148	62	47
	Unconditional coverage	<0.001	<0.001	0.032	0.305
	Independence	<0.001	<0.001	<0.001	<0.001
	Conditional coverage	<0.001	<0.001	<0.001	<0.001
	Average MRC	0.2429	0.2443	0.2374	0.2834
	Maximum # violations	22	23	14	15

Notes: This table shows the backtest results for 1-day VaR estimates and 10-day VaR estimates for the 20/80 portfolio of different historical simulation models, i.e. a model with equally weighted returns, a model with exponentially weighted returns and filtered historical simulation models with GARCH and EGARCH volatility. For both risk horizons, the first row indicates the expected number of violations, which corresponds to the 99% confidence level on which the VaR estimates are based. The second and third row show the observed numbers of violations and the observed number of consecutive violations, respectively. The p-values of the coverage tests are presented in rows 4 to 6. The last two rows give the average minimum required capital over the whole testing period and the maximum number of observed violations over the previous 250 trading days. The backtests are based on VaR estimates made on a daily basis from January 1st, 1981 to December 31st, 2010, giving a total of 7571 estimates to be evaluated for the 10-day VaR estimates and 7580 for the 1-day VaR estimates.

Table 20: Backtests on 99% VaR for the 20/80 portfolio - Monte Carlo simulation - Normal random numbers

		Unc. vol.	EWMA $\lambda = 0.94$	EWMA $\hat{\lambda}$	GARCH	EGARCH
1-day VaR	Expected # violations	76	76	76	76	76
	Observed # violations	190	162	169	151	145
	# consecutive violations	17	8	7	7	5
	Unconditional coverage	<0.001	<0.001	<0.001	<0.001	<0.001
	Independence	<0.001	0.034	0.134	0.046	0.228
	Conditional coverage	<0.001	<0.001	<0.001	<0.001	<0.001
	Average MRC	0.0704	0.0692	0.0694	0.0690	0.0682
	Maximum # violations	36	13	12	15	19
10-day VaR	Expected # violations	76	76	76	76	76
	Observed # violations	164	170	168	128	68
	# consecutive violations	107	108	102	87	45
	Unconditional coverage	<0.001	<0.001	<0.001	<0.001	0.365
	Independence	<0.001	<0.001	<0.001	<0.001	<0.001
	Conditional coverage	<0.001	<0.001	<0.001	<0.001	<0.001
	Average MRC	0.2132	0.2166	0.2172	0.2180	0.2804
	Maximum # violations	28	15	17	15	16

Notes: This table shows the backtest results for 1-day VaR estimates and 10-day VaR estimates for the 20/80 portfolio of different Monte Carlo simulation models with normal random numbers, i.e. a model with unconditional volatility (Unc. vol.), a model with EWMA volatility with λ equal to 0.94, a model with EWMA volatility with λ optimized and models with GARCH and EGARCH volatility. For both risk horizons, the first row indicates the expected number of violations, which corresponds to the 99% confidence level on which the VaR estimates are based. The second and third row show the observed numbers of violations and the observed number of consecutive violations, respectively. The *p*-values of the coverage tests are presented in rows 4 to 6. The last two rows give the average minimum required capital over the whole testing period and the maximum number of observed violations over the previous 250 trading days. The backtests are based on VaR estimates made on a daily basis from January 1st, 1981 to December 31st, 2010, giving a total of 7571 estimates to be evaluated for the 10-day VaR estimates and 7580 for the 1-day VaR estimates.

Table 21: Backtests on 99% VaR for the 20/80 portfolio - Monte Carlo simulation with Normal Copulas - Normal marginals

		Unc. vol.	GARCH	EGARCH
1-day VaR	Expected # violations	76	76	76
	Observed # violations	144	151	141
	# consecutive violations	13	7	5
	Unconditional coverage	<0.001	<0.001	<0.001
	Independence	<0.001	0.046	0.185
	Conditional coverage	<0.001	<0.001	<0.001
	Average MRC	0.0756	0.0691	0.0671
	Maximum # violations	30	14	18
10-day VaR	Expected # violations	76	76	76
	Observed # violations	166	129	66
	# consecutive violations	111	87	46
	Unconditional coverage	<0.001	<0.001	0.252
	Independence	<0.001	<0.001	<0.001
	Conditional coverage	<0.001	<0.001	<0.001
	Average MRC	0.2158	0.2174	0.2773
	Maximum # violations	27	16	14

Notes: This table shows the backtest results for 1-day VaR estimates and 10-day VaR estimates for the 20/80 portfolio of different Monte Carlo simulation models with normal copulas, based on normal marginals with unconditional volatility estimates, GARCH volatility estimates and EGARCH volatility estimates. For both risk horizons, the first row indicates the expected number of violations, which corresponds to the 99% confidence level on which the VaR estimates are based. The second and third row show the observed numbers of violations and the observed number of consecutive violations, respectively. The p-values of the coverage tests are presented in rows 4 to 6. The last two rows give the average minimum required capital over the whole testing period and the maximum number of observed violations over the previous 250 trading days. The backtests are based on VaR estimates made on a daily basis from January 1st, 1981 to December 31st, 2010, giving a total of 7571 estimates to be evaluated for the 10-day VaR estimates and 7580 for the 1-day VaR estimates.

Table 22: Backtests on 99% VaR for the 20/80 portfolio - Monte Carlo simulation with Student *t* Copulas - Normal marginals

		Unc. vol.	GARCH	EGARCH
1-day VaR	Expected # violations	76	76	76
	Observed # violations	142	152	141
	# consecutive violations	13	7	5
	Unconditional coverage	<0.001	<0.001	<0.001
	Independence	<0.001	0.049	0.185
	Conditional coverage	<0.001	<0.001	<0.001
	Average MRC	0.0755	0.0691	0.0672
	Maximum # violations	30	14	18
10-day VaR	Expected # violations	76	76	76
	Observed # violations	167	129	66
	# consecutive violations	112	87	46
	Unconditional coverage	<0.001	<0.001	0.252
	Independence	<0.001	<0.001	<0.001
	Conditional coverage	<0.001	<0.001	<0.001
	Average MRC	0.2159	0.2174	0.2774
	Maximum # violations	27	16	14

Notes: This table shows the backtest results for 1-day VaR estimates and 10-day VaR estimates for the 20/80 portfolio of different Monte Carlo simulation models with Student *t* copulas, based on normal marginals with unconditional volatility estimates, GARCH volatility estimates and EGARCH volatility estimates. For both risk horizons, the first row indicates the expected number of violations, which corresponds to the 99% confidence level on which the VaR estimates are based. The second and third row show the observed numbers of violations and the observed number of consecutive violations, respectively. The *p*-values of the coverage tests are presented in rows 4 to 6. The last two rows give the average minimum required capital over the whole testing period and the maximum number of observed violations over the previous 250 trading days. The backtests are based on VaR estimates made on a daily basis from January 1st, 1981 to December 31st, 2010, giving a total of 7571 estimates to be evaluated for the 10-day VaR estimates and 7580 for the 1-day VaR estimates.

Table 23: Backtests on 99% VaR for the 20/80 portfolio - Monte Carlo simulation with Clayton Copulas - Normal marginals

		Unc. vol.	GARCH	EGARCH
1-day VaR	Expected # violations	76	76	76
	Observed # violations	139	148	139
	# consecutive violations	13	7	5
	Unconditional coverage	<0.001	<0.001	<0.001
	Independence	<0.001	0.037	0.167
	Conditional coverage	<0.001	<0.001	<0.001
	Average MRC	0.0757	0.0690	0.0673
	Maximum # violations	29	14	17
10-day VaR	Expected # violations	76	76	76
	Observed # violations	166	129	66
	# consecutive violations	111	87	46
	Unconditional coverage	<0.001	<0.001	0.252
	Independence	<0.001	<0.001	<0.001
	Conditional coverage	<0.001	<0.001	<0.001
	Average MRC	0.2165	0.2178	0.2777
	Maximum # violations	27	16	14

Notes: This table shows the backtest results for 1-day VaR estimates and 10-day VaR estimates for the 20/80 portfolio of different Monte Carlo simulation models with Clayton copulas, based on normal marginals with unconditional volatility estimates, GARCH volatility estimates and EGARCH volatility estimates. For both risk horizons, the first row indicates the expected number of violations, which corresponds to the 99% confidence level on which the VaR estimates are based. The second and third row show the observed numbers of violations and the observed number of consecutive violations, respectively. The p-values of the coverage tests are presented in rows 4 to 6. The last two rows give the average minimum required capital over the whole testing period and the maximum number of observed violations over the previous 250 trading days. The backtests are based on VaR estimates made on a daily basis from January 1st, 1981 to December 31st, 2010, giving a total of 7571 estimates to be evaluated for the 10-day VaR estimates and 7580 for the 1-day VaR estimates.

Table 24: Backtests on 99% VaR for the 20/80 portfolio - Monte Carlo simulation - Student *t* random numbers

		Unc. vol.	EWMA $\lambda = 0.94$	EWMA $\hat{\lambda}$	GARCH	EGARCH
1-day VaR	Expected # violations	76	76	76	76	76
	Observed # violations	156	110	128	109	109
	# consecutive violations	12	6	4	4	4
	Unconditional coverage	<0.001	<0.001	<0.001	<0.001	<0.001
	Independence	<0.001	0.006	0.261	0.099	0.099
	Conditional coverage	<0.001	<0.001	<0.001	<0.001	<0.001
	Average MRC	0.07569	0.0706	0.0716	0.0721	0.0715
	Maximum # violations	32	8	11	11	17
10-day VaR	Expected # violations	76	76	76	76	76
	Observed # violations	137	161	158	121	70
	# consecutive violations	86	100	99	81	45
	Unconditional coverage	<0.001	<0.001	<0.001	<0.001	0.504
	Independence	<0.001	<0.001	<0.001	<0.001	<0.001
	Conditional coverage	<0.001	<0.001	<0.001	<0.001	<0.001
	Average MRC	0.2323	0.2239	0.2220	0.2169	0.2544
	Maximum # violations	23	16	16	14	15

Notes: This table shows the backtest results for 1-day VaR estimates and 10-day VaR estimates for the 20/80 portfolio of different Monte Carlo simulation models with Student *t* random numbers, i.e. a model with unconditional volatility (Unc. vol.), a model with EWMA volatility with λ equal to 0.94, a model with EWMA volatility with λ optimized and models with GARCH and EGARCH volatility. For both risk horizons, the first row indicates the expected number of violations, which corresponds to the 99% confidence level on which the VaR estimates are based. The second and third row show the observed numbers of violations and the observed number of consecutive violations, respectively. The *p*-values of the coverage tests are presented in rows 4 to 6. The last two rows give the average minimum required capital over the whole testing period and the maximum number of observed violations over the previous 250 trading days. The backtests are based on VaR estimates made on a daily basis from January 1st, 1981 to December 31st, 2010, giving a total of 7571 estimates to be evaluated for the 10-day VaR estimates and 7580 for the 1-day VaR estimates.

Table 25: Backtests on 99% VaR for the 20/80 portfolio - Monte Carlo simulation with Normal Copulas - Student *t* marginals

		Unc. vol.	GARCH	EGARCH
1-day VaR	Expected # violations	76	76	76
	Observed # violations	144	112	105
	# consecutive violations	13	5	3
	Unconditional coverage	<0.001	<0.001	0.001
	Independence	<0.001	0.033	0.253
	Conditional coverage	<0.001	<0.001	0.003
	Average MRC	0.0756	0.0734	0.0822
	Maximum # violations	30	11	15
10-day VaR	Expected # violations	76	76	76
	Observed # violations	130	121	69
	# consecutive violations	77	77	47
	Unconditional coverage	<0.001	<0.001	0.431
	Independence	<0.001	<0.001	<0.001
	Conditional coverage	<0.001	<0.001	<0.001
	Average MRC	0.2347	0.2185	0.2637
	Maximum # violations	22	14	14

Notes: This table shows the backtest results for 1-day VaR estimates and 10-day VaR estimates for the 20/80 portfolio of different Monte Carlo simulation models with normal copulas, based on Student *t* marginals with unconditional volatility estimates, GARCH volatility estimates and EGARCH volatility estimates. For both risk horizons, the first row indicates the expected number of violations, which corresponds to the 99% confidence level on which the VaR estimates are based. The second and third row show the observed numbers of violations and the observed number of consecutive violations, respectively. The p-values of the coverage tests are presented in rows 4 to 6. The last two rows give the average minimum required capital over the whole testing period and the maximum number of observed violations over the previous 250 trading days. The backtests are based on VaR estimates made on a daily basis from January 1st, 1981 to December 31st, 2010, giving a total of 7571 estimates to be evaluated for the 10-day VaR estimates and 7580 for the 1-day VaR estimates.

Table 26: Backtests on 99% VaR for the 20/80 portfolio - Monte Carlo simulation with Student *t* Copulas - Student *t* marginals

		Unc. vol.	GARCH	EGARCH
1-day VaR	Expected # violations	76	76	76
	Observed # violations	142	111	105
	# consecutive violations	13	5	3
	Unconditional coverage	<0.001	<0.001	0.001
	Independence	<0.001	0.031	0.253
	Conditional coverage	<0.001	<0.001	0.003
	Average MRC	0.0755	0.0734	0.0824
	Maximum # violations	30	11	15
10-day VaR	Expected # violations	76	76	76
	Observed # violations	131	121	69
	# consecutive violations	79	77	47
	Unconditional coverage	<0.001	<0.001	0.431
	Independence	<0.001	<0.001	<0.001
	Conditional coverage	<0.001	<0.001	<0.001
	Average MRC	0.2349	0.2185	0.2641
	Maximum # violations	22	14	14

Notes: This table shows the backtest results for 1-day VaR estimates and 10-day VaR estimates for the 20/80 portfolio of different Monte Carlo simulation models with Student *t* copulas, based on Student *t* marginals with unconditional volatility estimates, GARCH volatility estimates and EGARCH volatility estimates. For both risk horizons, the first row indicates the expected number of violations, which corresponds to the 99% confidence level on which the VaR estimates are based. The second and third row show the observed numbers of violations and the observed number of consecutive violations, respectively. The *p*-values of the coverage tests are presented in rows 4 to 6. The last two rows give the average minimum required capital over the whole testing period and the maximum number of observed violations over the previous 250 trading days. The backtests are based on VaR estimates made on a daily basis from January 1st, 1981 to December 31st, 2010, giving a total of 7571 estimates to be evaluated for the 10-day VaR estimates and 7580 for the 1-day VaR estimates.

Table 27: Backtests on 99% VaR for the 20/80 portfolio - Monte Carlo simulation with Clayton Copulas - Student *t* marginals

		Unc. vol.	GARCH	EGARCH
1-day VaR	Expected # violations	76	76	76
	Observed # violations	139	110	104
	# consecutive violations	13	5	3
	Unconditional coverage	<0.001	<0.001	0.002
	Independence	<0.001	0.029	0.242
	Conditional coverage	<0.001	<0.001	0.004
	Average MRC	0.0757	0.0736	0.0827
	Maximum # violations	29	11	15
10-day VaR	Expected # violations	76	76	76
	Observed # violations	128	119	68
	# consecutive violations	77	75	47
	Unconditional coverage	<0.001	<0.001	0.365
	Independence	<0.001	<0.001	<0.001
	Conditional coverage	<0.001	<0.001	<0.001
	Average MRC	0.2353	0.2187	0.2644
	Maximum # violations	22	14	14

Notes: This table shows the backtest results for 1-day VaR estimates and 10-day VaR estimates for the 20/80 portfolio of different Monte Carlo simulation models with Clayton copulas, based on Student *t* marginals with unconditional volatility estimates, GARCH volatility estimates and EGARCH volatility estimates. For both risk horizons, the first row indicates the expected number of violations, which corresponds to the 99% confidence level on which the VaR estimates are based. The second and third row show the observed numbers of violations and the observed number of consecutive violations, respectively. The *p*-values of the coverage tests are presented in rows 4 to 6. The last two rows give the average minimum required capital over the whole testing period and the maximum number of observed violations over the previous 250 trading days. The backtests are based on VaR estimates made on a daily basis from January 1st, 1981 to December 31st, 2010, giving a total of 7571 estimates to be evaluated for the 10-day VaR estimates and 7580 for the 1-day VaR estimates.

Table 28: Backtests on VaR for the 80/20 portfolio - Parametric linear models - Normal distribution

		Unc. vol.	EWMA $\lambda = 0.94$	EWMA $\hat{\lambda}$	Normal mixture
1-day VaR	Expected # violations	76	76	76	76
	Observed # violations	173	147	147	127
	# consecutive violations	16	8	8	14
	Unconditional coverage	<0.001	<0.001	<0.001	<0.001
	Independence	<0.001	0.011	0.011	<0.001
	Conditional coverage	<0.001	<0.001	<0.001	<0.001
	Average MRC	0.0773	0.0739	0.0736	0.0800
	Maximum # violations	36	12	11	25
10-day VaR	Expected # violations	76	76	76	76
	Observed # violations	152	156	157	61
	# consecutive violations	100	103	102	44
	Unconditional coverage	<0.001	<0.001	<0.001	0.079
	Independence	<0.001	<0.001	<0.001	<0.001
	Conditional coverage	<0.001	<0.001	<0.001	<0.001
	Average MRC	0.2344	0.2310	0.2286	0.3298
	Maximum # violations	35	15	16	13

Notes: This table shows the backtest results for 1-day VaR estimates and 10-day VaR estimates for the 80/20 portfolio of different parametric normal linear models, i.e. a model with unconditional volatility (Unc. vol.), a model with EWMA volatility with λ equal to 0.94, a model with EWMA volatility with λ optimized and a model based on a mixture of two normal distributions. For both risk horizons, the first row indicates the expected number of violations, which corresponds to the 99% confidence level on which the VaR estimates are based. The second and third row show the observed numbers of violations and the observed number of consecutive violations, respectively. The *p*-values of the coverage tests are presented in rows 4 to 6. The last two rows give the average minimum required capital over the whole testing period and the maximum number of observed violations over the previous 250 trading days. The backtests are based on VaR estimates made on a daily basis from January 1st, 1981 to December 31st, 2010, giving a total of 7571 estimates to be evaluated for the 10-day VaR estimates and 7580 for the 1-day VaR estimates.

Table 29: Backtests on VaR for the 80/20 - Parametric linear models - Student *t* distribution

		Unc. vol.	EWMA $\lambda = 0.94$	EWMA $\hat{\lambda}$
1-day VaR	Expected # violations	76	76	76
	Observed # violations	122	95	101
	# consecutive violations	10	4	3
	Unconditional coverage	<0.001	0.033	0.006
	Independence	<0.001	0.040	0.215
	Conditional coverage	<0.001	0.012	0.010
	Average MRC	0.0832	0.0770	0.0766
	Maximum # violations	28	10	9
10-day VaR	Expected # violations	76	76	76
	Observed # violations	114	115	115
	# consecutive violations	71	73	73
	Unconditional coverage	<0.001	<0.001	<0.001
	Independence	<0.001	<0.001	<0.001
	Conditional coverage	<0.001	<0.001	<0.001
	Average MRC	0.2571	0.2493	0.2470
	Maximum # violations	25	14	14

Notes: This table shows the backtest results for 1-day VaR estimates and 10-day VaR estimates for the 80/20 portfolio of different parametric Student *t* linear models, i.e. a model with unconditional volatility (Unc. vol.), a model with EWMA volatility with λ equal to 0.94 and a model with EWMA volatility with λ optimized. For both risk horizons, the first row indicates the expected number of violations, which corresponds to the 99% confidence level on which the VaR estimates are based. The second and third row show the observed numbers of violations and the observed number of consecutive violations, respectively. The *p*-values of the coverage tests are presented in rows 4 to 6. The last two rows give the average minimum required capital over the whole testing period and the maximum number of observed violations over the previous 250 trading days. The backtests are based on VaR estimates made on a daily basis from January 1st, 1981 to December 31st, 2010, giving a total of 7571 estimates to be evaluated for the 10-day VaR estimates and 7580 for the 1-day VaR estimates.

Table 30: Backtests on VaR for the 80/20 - Historical simulation models

		Equal weight	Exp. weight	FHS - GARCH	FHS - EGARCH
1-day VaR	Expected # violations	76	76	76	76
	Observed # violations	126	194	99	104
	# consecutive violations	10	9	4	5
	Unconditional coverage	<0.001	<0.001	0.011	0.002
	Independence	<0.001	0.102	0.053	0.018
	Conditional coverage	<0.001	<0.001	0.006	<0.001
	Average MRC	0.0834	0.0832	0.0783	0.0787
	Maximum # violations	25	10	11	14
10-day VaR	Expected # violations	76	76	76	76
	Observed # violations	148	220	102	69
	# consecutive violations	95	140	71	42
	Unconditional coverage	<0.001	<0.001	0.004	0.045
	Independence	<0.001	<0.001	<0.001	<0.001
	Conditional coverage	<0.001	<0.001	<0.001	<0.001
	Average MR	0.2610	0.2608	0.2555	0.2992
	Maximum # violations	22	23	14	15

Notes: This table shows the backtest results for 1-day VaR estimates and 10-day VaR estimates for the 80/20 portfolio of different historical simulation models, i.e. a model with equally weighted returns, a model with exponentially weighted returns and filtered historical simulation models with GARCH and EGARCH volatility. For both risk horizons, the first row indicates the expected number of violations, which corresponds to the 99% confidence level on which the VaR estimates are based. The second and third row show the observed numbers of violations and the observed number of consecutive violations, respectively. The *p*-values of the coverage tests are presented in rows 4 to 6. The last two rows give the average minimum required capital over the whole testing period and the maximum number of observed violations over the previous 250 trading days. The backtests are based on VaR estimates made on a daily basis from January 1st, 1981 to December 31st, 2010, giving a total of 7571 estimates to be evaluated for the 10-day VaR estimates and 7580 for the 1-day VaR estimates.

Table 31: Backtests on 99% VaR for the 80/20 - Monte Carlo simulation - Normal random numbers

		Unc. vol.	EWMA $\lambda = 0.94$	EWMA $\hat{\lambda}$	GARCH	EGARCH
1-day VaR	Expected # violations	76	76	76	76	76
	Observed # violations	177	162	166	149	145
	# consecutive violations	15	9	4	7	8
	Unconditional coverage	<0.001	<0.001	<0.001	<0.001	<0.001
	Independence	<0.001	0.011	1.000	0.040	0.009
	Conditional coverage	<0.001	<0.001	<0.001	<0.001	<0.001
	Average MRC	0.0757	0.0735	0.0732	0.0736	0.0726
	Maximum # violations	34	12	11	12	18
10-day VaR	Expected # violations	76	76	76	76	76
	Observed # violations	164	165	165	125	64
	# consecutive violations	107	104	104	86	44
	Unconditional coverage	<0.001	<0.001	<0.001	<0.001	0.165
	Independence	<0.001	<0.001	<0.001	<0.001	<0.001
	Conditional coverage	<0.001	<0.001	<0.001	<0.001	<0.001
	Average MRC	0.2317	0.2337	0.2316	0.2340	0.2965
	Maximum # violations	27	16	16	15	14

Notes: This table shows the backtest results for 1-day VaR estimates and 10-day VaR estimates for the 80/20 portfolio of different Monte Carlo simulation models with normal random numbers, i.e. a model with unconditional volatility (Unc. vol.), a model with EWMA volatility with λ equal to 0.94, a model with EWMA volatility with λ optimized and models with GARCH and EGARCH volatility. For both risk horizons, the first row indicates the expected number of violations, which corresponds to the 99% confidence level on which the VaR estimates are based. The second and third row show the observed numbers of violations and the observed number of consecutive violations, respectively. The *p*-values of the coverage tests are presented in rows 4 to 6. The last two rows give the average minimum required capital over the whole testing period and the maximum number of observed violations over the previous 250 trading days. The backtests are based on VaR estimates made on a daily basis from January 1st, 1981 to December 31st, 2010, giving a total of 7571 estimates to be evaluated for the 10-day VaR estimates and 7580 for the 1-day VaR estimates.

Table 32: Backtests on 99% VaR for the 80/20 portfolio - Monte Carlo simulation with Normal Copulas - Normal marginals

		Unc. vol.	GARCH	EGARCH
1-day VaR	Expected # violations	76	76	76
	Observed # violations	140	150	147
	# consecutive violations	12	8	7
	Unconditional coverage	<0.001	<0.001	<0.001
	Independence	<0.001	0.014	0.035
	Conditional coverage	<0.001	<0.001	<0.001
	Average MRC	0.0809	0.0736	0.0727
	Maximum # violations	31	14	18
10-day VaR	Expected # violations	76	76	76
	Observed # violations	159	124	69
	# consecutive violations	105	85	46
	Unconditional coverage	<0.001	<0.001	0.431
	Independence	<0.001	<0.001	<0.001
	Conditional coverage	<0.001	<0.001	<0.001
	Average MRC	0.2300	0.2346	0.2965
	Maximum # violations	28	15	16

Notes: This table shows the backtest results for 1-day VaR estimates and 10-day VaR estimates for the 80/20 portfolio of different Monte Carlo simulation models with normal copulas, based on normal marginals with unconditional volatility estimates, GARCH volatility estimates and EGARCH volatility estimates. For both risk horizons, the first row indicates the expected number of violations, which corresponds to the 99% confidence level on which the VaR estimates are based. The second and third row show the observed numbers of violations and the observed number of consecutive violations, respectively. The *p*-values of the coverage tests are presented in rows 4 to 6. The last two rows give the average minimum required capital over the whole testing period and the maximum number of observed violations over the previous 250 trading days. The backtests are based on VaR estimates made on a daily basis from January 1st, 1981 to December 31st, 2010, giving a total of 7571 estimates to be evaluated for the 10-day VaR estimates and 7580 for the 1-day VaR estimates.

Table 33: Backtests on 99% VaR for the 80/20 portfolio - Monte Carlo simulation with Student *t* Copulas - Normal marginals

		Unc. vol.	GARCH	EGARCH
1-day VaR	Expected # violations	76	76	76
	Observed # violations	140	150	147
	# consecutive violations	12	8	7
	Unconditional coverage	<0.001	<0.001	<0.001
	Independence	<0.001	0.014	0.035
	Conditional coverage	<0.001	<0.001	<0.001
	Average MRC	0.0809	0.0736	0.0727
	Maximum # violations	31	14	18
10-day VaR	Expected # violations	76	76	76
	Observed # violations	159	125	69
	# consecutive violations	105	85	46
	Unconditional coverage	<0.001	<0.001	0.431
	Independence	<0.001	<0.001	<0.001
	Conditional coverage	<0.001	<0.001	<0.001
	Average MRC	0.2300	0.2347	0.2965
	Maximum # violations	28	15	16

Notes: This table shows the backtest results for 1-day VaR estimates and 10-day VaR estimates for the 80/20 portfolio of different Monte Carlo simulation models with Student *t* copulas, based on normal marginals with unconditional volatility estimates, GARCH volatility estimates and EGARCH volatility estimates. For both risk horizons, the first row indicates the expected number of violations, which corresponds to the 99% confidence level on which the VaR estimates are based. The second and third row show the observed numbers of violations and the observed number of consecutive violations, respectively. The *p*-values of the coverage tests are presented in rows 4 to 6. The last two rows give the average minimum required capital over the whole testing period and the maximum number of observed violations over the previous 250 trading days. The backtests are based on VaR estimates made on a daily basis from January 1st, 1981 to December 31st, 2010, giving a total of 7571 estimates to be evaluated for the 10-day VaR estimates and 7580 for the 1-day VaR estimates.

Table 34: Backtests on 99% VaR for the 80/20 portfolio - Monte Carlo simulation with Clayton Copulas - Normal marginals

		Unc. vol.	GARCH	EGARCH
1-day VaR	Expected # violations	76	76	76
	Observed # violations	140	150	147
	# consecutive violations	12	8	7
	Unconditional coverage	<0.001	<0.001	<0.001
	Independence	<0.001	0.014	0.035
	Conditional coverage	<0.001	<0.001	<0.001
	Average MRC	0.0809	0.0736	0.0727
	Maximum # violations	31	14	18
10-day VaR	Expected # violations	76	76	76
	Observed # violations	159	124	69
	# consecutive violations	105	85	46
	Unconditional coverage	<0.001	<0.001	0.431
	Independence	<0.001	<0.001	<0.001
	Conditional coverage	<0.001	<0.001	<0.001
	Average MRC	0.2301	0.2346	0.2965
	Maximum # violations	27	15	16

Notes: This table shows the backtest results for 1-day VaR estimates and 10-day VaR estimates for the 80/20 portfolio of different Monte Carlo simulation models with Clayton copulas, based on normal marginals with unconditional volatility estimates, GARCH volatility estimates and EGARCH volatility estimates. For both risk horizons, the first row indicates the expected number of violations, which corresponds to the 99% confidence level on which the VaR estimates are based. The second and third row show the observed numbers of violations and the observed number of consecutive violations, respectively. The p-values of the coverage tests are presented in rows 4 to 6. The last two rows give the average minimum required capital over the whole testing period and the maximum number of observed violations over the previous 250 trading days. The backtests are based on VaR estimates made on a daily basis from January 1st, 1981 to December 31st, 2010, giving a total of 7571 estimates to be evaluated for the 10-day VaR estimates and 7580 for the 1-day VaR estimates.

Table 35: Backtests on 99% VaR for the 80/20 - Monte Carlo simulation - Student *t* random numbers

		Unc. vol.	EWMA $\lambda = 0.94$	EWMA $\hat{\lambda}$	GARCH	EGARCH
1-day VaR	Expected # violations	76	76	76	76	76
	Observed # violations	142	119	124	109	112
	# consecutive violations	10	5	3	4	5
	Unconditional coverage	<0.001	<0.001	<0.001	<0.001	<0.001
	Independence	<0.001	0.054	0.534	0.099	0.033
	Conditional coverage	<0.001	<0.001	<0.001	<0.001	<0.001
	Average MRC	0.0812	0.0766	0.0768	0.0770	0.0770
	Maximum # violations	31	10	8	9	14
10-day VaR	Expected # violations	76	76	76	76	76
	Observed # violations	123	147	160	117	67
	# consecutive violations	73	95	99	80	46
	Unconditional coverage	<0.001	<0.001	<0.001	<0.001	0.305
	Independence	<0.001	<0.001	<0.001	<0.001	<0.001
	Conditional coverage	<0.001	<0.001	<0.001	<0.001	<0.001
	Average MRC	0.2502	0.2359	0.2344	0.2308	0.2700
	Maximum # violations	23	15	16	14	16

Notes: This table shows the backtest results for 1-day VaR estimates and 10-day VaR estimates for the 80/20 portfolio of different Monte Carlo simulation models with Student *t* random numbers, i.e. a model with unconditional volatility (Unc. vol.), a model with EWMA volatility with λ equal to 0.94, a model with EWMA volatility with λ optimized and models with GARCH and EGARCH volatility. For both risk horizons, the first row indicates the expected number of violations, which corresponds to the 99% confidence level on which the VaR estimates are based. The second and third row show the observed numbers of violations and the observed number of consecutive violations, respectively. The *p*-values of the coverage tests are presented in rows 4 to 6. The last two rows give the average minimum required capital over the whole testing period and the maximum number of observed violations over the previous 250 trading days. The backtests are based on VaR estimates made on a daily basis from January 1st, 1981 to December 31st, 2010, giving a total of 7571 estimates to be evaluated for the 10-day VaR estimates and 7580 for the 1-day VaR estimates.

Table 36: Backtests on 99% VaR for the 80/20 portfolio - Monte Carlo simulation with Normal Copulas - Student *t* marginals

		Unc. vol.	GARCH	EGARCH
1-day VaR	Expected # violations	76	76	76
	Observed # violations	140	109	109
	# consecutive violations	12	5	4
	Unconditional coverage	<0.001	<0.001	<0.001
	Independence	<0.001	0.026	0.099
	Conditional coverage	<0.001	<0.001	<0.001
	Average MRC	0.0809	0.0766	0.0764
	Maximum # violations	31	11	16
10-day VaR	Expected # violations	76	76	76
	Observed # violations	130	122	72
	# consecutive violations	79	80	45
	Unconditional coverage	<0.001	<0.001	0.666
	Independence	<0.001	<0.001	<0.001
	Conditional coverage	<0.001	<0.001	<0.001
	Average MRC	0.2503	0.2316	0.2674
	Maximum # violations	25	14	15

Notes: This table shows the backtest results for 1-day VaR estimates and 10-day VaR estimates for the 80/20 portfolio of different Monte Carlo simulation models with normal copulas, based on Student *t* marginals with unconditional volatility estimates, GARCH volatility estimates and EGARCH volatility estimates. For both risk horizons, the first row indicates the expected number of violations, which corresponds to the 99% confidence level on which the VaR estimates are based. The second and third row show the observed numbers of violations and the observed number of consecutive violations, respectively. The *p*-values of the coverage tests are presented in rows 4 to 6. The last two rows give the average minimum required capital over the whole testing period and the maximum number of observed violations over the previous 250 trading days. The backtests are based on VaR estimates made on a daily basis from January 1st, 1981 to December 31st, 2010, giving a total of 7571 estimates to be evaluated for the 10-day VaR estimates and 7580 for the 1-day VaR estimates.

Table 37: Backtests on 99% VaR for the 80/20 portfolio - Monte Carlo simulation with Student *t* Copulas - Student *t* marginals

		Unc. vol.	GARCH	EGARCH
1-day VaR	Expected # violations	76	76	76
	Observed # violations	140	109	109
	# consecutive violations	12	5	4
	Unconditional coverage	<0.001	<0.001	<0.001
	Independence	<0.001	0.026	0.099
	Conditional coverage	<0.001	<0.001	<0.001
	Average MRC	0.0809	0.0766	0.0765
	Maximum # violations	31	11	16
10-day VaR	Expected # violations	76	76	76
	Observed # violations	131	122	72
	# consecutive violations	80	80	45
	Unconditional coverage	<0.001	<0.001	0.666
	Independence	<0.001	<0.001	<0.001
	Conditional coverage	<0.001	<0.001	<0.001
	Average MRC	0.2505	0.2315	0.2674
	Maximum # violations	25	14	15

Notes: This table shows the backtest results for 1-day VaR estimates and 10-day VaR estimates for the 80/20 portfolio of different Monte Carlo simulation models with Student *t* copulas, based on Student *t* marginals with unconditional volatility estimates, GARCH volatility estimates and EGARCH volatility estimates. For both risk horizons, the first row indicates the expected number of violations, which corresponds to the 99% confidence level on which the VaR estimates are based. The second and third row show the observed numbers of violations and the observed number of consecutive violations, respectively. The p-values of the coverage tests are presented in rows 4 to 6. The last two rows give the average minimum required capital over the whole testing period and the maximum number of observed violations over the previous 250 trading days. The backtests are based on VaR estimates made on a daily basis from January 1st, 1981 to December 31st, 2010, giving a total of 7571 estimates to be evaluated for the 10-day VaR estimates and 7580 for the 1-day VaR estimates.

Table 38: Backtests on 99% VaR for the 80/20 portfolio - Monte Carlo simulation with Clayton Copulas - Student *t* marginals

		Unc. vol.	GARCH	EGARCH
1-day VaR	Expected # violations	76	76	76
	Observed # violations	140	109	108
	# consecutive violations	12	5	4
	Unconditional coverage	<0.001	<0.001	<0.001
	Independence	<0.001	0.026	0.094
	Conditional coverage	<0.001	<0.001	<0.001
	Average MRC	0.0809	0.0766	0.0766
	Maximum # violations	31	11	16
10-day VaR	Expected # violations	76	76	76
	Observed # violations	130	122	73
	# consecutive violations	79	80	45
	Unconditional coverage	<0.001	<0.001	0.753
	Independence	<0.001	<0.001	<0.001
	Conditional coverage	<0.001	<0.001	<0.001
	Average MRC	0.2503	0.2315	0.2669
	Maximum # violations	25	14	15

Notes: This table shows the backtest results for 1-day VaR estimates and 10-day VaR estimates for the 80/20 portfolio of different Monte Carlo simulation models with Clayton copulas, based on Student *t* marginals with unconditional volatility estimates, GARCH volatility estimates and EGARCH volatility estimates. For both risk horizons, the first row indicates the expected number of violations, which corresponds to the 99% confidence level on which the VaR estimates are based. The second and third row show the observed numbers of violations and the observed number of consecutive violations, respectively. The *p*-values of the coverage tests are presented in rows 4 to 6. The last two rows give the average minimum required capital over the whole testing period and the maximum number of observed violations over the previous 250 trading days. The backtests are based on VaR estimates made on a daily basis from January 1st, 1981 to December 31st, 2010, giving a total of 7571 estimates to be evaluated for the 10-day VaR estimates and 7580 for the 1-day VaR estimates.