Coordination Of Lot-Sizing Decisions In A Game Theoretical Framework Part 1

Master Thesis

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Abstract

Individual decision making in a supply chain will often not lead to an outcome in which total profit is maximized. Additional action needs to be taken to increase overall efficiency. We investigate the coordinating effects of two mechanisms: a quantity discount schedule and an order bonus scheme. Each mechanism is analysed in an environment with continuous demand and an infinite planning horizon against the background of non-cooperative game theory. Whereas the traditional scheme fully aligns the supply chain, the alternative turns out to be completely ineffective.

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Chapter 1

Introduction

The main purpose of each supply chain is to organize the conversion of raw materials into end-consumer products via various stages of production and distribution. Upstream partners deliver inputs, while downstream partners buy the output. In an arbitrary configuration, the supplier's and buyer's base of different firms may overlap. The mutual relationships cause individual decisions to have system-wide implications. When each actor optimizes his own profit and disregards the effects of his decisions on others, total realized profit may be less than what it could be if all were to collaborate. However, full cooperation may not be possible and, instead, one has to resort to other, less extreme, remedies.

In this thesis we investigate the capabilities of two simple mechanisms to mitigate the negative aspects of anarchistically setting order sizes. For the much studied quantity discount model we provide a new detailed derivation of optimal behaviour by making use of non-cooperative game theory. The second approach is to allow (partial) reimbursement of the downstream order costs. Whereas the traditional scheme fully aligns the supply chain, the alternative turns out to be completely ineffective.

The remainder is organized as follows. In Chapter 2 we embed the purpose of the thesis in the more general concept of (counteracting) market inefficiencies. Chapter 3 discusses the relevant literature. An overview of the problem environment, including assumptions and notation, can be found in Chapter 4. Chapter 5 contains a short description of the game theoretical concepts applied in Chapter 6 for the derivation of sequentially optimal behaviour in the lot-sizing games. Chapter 7 concludes with a summary of our findings.

Chapter 2

Problem description

2.1 Introduction

Supply chain inefficiencies are a wide-ranging problem. In Section 2.2 we distinguish several ways in which myopically taking decisions can be bad for total profit. Optimization of the system by integrating all operations in one organization and two other, less extreme, solution approaches are discussed in Section 2.3. We formulate the goal of the thesis in Section 2.4.

2.2 Different supply chain inefficiencies

Double marginalization In the classic case of double marginalization, an upstream actor incorporates a mark-up above marginal costs in his price. Consequently, a downstream supply chain partner will set a higher price and will include a mark-up of his own. The overall result is an inflated price for consumers and a smaller quantity delivered. The realized total supply chain surplus will be less than in the welfare maximizing outcome, where the pricing decisions are solely based on the marginal costs of production and distribution at each stage of the supply chain (Pepall et al. (2002, pages 437-443)).

Non-optimal levels of advertising & product quality Apart from moving product to the end-consumer, other functions have to be performed in a supply chain. These have to do with the characteristics of a product, how and to what extent it is promoted and the amount of capital invested in developing new ones. For society, the benefits to all, including end-consumers, are relevant. In practice, advantageous aspects, like better informedness of consumers in case of advertising, will only be taken into account by an actor in as far as his profits are increased. Pepall et al. (2002, Chapters 10 & 11) shows that the price-sensitiveness of demand can be an important factor in this respect.

An equally important cause of socially inefficient decision making is free-riding. Less will be spent on promotion and product development, because one hopes someone else will make the necessary investments to increase demand.

Product and/or asset specificity (hold-up problem) Assets in the form of physical equipment or knowledge can be very hard or even impossible to employ in another business relationship as profitable as in the current one. When some supply chain member has acquired such a specific asset, the other party could behave opportunistically after the investment has been made. He may threaten to find another contracting party, unless the present one is willing to adjust the contract terms in the sense that the amount paid for the asset will merely be recovered up till the pay-offs in its second-best use. Rational actors anticipating this might not invest at all in situations where this opportunistic behaviour is not discouraged in some way.

Bullwhip effect Lee et al. (1997, pages 95-98) illustrate the well known logistical issue of the bullwhip effect. In a stochastic environment, deviations from the regular order pattern can be wrongfully interpreted as indications that the demand intensity has changed. When the upstream actor forecasts demand for production and inventory planning decisions, the error is amplified because of extrapolation into the future. Moreover, the resulting change of the order pattern will become more pronounced, when safety stocks are adjusted, if longer lead times apply and in case of higher demand variability.

Other problematic factors are order batching, which may translate small (expected) changes in the amount of product into larger ones upstream, and major undesirable swings in demand caused by buyers engaging in forward buying. Distorted information also spreads when some actor cannot meet demand and rations his supplies based on the size of the incoming orders. In response, customers will exaggerate their required amounts. The consequences are excessive inventories, more difficulties in smoothly organizing production and a necessity to hold available extra production capacity. The more one gets upstream the supply chain, the stronger the distortions will be.

Disproportionate risk exposure Problems may emerge as well if some supply chain member disproportionately carries the burden of negative economic developments. The exposed actor may refrain from actions beneficial to the entire chain. An example is the 2 echelon 1 period Newsboy problem in Axsäter (2006, pages 284-287). The downstream actor is confronted with stochastic end-consumer demand for which product must be purchased beforehand. While the upstream party is guaranteed to make a fixed profit equal to his margin times the number of units ordered in advance, the other actor's profit may vary considerably, and even be negative. The combination of the unilateral risk exposure, and the fact that the upstream price, including the margin, will be higher than the echelon cost of procurement, results in a smaller than jointly optimal downstream order size.

Production planning externalities Each firm has to schedule its production and value adding activities in order to satisfy demand on its output market. The resulting plan yields an order scheme, which, in turn, serves as the basis for the production and lot-sizing decisions taken at the upstream echelon(s). Because of positive or negative externalities to other actors, the individual planning activities may not minimize total costs in the supply chain.

2.3 Vertical integration & other remedies

Joint ownership by vertically integrating Combining activities in one firm has traditionally been regarded an important solution in all kinds of situations where market exchange is very costly or totally fails at all. Williamson (1971) discerns five classes of characteristics, which he deems important for the attractiveness of the approach. Among these are the monopolistic or oligopolistic contexts where double marginalization is likely to emerge. In Christy & Grout (1994) and Klein et al. (1978), it is indicated that, when there is much product and process/asset specificity, high costs of complex contracting make vertical integration a more preferred safeguard against mutual hold-up.

Despite the wide-ranging spectrum of problems vertical integration can remedy, the construction of a more sizable firm potentially has some major disadvantages. As described in Jeuland & Shugan (1983, page 250), a downstream actor may carry products from other manufacturers to exploit economies of scope, which becomes problematic after integration, a vertical merger may not be allowed by law, or each independent actor carries out his specialized function less efficiently in a larger organization.

Market solutions (closer collaboration) A less extreme approach is increasing the level of mutual cooperation in the supply chain. To counteract the bullwhip effect and coordinate buying practices across different actors, Lee et al. (1997, pages 98-100) put forward electronic data interchange (EDI) and vendor managed inventories (VMI). EDI means that supply chain partners share company specific information. Communicating details about stock levels and market forecasts enables anticipation on sudden drops or surges in demand, thereby decreasing the necessity to keep large safety stocks and making expensive overreactions less likely. Because of the relegation of all inventory related operations to the upstream partner, the VMI mechanism is somewhat more extreme.

Market solutions (aligning incentives) The autonomy of the supply chain members is respected most when coordination takes place by introducing contractual provisions which only influence the operating environment of another actor indirectly. For instance, the two-part tariff, as explained in Pepall et al. (2002, pages 481-483), is particularly suited to confront the problem of double marginalization. A lump sum payment and charging of the marginal cost of production for each unit bought will result in a lower price on the finished goods market and higher total surplus. The construction is typically found in a franchising agreement, and can, under circumstances, alleviate the problem of suboptimal product quality levels as well.

Methods to mitigate free-riding on someone else's (advertising) expenditures, are resale price maintenance and exclusive selling/dealing contracts. With resale price maintenance the price charged to consumers is no longer freely determined by the retailer. Exclusive selling and dealing agreements, on the contrary, restrict the number of downstream or upstream partners. The creation of (local) monopolies ensures that the exclusive supply chain member reaps all the benefits of his efforts without another supply chain member benefiting at his expense (Pepall et al. (2002, Chapter 9)).

For the 2 echelon Newsboy problem, Axsäter (2006, pages 286-287) proposes a buy-back contract. If demand is less than the quantity ordered in advance, the remaining products can be returned to the upstream partner. Adequately setting the wholesale and buy-back price results in an order size maximizing total expected supply chain profit.

Finally, a lot of supply chain efficiencies can be resolved by the profit sharing contract. Because each party gets a predetermined fraction of the pooled profit, operations will shift to the collectively most desired outcome. A major drawback is that it requires quite some monitoring resources. Success depends largely on the truthful revelation of individual revenues and costs.

2.4 Focus of research

Our analysis is restricted to the counteraction of production planning externalities, and more specifically to the negative consequences of anarchistic lot-sizing. As common ownership and a construction like VMI are quite rigorous forms of exerting vertical control, and integration of activities has some major problems of its own, we concentrate on two simple market solutions to align incentives.

The first mechanism is a quantity discount schedule, which may be used to directly reward the choice for certain order sizes. Under the alternative scheme, it becomes possible to reduce the other actor's costs of placing orders by granting an order bonus. We are interested in deriving Stackelberg (Nash) equilibria for 2 actor lot-sizing games, the degree to which a schedule enhances total profits and how the extra surplus, if any, benefits each partner.

Chapter 3

Literature review

3.1 Introduction

In their overview, Sarmah et al. (2006) roughly rubricate the supply chain coordination literature in categories like articles with a manufacturer's/seller's perspective, joint buyer/seller models, game theoretical literature and analyses with multiple buyers. In Section 3.2 we adhere to a somewhat different classification. We start with a system-wide view, and proceed with cooperative and non-cooperative contractual arrangements concentrating on the mitigation of production and inventory control externalities. The second part of the section contains a discussion of models where either the assumption of strategic certainty is relaxed, or where demand characteristics become more complex. Most articles assume continuous constant demand and an infinite planning horizon (the well known economic order quantity (EOQ) context). Section 3.3 relates our analysis to the literature.

3.2 Discussion

System-wide optimization An approach, closely linked to the concept of vertical integration, is the simultaneous optimization of the entire supply chain. The profits or costs of all actors are added and the objective becomes maximizing total surplus or minimizing total costs. In Goyal (1976), a standard EOQ context is used to establish the profits for a customer and a supplier. After inserting the expression for the optimal customer order cycle into the joint cost equation, the optimal lot-sizing integers can be easily determined. Goyal (1988) considers the more general case, in which the vendor/supplier has a finite production capacity and shipment to the customer/purchaser takes place at the end of a production run.

Central inventory control is probably even more important in complicated supply chain designs. Khouja (2003*a*) investigates the use of an equal cycle time for each actor and the application of an integer multiple or integer power of two policy at each stage of an arborescent type supply chain. In contrast to the integer multiple rule, the order intervals can differ among members in the same stage with integer powers of two. Under assumptions as in Goyal (1988), either a closed form expression or an algorithm to arrive at a solution (not necessarily optimal) is presented. The performance of the different policies is evaluated in a setting with seven retailers, three manufacturers and one supplier. The tabulated values and the results of a sensitivity analysis show that with integer multiples, quite substantial cost reductions can be realized compared to the equal cycle time option. Total cost can be further reduced, although to a smaller extent, with the integer power of two policy.

Another more complex configuration is analysed in Khouja (2003*b*). In a serial supply chain with $G + 1$ echelons, each supplier g delivers his output to actor $g + 1$ and adds some value, while the assembler in the last stage integrates the components into an end-product. To achieve mutual coordination, a synchronized solution is calculated. The author justifies the resulting lot-for-lot replenishment policies for all actors with a just in time (JIT) argument; they enable a quick reaction to changes in demand and product design. What seems especially strange in this respect, though, is that the inputs are assumed to come in some time before production actually starts. Total inventory carrying costs could be decreased if one would wait until the moment they are needed. The article lacks an explanation of why this early arrival of inputs should be useful.

From the individually most preferred order intervals it can be seen that, by synchronizing, the actors at both extremes of the supply chain are harmed most. This shifting of the cost balance might lead to a misalignment of incentives, even when all suppliers and the assembler are vertically integrated. To alleviate this problem, the author suggests the internal adjustment of the holding cost structures or the fixed costs of placing an order. Some doubt, however, is cast on the correctness and usefulness of the expressions stating how the individually preferred lot-sizing decisions can be made to correspond to the optimal joint JIT cycle time. In the numerical example, the entire chain is synchronized by fixing the selling price below the input price for some echelons, such that value is destroyed there.

Price-inelastic demand with cooperation When parties are autonomous cooperating decision makers, attention shifts to negotiation over the division of any efficiency gains arising from collaboration. A graphical illustration of the feasible set of seller prices and buyer order quantities is given in Dada & Srikanth (1987, page 1249). Because the authors assume price-independent inventory holding cost rates, total system costs are minimized at a fixed order quantity Q^{**}. The maximum total savings can be distributed in any possible way among the supply chain partners by appropriately setting the price charged to the buyer.

A second article, Kohli & Park (1989), contains a more extensive analysis. All Pareto efficient outcomes, where the resulting costs of one of the actors cannot be decreased without increasing those of the other, are located at the welfare maximizing order quantity Q^*, and only differ in terms of the supplier price. With the maximum price, all benefits accrue to the seller and the buyer is just compensated for his increased costs. Charging the minimum results in the opposite. Both the Nash and the Kalai & Smordinsky bargaining solution, see Kohli & Park (1989, page 700) for an explanation of these bargaining concepts, predict that, when the seller and the buyer are risk neutral or equally risk averse, an equal split of the efficiency gain will be agreed upon. The more some agent is risk averse, the smaller his share. Another bargaining model due to Eliashberg (Kohli & Park (1989, page 702)) points at the detrimental effects of a decrease in bargaining power. Comparable to Dada & Srikanth (1987), quantity discounts are proposed to enforce the agreement. Whether the discount scheme is of the all unit quantity discount (AQD) or the incremental unit quantity discount (IQD) type, is irrelevant.

Kim & Hwang (1989) show that an AQD and IQD schedule remain equally capable of realizing each price and order quantity combination in the feasible set if inventory holding costs directly depend on price. Pareto efficient outcomes are determined by minimizing the joint costs subject to the constraint that the savings for the parties are linearly related via a distribution parameter. Their parameter α equals $r/(1-r)$ in Chakravarty & Martin (1988), who, in addition, study the case with multiple buyers having a common order interval and a configuration with heterogeneous groups of buyers. Chiang et al. (1994) calculate Pareto bargaining solutions in an alternative way. An exponential transformation of variables is used to translate a mixed integer geometric programming formulation into a convex programming problem.

Price-inelastic demand without cooperation The article by Monahan (1984) may be seen as the starting point of a formal analysis of an AQD scheme in a non-cooperative environment. Saving on order processing and set-up costs, capturing transport discounts and cost of capital considerations might all motivate the vendor to change the buyer's lot-sizing behaviour. A buyer, willing to increase the standard EOQ order size by a factor K, needs to be compensated for extra costs. Incorporation of the 'practical' expression for the minimally acceptable discount, which does not take into account the effect of price on the buyer inventory holding cost rate, gives the vendor an extra decision variable. A relatively high vendor fixed order cost causes the optimal scaling factor K^* to be larger. Monahan (1988) defends the simple model by pointing out that it is merely meant to illustrate the use of quantity discount schedules to affect downstream inventory policies. Graphical illustrations of the 1 buyer and a multiple buyer scenario can be found in Drezner & Wesolowsky (1989).

In Joglekar (1988), it is argued that, if the buyer order size increases notably because of the discount scheme in Monahan (1984), production capacity can no longer be assumed unlimited. To accommodate to this complication, adjusted minimum discount and vendor profit expressions are derived. With a non-negligible time to produce, even a vendor using a lot-for-lot replenishment policy incurs carrying costs. The same finite production rate formulas are derived in Banerjee (1986*b*), where it is shown for the common order interval case, by referring to Banerjee (1986*a*), that a quantity discount schedule fully optimizes a 2 echelon serial supply chain.

When the assumption of an infinite production rate is maintained, order handling and manufacturing at the vendor should be decoupled according to Joglekar (1988). Since the order processing costs of the vendor and the buyer likely do not differ much, the optimal scaling factor will be very limited in magnitude, meaning that coordination of order cycles does not have a large impact. Instead, saving on large manufacturing set-up costs, by including different buyer batches in one production run, will be more cost effective.

The combination of a quantity discount schedule and a more flexible lot-sizing rule is analysed in Lee & Rosenblatt (1986). If the lot-for-lot assumption is set aside, the quantity discount offered to the buyer not solely depends on the fixed order and set-up cost, as in Monahan (1984), but also on the holding cost rates of both parties. The algorithm is criticised in Goyal (1987*a*) for being unnecessarily complex. By exploiting the characteristics of the problem, a much more elegant procedure, similar to that in Goyal (1976) and Goyal (1988), can be constructed. The reduced cost function, which results by substituting the expression for the optimal scaling factor, is convex in the supplier lot-sizing variable. The optimal integer(s) must satisfy a simple condition.

Goyal (1987*b*) introduces the parameter $a \geq 0$, reflecting the strength of the buyer in the supply chain. The more dominant the buyer, the larger a, the harder it is for the supplier to convince the buyer to choose the order quantity prescribed in the quantity discount scheme. If $a = 1$, the buyer is exactly compensated for the increase in costs, the model reduces to that in Goyal (1987*a*), and the supply chain is fully optimized. Other parameter values are less beneficial for total welfare: a approximating 0 leads to efficiency losses, while the non-coordination outcome is associated with a perfectly dominating buyer.

The nature of the Banerjee (1986*c*) model is quite different from the foregoing. The supplier's objective becomes the realization of a prespecified level of gross profit on each unit sold. The dependency of the customer inventory holding cost rate on price enables the supplier to affect downstream behaviour. A lower price decreases the supplier's gross margin and increases his incoming orders. There exists a unique solution, which yields the chosen profit margin. A numerical example and a sensitivity analysis are performed to provide more insight into the effect of changing parameters.

The already mentioned Chiang et al. (1994) explicitly recognizes game theory as the underlying behavioral mechanism. Next to cooperation, a non-cooperative scenario is elaborated on. Just like Banerjee (1986*c*), the buyer order size can be influenced indirectly because of a price-dependent holding cost rate. Knowing the buyer's lot-sizing response, the authors establish the price maximizing the seller's profit. A closer look at the analysis reveals that a price discount will merely be applied if the buyer's budget constraint is binding. The necessity of a limit on expenses, to ensure a positive optimal discount, seems somewhat artificial.

Munson & Rosenblatt (2001) is one of the few articles analysing coordination in a 3 echelon serial supply chain. The manufacturer, positioned in the middle of the chain, is the dominant actor, who decides on the quantity discount schedules he imposes on the downstream buyer and the upstream supplier. The authors present a tailored algorithm to solve the manufacturer's cost minimization problem. If savings can be extracted from the supplier, the discount schedules are configured to minimize total costs in the supply chain. It is theoretically shown that potential savings at the supplier or the manufacturer are largest when production capacity at either party is very small or very large. Several conditions state under which circumstances common order intervals across the supply chain are optimal. A lower bound on the realizable gains is given for the case with infinite production rates and lot-for-lot replenishment policies.

In a computational study, different combinations of parameter values are used to test the performance. Adding a third tier considerably adds to savings. On average, total cost is reduced by 42.81%. The manufacturer benefits most in two cases. When the supplier is confronted with an unfavorable cost structure, more savings can be appropriated. In situations where the inventory holding characteristics of the retailer and the manufacturer do not differ much, the relatively limited amount of compensation for the retailer adds to profitability. Neglection of the supplier in 29.48% of the cases does not matter much for total supply chain efficiency. The nearly perfect result of just 0.12% above the minimum total costs is explained by observing that the supplier is most likely to be left out when this actor's potential savings are small anyway.

Price-inelastic demand & uncertainty Coordination of the supply chain becomes a more challenging task, when there is a lack of knowledge about another actor's characteristics or decisions. Corbett & de Groote (2000) analyse the influence of strategic uncertainty on the effectiveness of a quantity discount scheme. Their seller is not fully informed about the buyer holding cost rate, but instead, is confronted with a probability distribution of buyer types ranging from $\underline{h}_b$ to $\bar{h}_b$. In an equilibrium, where the buyer truthfully reveals his type, the seller may find it profitable to exclude certain downstream actors by choosing a cut-off value $h_b^* < \bar{h}_b$. Compared to the full information scenario, buyers with a holding cost rate between h_b^* and $\bar{h}_b$ will find it too expensive to trade, while those with smaller rates are favoured by the presence of asymmetric information; their costs decrease. The higher the actual holding cost rate, the larger the difference between the jointly optimal order size and the buyer's lot-sizing decision. With less additional profit and a range of buyer types who keep some cost reductions to themselves, the seller cannot reap the maximum gain anymore.

Results, essentially the same as in a full information context, emerge in Li & Liu (2006) for a 2 echelon supply chain facing stochastic demand. Analogous to Kohli & Park (1989), and by implicitly making use of the approximation in Axsäter (2006, page 107) for the expected costs of the buyer, the minimum and maximum discounts are derived. The approximation is a reasonable one, if there are few backorders and the order quantity Q is large. The expected efficiency gain can be expressed as the difference between the maximum and the minimum wholesale price multiplied by the expectation of demand. $\alpha \in [0, 1]$ represents the fraction of the extra surplus which goes to the buyer after the (not modelled) negotiation process. Despite demand uncertainty, a quantity discount schedule is still capable to coordinate the supply chain towards the system-wide optimum.

Price-sensitive demand Another complicating factor arises when demand reacts to the price charged. In Section 2.2, we described how individualistic pricing causes double marginalization throughout the supply chain. Moorthy (1987) thinks that the profit sharing arrangement, by means of a quantity discount, proposed in Jeuland & Shugan (1983) is unnecessarily complex, may conflict with competition law because of price discriminatory aspects, and, above all, is not needed to achieve full coordination. Each mechanism making the adjusted retailer's effective marginal cost function intersect his marginal revenue function from below at the system-wide optimal quantity, will do the job. His alternative of a (unilaterally imposed) two-part tariff is, in turn, rejected in Jeuland & Shugan (1988), because it violates their equality of partners assumption.

Besides double marginalization, the familiar planning externalities emerge, when, next to pricing decisions, order sizes have to be set. Two articles, which to a large extent make use of the same model, are Li et al. (1995) and Li et al. (1996). The seller moves first by declaring his price charged to the buyer. Multiplication with a constant k (not a decision variable) yields the consumer price. With demand known, the buyer decides, under a budget restriction as in Chiang et al. (1994), on the lot-sizing policy. The resulting price and order size are respectively too high and too small to maximize total profits. To sustain mutually beneficial cooperation, the authors concentrate on the combination of a quantity discount schedule and equal profit sharing. A weakness in the analysis of Li et al. (1995) is the minimization of a 'quasi' joint cost function in which the effect of price on the buyer's gross margin is left out.

Unlike the previous articles, the buyer and seller each have to take pricing and lot-sizing decisions in Abad (1994). Much like Chiang et al. (1994), an algorithm, based on a set of rewritten first order conditions, is set out to calculate the Pareto efficient cooperative solutions. As a special case, the Pareto efficient Nash bargaining solution is obtained. In an extended configuration, the seller is confronted with multiple buyers. A distinction is made between a group composed of heterogeneous individuals and a group of buyers sharing some common cost and demand characteristics. Pricing (quantity discount) schedules are provided to support the different outcomes.

Any form of collaboration is absent in Weng (1995*a*). In contrast to Banerjee (1986*b*), it is proved that quantity discounts are not sufficient to achieve and enforce the most efficient 2 actor outcome in an environment with price-elastic demand. A second mechanism, like a franchise fee, is required. Weng (1995*b*) looks at the situation wherein such a complementary mechanism is lacking. A supplier has to decide for either an AQD or an IQD type of schedule on the discount fraction $d \in [0, 1)$ and the order size breakpoint. The prespecified minimum increase in profits to entice the buyer to accept the schedule leads to a lower bound ≥ 0 on the set of feasible discounts. Fractions above a certain upper bound ≤ 1 are not attractive enough from the supplier's point of view. After reducing the continuous interval of fractions to a discretized finite set, the supplier can, given the buyer's optimal lot-sizing response, determine his optimal discount. The equivalence of the two types of quantity discount schemes, established in Kohli & Park (1989) and Kim & Hwang (1989), is shown to continue to hold in both Weng (1995*a*) and Weng (1995*b*). This guarantees that, even with price-sensitivity, the optimal IQD can be directly inferred from the optimal AQD schedule, and vice versa.

Viswanathan & Wang (2003) deem quantity discounts insufficiently effective in boosting revenue, and propose the volume discount as a more direct way of stimulating sales at the downstream level. The authors implement an iterative search grid procedure to find the quantity and volume discounts maximizing the vendor's profit. The search range is repeatedly adjusted by concentrating on the most promising discount regions. The retailer is allowed to deny service to any customer, when he makes a loss given the characteristics of the scheme. Opting for the exit strategy results in a non-operating supply chain.

As expected by the authors, the volume discount indeed performs better in the simulation study. A higher vendor order cost, a higher vendor holding cost rate and less costly ordering for the retailer decrease the advantage of granting discounts on larger sales volumes. If the effect of price on demand diminishes, revenues are more difficult to influence, and the source of extra profits shifts to the cost economies associated with coordinated ordering. Consequently, as in the fixed demand literature, a quantity discount schedule is then the preferred mechanism. This preference is reinforced for larger values of the vendor order-processing cost. Maximum gain, i.e., a fully optimized supply chain, is achieved by combining the two schemes. With a volume discount instead of a franchise fee, the last result corroborates the two mechanism requirement in Weng (1995*a*).

Coordination and dynamic lot-sizing All foregoing literature assumes that demand occurs continuously over an infinite planning horizon. An alternative is to limit the number of time units, e.g. months or years, and subdivide these in different periods. Because the amount of product in each period need not be the same, a lumpy demand pattern emerges. In this last category of coordination models, we review two articles using mathematical programming techniques to solve the actors' dynamic lot-sizing (DLS) problems.

How total efficiency is affected by a gradual movement towards a more tightly integrated and more informed 2 actor DLS supply chain, is analysed in Sahin & Robinson (2005). In a rolling horizon environment, a vendor replenishes a manufacturer, who organizes production to fulfill consumer demand in K periods. Model outcomes are calculated for multiple sets of parameter combinations. Although the sharing of more information with the vendor can result in some cost reductions, vertically integrating operations to a smaller or larger extent is far more effective. Maximum savings up till 47.58% can be realized, once the manufacturer shares all information across a fully integrated supply chain. The value of joint decision making is larger if demand is less lumpy or increases, when transportation costs rise and in case the number of items grows.

Compared to the EOQ literature with reported percentages ranging from a few percent to just 35%, the magnitude of the reported cost savings is remarkable. The gap could, however, be due to the unexplained difference in interpretation of the costs of placing orders. The formal descriptions of the models in the appendix seem to imply that a single order may contain several types of product if integration is part of the coordination efforts, while set-up and invoice costs are incurred for each separate category otherwise. As the dramatic (more than 85%) savings in order costs are only realized the moment one goes beyond information sharing, and because exactly these cost reductions largely contribute to the substantial average total efficiency increases for the integration strategies, the authors' claim that coordination is much more important in make-to-order lumpy demand contexts seems a little premature.

In Dudek & Stadtler (2005), the authors check whether bargaining leads to a more profitably operating supply chain. The setting consists of a buyer and a supplier, each satisfying external demand for different items under capacity restrictions. The supplier, in addition, has to supply part of the buyer's requisites. Inputs can either be directly used, or may first have to be converted into intermediate products. Without collaboration, the buyer solves his production problem, and subsequently the supplier minimizes his own costs. If bargaining is possible, parties receive order proposals from one another and respond by suggesting updated plans containing minor modifications within some predefined bounds. A trade-off is made between higher personal benefits and the amount of deviations from the received proposal. The repetitive construction of new compromises continues until joint costs no longer decrease and another degradation of total net savings is not tolerated anymore. The outcome is made incentive compatible by offering the buyer a kind of quantity discount scheme adjusted to a DLS setting. The discount at least compensates for the extra costs at the downstream level.

The process is simulated for five different production structures across seven capacity utilization rates. When negotiation is introduced, all infeasibilities under anarchistic conditions disappear and a large part, on average 69.8%, of the gap between the non-coordination outcome and the joint optimum is closed. It remains unclear why, despite the collaborative context, parties are not able to coordinate on the joint optimum, reap the maximum efficiency gain and divide it among themselves. This would have the additional benefit of evading the rather cumbersome process of exchanging adjusted order schemes.

3.3 Link with thesis

The analysis of our first coordination mechanism in a 2 actor serial supply chain is closely related to much of the existing EOQ coordination literature. However, we deviate substantially from the mainstream by explicitly studying it in a non-cooperative game theoretical environment. Instead of concentrating on Pareto efficient bargaining outcomes, we look for subgame perfect Nash equilibria. Although several articles use other measures to streamline operations, like profit sharing, franchise fees and volume discounts, these are often solely considered in combination with quantity discounts. Results on some form of order bonus (OB) scheme seem to be lacking entirely.

Chapter 4

Modelling the supply chain

4.1 Introduction

Before deriving optimal behaviour in the supply chain, we present the underlying assumptions and our notation. Decision making takes place in a 2 actor serial supply chain facing continuous, constant and price-inelastic demand for an infinite period of time. Due to the exclusion of backordering (some of) the deterministic demand and the properties of the delivery and production processes, we can limit each actor's expenses to input prices and lot-sizing costs. Section 4.2 gives further details and an explanation of the strategic and informational aspects. We describe the model parameters, the coordination/lot-sizing decisions and some other notational elements in Section 4.3. Section 4.4 lays down the sequence of decision making.

4.2 Assumptions

Supply chain structure We look at a serial supply chain comprised of a Retailer serving end-consumer demand and a Wholesaler supplying him. Inputs originate from the Manufacturer, an otherwise passive agent in the 2 echelon setting. A serial configuration is relatively simple, and, as observed in Li et al. (1995, page 1456), avoids distraction from the main purpose of the analysis, namely, investigating the effects of measures to coordinate inventory policy decisions. Using multiple heterogeneous retailers, like Drezner & Wesolowsky (1989, pages 41-42) and Chakravarty & Martin (1988, pages 275-277), would already cause a lot of (unnecessary) complications.

Demand, planning horizon & prices The actors' operations are restricted to a single product, which is reasonable as long as no major cost synergies, so-called economies of scope, can be realized by combining several production and order policies. Demand occurs continuously for an infinite period of time at a constant rate not affected by the price charged to end-consumers. The process is completely deterministic. Because we normalize the number of inputs needed for 1 output, the upstream actors face the same demand intensity. Comparable to Khouja (2003*b*, page 986), value adding activities are reflected in increasing prices as one goes downstream. Apart from the price discount fraction associated with the first coordination mechanism, the actual determination of prices is exogenous to our models. A convenient consequence is that actors can restrict their attention to production and inventory related decisions.

Production & delivery rate In the normalized production structure, the Wholesaler and Retailer add value with, for example, promotional activities, tailored packaging and efficient distribution to their consumer(s). Although Munson & Rosenblatt (2001, page 377) show that capacity utilization may strongly influence savings, we impose infinite transformation and delivery rates: the time needed for value addition is negligible and inventories are replenished instantaneously upon arrival of the shipment. Infinite rates are a good approximation in case only a minor part of the resources in the supply chain is used. Introducing capacity constraints does not essentially change the results, but merely causes the mathematical expressions to be less tractable. Inputs are processed on a per order basis.

Backordering We do not allow shortages. Inventories are always maintained at levels such that all orders can be fulfilled entirely at the moment they come in. Excluding postponed delivery, so-called backordering, is reasonable in situations where stock-outs lead to disproportionately large losses.

Lead times The term lead time is used to designate the time between the moment an order is received from a customer and the moment that the product arrives there. Constant lead times are intrinsically linked to a deterministic demand environment without the possibility of backordering. To be absolutely sure that shortages will be avoided, an actor has to possess perfect knowledge about the demand pattern and about the time it takes to replenish him. A simple shift in a previously established order pattern suffices to accommodate for a change in the lead time. We might, therefore, just as well say that the product is delivered immediately upon ordering.

Costs Three categories of costs are assumed to be relevant. The first is the unit price an actor has to pay for acquiring an item from his upstream supply chain partner. The other two cost categories are related to the lot-sizing decisions: the fixed cost of ordering and the inventory holding cost rate. Each cost rate is linear in its driver, i.e., the number of items sold, the number of orders placed, or the stock level. Moreover, each is constant over time.

The order cost does not depend on the number of items in a batch. It covers administrative expenses and the costs of personnel to handle the incoming product. The properties of the delivery and production processes (constant (zero) lead time, infinite rates and conversion on a per order basis) imply that the production cycle is completely synchronized with the order cycle. Consequently, the order cost can be considered to include as well a fixed cost of setting up the value addition process/production.

As inputs are never kept in stock, the holding costs for each actor are limited to inventories of processed product. The components of a holding cost rate can be divided in two groups, depending on whether or not there is a direct relationship with the value (input price) of the product. Examples of value-related components are the interest foregone on the capital tied up and the losses resulting from obsolescence, damage and theft. On the other hand, costs of financing or renting storage space are at best remotely connected to price.

A major disadvantage of incorporating to some extent dependency on the input price is that the social optimum for a quantity discount scenario prescribes a 100 % discount to achieve minimum holding cost rates, see Weng (1995*b*, page 310). Zero prices are, however, not a realistic benchmark and are unacceptable to the actor offering the discount, so that, as formulated in Chakravarty & Martin (1988, page 274), there is "... no incentive for pursuit of the optimal 'social welfare' solution." Compared to the other mechanism, the incentive compatibility problem would lead to an underestimation of the efficiency enhancing effects of a quantity discount scheme. To avoid these peculiarities and to ensure that the most efficient outcome is the same, irrespective of whether or not coordination takes place, we assume price-independent holding cost rates. The coordination terms related to a mechanism drop out when the individual profit functions are aggregated.

Besides the fixed cost of preparing an outgoing shipment, a separate cost for handling the downstream partner's orders is used in Viswanathan & Wang (2003) and Weng (1995*a*). For reasons of mathematical clarity and to limit the number of parameter dimensions, we disregard, in accordance with Munson & Rosenblatt (2001), these *order processing* costs and, more generally, any other expenditure.

Strategic interaction & information The supply chain members cannot collaboratively agree on a joint lot-sizing policy. Unlike Kohli & Park (1989) and Dudek & Stadtler (2005), negotiations and bargaining are ruled out. Each maximizes profit without taking into account the possible beneficial or detrimental effects on others. From Corbett & de Groote (2000), it follows that the amount of available information is crucial for the nature of strategic interaction. We assume that the supply chain has been functioning for quite a while already.

Because of past order patterns, each participant is well-informed about the intensity of end-consumer demand and about the other actor's cost structure. Common knowledge is also the underlying principle with respect to the observability of decisions.

4.3 Notation

Environment & decisions The upper part of Table 4.1 presents an overview of all parameters describing the demand process and the actors' cost characteristics. The Retailer receives P_r for each item sold to end-consumers and pays a unit price P_w to the Wholesaler. With P_m the price paid to the Manufacturer, the gross margin per unit is then straightforwardly $P_w - P_m$ for the Wholesaler. In practice, the nature of production will often lead to downstream firms having a smaller order/set-up cost and a larger inventory holding cost rate than their upstream partner. Here, we do not impose $A_r < A_w$ and $h_r > h_w$, thereby maintaining a maximum degree of flexibility in inventory related cost patterns. In fact, this is yet another motivation for not directly linking holding cost rates to prices, which tend to increase as more value is added.

Parameter	**Description**
D	*Demand intensity per time unit*
P_i	*Price per unit charged by actor* $i \in \{r, w, m\}$
A_i	*Fixed order cost for actor* $i \in \{r, w\}$
h_i	*Holding cost rate for actor* $i \in \{r, w\}$
Variable	
Q	*Retailer order size*
n_w	*Wholesaler lot-sizing multiple*
α_w	*Quantity discount fraction set by Wholesaler*
R_w	*Quantity discount region set by Wholesaler*
$\bar{Q}_w$	*Order breakpoint belonging to* R_w
γ_w	*Order bonus fraction set by Wholesaler*

Table 4.1: *Parameters & decision variables*

The first variable in the remainder of the table, Q, represents the amount of product the Retailer orders each time to satisfy end-consumer demand. It forms the basis for the lot-sizing decision upstream. Because prescribing a lot-for-lot policy similar to Monahan (1984) and Khouja (2003*b*) would make the analysis far less interesting, the Wholesaler is allowed to deliver more than once during an order/production cycle. The zero-inventory property (see Axsäter (2006, pages 62 & 226)) ensures that his best course of action is to choose an integer multiple, making his order size equal $n_w Q$.

Mostly, a quantity discount scheme is of the following form: the domain of all possible order sizes is divided in several regions by specifying a set of order breakpoints. Each region is tagged with a certain price discount. Depending on whether the schedule is of the AQD or IQD type, the discount is granted on each item sold or merely on those units of the order falling into the associated region. In the previous chapter we have seen that Weng (1995*a*), Weng (1995*b*), Kohli & Park (1989) and Kim & Hwang (1989) prove the equivalence of both approaches in coordinating a 2 actor serial supply chain. Because a one-to-one transformation exists, there is no need to consider each. And, as observed in Weng (1995*b*, page 307), since an IQD scheme is more complex in nature, it is convenient to restrict attention to AQD schedules.

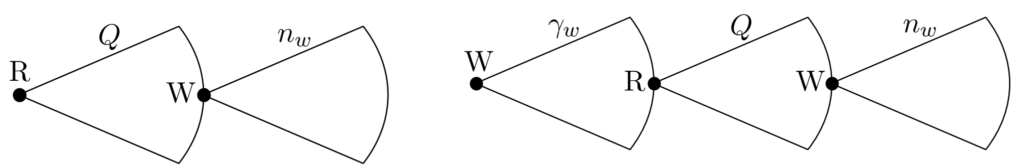
The Wholesaler, being the dominant actor, sets the terms of the schemes. The discount α_w is a fraction of the original pre-discount price. In accordance with the literature, one breakpoint $\bar{Q}_w$ is set. The discount region variable R_w, explicitly incorporating the flexibility suggested in Munson & Rosenblatt (2001, page 377), lets the Wholesaler determine whether the Retailer order size qualifies for the per unit discount $\alpha_w P_w$ in the region $\left(0, \bar{Q}_w\right]$ (quantity premiums) or $\left[\bar{Q}_w, \infty\right)$ (proper quantity discount schedule). The OB scheme result in an adjusted fixed order cost $(1 - \gamma_w)\, A_r$.

Demand, prices, cost parameters, Q and the order breakpoint are all positive (> 0). The variable n_w is restricted to the set of positive integers $\mathbb{N} = \{1, 2, 3, ...\}$. We have $\alpha_w \geq 0$ and $\gamma_w \in [0, 1)$. Negative values are excluded, because otherwise the downstream partner would be penalized, which is contrary to the compensating nature of both mechanisms. To guarantee that an actor continues to pay something for placing orders, a necessary requirement for an optimal lot-sizing decision to exist, the bonus fraction must be smaller than 1. We do not impose an upper limit on α_w In the unlikely scenario that total savings from the quantity discount scheme exceed or equal total gross revenue, the coordinating actor wants to select a value equal to or larger than 1. In practice though, this will not occur, as a member making a loss (negative revenue minus costs) will prevent the supply chain from operating.

Profits, optimal actions & bounds To distinguish among profits (Π, Π_r, or Π_w), optimal behaviour and bounds on some variables, we add (multi-element) superscripts. Optimality, lower and upper bounds are denoted by one of the following: $*$, $-$ or $+$. The other set of elements is made up of u, referring to an uncoordinated supply chain, s for the social optimum and a or c conveying the nature of coordination (which is superfluous if an optimum or bound for a coordination variable is described). When relaxing the requirement $n_w \in \mathbb{N}$, notation will instead be based on ν_w.

4.4 Overview of lot-sizing games

The two types of games are depicted in Figure 4.1. For convenience, we use the order bonus scheme to illustrate the typical set-up of a coordination game. Replacing γ_w by the variables α_w and R_w is sufficient to get the schematic overview for the other mechanism. Each figure satisfies the graphical conventions in Watson (2002). A node represents a decision for either the R(etailer) or the W(holesaler), while a pair of branches connected by an arc indicates that the domain for the decision variable at hand consists of an infinite number of elements. Strictly speaking, nodes situated on an arc stand for a multitude of points from which the game may continue: one for each value of the decision variable in the previous stage. Each node being in its own information set (the node itself) reflects the assumption that an actor taking a decision is fully informed about how the game has proceeded previously. The pay-offs are left out, as these will be established while analysing each game.



(a) Non-coordination (b) Coordination

Figure 4.1: *Extensive forms for 2 echelon games*

To be able to affect downstream behaviour, coordinative action must precede the lot-sizing stages. Contrary to standard practice in the 2 actor quantity discount literature (see for example Lee & Rosenblatt (1986)), we do not combine Wholesaler decision making. Not including the inventory policy decision in the coordination stage better reflects reality where the Retailer order size is observed before choosing an integer multiple. Although the more extensive set-up necessitates the derivation of a rule specifying how the Wholesaler responds to each (possibly irrationally chosen) Q, the difference in approach does not matter for the set of actions actually chosen by the supply chain members. The rewritten first stage optimization problems are constructed such that these also depend on n_w. Assuming a rational Retailer, the initially optimal integer will correspond to the action prescribed by the Wholesaler lot-sizing policy in the third phase of the game.

Chapter 5

Game theory

5.1 Introduction

All optimal behaviour for the actors will be derived using the tools from non-cooperative game theory. In Section 5.2, we briefly explain what is meant by strategies, strategy profiles, subgames and subgame perfect Nash equilibria. After clarifying these, we discuss the technique of backward induction. We end with the concept of an equilibrium path.

5.2 Relevant terminology & concepts

Strategies & strategy profiles Individual behaviour throughout the game is summarized by a strategy, which, in the words of Watson (2002, page 23), gives a complete contingent plan. Combining the individual strategies in a vector gives a strategy profile. Actions must be specified for all information sets belonging to a player. With respect to a representative node located on an arc in Figure 4.1, the player's strategy must thus prescribe an infinite number of actions (one for each actual decision node). Next to theoretical elegance, full contingency has its practical importance. Even though players do not anticipate to ever reach certain parts of the game, expected behaviour at information sets in later stages might be relevant for (optimal) decision making earlier on in the game. Moreover, it provides a means of dealing with an opponent's non-rational behaviour like mistakes (Watson (2002, page 27)).

Two classes of strategies may be discerned. We call a strategy mixed when for some information set an actor puts probability on different values of the decision variable and opts for one of these randomly. A pure strategy is just a special case; at each information set a particular value is selected with probability one, resulting in absolute certainty concerning an actor's decisions. In Chapter 6 we limit ourselves to the last strategy type.

Subgames Before clarifying our equilibrium concept, we note how each game can be subdivided in different subgames. Watson (2002, page 141) describes a subgame as the tree structure initiated by a decision node x where neither x nor any of its successors are part of an information set containing nodes that are not successors of x. The most comprehensive subgame is the game itself. In Figure 4.1, every decision node starts a new subgame.

Subgame perfect Nash equilibria In our games we will be looking for Nash equilibria. In general, these are defined as strategy profiles wherein each actor's strategy maximizes his profit given the other actors' strategies: each actor plays a best response. Sequential decision making necessitates a refinement of the Nash equilibrium concept, the *subgame perfect* Nash equilibrium, which incorporates the notion of sequential rationalizability in extensive form games. Citing Watson (2002, page 143), the idea behind subgame perfection is "..that a solution concept should be consistent with its own application from anywhere in the game where it can be applied." Upon entering a new subgame, the prescribed strategy must remain optimal in the sense that a party does not wish to deviate from it.

Backward induction To find subgame perfect Nash equilibria in pure strategies, we use backward induction as explained in Watson (2002, page 139). We start with the subgames in the last stage to determine the best action, which depends on how the game has evolved up till that point. Bearing in mind this characterization of optimal behaviour, we proceed in a similar manner with the preceding stage. The process of moving backwards, while anticipating subsequent profit maximizing behaviour, continues until the beginning of the game (the first decision node) is reached. Combining the optimal decision rules in all stages gives us the desired Nash strategy profile(s).

Equilibrium paths The resulting sequence of optimal actions constitutes an equilibrium path. In many games, a multitude of paths exists. Since our primary interest is in better aligning the supply chain, we let actors aim for an outcome with maximum social welfare in case of more than one solution.

Chapter 6

Nash equilibria for lot-sizing games

6.1 Introduction

In this chapter we analyse the sequentially rational behaviour for the different EOQ lot-sizing games, and formally describe the subgame perfect pure Nash equilibria. The assumption of one-time interaction implies that a player's actions remain the same for the entire infinite time horizon. The same order cycles will be repeated ad infinitum. Without any discounting for the time value of money, each actor simply maximizes average profit per time unit. In Section 6.2 we study the anarchy situation. After deriving the joint policy that maximizes total supply chain profit in Section 6.3, we introduce quantity discounts in Section 6.4, and the order bonus mechanism in Section 6.5.

6.2 Non-cooperative outcome

Pay-offs The total gross margin for the Retailer is $(P_r - P_w)D$. Subtracting his average lot-sizing costs yields

$$\Pi_r^u(Q) = (P_r - P_w)D - A_r\frac{D}{Q} - h_r\frac{Q}{2} \tag{6.2.1}$$

The costs of ordering equal the fixed order cost A_r times the average number of orders per time unit $\frac{D}{Q}$. If no shortages are allowed, it is most efficient to replenish when stocks have been depleted. Because of infinite delivery and production rates, the inventory level instantaneously becomes Q upon arrival of the replenishment order, and next, diminishes to 0 again at the constant demand rate. On average $\frac{Q}{2}$ is kept in stock during a typical order cycle. Multiplication by h_r gives the holding cost term.

The difference $(P_w - P_m)$ is the basis for the total gross margin per time unit in the Wholesaler non-coordination profit. The calculation of ordering costs makes use of A_w and the order size $n_w Q$. Similar to the Retailer, the product of the holding cost rate and the average stock level is the last component of

$$\Pi_w^u(Q, n_w) = (P_w - P_m) D - A_w \frac{D}{n_w Q} - h_w \frac{(n_w - 1) Q}{2} \tag{6.2.2}$$

Under our assumptions regarding shortages, the delivery process and the production technology, it is shown in Chiang et al. (1994, pages 156-157) and Joglekar (1988, Appendix) that the total inventory of converted product held upstream during each order cycle is $((n_w - 1) Q + (n_w - 2) Q + \cdots + Q) \frac{Q}{D} = \frac{(n_w-1) n_w Q^2}{2D}$. Dividing by the length of an order cycle $\frac{n_w Q}{D}$, gives the average $\frac{(n_w-1)Q}{2}$.

Wholesaler lot-sizing Using backward induction, we first take a look at the last stage in Figure 4.1(a). For a fixed Q, the Wholesaler has to solve

$$\begin{aligned} \max_{n_w} \Pi_w^u(Q, n_w) = (P_w - P_m) D - A_w \frac{D}{n_w Q} - h_w \frac{(n_w - 1) Q}{2} \\ \text{subject to:} \\ n_w \in \mathbb{N} \end{aligned} \tag{6.2.3}$$

The concave Wholesaler profit is maximized at

$$n_w^{*u}(Q) = \min \left\{ n_w : \Pi_w^u(Q, n_w + 1) \leq \Pi_w^u(Q, n_w) \,|\, n_w \in \mathbb{N} \right\}$$

In terms of the problem parameters we get

$$n_w^{*u}(Q) = \min \left\{ n_w : \frac{2 A_w D}{h_w Q^2} \leq n_w (n_w + 1) \,|\, n_w \in \mathbb{N} \right\}$$

Like Munson & Rosenblatt (2001, pages 375-376), an explicit functional form is obtained by rearranging the terms of the condition somewhat and applying the quadratic formula to $(n_w)^2 + n_w - \frac{2A_w D}{h_w Q^2} = 0$. Rounding up the positive (non-integer) solution gives the integer of interest. Alternatively, with $\Pi_w^u(Q, 0) = -\infty$, we can describe the best Wholesaler lot-sizing action as

$$n_w^{*u}(Q) = \max \left\{ n_w : \Pi_w^u(Q, n_w - 1) \leq \Pi_w^u(Q, n_w) \,|\, n_w \in \mathbb{N} \right\}$$

Combining both decision rules gives

$$n_w^{*u}(Q) = \begin{cases} \left\lceil -\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{2A_w D}{h_w Q^2}} \right\rceil \\ \text{or} \\ \left\lfloor \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{2A_w D}{h_w Q^2}} \right\rfloor \end{cases} \tag{6.2.4}$$

More demand and a higher fixed order cost force the Wholesaler to save on the costs of placing orders by increasing his integer multiple. Smaller integers, which lower the average quantity of product in store, become more attractive if the holding cost rate or the Retailer order size increases. As required, the minimum of each rule is 1. If the square root term times 2 is an odd number, no rounding is needed. Instead of a unique lot-sizing decision, two successive integers will be optimal. The smallest follows from the upper entier expression.

Retailer order size The Wholesaler lot-sizing rules are irrelevant for the Retailer. End-consumer demand is fulfilled most efficiently by solving, at the node initiating the game, the standard EOQ problem

$$\begin{aligned} &\max \Pi_r^u(Q) = (P_r - P_w) D - A_r \frac{D}{Q} - h_r \frac{Q}{2} \\ &\text{subject to:} \\ &\qquad Q > 0 \end{aligned} \tag{6.2.5}$$

As the objective function is concave in Q, it suffices to set the derivative with respect to Q equal to zero. The solution to the first order condition equals

$$Q^{*u} = \sqrt{\frac{2A_r D}{h_r}} \tag{6.2.6}$$

The influence of the parameters resembles the effects of A_w, D and h_w on $n_w^{*u}(Q)$.

Equilibrium strategies Our findings are summarized in Proposition 6.2.1. We observe that $Q^{*u} \times n_w^{*u}(Q)$ actually describes an infinite number of Nash equilibria. There is an unlimited number of (irrational) Retailer order sizes for which the Wholesaler can randomly choose among one of two optimal lot-sizing multiples returned by $n_w^{*u}(Q)$.

Proposition 6.2.1. *All subgame perfect pure Nash equilibria in the non-coordination game are given by the strategy profiles $Q^{*u} \times n_w^{*u}(Q)$ satisfying Equations (6.2.6) and (6.2.4).*

Proof. See foregoing. Since we do not neglect any optimal decision at some decision node, backward induction guarantees that all subgame perfect pure Nash equilibria are covered. □

Substitution of Q^{*u} in $n_w^{*u}(Q)$ results in the Wholesaler's action on the equilibrium path. A unique outcome exists when $\lceil \nu_w^{-u} \rceil = \lfloor \nu_w^{+u} \rfloor$ in

$$n_w^{*u} = \left\lceil \nu_w^{-u} \right\rceil \text{ or } \left\lfloor \nu_w^{+u} \right\rfloor \tag{6.2.7}$$

where:

$$\begin{aligned} \nu_w^{-u} &= -\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{A_w h_r}{A_r h_w}} \\ \nu_w^{+u} &= \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{A_w h_r}{A_r h_w}} \end{aligned} \tag{6.2.8}$$

The actors' profits become

$$\begin{aligned} \Pi_r^{*u} &= (P_r - P_w) D - \sqrt{2 A_r h_r D} \\ \Pi_w^{*u} &= (P_w - P_m) D - \left(\frac{A_w / n_w^{*u}}{2 A_r} + \frac{h_w (n_w^{*u} - 1)}{2 h_r} \right) \sqrt{2 A_r h_r D} \end{aligned} \tag{6.2.9}$$

The parameter D does not appear in the expression for n_w^{*u}. Therefore, the realized lot-sizing costs upstream are scaled by the same constant as those for the Retailer if demand changes. By construction, $\Pi_w^u(Q^{*u}, \nu_w^{-u}) = \Pi_w^u(Q^{*u}, \nu_w^{-u} + 1) = \Pi_w^u(Q^{*u}, \nu_w^{+u})$ holds. Since the concave function $\Pi_w^u(Q^{*u}, n_w)$ has its unrestricted maximum at $n_w = \nu_w^{*u}$, we get

$$\Pi_w^u\left(Q^{*u}, \nu_w^{-u}\right) = \Pi_w^u\left(Q^{*u}, \nu_w^{+u}\right) \leq \Pi_w^{*u} \leq \Pi_w^u\left(Q^{*u}, \nu_w^{*u}\right) \tag{6.2.10}$$

with:

$$\nu_w^{-u} < \nu_w^{*u} < \nu_w^{+u}$$

where:

$$\begin{aligned} \nu_w^{*u} &= \sqrt{\frac{A_w h_r}{A_r h_w}} \\ \Pi_w^u(Q^{*u}, \nu_w^{*u}) &= (P_w - P_m) D - \left(\sqrt{\frac{A_w h_w}{A_r h_r}} - \frac{h_w}{2 h_r} \right) \sqrt{2 A_r h_r D} \end{aligned} \tag{6.2.11}$$

6.3 Fully optimized supply chain

Joint profit To maximize total supply chain profit, we sum each actor's profit and decide on Q and n_w simultaneously. A combination of the approach in Goyal (1976) and the procedure to determine the individually optimal lot-sizing multiple(s) is used to solve:

$$\max \Pi^s(Q, n_w) = (P_r - P_m) D - (A_r + A_w/n_w) \frac{D}{Q} - (h_r + h_w(n_w - 1)) \frac{Q}{2}$$

$$\text{subject to:} \qquad Q > 0 \qquad n_w \in \mathbb{N} \tag{6.3.1}$$

Optimal collaborative Retailer action Given a particular $n_w \in \mathbb{N}$, we have the system-wide equivalent of Problem (6.2.5). With A_r replaced by $A_r + A_w/n_w$, and h_r appearing instead of the joint holding cost rate $h_r + h_w(n_w - 1)$, incurred for an average item moving through the supply chain, the more comprehensive solution becomes

$$Q^{*s}(n_w) = \sqrt{\frac{2(A_r + A_w/n_w) D}{h_r + h_w(n_w - 1)}}$$

The difference between $Q^{*s}(n_w)$ and Q^{*u} in Equation (6.2.6) concisely illustrates the potential for inefficient decision making under anarchy.

Optimal collaborative Wholesaler action Insertion of $Q^{*s}(n_w)$ results in a single variable objective function $\Pi^s(n_w) = \Pi^s(Q^{*s}(n_w), n_w)$ and a reduced problem

$$\max \Pi^s(n_w) = (P_r - P_m) D - \sqrt{2(A_r + A_w/n_w)(h_r + h_w(n_w - 1)) D}$$

$$\text{subject to:} \qquad n_w \in \mathbb{N}$$

As mere inspection does not reveal the behaviour of $\Pi^s(n_w)$, we relax the domain restriction from $n_w \in \mathbb{N}$ to $n_w \geq 1$ and take the derivative

$$\frac{\partial \Pi^s(n_w)}{\partial n_w}_{|n_w > 1} = -\frac{\sqrt{D}\left(A_r h_w + A_w(h_w - h_r)/(n_w)^2\right)}{\sqrt{2(A_r + A_w/n_w)(h_r + h_w(n_w - 1))}}$$

In case $h_r > h_w$, we solve $A_r h_w + A_w(h_w - h_r)/(n_w)^2 = 0$ by setting n_w equal to

$$\bar{\nu}_w^s = \sqrt{\frac{A_w(h_r - h_w)}{A_r h_w}}$$

The derivative is positive below and negative beyond $\bar{\nu}_w^s$. Comparable to the non-coordination setting, after defining $\Pi^s(0) = -\infty$, the optimal lot-sizing integer(s) is(are) characterized by

$$n_w^{*s} = \begin{cases} \min\left\{n_w : \Pi^s\left(n_w+1\right) \leq \Pi^s\left(n_w\right) | n_w \in \mathbb{N}\right\} \\ \text{or} \\ \max\left\{n_w : \Pi^s\left(n_w-1\right) \leq \Pi^s\left(n_w\right) | n_w \in \mathbb{N}\right\} \end{cases}$$

The minimum and maximum integer are found by rounding up or down the respective positive solutions to $(n_w)^2 + n_w - \frac{A_w(h_r-h_w)}{A_r h_w} = 0$ and $(n_w)^2 - n_w - \frac{A_w(h_r-h_w)}{A_r h_w} = 0$. Some further adjustment is necessary to take into account $h_r \leq h_w$, for which, with $\frac{\partial \Pi^s(n_w)}{\partial n_w}_{|n_w>1} < 0$, a lot-for-lot policy is optimal. Choosing the maximum of an arbitrary constant in the set $(0,2]$ ($[0,2)$) and the term $\frac{A_w(h_r-h_w)}{A_r h_w}$ makes that all scenarios are included in

$$n_w^{*s} = \begin{cases} \left\lceil -\frac{1}{2} + \sqrt{\frac{1}{4} + \max\left\{1, \frac{A_w\left(h_r-h_w\right)}{A_r h_w}\right\}} \right\rceil \\ \text{or} \\ \left\lfloor \frac{1}{2} + \sqrt{\frac{1}{4} + \max\left\{1, \frac{A_w\left(h_r-h_w\right)}{A_r h_w}\right\}} \right\rfloor \end{cases} \tag{6.3.2}$$

Retailer orders are combined in a single batch to a larger extent when ordering is relatively expensive for the Wholesaler ($\frac{A_w}{A_r}$ large) and when the cost structures favour holding inventory upstream ($\frac{h_r-h_w}{h_w}$ large). Smaller ratios let the optimum move towards a collaborative lot-for-lot policy. Again, two successive integers may be optimal.

Welfare maximizing outcome A solution to the joint optimization problem is comprised of n_w^{*s}, and Q^{*s}, which is calculated by substituting n_w^{*s} into $Q^{*s}\left(n_w\right)$. With these values, profits for each echelon are

$$\begin{aligned} \Pi_r^{*s} =& \left(P_r - P_w\right) D - \left(\frac{A_r}{2\left(A_r + A_w/n_w^{*s}\right)} + \frac{h_r}{2\left(h_r + h_w\left(n_w^{*s}-1\right)\right)}\right) \\ & \cdot \sqrt{2\left(A_r + A_w/n_w^{*s}\right)\left(h_r + h_w\left(n_w^{*s}-1\right)\right) D} \\ \Pi_w^{*s} =& \left(P_w - P_m\right) D - \left(\frac{A_w/n_w^{*s}}{2\left(A_r + A_w/n_w^{*s}\right)} + \frac{h_w\left(n_w^{*s}-1\right)}{2\left(h_r + h_w\left(n_w^{*s}-1\right)\right)}\right) \\ & \cdot \sqrt{2\left(A_r + A_w/n_w^{*s}\right)\left(h_r + h_w\left(n_w^{*s}-1\right)\right) D} \end{aligned}$$

Because Q^{*s} does not have to coincide with the individually optimal Q^{*u}, the inequality $\Pi_r^{*s} \leq \Pi_r^{*u}$ holds. By definition, we have $\Pi^{*s} = \Pi_r^{*s} + \Pi_w^{*s} \geq \Pi^{*u}$, and thereby $\Pi_w^{*s} \geq \Pi_w^{*u}$. As in the previous section, a change in demand merely scales the downstream order size and the resulting lot-sizing costs for both actors.

6.4 Quantity discount schedule

Overview of profits When a quantity discount scheme is introduced, the lot-sizing stages are preceded by the Wholesaler's decision on α_w and R_w. Both coordination variables appear in the extended gross margin expression for the Retailer. If the downstream order quantity is in the discount region, the indicator variable $1_{\{Q \in R_w\}}$ takes the value 1 and the Wholesaler charges $(1 - \alpha_w) P_w$. Otherwise, no discount is granted and the regular unit price applies. The remaining terms of Equation (6.2.1) do not change in

$$\Pi_r^a (Q, \alpha_w, R_w) = \left(P_r - \left(1 - \alpha_w 1_{\{Q \in R_w\}}\right) P_w\right) D - A_r \frac{D}{Q} - h_r \frac{Q}{2}$$

Incorporating the quantity discount policy similarly in Equation (6.2.2) gives

$$\Pi_w^a (Q, n_w, \alpha_w, R_w) = \left(\left(1 - \alpha_w 1_{\{Q \in R_w\}}\right) P_w - P_m\right) D - A_w \frac{D}{n_w Q} - h_w \frac{(n_w - 1) Q}{2}$$

Wholesaler lot-sizing stage At his lot-sizing decision nodes, the quantity discount characteristics and the Retailer order quantity have already been determined. As the objective function only differs by a constant $\alpha_w 1_{\{Q \in R_w\}} P_w D$ from $\Pi_w^u (Q, n_w)$ in Problem (6.2.3), Equation (6.2.4) remains optimal for

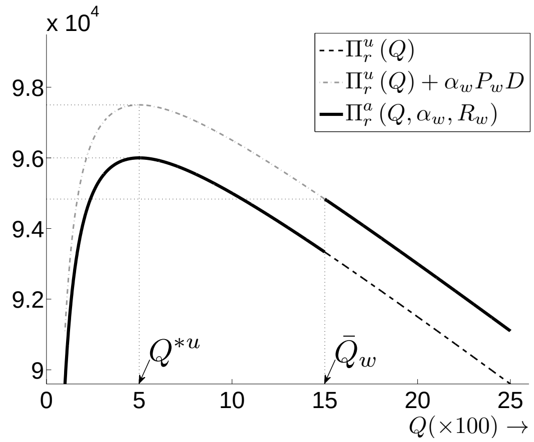
$$\begin{aligned} \max_{n_w} \Pi_w^a (Q, n_w, \alpha_w, R_w) = &\left(\left(1 - \alpha_w 1_{\{Q \in R_w\}}\right) P_w - P_m\right) D - A_w \frac{D}{n_w Q} - h_w \frac{(n_w - 1) Q}{2} \\ \text{subject to:} & \\ & n_w \in \mathbb{N} \end{aligned} \tag{6.4.1}$$

Retailer response The decision on the downstream inventory policy is no longer static, but depends on the vector (α_w, R_w). The reaction to a particular quantity discount layout follows from solving

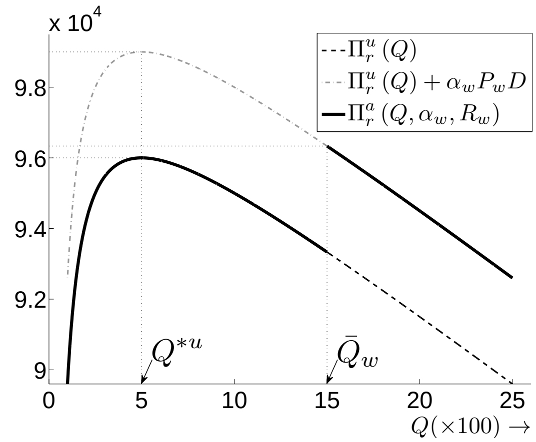
$$\begin{aligned} \max_{Q} \Pi_r^a (Q, \alpha_w, R_w) = &\left(P_r - \left(1 - \alpha_w 1_{\{Q \in R_w\}}\right) P_w\right) D - A_r \frac{D}{Q} - h_r \frac{Q}{2} \\ \text{subject to:} & \\ & Q > 0 \end{aligned}$$

The discontinuity of the profit function at $\bar{Q}_w$ makes finding the best order size somewhat more difficult than in case of Problem (6.2.5). Given an arbitrary set of price, demand and cost data, Figure 6.1 illustrates how altering the conditions of the AQD scheme influences the

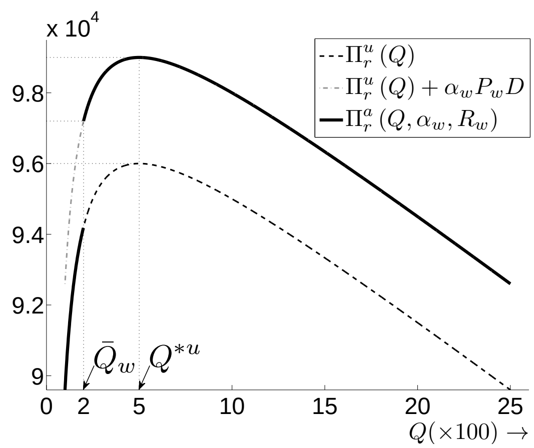
Retailer's options. In the no discount region ($Q \notin R_w$), profit just equals $\Pi_r^u(Q)$. For all order sizes in the discount region ($Q \in R_w$), $\Pi_r^a(Q, \alpha_w, R_w)$ coincides with the non-coordination profit shifted upwards by the average total discount per time unit. Because the two curves run parallel to one another, the unrestricted optimal order size for both is located at Q^{*u} in Equation (6.2.6).



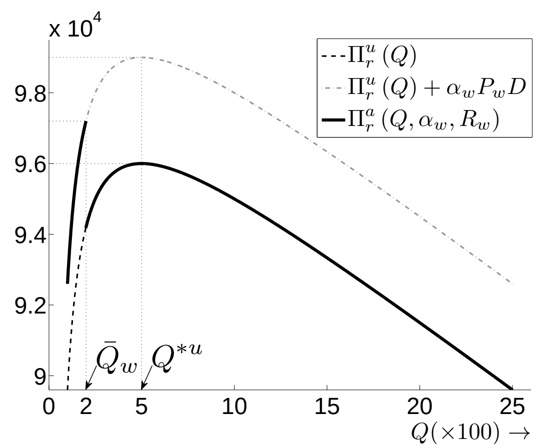
(a) $\alpha_w = 0.01$ and $R_w = [1500, \infty)$



(b) $\alpha_w = 0.02$ and $R_w = [1500, \infty)$



(c) $\alpha_w = 0.02$ and $R_w = [200, \infty)$



(d) $\alpha_w = 0.02$ and $(0, 200]$

Figure 6.1: *Optimization with $P_r = 25$, $P_w = 15$, $D = 10000$, $A_r = 100$ and $h_r = 8$ by the Retailer facing an AQD scheme*

Typical examples of the trade-off with $\bar{Q}_w \geq Q^{*u}$ are contained in the first two subfigures. The Retailer compares his profit at the breakpoint, which is the best among all discount order sizes, with the maximum non-coordination profit Π_r^{*u}. Figure 6.1(a) depicts a configuration wherein the Retailer will not deviate from his anarchy decision. After the discount has been increased to 2% in Figure 6.1(b), he chooses $\bar{Q}_w = 1500$. Figure 6.1(c) makes clear why a proper AQD schedule is incapable of provoking more frequent ordering downstream: the discount is received even when nothing changes at the Retailer. As shown in Figure 6.1(d), allowing quantity premiums removes this limitation.

The previous observations reveal the essence of the optimal response. The Retailer chooses $\bar{Q}_w$ if the default solution Q^{*u} is not part of the discount region minus the order breakpoint, and the profit at the breakpoint at least equals the maximum under anarchy:

$$Q^{*a}\left(\alpha_w, R_w\right) = \begin{cases} \bar{Q}_w & \text{if } Q^{*u} \notin \left(R_w \setminus \bar{Q}_w\right) \text{ and } \Pi_r^a\left(\bar{Q}_w, \alpha_w, R_w\right) \geq \Pi_r^{*u} \\ \text{and} & \\ Q^{*u} & \text{otherwise} \end{cases} \tag{6.4.2}$$

In view of reformulating the Wholesaler coordination problem, $Q^{*a}\left(\alpha_w, R_w\right)$ formally points at the breakpoint in the special case where both lot-sizing quantities are the same. We also assume that an indifferent Retailer accepts the schedule. Without the assumption, a Wholesaler not satisfied with Q^{*u}, could be forced to offer a scheme with a slightly higher discount to make the Retailer strictly prefer $\bar{Q}_w$. However, there is always a smaller discount that does the job, and, as a consequence, a Nash equilibrium might then not exist.

Coordination stage To take into account the indirect control over the Retailer order size, we rewrite the Wholesaler profit function as $\Pi_w^a\left(n_w, \alpha_w, R_w\right) = \Pi_w^a\left(Q^{*a}\left(\alpha_w, R_w\right), n_w, \alpha_w, R_w\right)$. The first stage problem is more easily solved by disregarding the rules in Equation (6.2.4), and instead include n_w in

$$\begin{aligned} \max \Pi_w^a\left(n_w, \alpha_w, R_w\right) = &\left(\left(1 - \alpha_w 1_{\{Q^{*a}(\alpha_w, R_w) \in R_w\}}\right) P_w - P_m\right) D \\ &- A_w \frac{D}{n_w Q^{*a}\left(\alpha_w, R_w\right)} - h_w \frac{\left(n_w - 1\right) Q^{*a}\left(\alpha_w, R_w\right)}{2} \\ \text{subject to:}& \\ & n_w \in \mathbb{N} \qquad \alpha_w \geq 0 \\ & R_w \in \left\{\left(0, \bar{Q}_w\right], \left[\bar{Q}_w, \infty\right)\right\} \qquad \bar{Q}_w > 0 \end{aligned} \tag{6.4.3}$$

Every downstream order size can be achieved by having the Retailer opt for the breakpoint of an AQD scheme. At the breakpoint it suffices that he is just as well of as under anarchy, i.e., the loss resulting from deviation of Q^{*u} is exactly compensated. A proper quantity discount should be chosen for $\bar{Q}_w > Q^{*u}$, while $\bar{Q}_w < Q^{*u}$ requires a quantity premium. In the special case where $\bar{Q}_w = Q^{*u}$ (with $\alpha_w = 0$), the remainder of the discount region may be located on either side of the breakpoint. In terms of n_w and $\bar{Q}_w$ we restate the problem as

$$\max \Pi_w^a \left(n_w, \bar{Q}_w\right) = \left(\left(1 - \frac{\Pi_r^{*u} - \Pi_r^u \left(\bar{Q}_w\right)}{P_w D}\right) P_w - P_m\right) D - A_w \frac{D}{n_w \bar{Q}_w} - h_w \frac{\left(n_w - 1\right) \bar{Q}_w}{2}$$

subject to: (6.4.4)

$$n_w \in \mathbb{N} \qquad \bar{Q}_w > 0$$

After substitution of the expression for $\Pi_r^u\left(\bar{Q}_w\right)$, and aggregating cost components per category, the reformulation becomes

$$\max \Pi_w^a \left(n_w, \bar{Q}_w\right) = \left(P_r - P_m\right) D - \left(A_r + A_w / n_w\right) \frac{D}{\bar{Q}_w} - \left(h_r + h_w \left(n_w - 1\right)\right) \frac{\bar{Q}_w}{2} - \Pi_r^{*u}$$

subject to:

$$n_w \in \mathbb{N} \qquad \bar{Q}_w > 0$$

In essence, apart from a constant and a change of variables, Problem (6.3.1) needs to be solved. The best integer n_w^{*a} equals n_w^{*s} in Equation (6.3.2), while the optimal breakpoint $\bar{Q}_w^*$ corresponds to the jointly optimal downstream order size Q^{*s}. Because the Retailer agrees to Q^{*s}, his pre-discount profit is Π_r^{*s}. From the solution we derive

$$\alpha_w^* = \frac{\Pi_r^{*u} - \Pi_r^{*s}}{P_w D} \tag{6.4.5}$$

and the discount region

$$R_w^* = \begin{cases} \left(0, Q^{*s}\right] 1_{\{Q^{*s} < Q^{*u}\}} + \left[Q^{*s}, \infty\right) 1_{\{Q^{*s} \geq Q^{*u}\}} \\ \text{or} \\ \left(0, Q^{*s}\right] 1_{\{Q^{*s} \leq Q^{*u}\}} + \left[Q^{*s}, \infty\right) 1_{\{Q^{*} > Q^{*u}\}} \end{cases} \tag{6.4.6}$$

By construction, two possible regions are defined if $Q^{*s} = Q^{*u}$. In fact, the location of R_w in a zero-discount scheme is not relevant at all. Moreover, for a Wholesaler satisfied with the non-coordination outcome, any schedule in which the Retailer is left with too little profit at the breakpoint, and therefore rejected by selecting Q^{*u}, is optimal.

Equilibrium strategies and efficiency increase Without further proof, we interpret the preceding analysis game theoretically in Proposition 6.4.1. Whether or not the supply chain needs the quantity discount scheme to operate more profitably, the presence of $n_w^{*u}\left(Q\right)$ makes that an infinite number of Nash strategy profiles is described.

Proposition 6.4.1. *The strategy profiles* $Q^{*a}(\alpha_w, R_w) \times [\alpha_w^*, R_w^*, n_w^{*u}(Q)]$ *satisfying Equations (6.4.2), (6.4.5), (6.4.6) and (6.2.4), are subgame perfect pure Nash equilibria in the quantity discount game. If* $Q^{*s} = Q^{*u}$*, every AQD set-up with* $\alpha_w = 0$*, or any schedule rejected by the Retailer, can be part of an equilibrium.*

Assuming the Retailer responds to α_w^* and R_w^* as anticipated (selecting the breakpoint $\bar{Q}_w^* = Q^{*s}$), the Wholesaler will be faced with a reduced Problem (6.4.4), which is solved by $n_w^{*u}(Q^{*s})$. The lot-sizing rule producing the same integer as in the coordination stage of the game shows that $n_w^{*a} = n_w^{*s}$ is situated on an equilibrium path. The number of outcomes (including those with rejected schedules) is infinite in case Q^{*u} is part of a jointly optimal inventory policy. Otherwise, because of the direct link with the solutions in Section 6.3, the maximum is 2.

While the Retailer does not improve upon his anarchy profit: $\Pi_r^{*a} = \Pi_r^{*u}$, the Wholesaler receives $\Pi_w^{*a} = \Pi^{*s} - \Pi_r^{*u}$ with $\Pi_w^{*u} \leq \Pi_w^{*a} \leq \Pi_w^{*s}$. The supply chain is fully aligned and any additional surplus accrues to the upstream actor. As mentioned in Chapter 3, complete synchronization of the supply chain has been observed before by Banerjee (1986*b*) in a context where a lot-for-lot replenishment policy is imposed.

6.5 Order bonus mechanism

Profits When coordination takes place by means of the second mechanism, the Wholesaler grants $\gamma_w A_r$ as a bonus each time he receives an order. Adjusting Equation (6.2.1) accordingly gives the Retailer profit

$$\Pi_r^c(Q, \gamma_w) = (P_r - P_w) D - (1 - \gamma_w) A_r \frac{D}{Q} - h_r \frac{Q}{2}$$

The implications for Equation (6.2.2) of awarding an order bonus are reflected in

$$\Pi_w^c(Q, n_w, \gamma_w) = (P_w - P_m) D - A_w \frac{D}{n_w Q} - \gamma_w A_r \frac{D}{Q} - h_w \frac{(n_w - 1) Q}{2}$$

Lot-sizing stages Because, as in Problem (6.4.1), the terms related to coordination drop out, Equation (6.2.4) remains optimal in the last stage of the game. Simply replacing A_r by $(1 - \gamma_w) A_r$ in Equation (6.2.6) yields the downstream lot-sizing response

$$Q^{*c}(\gamma_w) = \sqrt{\frac{2(1 - \gamma_w) A_r D}{h_r}} \tag{6.5.1}$$

The inequality $Q^{*c}(\gamma_w) \leq Q^{*u}$ means that, compared to non-coordination, the Retailer can only be stimulated to order in smaller batches.

Coordination stage At the beginning of the game, the Wholesaler incorporates $Q^{*c}(\gamma_w)$ in $\Pi_w^c(n_w, \gamma_w) = \Pi_w^c(Q^{*c}(\gamma_w), n_w, \gamma_w)$, but, analogous to Problem (6.4.3), he re-optimizes over n_w:

$$\begin{aligned}
\max \Pi_w^c(n_w, \gamma_w) = (P_w - P_m) D - &\left(\frac{A_w/n_w}{2(1-\gamma_w) A_r} + \frac{(1-\gamma_w) h_w (n_w - 1) + \gamma_w h_r}{2(1-\gamma_w) h_r} \right) \\
&\cdot \sqrt{2(1-\gamma_w) A_r h_r D}
\end{aligned}$$

$$\text{subject to:} \qquad n_w \in \mathbb{N} \qquad \gamma_w \in [0,1) \tag{6.5.2}$$

The derivative of the objective function with respect to γ_w is

$$\frac{\partial \Pi_w^c(n_w, \gamma_w)}{\partial \gamma_w} = \frac{Q^{*c}(\gamma_w)}{1-\gamma_w} \left(\frac{h_w(n_w-1) - h_r}{4} - \frac{(A_w/n_w + A_r) h_r}{4(1-\gamma_w) A_r} \right)$$

The term $\frac{Q^{*c}(\gamma_w)}{1-\gamma_w} = \sqrt{\frac{2A_r D}{(1-\gamma_w)h_r}}$ never equals 0, and is, therefore, neglected in the first order condition, which is solved by

$$\bar{\gamma}_w(n_w) = 1 - \frac{(A_w/n_w + A_r) h_r}{A_r(h_w(n_w-1) - h_r)}$$

If $n_w > \frac{h_w + h_r}{h_w}$, the solution constitutes a stationary point. Smaller lot-sizing multiples result in bonus levels for which the objective function and the derivative are not defined. In case $n_w = \frac{h_w + h_r}{h_w}$, or equivalently $h_w(n_w - 1) - h_r = 0$, a division by 0 precludes existence. The function $\bar{\gamma}_w(n_w)$ is increasing with $\lim_{n_w \to \infty} \bar{\gamma}_w(n_w) = 1$. In Figure 6.2, stationary points can be identified for $n_w \geq 4$.

Interesting multiples start at $n_w = 7$ where $\bar{\gamma}_w(n_w)$ equals 0. In general, we describe the smallest integer having $\bar{\gamma}_w(n_w) \in [0,1)$ as

$$n_w^{-c} = \min \left\{ n_w : \frac{(A_w/n_w + A_r) h_r}{A_r(h_w(n_w-1) - h_r)} \leq 1 | n_w \in \mathbb{N}, n_w > \frac{h_w + h_r}{h_w} \right\}$$

The positive solution to $(n_w)^2 - \left(2\frac{h_r}{h_w} + 1\right) n_w - \frac{A_w h_r}{A_r h_w} = 0$ is rounded up in order to get the explicit functional form

$$n_w^{-c} = \lceil \nu_w^{-c} \rceil$$

where:

$$\nu_w^{-c} = \frac{1}{2} + \frac{h_r}{h_w} + \sqrt{\left(\frac{1}{2} + \frac{h_r}{h_w}\right)^2 + \frac{A_w h_r}{A_r h_w}}$$

Ignoring $\frac{A_w h_r}{A_r h_w}$ and rearranging the remainder, we arrive at $n_w^{-c} \geq \left\lceil \frac{h_w + 2h_r}{h_w} \right\rceil > \frac{h_w + h_r}{h_w}$ (as required). From a global inspection of ν_w^{-c} and the expression in Equation (6.2.8), it can be seen that $\nu_w^{-c} > \nu_w^{+u}$ holds, and that the lower bound exceeds Equation (6.2.7): $n_w^{-c} > n_w^{*u}$.

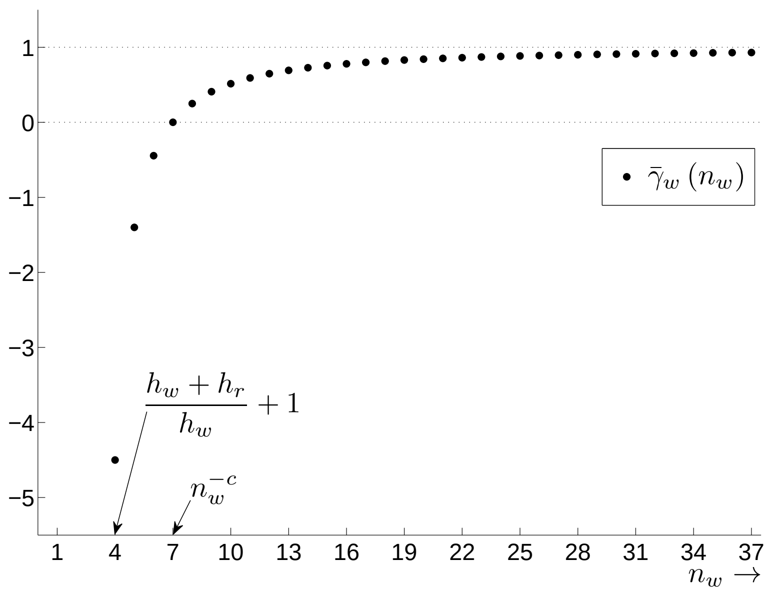


Figure 6.2: *Stationary points for relaxation of Wholesaler's OB problem with $A_r = 100$, $h_r = 8$, $A_w = 700$ and $h_w = 4$*

In the following lemma, we state the optimal bonus level per integer. Starting from n_w^{-c}, the Wholesaler tries to mitigate the negative effects of a growing integer multiple on inventory holding costs by tempting the Retailer to replenish more often. Cost structures favouring longer lot-sizing cycles upstream (A_w high, h_w low) and smaller orders downstream (A_r low, h_r high) tend to shift the optimal bonus level downwards.

Lemma 6.5.1. *For each lot-sizing integer $n_w \in \mathbb{N}$, the Wholesaler's optimal order bonus is given by $\gamma_w^*(n_w) = \bar{\gamma}_w(n_w)\, 1_{\{n_w \geq n_w^{-c}\}}$.*

Proof. We know that $\frac{Q^{*c}(\gamma_w)}{1-\gamma_w} > 0$. Three cases are distinguished:

(i) $n_w \leq \frac{h_w + h_r}{h_w}$: These integers lacking a stationary point satisfy $\frac{h_w(n_w - 1) - h_r}{4} \leq 0$. With

$\frac{(A_w/n_w+A_r)h_r}{4(1-\gamma_w)A_r} > 0$, the derivative $\frac{\partial\Pi_w^c(n_w,\gamma_w)}{\partial\gamma_w}$ is negative for all bonus levels in the relevant domain. Setting the bonus fraction at 0 maximizes profit;
(ii) $\frac{h_w+h_r}{h_w} < n_w < n_w^{-c}$: Here, $\frac{h_w(n_w-1)-h_r}{4} > 0$, and, hence, a stationary point exists. Its infeasibility and $\frac{(A_w/n_w+A_r)h_r}{4(1-\gamma_w)A_r}$ increasing on $\gamma_w < 1$ ensure $\frac{\partial\Pi_w^c(n_w,\gamma_w)}{\partial\gamma_w} < 0$ for $\gamma_w \in [0,1)$. Giving no bonus remains optimal;
(iii) $n_w \geq n_w^{-c}$: In contrast to the previous case, the stationary points are feasible. The derivative is positive for $\gamma_w \in [0, \bar{\gamma}_w(n_w))$, and negative on the set $(\bar{\gamma}_w(n_w), 1)$. The optimal bonus fraction is $\bar{\gamma}_w(n_w)$. □

Combination of $\gamma_w^*(n_w)$ and Problem (6.5.2) leads to the single variable Wholesaler profit $\Pi_w^c(n_w) = \Pi_w^c(n_w, \gamma_w^*(n_w))$ in

$$\begin{aligned}
\max \Pi_w^c(n_w) = {} & (P_w - P_m) D - 1_{\{n_w < n_w^{-c}\}} \left(\frac{A_w/n_w}{2A_r} + \frac{h_w(n_w-1)}{2h_r} \right) \sqrt{2A_r h_r D} \\
& - 1_{\{n_w \geq n_w^{-c}\}} \sqrt{2(A_w/n_w + A_r)(h_w(n_w-1) - h_r) D} \\
\text{subject to:} & \\
& n_w \in \mathbb{N}
\end{aligned} \tag{6.5.3}$$

For $n_w < n_w^{-c}$ the optimal bonus fraction is 0, the Retailer responds by setting Q^{*u} and the Wholesaler's profit is just $\Pi_w^u(Q^{*u}, n_w)$ (compare Equation (6.2.9)). Substitution of $\bar{\gamma}_w(n_w)$ yields the profit for larger integers. After relaxing the domain restriction from $n_w \in \mathbb{N}$ to $n_w \geq 1$, the objective function is made up of two continuous curves intersecting at $\nu_w^{-c} > 1$. The derivative for the second one equals

$$\frac{\partial\Pi_w^c(n_w)}{\partial n_w}_{|n_w > \nu_w^{-c}} = -\frac{\sqrt{D}\left(A_w(h_w+h_r)/(n_w)^2 + A_r h_w\right)}{\sqrt{2(A_w/n_w + A_r)(h_w(n_w-1) - h_r)}}$$

We now state which lot-sizing multiple and compensation level maximize profit in the first stage of the game.

Proposition 6.5.1. *The anarchy integer n_w^{*u} solves Problem (6.5.3), and the Wholesaler chooses $\gamma_w^* = 0$.*

Proof. Naturally, n_w^{*u} with profit Π_w^{*u} is optimal among all integers with $\gamma_w^*(n_w) = 0$. From Equation (6.2.10) we have the inequality $\Pi_w^{*u} \geq \Pi_w^u(Q^{*u}, \nu_w^{+u})$. By concavity, the first (non-coordination) curve declines beyond ν_w^{*u} in Equation (6.2.11), and thus declines between ν_w^{+u} and ν_w^{-c}. At the point of intersection we move to the other curve, where, since $\frac{\partial\Pi_w^c(n_w)}{\partial n_w}_{|n_w > \nu_w^{-c}} < 0$, profit further decreases. The zero bonus integer n_w^{*u} thus outperforms as well each integer with $\gamma_w^*(n_w) > 0$. □

Equilibrium Since the benefits of being able to adjust the incoming order size never outweigh the costs of the mechanism, the Wholesaler never grants a bonus. On an equilibrium path resulting from Proposition 6.5.2, the non-coordination inventory policies persist ($Q^{*c} = Q^{*u}$, $n_w^{*c} = n_w^{*u}$). Each actor earns his default profit: $\Pi_r^{*c} = \Pi_r^{*u}$ and $\Pi_w^{*c} = \Pi_w^{*u}$.

Proposition 6.5.2. *All subgame perfect pure Nash equilibria in the order bonus game are described by the strategy profiles $Q^{*c}(\gamma_w) \times [\gamma_w^* = 0, n_w^{*u}(Q)]$, which satisfy Equations (6.5.1) and (6.2.4). The value $\gamma_w^* = 0$ is the optimal bonus fraction associated with the integer $n_w^{*c} = n_w^{*u}$ solving Problem (6.5.3).*

Chapter 7

Conclusion

In order to investigate the potential of a quantity discount mechanism and an order bonus scheme to align lot-sizing decisions in a 2 actor serial supply chain, we derived subgame perfect pure Nash strategies using backward induction. It was shown that introduction of an AQD schedule leads to full optimization of the supply chain, as the Wholesaler reaps all the benefits of coordination, and, therefore, has an incentive to maximize surplus. For the OB scheme we proved that it is completely ineffective in mitigating lot-sizing externalities. The costs of the mechanism always exceed the benefits of adjusting the Retailer's order size, which precludes the Wholesaler from ever setting a positive order bonus.

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