# Crew scheduling for extreme winter days

# **BACHELOR THESIS**

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#### Abstract

In this thesis we will construct a new robust crew schedule at NS for extreme winter days. On such days there are often a lot of disruptions and rescheduling of the crew and rolling stock is in such case impossible. Therefore, it is necessary to construct a crew schedule which is easily to reschedule. In the robust crew schedule presented in this thesis, all crew members are commuting between two relief locations on a part of a line, but the rolling stock are assigned to the whole line. But implementing this idea needs about 20% more crew members. So in this thesis we discussed a mathematical model and a heuristic to select the optimal part of the tasks. In these methods we used some different sets of profit of tasks. The results of these methods and profits are compared.

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#### 1. Introduction

People are continuously looking for comfortable and easy ways of travelling. Attractive public transport, good connections and flowing transport junction points contribute to this. Public transport is also important for sustainable and reliable transport in the densely populated areas of Europe. One of the most densely populated areas of Europe is the Netherlands. In this country, Netherlands Railways (in Dutch: Nederlandse Spoorwegen or NS) is able to connect people with each other. NS is the main railway operator in the Netherlands, having the exclusive right to operate passenger trains on the so-called Dutch Main Railway Network until 2015.

Every weekday the NS transfers more than 1.1 million people to their work, to school or for recreation. NS operates about 4,800 trips per day.

Travelers expect, especially if they travel by train, to arrive at their destination around at the time published in the timetable. However, unforeseen events often take place, which cause delays or even cancellations of trains. As a result, passengers will arrive later than expected at their destination. In case of disruptions, it is the case to reschedule the crew and the rolling stock. Sometimes this is impossible, for example during the extreme weather circumstances in December 2009. The Dutch railway network incurs a lot of disruptions by these circumstances, leading to many canceled trains. During this whole period, dispatchers were far behind in rescheduling the crew, which made the situation even worse. Therefore, during the year 2010 an alternative timetable for days with extreme weather conditions is created. In this alternative schedule the trains and therefore the drivers and the conductors as well commute between two relief locations. This timetable differs a lot from the standard timetable, so when it is chosen to set up this schedule, it is necessary to follow the alternative schedule during the whole day. To test this timetable it is set up on a Sunday in October. Since the weather forecasts are sometimes incorrect, it was a too large risk to set up this alternative schedule during the winter period of 2010. Therefore, it is necessary to investigate for a robust crew schedule which does not differ so much from the normal timetable.

While investigating for robust rescheduling of railway crew schedules, Vlugt (2010) discussed the idea of assigning the drivers to a part of a line and let them commute, but assigning the trains to the whole line. In this timetable the passengers do not have to switch and the crew schedule is robust. When there are disruptions on a route between two bases, the drivers on that route have no tasks on other routes. Therefore other trips are not disrupted by the absence of a driver, which makes the crew schedule robust.

We will describe the problem of this thesis in Chapter 2. In Chapter 3 a mathematical model will be presented, which will be used to solve this problem. Afterwards, in Chapter 4, we will discuss some methods for the sets and parameters of the model. We will discuss a heuristic to solve the problem in Chapter 5. In the next chapter we will give the data description. We will finish with the results and conclusions in Chapter 7 and 8.

#### 2. Problem description

In this thesis we will focus on constructing a robust crew schedule at NS in case of an extreme winter weather forecast.

#### 2.1 Crew scheduling at NS

In this section we will discuss the way NS is normally dealing with crew scheduling.

Throughout the Netherlands, NS operates a set of lines, where a *line* is defined as a *route* between a start station and an end station. A line has a number of intermediate stops and is operated with a certain frequency.

Each train needs a driver and a number of conductors, depending on the length of the train for example. This means that crew planning can only be done when the timetable and the rolling stock schedule is determined. Constructing the timetable and scheduling the rolling stock are problems that will not be considered in this thesis.

Crew scheduling at NS is a complex problem. On a weekday, about 4,800 timetabled trips are scheduled. A *trip* is a train operating on a line between a start and end location having a departure and arrival time. For the operation of these trips, about 2,800 rolling stock carriages are used, and there are about 3,000 drivers and 3,500 conductors employed. There are 29 crew bases across the country from which the crew members operate. Each crew member has to perform tasks. A *task* is the smallest amount of work that has to be assigned to one crew member and starts and ends at a *relief location*, which is either a crew base or another location where a change of a crew member is allowed. At most relief locations it is possible for the crew to have a meal break. Besides trips or parts of trips, a task can also be a passenger task, a shunt task of a task where the crew member has to walk or taxi from one station to another. A passenger task means that a crew member travels as a passenger on a certain train. A sequence of tasks, possibly interrupted by breaks, is called a *duty*.

In this thesis we will only focus on train drivers and not on conductors, since the scheduling of conductors is done similarly to the scheduling of drivers.

For the duties there are some constraints that have to be satisfied. These constraints describe, for example, the length of the duties, the presence and the length of a (meal) break and the maximum working hours before and after the break. There are constraints at crew base level as well. These constraints consider a minimum or maximum percentage of duties which meet some constraint. In our case there are no constraints at crew base level. There are only some restrictions for the duties:

+ Duties have to start and finish at the same base.

+ The length of a duty is at most 9.5 hours and at least 6 hours.

+ A crew member starts with his first task 10 minutes after starting his shift and he finishes driving 5 minutes before the end of the shift, so his effective working time is at most 9.25 and at least 5.75 hours.

+ Each duty has a break of at least 30 minutes. This break must begin at most 5.5 hours after beginning

or before ending the shift.

+ Between two tasks in a duty there must be a break of at least 5 minutes.

To optimize the set of selected duties and schedule the crew, the problem is defined as a Crew Scheduling Problem (CSP), which is based on a Set Covering Problem (SCP). Since there are a lot of extra constraints compared to a standard SCP, the following problem with additional constraints for crew scheduling is introduced.

Sets

T: Set of all tasks to be covered, with  $t \in T$ 

- D: Set of all potential feasible duties, with  $d \in D$
- R: Set of all additional restrictions to be satisfied, with  $r \in R$

**Parameters** 

 $a_{td} = \begin{cases} 1, & \text{if task } t \text{ is covered by duty } d \\ 0, & \text{otherwise} \end{cases}$ otherwise

 $b_{rd} = \text{ parameter of restriction } r \text{ for duty } d$  $l_r$  = lower bound of restriction r

 $u_r$  = upper bound of restriction r

#### Decision variables

 $x_d = \begin{cases} 1, & \text{if duty } d \text{ is selected in the final selection} \\ 0, & \text{otherwise} \end{cases}$ 

(CSP):

$$\min\sum_{d\in D} c_d x_d \tag{2.1}$$

s.t. 
$$\sum_{d \in D} a_{td} x_d \ge 1$$
  $\forall t \in T$  (2.2)

$$l_r \le \sum_{d \in D} b_{rd} x_d \le u_r \quad \forall r \in R$$
(2.3)

$$x_d \in \{0,1\} \qquad \qquad \forall d \in D \qquad (2.4)$$

Restrictions

- The objective is to minimize the total costs (2.1)
- This restriction indicates that every task is covered by at least one duty. When more drivers are (2.2)on the same train, one of the crew members is the real driver, the others are passengers.
- (2.3) This restriction represents the constraints at crew base level, since the rules for individual duties are used for the generation of set *D*.
- (2.4)The decision variable  $x_d$ , is binary.

#### 2.2 Crew scheduling for extreme winter days

In this section we will develop a method to construct a robust crew schedule for extreme winter days.

In this alternative crew schedule drivers and therefore conductors as well, are assigned to tasks between two relief locations. One of these cities has to be their own base. After having meal break in their own base, it is possible to switch to another pair of relief locations. Again, one of these locations has to be their own base.

From previous research it has been turned out that to fulfill all trips of the original timetable, 1125 duties has to be scheduled. This means that about 20% more drivers and conductors are needed. Since it is impossible for NS to hire crew members for just one day, it is necessary to determine which part of the existing trips has to be operated. Therefore each task has a profit and the objective is to maximize the total profit. Furthermore, a crew member cannot be assigned to only one task; therefore it is necessary to select duties which cover the most unique tasks.

Another restriction on the selected part of trips is that the timetable must be cyclic. Since most of the lines have a frequency of an hour, all trips at that certain line operate or not. So when the intercity of 11:28 operates, the intercity of 12:28 at that station has to depart as well.

Furthermore, all tasks in the same trip have to be performed or the complete trip has to be canceled. When a train is driving from Rotterdam CS to Leeuwarden, it is not possible to operate only the parts from Rotterdam CS to Utrecht CS and from Zwolle to Leeuwarden, because the rolling stock is not available in Zwolle to operate this trip.

In this thesis we will investigate the decision which tasks to perform. The main research question in this thesis is: 'What is the maximal set of trips, which can be operated by the available drivers and conductors?' Therefore, we will determine an adjusted CSP, which is adjusted for this specific situation. After that, we will determine a value for the profit of each task which depends on for example of the mean number of passengers on that trip. Since a CSP is an NP-hard problem, we will propose a heuristic to solve this problem.

#### 3. Mathematical model

#### 3.1 **Adjusted CSP**

In this section we will describe the adjusted Crew Scheduling Problem (Adjusted CSP) and we will explain the restrictions.

To meet the restrictions, described in section 2.2, the following Mixed Integer Programming (MIP) model is formulated:

Sets

- T: Set of all tasks, which can be covered, with  $t \in T$
- *S*: Pseudo set of all tasks, which can be covered, with  $s \in S$
- *D*: Set of all potential feasible duties, with  $d \in D$
- *C*: Set of all base cities, with  $c \in C$

#### **Parameters**

 $a_{td} = \begin{cases} 1, & \text{if task } t \text{ is covered by duty } d \\ 0, & \text{otherwise} \end{cases}$  $s_{cd} = \begin{cases} 1, & \text{if city c is the start city of duty } d \\ 0, & \text{otherwise} \end{cases}$  $p_t = Profit of covering task t$ 

M = Big number

N = Number of available drivers

 $N_c$  = Number of available drivers with base city c

 $ST_t$  = Start time of task t

 $SC_t$  = Start city of task t

 $TC_t$  = Destination city of task t

 $TN_t = \text{trip number of task t}$ 

#### Decision variables

 $x_d = \begin{cases} 1, & \text{if duty } d \text{ is selected in the final selection} \\ 0, & \text{otherwise} \end{cases}$ 

$$v_t = \begin{cases} 1, & \text{if task t is covered by at least one duty in the final selection} \\ 0, & \text{otherwise} \end{cases}$$

(Adjusted CSP):

$$\max\sum_{t\in T} p_t v_t \tag{3.1}$$

s.t. 
$$\sum_{d \in D} a_{td} x_d \ge v_t \quad \forall t \in T$$
 (3.2)

$$\sum_{d\in D}^{d\in D} x_d \le N \tag{3.3}$$

$$\sum_{d \in D} s_{cd} x_d \le N_c \qquad \forall c \in C \tag{3.4}$$

$$v_t = v_s \qquad \forall (t,s) \in (T,S) \mid ST_t + 60 = ST_s \& SC_t = SC_s \& TC_t = TC_s \qquad (3.5)$$
  

$$v_t = v_s \qquad \forall (t,s) \in (T,S) \mid TN_t = TN_s \qquad (3.6)$$

 $v_t = v_s$   $\forall (t,s) \in (T,S) \mid TN_t = TN_s$ 

$$v_t, x_d \in \{0,1\} \qquad \forall t \in T, \forall d \in D \tag{3.7}$$

Restrictions

- (3.1)The objective is to maximize the total profit.
- This restriction indicates that if a task is covered by at least one duty in the final selection,  $v_t$ (3.2)could be one. When more drivers are on the same train, one of the crew members is the real driver, the others are passengers.
- This restriction provides that the total number of selected duties is at most equal to the number (3.3) of available drivers.
- (3.4)This restriction takes care of the fact that the number of selected duties for each base city is at most equal to the number of available drivers in that city.
- (3.5)The selected part of tasks has to be cyclic. Therefore, this restriction makes sure that every hour the same trips are covered.
- (3.6)A train has to go a trip, which is from a start station to an end station. So when tasks are at the same trip, all these tasks have to be covered or not covered.
- (3.7) The decision variables  $v_t$  and  $x_d$ , are binary.

#### 3.2 **Extra Restriction**

While executing the model given in section 3.1, it is possible to cover tasks in the final solution for which  $v_t = 0$ . Restriction (3.2) states that  $v_t$  could only be equal to one, when task t is covered by at least one of the selected duties. Although, when task t is covered by selected duties, restriction (3.3) does not require that  $v_t$  is equal to one.

Since the timetable has to be cyclic by restriction (3.5), it is necessary that  $v_t = v_s$ . So when it is not possible to cover  $v_s$ , it is profitable to set  $v_t$  equal to zero. Restriction (3.5) is in this case a soft constraint.

When it is necessary to have a strictly cyclic timetable, the following restriction has to be added to the model given in section 3.1.

$$\sum_{d \in D} a_{td} x_d \le M v_t \quad \forall t \in T$$
(3.8)

Restriction (3.8) requires that when task t is covered by at least one duty  $v_t$  has to be equal to one, since  $v_t$  is a binary variable. So by adding (3.8) to the adjusted CSP, restriction (3.5) is a hard constraint and the timetable will be cyclic.

#### 4. Methodology

In this chapter we will discuss the determination of some sets and parameters of the mathematical model, given in section 3.1.

#### 4.1 Tasks

In the alternative crew schedule a task is the shortest trip between two relief points. A trip could be from one station to another or from a base to another city (which is not a base) and then back to the base.

All these tasks are numbered on their starting time. Therefore, the first driving train has index number 1 and so on.

#### 4.2 Duties

The set of duties contains all possible duties with all tasks, which are discussed in section 4.1. These duties are restricted by the constraints for duties given in section 2.1. In our particular case there is an extra restriction for the duties; this restriction is based on the idea of Vlugt (2010).

+ A driver has to commute between two bases; therefore a duty contains only trips between two bases.

The set of all possible duties for each couple of bases is constructed, according to the following method. First all tasks between a given couple of bases are selected. These tasks are ordered on their start time. After that, for each task in the subset, it is determined by which of the tasks it can be followed. With these opportunities there are chains constructed. When the duration of a chain is too long, the chain is cut and thrown away. All chains with a too short duration of which the duration is too short, are also thrown away.

Finally, it is necessary to verify that there is a break of at least 30 minutes in each chain. When this is not the case, there is determined which tasks are located 5.5 hours after beginning and before ending of the chain. Each of these tasks can be removed from the chain for 30 minutes break, since the length of a task is in our case at least 20 minutes. So there are several possible duties with this chain of tasks. These adjusted chains and the remaining chains are all possible duties with the tasks in the subset between two bases. So for each couple of bases all possible duties are constructed.

When the total number of possible duties is too large, it is necessary to reduce it. Since a task can be operated several times, in that case one crew member is a driver and the others are passengers, we can assume that it is always possible to select a duty with more tasks. Therefore, when all tasks of a duty are in at least one other duty, we can throw this duty away. So for example Duty 1 contains task 1 and 3 and Duty 2 contains task 1, 2, 3 and 4, it is possible to remove Duty 1 of the set of all possible duties without loss of optimality, because the other tasks of Duty 2 can be covered twice.

#### 5. Heuristic

Since the standard CSP and also the adjusted CSP are NP-hard problems, it is useful to find a heuristic to solve the model. In this section we will propose a heuristic which is based on the optimal selection of duties.

#### 5.1 Basic idea

The main idea of this heuristic is, like the objective function of the mathematical model, to maximize the total profit. Therefore, we will search for the optimal selection of duties. For this heuristic we use the methods of Chapter 4 to create a reduced set of all possible duties. Every time the duty with the highest contribution to the total profit is chosen. If there are more duties with the same profit, the selected duty is randomly chosen. The duties which contain one or more of the new covered tasks are less profitable next time and duties which contain one or more tasks which are cyclic with the new covered tasks are more profitable next time.

Since the selected duties are randomly chosen, when their profit is equal, it could be useful to run the heuristic several times to optimize the set of selected duties.

The cyclic restriction is a soft constraint in this case, since it is not guaranteed that all tasks which are cyclic with the covered tasks are also covered.

#### 5.2 Algorithm

While there are drivers which are not yet performing a duty, do the following:

- 1. Determine for each task the additive profit to the total profit. This means that the profit is adjusted for the already chosen tasks and for the cyclic tasks.
- 2. Choose the duty with the highest additive profit and check there is still a driver available in the start base of this duty. When there are more duties with the same additive profit, one of these duties is randomly chosen. If there are no drivers left in that base, the additive profit of the chosen duty is set to zero and the again the duty with the highest additive profit is chosen.
- 3. If the additive profit of all duties is equal to zero, but there are drivers left. It is not necessary to select more duties, although it is still possible.

#### 6. Data

In this chapter we will discuss the data we use in this thesis. Furthermore, we select tasks and determine all possible duties and the number of available drivers.

#### 6.1 Data

There is data available about all the trips NS operates. For each trip the trip number, the starting and finishing station, the intermediary stations and the departure and arriving times for each intermediary station are given. In total there are 4726 trips. With the methods presented in this thesis, there are too many trips to calculate the optimal part under the restrictions given in section 2.2. Therefore, it is necessary to determine a test case of the total available trips. This can be a part of the lines or a part of the Netherlands. There is only one condition; the trips in the test case must influence each other.

#### 6.2 Test Case

For testing the mathematical model, given in section 3.1, we select a part of the trips. We have chosen to use the part of the trips operated in the southwest of the Netherlands. These are the trips between Vlissingen (Vs), Roosendaal (Rsd), Bergen op Zoom (Bgn), Tilburg (Tb), Dordrecht (Ddr), Breda (Bd), Den Bosch (Ht) and Eindhoven (Ehv). Breda, Bergen op Zoom and Tilburg are not bases, so in these cities crew members can neither change trains nor have a break.

In this area the following lines are operated:

- 1900-serie, between Ddr <-> Bd <->Tb <-> Ehv
- 2100-serie, between Ddr <-> Rsd <-> Vs
- 2200-serie, between Ddr <-> Bd <-> Ddr
- 2400-serie, from Vs -> Rsd -> Ddr (only during the morning rush-hours)
- 3600-serie, between Rsd <-> Bd <->Tb <-> Ht
- 5100-serie, between Ddr <-> Rsd <-> Bgn <-> Rsd
- 14600-serie, Rsd <-> Vs

There are some 70000-serie and 80000-serie trips operated in this area, which are covered in the test case as well. These trips are trips with empty rolling stock and they are not strictly necessary for the timetable.

In figure 6.1 the considered area of the Netherlands is shown.



Figure 6.1: The map of the south-west of the Netherlands with the covered stations.

#### 6.3 Tasks

Based on the definition in section 4.1, the total number of trips during a weekday in this area is 320. In the Table 6.2 one can see the distribution between the trips and the bases. All these trips are individual tasks in this set.

Bases	Number of trips
Ddr -> Bd -> Ddr	26
Ddr -> Ehv	37
Ddr -> Rsd	50
Ehv -> Ddr	37
Ehv -> Rsd	1
Ht -> Rsd	37
Rsd -> Bgn -> Rsd	8
Rsd -> Ddr	50
Rsd -> Ehv	1
Rsd -> Ht	39
Rsd -> Vs	15
Vs -> Rsd	19

Table 6.2: The bases in the test case with their incoming and outgoing trips.

#### 6.4 **Duties**

For each couple of bases is all possible duties are constructed with the method given in section 4.2. In total there are 1219797 possible duties which can be selected. In table 6.3 the numbers of possible duties per couple of bases are reported.

Bases	Number of	Reduced number			
	possible duties	of duties			
Ddr <-> Ddr	557	452			
Ddr <-> Ehv	11834	7143			
Ddr <-> Rsd	1171077	305640			
Ehv <-> Rsd	0	0			
Ht <-> Rsd	35868	17320			
Rsd <-> Rsd	0	0			
Rsd <-> Vs	461	340			
Total	1219797	330895			

Table 6.3: The couples of bases in the test case with the (reduced) number of possible duties.

After reducing, in according to the method described in section 4.2, the total numbers of duties, 330895 duties remain. The distribution of these duties over the routes is also given in shown table 6.3.

We have to remark that not all tasks of the subset are covered by at least one duty. There are no possible duties with the trips between Ehv and Rsd and no duties that start and end in Rsd.

#### 6.5 Profit

The profit of a trip is for example based to the mean number of passengers on that trip. We have not the data to determine profits for each trip. Therefore, we test our model with two different sets of profits for each task. In the first set, Profit 1, we assume that the profit of each task is the same and equal to one. The second set, Profit 2, makes a difference between Intercity tasks and tasks on slow trains. We assume that Intercity tasks are more important, so the profit of these tasks is equal to two and the profit of tasks on slow trains is equal to one.

In our test case tasks of the 1900, 2100, 2400 and 3600-series are Intercity tasks. Tasks of other series are slow train tasks.

#### 6.6 Number of drivers

The total number of drivers can be determined by counting the number of existing duties in the original crew schedule. When the existing duties are count by base, the number of drivers per base can be determined.

Since a test case is a part of the total number of trips, we cannot assume that the number of drivers for each base is the same as when we select all trips. So since we have covered all tasks which depart or finish from some bases, we may assume that all drivers with these cities as base are working on the covered duties. For the other bases we just cover a part of their trips, so we have to take a part of the drivers. To determine the number of available drivers per base, we count all tasks which depart from a

given base in all data. We also count the number of tasks which are covered by the test case. Furthermore, we determine the part of tasks which are covered by the test case. The number of available drivers of this base is the same part of the total number of drivers with this city as base. Since the number of drivers has to be integer, this number is rounded up. In formula:

Number of available drivers =  $\left[\frac{Tasks \ covered \ by \ test \ case}{Total \ number \ of \ tasks} \cdot Total \ number \ of \ drivers\right]$  (6.1)

Since we have covered all tasks which depart in Vlissingen and Roosendaal, all drivers from these cities are working on the covered duties. For the other three bases, Dordrecht, Eindhoven and Den Bosch, not all trips are covered by the test case, so we have to determine a reduced number of drivers.

According to formula (6.1), we have determined the number of available drivers. The results can be found in Table 6.4.

Base	Number of tasks covered by test case	Total number of tasks	Part of tasks covered by test case	Total number of drivers	Number of available drivers
Ddr	113	291	0.39	23	9
Ehv	38	322	0.12	46	6
Ht	37	351	0.11	18	2
Rsd	135	135	1.00	38	38
Vs	36	36	1.00	8	8

Table 6.4: The determining of the number of available drivers per base

In section 2.2 the statement is made that about 20% more drivers are needed when the drivers commute between two relief locations. In order to check this statement we also design a set of numbers of available drivers where there are in each base about 20% more drivers available.

So the two variations for the set of numbers of available drivers are displayed in Table 6.5. Since the number of available drivers +20% crosses the total number of drivers in some bases, namely Rsd and Vs, the second set of numbers is only used as a test and is not feasible in reality.

Base	Number of available drivers (Set 1)	Number of available drivers +20% (Set 2)				
Ddr	9	11				
Ehv	6	8				
Ht	2	3				
Rsd	38	46				
Vs	8	10				
Total	63	78				

Table 6.5: The numbers of available drivers in both sets per base

#### 7. Results

In this chapter we will define 12 different cases of the problem and for each case we will give the number of covered tasks and the distribution of these tasks over the given routes.

Given the remark in section 6.4 that there are ten tasks which are not covered by the defined duties, the maximum total number of covered tasks is 310 tasks.

There are two variations of the adjusted CSP model. The model given in section 3.1 with restrictions (3.1)-(3.7)(Model 1) and with (3.1)-(3.8)(Model 2). Model 2 is strictly cyclic and Model 1 is not.

There are also two sets with the numbers of available drivers. These sets are discussed in section 6.6. The difference between these two sets is that Set 1 is feasible and Set 2 not. Therefore, Set 2 is only used as a test for the determining of the number of available drivers.

Furthermore, we have discussed in section 6.5 two variations in determination of the profits. In Profit 1 the profit of all tasks is the same and equal to one and in Profit 2 the profit of the Intercity tasks is equal to two and for all other tasks it is equal to one.

Therefore, there are in total eight different cases. In Table 7.1 and 7.2 all possibilities are displayed.

Model 1	With Profit 1	With Profit 2
With Set 1	Case 1	Case 3
With Set 2	Case 2	Case 4

 Tables 7.1 & 7.2: The possible cases with the different models, sets of profits and sets number of available drivers

Besides these eight cases with the mathematical model there are also four cases with the heuristic, given in section 5.2. This heuristic can be combined with Set 1 and Set 2 and also with Profit 1 and 2. The possibilities are displayed in Table 7.3.

Heuristic	With Profit 1	With Profit 2
With Set 1	Case 9	Case 11
With Set 2	Case 10	Case 12

Table 7.3: The possible cases with the heuristic and the different sets of profit and number of available drivers

The total number of tasks covered in each case is displayed in Figure 7.4.

In Case 1 and 3 the number of covered tasks is not equal to the sum over all  $v_t$  from Model 1. This sum is the same as the number of covered tasks is Case 5 and 7. The number of covered tasks is these cases are equal to the number of tasks which is covered by the selected duties. This number is larger than the sum over all  $v_t$ , since there are tasks covered which are not cyclic with the other tasks and therefore is for these tasks  $v_t$  equal to zero. Since they are covered by the selected duties, we count them to the sum of all  $v_t$ .

We can see in Figure 7.4 that, as we had expected, with Model 1 not all tasks are covered with Set 1, but with the test set, Set 2, all tasks which can be covered, are covered. We also expected that when we require a strictly cyclic timetable, fewer tasks can be covered by the available drivers. That's also the case, but when we execute Model 2 with Set 2, all tasks are again covered. When we use Profit 2 with Set 1, like in Case 3 and 7, we see that with Model 1 there are more tasks covered then with Profit 1, but with Model 2 there are less tasks covered than in Case 5.

When we solve the problem with the heuristic with Set 1, we can see that there are more tasks covered than in Case 1, 3, 5 and 7, which have the same set of available drivers. Even if there are 20% more drivers available, like in Case 10 and 11, the heuristic cannot cover all tasks.



Figure 7.4: The number of covered tasks in different cases

When not all tasks which can be covered, are covered, it is interesting to know on which route there are less trains. In Table 7.5 on next page for the cases where not all tasks are covered, the number of trips covered on each route can be found.

Bases	Total Trips	Case 1	Case 3	Case 5	Case 7	Case 9	Case 10	Case 11	Case 12
Ddr -> Bd	26	26	8	26	0	21	22	12	24
-> Ddr									
Ddr -> Ehv	37	23	32	19	37	28	35	34	32
Ddr -> Rsd	50	50	50	50	50	50	50	50	50
Ehv -> Ddr	37	24	37	19	19	28	36	33	34
Ehv -> Rsd	1	0	0	0	0	0	0	0	0
Ht -> Rsd	37	37	37	37	37	37	37	37	37
Rsd -> Bgn	8	0	0	0	0	0	0	0	0
-> Rsd									
Rsd -> Ddr	50	50	50	50	50	50	50	50	50
Rsd -> Ehv	1	0	0	0	0	0	0	0	0
Rsd -> Ht	39	39	39	39	39	39	39	39	39
Rsd -> Vs	15	15	15	15	15	15	15	15	15
Vs -> Rsd	19	19	19	19	19	19	19	19	19

Table 7.5: The number of covered tasks per route in the cases where not all tasks are covered.

We can see in Table 7.5 that when all trips have the same profit, like in Case 1 and 5, first the trains on the route between Ddr and Ehv are not covered. In Case 1 there are 27 uncovered trips between Ddr and Ehv and in Case 5 there are 36 uncovered trips. In Case 9 and 10 there are uncovered trips between Ddr and Ehv and on the route from Ddr to Bd and back to Ddr. It seems that there are too less drivers available in Ddr.

When the Intercity tasks have a profit equal to two, like in Case 3, 7, 11 and 12, there are more trains scheduled on the route between Ddr and Ehv, but less on the route from Ddr to Bd and back to Ddr. In Case 3 there are only eight trips on the route between Ddr and Bd covered and in Case 7 none of these 26 trips are covered. In Case 11 and 12 there are respectively 12 and 24 of the 26 trips on this route covered.

#### 8. Conclusions

From our research we can concluded the following:

We can adjust the standard CSP-model to create an adjusted timetable for days with extreme winter weather. With this model we can create both a strictly cyclic timetable and a timetable which is not strictly cyclic. A heuristic is proposed which can be used to solve the problem.

After executing the adjusted CSP we can conclude that with the first set of numbers of available drivers per base not all tasks can be covered. When we execute the model with the set with 20% more drivers, all tasks in the test case are covered. So the statement in Vlught (2010) that there are about 20% more drivers needed to cover all tasks in the entire whole country seems to be right.

The solution of the heuristic cover more tasks than the solution of the mathematical model. Intuitively this seems to be wrong, because the results of the executing of the model are optimal. The main reason for this is that we do not strictly require that the selected part of tasks has to be cyclic. So since we relax the problem, the solution could be better than the cyclic solution.

When we consider the distribution of the selected tasks over the different routes, we can see that the route between Ddr en Ehv is the first route which is cancelled. It seems that there are too less drivers available in these bases. This is common in crew scheduling at NS. In the far corners of the country there are too many drivers and in the busy areas there are too less. When a driver has to commute between two bases where one of them has to be his home base, he stays in the area of his base and cannot perform tasks be useful in the other areas.

When we assume that the Intercity trains are more important than the slow trains and therefore the profit of the Intercity trains is larger equal to two, we can see that nearly all trains on the route between Ddr and Ehv are covered. In this case there are trains cancelled between Ddr and Bd. The reason for this is that the trains between Ddr and Bd are slow trains and between Ddr and Ehv the most trains are Intercity trains. So when the total profit is maximized, it is better to cancel slow trains than Intercity trains, when the Intercity trains are more profitable than the slow trains.

In total we can concluded that in our test case most of the trips are covered, but when we transfer the test area to the busy areas of the Netherlands, it will be more difficult to cover all trips.

### 9. References

Potthoff, D. (2010). Railway Crew Rescheduling: Novel Approaches and Extensions. PhD Thesis, ERIM. Vlught, D. (2010). Robust railway crew schedules. Master Thesis, Erasmus Universiteit Rotterdam.