COORDINATION THROUGH DISCOUNTS IN A SUPPLY CHAIN WITH THREE ACTORS

Abstract:
Nowadays most actors in a supply chain use discounts to maximize their own profit. Discounts are given to persuade other actors in a supply chain to order another quantity than their own optimal quantity. This thesis is about the efficiency of discounts in a supply chain with three actors. The efficiency gap is measured between the maximum total profit of the supply chain and the total profit of the supply chain when the actors do not give discounts. If all actors in a supply chain who can give discount, do give discount the efficiency gap is covered for 100%, given that all actors make rational decisions. If only the actor closest to the consumer receives discount, the total profit of the supply chain can decrease for a significant part of the times. The decrease in total profit will all go to the actor furthest away from the consumer.

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1. Introduction

Before any product reaches the end consumer it went through a supply chain from raw material to finished product. In a supply chain the product is modified by different actors before it is handed to the end consumer, all these actors attribute to the end value of the product.

At least three actors are common in a supply chain, a supplier of the raw materials, a manufacturer who changes the raw materials to the products and a retailer who delivers the end product to the consumer, see Figure 1. The demand for a product by the consumer is relatively fixed for a longer time period. It depends on the decision of the actors on order size and the number of times an order is placed how much profit can be gained, given that certain costs and prices are set in advance.

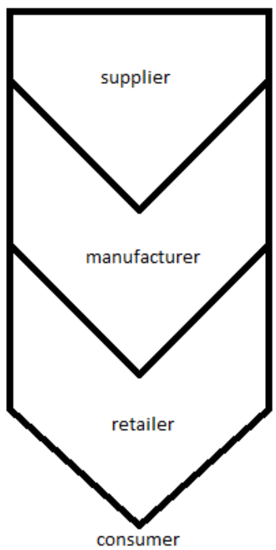

Figure 1: A supply chain with three actors. From upstream(supplier) to downstream(retailer).

Normally all actors in a supply chain maximize their profit by ordering and producing an optimal number of products according to their own profit functions. When an actor tries to create more profit, it might give discounts to the next actor in the supply chain to influence its decisions. The use of discounts is to close the efficiency gap. An efficiency gap is the gap between the total profit of a supply chain with full coordination, or profit maximization of the total supply, and the total profit with individual maximization where the profit per actor is maximized. When the total profit of the supply chain is maximized it is possible that some actors in that supply chain gain less profit than when they do not cooperate in maximization of the total profit of the supply chain. That is why discounts are needed to close the efficiency gap and make sure all actors in the supply chain will cooperate by compensation of the profit loss for certain actors in that supply chain.

Discounts are always used to let the buyer(or next actor in the supply chain) order a different order size, that will increase the profit of the seller and therefore the total profit of the supply chain. There are various ways of giving discounts to the next actor in the supply chain, for example: compensation of the holding costs when the seller wants a larger order, compensation of the set-up costs when the seller wants a smaller order or compensation on the order costs if the seller wants a different order size.

When only one of the actors tries to maximize its own profit by giving discounts to the buyer it might be possible that the profit of another actor will decrease and therefore the profit of the total supply chain might decrease as well. If all actors who are sellers as well try to maximize their own profit by giving discounts, the total profit of the supply chain might increase, decrease or remain the same.

In this thesis the effects of discounts in a supply chain with three actors are examined for the order size. The effects are measured by calculating the total profit and the profit per actor of the supply chain. Effects on discounts are examined for the case that one of the two actors who can give discount does give discount and for the case that both of them gives discounts. This comes to a total of three different ways that discount can be given in a supply chain with three actors. The optimal order size for maximization in (part of) the supply chain by discounts is gathered by calculating the optimal order sizes when full coordination is applied. When full coordination is applied the maximum profit for (part of) the supply chain is reached. By using these order sizes and discounts to divide the extra profit, the maximum profit might be reached.

These effects of discounts on the supply chain will be examined by varying parameters. Examples for parameters are: the set-up costs, the holding costs and the price for the product. For comparison between the different kinds of discounts the total profits are compared to each other, and the percentage of the efficiency gap that is covered will be given.

2. Research question

- What part of the efficiency gap can be covered by giving discounts on the order quantity in a supply chain with three actors?

The research question can be answered by some sub questions, described below. First of all the efficiency gap should be measured by finding the total profit functions when the actors do not coordinate and finding the profit functions when the actors fully coordinate. The functions that give the optimal values when (part of) the supply chain is fully coordinated are needed as well for the optimal discount quantity and discount fraction per unit. At last the total profits for discounts in (part of) the supply chain are gathered.

1 What is the total profit of a supply chain when no discount is given?

2 What is the maximum total profit of a supply chain when (part of) the supply chain is fully coordinated?

    2.1 Full coordination between retailer and manufacturer.

    2.2 Full coordination between manufacturer and supplier.

    2.3 Full coordination of the whole supply chain.

3 What is the total profit when the manufacturer gives discounts to the retailer? How does this discount influence the profit of the supplier?

4 What is the total profit when the supplier gives discounts to the manufacturer? Does this have any effects on the profit of the retailer?

5 What is the total profit when the supplier gives discounts to the manufacturer and the manufacturer gives discounts to the retailer?

3. Literature

There are few articles written about the effects of discounts in a supply chain. Most of these articles only include two actors, two of these article have some relevant information. The articles are Monahan (1984) and Goyal (1987).

In Monahan (1984) is written about a discount scheme that a supplier can make to maximize its profit. For this scheme the optimal order size for the supplier is calculated, together with the quantity discount needed to get the right order size. In this article the optimal order size is set as a multiple of the Economic Order Quantity (EOQ). Something that is not denoted in the article of Monahan (1984), is the production size of the supplier. In the article of Goyal (1987), a simple algorithm is given to get the optimal production size as multiple of the order size. In this article the discount function of Monahan (1984) is used as well.

In the previously discussed articles is shown how the optimal discount, order size and production size are determined for a supply chain with two actors, but most supply chains do have more than two actors. One of the few articles about a supply chain with three actors is Munson and Rosenblatt (2001).

This article is about a manufacturer who has a dominant position in the supply chain. The manufacturer can give discounts to the retailer and force discounts from the supplier. Munson and Rosenblatt (2001) give an algorithm that maximizes the total profit of the manufacturer and automatically minimize the total costs of the supply chain. In this thesis the supply chain has three actors, only the manufacturer does not have a dominant position. However the data that is used in this article will also be used in this thesis with some modifications.
In the thesis of Van den Hauwe (2011), is described how the order size of the retailer and the production size of the manufacturer can be determined through Nash-equilibriums. In the thesis of Van den Hauwe (2011) it is irrelevant what this does to the supplier and the total cost of the supply chain, because a supply chain with two actors is used.

This thesis will build on the thesis of Van den Hauwe (2011) by making use of the Nash-equilibriums for the order size. The difference will be that a supplier is added to the supply chain that already contains a manufacturer and a retailer.

4. The coordination models

The supply chain used in this thesis has three actors, a supplier(s), a manufacturer(m) and a retailer(r). These three actors all have their own profit function that may be combined. The profit functions consist of the parameters and variables as described below.

Parameters
D = demand intensity
P_i = price per unit charged by actor i ϵ {r ,m ,s}
S_i = fixed order costs for actor i ϵ {r ,m ,s}
h_i = holding cost rate for actor i ϵ {r ,m ,s}

Variables
Q = order size of the retailer
n_i = Integer lot-sizing multiple for actor i ϵ {m, s}
$\bar{Q}_i$ = optimal discount quantity according to actor i ϵ {m, s}
α_i = quantity discount fraction set by actor i ϵ {m, s}
v_i = relaxed version of n_i, $n_i \geq 1$ and i ϵ {m, s}

Denotations
There are different models used to define different kinds of coordination between the three actors in the supply chain. All different models have their own designation, and so does the optimal solution per model.

* = optimal solution
u = no optimization or discounts between the actors

c1 = full optimization between retailer and manufacturer
c2 = full optimization coordination between manufacturer and supplier
c3 = full optimization coordination between retailer, manufacturer and supplier
d1 = discount 1 = discount from manufacturer to retailer
d2 = discount 2 = discount from supplier to manufacturer
d3 = discount 3 = discount from supplier to manufacturer and from manufacturer to retailer

5. Sub question 1 - profit without coordination

To understand the effects of discounts in a supply chain, the total profit and the profit per actor in a supply chain without discounts should be gained for comparison and determination of the efficiency gap. When the actors all maximize their profit without mutual coordination, the profit functions for the three actors become as shown in (1.1) for the retailer, (1.2) for the manufacturer and (1.3) for the supplier.

$$\Pi^{\mathrm{u}}{}_{r}(Q) = (P_r - P_m)D - S_r\frac{D}{Q} - h_r\frac{Q}{2} \tag{1.1}$$

$$\Pi^{\mathrm{u}}{}_{m}(Q, n_m) = (P_m - P_s)D - S_m\frac{D}{n_m Q} - h_m\frac{(n_m - 1)Q}{2} \tag{1.2}$$

Other costs might be included in the profit function of the supplier, but are irrelevant for this thesis and there for kept out of the equation.

$$\Pi^{\mathrm{u}}{}_{s}(Q, n_m, n_s) = P_s D - S_s\frac{D}{n_s n_m Q} - h_s\frac{(n_s - 1)n_m Q}{2} \tag{1.3}$$

All three profit functions above consist of three parts. The first part is the profit per unit, the second part covers the costs for setting up the process of ordering and the last part covers the holdings costs, the costs for keeping a number of products in stock for a year.

The retailer maximizes the profit by optimizing over its order quantity, Q. When taking the derivative of the profit function of the retailer with respect to Q and set it equal to zero, the optimal number for Q is found. This is called the “Economic Order Quantity” and is given in (1.4).

$$Q^{*u} = \sqrt{\frac{2S_r D}{h_r}} \tag{1.4}$$

The manufacturer needs to optimize n_m for maximization of its own profit. This cannot be done by taking the derivative, because n_m is an integer. However in the thesis of Van den Hauwe (2011) is proven that the profit function of the manufacturer is concave and n_m is determined as described in (1.5).

$$n_m^{*u}(Q^u) = \begin{cases} min\{n_m: \Pi_m^u(Q^u, n_m + 1) \leq \Pi_m^u(Q^u, n_m) | n_m \epsilon \mathbb{N}\} \\ or \\ max\{n_m: \Pi_m^u(Q^u, n_m - 1) \leq \Pi_m^u(Q^u, n_m) | n_m \epsilon \mathbb{N}\} \end{cases} \tag{1.5}$$

When written in terms of the problem parameter, $min\{n_m: \Pi_m^u(Q^{*u}, n_m + 1) \leq \Pi_m^u(Q^{*u}, n_m) | n_m \epsilon \mathbb{N}\}$ becomes: $n_m^{*u}(Q^u) = min\left\{n_m: \frac{2S_m D}{h_m (Q^u)^2} \leq n_m(n_m + 1) | n_m \epsilon \mathbb{N}\right\}$. After rearranging this formulation the quadratic formula "$(n_m)^2 + n_m - \frac{2S_m D}{h_m (Q^u)^2} = 0$" appears. This can be rewritten to (1.6). The rewriting here is done in the thesis of Van den Hauwe (2011) as well, and will be used multiple times in this thesis.

$$n_m^{*u}(Q^u) = \begin{cases} \left\lceil -\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{2S_m D}{h_m (Q^u)^2}} \right\rceil \\ or \\ \left\lfloor \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{2S_m D}{h_m (Q^u)^2}} \right\rfloor \end{cases} \tag{1.6}$$

The supplier only needs to optimize the variable n_s for maximization of its own profit. Again this variable is an integer and cannot be attained when taking the derivative. Because of concavity of the suppliers profit function n_s can be found by the formulas below according to the thesis of Van den Hauwe (2011).

$$n_s^{*u}(Q^u, n_m^u) = \begin{cases} min\{n_s: \Pi_s^u(Q^u, n_m^u, n_s + 1) \leq \Pi_s^u(Q^u, n_m^u, n_s) | n_s \epsilon \mathbb{N}\} \\ or \\ max\{n_s: \Pi_s^u(Q^u, n_m^u, n_s - 1) \leq \Pi_s^u(Q^u, n_m^u, n_s) | n_s \epsilon \mathbb{N}\} \end{cases} \tag{1.7}$$

This will give:

$$n_s^{*u}(Q^u, n_m^u) = \begin{cases} \left\lceil -\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{2S_s D}{h_s (n_m^u)^2 (Q^u)^2}} \right\rceil \\ or \\ \left\lfloor \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{2S_s D}{h_s (n_m^u)^2 (Q^u)^2}} \right\rfloor \end{cases} \tag{1.8}$$

For both n_m and n_s , two solutions are given. Most of the times these two solutions are equal, only when they are not, the profits for the actor who sets the variable will be determined for both values of the variable. The value of the variable that gives the actor the highest profit is set as the only true value. When both profits are equal, the lowest value is set as the only value. For both n_m and n_s multiple solutions are given throughout this thesis. The decisions on which value to use are always the same as described above.

6. Sub question 2 – full coordination between (some of) the actors

This sub question is divided in three parts:

- Full coordination between the retailer and the manufacturer.
- Full coordination between the manufacturer and the supplier.
- Full coordination between the retailer, the manufacturer and the supplier.

The optimal values for the variables are needed to find the optimal solution when discounts are given for maximization of the supply chain's profit. That is why the formulations are written under here.

6.1 sub question 2.1 – full coordination between manufacturer and retailer

If there is full coordination between the retailer and the supplier the profit functions of both actors are combined as in (2.1.1). This makes sure that the total profit of these two is maximized.

$$\Pi^{c1}(Q, n_m) = (P_r - P_s)D - \left(S_r + \frac{S_m}{n_m}\right)\frac{D}{Q} - (h_r + h_m(n_m - 1))\frac{Q}{2} \qquad (2.1.1)$$

When this mutual profit function is maximized for a given n_m, the formulation for Q will become a variation of the normal EOQ as seen in (2.1.2).

$$Q^{*c1}(n_m) = \sqrt{\frac{2\left(S_r + \frac{S_m}{n_m}\right)D}{h_r + h_m(n_m - 1)}} \qquad (2.1.2)$$

Now the function of Q can be substituted in the mutual profit function, the New profit function is described in (2.1.3).

$$\Pi^{c1}(n_m) = (P_r - P_s)D - \sqrt{2\left(S_r + \frac{S_m}{n_m}\right)(h_r + h_m(n_m - 1))D} \qquad (2.1.3)$$

In the thesis of Van den Hauwe (2011) is proven that the function (2.1.3) unimodal is. Because of that the function for n_m^{*c1} can be written as shown in (2.1.4) and is rewritten to (2.1.5) as explained in sub question 1. This is also done in the thesis of Van den Hauwe (2011).

$$n_m^{*c1} = \begin{cases} min\{n_m : \Pi^{c1}(n_m + 1) \leq \Pi^{c1}(n_m) | n_m \epsilon \mathbb{N}\} \\ or \\ max\{n_m : \Pi^{c1}(n_m - 1) \leq \Pi^{c1}(n_m) | n_m \epsilon \mathbb{N}\} \end{cases} \qquad (2.1.4)$$

$$n_m^{*c1} = \begin{cases} \left\lceil -\frac{1}{2} + \sqrt{\frac{1}{4} + max\left\{1, \frac{S_m(h_r - h_m)}{S_r h_m}\right\}} \right\rceil \\ or \\ \left\lfloor -\frac{1}{2} + \sqrt{\frac{1}{4} + max\left\{1, \frac{S_m(h_r - h_m)}{S_r h_m}\right\}} \right\rfloor \end{cases} \tag{2.1.5}$$

6.2 sub question 2.2 - full coordination between supplier and manufacturer

In this case Q is already set according to optimization under uncoordinated conditions. Only $n_m$and $n_s$need to be determined. The profit function for full optimization between manufacturer and supplier becomes:

$$\Pi^{c2}(n_m, n_s) = P_m D - \left(S_m + \frac{S_s}{n_s}\right)\frac{D}{n_m Q} - (n_m(h_s n_s + h_m - h_s) - h_m)\frac{Q}{2} \tag{2.2.1}$$

Where Q is the EOQ according to sub question 1.

$$Q^{*c2} = \sqrt{\frac{2S_r D}{h_r}} \tag{2.2.2}$$

For the optimal multiplier n_m, the derivative is taken from the mutual profit function with respect to $n_m$and this derivative is set to zero. There is only one problem that n_m is an integer. However to find this integer, the continues function of n_m by relaxing from $n_m \epsilon \mathbb{N}$ to $n_m \geq 1$ should be examined, what comes back to the derivative according to (2.2.3).

$$v_m^{*c2}(n_s) = max\left\{1, \sqrt{\frac{S_m + \frac{S_s}{n_s}}{(h_s n_s + h_m - h_s)\frac{Q^2}{2D}}}\right\} \tag{2.2.3}$$

The fact that the derivative can be taken to n_m and given a certain number for n_s, there is only one solution for v_m^{*c2}, the conclusion can be drawn that the function is unimodal. The new function where the restriction $n_m \epsilon \mathbb{N}$ is given again can now be written as (2.2.4), what will become (2.2.5) after rewriting(explained in sub question 1).

$$n_m^{*c2}(n_s) = \begin{cases} min\{n_m : \Pi^{c2}(n_m + 1, n_s) \leq \Pi^{c2}(n_m, n_s) | n_m \epsilon \mathbb{N}\} \\ or \\ max\{n_m : \Pi^{c2}(n_m - 1, n_s) \leq \Pi^{c2}(n_m, n_s) | n_m \epsilon \mathbb{N}\} \end{cases} \tag{2.2.4}$$

$$n_m^{*c2}(n_s) = \begin{cases} \left\lceil -\frac{1}{2} + \sqrt{\frac{1}{4} + max\left\{1, \frac{S_m + \frac{S_s}{n_s}}{(h_s n_s + h_m - h_s)\frac{Q^2}{2D}}\right\}} \right\rceil \\ or \\ \left\lfloor \frac{1}{2} + \sqrt{\frac{1}{4} + max\left\{1, \frac{S_m + \frac{S_s}{n_s}}{(h_s n_s + h_m - h_s)\frac{Q^2}{2D}}\right\}} \right\rfloor \end{cases} \tag{2.2.5}$$

Now that the function for n_m is given the function for n_s can be derived. Again the derivative of the mutual profit function is taken now with respect to n_s , given that n_s is relaxed to $n_s \geq 1$, to proof that that the function for n_s is a unimodal function. This function is given in (2.2.6), and is a function of n_m because if the continuous function for n_m is substituted in it, the function will be very large.

$$v_s^{*c2}(n_m) = min\left\{1, \sqrt{\frac{S_s D}{n_m^2 Q^3 - \frac{n_m^2 Q^2 h_s}{2}}}\right\} \tag{2.2.6}$$

Here again is proven that this is a unimodal function with only one correct outcome. The real function for n_s given that $n_s \epsilon \mathbb{N}$ is shown in (2.2.7). The function can be rewritten as has been done in sub question 1, this is not shown because of the large size of this function.

$$n_s^{*c2} = \begin{cases} min\{n_s: \Pi^c(n_s + 1) \leq \Pi^c(n_s) | n_s \epsilon \mathbb{N}\} \\ \quad\quad\quad\quad or \\ max\{n_s: \Pi^c(n_s - 1) \leq \Pi^c(n_s) | n_s \epsilon \mathbb{N}\} \end{cases} \tag{2.2.7}$$

6.3 sub question 2.3 - full coordination between supplier, manufacturer and retailer

When the companies fully cooperate but do not give discounts to the next company in the supply chain another optimum can exist. Now the costs for the whole supply chain are minimized not just the costs per company. To minimize the costs of the whole supply chain a mutual profit function for the complete supply chain is needed and given by function (2.3.1).

$$\Pi^{c3}(Q, n_m, n_s) = P_r D - \left(S_r + \frac{S_m}{n_m} + \frac{S_s}{n_s n_m}\right)\frac{D}{Q} - (h_r + h_m(n_m - 1) + h_s(n_s - 1)n_m)\frac{Q}{2} \tag{2.3.1}$$

For a given, n_m and n_s the optimal Q is gained by taking the derivative with respect to Q and setting it equal to zero. The function for Q is shown in (2.3.2).

$$Q^{*c3} = \sqrt{\frac{2SD}{H}} \tag{2.3.2}$$

Where $S = S_r + \frac{S_m}{n_m} + \frac{S_s}{n_s n_m}$ and $H = h_r + h_m(n_m - 1) + h_s(n_s - 1)n_m$

When the function for Q is substituted in the mutual profit function the new profit becomes like in (2.3.3). The smaller functions for S and H are the as in (2.3.2).

$$\Pi^{c3} = P_r D - \sqrt{2SHD} \tag{2.3.3}$$

This gives a profit function with two variables left. When relaxing $n_s \epsilon \mathbb{N}$ to $n_s = v_s \geq 1$, the derivative of the profit function with respect of v_s and setting it to zero becomes (2.3.4).

$$v_s^{*c3}(n_m) = max\left\{1, \sqrt{\frac{S_s(\frac{(h_r-h_m)}{n_m}+h_m-1)}{(S_r+S_m)h_s n_m}}\right\} \quad (2.3.4)$$

This again gives an exclusive value to v_s given that $v_s \geq 1$, for a better explanation see sub question 2.2. Going back to the original restriction, $n_s \epsilon \mathbb{N}$, given that the continuous function is unimodal, (2.3.5) is needed to derive the optimal n_s. Again this function can be rewritten to (2.3.6).

$$n_s^{*c3}(n_m) = \begin{cases} min\{n_s: \Pi^{c3}(n_m, n_s+1) \leq \Pi^{c3}(n_m, n_s) | n_s \epsilon \mathbb{N}\} \\ or \\ max\{n_s: \Pi^{c3}(n_m, n_s-1) \leq \Pi^{c3}(n_m n_s) | n_s \epsilon \mathbb{N}\} \end{cases} \quad (2.3.5)$$

$$n_s^{*c3}(n_m) = \begin{cases} \left\lceil -\frac{1}{2} + \sqrt{\frac{1}{4} + max\left\{1, \frac{S_s(\frac{(h_r-h_m)}{n_m}+h_m-1)}{(S_r+S_m)h_s n_m}\right\}} \right\rceil \\ or \\ \left\lfloor \frac{1}{2} + \sqrt{\frac{1}{4} + max\left\{1, \frac{S_s(\frac{(h_r-h_m)}{n_m}+h_m-1)}{(S_r+S_m)h_s n_m}\right\}} \right\rfloor \end{cases} \quad (2.3.6)$$

When the derivative of the mutual profit function is taken to $n_m = v_m \geq 1$ and set to zero, (2.3.7) appears. n_s is not substituted in this function for a better overview.

$$(S_r - S_m)h_m + S_r h_s(n_s - 1) - S_m h_m = \frac{1}{v_m^2}\left(S_m h_r + \frac{S_s h_r}{n_s} - \frac{S_s h_m}{n_s}\right) - 2S_m h_m v_m \quad (2.3.7)$$

This derivative shows that there is not an exclusive value for v_m , and n_m can only be determined by enumeration of all possible values for n_m.

7. Sub question 3 - discount given by the manufacturer

<u>What is the total profit when the manufacturer gives discounts to the retailer? How does this discount influence the profit of the supplier?</u>

If the manufacturer tries to optimize its own profit it might be necessary to give quantity discounts to the retailer. When the retailers decision is rational, the order quantity will be equal to the EOQ. Mostly this value is not the optimal value if the mutual profit of the retailer and manufacturer is maximized. If the manufacturer wants the retailer to use the value for maximization of the mutual profit, discounts are needed to convince the retailer to distract from the EOQ. It makes sense that the retailer will not gain from that deviation without discounts, else he would have used this number anyway. The new profit functions with a discount factor(α_m) and an optimal discount Quantity($\bar{Q}_m$) are shown in (3.1) for the retailer, (3.2) for the manufacturer and (3.3) for the supplier. The discount factor is set in such a way that the loss for ordering the optimal discount quantity is exactly compensated.

$$\Pi_r{}^{d1}(Q,(\alpha_m,\bar{Q}_m)) = \left(P_r - (1-\alpha_m \mathrm{I}_{\{Q=\bar{Q}_m\}})P_m\right)D - S_t\frac{D}{Q} - h_r\frac{Q}{2} \tag{3.1}$$

$$\Pi_m{}^{d1}(Q,n_m,(\alpha_m,\bar{Q}_m)) = \left((1-\alpha_m \mathrm{I}_{\{Q=\bar{Q}_m\}})P_m - P_s\right)D - S_m\frac{D}{n_m Q} - h_m\frac{(n_m-1)Q}{2} \tag{3.2}$$

$$\Pi_s{}^{d1}(Q,n_m,n_s) = P_s D - S_s\frac{D}{n_s n_m Q} - h_s\frac{(n_s-1)n_m Q}{2} \tag{3.3}$$

First the optimal order quantity for the manufacturer is gathered to attain a number for α_m.
The formulation for the optimal discount quantity is given in (2.1.2) and is here indicated as $\bar{Q}_m$. When the optimal discount quantity is determined, the discount per unit(α_m^{d1}) given to the retailer to compensate its loss by differing from its EOQ can be determined according to (3.4).

$$\alpha_m^{d1} = \frac{\Pi_r^{*u} - \Pi_r(\bar{Q}_m)}{P_m D} \tag{3.4}$$

The loss for the retailer is exactly compensated according to (3.4), the retailer will not gain any extra profit by differing from its EOQ. In this thesis, the assumption is made that when the retailer is indifferent it will choose the best option for the manufacturer. In other parts of this thesis the same reasoning is used when an actor is indifferent. If both the optimal discount quantity for the manufacturer and the optimal discount per unit are determined, the retailer can determine whether it will order its EOQ or the optimal discount quantity.

$$Q^{*d1}\left(\alpha_m^{d1},\bar{Q}_m\right) = \begin{cases} \bar{Q}_m & if\ \ \Pi_r^{d1}(\bar{Q}_m\,,\alpha_m^{d1}) \geq \Pi_r^{u}(Q^{*u}) \\ & \qquad or \\ Q^u & otherwise \end{cases} \tag{3.5}$$

This will give the manufacturer the information it needs to set the right number for n_m and the supplier to set the right number for n_s.
The decision order is: $\bar{Q}_m$, $\alpha_m \rightarrow Q^{*d1} \rightarrow n_m \rightarrow n_s$. This means that first the optimal discount quantity with corresponding discount factor are determined by the manufacturer, after that the retailer can decide given the optimal discount quantity and the corresponding discount factor whether it will order the EOQ or the optimal discount quantity. At last the right numbers for n_m and n_s can be determined given the already determined values.

If the retailer orders $\bar{Q}_m$, the manufacturer uses the optimal multiplier that is set by (2.1.5), the new formulation for n_m is given by (3.6).

$$n_m^{*d1} = \begin{cases} n_m^{*c1}\ if\ Q = \bar{Q}_m \\ \qquad or \\ n_m^{u}(Q)\ \ otherwise \end{cases} \tag{3.6}$$

When Q and n_m are determined, the supplier can set n_s. n_s is set according to (1.8) only the numbers for Q and n_m might differ. The new formulation for n_s, when the manufacturer gives discount to the retailer is given in (3.7). The same formulation is used when there is no coordination between all three actors.

$$n_s^{*d1}\left(Q^{d1}, n_m^{d1}\right) = \begin{cases} \left\lceil -\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{2S_s D}{h_s (n_m^{d1})^2 (Q^{d1})^2}} \right\rceil \\ or \\ \left\lfloor \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{2S_s D}{h_s (n_m^{d1})^2 (Q^{d1})^2}} \right\rfloor \end{cases} \tag{3.7}$$

It might be possible that the supplier loses some profit compared to the uncoordinated version. The supplier cannot go back to the uncoordinated version, because the retailer and the manufacturer already have set their order size. This means that the loss of the supplier can be greater than the extra profit of the manufacturer and the total profit of the supply chain might decrease.

8. Sub question 4 - discount given by the supplier

What is the total profit when the supplier gives discounts to the manufacturer? Does this have any effects on the profit of the retailer?

When the supplier can give a discount to the manufacturer, and the manufacturer does not give any discount to the retailer, the retailers profit function is the same as when there is no coordination at all. The profit functions for the manufacturer and the supplier are different from the uncoordinated versions. An optimal discount quantity($\bar{Q}_s$) and a discount factor(α_s) are included in these functions. The profit function for the retailer is shown in (4.1), for the manufacturer it is shown in (4.2) and for the supplier in (4.3).

$$\Pi_r^{d2}(Q) = (P_r - P_m)D - S_r \frac{D}{Q} - h_r \frac{Q}{2} \tag{4.1}$$

$$\Pi_m^{d2}(Q, n_m, (\alpha_s, \bar{Q}_s)) = \left(P_m - (1 - \alpha_s \mathrm{I}_{n_m Q = \bar{Q}_s})P_s\right)D - S_m \frac{D}{n_m Q} - h_m \frac{(n_m - 1)Q}{2} \tag{4.2}$$

$$\Pi_s^{d2}(Q, n_m, n_s, (\alpha_s, \bar{Q}_s)) = (1 - \alpha_s \mathrm{I}_{n_m Q = \bar{Q}_s})P_s D - S_s \frac{D}{n_s n_m Q} - h_s \frac{(n_s - 1) n_m Q}{2} \tag{4.3}$$

When there is full coordination between the manufacturer and the supplier, the total profit of the manufacturer and the supplier is maximized. However, the extra profit will all go the supplier and the manufacturer might even lose some profit. This makes sense because, when no discounts are given the profit of the manufacturer does not depend on the decision of the supplier, but the suppliers profit does depend on the manufacturers decision. If the profit of the manufacturer can increase by ordering the number according to full coordination, it would do that without coordination as well. The optimal discount quantity will be the same as with full coordination and is set to $n_m^{*c2} * Q^{*d2}$, now called $\bar{Q}_s$. If this order size is actually ordered by the manufacturer the discount factor per unit exactly compensates for the loss of profit by differing from the optimal order size for the manufacturer and is given in (4.4).

$$\alpha_s^{*d2} = \frac{\Pi_m^u - \Pi_m^{c2}}{P_s D} \tag{4.4}$$

Because the profit function for the retailer does not change compared to the uncoordinated profit function, the retailer orders the EOQ according to (1.4).

$$Q^{*d2} = Q^{*u} = \sqrt{\frac{2S_r D}{h_r}}$$

When the functions for Q, α_s^{*d2} and $\bar{Q}_s$ are known, the manufacturer can decide whether or not it will take the discount and differ from its optimal number for n_m, according to (4.5). The decision of the supplier depends on whether or not the other actors in the supply have been rational in their decisions as shown in (4.6).

$$n_m^{*d2} = \begin{cases} n_m^{*c2} \; if \; \Pi_m(Q, n_m^{*c2}, (\alpha_s, \bar{Q}_s)) \geq \Pi_m^{*u} \\ \quad or \\ n_m^u(Q) \; otherwise \end{cases} \tag{4.5}$$

$$n_s^{*d2} = \begin{cases} n_s^{*c2} \; if \; \Pi_s(Q, n_m, n_s^{*c2}, (\alpha_s, \bar{Q}_s)) \geq \Pi_s^{*u} \\ \quad or \\ n_s^u(Q, n_m) \; otherwise \end{cases} \tag{4.6}$$

If the supplier gives discount to the manufacturer, the manufacturer will only accept when it does not lose any profit compared to the uncoordinated version. Also, the supplier will only give discount when it gains extra profit by doing so. A conclusion that can be drawn from this, is that the total supply chain will never lose some profit compared to the uncoordinated version, this also accounts for the three actors in the supply chain.

9. Sub question 5 - discount given by both the manufacturer and the supplier

What is the total profit when the supplier gives discounts to the manufacturer and the manufacturer gives discounts to the retailer?

In this case there is discount from the supplier to the manufacturer and from the manufacturer to the retailer, the corresponding profit functions are given in (5.1) for the retailer, (5.2) for the manufacturer and (5.3) for the supplier. For complete coordination through discounts two optimal discount quantities and two discount factors are needed, one for coordination between the manufacturer and the retailer($\alpha_m, \bar{Q}_m$) and one for coordination between the supplier and the manufacturer($\alpha_s, \bar{Q}_s$).

$$\Pi_r{}^{d3}(Q, (\alpha_m, \bar{Q}_m)) = \big(P_r - (1 - \alpha_m \mathrm{I}_{\{Q=\bar{Q}_m\}})P_m\big)D - S_t \frac{D}{Q} - h_r \frac{Q}{2} \tag{5.1}$$

$$\Pi_m^{d3}(Q, n_m, (\alpha_m, \bar{Q}_m), (\alpha_s, \bar{Q}_s)) = \big((1 - \alpha_m \mathrm{I}_{\{Q=\bar{Q}_m\}})P_m - (1 - \alpha_s \mathrm{I}_{n_m Q=\bar{Q}_s)})P_s\big)D - S_m \frac{D}{n_m Q} - h_m \frac{(n_m - 1)Q}{2} \tag{5.2}$$

$$\Pi_s^{d3}(Q, n_m, n_s, (\alpha_s, \bar{Q}_s)) = (1 - \alpha_s \mathrm{I}_{n_m Q = \bar{Q}_s}) P_s D - S_s \frac{D}{n_s n_m Q} - h_s \frac{(n_s - 1) n_m Q}{2} \tag{5.3}$$

The decision order becomes: (α_s, $\bar{Q}_s$) given ($\alpha_m^{d1}, \bar{Q}_m^{d1}$) → ($\alpha_m$, $\bar{Q}_m$)→Q^{d3}→n_m^{d3}→n_s^{d3}

As the discount from the supplier to the manufacturer is determined, the fact that the manufacturer tries to optimize its own profit by giving discount to the retailer should be included in the decision. The retailer cannot get more than the profit it gains using the EOQ however he can order a different amount than the EOQ when the right discount is offered. The optimal discount between the manufacturer and the retailer is already determined in sub question 3, where the manufacturer maximizes its own profit. If the supplier wants to maximize its profit it should include the profit of the manufacturer with discount to the retailer in its decision with the determination of the discount it should give to the manufacturer.

The maximum profit of the complete supply chain is found in sub question 2.3 where full coordination between all three actors is used. The number for Q, n_m and n_s that are found in these formulations are the optimal numbers for discount between all three actors. If the manufacturer and the supplier make rational decisions, $\bar{Q}_m$ becomes Q^{*c3} and $\bar{Q}_s$ becomes $n_m^{*c3} * Q^{*c3}$, with these numbers α_s and α_m are determined according to (5.4) and (5.5).

$$\alpha_m^{d3} = \frac{\Pi_r^{*u} - \Pi_r^{c3}}{P_s D} \tag{5.4}$$

$$\alpha_s^{d3} = \frac{\Pi_m^{*d1} - \Pi_m(\bar{Q}_m^{c3}, \alpha_m^{d3})}{P_s D} \tag{5.5}$$

Now that the right discounts and numbers for Q, n_m and n_s are gathered, the normal process is followed again by deciding the right Q, followed by the right number for n_m and at last the right number for n_s according to (5.6), (5.7) and (5.8). All actors will only choose the optimal values according to full coordination between all three when their profit will be higher than or equal to the maximum profit they can gain without full coordination with all three actors.

$$Q^{*d3}(\alpha_m^{d3}, \bar{Q}_m) = \begin{cases} \bar{Q}_m \quad if \;\; \Pi_r^{d3}(Q, (\alpha_m, \bar{Q}_m)) \geq \Pi_r^u \\ \quad or \\ Q^{*u} \;\; otherwise \end{cases} \tag{5.6}$$

$$n_m^{*d3}(Q^{d3}, (\alpha_m, \bar{Q}_m), (\alpha_s, \bar{Q}_s)) = \begin{cases} n_m^{*c3} \;\; if \quad \Pi_m\big(Q^{d3}, n_m^{*c3}, (\alpha_m, \bar{Q}_m), (\alpha_s, \bar{Q}_s)\big) \geq \Pi_m^{*d1} \\ \quad or \\ n_m^u\big(Q^{d3}\big) \;\; otherwise \end{cases} \tag{5.7}$$

$$n_s^{*d3}(Q, n_m^{d3}, (\alpha_s, \bar{Q}_s)) = \begin{cases} n_s^{*c3} \; if \; \Pi_s^{d3}\big(Q^{d3}, n_m^{d3}, n_s^{*c3}, (\alpha_s, \bar{Q}_s)\big) \geq \Pi_s^{*d1} \\ \quad or \\ n_s^u\big(Q, n_m^{d3}\big) \end{cases} \tag{5.8}$$

Naturally, maximization of the profit through discounts of the total supply gives a higher profit than all other versions of coordination with discount. This profit is divided between all three actors in such a way that the retailer will get the same profit as in the uncoordinated version, the manufacturer gets the same profit as when he optimizes its own profit by giving discount to the retailer and the supplier will get the rest of the total profit. The profit of the supplier will be higher than the profit without coordination and coordination through discounts between the manufacturer and the retailer, but might be lower than the version where only the supplier gives discounts to the manufacturer.

10. Simulation results

For the simulation in this thesis the following values were used for the parameters. All possible combinations are used in the simulation, that makes a total of 1 782 000 combinations. The data for the retailer and the manufacturer were also used in the thesis of van den Hauwe(2011), other data is obtained from Munson and Rosenblatt(2001) and supplemented with fabricated data that matches with the data of van den Hauwe (2011). The simulation was done on a 4.00 GB Installed memory (RAM) and AMD Phenom™ II X4 Processor and took 40 minutes.

D = 150 000
Pr = 25
Pm = 15
Ps = 5
S_r ϵ {20, 40, 60, 80, 100, 120, 140, 160, 180, 200}
S_m ϵ {50, 100, 150, 200, 250, 300, 350, 400, 450, 500}
S_s ϵ {50, 100, 150, 200, 250, 300, 350, 400, 450, 500}
h_r ϵ {5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60}
h_m ϵ {2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32, 35}
h_s ϵ {2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32, 35}

Details about the amount of discount per product for all three different versions of discounts in a supply chain are given in Table 1. As can be seen α never has a negative number, and the lowest number it can get is 0. This means that the optimal discount order quantity always larger is than the EOQ according to the retailer. Striking is that in Discount 1 α_m always larger is than zero, while in discount 3 α_m can be zero. This means that in some cases the best discount order quantity for discount 1 a larger number has than for discount 3.

	Discount from manufacturer to retailer (Discount 1)	Discount from supplier to manufacturer (Discount 2)	Discount from manufacturer to retailer and from supplier to manufacturer (Discount 3)	
	$\boldsymbol{\alpha_m}$	$\boldsymbol{\alpha_s}$	$\boldsymbol{\alpha_m}$	$\boldsymbol{\alpha_s}$
Mean	0.0013	0.0027	0.0020	0.0025
Standard deviation	0.0015	0.0039	0.0023	0.0028
Minimum	2.2477e-008	0	0	1.0849e-008
maximum	0.0106	0.0332	0.0170	0.0261
Percentage α = 0	0 %	37.73 %	1.1574e-006 %	0 %

Table 1: Information about the discounts given per unit for all three types of discounts used in this thesis.

For the simulation where the manufacturer gives discount to the retailer (Discount 1), the retailer does not lose or gain any profit and the manufacturer will gain some profit compared to the uncoordinated version. The supplier can sometimes lose some profit compared to no coordination and when this loss is high enough the complete supply chain loses profit. In this simulation 15.14% of all parameter combinations the total profit of the supply chain is decreased compared to the uncoordinated total profit. This decrease is in its entirety for the account of the supplier who loses profit in 33.81% of the parameter combinations compared to the uncoordinated profit of the supplier.

In the other simulations with discounts the total profit is always larger than or equal to the profit without coordination. This due to the fact that when the supplier gives discounts, both retailer and supplier have the power of ordering a different number when the profit is smaller compared to the uncoordinated version. The manufacturer also has the power to give discounts to the retailer in discount 3 to gain some extra profit and will not settle for a lower profit than the maximum profit it can get by giving discounts to the retailer. This can be seen in Table 2 where the percentages of extra profit for the manufacturer are the same for discount 1 and 3. Remarkable in Table 2 is that the minimum percentage of extra profit compared to the profit without coordination a negative number is. An explanation is that, the manufacturer will always give discount to the retailer to maximize its profit, this can cause a loss for the supplier. The only thing the supplier can do is minimize this loss by giving discount to the manufacturer.

Extra profit in percentages, compared to the profits without discount.		**Discount 1**	**Discount 2**	**Discount 3**
Total Profit	Mean	0.12 %	0.07 %	0.19 %
	Std	0.18 %	0.10 %	0.19 %
	Min	-0.54 %	0 %	0 %
	max	1.73 %	0.89 %	1.79 %
Profit retailer	Mean	0 %	0 %	0 %
	Std	0 %	0 %	0 %
	Min	0 %	0 %	0 %
	Max	0 %	0 %	0 %
Profit manufacturer	Mean	0.21 %	0 %	0.21 %
	Std	0.27 %	0 %	0.27 %
	Min	3.2e-006 %	0 %	3.2e-006 %
	Max	2.14 %	0 %	2.14 %
Profit supplier	Mean	0.20 %	0.33 %	0.54 %
	Std	0.53 %	0.51 %	0.68 %
	Min	-2.78 %	0 %	-0.51 %
	max	5.02 %	4.65 %	5.68 %

Table 2: The mean, standard deviation, minimum and maximum of the extra profits in percentages. Extra profit is the profit that is gained beyond the profit without coordination.

Table 3 shows the percentages of the efficiency gap that is covered by giving discounts. The efficiency gap is the gap between full maximization of the total profit of the supply chain and maximization of the individual profit per actor. Especially for discount 1 there is a great diversion between the different parameter sets. From a very large negative percentage to a 100 % fill of the gap. It depends on the parameter set whether it is good for the total profit of the supply chain to have discount between the manufacturer and the retailer. In figure 2 can be seen that the larger the holding costs for the retailer the smaller the percentage of the efficiency gap that is covered. In figure 3 and appendix 1 the average percentage of the efficiency gap that is covered are shown for the holding costs for the manufacturer and the supplier. Appendix 2, 3 and 4 show the average percentages of the filling of the efficiency gap for the set-up costs per actor. Figure 2, Figure 3 and Appendix 1 till 4 also show the percentage that the efficiency gap is covered for discount 2. For discount 3 it is unnecessary to give the filling of the efficiency gap because the values are always 100 %.

Percentage of efficiency gap that is covered by giving discounts.		**Discount 1**	**Discount 2**	**Discount 3**
Total Profit	Mean	40.85 %	40.20%	100 %
	Std	90.52 %	39.67 %	0 %
	Min	- 2 0853 %	0 %	100 %
	max	100%	100 %	100 %

Table 3: The percentage of the efficiency gap that is covered by giving discounts for all three discount versions.

The lower the holding costs and the set-up costs for the supplier, the smaller the efficiency gap gets if discount 1 is used. When the set-up and holding costs for the retailer and the manufacturer rise, the efficiency gap gets larger for discount 1. For discount 2 it is exactly the opposite as for discount 1.

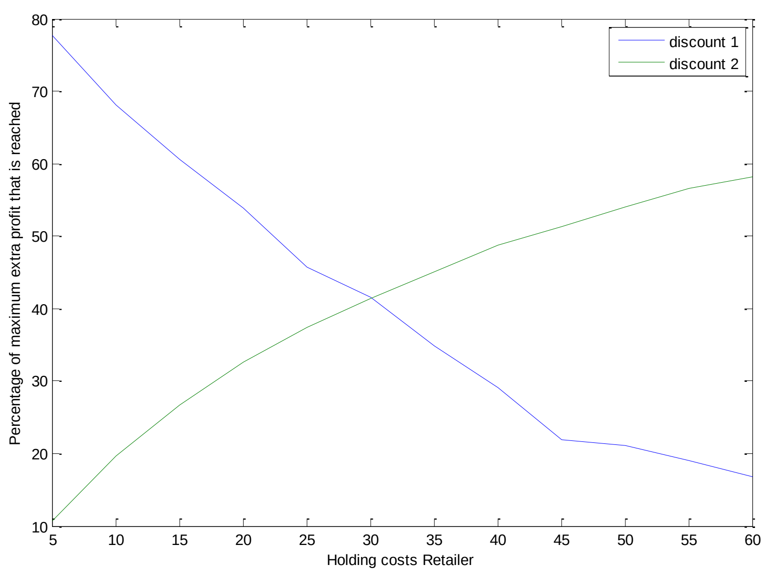

Figure 2: Average percentage of the efficiency gap that is covered by discount for different values of the retailer.

As said before, the efficiency gap for discount 3 can be larger than the efficiency gap without coordination. When looking at the average coverage of the efficiency gap per parameter, the graphs show that only when the holding costs for the manufacturer are low, the average percentage that the efficiency gap is covered goes below zero, as can be seen in figure 3.

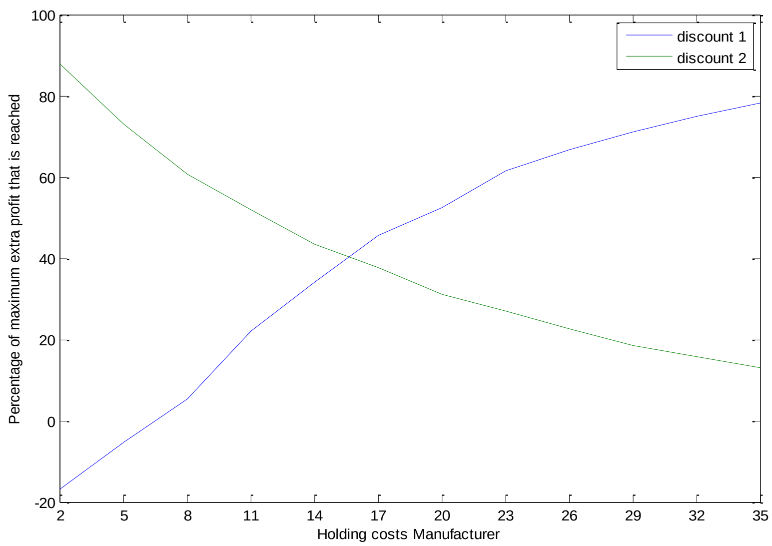

Figure 3: Average percentage of the efficiency gap that is covered for different values of the holding costs for the *manufacturer.*

11. Conclusion

When discounts are given most of the times the efficiency gap decreases, except for discount 1 where the manufacturer gives discount to the retailer, here the efficiency gap can increase. As shown in figure 3, especially the holding costs for the manufacturer have a large influence on the size of the efficiency gap. For the versions were the supplier gives discount, the efficiency gap will never increase and will mostly decrease. When both the supplier and the manufacturer give discount the efficiency gap is covered for 100% for all parameter sets. From this information the conclusion can be drawn that it is always better for the supplier to maximize its own profit by using discounts.

Another conclusion that can be drawn is that the retailer and the manufacturer will never lose profit compared to an uncoordinated supply chain, because they will not order a different quantity if they don’t get compensation. This will only account when both retailer and manufacturer make a rational decision. The supplier however can lose some profit compared to an uncoordinated supply chain because he is dependent on the decisions of the retailer and the manufacturer even if they make a rational decision.

12. Discussion

There have been given discounts on order size in this thesis, only it is also possible to give discounts on holdings costs for example. When both the supplier and the manufacturer give discount for maximization of the profit the efficiency gap is covered for 100%, but when one of them does not give discounts there will still be an efficiency gap. Maybe it is possible to decrease that efficiency gap by giving discount for other things than the order quantity.

It seems like that when discounts are given the supply chain can lose some profit when not all actors give discount, especially when the actors closest to the consumer give discounts while the actors further away from the consumer don't. What if there are more than three actors in a supply chain, is it necessary when the actor closest to the consumer gets discount, that the other actors also have to give discounts to be sure that they do not lose any profit compared to an uncoordinated supply chain?

And again for more than three actors in a supply chain, what happens to the efficiency gap when not the first or the last actors in a supply chain gives or receives any discount, but another actor in that supply chain does give discount?

13. References

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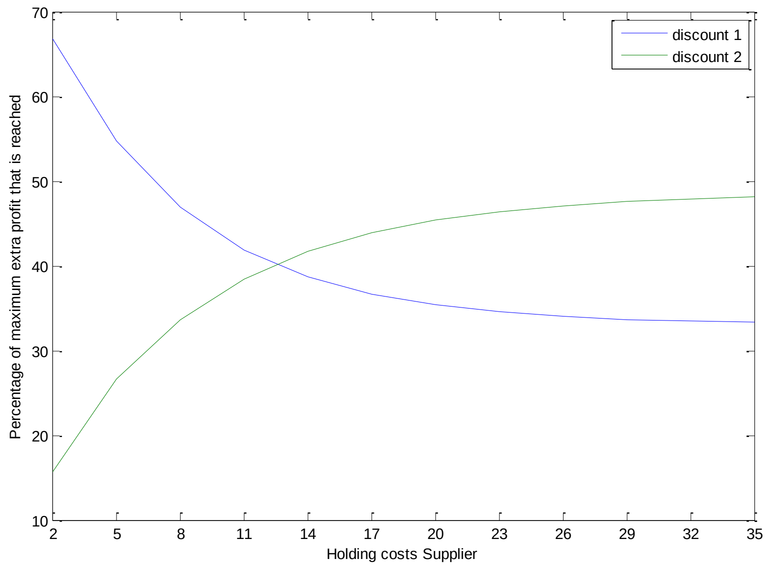
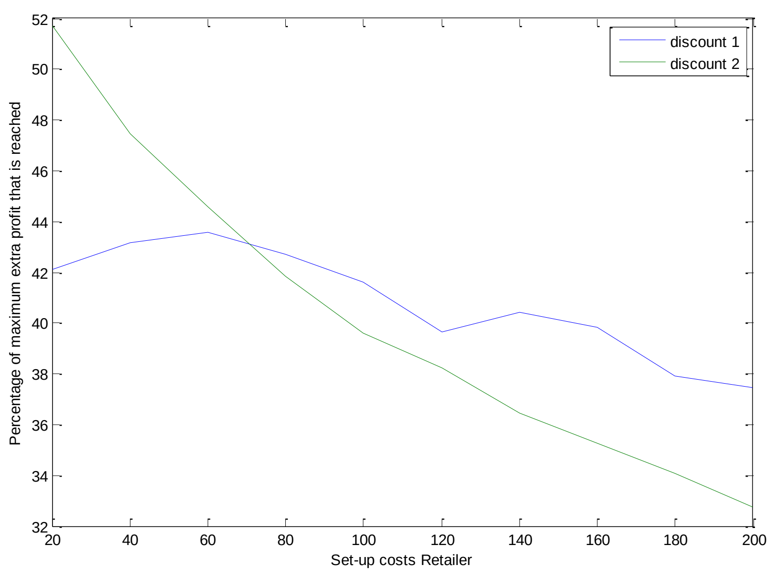
13. Appendix

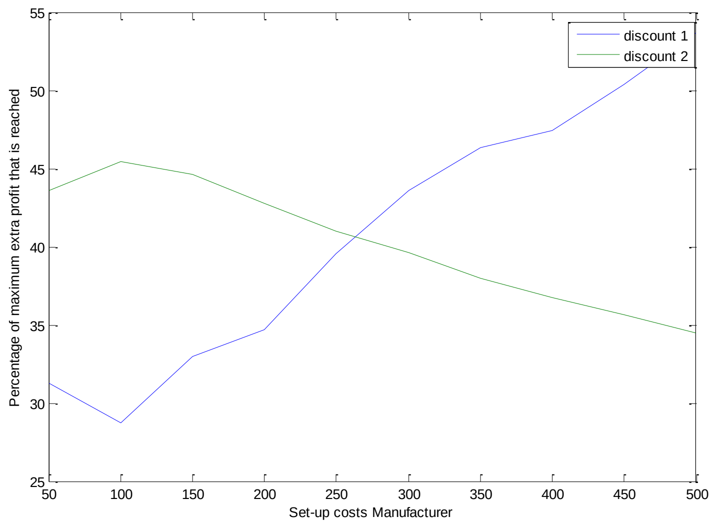
Appendix 1: Average percentage of the efficiency gap that is covered for different values of the holding costs for the supplier.



Appendix 2: Average percentage of the efficiency gap that is covered for different values of the set-up costs for the retailer.



Appendix 3: Average percentage of the efficiency gap that is covered for different values of the set-up costs for the manufacturer.



Appendix 4: Average percentage of the efficiency gap that is covered for different values of the set-up costs for the supplier.