Assignment Scheduling Rules:

Scheduling patients with different service times

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1. Introduction

Many economists believe that the collapse of the so called "housing bubble" in the United States was the trigger which resulted in the late-2000s financial crisis, for many of us this is referred as the Great Recession. Banking systems have to deal with liquidity shortfalls, large financial institution collapsed, the housing market declined and so forth. The financial crisis did not only 'hit' the United States, economies worldwide suffered from this recession. Governments all over the world tried everything to recover from this crisis and restore the economies. The Dutch government bastardized many subsidies on many fronts. The medical sector was not an exception; all the hospitals in the Netherlands have to reduce their budget with an amount of 314 million euro's. For this reason hospitals have to consider all kind of scenario's, from lowering wages to deleting expensive treatments. The latter suggestion could be catastrophic when it is realized; the lives of patients could depend on these treatments. Another option is to make 'things' more efficient than they are at the current situation.

One of the 'things' which can be improved is the so called appointment scheduling; every hospital or clinic has to deal with this. The managerial team does not want outpatients to walk-in whenever they want to. This will create chaos and uncertainty about the arrival time of the patients, which will result in more idle time of the medical team. On the point of view of the outpatients, they also prefer an appointment schedule. When simply reasoning, if all patients walk in at the same moment, then the average waiting time of these patients will be larger when compared to the alternative: usage of an appointment schedule. This alternative seeks the right balance between the idle time and the overtime of the medical team, especially the doctor or expensive equipments, and the waiting time of the outpatients. It is often the case that practitioners overestimate the value of their own time with respect to the patient's time, they will try to reduce the idle time, but simultaneously increase the patient's waiting time.

This thesis will handle this problem and try to find a general rule for appointment scheduling (ASR), in hope to reduce the costs of a hospital. Naturally, this rule could be implemented into other areas where they have to deal with appointments. In the past, various authors tried to find a 'good fitting' rule, many of them assumed that all patients are the same and have the same statistical properties, i.e., the same service time. This is not logical in the real world; we expect that there are differences between the patients, some with 'small' problems/questions and some with 'large' problems which require more time from the practitioner. This thesis assumes that there are two different groups of patients; the first group has a smaller service time and the second group has a larger service time.

Klassen Rohleder [12] and Hutzschenreuter [8] both already concluded that patients with a smaller expected service time should be scheduled during the beginning of the session. However, they only took the patient's waiting time and the practitioner's idle time into account and did not pay any attention to the so-called practitioner's overtime. This performance measurement should be investigated too; otherwise it is possible that a practitioner works a lot longer than the expected end time of a working day, resulting in more working hours. Furthermore, another difference between this research and previous researches is that we assume that the weights (unit costs) for the patient's waiting time and the practitioner's idle and overtime could be different. This means that the practitioner's idle or overtime could be more important than the patient's waiting time. Thus we want the practitioner's idle or overtime stays as low as possible. Or if the patient's waiting time is more important, than the waiting time should stay as low as possible. Hence different policies will result in different assignment scheduling rules. This thesis will try to find a general 'good' fitting rule and answer the following research question:

> *"What is a good assignment scheduling rule for two types of patients. Given the choice for the unit costs for the patient's waiting time and the practitioner's idle and overtime."*

Ho and Lau [6] already stated that there is not such a rule which performs better than other ASR's. Thus for a given environment one could find a good fitting rule. For this reason we expect to find more than one assignment scheduling rule for different experimental settings. We could easily change these settings with the use of simulations.

This thesis has the following structure: We will start with section 2 'Literature Review', this section contains a brief review of the researches and models performed by other authors. Next, we will discuss the nature of this assignment scheduling problem in section 3 'Problem Definition'. The simulation model used for this problem is described in section 4 'Simulation'. It illustrates the various parameters and assumptions needed for the problem. Furthermore, this section also contains a subsection about the validation of the simulation model. The different assignment scheduling rules will be introduced in section 5 'Assignment Scheduling Rules '. This section contains the results and analysis of the different ASR's. It also contains a subsection concerning the analysis of the unit costs, as described above, and a subsection regarding to the sensitivity analysis of the ASR's. Followed by section 6 'Variable Interval Concept' which is dedicated to the variable interval concept proposed by Ho and Lau [6]. See section 2 and 6 for a more detailed explanation concerning this phenomenon. Finally, we will end this thesis with the conclusions; an answer for this research in section 7 'Conclusions'.

2. Literature Review

In this section we give a brief literature review in Appointment Scheduling Rules (ASR). Soriano [1] mentioned three main classes of rules which focus on discrete time:

- Individual-block rule; every patient has a unique appointment time, the inter-arrival time between two consecutive patients is assumed to be equally spaced over the clinic session.
- Multiple-block rule; instead that every patient has its own unique appointment time, a group of m patients is scheduled at each possible arrival time. The inter-arrival time is also assumed to be equally spaced.
- Mixed Multiple-Individual block rule; the first possible arrival time has one group of n patients; other patients scheduled in the rest of session follow the individual-block rule. This is a combination of the first and second rule mentioned above. The Bailey-Welch rule is a well known ASR of this kind.

When a patient arrives at a hospital he or she will eventually be helped by the practitioner, the total amount of time to serve this patient is denoted by the service time. In the literature the service time follows a random distribution. Kaandorp and Koole [2] and Jansson [3] assumed that the service time is described by the exponential distribution. Other authors used other distributions to describe the service time e.g. Bosch and Dietz [4] assumed the service time to have an Erlang distribution and Vera Kusters [5] assumed it has a Weibull distribution. Hutzschenreuter [8] and Soriano [1] both used the Gamma distribution to portray the service time. Note that the distributions mentioned above all belong to the same exponential family. Furthermore, the so-called coefficient of variation cv is frequently used to measure the variability of the service time. Ho and Lau [6] defined the coefficient of variation as the result of dividing the standard deviation by the mean ($cv = \sigma/\mu$). In their wide ranging simulation study they were able to vary the distribution's cv, skewness and kurtosis independently and systematically and concluded that the performance of the ASR only is affected by the cv and not by the kurtosis nor the skewness. A service time with a relative high cv means that the uncertainty in the system is relative high. Hence the patients' waiting time, practitioner's idle and overtime will worsen with a higher cv. Ho and Lau also mentioned that the distribution of the service time is often not identical for all patients. Then the usage of simulation models is the only way to determine the suitable arrival times.

As mentioned above the ASR allocates every patient to an appointment time, whether the appointment time is unique or not depends on the nature of the ASR. In the literature many authors assumed that a fraction ρ of the scheduled patients is a 'no-show' patient. This means that the patient could not show up at the clinic or hospital due to unforeseen events. Hutzschenreuter [8] and Kusters [5] assumed this fraction to be 10 percent. Fetter Thompson [7] increased the fraction to 20 percent. Kaandorp Koole [2] experimented with the fraction ρ by considering two levels of no-show (0% and 5%). To consider different values for ρ seems logical in the reality. Since appointments with a common general practitioner are less important than appointments with a specialist. Because we expect the waiting list of specialists to be longer than that of the general practitioner. Thus patients do not want to fail to notice an 'important' appointment. Therefore we assume that the fraction ρ could take different values. Ho and Lau [6] showed that 'no-show' patients have the highest effect on the performance of an ASR, which means that the doctor's idle time and overtime and the patient's waiting time will increase, this is also shown in Bosch Dietz [4].

Punctuality of patients is also important for this study. Bailey Welch [10] discussed this matter in more detail. The overall patient will arrive early, but they should not arrive too early, this will result in overcrowded waiting rooms. Kusters [5] assumed that patients arrive precisely on time in contrast with Fetter Thompson [7]. White Pike [9] dedicated their study on the punctuality phenomenon. Not only patients but also the practitioner's punctuality is discussed in their study. They concluded that the waiting time of the patients do not differ very much when patients are punctual or unpunctual. This thesis will therefore assume that patients arrive punctual.

Another observable fact is that patients could walk-in without an appointment; this is often referred as emergency patients. In the literature an emergency patient has a higher priority than the other patients, because they need immediate attention. Swartz [11] showed that the unscheduled emergency patients can be represented by a Poisson process. The arrival rates do not differ from day to day, but are not the same within the same day. Fetter and Thompson [7] also included emergency patients in their study. However, this research does not take this feature into account.

Most of the studies were performed with simulations. Due to the flexibility of simulation models a researcher could easily change the experimental environment. Furthermore, the amount of patients needed to be scheduled in a session could vary. Bosch Dietz [4] experimented with only four or six patients needed to be scheduled, while Bailey [10] experimented with 10, 15, 20, 25 patients and Kaandorp Koole [2] did the same with 8, 10, 16, 20 patients. It turned out that the number of patients has an effect on the performance of an ASR. The more patients to be allocated the higher the expected waiting time, idle time and overtime.

Ho and Lau [6] also proposed the so-called ”Variable-Interval” concept. This concept is based on the fact that patients scheduled at the earlier part of the session tend to have a shorter waiting time than the patients scheduled in the latter part of the session. With the variable Interval concept they try to correct this “unfair” phenomenon. Ho and Lau included 50 different ASR's in their experiment and with use of simulations they tested every rule and concluded that no single rule was better than the others. The “goodness” of a rule depends on the environment. Despite the large variety of rules, the simple Bailey-Welch rule performed surprisingly well.

Hutzschenreuter [8] looked at the differences between the service times of patients in a detailed manner. Hutzschenreuter performed several interviews at health care institutions to create a better view of the reality. She found out that in practice the practitioners (doctors) need time to start a session; this is called the start-up period. In the simulation models different values are used for the start-up period and the other parameters. Hutzschenheuter used five different rules, and each of them included patients with short or long service times. The article concludes that scheduling patients with short expected service time in the beginning of the session is the best scheduling rule if patients could be characterized according to their service times. She also mentioned that certain rules are better for the practitioner's perspective and vice versa for the patient's perspective. For a detailed description of these rules see [8]. Klassen Rohleder [12] came with a similar conclusion, patients with a higher expected service times should be allocated towards the end of the session.

3. Problem Definition

This thesis uses the same notations as described in Ho and Lau (1992). Every patient i arrives punctually at the given schedule time S_i or is a 'no-show' patient. When patient i is not a 'no-show' patient then the corresponding schedule time is added to the vector A. Thus vector A only consists of schedule times of patients arriving punctually and excludes the 'no-show' patients. The fraction ρ denotes the probability that a random patient is a 'no-show' patient. Furthermore, the total amount of patients needed to be scheduled for a working day is referred as a session, denoted by N. As stated in the first section 'Introduction' this study seeks an ASR for two different groups of patients with different expected service times. With N_1 the amount of patients needed to be scheduled from the first group. This group has a smaller service time when compared to the second group. Similarly N_2 represents the patients needed to be scheduled from the second group. And the length of a session is denoted by T in minutes. Furthermore, t_i describes the real length of the service time of patient i. This research assumes that the service time follows a Gamma distribution with shape parameter α_1 and scale parameter γ_1 for the patients with a smaller expected service time. And shape parameters α_2 and scale parameter γ_2 for the second group with a larger expected service time. b_i and e_i represents the start time and end time of the service for patient i respectively. We do not know the service time, thus t_i, b_i and e_i are stochastic. The waiting time for patient i could be determined by

$$W_i = \max(0, b_i - A_i) \tag{1}$$

The idle time of the practitioner just before patient i is defined by

$$I_i = \max(0, A_i - e_{i-1}), \qquad with\ e_i = b_i + t_i \tag{2}$$

Then the total waiting time and idle time could be calculated with

$$W = \sum_{i=1}^{N} W_i \quad \text{and} \quad I = \sum_{i=1}^{N} I_i + \max(0, T - e_{last}) \tag{3}$$

e_{last} is the end time of the last not 'no-show' patient. Note that the total waiting time has an additional $\max(0, T - e_{last})$ within the formula. Because this problem assumes that the practitioner is on duty at the clinic until the session time T even when there are no patients present. On the other hand if the session time T is a smaller amount than the end time of the last not 'no-show' patient then this 'extra' time is denoted as the overtime. Thus the practitioner's overtime is calculated by

$$O = \max(0, e_{Last} - T) \tag{4}$$

As stated before, this study tries to find a rule which minimize the idle time and overtime of the practitioner and the waiting time of the patients. This rule will determine the appointment times S_i for every patient while minimizing the total costs:

$$E(C) = \alpha_W \cdot E(W) + \alpha_I \cdot E(I) + \alpha_O \cdot E(O) \tag{5}$$

Where α_W denotes the unit waiting cost of the patients and α_I denotes the unit idle cost of the practitioner and α_O the unit cost of the overtime.

4. Simulation

This study deals with patients with heterogeneous service times. We will use discrete event simulations to model the reality. One of the big advantages of using a simulation model is that it could compress the 'real' time. In such a way that it could only takes a few minutes to simulate the reality of a month. Furthermore, one could easily change the settings of the model without having too much problems.

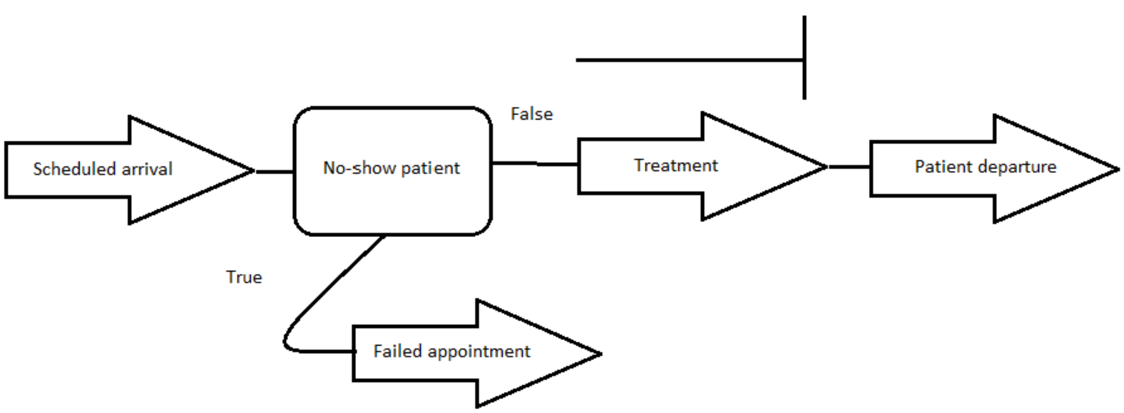
4.1 Simulation model

The simulation models a clinic/hospital with one practitioner. The length of a session is denoted by T (in minutes). Each patient i arrives punctually according to the schedule $S = \{S_1, S_2, \ldots, S_N\}$ unless the patient is a 'no-show' patient with probability ρ. The schedule S is generated with an assignment scheduling rule. When excluding all the 'no-show' patients from the schedule S we will obtain the simplified schedule A. The patients will be served according to the Gamma distribution. Furthermore, we assume that the service times of the patients are independent from each other. With this simulation model the costs of the schedule A could be estimated with the formula described in (5). Thus we have the following parameters:

- $\beta_1 = \alpha_1 * \gamma_1,$ $\text{expected service time (first group)}$
- $\beta_2 = \alpha_2 * \gamma_2,$ $\text{average service time (second group)}$
- N_1 $\quad$ *Number of patients with a short expected service time*
- N_2 $\quad$ *Number of patients with a long expected service time*
- N $\quad$ *Total number of patients,* $N = N_1 + N_2$
- A_i $\quad$ *Appointment time of patient i, given that he shows up*
- ρ $\quad$ *'no − show' probability*
- T $\quad$ *Planned length of the session*
- n $\quad$ *number of simulation replications for the session*
- cv $\quad$ *Coefficient of Variation*

In the next figure one could see a graphical representation of the hospital/clinic:

The research made the following assumption for the simulation model:



- $N_1 = N_2 = 12$

- $N = 24$
- $\rho \in \{0.0, \ 0.1, \ 0.2\}$
- $T = 480 \ minutes$
- $n = 1000$
- $cv \in \{0.2, \ 0.3, \ 0.5, \ 1.0\}$
- $\beta_1 = 10$
- $\beta_2 = 30$

As mentioned above, the service times follow the Gamma distribution. The mean of this distribution could be determined by multiplication between the shape and the scale parameter. Whereas the variation could be calculated by multiplying the shape parameter with the scaled version of the scale parameter. Note that the coefficient of variation and the mean for both types of patients are given. Furthermore, we know that $cv = \sigma/\mu$. Combining this information with the formula's for the mean and variation for the Gamma distribution, we could easily determine the mean and variation for the different cv's.

4.2 Model validation

In order to use the simulation model one should first validate the model. This step of the research is of utmost importance. When a researcher does not know that the model contains errors and continues to work with an incorrect model. He or she has to face the fact that the results could be wrong and completely useless. Therefore, we first have to validate the model described before. In order to do this, one could monitor every event of the simulation model. And check whether the statistical counters are correct or not. In this case one could check the patient's waiting time and practitioner's idle time after every event. Another way to validate the model is to replicate an existing study and compare the results. If the model is correct then the results should be close to each other.

The study of Ho and Lau [6] is slightly different from this study; they did not measure the practitioner's overtime and assumed that *all patients have the same expected service time*. Even so, both problems display a notable similarity. They considered the following environment for their simulations:

- Patients arrive punctual.
- Number of patients needed to be scheduled per session: $N = 10{,}20{,}30$.
- 'No-Show' probability: $\rho = 0.0, 0.1, 0.2$
- Service time distribution:
  - Uniformly distributed with $cv = 0.2$ and $\mu = 1$
  - Uniformly distributed with $cv = 0.5$ and $\mu = 1$
  - Exponentially distributed with $cv = 1.0$

Note that there are $3^3 = 27$ different environments to be tested for every rule. In [6] they considered nine rules which will be explained here in brief:

1. $A_1 = A_2 = 0; \quad for \ i > 2, set \quad A_i = A_{1-1} + \mu$
2. $A_1 = 0, A_2 = 0.2, A_3 = 0.6; \quad for \ i > 3, set \quad A_i = A_{1-1} + \mu$
3. $A_1 = 0, A_2 = 0.3, A_3 = 0.6, A_4 = 0.9; \quad for \ i > 4, set \quad A_i = A_{1-1} + \mu$
4. $A_1 = 0, A_2 = 0.5, A_3 = 1.0, A_4 = 1.5; \quad for \ i > 4, set \quad A_i = A_{1-1} + \mu$

5. $A_1 = A_2 = A_3 = A_4 = 0; \quad for\ i > 4, set \quad A_i = A_{1-1} + \mu$
6. $A_i = (i-1)\mu - k\sigma; \quad with\ k = 0.1$
7. $set\ A_i = (i-1)\mu; \quad for\ i \leq 5, modify \quad A_i = A_i - k_1(5-i)\sigma, \ with\ k_1 = 0.15$
   $for\ i > 5, modify \quad A_i = A_i - k_2(5-i)\sigma, \ with\ k_2 = 0.3$
8. $set\ A_i = (i-1)\mu; \quad for\ i \leq 5, modify \quad A_i = A_i - k_1(5-i)\sigma, \ with\ k_1 = 0.25$
   $for\ i > 5, modify \quad A_i = A_i - k_2(5-i)\sigma, \ with\ k_2 = 0.5$
9. $A_i = A_{i+1} = (i-1)\mu; \quad i = 1,3,5, \ldots$

The A_i for every rule represents the schedule time of patient i.
For the simplicity this section only presents the case where $cv = 0.5$, $N = 20$ and $\rho = 0.0$. The results of the expected patient's waiting time $E(W)$ and practitioner's idle time $E(i)$ for the rules 1 to 9 from Ho and Lau are given in Table 1. As well as the results from the replication, they are shown between brackets.

Table 1

Rule	1	2	3	4	5	6	7	8	9
E(W)	26.3 (26.4)	28.4 (29,1)	40.2 (40,6)	31.9 (31.7)	54.85 (54.84)	21.4 (20.9)	9.9 (9.45)	6.7 (5.42)	25.85 (25.49)
E(I)	0.76 (0.78)	0.65 (0,63)	0.31 (0,31)	0.53 (0,56)	0.12 (0.12)	1.08 (1.11)	2.81 (2.79)	4.03 (4.03)	1.33 (1.38)

For the first rule the $\{E(W), E(I)\} = \{26.3, 0.76\}$ and $\{26.4, 0.63\}$ for Ho and Lau and the replication respectively. These values can be interpreted as coordinates and are plotted in figure below. The corresponding values for the other rules are similarly plotted in the same figure. The rules 1 to 8 form an efficient frontier. Note that both lines are almost identical. From Table 1 and Figure 1 one could see that the differences between the results are relative small. And hereby one could say that the simulation model is valid.

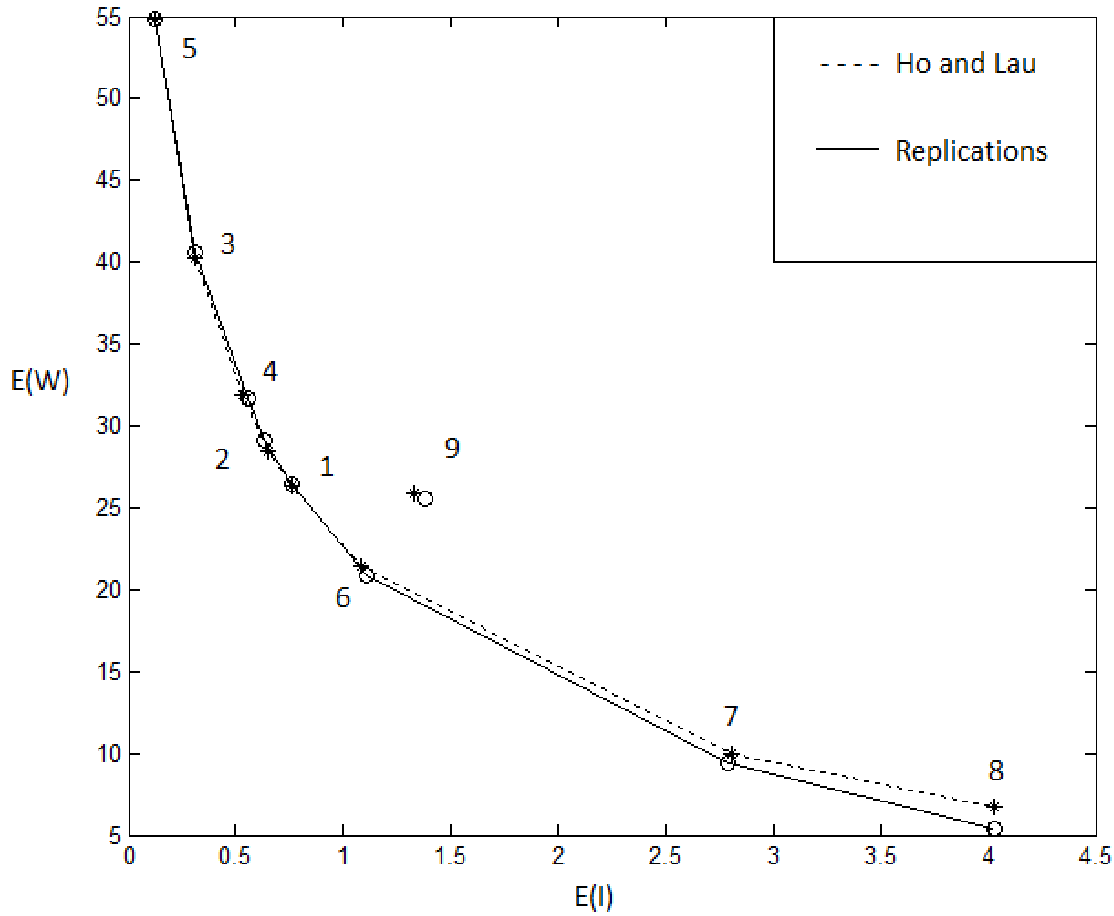


Figure :The Efficient Frontier for the Rules

5. Assignment Scheduling Rules

As mentioned before there are three main classes of Assignment Scheduling Rules. An *individual block rule* concentrates on unique appointment times for every patient. A *multiple block rule* allocates a group of patients to the same appointment time. And a *mixed multiple-individual block rule* is a combination of the previous two rules; first a group of n patients will be scheduled at the start of the session and the remaining patients will follow an individual block rule. This section will first introduce four rules from the individual block rule class, alongside with the analysis for the different $\rho's$ and cv's. Followed by six rules from the multiple block rule class. And subsequently three rules from the third class will be analyzed afterwards. Thus thirteen different rules will be introduced and analyzed. Furthermore, alongside the explanation of every rule, a visual representation will also be presented together with the explanation. All the visual representations from the thirteen different rules can also be found in Appendix A. Hereafter, a subsection will concentrate on the effects of different unit costs for α_W, α_I and α_O on the total costs, given in formula (5). Finally, the last subsection will dedicate itself to the so-called sensitivity analysis. Here we will investigate the effect of the changing ρ's and cv's on the performance measurements W, I and O.

As described above, Hutzschenreuter [8] and Klassen Rohleder [12] both investigated the assignment scheduling rules for two types of patients. Hutzschenreuter experimented with five different rules whereas Klassen Rohleder experimented with ten different rules. The ASR's discussed in this thesis are based on the assignment scheduling rules of the two previous mentioned articles. In such a way that $R1.1$, $R1.2$, $R1.3$, $R2.2$, $R2.5$ and $R3.3$ are also mentioned in the two previous mentioned articles. Whereas the other seven rules discussed in this thesis are modified versions. At this point, the various $RX.X$ rules are still unclear, but they will be introduced systematically in this section.

5.1 Individual Block Rules

Consider the following two rules:

$(R1.1)$ $\quad$ $Schedule\ first\ all\ the\ type\ 1\ patients.$

[1 1 1 1 1 1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 2 2 2 2 2]

$(R1.2)$ $\quad$ $Schedule\ first\ all\ type\ 2\ the\ patients.$

[2 2 2 2 2 2 2 2 2 2 2 2 2 1 1 1 1 1 1 1 1 1 1 1 1 1]

$R1.1$ allocates the patients with a shorter expected service time at the start of the session and those with a larger expected service time at the latter part of the session. $R1.2$ is similar to $R1.1$ but starts with patients with a larger expected service time. The results from the experiments with these ASR's for different environments are tabulated in Table 2. The left panel contains the expected total patient's waiting time, expected total practitioner's idle time and the expected practitioner's overtime for $R1.1$ and the right panel contains the results for $R1.2$. When looking at the ρ-values separately and concentrating on the fluctuation of the cv-values. It is apparent that a higher uncertainty in the service time results in worse performances for both ASR's. Thus the waiting time and both idle and overtime deteriorates when cv increases for every ρ. This worsening created by the fluctuation of the cv-values is the largest for $\rho = 0.0$. Since we know for certain that every patient will attend their clinic appointment, so that the impact of the cv will be the highest for $\rho = 0.0$. The results from $R1.1$ are plotted in Figure 1. For the experimented environmental settings the 'worsening' effect seems to increase in a constant manner. Whether this is correct or false, one should expand the range of the parameters ρ and cv and investigate this effect in more detail. Furthermore, similar graphs can be found for $R1.2$.

Table 2: R1.1 & R1.2

R 1.1				Wait	Idle	Over
ρ	0.0	cv	0.2	146.4762	14.6549	14.6338
	0.0		0.3	219.1980	21.9639	22.1512
	0.0		0.5	362.7365	36.4557	36.4826
	0.0		1.0	702.4050	70.9785	71.2651
	0.1		0.2	88.1702	56.7916	8.6043
	0.1		0.3	134.8829	64.4908	13.4291
	0.1		0.5	236.1516	71.8124	24.2627
	0.1		1.0	500.9489	101.4390	53.9489
	0.2		0.2	55.4780	101.3945	5.5902
	0.2		0.3	85.5371	104.3289	8.5645
	0.2		0.5	152.6894	112.2151	16.2323
	0.2		1.0	348.7784	136.2961	40.5505

R 1.2				Wait	Idle	Over
ρ	0.0	cv	0.2	258.7453	14.8094	14.6357
	0.0		0.3	390.1909	22.1757	22.1883
	0.0		0.5	648.5297	36.7817	36.9217
	0.0		1.0	1265.6663	72.7547	72.4598
	0.1		0.2	135.4545	54.1265	5.9112
	0.1		0.3	216.7880	58.5558	10.4254
	0.1		0.5	396.2978	69.1501	21.1609
	0.1		1.0	882.8180	100.1495	51.5211
	0.2		0.2	78.7342	98.5690	2.6809
	0.2		0.3	123.2900	100.8177	4.9310
	0.2		0.5	236.1880	107.6312	11.5201
	0.2		1.0	594.9177	131.6763	35.7132

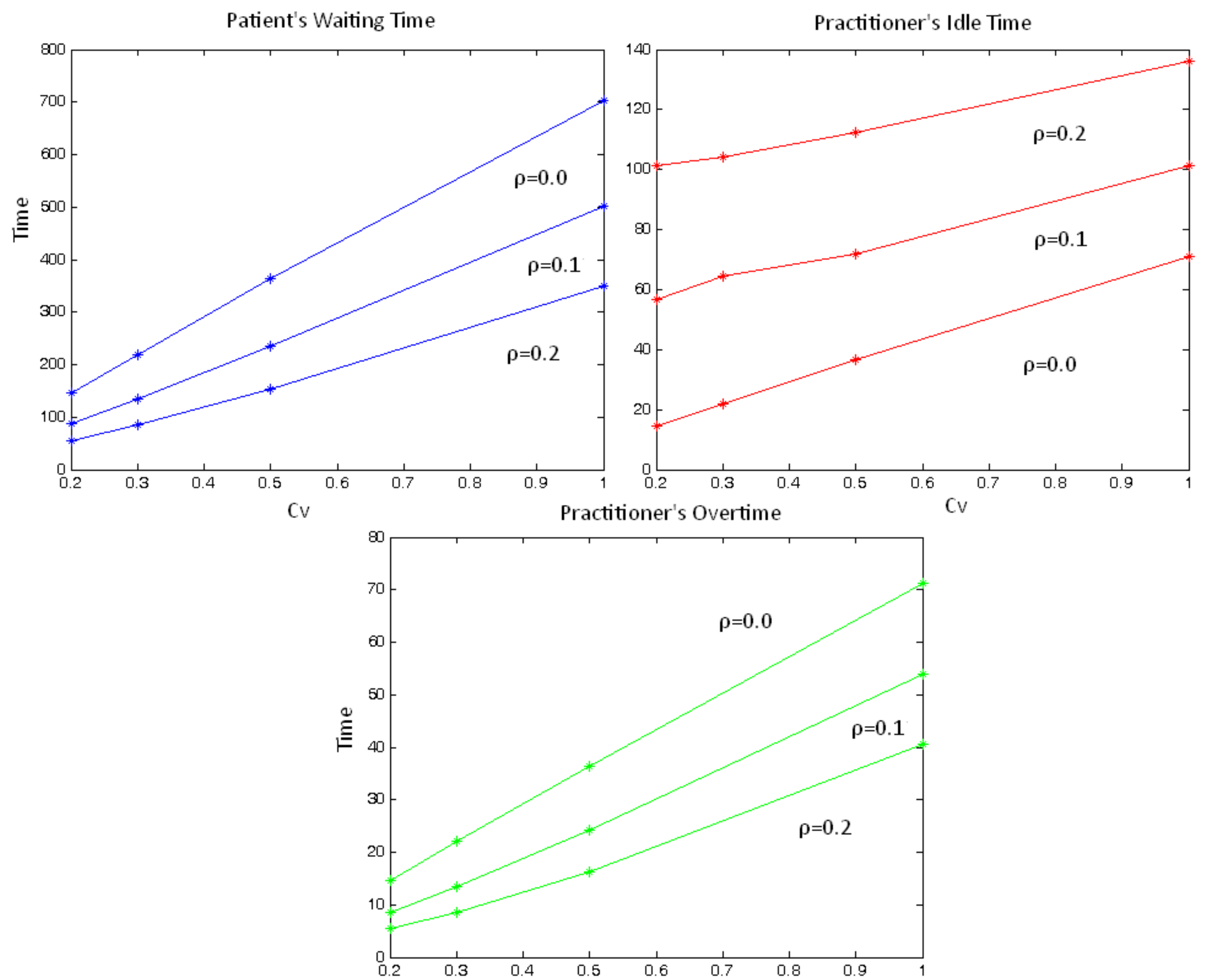


Figure 1: Trend cv's R1.1

The previous case looked at the ρ-values separately. The same analysis can be performed for the cv-values; look at these values separately and concentrate on the changes of the ρ-values. If the 'no-show' probability increases then the expected total waiting time of the patients decreases. This seems logical, because the ρ-value reflects the likelihood that a patient does not show up. In this case the practitioner will have more time for the other patients; which mean that the expected waiting time of a patient will decrease. The same logic could be applied to the fact that the practitioner's overtime decreases when ρ increases. However, this does not hold for the practitioner's idle time. The expected time that the practitioner has none patients at the clinic will increase when ρ increases. Since a higher probability for a 'no-show' will result in less expected patients for the practitioner to serve.

Table 2 also reveals that $R1.1$ has smaller values than $R1.2$ for the waiting time for all cases. The differences are most visible for $cv = 1.0$ and has a maximum difference of $1265.67 - 702.41 = 563.26$ for the situation where $\rho = 0.0$. While the minimum difference occurs at the environment setting $\rho = 0.2, cv = 0.2$ and obtains the value $78.73 - 55.48 = 23.26$. Regarding the practitioner's idle and overtime the maximum differences can be found for $\rho = 0.2$ and $cv = 1.0$. These differences are 4.62 and 4.84 for the idle and overtime respectively. Whereas the minimum differences occurs at $\rho = 0.0$ and $cv = 0.2$. Note that the differences for the waiting time are far greater than the differences occurring for the idle and overtime. Therefore the total costs formula (5) seems primarily to be influenced by the patient's waiting time. A more detailed analysis concerning this topic will be discussed in the subsection 'Unit Costs' of this section.

At this point, we merely analyzed two simple ASR's. Moreover, we assumed that the patients could be categorized into two parts of the session. In such a way that all patients with the same expected service time are allocated to the same part of the session. For example $R1.1$ allocates all patients with a short expected service time to the first part of the session and the patients with a long expected service time to the latter part of the session. One could question this assumption so that all patients need to be able to get an appointment time through the whole session. Therefore, consider the following two individual block rules; rules where patients with short and long service times alternate throughout the session. As a result, consider the following rules:

$(R1.3)$ $\quad Start\ with\ a\ type\ 1\ patient\ and\ alternate\ the\ remaining\ patients\ with\ short\ and\ long\ service\ times.$
[1 2 1 2 1 2 1 2 1 2 1 2 1 2 1 2 1 2 1 2 1 2 1 2]

$(R1.4)$ $\quad Start\ with\ three\ type\ 1\ patients\ and\ continue\ with\ three\ type\ 2\ patients.$
$Maintain\ this\ sequence\ until\ all\ N\ patients\ are\ scheduled.$
[1 1 1 2 2 2 1 1 1 2 2 2 1 1 1 2 2 2 1 1 1 2 2 2]

$R1.3$ initially schedules a short-service-time type patient, then it alternates to the long-service-time type patient. On the other hand, $R1.4$ schedules three patients of the same kind before alternating to three patients of the other type and so forth, so that both rules are similar to each other. The results are presented in Table 3.

Table 3: R1.3 & R1.4

R 1.3 ρ		cv	Wait	Idle	Over	**R 1.4** ρ		cv	Wait	Idle	Over
ρ	0.0	0.2	205.2758	14.7175	14.6033	ρ	0.0	0.2	195.3507	14.5920	14.5541
	0.0	0.3	306.5537	22.2201	21.8161		0.0	0.3	291.9529	22.1386	21.8822
	0.0	0.5	510.3989	36.4841	36.5253		0.0	0.5	482.9102	36.6328	36.3891
	0.0	1.0	991.9516	71.6267	71.2476		0.0	1.0	962.2554	70.2214	71.8217
	0.1	0.2	117.8615	55.3033	7.5411		0.1	0.2	110.4814	55.3072	7.8479
	0.1	0.3	179.6413	60.4766	12.3046		0.1	0.3	173.0170	60.3776	12.4261
	0.1	0.5	324.5920	70.8401	23.1454		0.1	0.5	310.7738	71.2708	23.0878
	0.1	1.0	695.2631	100.8206	52.3607		0.1	1.0	659.9963	101.3386	52.5481
	0.2	0.2	68.3944	100.7607	4.5737		0.2	0.2	64.8024	101.5156	4.7878
	0.2	0.3	109.1084	103.4591	7.2115		0.2	0.3	105.7910	104.1022	8.0073
	0.2	0.5	211.3223	110.4219	15.0081		0.2	0.5	193.7773	111.7169	14.8762
	0.2	1.0	486.6072	133.3849	39.2792		0.2	1.0	461.8614	134.4673	38.4624

This table reveals that $R1.3$ and $R1.4$ have the same properties as $R1.1$ and $R1.2$. In such a way that the performance measurements worsens in an upward trend if the coefficient of variation increases. Furthermore, it is apparent that the patient's waiting time and practitioner's overtime improve while the idle time deteriorates as ρ takes a higher value. When comparing the results from the third rule with the fourth rule. It is clear that $R1.3$ has bigger values than $R1.4$ for the expected waiting time with a maximum difference of 35.27 for $\rho = 0.1$ and $cv = 1.0$ and a minimum difference of 3.59 for $\rho = 0.2$ and $cv = 0.2$. Regarding to the idle and overtime the differences are rather insignificant compared to the differences of the waiting time. This is due to the fact that $R1.3$ and $R1.4$ are similar. In fact, the differences in the idle and overtime between the four previous rules are rather small compared to the differences in the waiting time. Hence, concentrating on the patient's waiting time and one ought to conclude that $R1.1$ has the smallest values for the waiting time followed by $R1.4$, $R1.3$ and $R1.2$. Use this rank from best to worst and subtract the results from the best rule ($R1.1$) from the second best rule ($R1.4$). In a similar way subtract the results from the second best rule from the third best rule ($R1.3$) and also do this for the worst rule. These results are presented in Table 4. Every column contains the differences of two consecutive ranked rules. For example, the first column contains the differences between $R1.1$ and R1.4 focused on the patient's waiting time. It is clear that the performances of the ranked ASR's improve for the waiting time, but it tends to deteriorate for the idle and overtime.

Table 4: Difference

		Wait			Idle			Over		
ρ	cv	4-1	3-4	2-3	4-1	3-4	2-3	4-1	3-4	2-3
0.0	0.2	48.87	9.93	53.47	-0.06	0.13	0.09	-0.08	0.05	0.03
0.0	0.3	72.75	14.60	83.64	0.17	0.08	-0.04	-0.27	-0.07	0.37
0.0	0.5	120.17	27.49	138.13	0.18	-0.15	0.30	-0.09	0.14	0.40
0.0	1.0	259.85	29.70	273.71	-0.76	1.41	1.13	0.56	-0.57	1.21
0.1	0.2	22.31	7.38	17.59	-1.48	0.00	-1.18	-0.76	-0.31	-1.63
0.1	0.3	38.13	6.62	37.15	-4.11	0.10	-1.92	-1.00	-0.12	-1.88
0.1	0.5	74.62	13.82	71.71	-0.54	-0.43	-1.69	-1.17	0.06	-1.98
0.1	1.0	159.05	35.27	187.55	-0.10	-0.52	-0.67	-1.40	-0.19	-0.84
0.2	0.2	9.32	3.59	10.34	0.12	-0.75	-2.19	-0.80	-0.21	-1.89
0.2	0.3	20.25	3.32	14.18	-0.23	-0.64	-2.64	-0.56	-0.80	-2.28
0.2	0.5	41.09	17.55	24.87	-0.50	-1.30	-2.79	-1.36	0.13	-3.49
0.2	1.0	113.08	24.75	108.31	-1.83	-1.08	-1.71	-2.09	0.82	-3.57

For this reason, we experimented with other variants of individual block rules in order to check this property and observed that all rules have approximately the same values for the practitioner's idle and overtime for the given environmental settings. While the patient's waiting time differs from rule to rule. Noteworthy to point out is the more an ASR looks like $R1.1$ the lower the expected waiting time will be. On the contrary, the more an ASR looks like $R1.2$ the higher the results tend to be. This feature can as well be found in the four assignment scheduling rules described before. Thus that $R1.4$ is the closest to $R1.1$, and indeed the results from $R1.4$ are the closest to $R1.1$.

Hence, if a practitioner or a clinic uses an individual ASR and has the primal goal to minimize the patient's waiting time, then he should consider $R1.1$ as his assignment scheduling rule, due to the fact that this rule has the lowest values for the patient's waiting time. And if the practitioner has the goal to minimize his own idle time or overtime, then he should allocate the patients according to $R1.2$.

5.2 Multiple block rule

A multiple block rule has the objective to allocate multiple patients at the same schedule time. Consider the following rules:

$(R2.1)$ $\quad Schedule\ first\ all\ the\ type\ 1\ patients\ with\ two\ patients\ at\ each\ possible\ appointment\ time.$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 & 2 & 2 \end{bmatrix}$$

$(R2.2)$ $\quad Schedule\ one\ type\ 1\ patient\ and\ one\ type\ 2\ patient\ at\ each\ possible\ appointment\ time.$ $Starting\ with\ a\ type\ 1\ patient.$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \end{bmatrix}$$

$(R2.3)$ $\quad Schedule\ one\ type\ 1\ patient\ and\ one\ type\ 2\ patient\ at\ each\ possible\ appointment\ time.$ $Starting\ with\ a\ type\ 2\ patient.$

$$\begin{bmatrix} 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$(R2.4)$ $\quad First\ schedule\ two\ type\ 1\ patients, alternate\ to\ scheduling\ two\ type\ 2\ patients\ at\ an\ appointment\ time. Maitain\ this\ chain\ until\ all\ N\ patients\ are\ scheduled.$

$$\begin{bmatrix} 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 \\ 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 \end{bmatrix}$$

$(R2.5)$ $\quad First\ schedule\ three\ type\ 1\ patients, alternate\ to\ scheduling\ three\ type\ 2\ patients\ at\ an\ appointment\ time. Maitain\ this\ chain\ until\ all\ N\ patients\ are\ scheduled.$

$$\begin{bmatrix} 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 \\ 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 \\ 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 \end{bmatrix}$$

$(R2.6)$ $\quad Schedule\ all\ N\ patients\ at\ the\ start\ of\ the\ session.$

Note that $R2.1$ is exactly the same as $R1.1$ with the only difference that it schedules two patients at an appointment time instead of one patient. Furthermore, $R2.2$ and $R2.3$ are basically identical, but one rule needs the practitioner to provide service to the type 1 patient before the type 2 patient while the other rule needs the practitioner to serve the type 2 patient first. $R2.4$ and $R2.5$ are variants from each other, in such a way that $R2.4$ schedules two patients at the same appointment time while $R2.5$ schedules three patients. $R2.6$ is probably the first ASR that existed, it schedules every patient at the start of the session. The results from these multiple block rules can be found in Appendix B.

The analysis procedures for the multiple block rules are the same as for the individual block rules. The worsening trend occurring between the performance measurements and the cv-values also holds for the multiple block rule. And again, a higher value for ρ results in improvements for the patient's waiting time and deteriorations for the practitioner's idle and overtime. The results from $R2.1$ differs considerable from $R2.2$. But is not clear which one is better, because some environments have better performances for $R2.1$ and other environments prefer $R2.2$. It appears that a higher variation in the service time tends to have better performances for the second rule. As stated above, the rules $R2.2$ and $R2.3$ are similar. But differences in the patient's waiting time is considerable large in favor of $R2.2$. For this reason, we created new ASR's from the rules described above and changed all type 1 patients into type 2 patients and vice versa all type 2 patients are changed into type 1 patients. The results are remarkable fascinating and similar to the comparison between $R2.2$ and $R2.3$. It states that when a multiple block rule starts with a type 2 patient instead of a type 1 patient, then the expected waiting time will be larger. And the expected idle and overtime have a tendency to decline. This is actually in line with the results found before where we stated that $R1.1$ has larger values for the expected waiting time than $R1.2$. The previous two rules allocate one patient from both types to the same appointment time. One could also investigate the effect of two patients from the same type on the same appointment time with $R2.4$. This rule results in higher expected waiting time but tends to have smaller values for the idle and overtime. Even a step further is to examine the influences of a multiple block of three patients. Appendix B shows that $R2.5$ has even higher values for the waiting time and smaller values for the idle and overtime. In the worst case scenario, the ASR schedules all patients in one block. It is realistic that the results from $R2.6$ show extraordinarily big values for the waiting time and relative small values for the idle and overtime.

As stated before, $R2.1$ is similar to $R1.1$ with the only difference that this rule allocates multiple patients to the same appointment time. In the same way, $R2.2$ and $R2.5$ are similar to $R1.3$ and $R1.4$ respectively. Table 5 shows the differences between $R2.1$ and $R1.1$, $R2.2$ and $R1.3$ and the differences between $R2.5$ and $R1.4$. In this fashion one could investigate the effect of a multiple block rule compared to an individual block rule. Since the 'Wait' columns only contain positive numbers one could say that a multiple block rule has a negative effect on the patient's waiting time. And in view of the fact that the 'Idle' and 'Over' columns only contain negative values, it is clear that a multiple block rule has a positive effect on the practitioner's idle and overtime. These outcomes are logical because a multiple block rule allocates multiple patients to the same appointment time. This means that some patients have to wait with certainty, which will result in larger expected waiting time. And consequently reducing the practitioner's expected idle and overtime.

Table 5: Differences between Individual and Multiple block rules

		R2.1-R1.1			**R2.2-R1.3**			**R2.5-R1.4**		
ρ	cv	Wait	Idle	Over	Wait	Idle	Over	Wait	Idle	Over
0,0	0,2	222,16	-1,18	-0,97	106,0413	-0,5362	-0,4297	438,9252	-1,6088	-1,2769
0,0	0,3	210,98	-1,60	-1,46	103,1484	-1,2993	-0,5188	419,2573	-2,7097	-2,5063
0,0	0,5	194,04	-2,61	-2,19	84,9021	-1,5113	-1,7493	366,8773	-3,8129	-4,8724
0,0	1,0	155,28	-4,17	-4,65	49,4979	-2,2661	-2,4817	286,2131	-7,1598	-6,7015
0,1	0,2	176,85	-1,31	-1,39	80,7681	-0,1487	-0,8049	355,3615	-1,5364	-1,8751
0,1	0,3	169,24	-5,43	-1,91	83,3462	-2,2525	-0,9403	334,0676	-3,1254	-2,7001
0,1	0,5	152,62	-2,73	-2,74	64,5598	-1,4922	-1,8990	300,1556	-5,0182	-4,6883
0,1	1,0	126,39	-4,93	-4,85	58,7477	-3,8111	-1,8643	244,7253	-9,3091	-6,7540
0,2	0,2	139,29	-1,03	-1,30	64,7001	-0,4542	-0,9320	277,9033	-1,2464	-1,7874
0,2	0,3	132,76	-2,12	-1,50	60,1668	-1,7784	-0,7670	264,2087	-3,5079	-2,8991
0,2	0,5	117,42	-3,16	-2,65	48,4049	-2,1684	-1,9180	241,8753	-6,4603	-4,4214
0,2	1,0	90,56	-2,66	-5,40	29,2302	-1,7342	-3,8360	160,6870	-5,8661	-7,9405

Thus if a practitioner uses a multiple block rule, and has the main goal to minimize the patient's waiting time. Then he should select the number of patients to be allocated for every appointment time to be as low as possible. For example, $R2.1$ allocates two different patients to every appointment time, whereas $R2.5$ allocates three different patients to every appointment time. Then he should select $R2.1$ over $R2.5$. And he should first schedule all the patients with a smaller expected service time before the patients with a larger expected service time. This rule is essentially summarized in $R2.1$. On the other hand, if the practitioner prefers to minimize his own idle or overtime. Then he should select the number of patients per block as high as possible and he should first schedule all the patients with a larger expected service time. However, if the practitioner is not restricted to multiple block rules, and is allowed to use individual block rules. Then the 'individual version' of an ASR generates smaller values for the patient's waiting time but higher values for the practitioner's idle and overtime.

5.3 Mixed multiple-individual block rule

A well-known mixed multiple-individual block rule is the Bailey-Welch rule. It schedules two patients at the start of a session on the same appointment time and continues allocating patients according the individual block rule. Although the Bailey-Welch rule assumed that there is only one type of patient, it is still possible to apply this rule to this research for two types of patients. Consider the following rules:

$(R3.1)$ $\quad Schedule\ first\ all\ the\ type\ 1\ patients.\ With\ the\ same\ appointment\ time\ for\ the\ first\ and\ second\ patient\ of\ the\ session.$

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 2\ 2\ 2\ 2\ 2\ 2\ 2\ 2\ 2\ 2\ 2\ 2\ \Big]$$

$(R3.2)$ $\quad Schedule\ first\ all\ the\ type\ 2\ patients.\ With\ the\ same\ appointment\ time\ for\ the\ first\ and\ second\ patient\ of\ the\ session.$

$$\begin{bmatrix} 2 \\ 2 \end{bmatrix} 2\ 2\ 2\ 2\ 2\ 2\ 2\ 2\ 2\ 2\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ \Big]$$

$(R3.3)$ $\quad Alternate\ the\ patients\ throughout\ the\ whole\ session, starting\ with\ a\ type\ 1\ patient.\ The\ first\ and\ second\ patient\ have\ an\ appointment\ time\ of\ '0'.$

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} 1\ 2\ 1\ 2\ 1\ 2\ 1\ 2\ 1\ 2\ 1\ 2\ 1\ 2\ 1\ 2\ 1\ 2\ 1\ 2\ 1\ 2\ \Big]$$

These rules are actually the Bailey-Welch rule applied on $R1.1$, $R1.2$ and $R1.3$ respectively. The corresponding results can be found in Appendix C.

It is clear that $R3.1$ has the smallest results for the patient's waiting time among the three mixed multiple-individual rules, while $R3.2$ has the biggest values. In a similar way one could see that $R3.2$ has the smallest values for the practitioner's idle and overtime and vice versa $R3.1$ has the largest values. $R3.3$ lies between the other two rules in terms of performances. The rules description above already pointed out that the $R3.X$ rules are variants from $R1.1$ through $R1.3$. In a similar way one could say that $R3.1$ is comparable with $R2.1$ and $R3.3$ comparable with $R2.2$. With this we can compare the results from similar rules from the three different classes. The differences between the performance measurements for these rules can be found in Appendix D. Firstly, concentrate on the differences between the individual block rule and the mixed multiple-individual block rule, see upper component of Appendix D. It is clear from the 'black' and positive numbers from the 'Wait' columns that the mixed multiple-individual block rules perform worse than the individual block rule for the waiting time. And the 'red' and negative numbers from the 'Idle' and 'Over' columns suggest that the mixed rules perform better than the individual rules for the idle and overtime. Now, concentrate on the lower component of Appendix D. Note that all values are negative, which means that the results from the multiple block rules are bigger than the results from the mixed variant. Thus regardless of the performances of the waiting time or the idle/overtime a mixed multiple-individual block rule outperforms its variant from the multiple block rule.

Thus if a practitioner is only allowed to select a rule from the mixed multiple-individual block rule class and given that his main goal is to minimize the patient's waiting time. Then he should use $R3.1$ as his ASR. However, he should use $R3.2$ if he wants to minimize the idle or overtime. And generally speaking, if the practitioner is not restricted to a specific class and is able to choose his ASR from any class. Then he should choose $R1.1$ in order to minimize the patient's waiting time. And he should select $R3.2$ in order to minimize his own idle or overtime. Note that the multiple block rule class is inferior to the individual and the mixed classes.

5.4 Unit Costs

Thus far only the expected patients' waiting time and the practitioner's idle and overtime has been presented. However, it is very likely that the different performance measurements have different weights. For example, a practitioner prefers to minimize his own idle time because 'time is money'. Hence, he shall increase the weight (or unit cost) for the idle time. The same reasoning can be applied to the waiting time and the overtime. Consequently different unit costs will result into different total costs. This reasoning has already been presented in section 3 ´Problem definition´ with the Total Costs formula (5):

$$E(C) = \alpha_W \cdot E(W) + \alpha_I \cdot E(I) + \alpha_O \cdot E(O) \tag{5}$$

The unit costs parameters are α_W, α_I and α_O for the waiting time, idle time and overtime respectively.

It is in our interest to reduce the total costs for a given set of α's. This can be achieved by choosing the 'right' ASR with the smallest total costs. We will start with holding α_I and α_O constant on the value '1' and analyze the effect of α_W. Use the following table to see the effect of the choices of α_W.

Table 6: Best ASR with Idle & Over = 1,0

		W =					
ρ	cv	5,0	1,0	0,3	0,2	0,1	0,0
0,0	0,2	R1.1	R1.1	R1.1	R1.1	R3.2	R3.2
0,0	0,3	R1.1	R1.1	R1.1	R1.1	R3.2	R3.2
0,0	0,5	R1.1	R1.1	R1.1	R1.1	R3.2	R3.2
0,0	1,0	R1.1	R1.1	R1.1	R3.1	R3.2	R3.2
0,1	0,2	R1.1	R1.1	R1.1	R1.1	R3.2	R3.2
0,1	0,3	R1.1	R1.1	R1.1	R3.1	R3.2	R3.2
0,1	0,5	R1.1	R1.1	R1.1	R3.1	R3.2	R3.2
0,1	1,0	R1.1	R1.1	R3.1	R3.1	R3.2	R3.2
0,2	0,2	R1.1	R1.1	R1.1	R3.1	R3.2	R3.2
0,2	0,3	R1.1	R1.1	R1.1	R3.1	R3.2	R3.2
0,2	0,5	R1.1	R1.1	R3.1	R3.1	R3.2	R3.2
0,2	1,0	R1.1	R1.1	R3.1	R3.1	R3.2	R3.2

If all performance measurements have the same weight ($\alpha_W = \alpha_I = \alpha_O = 1$) then the practitioner or clinic should allocate the patients according $R1.1$, see Table 6 column 2, for the given environmental settings. This is reasonably because the value of the total costs will be mainly caused by the waiting time. Considering $R1.1$has the least overall waiting time. Hence, the preferred assignment scheduling rule will be $R1.1$. As a matter of fact, if α_W is larger than α_I and α_O then the weight on the waiting time will be larger thus the choice for $R1.1$ will again be made. This can be concluded from Table 6 column 1, with an α_W five times bigger than α_I and α_O. However, what choices will be made if α_W is smaller than the other two unit costs. In this research if α_W is 0.3 to 0.2 times smaller than α_I and α_O then it is very likely to choose for the Bailey Welch modified $R1.1$, namely $R3.1$, when ρ or cv increases. See columns three and four from Table 6. When ρ or cv increases, then it is more likely to select $R3.1$ as the ASR. Furthermore, in the case that the unit cost of the waiting time is insignificant to the other two unit costs then the choice for $R3.2$ will be made. Since that this rule has the lowest value for the idle and overtime, see the columns five and six of Table 6. Thus summarizing this, if α_W is larger or equal to the other two unit costs then the preferred assignment scheduling rule will be $R1.1$. And if α_W is significant smaller than the other two unit costs then the choice will be made for $R3.2$. Furthermore, if α_W is three to five times smaller than the other two unit costs then the practitioner should select $R3.1$ as his ASR when ρ of cv is high.

The analysis described above can also be applied to the situation where the unit costs of the waiting time (α_W) and overtime (α_O) are held constant on the value '1'. See Appendix E upper table for the results of the effects of α_I. Yet again, if the unit cost of the idle time is the same as the other two unit costs then the preferred ASR will be $R1.1$, as confirmed above. This also holds for the situation when α_I is smaller than the other two weights. Since that the waiting time still has the most influence on the total costs. As a matter of fact, the unit cost of the idle time may be five times larger than the other two unit costs without changes the choice of the ASR. However, if the increase is more than five times then the preferred ASR will be adjusted to $R3.1$. This means that the effect from the idle time on the total cost is larger than the effect from the waiting time. Thus, the assignment scheduling rule with the lowest expected idle time will be selected, to be precise $R3.1$. But if the practitioner's time is incredible important, then the weight on the idle time will be supreme. If it is approximating 100 times or more, then it is advisable to choose for $R3.2$. This conclusion differs from the situation when α_I is approximately five to fifteen times larger than the other two unit costs. Because in this case, the waiting time and overtime still have a considerable effect on the total costs. Whereas these effects are irrelevant in the case where α_I is 100 times larger.

A further analysis for the situation where the unit costs for the patient's waiting time (α_W) and practitioner's idle time (α_I) are held constant on the value '1' can also be applied. These results can be found in Appendix E lower table. Due to the fact that the results are almost identical to the situation where α_W and α_O are held constant, we may conclude that these analysis have the same properties.

Until now we performed the analysis with the assumption that two unit costs are held constant on the value '1'. However, a practitioner could decide to have different values for the unit costs. If the α's are chosen in such a way that they are relatively close to each other, then $R1.1$ is preferred as the scheduling rule. Again, because the waiting time will the most influential performance measurement for the total costs. In other cases when α_I or α_O are chosen in such a way that the cost of the idle or the overtime are the most influential for the total costs. Then is it advisable to choose $R3.2$. Hence it is logical that if the combination of the costs of the idle and overtime have the biggest influence, then it is wise to choose $R3.2$ for lowering the total costs. Conversely, if α_W is chosen in such a way that the cost for the waiting time is most significant for the total costs, then the practitioner should choose $R1.1$. Finally, the last situation where the α's are chosen in such a way that the costs for the waiting time, idle and overtime approximately the same. This means that the influence from every performance measurement on the total costs is approximately the same, then the costs for the performance measurements will be roughly the same. In this situation it will be wise to choose for $R3.1$. All these results are in line with the conclusions stated in section 5.1, 5.2 and 5.3. Where the practitioner should select $R1.1$ when he aims to minimize the patients waiting time. And selects $R3.2$ when his own idle or overtime is most important.

In the reality, a practitioner is every likely to prefer only one performance measurement to be minimized. This could be the patients waiting time or his own idle and overtime. He should then put more weight on that specific measurement. But unfortunately, there are infinite many possibilities for the choices for α_W, α_I and α_O. As a guidance, he could use the previous mentioned guidelines for selecting his ASR.

5.5 Sensitivity Analysis

The previous subsections looked at different assignment scheduling rules and different weights for the performance measurements for the given environmental settings. But how exactly will the different ASR's react on the different environmental settings. Therefore, this subsection will be dedicated to find the robustness of the different classes and rules. This research assumed that ρ takes the following values: 0.0, 0.1 and 0.2 while cv could vary between the following values: 0.2, 0.3, 0.5 and 1.0. Together they formed twelve distinct environments for us to experiment. The next question to ask is how did the performance measurements react to the changes of the environmental settings. For the simplicity, this thesis will only discuss this matter for $R1.1$ in more detail. Followed by a more general sensitivity analysis for the individual block rules, multiple block rules and mixed multiple-individual block rules.

Table 7: R1.1 Percentage Change

ρ	cv	cv	Wait	Idle	Over
0,0	0,2	-	-	-	-
0,0	0,3	0,500	0,496	0,499	0,514
0,0	0,5	0,667	0,655	0,660	0,647
0,0	1,0	1,000	0,936	0,947	0,953
0,1	0,2	-	-	-	-
0,1	0,3	0,500	0,530	0,136	0,561
0,1	0,5	0,667	0,751	0,114	0,807
0,1	1,0	1,000	1,121	0,413	1,224
0,2	0,2	-	-	-	-
0,2	0,3	0,500	0,542	0,029	0,532
0,2	0,5	0,667	0,785	0,076	0,895
0,2	1,0	1,000	1,284	0,215	1,498

But first note that the cv starts with the value 0.2 and the next cv value is 0.3, this is an increase of 50%. In a similar way the cv increases from 0.3 to 0.5, this is an increase of 66%. The last increment has an increase of 100% (from 0.5 to 1.0). Consider the adjacent table. This table contains the percentage changes of $R1.1$, see Table 2 left panel for the actual data of $R1.1$. The cv column has already been discussed. In a comparable way, the values within the 'Wait', 'Idle' and 'Over' columns are the percentage changes. For example, the first value under the 'Wait' column is 0.496. This means that when cv changes from 0.2 to 0.3 the expected waiting time will increase with approximately 49.6%. We will analyze how $R1.1$ reacts to the changes of cv. Note that when $\rho = 0.0$ the percentage changes for all columns are roughly the same. Unfortunately, this property does not hold for the other ρ's. Some of the values are relative small, while other values relative large. For example, consider the setting where $\rho = 0.2$ and $cv = 0.3$. The cv-column returns an increase of 50% but the corresponding 'Idle' value only is as small as 2.9%. This means that the idle time is relatively robust for *this* given environmental setting. So the next step is to compare the percentage changes of expected waiting time, idle and overtime with the changes of cv, see Table 8. This time the values from the 'Wait', 'Idle' and 'Over' columns are given in proportion to the 'cv' column. For example,

Table 8: R1.1 Compare Change

ρ	cv	cv	Wait	Idle	Over
0,0	0,2	-	-	-	-
0,0	0,3	0,500	0,993	0,997	1,027
0,0	0,5	0,667	0,982	0,990	0,970
0,0	1,0	1,000	0,936	0,947	0,953
0,1	0,2	-	-	-	-
0,1	0,3	0,500	1,060	0,271	1,121
0,1	0,5	0,667	1,126	0,170	1,210
0,1	1,0	1,000	1,121	0,413	1,224
0,2	0,2	-	-	-	-
0,2	0,3	0,500	1,084	0,058	1,064
0,2	0,5	0,667	1,178	0,113	1,343
0,2	1,0	1,000	1,284	0,215	1,498

compare the 0.496 value from Table 7 with the corresponding cv value (0.500). So that one ought to conclude that 0.496 is 99.3% of 0.500. This 99.3% is displayed in Table 8 as 0.993. This way one could compare the relative percentage changes. And yet again Table 8 shows that for $\rho = 0.0$ the changes are fairly close to each other. For $\rho = 0.1$ the percentage changes of the waiting time is higher than the percentage changes of cv. In fact, it is around 10% bigger. This means that when the real values of cv varies then the changes for the expected waiting time will change relatively even more. The same could be said about the practitioner's overtime. And because the corresponding values from Table 8 are bigger one could induce that the overtime reacts more intense than the waiting time. On the other hand, the percentage changes of the 'Idle' column are only a 'small' fraction of the cv changes. Thus the practitioner's idle time is quit robust when compared to the other two performance measurements. For the case where $\rho = 0.2$ the same conclusions can be drawn. However, notice that the corresponding values fluctuates more than the values from $\rho = 0.0$ and $\rho = 0.1$. So it seems that the robustness of the performance measurements tend to deteriorate when ρ increases.

The previous analysis concentrated on $R1.1$. The same analysis are performed with $R1.2$ and it appears that this rule has approximately the same properties as $R1.1$, see Appendix F. In such a way that the expected waiting time and overtime are not as robust as the idle time. Furthermore, the waiting time and overtime $R1.2$ are worst in the sense that they respond more to changes of cv, but are still relatively close to the results found in Table 7 for $R1.1$. And surprisingly, the percentage changes of the idle time are approximately the same as the values given in Table 7. Again this suggests that the idle time is the most robust performance measurement for an individual block rule. As a matter of fact, these properties are found for every rule from the individual block rule class. Until now, we only discussed the individual class. Next, we will analyze the multiple class, see Appendix G for the percentage change of $R2.1$ and $R2.2$. It appears that the robustness for the practitioner's idle and overtime is approximately the same for the individual and multiple classes. But the values for the patient's waiting time of the multiple class are more constant than the values from the individual class. Recall from section 5.2 that the individual class version of an ASR has lower values for the waiting time than the multiple class version. However, it seems that the multiple class version is more robust. Finally, see Appendix H for the percentage changes of $R3.1$ and $R3.2$ from the mixed class. As mentioned before, $R3.1$ is A Bailey-Welch modified version of $R1.1$ and $R3.2$ is the Bailey-Welch modified version of $R1.2$. And together with Appendix G we could compare the robustness from different classes. But first note that the percentage changes from idle time are approximately the same for all classes. Which means that the practitioner's idle time is quite robust when compared to the other two performance measurements. Whereas the percentage change for the patient's waiting time for the multiple and mixed classes are approximately the same. However, the percentage changes for the overtime of a mixed class are considerably larger than the other two classes. This means that this class reacts more to the changes of cv. Summarizing these results: The practitioner's idle and overtime from the individual and multiple classes react evenly to the changes of cv, while the patient's waiting time from the multiple class is more robust than the individual class. In a similar way, we could conclude that the waiting time and idle time from the mixed and multiple classes reacts evenly to the changes of cv. While the changes of the overtime for the mixed class is less robust than the multiple class. This means that the multiple block rule class is the most robust to changes of cv. Nevertheless, in subsection 5.4 'Unit Costs' we concluded that the practitioner should select $R1.1$ if the patient's waiting time has the priority to be minimized, and he should select $R3.2$ when he wants to minimize his own idle or overtime. Note that he should not select any rule from the multiple class, although this class has the highest robustness, when cv changes.

The next question is to ask how will the performance measurements react to changes of ρ instead of cv. The sensitivity analysis for this part is a bit different from the analysis for cv. We only concentrate on the percentage change of the actual data. We do not compare this with the percentage changes of the ρ's. Because this research assumes that ρ could be 0.0, 0.1 or 0.2. And due to the fact that it is impossible to calculate a percentage change from zero to 0.1 which will result in only one usable percentage change, namely from 0.1 to 0.2. Fortunately, the 'interval' between two consecutive values of ρ is 0.1. We used this fact to allow us to compare the percentage changes. So consider Table 9 which contains the percentage changes for $R1.1$. For example, concentrate on $cv = 0{,}2$ and $\rho = 0.0$. If ρ increases to 0.1 then the corresponding expected waiting time will drop with approximately 39.8%. This value can be

Table 9: Rho change

cv	ρ	Wait	Idle	Over
0,2	0,0	-	-	-
0,2	0,1	-0,398	2,875	-0,412
0,2	0,2	-0,371	0,785	-0,350
0,3	0,0	-	-	-
0,3	0,1	-0,385	1,936	-0,394
0,3	0,2	-0,366	0,618	-0,362
0,5	0,0	-	-	-
0,5	0,1	-0,349	0,970	-0,335
0,5	0,2	-0,353	0,563	-0,331
1,0	0,0	-	-	-
1,0	0,1	-0,287	0,429	-0,243

1,0	0,2	-0,304	0,344	-0,248

found in Table 9 as -0.398. Because cv has four different values we have four different cases. The percentage changes within the 'Wait' and 'Over' columns are quite the same for every case. So that one could say that the waiting time and overtime change along with the changes of ρ. But regarding to the idle time, the percentage changes differ significant from each other. It tends to change relatively more when ρ is small. So with the given environmental parameters it can be stated that the idle time is not as robust as the other two measurements.

These properties for $R1.1$ can be found for all individual block rules. Thus when the interval between two consecutive ρ's stay constant then the percentage drop for the waiting time and overtime is approximately the same. But the idle time on the other hand shows different values. The percentage changes are bigger when ρ is small. As a matter of fact, these properties can also be found for the multiple block rules and the mixed multiple-individual block rules. See Appendix I for the percentage changes for $R1.2$, $R2.1$ and $R3.1$ when ρ changes. Similar results can be found for the other $RX.X$ rules. Hence it does not really matter which class of rule a practitioner or clinic chooses. They all seem to react the same way to changes of ρ.

6. Variable Interval Concept

In section 5 'Assignment Scheduling Rules' we introduced and analyzed different rules from different classes. This section will try to improve the patient's waiting time from the different ASR's with the so-called "Variable Interval (*VI*)"-concept. This concept is first introduced by Ho and Lau [6] because patients allocated to the earlier part of the session tend to have shorter expected waiting time than patients allocated to the latter part of the session. To counterpart this "unfairness" Ho and Lau scheduled the earlier patients a fragment earlier than originally planned and scheduled the latter patients a fragment later than planned. Furthermore, Appendix J illustrates this unfairness introduced by Ho and Lau. Both figures from Appendix J represents the expected waiting time for a patient scheduled at a given time. The upper figure contains the results when patients were allocated according to the $R1.1$ rule and the lower figure follows the $R1.3$ rule. Note that all lines seem to have an upward trend, thus the expected waiting time is not the same for the patients during a session. Subsequently is clear that the expected waiting time per patient increases during a session. This increase is most visible for $\rho = 0.0$, where all patients show up at the clinic. However, when $\rho = 0.2$ the increase in the expected waiting time seems to decrease during the session, but nevertheless it is apparent that patients scheduled in the earlier part of the session tend to wait shorter. Generally speaking, this unfairness can be found for each and every rule discussed in this section. For this reason we apply the VI concept to the following rules: $R1.1, R1.3, R2.1, R2.2, R3.1$ and $R3.3$. The first twelve patients from the ASR are considered as the patients from the earlier part of the session and vice versa the last twelve patients are considered as the patients from the latter part of the session. Furthermore, patients from the earlier part will start 2 minutes earlier than originally planned while patients from the latter part will start 2 minutes later.

The six modified ASR's are denoted by $R1.1^*, R1.3^*, R2.1^*, R2.2^*, R3.1^*$ and $R3.3^*$. Appendix K shows the results from these modified rules. This table also contains the percentage differences with the original rules between brackets. For example, $R1.1^*$ has an expected waiting time of 140.66 minutes, this is about 0.04% less than the expected waiting time from $R1.1$. In a similar way one could concentrate on the 'Wait' columns and notice that the bracket values are mainly negative numbers, except for $R3.1^*$ and $R3.3^*$. This means that the VI concept is likely to reduce the patient's waiting time for an individual block rule and a multiple block rule as anticipated. But a mixed multiple-individual block rule seems to perform worse with the variable interval concept. Since $R3.1^*$ has eight from the twelve cases where the expected waiting time is longer than the original rule. But this does not hold for $R3.3^*$ where only two cases perform worse than the original. This means that the VI concept seems to have a positive effect on some rules from the mixed multiple-individual block class, but a negative effect on other rules within the same class. For this reason we experimented with the variable interval concept on other mixed multiple-individual rules. And it turned out that the more a mixed multiple-individual ASR looks like $R3.1$ the more likely the VI concept increases the expected waiting time. This is due to the fact that $R3.1$ on its own performs quite well, as stated in Ho and Lau [6]. Again, the main goal of the variable interval is to counter the 'unfairness' that patients have different expected waiting times during a session. The following figure illustrates that the variable interval concept indeed softens this 'unfairness'. The solid line represents the expected waiting time for a patient at a given schedule time when using the original $R1.1$ rule. While the dotted line corresponds to the modified $R1.1^*$ rule. It is clear that both lines differ from each other. The modified rule has a higher expected waiting time for the first twelve patients and a lower expected waiting time for the latter twelve patients when compared to the original rule, which means that the differences between the various waiting times during a session is reduced. However the difference between the first twelve patients and the last twelve patients is still significant.

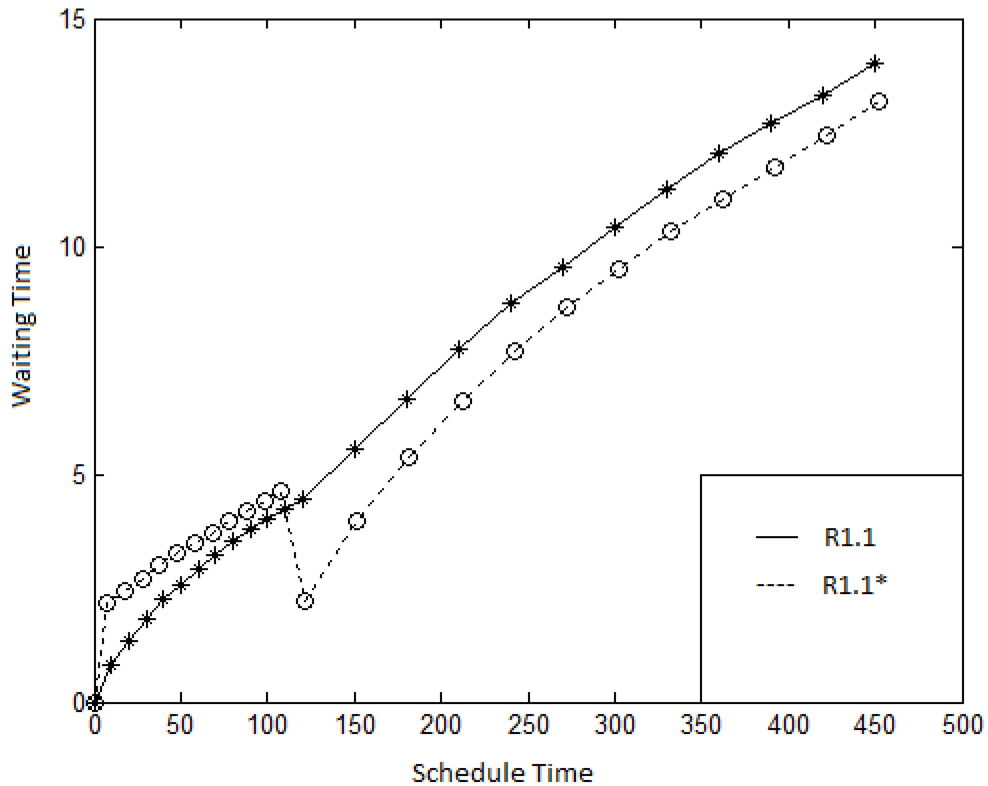


Figure 2: Different reduction/increase
Both lines correspond to the expected waiting time per patient with the following environmental setting: cv =0.2 and rho = 0.0

In our case, we chose two minutes for the 'interval' reduction/increase for both types of patients. However, it seems more logical for the patients with a larger expected service time to have a greater 'interval' reduction/increase. For this reason we increased/reduced the interval for a type 1 patient with two minutes and for a type 2 patient with 6 minutes. These new VI modified rules are denoted by $R1.1^{**}, R1.3^{**}, R2.1^{**}, R2.2^{**}, R3.1^{**}$ and $R3.3^{**}$, see Appendix L for the results. If we compare Appendix K with Appendix L then we conclude that the results from Appendix L have smaller values for the waiting time but tend to have larger values for the idle and overtime. In other words, the usage of different 'intervals' for different types of patients seems to improve the patients waiting time, but tends to increase the practitioner's idle and overtime. This property is most clear when $R1.1^{**}$ is compared with $R1.1^{*}$. The main goal of using different intervals for different types of patients is to make the VI-concept more efficient. This means that it should smoothen the expected waiting time per patient. Figure 3 shows that this is indeed the case. In this figure the solid line represents $R1.1^{*}$ and the dotted line represents $R1.1^{**}$. One could clearly see that that the dotted is lower than the solid line for the second part of the figure. However, the expected waiting time for patients during the first part of the session is less than the last part of the session. Thus one should find the optimal 'interval' reduction/increase for every single patient to make sure that every patient has approximately the same expected waiting time.

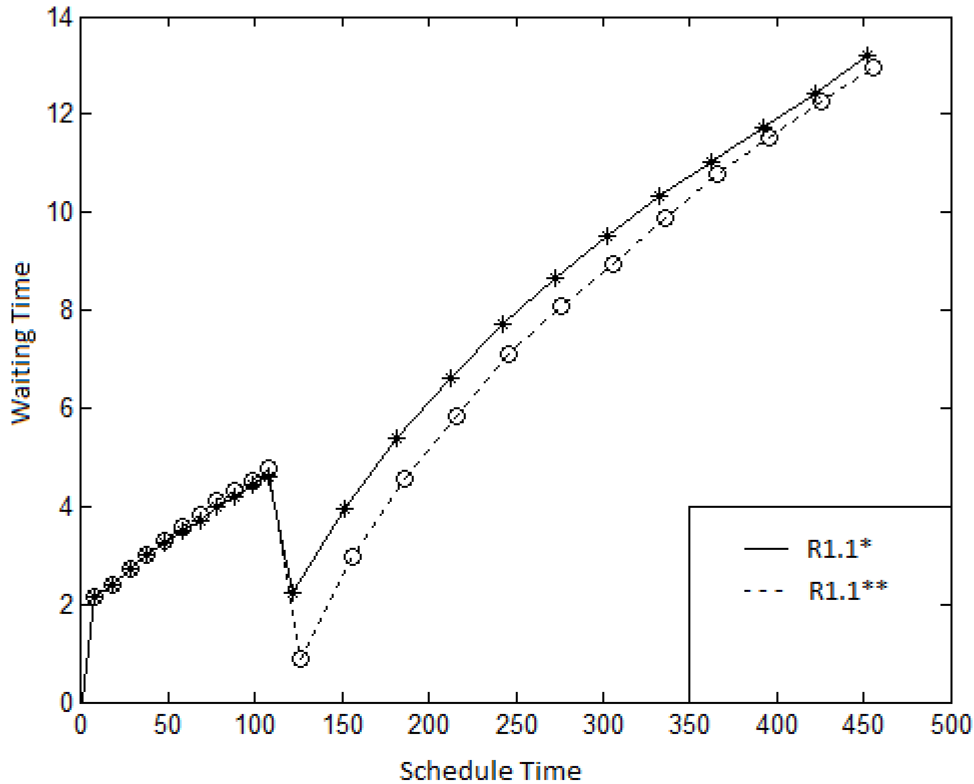


Figure 3: Different reduction/increase
Both lines correspond to the expected waiting time per patient with the following environmental setting: cv =0.2 and rho = 0.0

Generally speaking, the VI concept reduces the expected waiting time, at the same time it increases the idle and overtime. This can be stated from Appendix K and Appendix L, where the 'Idle' and 'Over' columns mostly have positive bracket values. This means that the original rules have better performances than the VI modified rules for the idle and overtime. This has already been confirmed by Ho and Lau for one type of patient. With this section the same can be concluded for two patients. Hence a practitioner should only apply the VI-concept if it is his goal to minimize the patient's waiting time. If this is not the case he should not apply the VI-concept. Taking a closer look at Appendix K one could observe that the values are bigger if cv is small. For example, look at the 'Idle' column from $R1.1^{*}$ and focus on $\rho = 0.0$. For the situation where $cv = 0.2$ it returns a percentage of 7%. And when cv increases to 0.3 then the percentage will drop to 4%. Thus if cv continues to climb then the percentage tends to decrease to zero. This means that the difference between an ASR and its VI modified version becomes smaller. This is logical since the larger the uncertainty about the service time, the less effect the variable interval concept has.

7.Conclusions

The main objective of this thesis is to find the 'best' assignment scheduling rule for two different types of patients, given the choice for the unit costs. The so-called 'best' rule is the rule with the lowest total costs, see formula (5). As mentioned before, there are three main classes of assignment scheduling rules. And when the unit cost for the patient's waiting time is greater or equal to the other two unit costs. Then the practitioner or clinic should allocate the patients according the individual block rule $R1.1$. This means that the practitioner or clinic should first allocate all the patients with a shorter expected service time before allocating the patient with a longer expected service time. However, if the unit costs for the practitioner's idle or overtime are chosen in such a way that the idle or overtime has the most effect on the total costs. Then the practitioner or clinic should choose the mixed multiple-individual block rule $R3.2$. This ASR first schedules all the patients with a long expected service time before the patients with a short expected service time. Furthermore, this ASR schedules the first and second patient on the same appointment at the start of the session (Bailey-Welch). For the case that all three performance measurements have approximately the same effect on the total costs. Then the practitioner or clinic should select the mixed multiple-individual block rule $R3.1$. This rule is actually the Bailey-Welch modified version of $R1.1$.

Thus, there are three 'best' rules for the given environmental settings, originating from the individual block rule class and the mixed multiple-individual block rule class. This means that none of the rules originating from the multiple block rule class will be chosen. Due to the fact that this class is inferior to the other two classes. In the sense that the other two classes return smaller values for the performance measurements. However, the multiple block rule class is more robust than the other two classes. Hence, if the practitioner or clinic prefers stability for the performance measurements, then he should select a rule from the multiple block rule class.

The 'Variable-Interval'-concept introduced by Ho and Lau indeed reduced the 'unfairness for two types of patients. With this concept, the expected waiting time for patients in the first part of the session tends to be longer than the original waiting time, while the expected waiting time for patients in the latter part of the session tends to be shorter. Furthermore, this VI-concept reduced the patients' expected total waiting time. On the other hand, it increases the practitioner's expected idle and overtime. For this reason, the practitioner or clinic should only apply the VI-concept if he wants to minimize the patients waiting time or if he wants to reduce the 'unfairness'. If this is not the case then it is wise not to apply the Variable Interval concept.

8. Further Research

This thesis made several assumptions; the probability ρ that a random patient does not show up at the appointment time could be 0.0, 0.1 or 0.2. Whereas the coefficient of variation cv varies between the values 0.2,0.3,0.5 and 1.0. One could expand the range of these assumptions in order to understand the effects of these parameters better. Furthermore, this thesis assumed that the total number of patients to be scheduled for a session is equal to 24. One could experiment with more or less patients, with the purpose to investigate properties of all the different ASR. Moreover, we held the total number of patients to be allocated for type 1 and 2 patients ($N_1 \& N_2$) to be 12. This means that the number of patients for type 1 patients is the same for type 2 patients. One could change this ratio. Thus we could schedule more type 1 patients than type 2 patients or vice versa.

9. Appendix

Appendix A: ASR representation

Appendix A: ASR representation													
Patient	**R1.1**	**R1.2**	**R1.3**	**R1.4**	**R2.1**	**R2.2**	**R2.3**	**R2.4**	**R2.5**	**R2.6**	**R3.1**	**R3.2**	**R3.3**
1	1	2	1	1	1	1	2	1	1	1	1	2	1
2	1	2	2	1	1	2	1	1	1	1	1	2	2
3	1	2	1	1	1	1	2	2	1	1	1	2	1
4	1	2	2	2	1	2	1	2	2	1	1	2	2
5	1	2	1	2	1	1	2	1	2	1	1	2	1
6	1	2	2	2	1	2	1	1	2	1	1	2	2
7	1	2	1	1	1	1	2	2	1	1	1	2	1
8	1	2	2	1	1	2	1	2	1	1	1	2	2
9	1	2	1	1	1	1	2	1	1	1	1	2	1
10	1	2	2	2	1	2	1	1	2	1	1	2	2
11	1	2	1	2	1	1	2	2	2	1	1	2	1
12	1	2	2	2	1	2	1	2	2	1	1	2	2
13	2	1	1	1	2	1	2	1	1	2	2	1	1
14	2	1	2	1	2	2	1	1	1	2	2	1	2
15	2	1	1	1	2	1	2	2	1	2	2	1	1
16	2	1	2	2	2	2	1	2	2	2	2	1	2
17	2	1	1	2	2	1	2	1	2	2	2	1	1
18	2	1	2	2	2	2	1	1	2	2	2	1	2
19	2	1	1	1	2	1	2	2	1	2	2	1	1
20	2	1	2	1	2	2	1	2	1	2	2	1	2
21	2	1	1	1	2	1	2	1	1	2	2	1	1
22	2	1	2	2	2	2	1	1	2	2	2	1	2
23	2	1	1	2	2	1	2	2	2	2	2	1	1
24	2	1	2	2	2	2	1	2	2	2	2	1	2

Appendix B: Multiple Block Rule

Appendix B: Multiple Block Rule

		R2.1			R2.2			R2.3			R2.4		
ρ	cv	Wait	Idle	Over	Wait	Idle	Over	Wait	Idle	Over	Wait	Idle	Over
0,0	0,2	368,64	13,48	13,67	311,32	14,18	14,17	551,01	14,14	14,30	414,34	13,82	13,66
0,0	0,3	430,18	20,37	20,69	409,70	20,92	21,30	650,13	21,09	21,40	503,13	20,49	20,69
0,0	0,5	556,78	33,84	34,30	595,30	34,97	34,78	825,28	35,65	34,89	684,79	33,95	34,71
0,0	1,0	857,69	66,81	66,61	1041,45	69,36	68,77	1261,52	69,79	67,42	1097,73	66,83	68,47
0,1	0,2	265,02	55,48	7,22	198,63	55,15	6,74	394,29	54,13	6,79	287,71	54,22	6,59
0,1	0,3	304,12	59,06	11,52	262,99	58,22	11,36	454,34	59,55	11,45	338,35	58,95	10,53
0,1	0,5	388,77	69,08	21,52	389,15	69,35	21,25	578,63	70,02	21,01	462,71	68,09	20,44
0,1	1,0	627,34	96,51	49,10	754,01	97,01	50,50	944,96	97,03	50,60	784,85	96,40	47,85
0,2	0,2	194,76	100,36	4,29	133,09	100,31	3,64	286,73	99,75	3,85	201,52	100,41	3,72
0,2	0,3	218,30	102,20	7,07	169,28	101,68	6,44	324,22	101,98	6,25	235,52	102,36	6,00
0,2	0,5	270,11	109,06	13,58	259,73	108,25	13,09	411,16	109,27	12,96	309,56	108,58	12,32
0,2	1,0	439,33	133,63	35,15	515,84	131,65	35,44	672,16	132,29	35,90	541,67	130,13	33,78
		R2.5			R2.6								
0,0	0,2	634,28	12,98	13,28	4080,37	8,79	8,72						
0,0	0,3	711,21	19,43	19,38	4085,27	12,93	13,28						
0,0	0,5	849,79	32,82	31,52	4078,37	21,96	22,35						
0,0	1,0	1248,47	63,06	65,12	4086,05	43,16	43,57						
0,1	0,2	465,84	53,77	5,97	3300,80	49,87	1,46						
0,1	0,3	507,08	57,25	9,73	3296,03	51,79	2,97						
0,1	0,5	610,93	66,25	18,40	3320,41	54,83	8,55						
0,1	1,0	904,72	92,03	45,79	3305,03	72,76	24,36						
0,2	0,2	342,71	100,27	3,00	2604,78	96,67	0,19						
0,2	0,3	370,00	100,59	5,11	2610,34	96,59	0,60						
0,2	0,5	435,65	105,26	10,45	2610,23	98,54	2,52						
0,2	1,0	622,55	128,60	30,52	2615,38	108,96	12,54						

Appendix C: Mixed Multiple-Individual Rule

Appendix c: Mixed Multiple-Individual Rule										
		R3.1			**R3.2**			**R3.3**		
ρ	cv	Wait	Idle	Over	Wait	Idle	Over	Wait	Idle	Over
0,0	0,2	271,11	9,92	9,97	703,97	8,80	8,66	305,12	9,99	9,94
0,0	0,3	321,80	16,24	16,16	756,58	13,36	13,30	388,66	16,59	16,50
0,0	0,5	439,62	30,31	29,97	917,87	23,72	23,59	560,36	30,74	29,63
0,0	1,0	751,57	64,22	64,60	1442,08	54,63	54,66	1037,13	65,15	64,67
0,1	0,2	143,35	51,85	3,94	328,27	49,92	1,53	161,08	51,44	3,33
0,1	0,3	182,60	56,22	7,97	391,63	50,96	3,55	221,88	55,41	7,28
0,1	0,5	278,51	65,47	18,14	543,91	58,76	9,90	357,47	65,60	16,94
0,1	1,0	535,40	94,18	48,37	980,74	83,31	34,46	719,48	95,61	45,99
0,2	0,2	82,99	97,70	1,98	167,37	96,19	0,22	91,85	97,24	1,29
0,2	0,3	110,00	101,24	4,46	205,76	97,34	0,71	129,80	99,87	3,54
0,2	0,5	177,19	107,01	11,79	320,04	99,96	3,71	229,84	105,28	10,30
0,2	1,0	374,43	130,33	35,11	663,94	117,79	21,51	499,42	128,35	32,09

Appendix D: Differences between the three classes

Appendix D: Differences between the three classes										
		R3.1-R1.1			**R3.2-R1.2**			**R3.3-R1.3**		
ρ	cv	Wait	Idle	Over	Wait	Idle	Over	Wait	Idle	Over
0,0	0,2	124,64	-4,74	-4,66	445,23	-6,01	-5,98	99,84	-4,72	-4,66
0,0	0,3	102,60	-5,72	-5,99	366,39	-8,81	-8,89	82,11	-5,63	-5,32
0,0	0,5	76,88	-6,15	-6,51	269,34	-13,06	-13,33	49,96	-5,74	-6,90
0,0	1,0	49,17	-6,76	-6,67	176,42	-18,12	-17,80	45,18	-6,48	-6,58
0,1	0,2	55,18	-4,94	-4,66	192,82	-4,21	-4,39	43,22	-3,86	-4,21
0,1	0,3	47,72	-8,27	-5,46	174,84	-7,59	-6,88	42,24	-5,06	-5,02
0,1	0,5	42,36	-6,34	-6,12	147,61	-10,39	-11,26	32,87	-5,24	-6,20
0,1	1,0	34,45	-7,25	-5,58	97,92	-16,84	-17,06	24,22	-5,21	-6,37
0,2	0,2	27,51	-3,70	-3,61	88,63	-2,38	-2,46	23,45	-3,53	-3,29
0,2	0,3	24,46	-3,09	-4,10	82,47	-3,48	-4,22	20,69	-3,59	-3,67
0,2	0,5	24,50	-5,21	-4,45	83,85	-7,68	-7,81	18,52	-5,14	-4,71
0,2	1,0	25,66	-5,97	-5,44	69,02	-13,88	-14,21	12,81	-5,04	-7,19
ρ	cv	**R3.1-R2.1**			**R3.3-R.2.2**					
0,0	0,2	-97,52	-3,56	-3,69	-6,20	-4,19	-4,23			
0,0	0,3	-108,38	-4,12	-4,52	-21,04	-4,33	-4,80			
0,0	0,5	-117,16	-3,53	-4,32	-34,94	-4,23	-5,15			
0,0	1,0	-106,12	-2,59	-2,01	-4,32	-4,21	-4,10			
0,1	0,2	-121,68	-3,62	-3,27	-37,55	-3,72	-3,40			
0,1	0,3	-121,52	-2,84	-3,55	-41,11	-2,81	-4,08			
0,1	0,5	-110,26	-3,61	-3,38	-31,69	-3,75	-4,30			
0,1	1,0	-91,94	-2,33	-0,73	-34,53	-1,40	-4,50			
0,2	0,2	-111,78	-2,66	-2,31	-41,25	-3,07	-2,35			
0,2	0,3	-108,30	-0,96	-2,61	-39,48	-1,81	-2,90			
0,2	0,5	-92,93	-2,05	-1,80	-29,88	-2,97	-2,79			
0,2	1,0	-64,90	-3,31	-0,05	-16,42	-3,30	-3,35			

Appendix E: Best ASR

Best ASR with Wait & Over = 1,0								
		I =						
ρ	cv	0,0	1,0	4,0	8,0	12,0	50,0	150,0
0,0	0,2	R1.1	R1.1	R1.1	R1.1	R1.1	R3.1	R3.1
0,0	0,3	R1.1	R1.1	R1.1	R1.1	R1.1	R3.1	R3.2
0,0	0,5	R1.1	R1.1	R1.1	R1.1	R3.1	R3.1	R3.2
0,0	1,0	R1.1	R1.1	R1.1	R3.1	R3.1	R3.1	R3.2
0,1	0,2	R1.1	R1.1	R1.1	R1.1	R3.1	R3.3	R3.2
0,1	0,3	R1.1	R1.1	R1.1	R3.1	R3.1	R3.2	R3.2
0,1	0,5	R1.1	R1.1	R1.1	R3.1	R3.1	R3.2	R3.2
0,1	1,0	R1.1	R1.1	R3.1	R3.1	R3.1	R3.2	R3.2
0,2	0,2	R1.1	R1.1	R1.1	R3.1	R3.1	R3.3	R3.2
0,2	0,3	R1.1	R1.1	R1.1	R3.1	R3.1	R3.2	R3.2
0,2	0,5	R1.1	R1.1	R3.1	R3.1	R3.1	R3.2	R3.2
0,2	1,0	R1.1	R1.1	R3.1	R3.1	R3.1	R3.2	R3.2

Best ASR with Wait & Idle = 1,0							
		O =					
ρ	cv	0,0	5,0	8,0	12,0	50,0	150,0
0,0	0,2	R1.1	R1.1	R1.1	R1.1	R3.1	R3.1
0,0	0,3	R1.1	R1.1	R1.1	R1.1	R3.1	R3.1
0,0	0,5	R1.1	R1.1	R1.1	R3.1	R3.1	R3.2
0,0	1,0	R1.1	R1.1	R3.1	R3.1	R3.1	R3.2
0,1	0,2	R1.1	R1.1	R1.1	R3.1	R3.3	R3.2
0,1	0,3	R1.1	R1.1	R3.1	R3.1	R3.2	R3.2
0,1	0,5	R1.1	R1.1	R3.1	R3.1	R3.2	R3.2
0,1	1,0	R1.1	R3.1	R3.1	R3.1	R3.2	R3.2
0,2	0,2	R1.1	R1.1	R3.1	R3.1	R3.3	R3.2
0,2	0,3	R1.1	R1.1	R3.1	R3.1	R3.2	R3.2
0,2	0,5	R1.1	R3.1	R3.1	R3.1	R3.2	R3.2
0,2	1,0	R1.1	R3.1	R3.1	R3.1	R3.2	R3.2

Appendix F: Percentage changes R1.2

R1.2 Percentage Change					
ρ	cv	cv	Wait	Idle	Over
0,0	0,2	-	-	-	-
0,0	0,3	0,500	0,508	0,497	0,516
0,0	0,5	0,667	0,662	0,659	0,664
0,0	1,0	1,000	0,952	0,978	0,963
0,1	0,2	-	-	-	-
0,1	0,3	0,500	0,600	0,082	0,764
0,1	0,5	0,667	0,828	0,181	1,030
0,1	1,0	1,000	1,228	0,448	1,435
0,2	0,2	-	-	-	-
0,2	0,3	0,500	0,566	0,023	0,839
0,2	0,5	0,667	0,916	0,068	1,336
0,2	1,0	1,000	1,519	0,223	2,100

Appendix G: Percentage change R2.1 & R2.2

R2.1 Percentage Change					
ρ	cv	cv	Wait	Idle	Over
0,0	0,2	-	-	-	-
0,0	0,3	0,500	0,167	0,511	0,514
0,0	0,5	0,667	0,294	0,662	0,658
0,0	1,0	1,000	0,540	0,974	0,942
0,1	0,2	-	-	-	-
0,1	0,3	0,500	0,148	0,065	0,596
0,1	0,5	0,667	0,278	0,170	0,869
0,1	1,0	1,000	0,614	0,397	1,281
0,2	0,2	-	-	-	-
0,2	0,3	0,500	0,121	0,018	0,648
0,2	0,5	0,667	0,237	0,067	0,922
0,2	1,0	1,000	0,626	0,225	1,588

R2.2 Percentage Change					
ρ	cv	cv	Wait	Idle	Over
0,0	0,2	-	-	-	-
0,0	0,3	0,500	0,316	0,475	0,503
0,0	0,5	0,667	0,453	0,672	0,633
0,0	1,0	1,000	0,749	0,983	0,977
0,1	0,2	-	-	-	-
0,1	0,3	0,500	0,324	0,056	0,687
0,1	0,5	0,667	0,480	0,191	0,870

0,1	1,0	1,000	0,938	0,399	1,377
0,2	0,2	-	-	-	-
0,2	0,3	0,500	0,272	0,014	0,770
0,2	0,5	0,667	0,534	0,065	1,031
0,2	1,0	1,000	0,986	0,216	1,708

Appendix H: Percentage change R3.1 & R3.2

R3.1 Percentage Change					
ρ	cv	cv	Wait	Idle	Over
0,0	0,2	-	-	-	-
0,0	0,3	0,500	0,187	0,638	0,621
0,0	0,5	0,667	0,366	0,866	0,854
0,0	1,0	1,000	0,710	1,119	1,155
0,1	0,2	-	-	-	-
0,1	0,3	0,500	0,274	0,084	1,020
0,1	0,5	0,667	0,525	0,165	1,277
0,1	1,0	1,000	0,922	0,439	1,666
0,2	0,2	-	-	-	-
0,2	0,3	0,500	0,326	0,036	1,258
0,2	0,5	0,667	0,611	0,057	1,642
0,2	1,0	1,000	1,113	0,218	1,979

R3.2 Percentage Change					
ρ	cv	cv	Wait	Idle	Over
0,0	0,2	-	-	-	-
0,0	0,3	0,500	0,075	0,519	0,536
0,0	0,5	0,667	0,213	0,775	0,774
0,0	1,0	1,000	0,571	1,303	1,317
0,1	0,2	-	-	-	-
0,1	0,3	0,500	0,193	0,021	1,326
0,1	0,5	0,667	0,389	0,153	1,791
0,1	1,0	1,000	0,803	0,418	2,480
0,2	0,2	-	-	-	-
0,2	0,3	0,500	0,229	0,012	2,286
0,2	0,5	0,667	0,555	0,027	4,223
0,2	1,0	1,000	1,075	0,178	4,800

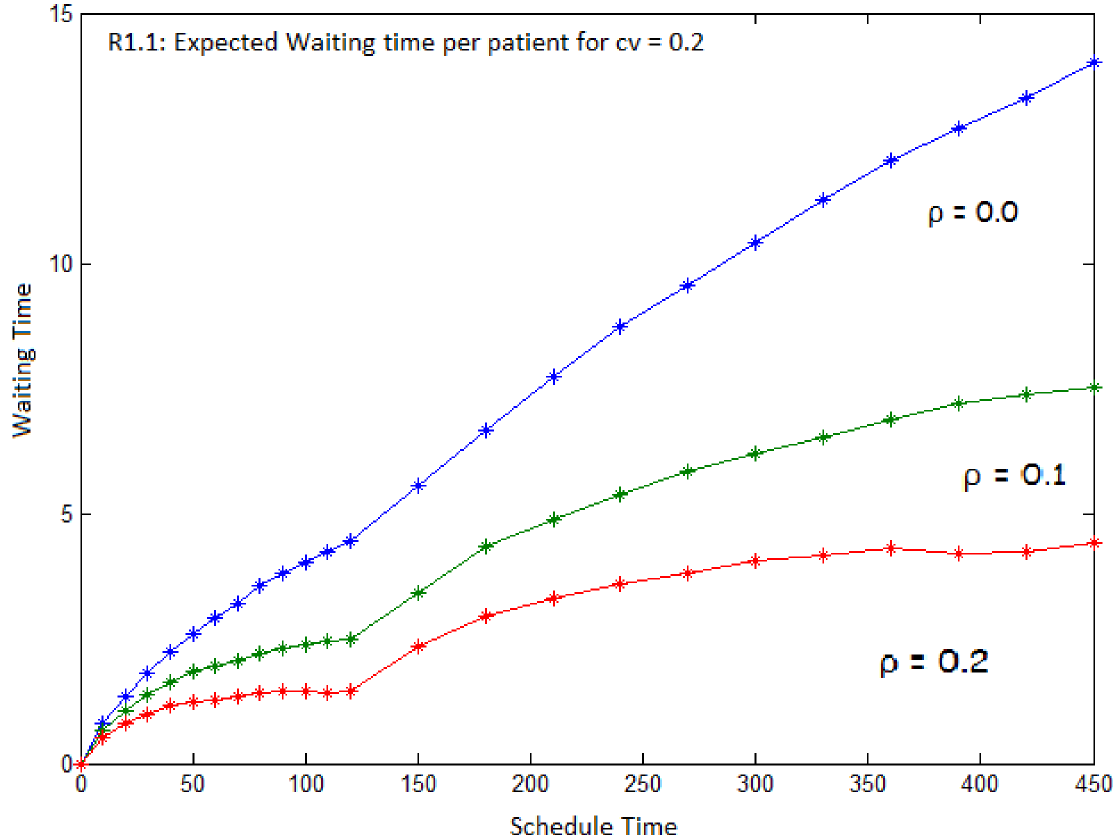
Appendix I: Percentage change R1.2 & R2.1 & R3.1

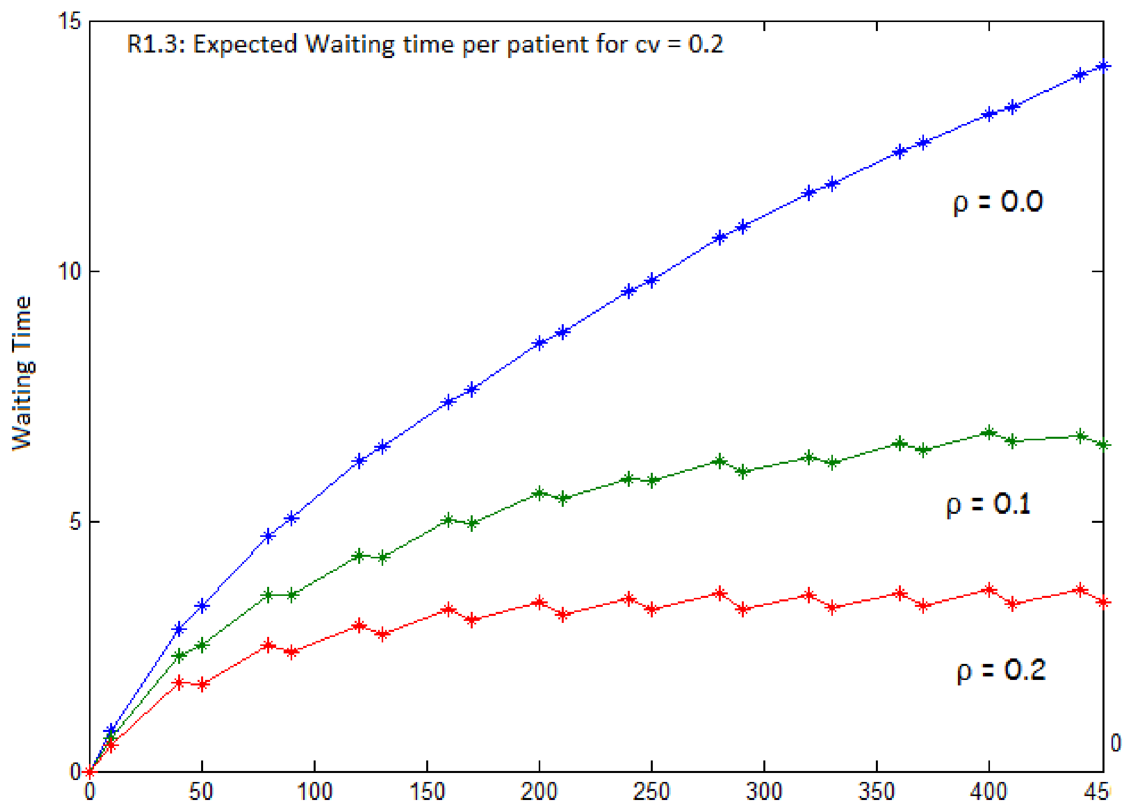
R1.2 Rho change				
cv	ρ	Wait	Idle	Over
0,2	0,0	na	na	na
0,2	0,1	-0,476	2,655	-0,596
0,2	0,2	-0,419	0,821	-0,546
0,3	0,0	na	na	na
0,3	0,1	-0,444	1,641	-0,530
0,3	0,2	-0,431	0,722	-0,527
0,5	0,0	na	na	na
0,5	0,1	-0,389	0,880	-0,427
0,5	0,2	-0,404	0,556	-0,456
1,0	0,0	na	na	na
1,0	0,1	-0,302	0,377	-0,289
1,0	0,2	-0,326	0,315	-0,307

R2.1 Rho change				
cv	ρ	Wait	Idle	Over
0,2	0,0	na	na	na
0,2	0,1	-0,281	3,116	-0,472
0,2	0,2	-0,265	0,809	-0,406
0,3	0,0	na	na	na
0,3	0,1	-0,293	1,900	-0,443
0,3	0,2	-0,282	0,731	-0,386
0,5	0,0	na	na	na
0,5	0,1	-0,302	1,041	-0,372
0,5	0,2	-0,305	0,579	-0,369
1,0	0,0	na	na	na
1,0	0,1	-0,269	0,445	-0,263
1,0	0,2	-0,300	0,385	-0,284

R3.1 Rho change				
cv	ρ	Wait	Idle	Over
0,2	0,0	na	na	na
0,2	0,1	-0,471	4,230	-0,605
0,2	0,2	-0,421	0,884	-0,499
0,3	0,0	na	na	na
0,3	0,1	-0,433	2,461	-0,507
0,3	0,2	-0,398	0,801	-0,440
0,5	0,0	na	na	na
0,5	0,1	-0,366	1,160	-0,395
0,5	0,2	-0,364	0,634	-0,350
1,0	0,0	na	na	na
1,0	0,1	-0,288	0,467	-0,251
1,0	0,2	-0,301	0,384	-0,274

Appendix J: Variable Interval Concept





Appendix E: Variable Interval Concept

Appendix K: Variable Interval Concept

Variable Interval Concept																			
		R1.1*						**R1.3***						**R2.1***					
ρ	cv	Wait	%	Idle	%	Over	%	Wait	%	Idle	%	Over	%	Wait	%	Idle	%	Over	%
0,0	0,2	140,66	-0,04	15,72	0,07	15,49	0,06	190,69	-0,07	15,05	0,02	14,97	0,02	361,94	-0,02	14,73	0,09	14,46	0,06
0,0	0,3	213,76	-0,02	22,95	0,04	22,96	0,04	291,16	-0,05	22,11	0,00	22,30	0,02	422,73	-0,02	21,26	0,04	21,42	0,04
0,0	0,5	355,05	-0,02	37,25	0,02	37,64	0,03	491,33	-0,04	36,73	0,01	36,91	0,01	544,38	-0,02	34,62	0,02	34,59	0,01
0,0	1,0	691,47	-0,02	72,03	0,01	72,41	0,02	967,85	-0,02	72,09	0,01	71,53	0,00	848,84	-0,01	66,80	0,00	67,39	0,01
0,1	0,2	87,39	-0,01	57,48	0,01	9,76	0,13	107,37	-0,09	56,25	0,02	8,56	0,14	264,79	0,00	55,95	0,01	8,46	0,17
0,1	0,3	130,70	-0,03	62,69	-0,03	14,28	0,06	172,92	-0,04	60,57	0,00	13,01	0,06	299,61	-0,01	60,28	0,02	12,45	0,08
0,1	0,5	228,49	-0,03	72,96	0,02	24,99	0,03	313,38	-0,03	71,22	0,01	23,02	-0,01	381,60	-0,02	69,92	0,01	21,73	0,01
0,1	1,0	498,45	0,00	102,61	0,01	55,63	0,03	676,36	-0,03	101,19	0,00	52,04	-0,01	602,53	-0,04	98,81	0,02	49,33	0,00
0,2	0,2	54,87	-0,01	102,41	0,01	6,60	0,18	65,74	-0,04	101,88	0,01	5,53	0,21	193,11	-0,01	102,00	0,02	5,20	0,21
0,2	0,3	83,46	-0,02	105,91	0,02	9,73	0,14	104,80	-0,04	104,75	0,01	8,49	0,18	216,52	-0,01	103,73	0,01	7,71	0,09
0,2	0,5	147,45	-0,03	114,43	0,02	17,08	0,05	201,85	-0,04	110,71	0,00	15,49	0,03	265,28	-0,02	111,68	0,02	14,19	0,04
0,2	1,0	343,29	-0,02	137,63	0,01	41,45	0,02	476,90	-0,02	134,94	0,01	39,24	0,00	434,28	-0,01	132,49	-0,01	35,54	0,01
		R2.2*	%		%		%	**R3.1***	%		%		%	**R3.3***	%		%		%
0,0	0,2	140,66	-0,04	15,72	0,03	15,49	0,00	190,69	0,02	15,05	0,05	14,97	0,04	361,94	0,00	14,73	0,02	14,46	0,03
0,0	0,3	213,76	-0,05	22,95	0,03	22,96	0,00	291,16	0,01	22,11	0,04	22,30	0,04	422,73	-0,02	21,26	0,01	21,42	0,01
0,0	0,5	355,05	-0,03	37,25	0,01	37,64	0,02	491,33	0,01	36,73	0,00	36,91	0,03	544,38	-0,01	34,62	0,00	34,59	0,02
0,0	1,0	691,47	-0,02	72,03	0,00	72,41	0,00	967,85	-0,01	72,09	0,02	71,53	0,01	848,84	-0,02	66,80	0,01	67,39	0,01
0,1	0,2	87,39	-0,03	57,48	0,02	9,76	0,12	107,37	0,03	56,25	0,02	8,56	0,13	264,79	-0,02	55,95	0,01	8,46	0,09
0,1	0,3	130,70	-0,05	62,69	0,03	14,28	0,04	172,92	0,03	60,57	0,02	13,01	0,09	299,61	-0,02	60,28	0,00	12,45	0,07
0,1	0,5	228,49	-0,01	72,96	0,00	24,99	0,04	313,38	-0,01	71,22	0,02	23,02	0,04	381,60	-0,01	69,92	-0,01	21,73	0,05
0,1	1,0	498,45	-0,03	102,61	0,01	55,63	-0,01	676,36	-0,02	101,19	0,03	52,04	-0,01	602,53	0,01	98,81	-0,01	49,33	0,03
0,2	0,2	54,87	-0,02	102,41	0,00	6,60	0,27	65,74	0,05	101,88	0,01	5,53	0,23	193,11	0,00	102,00	0,00	5,20	0,32
0,2	0,3	83,46	-0,02	105,91	0,02	9,73	0,11	104,80	0,04	104,75	0,00	8,49	0,18	216,52	-0,02	103,73	0,01	7,71	0,10
0,2	0,5	147,45	-0,04	114,43	0,01	17,08	0,05	201,85	0,01	110,71	0,01	15,49	0,02	265,28	-0,04	111,68	0,01	14,19	0,02
0,2	1,0	343,29	-0,01	137,63	0,01	41,45	0,02	476,90	-0,01	134,94	0,01	39,24	0,03	434,28	-0,01	132,49	0,01	35,54	0,04

Appendix L: VI-concept: Type1 (2), Type2 (6)

VI-Concept: Type 1 (2), Type 2 (6)										
		R1.1**			**R1.3****			**R2.1****		
ρ	cv	Wait	Idle	Over	Wait	Idle	Over	Wait	Idle	Over
0,0	0,2	134,49	18,99	19,14	203,91	16,81	16,69	352,25	17,90	17,45
0,0	0,3	202,40	26,10	26,22	296,60	23,68	23,64	410,07	24,11	24,21
0,0	0,5	338,26	40,36	40,11	490,06	37,56	38,02	529,64	37,59	36,74
0,0	1,0	676,49	74,26	74,45	935,36	73,52	70,97	830,20	69,48	69,61
0,1	0,2	84,37	60,77	12,89	122,36	58,92	10,84	261,59	59,02	10,96
0,1	0,3	128,25	65,03	17,77	178,61	63,50	14,67	293,93	63,27	14,90
0,1	0,5	224,18	75,27	28,09	311,56	72,97	24,51	374,86	72,72	24,62
0,1	1,0	487,62	104,69	58,13	675,30	102,48	53,27	604,57	100,17	53,20
0,2	0,2	54,17	105,49	9,25	77,88	103,96	7,87	193,30	103,56	7,15
0,2	0,3	82,29	108,94	12,39	113,95	106,00	10,14	213,69	106,18	9,52
0,2	0,5	145,57	115,66	19,21	200,26	113,37	16,88	263,67	112,91	16,19
0,2	1,0	341,01	139,65	43,88	470,28	135,97	39,31	428,07	135,73	38,60
		R2.2**			**R3.1****			**R3.3****		
0,0	0,2	140,66	15,72	15,49	252,28	12,00	11,63	310,75	11,29	10,91
0,0	0,3	213,76	22,95	22,96	306,91	18,93	18,87	389,71	17,35	17,72
0,0	0,5	355,05	37,25	37,64	426,03	33,02	32,98	559,26	31,21	30,82
0,0	1,0	691,47	72,03	72,41	728,50	67,35	65,35	1007,75	65,31	65,14
0,1	0,2	87,39	57,48	9,76	143,70	53,68	6,24	173,86	52,32	4,69
0,1	0,3	130,70	62,69	14,28	178,76	59,22	10,59	226,32	56,69	8,81
0,1	0,5	228,49	72,96	24,99	272,58	68,42	21,60	354,56	66,47	18,86
0,1	1,0	498,45	102,61	55,63	518,73	97,56	50,30	709,56	96,03	47,10
0,2	0,2	54,87	102,41	6,60	85,30	99,32	3,60	103,61	98,70	2,51
0,2	0,3	83,46	105,91	9,73	110,86	103,00	6,50	137,35	101,33	4,90
0,2	0,5	147,45	114,43	17,08	172,97	110,18	13,66	221,07	108,42	11,39
0,2	1,0	343,29	137,63	41,45	360,14	133,03	37,47	476,54	131,79	33,55

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