# Psychological barriers in individual stocks in the United States

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**Abstract**

This paper examines the existence of psychological barriers in the closing price levels of individual stocks in the United States for the past twenty years. Due to the different pricing methods used in the US, the dataset will be divided into three periods. In each period the barriers will be examined at two different levels and several ranges; the higher level where barriers are defined as multiples of ten and five dollars, and the lower level for every whole dollar price. I find no evidence supporting the existence of psychological barriers in the dataset, neither for the higher nor the lower level. Instead price clustering is found for the lower level. Investors appear to have a strict preference for whole dollars above any other price.

## 1. Introduction

Psychological barriers have been a subject of discussion in recent years. While some (financial)journalists and market practitioners are convinced of their existence, the academic world remains sceptic. Psychological barrier is the phenomenon whereby a symbolic, round or interesting number is seen as difficult to cross; this could for instance be a certain stock index level or exchange rate. These particular numbers are not different from any other numbers except that people find them interesting. But when many people believe they are difficult to cross, the numbers become psychological barriers which can affect the course of the relevant index or exchange rate. Some articles (Donaldson and Kim (1993), Koedijk and Stork (1994)) have provided statistic evidence, but recent studies including Dorfleitner and Klein (2009) have shown the effect is diminishing or has already disappeared to a large extent.

Psychological barriers can be seen both as a “supporting level” and “resisting level”, since they have a trend disrupting effect. In particular they have the natural tendency of “pushing” prices away. Imagine the price of a certain stock is rising quickly; investors will buy the stock to profit from a further price increase. However, when the price is closing to a psychological barrier, the chance of a further increase becomes smaller. Assuming it is not possible to cross the barrier, the chance of a further increase will eventually be very close to zero. At a certain price level, before the barrier is reached, the probability of a further increase is so small that investors will start selling more than buying. The price, as a consequence, will start dropping. Because of these effects, trades do not happen often at and around the psychological barriers. The same but reversed happens when a price is dropping towards a psychological barrier.

The focus of many researches has been on the indices of the stock exchanges. For the existence of psychological barriers it is needed that many people believe in them. Since stock indices represent the market, investigating them is a good way to look for psychological barriers. The research on individual stocks has not been done much however and I will do so in this article. Since stock indices are constructed by accumulating the individual stocks, presence of psychological barriers in the former should also mean presence in the latter. For investors, individual stocks have two benefits over the index. It is much easier for them to influence individual stocks than stock indices, meaning that investors are relatively more in control. And the stock price should give a much more precise indication of how the company is performing and how the market practitioners think about the particular company.

My hypothesis is: *Psychological barriers are present in individual stocks in the United States*

This research focuses on whether psychological barriers exist in individual stocks. Presence of the phenomenon implies that the market is not efficient, because there should be no reason to prefer one price over another according to the market efficiency hypothesis. If they do exist, profit strategies can be applied to anticipate on their occurrences.

The paper will examine the daily closing price of ten individual stocks from the United States for the past twenty years. Much effort has been put in defining barriers, because they are vital for testing the data and interpreting the results. There are no standard measurements for psychological barriers however; this is solved by making use of the so called $M$-values, which is the most widely used method. It involves a transformation of the data of stock prices into comparable and testable values. Psychological barriers, represented by certain $M$-values after the transformation, are then characterized by a significant diminished trade frequency compared to non-barrier $M$-values. Understanding of the definition of barriers is very much needed to read the paper.

Another topic which requires attention is the fact that the United States have been using three different pricing mechanisms in the sample period. In particular these pricing methods are eighth fractional pricing, sixteenth fractional pricing and decimal pricing. It appears that these methods have important consequences for the research. Firstly, it leads to the division of the sample into three sub-periods. Secondly, this means that a few conventional testing techniques cannot be applied due to the complication of the pricing. I therefore introduce the conditional coverage testing as a new and alternative way of testing for psychological barriers. Finally, the different pricing means that results of the three periods cannot be compared to each other.

Furthermore, the examination has been performed on two different levels; the *higher level* which considers the second last and third last trailing digit of the prices, and the *lower level* where the focus lies with the last and second last trailing digit. The higher level examinations do not lead to any clear conclusions. In fact, the results are contradicting to the hypothesis. Nevertheless it is clear that at least for the considered initial barriers no abnormalities are found. The results of the lower level are surprising however. Instead of psychological barriers, price clustering is found. These findings oppose the hypothesis since psychological barrier means significant fewer trading at the barrier level, while price clustering means significant more. I conclude that no psychological barriers are observed in the investigated individual stocks in the United States. Instead a price clustering is observed for all the periods at the lower level. This however does not necessarily mean psychological barriers do not exist at all. It is possible that both price clustering and psychological barriers occur at similar price levels. It can then very well be that the effects of the former are greater than that of the latter.

Section 2 gives a brief overview of the past literature and point out some major findings and conclusions. Especially the debate around the assumptions of psychological barriers used in earlier papers is important. The final conclusion is that these assumptions do hold. Section 3 provides the description of the data used in this research. Also the different pricing methods and some of its consequences are described in more detail. Since it is not straightforward to see what psychological barriers are, I formally define them in section 4. The transformations from closing prices to M-values are introduced here as well. I begin section 5 by explaining a few traditional testing methods used in earlier paper, followed by the introduction of the conditional coverage testing. The results of the tests for both the higher level and the lower level and any other findings are presented in section 6. Section 7 concludes and section 8 discusses some flaws of the research and possible improvements.

## 2. Literature

Donaldson and Kim (1993) and Koedijk and Stork (1994) were some of the first to show the existence of psychological barriers among stock indices. Both articles have shown the existence of psychological barriers in the more publically known indices. This is not the case for the less famous ones.

Donaldson and Kim (1993) have examined the Dow Jones Industrial Average (DJIA) and the Wilshire Associates 5000's (WA) for psychological barriers. They have used the closing price of fifteen years of data starting from 1974. WA has been indicated as a less important index and no signs of psychological barriers were found here. The DJIA on the other hand showed evidence for psychological barriers at multiples of hundred (2000, 2100, 2200 etc.). Their conclusion is that only the important indices, like DJIA, contains psychological barriers since they give a better representation of the market.

Five indices were researched in Koedijk and Stork (1994). Specifically they were the FAZ General, Brussels Stock Index, Nikkei Stock Average 225, Standard and Poor's Composite and the FTSE 100. The data covered a period of twelve years starting from 1980. They have found psychological barriers in the FAZ, S&P and FTSE, but almost none in the Brussels and Nikkei index. Again it seemed that the more widely known indices contain psychological barriers.

The conclusions of the articles named above are based on a crucial assumption: the frequency of the price levels should be uniformly distributed for non-psychological barriers and significantly less for psychological barriers. This assumption has been much of a dispute however and has been rejected by De Ceuster, Dhaene and Schattema (1998) by using the Benford's Law. In essence the Benford's Law states that numbers (1,2,3 etc) are not increasing at a constant percentage rate which leads to unequal probabilities ( Pr(1)>Pr(2)>Pr(3) etc ). This implies that the previous articles have wrong conclusions since the most important assumption does not apply. The conclusion of De Ceuster etal (1998) has been rejected again in Mitchell (2001) who argues that such relationships are bound to many restrictions and appears to be invalid for financial data and markets. In other words, the Benford's Law cannot be applied to financial data and the assumption of uniformity holds true.

Dorfleitner and Klein (2009) were one of the first to describe the psychological barrier effect for individual stocks. They used the data of European stock indices and German stocks. They only found evidence for two of the eight companies, but do argue that the choice of their stocks might have caused this and suggest further research to take companies that are frequently traded by investors.

## 3. Data

Following the suggestion of Dorfleitner and Klein (2009) I focus on stocks with high average trading volume. This means that the average number of times a share is traded on a daily basis has to be high. Companies from different sectors are selected to have a better representation of the market. For example the financial, IT and production sector are taken into account. The closing price[1] level, which is the price of the last trade on the given day, of each stock is taken as the price level in this paper. Consideration is taken for the period January 2$^{nd}$, 1991 to May 11$^{th}$, 2011 providing over five thousand observations. A long stable period of growth is present between 1991 and 1997 and a relative turbulent period after 2000. Especially two recession periods, defined as two consecutive quarters of negative GDP growth, have been observed after 2000. The Internet crisis (Dot-com bubble) and September 11 attacks caused the first one lasting from March 2001 to November 2001 and the second caused by the Financial crisis lasting from December 2007 to June 2009.

**Table 1 Summary statistics stocks 1991-2011**

| Stocks | Average Trading Volume | Average Price | Max Price | Min Price |
|---|---|---|---|---|
| C | 7,798,054 | 40.81 | 77.31 | 1.02 |
| BAC | 39,291,972 | 51.65 | 124.00 | 3.14 |
| GE | 29,823,272 | 55.46 | 166.00 | 6.66 |
| MSFT | 68,085,759 | 63.68 | 179.94 | 15.15 |
| JPM | 14,330,107 | 48.83 | 147.00 | 10.50 |
| F | 22,186,008 | 26.31 | 69.50 | 1.26 |
| T | 10,328,791 | 40.90 | 83.75 | 19.34 |
| PFE | 22,376,315 | 48.99 | 150.13 | 11.66 |
| INTC | 68,113,987 | 50.32 | 169.31 | 12.08 |
| LOW | 8,303,260 | 39.52 | 85.25 | 13.39 |

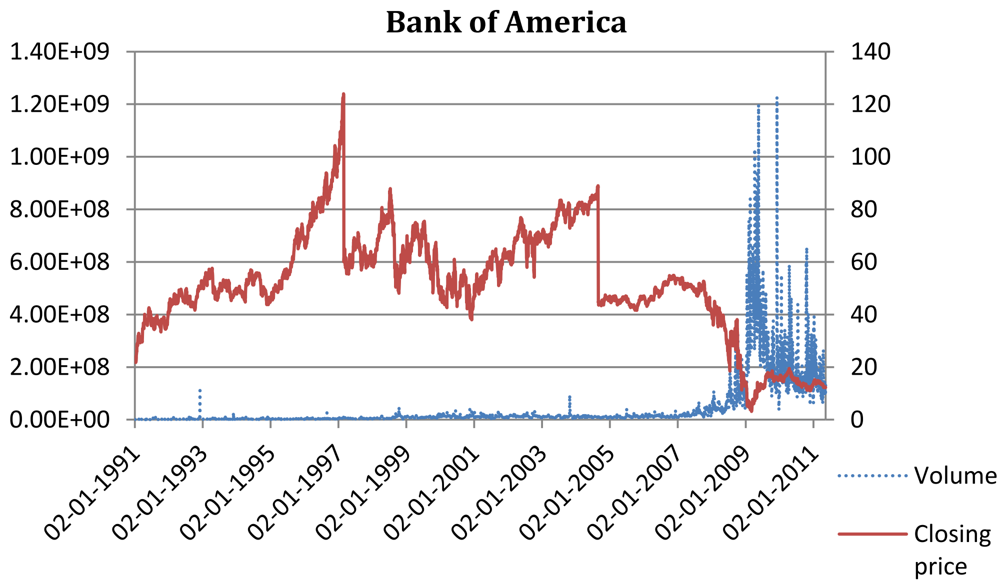
*Note: Summary statistics from the stocks examined in the period January 2$^{ne}$, 1991 to May 11$^{th}$, 2011.*

[1] The data is retrieved from finance.yahoo.com

For this research the daily stock closing prices of the following ten stocks have been taken: Citigroup (C), Bank of America (BAC), General Electric (GE), Microsoft (MSFT), JP Morgan Chase (JPM), Ford (F), AT&T (T), Pfizer (PFE), Intel (INTC) and Lowe's Companies (LOW).

In Table 1 above the summary statistics are given of the stocks used in this article, it is remarkable to see that several maximum closing prices are equal to a round number. The overall minimum price is 1.02 for Citigroup and the overall maximum is for Microsoft standing at 179.94. The volume is measured at a daily basis meaning our stock with the lowest average trading volume is close to 8 million and the highest over 68 million. There are fluctuations herein however, especially for financial stocks after 2007. Figure 1 presents the volume traded and the price level over the relevant period; it is observed that the trade volume has increased enormously after 2007.

**Figure 1 Bank of America closing value and daily volume**




*Note: the figure presents the daily volume and closing price of the Bank of America for the period January 2ne, 1991 to May 11th, 2011. The left axis represents the volume and the right axis the price. Note that the volume increased very much starting from around 2008.*

The most important remark about the data is the pricing mechanisms used in the United States for the past twenty years. Roughly until 2001 fractional pricing was used in stock exchanges, especially eighth fractional pricing until 1997 and sixteenth fractional pricing until 2001. The principle of fractional pricing is that a stock cannot be bought with a price accurate to the cents (hundred intervals), but only eighth (.00;.12/.13;.25;.37/.38 etc) or sixteenth (.00;.06;.12/.13;.19;.25 etc). The use of this pricing system is mainly due to the trading tradition originating from the eighteenth century when the first formal stock markets were established and the Spanish dollar, the piece of eight, was the smallest currency. As the United States Security and Exchange Commission (SEC) eventually wanted to convert the stock markets to decimal pricing, sixteenth pricing has been implemented as a transition stage, hence the relative short period of use.

The use of different pricing methods has caused a few complications for this research. The first problem concerns the division of the sample period. The second has to do with a proper definition of psychological barriers. Finally, the different pricing mechanisms cause difficulties with choosing appropriate testing methods; especially some conventional techniques are very difficult to apply. In this section I will only explain the first problem. Problem two and three will be dealt with in the remaining part of the paper.

A both natural and forced consequence of the pricing mechanisms is the division of the sample period into three sub-periods. The first period covers the period with eighth fractional pricing, the second with the use of sixteenth fractional pricing and the third with the implementation of decimal pricing. The consequence is natural because it is very logical to determine the sub-periods by the pricing method, but forced because of the following reasons. Note that for the determination of a psychological barrier, the frequency of trade at a certain price level plays the most important role. Specifically it is expected that trade around barriers occur less often than other prices. When the pricing mechanisms differ, so do the possible price levels. A consequence is that investors are limited in their choices in earlier periods. With eighth fractional pricing, a market practitioner could for example either choose $34.25 or $34.37. After the introduction of decimal pricing on the other hand, he can choose any price ranging from $34.25 to $34.37, which leads to considerably more options. This means that the frequencies of the same price level cannot be compared between different sub-periods, because some of the price levels will not be chosen at all due to the pricing. Any kind of comparison tests between sub-periods will lead to misguiding results and conclusions.

Another complication with the data is the transition date from one kind of pricing method to another. Though a guideline was given by the SEC which states that the stock exchanges should change their pricing methods within a certain time span, it was not required to make the change for all the stocks on one single day. This had the consequence that the pricing methods of some stocks were adapted earlier and others later. The transition date, for both sixteenth fractional pricing and decimal pricing, therefore has to be determined for each stock individually by looking at the occurrence of price levels.

## 4. Definition of barriers

Psychological barriers do not have a universal standard measurement like prices in dollars, distance in miles or weight in kilograms. Instead, they are characterized by the relative amount of trade. Without barriers, the frequency of trade should be uniformly distributed. $27.01 should for example be statistically traded equally often as $380.45. However, due to the nature of psychological barriers, the frequency of trade will be less compared to non-psychological barrier prices. This means that if $400.00 is seen as a barrier, the trade frequency at this price will be less than that of $387.40, which is not a barrier. Testing for barriers therefore comes to testing for the frequency of trade. Previous researches mainly analysed stock indices to determine the existence of psychological barriers. The definition of barriers, however, appeared to be difficult since they tend to be relative and not absolute. As the indices often range from a few hundred to a few thousand, several problems arise. First of all the values are not multiplicatively regenerative, this means that when for example 1600 is a barrier 160 does not necessarily have to be one too. Secondly, the gap between each following barrier is relative. This is clear when 5 and 10 are seen as barriers but not 500 and 505. Although, these problems reduce when the range considered is relatively small, which is the case for this paper, they will not completely be absent.

A way to deal with the above problems and to make the prices comparable and testable is to examine on different levels. The most widely used method here for is to transform the prices into *M*-values. The *M*-value is a transformation of the considered price into a two digit number ranging from 00 to 99 by cutting of all the numbers before and after the relevant level. This means that tough prices may differ; they can still receive the same *M*-value. The prices usually consist of three or more trailing digits. By taking different sequential trailing digits, they can be observed as trades on different levels. Take for example $132.89 which is constructed by five trailing digits. From left to right the following sequential numbers can be extracted: 13, 32, 28 and 89. These are the *M*-values, in particular they will be $M_{13}$, $M_{32}$, $M_{28}$ or $M_{89}$ depending on the examined level.

The examination at different levels is needed to define barriers of different magnitudes and to count the frequency of relevant trades. If all the multiples of $100,- ($100, $200, $300 etc.) are for example seen as barriers, it will be very least useful to extract the *M*-values from the last and second last trailing digits of the prices. For this research two levels will be examined: the *higher level* where the second-last and the third-last trailing digit are considered and the *lower level* where the second-last and last trailing digits are taken into account.

The idea behind the use of *M*-values to test for psychological barriers is as follows. If barriers or any other abnormalities are absent, the distribution of frequency of trade over the *M*-values should be uniform, since there are no reasons to prefer one price over another. With psychological barriers however, the relevant price levels will be crossed less and thus will lead to a reduced frequency of trades.

To examine the existence of psychological barriers, it first needs to be determined which *M*-values represent them. These *M*-values are called *initial barrier values*; it would be for example fairly straightforward to see $M_{00}$ as a psychological barrier. However, as described in the literature, the barrier starts influencing trades before the actual barrier level is reached. This is straightforward since market practitioners will anticipate and therefore take action in an earlier stadium. This is known as *band technique,* Dorfleitner and Klein (2009) said: "*a barrier is defined as an interval with a certain length around the actual barrier number*". I will define this "*certain length"* as *range* and all the *M*-values inside this range will be considered as *barrier values.* So for an initial barrier level of $M_{00}$ with range 1; $M_{99}$, $M_{00}$, and $M_{01}$ will be defined as barrier values.

The existence of psychological barriers should lead to significant fewer observations of the barrier values. This is the most critical assumption and will be used throughout the paper.

*Higher level*

Following Sonnemans (2006) psychological barriers are defined as multiples of ten dollars ($10, $20, $30 etc.) and to a lesser extent multiples of five dollars ($5, $15, $25 etc.). The corresponding initial barrier values will then be $M_{00}$ and $M_{50}$. To transform the closing prices into higher level *M*-values the following formula is used. The *Integer* in (1) means that everything behind the decimal point is cut off and not rounding to the nearest integer.

$$Integer\left(\left(P_{n,t} - Integer\left(\frac{P_{n,t}}{10}\right)\cdot 10\right)\cdot 10\right) \qquad (1)$$

Three ranges will be defined for each initial barrier for each period. Range 0, which equals the initial barrier value itself, range 2 and range 5. The corresponding barrier values are $M_{98}$ to $M_{01}$ and $M_{48}$ to $M_{51}$ for range 2 and $M_{95}$ to $M_{04}$ and $M_{45}$ to $M_{54}$ for range 5. Note that I have taken 98-01 instead of 98-02; this is because the M-values only represents the first number before and after the decimal point at the higher level. The second decimal number is left aside but should be taken into account as XY0.1Z ranges from XY0.10 to XY0.19, where X, Y and Z can take the value of any single digit decimal.

*Lower level*

As mentioned before, the lower level considers the last two trailing digits of the closing prices. With the initial barrier value set at $M_{00}$ it means that all the round prices are seen as potential psychological barriers. For the transformation a slightly different formula is used. Again the *Integer* means everything behind any round number is cut off and not rounded to the nearest round number.

$$Integer\left(\left(P_{n,t} - Integer(P_{n,t})\right)\cdot 100\right) \qquad (2)$$

For the lower level period one and period two only consider the initial barrier value with range 0. For period three, range 2 and range 5 will also be evaluated. The range for the lower level works a bit different however. For the higher level I have shown that caution must be taken when defining ranges because the last trailing digit is not taken into account. For the lower level, this problem is no longer present. A range of 2 for the initial barrier value $M_{00}$ thus means that al the *M*-values ranging from $M_{98}$ to $M_{02}$ are considered as barrier values. Similar results hold for range 5.

## 5. Methods

This section describes the methods used to test for the existence of psychological barriers. Firstly, three commonly used tests by earlier studies are introduced. But due to the different pricing methods used in the US, it is rather difficult to apply these tests to all the periods and levels. Specifically, period one and two for the higher level forms a complication. A more detailed explanation is provided in section 6. As a result, the tests are only applied to the research of the whole period described in Appendix A. The barrier proximity test is only used for the lower level.

The conditional coverage testing is then introduced. It consists of the unconditional coverage test, the independence test and the conditional coverage test. These tests will be applied to both the higher and the lower level and will be the most important tests to be relied on.

### *5.1. Uniformity*

Following earlier literature I describe three commonly used methods to test for psychological barriers. First of all the test for uniform distribution is proposed. By dividing the *M*-values into equal groups, for example each group consisting ten different *M*-values, it can be tested whether they are uniformly distributed. The total frequency of the ten *M*-values accumulated should be equal to ten per cent, if no psychological barriers are present in the data.

$$\chi^2 = \sum_{i=1}^{I} \frac{(O_i - E_i)^2}{E_i}. \tag{3}$$

The formula presented by equation (3) gives the Pearson's chi-square test. In the formula $O_i$ stands for the actual frequency observed in the sample and $E_i$ the expected observations in group *i*. The results are added together and form the test statistic which is $\chi^2(n-1-p)$ distributed, where *n* is the number of groups and *p* the number of parameters which equals one. The null hypothesis is $H_0$: *p*(Group1) = *p*(Group2) = *p*(Group3)...etc, where *p*(Group1) stands for the probability of an observation of an *M*-value belonging to group 1. The amount of groups can be varied between 2 and *N*, where *N* stands for the total observations. Dividing both in too many or too few groups may lead to misleading results however. Following Dorfleitner and Klein (2009), I will divide the *M*-values in ten groups where *group 1* = $M_{00}$- $M_{09}$, *group 2* = $M_{10}$ –$M_{19}$ etc.

Secondly I will apply the *barrier proximity* test which is described by Donaldson and Kim (1993) given by equation (4). The main goal of this test is to see how much the frequency of barrier values differ from a uniform distribution. The independent variable is the dummy *D* which can take the value on one or zero. The dependent variable *f(M)* is the amount of observations or put otherwise, the frequency. *D* takes the value of one for barriers and zero for non-barriers. Under the null hypothesis all *M*-values will have statistically the same amount of observations, that is $H_0$: p($M_{00}$)= p($M_{01}$)= p($M_{02}$)...etc. The $\alpha$ will then be equal to 0.01 as *M* can take one hundred different values. Since the frequencies should not differ for barrier and non-barrier values, $\beta$ should equal zero. When psychological do exist however, it is expected that there are less observations for the barrier values. Especially it means that the *f(M)* is lower when the dummy equals one. The slope of the Ordinary Least Squares (OLS), represented by $\beta$, should then be significantly negative. Positive $\beta$ then means a significantly higher frequency for the barrier values.

$$f(M) = \alpha + \beta D + \varepsilon \tag{4}$$

Finally Donaldson and Kim (1993) suggest the use of the *barrier hump* test. While the *barrier proximity* test focuses on the uniformity of the barriers, the *barrier hump* is applied to find out how the entire price level is distributed. The test suggests that with the absence of uniformity, the existence of a different shape should be present. In equation (5) the *f(M)* is again the frequency of the $M$-values, it is then regressed on a constant $\alpha$, the $M$-values and the square of the $M$-values. Under $H_o$: $\gamma$ is zero meaning there are no barriers, while it should be negative with the presence of barriers.

$$f(M) = \alpha + \phi M + \gamma M^2 + \eta \tag{5}$$

*5.2. Conditional coverage*

The previous methods for testing for psychological barriers are based on a clear definition and distinction of the $M$-values. This is however not always possible, I therefore introduce conditional coverage testing. Which consists of three parts. The first part, unconditional coverage testing, looks whether the observed frequency of barriers corresponds with the theoretical one. The second part tests for independence under barriers among sequential observations. Put it differently it tests whether the barrier $M$-value of today has any significant influence on the $M$-value of tomorrow. Part one and part two combined forms the conditional coverage test.

First of all the following indicator variable is introduced,

$$I_{n,t} = \begin{cases} 1, & if\ M_{n,t} \in barrier\ value \\ 0, & if\ M_{n,t} \notin barrier\ value \end{cases}$$

$M_{n,t}$ stands for the $M$-value of stock $n$ at time $t$. The indicator takes the value of one when the considered $M$-value is a barrier value and zero if not. Note that the one and the zero can be seen as two different stages.

*5.2.1. Test of Unconditional Coverage*

The unconditional coverage test is used to research whether $M_{00}$ is distributed as expected by testing $H_0$: $E[I_{n,t}] = p_n$ against the alternative $E[I_{n,t}] \neq p_n$, where $p_n$ is the proportion the $I_n$ should have under uniform distribution. For example: period one consists of eight $M$-values, if the barrier value is set as $M_{00}$ with range 0, $p_n$ will get a value of 0.125 (one divided by eight). Similarly, $p_n$ will get a value of 0.04 if consideration is taken for period three range 2 at the higher level (four divided by hundred). Note that the null hypothesis implies the barrier is uniformly distributed, and should thus lead to similar conclusions as (4). The likelihood under $H_0$ and $H_a$ are respectively as follows

$$L(p_n; I_{n,1}, I_{n,2}, \dots, I_{n,T}) = (1 - p_n)^{n_0} \, {p_n}^{n_1},$$

$$L(\pi_n; I_{n,1}, I_{n,2}, \dots, I_{n,T}) = (1 - \pi_n)^{n_0} \, {\pi_n}^{n_1},$$

where $n_o$ and $n_1$ stands for the amount of $I_{n,t} = 0$ and $I_{n,t} = 1$ respectively and $\pi_n$ is estimated by $n_1/(n_0 + n_1)$. The likelihood ratio test for unconditional coverage is then defined by

$$LR_{uc} = -2log\left[\frac{L(p_n; I_{n,1}, I_{n,2}, \dots, I_{n,T})}{L(\pi_n; I_{n,1}, I_{n,2}, \dots, I_{n,T})}\right] \;\; \widetilde{asy} \;\; \chi^2(s-1). \qquad (6)$$

Acceptance of the null hypothesis implicates the observed percentage of frequency of the barrier values actually corresponds with the expected percentage. Rejection of the null hypothesis therefore means that the observed percentage differs significantly from the theoretical one. The unconditional coverage test however lags the ability to detect clustering of zeros and ones in a certain timeframe, or in other words it lacks the ability to detect dependency among observations.

### *5.2.2. Test of Independence*

The purpose of the Independence test is to find out whether the barrier values are distributed independently. That is the occurrences of barrier and non-barrier values should be, though with some underlying generating function, random. Observing a (non)barrier value today, thus will not lead to a higher or lower probability of observing a (non)barrier value tomorrow. In a nutshell, the test looks for concentration of ones and zeroes; this is done by counting the frequency of consecutive ones. If there are no significant concentrations present, the sample is assumed to be independently distributed. A first-order binary Markov chain is defined as below, with $\pi_{ij} = \Pr(I_t = j | I_{t-1} = i)$ the actual probability of transitions between stages derived from the data; also the corresponding likelihood function is given.

$$\Pi_1 = \begin{bmatrix} 1 - \pi_{01} & \pi_{01} \\ 1 - \pi_{11} & \pi_{11} \end{bmatrix},$$

$$L(\Pi_1; I_{n,1}, I_{n,2}, \dots, I_{n,T}) = (1 - \pi_{01})^{n_{00}} \, \pi_{01}^{n_{01}} \, (1 - \pi_{11})^{n_{10}} \, \pi_{11}^{n_{11}}.$$

$\Pi_1$ can be estimated by making use of the following matrix with $n_{ij}$ indicating the number of transitions from stage $i$ to $j$. Note that $\pi_{11}$ represents the frequency of consecutive ones. Both too many or too few observations can lead to the rejection of the null hypothesis. Too many indicate a concentration of ones and too few a concentration of zeroes. Since there are only two states, the amount of transitions from state zero to state one should be (almost) equal to state of state one back to state zero. Put it otherwise, $\pi_{01}$ should be (almost) equal to $\pi_{10}$.

$$\widehat{\Pi}_1 = \begin{bmatrix} \frac{n_{00}}{n_{00}+n_{01}} & \frac{n_{01}}{n_{00}+n_{01}} \\ \frac{n_{10}}{n_{10}+n_{11}} & \frac{n_{11}}{n_{10}+n_{11}} \end{bmatrix}.$$

When independency holds: $\pi_{00}$ equals $\pi_{11}$. This is represented by $\Pi_2$, the Markov chain for independency

$$\Pi_2 = \begin{bmatrix} 1-\pi_2 & \pi_2 \\ 1-\pi_2 & \pi_2 \end{bmatrix},$$

and it is estimated by

$$\widehat{\Pi}_2 = \frac{n_{01}+n_{11}}{n_{00}+n_{10}+n_{01}+n_{11}}.$$

The corresponding likelihood function for independency is given by

$$L\left(\Pi_1; I_{n,1}, I_{n,2}, \dots, I_{n,T}\right) = (1-\pi_2)^{(n_{00}+n_{10})}\, \pi_2^{(n_{01}+n_{11})} .$$

Combining the two aforementioned functions, the likelihood ratio test can be performed as follows

$$LR_{ind} = -2log\left[\frac{L\left(\widehat{\Pi}_2; I_{n,1}, I_{n,2}, \dots, I_{n,T}\right)}{L\left(\widehat{\Pi}_1; I_{n,1}, I_{n,2}, \dots, I_{n,T}\right)}\right] \quad \widetilde{asy} \;\; \chi^2((s-1)^2), \tag{7}$$

where in this case $s$=2 implying a $\chi^2(1)$ distribution. Rejection of the null hypothesis leads to the conclusion that significant evidence is found to support the idea of dependency among barrier and non-barrier values.

### *5.2.3. Test of Coverage and Independence*

A combination of the unconditional coverage test and independence test can be made which leads to the test of coverage and independence presented by equation (8). Note that the test statistic can be obtained by simply adding the unconditional test statistic to that of the independence test statistic.

$$LR_{cc} = -2log\left[\frac{L\left(p_n; I_{n,1}, I_{n,2}, \dots, I_{n,T}\right)}{L\left(\pi_n; I_{n,1}, I_{n,2}, \dots, I_{n,T}\right)}\right] - 2log\left[\frac{L\left(\widehat{\Pi}_2; I_{n,1}, I_{n,2}, \dots, I_{n,T}\right)}{L\left(\widehat{\Pi}_1; I_{n,1}, I_{n,2}, \dots, I_{n,T}\right)}\right] \tag{8}$$

$$= LR_{uc} + LR_{ind} \ \ \widetilde{asy} \ \ \chi^2(s(s-1)^2).$$

The null hypothesis here is $H_0$: $E[I_{n,t+1}|I_{n,t}] = p_n$ , that is given the barrier value of the previous day, the expected value of the barrier value of today should be equal to the theoretical one. The test is $\chi^2(2)$ distributed since the $s$ is equal to two.

## 6. Results

This section presents and discusses the results obtained from the research. It has been divided in two parts where the first part evaluate the higher level, and the second part the lower level. Before I start with the results however, a brief overview shall be given for the prices occurred in the different sub-periods. Knowledge of these will be important to understand certain steps made during the testing. Table 2 presents the possible values for the cents. Period one is featured by eighth fractional pricing, but more numbers are listed in the table. This is because a price level of .125, which corresponds with one eighth of a dollar, cannot be represented by using two decimals; instead .12 and .13 are both used. Similar results hold for other fractions. Sixteenth pricing method has the same price distribution as period one with a few extra numbers. Basically the number of intervals has been doubled. Since it is not needed to use several numbers to represent any of these extra fractions, period two has eight more possible values than period one. Finally, all possible prices accurate to the cent are allowed in period three.

**Table 2 Possible values of prices in different sub-periods**

| Period | *M*-values |
|---|---|
| Period one | 00, 12, 13, 25, 37, 38, 50, 62, 63, 75, 87, 88 |
| Period two | Period one + 06, 19, 31, 44, 56, 69, 81, 94 |
| Period three | 00, 01, 02, …50, 51, 52, …97, 98, 99 |

### *6.1. Higher level*

The testing for the higher level has encountered some complications which have to be explained first. Note that for the higher level, the last three trailing digits of the closing prices are considered relevant. It can take thousand different values with decimal pricing from 000 to 999, though the last digit is ignored and the first two digits forms the *M*-value. With eighth and sixteenth fractional pricing, however, the price level can only take eighty and hundred sixty values respectively. Especially in period one, all the 4 and 9 ending *M*-values do not have any observations at all as they are continually skipped by the pricing mechanism. For period two similar problems arise because some ending values contain more fractions than others. For example the price level of $23.12 corresponds with $M_{31}$, but the price level of $23.19 receives the same *M*-value. $M_{39}$ on the other hand can only take one kind of price level, that is of $X3.94. Due to these irregularities the uniformity tests are very hard to execute and therefore left out of the research. Instead the conditional coverage test is applied.

The problems with the fractional pricing do not end with the exclusion of the barrier proximity test however. The range of both period one and two are also affected. Remember that the same range can include different amount of barrier values for the higher and lower level. In particular the higher level ranges contain one $M$-value less due to the unconsidered last trailing digit of the prices. For period one and two in the higher level, I have to deviate from this conclusion for range 5. Instead of $M$-values 95 to 04, I set 95 to 05 all as barrier values. This is done because $M_{04}$ does not contain any observations; as a consequence more price levels are included on the left and side than on the right hand side of $M_{00}$. To correct this, the definition of ranges has been revised. However, for range 2 this is not the case and the barrier values, $M_{98}$-$M_{01}$ will thus remain unchanged.

Finally the unconditional coverage test is a chi-square test, frankly this means that it can only test if the frequency is statistically the same as the theoretical one or not. No conclusions can be made whether there are more or less observations. This is solved by putting a table in the Appendix which shows whether the actual observations are higher (H), lower (L) or equal (E) to the theoretical amount. Consequently a rejection combined with an H for the considered stock means that there are significantly more observations. It should be noted that for the initial barrier level of $M_{00}$, periods one and two are mainly dominated by H and period three by L. The domination by H in the first two periods resembles price clustering, which is contrary to the hypothesis. For the initial barrier level of $M_{50}$ period one is slightly dominated by L and periods two and three mainly by H.

**Table 3 Unconditional coverage higher level results with $M_{00}$**

| Stocks | Per 1 (0) | Per 1 (2) | Per 1 (5) | Per 2 (0) | Per 2 (2) | Per 2 (5) | Per 3 (0) | Per 3 (2) | Per 3 (5) |
|---|---|---|---|---|---|---|---|---|---|
| C | 1.40 | 0.22 | 0.16 | 0.23 | 0.11 | 0.23 | 1.46 | 2.89 | 9.06** |
| BAC | 4.75* | 1.49 | 0.47 | 0.58 | 0.46 | 0.03 | 0.14 | 7.72** | 14.82** |
| GE | 0.63 | 10.67** | 17.53** | 6.65** | 12.38** | 1.49 | 0.62 | 1.91 | 9.06** |
| MSFT | 14.07** | 0.47 | 2.11 | 4.31* | 20.51** | 13.79** | 0.08 | 0.00 | 4.03* |
| JPM | 0.30 | 1.48 | 3.89* | 2.47 | 0.46 | 0.35 | 0.62 | 2.21 | 3.66 |
| F | 0.58 | 0.01 | 1.13 | 0.53 | 0.72 | 1.07 | 0.17 | 0.02 | 18.89** |
| T | 6.55* | 5.91* | 4.83* | 14.04** | 18.07** | 5.52* | 2.71 | 17.10** | 46.73** |
| PFE | 0.95 | 0.01 | 0.14 | 0.53 | 1.19 | 5.20* | 0.14 | 2.89 | 24.20** |
| INTC | 11.66** | 0.03 | 0.42 | 2.59 | 1.73 | 5.95* | 4.71* | 15.02** | 19.47** |
| LOW | 1.91 | 0.10 | 0.02 | 0.04 | 0.11 | 0.14 | 0.00 | 4.99* | 8.65** |

*Note: the table presents the test statistics of equation (6) at initial barrier level $M_{00}$. The */** denote the coefficient is significant at the 5% and 1% significance level respectively.*

The unconditional coverage test results for the initial barrier level of $M_{00}$ are given in the table above. For each of the three periods the ranges 0, 2 and 5 are taken for the initial barrier level. In period one only two or three stocks reject the null hypothesis of $E[I_{n,t}] \neq p$, there is thus not much evidence for deviation from the expected amount. For period two I come to similar conclusions because not many stocks reject the null hypothesis here either.

Period three starts off with only one significant stock, but as the range increases, so does the amount of rejection of the null hypothesis. Combined with the results of B. 2 it can be concluded that, at range 5 nine of the ten stocks have significantly lower frequencies.

On first sight, these results might seem encouraging, since they look similar to what is expected to be found for psychological barriers. Note however that for range 0 the frequency is not significantly different. If a figure is plotted with the barrier values on the horizontal axis and frequency on the vertical axis, a bell shape would be observed. The maximum will be at $M_{00}$ meaning that price clustering is present. This is the opposite of what is expected to be found. For a psychological barrier, a reversed bell shape should be observed with a minimum at the initial barrier value. The frequency at $M_{00}$ should be low, and increasing when the $M$-values are situated farther away.

As can be observed in the table below, the results of $M_{50}$ follow a relative similar pattern as that of $M_{00}$. Period one has very few rejections of the null hypothesis and many in period three. Period two is however different from the previous table, as it contains more rejections. Period three range 0 starts with only three rejections; it increases to six for range 2 and even seven for range 5. This is similar to the results at $M_{00}$, but combining with B. 3 I reach opposite conclusions. B. 2 and B. 3 show that while L dominates for $M_{00}$ period three, H dominates for $M_{50}$. Thus compared to the former case, an increase in the range leads to relatively more observations of the barriers. Note that a reversed bell shape is observed now. $M_{50}$ neither has significantly less observations and the null hypothesis cannot be rejected. It could therefore be argued that a similar distribution is observed here as for the psychological barrier except for the fact that this reversed bell shape lies higher.

**Table 4 Unconditional coverage higher level results $M_{50}$**

| Stocks | Per 1 (0) | Per 1 (2) | Per 1 (5) | Per 2 (0) | Per 2 (2) | Per 2 (5) | Per 3 (0) | Per 3 (2) | Per 3 (5) |
|---|---|---|---|---|---|---|---|---|---|
| C | 3.19 | 0.00 | 1.89 | 0.23 | 6.81** | 3.78 | 0.97 | 15.49** | 29.27** |
| BAC | 7.57** | 5.93* | 6.28** | 4.34* | 9.41** | 9.94** | 1.36 | 4.38* | 4.32* |
| GE | 12.16** | 11.38** | 7.45** | 0.01 | 0.52 | 2.20 | 0.62 | 3.26 | 0.75 |
| MSFT | 0.01 | 0.04 | 1.30 | 0.81 | 5.21* | 1.80 | 0.50 | 1.09 | 16.84** |
| JPM | 0.32 | 3.82 | 9.05 | 1.71 | 0.46 | 1.25 | 2.93 | 24.15** | 42.84** |
| F | 0.32 | 1.58 | 1.88 | 15.77** | 27.50** | 11.90** | 4.27* | 3.62 | 0.00 |
| T | 0.32 | 2.80 | 7.61 | 3.35 | 3.33 | 0.24 | 4.27* | 17.68** | 58.71** |
| PFE | 0.95 | 0.20 | 0.01 | 12.39** | 12.38** | 10.57** | 8.55** | 25.02** | 28.63** |
| INTC | 0.26 | 3.02 | 0.29 | 6.51* | 3.64 | 5.26* | 1.77 | 9.92** | 27.79** |
| LOW | 0.11 | 0.04 | 0.03 | 6.65** | 10.36** | 7.04** | 1.83 | 1.52 | 3.57 |

Note: the table presents the test statistics of equation (6) at initial barrier level $M_{50}$. The */** denote the coefficient is significant at the 5% and 1% significance level respectively.

A possible explanation for the bell shape distribution of the results in period three for $M_{00}$ lies within the investor preference to use round numbers. Although psychological barriers might be present in indices, they do not necessarily have to be observed among individual stocks because the effect of barriers might be outweighed by the investors' preferences. One underlying theory is the aspiration level hypothesis. This theory argues investors often have a target price in mind and are often set at round numbers as they are easy to use. However, observing the frequency and volume of today's trade where every cent could make the difference between huge losses and gains, it is hard to believe that investors only use round numbers as target prices. Especially if it is taken into account that the bell shape is only observed for $M_{00}$ and not for $M_{50}$.

Nevertheless, the two results lead to one important conclusion since they have one thing in common; there is very little proof for rejection of the initial barrier levels. It can thus be concluded that psychological barriers are not present, at least not for these two barriers at this level. The different shapes of range 5 for the two $M$-values are however odd as they lead to opposite conclusions, therefore I will try to give an explanation based on the independence test.

**Table 5 Independence test higher level results**

| **Barrier** | **Barrier = $M_{00}$** | | | **Barrier = $M_{50}$** | | |
|---|---|---|---|---|---|---|
| **Period** | **Per 3 (0)** | **Per 3 (2)** | **Per 3 (5)** | **Per 3 (0)** | **Per 3 (2)** | **Per 3 (5)** |
| C | 2.11 | 71** | 332** | 7.74** | 246** | 752** |
| BAC | 5.50* | 19** | 225** | 3.44 | 63** | 489** |
| GE | 26.56** | 128** | 461** | 6.16* | 185** | 625** |
| MSFT | 1.47 | 112** | 438** | 13.57** | 135** | 648** |
| JPM | 1.79 | 39** | 297** | 0.46 | 99** | 390** |
| F | 49.81** | 313** | 930** | 19.93** | 306** | 844** |
| T | 7.74** | 51** | 336** | 5.81* | 189** | 746** |
| PFE | 16.82** | 182** | 556** | 16.89** | 283** | 1001** |
| INTC | 14.49** | 141** | 652** | 2.26 | 60** | 420** |
| LOW | 1.25 | 46** | 344** | 16.94** | 108** | 305** |

*Note: the table presents the test statistics of equation (7) for all ranges in period three. The */** denote the coefficient is significant at the 5% and 1% significance level respectively.*

One possible explanation for the contradicting results lies with the lack of independency among the $M$-values. The results for the independence test of period three for both initial barrier values are presented in the table above. B. 4 and B. 5 present all the independency test results. There are many Not a Number results in these tables, this is because no consecutive observations of barrier levels are found regarding the periods. Normally speaking both an overflow and lack of these observations will lead to the rejection of independency. But due to the limited amount of trades for each period, only a few observations are enough for rejection. NaN can therefore, in this case, be viewed as prove of independency among stock prices.

A high dependency that is, a rejection of the null hypothesis, results in concentration of zeroes and ones. It is thus possible that the significant more observations for $M_{50}$ and significant fewer observations for $M_{00}$ are caused by the high dependency among stock prices. Once an observation is categorized as a barrier value, it is very likely it will be a barrier again for the next day. The same holds for non-barrier values. The results of the unconditional coverage tests then might be a consequence of high concentration of ones and zeroes in the particular periods.

This explanation however does not seem probable for three reasons. First, the independence test results of all the periods are similar for both $M_{00}$ and $M_{50}$. The results from the unconditional tests are on the other hand far from comparable at all. Specifically, period three gives completely opposite conclusion. It is therefore not likely that the lack of independency is the main underlying factor. Secondly, as the range of barrier values increases, so does the probability of any observation being a barrier. This is straight forward as I start with one barrier value for range 0 and increase to ten barrier values for range 5. Assuming the *M*-values are uniformly distributed, the probability of any observation being a barrier value ten folds from 1% to 10%. Finally, the ten consecutive *M*-values covers a range of one dollar at the higher level. On average, it means that a barrier value has to change more than $0.50 to become a non-barrier value. This is quite a long range to cross, assuming the price levels do not fluctuate too much on sequential days. Combining the above reasons, it seems no more than logical for the independence test to be rejected among longer ranges. In fact, this is something I would certainly expect; it would be a surprise if the opposite is found.

Although I have divided the periods according to their different pricing mechanisms, the results of this part of the research are contradicting. While a price clustering seems to be present around $M_{00}$ the opposite is true for $M_{50}$. Though on first sight the strong dependency of the price levels might seem to be the cause of the results observed, it cannot be more wrong. Nevertheless, I have come to the important conclusion that the initial barrier values proposed here do not show any sign of deviating from the theoretical amount of observations. Psychological barriers are thus not present at these values. I therefore continue the research by focusing on the lower level.

### *6.2. Lower level*

For the higher level I did not find any evidence supporting the existence of psychological barriers. To eliminate the possibility that this is caused by the selection of wrong initial barriers at the lower level, I use the barrier proximity test as a selection mechanism. That means all the eight *M*-values of period one will undergo the barrier proximity test, the same holds for the sixteen *M*-values for period two and hundred *M*-values for period three. The *M*-values which satisfy two requirements will be set as initial barriers. First, many of the ten stocks have to be rejected. Second, the number of rejections has to be relative constant over all sub-periods. Of course, the definition of “all sub-periods” differs per *M*-value. $M_{99}$ for example will only occur in period three and not in the first two. After the selection of the initial barrier value, the conditional coverage tests will be applied to gain more information.

In section 4 it has been explained that for period one and two only range 0 will be tested. This is because of the large gaps between the *M*-values produced by the fractional pricing. The nearest *M*-value to any initial barrier value is at least twelve cents for period one and six cents for period two. Both of them thus fall outside the range of 5. It is hard to argue that these "neighbouring" *M*-values will be of any significant influence, they are therefore left out of the research and only range 0 is considered.

**Table 6 Barrier Proximity lower level results with $M_{00}$**

| Stocks | Per 1 | Per 2 | Per 3 (0) | Per 3 (2) | Per 3 (5) |
|---|---|---|---|---|---|
| C | 0.0353** | 0.0582** | 0.0114** | 0.0007 | 0.0007 |
| BAC | 0.0453** | 0.0634** | 0.0149** | 0.0032** | 0.0015 |
| GE | 0.0354** | 0.0722** | 0.0188** | 0.0030 | 0.0021 |
| MSFT | 0.0964 | 0.0446 | 0.0049* | 0.0024** | 0.0007 |
| JPM | 0.0218 | 0.0438** | 0.0083** | 0.0021 | 0.0013 |
| F | 0.0136 | 0.0399* | 0.0086** | 0.0010 | 0.0010 |
| T | 0.0359** | 0.0708** | 0.0090** | 0.0018 | 0.0016 |
| PFE | 0.0387** | 0.0797** | 0.0118** | 0.0027* | 0.0017 |
| INTC | 0.0849 | 0.0440** | 0.0034 | 0.0017** | 0.0004 |
| LOW | 0.0316** | 0.0466** | 0.0161** | 0.0026 | 0.0010 |

*Note: the table presents the test statistics of equation (4) for all periods at range 0 and range 2 and 5 for period three. The */** denote the coefficient is significant at the 5% and 1% significance level respectively.*

The above table contains the results of the barrier proximity test for $M_{00}$ for the three sub-periods and the corresponding ranges. Merely the initial barrier value of $M_{00}$ is shown, because it is the only one which satisfies both the aforementioned requirements. Though some of the other *M*-values showed enough rejections, they were not consistent over the periods. First of all notice that all the coefficients are positive, meaning that price clustering is observed among the individual stocks for all periods. For period one, six of the ten stocks are rejected at 1% significance level; this gives some indication of price clustering. Period two and period three range 0 are strongly rejected, since nine of the ten stocks are significant and eight of the ten even significant at 1%. It is interesting to observe that although $M_{00}$ is very significant, other *M*-values close to it do not show any sign of similar results. This means that investors very much concentrate on trading at round prices while other prices are distributed uniformly.

Furthermore it is remarkable to see that the number of significant stocks reduces for longer ranges in period three and disappears even completely at the range of five. This should however be straightforward with the following reasoning. As mentioned before, only the coefficients of $M_{00}$ are significantly positive while the neighbouring *M*-values are uniformly distributed. When the ranges increase, the frequency of $M_{00}$ will play a less significant role in the total amount of observations and will thus be more uniformly distributed. The observation of a reduced number of significant stocks at range 2 and complete disappearance at range 5 supports this explanation.

The results indicate that a price clustering is present and not so much a psychological barrier. $M_{00}$ will be set as the initial barrier level and the research will continue focusing hereon. Specifically, the unconditional coverage, independence and conditional coverage tests will be applied.

**Table 7 Unconditional coverage lower level results $M_{00}$**

| Stocks | Per 1 | Per 2 | Per 3 (0) | Per 3 (2) | Per 3 (5) |
|---|---|---|---|---|---|
| C | 13.36** | 36.80** | 25.04** | 0.60 | 1.32 |
| BAC | 21.63** | 42.87** | 40.25** | 11.72** | 5.33* |
| GE | 13.43** | 54.51** | 60.19** | 10.07** | 10.71** |
| MSFT | 47.57** | 43.42** | 5.50** | 7.01** | 1.40 |
| JPM | 5.23* | 21.82** | 14.04** | 5.05* | 4.02* |
| F | 2.07 | 18.39** | 15.27** | 1.26 | 2.30 |
| T | 13.80** | 52.75** | 16.55** | 3.96* | 6.20* |
| PFE | 15.96** | 63.90** | 26.59** | 8.54** | 7.14** |
| INTC | 36.28** | 42.31** | 2.67** | 3.51 | 0.38 |
| LOW | 10.78** | 24.65** | 45.92** | 7.59** | 2.69 |

*Note: the table presents the test statistics of equation (6) at initial barrier level $M_{00}$ for all periods at range 0 and range 2 and 5 for period three. The */** denote the coefficient is significant at the 5% and 1% significance level respectively.*

Table 7 gives the results of the unconditional coverage test for the lower level. The null hypothesis of $E[I_{n,t}] = p_n$ is rejected at 1% significance level by all stocks in the second and third column. The first column gives nine rejections, which also indicates strong overall rejection. This corresponds with the findings in Table 6 which means that the frequency at $M_{00}$ is not uniformly distributed. Combining with the positive coefficients from the *barrier proximity* test, it can be concluded that the frequency of barrier *M*-values are significantly higher. Note that the unconditional coverage test rejects the null hypothesis more easily than the barrier proximity test. Column one holds nine significant test statistics while there are only six for the barrier proximity test. Columns four and five both have more significant values than the results from equation (4). A similar trend can be observed in period three, that is the longer the range the more the barrier values are uniformly distributed. This is, as mentioned before, caused by the uniform distribution of the *M*-values neighbouring $M_{00}$.

Overall, it can be concluded that the frequency of barriers are not uniformly distributed which is in line with the expectation. However, instead of a psychological barrier, which should lead to significant fewer observations, significant more is found. This indicates price clustering at $M_{00}$, representing every possible round price level. Especially investors seem to have a strict preference for round numbers. While clustering is observed for $M_{00}$, no such conclusions can be made for neighbouring *M*-values.

But it is too soon to draw the final conclusion yet; dependency among stocks can still play a significant role. The sample period may very well consist of concentration of ones and zeroes and therefore contaminate our findings for uniformity. It is only possible to conclude price clustering when independency holds true.

Table 8 gives the results of the Independence test for the lower level. Period one and two shows no significant test results at all implying that independency holds and the value of a barrier is thus not determined by the preceding day. Significant results are however observed for period three. Two rejections for both range 0 and 2 and four for range 5. For three stocks at range 0 the result are NaN, this is again due to the lack of any observations of two consecutive barrier value days. Like explained before, in this particular case the NaN can be interpreted as independency among stock prices and it can therefore be concluded that only two of the ten stocks are significant at range 0. The increase in significant stocks for range 5 is expected. This is explained by following the reasoning for the independence test for the higher level; as the range becomes longer, the probability of price level staying a barrier increases. Tough there are some rejections for period three, but looking at the bigger picture there is much stronger evidence supporting independency among individual stocks. The overall conclusion is then that the stock prices are independently distributed. This is a very important as it thus can be concluded that the price clustering found in Table 7 are caused by the underlying distribution. Investors therefor have a strict preference for round number prices.

**Table 8 Independence test lower level results**

| Stocks | Per 1 | Per 2 | Per 3 (0) | Per 3 (2) | Per 3 (5) |
|---|---|---|---|---|---|
| C | 0.81 | 0.34 | 0.51 | 0.96 | 11.46** |
| BAC | 0.34 | 1.39 | NaN | 3.03 | 0.02 |
| GE | 0.96 | 1.95 | 7.66** | 6.23* | 6.19* |
| MSFT | 0.01 | 0.03 | 0.28 | 1.16 | 2.22 |
| JPM | 0.62 | 0.03 | NaN | 0.83 | 0.26 |
| F | 0.80 | 1.37 | NaN | 3.31* | 27.38** |
| T | 1.64 | 2.18 | 0.01 | 0.51 | 1.46 |
| PFE | 0.06 | 0.87 | 10.48** | 0.77 | 11.70** |
| INTC | 1.18 | 0.09 | 2.97 | 1.34 | 1.66 |
| LOW | 0.12 | 1.36 | 0.04 | 0.93 | 8.55** |

*Note: the table presents the test statistics of equation (7) at initial barrier level $M_{00}$ for all periods at range 0 and range 2 and 5 for period three. The */** denote the coefficient is significant at the 5% and 1% significance level respectively.*

Finally B. 6 lists the results of the conditional coverage test. The main part of these test statistics consists of the unconditional coverage test results, since the independent test was generally not rejected. Equation (8) basically tests whether the unconditional coverage test statistic is also enough to reject the null hypothesis at a $\chi^2(2)$ distribution. It can be observed that rejections are the result for (almost) all stocks in period one and two. Period three range 0 contains three NaN, however these NaN are because of the lack of test results for the independence test. It can be proved that for these three stocks the conditional coverage test would be rejected at 1% significance level solely based on the unconditional coverage test results. There are therefore eight rejections for period three range 0. Not surprisingly, range 2 and 5 show similar results too. It can be concluded that the conditional coverage test is generally rejected for all of the three periods.

Combining the barrier proximity, unconditional coverage, independence and conditional coverage tests I come to the conclusion of price clustering among individual stocks at round price levels. Investors have a strict preference for round prices as neighbouring *M*-values do not show tendency of deviation from the theoretical frequency. This however does not necessarily mean psychological barriers are completely absent; they are only not observable. Since price clustering and psychological barriers may both occur at round price levels, it is possible that the effect of price clustering is more present and thus outweighs that of the barriers. There is one implication for the absence of psychological barriers however. Notice that the band technique effect should be present; that is *M*-values around the initial barrier should also show reduced frequency. Remember that this is not the case since no deviations were found for these *M*-values.

## 7. Conclusion

In this paper I have researched the existence of psychological barriers in ten frequently traded stocks in the United States. This was done by first redefining and reclassifying the observed daily closing price values of the stocks of the past twenty years into *M*-values and then testing for uniformity and independence using several methods. Especially both the higher level (concerning the second- and third last trailing digit) and the lower level (concerning the second last and the last trailing digit) have been examined.

The higher level was tested for the initial barrier level of $M_{00}$ and $M_{50}$. These *M*-values correspond with the multiples of ten and five dollars. Due to the pricing mechanisms however, traditional testing methods could not be used. Instead the unconditional coverage test and independence test were introduced. The results of the unconditional tests were contradicting since a bell shape distribution for $M_{00}$ and a reversed bell shape for $M_{50}$ were observed with increasing ranges. However, both the initial barrier values could not be rejected meaning that a psychological barrier is not present. The independence test further showed high dependency among stocks, but this is logical and was expected for the higher level.

The research continued by looking at the lower level because the higher level results did not lead to any satisfying conclusions. The barrier proximity test is first performed for every possible *M*-value to select the initial barrier values. It appeared that unlike other *M*-values, $M_{00}$ was the only one which gave consistent and strong rejections. The coefficients of the tests were however all positive indicating presence of price clustering. The unconditional coverage test confirms these results. The independency test shows that independency holds for all periods. Combined with the fact that all other *M*-values are uniformly distributed, it can be concluded that investors have a strict preference for round prices. This also meant that the band technique effect is absent, one that should be observed with the presence psychological barriers.

Nonetheless, it cannot be concluded that psychological barriers are completely absent in individual stocks. It is possible that both price clustering and psychological barriers are situated at the same initial barrier $M_{00}$. In this case the observation might be caused by a stronger effect of price clustering.

I conclude that psychological barriers cannot be found in individual stocks in the United States for both the higher and the lower level. Price clustering appears however at the lower level, especially at $M_{00}$ which represents round price levels. This is the opposite of what I have expected to find. Investors seem to have a strict preference for round prices as no other $M$-values reveals similar results. It is however not possible to conclude the non-existence of psychological barriers only based on price clustering; this is left to further research.

## 8. Discussion

This research has found statistical evidence for price clustering at round dollar prices. There is room for improvement however and especially flaws might be present in the use of data and methods, I discuss some of them here.

First of all the use of ten stocks in this research should be enough to represent the vast majority of stocks. However the use of different stocks both in amount, period and location could affect the outcome. Especially including more companies representing different industries will lead to more accurate results.

Due to the different pricing mechanisms used in the United States the amount of observations for each period is very limited. This can influence the test results of sixteenth fractional pricing. The use of stocks from other countries or the use of high frequency data is a solution to this problem and will lead to a more robust outcome.

The lack of data has furthermore caused NaN results for the independence test in some periods. Though I have argued that in this particular case it can be viewed as support for the null hypothesis, it cannot be proved. The only statement that can be made is: no rejection is more probable than rejection of the null hypothesis. Especially the conclusion of independency among $M$-values for the lower value is based on the previous assumption and might thus be less convincing.

The currently used definition for psychological barriers is the most accepted one. However, as it appeared from this research, one definition is not enough to cover all circumstances. If it is possible to give a new or alternative definition of psychological barriers, this will greatly improve future research.

## References


Aggarwal, R., & Lucey, B. M. (2007). Psychological barriers in gold prices? *Review of Financial Economics*, 217-230.

Christoffersen, P. F. (1998). Evaluating interval forecasts. *International Economic Review*, 841-862.

De Ceuster, M. J., Dhaene, G., & Schatteman, T. (1998). On the hypothesis of psychological barriers in stock markets and Benford's Law. *Journal of Empirical Finance*, 263-279.

Donaldson, R. G., & Kim, H. Y. (1993). Price barriers in the Dow Jones Industrial Average . *Journal of Financial and Quantitative Analysis*, 313-330.

Dorfleitner, G., & Klein, C. (2009). Psychological barriers in European stock markets: Where are they? *Global Finance Journal*, 268-285.

Koedijk, K. G., & Stork, P. A. (1994). Should we care? Psychological barriers in stock markets. *Economics Letters* , 427-432.

Ley, E., & Varian, H. R. (1994). Are there psychological barriers in the Dow-Jones index? *Applied Financial Economics*, 217-224.

Mithchell, J., & Izan, H. (2006). Clustering and psychological barriers in exchange rates. *International Financial Markets, Institutions and Money*, 318-344.

Sonnemans, J. (2006). Price clustering and natural resisitance points in the Dutch stock market: A natural experiment. *European Economic Review*, 1937-1950.

# Appendix A

This part gives a brief overview of what would happen when conventional testing methods are applied to the dataset without taking the price mechanisms into account. For this part the initial barrier has been set at $M_{00}$ and $M_{50}$ with each a range of 2 and 5. Since no considerations are taken for the pricing mechanisms the dataset will be divided in a few ways. First of all the whole period is tested by the Pearson's chi square test. Then the barrier proximity test is performed for the whole period and two sub-periods. Sub-period one ranges from 1991 to 1999 which is a relevant stable period of growth. Sub-period two is from 2000 to 2011 and is featured as a turbulent period. The point of view is that the results given in the paper are not yet known.

Following Dofrleitner and Klein (2009) the test for uniform distribution in the *M*-values is conducted by dividing them into ten equal parts with the first ranging from $M_{00}$ to $M_{09}$, second from $M_{10}$ to $M_{19}$ and so on. The Pearson's chi-square test, given by equation (3), is then applied to see whether the ten parts are equally distributed. The null hypothesis is then $H_0$: $p(M_{00\text{-}09}) = p(M_{10\text{-}19}) = p(M_{20\text{-}29})$... meaning that all the ten parts are equally distributed which corresponds to ten per cent of the total observations for each group.

The results of the Pearson's chi-square test in equation (3) for each stock are presented in A. 1. Both the test statistics and the corresponding *p*-values are given. As can be seen, all the tests for all the stocks are rejected at 1% significance level, this confirms that the *M*-values are not uniform. However, no conclusions can be made about psychological barriers yet.

**A. 1 Chi-square test results equation (3)**

| Stocks | $\chi^2$ for 10 groups | *p*-value |
|---|---|---|
| C | 355.32 | 0.000 |
| BAC | 60.74 | 0.000 |
| GE | 188.05 | 0.000 |
| MSFT | 184.62 | 0.000 |
| JPM | 61.97 | 0.000 |
| F | 137.54 | 0.000 |
| T | 106.52 | 0.000 |
| PFE | 140.47 | 0.000 |
| INTC | 70.72 | 0.000 |
| LOW | 23.14 | 0.003 |

*Note: the table presents the test statistics and the corresponding p-value for ten groups and hundred groups.*

I continue to test for uniformity by following Donaldson and Kim (1993), they suggest the *barrier proximity* and the *barrier hump* represented by equation (4) and (5) respectively. Table A. 2 presents the estimations of $\beta$ described by the barrier proximity test. Since the $\alpha$'s are all equal to 0.10, they are not presented. When a closer look is taken at A. 2, it can be observed that the coefficients of five $\beta$'s are significantly different from zero, only one of them is significantly negative however. It is therefore concluded that no hard evidence can be found which supports the existence of psychological barriers, even more it seems the price levels rather concentrates around the barrier which is surprising.

**A. 2 Results $\beta$ estimation barrier proximity equation (4) whole period**

| Stocks | $M_{00}$ (2) | $M_{50}$ (2) | $M_{00}$ (5) | $M_{50}$ (5) |
|---|---|---|---|---|
| C | -0.0008 | 0.0026 | -0.0012 | 0.0025* |
| BAC | -0.0010 | 0.0027 | -0.0010 | 0.0016 |
| GE | -0.0011 | 0.0000 | -0.0017 | 0.0006 |
| MSFT | 0.0018 | 0.0012 | 0.0002 | 0.0017 |
| JPM | -0.0004 | 0.0018 | -0.0004 | 0.0014 |
| F | -0.0001 | 0.0019 | -0.0015 | 0.0004 |
| T | 0.0000 | 0.0019 | -0.0010 | 0.0017 |
| PFE | -0.0014 | 0.0033* | -0.0020* | 0.0024* |
| INTC | 0.0023 | -0.0015 | 0.0020** | -0.0012 |
| LOW | -0.0014 | 0.0014 | -0.0010 | 0.0011 |

*Note: the table presents the estimation of $\beta$ of equation (4). The */** denote the coefficient is significant at the 5% and 1% significance level respectively.*

Table A. 3 presents the results of the barrier proximity test for the sub-periods. It is remarkable to see that at 5% significance level no significant $\beta$ is found at all between 1991 and 1999, while sixteen $\beta$'s are significant different from zero between 2000 and 2011. However, when a closer look is taken, I come to the conclusion that only six out of the sixteen coefficients are negative. That is, even though significant coefficients are found for many stocks, it cannot be said for certain a barrier effect is present. Even more it seems the frequency is higher for barrier values.

**A. 3 Results $\beta$estimation barrier proximity equation (4) 1991-1999**

| Stocks | 1991-1999 | | | | 2000-2011 | | | |
|---|---|---|---|---|---|---|---|---|
| | $M_{00}$ (2) | $M_{50}$ (2) | $M_{00}$ (5) | $M_{50}$ (5) | $M_{00}$ (2) | $M_{50}$ (2) | $M_{00}$ (5) | $M_{50}$ (5) |
| C | 0.0001 | 0.0008 | -0.0007 | 0.0014 | -0.0016 | 0.0040 | -0.0016 | 0.0034* |
| BAC | 0.0014 | 0.0031 | 0.0008 | 0.0019 | -0.0029* | 0.0024 | -0.0024** | 0.0015 |
| GE | -0.0018 | 0.0027 | -0.0017 | 0.0012 | -0.0005 | -0.0022 | -0.0017 | 0.0000 |
| MSFT | 0.0030 | 0.0016 | 0.0015 | 0.0011 | 0.0009 | 0.0008 | -0.0008 | 0.0022 |
| JPM | 0.0003 | -0.0013 | 0.0007 | -0.0015 | -0.0010 | 0.0043* | -0.0012 | 0.0037** |
| F | 0.0002 | 0.0005 | 0.0004 | -0.0001 | -0.0004 | 0.0031 | -0.0030* | 0.0008 |
| T | 0.0041 | -0.0014 | 0.0027 | -0.0023 | -0.0032 | 0.0044* | -0.0040** | 0.0049** |
| PFE | -0.0006 | -0.0005 | -0.0002 | 0.0002 | -0.0021 | 0.0064** | -0.0034** | 0.0043** |
| INTC | 0.0001 | -0.0003 | 0.0013 | 0.0009 | 0.0041* | -0.0024 | 0.0025** | -0.0028** |
| LOW | -0.0009 | 0.0016 | 0.0000 | 0.0012 | -0.0018 | 0.0013 | -0.0017* | 0.0011 |

*Note: There are no significant $\beta$ at all at 5% significance level for 1991 - 1999. There are however many more for 2000 – 2011. However the amount of significant negative $\beta$ is only 8 out of 17. This means that it is approximately half to half negative and positive.*

In table A. 4 the outcomes of equation (5) are given for the whole period, period 1 and period 2. $H_0$ is only rejected for two stocks in period one, with one positive and one negative coefficient. This is very unlike period 2 where the $\gamma$ is significant different from zero for all stocks and with only one positive coefficient. Rejection of the $H_0$ with a negative coefficient implies the existence of barrier which is a contradiction of the results of equation (4).

**A. 4 Results $\gamma$ estimation barrier hump equation (5) whole period**

| Stocks | Whole period | 1991-1999 | 2000-2011 |
|---|---|---|---|
| C | -0.0000016** | -0.0000006 | -0.0000025** |
| BAC | -0.0000011** | -0.0000002 | -0.0000018** |
| GE | -0.0000016** | -0.0000014 | -0.0000017** |
| MSFT | -0.0000004 | 0.0000004 | -0.0000011** |
| JPM | -0.0000004 | 0.0000007 | -0.0000013** |
| F | -0.0000005 | 0.0000007 | -0.0000014** |
| T | -0.0000013** | 0.0000018** | -0.0000037** |
| PFE | -0.0000018** | -0.0000003 | -0.0000029** |
| INTC | 0.0000013** | 0.0000002 | 0.0000022** |
| LOW | -0.0000004 | 0.0000000 | -0.0000008** |

*Note: the table presents the estimation of $\alpha$ of equation (5). The */** denote the coefficient is significant at the 5% and 1% significance level respectively.*

The test results described above are very contradicting. The Pearson's chi square test confirms that the stocks are not uniformly distributed. The barrier proximity test however found little evidence to support this conclusion. The barrier values do not seem to deviate much from the theoretical value. Special attention should be paid for the barrier proximity test results of the two sub-periods. While no rejections were found at all for the period from 1991 to 1999, sixteen were found for 2000 to 2011. More importantly, the test results seem to point in the direction of price clustering instead of psychological barriers. The most significant coefficients tend to be positive instead of negative. On first sight the barrier hump test looks promising as the most test statistics are negative. However a separate test on the sub-periods shows similar result as before: very few rejections for sub-period one and many for sub-period two.

Obviously, the period of the observations significantly influence the test results. A thorough examination on the dataset should be performed before any further research is executed.

# Appendix B

**B. 1 Abbreviations of stocks**

| Abbreviations | Stocks | Abbreviations | Stocks |
|---|---|---|---|
| C | Citigroup | F | Ford |
| BAC | Bank of America | T | AT&T |
| GE | General Electric | PFE | Pfizer |
| MSFT | Microsoft | INTC | Intel |
| JPM | JP Morgan Chase | LOW | Lowe's Companies |

**B. 2 Higher level amount of observations compared to average for $M_{00}$**

| Stocks | Per 1 (0) | Per 1 (2) | Per 1 (5) | Per 2 (0) | Per 2 (2) | Per 2 (5) | Per 3 (0) | Per 3 (2) | Per 3 (5) |
|---|---|---|---|---|---|---|---|---|---|
| C | H | H | L | H | H | H | L | L | L |
| BAC | H | H | H | H | H | H | L | L | L |
| GE | L | L | L | H | H | H | L | L | L |
| MSFT | H | H | H | H | H | H | L | L | L |
| JPM | H | H | H | H | H | L | L | L | L |
| F | H | H | H | L | H | L | H | L | L |
| T | H | H | H | H | H | H | L | L | L |
| PFE | H | H | H | L | L | L | L | L | L |
| INTC | H | H | H | H | H | H | H | H | H |
| LOW | H | L | H | H | H | L | E | L | L |

*Note: the table presents whether the observed amount of observations is higher(H), lower(L) or equal(E) to the theoretical one.*

**B. 3 Higher level amount of observations compared to average for $M_{50}$**

| Stocks | Per 1 (0) | Per 1 (2) | Per 1 (5) | Per 2 (0) | Per 2 (2) | Per 2 (5) | Per 3 (0) | Per 3 (2) | Per 3 (5) |
|---|---|---|---|---|---|---|---|---|---|
| C | H | E | H | H | H | H | H | H | H |
| BAC | H | H | H | H | H | H | H | H | H |
| GE | H | H | H | E | L | L | L | L | L |
| MSFT | E | L | H | H | H | H | H | H | H |
| JPM | L | L | L | H | H | H | H | H | H |
| F | L | L | L | H | H | H | H | H | E |
| T | L | L | L | H | H | L | H | H | H |
| PFE | H | L | L | H | H | H | H | H | H |
| INTC | H | L | H | H | H | H | L | L | L |
| LOW | L | H | L | H | H | H | H | H | H |

*Note: the table presents whether the observed amount of observations is higher(H), lower(L) or equal(E) to the theoretical one.*

### B. 4 Independence test higher level results for $M_{00}$

| Stocks | Per 1 (0) | Per 1 (2) | Per 1 (5) | Per 2 (0) | Per 2 (2) | Per 2 (5) | Per 3 (0) | Per 3 (2) | Per 3 (5) |
|---|---|---|---|---|---|---|---|---|---|
| C | 12.06** | 51.46** | 196** | NaN | 9.36** | 54** | 2.11 | 71** | 332** |
| BAC | NaN | 24.36** | 209** | 1.59 | 5.29* | 25** | 5.50* | 19** | 225** |
| GE | NaN | 10.07** | 161** | 0.44 | 3.29 | 19** | 26.56** | 128** | 461** |
| MSFT | 0.10 | 0.21 | 24** | 0.25 | 6.56* | 40** | 1.47 | 112** | 438** |
| JPM | 8.68** | 41.49** | 251** | 4.34* | 5.50* | 18** | 1.79 | 39** | 297** |
| F | 8.18** | 31.66** | 336** | NaN | 18.73** | 60** | 49.81** | 313** | 930** |
| T | 8.40** | 53.48** | 312** | 1.56 | 37.94** | 122** | 7.74** | 51** | 336** |
| PFE | 7.71** | 16.05** | 112** | NaN | 3.54 | 21** | 16.82** | 182** | 556** |
| INTC | 2.01 | 1.99 | 47** | 0.38 | 0.13 | 24** | 14.49** | 141** | 652** |
| LOW | 3.09 | 25.49** | 232** | NaN | 9.36** | 47** | 1.25 | 46** | 344** |

*Note: the table presents the estimation of α of equation (7). The */** denote the coefficient is significant at the 5% and 1% significance level respectively.*

### B. 5 Independence test higher level results for $M_{50}$

| Stocks | Per 1 (0) | Per 1 (2) | Per 1 (5) | Per 2 (0) | Per 2 (2) | Per 2 (5) | Per 3 (0) | Per 3 (2) | Per 3 (5) |
|---|---|---|---|---|---|---|---|---|---|
| C | NaN | 16.34** | 197** | NaN | 0.05 | 10.93** | 7.74** | 246** | 752** |
| BAC | 4.42* | 62.50** | 237** | 7.68** | 8.88** | 51.34** | 3.44 | 63** | 489** |
| GE | 1.11 | 22.20** | 195** | NaN | 2.91 | 28.73** | 6.16* | 185** | 625** |
| MSFT | NaN | 0.49 | 41** | NaN | 0.22 | 24.99** | 13.57** | 135** | 648** |
| JPM | 6.03* | 48.07** | 302** | NaN | 8.23** | 11.10** | 0.46 | 99** | 390** |
| F | NaN | 41.87** | 289** | 0.05 | 40.88** | 123** | 19.93** | 306** | 844** |
| T | 1.71 | 34.51** | 219** | 3.92* | 17.52** | 55.79** | 5.81* | 189** | 746** |
| PFE | 7.71** | 26.29** | 130** | 4.66* | 7.37** | 53.20** | 16.89** | 283** | 1001** |
| INTC | NaN | NaN | 29** | 4.49* | 1.19 | 23.73** | 2.26 | 60** | 420** |
| LOW | NaN | 30.77** | 273** | 2.86 | 8.36** | 25.65** | 16.94** | 108** | 305** |

*Note: the table presents the estimation of α of equation (7). The */** denote the coefficient is significant at the 5% and 1% significance level respectively.*

**B. 6 Table conditional coverage test**

| Stocks | Per 1 | Per 2 | Per 3 (0) | Per 3 (2) | Per 3 (5) |
|---|---|---|---|---|---|
| C | 14.17** | 37.14** | 25.55** | 1.55 | 12.78** |
| BAC | 21.96** | 44.26** | NaN | 14.75** | 5.35 |
| GE | 14.39** | 56.45** | 67.86** | 16.30** | 16.90** |
| MSFT | 47.58** | 43.45** | 5.77 | 8.17* | 3.62 |
| JPM | 5.85 | 21.85** | NaN | 5.88 | 4.29 |
| F | 2.87 | 19.76** | NaN | 4.57 | 29.68** |
| T | 15.45** | 54.93** | 16.55** | 4.47 | 7.67* |
| PFE | 16.02** | 64.77** | 37.07** | 9.31** | 18.83** |
| INTC | 37.46** | 42.40** | 5.65 | 4.85 | 2.04 |
| LOW | 10.90** | 26.01** | 45.96** | 8.52* | 11.24** |

*Note: the table presents the test statistics of equation (8) at initial barrier level $M_{00}$ for all periods at range 0 and range 2 and 5 for period three. The */** denote the coefficient is significant at the 5% and 1% significance level respectively.*