

# The joint optimization of a replacement policy and inventory control policies using response surface methodology

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## Abstract

This thesis considers the joint optimization of a replacement policy and inventory control policies. For the replacement policy the modified block replacement policy (MBRP) is used. As inventory policy the  $(s,S)$  policy is used. Joint ordering policies and separate ordering policies for both the orders for failure replacement and orders for block replacement are compared. Two new ordering policies are proposed and compared with two existing policies. Response surface methodology (RSM) is used with experimental design in order to determine the response surface function, which gives the relationship between the average costs and the input parameters. Using RSM for finding the optimal parameter settings led to some difficulties. The common used fractional factorial design did not give a model with high explanatory power and one of the main parameters did not have a significant influence on the costs. The results show that the proposed policies are an improvement, since they are more practical to implement than the existing policies and perform the same or only slightly poorer.

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## 1. Introduction

This thesis considers a case study from the soft drinks and juice manufacturer Menken Drinks. In their factory in Bodegraven, The Netherlands, this company has several bottling machines which fill carton packages with soft drinks. Such machines consist of many different components, which are all needed in order to let the bottling machine do its work. However if one or more of the components of the bottling machine fails, the machine cannot work anymore at its normal level. Menken Drinks has a service contract with the producer of their bottling machines, which states that maintenance is performed every 500 days. Components are replaced during those maintenance actions and the needed components are included in the service contract. If components fail before the preventive maintenance takes place, the company takes care of the replacement of the components itself. The company uses this replacement policy in order to prevent failure of the components and stagnation of the machine. This thesis will look at one specific part of the bottling machine, the filling pipe, which costs €6000 per filling pipe. The term 'item' will be used to refer to the filling pipe from now on. The other components of the machine together have a total value of €2 million. The company only keeps a safety stock of relatively cheap components that have an expected lifetime of shorter than one year. The item considered in this thesis has a longer expected lifetime and is too expensive to keep in stock. If the filling pipe fails, a new item must be ordered and the machine breaks down, which costs €960 per hour. Therefore taking a close look at the maintenance and inventory policies and trying to find improvements is an interesting topic for companies like Menken Drinks.

This thesis will consider the maintenance policy and inventory control of items. There are different costs involved in this problem. In a production environment failure of one of the items of a machine causes the machine to break down, which gives shortage costs. Failed items therefore must be replaced quickly by new items from stock. Companies need to determine a maintenance policy to replace items at failure as well as replace items in order to prevent failure. Both types of replacements have different replacement costs. The moment of failure of an item is unknown and therefore a company can decide to keep safety stock of items. Keeping items in stock results in inventory costs and therefore a large safety stock is expensive. An inventory control policy which controls the ordering and stocking of items can be helpful. The total costs for the maintenance contain replacement and shortage costs and the inventory costs can be split into holding costs and ordering costs.

Usually the maintenance policy and the inventory control are considered and optimized separately. The assumption in maintenance policies then is that items are always available. This assumption however does not reflect reality and therefore joint optimization is needed. This thesis will focus on the joint optimization of replacement policy and inventory control in order to minimize the total average costs.

The modified block replacement policy (MBRP) of Archibald and Dekker [4] will be used as maintenance policy. This policy replaces items at failure or periodically during block replacements. The interval between two block replacements is  $T$  and the block replacements are performed every time  $kT$  ( $k=1,2,3,\dots$ ). During a block replacement only items with an age above a specified age threshold will be replaced. This age threshold is defined as a fraction of the time between two block replacements. The age threshold factor is  $h$  and the age threshold then becomes  $hT$ . The  $(s,S)$  model will be used for

inventory control. This model orders new items when the inventory position drops below  $s$ . The inventory position is defined as the inventory level together with the outstanding orders. The maximum number of items in stock is  $S$  and therefore when items are ordered to raise the inventory level a number of  $S - \text{inventory level items}$  is ordered. The following parameters will be considered:

- optimal interval between block replacements ( $T$ )
- the age threshold factor ( $h$ ), where  $hT$  is the age threshold
- reorder point ( $s$ )
- maximum stock level ( $S$ )

Mardin [10] showed that more advanced coordination between inventory and maintenance can lower costs. The best performing policy is the  $(s,S,T)$  Skip – Separate, BR order  $N - \#skip$  policy. In this policy the items needed for block replacement and those for failure replacement are ordered separately. However this policy has a large drawback: it uses the information about the life time of the items which can be known beforehand in a simulation model, but is unknown in practice. Thus although this policy performs well on paper, it cannot be implemented in practice. This thesis will consider two policies from literature: the joint optimization based on the MBRP with joint ordering and the mentioned policy of Mardin [10]. Also two new policies with MBRP and separate ordering will be proposed, which are more practical to implement. These policies will be based on the best policy of Mardin [10], but will be adjusted such that they can be used in practice.

The different policies will be compared and the parameters will be optimized in order to minimize the long run average costs, which are equal to the total cost per time period.

This thesis will start with a literature review about maintenance and inventory policies in chapter 2. The chapters 3 and 4 will discuss respectively the case study of Menken Drinks and the simulation model and methodology which are used to find the optimal parameter setting. In Chapter 5 the proposed policies are described and compared with existing policies. In the final chapter, chapter 6, the conclusions will be presented.

## 2. Literature review

Maintenance and inventory of items of machines are two closely related topics. Replacement is only possible if the needed items are available. The optimization of maintenance policies and the optimization of inventory control have been considered as two separate problems. More recently these problems are combined and joint optimization models have been developed.

Barlow and Hunter [1] introduced two replacement policies: the ARP and the BRP. The age replacement policy (ARP) consists of replacing the items upon failure or when they reach a pre-defined age. The block replacement policy (BRP) on the other hand replaces the items at failure and periodically at  $kT$  ( $k=1,2,\dots$ ), irrespective of the age. Barlow and Proschan [2] showed that the ARP performs better than the BRP does. The BRP however seems to be more practical to implement, since keeping track of the age of all the items is not necessary.

Berg and Epstein [3] introduced a modified block replacement policy (MBRP). Archibald and Dekker [4] extended this policy for multiple components. The MBRP replaces the items at failure and preventively at regular time intervals. During preventive maintenance however not all items are replaced, but only those which are older than a fixed age threshold. Archibald and Dekker [4] showed that the MBRP performs significantly better than the BRP and only slightly poorer than the ARP.

Tango [5] extended the BRP by replacing items with new items if they fail in  $[(k-1)T, kT-v)$  and replacing them with used items if they fail in  $[kT-v, kT)$ . Tango [6] modified Tango [5] by using less reliable items for replacement if an item fails in  $[kT-v, kT)$ .

Previous maintenance policies assumed that the items needed are always available. However the assumption of infinite inventory does not reflect reality. Therefore models have been developed in which the maintenance policy and the inventory control are jointly optimized.

Kabir and Al-Olayan [7] introduced a joint optimization model based on the age replacement policy. Their model is known as the  $(s,S,T)$  model, where  $s$  is the reorder level,  $S$  is the maximum stock level and  $T$  is the preventive replacement time. Sarker and Haque [8] also developed a  $(s,S,T)$  model, but now based on the block replacement. In that model parameter  $T$  stands for the time interval length between two block replacements. Both studies use simulations to determine the optimal values for the variables  $(s, S$  and  $T)$  and conclude that joint optimization performs better than the separate optimization of Barlow and Proschan [2].

Regz et al [9] constructed a joint optimization model based on the ARP policy and an inventory control policy. This model uses two approaches to determine the optimal parameter values. The model is analytically analyzed and a solving method based on simulation and experimental design is used. The results show that both approaches give results of the same order and that the second approach can be used in situations which are difficult to study analytically.

Mardin [10] considers the  $(s,S,T)$  model based on MBRP, with a so called BR order  $N - \#skip$  policy for the inventory control. When there are  $N$  items in total and during the block replacement  $\#skip$  items have

an age below the age threshold, only  $N - \#skip$  items need to be replaced. Those items are ordered one lead time before the block replacement and orders for failure replacement and block replacement are separated. This policy is referred to as the  $(s,S,T)$  Skip – Separate, BR order  $N - \#skip$  policy and performs better than both the  $(s,S,T)$  model based on MBRP with a joint ordering policy and the  $(s,S,T)$  model based on BRP with the ordering of  $N$  items one lead time before the block replacement.

Mardin [10] compares the performance of policies by using an optimization package in order to obtain the optimal parameter setting. Other papers, which search for the optimal parameter values for a specific policy, assume that the relation between the parameters in models and the objective function follows a response surface function. Response surface methodology is the process in which the response surface function is determined via ANOVA and regressions. With the response surface function, the optimal parameter values can be determined. This method is used in the joint optimization of maintenance and inventory by Rezg et al [9], Boulet et al [11] and Berthaut et al [12] among others.

Boulet et al [11] consider a joint optimization with the BRP policy for maintenance and the  $(s,Q)$  inventory model. The article uses a multiobjective optimization with both minimizing the costs and maximizing the availability of the system. A model is proposed in which experimental design, simulation and response surface methodology are used to simultaneously optimize the objectives. They show that this multiobjective model can be used to determine the best parameters.

Berthaut et al [12] proposed a model in which the inventory of the final product is taken into account. An inventory policy based on hedging point policy (HPP) is used, where a safety stock of the final product is built up in order to avoid shortages during maintenance of the machines. Simulation, experimental design and response surface methods are used to determine the optimal joint parameters. The authors showed that MBRP/HPP performs better than BRP/HPP.

This thesis will focus on joint optimization of maintenance and inventory control. The  $(s,S,h,T)$  model based on MBRP will be considered. The parameter  $h$  in this model is the age threshold factor and  $hT$  is the age threshold. Only items with an age higher than the age threshold will be replaced during block replacements. The joint ordering policy and the best policy of Mardin [10] will be analyzed. The last policy is a separate order policy in which the items needed for the block replacement are ordered one lead time before the block replacement takes place and items needed for failure are ordered when the inventory position drops below a given level. This policy, which is called the  $(s,S,T)$  Skip – Separate, BR order  $N - \#skip$  policy however uses information about the life time of items from the simulation model, which is not known in reality. Therefore this policy cannot be used in practice. This thesis proposes two new policies which have the same ordering method as the best policy of Mardin [10], but are adjusted such that they are more practically to implement. The two proposed policies are compared with the MBRP with joint ordering and with the  $(s,S,T)$  Skip – Separate, BR order  $N - \#skip$  policy from Mardin [10]. For these four policies the optimal value of the following parameters need to be determined:

- T optimal time interval between two consecutive block replacements
- h age threshold factor, where  $hT$  is the age threshold
- s reorder point
- S maximum stock level

The method of Boulet et al [11] with experimental design, simulation and response surface methodology will be used in order to determine the optimal parameter values. The commonly used designs, such as the Box-Behnken design, assume that there are no constraints on combinations of parameters. However in this problem the parameters  $s$ ,  $S$ ,  $h$  and  $T$  are not completely independent. The reorder point cannot be larger than the maximum stock level, which means that  $s$  must be smaller than  $S$ .

Jia and Chinnam [15] investigate the  $(s,S,T)$  model with response surface methodology and experimental design. They choose central composite design to generate experiments. The levels of their parameters are chosen such that the lowest value for  $S$  is smaller than the largest value for  $s$ . Therefore in a specific experiment, it is possible that the constraint  $s < S$  is violated. In their article Jia and Chinnam [15] however do not mention anything about possible solutions to prevent this or about adjustments to the regular procedure.

This thesis will change the parameters slightly so that the common designs can be used. The parameters  $h$  and  $T$  will remain unchanged. However instead of using  $s$  and  $S$ , the parameters  $s$  and  $k$  will be used, where  $k$  is defined as  $S-s$ . Now there is no restriction on the parameters any more.

Most papers about joint optimization of maintenance policies and inventory control use simple self constructed problems for their analysis. This thesis will use a case study from the soft drinks and juice manufacturer Menken Drinks.

### 3. Problem description

This thesis considers a case study from the soft drinks and juice manufacturer Menken Drinks. This company has a bottling machine, the A3 speed, with a large number of components. For this study only one item, namely the filling pipe, is considered. This item is one of the most expensive components of the machine and costs € 6000. Every machine contains one filling pipe and the machine cannot work without this item. For this thesis we assume that the manufacturer has eight bottling machines.

There are different costs involved in this maintenance and inventory problem: the ordering, replacement, shortage and storage costs. Keeping one item in stock for one year costs € 1110 per year per item. The shortage costs are very high, since the entire machine breaks down at the failure of the filling pipe. A machine that is not running costs € 23040 per day. Replacements of the items cost € 555 per replacement per item if it is during a block replacement and € 1665 per replacement per item if the item is replaced at failure. In these replacement costs the costs for stagnation of the machine during the replacement is incorporated. There are two types of orders: normal orders and emergency orders. The normal order set up costs are € 0 independent of the size of the order and a normal order arrives after three weeks. The price for transport and the ordering costs are already incorporated in the price of the item. The set up costs for emergency orders are € 600 and emergency orders arrive after 24 hours. The time unit of a day is used and the cost values can be found in table 1.

There are two different orders which can be placed, with each different order set up costs and a different lead time. Emergency orders arrive quickly after ordering and are used if the inventory position drops below  $s$ . Normal orders have a larger lead time and are used if items are ordered in advance for the block replacement.

		Value
<b>Holding costs</b>		€ 3.04 / day / item
<b>Shortage costs</b>		€ 23040 / day / item
<b>Price</b>		€ 6000 / item
<b>Order set up costs</b>	<i>Normal</i>	€ 0 / order
	<i>Emergency</i>	€ 600 / order
<b>Lead time</b>	<i>Normal</i>	21 days
	<i>Emergency</i>	1 day
<b>Replacement costs</b>	<i>Failure</i>	€ 1665 / replacement / item
	<i>Block</i>	€ 555 / replacement / item

**Table 1** - Input for the case study.

The company has historical data about the life time of the filling pipe, which states that the average lifetime is 2 years. After one year almost none of the items have failed, but after three years this is almost 100%. The company uses for simplicity as an approximation of the distribution of the life time of the filling pipe a normal distribution with mean 2 years and variance 1/3 year. In literature however usually the Weibull distribution is used as an approximation for the life time distribution. Therefore the



corresponding parameters of the Weibull distribution are determined (table 2). A list of 10000 lifetimes from the given normal distribution is generated and the fitted Weibull parameters are estimated.

<b>Parameter</b>	<b>Value</b>	<b>Lowerbound</b>	<b>Upperbound</b>
<b>Scale</b>	779.9334	777.4791	782.3954
<b>Shape</b>	6.5760	6.4810	6.6724

**Table 2** - Weibull parameters for the distribution of the life time and the bounds for the 95% confidence interval.

## 4. Simulation

To study the problem of maintenance policies and inventory control a simulation model is designed. This model simulates the replacements and ordering of the items and determines the average costs when parameter values are given. This chapter will first describe the parameters which are used and then explain the simulation model. This model is validated and the simulation model is used in the determination of the optimal parameter setting. The experimental design and response surface methods, which are used to find the optimal parameter values, are explained.

### 4.1 Parameters

The problem in this thesis has four parameters, which can all take many different values. Therefore many combinations of parameter values are possible. The problem is described in the following parameters:

T	optimal time interval between two consecutive block replacements
h	age threshold factor, where $hT$ is the age threshold
s	reorder point
S	maximum stock level

This thesis will use experimental design in order to compare the average costs when the parameters take different values. This method, which will be described in detail in section 4.3, assumes that the different parameters can vary independently. Restrictions on parameter combinations are not taken into account. However the four parameters above, T, h, s and S, cannot vary completely independently from each other. Items are ordered if the inventory positions drops below the reorder point. This reorder point cannot be larger than the maximum stock level and therefore s must be smaller than S. Due to this restriction the parameters s and S cannot vary independently and the normal experimental design approach cannot be used.

In Myers et al [14] designs are discussed with multicomponent constraints, constraints in which at least two parameters occur. Two software packages, namely Design Expert and JMP, are mentioned which are able to generate designs for problems with multicomponent constraints. However because of the ease of use of the experimental design without restrictions this approach is preferred. The parameter values in the different experiments are much easier to determine in that approach than with the software packages. Therefore this thesis uses experimental design and cannot handle multicomponent constraints.

In order to vary the parameters without restrictions, a parameter translation can be used. The parameter k is introduced, which is defined as the difference between the maximum stock level and the reorder point,  $k = S - s$ . If two of the three parameters s, S and k are known, the third one can be calculated. The parameters s and S together cannot be used, since the restriction  $s < S$  must hold. The same holds for the parameters k and S. Since s cannot be smaller than 0, the restriction must  $k < S$  hold as well. However if the parameters s and k are chosen, there is no restriction with more than one

parameter in it. The parameters  $s$  and  $k$  can take values above 0 independent from each other and  $S$  is then calculated as the sum of  $s$  and  $k$ . Therefore the problem is this thesis will be expressed in the parameters  $T$ ,  $h$ ,  $s$  and  $k$ :

- $T$  optimal time interval between two consecutive block replacements
- $h$  age threshold factor, where  $hT$  is the age threshold
- $s$  reorder point
- $k$  difference between maximum stock level and reorder point ( $k = S - s$ )

## 4.2 Simulation model

This thesis compares and analyses four different inventory policies, which are explained in detail in section 5.1. For each of the four policies a simulation model is constructed based on the same approach. The pseudo code of the basic simulation can be found in figure 1. All policies have small modifications in the ordering of the items. More detailed information about the simulation program and the differences for different policies can be found the simulation program algorithms in Appendix A.

```

Initialize
t=0
While t ≤ T_end
- check whether there are ordered items arriving
- check if there are outstanding replacements and perform them if there are items available
- check whether items have failed
  - replace the failed items, update outstanding replacements if necessary
  - update replacement costs
- check whether the block replacement must be performed
  - replace all items older than hT, update outstanding replacements if necessary
  - update replacement costs
- check whether inventory position < s, if so order (S – inventory level) items
  - update ordering costs
- determine inventory level and shortage and update corresponding costs
- t=t+1
end
Determine average costs: total costs / (T_end + 1)
end of simulation

```

**Figure 1** - Pseudo code of the simulation.

## 4.3 Simulation validation

The simulation is tested with a simulation trace and a small deterministic case. For the deterministic case the following case is considered: 2 items with price 10 per item, lead time = 2, life time = 2, total simulation length = 10. The costs are: order costs = 120 per order, shortage costs = 45 per item per time unit, holding costs = 15 per item per time unit, replacement costs = 30 per item per replacement for block replacements and 90 per item per replacement for failure replacements. As initial position is chosen that item 1 is new and item 2 has an age of one time period, both the inventory level and the inventory position are initialized at zero. The parameters that are used are  $s=1$ ,  $k=1$ ,  $h=0$ ,  $T=3$ .

The total costs for the deterministic case can be calculated by hand. At the beginning the inventory position = 0 and therefore below  $s$ , which means that two items are ordered. At  $t=1$  item 2 fails but cannot be replaced, thus there is 1 period of shortage. The inventory level is -1, since item 2 must be replaced. There are outstanding orders of size 2 and therefore the inventory position is 1, which means that three items are ordered. At  $t=2$  the two ordered items arrive and both item 1 and 2 can be replaced for failure, zero items remain in stock. At  $t=3$  the three ordered items arrive and the block replacement takes place, only one item remains in stock and therefore one item is ordered. At  $t=4$  nothing happens. At  $t=5$  both items fail and are replaced from stock and from the arriving item, zero items remain in stock and two items are ordered. At  $t=6$  the block replacement should take place, but there are no items available. The inventory level drops to -2 and four items are ordered. At  $t=7$  the outstanding block replacement is performed with the two incoming items. At  $t=8$  the outstanding order of four items arrives. At  $t=9$  the block replacement is performed which results in two items in stock. At  $t=10$  nothing happens.

To summarize, the deterministic case results in 1 period of shortage of 1 item, 5 orders with in total 12 ordered items, 4 items are replaced for failure and 6 items are replaced during a block replacement. In total 10 items are kept in stock for 1 period. The total costs therefore are  $45 + 5*120 + 12*10 + 4*90 + 6*30 + 10*15 = 1455$ . Table 3 shows both the simulation trace and the total costs from the simulation. The last are the same as the total costs from the calculation by hand.

Time	Item 1	Item 2	Inventory level	Inventory position	Order	Replacement (block)	Replacement (failure)	Total costs
0			0	0	2	0	0	140
1		F	-1	1	3	0	0	335
2	F+R	R	0	3	0	0	2	515
3	B+R	B+R	1	1	1	2	0	720
4			1	2	0	0	0	735
5	F+R	F+R	0	0	2	0	2	1055
6			-2	0	4	0	0	1215
7	B+R	B+R	0	4	0	2	0	1275
8			4	4	0	0	0	1335
9	B+R	B+R	2	2	0	2	0	1425
10			2	2	0	0	0	1455

**Table 3** - Simulation results of the deterministic case with parameters  $s=1$ ,  $k=1$ ,  $h=0$  and  $T=3$ .

A deterministic case with an age threshold larger than zero is considered as well. The results can be found in table 4. The problem setting is the same as described before. The parameters are changed to  $s=1$ ,  $k=1$ ,  $h=0.5$  and  $T=3$ .

This problem can be calculated by hand as well, which results in 1 period of shortage of 1 item, 6 orders with in total 12 ordered items, 2 items replaced in a block replacement and 8 items replaced for failure. In total 5 items are kept in stock for 1 time period. The total costs then are  $45 + 6*120 + 12*10 + 2*30 + 8*90 + 5*15 = 1740$ . In table 4 can be seen that the block replacement at  $t=3$  is not performed, since

both items then have an age of only 1 time period, which is smaller than the age threshold of  $0.5 \cdot 3 = 1.5$ . The total costs from the simulation are the same as the total costs calculated by hand.

Time	Item 1	Item 2	Inventory level	Inventory position	Order	Replacement (block)	Replacement (failure)	Total costs
0			0	0	2	0	0	140
1		F	-1	1	3	0	0	335
2	F+R	R	0	3	0	0	2	515
3			3	3	0	0	0	560
4	F+R	F+R	1	1	1	0	2	885
5			1	2	0	0	0	900
6	B+R	B+R	0	0	2	2	0	1100
7			0	2	0	0	0	1100
8	F+R	F+R	0	0	2	0	2	1420
9			0	2	0	0	0	1420
10	F+R	F+R	0	0	2	0	2	1740

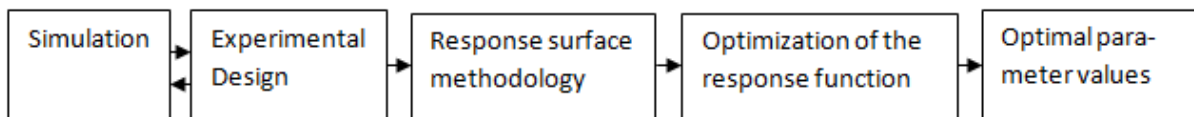
**Table 4** - Simulation results of the deterministic case with parameters  $s=1$ ,  $k=1$ ,  $h=0.5$  and  $T=3$ .

#### 4.4 Experimental design and Response Surface Methodology

In order to compare different inventory and maintenance policies, the average costs for each policy can be determined for given parameters. However one specific parameter setting may be optimal for one policy and far from that for another policy. Comparisons based on specific parameter settings can therefore lead to the wrong conclusions. To compare the best performance of all policies, the optimal parameters settings need to be determined. It would take a lot of computation time if the optimal parameter values should be determined via trying all possible parameter combinations with simulations. Experimental design and response surface methodology can be used to find domains of interest for the parameters and determine the optimal values. Not all combinations of parameter values need to be checked if these methods are used.

The total process of the methods used to determine the optimal parameter values is represented in figure 2. First for each of the parameters a region of interest is determined. Off line simulations where all parameter except one are fixed are used. For each of the four parameters three values will be determined: a low level, a high level and a center level, where the center level is exactly in the middle of the high and the low level. Then there are in total  $3^4 (=81)$  different combinations of parameter levels. Experimental design is used to determine which combinations of the parameter levels need to be analyzed. One combination of parameter levels is one experiment. It is possible to analyze all 81 experiments in a full factorial design or to use only a part of these experiments in a fractional factorial design. The Box-Behnken design is a commonly used fractional factorial design in which a number of experiments is selected, which can be analyzed instead of all experiments. More information about the comparison of the Box-Behnken design and the full factorial design can be found in the last part of this section.

When the experiments which need to be analyzed are determined, the simulations with the parameter values from the experiments are performed. For every experiment with given parameter values, the average costs for the maintenance and inventory is determined. With response surface methodology the relation between the parameter values and the average costs can be determined. This methodology assumes a response surface function which is the relation between the average costs and the parameters. The response surface methodology is performed with the software package Minitab. First the significance of the influence of the parameters on the average costs is investigated. An ANOVA is performed in which the influence of variation in the parameter values on the variation in the average costs is analyzed. The data from the experiments is used for the ANOVA. A full factor second order model is used for the relation between the parameters and the average costs. At a significance level of 5% is determined which factors can significantly explain the variance in the average costs. With this information the response surface function can be constructed. A second order equation with only the significant terms from the ANOVA is used. With linear regression the coefficients for this function can be estimated and the response surface function can be determined. The objective is to minimize the average costs, thus the optimal parameter values can be found by minimizing the response surface function. Afterwards response surface contour plots can be made to control whether the response surface function is actually convex in the optimum.



**Figure 2** - Systematical representation of the process of determining the optimal parameter values.

### Experimental Design

The Box-Behnken design for a problem with four parameters and three levels for every parameter needs 27 different experiments. The full factorial design, which combines all parameter levels, has 81 different experiments. Because of reducing the computation time, the Box-Behnken design is preferred to the full factorial design. However in the response surface methodology both designs can give different results. The chosen experiments in the Box-Behnken design must give enough information for the entire problem to be able to give reliable results. Specifically the estimation of the coefficients for the response surface function with linear regression will give problems. The value of the adjusted  $R^2$  will be low if the Box-Behnken design is not a good alternative to the full factorial design. A low adjusted  $R^2$  means that the explanatory power of the model is very low and that it cannot be used to make reliable statements about the optimal parameter values.

Low values for the adjusted  $R^2$  can also occur if the variance in the simulation output is large. In the experimental design the result of one simulation run for every experiment is used as the response for that experiment. If the variance in the simulation output is large, the confidence interval for the response of an experiment is large as well. In that case the value of the specific simulation run does not give so much information about the actual response for that experiment. This thesis considers four different inventory policies, which will be described in further detail in chapter 5. For every policy the variance of the simulation output is determined after simulating 500 runs with the same parameter values. The results can be found in table 5.

Policy	Joint order	Separate BR order N-#skip	Separate BR order N-average#skip	Separate BR order N-average#failure
Mean (€/day)	106.6642	117.0966	105.7647	105.5667
Variance ((€/day) <sup>2</sup> )	0.2204	3.2819	0.2022	0.2356
Length of 95% confidence interval (1 simulation run)	1.8403	7.1015	1.7627	1.9027
Length of 95% confidence interval (5 simulation runs)	0.8230	3.1759	0.7883	0.8509

**Table 5** - Variances in simulation output over 500 simulation runs with parameters  $s=4$ ,  $k=3$ ,  $h=0.6$  and  $T=638$ .

The influence of the variance of the simulation output can be reduced if for every experiment more simulation runs are performed and the mean average costs are used as the response for that specific experiment. In table 6 both the Box-Behnken design (BB design) and the full factorial design are compared and using one simulation run or five simulation runs per experiment. The table shows that the increase of the number of simulation runs per experiment indeed gives a higher adjusted  $R^2$ . This increase however is small compared to the increase in adjusted  $R^2$  when the full factorial design is performed instead of the Box-Behnken design. This leads to the conclusion that the fractional factorial design, the Box-Behnken design in this case, is the cause of the low adjusted  $R^2$ . In the last column an indication of the calculation time is given. Since the full factorial design has a larger number of experiments, this increases the computation time. Also using more simulations runs per experiment leads to a higher computation time, as expected.

To be able to use the response surface function for the determination of the optimal parameter values, a high adjusted  $R^2$  is necessary. Therefore this thesis will use the full factorial design. Both performing one simulation run and five simulation runs gives high values of the adjusted  $R^2$ . In this thesis every experiment is performed with 5 runs in order to be able to make good statements about the optimal parameter values. Therefore every of the 81 experiments is performed five times and the average costs are used as the response for that specific experiment. The full factorial design is replicated three times, which resulted in 243 simulated experiments in total. For every experiment a simulation length of 10000 days is used.

Policy	Joint order	Separate BR order N-#skip	Separate BR order N-average #skip	Separate BR order N-average # failure	Indication calculation time
Adj. $R^2$ in BB design with 1 run	34.93%	29.99%	32.31%	27.56%	300 (s)
Adj. $R^2$ in BB design with 5 runs	36.34%	36.81%	34.21%	26.99%	1400 (s)
Adj. $R^2$ in full factorial design with 1 run	97.38%	97.36%	84.16%	92.21%	800 (s)
Adj. $R^2$ in full factorial design with 5 runs	97.44%	99.17%	93.93%	93.70%	5000 (s)

**Table 6** - The adjusted  $R^2$  for the four policies using different designs and a different number of simulation runs per experiment.

## 5. Comparing inventory policies

For the maintenance and inventory of items different policies can be considered. This thesis will use the modified block replacement policy for the maintenance and replacement of items. For the inventory control two policies from literature are used and two other policies are proposed. The performance of these four policies is compared.

### 5.1 Policies

This thesis considers four different policies for the replacement and ordering of items. All policies are based on the modified block replacement policy (MBRP), which means that items are replaced at failure or during block replacements which take place at times  $nT$  (with  $n=1, 2, \dots$ ). Only items older than the age threshold  $hT$  are replaced during block replacements. As inventory control policy the  $(s,S)$  policy is used. New items are ordered if the inventory position drops below  $s$ , where the inventory position is defined as the inventory level together with the outstanding orders. The maximum inventory level is  $S$  and is calculated as  $k+s$ . The policies considered in this thesis differ in the ordering of the items. There are two different ordering methods: joint ordering and separate ordering.

Joint ordering means that both the items needed for failure replacement and the items needed for block replacement are ordered together. When the inventory position drops below a predetermined level ( $s$ ), items are ordered with an emergency order. Separate ordering policies order the items for block replacements and for failure replacements separately. Since the time of failure of an item is not known, companies keep a safety stock to be able to replace items at failure. If the inventory position drops below  $s$ , items are ordered with emergency orders. The time of performance of a block replacement however is known beforehand and therefore items can be ordered in advance. The lead time of the items is also known and assumed to be constant in this thesis. Therefore items for the block replacement can be ordered one lead time before the block replacement takes place. Items then do not need to be stored, but can be used immediately which reduces the inventory costs. For the orders for the block replacements, normal orders are used.

The number of items to order for a block replacement is difficult to determine. Only items that are older than a specific age threshold will be replaced during the block replacement. However one lead time before the block replacement takes place the number of items older than the age threshold during the block replacement is not known. Items can fail between the time of the order and the time of the block replacement.

This thesis considers two existing policies from literature and two new policies are proposed. The existing policies are the MBRP with joint ordering and the MBRP with a separate ordering policy:

1. MBRP with joint order
2. MBRP with separate BR order  $N - \#skip$

1. *MBRP with joint order*: items for block replacements and failure replacements are ordered jointly when the inventory position drops below  $s$ .



2. *MBRP with separate BR order N - #skip*: this policy is described in Mardin [10] and is concluded to be the best among different inventory policies. The policy is based on the fact that the life time of the items is generated beforehand in the simulation. Therefore the number of items that will be below the age threshold during the block replacement can be determined exactly. These items do not have to be replaced during the block replacement and can therefore be skipped. The policy determines this number and orders exactly the amount of items needed for the block replacement one lead time in advance with a normal order. This number is equal to the total number of items (=N) minus the number of items which can be skipped (N - #skip). The disadvantage of this policy is that it cannot be used in practice. Where in the simulation the lifetime is exactly known, this is not the case in real life and therefore the exact number to order cannot be determined so easily. Items needed for failure are ordered with emergency orders if the inventory position drops below s.

The MBRP with separate BR order N - #skip policy has its advantage in lower costs, but cannot be implemented in practice. Therefore this thesis proposes two new policies, which are based on the MBRP with separate BR order N - #skip policy. The number of items to order one lead time before the block replacement takes place is calculated differently. Information which is in reality not known beforehand will not be used. These new policies, which do have a practical implementation, are:

3. MBRP with separate BR order N - average#skip
4. MBRP with separate BR order N - average#failure

3. *MBRP with separate BR order N-average#skip*: This policy is based on the separate BR order N - #skip policy, but now takes care of the fact that the number of items to skip during the block replacement is not known beforehand. Every block replacement the number of items to skip is determined and the average of the #skip is updated. The following formula is used, where n is the number of block replacements that is already performed and #skip<sub>n+1</sub> is the number of items that were skipped during the (n + 1)<sup>th</sup> block replacement.

$$\#skip_{average} = \frac{\#skip_{average} * n + \#skip_{n+1}}{(n+1)}$$

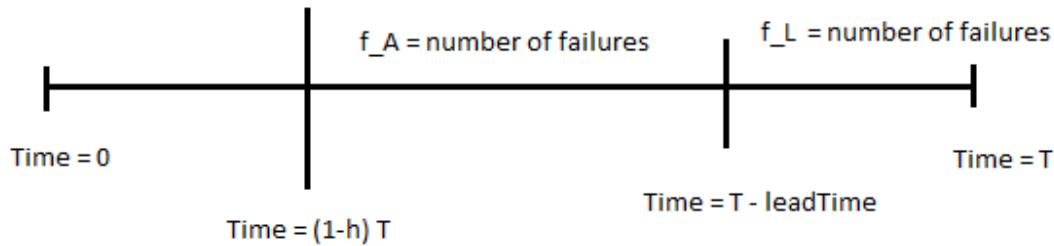
At the first block replacement n is equal to 0 and #skip<sub>average</sub> is not known. To be able to use the formula above #skip<sub>average</sub> is initialized at 0. One lead time before the block replacement N – average#skip items are ordered with normal orders, where N is the total number of items in the machine. Again if the inventory position drops below s items are ordered for the failure replacements.

4. *MBRP with separate BR order N – average#failure*: This policy also orders items one lead time before the block replacement. Items younger than the age threshold are not replaced during the block replacement. When the block replacement is performed every T time units and the age threshold is hT, which means that only items older than hT will be replaced during block replacements, items that have failed and are replaced between (1-h)T and T do not have to be replaced then. Figure 3 is the representation of the idea behind this method. The block replacement takes place at time T and one lead time before the block replacement the items are ordered. The number of items that have failed between (1-h)T and T – leadtime is known at that time and is called f<sub>A</sub>. The number of items that will fail between the ordering time and the block replacement time is however not known. This policy

determines the average number of failures and updates that average using a similar formula as the previous policy. The value for  $\#failures_{L_{average}}$  is initialized at 0.

$$\#failures_{L_{average}} = \frac{\#failures_{L_{average}} * n + \#failures_L}{(n+1)}$$

When the company has in total N items in the machine, one lead time before the block replacement a number of  $N - f_A - f_L$  items are ordered with a normal order. The emergency orders are placed if the inventory position drops below s.



**Figure 3** - Graphical representation of the determination of the number of items to order.

## 5.2 Comparison

For determining the optimal parameter values experimental design is used and for every policy regions of interest for the parameters are determined. The response surface methodology determines the response surface function, optimizes this function and checks whether the optimal parameter values are actually in the regions of interest. When the optimal parameter values seemed to be outside the original region of interest, the region is moved and the response surface method is performed again. The final regions of interest, with high levels, center values and low levels for the parameters, can be found in table 7.

Policy	s	k	h	T
Joint order	[0,1,2]	[1,2,3]	[0.6,0.8,1]	[455,516,577]
Separate BR order N-#skip	[0,1,2]	[1,2,3]	[0.6,0.8,1]	[424,516,608]
Separate BR order N-average#skip	[0,2,4]	[1,3,5]	[0.4,0.7,1]	[516,577,638]
Separate BR order N-average#failure	[0,2,4]	[1,3,5]	[0.5,0.6,0.7]	[485,561,638]

**Table 7** - Regions of interest in different policies for the four parameters.

Only the values in table 7 will be used to determine the response surface function, which is the relation between the parameters and the average costs. All possible combinations between the three levels of the four parameters are simulated. To determine which parameters have a significant influence on the average costs an ANOVA is performed in a full factorial second order model. The insignificant terms and the corresponding p-values for the different policies can be found in table 8. The complete ANOVA tables can be found in Appendix B.

Policy	Insignificant terms (p-value)
Joint order	T(0.945), T*T(0.909), s*T(0.946), k*T(0.960), h*T(0.892)
Separate BR order N-#skip	T(0.243), s*s(0.408), T*T(0.954), s*k(0.458), s*T(0.993), k*T(0.834), h*T(0.703)
Separate BR order N-average#skip	T(0.670), T*T(0.866), s*T(0.213), k*T(0.315), h*T(0.649)
Separate BR order N-average#failure	T(0.431), k*k(0.484), T*T(0.919), s*T(0.573), k*T(0.717), h*T(0.841)

**Table 8** - Insignificant terms with their p-value for the different policies.

Table 8 shows that all terms with parameter T are not significant in each of the four policies. This means that the time between two block replacements does not have a significant influence on the average costs. This result is unexpected and section 5.3 will take a closer look at parameter T and will try to find explanations for the insignificance of this parameter.

To determine the response surface function only the significant terms are used in a linear regression. The regression tables can be found in Appendix B. The coefficients for the response surface functions are estimated and the functions can be found in figure 4.

$$\begin{aligned} \text{Costs}_{\text{JointOrder}} &= 110.135 + 2.020 s - 6.575 k - 8.401 h + 0.463 s * s - 6.558 k * k + 1.631 h * h - 0.786 s * k - 0.554 s * h + 3.779 k * h \\ \text{Costs}_{\text{Separate N-#skip}} &= 115.706 + 3.532 s + 0.974 k - 14.381 h - 0.526 k * k + 5.747 h * h - 2.033 s * h + 0.302 k * h \\ \text{Costs}_{\text{Separate N-av\#skip}} &= 101.246 - 5.112 s + 1.607 k + 3.264 h + 4.349 s * s + 0.967 k * k + 0.5087 h * h - 1.450 s * k - 5.272 s * h + 1.487 k * h \\ \text{Costs}_{\text{Separate N-av\#failure}} &= 108.861 - 3.521 s - 0.824 k - 1.384 h + 3.316 s * s + 6.152 h * h + 0.544 s * k - 5.137 s * h - 0.835 k * h \end{aligned}$$

**Figure 4** - Response surface functions for the different policies.

The objective is to minimize the average costs. The optimal parameter values can be determined by optimizing the response surface functions and considering the regions of interest from table 7. Table 9 shows the optimal parameter values.

Policy	s*	k*	h*	T*	Costs	Adj. R <sup>2</sup>
Joint order	0.2626	3 <sup>1</sup>	1	- <sup>2</sup>	93.7624	97.44%
Separate BR order N-#skip	0	1	1	- <sup>2</sup>	103.7719	99.17%
Separate BR order N-average#skip	3.0303	2.0909	0.8020	- <sup>2</sup>	99.5747	93.93%
Separate BR order N-average#failure	3.7576	5 <sup>1</sup>	0.6556	- <sup>2</sup>	106.1396	93.70%

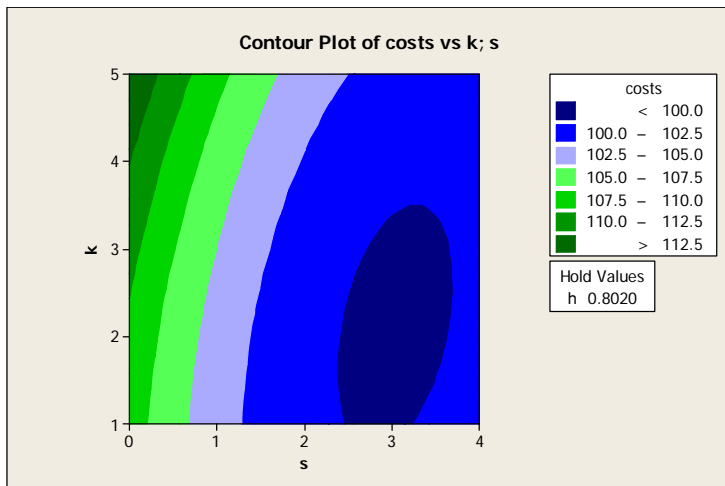
**Table 9** – Optimal parameters for the different policies.

<sup>1</sup> This parameter value could be raised, but if the region of interest moves upwards, the parameter s then reaches 0 and k needs to be raised even further.

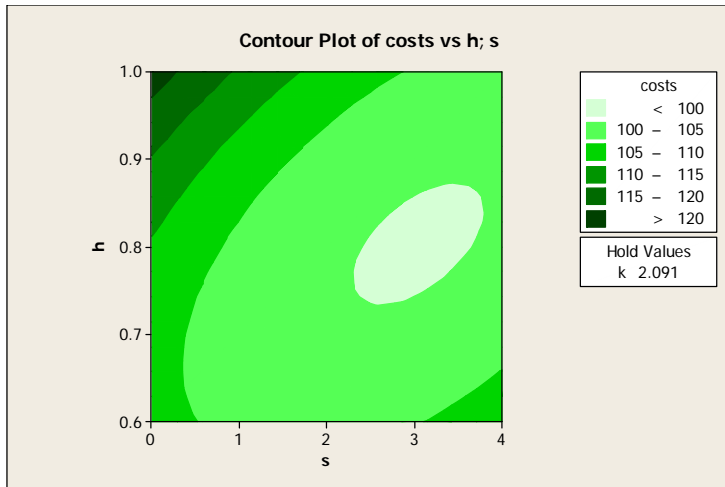
<sup>2</sup> The influence of parameter T on the average costs is insignificant and therefore an optimal value cannot be determined.

The value for the adjusted  $R^2$  for all policies is higher than 90 %. This means that all response surface methods have a high explanation power. Also table 9 shows that the separate order policies give slightly higher costs than the joint ordering policy. This could be caused by the fact that the joint ordering policy orders in total less items during the simulation length, namely around the 1270 items. The separate ordering policies all order around the 1550 items during the simulation length. The number of block replacements and the life time of items is the same for both ordering policies. However the joint or separate ordering method can make a difference in the replacement of items. The separate ordering policies order the needed items for the block replacement in advance. Therefore the number of items available for replacements during a block replacement is higher than with joint ordering. Also simulation results show that the average number of replaced components during a block replacement is lower for joint ordering than for separate ordering. Further research is needed to investigate the reason for the differences concerning the number of needed items. Of all the three separate order policies the Separate BR order N-average#skip has the lowest cost. The results in table 9 should however be examined critically. The fact that parameter T is not significant has also influence on parameter h. The age threshold for the block replacement is  $hT$  and when T does not have an optimal value, the optimal value for h as no meaning. Therefore the results from the response surface methodology in table 9 do not give so much useful information, because of the insignificance of parameter T.

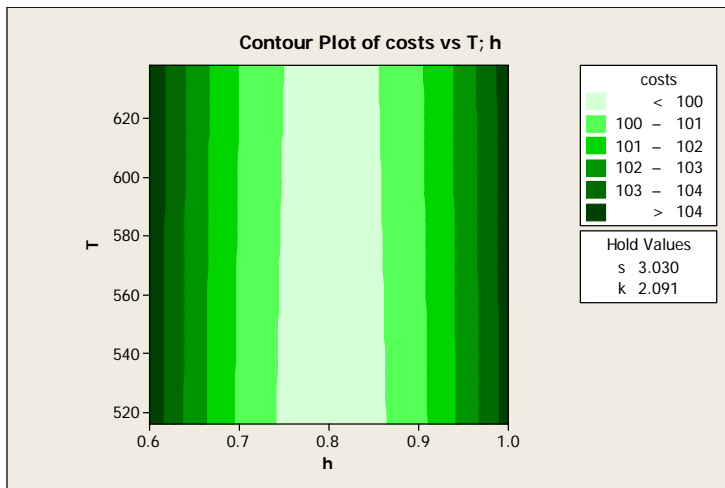
Contour plots of the response surface function can be made in order to check whether the function is actually convex in the given optimal parameters. Figure 5, 6 and 7 show contour plots for the Separate BR order N – average#skip policy. Similar contour plots are made for the other policies and can be found in Appendix C. The figures show that the response surface function is convex in the optimal parameters. Figure 7 also shows that the value of parameter T has a very small and almost no influence on the average costs.



**Figure 5** - Contour plot of the parameters k and s.



**Figure 6** - Contour plot of the paramters h and s.



**Figure 7** - Contour plot of the paramters T and h.

It is difficult to compare the results for the different policies obtained with the response surface methodology, because of the insignificance of parameter T. To be able to compare the policies and to make statements about the differences, the policies are compared with the same parameter settings. These parameter settings are chosen based on the optimal parameter values in table 9. The value for parameter T is chosen randomly. Two cases will be considered:

- case 1:  $s=0$ ,  $k=1$ ,  $h=1$ ,  $T=638$
- case 2:  $s=3$ ,  $k=2$ ,  $h=0.8$ ,  $T=638$

For every case and for every policy 500 simulation runs are preformed. The mean average costs and variation in average costs can be found in table 10.

Policy	Joint order	Separate BR order N-#skip	Separate BR order N-average #skip	Separate BR order N-average # failure
<b>Mean (€/day) case 1</b>	94.3250	104.5467	113.9822	104.6811
<b>Variance ((€/day)<sup>2</sup>) case 1</b>	0.5792	3.6292	42.3139	4.1994
<b>Mean (€/day) case 2</b>	103.5640	113.9300	97.0032	102.7308
<b>Variance ((€/day)<sup>2</sup>) case 2</b>	0.1825	5.0935	0.4206	0.3079

**Table 10** – Mean and variances in simulation output over 500 simulation runs for two cases of parameter settings.

Table 10 shows that the two proposed policies: separate BR order N - average#skip and separate BR order N - average#failure have similar or even lower average costs than the separate BR order N - #skip policy, which was proposed in Mardin [10]. The joint order policy has low costs which is not supported by Mardin[10], there separate ordering reduces costs. However in their research both policies lead to the same number of ordered items. In the case study in this thesis for the parameters in case 2 all policies order about 1255 items during one simulation length, but for the parameters in case 1 the joint ordering method orders about 100 items less, which is a reason for the lower costs. Further research is needed to find reasons for this lower number of ordered items in the joint ordering policy. Mardin [10] concludes that the separate BR order N - #skip policy is the policy with the lowest cost. This policy has one large disadvantage: it cannot be implemented in practice. This thesis has shown that the two proposed adjustments to the best policy of Mardin [10] have similar average costs, but do not have the disadvantage and can therefore be used in practice.

## 5.3 Discussion

After performing the ANOVA in the response surface methodology the results showed that all terms with parameter T could not significantly explain the variance in the average costs. Table 8 and 9 show that this holds for all policies. Intuitively the time between two block replacements should influence the average costs. Performing block replacements very often will give higher replacement costs and higher order costs. On the other hand performing block replacements very rarely will lead to failure of the items, stagnation of the machine and high shortage costs.

This section will try to find explanations for the fact that the influence of parameter T on the average costs is insignificant. Adjustments in the case study are tested to see whether these changes can result in a significant influence of parameter T. Also the basic case in Mardin [10] is considered to see whether analyzing that case gives the same results.

### 5.3.1 Adjustments in case study

There could be different reasons for the fact that parameter T does not have a significant influence on the average costs. Three adjustments are made in the case study of this thesis in order to test three explanations for the insignificant influence of parameter T. First the considered region of interest could be too small. In that case the different parameter values for T within the region of interest could all have the same influence on the costs. To verify this result, the response surface method is performed again. The regions of interest for the parameters s, k and h are the same as stated in table 7, but for parameter T the region of interest is enlarged to [30, 548, 1126]. The p-values for estimated coefficients containing parameter T in the determining of the response surface function with regression can be found in table

11. Only the MBRP with Separate BR order N - average#skip is considered. However since all policies are applied to the same case study and have almost the same simulation model, assumed is that the reason for the insignificant influence of parameter T on the average costs is the same for all policies.

A second possible reason for the insignificant influence of T, could be the ratio between the block replacement costs and the failure replacement costs. In the case study considered in this thesis these costs are respectively € 555 and € 1665 per replacement per item. The block replacement could be relatively expensive compared to the failure replacement, such that performing block replacements is not advantageous if costs are minimized. To test for this explanation, the response surface methodology is performed with block replacement costs of € 5 per replacement per item. The failure replacement costs remain € 1665 per replacement per item and the regions of interest are the same as in table 7. Again the results for the significance of the influence of parameter T on the average costs for the MBRP with Separate BR order N - average#skip can be found in table 11.

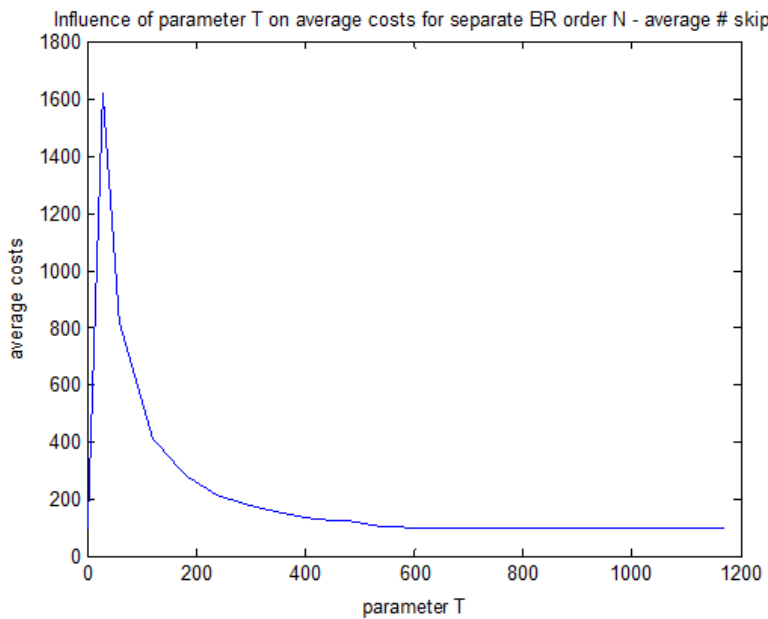
A third explanation can be that for the setting of the case study in this thesis performing a block replacement is not advantageous. The case states that the company has 8 machines with only one filling pipe per machine. This means that it is a single-component system and that failure of one of the items has no influence on the other items. To analyze whether this explains the insignificant influence of parameter T, the case is adjusted to a multicomponent system. This is done by adjusting the shortage costs. In the case study the shortage costs are € 23040 per day per item. Now the costs are € 23040 per day if one or more of the items is not working. This means that the machine falls short as soon as at least one of the items fails. Performing block replacements could be advantageous now, since it prevents stagnation of the machine. The results for the significance of the influence of parameter T on the average costs for the MBRP with Separate BR order N - average#skip can be found in table 11.

	Terms (p-value)
<b>Enlarged region of interest for parameter T</b>	T(0.927), T*T(0.929), s*T(0.972), k*T(0.900), h*T(0.994)
<b>Reduced block replacement costs</b>	T(0.794), T*T(0.936), s*T(0.543), k*T(0.423), h*T(0.544)
<b>Adjusted to multicomponent system</b>	T(0.870), T*T(0.115), s*T(0.908), k*T(0.997), h*T(0.958)

**Table 11** - Terms containing parameter T with the corresponding p-value in the regression for the response surface curve for three different adjustments in order to investigate the influence of parameter T on the costs in the MBRP with Separate BR order N - average#skip policy.

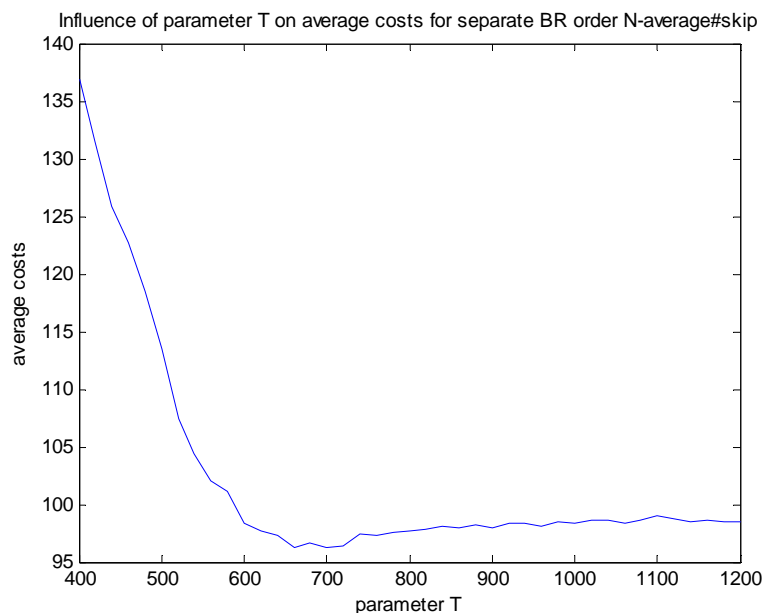
Table 11 shows that all adjustments do not lead to a significant influence of parameter T on the average costs. All coefficients for the terms with this parameter do still not differ significantly from zero in the regression. To investigate how the average costs are related to parameter T, twenty different experiments are performed in which parameter T takes values between 0 and 1160. The other parameters stay constant. The separate BR order N – average#skip policy is used and the parameters s, k and h take the values of 3, 2 and 0.8020 respectively. These are the optimal parameter values for this policy which can be found in table 9. Every experiment is performed with five simulation runs and the mean average costs are used as the response. Figure 8 is the result of this analysis and the graph shows

that the relation between the average costs and parameter T is not convex. There is no obvious minimum and the average costs seem to decrease when the value for T becomes larger.



**Figure 8** – The graphical relation between parameter T and the average costs for the separate BR order N – average#skip policy.

The average life time of a component is around 730 days. Therefore it is reasonable to assume that if an optimal value for T exist, it is not larger than 730. The area around 730 is investigated and again the other parameters are kept constant at the same values ( $s=3$ ,  $k=2$ ,  $h=0.8020$ ) while parameter T varied. For the different values of T the mean average costs over five simulation runs can be found in figure 9. The graph does not show an obvious convex pattern, although it seems to have a minimum around 690.



**Figure 9** - The graphical relation between parameter T and the average costs for the separate BR order N – average#skip policy.



Investigated is whether the optimal parameter values for the parameters  $s$ ,  $k$  and  $h$  could be determined by setting  $T$  to 690 and then performing response surface methodology for 3 parameters. However the results, which can be found in Appendix D, showed that influence of parameter  $h$  on the average costs was insignificant. Therefore the optimal parameter values could not be determined this way.

The response surface methodology determines the response surface function and uses that as an approximation of the relation between the average costs and the parameter values. Both figure 8 and 9 show that the graph of the relation between the average costs and parameter  $T$  is very flat. Probably this is too flat for the response surface methodology to be able to say that the influence of parameter  $T$  on the average costs is significant.

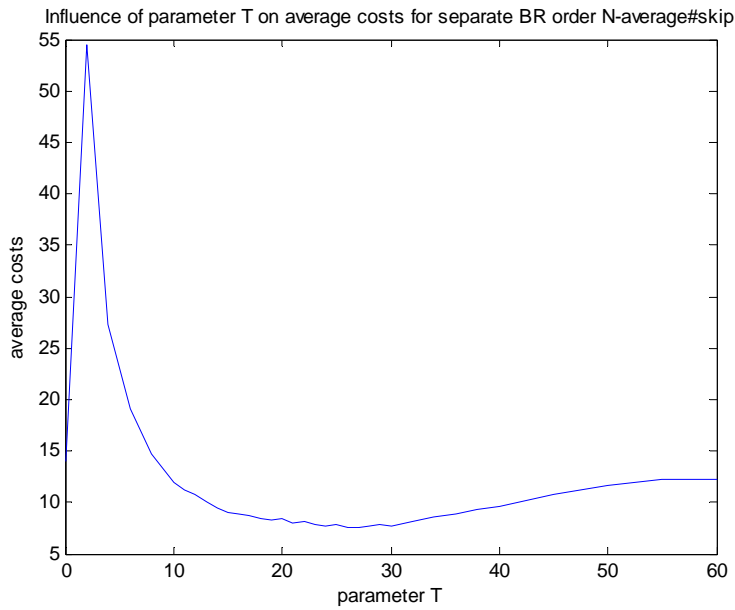
### 5.3.2 Comparison with case from literature

Trying to find explanations for the insignificant influence of parameter  $T$  on the average costs and adjusting the case study of this thesis, did not result in a proper explanation. The several adjustments did not lead to a significant influence of parameter  $T$  on the average costs. In this part of the discussion a basic case from Mardin [10] is considered. The response surface function is determined to see what the influence of parameter  $T$  is in that case. The objective is to find out whether the response surface method will be able to find significant influence of parameter  $T$  on the average costs in a different case. Table 12 shows the problem description of the basic case used in Mardin [10].

	Value		Value
<b>Number of items</b>	5	<b>Order set up costs</b>	<i>Normal</i> 5 / order
<b>Price</b>	0 / item		<i>emergency</i> 30 / order
<b>Lifetime: Weibull distribution</b>	shape parameter = 3	<b>Lead time</b>	<i>Normal</i> 5 unit of time
	scale parameter = 50		<i>emergency</i> 1 unit of time
<b>Shortage costs</b>	20 / unit of time / item	<b>Replacement costs</b>	<i>Failure</i> 100 / replacement / item
<b>Holding costs</b>	1 / unit of time / item		<i>Block</i> 20 / replacement / item

**Table 12** – Input parameters for the case study of Mardin.

The relation between the average costs and parameter  $T$  is determined. The other parameters are kept constant at  $s=0$ ,  $k=1$ , and  $h=0.509091$ . The simulation length for this case is set at 6200 time units and for every experiment five simulation runs are used to determine the mean average costs.



**Figure 10** - The graphical relation between parameter T and the average costs for the separate BR order N – average#skip policy with the case study from Mardin.

Figure 10 shows that for the case of Mardin[10] the relation between the average costs and parameter T seems to be slightly convex. The minimum costs are obtained when parameter T takes values around 25. Response surface methodology is used to see whether the influence of parameter T on the average costs is actually significant. The four policies, described in section 5.1, are considered and for every policy the regions of interest in table 13 are used.

Parameter	s	k	h	T
<b>Joint order</b>	[0,1,2]	[1,2,3]	[0.2,0.5,0.8]	[15,25,35]
<b>Separate BR order N-#skip</b>	[0,1,2]	[1,2,3]	[0.2,0.5,0.8]	[15,25,35]
<b>Separate BR order N-average#skip</b>	[0,1,2]	[1,2,3]	[0.2,0.5,0.8]	[5,25,45]
<b>Separate BR order N-average#failure</b>	[0,1,2]	[1,2,3]	[0.2,0.5,0.8]	[5,25,45]

**Table 13** - Region of interest for the four parameters and the different policies. For every parameter the low value, the center value and the high value are given.

With these regions of interest the response surface methodology is performed: the terms with a significant influence on the variation of the average costs are determined, the response surface function is determined and optimized. The ANOVA and regression tables can be found in Appendix E. The optimal parameter values can be found in table 14.

Policy	s*	k*	h*	T*	Costs	Adj. R <sup>2</sup>
<b>Joint order</b>	0	1	0.551551	15 <sup>3</sup>	8.60513	96.31%
<b>Separate BR order N-#skip</b>	0	1	0.509091	15 <sup>3</sup>	7.31946	85.99%
<b>Separate BR order N-average#skip</b>	0	1	0.6	45 <sup>3</sup>	6.1701	99.17%
<b>Separate BR order N-average#failure</b>	0	1.28283	0.6	- <sup>4</sup>	6.50151	99.51%

**Table 14** - The optimal parameters for the different policies.

<sup>3</sup> These values for T are either lower or upper bounds of the region of interest for parameter T. When the region is moved, the influence of parameter T on the average costs becomes insignificant.

<sup>4</sup> The influence of parameter T on the average costs is insignificant.

Table 14 shows that in contrast with the results of the case study in section 5.2, in this case the joint ordering method results in higher average costs than the separate ordering methods. After analysis, the number of ordered and replaced items for the case of Mardin [10] was the same for all policies. Since this number differed in section 5.2, this could explain the results. Table 14 also shows that the influence of parameter T on the average costs is significant when a very large region of interest is chosen for parameter T. Only for the separate BR order N - average#failure policy the influence of parameter T on the average costs remains insignificant. However the optimal values that are determined are all located on the edges of the regions of interest. If the region of interest is moved in the right direction, the influence of parameter T on the average costs becomes insignificant. Also the region of interest for parameter T is very large. In those cases the response surface function cannot give such a good approach for the relation between the average costs and the parameters. If the response surface methodology is performed properly, the region of interest for parameter T should be moved and reduced. However if that procedure is followed, the difference in average costs for different parameter values of T becomes very small, which can be seen in the flat part of figure 10. Performing an ANOVA then leads to the conclusion that the influence of parameter T on the average costs is not significant any more.

Intuitively the time between two block replacements, and thus the value of parameter T, would have a large influence on the average costs. Since performing block replacements prevents the machine to stagnate and reduces therefore shortage costs. However using response surface methodology and determining the response surface function, does not lead to the conclusion that the influence of parameter T on the average costs is significant. Both the case study in this thesis and the basic case from Mardin [10] show these results. Response surface methodology is therefore not able to determine optimal values for the parameter T and it cannot be used to describe the influence of parameter T on the average costs. Further research is needed to find out whether this holds for the joint optimization of the modified block replacement policy and ordering policies in general or that only these specific cases lead to this conclusion.

## 6. Conclusion

In this thesis different ordering policies for the joint optimization of inventory control and maintenance of items under block replacement are compared. The optimal parameter values are determined with experimental design and response surface methodology.

Two existing ordering policies from literature are considered and two new ordering policies are proposed. One of the existing policies uses information about the lifetime of items which is only available in the simulation model. The new proposed policies only use information which is available in practice and are therefore more practical to implement. The performance of the new policies is compared with the existing policies, which shows that the adjusted policies perform the same or only slightly poorer than the existing policies. Since the existing separate ordering policy cannot be implemented in practice, these new proposed policies with practical implementation are an improvement.

In order to find the optimal parameter setting response surface methodology is used. The experimental design approach cannot imply constraints on the parameters. Since the restriction  $s < S$  must hold, the parameters  $s$ ,  $S$ ,  $h$  and  $T$  cannot be used. To be able to determine the optimal parameter values without taking care of restrictions the parameter  $S$  can be replaced by the parameter  $k$ , which is defined as  $S - s$ .

It is found that the Box-Behnken design results in low values for the adjusted  $R^2$  and thus in a model with low explanatory power. Using a full factorial design instead of a fractional factorial design led to a higher adjusted  $R^2$ . Also using more simulation runs per experiment and taking the mean average costs as the response reduced the variance and increased the explanatory power.

The results show that the terms with the parameter  $T$  do not have a significant influence on the average costs. This is not caused by the ratio between the costs for block replacement and the costs for failure replacement, by the size of the region of interest or by the fact that the case study describes a single component system.

The results show that the joint ordering policy and the separate order policies have about the same costs, the joint ordering policy costs are slightly lower in some cases. This could have to do with the fact that the joint ordering policy orders less items. Since the items are very expensive this saves costs.

### Further research

Further research is needed to determine why the influence of parameter  $T$  on the costs is not significant. The response surface methodology cannot be used to make reliable statements about optimal parameter settings if one of the parameters does not have a significant influence on the response.

Also the difference in number of ordered items between the joint ordering method and the separate ordering methods need to be investigated more closely. For only one of the two case studies considered in this thesis the number of ordered items really differed. Further research on the influence of the joint or separate ordering policy on the number of ordered items is needed.

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## 8. Appendices

### 8.1 Appendix A

#### *Simulation Program Algorithms*

##### **Simulation Program Algorithm – MBRP with joint ordering**

1. Set the number of components (N), the distribution of the lifetime, the price per item, the different costs (holding costs, shortage costs, order set up costs for emergency orders and replacement costs for block replacement and for failure replacement), the lead time for emergency orders, the simulation length, the initial InventoryLevel, the initial InventoryPosition, the outstanding orders and the outstanding replacements. Generate the lifetimes for the N components. Store the failure time and the start time of the components in ItemList. Start time for the initial components is 0. Set time  $t = 0$ .
2. If time  $t \leq$  simulation length, go to step 3. Else go to step 22.
3. Compare the outstanding order list with time  $t$ , if there are ordered items with arrival time  $t$ , go to step 11.
4. Check if there are outstanding replacements that still need to be performed, this is the case if the list with outstanding replacements is not empty. If so, go to step 14. Else, go to step 5.
5. Check if the block replacement needs to be performed, this is the case if  $\text{mod}(\text{time } t, \text{parameter } T) = 0$  and time  $t$  is not 0. If so, go to step 13. Else, go to step 6.
6. Check if there are failed items. Compare the failure time of the items with the time  $t$ . If the failure time  $<$  time  $t$ , the items have failed. Add the failed items to the vector ToReplace. If ToReplace is not empty, go to step 12. Else, go to step 7.
7. Update InventoryPosition: if there are no outstanding orders, InventoryPosition = InventoryLevel. Else, InventoryPosition = InventoryLevel + Total number of items from outstanding orders. Go to step 8.
8. If inventoryPosition  $<$   $s$ , go to step 20. Else, go to step 9.
9. Determine the inventory level and the shortage level. The shortage level is the number of items in the ItemList, with both a start time and a failure time of Inf. Go to step 21.
10. Do time  $t = \text{time } t + 1$  and return to step 2.
11. Remove the arriving items from the outstanding order list and add the number of arriving items to the inventory level. If the inventory level was negative before the order arrived, go to step 15. Else go to step 5.
12. Compare the number of items from the vector ToReplace with the InventoryLevel. If the InventoryLevel  $>$  number of items to replace, go to step 17. Else set the number of items that can be replaced = InventoryLevel and go to step 18.
13. Determine which items need to be replaced in the block replacement. All items with an age  $\geq hT$  need to be replaced, place those items in ToReplace. Age of an item is time  $t -$  start time of that item. If ToReplace is not empty, go to step 12. Else, go to step 6.
14. Determine which items need to be replaced, these are the items which are stored in the outstanding replacement list. Add these items to the vector ToReplace. If ToReplace is not empty, go to step 16. Else, go to step 5.
15. Determine the number of outstanding replacements (Z) that can be performed by the incoming items. If the size of the incoming order is larger than the absolute value of the inventory level before the order arrived,  $Z = -$  InventoryLevel, else  $Z =$  size of the incoming order. Go to step 4.

16. If  $Z \geq$  items in the vector ToReplace, all items from the vector ToReplace can be replaced. Else, only Z items can be replaced. Remove the items that can be replaced from the ItemList and add new items with time t as the start time and a failure time of time t + a generated life time. Place the other items from ToReplace back to the list of outstanding replacements. Empty the vector ToReplace. Go to step 19.
17. Remove all items that need to be replaced from the ItemList. Add new items, with time t as the start time and a failure time of time t + generated life time. Update the InventoryLevel and the InventoryPosition.  $\text{InventoryLevel} = \text{InventoryLevel} - \text{number of replaced items}$ .  $\text{InventoryPosition} = \text{InventoryPosition} - \text{number of replaced items}$ . Empty the vector ToReplace. Go to step 19.
18. Only replace the items from ToReplace until the number of items that could be replaced from the inventory level is reached. Remove those items from the ItemList and add new items with time t as the start time and a failure time of time t + generated life time. Add the other items from ToReplace to the list of outstanding replacements. If an item is already in the list of outstanding replacements, do not add it for the second time. Update the ItemList: if the not replaced items should have been replaced because of failure, set the start and the failure times of those items to Inf. Update InventoryLevel:  $\text{InventoryLevel} = \text{InventoryLevel} - \text{number of items in ToReplace}$ . If  $\text{InventoryLevel} < -N$ , set  $\text{InventoryLevel} = -N$ . Empty the vector ToReplace. Go to step 19.
19. Determine the number of replaced items per type of replacement (block replacement or failure replacement).  $\text{Replacement Costs} = \text{Replacement Costs} + \text{number of block replacements} * \text{Block Replacement Costs per item} + \text{number of failure replacements} * \text{Failure Replacement Costs per item}$ . If the replacement was an outstanding replacement, go to step 5. If the replacement was a block replacement, go to step 6. If the replacement was a failure replacement, go to step 7.
20. Order  $S - \text{InventoryLevel}$  items. Add these items to the outstanding order list with arrival time = time t + emergency lead time. Update Order Costs:  $\text{Order Costs} = \text{Order Costs} + \text{order set up costs} + \text{emergency} * \text{number to order} * \text{price per item}$ . Go to step 9.
21. Update the inventory costs and the shortage costs with the payment for the next day.  $\text{Inventory Costs} = \text{Inventory Costs} + \text{Inventory Level} * \text{Holding costs per item per day}$ .  $\text{Shortage Costs} = \text{Shortage Costs} + \text{Number of items shortage} * \text{Shortage costs per item per day}$ . Go to step 10.
22. Determine  $\text{Total Costs} = \text{Order Costs} + \text{Replacement Costs} + \text{Inventory Costs} + \text{Shortage costs}$ . Determine  $\text{Average Total Costs} = \text{Total Costs} / (\text{Simulation Length} + 1)$ . Go to step 23.
23. End.

### Simulation Program Algorithm – MBRP with separate ordering

1. Set the number of components (N), the distribution of the lifetime, the price per item, the different costs (holding costs, shortage costs, order set up costs for normal and emergency orders and replacement costs for block replacement and for failure replacement), the lead time for normal orders and for emergency orders, the simulation length, the initial InventoryLevel, the initial InventoryPosition, the outstanding orders and the outstanding replacements. Generate the lifetime for the N components. Store the failure time and the start time of the components in ItemList. Start time for the initial components is 0. Set time  $t = 0$ .
2. If time  $t \leq$  simulation length, go to step 3. Else go to step 23.
3. Compare the outstanding order list with time  $t$ , if there are ordered items with arrival time  $t$ , go to step 11.
4. Check if there are outstanding replacements that still need to be performed, this is the case if the list with outstanding replacements is not empty. If so, go to step 14. Else, go to step 5.
5. Check if the block replacement needs to be performed, this is the case if  $\text{mod}(\text{time } t, \text{parameter } T) = 0$  and time  $t$  is not 0. If so, go to step 13. Else, go to step 6.
6. Check if there are failed items. Compare the lifetimes of the items with the time  $t$ . If the lifetime  $<$  time  $t$ , the items have failed. Add the failed items to the vector ToReplace. If ToReplace is not empty, go to step 12. Else, go to step 7.
7. Update InventoryPosition. If there are no outstanding orders, InventoryPosition = InventoryLevel. Else, InventoryPosition = InventoryLevel + Total number of items from outstanding orders. Go to step 8.
8. If the next block replacement is one lead time for normal orders ahead, which means that  $\text{mod}(\text{time } t + \text{lead time for normal orders, parameter } T) = 0$ , go to step 20. Else and if InventoryPosition  $<$   $s$ , go to step 21. Else, go to step 9.
9. Determine the inventory level and the shortage level. The shortage level is the number of items in the ItemList, with both a start time and a failure time of Inf. Go to step 22.
10. Do time  $t = \text{time } t + 1$  and return to step 2.
11. Remove the arriving items from the outstanding order list and add the number of arriving items to the inventory level. If the inventory level was negative before the order arrived, go to step 15. Else go to step 5.
12. Compare the number of items from the vector ToReplace with the InventoryLevel. If the InventoryLevel  $>$  number of items to replace, go to step 17. Else set the number of items that can be replaced = InventoryLevel and go to step 18.
13. Determine which items need to be replaced in the block replacement. All items with an age  $\geq hT$  need to be replaced, place those items in ToReplace<sup>5</sup>. Age of an item is time  $t - \text{start time of that item}$ . If ToReplace is not empty, go to step 12. Else, go to step 6.
14. Determine which items need to be replaced, these are the items which are stored in the outstanding replacement list. Add these items to the vector ToReplace. If ToReplace is not empty, go to step 16. Else, go to step 5.
15. Determine the number of outstanding replacements (Z) that can be performed by the incoming items. If the size of the incoming order is larger than the absolute value of the inventory level before the order arrived,  $Z = - \text{InventoryLevel}$ , else  $Z = \text{size of the incoming order}$ . Go to step 4.

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<sup>5</sup> For the separate BR order  $N - \text{average \# failure}$  and Separate BR order  $N - \text{average \# skip}$ , update respectively the average number of failures and the average number of skipped items according to the update formulas in section 5.1.



16. If  $Z \geq$  items in the vector ToReplace, all items from the vector ToReplace can be replaced. Else, only  $Z$  items can be replaced. Remove the items that can be replaced from the ItemList and add new items with time  $t$  as the start time and a failure time of time  $t +$  generated life time. Place the other items from ToReplace back to the list of outstanding replacements. Empty the vector ToReplace. Go to step 19.
17. Remove all items that need to be replaced from the ItemList. Add new items, with time  $t$  as the start time and a failure time of time  $t +$  generated life time. Update the InventoryLevel and the InventoryPosition.  $\text{InventoryLevel} = \text{InventoryLevel} - \text{number of replaced items}$ .  $\text{InventoryPosition} = \text{InventoryPosition} - \text{number of replaced items}$ . Empty the vector ToReplace. Go to step 19.
18. Only replace the items from ToReplace until the number of items that could be replaced from the inventory level is reached. Remove those items from the ItemList and add new items with time  $t$  as the start time and a failure time of time  $t +$  generated life time. Add the other items from ToReplace to the list of outstanding replacements. If an item is already in the list of outstanding replacements, do not add it for the second time. Update the ItemList: if the not replaced items should have been replaced because of failure, set the start and the failure times of those items to Inf. Update InventoryLevel:  $\text{InventoryLevel} = \text{InventoryLevel} - \text{number of items in ToReplace}$ . If  $\text{InventoryLevel} < -N$ , set  $\text{InventoryLevel} = -N$ . Empty the vector ToReplace. Go to step 19.
19. Determine the number of replaced items per type of replacement (block replacement or failure replacement).  $\text{Replacement Costs} = \text{Replacement Costs} + \text{number of block replacements} * \text{Block Replacement Costs per item} + \text{number of failure replacements} * \text{Failure Replacement Costs per item}$ . If the replacement was an outstanding replacement, go to step 5. If the replacement was a block replacement, go to step 6. If the replacement was a failure replacement, go to step 7.
20. Order the number of items needed for the next block replacement<sup>6</sup>. Add these items to the outstanding order list with arrival time = time  $t +$  lead time normal. Update Order Costs:  $\text{Order Costs} = \text{Order Costs} + \text{order set up costs normal} + \text{number to order for next block replacement} * \text{price per item}$ . If  $\text{InventoryLevel} < s$ , go to step 21. Else, go to step 9.
21. Order  $S - \text{InventoryLevel}$  items. Add these items to the outstanding order list with arrival time = time  $t +$  lead time emergency. Update Order Costs:  $\text{Order Costs} = \text{Order Costs} + \text{order set up costs emergency} + \text{number to order} * \text{price per item}$ . Go to step 9.
22. Update the inventory costs and the shortage costs with the payment for the next day.  $\text{Inventory Costs} = \text{Inventory Costs} + \text{Inventory Level} * \text{Holding costs per item per day}$ .  $\text{Shortage Costs} = \text{Shortage Costs} + \text{Number of items shortage} * \text{Shortage costs per item per day}$ . Go to step 10.
23. Determine  $\text{Total Costs} = \text{Order Costs} + \text{Replacement Costs} + \text{Inventory Costs} + \text{Shortage costs}$ . Determine  $\text{Average Total Costs} = \text{Total Costs} / (\text{Simulation Length} + 1)$ . Go to step 24.
24. End.

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<sup>6</sup> The number of items to order is different for the different separate ordering policies. For the Separate BR order  $N - \#$  skip policy:  $N - \#$  skip items are ordered, for the Separate BR order  $N - \text{average} \#$  skip policy:  $N - \text{average} \#$  skip items are ordered and for the Separate BR order  $N - \text{average} \#$  failure policy:  $N - \text{average} \#$  failure items are ordered.

## 8.2 Appendix B

### *ANOVA and regression tables*

All ANOVA tables below are the results of the full factorial design with 81 experiments with the case study described in this thesis. For every experiment five simulations runs are used and the design is replicated three times. The tables with information about the regressions only contain the terms that had a significant influence on the costs in the ANOVA of the full factorial second order model.

#### Analysis of Variance for costs

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Regression	14	23217.4	23217.4	1658.39	645.32	0.000
Linear	4	19097.3	19097.3	4774.32	1857.82	0.000
Square	4	2477.4	2477.4	619.36	241.01	0.000
Interaction	6	1642.7	1642.7	273.79	106.54	0.000
Residual Error	228	585.9	585.9	2.57		
Lack-of-Fit	66	579.6	579.6	8.78	225.80	0.000
Pure Error	162	6.3	6.3	0.04		
Total	242	23803.4				

**Table B.1** - The ANOVA table for the joint ordering policy.

#### Analysis of Variance for costs

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Regression	14	37936.0	37936.0	2709.71	2039.85	0.000
Linear	4	35679.3	35679.3	8919.83	6714.79	0.000
Square	4	1799.3	1799.3	449.82	338.62	0.000
Interaction	6	457.4	457.4	76.23	57.39	0.000
Residual Error	228	302.9	302.9	1.33		
Lack-of-Fit	66	223.0	223.0	3.38	6.86	0.000
Pure Error	162	79.8	79.8	0.49		
Total	242	38238.9				

**Table B.2** - The ANOVA table for the Separate BR order N – #skip policy.

#### Analysis of Variance for costs

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Regression	14	12325.0	12325.0	880.36	266.26	0.000
Linear	4	6378.5	6378.5	1594.63	482.28	0.000
Square	4	2469.6	2469.6	617.41	186.73	0.000
Interaction	6	3476.9	3476.9	579.48	175.26	0.000
Residual Error	228	753.9	753.9	3.31		
Lack-of-Fit	66	586.7	586.7	8.89	8.61	0.000
Pure Error	162	167.2	167.2	1.03		
Total	242	13078.9				

**Table B.3** - The ANOVA table for the Separate BR order N – average#skip policy.

Analysis of Variance for costs

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Regression	14	8027.53	8027.53	573.395	252.93	0.000
Linear	4	2430.37	2430.56	607.640	268.03	0.000
Square	4	2638.54	2638.54	659.634	290.97	0.000
Interaction	6	2958.62	2958.62	493.104	217.51	0.000
Residual Error	228	516.88	516.88	2.267		
Lack-of-Fit	66	450.83	450.83	6.831	16.75	0.000
Pure Error	162	66.05	66.05	0.408		
Total	242	8544.41				

**Table B.4** - The ANOVA table for the Separate BR order N – average#failure policy.

Estimated Regression Coefficients for costs

Term	Coef	SE Coef	T	P
Constant	110.135	0.2692	409.162	0.000
s	2.020	0.1246	16.210	0.000
k	-6.575	0.1246	-52.770	0.000
h	-8.401	0.1246	-67.419	0.000
s*s	0.463	0.2158	2.143	0.033
k*k	-6.558	0.2158	-30.386	0.000
h*h	1.631	0.2158	7.556	0.000
s*k	-0.786	0.1526	-5.152	0.000
s*h	-0.554	0.1526	-3.633	0.000
k*h	3.779	0.1526	24.766	0.000

S = 1.58593      PRESS = 636.550  
R-Sq = 97.54%    R-Sq(pred) = 97.33%    R-Sq(adj) = 97.44%

**Table B.5** - The regression table for the joint ordering policy.

Estimated Regression Coefficients for costs

Term	Coef	SE Coef	T	P
Constant	115.706	0.16384	706.200	0.000
s	3.532	0.08974	39.356	0.000
k	0.974	0.08974	10.849	0.000
h	-14.381	0.08974	-160.249	0.000
k*k	-0.526	0.15544	-3.387	0.001
h*h	5.747	0.15544	36.972	0.000
s*h	-2.033	0.10991	-18.501	0.000
k*h	0.302	0.10991	2.747	0.006

S = 1.14221      PRESS = 327.045  
R-Sq = 99.20%    R-Sq(pred) = 99.14%    R-Sq(adj) = 99.17%

**Table B.6** - The regression table for the Separate BR order N – #skip policy.

Estimated Regression Coefficients for costs

Term	Coef	SE Coef	T	P
Constant	101.246	0.3073	329.486	0.000
s	-5.112	0.1422	-35.939	0.000
k	1.607	0.1422	11.298	0.000
h	3.264	0.1422	22.945	0.000
s*s	4.349	0.2464	17.653	0.000
k*k	0.967	0.2464	3.925	0.000
h*h	5.087	0.2464	20.649	0.000

s*k	-1.450	0.1742	-8.323	0.000
s*h	-5.272	0.1742	-30.261	0.000
k*h	1.487	0.1742	8.538	0.000

S = 1.81049      PRESS = 832.638  
R-Sq = 94.16%    R-Sq(pred) = 93.63%    R-Sq(adj) = 93.93%

**Table B.7** - The regression table for the Separate BR order N – average#skip policy.

Estimated Regression Coefficients for costs

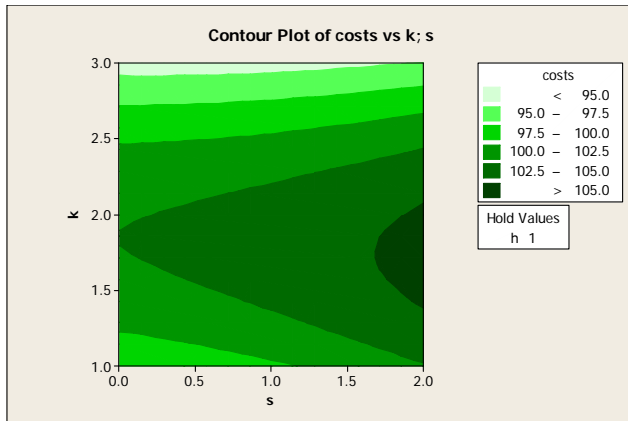
Term	Coef	SE Coef	T	P
Constant	108.861	0.2139	508.828	0.000
s	-3.521	0.1172	-30.049	0.000
k	-0.824	0.1172	-7.031	0.000
h	-1.384	0.1172	-11.812	0.000
s*s	3.316	0.2030	16.337	0.000
h*h	6.152	0.2030	30.310	0.000
s*k	0.544	0.1435	3.788	0.000
s*h	-5.137	0.1435	-35.795	0.000
k*h	-0.835	0.1435	-5.816	0.000

S = 1.49149      PRESS = 562.215  
R-Sq = 93.91%    R-Sq(pred) = 93.42%    R-Sq(adj) = 93.70%

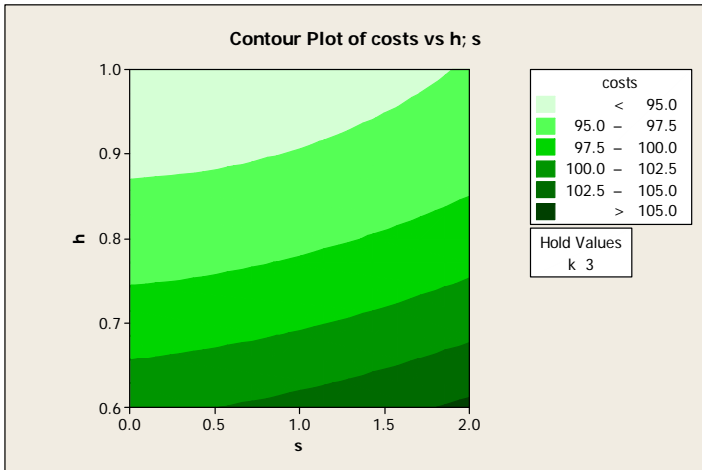
**Table B.8** - The regression table for the Separate BR order N – average#failure policy.

## 8.3 Appendix C

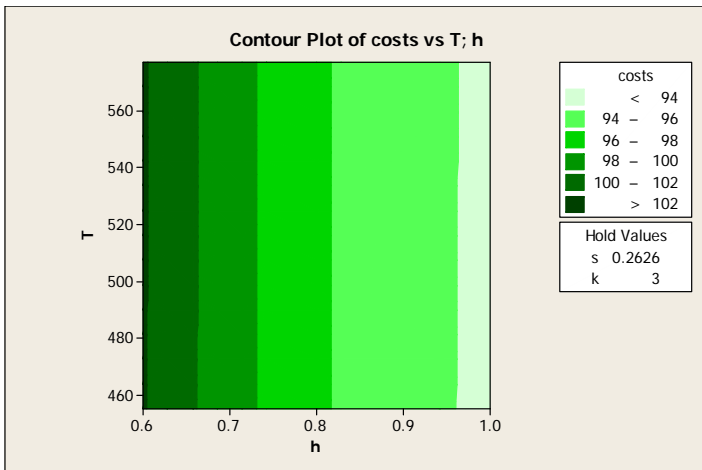
### Contour plots



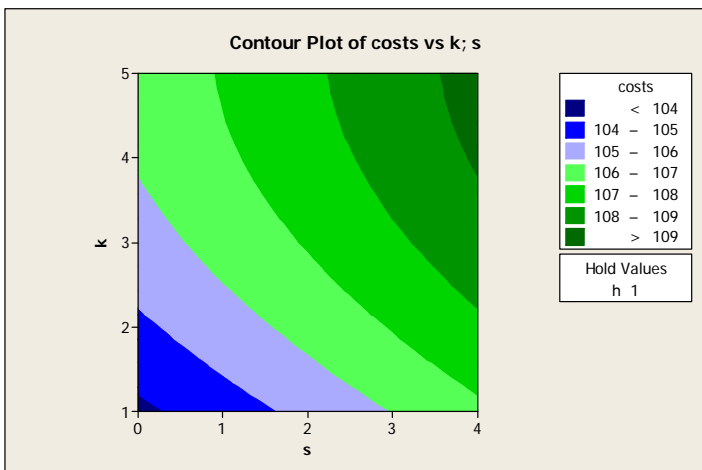
**Figure C.1** – Contour plot for the joint ordering policy, parameters k,s.



**Figure C.2** – Contour plot for the joint ordering policy, parameters h,s.



**Figure C.3** – Contour plot for the joint ordering policy, parameters T,h.



**Figure C.4** – Contour plot for the Separate BR order N - #skip policy, parameters k,s.

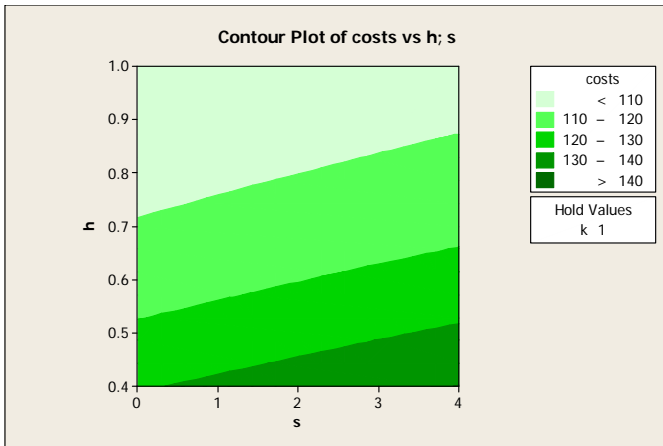


Figure C.5 – Contour plot for the Separate BR order N - #skip policy, parameters h,s.

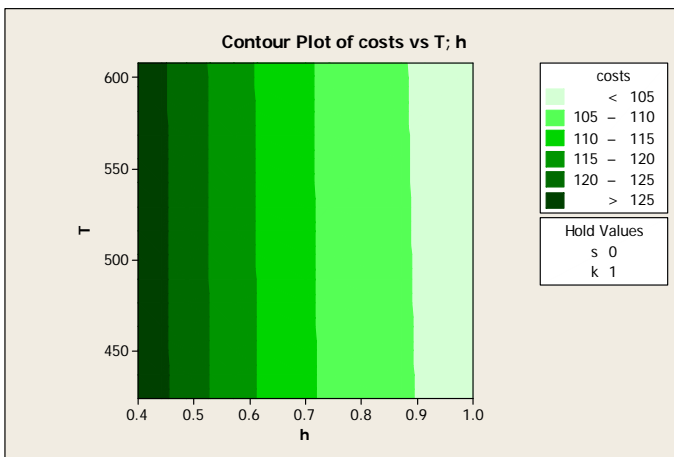


Figure C.6 – Contour plot for the Separate BR order N - #skip policy, parameters T,h.

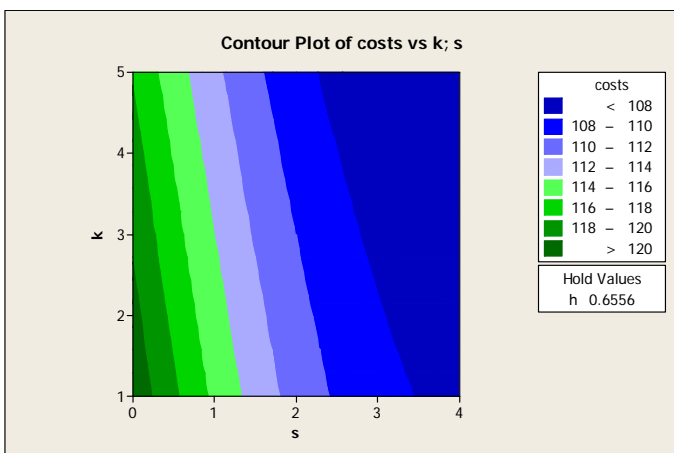
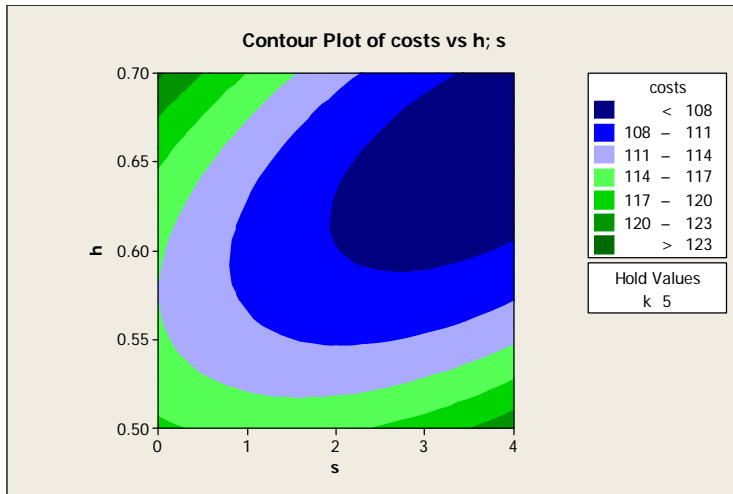
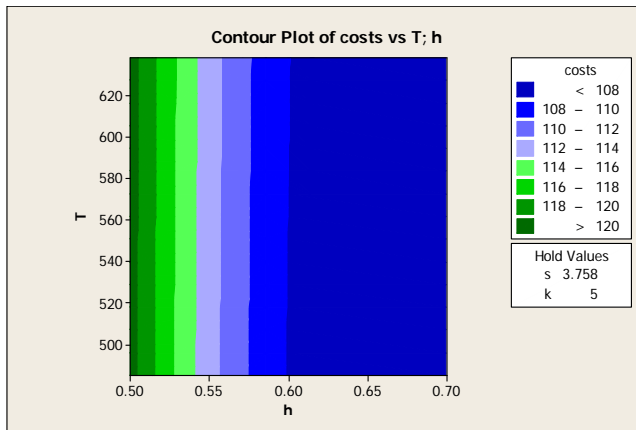


Figure C.7 – Contour plot for the Separate BR order N - average#failure policy, parameters k,s.



**Figure C.8** – Contour plot for the Separate BR order N - average#failure policy, parameters h,s.



**Figure C.9** – Contour plot for the Separate BR order N - average#failure policy, parameters T,h.

## 8.4 Appendix D

*Response surface methodology for the three parameters s, k and h, when T = 690.*

Analysis of Variance for costs

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Regression	9	16399.8	16399.8	1822.20	596.13	0.000
Linear	3	15730.2	15730.2	5243.40	1715.38	0.000
Square	3	483.9	483.9	161.28	52.76	0.000
Interaction	3	185.7	185.7	61.91	20.26	0.000
Residual Error	71	217.0	217.0	3.06		
Lack-of-Fit	17	63.1	63.1	3.71	1.30	0.226
Pure Error	54	153.9	153.9	2.85		
Total	80	16616.8				

**Table D.1** - The ANOVA table for the separate BR order N – average#skip policy.

Estimated Regression Coefficients for costs

Term	Coef	SE Coef	T	P
Constant	131.117	0.5140	255.110	0.000
s	-16.211	0.2379	-68.137	0.000
k	5.337	0.2379	22.432	0.000
h	0.122	0.2379	0.515	0.608
s*s	4.963	0.4121	12.044	0.000
k*k	1.483	0.4121	3.600	0.001
h*h	0.213	0.4121	0.517	0.607
s*k	-2.268	0.2914	-7.785	0.000
s*h	0.081	0.2914	0.277	0.783
k*h	-0.084	0.2914	-0.290	0.773

S = 1.74834 PRESS = 279.763  
R-Sq = 98.69% R-Sq(pred) = 98.32% R-Sq(adj) = 98.53%

**Table D.2** - The regression table for the separate BR order N – average#skip policy.

## 8.5 Appendix E

### *ANOVA and regression tables*

The ANOVA and regression tables for the four different policies applied to the case study from the thesis of Mardin are given below. The regression tables only contain the significant terms from the ANOVA.

Analysis of Variance for costs

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Regression	14	229.584	229.584	16.3989	459.42	0.000
Linear	4	118.264	118.264	29.5659	828.30	0.000
Square	4	99.225	99.225	24.8062	694.96	0.000
Interaction	6	12.096	12.096	2.0160	56.48	0.000
Residual Error	228	8.138	8.138	0.0357		
Lack-of-Fit	66	2.953	2.953	0.0447	1.40	0.046
Pure Error	162	5.186	5.186	0.0320		
Total	242	237.723				

**Table E.1** - The ANOVA table for the joint order policy.

Analysis of Variance for costs

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Regression	14	196.985	196.985	14.0704	106.68	0.000
Linear	4	114.889	114.889	28.7221	217.76	0.000
Square	4	76.042	76.042	19.0105	144.13	0.000
Interaction	6	6.055	6.055	1.0091	7.65	0.000
Residual Error	228	30.073	30.073	0.1319		
Lack-of-Fit	66	9.853	9.853	0.1493	1.20	0.183
Pure Error	162	20.220	20.220	0.1248		
Total	242	227.058				

**Table E.2** - The ANOVA table for the separate BR order N - #skip policy.



Analysis of Variance for costs

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Regression	14	9979.7	9979.69	712.84	2078.91	0.000
Linear	4	5901.1	5901.11	1475.28	4302.50	0.000
Square	4	4061.5	4061.51	1015.38	2961.25	0.000
Interaction	6	17.1	17.07	2.84	8.30	0.000
Residual Error	228	78.2	78.18	0.34		
Lack-of-Fit	66	37.1	37.10	0.56	2.22	0.000
Pure Error	162	41.1	41.08	0.25		
Total	242	10057.9				

**Table E.3** - The ANOVA table for the separate BR order N - average#skip policy.

Analysis of Variance for costs

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Regression	14	9020.10	9020.10	644.29	3570.11	0.000
Linear	4	5253.71	5253.71	1313.43	7277.87	0.000
Square	4	3760.08	3760.08	940.02	5208.77	0.000
Interaction	6	6.32	6.32	1.05	5.83	0.000
Residual Error	228	41.15	41.15	0.18		
Lack-of-Fit	66	11.98	11.98	0.18	1.01	0.473
Pure Error	162	29.17	29.17	0.18		
Total	242	9061.25				

**Table E.4** - The ANOVA table for the separate BR order N - average#failure policy.

Estimated Regression Coefficients for costs

Term	Coef	SE Coef	T	P
Constant	9.08428	0.02730	332.729	0.000
s	0.54735	0.01495	36.602	0.000
k	0.03428	0.01495	2.293	0.023
h	-0.65477	0.01495	-43.785	0.000
T	0.02290	0.01495	1.531	0.127
s*s	0.20604	0.02590	7.955	0.000
h*h	1.33857	0.02590	51.680	0.000
s*h	-0.30964	0.01831	-16.906	0.000
s*T	-0.04108	0.01831	-2.243	0.026
k*h	0.11521	0.01831	6.291	0.000

S = 0.190335    PRESS = 9.17354  
R-Sq = 96.45%    R-Sq(pred) = 96.14%    R-Sq(adj) = 96.31%

**Table E.5** - The regression table for the joint ordering policy.

Estimated Regression Coefficients for costs

Term	Coef	SE Coef	T	P
Constant	8.58053	0.05200	165.000	0.000
s	0.48896	0.02848	17.166	0.000
k	0.67886	0.02848	23.833	0.000
h	-0.02640	0.02848	-0.927	0.355
T	0.09258	0.02848	3.250	0.001
h*h	1.18216	0.04933	23.962	0.000
T*T	-0.10294	0.04933	-2.087	0.038
s*k	0.10279	0.03488	2.947	0.004
s*h	-0.12808	0.03488	-3.672	0.000
k*h	0.15502	0.03488	4.444	0.000

S = 0.362534 PRESS = 33.2983  
R-Sq = 86.51% R-Sq(pred) = 85.33% R-Sq(adj) = 85.99%

**Table E.6** - The regression table for the separate BR order N - #skip policy.

Estimated Regression Coefficients for costs

Term	Coef	SE Coef	T	P
Constant	8.1737	0.08450	96.730	0.000
s	0.3554	0.04628	7.679	0.000
k	0.3418	0.04628	7.385	0.000
h	-6.0142	0.04628	-129.944	0.000
T	-0.1147	0.04628	-2.477	0.014
k*k	-0.2430	0.08016	-3.032	0.003
h*h	8.6691	0.08016	108.142	0.000
s*h	-0.2830	0.05668	-4.993	0.000
k*h	0.1658	0.05668	2.926	0.004
h*T	0.1609	0.05668	2.839	0.005

S = 0.589085 PRESS = 88.2115  
R-Sq = 99.20% R-Sq(pred) = 99.12% R-Sq(adj) = 99.17%

**Table E.7** - The regression table for the separate BR order N - average#skip policy.

Estimated Regression Coefficients for costs

Term	Coef	SE Coef	T	P
Constant	7.9113	0.06139	128.877	0.000
s	0.3345	0.03362	9.948	0.000
k	0.4401	0.03362	13.088	0.000
h	-5.6677	0.03362	-168.569	0.000
k*k	0.3326	0.05824	5.710	0.000
h*h	8.3374	0.05824	143.165	0.000
s*h	-0.1831	0.04118	-4.446	0.000
k*h	0.1210	0.04118	2.937	0.004

S = 0.427946 PRESS = 46.2014  
R-Sq = 99.53% R-Sq(pred) = 99.49% R-Sq(adj) = 99.51%

**Table E.8** - The regression table for the separate BR order N - average#failure policy.