Modeling and Optimization of Maintenance Policies for Computers

Update or replace a computer?

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Abstract

The performance of a computer deteriorates over time. It is important that the computer performance stays above a required minimum performance level, P_{min}, in order not to affect the working efficiency. Therefore, a good maintenance policy is necessary. In this thesis we determine the optimal maintenance policy for computers for a period of T years, in the sense of guaranteeing a computer performance above or equal to P_{min} at minimized costs. We assume that there are 3 types of computers available and that each type of computer consists of 4 different components. Because a computer performance mainly depends on the performance of its components, we first model the component performance. Study showed that the components exponentially deteriorate [5], which is therefore taken into account. We propose to model the computer performance at time t as the average of all component performances at time t. To determine the optimal maintenance policy, two types of policies are implemented, namely deterministic and stochastic policies. Each type of policy considers replacing components, which is called updating, or replacing the computer. The deterministic policies execute maintenance in a fixed order, while the stochastic policies determine a maintenance policy with a decision rule. We propose 2 decision rules: the performance – cost ratio and the lifetime extension – cost ratio. Stochastic policies in general provide better maintenance policies than deterministic policies. Therefore, we determine which decision rule delivers the optimal maintenance policy to make a recommendation which type of computer the best option to buy is. At last sensitivity analysis is implemented to test whether the choice of the optimal decision rule changes when the settings change.

Key words: maintenance optimization model, maintenance policy, computer, component, exponential deterioration.

Contents

1 Introduction

Nowadays it is impossible to imagine to work without computers. Computers are subjected to deterioration with usage and age. Failure of computers during operating can affect the working efficiency of employees. It is important to avoid failure since such an event can be costly and therefore, a good maintenance policy is necessary. The deterioration of the computer depends mainly on the deterioration of its components. Earlier study showed that the components exponentially deteriorate [5].

Company X wants us to determine the optimal maintenance policy for computers at minimized costs while guaranteeing a minimum computer performance level, P_{min}. Hence, a computer performance below P_{min} can be seen as failure. We will determine the optimal maintenance policy while taking into account that the computer performance depends on the performances of its components. Many maintenance problems have been investigated in the past several decades however, no such problem as ours has been investigated.

In this research we assume that components and computers can be replaced. Replacing components is called updating/upgrading throughout this research and replacement throughout indicates the replacement of a computer. We will first model the component performances. Thereafter the computer performance will be modeled. We will then implement deterministic policies to check whether updating is more beneficial than replacement. Next stochastic policies will be used to determine the optimal maintenance policy. The stochastic method uses a decision rule to determine a maintenance policy. We propose 2 decision rules: the performance – cost ratio and lifetime extension – cost ratio. Our research question is:

What will be the best decision rule which yields the maintenance policy with the minimum costs while guaranteeing that the computer performance stays above the minimum performance level (P_{min})?

This thesis is structured as follows: first a short literature overview about maintenance policies is given. Section 2 contains the model descriptions. The deterministic and stochastic policies are discussed in section 3. In section 4 the results are presented. Also, sensitivity analysis is implemented to check whether our obtained results remain the same when the settings change. The results of the sensitivity analyses are discussed in section 5. Finally, section 6 concludes.

1.1 Literature overview

Maintenance can be defined as the combination of all technical and associated administrative actions intended to retain an item or system in, or restore it to, a state in which it can perform its required function [2]. From literature surveys [3, 4, 6, 8, 10] it appears that most papers deal with maintenance optimization models. [4] defines a maintenance optimization model as a mathematical model in which both costs and benefits of maintenance are quantified and in which an optimum balance between both is obtained.

In many papers a maintenance strategy is made by optimizing a certain type of policy. Well known maintenance policies are the age and block replacement policies [1]. The vast majority of literature uses or extends these policies. Under the standard age replacement policy, it is assumed that a planned replacement is made when the age of the system reaches a predetermined age T, or a replacement is made whenever the system fails. Under the standard block replacement policy, it is assumed that a planned replacement will be made at time epochs T, $2T$, $3T$…, irrespective of the age of the system and that a replacement is made whenever the system fails.

In [7] policies of the block replacement type are extended for two-component systems. The authors group block replacement in the following way: replace failed components and replace the system at times T, $2T$, $3T$,… . They also propose a combined policy: replace both components (whether failed or not) on failure of the system and replace the system at times T, $2T$, $3T$,… . Also a modified block replacement policy [7] for a two-component system is proposed where a component is only replaced at the block replacement times if its age is greater than a determined critical value.

In [9] the problem of replacing light bulbs in traffic control signals is examined. It is a multi-component problem since each installation consists of three compartments for the green, red, and yellow lights. The failure of individual bulbs is an opportunity for doing preventive maintenance on other bulbs. The authors propose an age-based grouping policy. Upon failure of a light bulb, the failed bulbs and all other bulbs older than a certain age are replaced.

As noted in [3] it must be taken into account that there can exist some dependence in multi-components systems, for example economic or stochastic dependence. Economic dependence means performing maintenance on several components jointly, costs less money and/or time than on each component separately. We will discuss economic dependence in section 2. In this research we assume that components do not influence each other's performance, so no stochastic dependence is present.

2 Model description

In this section we model component and computer performances and prices. We will analyze all performances over a period of T years. In section 2.1 the component performance model is introduced. The computer performance model, which depends on the components performance models, is introduced in section 2.2. In the last section the costs of all components and computers are presented.

2.1 Component performance

In this research we assume that 3 types of computers are available: a low end, medium end and a high end computer. A computer consists of multiple components. We assume that a computer consists of 4 components. These components are the central processing unit (CPU), graphics processing unit (GPU), hard disk drive (HDD) and the remaining system (REM). The CPU is the portion of a computer system that carries out the instructions of a computer program. It is the primary element carrying out the functions of the computer or other processing device [11]. The GPU is a specialized circuit designed to rapidly manipulate and alter memory in such a way so as to accelerate the building of images in a frame buffer intended for output to a display [12]. The HDD is a non-volatile, random access device for digital data [13]. We assume that the REM consist of all remaining components, for example the RAM-memory, sound cards, USB ports etc..

The component performance model should take into account that the component performance exponentially deteriorates [5]. Therefore, the following performance function for a component j of a type i computer is proposed:

$$P_{ij}(t) = P_{ij}(0)\, e^{(-\lambda_{ij} t)} \tag{1}$$

With

- $P_{ij}(t)$: performance level of component j of a type i computer at time t;
- $P_{ij}(0)$: initial performance level of component j of a type i computer;
- λ_{ij}: deterioration rate of component j of a type i computer.

And indices
j = components, 1, 2, 3, 4 (1 = CPU, 2 = GPU, 3 = HDD, 4 = REM)
i = type computer, 1, 2, 3 (1 = Low, 2 = Medium, 3 = High end computer)

Thus, (1) computes the component performance at time t, with $t \leq T$. Note that $P_{ij}(t)$ only takes values between 0 and $P_{ij}(0)$. The component performance is expressed in percentages and indicates the fitness of the component.

To make computations a bit easier we assume that every new component of a type i computer has the same initial performance level as a type i computer. This means that $P_{ij}(0) = P_i(0)$. We take the average of the initial performance levels of the components, which are given in [5], to compute the initial performance level per type of computer. This average is then rounded off to the nearest tenfold. The obtained initial performance levels per type of computer are shown in table 1.

	Type of computer		
	Low	*Medium*	*High*
$P_i(0)$	60	80	100

Table 1 – Initial performance level, $P_i(0)$, per type of computer

The deterioration rates of the CPU, GPU and HDD are computed in [5] and are shown in table 2. The CPU of each type of computer deteriorates the fastest. What is strikingly, is that the HDD deterioration rate of the medium end computer is very low compared to the HDD deterioration rates of the low and high end computer. Though, no explanation for this can be found or is given in [5]. The deterioration rate of the REM is not known. We assume that the REM slowly deteriorates so replacement of the REM is excluded. Thus, to take this assumption into account, the REM deterioration rate, λ_{i4}, should be low. We also assume that the REMs of all types of computers differ, so all should have different λ_{i4} rates, otherwise we would assume that the REMs of all computers are the same. Notice that the deterioration rates of the other components can be the same but because the REM differs in many ways, as mentioned before, we assume that λ_{i4} of each type of computer cannot be the same. Furthermore we assume that the REM of the high end computer has the most advanced parts of the 3 computers. By this we mean that a high end computer contains for example more USB ports than a medium or low end computer. Because of this, more components will simultaneously deteriorate which leads to a higher deterioration than the other two types of computers. The REM deterioration rate per type of computer is also shown in table 2.

	Type of computer		
Component	*Low*	*Medium*	*High*
CPU	0.65	0.72	0.65
GPU	0.45	0.45	0.67
HDD	0.43	0.14	0.44
REM	0.01	0.02	0.03

Table 2 – Deterioration rates per component and type of computer, λ_{ij}

2.2 Computer performance

The computer performance depends on the performances of the components. Therefore, the computer performance model must involve the models of the components performances. Note that a type *i* computer only consists of type *i* components thus can only be upgraded with similar type *i* components.

The computer performance can be modeled in multiple ways. For instance as the average, the weighted average or as the minimum performance of all components performances. In this research we propose to model the computer performance as the average performance. The average computer performance is computed by taking the average of all component performances at time *t*. This is expressed with the following formula:

$$P_i(t) = \frac{1}{4}\sum_{j=1}^{4} P_{ij}\,(t) \tag{2}$$

Thus, (2) computes the computer performance at time t, with $t \leq T$. $P_{ij}(t)$ in (2) is computed as in (1). Also, $P_i(t)$ can only take values between 0 and $P_i(0)$. This formulation of the computer performance assumes that all components performances are equally important. Note that, because we take the average of all components, it can be possible that although $P_i(t) \geq P_{min}$, $P_{ij}(t)$ can be below P_{min}. The computer performance is expressed in percentages and indicates the fitness of the computer.

2.3 Costs

Prices of computers decrease over time due to technological progress. Therefore, we take this price decrease into account in our computations. For now we will assume that the prices of the components stay the same over time. This is a reasonable assumption as the producer has a limited inventory of different types of computers and components. And since he is more willing to sell computers than components, he will not lower the prices of the components, so people will be more inclined to buy a new computer. The following formula is used to take the price decrease of computers into account:

$$C_i(t) = \frac{C_i(0)}{\left(1 + \frac{\alpha}{100}\right)^t} \tag{3}$$

With

- $C_i(t)$: the price of a type i computer at time t
- $C_i(0)$: the initial price of a type i computer
- α: discount rate

Thus, (3) computes the discounted computer price at time t. The prices of the components and computers are shown in table 3 and are based on www.tigerdirect.com. As mentioned in section 1.1, economic dependence can exist. Though for now we will assume that a maintenance action is immediate, complete and costless, so no economic dependence is present.

	Type of computer		
	Low	*Medium*	*High*
Computer	€ 700	€ 1095	€ 1500
Component			
CPU	€ 120	€ 170	€ 240
GPU	€ 115	€ 155	€ 200
HDD	€ 40	€ 55	€ 90
REM	€ 500	€ 800	€ 1300

Table 3 – Prices of new computers and components

3 Policies

Maintenance policies in general can be divided into 2 categories: deterministic policies and stochastic policies. We first implement deterministic policies to test if updating is more beneficial than replacement. In section 3.2 the stochastic policy is presented which determines a maintenance policy with a decision rule. We propose 2 decision rules: the performance – cost ratio and the lifetime extension – cost ratio.

3.1 Deterministic policy

In order to get an impression whether updating is more beneficial than replacement in terms of costs, different predetermined maintenance policies are examined in a deterministic setting. A deterministic maintenance policy (DMP) indicates which maintenance action will take place whenever $P_i(t) < P_{min}$. Hence, the DMPs can be seen as recursive updates till T is reached.

Many DMP combinations exist but we will only examine 17 DMPs, which have been split up in three categories that can be seen in table 4. The first category, DMP 1 - 10, assumes that after a computer is purchased it will only be upgraded. The first 6 DMPs only update one component per maintenance. DMPs 7 - 9 update one or two components per maintenance while DMP 10 updates 3 components per maintenance. The second category, DMP 11, assumes that after a computer is purchased it will only be replaced. The third category, DMP 12 - 17, assumes that after a computer is purchased both upgrading or replacement can take place. We should keep in mind that the DMPs presented here do not guarantee to be the best MP, since not all possible DMP combinations will be examined. The algorithm of a DMP is given in table A1 in appendix A.

The expectation is that (a mixture of) updating components (and replacement of computers) will be more cost effective than replacing a computer whenever $P_i(t) < P_{min}$. Because, looking at the prices, it seems more beneficial to upgrade components than to replace a computer whenever $P_i(t) < P_{min}$.

The results of this policy will be discussed in section 4.1.

	Category	
1	*2*	*3*
1. CPU, GPU, HDD	11. PC	12. CPU, PC
2. CPU, HDD, GPU		13. GPU, PC
3. GPU, HDD, CPU		14. HDD, PC
4. GPU, CPU, HDD		15. CPU, GPU, PC
5. HDD, CPU, GPU		16. CPU, HDD, PC
6. HDD, GPU, CPU		17. GPU, HDD, PC
7. CPU & GPU, HDD		
8. CPU & HDD, GPU		
9. HDD & GPU, CPU		
10.CPU & GPU & HDD		

Table 4 – Predetermined maintenance policies

3.2 Stochastic policy

The stochastic policy determines the optimal maintenance policy with a decision rule. The decision rule determines if an update or replacement should take place whenever $P_i(t) < P_{min}$.

We assume that there are 8 different maintenance actions possible, of which 7 update components and 1 replaces the computer. These maintenance actions are shown in table 5. Updating the REM is excluded.

1. CPU	4. CPU & GPU	7. CPU & GPU & HDD
2.GPU	5. CPU & HDD	8. Computer
3.HDD	6. HDD & GPU	

Table 5 – Possible maintenance actions

We propose 2 different decision rules to decide what maintenance action should be executed whenever $P_i(t) < P_{min}$ namely, the performance - cost ratio and the lifetime extension - cost ratio. We will compare the results of the 2 decision rules and decide which decision rule performs better, within the meaning of reaching T at the lowest possible cost while guaranteeing $P_i(t) \geq P_{min}$. We will also make recommendations which type of computer the best option to buy is using the obtained results. The following subsections explain the decision rules in more detail.

3.2.1 Performance - cost ratio

We will compute the performance – cost ratio, ΔP/C, as formulated in (4) for every maintenance action of table 5. The performance improvement, ΔP, is calculated by subtracting the computer performance level after maintenance from the computer performance level before maintenance. After this the performance improvement is divided by the cost of this maintenance. This procedure will yield 8 ΔP/C ratios, of which the maintenance with the highest ΔP/C ratio will be executed. We repeat this procedure whenever $P_i(t) < P_{min}$ and until T is reached. Eventually we will get a maintenance policy indicating what maintenance action should take place at what time. The algorithm of this decision rule is given in table A2 in appendix A.

$$Performance - cost\ ratio = \frac{\Delta P_i^k(t)}{C_k} \tag{4}$$

With:

- $\Delta P_i^k(t)$: Performance improvement of a type i computer due to maintenance k at time t
- C_k: Cost of maintenance k

K indicates a maintenance possibility from table 5, hence it can take a value from 1 to 8.

The results of this decision rule will be discussed in section 4.2

3.2.2 Lifetime extension - cost ratio

We will compute the lifetime extension – cost ratio, ΔL/C, as formulated in (5) for every maintenance action of table 5. The lifetime extension, ΔL, is calculated by subtracting the time until new maintenance is needed from the time maintenance is executed. We calculate the time until a new maintenance is needed by running a simulation where a maintenance action has been executed and note the time when $P_i(t) < P_{min}$. After this, the lifetime extension is divided by the cost of this maintenance. This procedure will yield 8 ΔL/C ratios, of which the maintenance with the highest ΔL/C ratio will be executed. We repeat this procedure whenever $P_i(t) < P_{min}$ and until T is reached.

$$\textit{Lifetime extension} - \textit{cost ratio} = \frac{\Delta L_i^k(t)}{C_k} \quad (5)$$

With:

- $\Delta L_i(t)$: Lifetime extension of a type i computer due to maintenance k
- C_k: Cost of maintenance k

K indicates a maintenance possibility from table 5, hence it can take a value from 1 to 8.

This decision rule provides the possibility to take into account that near the end of a planning period T, an expensive last maintenance can be selected while this maintenance is not necessarily needed. We give a example to illustrate this idea. Suppose we must determine a maintenance policy for 10 years and a maintenance at t = 9.10 is needed. The hypothetical maintenance actions are given in table 6. According to our criteria update A will be selected, since it has the highest ΔL/C ratio. However maintenance actions B and C would also be sufficient since 9.10 + 1.00 = 10.10 and 9.10 + 0.95 = 10.05. Also, executing maintenance action B or C would save €150. To take this situation into account we therefore check whether a cheaper maintenance action is possible when the last maintenance is selected, that is when $\Delta L + t > T$ were t denotes the time at which $P_i(t) < P_{min}$ is. Maintenance actions that at least reach T, so satisfy $\Delta L + t > T$, must only be considered as a substitution option of the current selected maintenance. Among all substitution options, the cheapest substitution option will be selected to be executed. So in our previous example we would only consider maintenance B and C as substitution options because maintenance D, 9.10 + 0.25 = 9.35, does not satisfy $\Delta L + t > T$.

Maintenance	*Cost*	$\Delta L_i(t)$	$\Delta L_i(t)/C$
A	€ 300	2.50	0.0083
B	€ 150	1.00	0.0067
C	€ 150	0.95	0.0063
D	€ 100	0.25	0.0025

Table 6 – Hypothetical maintenance possibilities

It can also be that 2 or more substitutions will be selected. If that is the case, the maintenance action with the highest ΔL/C ratio will be selected to be executed. In our previous example this would mean that maintenance B instead of C would be executed. The algorithm of this decision rule is given in table A3 in appendix A. The results of this policy will be discussed in section 4.3.

4 Results

In this section the results of all proposed policies are presented. The simulations of the policies have been done with Matlab. We used a fixed-increment time advance mechanism to analyze the performance functions. We analyzed the computer and components performances over a period of 10 years, that is $T = 10$. The parameters presented in section 2 were used and P_{min} was set to 40%. We discounted the computer prices at a rate of 10%, which is $\alpha = 10$ in formula (3). As mentioned before, a maintenance action was executed whenever $P_i(t) \leq P_{min}$, where $P_i(t)$ was computed as formulated in (2). We further assumed that a maintenance action was immediate, complete and costless. Replacement of the REM was excluded.

In section 4.1 the results of the deterministic policy are presented. In the next section we will discuss the results of the performance – cost ratio. In section 4.3 the results of the lifetime extension cost – ratio will be discussed. At last we will compare the results of all policies in section 4.4, to determine the best decision rule.

4.1 Deterministic policy

The execution of all 17 proposed DMPs from table 4 took less than 7 seconds per type of computer. The results are given in table B1-B3 in appendix B. Table 7 summarizes the results. It shows the total costs of the most and least expensive DMPs. The total costs include the purchase of a computer at the start and the maintenance costs. DMP nr indicates the regarding DMP from table 4.

It turns out that all least expensive DMPs are from category 1. More specifically, the DMPs which only upgrade one component whenever $P_i\,(t) < P_{min}$ appear to be the least expensive maintenance policies. DMP 11 from category 2, which represents replacing a computer whenever $P_i\,(t) < P_{min}$, turns out to be the most expensive maintenance policy. This is in accordance with our expectations. The difference between the least and most expensive DMP range from € 1341,72 to € 2100,30, so we can say that a lot of money can be saved when we do not replace a computer whenever $P_i(t) < P_{min}$.

	Type of computer		
	Low	*Medium*	*High*
Least expensive DMP	€ 2115,00	€ 1685,00	€ 2560,00
DMP nr	5, 6	3, 6	1, 2, 3, 4, 5, 6
Most expensive DMP	€ 4215,30	€ 3026,72	€ 4292,90
DMP nr	11	11	11
Absolute difference	€ 2100,30	€ 1341,72	€ 1732,90
% Difference	99.31%	79.63%	67.69%

Table 7 – The least and most expensive DMPs

The total costs of the DMPs category 3, which are shown in table B1-B3, are also pretty high. This can also be explained due to the high price of the computer when it is replaced. For instance, the maintenance times of DMP 15 - 17 of the medium and high end computer show that after approximately 5 years a computer is replaced. The prices of the medium and high end computer are then respectively, € 679,91 and € 931,38. These are high compared to the price of a maintenance where only 1, 2 or even 3 components would be upgraded.

We cannot conclude that all policies from category 1 are better than the policies from category 3 as DMP 17 is cheaper than DMP 10 for all types of computers. This means that we cannot conclude that updating components is always more beneficial than considering both updating or replacement. We can only conclude that updating and considering both updating or replacement are more beneficial than replacement only.

As noted before, we see that the proposed DMPs do not have to be the most efficient maintenance policies. For instance, the last maintenance of DMP 15 of the medium end computer is executed at 9.97 and the GPU is updated. Lower maintenance costs could be achieved when instead of the GPU the HDD would be updated, given that this maintenance would provide sufficient improvement in the computer performance. Because it is not possible to alter the maintenance pattern of a DMP we therefore conclude that the best DMP of our proposed DMPs does not have to be to the most efficient maintenance policy. Also, the computer performance at the end, $P_i(10)$, of some DMPs are higher than 50% while a computer performance of at least 40% is required. The maintenance times of these DMPs show that the last update has been carried out in the very end causing a high computer performance at the end.

In table 8, four features are shown of the least expensive DMPs per type of computer. The low end computer needs the most maintenance of all types of computers. This is not surprising since the initial performance level is the closest to P_{min} 40% of all types of computers. You would therefore expect that the low end computer would quicker need maintenance than the other two types of computers. However, remarkable is that the medium end computer needs less maintenance than the high end computer. You would expect that the high end computer would need the least maintenance since it has the highest initial performance level. We take a look at figure 1, which displays the development of the computer and components performances of DMP 3, to give an explanation for this. It can be seen that whenever maintenance is executed the performance level of the component jumps to the initial performance level and the computer performance improves. The figure also shows that the medium end computer, despite its lower initial performance level, has its first maintenance later executed than the high end computer. The subsequent maintenances also follow later than the high end computer. The explanation for this appearance is that the deterioration rates of the GPU and HDD of the medium end computer are much lower than the deterioration rates of the GPU and HDD of the high end computer, causing it to take more time whenever maintenance is needed for a medium end computer.

Another striking thing that can be seen in table 8 is that the low end computer, despite the highest maintenance costs, is a more beneficial option than the high end computer. This is because the purchase of a low end computer is cheaper than the purchase of a high end computer at the start.

We conclude that the medium end computer is best option to buy at P_{min} 40% as it has the lowest total cost namely, € 1685,00.

	Type of computer		
Least expensive DMPs	*Low*	*Medium*	*High*
End computer performance $P_i(10)$	42.77 - 43.28	40.53 - 40.99	43.11 - 44.93
Total # maintenances	16	5	6
Total maintenance cost	€ 1415,00	€ 590,00	€ 1060,00
Total cost	€ 2115,00	€ 1685,00	€ 2560,00

Table 8 – Least expensive DMP per type of computer

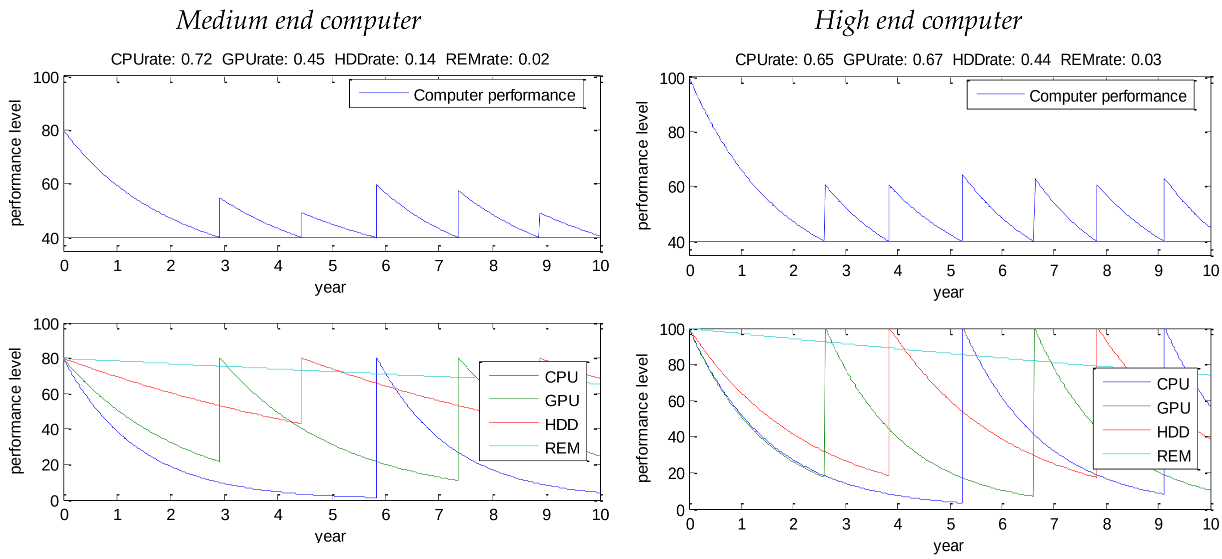


Figure 1 - Development of the computer and component performances of a medium and high end computer: The top figures display the computer performance and the bottom figures display the components performance.

4.2 Performance - cost ratio

The execution of the performance – cost ratio algorithm approximately took 0.24 seconds per type of computer. The results are shown in table 9. The table shows the performance level at time T = 10, $P_i(10)$, the number of maintenances executed and its related costs. The total cost includes the purchase of a computer at the start and the maintenance costs. The maintenance policy indicates which maintenances are executed. The corresponding numbers indicate a maintenance action from table 5. For instance, maintenance policy 7 3 1 indicates that first the CPU&GPU&HDD are upgraded after that the HDD and last the CPU. The upgrade times corresponding to the maintenance policies are given in table C1 in appendix C.

All maintenance policies in table 9 show that per maintenance only one component is upgraded. The HDD (nr. 3) is the most frequently updated component. The HDD has the lowest price of all components which leads to a high ΔP/C ratio, given that there is a sufficient computer performance improvement, which explains why the HDD is so frequently updated. The CPU (nr. 1), which is the most expensive component, is the least frequently updated.

Remarkable about the maintenance policy of the high end computer is that the CPU is never updated. Figure C1 in appendix C shows the performance development of the high end computer. It can be seen that even though the computer performance stays above 40%, the CPU performance approaches zero. We note that it is not realistic to assume that the computer works fine while a component performance approaches zero.

Another thing we note is that this decision rule does not have to lead to the most efficient maintenance policy. The maintenance times and policy of the low end computer show that the last maintenance takes place at $t = 9.96$ and that the CPU is updated. Lower costs could be achieved if instead of the CPU, the GPU *or* HDD (both are cheaper than the CPU) would be updated, given that this update would provide sufficient improvement in the computer performance. Also, the update near the end of T of the low end computer leads to a high end performance, $P_i(10)$, while a P_{min} of 40% is required.

The low end computer needs the most maintenance and due to that has the highest maintenance cost. The medium end computer needs less maintenance than the high end computer. The explanation given in 4.1 also applies here. The medium end computer has the lowest total cost namely € 1755,00. This makes the medium end computer the best option to buy when a P_{min} of 40% is required.

	Type of computer		
ΔP/C ratio	*Low*	*Medium*	*High*
End computer performance $P_i(10)$	50.34	45.40	45.86
Total # maintenances	18	6	7
Total maintenance cost	€ 1420,00	€ 660,00	€ 850,00
Total cost	€ 2120,00	€ 1755,00	€ 2350,00
Maintenance policy	3 1 3 2 3 1 3 2 3 1 3 2 3 1 3 2 3 1	3 1 2 3 1 3	3 3 2 3 3 2 3

Table 9 – Results performance - cost ratio

4.3 Lifetime extension - cost ratio

The execution of the lifetime extension – cost ratio algorithm approximately took 1.65 seconds per type of computer. This is relatively longer than execution of the performance – cost ratio algorithm or a DMP but this is because the lifetime extension, ΔL, per maintenance action has to be calculated whenever $P_i(t) < P_{min}$. The results of this decision rule are shown in table 10.

All maintenance policies in table 10 show that per maintenance only one component is updated. The upgrade times corresponding to the maintenance policies are given in table C1 in appendix C. The policies show that the HDD is the most frequent updated component whereas the CPU the least frequently. The explanation given in 4.2 also applies here. Remarkable is that the CPU of the medium end and high end computer is never updated. Due to this, the CPU performance approaches zero as figures C2 and C3 in appendix C show. The computer performance however stays above 40%. Again we note that it is not realistic to assume that the computer works fine while a component performance approaches zero.

The last maintenance action of each maintenance policy updates the cheapest component. The simulation showed that sometimes indeed a cheaper maintenance action near the end can be chosen instead of the maintenance with the highest ΔL/C ratio. Also, all computer performances at the end are close to P_{min} 40%. These observations make the obtained maintenance policies efficient.

The low end computer needs the most maintenances and has the highest maintenance costs but is a cheaper option than the high end computer. The medium end computer has the lowest total cost which makes it the best option to buy when a P_{min} level of 40% is required. However, since the CPU of the medium end computer is never updated, this would be an inadequate recommendation. Therefore, we recommend purchasing the low end computer.

	Type of computer		
ΔL/C ratio	Low	Medium	High
End computer performance $P_i(10)$	41.56	40.52	40.98
Total # maintenances	18	6	8
Total maintenance cost	€ 1340,00	€ 530,00	€ 830,00
Total cost	€ 2040,00	€ 1625,00	€ 2330,00
Maintenance policy	3 3 1 3 2 3 1 3 2 3 1 3 2 3 1 3 2 3	3 2 3 2 3 3	3 3 3 3 2 3 3 3

Table 10 – Results of the lifetime extension - cost ratio

4.4 Comparison

Table 11 summarizes some features of the optimal maintenance policies of the deterministic and stochastic policies per type of computer. The given costs represent the total costs.

Type of computer	*Deterministic Total cost*	*ΔP/C ratio Total cost*	*ΔL/C ratio Total cost*	*Deterministic #updates*	*ΔP/C ratio # updates*	*ΔL/C ratio # updates*	*Deterministic $P_i(T)$*	*ΔP/C ratio $P_i(T)$*	*ΔL/C ratio $P_i(T)$*
Low	€ 2115,00	€ 2120,00	*€ 2040,00	16	18	18	42.63 - 43.13	50.34	41.56
Medium	€ 1685,00	€ 1755,00	*€ 1625,00	5	6	6	40.46 - 40.91	45.40	40.52
High	€ 2560,00	€ 2350,00	*€ 2330,00	6	8	7	43.01 - 44.56	45.86	40.98

Table 11 – Summary of the optimal maintenance policies of the deterministic and stochastic policies

Under the setting P_{min} = 40%, α = 10, T = 10 and the parameters and costs presented in section 2, all policies show that the medium end computer is the best option to buy. The low end computer is the next best option to buy and last the high end computer. It can also be seen that the ΔL/C ratio provides the cheapest maintenance policies per type of computer which makes it the best decision rule.

Remarkable is that deterministic policy provides cheaper maintenance policies than the ΔP/C ratio for the low and medium end computer. You would expect that the ΔP/C ratio would provide better maintenance policies than updating components in a fixed order. We will perform sensitivity analyses to check whether the deterministic policy also provides better maintenance policies than the ΔP/C ratio when the settings change.

Although the maintenance policies of the ΔL/C ratio execute more maintenances than the maintenance policies of the best DMPs, they still achieve the lowest total costs. This is because the ΔL/C ratio more often selects to update the HDD, which is the cheapest component. The updating of components in a fixed order leads to higher maintenance costs. The maintenance policies of the ΔL/C ratio also cause the computer performance to end the closest to P_{min} 40%. This makes the maintenance policies of the ΔL/C ratio the most efficient.

Overall we conclude that the ΔL/C ratio provides the most cheapest and efficient maintenance policies under the previously described setting. Thus, the ΔL/C ratio is the best decision rule.

Another method, to give a recommendation which computer to buy, is to compute the average computer performance – total cost ratio. We use the results of the ΔL/C ratio and calculate the average computer performance by taking the average of the computer performance over T = 10 and divide this by the total costs. The results are given in table 12. According to this criterion the medium end computer is the best option to buy. This result is consistent with our result in section 4.3 where we only used the total costs as criterion to make a recommendation which computer to buy. Also, according to this method, the low and high end computer are equal likely the next best option to buy.

	Type of computer		
ΔL/C ratio	*Low*	*Medium*	*High*
Average computer performance	44.54	48.03	50.73
Total costs	€ 2040,00	€ 1625,00	€ 2330,00
Average computer performance - cost ratio	0.0218	0.0296	0.0218

Table 12 – Results of the average computer performance - total cost ratio

5 Sensitivity analysis

We have implemented sensitivity analyses to check whether the lifetime extension – cost ratio is indeed the best decision rule to use when we change the settings. In this section the results of the sensitivity analyses are presented.

We implemented 3 sensitivity analyses on the deterministic and stochastic policies. Again we analyzed the computer and components performances over a period of 10 years, that is T = 10. First we examined the effect of changing P_{min}. The results of this sensitivity analysis will be discussed in section 5.1. Section 5.2 discusses the results of the sensitivity analysis where we examined the effect of altering the discount rate. Finally, we discuss the results of the sensitivity analysis where the effect of varying the deterioration rates is examined.

5.1 P_{min}

In section 4 we have determined the optimal maintenance policy per type of computer when a P_{min} of 40% was required. The best option for the company to do was, to buy the type of computer with the lowest total cost. However, it not necessarily has to be true that the chosen type of computer at P_{min} 40% will also be the best choice at P_{min} 50%. This is why we examined the total costs of the maintenance policies at different P_{min} levels. This allowed us to make a P_{min} – total cost tradeoff. From this tradeoff it instantly can be seen which type of computer the best option to buy is at a certain P_{min} or budget.

This sensitivity analysis used the parameters and costs presented in section 2 and α = 10%. We had to take into account that, for example, a low end computer cannot achieve a computer performance of 70% as the maximum computer performance is, $P_1(0)$, 60%. So, all P_{min} levels above 60% were excluded for the low end computer. Also, it would also not be realistic to assume that a low end computer would maintain a computer performance level of 60% through time. Similar assumptions for the medium and high end computer were made. We took all these assumptions into account and tested the integer P_{min} levels, as given in table 13. So we analyzed 31, 51 and 66 different P_{min} levels for, respectively, the low, medium and high end computer.

Type of computer	*Considered P_{min} levels*
Low	$20 \leq P_{min} \leq 50$
Medium	$20 \leq P_{min} \leq 70$
High	$20 \leq P_{min} \leq 85$

Table 13 – Considered P_{min} levels per type of computer

The analysis was performed in the following way: first we determined the best DMP per type of computer, which is the DMP with the lowest total cost. Next we compared the results of the best DMP to the results of the ΔP/C ratio and ΔL/C ratio per type of computer. From this comparison we determined what decision rule yielded the best maintenance policies per type of computer, in the sense of reaching T at the lowest possible total costs.

So we made 3 comparisons:
1. Best DMP – ΔP/C ratio
2. Best DMP – ΔL/C ratio
3. ΔP/C ratio – ΔL/C ratio

At last we compared the results of the selected best decision rules per type of computer, to recommend what type of computer the best option is to buy. Table 14 summarizes the algorithm that is used.

For a type *i* computer:
1. Compare the total costs of all DMPs 2. Choose the best DMP 3. Compare the total costs of the best DMP to the total costs of the ΔP/C ratio 4. Compare the total costs of the best DMP to the total costs of the ΔL/C ratio 5. Compare the total costs of the ΔP/C ratio to the total costs of the ΔL/C ratio 6. Choose best decision rule for a type *i* computer
----
Compare the total costs of the best decision rule of all types of computers

Table 14 – Summary algorithm to determine the best decision rule for a type i computer

Figures D1 and D2 in appendix D show the comparison of the total costs of all DMPs for the low end computer. It appears that DMP 6 is the best DMP at most considered P_{min} levels for the low end computer. The best DMPs for the medium and high end computer were respectively DMP 5 and 6 at most considered P_{min} levels. Tables D1 - D3 in appendix D, show the comparisons between the total costs of all decision rules at different P_{min} levels for all types of computers. The tables show at what P_{min} level, what decision rule performs better and gives the difference in the total costs per comparison. Table 15 summarizes the results of tables D1 - D3. The numbers in the table indicate how often a deterministic or stochastic policy provide a maintenance policy with lower or equal total costs than the other considered policy.

	Type of computer		
Comparison	*Low*	*Medium*	*High*
Total # analyses	*31*	*51*	*66*
Costs ΔP/C ratio ≤ costs deterministic policy	29	40	58
Costs ΔL/C ratio ≤ costs deterministic policy	31	46	59
Costs ΔL/C ratio ≤ costs ΔP/C ratio	29	41	56

Table 15 – Summary results P_{min} sensitivity analysis

Table 11 showed the deterministic policy provided cheaper maintenance policies than the ΔP/C ratio for the medium and high end computer at P_{min} 40%. However, table 15 and D1-D3 show that the ΔP/C ratio equally or better performs than the deterministic policy at most considered P_{min} levels for all types of computers. For example, table D3a shows that at 8 of the 66 considered P_{min} levels, DMP 6 provides cheaper maintenance polices, while the ΔP/C ratio provides cheaper maintenance polices at 54 P_{min} levels for the high end computer. This comes down to a cost saving of respectively € 1398,30 and € 9234,90 which makes the ΔP/C ratio the better decision rule. Similar calculations and conclusions can be made for the medium and low end computer. Hence, we conclude that the ΔP/C ratio is a better decision rule than the deterministic policy.

The analysis also shows that the ΔL/C ratio equally or better performs than the deterministic policy at most considered P_{min} levels for all types of computers. In case of the low end computer, the ΔL/C ratio provides cheaper maintenance policies than DMP 6 at all 31 considered P_{min} levels. We thus conclude that the ΔL/C ratio is a better decision rule than the deterministic policy.

At last we check whether the ΔP/C ratio or the ΔL/C ratio is a better decision rule. The analysis shows that the ΔL/C ratio equally or better performs than the ΔP/C ratio at most considered P_{min} levels for all types of computer. Table D2c shows that at 7 of the 51 considered P_{min} levels the ΔP/C ratio provides cheaper maintenance polices while the ΔL/C ratio provides cheaper maintenance polices at 26 P_{min} levels for the medium end computer. This comes down to a cost saving of respectively € 625,23 and € 2292,30 which makes the ΔL/C ratio the better decision rule. Similar calculations and conclusions can be made for the low and high end computer. We thus conclude that the ΔL/C ratio is a better decision rule than the ΔP/C ratio.

Overall we conclude that the ΔL/C ratio is the best decision rule which is in accordance with our earlier results. The ΔP/C ratio is the next best decision rule, which is not in accordance with our earlier results but is justified if looked at the cost savings.

We continue to give recommendations which computer to buy at which required P_{min} level using the ΔL/C ratio, since it is the best decision rule.

The results of the ΔL/C ratio show that the higher P_{min} level is required, the more maintenance is needed, which leads to higher maintenance costs. This makes sense, since P_{min} is reached quicker when a high P_{min} is required. Remarkably, the policies of all types of computer show that most of the time only 1 component per maintenance is updated. The maintenance policies of the high and the medium end computer have exceptions at respectively, P_{min} 82% - 85% and P_{min} 70% where a computer is replaced once. The maintenance policies show that at all considered P_{min} levels, the HDD is by far the most frequent updated component, followed by respectively the GPU and CPU. But other than that, no specific pattern can be found in the maintenance policies. This makes sense, since different P_{min} levels require maintenance at different times by which different maintenance actions are selected.

Figure 2 shows the total cost - P_{min} tradeoff. It can be seen that if a P_{min} between 20% - 28% is required, the low end computer is the best option to purchase. The medium end computer should be purchased when a P_{min} between 28% - 63% is required and if a P_{min} level above 63% is required, the high end computer should be purchased.

However, in section 4.3 we made a comment that the CPU of the medium and high end computer is never updated at P_{min} 40%. This makes the best maintenance policy somewhat unrealistic, since the CPU performance of the medium end computer approached zero. The maintenance policies at different P_{min} levels show that the CPU of the low, medium and high end computer are at least once updated from respectively, P_{min} 23%, 44% and 46%. A realistic recommendation would therefore be to purchase a low end computer if a P_{min} between 23% - 44% is required while a medium end computer should be purchased when a P_{min} between 44% - 63% is required. The high end computer should be purchased when a P_{min} above 63% is required.

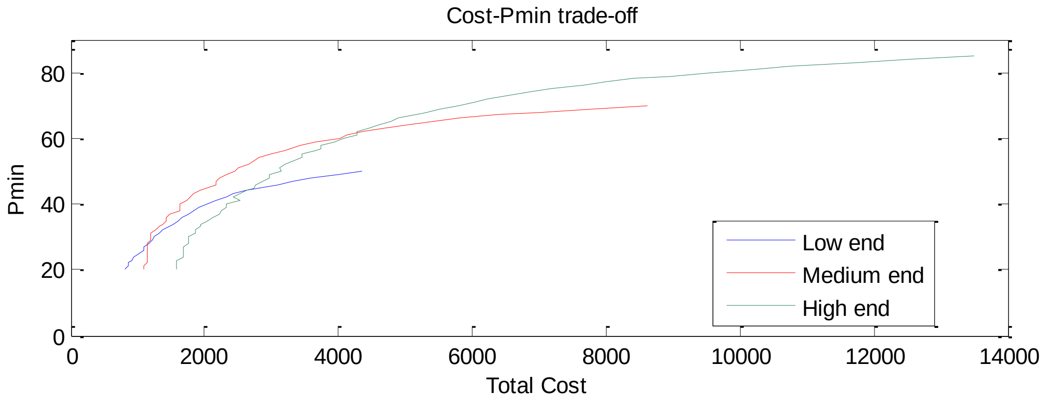


Figure 2– Total cost-P_{min} tradeoff of the L/C ratio

Summarizing the results of the P_{min} sensitivity analysis, we concluded that the ΔL/C ratio is the best decision rule at most considered P_{min} levels. We recommend purchasing a low end computer if a P_{min} between 23% - 44% is required. The medium end computer should be purchased when a P_{min} level between 44% - 63% is required while the high end computer should be purchased if a P_{min} level above 63% is required.

5.2 Discount rate

The analyses in section 4 assumed that only the computer prices are discounted over time. We could also discount the prices of the components with the reasoning that the producer has limited inventory space and cannot store all components and computers forever, since inventory costs are assumed to be high. To get rid of the inventory he will therefore, sooner or later lower the prices of components and computers. However, the results of section 4 showed that the decision rules already choose to update components per maintenance while replacement of computers is not considered. Discounting the prices of the components would not change the maintenance policies as the components would only become cheaper and replacement of computers will still not be considered. That is why we did not discount the prices of the components. Instead we increased the discount rate of the computer, α.

This sensitivity analysis used the parameters and costs presented in section 2, except for the discount rate which we varied. This analysis started with a discount rate of α = 10%, and per analysis it increased with 1% until α = 60% was reached. Note that increasing the discount rate of the computer prices has no influence on the total costs of the DMPs from category 1, since only upgrading of components is considered in this category. A similar algorithm as described in section 5.1 was used to determine the best decision rule.

First we determined the best DMP per type of computer. Figure E1 in appendix E, shows the comparison of the total costs of all DMPs for the low end computer. It shows that, at first

DMP 6 provides the cheapest maintenance policies but from α = 33%, DMP 14 provides the cheapest maintenance policies. Figure E2 shows the comparison of the total costs of the best deterministic and stochastic policies for the low end computer. We considered both DMP 6 and 14 in the comparison. It appears that the ΔL/C ratio provides the cheapest maintenance policies at most discount rates. This makes the ΔL/C ratio the best decision rule for the low end computer. The best DMP for the medium and high end computer were DMP 6. Again, the comparison of the total costs of the best deterministic and stochastic policies for the medium and high end computer revealed that the ΔL/C ratio is the best decision rule at most considered discount rates. Thus, we conclude that the ΔL/C ratio is the best decision rule for all types of computers.

We continue to give recommendations which computer to buy at which discount rate using the ΔL/C ratio, since it is the best decision rule.

Table 16 shows how many maintenance actions are executed when a certain discount rate is used. Table 17 shows from what discount rate computers are being replaced. The results show that the higher the discount rate gets, the less maintenance is executed and the more replacement instead of updating is carried out. The reason for this is that the higher the discount rated gets, the cheaper a computer becomes over time, making it more beneficial to replace a computer instead of updating its components. Also, when a computer is replaced it will take longer to reach P_{min} than if a component would be upgraded, which explains why less maintenance is needed. For example, the low end computer 18 times needs maintenance to be executed if α = 13%. Table 17 shows that if α = 14%, once a computer is replaced and table 16 shows that at this discount rate, indeed less maintenance needs to be executed than at α = 13%.

Type of	# Maintenance executed									
computer	*4*	*5*	*6*	*7*	*8*	*12*	*13*	*15*	*16*	*17*	*18*
Low	-	-	-	-	-	40-60	28-38	24-27	19-23	14-18	10-13
Medium	42-60	22-41	10-21	-	-	-	-	-	-	-	-
High	40-60	30-39	25-29	20-24	10-19	-	-	-	-	-	-

Table 16 – The number of maintenances executed when discount rate α is used

	# times computer replaced					
Type of computer	*1*	*2*	*3*	*4*	*5*	*6*
Low	14%	19%	24%	28%	39%	46%
Medium	22%	36%	46%	-	-	-
High	24%	30%	59%	-	-	-

Table 17– Number of replacements when discount rate α is used

Figure 3 shows the total cost – discount rate tradeoff of the ΔL/C ratio per type of computer. The total costs of all types of computers do not change in the very beginning. This is because the discount rate is not high enough to consider replacement of a computer causing the maintenance policies not to change. Furthermore it can be seen that the medium end computer is the best option to buy until a discount rate of 53%. The low end computer becomes the best option to buy at higher discount rates.

Taking our earlier comment about the CPU performance approaching zero into account, we recommend buying a low end computer if the discount rate is between 10% - 21%. Between a discount rate of 22% - 53% the medium end computer is the best option to buy and at higher discount rates the low end computer again becomes the best option to buy. These recommendations hold under the setting we described in the beginning.

Summarizing the results of the discount rate sensitivity analysis, we concluded that the ΔL/C ratio is the best decision rule at most considered discount rates. We have seen that the higher the discount rate gets, the lower the computer prices become over time which causes more replacement instead of upgrading. Due to the replacement less maintenance is needed which results in lower maintenance costs. We recommend purchasing the medium end computer if a discount rate between 22% - 53% is used. If other discount rates are used the low end computer should be purchased.

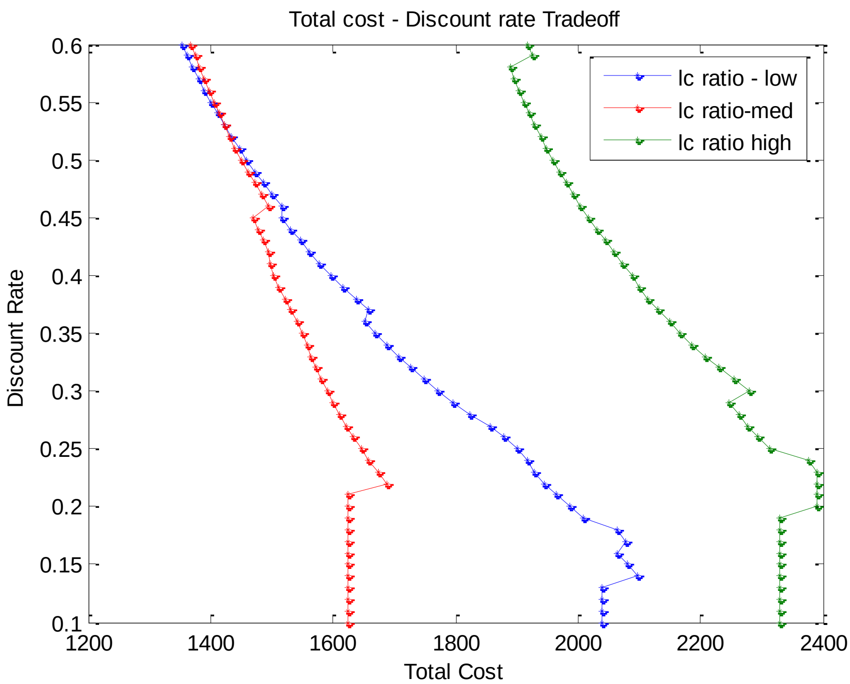


Figure 3 – Total cost – discount rate tradeoff

5.3 Deterioration rates

The computer performance depends on the performance of the components, which in turn mainly depends on the deterioration rate. In this section the result of the sensitivity analyses, where the deterioration rates have been varied, are presented. Only the deterioration rates of the CPU, GPU and HDD have been varied. First we will discuss the results of the sensitivity analysis where the CPU deterioration rate has been varied and subsequently discuss the results of the sensitivity analysis where respectively the GPU and HDD deterioration rates have been varied.

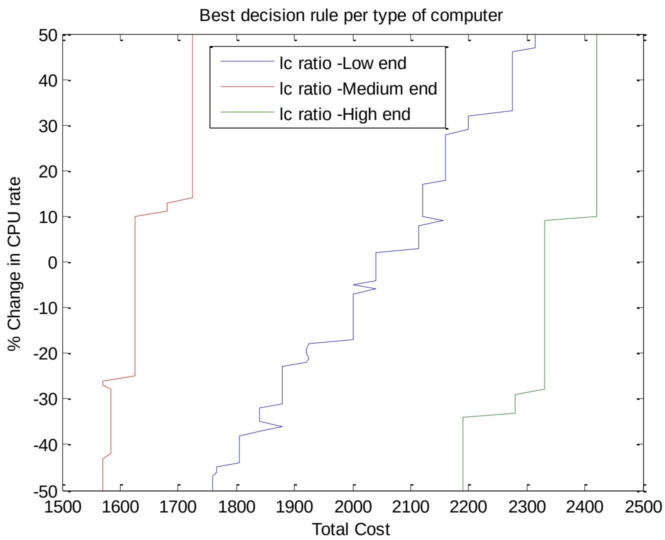
One deterioration rate per sensitivity analysis was reduced/increased with 1, 2,…, 50%, while the other 3 deterioration rates remained the same. In these analyses P_{min} was set to 40%, α = 10 and the initial performance levels and costs remained the same as presented in section 2.

A similar algorithm as described in section 5.1 was used to determine the best decision rule.

5.3.1 CPU

In this sensitivity analysis only the CPU deterioration rate of all types of computers has been varied.

First we determined the best DMP per type of computer. Figure F1 in appendix F, shows the comparison of the total costs of all DMPs of the high end computer. DMP 6 provides the cheapest maintenance policies at most considered variations in the CPU deterioration rate. Figure F2 shows the comparison of the total costs of all policies of the high end computer. It appears that the ΔL/C ratio provides the cheapest maintenance policies at most variations in the CPU deterioration rate. This makes the ΔL/C ratio the best decision rule for the high end computer. Similar results were found for the low and medium end computer. Thus, we conclude that the ΔL/C ratio is the best decision rule at most considered variations in the CPU deterioration rate.



Figuur 4 - Total Cost – deterioration rate change tradeoff

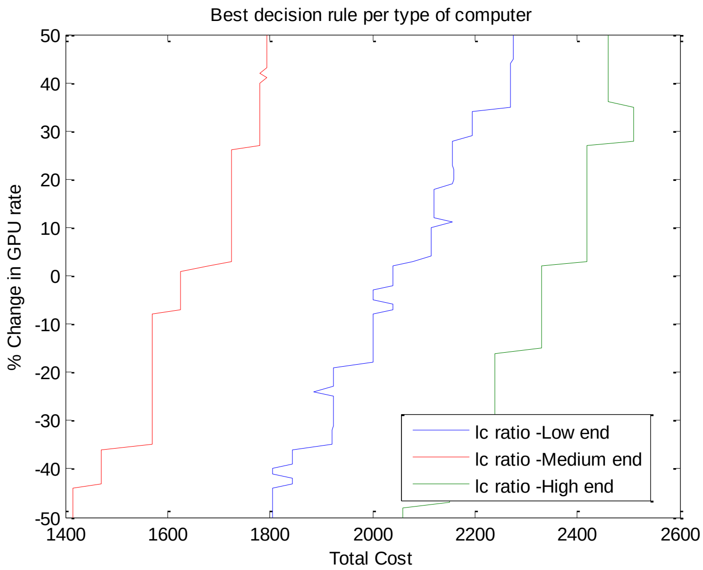
Figure 4 shows the comparison of the results of the ΔL/C ratio of all types of computers. The higher the change in the deterioration rate gets, the higher the total costs gets. The maintenance policies show that the higher the change in the deterioration rate gets, the more updates are needed. These results make sense because the higher the deterioration rate gets, the faster the component performance (and thus computer performance) deteriorates, hence reaching P_{min} more quickly. The figure also shows that the medium end computer is the best option to buy when the CPU deterioration rate changes. However, the maintenance policies show that the CPU of the medium end computer is never updated which would make this recommendation inadequate. Therefore, we recommend the low end computer as the best option to buy. Table F1 in appendix F shows the differences in the total costs from this recommendation. The differences in the costs range from € 190 - € 590.

Summarizing the results of this sensitivity analysis, we concluded that the ΔL/C ratio is the best decision rule when the only the CPU deterioration rate changes. Also, the low end computer is the best option to buy.

5.3.2 GPU

In this sensitivity analysis only the deterioration rate of the GPU of all types of computers has been varied.

First we determined the best DMP per type of computer. Figure F3 in appendix F, shows the comparison of the total costs of all DMPs for the high end computer. DMP 6 provides the cheapest maintenance policies at most considered variations in the GPU deterioration rate. Figure F4 shows the comparison of the total costs of all policies for the high end computer. It appears that the ΔL/C ratio provides the cheapest maintenance policies at most variations in the GPU deterioration rate. This makes the ΔL/C ratio the best decision rule for the high end computer. We got similar results for the low and medium end computer. Thus, we conclude that the ΔL/C ratio is the best decision rule at most considered variations in the GPU deterioration rate.



Figuur 5 - Total Cost – deterioration rate change tradeoff

Figure 5 shows the comparison of the results of the ΔL/C ratio of all types of computers. Again we see that the higher the change in the deterioration rate gets, the higher the total cost gets. Also, the maintenance policies again show that the higher the change in the deterioration rate gets, the more updates are needed. The explanation given in 5.3.1 also applies here. The figure shows that the medium end computer is the best option to buy when the GPU deterioration rate changes. However, again, the maintenance policies show that the CPU of the medium end computer is never updated which again, would make this recommendation inadequate. Therefore we recommend the low end computer as the best option to buy. Table F2 in appendix F, shows the differences in the total costs from this recommendation. The differences in the costs range from € 355 - € 490.

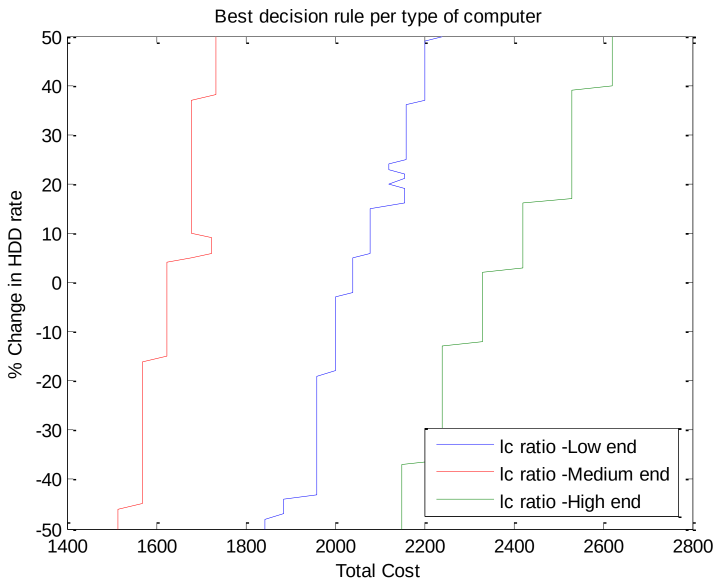
Summarizing the results of this sensitivity analysis, we concluded that the ΔL/C ratio is the best decision rule when the only the GPU deterioration rate changes. Also, the low end computer is the best option to buy.

5.3.3 HDD

In this sensitivity analysis only the deterioration rate of the HDD of all types of computers has been varied.

First we determined the best DMP per type of computer. Figure F5 in appendix F, shows the comparison of the total costs of all DMPs for the high end computer. DMP 6 provides the cheapest maintenance policies at most considered variations in the HDD deterioration rate. Figure F6 shows the comparison of the total costs of all policies for the high end computer. It appears that the ΔL/C ratio provides the cheapest maintenance policies at most variations in the HDD deterioration rate. This makes the ΔL/C ratio the best decision rule for the high end computer. Similar results were found for the low and medium end computer. Thus, we conclude that the ΔL/C ratio is the best decision rule at most considered variations in the HDD deterioration rate.

Figure 6 shows the comparison of the results of the ΔL/C ratio of all types of computer. Again we see that the higher the change in the deterioration rate gets, the higher the total cost gets. The maintenance policies also show that the higher the change in the deterioration rate gets, the more updates are needed. The reason given in 5.3.1 also applies here. The figure also shows that the medium end computer is the best option to buy when the HDD deterioration rate changes. However, the maintenance policies again show that the CPU of the medium end computer is never updated which again, would make this recommendation inadequate. Therefore we recommend the low end computer as the best option to buy. Table F3 in appendix F, shows the differences in the total costs from this recommendation. The differences in the costs range from € 330 - € 505.



Figuur 6 - Total Cost – deterioration rate change tradeoff

Summarizing the results of this sensitivity analysis, we concluded that the ΔL/C ratio is the best decision rule when only the HDD deterioration rate changes. Also, the low end computer is the best option to buy.

Conclusion

The goal of this research was to determine the best decision rule which yields the maintenance policy with the minimum costs while guaranteeing that the computer performance stays above a minimal performance level, P_{min}.

First we modeled the component and computer performance and costs. We proposed a component performance model which took into account that the component performance exponentially deteriorates [5]. Next we proposed the average performance model to compute the computer performance at time t by taking the average of all component performances at time t. We also proposed a model which discounted the computer prices at rate α%.

Secondly, two policies were proposed namely, the deterministic policy and the stochastic policy. The deterministic policy examined 17 predetermined maintenance policies which had been split up in 3 categories. They respectively considered upgrading, replacement or both. The stochastic policy determined a maintenance policy with a decision rule. Two decision rules were proposed: the performance – cost ratio and the lifetime extension – cost ratio. The former took the performance improvement of a maintenance action into account while the latter took the lifetime extension of a maintenance action into account. Both decision rules considered 8 possible maintenance actions. Whenever the computer performance dropped below the minimal performance level, P_{min}, the ratio of each maintenance possibility was calculated. The maintenance action with the highest ratio would be executed. The lifetime extension - cost ratio also took into account that near the end of a planning period T, a cheaper maintenance action could perhaps be carried out, instead of the maintenance with the highest $\Delta L/C$ ratio.

We implemented the deterministic and stochastic policies for a period of T = 10 years, α = 10% and a P_{min} of 40%. We concluded that the lifetime extension - cost ratio was the best decision rule. Although the medium end computer had the lowest total cost, we recommended purchasing the low end computer since the CPU of the medium end computer was never updated.

At last we performed 3 sensitivity analyses to check whether the obtained results also held when the settings changed. We implemented the deterministic and stochastic policies for a period of 10 years and separately considered changing the P_{min}, discount rate and deterioration rates. All sensitivity analyses revealed that the lifetime extension - cost ratio was the best decision rule. The P_{min} sensitivity analysis showed that the low end computer was the best option to purchase when P_{min} between 23% - 44% is required. The medium end computer should be purchased when a P_{min} level between 44% - 63% is required while the high end computer should be purchased if a P_{min} level above 63% is required. The discount rate sensitivity analysis showed that the medium end computer should be purchased when a discount rate between 22% - 53% is used. If other discount rates are used the low end computer should be purchased. The results of the deterioration rate sensitivity analysis indicated the medium end computer as the best option to buy. However, the CPU was never updated and therefore we recommended the low end computer as the best option to buy.

Discussion

In achieving our goal of determining the best decision rule, we encountered a few boundaries. In this section we will mention them and provide possibilities for improvement. We will also give recommendations for further research.

The results of the lifetime extension - cost ratio showed that the medium end computer was the best option to buy. However, the maintenance policy revealed that the CPU was never updated which caused the CPU performance to approach zero. This made it unrealistic to assume that the computer worked fine. A solution to this problem would be to use separate P_{min} levels for the computer and component performance. In this research for example a P_{min} of 40% could be set for the computer performance while a P_{min} of 20% could be set for the CPU performance. Another possibility to avoid the component performance approaching zero, would be to compute the computer performance as the minimum performance of the components. This would mean that no component performance is allowed to drop below P_{min}.

In this research we assumed that the REM is not considered for updating. Further research could investigate what influence including the REM in some maintenance actions would have on the maintenance policies and costs.

Also, we assumed that a maintenance action is costless. It would be interesting to examine if maintenance costs would have an impact on the maintenance policies. Also, economic dependence could then be taken into account.

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Appendix A

```
Initialize t, cpu, gpu, hdd, rem, update

While t ≤ T

Compute P_ij(t)
Compute P_i(t)

If P_i(t) < P_min & update = 0 or 3 or 6 etc.        (1st , 4th ,7th etc. update when P_i(t) < P_min)
Update component(s) X → initialize X
update = update+1
End if

Next if P_i(t) < P_min & update =1 or 4 or 7 etc     (2nd, 5th, 8th etc. update when P_i(t) < P_min)
Update component(s) Y → initialize Y
update = update+1
End if

Next if P_i(t) < P_min update = 2 or 5 or 8 etc.     (3rd , 6th, 9th etc. update when P_i(t) < P_min)
Update component(s) Z → initialize Z
update = update+1
   If P_i(t) < P_min
   Update component(s) X → initialize X
   update = update+1
   End if
End if

t+, cpu+, gpu+, hdd+, rem+

End While
```

Table A1- Algorithm deterministic maintenance policy with 3 type of maintenances

cpu,gpu,hdd and *rem* indicate the age of the concerning component.
t denotes the time
updates denotes the number of executed maintenances

X, Y,Z indicate the set of components which should be upgraded.
For example X, Y and Z respectively are the CPU, GPU and HDD in DMP 1.

```
Initialize t, cpu, gpu, hdd, rem, update
While time t ≤ T
Compute Pij(t)
Compute Pi(t)
If Pi(t) < Pmin
        Compute ΔP/C ratio per maintenance action
        Pbefore_maintenance = Pi(t)
        Execute maintenance →
        initialize concerning component(s)
        Compute Pij(t)
        Compute Pi(t)
        Pafter_maintenance = Pi(t)
        calculate performance improvement
        ΔP = Pafter_maintenance – Pbefore_maintenance
        C = Cost maintenance
        ΔP/C ratio = ΔP/C
        End computation ΔP/C ratio per maintenance action
Compare the 8 ΔP/C ratio's →
Execute maintenance with the  highest ΔP/C ratio →
initialize concerning components
Update=update+
End if
t+, cpu+, gpu+, hdd+, rem+
End while
```

Table A2 - Algorithm ΔP/C ratio

```
Initialize t, cpu, gpu, hdd, rem,update

While time t ≤ T

Compute Pij(t)
Compute Pi(t)

  If Pi(t) < Pmin

        Compute ΔL/C ratio per maintenance action
        timebound_beforemaintenance = t

        Execute maintenance →
         initialize concerning components and calculate lifetime extension:

        While  Pi(t) ≥ Pmin
        t+, cpu+, gpu+, hdd+, rem+

        Compute Pij(t)
        Compute Pi(t)

        End while

        Timebound_aftermaintenance (ΔL +t) = t

        ΔL = Timebound_aftermaintenance -timebound_beforemaintenance
        C = Cost maintenance
        ΔL/C ratio =  ΔL/C
        End computation ΔL/C ratio per maintenance action

 Compare the 8 ΔL/C ratio's→
 Select  maintenance with the highest LC ratio

   If Timebound_aftermaintenance of the selected maintenance ≥ T
      Check whether cheaper maintenance is available →
      Timebound_aftermaintenance of maintenance action must be  ≥ T
     → Select cheapest substitution

           If more than one substitution available→
                 Select the  substitution with the highest  ΔL/C ratio among all
           End if

     End if

  Execute selected maintenance action → initialize concerning components
  Update=update+

  End if

t+, cpu+, gpu+, hdd+, rem+

End while
```

Table A3 - Algorithm ΔL/C ratio

Appendix B

		Low end computer				
Cat.	DMP	*Maintenance Cost*	*Total Cost*	*# maintenance*	*End P(t)*	*Maintenance at time*
1	1	€ 1495,00	€ 2195,00	16	44.26	1.16, 1.67, 2.20, 2.83, 3.46, 4.01, 4.58, 5.21, 5.76, 6.31, 6.94, 7.47, 8.01, 8.63, 9.15, 9.68
	2	€ 1495,00	€ 2195,00	16	44.43	1.16, 1.67, 2.19, 2.84, 3.46, 4.01, 4.59, 5.22, 5.76, 6.32, 6.94, 7.47, 8.02, 8.64, 9.15, 9.69
	3	€ 1490,00	€ 2190,00	16	42.91	1.16, 1.60, 2.14, 2.84, 3.38, 3.95, 4.59, 5.14, 5.69, 6.32, 6.86, 7.40, 8.02, 8.55, 9.08, 9.69
	4	€ 1490,00	€ 2190,00	16	42.80	1.16, 1.60, 2.21, 2.83, 3.38, 4.02, 4.57, 5.14, 5.77, 6.30, 6.86, 7.48, 8.00, 8.55, 9.16, 9.67
	5	€ 1415,00	€ 2115,00	16	42.63	1.16, 1.59, 2.20, 2.83, 3.37, 4.01, 4.57, 5.13, 5.76, 6.30, 6.85, 7.47, 8.00, 8.54, 9.15, 9.67
	6	€ 1415,00	€ 2115,00	16	43.13	1.16, 1.59, 2.14, 2.84, 3.37, 3.95, 4.59, 5.13, 5.70, 6.33, 6.86, 7.42, 8.04, 8.56, 9.11, 9.72
	7	€ 1885,00	€ 2585,00	13	51.50	1.16, 2.01, 2.60, 3.56, 4.07, 5.04, 5.53, 6.49, 6.97, 7.91, 8.39, 9.32, 9.79
	8	€ 1810,00	€ 2510,00	13	51.80	1.16, 2.00, 2.61, 3.56, 4.09, 5.05, 5.55, 6.50, 6.99, 7.93, 8.41, 9.34, 9.81
	9	€ 1650,00	€ 2350,00	12	40.27	1.16, 1.97, 2.64, 3.55, 4.15, 5.06, 5.65, 6.55, 7.13, 8.02, 8.59, 9.46
	10	€ 2475,00	€ 3175,00	9	57.66	1.16, 2.30, 3.43, 4.55, 5.66, 6.75, 7.83, 8.90, 9.96
2	11	€ 3515,30	€ 4215,30	8	46.14	1.16, 2.32, 3.48, 4.64, 5.80, 6.96, 8.12, 9.28
3	12	€ 2946,56	€ 3646,56	11	40.18	1.16, 1.67, 2.83, 3.34, 4.50, 5.01, 6.17, 6.68, 7.84, 8.35, 9.51
	13	€ 3237,27	€ 3937,27	12	51.66	1.16, 1.60, 2.76, 3.20, 4.36, 4.80, 5.96, 6.40, 7.56, 8.00, 9.16, 9.60
	14	€ 2794,71	€ 3494,71	12	50.56	1.16, 1.59, 2.75, 3.18, 4.34, 4.77, 5.93, 6.36, 7.52, 7.95, 9.11, 9.54
	15	€ 2763,57	€ 3463,57	13	47. 13	1.16, 1.67, 2.20, 3.36, 3.87, 4.40, 5.56, 6.07, 6.60, 7.76, 8.27, 8.80, 9.96
	16	€ 2467,21	€ 3167,21	13	46.44	1.16, 1.67, 2.19, 3.35, 3.86, 4.38, 5.54, 6.05, 6.57, 7.73, 8.24, 8.76, 9.92
	17	€ 2460,57	€ 3160,57	13	41.97	1.16, 1.60, 2.14, 3.30, 3.74, 4.28, 5.44, 5.88, 6.42, 7.58, 8.02, 8.56, 9.72

Table B1- Results per DMP for a low end computer

			Medium end computer			
Cat.	DMP	*Maintenance Cost*	*Total Cost*	*# maintenance*	*End P(t)*	*Maintenance at time*
	1	€ 705,00	€ 1800,00	5	40.70	2.92, 4.26, 5.71, 7.17, 8.62
	2	€ 760,00	€ 1855,00	6	53.32	2.92, 4.26, 5.42, 7.27, 8.63, 9.72
	3	€ 590,00	€ 1685,00	5	40.46	2.92, 4.44, 5.83, 7.35, 8.85
	4	€ 705,00	€ 1800,00	5	41.35	2.92, 4.44, 5.77, 7.10, 8.81
1	5	€ 760,00	€ 1855,00	6	55.49	2.92, 4.02, 5.56, 7.19, 8.44, 9.90
	6	€ 590,00	€ 1685,00	5	40.91	2.92, 4.02, 5.88, 7.29, 8.42
	7	€ 1085,00	€ 2180,00	5	67.16	2.92, 5.05, 6.40, 8.67, 9.73
	8	€ 760,00	€ 1855,00	4	41.95	2.92, 4.95, 6.66, 8.72
	9	€ 760,00	€ 1855,00	4	42.90	2.92, 5.24, 6.70, 8.95
	10	€ 1140,00	€ 2235,00	3	47.33	2.92, 5.68, 8.30
2	11	€ 1931,72	€ 3026,72	3	55.96	2.92, 5.84, 8.76
	12	€ 1555,72	€ 2650,72	4	52.84	2.92, 4.26, 7.18, 8.52
	13	€ 1496,91	€ 2591,91	4	57.67	2.92, 4.44, 7.36, 8.88
3	14	€ 1365,36	€ 2460,36	4	47.59	2.92, 4.02, 6.94, 8.04
	15	€ 1285,42	€ 2380,42	5	56.52	2.92, 4.26, 5.71, 8.63, 9.97
	16	€ 1103,23	€ 2198,23	5	45.97	2.92, 4.26, 5.42, 8.34, 9.68
	17	€ 993,20	€ 2088,20	4	41.94	2.92, 4.44, 5.83, 8.75

Table B2- Results per DMP for a medium end computer

			High end computer			
Cat.	DMP	*Maintenance Cost*	*Total Cost*	*# maintenance*	*End P(t)*	*Maintenance at time*
	1	€ 1060,00	€ 2560,00	6	43.04	2.61, 3.85, 5.10, 6.53, 7.82, 9.00
	2	€ 1060,00	€ 2560,00	6	44.77	2.61, 3.85, 5.26, 6.61, 7.83, 9.12
	3	€ 1060,00	€ 2560,00	6	44.55	2.61, 3.84, 5.24, 6.60, 7.81, 9.10
	4	€ 1060,00	€ 2560,00	6	42.98	2.61, 3.84, 5.09, 6.53, 7.80, 8.99
1	5	€ 1060,00	€ 2560,00	6	44.70	2.61, 3.93, 5.30, 6.58, 7.90, 9.16
	6	€ 1060,00	€ 2560,00	6	44.56	2.61, 3.93, 5.28, 6.57, 7.90, 9.15
	7	€ 1500,00	€ 3000,00	5	54.38	2.61, 4.48, 5.91, 7.80, 9.03
	8	€ 1390,00	€ 2890,00	5	55.11	2.61, 4.62, 5.94, 7.91, 9.08
	9	€ 1350,00	€ 2850,00	5	55.08	2.61, 4.61, 5.94, 7.90, 9.08
	10	€ 2120,00	€ 3620,00	4	76.86	2.61, 5.06, 7.37, 9.57
2	11	€ 2792.90	€ 4292.90	3	44.99	2.61, 5.22, 7.83
	12	€ 2239,33	€ 3739,33	4	43.38	2.61, 3.85, 6.46, 7.70
	13	€ 2161,70	€ 3661,70	4	43.14	2.61, 3.84, 6.45, 7.68
3	14	€ 1920,54	€ 3420,54	4	45.38	2.61, 3.93, 6.54, 7.86
	15	€ 1802,55	€ 3302,55	5	42.49	2.61, 3.85, 5.10, 7.71, 8.95
	16	€ 1568,59	€ 3068,59	5	45.89	2.61, 3.85, 5.26, 7.87, 9.11
	17	€ 1490,32	€ 2990,32	5	45.40	2.61, 3.84, 5.24, 7.85, 9.08

Table B3- Results per DMP for a high end computer

Appendix C

Type of computer	Maintenance at time *t*
ΔP/C: Low	1.16, 1.59, 2.20, 2.58, 3.28, 3.69, 4.38, 4.78, 5.42, 5.82, 6.51, 6.90, 7.52, 7.91, 8.59, 8.97, 9.58, 9.96
ΔL/C: Low	1.16, 1.59, 1.79, 2.44, 2.76, 3.48, 3.88, 4.57, 4.96, 5.60, 6.00, 6.69, 7.08, 7.70, 8.09, 8.76, 9.14, 9.75
ΔP/C:Medium	2.92, 4.02, 5.56, 7.19, 8.44, 9.90
ΔL/C:Medium	2.92, 4.02, 5.88, 6.98, 8.63, 9.63
ΔP/C: High	2.61, 3.93, 4.89, 6.27, 7.41, 8.21, 9.45
ΔL/C: High	2.61, 3.93, 4.89, 5.68, 6.36, 7.74, 8.79, 9.52

Table C1- Maintenance times per decision rule and type of computer

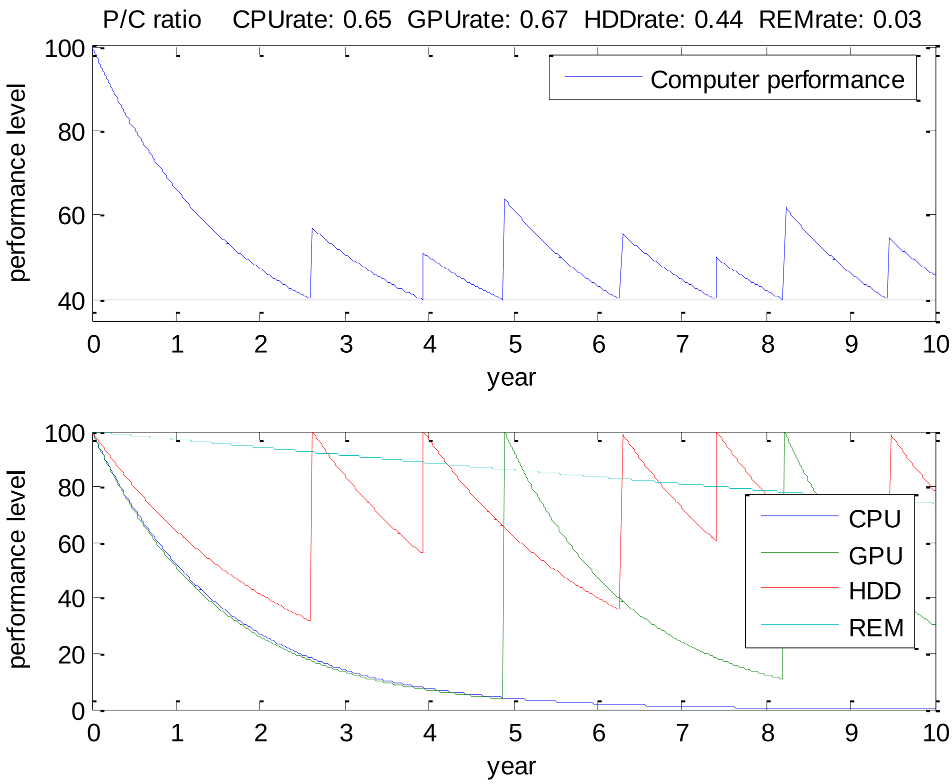

Figure C1- Development of the computer and component performance of the high end computer using the ΔP/C ratio

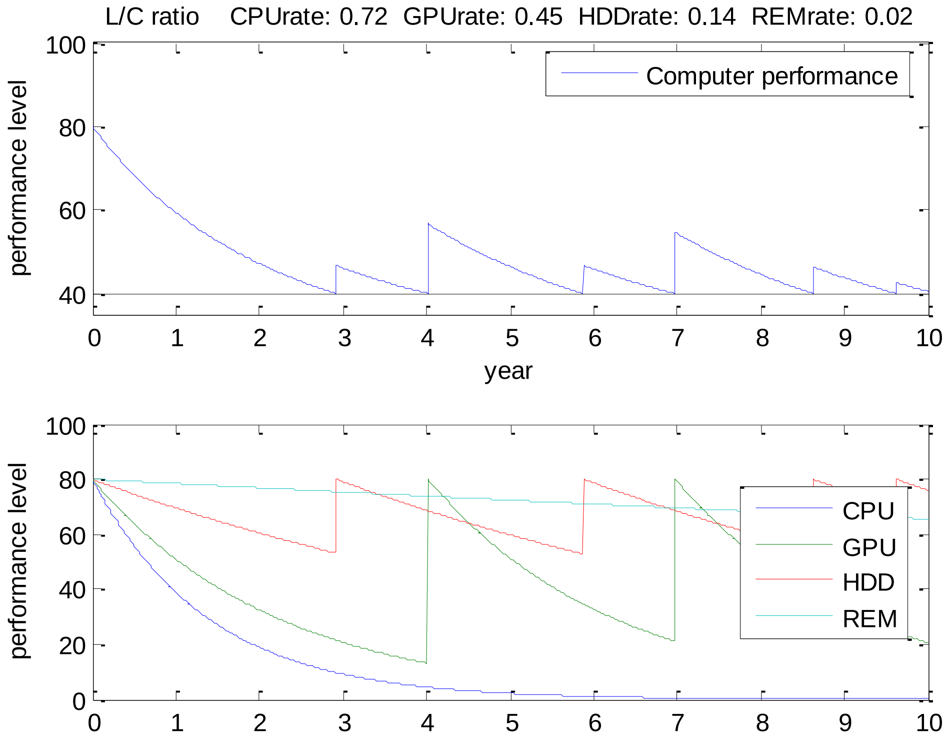

Figure C2- Development of the computer and component performance of the medium end computer using the ΔL/C ratio

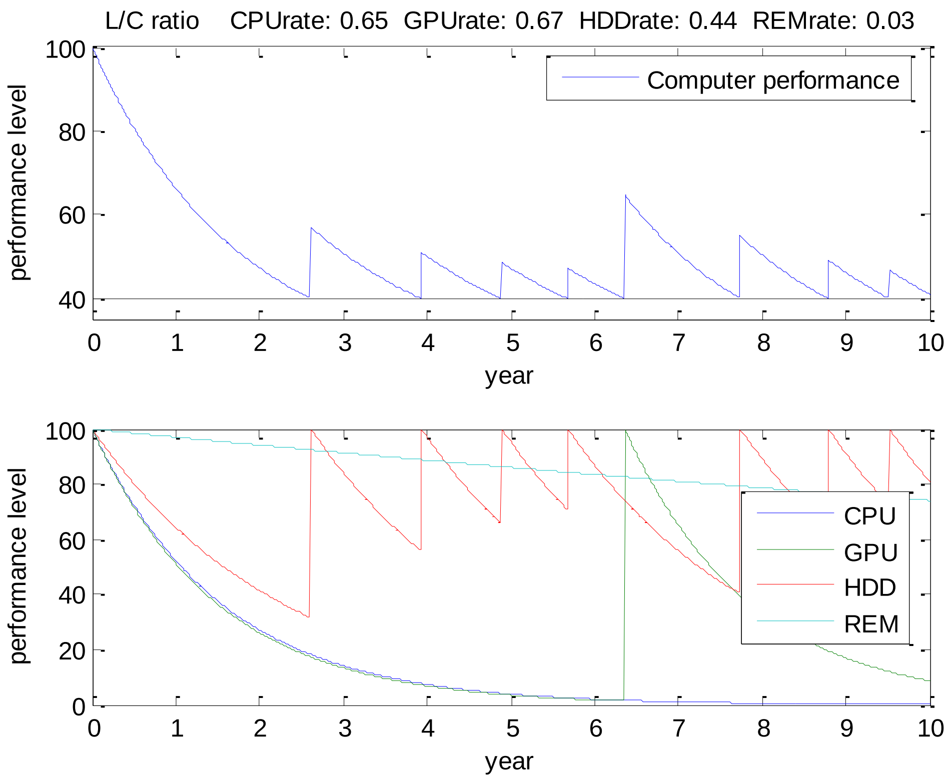

Figure C3- Development of the computer and component performance of the high end computer using the ΔL/C ratio

Appendix D

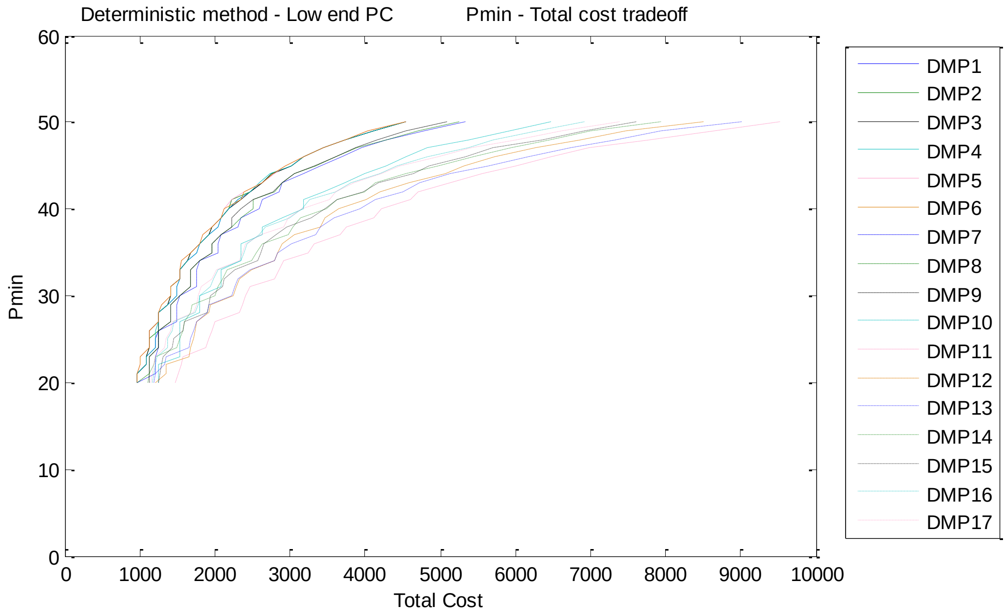

Figure D1 – Comparison of all DMPs - low end computer

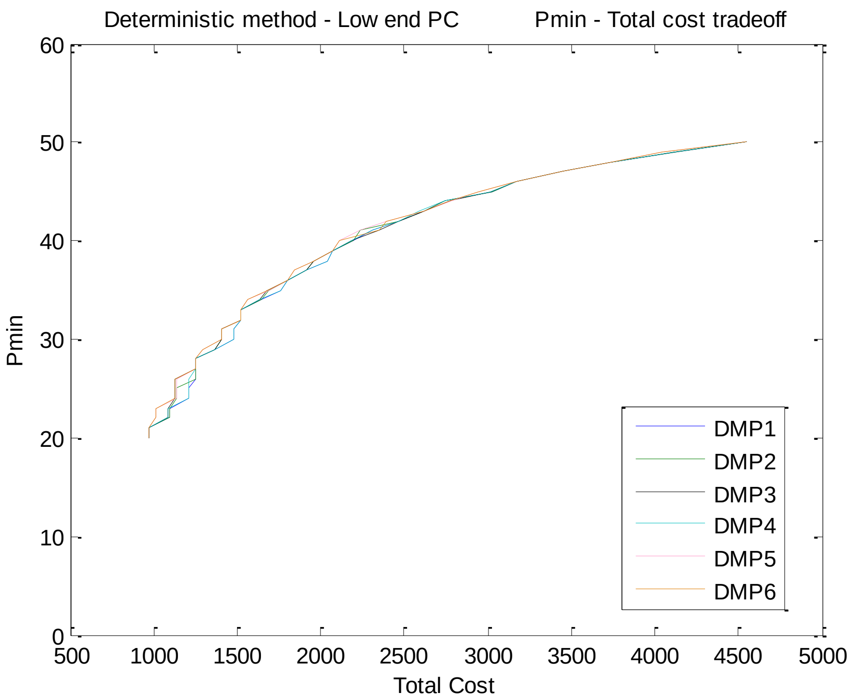

Figure D2– Comparison of the cheapest DMPs - low end computer

Comparison low end computer

Deterministic policy better than PC ratio(2x)	
P_{min}	33 40
Difference in costs (€)	5,00 5,00

PC ratio better than deterministic policy (27x)	
P_{min}	20 21 22 23 24 25 27 28 29 30 32 34 35 36 37 38 39 41 42 43 44 45 46 47 48 49 50
Difference in costs (€)	-155,00 -115,00 -155,00 -115,00 -120,00 -120,00 -115,00 -75,00 -35,00 -115,00 -115,00 -35,00 -40,00 -115,00 -35,00 -115,00 -115,00 -75,00 -75,00 -190,00 -155,00 -150,00 -110,00 -70,00 -150,00 -30,00 -185,00

PC ratio equal to deterministic policy (2x)	
P_{min}	26 31

a.

Deterministic policy better than LC ratio(0x)

LC ratio better than deterministic policy (31x)	
P_{min}	20 – 50
Difference in costs (€)	-155,00 -115,00 -155,00 -115,00 -195,00 -115,00 -40,00 -155,00 -80,00 -80,00 -160,00 -80,00 -155,00 -40,00 -35,00 -80,00 -155,00 -75,00 -120,00 -150,00 -75,00 -75,00 -75,00 -190,00 -155,00 -150,00 -75,00 -150,00 -150,00 -35,00 -185,00

LC ratio equal to deterministic policy (0x)

b.

PC ratio better than LC ratio(2x)	
P_{min}	25 46
Difference in costs (€)	5,00 35,00

LC ratio better than PC ratio (17x)	
P_{min}	24 26 27 28 29 30 31 32 33 35 36 37 38 39 40 47 49
Difference in costs (€)	-75,00 -40,00 -40,00 -5,00 -45,00 -45,00 -80,00 -40,00 -45,00 -40,00 -40,00 -40,00 -5,00 -35,00 -80,00 -80,00 -5,00

LC ratio equal to PC ratio (12x)	
P_{min}	20 21 22 23 34 41 42 43 44 45 48 50

c.

Table D1 – Comparison of all decision rules for the low end computer

Comparison medium end computer

Deterministic policy better than PC ratio(11x)	
P_{min}	39 40 44 45 46 48 50 54 58 62 67
Difference in costs (€)	15,00 70,00 15,00 15,00 15,00 170,00 15,00 15,00 115,00 15,00 166,94

PC ratio better than deterministic policy (18x)	
P_{min}	29 30 32 33 34 38 41 49 53 56 60 63 64 65 66 68 69 70
Difference in costs (€)	-100,00 -100,00 -115,00 -115,00 -115,00 -100,00 -100,00 -40,00 -40,00 -40,00 -40,00 -40,00 -40,00 -40,00 -40,00 -29.6900 -181,500 -424,34

PC ratio equal to deterministic policy (22x)	
P_{min}	20 21 22 23 24 25 26 27 28 31 35 36 37 42 43 47 51 52 55 57 59 61

a.

Deterministic policy better than LC ratio(5x)	
P_{min}	46 50 52 60 62
Difference in costs (€)	110,00 10,00 40,00 55,00 15,00

LC ratio better than deterministic policy (31x)	
P_{min}	29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 47 48 49 51 53 57 59 63 64 65 66 68 69 70
Difference in costs (€)	-100,00 -100,00 -100,00 -215,00 -160,00 -105,00 -115,00 -115,00 -60,00 -60,00 -60,00 -60,00 -130,00 -75,00 -75,00 -115,00 -45,00 -60,00 -5,00 -115,00 -115,00 -100,00 -100,00 -100,00 -40,00 -55,00 -40,00 -40,00 -210,00 -155,00 -280,61

LC ratio equal to deterministic policy (15x)	
P_{min}	20 21 22 23 24 25 26 27 28 54 55 56 58 61 67

b.

PC ratio better than LC ratio(7x)	
P_{min}	34 38 46 52 56 60 69 70
Difference in costs (€)	10,00 40,00 95,00 40,00 40,00 95,00 26,50 143,73

LC ratio better than PC ratio (26x)	
P_{min}	31 32 33 35 36 37 39 40 41 42 43 44 45 47 48 49 50 51 53 54 57 58 59 64 67 68
Difference in costs (€)	-100,00 -100,00 -45,00 -115,00 -115,00 -60,00 -75,00 -130,00 -30,00 -75,00 -75,00 -130,00 -60,00 -60,00 -175,00 -75,00 -5,00 -115,00 -60,00 -15,00 -100,00 -115,00 -100,00 -15,00 -166,94 -180,31

LC ratio equal to PC ratio (15x)	
P_{min}	20 21 22 23 24 25 26 27 28 29 30 55 61 62 63 65 66

c.

Table D2 – Comparison of all decision rules for the medium end computer

Comparison high end computer

Deterministic policy better than PC ratio(8x)	
P_{min}	61 67 70 71 74 77 78 80
Difference in costs (€)	30,00 10,00 10,00 100,00 439,42 358,32 10,00 440,52

PC ratio better than deterministic policy (54x)	
P_{min}	24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 62 63 64 65 66 68 69 72 73 75 76 79 81 82 83 84 85
Difference in costs (€)	-110,00 -110,00 -110,00 -350,00 -260,00 -260,00 -260,00 -260,00 -260,00 -60,00 -60,00 -260,00 -260,00 -60,00 -300,00 -210,00 -210,00 -60,00 -150,00 -60,00 -110,00 -110,00 -20,00 -260,00 -60,00 -150,00 -150,00 -60,00 -20,00 -20,00 -170,00 -170,00 -60,00 -170,00 -170,00 -170,00 -80,00 -170,00 -80,00 -170,00 -80,00 -80,00 -190,00 -230,00 -140,00 -10,00 -140,00 -100,00 -53,55 -57,07 -224,0800 -431,02 -616,41 -842,80

PC ratio equal to deterministic policy (4x)	
P_{min}	20 21 22 23

a.

Deterministic policy better than LC ratio(7x)	
P_{min}	61 76 77 79 80 81 83
Difference in costs (€)	30,00 40,00 40,00 20,00 70,00 110,00 70,00

LC ratio better than deterministic (55x)	
P_{min}	24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 62 63 64 65 66 67 68 69 70 71 72 73 74 75 78 82 84 85
Difference in costs (€)	-110,00 -110,00 -110,00 -350,00 -260,00 -260,00 -260,00 -260,00 -260,00 -170,00 -170,00 -280,00 -190,00 -100,00 -320,00 -230,00 -230,00 -120,00 -210,00 -120,00 -210,00 -120,00 -80,00 -230,00 -120,00 -210,00 -30,00 -60,00 -170,00 -60,00 -170,00 -170,00 -60,00 -170,00 -170,00 -210,00 -80,00 -170,00 -230,00 -170,00 -190,00 -80,00 -30,00 -190,00 -230,00 -30,00 -10,00 -140,00 -10,00 -50,00 -160,00 -70,00

LC ratio equal to deterministic (4x)	
P_{min}	20 21 22 23

b.

Table D3 – Comparison of all decision rules for the high end computer (continued on next page)

PC ratio better than LC ratio(10x)	
P_{min}	36 47 50 76 79 81 82 83 84 85
Difference in costs (€)	70,00 30,00 120,00 140,00 73,55 167,07 187,03 501,02 511,69 643,51

LC ratio better than PC ratio (28x)	
P_{min}	33 34 35 37 38 39 40 41 42 43 44 45 46 48 49 52 53 59 63 65 67 70 71 74 75 77 78 80
Difference in costs (€)	-110,00 -110,00 -20,00 -40,00 -20,00 -20,00 -20,00 -60,00 -60,00 -60,00 -100,00 -10,00 -60,00 -60,00 -60,00 -150,00 -40,00 -40,00 -150,00 -110,00 -40,00 -40,00 -110,00 -489,42 -20,00 -318,32 -80,00 -370,52

LC ratio equal to PC ratio (28x)	
P_{min}	20 21 22 23 24 25 26 27 28 29 30 31 32 51 54 55 56 57 58 60 61 62 64 66 68 69 72 73

c.

Table D3 – Comparison of all decision rules for the high end computer

Appendix E

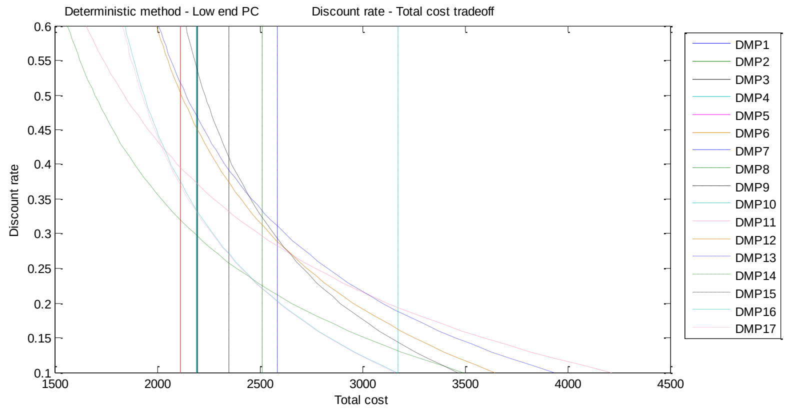

Figure E1- Comparison of all DMPs - low end computer

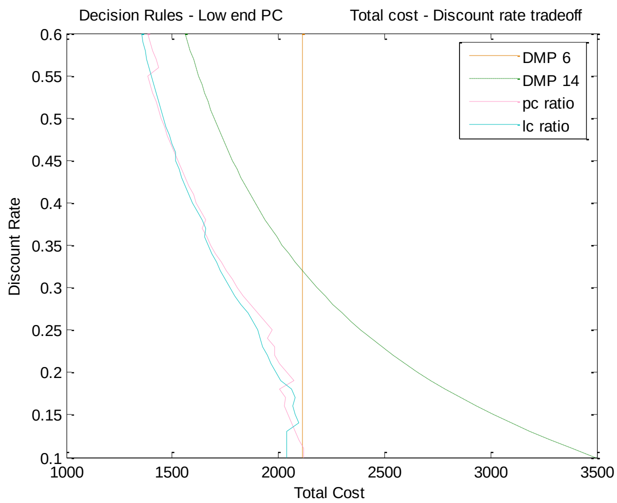

Figure E2- Comparison of all policies - low end computer

Appendix F

CPU

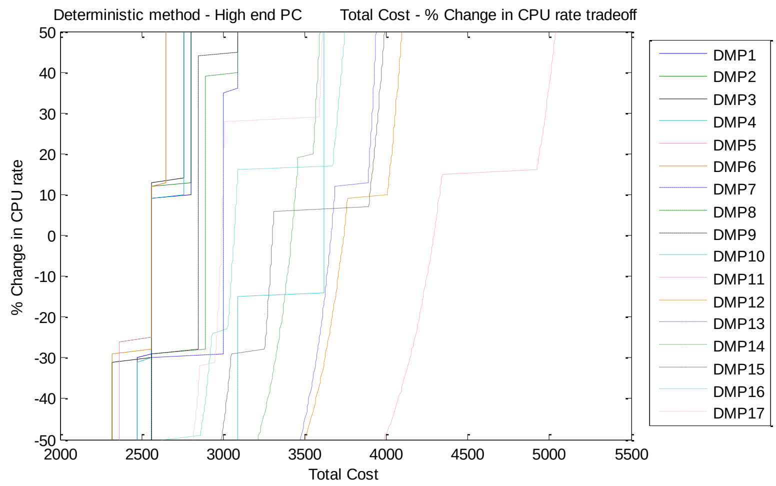

Figure F1- comparison of all DMPs - high end computer

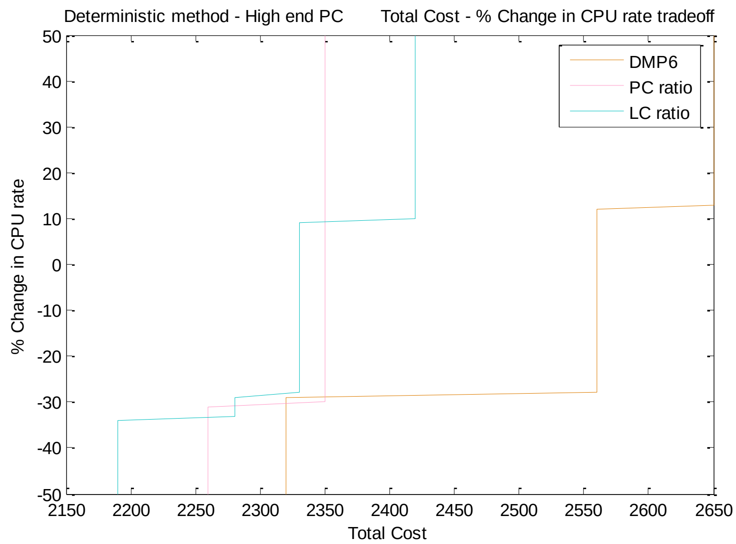

Figure F2- Comparison of all policies - high end computer

GPU

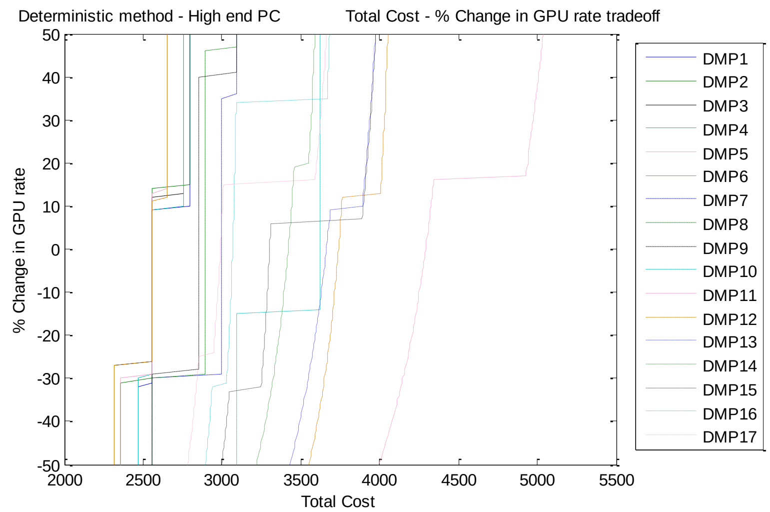

Figure F3- comparison of all DMPs - high end computer

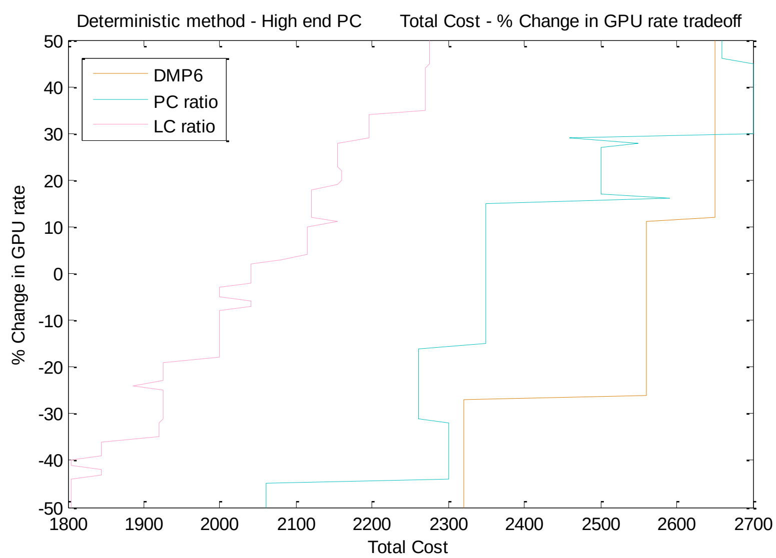

Figure F4- Comparison of all policies - high end computer

HDD

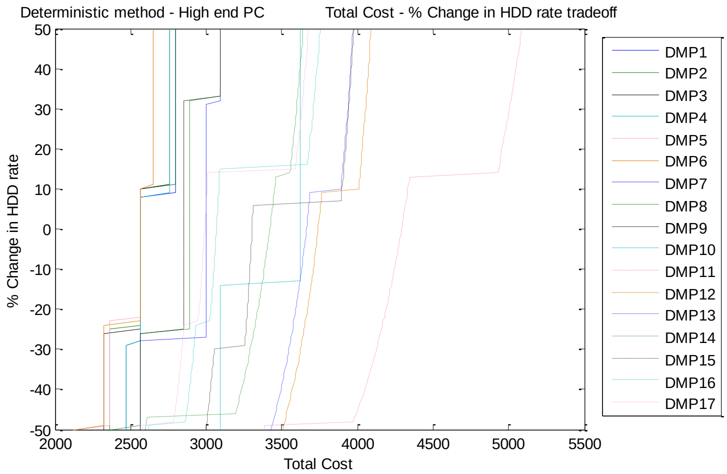

Figure F5- comparison of all DMPs - high end computer

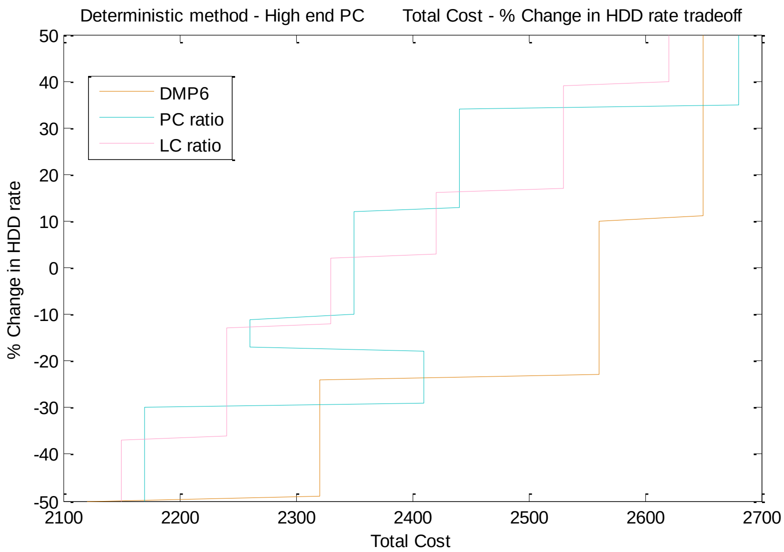

Figure F6- Comparison of all policies - high end computer

CPU % change in det. rate	Type of computer Low	Medium	Difference total costs	% difference
-50	€ 1760	€ 1570	€ 190	12.10
-40	€ 1805	€ 1585	€ 220	13.88
-30	€ 1880	€ 1585	€ 295	18.61
-20	€ 1920	€ 1625	€ 295	18.15
-10	€ 2000	€ 1625	€ 375	23.08
0	€ 2040	€ 1625	€ 415	25.54
10	€ 2120	€ 1625	€ 495	30.46
20	€ 2160	€ 1725	€ 435	25.21
30	€ 2200	€ 1725	€ 475	27.54
40	€ 2275	€ 1725	€ 550	31.88
50	€ 2315	€ 1725	€ 590	34.20

Table F1 – Difference total cost medium and low end computer

GPU % change in det. rate	Type of computer Low	Medium	Difference total costs	% difference
-50	€ 1805	€ 1415	€ 390	27.56
-40	€ 1805	€ 1470	€ 335	22.79
-30	€ 1925	€ 1570	€ 355	22.61
-20	€ 1925	€ 1570	€ 355	22.61
-10	€ 2000	€ 1570	€ 430	27.39
0	€ 2040	€ 1625	€ 415	25.54
10	€ 2115	€ 1725	€ 390	22.60
20	€ 2160	€ 1725	€ 435	25.21
30	€ 2195	€ 1780	€ 415	23.31
40	€ 2270	€ 1780	€ 490	27.53
50	€ 2275	€ 1795	€ 480	26.74

Table F2– Difference total cost medium and low end computer

HDD % change in det. rate	Type of computer Low	Medium	Difference total costs	% difference
-50	€ 1845	€ 1515	€ 330	21.78
-40	€ 1960	€ 1570	€ 390	24.84
-30	€ 1960	€ 1570	€ 390	24.84
-20	€ 1960	€ 1570	€ 390	24.84
-10	€ 2000	€ 1625	€ 375	23.08
0	€ 2040	€ 1625	€ 415	25.54
10	€ 2080	€ 1680	€ 400	23.81
20	€ 2120	€ 1680	€ 440	26.19
30	€ 2160	€ 1680	€ 480	28.57
40	€ 2200	€ 1735	€ 465	26.80
50	€ 2240	€ 1735	€ 505	29.11

Table F3 – Difference total cost medium and low end computer