Constructing portfolio using Jurek and Viceira

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Abstract

In this paper we investigate whether the portfolio policy creating method, called the Jurek & Viceira method, performs optimal. We calculate optimal portfolio weights for investors with long investment horizons with this method. Particularly, our aim is to find how optimal this method performs for longer horizons. We construct different scenarios concerning different risk aversion levels, horizons and different combinations of state variables. The performance of this method is measured using several benchmarks, including the out-of-sample performance of the Jurek & Viceira method. Using these benchmarks and the results of these, we find that the Jurek & Viciera method does not perform optimal for investors with long investment horizons.

Contents

1 Introduction

In recent years research on portfolio strategies (how much to invest in equities, bonds, stocks, etc.) of long-term investors such as pension funds is a popular topic in the world of long-term investors. There are several reasons for this. The first reason is the finding that stock returns are may be predictable using several variables such as the dividend-price ratio. If this is the case, then the optimal portfolio weights vary over time and the long-term investors need to invest more in stocks on average. The second reason are the faster computers. These can solve more complex problems in less time.

There are many financial economists who have researched many solution methods for the long-term portfolio choice problem. One of them are Campbell and Viceira (2002), who developed a analytical method. They also showed that the standard method to obtain the optimal weights of the assets in the portfolio is to first specify an econometric model and to calculate the weights according to the implications of the specified model afterwards. Usually, a Vector Autoregressive (VAR) model is used for this.
Jurek & Viciera (2010) provide an analytical recursive solution to the dynamic portfolio choice problem of an investor whose utility is defined over wealth at a future date.
However, the method extended by Jurek & Viciera (JV-method henceforth) is said to be powerful in predicting the optimal portfolio weights if the econometric model is well specified, but very sensitive to misspecifications.

Because of the stated sensitivity of this JV-method, our main purpose of this research is to investigate whether this method performs optimal for long investment horizons. Thus our main research question is formulated as: Is the JV-method an optimal method to solve long-term portfolio problems?

This research is relevant and the research question is important to answer because of several reasons. A person is not able to work forever. What will a person do when the time comes to retire? If we look at the recent news, we see that our health insurance is in trouble. It is uncertain what happens in the future. Long term investments gives the security to know that in bad circumstances, money is there.

This paper has been organized in the following way. First, we describe the JV-method in detail in the section named 'Methods'. In this section we also explain some benchmarks. This research is about constructing portfolio weights and as a portfolio consists of a certain amount of assets there is data needed for this. These data is described in the section named 'Data'. Then we have a section in which the JV-method in order to construct portfolio weights is applied.
The results of our research are provided in the section called 'Results'. And finally we end up with the section 'Conclusion', in which we summarize our results and draw our conclusions.

2 Methodology

As the purpose of this paper is to discover if the JV-method performs optimal for long investment horizons, we will describe the JV-method in detail in the first subsection of this section. And in the second subsection we will explain the benchmarks to see how sensitive the JV-method is and how optimal this method performs in the long run.

2.1 Weight construction

In this section the method that we will use in this research will be explained. As said before, we will focus on the recently derived analytical procedure by Jurek & Viciera (2010) to solve portfolio choice problems. This method is basically an extension of the classical approach for estimating portfolio weights.
The main feature of the classical approach is that it assumes an econometric model for assets and state-variables. Campbell & Shiller (1987) already used Vector-Autoregressive (VAR) models in this context, but their work was not especially about portfolio choice problems. Nowadays the VAR-approach is still the most common approach in modeling returns, in this way contributing to the solving of portfolio choice problems. Not surprisingly, the JV-method also assumes a VAR-model for the returns and state-variables. This model is our first step in calculating the optimal portfolio weights.

A VAR(1)-model estimates z_{t+1}, which is a $(1+n+m) \times 1$ vector with asset returns and state variables on time $t+1$, in the following way:

$$z_{t+1} = \Phi_0 + \Phi_1 z_t + v_{t+1}, \tag{1}$$

where Φ_0 is a $(1+n+m) \times 1$ vector of intercepts. n denotes the number of risky assets and m denotes the number of state variables. Φ_1 is a $(1+n+m) \times (1+n+m)$ square matrix of slope coefficients and v_{t+1} is a $(1+n+m) \times 1$ vector with error terms which are assumed to be homoskedastic and normally distributed:

$$v_{t+1} \overset{i.i.d.}{\sim} \mathcal{N}(0, \Sigma_v), \tag{2}$$

The vector z_{t+1} is structured as

$$z_{t+1} = \begin{bmatrix} r_{tbill,t+1} \\ r_{t+1} - r_{tbill,t+1}\iota \\ s_{t+1} \end{bmatrix} = \begin{bmatrix} r_{tbill,t+1} \\ x_{t+1} \\ s_{t+1} \end{bmatrix}. \tag{3}$$

Here, $r_{tbill,t+1}$ denotes the *log* real return on the T-bill that is used as a benchmark in excess return computations, x_{t+1} is a vector of excess *log* returns on all other assets with respect to the benchmark and s_{t+1} is a vector with the realizations of the state variables. Now the covariance matrix Σ_v can be written as

$$\Sigma_v = \begin{bmatrix} \sigma^2_{tbill} & \sigma^\top_{tbill,x} & \sigma^\top_{tbill,s} \\ \sigma_{tbill,x} & \Sigma_{xx} & \Sigma^\top_{xs} \\ \sigma_{tbill,s} & \Sigma_{xs} & \Sigma_s \end{bmatrix}. \tag{4}$$

The elements on the main diagonal are the variances of the real return on the benchmark asset (σ^2_{tbill}), Σ_{xx} is the variance-covariance matrix of unexpected excess returns and Σ_s denotes the variance-covariance matrix of the shocks to the state variables. $\sigma_{tbill,x}$ and $\sigma_{tbill,s}$ are the covariances of the real return and Σ_{xs} is the covariance of excess returns with shock to the state variables.

Before we proceed to the next step, let us first briefly summarize the most important outcomes of the VAR-model. In the first way, the model generates the vector Φ_0 and the matrix Φ_1 as output. These elements contain information about the evolution of the returns and the state-variables, which are important determinants for explaining the dynamics of the weights. Secondly, the residuals matrix Σ_v also contains information that is crucial for estimating the optimal portfolio weights. Finally, the structure of the z vector is defined in (3).
Now we will discuss the objective of an investor or portfolio manager. The JV-method considers an investor with initial wealth W_t at time t who chooses portfolio weights in such a way that the expected utility of wealth H periods ahead is maximized. After these H periods, the investor will abandon the portfolio and consume the final wealth at the terminal date, $t + H$. The investor's wealth evolves over time as

$$W_{t+1} = W_t(1 + R_{p,t+1})\forall t \tag{5}$$

where $R_{p,t+1}$ is the portfolio return at $t + 1$.
Further on, the investor has a constant coefficient of risk aversion γ. Formally, the investor chooses a sequence of portfolio weights $\alpha_{t+H-\tau}$ between time t and $(t + H - 1)$ such that

$$\left\{\alpha^{(\tau)}_{t+H-\tau}\right\}^{\tau=1}_{\tau=H} = arg\max E_t\left[\frac{1}{1-\gamma}W^{1-\gamma}_{t+H}\right]. \tag{6}$$

The function between brackets is called the utility function. The form of utility presented here is called power utility. Keeping the investor's objective in mind, our next step is to present the closed-form formula to calculate the weights.

Jurek & Viceira (2010) derived a iterative solution for the weights. Their method computes the optimal portfolio weights by first optimizing the power utility function over a 1-period horizon. Secondly, they solved the problem for two periods remaining and by using these solutions, they developed a general recursive solution for horizons with arbitrary lengths. As the derivations are beyond the scope of this paper, we will just present the formula. It is given by

$$\alpha^{(\tau)}_{t+H-\tau} = A^{(\tau)}_0 + A^{(\tau)}_1 z_{t+H-\tau}. \tag{7}$$

From this formula we see that the weights at time $t + H - \tau$, which means the weights at τ time periods prior to the terminal time period, depend on $A^{(\tau)}_0$, $A^{(\tau)}_1$ and $z_{t+H-\tau}$. The derivations of $A_0^{(\tau)}$ and $A_1^{(\tau)}$ can be found in the Supplementary Appendices of Jurek & Viceira (2010).

2.2 Sensitivity of the JV-method

2.2.1 Parameters

In order to look how sensitive the JV method is and how optimal this method performs for longer periods, we will construct several scenarios concerning the horizon (H), the coefficient of relative risk aversion (γ) and state variables.

The parameter γ is the relative risk aversion, for this parameter we choose the values $\gamma = 2$, $\gamma = 5$ and $\gamma = 10$ for our research.
Next, we will look at the parameter horizon H which is the amount of periods for which we will construct a strategy. As already said in the previous sections, we will take especially strategies for long horizons into account. For this reason we will look at the values $H = 4$, $H = 8$ and $H = 20$. This comes down to strategies for 1-year, 2-year and 5-year horizon.
The last parameter on which will be focussed are different combinations of state variables. This will be done by making combinations of 1, 3 and 6 randomly chosen state variables. We choose the sets of state variables about randomly because we did not investigated variable selection. This because this topic is beyond the scope of our research.

2.2.2 Performance measures

In order to find how well our JV-method performs for investors with a long investment horizon, we need to focus on the performance of this method in the long run. In this section we will explain in detail the performance measures which we will consider.

Utility
The utility is a measure to decide the performance of the model because the weights are computed by optimizing a utility function. To make some results easy to interpret, we will also look at the certainty equivalent of utility. The certainty equivalent of utility can be interpreted as the risk-free return an investor needs to receive in order to obtain the same utility. The certainty equivalent for the JV-method, which assumes a power utility, is defined as:

$$\frac{1}{1-\gamma}\left(CE_U\right)^{1-\gamma} = E\left[\frac{1}{1-\gamma}W_{t+H}^{1-\gamma}\right], \tag{8}$$

where we replace the expectation by the sample mean to come to

$$\frac{1}{1-\gamma}\left(CE_U\right)^{1-\gamma} = \sum_{t=0}^{T-H}\left(\frac{1}{1-\gamma}W_{t+H}^{1-\gamma}\right), \tag{9}$$

thus

$$CE_U = \left[(1-\gamma)\,\overline{U}\right]^{\frac{1}{1-\gamma}} \tag{10}$$

Interpetation of the weights
In this research we will look at the JV-method for long investment horizons. So it is very important to see how the portfolio weights behave in this case. In order to do this we will look at the mean stock weights for long horizons.

Improvement in relation to static weights
To see if it is important to include predictors when an investor has a long investment horizon, we will compare the certainty equivalents of utility where we include state variables to the certainty equivalents of utility where no predictors are included. This will be done by computing percentage improvements of the dynamic strategies over the static strategies. From the percentage improvements we can see if there is added value when including predictors.

Out-of-sample performance
The out-of-sample performance of the JV-method is also a benchmark to see how well the JV-method performs. This means that we use only a part of our data to estimate the weights out-of-sample. This because by looking at the out-of-sample performance, we can see how efficient the portfolio weights are for longer horizons. Here we will also focus on the certainty equivalent of utility.

We will also try to improve the initial out-of-sample performances. This will be done by imposing zero-coefficient constraints on the VAR model. A VAR model with zero coefficient restrictions is formulated as a Seemingly Unrelated Regressions (SUR) model. This algorithm is used to choose a subset of the most statistically-significant variables of a VAR model. We will continue by explaining the SUR-model.

The VAR(1)-model in (1) can be written in the compact form

$$Y = XB + U \tag{11}$$

where Y is the vector of observations on the dependent variable, X is the lagged exogenous data matrix, B is the coefficient matrix and U is the error term.
Now let $B = (b_1 \; . \; . \; . \; b_G)$ and S_i denote a selection matrix such that $\beta_i = S_i^T b_i$ corresponds to the non-zero coefficients of b_i $(i = 1, \; . \; . \; . \; , G \;\&\; G = 1 + n + m)$.

Further, let $X_i = XS_i$ which are the columns of X that correspond to the non-zero coefficients of b_i. Then the SUR-model is defined as

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_G \end{bmatrix} = \begin{bmatrix} XS_1 & & & \\ & XS_2 & & \\ & & \ddots & \\ & & & XS_2 \end{bmatrix} \begin{bmatrix} S_1^T b_1 \\ S_2^T b_2 \\ \vdots \\ S_G^T b_G \end{bmatrix} + \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_G \end{bmatrix} \tag{12}$$

where the error term has mean zero and the variance matrix is given by var(U) $= \Sigma \otimes I_M$. It is important to note that the regression equations in a SUR-model need to be estimated simultaneously in order to obtain effcient estimates.

3 Data

This section explains the data that we will use in this research for the empirical work. This research is about constructing portfolio weights and as a portfolio consists of a certain amount of assets there is data needed for this. The data is in line with Goyal and Welch (2008) and Rapach, Strauss and Zhou (2010). The data consist of 332 quarterly observations of 3 assets and 15 predictors from 1926 to 2008.

3.1 Assets

Real risk-free rate: The risk-free rate is the Treasury bill rate. The real risk-free rate is modeled as $ln(1 + R_f) - ln(1 + INFL)$, where R_f is the risk-free rate and $INFL$ is the Consumer Price Index from the Bureau of Labor Statistics.

Excess stock return: S&P 500 index returns from the Center for Research in Security Press (CRSP) month end values is used. As Goyal and Welch (2008) mention, the stock returns are the continuously compounded returns on the S&P 500 index including dividends. The excess stock return is modeled as $ln(1 + R_s) - ln(1 + R_f)$, where R_s is the stock return and R_f is the risk-free rate.

Excess bond return: long-term government bond returns R_b is used to model the excess bond return as $ln(1 + R_b) - ln(1 + R_f)$.

3.2 State variables

Dividend-price ratio: The *dividend-price ratio* is the difference between the *log* of dividends and the *log* of prices. In order to compute the dividend-price ratio dividends that are 12-month moving sums of dividends paid on the S&P 500 index is used. Furthermore the price of the S&P 500 index is used. Then the dividend-price ratio can be computed, which has the following form: $ln(D/P)$. Here, D is the dividend and P is the price.

Dividend-yield ratio: The *dividend-yield ratio* is the difference between the *log* of dividends and the *log* of lagged prices of the S&P Index. The dividend-yield ratio has the following form: $ln(D/P^*)$, where P^* is the lagged price of the S&P 500 Index.

Earnings-price ratio: The *earnings-price ratio* is the difference between the *log* of earnings and the *log* of prices. Earnings are 12-month moving sums of earnings on the S&P 500 index. The earnings-price ratio has the following form: $ln(E/P)$. Here, E is the earning and P is the price.

Dividend-payout ratio: The *Dividend-payout ratio* is the difference between the *log* of dividends and the *log* of earnings. It has the following form: $ln(D/E)$, where D is the dividend and E represents the earnings.

Stock variance: Stock variance is computed as the *log* of the sum of squared daily returns on the S&P 500. These daily returns are from The Center for Research in Security Press (CRSP). The stock variance has the following form: $ln(SVAR)$, where $SVAR$ is the sum of squared daily returns.

Book-to-Market ratio: The *book-to-market ratio* is the ratio of book value to market value for the Dow Jones Industrial Average. It has the following form: $ln(B/M)$. B represents the book value of Dow Jones, whereas M represents the market value of the Dow Jones.

Net Equity Expansions: The *Net Equity Expansion* is the ratio of 12-month moving sums of net issues by NYSE listed stocks divided by the total end-of-year market capitalization of NYSE stocks. It has the following form: $NTIS$.

Treasury Bill Rate: The *Treasury bill rate* has the following form:
$ln(1 + TBL)$, where TBL is the annualized interest on 3-month Treasury bill rate.

Long Term Yield: The *Long term yield* has the following form:
$ln(1 + LTY)$. Here, LTY is the annualized long-term government bond yield.

Long Term Return: The *Long term return* has the following form:
$ln(1 + LTR)$, where LTR is the annualized long-term government bond return.

Term Spread: The *Term Spread* is the difference between the *log* of long term yield on government bonds and the *log* of Treasury-bills. The *Term Spread* is modeled as $ln(1 + LTY) - ln(1 + TBL)$. LTY represents the long term yield and TBL represents the treasury bill rate.

Default Yield Spread: The *Default Yield Spread* is the difference between the *log* of

BAA and the *log* of AAA-rated bond yields, which is modeled as $ln(1+BAA)-ln(1+AAA)$.

Default Return Spread: The *Default Return Spread* is the difference between long-term corporate bond and long-term government bond returns. This is represented as $-ln(1+LTCR)-ln(1+LTR)$, where $LTCR$ is the long-term corporate bond return and LTR is the long-term government bond return.

Inflation: Inflation is the Consumer Price Index from the Bureau of Labor Statistics. It has the following form: $ln(1+INFL)$, where $INFL$ is the inflation.

Investment-to-Capital ratio: The *Investment-to-Capital ratio* is the ratio of aggregate (private nonresidential fixed) investment to aggregate capital for the whole economy,which has the following form: I/K. This is the variable proposed in Cochrane (1991), who provided the updated data to Goyal and Welch.

4 Application of weights

In this section we will describe how the JV-method in order to construct portfolio weights is applied. In the section 'Methods' we have already explained how to construct the portfolio weights.

We will only explain one-period ahead weights because our goal is only to describe how the method is applied. We first discuss the VAR-model describing the evolution of the risky asset returns and the state variables used to forecast them, specified in (1). Our risky assets are the familiar stock index and bond index returns and we use DP as the state-variable. As the risky returns are excess returns, we naturally choose the real return on the T-bill as the benchmark asset.
With these parameter values, (3) is in this case defined as

$$z_{t+1} = \begin{bmatrix} r_{tbill,t+1} \\ r^s_{t+1} \\ r^b_{t+1} \\ DP_t \end{bmatrix}. \tag{13}$$

After estimation, the VAR-model yields us ϕ_0, ϕ_1 and Σ_v as defined in (4). At this point, we have obtained all the information we need to calculate $A_0^{(1)}$ and $A_1^{(1)}$. These two elements in combination with z_{t+1} are sufficient for calculating the weights which are defined in (7).

5 Results

Using our methods, performance measures and data, we managed to obtain our empirical results which are necessary for this research to answer our research question. In this section we will provide these results. First, we will give a briefly interpretation of the portfolio weights constructed by our JV-method. Then we will investigate the utilities. And in the final part of this section we will evaluate the out-of-sample performance of our method.

5.1 Portfolio weights for the Long Run

We start with portfolio allocation exercises by clearly looking at the optimal portfolio weights for longer horizons, namely $H = 4$, 8 and 20. The exercise is repeated for three alternative values of the coefficient of relative risk aversion, namely $\gamma = 2$, 5, and 10. And 3 combinations of state-variables, namely DP, DFY DFR LTR and EP BM SVAR DE LTY DFY.

Figures 1, 2 and 3 graphically shows us the mean optimal percentage allocated to the stock index for a 5-year en 1-year horizon with only DP as state variable. The figures are respectively based on a risk-aversion of $\gamma = 2$, 5 and 10.

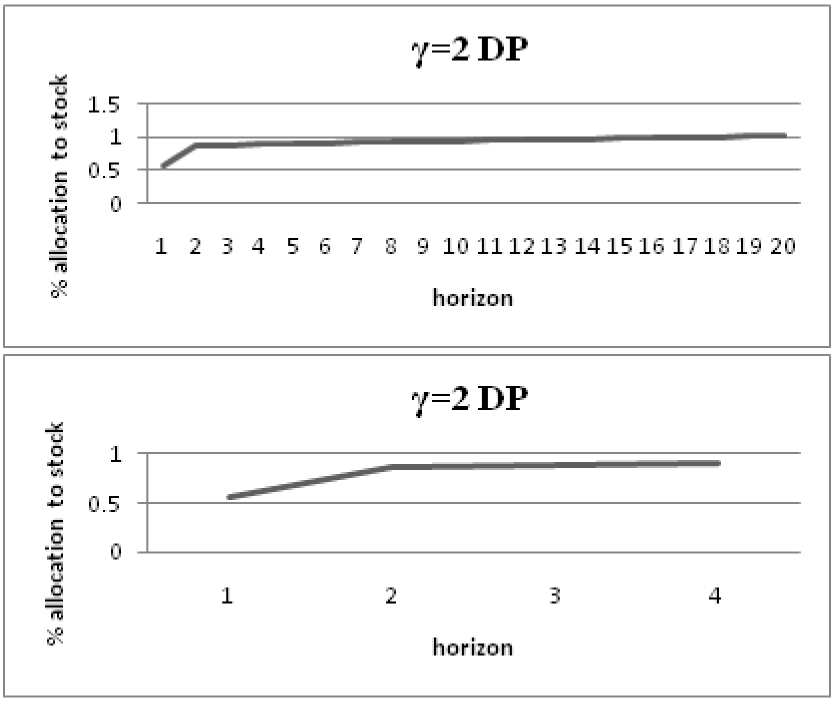

Figure 1: portfolio weights with $H = 20$ and 4, $\gamma = 2$ and DP as state-variable

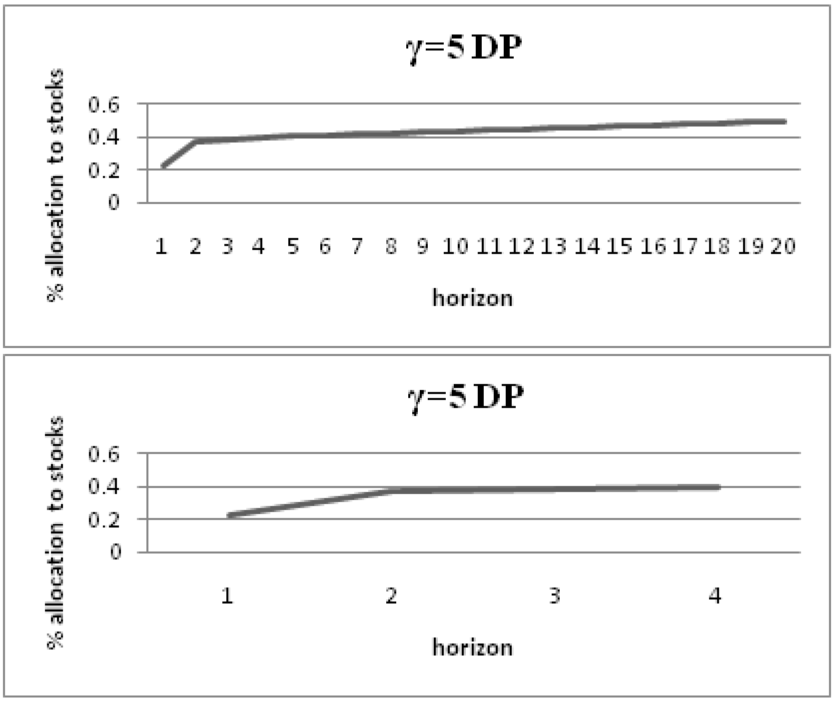

Figure 2: portfolio weights with $H = 20$ and 4, $\gamma = 5$ and DP as state-variable

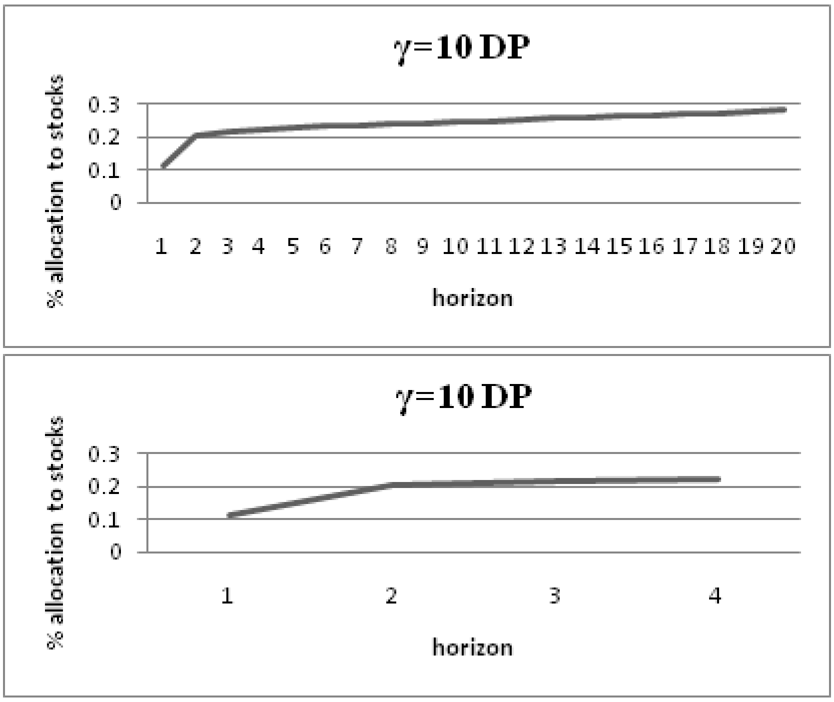

Figure 3: portfolio weights with $H = 20$ and 4, $\gamma = 10$ and DP as state-variable

From this figures we can make several observations. Interestingly, figures 1, 2 and 3 shows us that in this cases the stock allocation rises as the investment horizon increases. We also see that an investor with a horizon of 5 years allocates significantly more to stocks (than bonds) than someone with a 1-year horizon.
We conclude that a long-horizon investor allocate his wealth differently from a short-horizon investor, namely that investors with long investment horizons allocate more heavily to stocks. So when using the JV-method to estimate portfolio weights a long-horizon investor may overallocate to stocks.
In figure A.1 we can see the optimal mean percentage allocated to the stock index for the 2-year horizon, these figures also confirm this conclusion.

In the preceding we graphically saw the mean optimal percentage allocated to the stock index for a 5-year en 1-year horizon with only DP as state variable. Now we will discuss the mean weights on the stock index and on the bond index with all combinations state variables. Table 1 shows us the mean optimal portfolio weights for 5-year horizon with $\gamma = 5$. From this table we can see that the mean weights on the stock index and on the bond index are fairly constant over time, but not over the different combinations of state variables. Particularly we see that when the state variable combination EP BM SVAR DE LTY DFY is included, the weights become extremely high compared to the other state variables combinations. For example, if we look at the mean weights on the bond index when there are 11 periods remaining, we see that when we include only the state variable DP the mean weight equals 0.405. But when we include the state variable combination EP BM SVAR DE LTY DFY the mean weight equals 0.626. This means that the mean weights increases as the number of state variables increases.
In Table A.1 we can see the same results for $\gamma = 10$.

Table 1: mean 5-year weights with $\gamma = 5$

H=20	$\gamma = 5$									
	Stocks									
State variables	τ=1	τ=2	τ=3	τ=4	τ=5	τ=6	τ=7	τ=8	τ=9	τ=10
No state variables	0.225	0.370	0.379	0.383	0.385	0.386	0.387	0.387	0.388	0.388
DP	0.227	0.375	0.388	0.398	0.406	0.413	0.419	0.424	0.430	0.436
DFY DFR LTR	0.227	0.373	0.385	0.395	0.404	0.410	0.416	0.422	0.427	0.431
EP BM SVAR DE LTY DFY	0.237	0.396	0.421	0.444	0.465	0.487	0.508	0.529	0.549	0.569
	τ=11	τ=12	τ=13	τ=14	τ=15	τ=16	τ=17	τ=18	τ=19	τ=20
No state variables	0.388	0.388	0.388	0.388	0.388	0.388	0.388	0.388	0.388	0.388
DP	0.442	0.447	0.453	0.459	0.465	0.471	0.477	0.483	0.489	0.494
DFY DFR LTR	0.435	0.438	0.441	0.444	0.447	0.449	0.451	0.452	0.454	0.455
EP BM SVAR DE LTY DFY	0.589	0.609	0.628	0.647	0.666	0.685	0.703	0.721	0.738	0.756
	Bonds									
	τ=1	τ=2	τ=3	τ=4	τ=5	τ=6	τ=7	τ=8	τ=9	τ=10
No state variables	0.458	0.430	0.417	0.409	0.405	0.403	0.402	0.402	0.401	0.401
DP	0.456	0.427	0.414	0.407	0.404	0.403	0.403	0.403	0.403	0.404
DFY DFR LTR	0.456	0.431	0.413	0.406	0.402	0.401	0.400	0.400	0.400	0.400
EP BM SVAR DE LTY DFY	0.481	0.503	0.508	0.520	0.534	0.550	0.566	0.582	0.597	0.612
	τ=11	τ=12	τ=13	τ=14	τ=15	τ=16	τ=17	τ=18	τ=19	τ=20
No state variables	0.401	0.401	0.401	0.401	0.401	0.401	0.401	0.401	0.401	0.401
DP	**0.405**	0.405	0.406	0.407	0.408	0.409	0.409	0.410	0.411	0.412
DFY DFR LTR	0.401	0.402	0.403	0.405	0.406	0.408	0.409	0.411	0.413	0.414
EP BM SVAR DE LTY DFY	**0.626**	0.640	0.654	0.668	0.681	0.694	0.707	0.719	0.731	0.743

5.2 Utility for the Long Run

In this section we will first investigate what happens to the utility for the long run when varying the different parameters. Here we will directly look at the utility values. Secondly, we will look what happens to the performance of our JV-method when including predictors. We will especially look if there is improvement when adding predictors to the model. For this we will compare the utility when including state variables in the model with the utilities when not including any state variable at all in the model. For this we will use the percentage improvement in the certainty equivalent of the utility.

We start by looking at mean utilities for the different state variable combinations, horizons and values of γ. The results are given in table 2. For all combinations of state

Table 2: mean utility

State variables	**H**	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$
No state variables	4	-0.935	-0.234	-0.114
	8	-0.860	-0.222	-0.246
	20	**-0.662**	**-0.484**	**-637.266**
DP	4	-0.959	-0.267	-0.129
	8	-0.889	-0.288	-0.268
	20	**-0.631**	**-0.351**	**-247.961**
DFY DFR LTR	4	-0.933	-0.237	-0.115
	8	-0.860	-0.233	-0.251
	20	**-0.612**	**-0.437**	**-560.572**
EP BM SVAR DE LTY DFY	4	-0.804	-0.191	-0.094
	8	-0.621	-0.136	-0.176
	20	**-0.304**	**-0.127**	**-139.060**

variables when $\gamma = 2$ the utility rises as the horizon increases. And for this γ the utilities are strikingly high when $H = 20$. This is not the case for $\gamma = 5$ and $\gamma = 10$, in this case the utility clearly falls as the horizon increases. So we can easily conclude that according to the JV-method the utilities of more risk-averse investors falls as the investment horizon increases.

We continue by discussing whether the performance of our JV-method improves when including predictors. In table 3 we can see results of the improvements when including predictors.

Out of the results we can clearly see that for these predictor combinations, it does make

Table 3: CE_U percentage improvement in relation to static weights

State variables	**H**	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$
DP	4	**-2,49%**	**-3,28%**	**-1,37%**
	8	**-3,32%**	**-6,24%**	**-0,93%**
	20	5,01%	8,34%	11,06%
DFY DFR LTR	4	0,20%	**-0,34%**	**-0,13%**
	8	0,04%	**-1,20%**	**-0,22%**
	20	8,23%	2,55%	1,43%
EP BM SVAR DE LTY DFY	4	16,25%	5,25%	2,22%
	8	38,47%	13,04%	3,76%
	20	118,14%	39,69%	18,43%

sense to add predictors. When there are six predictors added, the method performs much better than when there are no predictors added for all horizons and values of γ. This is reflected from the positive values for this combination of predictors.
When adding only one predictor (in this case DP), the method performs worser than the other predictor combinations for all γ. But this is only the case when $H = 4$ and $H = 8$. Further on, we see a striking result, namely for $H = 20$ when adding one or more predictors for all γ the method performs always better than when there are no predictors added. So when predictors are added in the model for long term investors, the method performs very optimal.

5.3 Out-of-sample performance evaluation

In this section we will evaluate the out-of-sample performance of our JV-method. The set up of our out-of-sample experiment is as follows. We examine out-of-sample performance for the in-sample period 1926Q1-1965Q4 containing 160 observations (40 years) and the out-of-sample period 1966Q1-1995Q4 containing 120 observations (30 years). Our starting date is equal to 1966Q1 in order to have enough initial observations (40 years) to estimate the model and to have a relative long out-of-sample period. As already said, we will use the certainty equivalent of utility (CE_U) as performance criterium. Furthermore, we use an expanding in-sample window for generating the out-of-sample estimates.
In the first part of this section we will provide the out-of-sample performance results of the method and will discuss it. In the second part of this section we will try to improve these out-of-sample performances and will supply the percentages of improvement.

The out-of-sample performance result are given in table 4. We have done the out-of-sample experiment with $\gamma = 2$, 5 and 10, $H = 4$, 8 and 20 and different combinations of state variables.
We can clearly see that the most out-of-sample performances for $\gamma = 10$ are higher than

Table 4: CE_U performance of in-sample period 1926Q1-1965Q4 and out-of-sample period 1966Q1-1995Q4

		$\gamma = 2$		$\gamma = 5$		$\gamma = 10$	
State variables	**H**	**In-sample**	**Out-sample**	**In-sample**	**Out-sample**	**In-sample**	**Out-sample**
No state variables	4	1,085	0,915	1,030	0,950	1,017	0,971
	8	1,226	0,873	1,079	0,921	1,039	0,945
	20	1,694	**0,677**	1,198	**0,819**	1,060	**0,864**
DP	4	1,105	0,952	1,038	0,957	1,021	0,972
	8	1,301	0,988	1,109	0,964	1,055	0,965
	20	2,148	**1,019**	1,330	**0,954**	1,119	**0,935**
DFY DFR LTR	4	1,105	0,605	1,041	0,701	1,023	0,825
	8	1,277	0,493	1,102	0,635	1,052	0,784
	20	2,037	**0,300**	1,303	**0,551**	1,101	**0,759**
EP BM SVAR DE LTY DFY	4	1,196	0,706	1,056	0,786	1,027	0,876
	8	1,634	0,579	1,198	0,713	1,091	0,830
	20	3,595	**0,402**	1,585	**0,639**	1,212	**0,757**

the performances of $\gamma = 5$, and the most out-of-sample performances for $\gamma = 5$ are higher than the performances of $\gamma = 2$. We conclude that the out-of-sample performance is better for higher γ values and that this method is recommended to risk-averse investors. But only the DP-model concludes the reverse. For example, when we look at the DP-model with $H = 20$, we can see that the out-of-sample performance for $\gamma = 10$ (0,935) is lower than the performance of $\gamma = 5$ (0,954), and the out-of-sample performance for $\gamma = 5$ is lower than the performance of $\gamma = 2$ (1,019).
Further we observe that for every state variable combination with the corresponding γ, except the DP-model, the longest investment horizon (in this case $H = 20$) has the lowest out-of-sample performance. This means that the out-of-sample performance for short investment horizons are generally better than for very long investment horizons. The malfunctioning of the DP-model is also stated by Goyal and Welch (2008).

Table 5: CE_U improved performance of in-sample period 1926Q1-1965Q4 and out-of-sample period 1966Q1-1995Q4

		$\gamma = 2$		$\gamma = 5$		$\gamma = 10$	
State variables	**H**	**In-sample**	**Out-sample**	**In-sample**	**Out-sample**	**In-sample**	**Out-sample**
No state variables	4	1,085	0,943	1,030	0,957	1,017	0,975
	8	1,229	0,927	1,080	0,933	1,039	0,960
	20	1,717	1,079	1,203	0,996	1,061	**0,968**
DP	4	1,105	0,934	1,038	0,950	1,021	0,971
	8	1,303	0,911	1,110	0,922	1,055	0,954
	20	2,172	1,058	1,335	0,986	1,120	**0,963**
DFY DFR LTR	4	1,106	4,316	1,041	1,136	1,023	0,978
	8	1,281	1,929	1,103	0,818	1,052	0,809
	20	2,068	**3,034**	1,311	**1,129**	1,103	**0,892**
EP BM SVAR DE LTY DFY	4	1,197	0,919	1,056	0,954	1,027	0,975
	8	1,632	0,805	1,197	0,873	1,091	0,921
	20	3,611	**0,564**	1,586	**0,714**	1,212	**0,795**

We will continue with investigating whether the initial out-of-sample performance of above can be improved.
We do this by using the SUR-model as described in the section 'Methods'. The resulting out-of-sample performances are given in table 5. The improvement percentages are given in table 6. We draw several conclusions out if these tables.

From table 5 we can see that also after improvement the longest investment horizon has a lower out-of-sample performance than the shortest investment horizon for the most cases. But fortunately, we can see from table 6 that there are much out-of-sample performance improvements. For most state variable combinations, except the DP-model, there are performance improvements for all γ and all horizons. For the DP-model again there is no performance improvement when $H = 4$ and $H = 8$. We can also see that in most of the cases the improvement increases as the horizon increases. A very important result is that for the longest horizon, $H = 20$ (5-year horizon), there is improvement for all cases. With all these results we can definitely say that we managed to improve the initial out-of-sample performance, especially for the longest investment horizon.

Table 6: CE_U percentage improvement out-of-sample performances

State variables	**H**	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$
No state variables	4	3,10%	0,74%	0,42%
	8	6,23%	1,29%	1,59%
	20	59,35%	21,69%	12,10%
DP	4	**-1,85%**	**-0,73%**	**-0,17%**
	8	**-7,85%**	**-4,36%**	**-1,17%**
	20	3,87%	3,36%	2,97%
DFY DFR LTR	4	613,26%	62,10%	18,50%
	8	291,28%	28,81%	3,25%
	20	910,26%	104,93%	17,56%
EP BM SVAR DE LTY DFY	4	30,28%	21,38%	11,20%
	8	39,05%	22,45%	11,01%
	20	40,28%	11,87%	5,00%

6 Conclusion

In this paper a portfolio policy creating method named the Jurek & Viceira method is fully researched. In this section we will draw our conclusions of this research. In order to do this we will first repeat the main research question which we mentioned early in the section 'Introduction': Is the JV-method an optimal method to solve long-term portfolio problems?

The method of Jurek & Viceira is basically an extension of the classical approach for creating portfolio policies. The main feature of this classical approach is that it assumes an econometric model for assets and state-variables. That is why the JV-method uses a VAR(1)-model. The JV-method computes the optimal portfolio weights by first optimizing the power utility function. Finally, we managed to develop a general recursive solution for horizons with arbitrary lengths.

We described the several outcomes of different parameters and different performance measures in order to investigate how optimal our JV-method performs for long investment horizons. The parameters are the investment horizon and the relative risk-aversion. We have also described the outcomes of different combinations of state variables.

As mentioned, we investigated different performance measures. First we investigated the resulting optimal portfolio weights for longer horizons. These showed that when using the JV-method to estimate portfolio weights, a long-horizon investor may overallocate to stocks by a sizeable amount. We also found that the mean weights on the stock index and on the bond index are not constant over the different combinations of state variables. Secondly, we looked at the performance of our JV-method in terms of utility and its certainty equivalent. While looking straight to the mean utility values we concluded that the utilities of more risk-averse investors falls as the investment horizon increases. And the utilities of investors with a lower coefficient of risk-aversion rises as the horizon increases.
Further on, we investigated whether it makes sense at all to include state variables to increase performance. We found that it does make sense to add predictors to increase performance. We also found that when predictors are added in the model, the method performs optimal especially for long investment horizons.
We also investigated the out-of-sample performance of the JV-method and tried to improve these out-of-sample performances. The results showed that the initial and improved out-of-sample performances for short horizons were generally better than for very long horizons.
The out-of-sample improvement percentages showed that for most state variable combinations there were performance improvements for all levels of risk-aversion and all horizons. And in most cases the improvement increased as the horizon increased.

Now that we have investigated our results of our research thoroughly, we are finally able to answer our main research question. As said before, our goal is to find whether the JV-method is a optimal method to solve long term portfolio problems. From the results we conclude that the JV-method is not a optimal method to solve long-term portfolio problems. Summarizing, we draw this conclusion because for long investment horizons: this method overallocate to stocks, investors which are not very risk-averse have higher utilities while there are much investors who are risk-averse, the out-of-sample performance is worser than for short investment horizons. Only after the improvement, the out-of-sample performance for long investment horizon became better.

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A Appendix

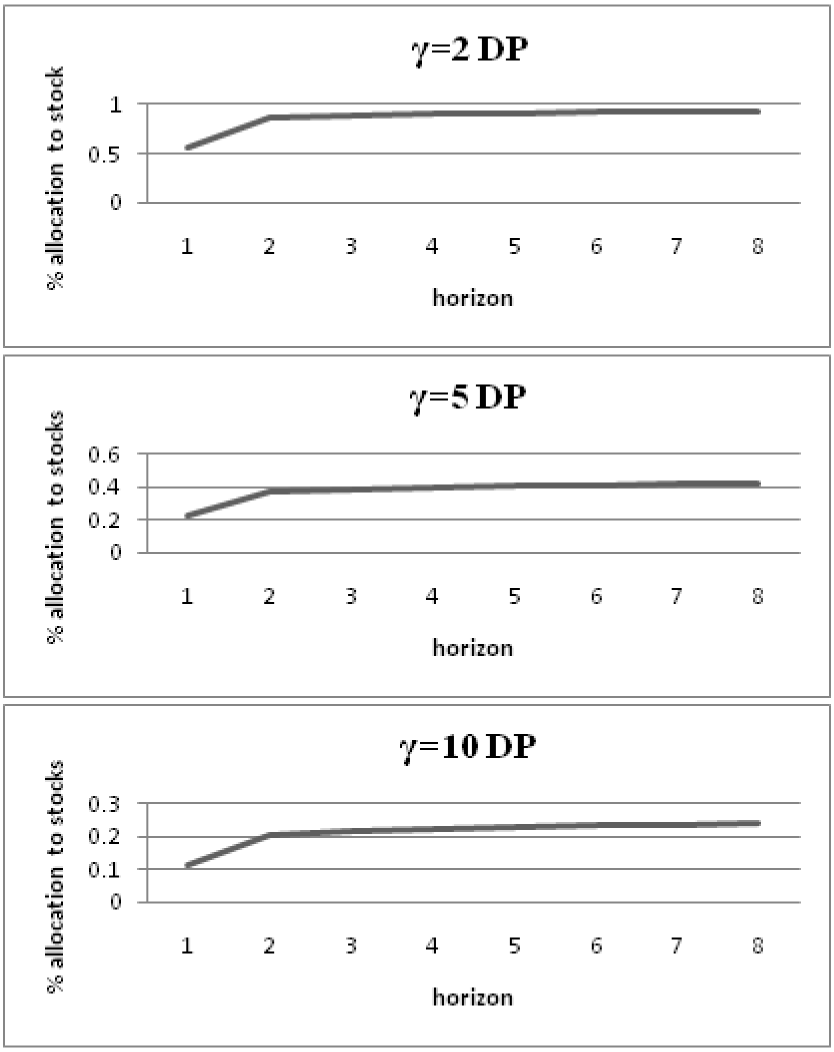

Figure A.1: portfolio weights with H=8, γ=2, 5 and 10 and DP as state-variable

Table A.1: mean 5-year weights with $\gamma = 10$

H=20	$\gamma = 10$									
	Stocks									
State variables	τ=1	τ=2	τ=3	τ=4	τ=5	τ=6	τ=7	τ=8	τ=9	τ=10
No state variables	0.113	0.201	0.210	0.214	0.217	0.218	0.218	0.219	0.219	0.219
DP	0.114	0.203	0.215	0.222	0.228	0.232	0.235	0.239	0.242	0.245
DFY DFR LTR	0.114	0.201	0.212	0.221	0.228	0.233	0.238	0.242	0.246	0.249
EP BM SVAR DE LTY DFY	0.119	0.211	0.228	0.243	0.256	0.270	0.283	0.295	0.308	0.320
	τ=11	τ=12	τ=13	τ=14	τ=15	τ=16	τ=17	τ=18	τ=19	τ=20
No state variables	0.219	0.219	0.219	0.219	0.219	0.219	0.219	0.219	0.219	0.219
DP	0.249	0.252	0.255	0.259	0.262	0.266	0.269	0.273	0.277	0.280
DFY DFR LTR	0.252	0.254	0.256	0.258	0.259	0.261	0.262	0.263	0.264	0.264
EP BM SVAR DE LTY DFY	0.332	0.344	0.355	0.367	0.378	0.389	0.400	0.410	0.421	0.432
	Bonds									
	τ=1	τ=2	τ=3	τ=4	τ=5	τ=6	τ=7	τ=8	τ=9	τ=10
No state variables	0.229	0.165	0.150	0.141	0.137	0.135	0.134	0.133	0.133	0.133
DP	0.228	0.164	0.148	0.140	0.136	0.135	0.134	0.134	0.134	0.134
DFY DFR LTR	0.228	0.167	0.152	0.147	0.147	0.148	0.150	0.153	0.156	0.159
EP BM SVAR DE LTY DFY	0.240	0.214	0.215	0.226	0.240	0.255	0.271	0.286	0.302	0.317
	τ=11	τ=12	τ=13	τ=14	τ=15	τ=16	τ=17	τ=18	τ=19	τ=20
No state variables	0.133	0.133	0.132	0.132	0.132	0.132	0.132	0.132	0.132	0.132
DP	**0.134**	0.134	0.135	0.135	0.136	0.136	0.136	0.137	0.137	0.138
DFY DFR LTR	0.163	0.166	0.170	0.174	0.178	0.181	0.185	0.189	0.193	0.197
EP BM SVAR DE LTY DFY	**0.333**	0.348	0.362	0.377	0.391	0.405	0.419	0.432	0.445	0.458