Repair priorities in a two-product inventory model

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Abstract

This report considers a repairable item inventory system with two item types and a single repair shop. Failures occur according to two Poisson processes with rates λ_1 and λ_2. Repair times are exponentially distributed with identical mean μ and items are managed according to a (s-1,s) policy. Costs are incurred for backordering items and may differ between both product types. We search for priority policies that lead to minimal backordering costs. We find that policies that include failure rates, backordering costs, stock levels and the variability of the failure process lead to the lowest costs and outperform simplistic rules by percentages up to 20 %.

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1 Introduction

Studies of inventory systems often concern the minimization of the total costs associated with these systems. These costs mainly consist of holding costs and backordering costs. Holding costs are costs required to keep and maintain items in storage and include space rent, materials, insurance, opportunity costs, and etcetera. Backordering costs are costs incurred by businesses when they are unable to fill and order and must complete it later. In this research, we consider an inventory system that consists of repairable products of two different types with base stock levels s_1 and s_2. We assume that broken items are returned according to two Poisson processes with rates $\lambda_1$and λ_2. When a broken item is returned, the customer immediately receives a new one, if available. If not, the item is backordered with backordering costs per time unit equal to b_1 and b_2. In a system like this, the total amount of products in inventory, either broken or repaired, is constant. We assume that the initial stock levels, and therefore holding costs, are fixed, so that only backordering costs can vary. The items are managed according to a (S-1, S)-policy. This means that when an item is given to a customer, an order is issued to the repair shop for an item of the same type. Our system consists of a single repair shop, which implies that only one order a time can be fulfilled. The total amount of backordering costs depends on the sequence with which orders to the repair shop are fulfilled. Our goal is to design a priority scheme for repairing items that will result in minimal backordering costs.

Where most papers on inventory control deal with optimizing stock levels, also repair priorities have been studied in a variety of settings. Most of these settings include failure times that are Poisson distributed (and thus inter-failure times that are exponentially distributed) and repair times that follow an exponential distribution. Hausman and Scudder (1982) examine a large amount of priority policies by means of simulation in an indentured product structure. They find that dynamic rules, which use work-in-process inventory information, outperform static rules that are based on fixed characteristics of the product types. Perez and Zipkin (1997) develop an effective dynamic heuristic policy in a Make-To-Stock inventory system, where several products share a single processor of limited capacity. Numerical experiments in a two-product environment suggest that their dynamic heuristic performs better than static heuristics and is close to optimality. Dynamic priorities are also proposed by Caggiano et al. (2006), in a two-echelon repairable item system with a single repair shop. Adan et al. (2006) evaluate the performance of static priority rules in a single-echelon, single repair shop repairable item system, not only by means of simulation, but also analytically. They simultaneously optimize base stock levels and find that with static priorities, large costs savings can be made compared to a First Come First Served policy. Tiemessen and van Houtum (2010) consider the same problem as Adan et al. and use the same optimized base stock levels, but plug in dynamic policies to evaluate their performance compared to static policies. They show that dynamic scheduling rules often reduce total costs by more than 10%.

Our work is most closely related to the last reference. Our system set up is similar to that of Tiemessen and van Houtum in most aspects, limited to a situation with two product types. That is, we deal with a single-echelon single-repair shop system in which failures are modeled according to two Poisson processes and repair times are exponentially distributed with equal mean repair times for both types. However, we include the possibility that backordering costs differ for both product types, which has important policy implications. Inclusion of different backordering costs has been done before in a Make-To-Stock environment, but not in a repairable item system. Since repairable item systems in which different products have different backordering costs are common, low-cost priority policies for these systems have many practical applications. Therefore, the goal of this research is to design priority repair schemes in a single-repair shop inventory system consisting of two asymmetric-cost products that lead to minimal backordering costs.

To achieve this goal, we will first state our assumptions and describe the system as a Markov decision process (Puterman (1994)). Thereafter, several priority policies will be proposed and their rationales will be explained. Evaluation of these policies will be done by means of simulation. The simulation results are reported in chapter 4. The performance of the different policies will be evaluated and conclusions will be given in chapter 5.

2 System description

In this section, our assumptions will be stated and the system will be described as a Markov decision process. This will help us obtain a better understanding of the problem and its possible solutions.

2.1 System description

We consider a system with two types of repairable items. Both types fail (and are returned) according to Poisson processes with rates λ_1 and λ_2 respectively. Repair times of both types follow the same distribution with mean parameter μ. Backordering costs are defined per product per time unit and are denoted by b_1 and b_2. Initial stock levels are given and denoted by s_1 and s_2. Because of the fact that no new products can be produced and broken items are always replaced if possible, the stock level can never exceed these initial levels. As mentioned in the introduction, only one item a time can be repaired in the single repair shop that we consider. When a repair has started, it will not be interrupted by another repair, which is called non pre-emption. If broken items are present, the repair shop is always occupied. This assumption is based on the assumption that holding costs are the same for either broken or repaired items. Second, it is based on the assumption that with equal mean repair times it is (almost) never beneficial to keep the repair shop free in case a broken product of the other type comes in. Therefore, the priority rules that are considered, only decide *which* item to repair and not *if* an item should be repaired. Allowing the repair shop to be empty during certain states, would require more complex priority rules. A schematic overview of the system can be found in figure 1.

Assume that initially, an amount of M products is operative and S products are in stock. Then:

- The total number of products in the system equals M+S;
- The maximum number of operative products is M;
- The maximum number of products in the stock base is S, otherwise there would be non-operative products that are not replaced;
- The maximum number of products in the repair queue is M+S-1, since repair is always performed if possible;
- The maximum number of items in the repair shop is 1, and equals zero only if the number of items in the repair queue is zero.

Further, we assume that the number of operative products is infinite, such that $M = \infty$. As described in the introduction, our goal is to design priority repair schemes that lead to minimal backordering costs. This is an

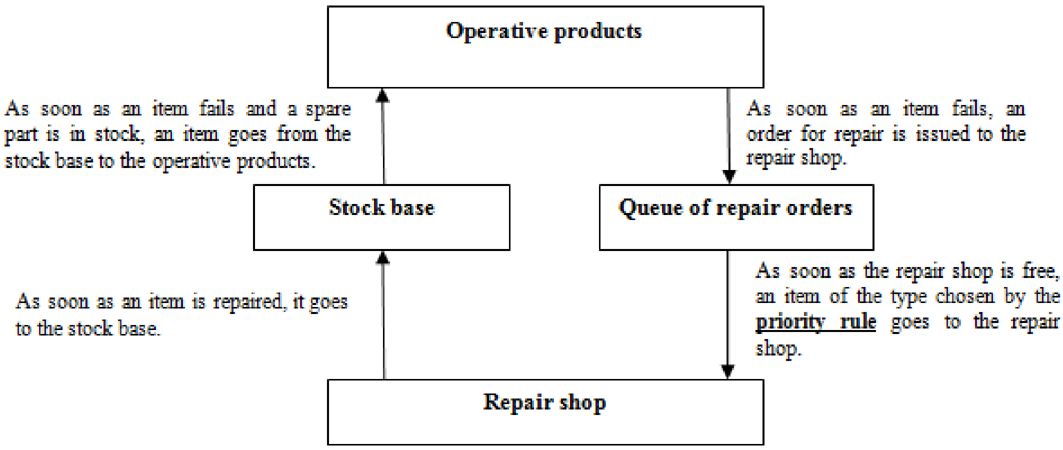

Figure 1: Schematic overview of the system.

optimization problem that is mathematically formulated as follows:

$$\min_{\pi} \sum_{n=1}^{2} b_n \cdot EB_n(\pi) \tag{1}$$

In which π indicates the priority policy that is implemented, b_n stands for the backordering costs per time unit for product type n and EB_n stands for the expected backordering costs for policy π. In the steady state, assuming that it exists, the probabilities to arrive at a certain state, equal the limiting probabilities when time goes to infinity. The expected number of backorders depends on the rates at which parts fail, on the mean repair time, on the initial stock levels and on the repair policy.

2.2 Analysis

The system that is described above can be modeled as a Markov decision process. A rigourous treatment of this kind of processes is given by Puterman (1994). A Markov decision process is similar to a Markov chain, except that transition probabilities depend on the actions of the decision maker and that for each state, the decision maker receives a reward. Just as with Markov chains, the system undergoes transitions from one state to another in a chainlike manner. The state in which the system will be next, is only influenced by the current state and action, but is independent of previous states and actions, which is called the Markov property. The general goal of a Markov decision process is to find a policy, that is, a specification of which action to take in each state, that maximizes some function (in our case the mean) of the sequence of rewards. We will start by giving a description of the states that are present in the system of study. Since the system consists of only two types of products, we deal with a two-dimensional state space. We define the state $x = [x_1 \quad x_2]$ as a two-dimensional vector that holds the net stock level for both products. A negative net stock level means that backorders are standing out for the concerning product type. Since we assume that the number of operative products is unbounded, the same holds for the possible number of outstanding backorders, implying that x_1 and x_2 range from $-\infty$ to s_1 and s_2 respectively. The steady state probabilities to go from one state to another by one product less in stock, will, irrespective of the priority policy, be smaller than one, since we always have the possibility that an item will be repaired before another item fails. Therefore, the probability to arrive at a state with a very large number of backorders is only small. So, unless failure rates are high and repair times low enough, the vast majority of time the system is in states that do not exceed a certain amount of backorders. The next step in describing the Markov decision process, is defining the action space A, and the set of admissible actions $A(x)$. We only have two products, and thus the action space in our model consists of $A = \{a_0, a_1, a_2\}$, with a_0, a_1 and a_2 the possible actions. Assuming non pre-emption, which means that we do not interrupt current repairs, we only need to make a scheduling decision when a repair job is finished. Action a_0 is only performed if there are no broken items, which is the case when $x_1 = s_1$ and $x_2 = s_2$, where s_1 and s_2 are the initial stock levels. Since the repair shop is never empty when broken items are present, action a_1 will only be performed with certainty if there *are* broken items of type 1 and none of type 2. Action a_2 will only be performed with certainty if there *are* broken items of type 2 and none of type 1. If we can only say with certainty which action should be performed if there are no broken items or broken items of one type only, how should we find the optimal action for *every* state x that leads to the lowest costs per time unit? One method that is often used for calculating the optimal action for every state x, is by solving the Bellman equations for an optimal policy for a Continuous Time MDP. A Bellman equation (Bellman (1953)), writes the value of a decision problem at a certain point in time in terms of the payoff from initial choices and the value of the remaining decision problem that results from the initial choices. The value of our decision problem, coincides with the height of the backordering costs. These should be minimized over all actions, leading to the following recursive equation, called the Bellman optimality equation:

$$B^{*}\left(\mathbf{x}\right)=\min_{a\in A(\mathbf{x})}\left[\left(-b_{1}\cdot\min\left(0,\mathbf{x_{1}}\right)-b_{2}\cdot\min\left(0,\mathbf{x_{2}}\right)\right)\cdot\tau\left(\mathbf{x},\mathbf{a}\right)\right.$$
$$\left.+\sum_{\mathbf{y}\in\mathbf{S}}p_{\mathbf{xy}}\left(a\right)B^{*}\left(\mathbf{y}\right)\right]\ \forall x \qquad (2)$$

In this equation, $\mathbf{y}$ is the state to which a transition from $\mathbf{x}$ is possible. $B^{*}(\mathbf{x})$ represents the lowest costs possible, when the initial state is $\mathbf{x}$. That the Bellman equation is a recursive one, can be seen from the fact that $B^{*}(\mathbf{y})$, present on the right hand side, represents the lowest cost possible when the initial state is state $\mathbf{y}$. Since there are several possible transitions from state $\mathbf{x}$ to state $\mathbf{y}$, we multiply the values of $B^{*}(\mathbf{y})$ with the probabilities that state $\mathbf{x}$ transits into state $\mathbf{y}$, given action a is performed, denoted by $p_{\mathbf{x},\mathbf{y}}(a)$. For all states $\mathbf{x}$, only a limited number of transitions has a probability higher than zero. To specify the transition probabilities we introduce the transition operators T_1^+, T_1^-, T_2^+ and T_2^-, that stand for respectively the transition in which one product of type 1 is repaired, the transition in which one product of type 1 fails, the transition in which one product of type 2 is repaired and the transition in which one product of type 2 fails. If we assume that mean repair times are different for both product types, this results in the transition probabilities that are specified in figure 2.
After the values of $V^{*}(\mathbf{y})$ have been multiplied with the transition probabilities, we sum over all $\mathbf{y}$. The first part of the Bellman equation, $(-b_1 \cdot \min(0,\mathbf{x_1}) - \mathbf{b_2}\cdot\min(\mathbf{0},\mathbf{x_2})\cdot\tau(\mathbf{x},\mathbf{a}))$, consists of the backordering costs per time unit that result from the system being in state $\mathbf{x}$ multiplied by the time the system is in state $\mathbf{x}$, given action a is performed. $\tau(\mathbf{x},a)$ are specified in figure 3.
As described above, the system of study counts a large number of states, an infinite number to be precise. Since this number is infinite, it is impossible to define an optimal action for every possible state. However, as mentioned before, the vast majority of time, the system is in states that do not exceed a certain amount of backorders. Therefore we could truncate the state space by ignoring the states with a large number of backorders. For a truncated state space, we could solve the Bellman equations and consequently give the optimal action for every evaluated state. However, this does not provide us with the real optimal solution, which would require evaluation of all states and not only those after truncation. Instead of searching for the optimal solution, we study general policies that are applicable to every problem instance.

In this section we gave a detailed description of our system and problem. In the next chapter we will first examine some possible general priority policies and thereafter explain how to evaluate their performance by means

$$p_{xy}(a_0) = \begin{cases} \dfrac{\lambda_1}{\mu_1 + \lambda_1 + \lambda_2} & if\ y = T_1^- \\ \dfrac{\lambda_1}{\mu_1 + \lambda_1 + \lambda_2} & if\ y = T_1^- \\ 0 & otherwise \end{cases}$$

$$p_{xy}(a_1) = \begin{cases} \dfrac{\mu_1}{\mu_1 + \lambda_1 + \lambda_2} & if\ y = T_1^+ \\ \dfrac{\lambda_1}{\mu_1 + \lambda_1 + \lambda_2} & if\ y = T_1^- \\ \dfrac{\lambda_2}{\mu_1 + \lambda_1 + \lambda_2} & if\ y = T_2^- \\ 0 & otherwise \end{cases}$$

$$p_{xy}(a_2) = \begin{cases} \dfrac{\mu_2}{\mu_2 + \lambda_1 + \lambda_2} & if\ y = T_2^+ \\ \dfrac{\lambda_1}{\mu_2 + \lambda_1 + \lambda_2} & if\ y = T_1^- \\ \dfrac{\lambda_2}{\mu_2 + \lambda_1 + \lambda_2} & if\ y = T_2^- \\ 0 & otherwise \end{cases}$$

Figure 2: transition probabilities

of simulation. For the simulation we use the assumptions, state space and events as described in this chapter.

3 Methods

In this section, we introduce different priority policies and explain the rationale behind them. First, a general overview of all policies will be given, after which they will be explained separately. The second part of this chapter will be about the method we use to test our policies.

3.1 Policies

From previous section, we know that the system of interest can be described as a Markov decision process, possessing the Markov, or memoryless property. This means that, given the current state x, and the current action a,

$$\tau(x, a_n) = \begin{cases} \dfrac{1}{\lambda_1 + \lambda_2} & if\, n = 0 \\ \dfrac{1}{\mu_n + \lambda_1 + \lambda_2} & if\, n = 1\, or\, n = 2 \end{cases}$$

Figure 3: Time lengths the system is in state x, given action a is performed

the next state is conditionally independent of all previous states and actions. Therefore, we only have to take into account the current state of the system when we make a priority decision. Further, we know from previous chapters that mean repair times are assumed to be equal for both types. This leaves us with three remaining factors on which we can base our repair policies. These are the backordering costs b_n, the failure intensity λ_n and the current stock level s_n. When we consider those factors in isolation, we prioritize types with high backordering costs over types with low backordering costs, types with high failure rates over types with low failure rates, and types with low stock levels over types with high stock levels. To get a clearer picture of the problem, we will first examine some hypothetical states in which the system can be.

Imagine a state in which the only difference between both product types is the height of the backordering costs (i.e. equal stock levels, equal failure rates and equal mean repair times). In that case, we act optimally by repairing a product of the type for which backordering costs are highest. In case a backorder occurs, this is worse for the high-cost type, whereas there is no reason, like a higher stock-out probability or longer occupation time of the repair shop, to choose for the low-cost type. This kind of states, however, in which the only difference between both product types are the backordering costs, will only seldom occur. So what should we do when the type with higher backordering costs, also has a higher stock level? Should we repair the type with the lower stock level, since it has a higher probability of reaching a negative stock level (and thus of causing backordering costs), or still the type with the higher backordering costs per time unit, since the consequences of reaching a negative net stock level for this type are more costly? Obviously, we need more information about stock levels and backordering costs than just the statements 'higher' and 'lower'. But even then, if we were to make a decision based on a trade-off between backordering costs and stock levels, how important should each factor be? By similar reasoning, we should always repair an item of the type with the lowest stock level when we consider a state in which the only difference between both product types is the current stock level. Because then, the low-stock-

Name of policy	Incorporated factors	Static/Dynamic
Random	none	neither
B	b	static
S	s	dynamic
Lab	λ	static
Diff	s	dynamic
Blab	b, λ	static
EBT	s, λ	dynamic
Myopic	s, λ	dynamic
SB	b, s	dynamic
EBT+B	b, s, λ	dynamic
Myopic+B	b, s, λ	dynamic
Myopic+B approximation	b, s, λ	dynamic
Presbyobic(p)	b, s, λ	dynamic

Table 1: overview of policies; b=backordering costs, λ = failure rates, s = stock levels.

type is more likely to reach a negative net stock level than the high-stock-type. Situation changes when we also have different failure rates. If one type has a smaller current stock level, but the other type has a higher mean number of failures per time unit, we have an incentive to repair the type with the higher failure rate instead of the type with the lower stock level. But then again, how much should the difference in failure rates be, compared to the difference in stock levels, to prioritize the high-failure-rate-type over the low-stock-type? It looks as if policies that include all three factors (b_n, λ_n, and s_n), are more likely to lead to low costs than policies that include only one, two or zero of these factors. Priority policies that include dynamic information about the current state of the system, like the current net stock level, are called dynamic policies. Policies that only include factors that are unchanging over time, like backordering costs or failure rates, are called static policies. Earlier work in the area of priority policies in inventory models has shown that dynamic policies outperform static policies. Therefore, our focus is on dynamic policies. However, we also investigate some static policies as a comparison. An overview of the policies that are investigated is shown in table 1. Especially the first few policies speak for themselves, but still, we will shortly explain them and state expectations about their quality.

Random-policy
The Random-policy randomly prioritizes either type 1 or type 2. This policy does not take into account backordering costs, failure rates or stock levels.

Therefore we do not expect this policy to provide good results.

B-policy
The B-policy always repairs the item with highest backordering costs, but does not take into account failure rates or stock levels. So according to the B-policy, products of the low-cost type are repaired only if there are no broken items of the high-cost type. We expect this rule to provide good results only in case cost differences are high enough and the utilization rate $((\lambda_1 + \lambda_2)/\mu)$ is low enough, so that sometimes the repair shop is free to repair the low-cost-type.

S-policy
The S-rule repairs the item of which stock is lowest, but does not take into account failure rates or backordering costs. Despite of the fact that this is a dynamic rule, we only expect it to yield good results when backordering costs and failure rates of both types are equal. Imagine the case in which there are 3 products left in stock of type 1, with a failure rate of 0.7 and backordering costs of 10 per time unit, and there are 2 products left in stock of type 2, with a failure rate of 0.2 and backordering costs of 1 per time unit. Whereas the S-rule would prioritize type 2, it seems much more rational to prioritize type 1, since it has a higher probability of reaching a negative stock level (0.0058 versus 0.0011). Another reason to prioritize type 1, is that reaching a negative stock level is more costly for type 1, due to the high backordering costs per time unit.

Lab-policy
The Lab-rule repairs the item which has the highest failure rate, but does not take into account the stock level or the backordering costs. The low-failure-type is repaired only if there are no broken high-failure-types. We expect this rule to provide good results only if backordering costs are equal for both types or when the utilization rate is sufficiently low, such that in some periods the repair shop is empty to repair the low-failure-type.

Diff-policy
The Diff-rule repairs the item of which the difference between initial stock and current stock is highest. We expect this rule to provide good results only in cases as described by the S-rule and is expected to outperform the S-rule only in case the initial stock level is chosen by an optimization procedure. Otherwise, aiming at the initial stock level has no function.

Blab-policy
The Blab-rule prioritizes the item which has the highest product $b_n \cdot \lambda_n$. This rule is expected to lead to lower costs than both the B-rule as the lab-rule in cases in which the B-rule and lab-rule lead to opposite decisions. In case

that one type has higher costs as well as a higher failure rate than the other type, the B-,lab- and Blab-rule lead to exactly the same priority decisions.

EBT-policy
The EBT rule (Equalization of Backorder Times rule), was proposed by Tiemessen and van Houtum (2010) and turned out to be a good rule in case of equal backordering costs. It tries to maximize the expected time until the next backorder by computing the prioritized item as follows:

$$\text{prioritized item} = \arg\min \left\{ \frac{s_n + 1}{\lambda_n} \right\} \tag{3}$$

We add +1 to the current stock level, to incorporate for the fact that backordering costs only occur from the moment the number of failures *exceeds* the current stock level with one. If we would apply this rule in case net stock levels are negative, we would obtain inadequate results, since high failure rates would then imply a low priority, which is the adverse of what we want. Therefore, in case of negative stock levels, priority is given as follows:

$$\text{prioritized item} = \arg\min \{ s_n \cdot \lambda_n \} \tag{4}$$

The EBT- rule does not take into account the severity of stock-out in terms of backordering costs, and is therefore expected to provide bad results as the difference in backordering costs between both product types gets larger. A second shortcoming of this method is that it prioritizes the type with the highest expected run-out time, whereas at the same time, this type might have a much larger probability of running out. So, this rule does not take into account the variability of the process.

Myopic policy
Just as the EBT-policy, the myopic policy is based on the failure rate and the current stock level. However, unlike the EBT-policy, the myopic policy takes into account the variability of the failure process. The myopic rule tries to minimize the expected number of backorders in the near future. As 'near future' we choose the expected future time period in which we can not influence the system by making a priority decision. Assuming non pre-emption, that is, we do not interrupt repair for another repair, this period equals the expected repair time. Minimizing the expected number of backorders during the next repair time is equivalent with choosing the type which has the largest probability of reaching a negative next stock level, during the next service time. Therefore, in this rule, priority is given as follows:

$$\text{prioritized item} = \arg\max \{ 1 - P_{\lambda_n \mu_n}(N_n \leq s_n) \} \tag{5}$$

In which N_n denotes the number of failures for type n and s_n denotes the stock level of type n at the beginning of the repair time. $P_z(Y < y)$ denotes the probability that $Y \geq y$, where Y follows a Poisson distribution with mean z.

SB-policy
The SB-rule prioritizes the item with the lowest ratio s_n/b_n in case of positive stock levels, and the item with the lowest $s_n \cdot b_n$ in case of negative stock levels. This rule implies that getting priority is more likely the lower the stock level and the higher the backordering costs. The measure s_n/b_n as 'the backordering costs per item in stock' in itself has no practical meaning, since items present in stock do not impose backordering costs. The SB-rule does not take into account failures rate and is therefore expected to provide good results only in case of equal failure rates.

All of the priority rules described above ignore one or more aspects of the repairable item inventory system, and therefore we assign most credit to rules that incorporate as well b_n, λ_n as s_n, as the ones described below.

EBT+B-policy
The EBT+B-policy, can be seen as an extension of the EBT rule with backordering costs (or as an extension of Blab with stock levels, or as an extension of SB with failure rates). In this rule, the prioritized item is computed as:

$$\text{prioritized item} = \arg\min \left\{ \frac{s_n + 1}{\lambda_n \cdot b_n} \right\} \tag{6}$$

Whereas $(s_n + 1)/(\lambda_n)$ denotes the expected run-out time, division by the backordering costs does not have a real meaning, since costs per item in stock do not exist. What division by backordering costs only does, is assigning a higher priority to high-cost types. When backordering costs of type 1 are four times as high as those of type 2, item 1 is prioritized unless the run-out time for item 2 is more than four times as low as the run-out time for item 1. Or, another example: When type 1 has a run-out time of 8 time units, and type 2 has a run-out time of 2 time units, and costs per time unit are 4 and 1 respectively, the EBT+B rule gives both items equal priority. Starting from the moment of the priority decision, table 2 shows an overview of the backordering costs per item, t time units after the moment of decision. From this table, we see that whether giving both types equal priority is a good decision depends on the time interval we consider. If we consider a time interval of 5 seconds after the priority decision, we might have performed better by giving priority to type 2. If we consider a time interval of 11 seconds after the moment of decision we might have acted better by prioritizing type 1. So, besides the expected run out time, also the probability of stock out during a certain interval is important, which leads us to our second three-factor policy, the myopic+B policy.

Time units after priority decision	**costs type 1**	**costs type 2**
1	0	0
2	0	0
3	0	1
4	0	2
5	0	3
6	0	4
7	0	5
8	0	6
9	4	7
10	8	8
11	12	9

Table 2: Costs for both types, during the next 11 time units, assuming only one item fails, and no item is repaired.

Myopic+B-policy

The Myopic+B policy is an extension of the myopic policy that includes backordering costs. It tries to minimize the expected backordering *costs*, instead of the expected *number* of backorders in the myopic rule, during the next repair time. This rule is based on the myopic rule in Perez and Zipkin where it is used in a Make-To-Stock environment, with different backordering and holding costs for each type. The prioritized type is the type with the highest expected costs during the next repair. To compute these costs, we need to know the expected number of backorders during one repair time. This can be expressed as follows:

$$E[N_B] = \sum_{n=s}^{\infty} (n-s) \cdot P(N=n) \tag{7}$$

in which N_B denotes the number of backorders, N denotes the number of failures and s the number of items in stock at the beginning of the repair. The probability $P(N = n)$ of having n failures during one repair time can be computed conditioning on the repair time. This leads to the following expression (see Appendix 1):

$$P(N=n) = \frac{(\lambda\mu)^n}{(\lambda\mu+1)^{n+1}} \tag{8}$$

in which λ is the failure rate and μ is the mean service time. Using geometric series, we find that the expected costs during one repair time (eq. 7) can be computed as follows:

$$E[N_B] = \frac{1}{\lambda\mu+1} \cdot \frac{r^{-s+1} \cdot (1+s(r-1))}{(r-1)^2} - s \cdot \frac{r^{-s}}{r-1} \tag{9}$$

$$with \quad r = \frac{\lambda\mu + 1}{\lambda\mu}$$

After we have incorporated the backordering costs, we prioritize the type with the highest expected backordering costs during one repair time. We expect the myopic+B policy to provide better results than the EBT+B policy, since it incorporates the backordering costs in a more meaningful manner and since it takes into account the variability of the failure process.

Myopic+B approximation policy
In the myopic policy (without backordering costs), we use a measure that is equivalent to choosing the type with the highest expected number of backorders during one repair time. Namely, choosing the type with the highest probability of reaching a negative net stock level during one repair time. In case of the myopic*+B*-policy, there is no such equivalent, since a higher probability of reaching a negative net stock level for one type does not necessarily lead to higher expected backordering costs if those costs per time unit are higher for the other type. However, it is still possible to use an approximation, that leads to (almost) equivalent results. In this approximation we choose the prioritized item in the following way:

$$\text{Prioritized item} = \arg\max\left\{(1 - P_{\lambda_n\mu_n}(N_n \leq s_n)) \cdot b_n\right\} \tag{10}$$

In which N_n denotes the number of failures for type n, s_n denotes the stock level of type n at the beginning of the repair time, and b_n denotes the backordering costs per time unit for type n. $P_z(Y < y)$ denotes the probability that $Y \geq y$, where Y follows a Poisson distribution with mean z. The rule prescribes to prioritize the item with the highest probability of reaching a negative net stock multiplied with the backordering costs per item per time unit. For this approximation to be a valid tool in reaching our goal (that is, prioritizing the item with the highest expected backordering costs during one repair time), we have to assume the following: The probability of stock-out multiplied by the backordering costs, can for one type never be higher or lower than for the other type, when the expectation of the backordering costs during one repair time is not higher or lower than for the other type. This assumption holds for (almost) all states that we consider. Even the ratio measure$_{type_1}$/measure$_{type_2}$ is close to the one obtained by computing the exact expectations.

Presbyopic(p)-policy
The myopic+B (approximation) policy makes priority decisions based on changes in the system during one repair time ahead. However, it might be beneficial to look more than one repair time ahead. That is why we introduce the presbyopic rule. This rule prioritizes the item with the highest expected costs during the horizon of p expected repair times. Since the approximation as used in the previous rule is easier to compute and probably

leads to the same priority decisions, we use it again in this rule. Therefore, the presbyopic policy comes down to the following:

$$\text{prioritized item} = \arg\max \{(1 - P_{\lambda_n p \cdot \mu_n}(N_n \leq s_n)) \cdot b_n\} \quad (11)$$

in which N_n denotes the number of failures for type n, s_n denotes the stock level of type n at the beginning of the repair time, b_n denotes the backordering costs per time unit for type n and p denotes number of repair times that is evaluated. Note that for a horizon (p) of 1, the presbyopic policy coincides with the myopic+B policy. $Pz(Y \leq y)$ denotes the probability that $Y \leq y$, where Y follows a Poisson distribution with mean z. Whereas the myopic+B(approximation) policy only tries to minimize the expected costs during the next repair time, the presbyopic rule takes into account the fact that also during later repair times, costs should be kept low. Therefore, we expect this rule to lead to better results than the Myopic+B rule.

3.2 Simulation

To evaluate the performance of different policies we use discrete event simulation. Possible events are a failure of type 1, a failure of type 2, a repair of type 1 and a repair of type 2. Every time an event occurs, a vector with net stock levels is updated and the backordering costs of the previous state are added to the total backordering costs. To obtain a measure of the variability of the results and to consider the system in its steady state, we use the method of batch means. According to this method, we simulate a large number of failures and split them in equal parts. Since the first batch starts with an empty system (and thus is not an adequate representation of the steady state), we ignore the results from this first batch. The number of failures used for each simulation instance, ranges from 200.000 to 300.000. For each test instance, we use the same failure and repair times to drive the simulation for all policies. This gives us an accurate approximation of the cost-differences between methods. The test bed consists of a single mean repair time μ, equal to 1. We use four different utilization rates ($\rho = (\lambda_1 + \lambda_2)/\mu$), ranging from heavy to low traffic, of respectively 0.99, 0.95, 0.8 and 0.7. Failure rates are chosen such that the ratio λ_2/λ_1 equals 1 or 4. Further, we use minimum base stock levels s_1 and s_2 as obtained by Adan et al. that lead to a target fill rate of 0.8, 0.8 assuming that a FCFS policy is used. This choice of parameters allows us to compare our results with those from earlier research and prevents us from choosing an unreasonable/unrealistic number of items in stock to begin with. Since the most important contribution of our policies is that they take into account differences in backordering costs, that is where we apply most variation. The backordering costs ratio's b_2/b_1, range from 8 to 0.5. We will investigate what the different priority policies imply for the average number of backorders in the system and compare the policies based on the average costs per time unit. We will use multiple comparisons to test the statistical significance of the difference between policies.

4 Results

This section reports the results obtained by simulation using the parameters as defined in section 3.2. Table 3 and 4 in Appendix 2 contain the average costs per time unit for every method. Table 4 and 5 in Appendix 2 contain the average number of backorders per time unit for every method and type. Our estimates are reasonably accurate, but not perfect, due to variation. However, we are not directly interested in the exact policy costs, but more in the cost differences between policies. The estimates of the costs differences are considerably more accurate, since we used the same failure and repair times for each method.

4.1 Comparison

The first four columns of tables 3,4,5 and 6 contain the system parameters. The subsequent columns contain the average costs per time unit for 14 methods. Estimates from the myopic+B policy are omitted, since they are equal to those of the myopic+B approximation policy. We notice several things:

- The simplistic random-, S-, diff-, EBT-, myopic- and SB-policy are never (among the) best. This is in accordance with our expectations, since these policies all lack one or more of the three factors (λ,b and s) that we could use. However, also the simplistic B-, lab- and Blab-policy lack one or more factors, but sometimes they *do* belong to the best policies. This leads to the following remark;

- The B-, lab- and Blab-policy are sometimes (among the) best. This is not in accordance with our expectations, since these policies all lack one of the three factors (λ,b and s) that we could use. An explanation is found after a closer inspection of the results. We see that the B-,lab- and Blab-policies lead to good results only in highly asymmetric-cost instances with a high utilization rate. Due to the high utilization rate, the total amount of backorders is large (see table 5 and 6), and due to the high difference in backordering costs per time unit, it is cost effective to repair only the high-cost-type, unless there are no broken items of this type. The reason that sometimes the lab-rule performs well, is that in those cases, it equals the B-rule. This happens if a type has a high failure rate as well as high backordering costs. The lab-rule in itself does not lead to good results, which can be seen for example in case of a utilization rate of 0.99, a failure-rate-ratio of 1 and a cost-ratio of 0.125. The policies mentioned in this remark, are not the only good performers in the described instances (asymmetric-cost instances with a high utilization rate). Also the more refined policies that incorporate all three factors lead to the same good results. This leads to the next remark;

- Three-factor policies (EBT+B, myopic+B, presbyopic(2), presbyopic(4) and presbyopic(6)) belong to the best performers in every instance. The last column of table 3 and the last column of table 4, display the percentage difference between the best 'simplistic' rule (lacking one or more factors) and the best rule incorporating all three factors. In the highly asymmetric cost instances with high failure rates, they perform identically to the B-policy (% difference = 0), which means that the high-cost type is in all states prioritized unless there are no broken products of that type. Whereas the B-policy works well by 'coincidence', the myopic+B or presbyopic rule prioritize the high-cost type based on a more weighted decision, namely after comparison of the expected costs for both types. Due to this weighted decision, these policies also perform well in case the cost differences are low and in case the high-cost type also has a high failure rate. Another remark regarding the three-factor policies, is that the EBT+B performs least well. The bad performance compared to the other three-factor policies becomes more pronounced when both types have different failure rates. This confirms our expectation that policies should include the variability of the failure process.

- At last, we examine the effect of the length of the time period for which we compute the expected backordering costs. This we do by comparing the myopic+B- and presbyopic-rules with each other. We conclude that in most cases, choosing $p = 4$ leads to the lowest cost. The few instances in which it might be beneficial to choose a larger p, are the highly asymmetric cases in which one type has a higher failure rate and higher backordering costs, in combination with a high utilization rate. These are the same instances as in remark two, in which the B-rule performs well and prioritizing the high-cost-type in every state of the system is optimal. However, the difference between the presbyopic(4) and presbyopic(6) rule might not be significant, which we will investigate next.

4.2 Statistical significance of the differences

To examine the statistical significance of the cost differences we use multiple comparisons as used in Koning et al. (2005). After we have rejected the null-hypothesis that the effects of every method are equal, by means of the Friedman test, we want to know which components of these hypothesis can also be rejected. The component hypotheses take the following form:

$H_0, k_1 k_2 : \tau_{k_1} = \tau_{k_2}$,

where $k_1 = 1, 2, ..., k_2 - 1$ and $k_2 = 1, 2, ..., 14,$, k are the methods and τ stands for the policy effect. Each component hypothesis $H_{0,k_1 k_2}$ is rejected if and only if:

$|\overline{R}_{k_1} - \overline{R}_{k_2}| \geq r_{\alpha,14,100}$,

in which 14 is the number of methods, 100 the number of simulations and $\overline{R_k}$ is the mean rank of policy k. For $\alpha = 0.05$, r equals 4.743 (see Harter(1960)). A visualization of the multiple comparisons can be found in figure 4, in which for every method an interval is drawn of length r, around the mean rank. Methods for which these lines do not overlap, significantly differ from each other. A bold line is drawn at the upper bound of the best method, such that all policies for which the interval lies above the bold line, perform significantly worse than the best method. We distinguish three cases:

The upper figure displays an example of the pattern that is found in the highly asymmetric cases, in which product 2 has high costs as well as a high failure rate in combination with a high utilization rate. We see that, as explained in remark 2, the B-, lab-, Blab-, presbyopic(4) and presbyopic(6) rule perform equally well, and that every other method performs significantly worse.

The mid figure represents the typical pattern we find in case of highly asymmetric product types in combination with a low utilization rate. Here, the presbyopic(4)- and presbyopic(6)-policy perform significantly best, where now the B-, lab- and Blab-policies perform significantly worse. This can be explained by the fact that the utilization rate is now low enough, such that there is more room to *prevent* backorders, instead of choosing the lowest backordering costs.

The bottom figure represents a typical example of a case in which product 1 has a low failure rate but high costs. A balanced decision has to be made, which leads to the three-factor policies, EBT+B, myopic+B, presbyopic(2) and presbyopic(4), being significantly better than the other methods. The presbyopic(4) policy never performed significantly worse than the best policy, which makes presbyopic(4) the best policy to choose in every evaluated instance.

In this section we evaluated the performance of all policies and examined the statistical significance of the differences between policies. In the next section we will summarize the conclusions following from these results and give recommendations for further research.

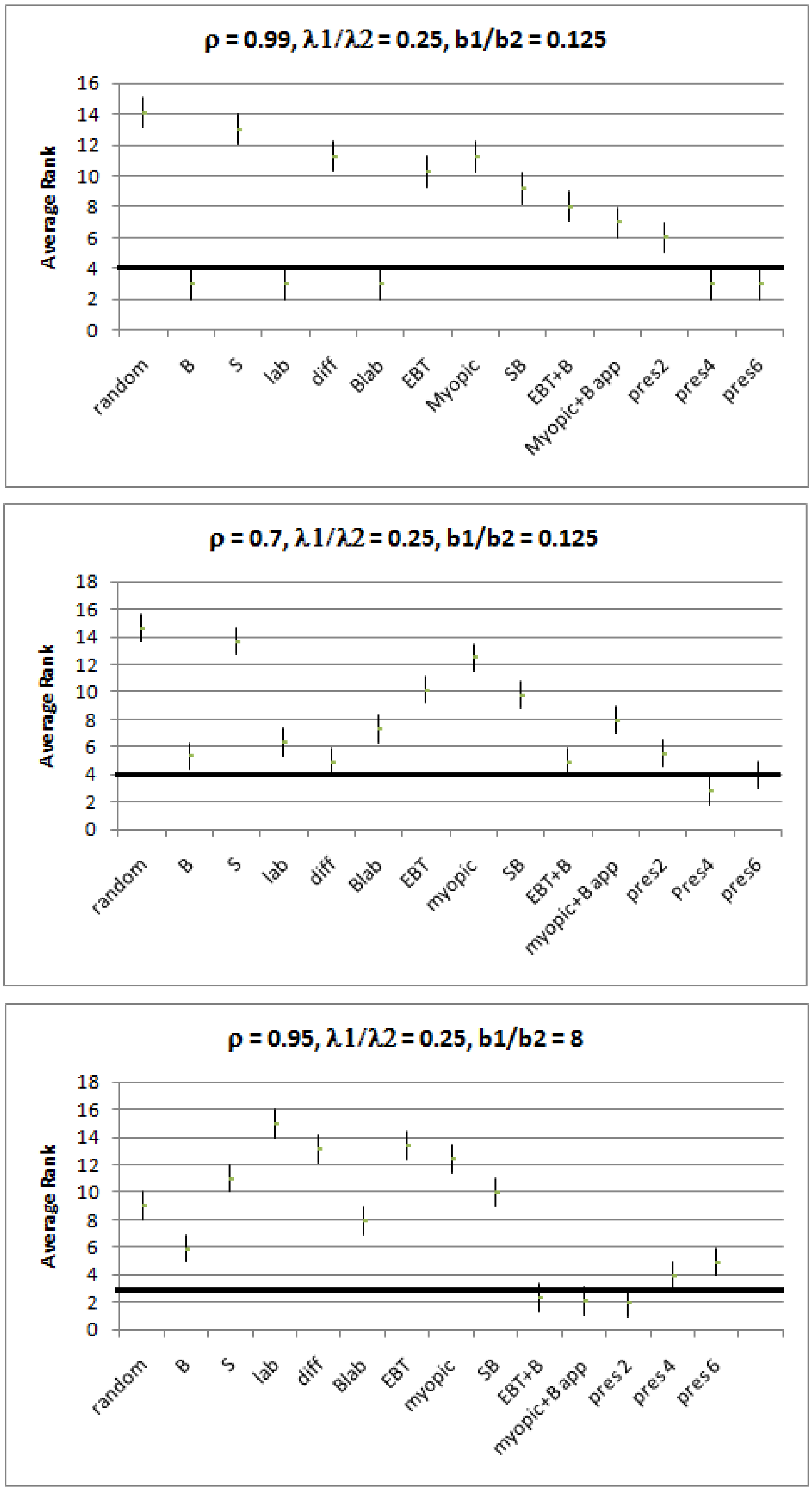

Figure 4: Average ranks of 14 methods over 100 simulations, intervals compare with the best method

5 Conclusion

We studied the use of priority policies in a repairable item inventory system with two product types. Products fail according to two Poisson processes with rates λ_1 and λ_2, and repair times are exponentially distributed with mean μ. We assume that both product types have different backordering costs, b_1 and b_2 and that products can be repaired in a single repair shop. We searched for a priority policy that leads to minimal costs. Differing backordering costs and repairable item inventory systems have been included in studies of priority policies before. However, they were never examined in combination. Since repairable item systems with different backordering costs are common, good priority policies for these systems have many practical applications. We proposed 14 different policies based on one or more factors describing the current state of the system (s,b and λ). We evaluated the quality of those policies by means of simulation in 36 problem instances. Simulation showed that policies that include all three factors, significantly lead to better results than policies that include zero, one or two factors. Their superior quality becomes more pronounced when the utilization rate is low, with percentage improvements around 15 %. The presbyopic(4) policy, that prioritizes the type with the highest probability of stock-out during the next 4 repair times, multiplied by the backordering costs per time unit, leads in every problem instance to the lowest costs.

Possibilities for further research

Our results and recommendations are based on simulations. It would be good to examine whether the same results can be derived analytically. Also, the Bellman equations from section 2 could be solved to evaluate how far our best results (obtained by presbyopic(4)) are from optimality. Also analytical proof could be given for the equivalence between the myopic+B-policy and the myopic+B-approximation-policy. A last, but certainly not unimportant recommendation for further research is to make less strict assumptions regarding the distribution and mean of the repair times. In this research, we assumed that repair times are exponentially distributed with equal mean repair times. The assumption of equal mean repair times might not be very realistic if we deal with unequal product types and consequently unequal failure types. Incorporating different mean repair times, would require small adjustments in our simulation, since our best policies already incorporate the effect of different mean repair times. Second, the assumption of exponentially distributed repair times is somewhat too strict. Whereas it is likely that failures can be modeled according to a Poisson process, the distribution of repair times is less secure. Instead of an exponential distribution, a Gamma distribution could be used to model repair times. In this case, we would probably need new policies leading to optimal results, that not only include mean repair times, but also the variance of the repair times.

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6 Appendix

6.1 Derivation

Expected number of backorders during one repair time.
N_B is the number of backorders during one repair time, N is the number of failures, s is the stock level, $P(N = n)$ is the probability of n failures during 1 repair time. N follows a poisson distribution with rate λ, repair times are exponentially distributed with mean μ.

$$E[N_B] = \sum_{n=s}^{\infty}(n-s) \cdot P(N=n)$$

$$\begin{aligned} P(N=n) &= \int P(N=n|T=t) \cdot f_T(t)dt \\ &= \int \frac{e^{-\lambda t}(\lambda t)^n}{n!} \cdot \frac{1}{\mu} e^{-\frac{t}{\mu}} dt \\ &= \int t^n \cdot \frac{e^{-(\lambda+\frac{1}{\mu})t} \cdot \lambda^n}{n!\mu} dt \end{aligned}$$

Write as gamma distribution with $k = n+1$ and $\theta = \frac{1}{\lambda+\frac{1}{\mu}}$

$$\begin{aligned} &= \int t^n \cdot \frac{e^{-(\lambda+\frac{1}{\mu})}}{(\frac{1}{\lambda+\frac{1}{\mu}})^{n+1}} \cdot \frac{1}{(\lambda+\frac{1}{\mu})^{n+1}} \cdot \frac{\lambda^n}{\mu} dt \\ &= \int 1 \cdot \frac{1}{(\lambda+\frac{1}{\mu})^{n+1}} \cdot \frac{\lambda^n}{\mu} dt \\ &= \frac{(\lambda\mu)^n}{(\lambda\mu+1)^{n+1}} \end{aligned}$$

$$E[N_B] = \sum_{n=s}^{\infty}(n-s) \cdot P(N=n)$$

This is a known series (Wolfram Mathematica) of which the result is as follows:

$$= \frac{1}{\lambda\mu+1} \cdot \sum_{n=s}^{\infty} n \cdot (\frac{\lambda\mu}{\lambda\mu+1})^n - s\sum_{n=s}^{\infty}(\frac{\lambda\mu}{\lambda\mu+1})^n$$

Say, $\frac{\lambda\mu+1}{\lambda\mu} = r$,

$$= \frac{1}{\lambda\mu+1} \cdot \frac{r^{-s+1}(1+s(r-1))}{(r-1)^2} - s(\frac{r^{-s}}{r-1})$$

6.2 Tables of average costs and average number of backorders

Simulation parameters (μ = 1)					average costs of policies															
ρ	$\lambda 1/\lambda 2$	b1/b2	s1	s2	Random	B	S	lab	diff	Blab	EBT	Myopic	SB	EBT+B	myopic+B	myopic+B approx	pres2	pres4	pres6	% difference
0.99	1.00	0.500	4	4	34.08	24.25	33.18	33.58	33.18	24.25	33.37	33.37	30.07	23.01	25.35	23.23	23.01	23.18	23.63	5.11%
		0.250			51.01	22.90	51.41	51.14	51.40	22.90	51.96	51.96	35.58	22.68	23.10	22.73	22.33	22.46	22.90	2.46%
		0.125			104.69	25.59	102.41	107.59	102.41	25.59	103.78	103.78	48.72	27.48	33.37	27.53	25.99	25.59	25.59	0.00%
	0.25	0.500	2	6	32.57	18.16	24.19	18.16	22.89	18.16	20.88	20.91	22.34	18.06	33.12	18.12	18.09	17.70	17.90	2.55%
		0.250			63.74	19.99	40.77	19.99	36.69	19.99	31.41	31.50	31.41	23.11	64.78	23.22	22.02	20.49	19.99	0.00%
		0.125			158.44	30.11	85.89	30.11	85.91	30.11	59.29	59.37	51.92	36.96	159.50	34.17	31.85	30.11	30.11	0.00%
		2.000			18.62	18.94	23.51	32.47	23.51	32.47	26.90	26.90	21.39	16.82	18.94	16.60	16.60	16.82	16.82	10.85%
		4.000			20.05	20.32	41.18	68.45	41.17	20.32	52.48	52.45	28.35	18.39	20.32	18.58	18.37	18.37	18.83	8.39%
		8.000			17.93	18.14	62.62	118.25	62.63	18.14	84.77	84.72	29.46	16.54	18.14	16.49	16.48	16.72	16.73	8.08%
0.95	1.00	0.500	4	4	15.49	11.51	14.66	15.42	14.66	11.51	14.81	14.81	13.46	10.46	11.40	10.60	10.46	10.61	10.99	9.13%
		0.250			27.91	12.30	26.09	27.61	26.10	12.30	26.53	26.53	18.70	12.03	12.79	12.08	12.04	11.92	12.30	3.10%
		0.125			45.21	12.00	44.05	48.06	44.05	12.00	45.07	45.07	23.34	13.18	12.75	13.25	12.20	12.00	12.00	0.00%
	0.25	0.500	2	6	18.69	9.34	12.09	9.34	12.09	9.34	10.73	10.75	11.31	9.41	19.23	9.45	9.42	9.25	9.34	0.95%
		0.250			36.87	11.61	21.03	11.61	21.03	11.61	17.06	17.10	17.06	13.07	37.36	13.13	12.39	11.61	11.61	0.00%
		0.125			79.76	17.07	42.17	17.07	42.18	17.07	31.60	31.70	28.85	21.23	79.54	19.50	18.06	17.07	17.07	0.00%
		2.000			10.21	10.47	11.74	16.71	11.74	16.71	13.18	13.17	10.84	8.72	10.47	8.54	8.54	8.72	8.72	16.38%
		4.000			11.68	11.92	21.76	37.36	21.77	11.92	27.12	27.09	15.54	10.27	11.92	10.39	10.24	10.24	10.62	12.33%
		8.000			9.64	9.82	29.81	58.51	29.79	9.82	39.29	39.23	15.07	8.59	9.82	8.50	8.50	8.68	8.69	11.80%

Table 3: Average costs per policy; λ = failure rate ρ = utilization rate b = backordering costs per time unit, s = stock level, 1 = type 1 and 2 = type 2.
The lowest average costs per instance are marked.
% difference = difference between best policy including 0,1 or 2 factors and presbyopic(4) policy.

Simulation parameters (μ = 1)					average costs of policies															
ρ	$\lambda 1/\lambda 2$	b1/b2	s1	s2	Random	B	S	lab	diff	Blab	EBT	Myopic	SB	EBT+B	myopic+B	myopic+B approx	pres2	pres4	pres6	% difference
0.80	1.00	0.500	4	4	1.36	1.23	1.10	1.36	1.10	1.23	1.14	1.14	1.07	0.89	1.01	0.90	0.89	0.89	0.94	16.75%
		0.250			2.38	1.29	1.86	2.35	1.86	1.29	1.98	1.98	1.58	1.10	1.26	1.12	1.11	1.08	1.15	16.37%
		0.125			3.93	1.35	3.04	4.04	3.04	1.35	3.28	3.28	2.33	1.39	1.35	1.43	1.28	1.25	1.35	7.02%
	0.250	0.500	2	6	1.54	1.16	1.15	1.16	0.97	1.16	1.04	1.07	1.09	0.92	1.65	0.98	0.92	0.90	0.97	7.04%
		0.250			3.30	1.49	2.27	1.49	1.67	1.49	1.97	2.05	1.97	1.52	3.28	1.62	1.52	1.39	1.42	6.45%
		0.125			6.73	2.16	4.50	2.16	2.87	2.16	3.90	4.11	3.79	2.76	5.71	2.73	2.42	2.20	2.16	0.00%
		2.000			0.99	1.00	1.00	2.22	1.30	2.22	1.06	1.04	0.97	0.86	1.00	0.82	0.82	0.82	0.86	18.23%
		4.000			0.95	0.89	1.19	3.89	1.98	0.89	1.35	1.28	1.02	0.79	0.89	0.80	0.79	0.79	0.81	11.15%
		8.000			1.12	0.95	1.90	7.54	3.68	0.95	2.29	2.14	1.39	0.89	0.95	0.89	0.89	0.95	0.95	6.67%
0.700	1.000	0.500	4	4	0.37	0.34	0.26	0.37	0.26	0.34	0.27	0.27	0.25	0.22	0.27	0.22	0.22	0.22	0.24	14.46%
		0.250			0.62	0.38	0.41	0.62	0.41	0.38	0.45	0.45	0.38	0.29	0.36	0.29	0.29	0.28	0.32	24.17%
		0.125			1.09	0.42	0.71	1.11	0.71	0.42	0.80	0.80	0.60	0.40	0.42	0.42	0.37	0.37	0.42	11.39%
	0.250	0.500	2	6	0.42	0.37	0.32	0.37	0.27	0.37	0.29	0.30	0.30	0.26	0.46	0.28	0.27	0.26	0.26	10.20%
		0.250			0.77	0.45	0.57	0.45	0.39	0.45	0.49	0.53	0.49	0.39	0.69	0.42	0.40	0.38	0.38	16.48%
		0.125			1.52	0.63	1.13	0.63	0.66	0.63	0.94	1.04	0.92	0.67	1.04	0.75	0.68	0.59	0.60	6.62%
		2.000			0.24	0.23	0.21	0.59	0.30	0.59	0.21	0.20	0.21	0.19	0.23	0.18	0.18	0.18	0.19	12.92%
		4.000			0.29	0.25	0.26	1.11	0.52	0.25	0.30	0.26	0.25	0.22	0.25	0.22	0.22	0.22	0.22	14.25%
		8.000			0.39	0.29	0.41	2.22	1.01	0.29	0.52	0.43	0.36	0.27	0.29	0.27	0.27	0.26	0.29	7.58%

Table 4: Average costs per policy; λ = failure rate ρ = utilization rate b = backordering costs per time unit, s = stock level, 1 = type 1 and 2 = type 2.
The lowest average costs per instance are marked.
% difference = difference between best policy including 0,1 or 2 factors and presbyopic(4) policy.

Simulation parameters (μ = 1)					average number of backorders per type for every policy																	
ρ	$\lambda 1/\lambda 2$	b1/b2	s1	s2	Random		B		S		lab		diff		Blab		EBT		Myopic		SB	
					type 1	type 2	type 1	type 2	type 1	type 2	type 1	type 2	type 1	type 2	type 1	type 2	type 1	type 2	type 1	type 2	type 1	type 2
0.99	1.00	0.500	4	4	10.91	11.58	24.04	0.11	11.06	11.06	11.42	11.08	11.06	11.06	24.04	0.11	10.87	11.25	10.87	11.25	14.19	7.94
		0.250			10.93	10.02	22.45	0.11	10.28	10.28	10.88	10.06	10.28	10.28	22.45	0.11	10.09	10.47	10.09	10.47	15.59	5.00
		0.125			11.49	11.65	24.72	0.11	11.39	11.38	11.06	12.07	11.39	11.38	24.72	0.11	11.20	11.57	11.20	11.57	19.10	3.70
	0.25	0.500	2	6	0.13	16.22	16.16	1.00	6.83	8.68	16.16	1.00	8.20	7.35	16.16	1.00	10.10	5.39	10.08	5.42	8.65	6.85
		0.250			0.13	15.90	15.82	1.04	6.63	8.54	15.82	1.04	8.03	7.16	15.82	1.04	9.72	5.42	9.70	5.45	9.72	5.42
		0.125			0.02	19.80	16.59	1.69	7.76	9.77	16.59	1.69	7.75	9.77	16.59	1.69	11.52	5.97	11.51	5.98	12.57	4.92
		2.000			0.02	18.59	0.00	18.93	7.18	9.16	15.39	1.69	7.18	9.16	15.39	1.69	10.60	5.70	10.59	5.71	4.97	11.45
		4.000			0.02	19.96	0.00	20.32	7.83	9.85	16.67	1.78	7.83	9.86	0.00	20.32	11.61	6.04	11.60	6.05	3.51	14.29
		8.000			0.02	17.79	0.00	18.13	6.73	8.75	14.57	1.67	6.74	8.75	0.00	18.13	9.90	5.55	9.90	5.56	1.98	13.64
0.95	1.00	0.500	4	6	4.93	5.28	11.32	0.09	4.88	4.89	4.99	5.21	4.88	4.89	11.32	0.09	4.73	5.04	4.73	5.04	6.09	3.69
		0.250			5.17	5.69	11.98	0.08	5.22	5.22	5.27	5.59	5.22	5.22	11.98	0.08	5.08	5.36	5.08	5.36	7.74	2.74
		0.125			5.19	5.00	11.31	0.09	4.88	4.90	4.79	5.41	4.88	4.90	11.31	0.09	4.74	5.04	4.74	5.04	7.90	1.93
	0.25	0.500	4	6	0.01	9.34	6.92	1.21	3.01	4.54	6.92	1.21	3.01	4.54	6.92	1.21	4.28	3.23	4.27	3.24	3.71	3.80
		0.250			0.01	9.21	6.86	1.19	3.00	4.51	6.86	1.19	3.00	4.51	6.86	1.19	4.26	3.20	4.26	3.21	4.26	3.20
		0.125			0.02	9.97	7.58	1.19	3.34	4.85	7.58	1.19	3.34	4.85	7.58	1.19	4.80	3.35	4.79	3.36	5.20	2.96
		2.000			0.02	10.17	0.00	10.47	3.39	4.96	7.76	1.19	3.39	4.95	7.76	1.19	4.87	3.43	4.86	3.45	2.39	6.07
		4.000			0.02	11.60	0.00	11.91	4.02	5.68	9.01	1.31	4.02	5.68	0.00	11.91	5.82	3.84	5.81	3.85	1.89	7.98
		8.000			0.01	9.53	0.00	9.81	3.14	4.66	7.17	1.18	3.14	4.66	0.00	9.81	4.50	3.25	4.50	3.26	1.01	6.97

Table 5: average number of backorders per policy; ; λ = failure rate ρ = utilization rate b = backordering costs per time unit, s = stock level, 1 = type 1 and 2 = type 2.

Simulation parameters (μ = 1)					average number of backorders per type for every policy											
ρ	$\lambda 1/\lambda 2$	b1/b2	s1	s2	EBT+B		Myopic+B		Myopic+B approx		pres2		Pres4		pres6	
					type 1	type 2	type 1	type 2	type 1	type 2	type 1	type 2	type 1	type 2	type 1	type 2
0.99	1.00	0.500	4	4	21.73	0.64	22.51	1.42	21.01	1.11	21.71	0.65	22.47	0.36	23.24	0.20
		0.250			20.22	0.62	22.38	0.18	20.16	0.64	20.90	0.36	21.66	0.20	22.45	0.11
		0.125			22.43	0.63	23.61	1.22	22.39	0.64	23.14	0.36	24.72	0.11	24.72	0.11
	0.25	0.500	2	6	13.06	2.50	0.03	16.55	12.97	2.57	13.00	2.55	14.21	1.75	15.48	1.21
		0.250			12.58	2.63	0.03	16.19	12.53	2.67	13.14	2.22	14.47	1.50	15.82	1.04
		0.125			14.78	2.77	0.03	19.93	15.36	2.35	15.97	1.98	16.59	1.69	16.59	1.69
		2.000			0.09	16.65	0.00	18.93	0.31	15.99	0.31	15.99	0.09	16.65	0.09	16.65
		4.000			0.08	18.06	0.00	20.32	0.31	17.35	0.09	18.02	0.09	18.02	0.02	18.74
		8.000			0.08	15.94	0.00	18.13	0.08	15.83	0.08	15.84	0.02	16.55	0.02	16.56
0.95	1.00	0.500	4	6	9.48	0.49	10.82	0.29	8.93	0.84	9.46	0.50	10.05	0.28	10.67	0.16
		0.250			10.22	0.45	11.80	0.25	10.14	0.49	10.18	0.46	11.33	0.15	11.98	0.08
		0.125			9.58	0.45	11.20	0.19	9.51	0.47	10.06	0.27	11.31	0.09	11.31	0.09
	0.25	0.500	4	6	5.69	1.86	0.00	9.62	5.65	1.90	5.68	1.87	6.50	1.37	6.92	1.21
		0.250			5.65	1.85	0.01	9.34	5.62	1.88	6.03	1.59	6.86	1.19	6.86	1.19
		0.125			6.33	1.86	0.05	9.94	6.72	1.60	7.15	1.36	7.58	1.19	7.58	1.19
		2.000			0.07	8.58	0.00	10.47	0.24	8.06	0.24	8.06	0.07	8.57	0.07	8.57
		4.000			0.07	10.00	0.00	11.91	0.24	9.43	0.07	9.95	0.07	9.95	0.02	10.55
		8.000			0.05	8.18	0.00	9.81	0.06	8.03	0.06	8.04	0.02	8.55	0.01	8.57

Table 5: average number of backorders per policy; ; l = failure rate r = utilization rate b = backordering costs per time unit, s = stock level, 1 = type 1 and 2 = type 2.

Simulation parameters (μ = 1)					average number of backorders per type for every policy																	
ρ	$\lambda 1/\lambda 2$	b1/b2	s1	s2	Random		B		S		lab		diff		Blab		EBT		Myopic		SB	
					type 1	type 2	type 1	type 2	type 1	type 2	type 1	type 2	type 1	type 2	type 1	type 2	type 1	type 2	type 1	type 2	type 1	type 2
0.80	1.00	0.500	4	4	0.49	0.43	1.18	0.03	0.37	0.37	0.49	0.44	0.37	0.37	1.18	0.03	0.33	0.40	0.33	0.40	0.43	0.32
		0.250			0.46	0.48	1.17	0.03	0.37	0.37	0.46	0.47	0.37	0.37	1.17	0.03	0.33	0.41	0.33	0.41	0.52	0.27
		0.125			0.41	0.44	1.08	0.03	0.33	0.34	0.41	0.45	0.33	0.34	1.08	0.03	0.30	0.37	0.30	0.37	0.50	0.23
	0.25	0.500	2	6	0.04	0.75	0.89	0.13	0.18	0.49	0.89	0.13	0.41	0.28	0.89	0.13	0.24	0.40	0.22	0.43	0.20	0.45
		0.250			0.04	0.81	0.96	0.13	0.21	0.52	0.96	0.13	0.44	0.31	0.96	0.13	0.28	0.42	0.26	0.45	0.28	0.42
		0.125			0.05	0.84	0.99	0.15	0.19	0.54	0.99	0.15	0.46	0.30	0.99	0.15	0.25	0.46	0.23	0.48	0.29	0.44
		2.000			0.04	0.90	0.01	0.98	0.21	0.58	1.03	0.16	0.48	0.34	1.03	0.16	0.29	0.48	0.26	0.51	0.16	0.64
		4.000			0.05	0.76	0.01	0.84	0.17	0.49	0.94	0.13	0.43	0.28	0.01	0.84	0.23	0.42	0.21	0.45	0.11	0.57
		8.000			0.04	0.78	0.01	0.85	0.17	0.51	0.93	0.14	0.42	0.29	0.01	0.85	0.23	0.42	0.21	0.45	0.10	0.60
0.70	1.00	0.500	4	4	0.12	0.12	0.31	0.02	0.08	0.09	0.12	0.12	0.08	0.09	0.31	0.02	0.07	0.10	0.07	0.10	0.10	0.08
		0.250			0.12	0.13	0.31	0.02	0.08	0.08	0.12	0.12	0.08	0.08	0.31	0.02	0.07	0.10	0.07	0.10	0.12	0.06
		0.125			0.11	0.12	0.30	0.01	0.08	0.08	0.11	0.13	0.08	0.08	0.30	0.01	0.07	0.09	0.07	0.09	0.13	0.06
	0.25	0.500	2	6	0.02	0.20	0.26	0.05	0.03	0.14	0.26	0.05	0.11	0.08	0.26	0.05	0.05	0.12	0.04	0.13	0.04	0.13
		0.250			0.02	0.19	0.27	0.05	0.03	0.14	0.27	0.05	0.11	0.07	0.27	0.05	0.05	0.11	0.04	0.12	0.05	0.11
		0.125			0.03	0.19	0.28	0.04	0.03	0.14	0.28	0.04	0.12	0.07	0.28	0.04	0.05	0.11	0.04	0.12	0.07	0.11
		2.000			0.02	0.19	0.01	0.22	0.03	0.14	0.27	0.04	0.12	0.07	0.27	0.04	0.05	0.11	0.04	0.12	0.03	0.15
		4.000			0.03	0.19	0.01	0.22	0.03	0.14	0.27	0.05	0.11	0.07	0.01	0.22	0.05	0.12	0.03	0.13	0.02	0.15
		8.000			0.02	0.20	0.01	0.22	0.03	0.14	0.27	0.05	0.12	0.07	0.01	0.22	0.05	0.12	0.04	0.13	0.03	0.16

Table 5: average number of backorders per policy; ; λ = failure rate ρ = utilization rate b = backordering costs per time unit, s = stock level, 1 = type 1 and 2 = type 2.

Simulation parameters (μ = 1)					average number of backorders per type for every policy											
ρ	$\lambda 1/\lambda 2$	**b1/b2**	**s1**	**s2**	**EBT+B**		**Myopic+B**		**Myopic+B approx**		**pres2**		**Pres4**		**pres6**	
					type 1	**type 2**	**type 1**	**type 2**	**type 1**	**type 2**	**type 1**	**type 2**	**type 1**	**type 2**	**type 1**	**type 2**
0.99	1.00	0.500	4	4	0.68	0.10	0.86	0.07	0.57	0.16	0.66	0.11	0.68	0.10	0.81	0.06
		0.250			0.74	0.09	1.13	0.03	0.67	0.11	0.69	0.10	0.84	0.06	0.99	0.04
		0.125			0.69	0.09	1.08	0.03	0.62	0.10	0.74	0.07	0.90	0.04	1.08	0.03
	0.25	0.500	2	6	0.41	0.25	0.01	0.82	0.29	0.34	0.36	0.28	0.44	0.23	0.64	0.16
		0.250			0.46	0.26	0.02	0.81	0.39	0.31	0.47	0.26	0.67	0.18	0.83	0.15
		0.125			0.44	0.29	0.12	0.70	0.45	0.29	0.57	0.23	0.71	0.19	0.99	0.15
		2.000			0.02	0.81	0.01	0.98	0.05	0.71	0.05	0.71	0.05	0.72	0.03	0.81
		4.000			0.02	0.71	0.01	0.84	0.04	0.62	0.02	0.70	0.02	0.70	0.02	0.73
		8.000			0.02	0.75	0.01	0.85	0.02	0.71	0.02	0.71	0.01	0.85	0.01	0.85
0.95	1.00	0.500	4	6	0.15	0.04	0.21	0.03	0.12	0.05	0.14	0.04	0.15	0.04	0.18	0.03
		0.250			0.17	0.03	0.29	0.02	0.13	0.04	0.14	0.04	0.18	0.03	0.24	0.02
		0.125			0.18	0.03	0.30	0.01	0.14	0.04	0.18	0.02	0.23	0.02	0.30	0.01
	0.25	0.500	4	6	0.10	0.08	0.01	0.23	0.05	0.11	0.07	0.10	0.09	0.08	0.12	0.07
		0.250			0.10	0.07	0.02	0.17	0.07	0.09	0.09	0.08	0.12	0.06	0.17	0.05
		0.125			0.11	0.07	0.06	0.12	0.07	0.09	0.10	0.07	0.18	0.05	0.23	0.05
		2.000			0.01	0.17	0.01	0.22	0.02	0.14	0.02	0.14	0.02	0.14	0.01	0.16
		4.000			0.01	0.18	0.01	0.22	0.02	0.15	0.02	0.15	0.01	0.17	0.01	0.17
		8.000			0.01	0.19	0.01	0.22	0.01	0.18	0.01	0.18	0.01	0.18	0.01	0.22

Table 5: average number of backorders per policy; ; l = failure rate r = utilization rate b = backordering costs per time unit, s = stock level, 1 = type 1 and 2 = type 2.