

Title Optimizing the marketing budget allocation problem using Lagrangean based techniques

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Abstract

In this Master Thesis project we present a Lagrangean relaxation optimization approach to the marketing budget allocation problem. This approach is aiming to allocate the marketing budget among the various channels such that the effect to the consumers will be maximized. Taking under consideration the cost parameters, the problem is formulated as a Mixed Integer Non-Linear Programming problem. The Upper Bound solution values are obtained by relaxing the problem using the traditional Lagrangean relaxation technique as well as a series of Lagrangean. The Lower Bound for the problem is obtained through a heuristic algorithm which is based on the Lagrangean method solutions and its values are compared to the solution values produced by a random algorithm.

The optimization approach has been implemented for 34 marketing channels. The Upper Bound values derived by the Lagrangean heuristic method has shown to be improved compared to the traditional method, approaching the corresponding Lower Bound values. Although the Upper Bound values were lowering their minimum values while the constraints of the problem were being modified, they maintained a considerable distance from the Lower Bound values. The optimal solution though has not been met, despite the improved results,. To obtain the optimal solution value of the marketing budget allocation problem, further research recommendations are presented in the end of the paper.

Keywords

Lagrangean relaxation, optimization, marketing budget allocation problem, MINLP problem

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1. Introduction

“Marketing is the management process that identifies, anticipates and satisfies customers’ requirements profitably” (CharteredInstituteMarketing). Based on this definition, companies and corporations annually invest a respectable amount of their budget in marketing campaigns aiming to meet customers’ needs and subsequently increase their sales. In order to achieve this goal and to reach the target market, they are using different ways called *marketing channels*. In fact, a marketing channel is defined as *“the set of practises or activities necessary to transfer the ownership of goods, and to move goods, from the point of production to the point of consumption and, as such, consists of all the institutions and all the marketing activities in the marketing process”* (AmericanMarketingAssociation).

Traditionally the number of channels was restricted only to outdoors advertising, newspaper, magazine, television and radio while marketing activities in total were not considered to be as applications of a broad scientific field but more as an empirical procedure. As such, they were not planned and organised as they should and the needed marketing expenditures were only considered as an indispensable cost of simply doing business. Nevertheless, the technological evolution that occurred during the last two decades added many more channel options (Online advertising, Internet search and advertising in social networks) to those already available, creating a multichannel environment and contributing to audience fragmentation. Additionally, the marketing expenses are nowadays treated as a well-planned investment with expected returns like every investment. The most effective allocation of the marketing budget across all the available channels therefore becomes an important issue for competitive companies. It is simultaneously a highly complex problem for their marketing executives who have to face a great challenge.

Pointlogic is a leading provider of integrated marketing communications as well as marketing Return On Investment (ROI) solutions while it is active in several different industries, including media, marketing, human resource management and logistics. Pointlogic combines cutting-edge research methods and advanced mathematical and econometrical modelling techniques in flexible software tools, enabling its clients to make smart decisions. Specifically, they apply analytical thinking to approach the various problems and they use complex mathematics, econometrics and operational research background to offer to their clients a deeper insight into their choices. In the end, they mostly deliver their solution implemented in an easily understood and concrete software platform helping their clients to improve their tactical and strategic business decisions. (Pointlogic, 2010)

More specifically, Pointlogic, acknowledging that analytics are a powerful tool to understand and optimize the marketing functions, has developed the Integrated Marketing Communications (IMC) software suite. IMC is a set of software applications, offering the know-how for effective channel planning, faster and improved communication planning and the appropriate combination of messages and media. One of the basic roles of the IMC suite is what has been described above as the effective allocation of the marketing budget in a multichannel environment. Indeed, by applying

complex calculations and analysis based on high level mathematics, it concludes to such combination of channels, products and markets that contributes to the optimal communication of the desired brand. Although the current version of the software is designed to produce an optimal channel allocation using a heuristic approach, the marketing channels field is improving so fast that it is expected that soon the obtained solutions would become inaccurate and the computational time will be highly increased. Hence the need of developing and applying a structured methodology, adaptive to alterations, is increasing. (Pointlogic, 2010)

1.1 Mixed Integer Non Linear Programming problems

Mixed Integer Nonlinear Programming (MINLP) problems refer to optimization problems including both continuous and integer variables as well as nonlinearities to the objective function and the constraints (or at least to one of them). This type of problems is used to applications where it is important to simultaneously optimize both its discrete and its continuous variables. This ability enabled their implementation in diverse scientific areas such as (Li, et al., 2006):

- Finance: capital budgeting
- Engineering: production planning, computer networks
- Management: portfolio selection
- Operations Research: reliability networks, resource allocation

These are usually large scale, highly combinatorial and nonlinear complex problems (R.Bussieck, et al., 2003). A typical formulation for the MINLP problem is as following:

$$\begin{array}{l}
 \text{P} \left\{ \begin{array}{ll}
 \text{Minimize} & f(x,y) \\
 \text{Subject to} & g(x,y) \leq 0 \\
 & x \in X \\
 & y \in Y, \text{integer}
 \end{array} \right. \quad (1)
 \end{array}$$

Where the function $f(x,y)$ is a nonlinear objective function, $g(x,y)$ is the constraint function that can be nonlinear (but it can be linear as well) and x and y are the decision variables where y is required to be integer. It is also observed that if the integer variables y are fixed, the problem P is transformed into a nonlinear programming (NLP) problem while if the functions f and g were linear, the problem P would have been a mixed integer linear programming (MILP) problem. Noticing that in the MINLP problem are embedded both NLP and MILP problems and taking into consideration that some subclasses of both of these problems are NP- complete then it is concluded that the MINLP problem is an NP-hard problem, thus difficult to solve (Nzengang, 2009).

The solution approaches developed for the MINLP problems are based on their embedded nature and they approach the global optimal solution by solving each problem separately (NLP and MILP), aiming to find the converging solution that will simultaneously satisfy both of them (thus the original

MINLP). As it is expected, there are numerous approaches due to the big variety of MINLP applications but they are mainly divided into two categories (Nzengang, 2009) (Li, et al., 2006):

- Exact

They usually require some conditions from the objective function and the constraints (i.e. convexity) and they terminate either when the global optimal solution has been obtained or when infeasibility has occurred. Some representative deterministic solution approaches are:

- Branch and Bound (Quesada, et al., 1992)
- Outer-approximation (Fletcher, et al., 1994)
- Genetic algorithms (Yokota, et al., 1996)
- Lagrange relaxation (Beasley, 1993), (Kurt, et al., 1986)

- Heuristic

Heuristic methods are preferred when an exact approach is not possible to be applied or its computational time is relatively high while they only produce suboptimal solutions. These methods include routing procedures, random searches, gradient function and penalty function methods.

1.2 Problem description

In principle, what would have been the desired result for every marketing planner is to be able to influence all the people exposed to his marketing campaign. In order to achieve this goal and reach the complete target audience, given its fragmentation, they are called to divert their campaign to a variety of channels aiming to maximize the result while minimizing the expenditures as much as possible. Additionally, they have to derive criteria which would first group and value the consumers and then would lead them to the appropriate, out of a wide range, available marketing channel so as to target them effectively. Maximizing the total effect, though, is not a trivial task and this is becoming more obvious when taking into consideration the following:

- Calculation of the overall effect

The overall effect is hard to be defined and hence challenging to be computed. Except from the principal difficulty of quantifying marketing measures, there are also noticed interrelations among the variety of the channels called overlapping effects. Specifically, the overlapping effect could be described as the effect caused by a share of the audience that is attending multiple channels. Indeed, when applying a marketing campaign in two channels, for example radio and television, the resulting effect cannot be the sum of the individual effects because there is a common audience that is both watching television and listening to the radio. In other words, it is not possible to affect this part of the audience twice because it has already been affected by one of the two channels. Hence, the effect of every channel is not an independent variable.

- Budget limitation

As it is expected, the amount of money available to be invested in a marketing campaign is limited, thus it is not possible to use every channel up to its maximum effect which would be the ideal condition. Especially during times of economic recession when the earnings are suppressed, the marketing budget is faced with such scepticism that it is from the first expenditures to be reduced. However, the limited budget is the greatest challenge modern marketers have to face and is turning the problem of maximizing the resulting effect into a great challenge.

- Expenditures parameters

The total cost of the marketing campaign consists of the summation of the setup cost and the operational cost times the amount of channel usage over all channels. What it would be useful to be mentioned here is, that the values of both setup and operational cost highly differentiate among the various channels, markets, products or brands. For this reason, it is not possible to retrieve cost data from a data base and use them for the scope of the thesis project. Instead, we are generating multiple random cost values using as a mean the average value of the data costs per channel. As a result, it is expected that since the cost values would vary depending on the case that the solution approach is implemented, there would also be a variation to the result of the overall problem as well as to the methodology followed.

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- Market restrictions

Apart from the budget constraint, there are series of other constraints that should also be taken into consideration when developing and applying a realistic solution approach since they add complexity to the problem. For example, there is a minimum amount of money (apart from the setup cost) that has to be invested in each channel in order to use it. Also, there are groups of channels that can only be combined with each other and any other combination of channels from different bunches is not applicable.

As we will see, the optimal budget allocation problem could be described as a Nonlinear Programming (NLP) problem with non-linear objective function and linear constraints which in general is hard to solve. As we will show later on the project, the problem becomes more complex when there are also included binary variables to introduce the setup costs, turning it into a Mixed Integer Nonlinear Programming (MINLP) problem.

1.3 Motivation

The problem described above is not only of exclusive scientific relevance but it is also of high interest for media planning applications as well. Although from a rather complex since it involves the modification and implementation of already developed optimization methods as well as the development of new optimization heuristics, the contribution to the media planning of an advertising campaign is important. The use of the obtained solution is expected to help the media planners:

- to reach the various target audiences
- to decide how to divert the available budget over the multiple products or brands

- to decide how to divert the available budget across the available media and to define the time periods of the planning horizon

Additionally, it enables them to influence even more customers keeping their budget limited.

From practical point of view, for example in the case of Pointlogic, it will enable it to reform the algorithm already applied and to implement a more complete and concrete methodology. In this way, the IMC suite will be able to produce a high quality solution to the budget allocation problems in a reasonable time frame while its algorithm will be able to be further developed in case of additional channels, restrictions and to reform it according to its needs.

Currently, Pointlogic has developed a solution method which is solving to optimality the Nonlinear Programming problem without taking into consideration the setup cost parameters. Therefore, the solutions obtained do not represent real life conditions while they are actually neglecting a high cost parameter which is influencing the final results. So, in order to include setup costs into the problem formulation, it would be necessary to introduce binary variables transforming our problem to a Mixed Integer Nonlinear Programming (MINLP).

In more details, the suggested solution approach would:

- accurately describe the actual problem, taking into consideration:
 - the actual formulation of the objective function as estimated by the historic data
 - the interrelations between the independent marketing channels
 - vital parameters such as setup and operational cost
- be based on a structured mathematical methodology, in order to be:
 - understandable and easily applicable
 - able to be maintained and further developed in case additions are needed
- obtain a good quality solution within a limited timeframe

As a result, its implementation to the IMC suite would result to a combination of channels which would derive a realistic maximum total effect using the available marketing budget.

1.4 Objective

The main objective of this master thesis project is:

to develop a methodology using advanced mathematical programming techniques which would allocate the budget of a marketing campaign across all the available channels such that the resulting effect to be maximized.

1.5 Structure

The Master Thesis project is organised as follows. Chapter 2 includes the solutions methods found during the literature research. Although the problem in its full complexity has not been solved in the literature, there are some notable methodologies facing the problems of media planning and budget

allocation. Chapter 3 presents the theoretical knowledge essential to comprehend the problem's environment and the necessity of obtaining the optimal solution. For this reason, the main terminology used during the project will be explained, the problem's modelling process will be analysed step by step and finally the formulation of the problem will be developed. Afterwards, in Chapter 4, we will analytically present the solution procedure and the methodologies on which it is based. In order to thoroughly explain the procedure and to gain a better insight, the necessary theory for each of the methodologies will be outlined. The topic of Chapter 5 is the implementation of the solution approach. As such, it will contain the strategy behind initializing and updating the parameters in the most appropriate way. Chapter 6, presents the results obtained by the implementation of the various policies together with the necessary argumentation and justification while Chapter 7 includes the conclusions and recommendations for further research.

2. Literature Review

The problem addressed in this master thesis project has not been solved in the literature yet. Instead, there are numerous solution approaches regarding the optimal allocation of marketing resources as well as the optimal planning of the marketing mix as separated problems. Due to the lack of literature references and facing the initial problem as the combination of resource allocation and planning the marketing mix, the literature research has been concentrated on methods and techniques for these two problems described above and transforming them in order to fit to this particular occasion. Over the past decades, these problems have received a lot of attention from scientists with diverse background, which resulted to an impressive amount of literature references. Due to the diverse nature of marketing science in general and the different ways companies implement it to their strategic and operational functions, scientists approached those problems from different perspectives. This resulted to the development of many different solution methods which are based on a variety of techniques (both qualitative and quantitative).

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2.1 Empirical methods

The simplest approach for optimizing the marketing mix problem does not use any mathematical model or economic analysis but it is mainly based on an empirical methodology (Freimer, et al., 1967). In fact, the methodology suggests to scale the factors that are important when deciding the optimal marketing mix (Richman, 1972) and to assign a weight to each of them to describe their importance. In the end, the final decision for the optimal marketing mix would be made based on that combination of channels which will gather the highest score for the factors that are most important for every occasion. As it is becoming understood, this methodology has a number of serious weaknesses from identifying which factors are the most important and weighting them accordingly to the range of applications that it could be used. Finally, this type of empirical methods is designed and applied only to specific problems, thus it is not feasible to construct a general empirical methodology which could be used in many different problems.

2.2 Multiple regression

In more details, (Bhattacharya, 2008) is facing the problem of allocating the limited resources across the marketing channels such that the objective function is maximized within the limitation of the media mix modelling constraints. In fact, (Bhattacharya, 2008) has developed an alternative solution approach which was deployed with the use of a SAS macro and it is used to produce a close-to-optimal global solution for the channel allocation problem. The proposed methodology is capable of incorporating nonlinear and non-convex structures making the whole suggestion more realistic, offering a representative solution to the real problem. Specifically, it is introduced a nonlinear objective function (media response function) representing the total sales as a function of the different media used in the media mix. Additionally, it is accepted that there exist threshold effects according to which the marketing efforts are not effective until they exceed a certain minimum level

(Corkindale, et al., 1978). In order to include this property into the objective function, it has to be S-shaped.

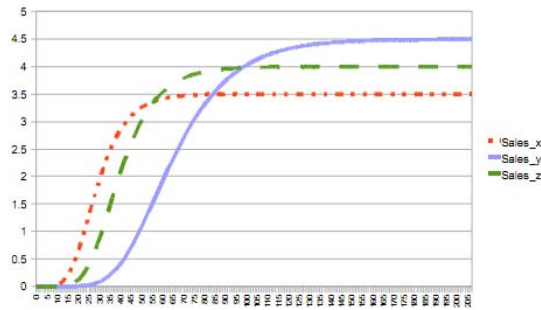


Figure 1: Response curves

Most of the marketing managers and practitioners agree that such advertising thresholds exist and that there are levels of advertising below which there is no sales response. Although the existence of threshold effects has been tested for various demand curves and it has concluded that in some occasions there is a minimum threshold level on the advertising share, it has not been proven to be consistent with the most of the markets in the majority of occasions (Bemmaor, 1984).

Apart from the S-shape of the response curve, the marketing carry-over effect, as a result of the dynamic nature of marketing, is also implemented in the objective function. The term carry-over effect (also known as lagged effects) is used to describe the effects of the marketing campaigns which are active even when the campaign is over and part of them can also be perceived in future periods. In order to implement this effect to the objective function, it is assumed that only a fraction of the customers, who became aware of the marketing campaign, will express directly their interest in the product or service while others will do so during the following weeks. Although the carry over effect is time dependent and the problem as defined here does not include the time dimension, these effects have been modelled for the time frame that the model is implemented and they are included into the objective function.

The implementation of the two characteristics mentioned above to the objective function, explains sufficiently the reason why the nonlinear formulation of the objective function represents better the real life conditions. In fact, the majority of the studies have focused on effect curves that are linear and have assumed that the demand functions are such that can be easily solved to optimality using simple algebraic methods. However, there are some approaches in literature that use nonlinear effect functions assuming though that it is convex or it can be transformed into a convex one.

(Bhattacharya, 2008) is also assuming that there are no interaction effects between the media variables, thus the total resulting effect equals to the summation of the effect of the individual channels. But as it is described in the introduction, in reality there are such interaction effects and by modelling them, the problem is becoming more complex (since this assumption is increasing the nonlinearities and non-convexities of the problem). As a result, a deeper and more advanced analysis to conclude to the optimal solution is needed. For the sake of simplification and in order to use an

algebraic approach, this interrelation between the different marketing channels is ignored. However, it has to be mentioned that in a more theoretical approach (Bhattacharya, 2008), he is supporting that there are actually marketing efforts that take place at the same time that result to co-linearities and strongly relate the independent variables to each other.

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In the end, it is concluded that the final model would attempt to maximize the total Sales by obtaining the optimal marketing mix, the optimal values for the amount of GRP invested to each of the participating channels (Chan1, Chan2, Chan3). As an objective function has been applied a nonlinear Gompertz function with the following formulation:

$$\text{Sales}(\text{Chan1}, \text{Chan2}, \text{Chan3}) = A \cdot b \cdot a^{f(\text{Chan1})} + C \cdot d \cdot c^{g(\text{Chan2})} + E \cdot f \cdot h(\text{Chan3}) \quad (2)$$

Where: A, b, C, d, E, f are coefficients parameters

f, g, h are the effect functions for each of the three channels participating into the channel mix

Chan1, Chan2, Chan3 are the decision variables, the amount of GRPs invested to each of the three channels

The optimal values for the coefficient parameters are determined through solving the nonlinear regression model above.

It has to be noted that although this solution was derived by the use of optimization techniques, it deviates significantly from the real solution and the real optimal combination of channels might be different from the one produced. The reason for this rests on the assumptions made during the formulation of the problem aiming to make it simpler and to easily obtain the optimal solution.

2.3 Geometric programming

Another solution approach to the marketing mix problem was introduced by (Balanchandran, et al., 1974). In their paper, they have attempted to deal with the full complexity of the problem instead of applying assumptions and simplifications which may reduce its difficulty and complexity but they reduce the accuracy of the results. In order to make it more understandable the whole methodology is also applied in a specific example. Based on this prospective, they have created an objective function describing the total sales which was derived by raw historical data while taking into consideration the interactions among a variety of measurements. As a result, a nonlinear objective function it is constructed, which is optimized using geometric programming, aiming to maximize the sales using a certain combination of media channels.

First, the methodology is attempting to construct the sales function. Using the raw historical sales data and with respect to the interrelations developed, it is developed a two-state regression consisting of four steps:

- The sales data are logged and regressed with the logged predictor variables under the limitations set by the combination of variables. This step results in the elasticity coefficients of the function describing the developed influences
- The results that yielded useless conclusions were then cleaned to avoid false interpretation of the formulation
- The useful parameters that are left after the screening are combined into a linear model in order to validate the independence among the participating variables
- Apply multiple regression to figure out which parameters are strongly correlated.

After the screening of the sales data, keeping those that are considered to be important, identifying their correlations and getting which of them fit together, they construct the final formulation. This formulation represents the total sales as a function of the amount of money invested in each of the channels participating to the marketing mix. Hence, the objective function of the problem will be to maximize the total sales by obtaining the optimal amount of money invested to each of the channel into the mix.

The application of the Geometrical Programming as a solution method for obtaining a global optimal solution demands the formulation of the objective function to be of an acceptable format. Specifically, the Sales function which was estimated in the four-step procedure described above, consists of regression parameters which, depending on the objective of the problem (maximization or minimization), have to fulfil certain requirements. If the requirements are satisfied then it is guaranteed that the Geometric Programming approach would provide a global optimal solution.

However, it is possible that the formulation of the problem (sales function and the constraints) is incompatible with Geometrical Programming. Due to the violated independence assumption, the multiple regression described in the fourth step cannot take place. Instead, the sales function is possible to be estimated all in one step by estimating the elasticities and the coefficients of each term all at one stage. Thus, a polynomial sales function would be estimated. In case that the regression coefficients are both positive and negative (sigmoidal terms), then the Geometric Programming requirements that would lead to a global optimal solution value are not satisfied. For this category of problems, there is developed an alternative methodology (Avriel, et al., 1970). If this is not the case, the Geometric Programming approach obtaining a global optimal solution could be applied

2.4 Lancaster model

(Naik, et al., 2005) adopted a different approach in their article "Planning Marketing Mix Strategies in the Presence of Interaction Effects" where they recognized the existence of interaction effects among the different marketing activities and between competitive brands. They developed a dual methodology which allows the optimal planning by looking forward into sales forecasts and decisions need to be made and backwards to compare their own strategy with others. Indeed, two models are introduced: the former is calculating the optimal combination of marketing mix and the latter estimates the resulting effect. In other words, this methodology concludes to the optimal amount of

marketing budget and its allocation decisions while it estimates the efficiency and the interaction of the advertising and the promoting for each of the five brands of their example as well.

In order to illustrate this methodology, it is used the Lancaster model (J.Lancaster, 1966) extended by introducing multiple brands, multiple marketing activities and the interactions between them. Thus, the extension of the Lancaster problem formulation results to a mathematical problem which cannot be solved by the already developed algorithms. So, a two-point marketing mix algorithm was constructed which includes the nonlinearities and the interrelations among the variables.

The formulation of the problem is rather complicated and its detailed presentation does not offer much to the reader, however it is worth mentioning that it applies a dual methodology optimizing advertising and promoting effects by incorporating strategic foresight (looking forward) and reasoning backwards. This problem is also called the two-point boundary value problem (TPBVP). The solution algorithm is not only restricted to the extended Lancaster model but it be applied to every TPBVP as well producing the optimal marketing strategy (in terms of budget and channel allocation).

2.5 MINLP optimization methods

In addition to the solution methodologies described above, it would be efficient to search for a solution method among the variety of approaches developed for MINLP problems. As mentioned in Chapter 1, there are two main categories of solution approaches made for MINLP problems and it would not be applicable to present all of them. Adopting computational efficiency as a key point for the development of the algorithm, (Beasley, 1992), we will focus on Lagrangean relaxation techniques.

➤ Lagrangean Relaxation

The Lagrangean relaxation approach due to its plenty alterations (heuristics) and its low computational expense of solving problems, has become an indispensable technique for obtaining bounds for use in algorithms designed from solving combinatorial optimization problem (Beasley, 1992). As a result, many scientists have developed their research on developing and optimizing this procedure.

Briefly, the Lagrangean relaxation (Beasley, 1992), (Wolsey, 1998) and (Nemhauser, et al., 1999) for a given problem:

$$P \left\{ \begin{array}{l} \text{Maximize } (c \cdot x) \\ \text{Subject to:} \\ D \cdot x \leq d \\ x \in X \end{array} \right. \quad (3)$$

with respect to $Dx \leq d$ (n complicating constraints), is defined by introducing a (Lagrangean) parameter:

$$W = (W_1, W_2, \dots, W_n) \geq 0$$

which is attached to this constraint set and it is introducing it to the objective function.

By this way it is defined a new problem formulation:

$$\text{RP} \left\{ \begin{array}{l} \text{Maximize } c \cdot x + u \cdot (d - D \cdot x) \\ \text{Subject to:} \end{array} \right. \quad (4)$$

known as the relaxation of the original problem P.

From this point and on, the solution approach is focused on obtaining such values for the parameter u such that the overall result would be as minimum as possible, creating an Upper Bound for the optimal solution. Indeed, the Lagrangean relaxation succeeds by introducing some of the constraints in the problem's objective function, to widen the solution space and to conclude to solution values that would become the bound for the optimal solution. Then by updating the Lagrangean parameter, it will be attempted to lower this bound until it meets the optimal solution for the original problem.

There have been developed two basic approaches for updating the Lagrangean parameters (Beasley, 1992):

a) Subgradient optimisation

It is an iterative procedure that starting from some initial values for the parameters, it is generating further Lagrangean multipliers in a systematic fashion. (Wolsey, 1998)

b) Multiplier adjustment

It a heuristic methodology which starting from some initial values for the parameters, it is updating them using an iterative procedure until the best value is achieved.

Concluding, it has to be noted that the efficiency, the accuracy and the computational time the Lagrangean approach will need to optimize a certain problem depends on the way the Lagrangean multipliers are going to be updated.

Summarizing:

- the empirical method although it is very simple to implement it in a specific problem, its results are expected to be poor. Also, it is not able to construct a solution method which could be applied in general problem, thus it is not of our preference.
- the geometric programming and the Lancaster methods, although based on different principles and different development methods, they both have a complex approach and difficult mathematical formulations. In addition, in order to apply them in specific problems, there are series of criteria that need to be met and as such they are not easily applied to general problems.
- the multiple regression approach would fit the best to our problem. In fact, it is describing the same marketing problem as the one we are studying in the current thesis project and it is

making a similar approach to ours. However, the simplifications and the assumptions made in order to make the modelling and the calculations simpler, result to lack of accuracy and precision of the solution. Hence, the overall result does not approach the optimal solution of the original problem as described in Chapter 1.

- the Lagrangean relaxation method although it has not been widely applied in marketing mix problems, it offers the prospective of a relatively simple approach without complex formulations. The main difficulty is to derive a way of updating the Lagrangean parameters in such a way that it would be representative of real life conditions. Hence, since the computations taking place are relative simple, the computational time is restricted and the computational efficiency increased. Additionally, as a MINLP problem optimization method, it can be easily applied in a variety of problems without needed any alterations or adjustments.

3. Theoretical Background

In this chapter, we will demonstrate the theoretical knowledge necessary to comprehend the modelling of the problem. So, there is going to be a detailed description of the problem's environment, defining the decision variables, the parameters of the problem, the objective function as well as its constraints. Additionally, there would be presented the procedure until the final mathematical formulation of the problem is obtained

3.1 Definitions

In the beginning, it would be necessary to introduce the brief definitions of some marketing terms that will be extensively used in the project:

- As *marketing communication channels* are defined the various sources used by marketers to send marketing messages to potential consumers. These might be personal, involving two or more persons communicating directly with each other, such as a customer/salesperson relationship, or impersonal like billboard, outdoor advertising or any other form of mass communication where there is no personal contact. (CharteredInstituteMarketing)
- *Respondent* is the individual that provides information to be collected during the research process (poll, study etc). (CharteredInstituteMarketing)
- The term *reach* is defined as the estimated percentage of the potential customer who has been reached through an advertising medium or a promotional campaign over a specific time period. (MarketingAssociationAustralia)
- *Frequency* is average number of times a commercial or advertisement has been viewed per person (or per household) during a specific time period. (MarketingAssociationAustralia)

An important property of the marketing channels is how they will generate contacts with the audience and which part of the audience are they able to reach. Some channels can only reach a small part of the audience (like instore advertising) while other channels can reach almost the entire population (television). So, the reach of every channel is highly correlated to the frequency this reach is achieved.

- *Gross Rating Point (GRP)* is the term used to measure the size of an audience reached by a specific marketing channel. It is described as the product of frequency and reach:

$$GRP = freq \cdot reach \tag{5}$$

and expresses the “gross” duplicate percentage of audience that will be reached by a proposed plan. In other words, GRP represents the average percentage of the audience reached and as such a 100 GRPs equal to reaching the complete audience on average exactly once. Alternatively, it could be al

claimed that 100 GRPs equal to reaching 50% of the audience on average two times. Also, the budget is directly related to the GRP because the higher the desired GRP is, the more budget will be needed and because of that, we are usually referred to GRP using the verbs “spent” or “invested”. (AmericanMarketingAssociation)

3.2 Problem modelling

The optimal allocation of the budget over the marketing channels implies that the available amount of money (in terms of GRPs) should be invested in so many channels and diverted among these channels in such way that the total effect of the campaign would be maximized. The effect is initially calculated per channel and respondent and it is used to describe the effective reach. The effective reach represents the part of the targeted audience which has been exposed to the message and it has been positively affected towards it. In case of a product or a service, this means that it has either raised their awareness towards it, or affected their behaviour or convinced them to purchase it.

Mathematically, we can denote the effect of channel c for respondent r as follows:

$$f_{c,r}(x_c) \tag{6}$$

- Where x_c : GRP per channel c
- f : the effect per channel and respondent
- r : the index of respondents
- c : the index of channels

The total effect over all respondents and channels is a percentage, thus it is taking values from zero to one and it is represented by a different curve for each channel. The shape of the effect function should be such that would also take into account diminishing returns since a constantly increasing number of GRPs in a specific channel does not lead to a continuous increase of the effect. Instead, there is a limit of GRPs for each channel where the audience is becoming “satisfied” and from that point and on, it is ineffective to add more GRPs to that channel. So, it is expected that the effect curve for different channels would have the following shape:

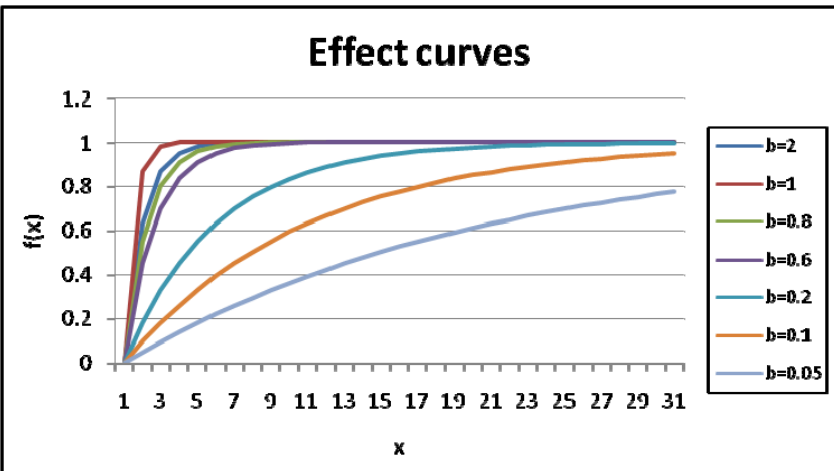


Figure 2: Effect curves

Finally, in order to conclude to an overall effect value there is the need of an aggregation procedure for the obtained effect value over all the multiple channels and the number of respondents. L

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➤ **Aggregation over all channels**

In case of a marketing campaign which is using multiple channels and the effect of each channel is independent of the effects of the other channels, the total effect for every respondent would simply be the sum of all separate effects. In general, channels are sharing the same audience though, resulting to an overlap in their effects and making it impossible to simply sum them up. For example, in a campaign where we use two different channels, the resulting effect cannot be the sum of the individual effects because there is a common audience that attends both media. So, it is not possible to affect this part of the audience twice because it has already been affected by one of the two channels. The formulation that approximates this behaviour would be:

$$f(x)_r = 1 - \prod_C (1 - f_{r,c}(x_r)) \tag{7}$$

Where f_r : the effect per respondent
 C : the total number of channels

➤ **Aggregation over all respondents**

To conclude to a final effect for all the respondents, it is calculated their weighted sum:

$$f(x)_{total} = \frac{\sum_R (f(x)_r \cdot w_r)}{\sum_R w_r} \tag{8}$$

Where w_r : the weight parameter for every respondent

R: the total number of all respondents

3.3 Problem formulation

In the beginning, let us introduce the following notation:

Decision variables:

- x_c : number of GRPs invested in a marketing channel (c)
- y_c : indicator variable showing when a channel (c) is operating or not

Parameters:

- sc_c : setup cost
- oc_c : operational cost
- B: budget
- LB_c : artificial Lower Bound of x_c
- UB_c : artificial Upper Bound of x_c
- R: set of respondents
- r: index of respondents
- C: set of channels
- c: index of channels

Next to that, the formulation of the problem consists of the objective function

$$\max \frac{\sum_R [1 - \prod_C (1 - f(x_{r,c}))] \cdot w_r}{\sum_R w_r} \tag{9}$$

and the constraints:

Budget Constraint: The amount of money spent in order to activate some specific channels (setup cost) and invest some GRPs in them (operational cost) should not exceed the available budget.

$$\sum_C (sc_c \cdot y_c + oc_c \cdot x_c) \leq B \tag{10}$$

Feasibility constraint:

$$x_c \geq 0 \quad \forall c \in C \tag{11}$$

Binary constraint:

$$y_c \in \{0,1\} \quad \forall c \in C \tag{12}$$

Originally, the only constraint limiting the values of x variables is the available budget. However, like in the majority of mixed integer problems, it must be ensured that the values of x- and y- variables are aligned. More specifically, when x variables are taking positive values the corresponding y

variables have to take value one and the opposite. For this reason, we introduced two additional constraints which also include some “artificial” bounds for x variables. Indeed:

GRP constraints: Although theoretically there is not a rule limiting the number of GRPs spent in every channel, in practice there are applied some “artificial” lower and an Upper Bounds. Indeed, in order to achieve an effective use of a specific channel, there is a minimum amount of GRPs needed to be spent. This is reasonable, since from consumers’ effect point of view, spending only one or two GRPs in a channel will most probably have negligible results and increased cost.

$$x_c \geq LB_c y_c \quad \forall c \in C \quad (13)$$

The same constraint also holds for the Upper Bound of the GRPs. In figure 2, it is observed that the effect curve has a concave function approaching 1 when the invested GRPs go to infinity. In fact, from a number of GRPs and on, every additional GRP that is invested has a minor influence to the total effect. So, it is preferred not to invest GRPs more that those resulting to an effect of 95% (Chapter 6).

$$x_c \leq UB_c y_c \quad \forall c \in C \quad (14)$$

Briefly, the objective function will be estimated based on historic effect data using the least square estimates method and inserting the minimum possible error. The detailed procedure followed to conclude to the formulation of the objective function is presented in the Appendix (A2). Finally, it is formulated as:

$$f_{r,c}(x_c) = L_c \cdot (1 - e^{-b_{r,c} x_c}) \quad (15)$$

Where

L_c : the potential, the maximum effect channel c is able to achieve

$b_{r,c}$: the speed of the channel

Overview

$$\text{Max} \quad \frac{\sum_c [1 - \prod_c (1 - L_c (1 - e^{-b_{r,c} x_c}))]}{\sum_c w_c} \quad (16)$$

Subject to

$$\sum_c (sc1_c \cdot y_c + oc_c \cdot x_c) \leq B$$

$$x_c \geq LB_c y_c \quad \forall c \in C$$

$$\begin{array}{ll}
 x_c \leq UB_c & \forall c \in C \\
 y_c \in \{0,1\} & \forall c \in C \\
 x_c \geq 0 & \forall c \in C
 \end{array}$$

Summarizing, in this chapter:

- we thoroughly explained the procedure that was followed to derive the objective function of the problem
- we presented in detail all the constraints applied to the problem together with their formulation
- we demonstrated the final formulation of the problem as it is going to be used in the following chapters

4. Solution approach

The problem addressed in this Master Thesis Project is an NP-hard problem (Chapter 1) and as such there has not been developed a polynomially bounded algorithm which in a specific number of steps would be able to produce the optimal solution. Conceptually, a solution methodology that would lead to the optimal solution value could be based on the following dual approach:

- Obtain Upper Bounds which are values above of the optimal solution value
- Obtain Lower Bounds which are values lower than the optimal solution value

In other words, it is produced a solution space defined by the Upper and the Lower Bound inside which would be located the optimal solution value.

The challenge for the project is to define the bounds in such a way that the optimal solution will be limited to a certain value (accepting a margin inside which the optimal value is will be located). In such way, it could be argued that it is produced a high quality solution by reducing the Upper Bound and increasing the Lower Bound as much as possible bringing both of them closer to a specific value which would approach (and ideally meet) the optimal solution value. In the end, the computational success of the solution algorithm depends on the quality of the solution produced, thus of the margin inside which the optimal value is located.

In this chapter, we will present the solution approach based on the Lagrangean heuristic method to derive an Upper Bound for the solution values and on a heuristic algorithm to construct a Lower Bound. As it was mentioned in Chapter2, the Lagrangean method was chosen based on the criteria mentioned above, that indeed is expected to derive a solution which would be of a good quality during a limited computational time fulfilling the need for computational efficiency. Finally, there is going to be developed a random algorithm which would provide a solution to the problem based not on a mathematical technique but on an empirical-random way.

4.1 Upper Bound

A widely used technique which is available to derive Upper Bounds (in case of a maximization problem) is Lagrangean relaxation:

4.1.1 Lagrangean relaxation

With the use of the Lagrangean relaxation, it is attempted to create an easier to solve problem and to obtain an Upper Bound of the optimal solution of the original problem. In order to achieve this:

- there are chosen some set (or all) of the constraints in the problem for relaxation
- each of them is multiplied with a non-negative Lagrangean parameter (taking any non-negative value)
- they are inserted into the objective function
- the relaxed problem is solved to optimality

As a result, a relaxed problem is created which includes an objective function containing the relaxed constraints multiplied with the Lagrangean parameters and having as constraints only those that are not relaxed. By relaxing some (or all) of the constraints, the solution space of the relaxed problem is becoming wider than the original's, thus a feasible solution is easier to be obtained. However, a feasible solution for the relaxed problem does not necessarily mean that it is also feasible for the original problem. In fact, any solution obtained by the relaxed problem will always have a higher objective value than the optimal solution giving an Upper Bound on the optimal solution of the original problem. In particular, we are interested in finding these values for the Lagrangean parameters that would produce the minimum Upper Bound, thus the solution which would be as close as possible to the optimal solution value.

To apply Lagrangean relaxation to a maximization problem like the current one, all the constraints have to be of \geq form and then to introduce their Lagrangean parameters. So:

$$B - \sum_c (l_c \cdot y_c + u_c \cdot x_c) \geq 0 \quad \cdot \lambda \quad (17)$$

$$x_c - LB_c y_c \geq 0 \quad \cdot \mu_c \quad (18)$$

$$UB_c y_c - x_c \geq 0 \quad \cdot v_c \quad (19)$$

Where $v_c, \mu_c, \lambda_c \geq 0$

Inserting the relaxed constraints to the objective function:

$$\max \{ f(x)_{\text{total}} + \lambda \cdot \left(B - \sum_c (l_c \cdot y_c + u_c \cdot x_c) \right) + \mu_c \cdot (x_c - LB_c y_c) + v_c \cdot (UB_c y_c - x_c) \Rightarrow \quad (20)$$

$$\max f(x)_{\text{total}} + \sum_c (x_c - \lambda \cdot u_c - v_c) \cdot x_c + \sum_c (v_c \cdot UB_c - \lambda \cdot l_c - \mu_c \cdot LB_c) \cdot y_c + \lambda \cdot B \Rightarrow \quad (21)$$

$$\text{Max } g(x) + h(y) + \text{Const}$$

While the constraints are limited to:

$$x_c \geq 0$$

$$y_c \in \{0,1\}$$

In principle, the solution obtained by the relaxation is not always feasible to the original problem. In case that the solution obtained by the relaxed problem is feasible to the original problem, it is not necessarily the optimal solution for the original problem. Note that in order to conclude to the optimal solution value of the original problem though, it is essential the solution of the relaxed problem to be feasible for the original problem, thus to fulfil its constraints. The

procedure which is attempting to construct feasible solutions to the original problem based on the solution of the relaxed problem is called Lagrangean heuristic.

4.1.2 Lagrangean heuristic-multipliers' adjustment

The aim of the Lagrangean heuristic is to create an Upper Bound as low as possible which would approach and ideally meet the optimal solution value of the original problem. To achieve this, an iterative procedure for updating the Lagrangean parameters will be developed, based on the method of the multipliers' adjustment. By this way, the Upper value would be continually improving, approaching the optimal solution.

The original problem addressed in this master thesis project is a maximization problem, thus in order to apply Lagrangean heuristic, all the constraints have to be of the \geq form. Then the optimization algorithm should be as following:

- 1) Begin with the Lagrangean parameters λ , μ_i and v_i equal to zero. Also define a step size for each of the parameters (κ , ι and θ accordingly).
- 2) Then we solve the relaxed linear problem and we obtain a solution for x 's and y 's.
- 3) For every violated constrained, we increase the corresponding parameter by its step size.
- 4) For every constraint with positive slack, we decrease the corresponding parameter by its step size.
- 5) Then using the updated parameters, we return at step two and we repeat the procedure.
- 6) If for r iterations the relaxation value fluctuates between the values of a margin m , we reduce the step size of the parameters by half and we return to step two.
- 7) Finally, if the relaxation value remains steady below a margin m' for more than r' iterations then it is concluded that the obtained value is the optimal one.

The key is to choose both the initial values of the Lagrangean parameters and their step size such that the iteration procedure to conclude to those values that will lower the Upper Bound as much as possible (Beasley, 1992).

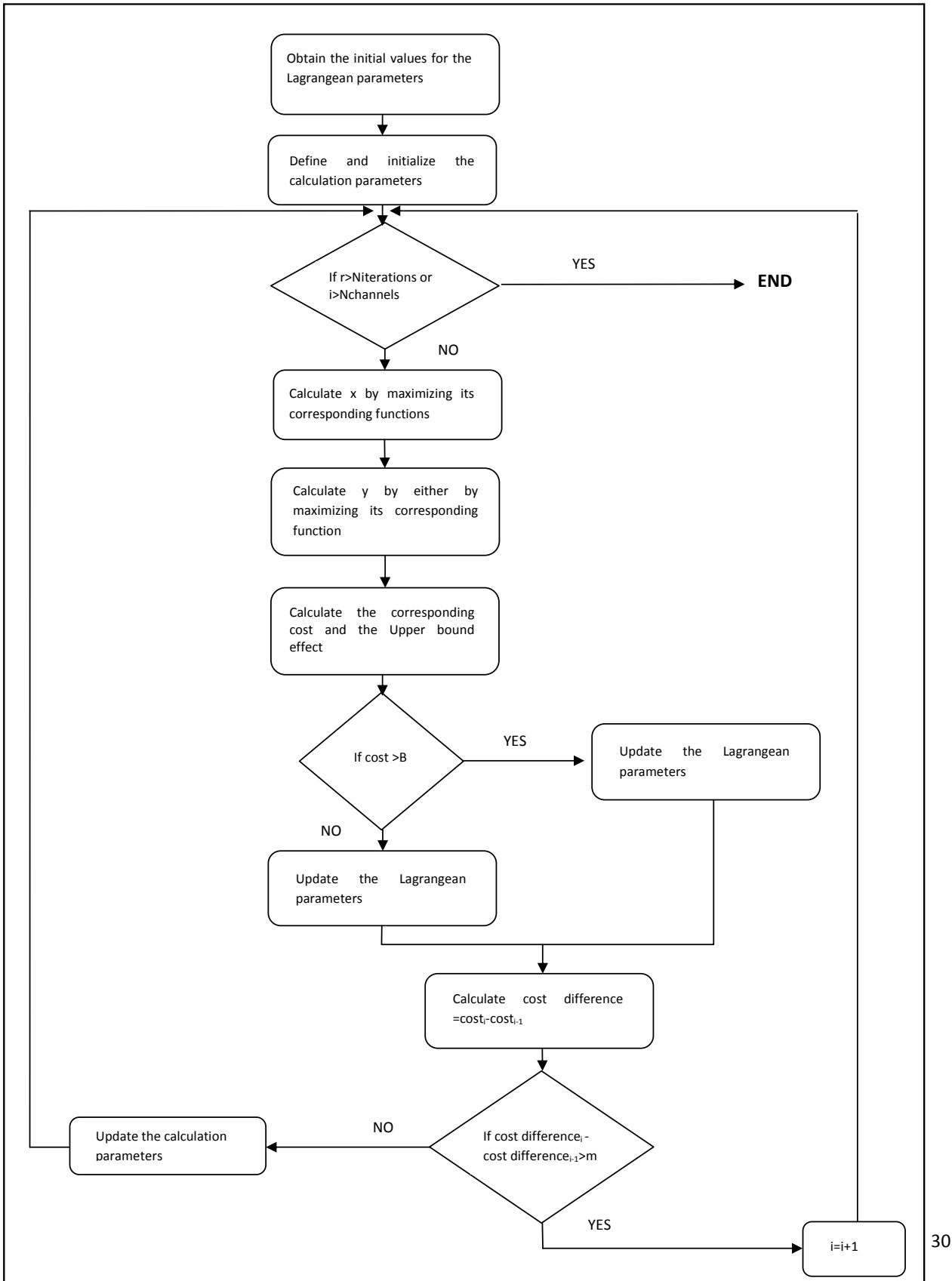


Figure 3: Lagrangean heuristic-multipliers' adjustment work-flow

4.2 Lower Bound

There are many techniques generating a Lower Bound (in case of a maximization problem) which are based on searching for a feasible solution which will satisfy the constraints.

4.2.1 Lower Bound Heuristic

The Lower Bound effect values are generated by a heuristic method which is using as initial values the solutions produced by the Lagrangean heuristic and based on a mathematical procedure it is turning them into feasible for the original problem. The aim is to achieve such solutions that they will be feasible and they will produce an effect as closer to the optimal effect value as possible. Like in the Lagrangean relaxation, the key lies to the initial values of x as to the updating procedure to be such that the Lower Bound would approach a maximum value.

It should be noted that using the solution values as produced by the Lagrangean heuristic, it has already been decided which channels will be included into the channel mix and only the amount of GRPs invested in these channels is being differentiated in order to comply with the constraints of the original problem. As such, the Lower Bound heuristic will not enable any alteration to the already selected channels and it will be concentrated on modifying the solution of the Lagrangean problem which although is infeasible for the original problem, it can be used to derive a lower Bound which would approach to the optimal value.

In more details, the methodology behind the heuristic method is the following:

- Using as an initial solution the one obtained by the Lagrangean heuristic method (x_{LH}), it sets to zero all the x -variables that have values lower than their Lower bounds. The amount of money withdrawn will be subtracted by the total cost when it will be recalculated.

- Recalculating the total cost (expenditures) after the above modification:

$$TE = \sum_c (a_{1c} \cdot y_c + a_{2c} \cdot x_{LHC})$$

- It is defining a small amount of money (dx) which is invested or withdrawn in every iteration equal to the minimum operations cost and an updating parameter which will depend on the total cost. In such way, the amount of money and consequently the amount of GRPs invested or withdrawn in every iteration will be proportionate to the change needed for the total cost to approach the budget:

$$da = \frac{|TE - B|}{B} \quad (22)$$

- Then, it is calculating the gradient of the effect-curve for every channel:

$$grad_c(x) = \frac{df}{dx_c} \quad (23)$$

- It is quantifying how expensive it is to change the amount of GRPs invested in every channel compared to the change it will cause to the total effect. Using the operational cost, there are created the ratios:

$$r_{cost,c} = \frac{grad_c(x)}{OC_c} \quad (24)$$

- Based on the ratios and the total cost, it will be attempted either to increase the amount of GRP invested in those channels that result to the highest effect with the lowest cost or to decrease it in those channels that result to the lowest effect with the highest cost. The only restriction will be that the invested GRPs have to be between the Upper and the Lower bound. Thus:

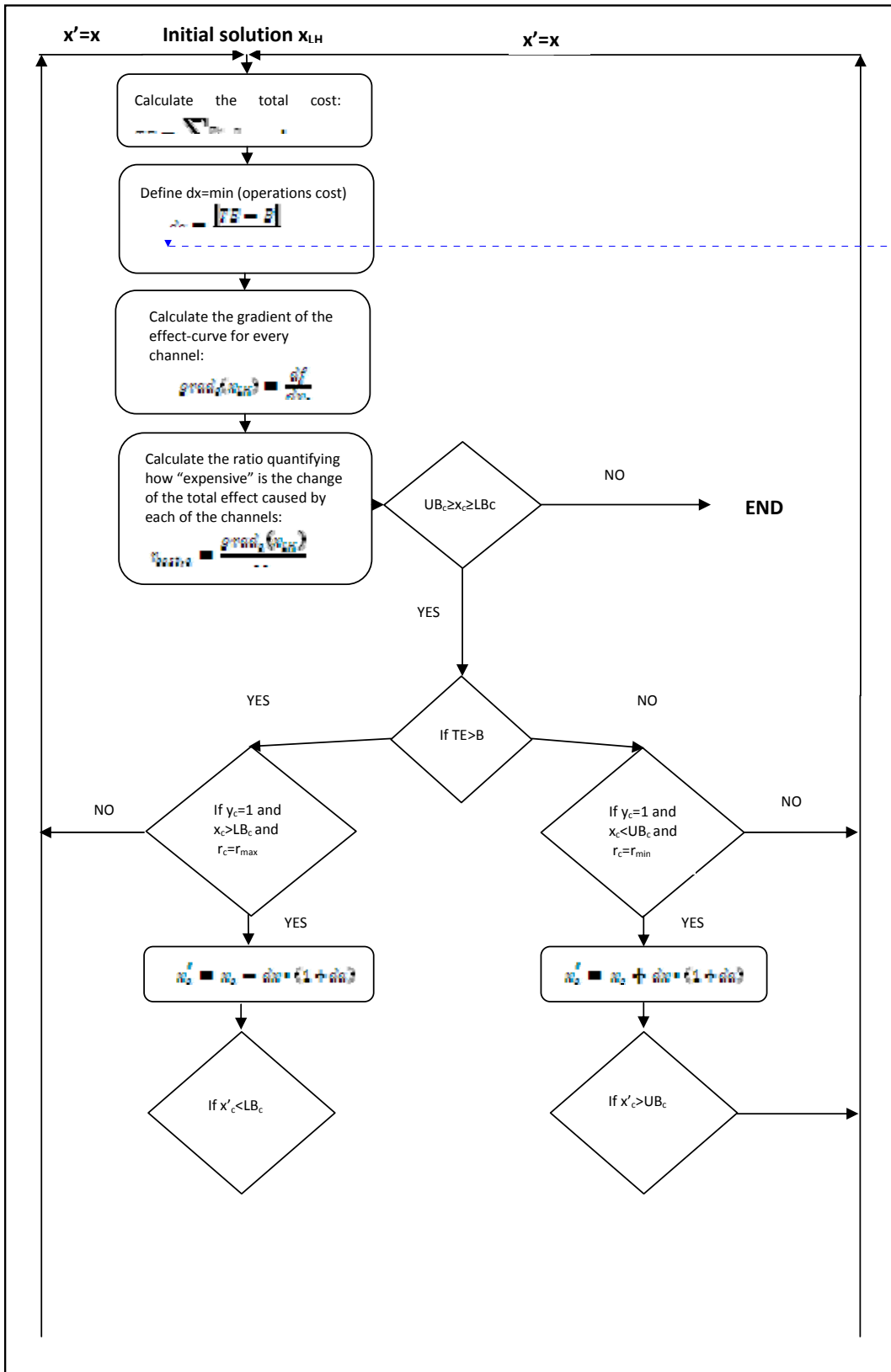
- If the total cost is larger than the budget:
Then the cost has to be decreased, limiting as much as possible the loss of effect. To achieve this, if the channel with the minimum $r_{cost,c}$ has an $x_c \geq LB_c$ then it reduces its invested GRP to $x' = x - dx \cdot (1 + da)$.
- In case that the overspending is greater than the number of GRP subtracted by the most expensive channel, the same procedure will be repeated with the other channels following a cost-descending order until the whole amount is covered. To avoid choosing the same channel, the r_c for the channel that has reached its Lower Bound would receive the artificial value MAXIMUM (which in this case is set to 1000).
- If the total cost is less than the budget:
Then there is a surplus that can be invested in one of the channels already operating. To achieve this, if the channel with the maximum $r_{cost,c}$ has a $x_c \leq UB_c$ then it increases its invested GRP to $x' = x + dx \cdot (1 + da)$.
- However, it is possible that the surplus is not sufficient enough to invest one GRP to the channel with the minimum ratio. So, first it has to be checked whether the surplus is at least equal to the minimum operational cost of the chosen channels and then to invest it to one of those channels with the minimum ratio.
- In case that the surplus is greater than the number of GRP invested to the cheapest channel, the same procedure will be repeated with the other channels following a cost-ascending order until the whole amount is covered. To avoid choosing the same channel, the r_c for the

channel that has reached its Upper Bound would receive the artificial value MINIMUM (which in this case is set to -1).

- The procedure is repeated either until the total cost is limited at least equal to the budget (but always lower than it) or the amount of GRPs to the chosen channels have reached the Upper or the Lower Bound.

The approach of the heuristic though, is not fully applicable in all the budget range, becoming insufficient when the available budget is either too low or too high. In these cases, the assumption made in the beginning that the combination of channel chosen by the Lagrangean heuristic will be used without any alteration is becoming a drawback and the algorithm is losing in precision. In fact, when the budget is too low, it would be preferable that the Lower Bound heuristic would be able to completely close the channels with the lowest ratio and the money earned to be invested in the already open channels. By this option, it would be possible to save the money invested for opening one channel (mainly the setup cost and the operations costs) and use them to increase the GRPs of the other channels contributing more to the increase of the total effect. In case that the available budget is

An option like the one described above, although it is expected to improve the performance of the algorithm, it will make it very complex increasing exponentially the computation time. That is the reason why we have chosen not to implement it but to mention it as a further recommendation for improved results since in case of extreme budget values, the optimization of the investment is not a common phenomenon.



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4.3 Random algorithm

Next to the above, there is also going to be developed a random algorithm, a non-structured way, for solving the original problem. Indeed, we will model an intelligent way of investing the budget among the available channels mainly based on expert knowledge, including randomization and avoiding the use of a structured mathematical procedure. The scope of this algorithm is to prove that any solution derived by a random algorithm is not a sufficient quality approach compared to the one produced by the developed methodology. In other words, the solution of the random algorithm is used as a comparison for the quality of the result of the developed methodology. Additionally, it is a threshold for the minimum quality result that could be achieved in order our presented complicated methodology to be worth applying.

In more details, the random algorithm follows the intelligent procedure below:

- First, it is defining the number of channels which will be included in the investment. The number of channels that the marketing mix will consist of is derived by the ratio of the 30% of the available budget with the average setup cost. By this way, it is implied that this amount of the budget (30%) will be spent in order to cover the setup costs for opening some channels and the rest of it will be invested, putting GRPs in them. To insert some variation in the result since the percentage of the budget was defined arbitrary and it is highly possible to be different, the final number of channels will be obtained by generating a random value in a range ± 2 channels and using as a mean the number produced above.
- It should also be determined, in which channels we are going to invest the available budget. So it is also used a random number generator from 1 to 34, obtaining as many channels as calculated in the previous step and avoiding any duplicates.
- The procedure, according to which the GRPs are assigned to the chosen channels, is the same as in the Lower Bound heuristic only applied to the specific channels chosen above.

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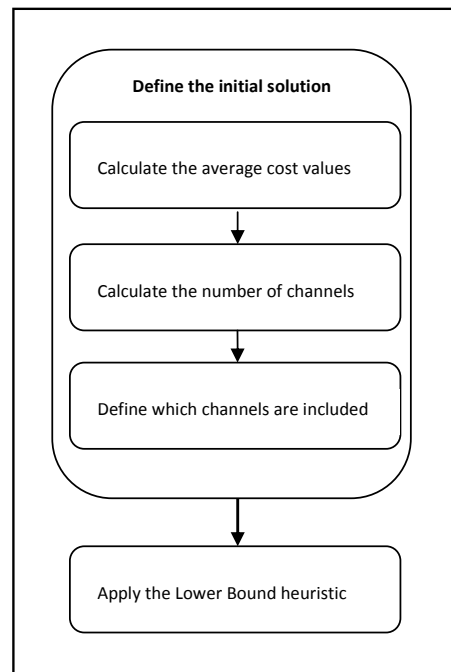


Figure 5 Random algorithm work-flow

Concluding, there were developed:

- A Lagrangean heuristic approach which is calculating an Upper Bound of the problem's solution
- A heuristic algorithm deriving a Lower Bound of the problem's solution
- A random algorithm producing a solution based on empirical methods and implementing randomization.

In the following chapters, these solution approaches will be implemented and their results will be evaluated.

5. Implementation

In this chapter, we will describe the necessary steps to apply the solution approach to the current problem. So, we will present a detail description of the initialization of the problem, explaining the strategy behind assigning values to the Lagrangean parameters and we will also develop the updating procedure implemented to the Lagrangean heuristic.

5.1 Problem parameters

In the beginning, it would be essential to present the basic parameters used during the implementation of the solution approach such as the number of channels and iterations as well as the available budget. For this purpose, it is constructed a summarized table:

Number of channels	Budget (€)	Number of iterations
34	350,000	50

Table 1: Problem's parameters

The number of iteration had to be such that would enable the system to reach a steady state and at the same time to maintain the computational time in reasonable levels. We concluded to use 50 iterations after implementing the algorithms a number of times and estimating when the system is becoming stable. The budget was chosen to be 350,000€ which is approaching real life investments size and it ensuring that the Lower Bound heuristic will operate sufficiently. The setup and the operational costs (as mentioned in Chapter 1) were estimated calculating the average costs produced applying a random number generator and using as means the estimates provided by Pointlogic.

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5.2 Obtaining initial values

As described in the solution approach, applying Lagrangean relaxation is all about assigning values to the Lagrangean parameters and calculating an Upper Bound value for the total effect. In fact, it applies an iterative procedure starting from some initial values of the parameters and it is continually updating them until the global minimum Upper Bound value is obtained, given a maximization problem. Depending on the initial values of the Lagrangean parameters though, the number of iterations and subsequently the computational time until the optimal solution is produced, may significantly vary. For this reason, the solution approach begins by solving the diagonal problem (Appendix A2), obtaining the values for the Lagrangean parameters that correspond to its optimal solution and then they are used to estimate the initial values of the Lagrangean parameters for the original relaxed problem. The diagonal problem is a simplified version of the original problem according to which the interaction effects among different channels do not exist. Instead, the effect of every channel is independent of the effect of the other channels and when the total effect is described using a matrix with the independent effect of all the channels, this would contain elements only on its diagonal. In such way, the number of iterations needed until the optimal global result is obtained, will be reduced because they would initially approach realistic values. (figure 6)

The relaxation of the diagonal problem (Appendix A2) includes only the budget constraint since the artificial constraints (which are restricting the values of GRP between an Upper and a Lower Bound) are kept as they are. This means that the initial values for the other Lagrangean parameters have to be generated based on this value.

So, given the alternative formulation of the objective function as developed in Chapter 2:

$$\max f(x)_{\text{total}} + \sum_c (\mu_c - \lambda \cdot \sigma c_c - \nu_c) \cdot x_c + \sum_c (\mu_c \cdot UB_c [-\lambda \cdot \sigma c_c - \mu_c \cdot LB_c] \cdot y_c + \lambda \cdot B$$

$$\max f(x)_{\text{total}} + A \cdot x_c + B \cdot y_c + C$$

where $f(x)_{\text{total}}$: actual effect

A: parameter of the decision variable x_c

B: parameter of the binary decision variable y_c

C: constant part

The initial values of the Lagrangean parameters would be defined as following:

λ parameter:

As mentioned above, the initial value for the λ -parameter will be derived based on the value of the Lagrangean parameter of the diagonal problem. So, having calculated the optimal value for the diagonal problem, there is also obtained the corresponding optimal value of the Lagrangean parameter which is used to conclude to the initial value of the λ -parameter for the relaxed problem. However, it would not be effective to use the same value as an initial value of the λ -parameter. Instead, we developed a methodology with the necessary argumentation explaining how the initial value of the parameter should be calculated. So, the methodology behind the initial value of λ parameter is the following:

- Solving the relaxed simplified problem, there is obtained a global minimum Upper Bound which is the optimal solution of the Lagrangean relaxation and an optimal value for the Lagrangean parameter
- Regarding the results of the simplified problem, it was expected that the effect value would be higher since by definition there are no interaction effects between different channels and the total effect equals to the sum of the individual effects. But as it was mentioned to Chapter 3, there are interaction effects among different channels which results to a final effect lower than the one obtained by the diagonal problem. As a result, it would not be favored to use exactly the value of its Lagrangean parameter as an initial value for λ -parameter since this would lead to an increased number of iterations until it approaches its appropriate size.
- To adjust the value of the Lagrangean parameter produced by the simplified problem to become an initial value for the λ -parameter, there were compared the effects produced by

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the diagonal problem to the effect values obtained by the raw data files available. As a result, it was observed that the values of the diagonal problem were constantly larger than the real values. Then, taking into consideration that the role of λ parameter is to penalize the exceeding of the budget constraint (overspending), it is concluded that the size of the parameter should be proportionately adjusted to the difference of the effects. More specifically, it is expected that the solution of the simplified problem would exceed the budget constraint way more than in the current problem thus the penalty about the constraint violation is expected to be higher. To adjust the difference of the influence size, it is suggested that the initial value of the λ parameter would equal the one of the diagonal problem multiplied by the ratio of the resulting effects.

In order to demonstrate that the methodology described above results to improved starting values of the λ -parameter which are indeed adjusted in a limited number of iterations in the beginning of the procedure, there will be presented a graph showing how the parameter is evolving given different initial values:

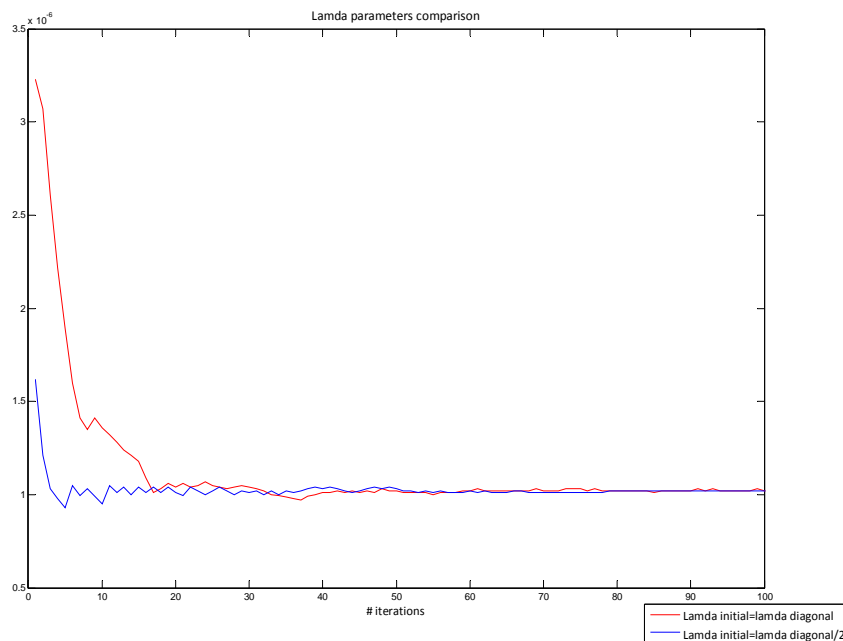


Figure 6: λ -parameters with different GRPs Upper Bound

μ_c and ν_c parameters:

The initial values of the artificial constraint Lagrangean parameters will be based on the λ parameter and they will be adjusted depending on the penalty implied whenever each of the constraints is violated. In more details:

- μ_c parameter which is used to penalize the exceeding of x_c variable from the Upper Bound, would be proportionate to the penalty implied to the budget and the Upper Bound. So:

$$\mu_c = \frac{\lambda \cdot B}{R \cdot UB_c} \quad (25)$$

Where R equals a parameter which value depends on how big the influence of the Upper Bound compared to the penalty in budget will be.

- ν_c parameter which is used to penalize the exceeding of x_c variable from the Lower Bound, would be defined in the same way as the μ parameter.

5.3 Updating procedure

The objective function is transformed and presented in such way that the two decision variables are separated and followed by parameters (A and B part) which both include all the Lagrangean parameters.

$$A - part = \sum_c (\mu_c - \lambda \cdot \sigma c_c - \nu_c)$$

$$B - part = \sum_c (\nu_c \cdot UB_c [-\lambda \cdot \rho_c - \mu_c \cdot LB_c])$$

The more positive each part is becoming, the more likely it is its corresponding variable to take positive values. Taking into consideration that both variables refer to the same channel, it is expected that there should be a corresponding trend in obtaining positive values. This means that when the y_c variable is receiving value one, this specific channel is chosen and the corresponding x_c variable should also take positive values. In order this to occur, the A-and B-parts should have aligned signs and receive simultaneously positive values and taking into consideration that these two parts consist of the Lagrangean parameters, it is becoming clear that their values are influencing the whole procedure. However, it is observed that the Lagrangean parameters appearing to these two parts with opposite signs and given the fact that they are multiplied with different constants, it results to a complicated updating procedure which is crucial for the quality of the result obtained in the end.

In literature, there are mentioned two different ways for updating the Lagrangean parameters, the subgradient optimization and the multiplier adjustment. Due to the fact that the multiplier adjustment updating technique receives the complexity the user is assigning to it, depending on the desired quality of results, it can be modified accordingly receiving from a very simple to a very complex form. So, since we would like to derive a solution applied to a general problem, it will be preferred.

There are developed two different policies for updating the Lagrangean parameters:

- The simultaneous update of all parameters based on Lagrangean heuristic with multipliers' update.
- Updating only two of the Lagrangean parameters and assigning to the third one such values that B-part equals to zero.

- **Updating all the three Lagrangean parameters simultaneously using multipliers' update**

Applying the Lagrangean heuristic methodology, the Lagrangean parameters are updated according to their corresponding constraints.

- Budget constraint

- If the combination of channels in the marketing mix results to a total cost greater than the available budget, the λ -parameter is increased.
- If the combination of the channels in the marketing mix results to a total cost less than the available budget, the λ -parameter is decreased.

- Artificial constraints (see Chapter 3)

As explained, the artificial constraints are set in order to ensure that the values of the GRPs spent will be located between certain values picturing real life conditions. As such:

- If the resulting x_c (GRPs) is lower than the Lower Bound then the μ_c - parameter is increased and when it is higher than the Lower Bound it is decreased.
- If the resulting x_c (GRPs) is higher than the Upper Bound, then the ν_c parameters is increased and when it is lower than the Upper Bound it is decreased.

The size of the adjustment (increase or decrease) of all the Lagrangean parameters is weighted depending on the size of the difference between variable's value and its corresponding constraint value. In this way, it is avoided disproportionate changes to the size of the measurements. (Beasley, 1992)

In the following chapter, there will be a detailed presentation of the results obtained when we applied this updating technique. However, it would be useful at this point to highlight in advance that the behavior of the system during the simultaneous updating was rather unstable. Indeed, it proved to be hard from computational point of view to conclude to a certain set of values for the parameters that would result to lower the values of the Upper Bound, mainly due to the opposite signs with which they are participating to the two parts of the relaxed objective function. As a result, the obtained values of the Upper Bound effect were fluctuating following the continuous change of the parameters.

Deleted:

- **Updating only two of the Lagrangean parameters setting B-part =0**

The drawback of the simultaneous updating of the Lagrangean parameters could be overtaken if it were developed a methodology according to which at least one of the parameters would receive values based on the others. In such way, there would be avoided the fluctuations noticed in the previous policy and an improved value for the Upper Bound effect would be obtained.

Actually, it is observed that in the obtained solution of the 1st policy, the x and y variables were not aligned. Indeed, they were completely opposite, assigning values to y-variables when x variables were equal to zero and vice versa. The reason behind this is that the Lagrangean parameters of the artificial constraints appear with opposite signs to the two parts and every time the updating procedure is adjusting them according to one of the decision variables, the other decision variable is influenced in the opposite way.

For example, if a given solution of x- and y- variables results to overspending while x values remain below the Upper Bound and above the Lower Bound, the updating of the parameters would be the following:

- λ -parameter will increase to penalize the exceeding of the budget constraint. This would lead both A- and B-part to decrease because λ always appear with a negative sign.
- v -parameter will decrease to promote the increase of x values to meet the Upper Bound. This would result the two parts of the function to move to opposite direction, decreasing the A-part and increasing the B-part.
- μ -parameter, similar to the v -parameter, will decrease promoting the decrease of x values to meet the Lower Bound. But this would tend to decrease A-part and increase B-part

As a result the A-part will tend to decrease, making it less attractive for some x variables to be chosen and contrary the corresponding B-part will tend to increase, making it more attractive to choose those channels.

Consequently, there was the need of an alternative updating procedure which would overcome this phenomenon. So, it was developed the following methodology:

- Parameters λ and μ are updated as described above.
- Parameter v is taking such values that B-part equals to zero.
The reasoning behind this assumption is that by keeping B-part equal to zero, all the y-variables are potential candidates to receive positive values. As explained above, the system tends to assign positive values to the decision variables with positive parameters therefore by adjusting the parameters such that B-parts equal to zero, the corresponding decision variables are taking zero value. This enables the user to choose which of the y variables would receive a value and to adjust them in such way that they correspond to the x-variables. In more details, by assigning values to those y variables that have x-variables with value higher than their Lower Bound, it is

ensured a complete match between the variables of the same channel and consequently the feasibility of the obtained solution to the original problem.

- **Setting the Upper Bound in a lower value**

The Upper Bound constraint was defined in Chapter 3 and it was set such as the value of GRPs invested in every channel will not over-exceed the value of GRP that results to an effect equal to 95%. However, the implementation of the first two policies proved (presented in detail in Chapter 6) that the Upper Bound was set in high values failing to restrict the values of x variables and resulting to a great variance. In an attempt to further improve the Upper Bound effect obtained by the second policy and to approach the values of the actual effect, the Upper Bound of x variable will be reduced such that the amount of GRP finally invested in every channel to approach it more without hitting it.

In order to redefine the effect up to which the amount of GRP in every channel could raise, we thoroughly examined the solution derived by the second policy ensuring that the reduction of the Upper Bound of GRPs will not influence the final outcome. Indeed, the Upper Bound effect derived by the second policy was based on such values of GRPs that were significantly lower than their Upper Bound, thus the constraint was actually inactive. So, by reducing the Upper Bound of GRPs it is expected that the values of the Upper Bound effect will be improved since the fluctuations of the x variables will be restricted.

Apart from the reduction of GRPs' Upper Bound, the rest of the procedure remains the same as in the second policy and it is briefly described in the previous section.

6. Results

In this chapter, we will present the results derived by the implementation of the solution approach as described in Chapter 5. Indeed, we succeeded, by applying the alternative updating techniques, thus differentiating our approach from the traditional Lagrangean heuristic methodology, to produce improved results.

The results of every solution approach will be demonstrated using summarizing graphs followed by the necessary argumentation, mainly emphasizing on the weaknesses occurred by each technique and the suggested improvements. Next to these, the complete analytical results will be available in the Appendix.

Overall, it should be highlighted that we succeed to produce an Upper Bound and a Lower Bound effect close to each other deriving a solution which is close to optimality.

6.1 Lagrangean heuristic

It is useful to remind the different formulations of the problem as developed in Chapter 3:

1st formulation (Original formulation):

$$\text{Max } f(x)_{\text{total}}$$

Subject to

$$\sum_c (sc_c \cdot y_c + oc_c \cdot x_c) \leq B$$

$$x_c \geq LB_c \cdot y_c \quad \forall c \in C$$

$$x_c \leq UB_c \cdot y_c \quad \forall c \in C$$

$$y_c \in \{0,1\} \quad \forall c \in C$$

$$x_c \geq 0 \quad \forall c \in C$$

2nd formulation (Lagrange formulation):

$$\max f(x)_{\text{total}} + \sum_c (\mu_c - \lambda \cdot \theta c_c - \nu_c) \cdot x_c + \sum_c (\nu_c \cdot UB_c [-\lambda \cdot sc_c - \mu_c \cdot LB_c]) \cdot y_c + \lambda \cdot B$$

$$\max f(x)_{\text{total}} + A \cdot x_c + B \cdot y_c + C$$

where $f(x)_{\text{total}}$: actual effect

A: parameter of the decision variable x_c

B: parameter of the binary decision variable y_c

C: constant part

6.2 1st. Policy “Updating all the three Lagrangean parameters simultaneously”

The implementation of the first policy introduces the updating of all the Lagrangean parameters simultaneously. The results will be summarized to the following graphs which will include the Upper Bound effect together with the relaxed constraints as well as with every part of the relaxed objective function individually. By this way, it is becoming more comprehensive how every substantial part of the system reacts when the values of the Lagrangean parameters are changing and how this reaction influences the behaviour of the system.

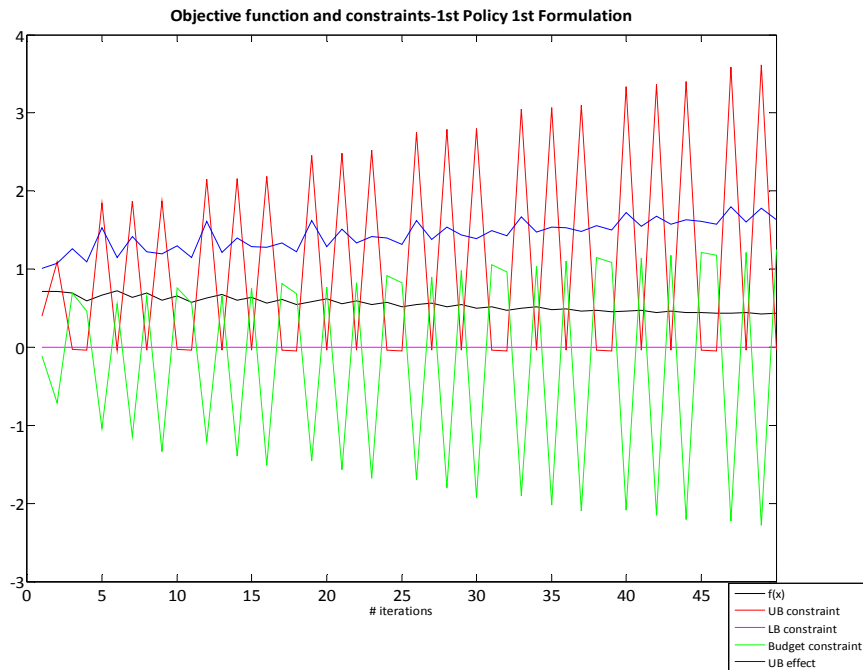


Figure 7: Objective function and constraints-1st policy

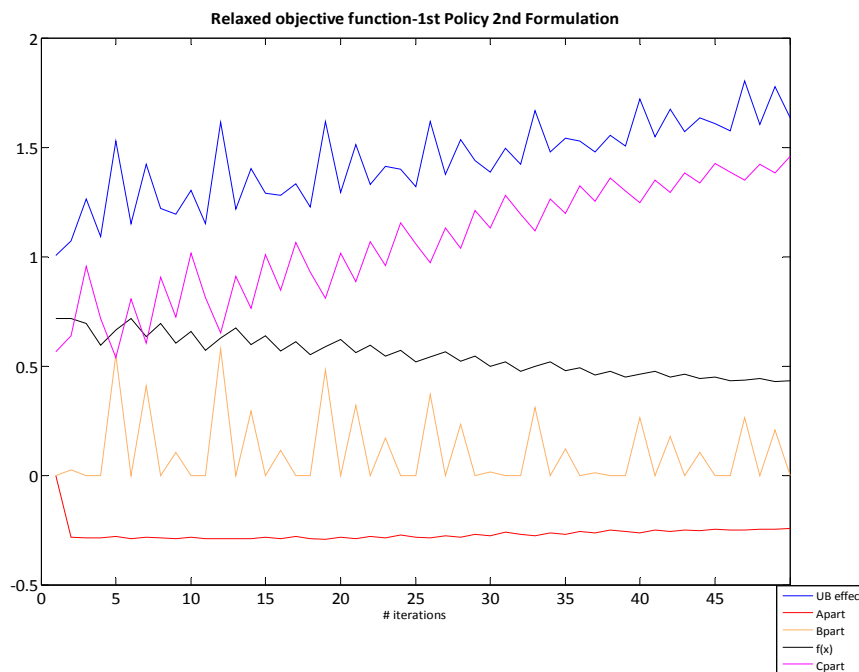


Figure 8: Relaxed objective function and substantial parts-1st policy

It is obvious that the Lagrangean relaxation does not produce an Upper Bound that could be useful in obtaining an optimal solution of the original problem. The main reason is that the obtained Upper Bound exceeds the value one which is interpreted as influencing more than 100% of the audience. But by definition any percentage above 100% is not realistic and in this case the Upper Bound does not provide any improved value other than the known “Upper Bound” of the 100%.

In more details, it is observed that:

- There is a diverging tendency between the Upper Bound effect and the actual effect values. In fact, the Upper Bound effect is continually increasing while the actual effect continually decreasing. It would be expected the Upper Bound to at least approach the actual values of the effect.
- There are fluctuating effects appearing to all the substantial parts of the problem’s formulation.
 - The budget constraint is fluctuating around zero without approaching it or any other value close to it. This means that the money spent in every iteration is either way above or way below the budget and the updating of the parameters is proved to be insufficient to bring it to a steady state close to the available budget.

- The artificial Upper Bound constraint is also highly fluctuating, ranging from values below zero to even above 1.5. In other words, x variables are receiving values that sometimes exceed the Upper Bound and others are reduced way below it.
- The artificial Lower bound constraint shows negligible differentiation from zero value. This could be interpreted as a result of x moving close to the Lower Bound.

A better insight of the problem's behaviour is obtained when recalling the formulation of the relaxed objective function and the procedure description in Chapter 5:

$$\max f(x)_{\text{total}} + \sum_i (w_i - \lambda \cdot \rho_i - v_i) \cdot x_i + \sum_j (v_j \cdot UB_j - \lambda \cdot \rho_j - \mu_j \cdot LB_j) \cdot y_j + \lambda \cdot B$$

- Fluctuations

- Every time that the budget constraint is violated, the λ -parameter is increased in order to penalize this behaviour and to push some specific channels to reduce their values in x and y variables, making it less likely to pick them. In other words, the amount of GRPs invested in these channels will be reduced and the channels will not be chosen, assigning to the binary variable y the zero value.
- This tendency to reduce the values in the variables will also tend to lower the x-variables which eventually some of them will be driven below their GRP Upper bounds. As a result, the corresponding v-parameter will be decreased in an attempt to increase the GRPs of these variables to push them to meet their Upper bounds. Given the fact that that this change will affect A-part positive because v-parameter is appearing with a negative sign and the B-part negative since it has a positive sign there, these channels are becoming more likely to be picked increasing their amount of GRPs and placing their binary variable to one. Consequently, the total cost will be increased having an opposite effect to the budget constraint.
- Finally, the μ -parameter would be adjusted according to the Lower Bound constraint, thus this parameter will also be decreased pushing the GRPs to lower values. Contrary, because of μ -parameter's negative sign at B-part, the y-variables will tend to get value one picking those channels although having limited (or even zero) GRPs.

- Diverging

The reason behind the increasing tendency of the Upper Bound effect is that in every iteration:

- That the budget constraint is violated, the λ -parameter is increased even more in order to lower the cost below the available budget by reducing the GRPs to lower values and closing some of them completely (by setting $y_i=0$).

- As follows, there is a slack appeared to the Upper Bound constraint due to the limited number of GRPs invested to the channels that are left open. So, the v -parameter will also increase more to overcome this effect and to bring the amount of the invested GRPs closer to their Upper Bound value.
- Consequently, more channels will be picked by making it more likely for y -variables to receive value one and thus increasing the cost.

The whole approach is creating an iterative procedure which tends to continually increase the values of λ - and v - parameters increasing the Upper Bound effect.

On the other hand, the amount of GRPs invested in every channel is gradually decreased while the corresponding binary variable tend to receive value one. This results to misaligned variables where the amount of GRPs invested in a channel is limited (x -variables close to zero) while the channel is actually open (y -variables equal to one).

Summarizing, the conclusions that are drawn from the above notifications are:

- o The whole system is not able to reach a stable condition and is presenting an unstable behaviour, mainly based on the response of its substantial parts to the updating of the parameters.
- o The Upper Bound fails to reduce itself and to converge to a certain value close to the original effect
- o The updating procedure of the Lagrangean parameters is not sufficient to conclude to a combination of channels which would result to an effect close to the real effect
- o An improved way of updating the Lagrangean parameters could have potentially led to reduce the diverging phenomena but it would not be sufficient to have the same result to the fluctuation effects.

For these reasons and in order to avoid the resulting viscous circle of opening and closing the same channels, it is suggested an alternative methodology. Instead of updating all the parameters applying the Lagrangean heuristic rule, it was suggested to update only the budget and the Lower Bound constraint Lagrangean parameters and to the Upper Bound constraint parameter to be assigned values such that the B-part of the alternative formulation of the objective function equals to zero.

6.3 2nd. Policy “Updating only two of the Lagrangean parameters setting B-part =0”

The conclusions derived by the implementation of the first policy led to the development of a second policy which suggested updating only two of the Lagrangean parameters and adjusting the third one in such a way that B-part (see 2nd formulation of the problem) equals to zero. The results are presented in the following graphs:

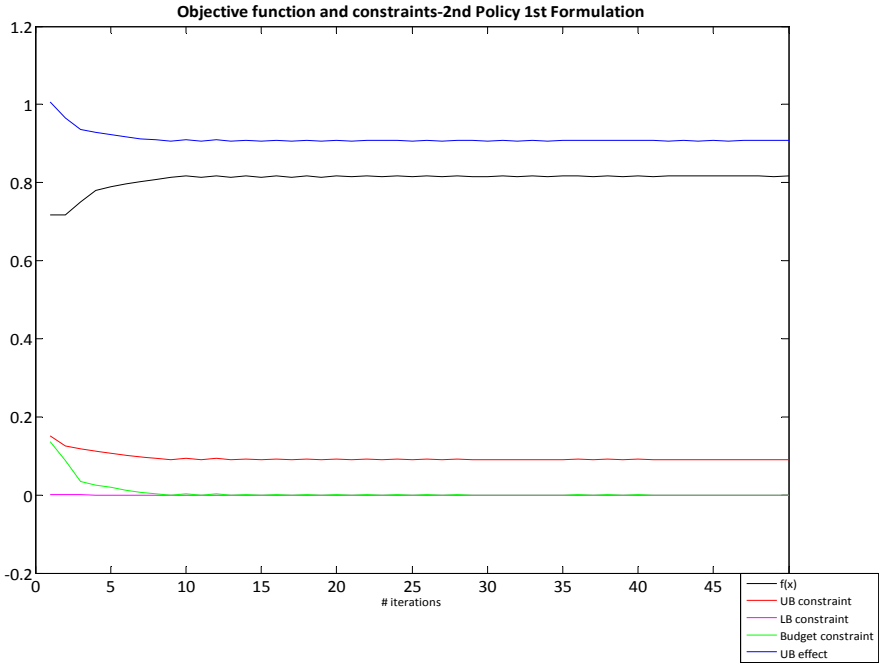


Figure 9: Objective function and constraints-2nd policy

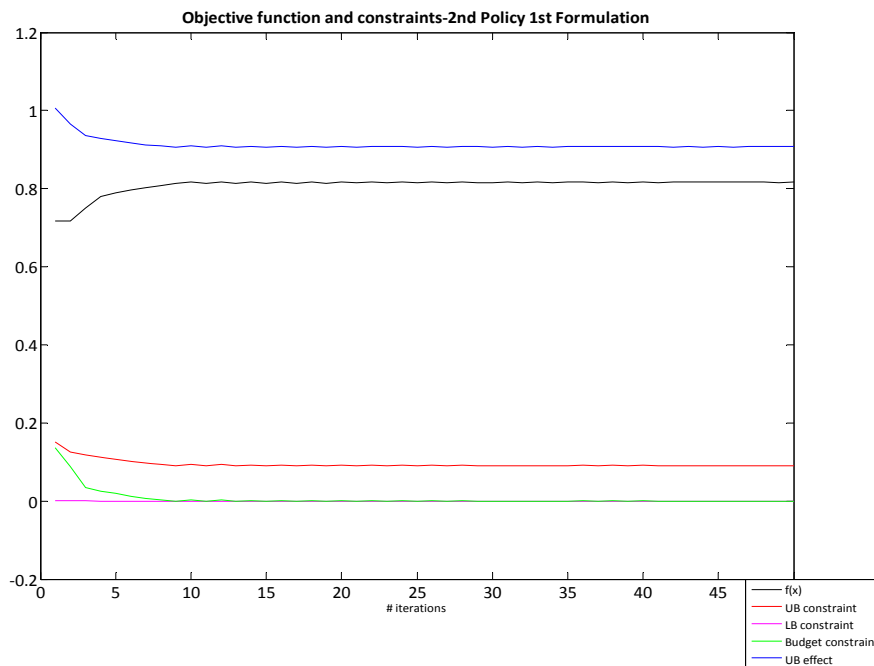


Figure 10: Relaxed objective function and substantial parts-2nd policy

It is noticeable that:

- Both the Upper Bound and the effect do not appear to have an increasing trend and they have become more stable approaching reasonable values, thus they do not diverge any more. However, they are still not converging but they are remaining parallel maintaining a high value distance from each other that it would not be safe to draw conclusions regarding the optimal solution.
- There is a significantly more stable behaviour to all the parts of the relaxed objective function with all the fluctuating effects reduced to the minimum.
- The budget constraint has reduced its fluctuations and in fact after a small number of iterations it is converging to the available budget
- The Upper Bound constraint has been restricted receiving positive values of a limited range, although it is still deviating from the zero value.

Next to these, we are going to present a comparison graph between the Upper Bound values of the GRP and the values derived as a solution by the 2nd policy, in order to fully explain the reason why the Upper Bound constraint is receiving such high values.

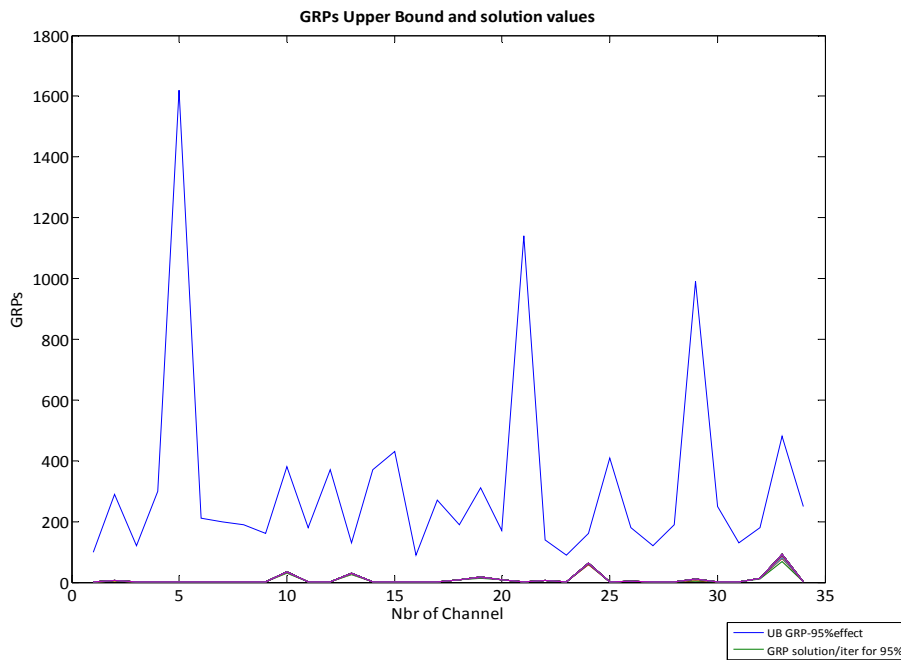


Figure 11: GRPs Upper Bound compared to GRPs actual solution values

It is worth mentioning that the Upper Bound values for the GRP have been set extremely high expressing an unrealistic limitation. It is proved that the GRP values produced by the solution method will never approach the Upper Bound values, thus the Upper Bound constraint is remaining practically inactive since it is not restricting them. For this reason, the improvement of the 2nd policy will introduce a decrease of these Upper Bounds values aiming to Lower the Upper Bound constraint closer to zero and consequently to reduce the Upper Bound effect.

6.4 Lower Bound heuristic

It could be claimed that the implementation of the second policy succeeded in producing values for the Upper Bound effect which are more stable, without significant fluctuations and not diverging, thus of a good quality. Therefore, the values of the x-and y-variables produced by the Lagrangean heuristic will be used to derive a Lower Bound according to the Lower Bound heuristic algorithm described in Chapter 4. Hence, it is expected to produce Lower Bound values that will approach and ideally meet with the Upper Bound.

Consequently, the Upper Bound, the Lower Bound and the actual effect are plotted in the following graph:

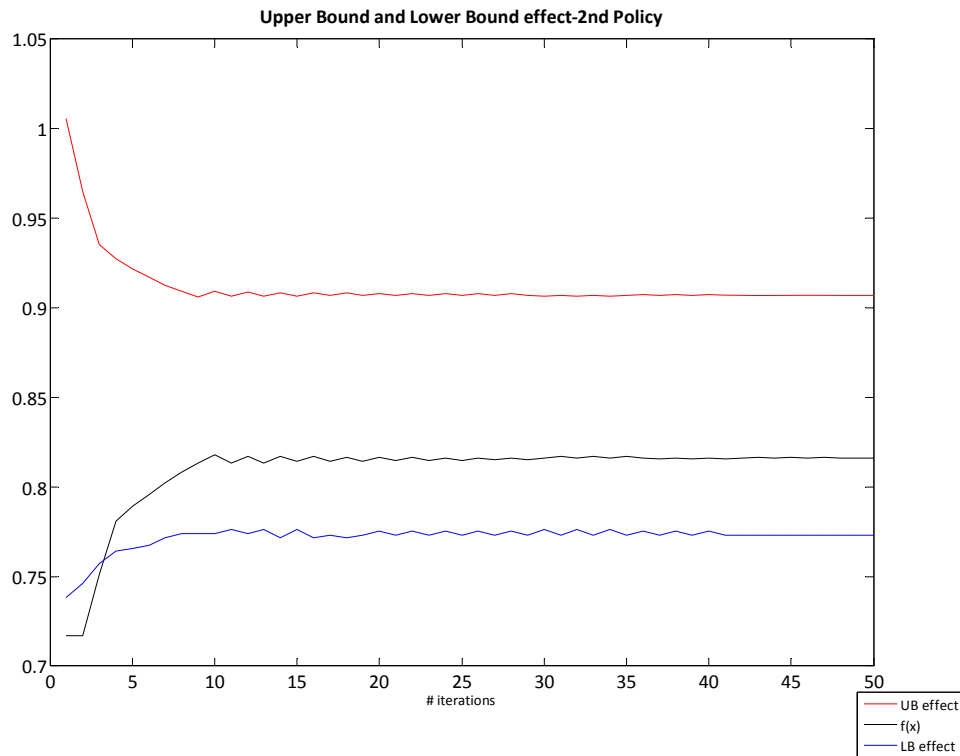


Figure 12: Upper Bound and Lower Bound effect-2nd policy

It should be noted that the values of actual effect ($f(x)$) are included in the graph only to enable their comparison to the calculated effects (Upper and Lower Bound) and to evaluate whether these values approach the reality sufficiently or not. To avoid any misunderstanding, it would be useful to mention that the computation of the optimal solution value will be obtained by the converging of the two bounds and it is irrelevant to the values of the actual effect.

The Upper and the Lower Bound effect while they are converging, after a small number of iterations (9) are becoming stable and they are negligibly fluctuating. However, the distance between them is remaining equal to 13% in absolute value without any indication of further reduction. This distance between the two bounds has as a result a solution which is although it is feasible, it is not possible to draw any further conclusions regarding the optimal solution value.

On the other hand, it would be demonstrate that the results obtained by the Lower Bound heuristic, which are based on the solution of the Lagrangean heuristic, are significantly improved compared to any other random results. Therefore, there is applied the random algorithm as developed in Chapter 4 and the effect solutions that are produced, are compared to the solutions of the Lower Bound. By this way, it is ensured that the heuristic algorithm developed to purchase a Lower Bound effect has an intelligent script behind it and its solutions would not be able to be approached by any other random technique. In fact, we present the effect values of the random algorithm compared to those derived by the Lower Bound heuristic during 50 iterations:

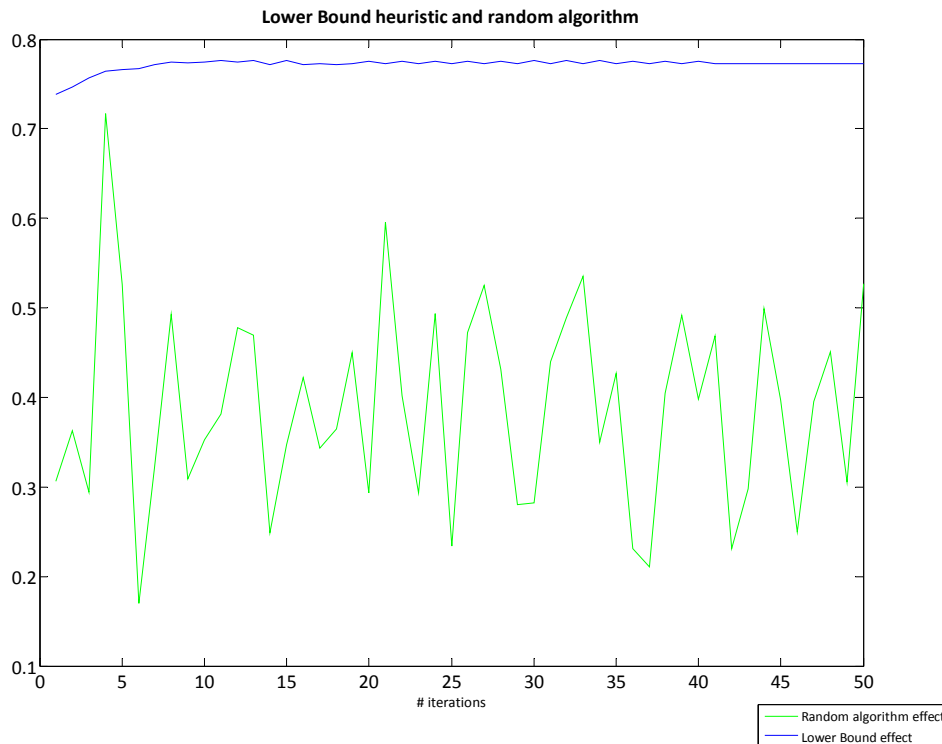


Figure 13: Lower Bound effect compared to random effect

It is clear that the effect derived by the Lower Bound heuristic is of a significantly better quality compared to the one derived using a random algorithm. So, it is concluded that indeed, the heuristic algorithm is producing an improved solution that it would not be possible to be derived using a simpler methodology.

The conclusions drawn by the implementation of the second policy could be summarized:

- The whole system is having a more stable behaviour with all its substantial parts reacting positive to the new policy of updating the Lagrangean parameters.
- The Upper Bound effect and the actual values of the effect are neither diverging nor obtaining completely different values as in the first policy. In fact, the two effects are converging to a certain value without fluctuating considerably, although it is still maintained a large value gap between them.
- Setting B-part equal to zero has resulted to aligned x- and y-variables. In such a way, every time that a channel is picked ($y_c=1$) it is invested in it an amount of GRPs between its Lower and Upper Bound ($UB > x_c > LB$)

- The Upper Bound constraint although it has reduced its instabilities, it continues to mainly affect the behaviour of the Upper Bound effect and maintaining it in values higher than the actual effect. As it was mentioned above, the values that were assigned as Upper Bounds to x variables are never met and as a result every time that the parameters are updated, the channels which are chosen raise the values of the x variables so much that the budget constraint is violated.

Although the adjustment to the Lagrangean heuristic by setting artificially the B-part to zero has overall improved the results and the Upper Bound effect is not diverging, it is still not approaching the Lower Bound values. In fact, it would have been expected to obtain an Upper Bound which would approach and finally meet the Lower Bound, providing good quality values which could be used to obtain the global optimal solution value of the original problem. However, this is not the case since the Upper Bound effect values do not demonstrate any reducing tendencies, thus any conclusion regarding the optimal solution value would be misleading.

6.5 2nd.Policy – Improvement “Setting the Upper Bound in a lower value”

As described in Chapter 5, this policy improvement involves the adjustment of the Upper Bound in lower values such that the Upper Bound effect will approach more the Lower Bound and it will finally conclude to an optimal solution of the original problem. The decrease to the GRPs’ Upper Bound values though, has to be such that the produced solution will not meet or exceed these values. In order to conclude to the Upper Bound of GRP values that result to an effect of 75%, there were studied the Upper Bound effects using different Upper Bound values for the GRPs (from 25% to 95% of the total effect). At the following graph, we are presenting these curves highlighting (with red circles) the iterations when the GRPs Upper Bound has been met:

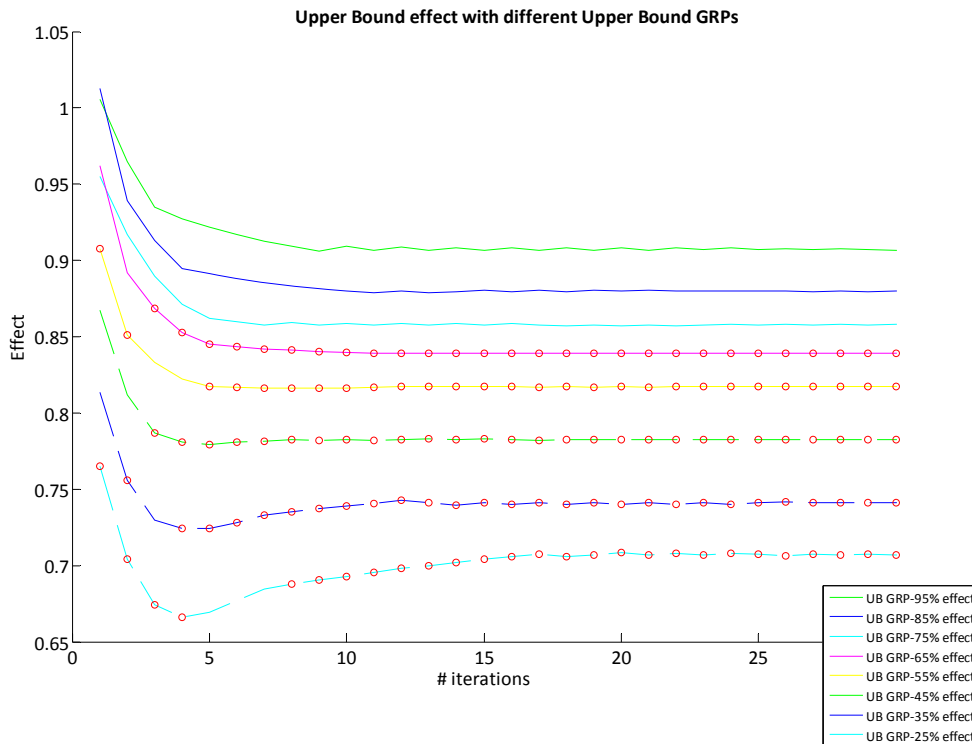


Figure 14: Upper Bound effect derived from different GPRs Upper Bound

It is obvious that decreasing the Upper Bound of GRPs to be such that the individual effect of every channel is less than 75% , leads to a violation of the Upper Bound constraint for at least one channel in every iteration. As it was described above, violation of the Upper Bound constraint due to lowering the artificial Upper Bound values results to manipulation of the solution values and that is the reason why the Upper Bound values are not reduced further than 75% of the effect.

Moreover, both the Upper Bound values for the GRPs have been defined arbitrary as a percentage of the individual effect of every channel. So, both the 95% and the 75% of the effect were derived as an Upper Bound through try and error and they could as well vary depending on the available. In fact, while examining the behaviour of the Upper Bound effect for different GRPs Upper Bound and different budgets, it is concluded that:

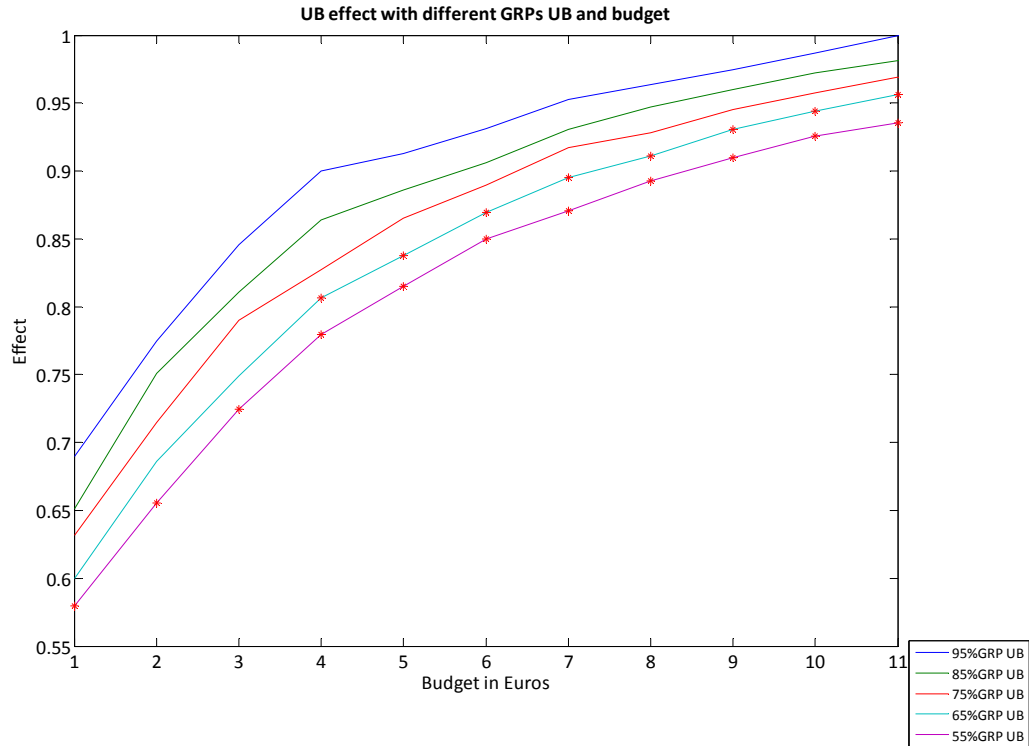


Figure 15: Upper Bound effect for different GRPs Upper Bound and budget

- For GRP Upper Bound values higher or equal to 75%, the solution for x-variables never meets the Upper Bound set, regardless the available budget. The reason is that the Upper Bounds are set in very high values and although the available budget reaches up to 600.000€, it is not enough to raise the GRPs invested in any of the channels in values higher than these bounds.
- For GRP Upper Bound equals to 65% of the effect and for budget greater of equal to 400000, the Upper Bound is met at least by one channel during at least one iteration. This is expected since the more the available budget, the more GRPs tend to be invested in the chosen channels. Combined to low GRPs Upper Bound, it has as a result the invested GRPs to meet or over-exceed these bounds.
- For GRP Upper Bound equal to 55%, it is observed that the solution derived meets the Upper Bound at least for one channel during one iteration for every available budget. In this case, the reasoning is the opposite as in Upper Bounds higher than 75%. In other words, the Upper Bound is set in low values, thus regardless how low the available budget is, the amount of GRPs invested in the channels will always be higher than the bounds set.

Overall, it has to be highlighted that the values for the Upper Bound of GRP are highly related to the available budget and as such they should be adjusted accordingly in a way that the solution of the x-variables will neither meet the bounds nor lay way beyond it.

The implementation of this improvement to the 2nd policy has the following results:

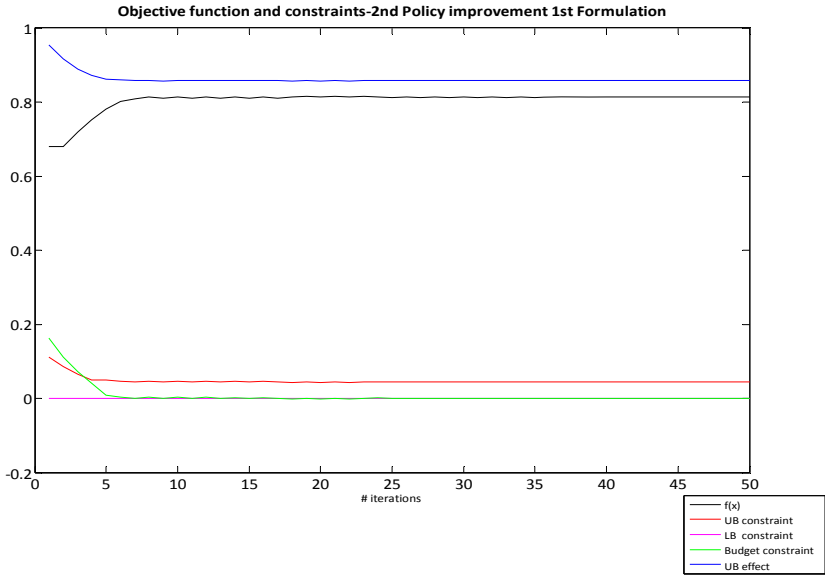


Figure 16: Objective function and constraints-2nd policy improvement

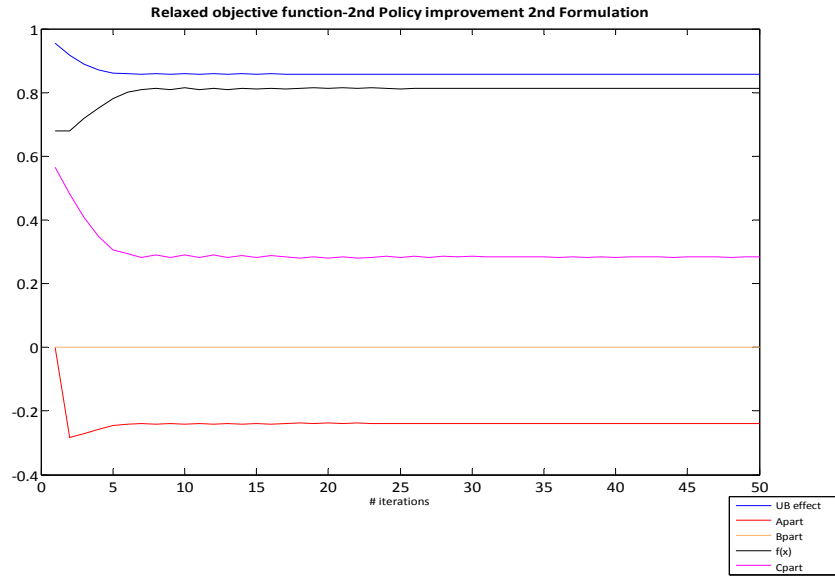


Figure 17: Relaxed objective function and substantial parts-2nd policy improvement

- The Upper Bound effect has reduced its fluctuations even more, demonstrating a more stable behaviour.
- It is also accomplished to reduce its values by approaching closer to the values of the actual effect. In fact, after a small number of iterations (5 iterations), the Upper Bound effect is becoming equal to $0.857 (\pm 10^{-4})$
- Accordingly, the total cost following a reducing tendency, after 5 iterations meets the budget constraint without deviating considerably from then and onwards (-300€ from the available budget)
- The Upper Bound constraint despite the fact that it has been reduced, it is failing to meet zero borderline remaining continually positive without presenting any fluctuations to its behaviour.

Next to the Upper Bound solution, it is also applied the Lower Bound heuristic producing the following solution:

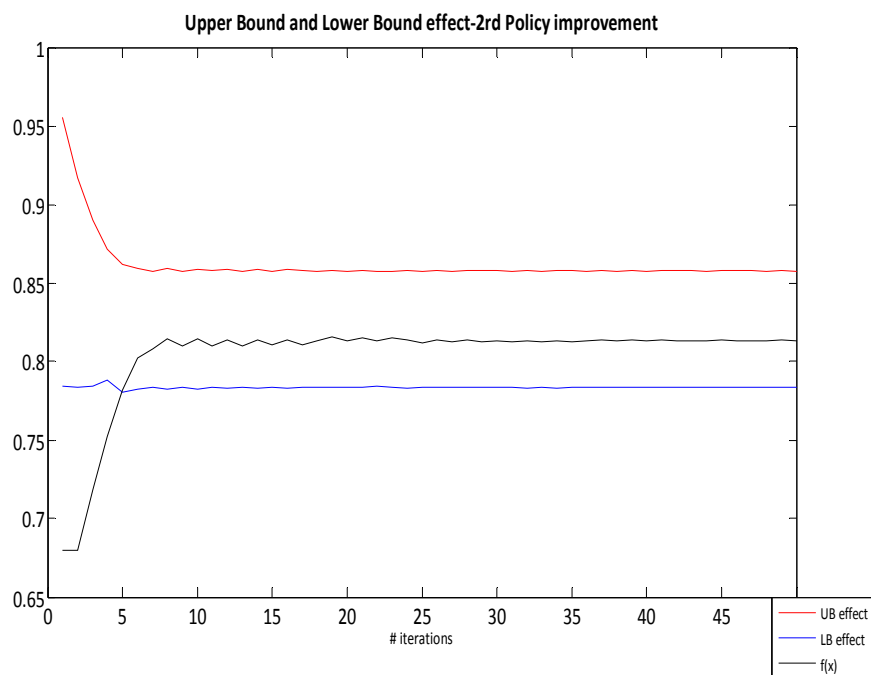


Figure 18: Upper Bound and Lower Bound effect-2nd policy improvement

It is worth highlighting that

- The Lower Bound has been improved producing higher, more stable effect values. The reason lies to the fact that the values for the decision variables provided by the Lagrangean heuristic are improved.
- The Upper Bound effect has also improved obtaining lower values and resulting to reducing the gap with the Lower Bound effect to 8 % (absolute value).

Comparing the results of the 2nd policy before and after lowering the Upper Bound of the GRPs:

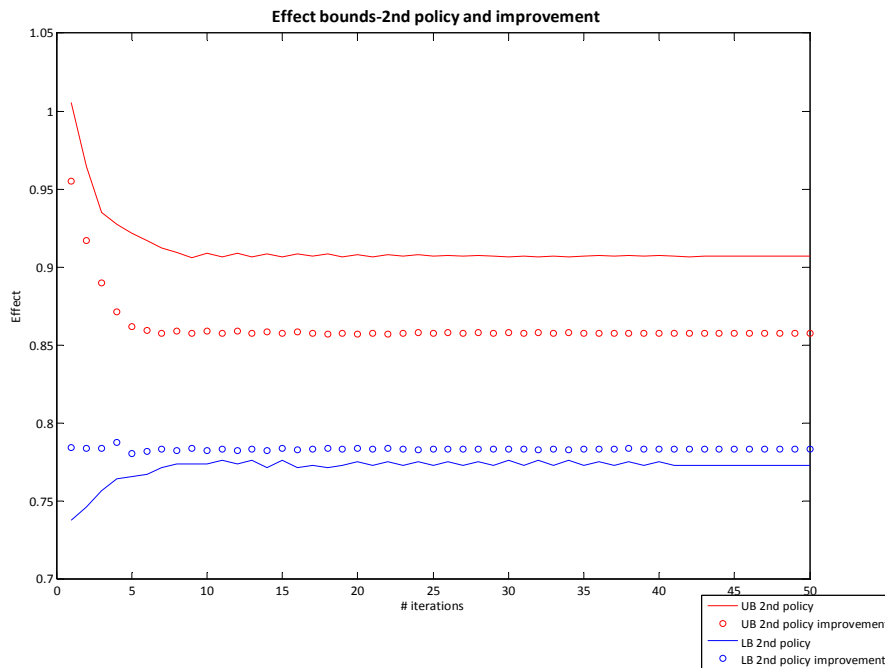


Figure 19: Upper and Lower Bound effect comparison between the 2nd policy and its improvement

It is noticed that indeed both the bounds have been improved with the Upper Bound demonstrating significantly lower values. Due to lowering the Upper Bound values of the GRP, the Lower Bound constraint was able to reduce itself even more coming closer to zero which directly affected the values of the Upper Bound. Also, the Lower Bound has increased given the improved solutions of the Lagrangean heuristic which are used as initial solutions.

Summarizing:

- The diverging and fluctuating effects observed in the 1st policy have disappeared, resulting to a stable and converging behaviour.
- The solution obtained by the Lagrangean heuristic has achieved to reduce the Upper Bound effect to values close to the actual effect.
- The Lower Bound heuristic using the solution of the Lagrangean heuristic has also been improved increasing the values of the Lower Bound effect and approaching more the Upper Bound effect.
- There is still a gap of 8% between the two bounds (Upper and Lower) without any indication of a further reduction. It could be claimed that the attempt to lower the Upper Bound of the invested GRPs indeed improved the effect bounds however a possible further research on this direction is expected to improve the insight regarding the optimal solution.

6.6 Multiple budgets

The implementation of the improved 2nd solution policy has produced the best (among the other policies) results, obtaining a good feasible solution for the original problem. Ensuring that the obtained solution is significant and that it could not be derived by any random algorithm, it would also be interesting to see how the solution approach is performing for different budgets:

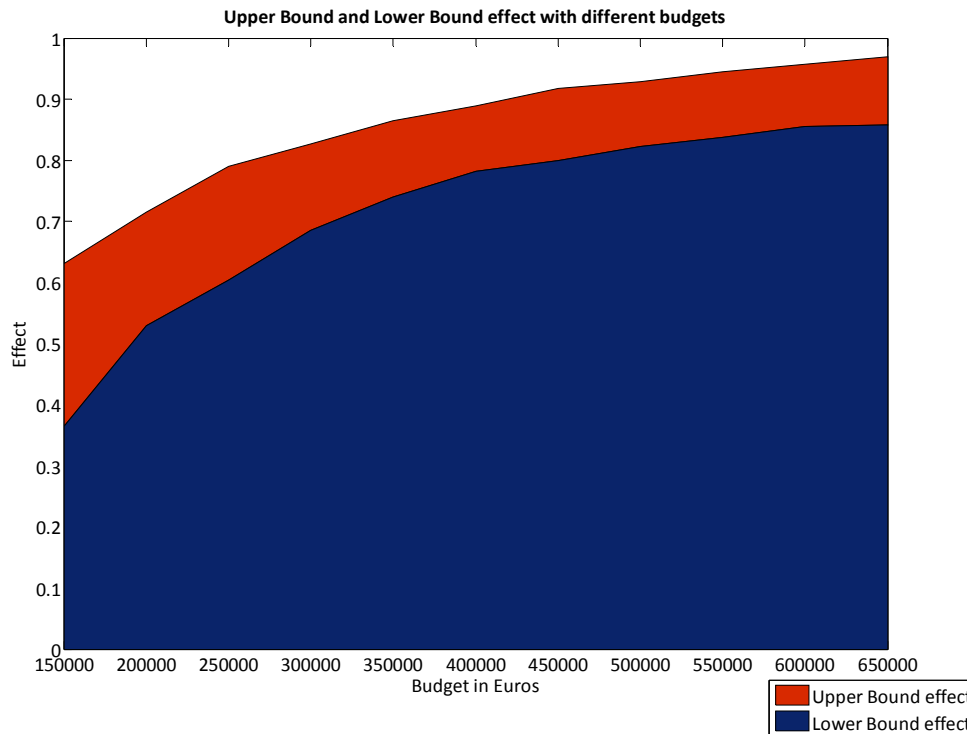


Figure 20: Upper and Lower Bound effects for various available budgets

- **Budget [150000-250000]**

The Lower and Upper Bound are demonstrating a deviating behaviour when the available budget is limited, especially for budgets below 250000€. In more details, the solution obtained by the Lagrangean heuristic is infeasible for the original problem presenting relative high values for the Upper Bound effect. In addition to this, the Lower Bound heuristic is developed in such way that it does not open or close any of the channels picked by the Lagrangean heuristic considering this channel combination as the optimal one. So, due to the restricting limited budget, it is becoming impossible to raise the number of GRPs of the chosen channels above their Lower bound resulting to low values effects. In case that the Lower Bound heuristic was extended in order to be able to close some of the channels derived by the Lagrangean heuristic, the amount of money earned by their setup costs and the GRPs invested in them could be converted into GRPs for the other channels increasing the values of the Lower bound effect.

- **Budget [300000-450000]**

The budget range between 300000€ and 450000€ results to solutions similar to the one described in this Chapter. The distance between the Upper and the Lower Bound effect values has been reduced but it is still remaining high enough not to produce the optimal solution values. The gap between the two bounds is due to both the relative high GRPs Upper Bounds and the way the Lower Bound heuristic is structured. Despite that, It is becoming obvious that the more the available budget, the more the two bounds are converging approaching each other.

- **Budget [500000-650000]**

For a budget higher than 500000€, the distance between the two bounds has reduced to the minimum possible value. However, we could not avoid mentioning that the reason that maintains this distance in such high values is the artificial Upper Bound values for the GRPs. Since there is a sufficient amount of money available to be invested, the drawback of the Lower Bound heuristic is overcome because there is no need of altering the channels by closing some of those participating in the mix.

7. Conclusions and Recommendations

The implementation of the various solution approaches led to a number of conclusions which are summarized below:

- The Lagrangean heuristic algorithm, as developed in literature by updating the parameters using the method of the multipliers adjustment, has proved to be an insufficient solution approach. In fact, the system has been completely unstable throughout the iterative procedure failing to reach a combination of parameters that would lead to a stable and diverging Upper Bound effect.
- The implementation of the second Lagrangean policy introduced the update of only two of the parameters and related the third one to them leading to manually define the values of the binary variables. The overall result was to avoid diverging unstable values producing an improved solution.
- The Lower Bound heuristic was implemented based on the solutions derived by the second policy. This heuristic algorithm however, is developed in such way that it is not able to alter the channels participating in the channel mix that is derived by the Lagrangean heuristic. Given that, the solutions obtained as a Lower Bound effect values, approached the Upper Bound values and remained stable after a small number of iterations, failing to meet them. As a result, the solution derived by the two bounds was not sufficient enough to draw conclusions regarding the optimal solution values.
- Next to the Lower Bound heuristic, it was applied a random algorithm. Its results were used to compare the performance of the Lower Bound heuristic and to ensure that the solution that was produced was indeed of a quality that could not be obtained by any random method.
- The best results obtained by the improvement of the second policy led to converging Upper and Lower Bound effects, although the distance between them remained equal to 8% (absolute value). This leads to the conclusion that despite the improved quality of the results, there were not drawn any conclusions regarding the optimal solution values.

So, it would be recommended a series of recommendations for further research, aiming either to close the gap between the bounds and to obtain the optimal solution value for the problem or to conclude that the solution already obtained is not possible to further improve, thus it is the optimal solution.

It could be claimed that the distance between the two bounds is due to both the inability of the Lower Bound heuristic to rearrange the number of channels participating in the mix and of the high Upper Bound values set for the GRPs. Hence it is recommended:

- To further expand the Lower Bound heuristic by enabling in to control the channels that will be including in the marketing mix. Using as an initial solution the one obtained by the Lagrangean heuristic, it will be able to close channels at which are invested a low number of

7. CONCLUSIONS AND RECOMMENDATIONS

- GRPs and invested these money in other – already open- channels. By this way, it is overtaken the basic drawback of the heuristic algorithm and it is expected that the resulting Lower Bound effect will be further increased. However, it is also expected that this addition will increase the computational time of the heuristic exponentially and then it will open to argumentation whether the overall result is beneficial enough to justify such a time-sacrifice or the Lagrangean based technique that it is developed in the thesis is the best technique to be applied in such a problem.
- To critically further reduce the Upper Bound GRP values for some channels. It was proven that the reduction of these Upper Bounds resulted to improved effect bounds but to proceed to further reduction, it has to be mentioned that: further overall reduction for the Upper Bounds will lead to violation of the constraint (as it was proved in figure and figure). So, it is recommended to further reduce the Upper Bound for those channels that either they do not participate into the channel mix or they demonstrate a low amount of GRPs. On the other hand, to these channels that they have a significant amount of GRPs invested in them and a further reduction of their Upper Bound might lead to constraint violation, their Upper Bound will not be modified. As a result, it is expected that the Upper Bound would reduce even more approaching and (improved) Lower Bound.

APPENDIX

A1. Development of the objective function

The main goal when developing a new objective function for the problem is to have such a formulation that:

- will closely describe the original effect curve
- would include parameters so as to be easily adaptive to changes
- would have those characteristics allowing the implementation of advanced mathematical techniques

Based on the above, it was concluded that the most appropriate formulation that fits to the resulting effect would be:

$$f(x) = L \cdot (1 - e^{-bx})$$



Figure A1: Characteristics of an effect curve

The parameters were estimated for every respondent and channel as following:

- L: Since it describes the potential of the curve, it will receive the maximum value of the effects for every channel
- b: They were estimated using the Least Square Estimates method

Distribution fit

It is expected that the estimated effect curve would produce effect values that will not be exactly the same as the original values. In order to quantify this deviation of the estimated curve from the original one, in other words to evaluate how good does the estimated curve represent reality, there is going to be a mean square error (MSE) calculation.

First, it would be helpful to create a graph with the original effect values and the estimated curve to visualize the deviation:

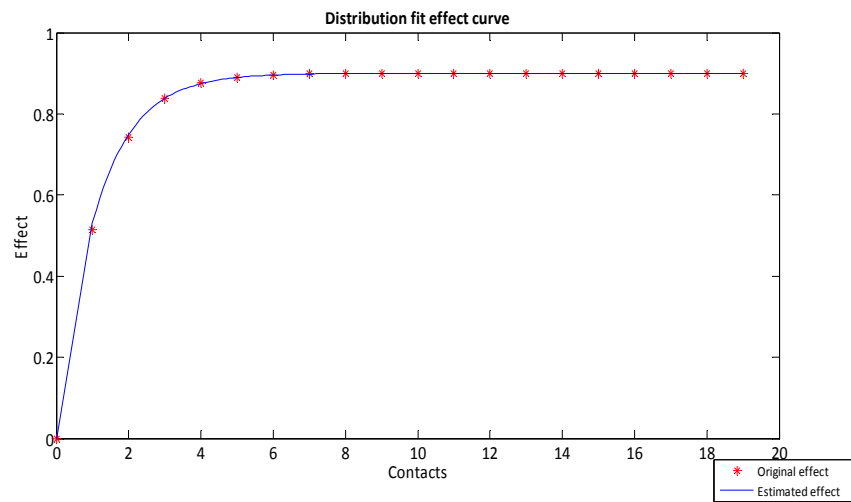


Figure A2: Fit of the estimated effect curve to the original effect curve

With a coefficient of determination:

$$R^2 = 0.999627$$

A2. Simplified Problem

One way to get an overall idea about the behaviour of the problem and how it is affected regarding changes to the decision variable is to solve a simplified version of it. Also, by this way there can be obtained initial values for the Lagrangean parameters which will be close, or at least of the same size, to the original thus the iterative procedure will not spend time until it adjusts them and brings the problem to a "steady-state". So, if the interrelation effect among channels is reconsidered and it is assumed that the effects of the channels are independent from each other, then the total effect would be transformed into:

$$f(x)_{total} = \frac{\sum_R (f(x)_{R,c}) \cdot W_R}{\sum_R W_R}$$

It is observed that the aggregated effect across every channel is substituted by the sum of the independent effects of every channel. Moreover by changing the constraints for x values from the artificial bounds to actual values, the problem is further simplified. So, instead of using LB it will be used zero since it is not logical for GRPs to take negative values. Also, instead of UB it will be defined an Upper value for GRPs (limGRP) above which any increase to GRP has a negligible result to the effect. These Upper values could be defined as the value of GRPs that result to the 95% of the effect for each channel. (It has to be mentioned that the limit 95% of the effect is an arbitrary one and due to computational reasons that may occur, it could be altered to around 90% or even lower)

As a result, the simplified problem will have the following formulation:

Max

$$f(x)_{total} = \frac{\sum_R (f(x)_{R,c}) \cdot W_R}{\sum_R W_R}$$

Subject to:

$$\sum_C (sc_{c,q} \cdot y_c + oc \cdot x_c) \leq B$$

$$limx \leq x \leq 0$$

$$y \in \{0,1\}$$

A2.1 Lagrangean Relaxation to the simplified problem

Applying the relaxation method described above, the original simplified problem is transformed into:

$$\text{Max } f(x)_{\text{total}} = \lambda \cdot \sum_c a c_c \cdot x_c - \lambda \cdot \sum_c s c_c \cdot y_c + \lambda \cdot B$$

Subject to:

$$\text{Min } x \geq 0$$

$$y \in \{0,1\}$$

To get an initial solution, it is necessary to give some initial values to the Lagrangean parameter λ . Then, there are calculated the values of x such that the first derivative of the objective function is zero.

$$\frac{df_{\text{total}}}{dx} = 0 \Rightarrow$$

$$L_c \cdot b_{m,c} \cdot e^{b_{m,c} x_c} - \lambda \cdot a c_c = 0$$

Accordingly, y 's are given value one when x 's are positive and then it is calculated the total cost. The λ parameter updating procedure is the same as developed for the original problem.

By this way, apart from a detailed insight on how the procedure is applied it is also obtained a reasonable starting value for λ Lagrangean parameter which is the one that actually penalizes the overspending and mainly influences the final result.

A3. Results

1st Policy: Updating all the three Lagrangean parameters simultaneously

- Graphs and remarks

λ parameter

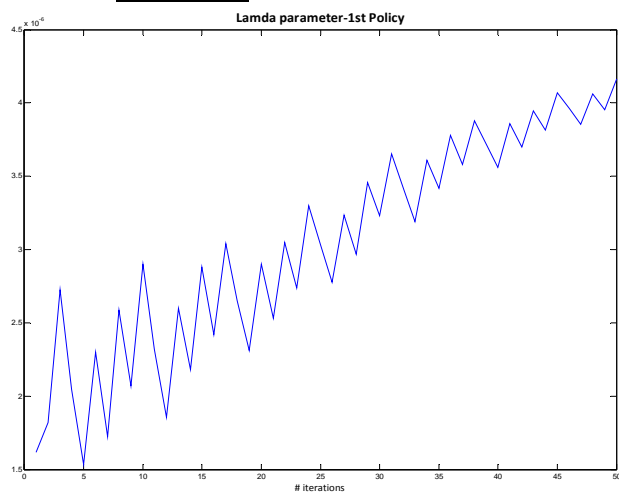


Figure A3: λ -parameter-1st policy

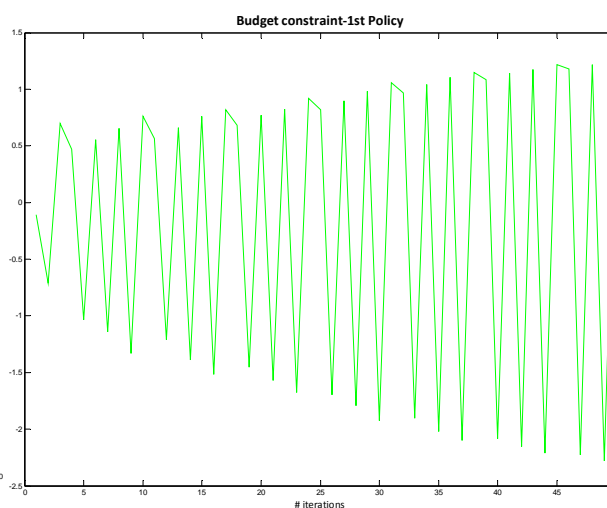
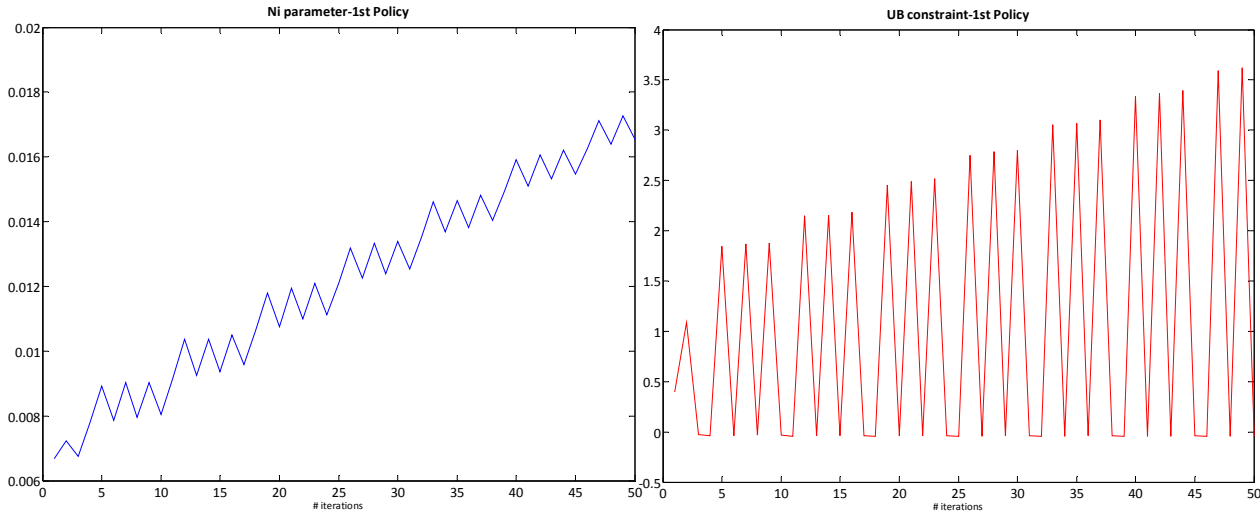


Figure A4: Budget constraint-1st Policy

Studying the two figures, it is worth mentioning that:

- The λ parameter is fluctuating failing to approach a certain value while it is showing a clear increasing trend. A more detailed observation would lead to the conclusion that the readjustment of the λ -parameter is insufficient leading from a positive extreme result to a negative one.
- The budget constraint also demonstrates the same behavior with values both above and below the zero borderline. This is interpreted as a constant violation of the budget constraint followed by a cost value much lower than the available budget.
- It is also noticeable that the shape of the budget constraint could be described as trapezoidal receiving more negative than positive values. Despite that the available budget is set to 350.000, it is observed that the money spent highly fluctuate from values 32.300 to over 1.000.000 with the cost values over-exceeding budget to increase as the number of iterations increase.

v parameter**Figure A5: v-parameter-1st policy****Figure A6: Upper Bound constraint-1st policy**

The behavior of the Upper Bound constraint and its corresponding Lagrangean parameter could be summarized to the following remarks:

- The v parameter is presenting an unstable behavior increasing itself periodically after a number of iterations. Although it seems to be restricted between two values, after a number of iterations its values are increased and the same procedure is repeated. In fact this recurrent phenomenon results to the increasing tendency of the v -parameter without showing any signs of approaching a certain value.
- Accordingly, the Upper Bound constraint is also demonstrating an unstable behavior fluctuating between slightly negative and positive values while also showing an overall increasing trend. In fact, the same periodical phenomenon appeared to the v -parameter is also obvious at the Upper Bound constraint increasing its positive values after a certain number of iterations.
- Since the behavior of the Lagrangean parameter is influenced by its corresponding constraint, every increase to the value of the Upper Bound constraint affect the v -parameter which tend to further increase to promote the increase to values of x -variables.

Aiming to highlight the reasons that led to the abnormal behavior mentioned above, there will be a brief description of the v -parameter's updating procedure.

So, having as a starting point iteration number 35:

- the x variables have received values that are significantly lower than Upper bound and that is the reason the Upper Bound constraint is positive with a value almost equal to three.
- During the following iteration (36), v parameter is decreased in order to assign higher values to the x variables which result them to over-exceed the Upper bound bringing the Upper Bound constraint to slightly negative values.
- Consequently, the next iteration (37) will increase the v-parameter more to avoid assigning values to the x-variables which would violate the Upper Bound constraint.
- At iterations 38 and 39, the updating procedures fails to improve the values of x-variables and the constraint is constantly violated. As a result, at iteration 40 the v parameter is increased even more to force the x-variables to reduce their values resulting to a combination of values that is even lower than the Upper bound than the previous iterations.

Number of iterations											
	35	36	37	38	39	40	41	42	43	44	45
v parameter	0.04086	0.03827	0.041321	0.038964	0.041759	0.044688	0.042138	0.045161	0.042584	0.045334	0.043069

Table 2: v-parameter values

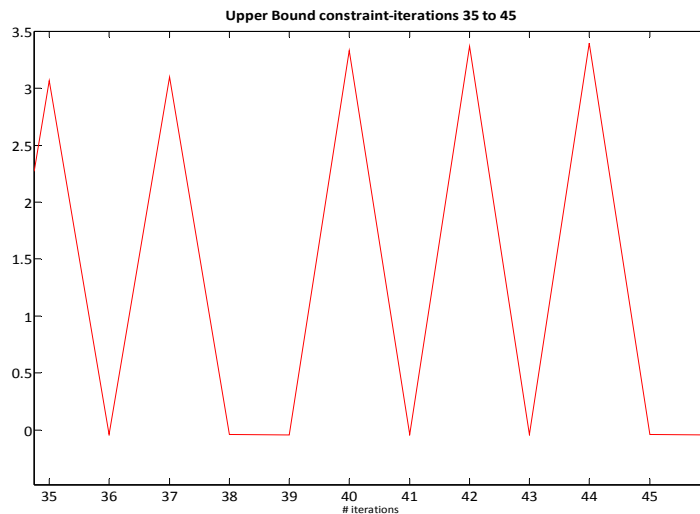


Figure A7: Upper Bound constraint behavior

This phenomenon as described above is repeated during the iterative procedure resulting to the increasing trend of both v-parameter and Upper Bound constraint.

Recalling that the values for the Upper bounds are artificially set, it is highly possible that the chosen values do not represent reality thus they are not truly restricting x-values. As a result, it could be argued that the artificial Upper bound constraint is inactive allowing the variable to receive any value. This statement also leads to the conclusion that the current updating procedure is not sufficient enough to result to an optimal (as lower as possible) Upper bound.

μ parameter

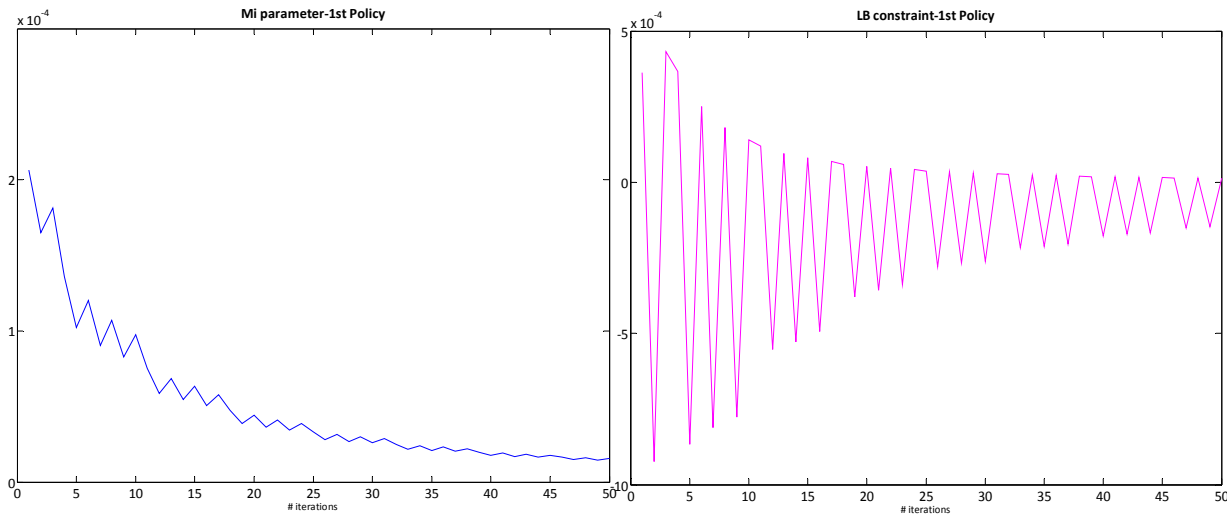


Figure A8: μ -parameter-1st policy

Figure A9: Lower Bound constraint-1st policy

- It is observed that after a number of iterations, the Lower bound constraint is reaching a steady state fluctuating around zero value in a restricted range of values ($\approx 10^{-6}$). This could be interpreted by the fact that in the beginning of the iteration procedure, x-variables are receiving values that are way above or below the Lower bound and while the procedure is evolving they are approaching the Lower bounds. So, this explains why during the first iterations, the Lower bound constraint is highly fluctuating between positive and negative values while at the following iterations it converges to the zero value.

Similarly to its corresponding constraint, the Lagrangean parameter (μ_i) is gradually declining while the constraint is reducing its value range and it is also converging to a certain value when the constraint has stabilized and is fluctuating with a small margin. This decreasing trend of the μ parameter could also be explained by referring to the same phenomenon as in the Upper bound constraint with a reduced effect. The values assigned to the x variables are such that after a small number of iterations, the Lower Bound constraint is never met.

As a result, the constraint parameter is continually decreasing reaching a value where the corresponding Lower Bound constraint is becoming stable though remaining to slightly negative values.

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• **Analytical results**

To obtain a better insight of the system’s behavior, we will also present the solution values of the Upper Bound effect, the budget together and the decision variables. So:

- y-variables

		Number of iterations										
		15	16	17	18	19	20	21	22	23	24	25
Number of channels	18	0	1	0	0	1	0	1	0	1	0	0
	21	0	1	0	0	1	0	1	0	1	0	0
	23	0	1	0	0	1	0	1	0	1	0	0
	33	0	1	0	0	1	0	1	0	1	0	0

Table 3: Example of channels with their binary variable values during 10 consequent iterations-1st policy

- x-variables

		Number of iterations										
		15	16	17	18	19	20	21	22	23	24	25
Number of channels	18	3.875989	4.046661	3.771108	3.897287	3.898761	3.674639	3.7665	3.523153	3.597815	3.39527	3.479715
	21	0.000857	0.00133	0.00078	0.001077	0.001626	0.000887	0.00128	0.000744	0.000965	0.000647	0.000822
	23	5.50E-05	5.52E-05	5.30E-05	5.29E-05	5.21E-05	5.06E-05	5.01E-05	4.84E-05	4.79E-05	4.64E-05	4.61E-05
	33	17.88552	28.87638	14.58282	23.25518	31.07017	17.08476	25.52258	11.4259	18.1499	6.524214	13.34535

Table 4: Example of channels with their GRP values during 10 consequent iterations-1st policy

- Upper Bound effect and budget

		Number of iterations										
		15	16	17	18	19	20	21	22	23	24	25
Upper Bound	1.24472	1.22756	1.28656	1.17568	1.55649	1.24217	1.45319	1.31891	1.26655	1.392686	1.28611	
Budget	73112	960158	68174	90771	961078	69842	953577	62126	943985	55452	75760	

Table 5: Upper effect values and their corresponding costs during 10 consequent iterations-1st policy

It is remarkable that:

- y variables are presenting an irregular behaviour following a pattern of continually fluctuating between values zero and one without any indication of concluding to a value.
- x variables differentiate themselves from the y variables thus following an unsteady behavioural pattern. Indeed, there are channels (33 for example) to which there are assigned a certain number of GRPs greater than zero and which are expected to have a considerable effect. Also, there are channels like channel number 21 and 23 to which are assigned a negligible number of GRPs very close to zero which do not actively affect the overall effect and this practically means that the particular channels are never going to be included to the channel mix. However, there are channels which have a slightly different behavior and which are picked and they receive an amount of GRPs but it is that low that in the end they do not actively influence the overall effect. As an example, we present channel 18 which is chosen to be included in the channel mix but its contribution (its number of GRPs) are lower than its Lower Bound and it does not result to a considerable effect.
- x and y variables they are not aligned. Searching for a feasible solution, it would be expected that when y variable is receiving value one, x variable would take positive values and when it has a zero value, x variable would also receive zero value. But this is not the case, since in the majority of the iterations there is noticed a complete mismatching. As explained in chapter 5 such a behavior was expected due to the opposite signs of the Lagrangean parameters in the two parts of the objective function. However, it was expected that during the iterative procedure the values of the parameters which would have been continually updated, they would have succeed to overcome this drawback.
- Regarding the feasibility of the produced solution, the budget values have to be examined. As it was also observed by the figure presented above, the total amount of money spent is fluctuating around the budget receiving extreme values resulting to a series of solutions ranging from feasible to infeasible in every iteration.
- The values of the Upper Bound effect, as it was also mentioned in the beginning of the paragraph, are higher than one and thus they do not contribute to the optimization of the problem. However, it is worth mentioning that the Upper Bound is also fluctuating with an increasing tendency and without any indication of approaching a certain value.

2nd Policy: Updating only two of the Lagrangean parameters setting B-part =0

- **Graphs**

- **λ parameter**

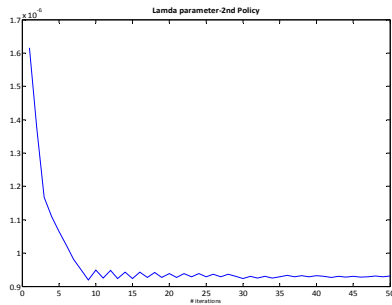


Figure A10: λ-parameter-2nd policy

- **Budget Constraint**

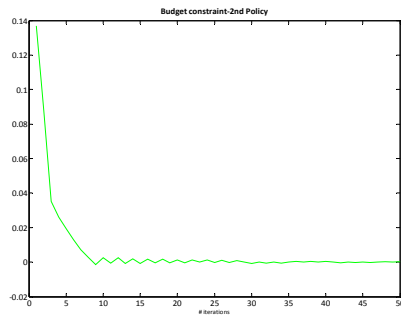


Figure A11: Budget constraint-2nd policy

- **v parameter**

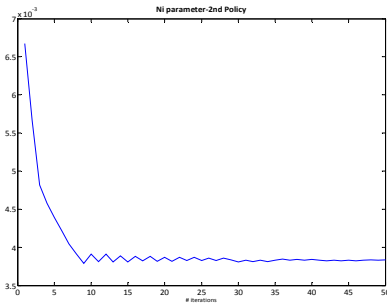


Figure A12: v-parameter-2nd policy

- **Upper Bound constraint**



Figure A13: Upper Bound constraint-2nd policy

- **μ parameter**

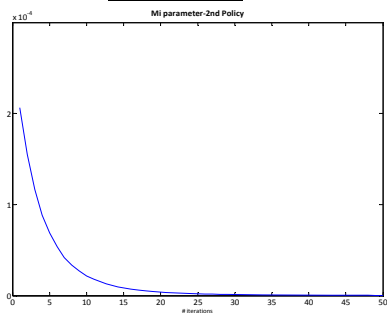


Figure A14: μ-parameter-2nd policy

- **Lower Bound constraint**

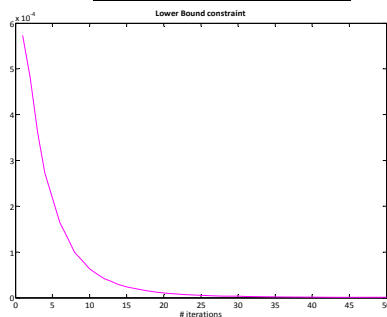


Figure A15: Lower Bound constraint-2nd policy

- **Graphs remarks**

- The λ parameter after a number of initial iterations is becoming more stable and is finally converging to a certain value. Indeed, after the 35th iterations the values of λ -parameter are remaining steady at 0.00000093 only deviating in a range of 10^{-8} .
- The v parameter is following a similar trend to λ parameter obtaining a more stable form and reducing the fluctuations avoiding extreme values.
- The Upper Bound constraint keeps fluctuating but in a reduced range of values which are always positive. This means that the values of x variable never exceed the Upper Bound and in fact they are closer to the Upper Bound set though still not meeting it. As a result, from the time that bounds are defined so high that they do not restrict the values assigned to the parameters it could be mentioned that the constraint is not actually active. This could be a starting point for future improvement.
- It should not be disregarded though, that practically the v parameter is compensating for the effect of λ parameter. So, assigning values to the x variable which are always below the Upper Bound tends to increase the v parameter which results to a decreasing λ parameter and as such the Upper Bound effect is not possible to decrease and meet the actual effect.
- The Lower Bound constraint is highly affected by setting B-part equals to zero. This is happening due to the opportunity given to assign values to those y variables that correspond to x variables higher than the Lower Bound. In this case, the Lower Bound is becoming steady after a small number of iterations and remains like this during the whole procedure. Since x and y variables are aligned and x variables are receiving values very close to the Lower Bound, the constraint remains close to zero with a few negligible deviations.
- Similarly, μ parameter is decreasing until x variables receive the values they will maintain until the end of the procedure and correspond to the steady behaviour of the constraint.

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- **Analytical results**

In order to support the remarks made above and to enable the comparison with the previous methodology, there have been chosen the same channels during the same iterations to study their behaviour. So:

- y -variables

		Number of iterations										
		15	16	17	18	19	20	21	22	23	24	25
Number of channels	18	0	0	0	0	0	0	0	0	0	0	0
	21	0	0	0	0	0	0	0	0	0	0	0
	23	0	0	0	0	0	0	0	0	0	0	0
	33	1	1	1	1	1	1	1	1	1	1	1

Table 6: Example of channels with their binary variable values during 10 consequent iterations-2nd policy

- x-variables

		Number of iterations										
		15	16	17	18	19	20	21	22	23	24	25
Number of channels	18	8.615915	8.569878	8.617911	8.553445	8.594962	8.553324	8.604324	8.580482	8.575824	8.538611	8.595914
	21	0.010625	0.009702	0.010457	0.009577	0.010429	0.009659	0.010548	0.010074	0.010424	0.009822	0.010348
	23	0.000247	0.000242	0.000246	0.000242	0.000246	0.000243	0.000246	0.000243	0.000246	0.000243	0.000246
	33	92.7166	91.46542	92.54503	91.5298	92.54673	91.72748	92.57617	91.7914	92.37279	91.74893	92.42206

Table 7: Example of channels with their GRP values during 10 consequent iterations-2nd policy

- bounds and costs

		Number of iterations										
		15	16	17	18	19	20	21	22	23	24	25
Upper Bound	0.906539	0.908392	0.906864	0.908357	0.906831	0.908022	0.90681	0.908001	0.90703	0.907987	0.907017	
Lower Bound	0.77632	0.771639	0.772926	0.771665	0.772924	0.775218	0.772928	0.775222	0.772856	0.775233	0.772866	
UB Budget	350891.6	348122.7	350376	348152.4	350403.3	348638.5	350431.7	348668.9	350081.2	348658.8	350088.3	
LB Budget	349486.3	347224	346853.1	347030.3	346839.8	349881.3	346770.5	349756	347443.7	349791.9	347321.8	

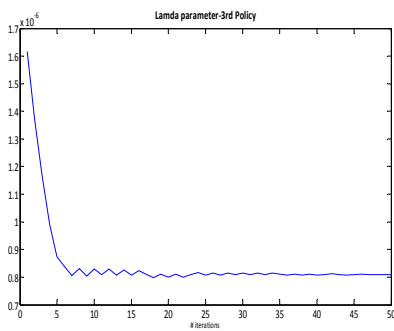
Table 8: Upper and Lower Bound effect values and their corresponding costs-2nd policy

- y variables are receiving either value one or zero avoiding the phenomenon of opening and closing the same channel.
- x-variable is receiving values which are not fluctuating but in many cases are lower than the Lower Bound. This explains why in some iterations there are values assigned to x variable when y variable equals to zero.

2nd Policy - Improvement: Setting the Upper Bound in a lower value

- Graphs

λ parameter



Budget Constraint

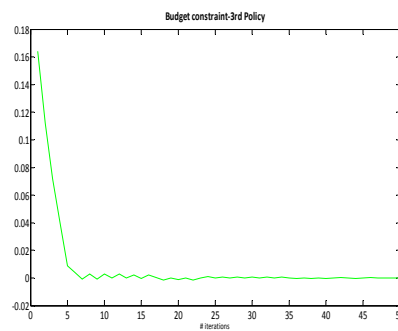


Figure A17: Budget constraint-2nd policy improvement

v parameter

Upper Bound constraint

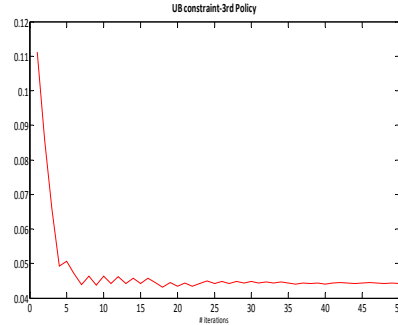
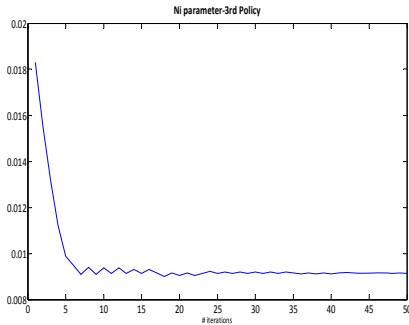


Figure A18: v-parameter-2nd policy improvement

Figure A19: Upper Bound constraint-2nd policy improvement

μ parameter

Lower Bound Constraint

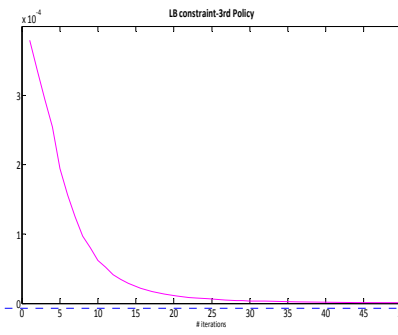
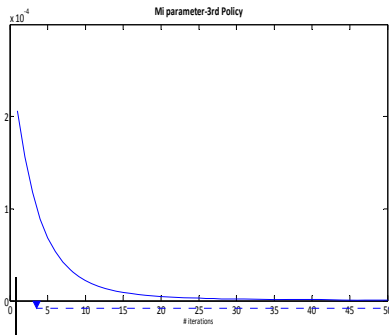


Figure A20: μ-parameter-2nd policy improvement

Figure A21: Lower Bound constraint-2nd policy improvement

made above and

methodology, there have been chosen the same channels during the same iterations to study their behaviour. So:

- o y-variables

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		Number of iterations										
		15	16	17	18	19	20	21	22	23	24	25
er	of	0	0	0	0	0	0	0	0	0	0	0
cha	18	0	0	0	0	0	0	0	0	0	0	0

	21	0	0	0	0	0	0	0	0	0	0	0
	23	0	0	0	0	0	0	0	0	0	0	0
	33	1	1	1	1	1	1	1	1	1	1	1

Table 9: Example of channels with their binary variable values during 10 consequent iterations-2nd policy improvement

- x-variables

		Number of iterations										
		15	16	17	18	19	20	21	22	23	24	25
Number of channels	1	7.450802	7.36312	7.427436	7.48294	7.432018	7.472853	7.43938	7.4904	7.451614	7.403054	7.453928
	2	2.273019	1.979098	2.21541	2.471362	2.241835	2.418903	2.242115	2.406332	2.267218	2.104169	2.298848
	3	0.000169	0.000165	0.000168	0.000171	0.000168	0.00017	0.000168	0.00017	0.000169	0.000167	0.000169
	3	97.90686	96.63535	97.66082	98.69635	97.65871	98.45302	97.68811	98.50413	97.8611	97.19979	97.83681

Table 10: Example of channels with their GRP values during 10 consequent iterations-2nd policy improvement

- bounds and costs

		Number of iterations										
		15	16	17	18	19	20	21	22	23	24	25
Upper Bound		0.857635	0.858559	0.857792	0.857076	0.857773	0.857199	0.857761	0.857188	0.857635	0.858104	0.857628
Lower Bound		0.783703	0.782791	0.783589	0.783974	0.78359	0.783949	0.783624	0.783994	0.783614	0.783157	0.783611
UB Budget		350317.6	347546.5	349805	352073.3	349842.2	351635	349863.4	351664.9	350232.8	348804.4	350250.6
LB Budget		349927.8	349691.2	349955.9	349860.5	349954.6	349955.2	349987.2	349991.8	349854.9	349726	3498

Table 11: Upper and Lower Bound effect values and their corresponding costs-2nd policy improvement

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